Non-Gaussianity in Heavy Ion Collisions

Masakiyo Kitazawa
（Osaka U.）

MK, Asakawa, Ono, to be submitted soon

RIKEN Seminar, 2013/Jul./8

Baryon vs Proton Number Fluctuations

MK, Asakawa, PRC85,021901C(2012); PRC86, 024904(2012)

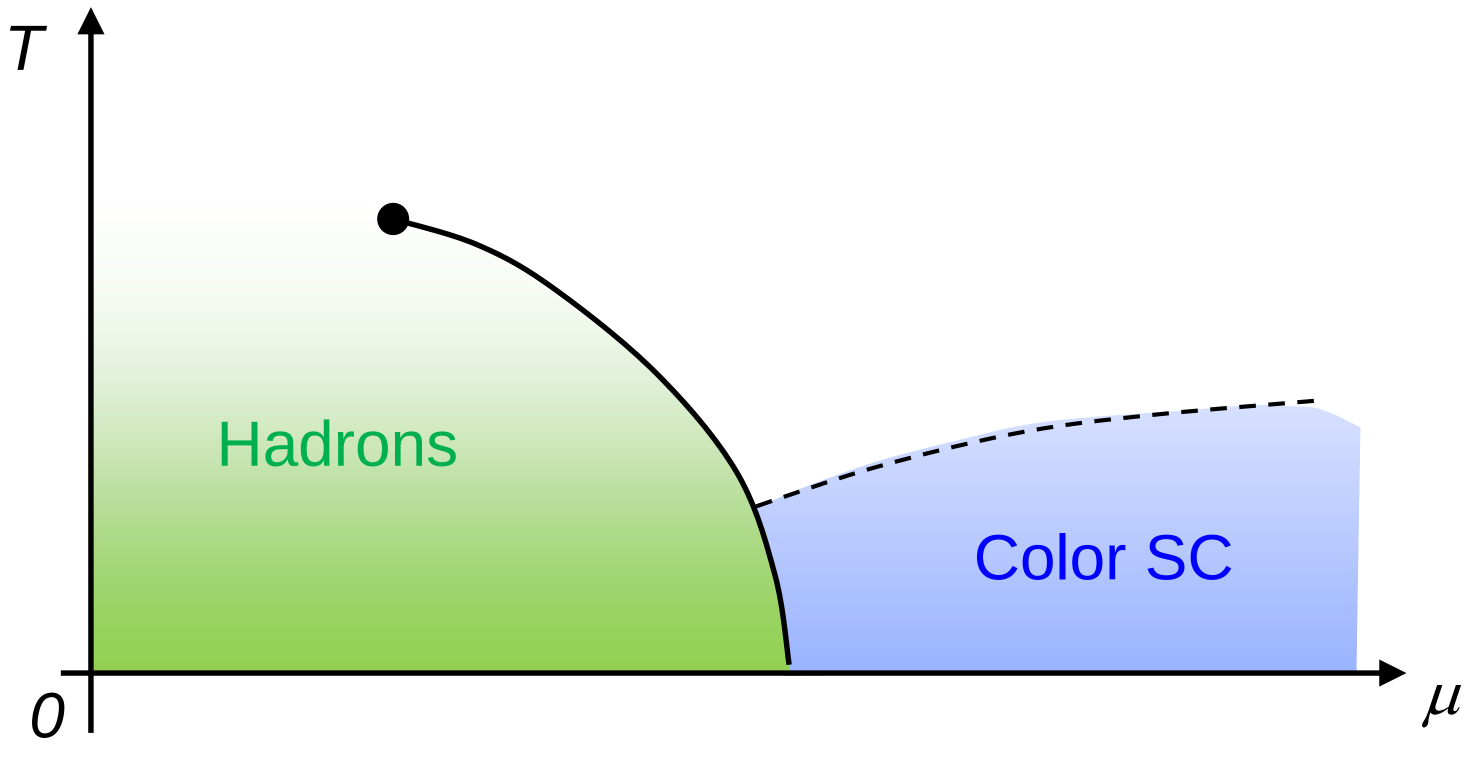
10min

Time Evolution of Higher Order Cumulants

MK, Asakawa, Ono, arXiv:1307.xxxx

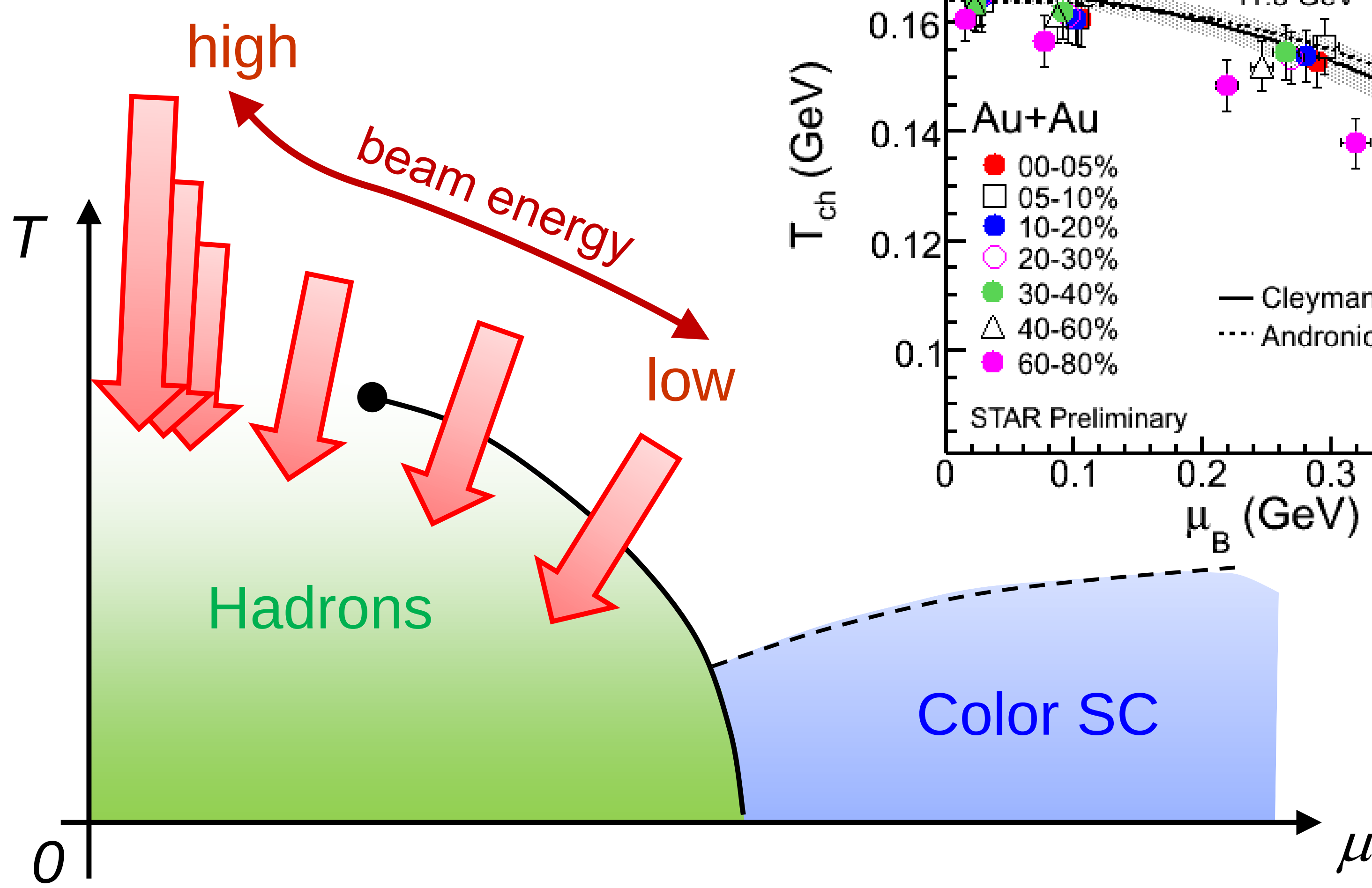
30min

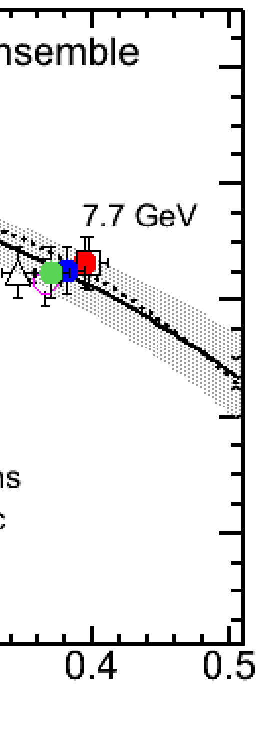
Beam-Energy Scan



Beam-Energy Scan

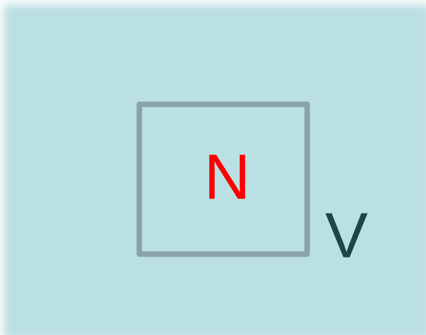
STAR 2012

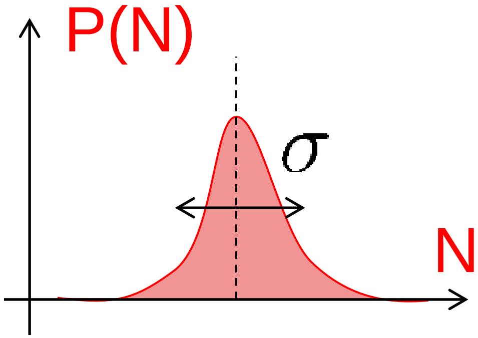




Fluctuations

Observables in equilibrium are fluctuating.





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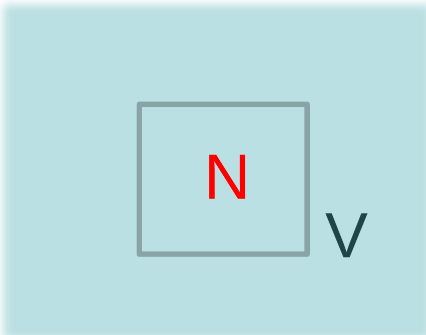
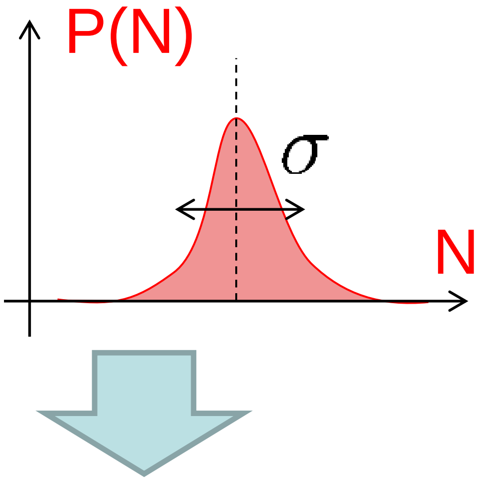


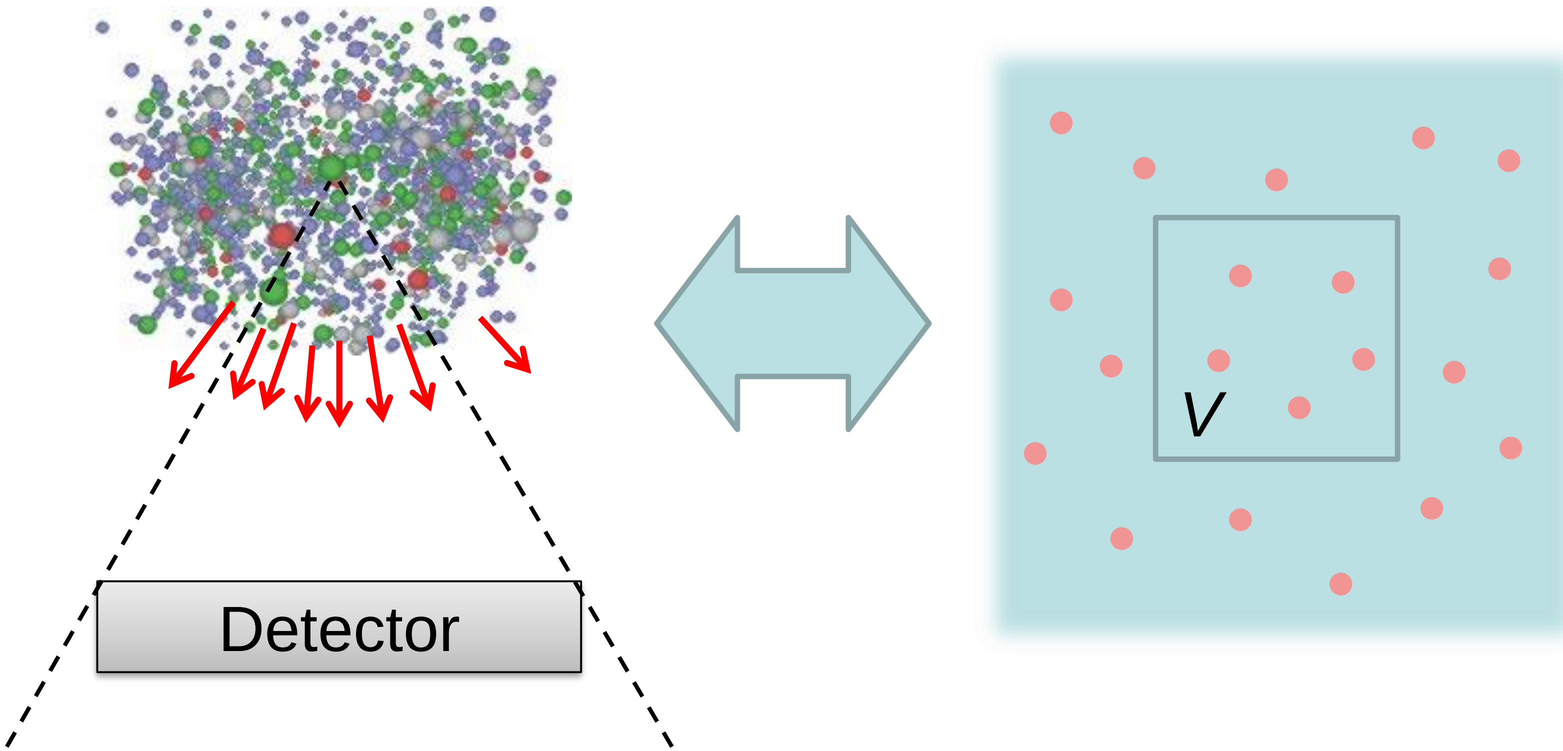


- Variance: $\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \dfrac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

$$\delta N = N - \langle N \rangle$$

Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.



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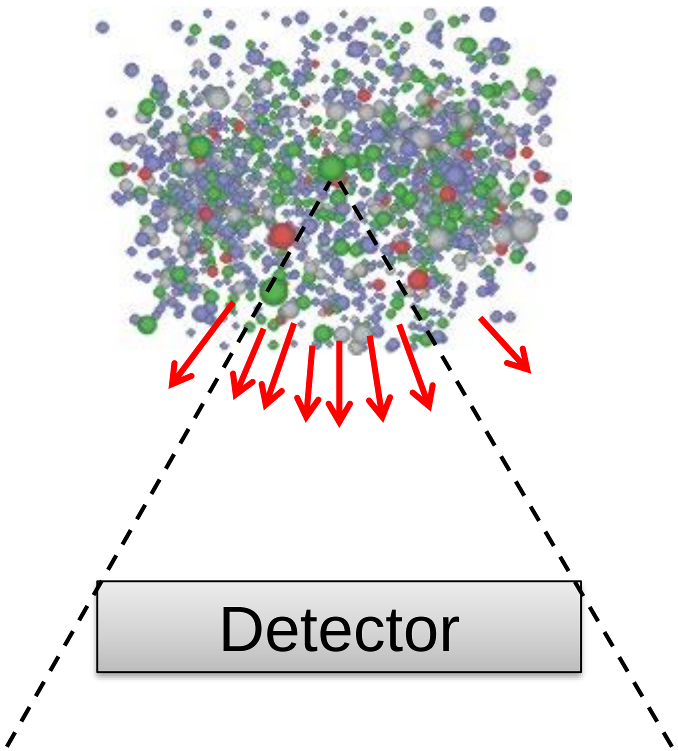
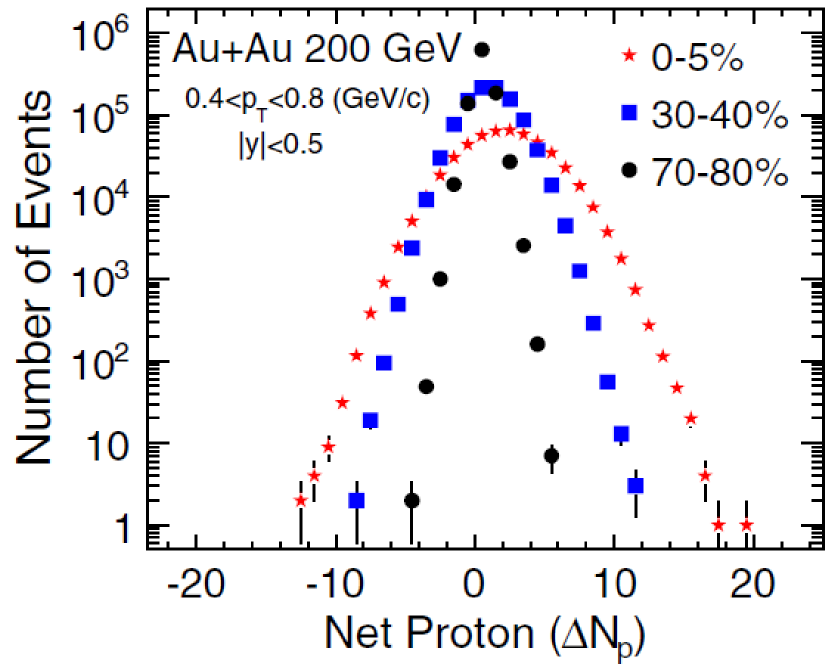
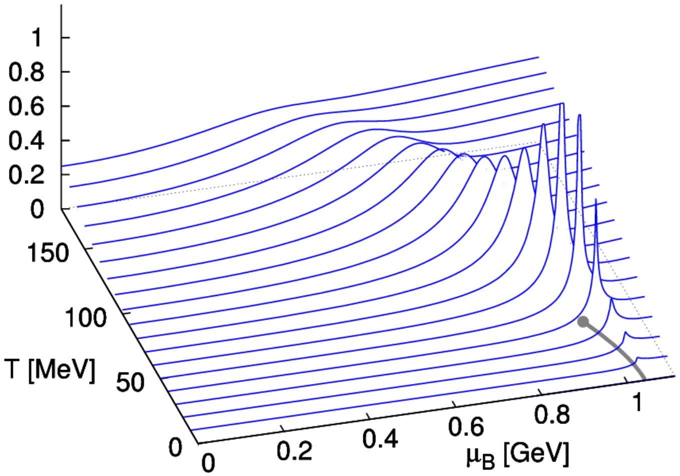


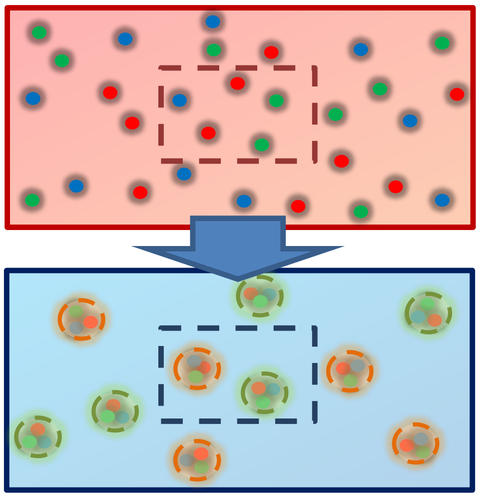
STAR, PRL**105** (2010)

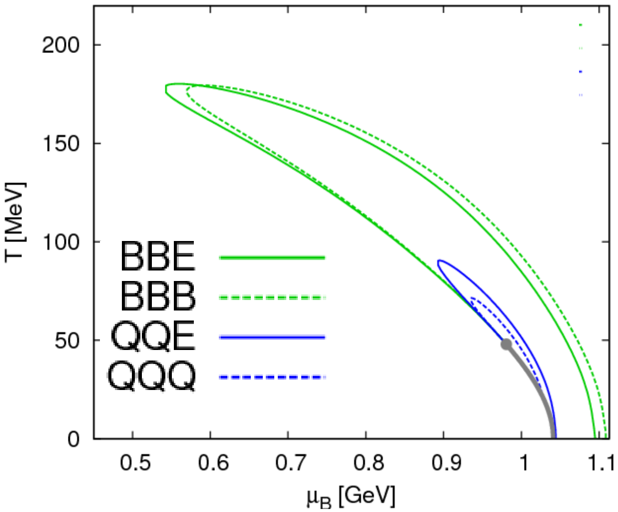


Fluctuations

- Fluctuations reflect properties of matter.
  - Enhancement near the critical point
    Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);…
  - Ratios between cumulants of conserved charges
    Asakawa,Heintz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)
  - Signs of higher order cumulants
    Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)



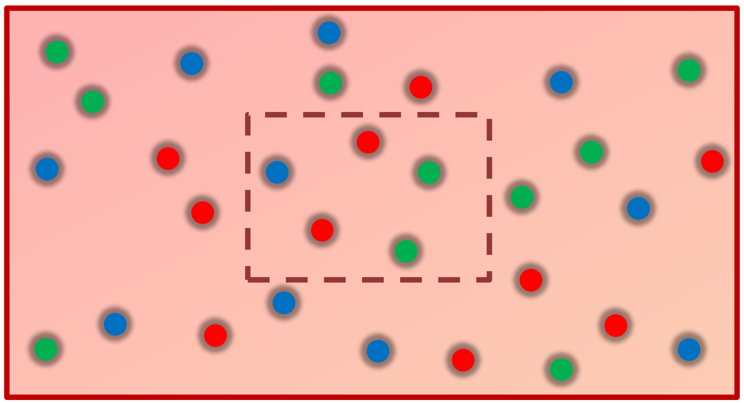




Fluctuations

Free Boltzmann → Poisson

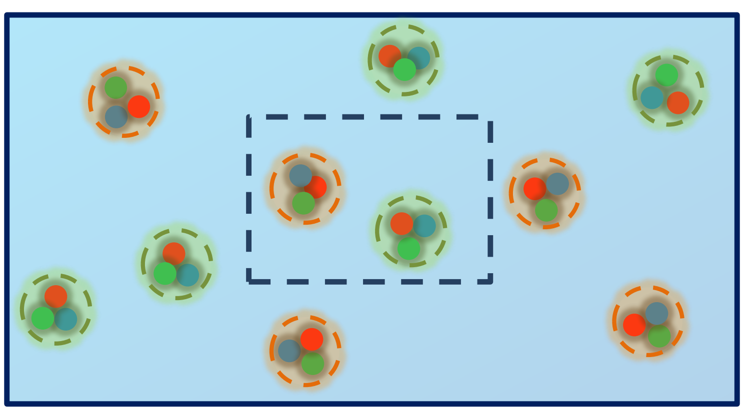
$$\langle \delta N^n \rangle_c = \langle N \rangle$$



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$



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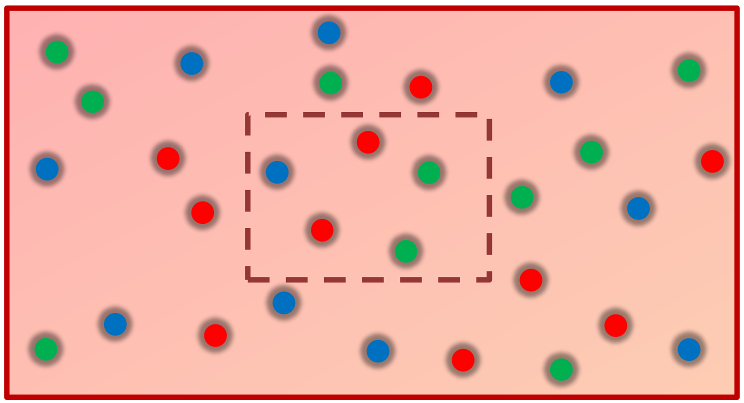

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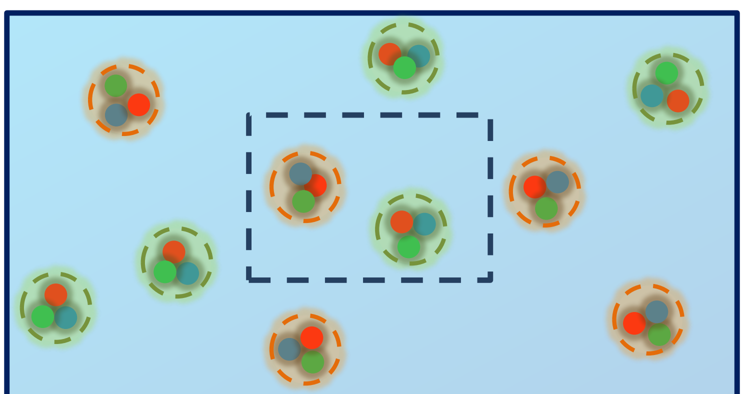
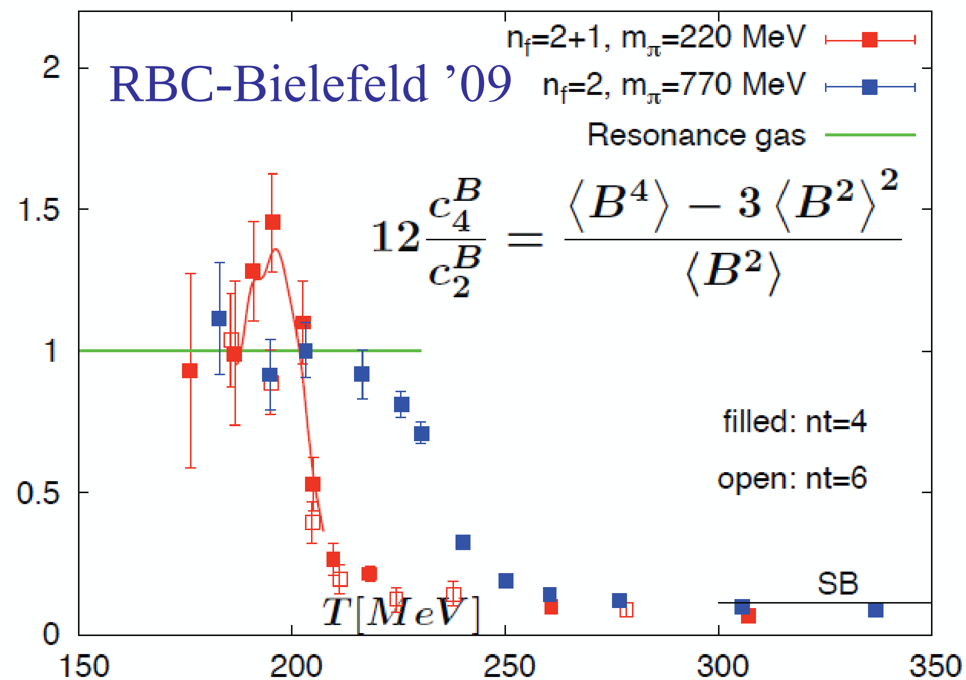
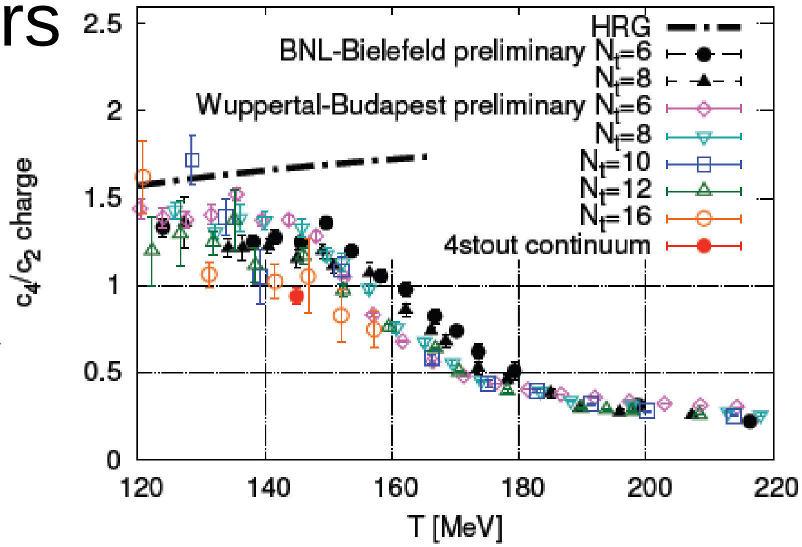




Conserved Charges : Theoretical Advantage

- Definite definition for operators
  - as a Noether current
  - calculable on any theory

ex: on the lattice



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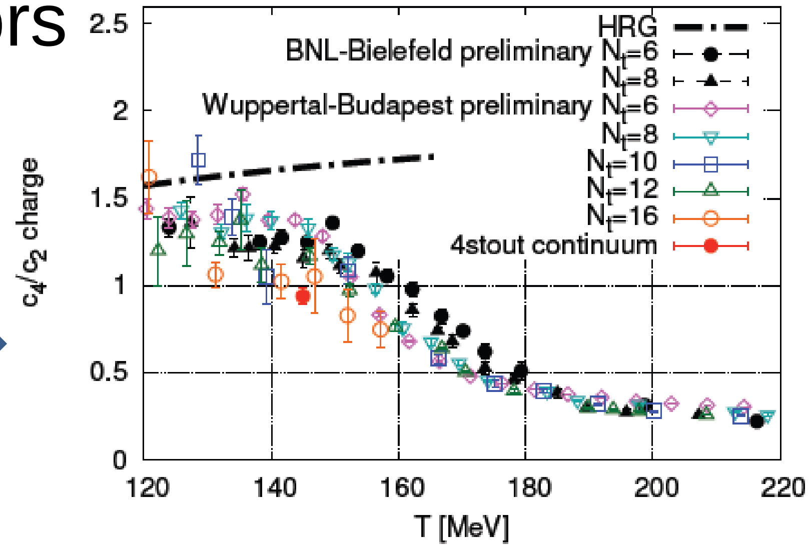


- **Simple thermodynamic relations**

$$\langle \delta N_c^n \rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}$$

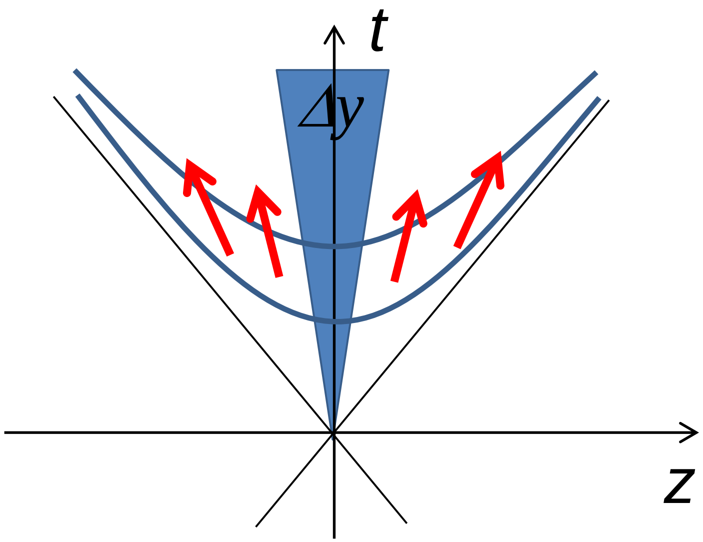
  - Intuitive interpretation for the behaviors of cumulants

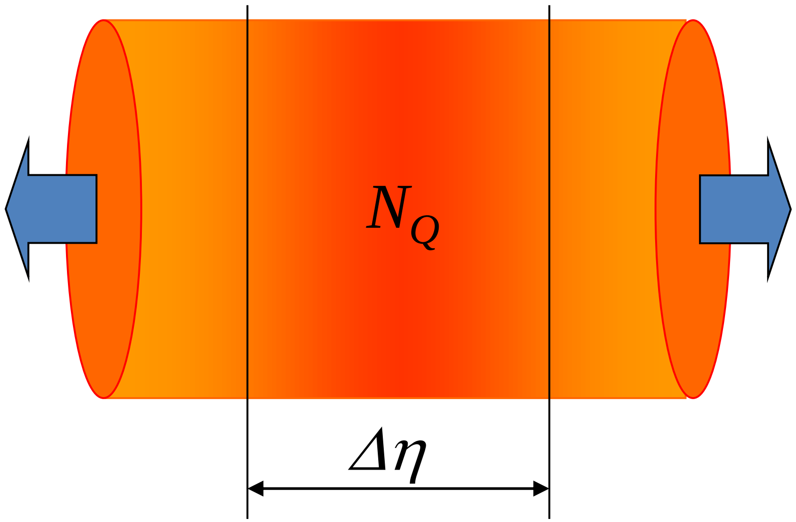
ex: $$\langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B}$$

Fluctuations of Conserved Charges

- Under Bjorken expansion

Asakawa, Heintz, Muller, 2000
Jeon, Koch, 2000
Shuryak, Stephanov, 2001

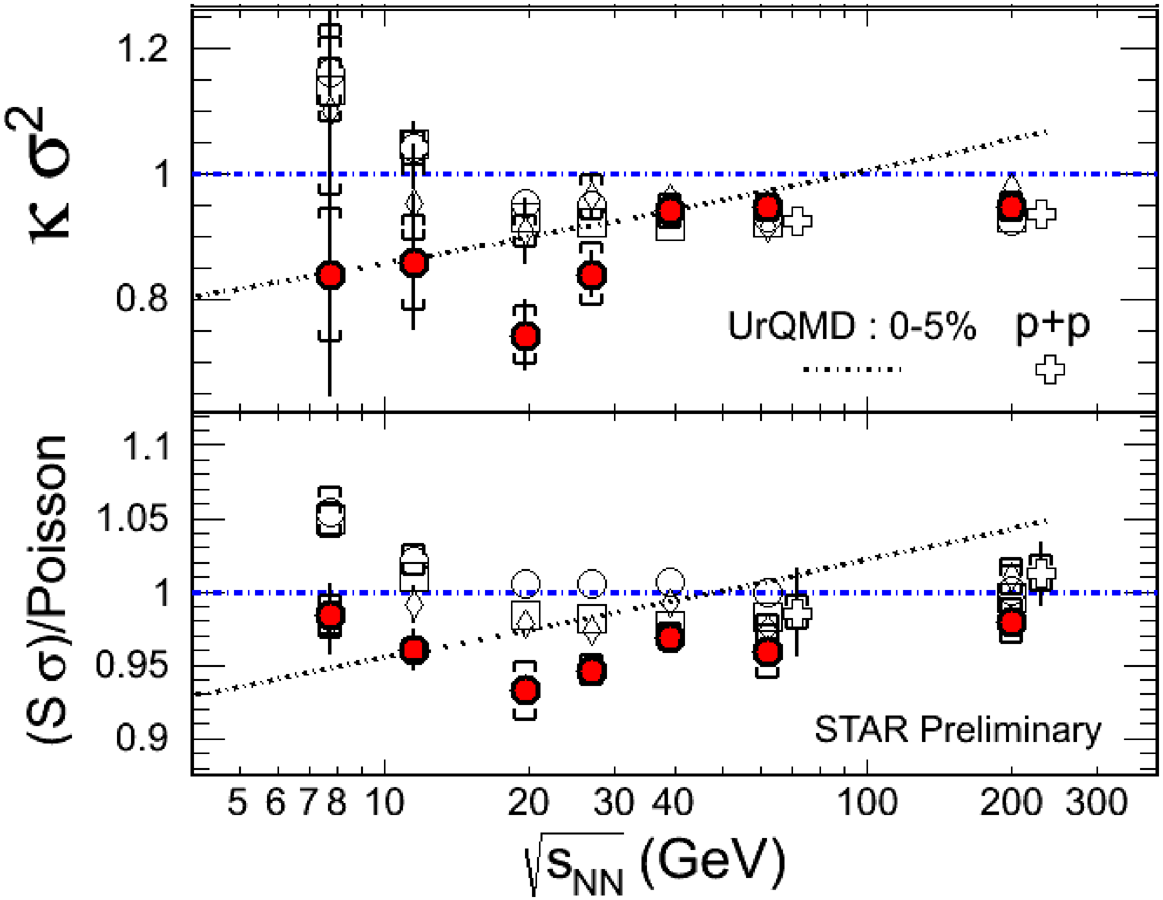




- Variation of a conserved charge in $\Delta\eta$ is **slow**, since it is achieved only through diffusion.

➡ Primordial values can survive until freezeout.
The wider $\Delta\eta$, more earlier fluctuation.

Proton # Cumulants @ STAR-BES



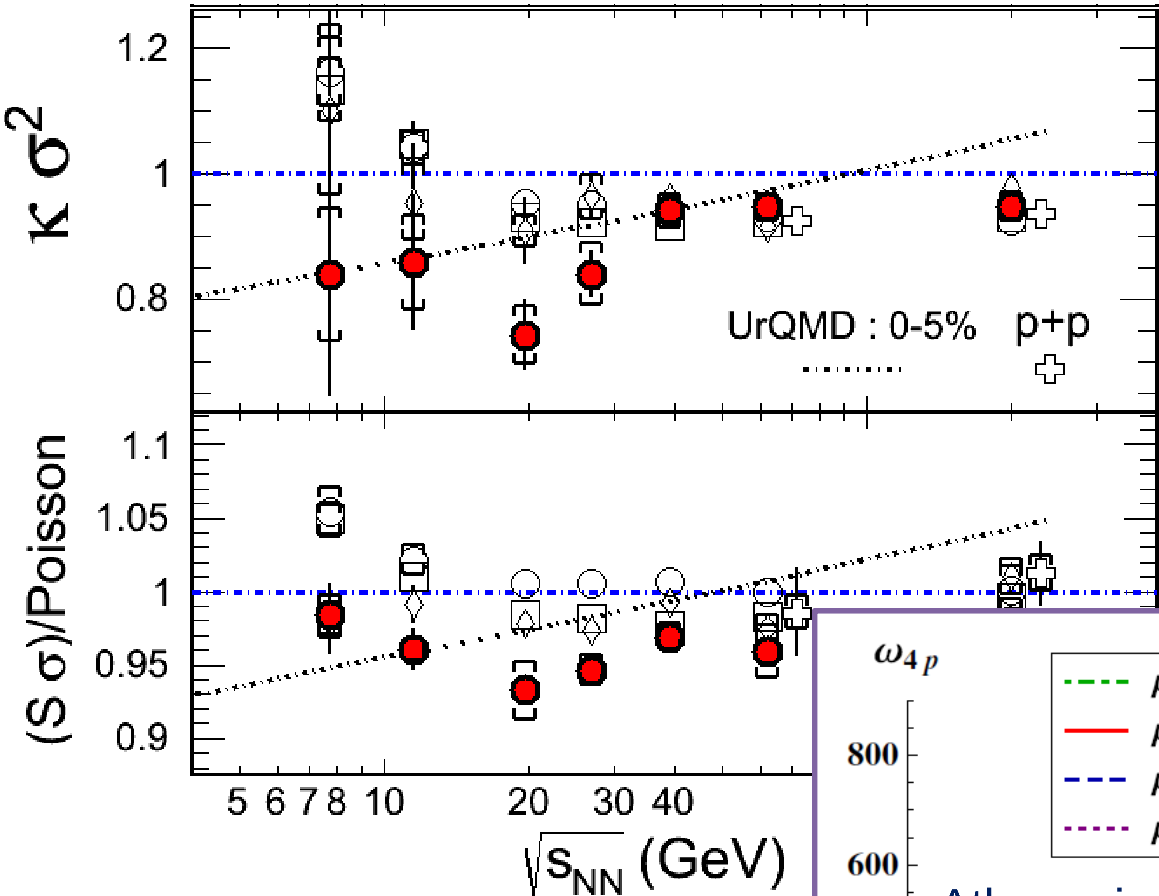
STAR,QM2012

$$\frac{C_4}{C_2}$$

$$\frac{C_3}{C_1} = \frac{C_3/C_2}{\text{Poissonian}}$$

No characteristic signals on
phase transition to QGP nor QCD CP

Proton # Cumulants @ STAR-BES



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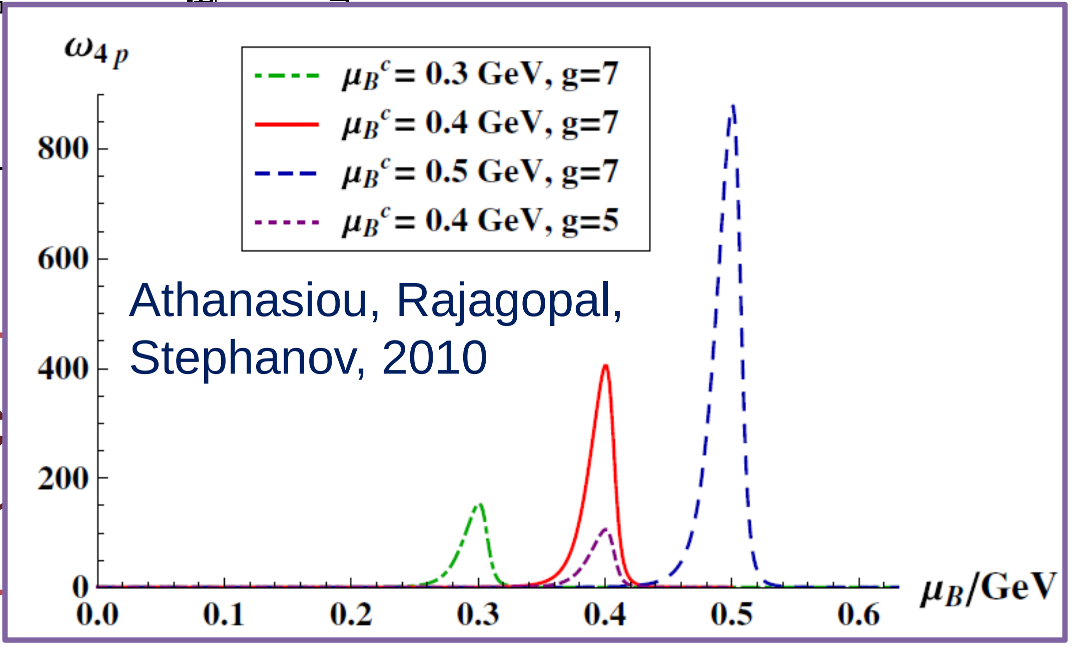
$$\frac{C_4}{C_2}$$

$$\frac{C_3}{\ } = \frac{C_3/C_2}{\ }$$



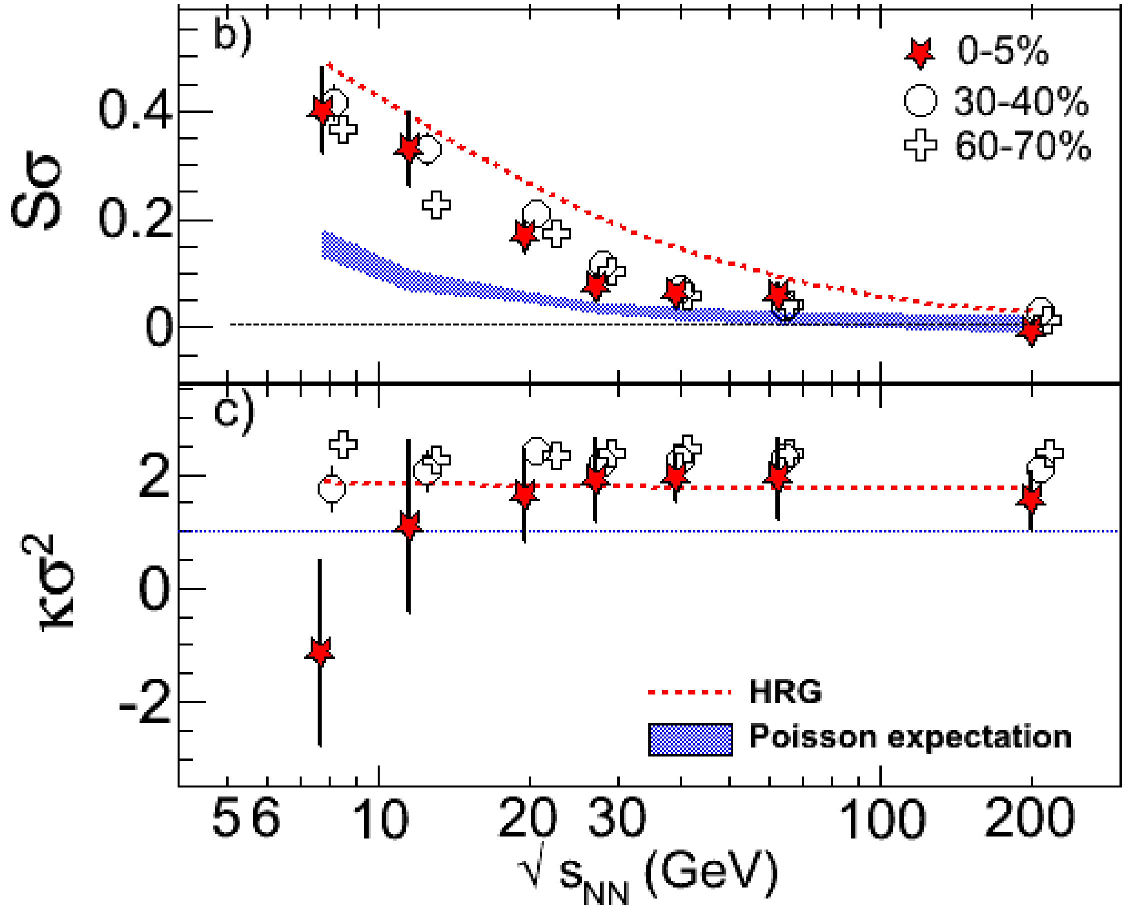
No charac

phase transitior

Charge Fluctuations @ STAR-BES

$$\frac{\langle \delta N_Q^3 \rangle}{\langle \delta N_Q^2 \rangle}$$

$$\frac{\langle \delta N_Q^4 \rangle_c}{\langle \delta N_Q^2 \rangle}$$



STAR, QM2012

No characteristic signals on
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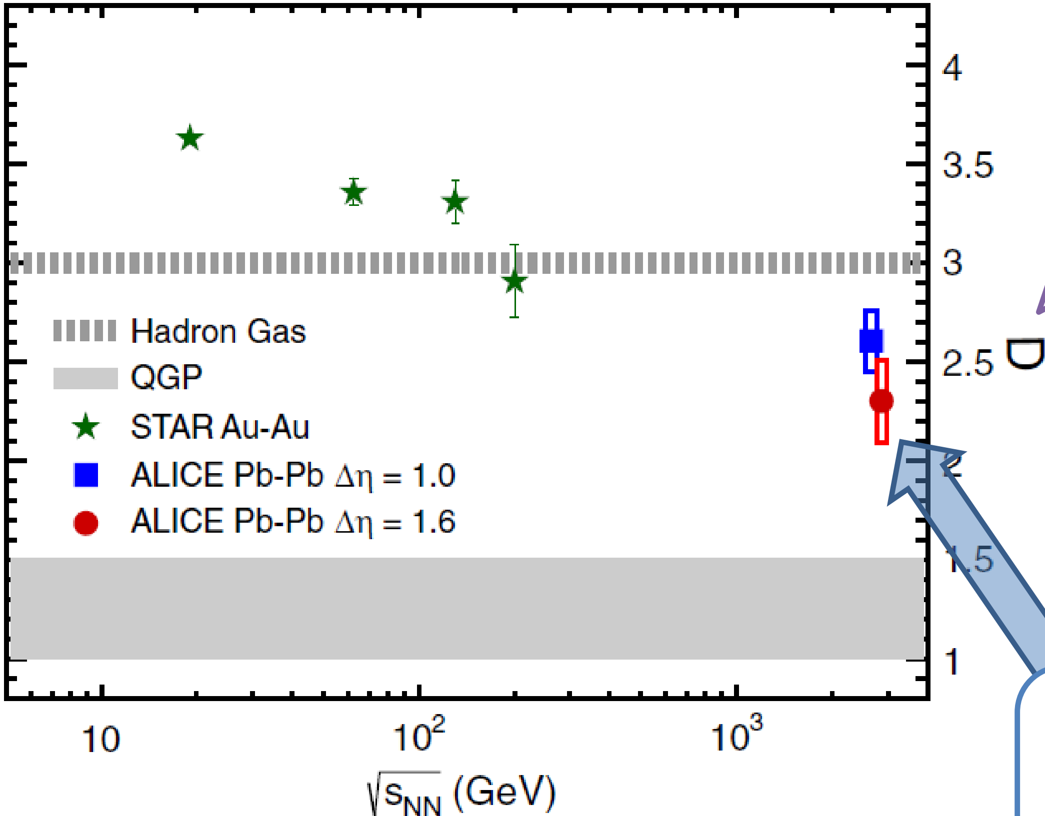
Charge Fluctuation @ LHC



ALICE, PRL110,152301(2013)

D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

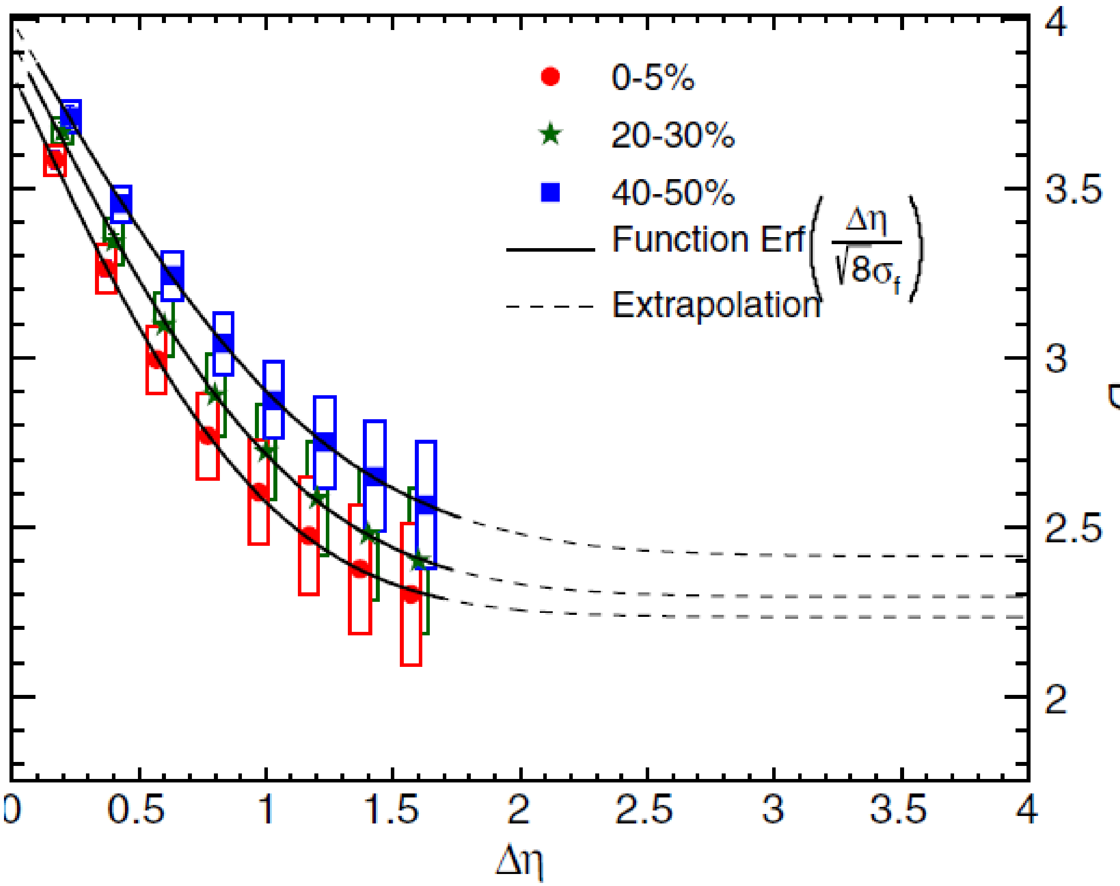
- $D \sim$ 3-4 Hadronic
- $D \sim 1$ Quark

LHC:
significant suppression from hadronic value

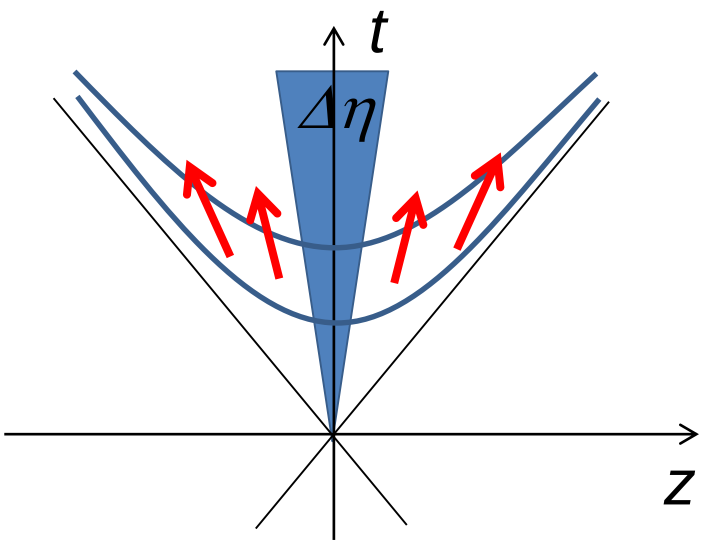
$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

Δη Dependence @ ALICE

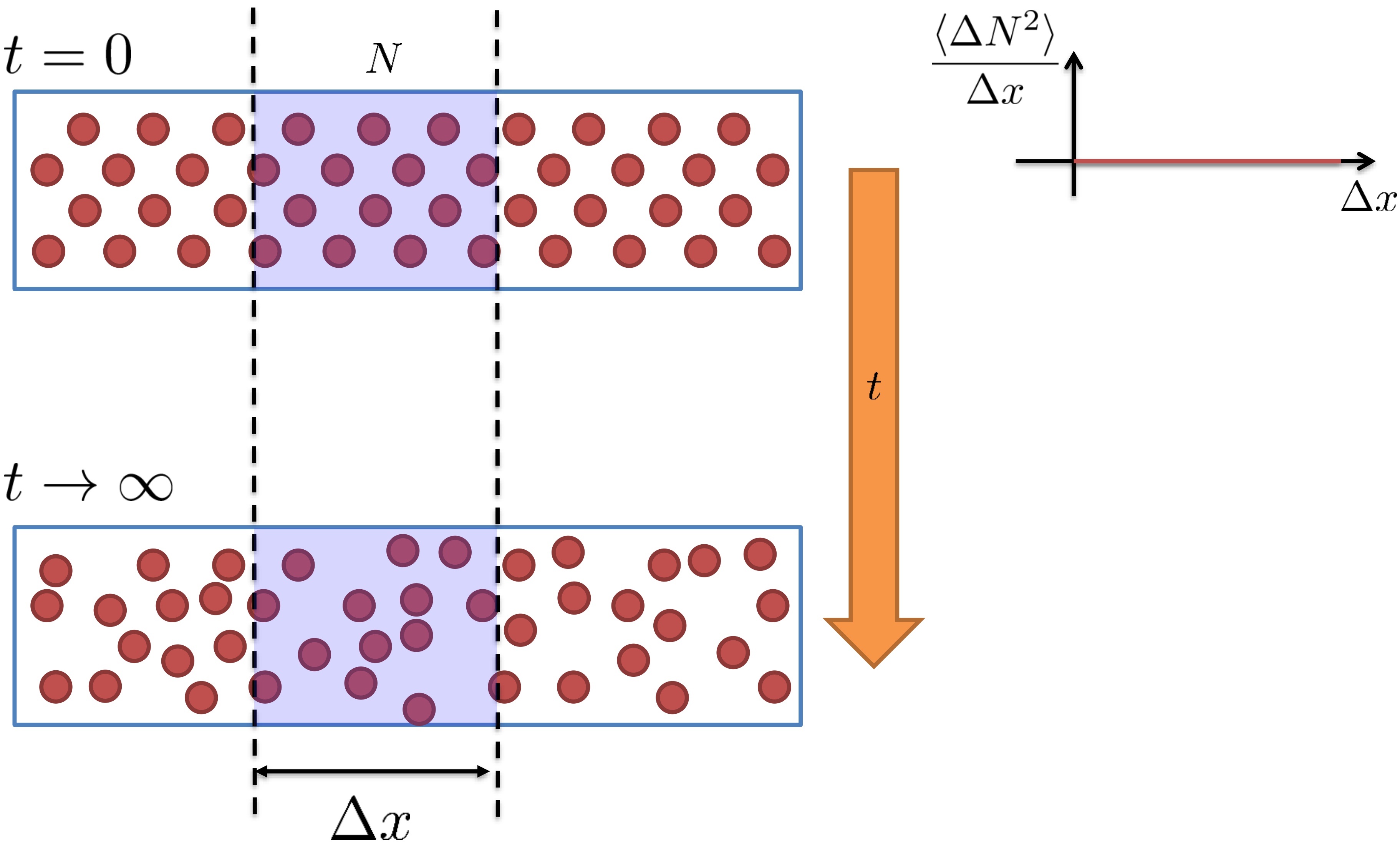
ALICE
PRL 2013



rapidity window



Dissipation of a Conserved Charge



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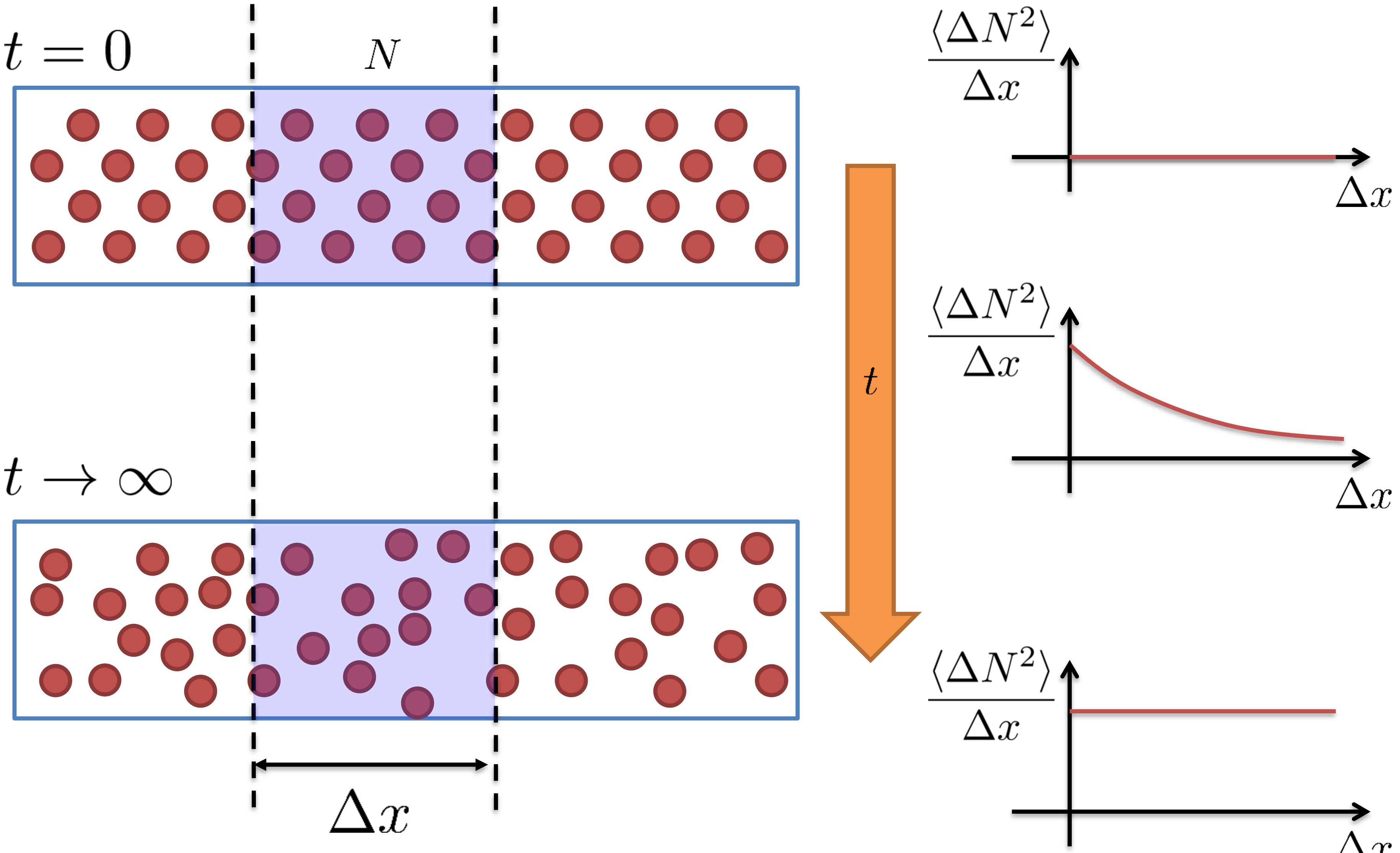
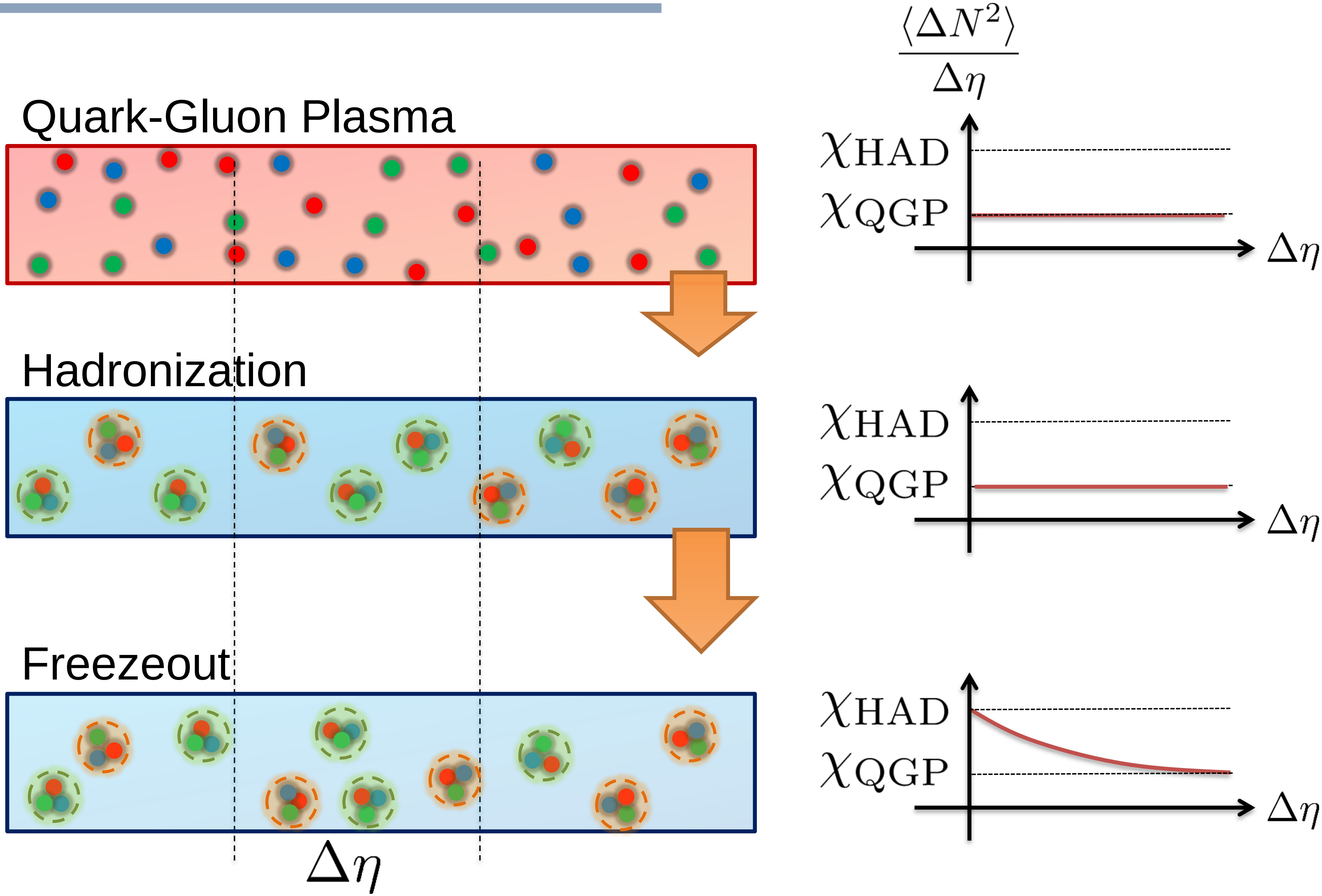


Time Evolution in HIC



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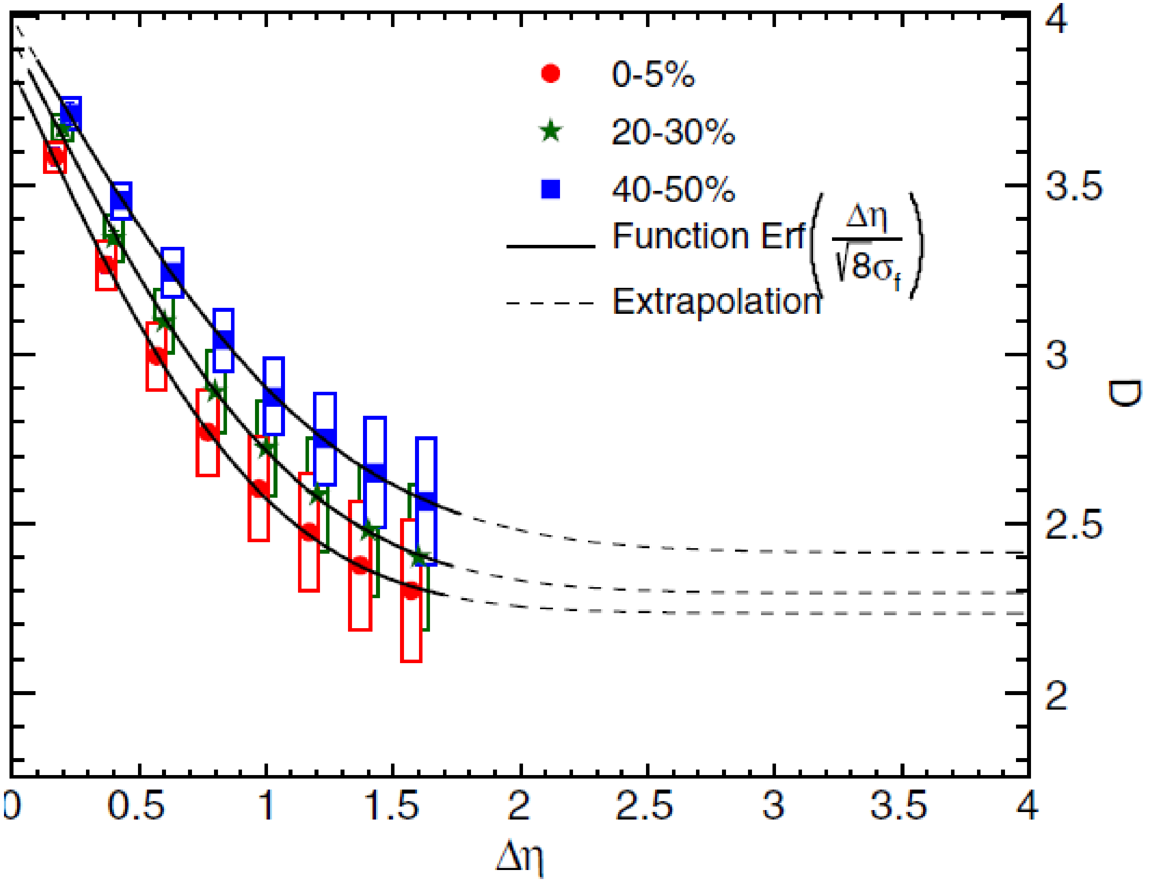
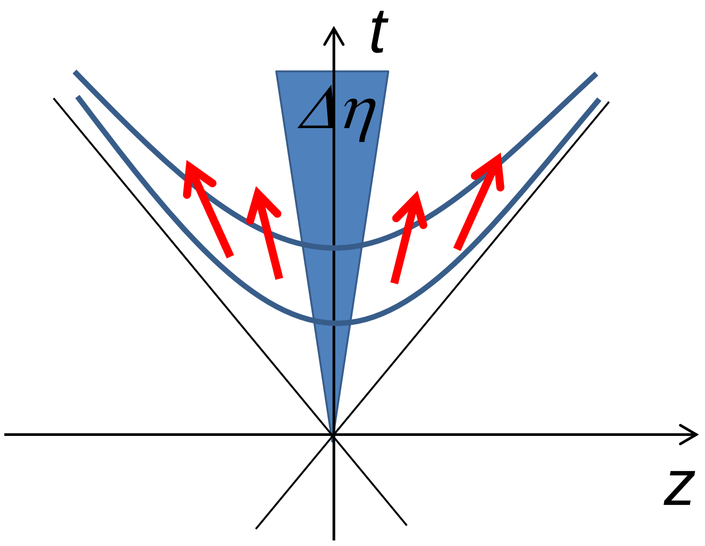
PRL 2013

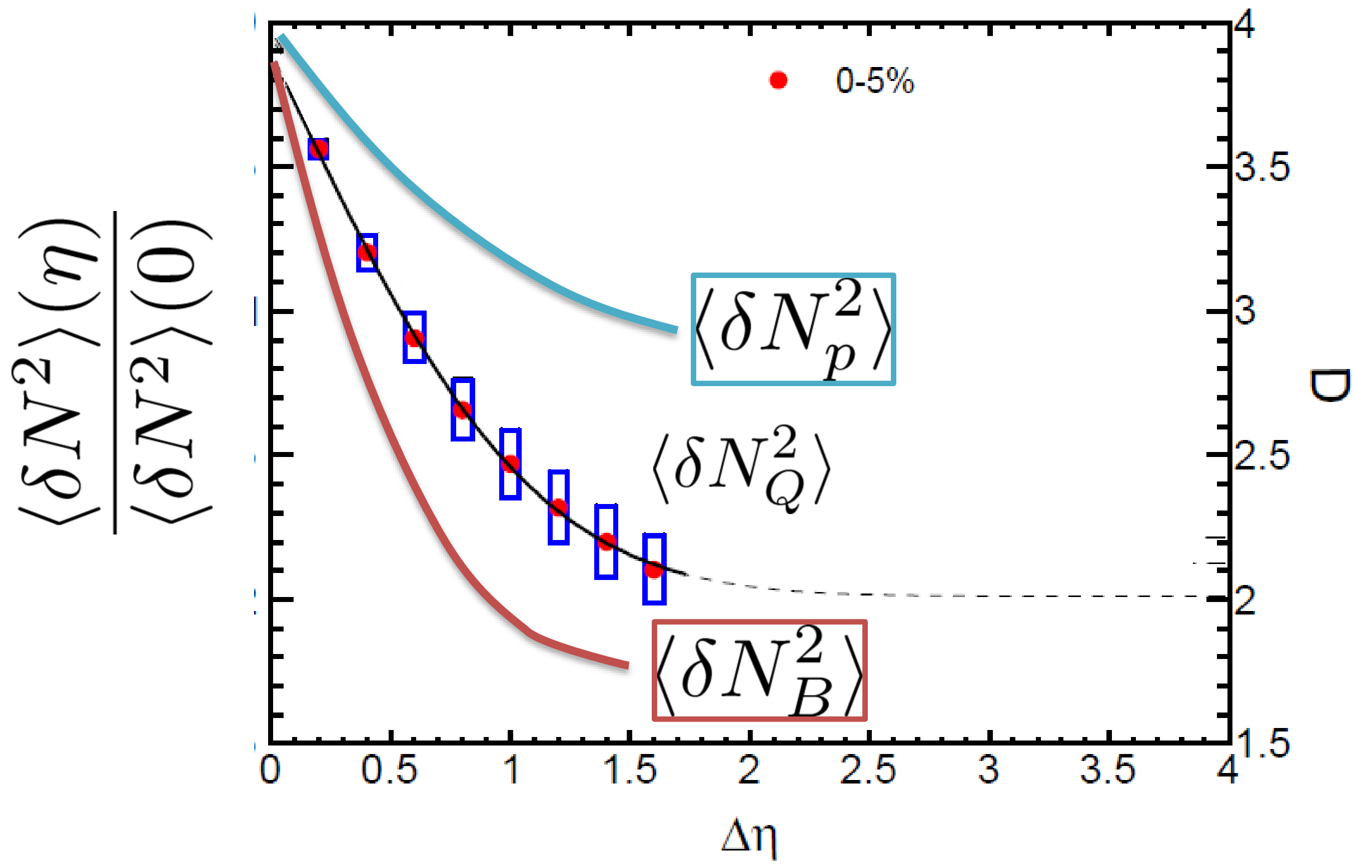




Δη dependences of fluctuation observables encode history of the hot medium!

$<\delta N_B{}^2>$ and $<\delta N_p{}^2>$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
should have different Δη dependence.

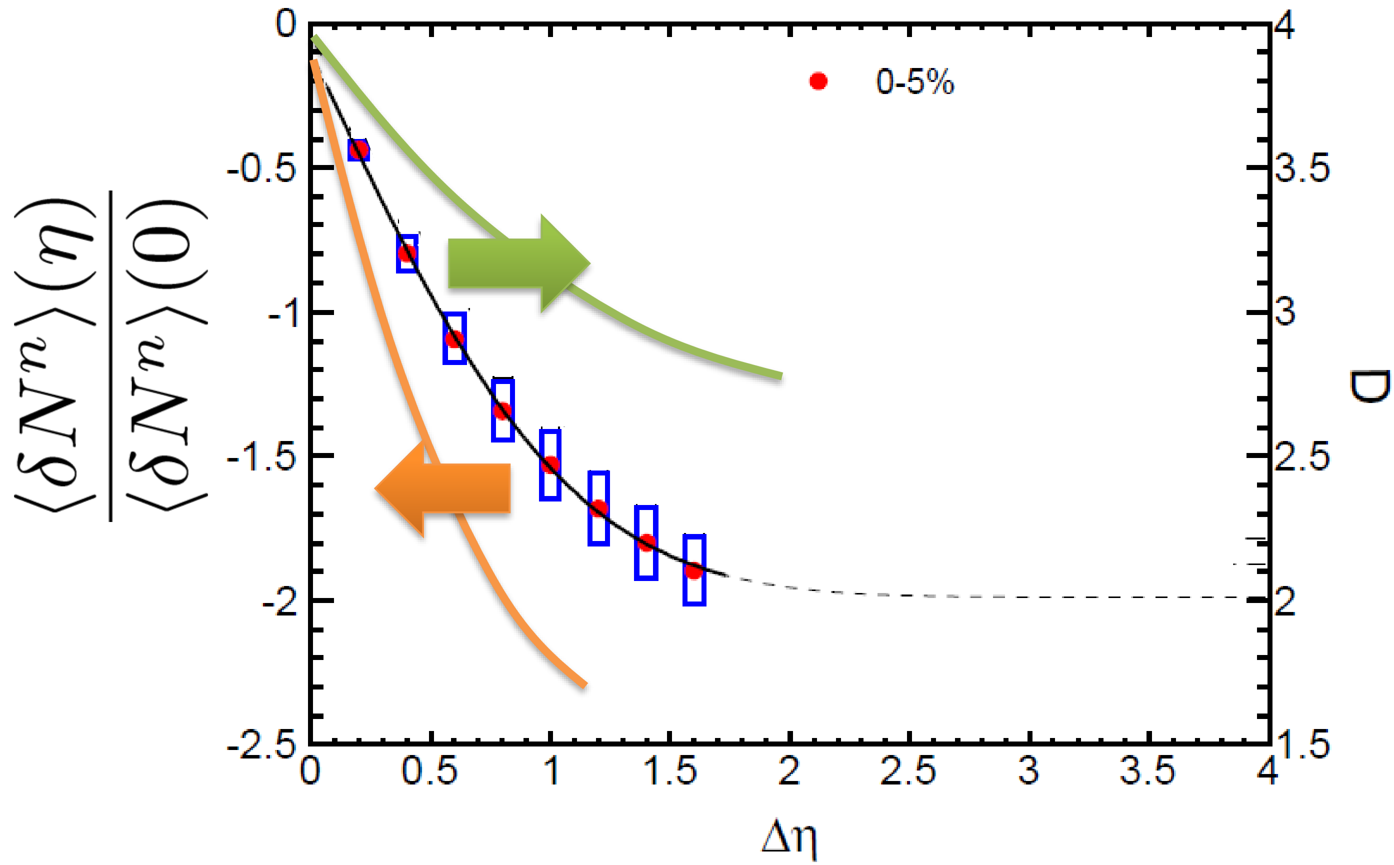


$$\langle (\delta N_p^{(\mathrm{net})})^2 \rangle = \frac{1}{4} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^2 \rangle + \frac{1}{4} \langle N_{\mathrm{B}}^{(\mathrm{tot})} \rangle$$

$\langle\delta N_Q^4\rangle$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

Left (suppression) or Right (hadronic)



Baryon vs Proton Number Fluctuations

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10min

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30min

Baryon vs Proton Number Fluctuations

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- $$\frac{\langle \delta N_B^n \rangle_c}{\langle \delta N_B^m \rangle_c} \neq \frac{\langle \delta N_p^n \rangle_c}{\langle \delta N_p^m \rangle_c}$$
- $\langle \delta N_B^n \rangle_c$ are experimentally observable

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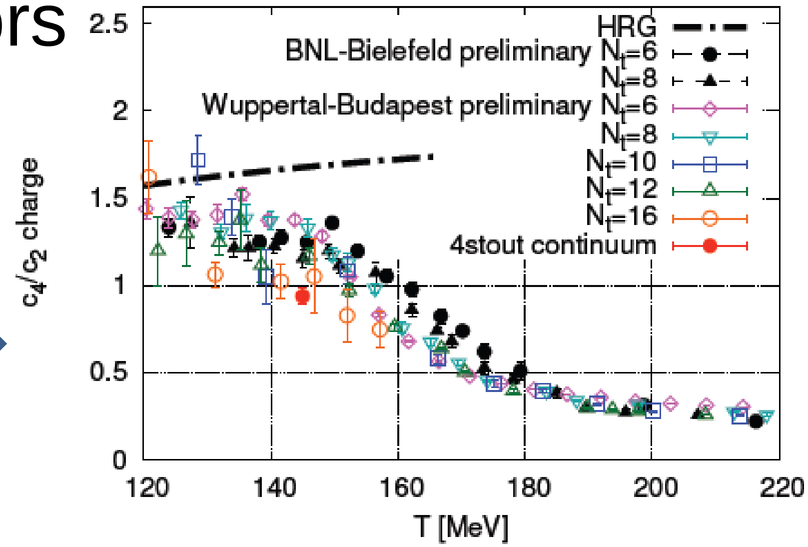
ex: on the lattice



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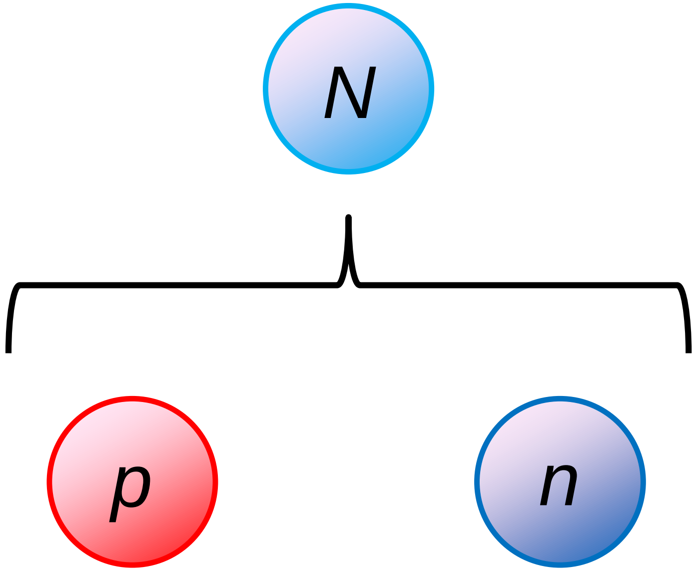
Nucleon Isospin as Two Sides of a Coin



Nucleons have
two isospin states.

MK, Asakawa,2012

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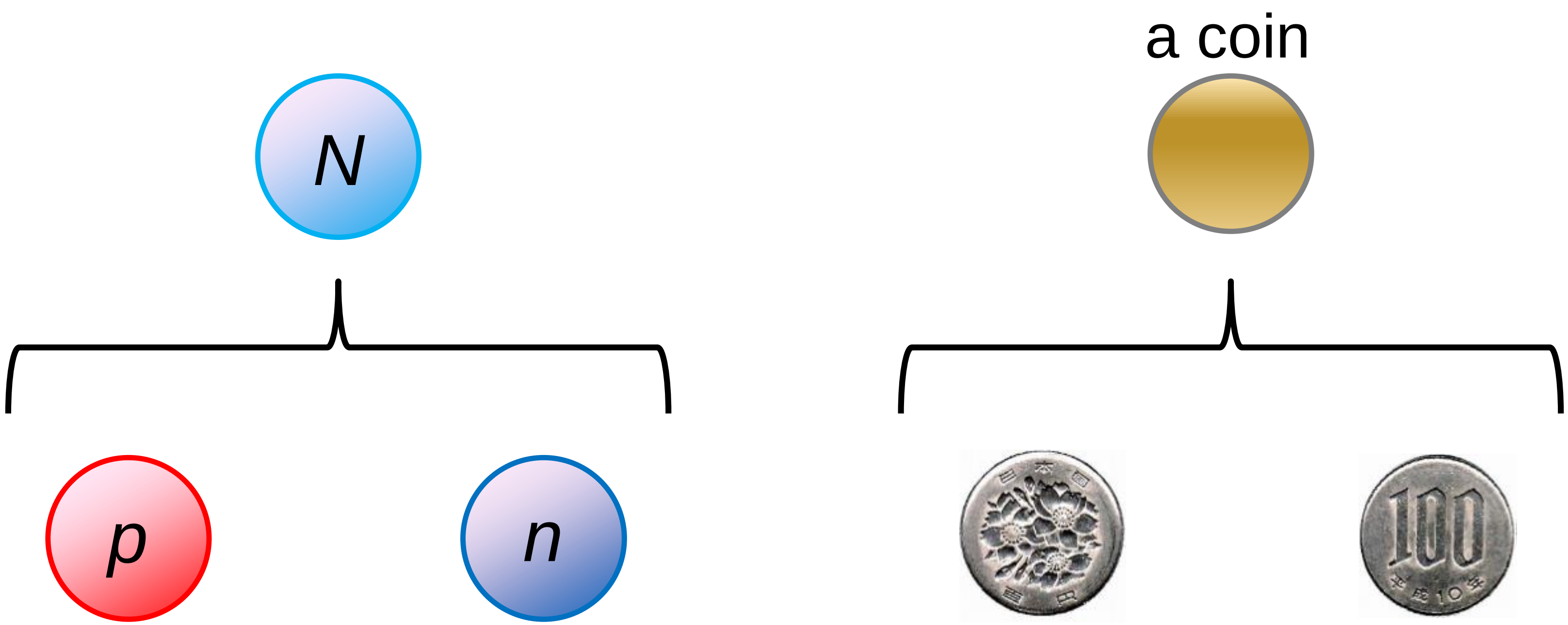

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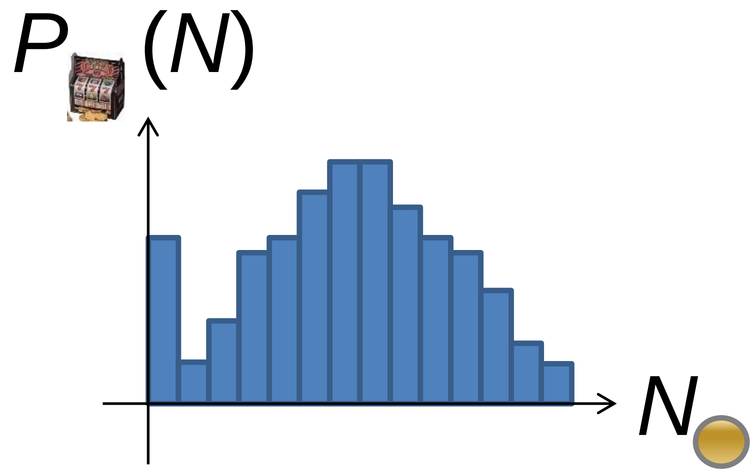
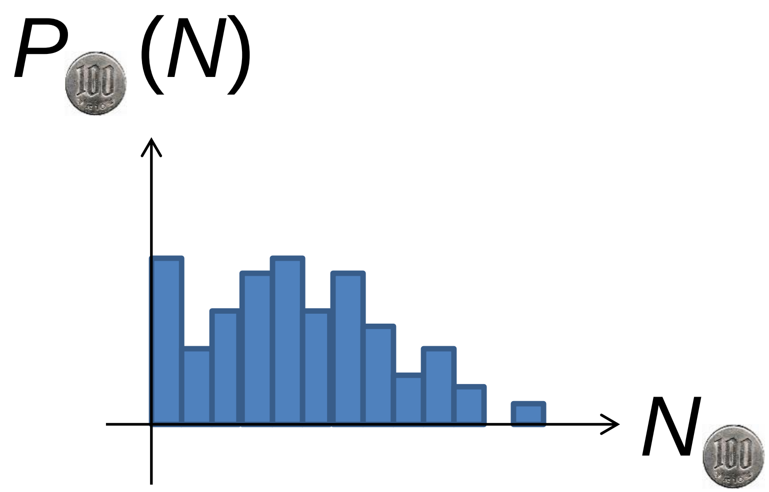
Coins have two sides.

MK, Asakawa,2012

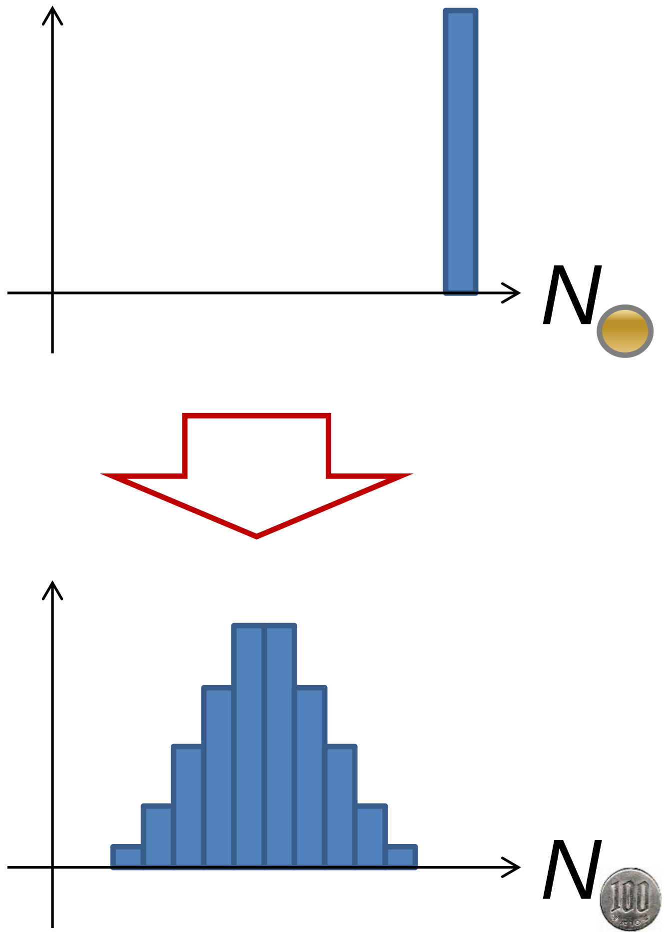
Slot Machine Analogy



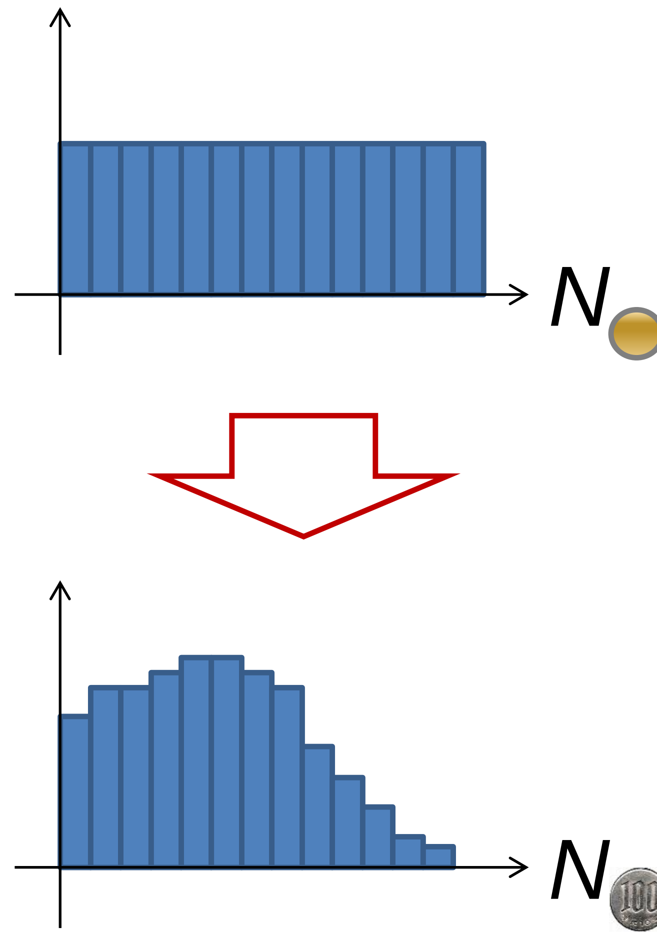


Extreme Examples

Fixed # of coins



Constant probabilities



Reconstructing Total Coin Number

$$P(N) = \sum P(N)\,B_{1/2}(N\,;N)$$

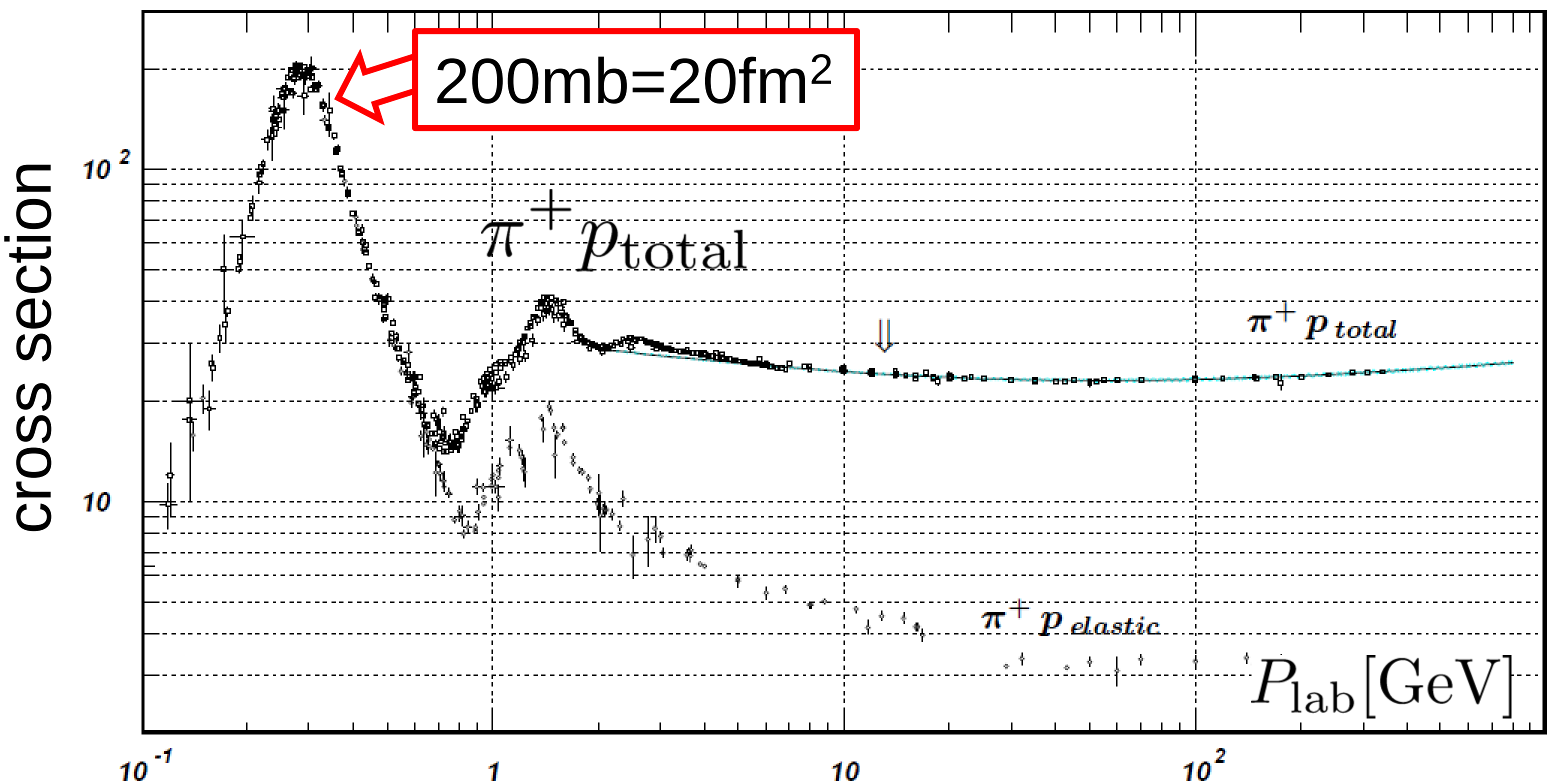
$B_p(k;N) = p^k(1-p)^{N-k}\ {}_kC_N$:binomial distr. func.

Nucleon Isospin in Hadronic Medium

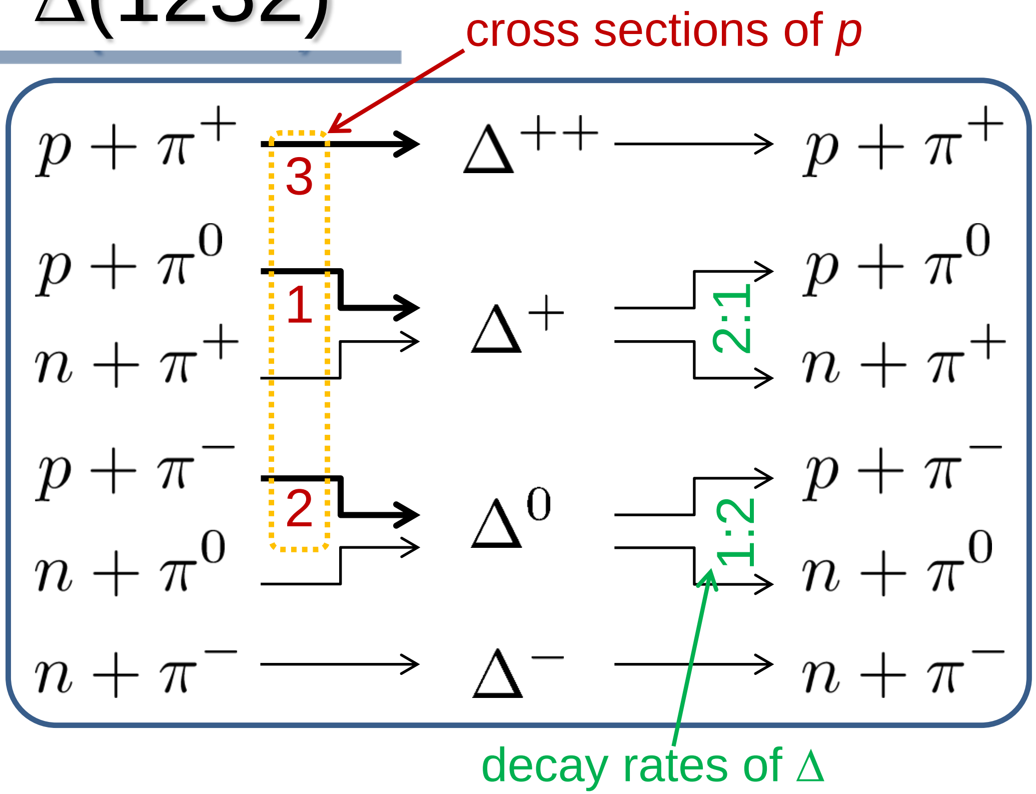
- Isospin of baryons can vary after chemical freezeout via charge exchange reactions mediated by Δ(1232):

$$p, n \rightarrow \Delta(1232) \rightarrow p, n$$
$$\pi \rightarrow \qquad \qquad \rightarrow \pi$$

$$I = 3/2$$
$$\Gamma \simeq 1.8 \text{ [fm]}$$



Δ(1232)



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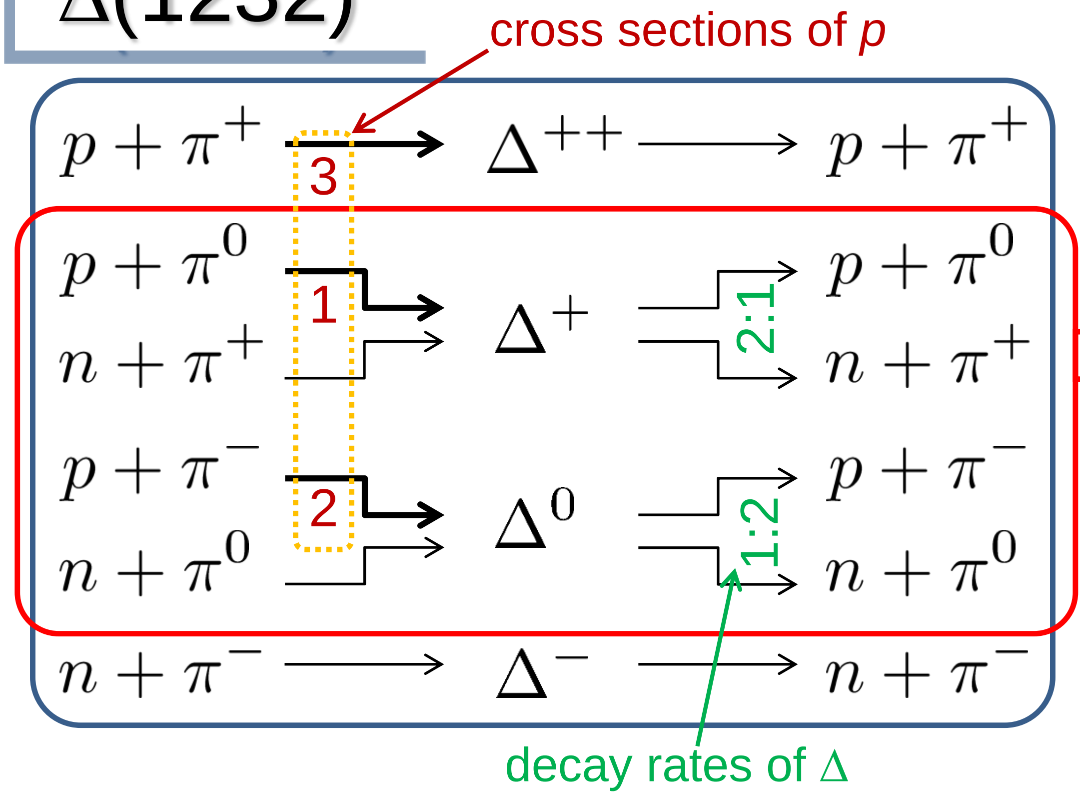
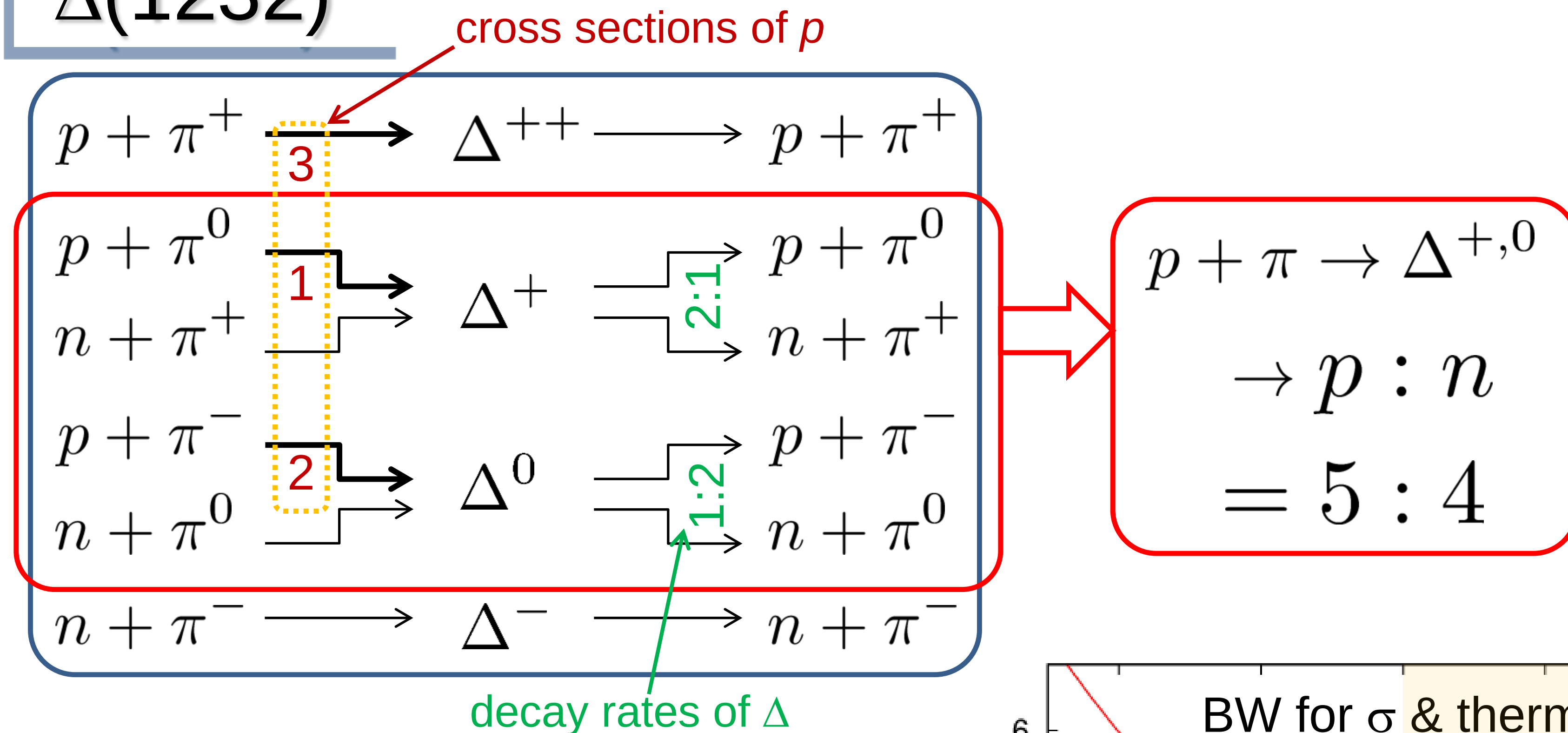


$$p+\pi\to\Delta^{+,0}$$
$$\to p:n$$
$$=5:4$$

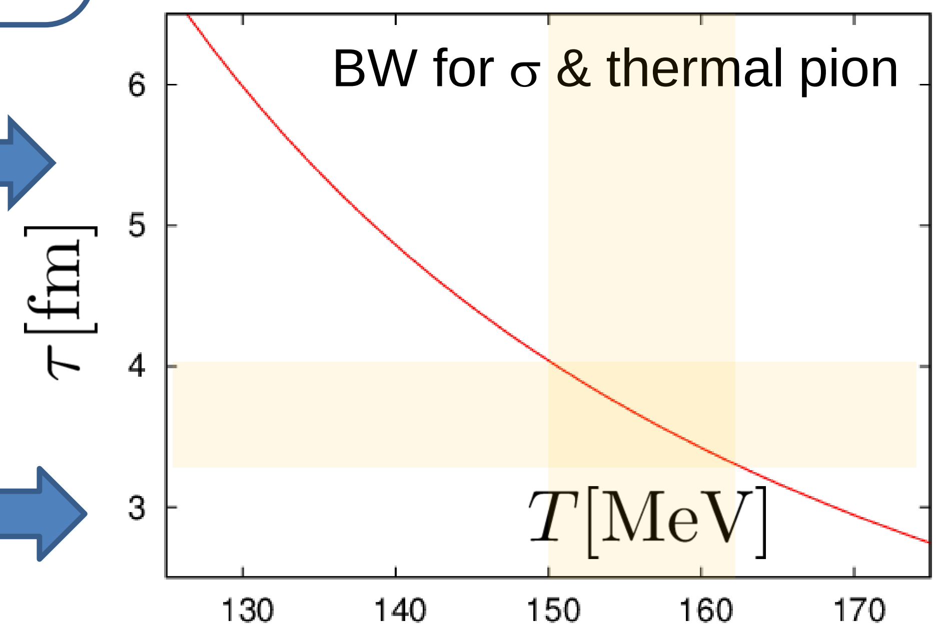
Δ(1232)



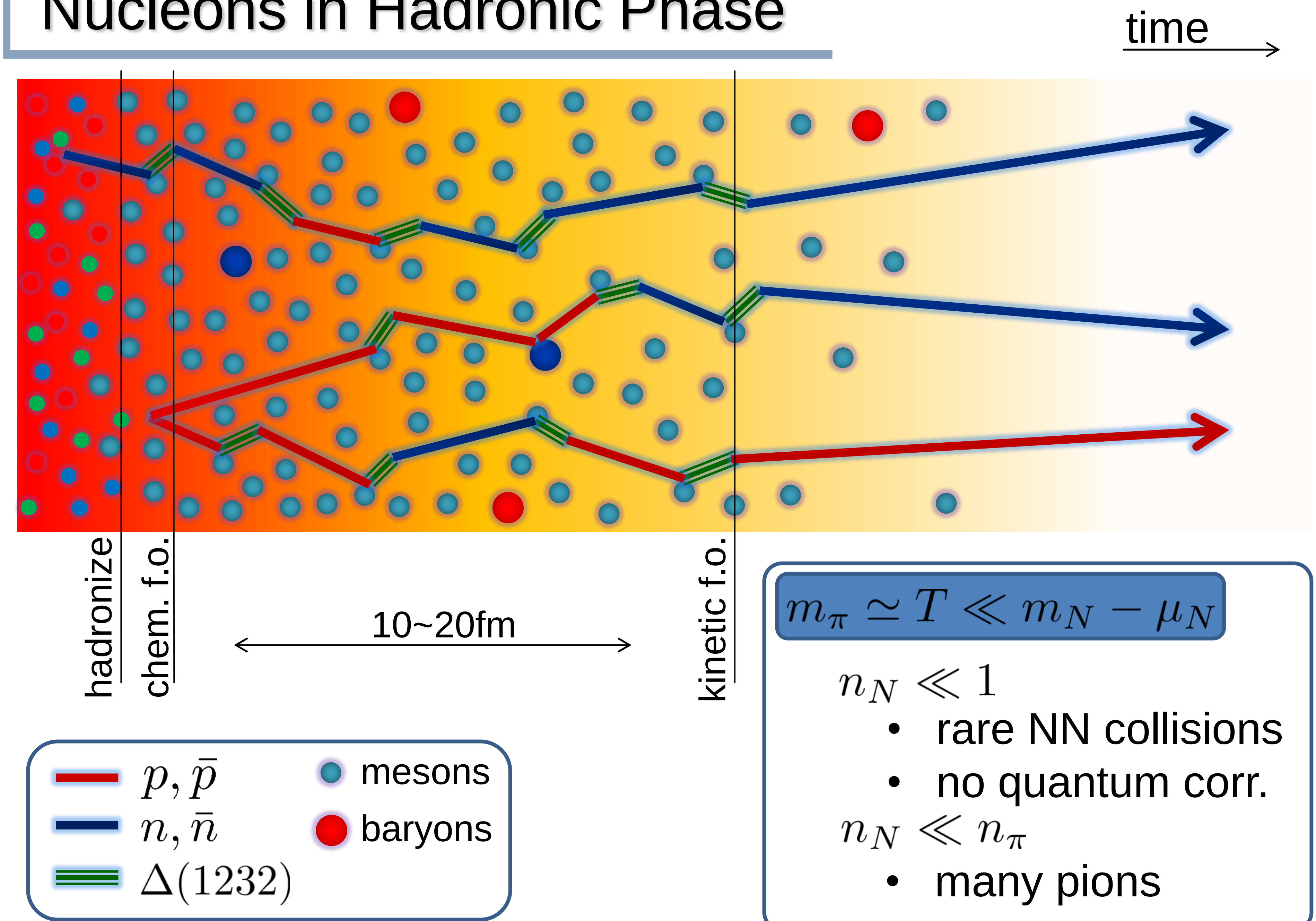
Lifetime to create Δ^+ or Δ^0

$$\tau^{-1} = \int \frac{d^3k_\pi}{(2\pi)^3}\sigma(E_{\rm cm})v_\pi n(E_\pi)$$

$$(\text{freezeout time}) \simeq 20[\text{fm}]$$



Nucleons in Hadronic Phase



$$m_\pi \simeq T \ll m_N - \mu_N$$

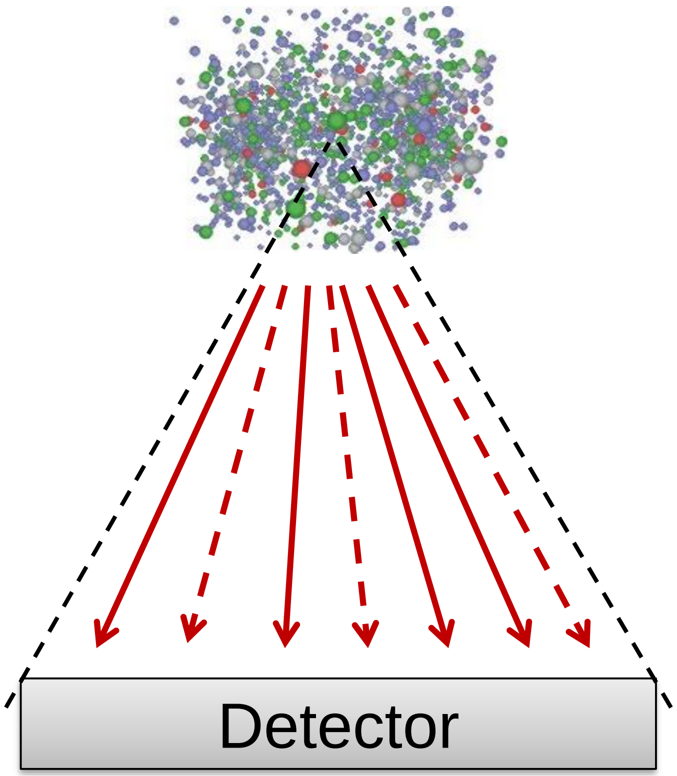
$n_N \ll 1$

- rare NN collisions
- no quantum corr.

$n_N \ll n_\pi$

- many pions

Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$



- $\left\{ \begin{array}{l} \rightarrow N_N \text{ nucleons} \\ \dashrightarrow N_{\bar{N}} \text{ anti-nucleons} \end{array} \right.$

  ⇨ $F(N_N, N_{\bar{N}})$

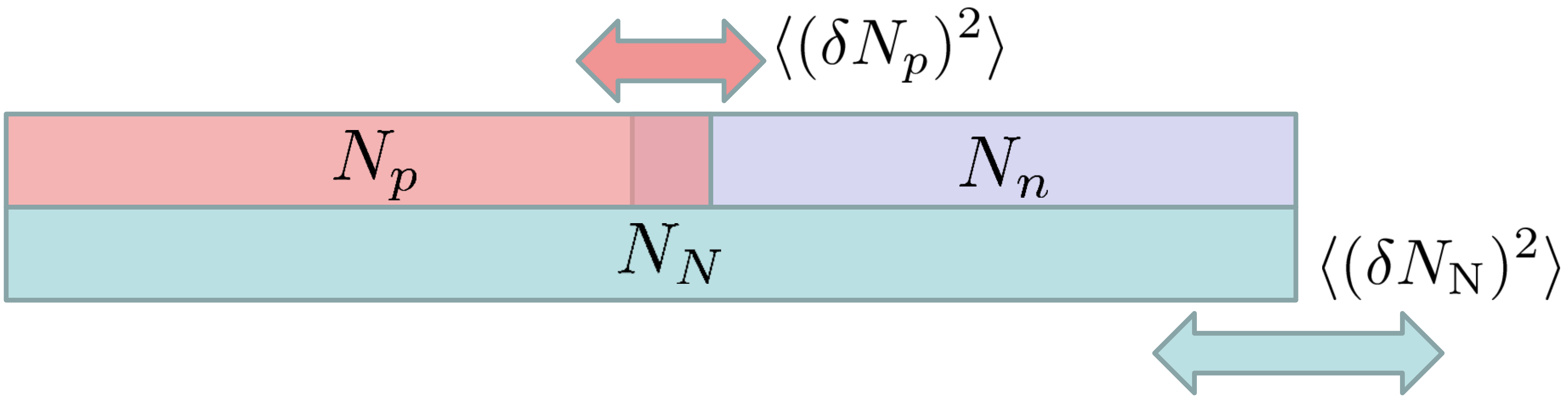
- $N_N \left\{ \begin{array}{l} N_p \text{ protons} \\ N_n \text{ neutrons} \end{array} \right.$

  ⇨ $B(N_p; N_N)$ — binomial distribution func.

$$\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$
$$= F(N_N, N_{\bar{N}}) B(N_p; N_N) B(N_{\bar{p}}; N_{\bar{N}})$$

- for any phase space in the final state.

Nucleon & Proton Number Fluctuations



$$\left\{ \begin{aligned} \langle(\delta N_p^{(\mathrm{net})})^2\rangle &= \frac{1}{4}\langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^2\rangle + \frac{1}{4}\langle N_{\mathrm{N}}^{(\mathrm{tot})}\rangle \\ \langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^2\rangle &= 4\langle(\delta N_p^{(\mathrm{net})})^2\rangle - 2\langle N_p^{(\mathrm{tot})}\rangle \end{aligned} \right.$$

- for isospin symmetric medium
- effect of isospin density <10%
- Similar formulas up to any order!

For free gas

$$\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^2\rangle$$

3^{rd} & 4^{th} Order Fluctuations

$N_{\rm B} \to N_p$

$$\langle (\delta N_p^{\rm (net)})^3 \rangle = \frac{1}{8}\langle (\delta N_{\rm B}^{\rm (net)})^3 \rangle + \frac{3}{8}\langle \delta N_{\rm B}^{\rm (net)} \delta N_{\rm B}^{\rm (tot)} \rangle,$$

$$\langle (\delta N_p^{\rm (net)})^4 \rangle_c = \frac{1}{16}\langle (\delta N_{\rm B}^{\rm (net)})^4 \rangle_c + \frac{3}{8}\langle (\delta N_{\rm B}^{\rm (net)})^2 \delta N_{\rm B}^{\rm (tot)} \rangle + \frac{3}{16}\langle (\delta N_{\rm B}^{\rm (tot)})^2 \rangle - \frac{1}{8}\langle N_{\rm B}^{\rm (tot)} \rangle,$$

$N_p \to N_{\rm B}$

$$\langle (\delta N_{\rm B}^{\rm (net)})^3 \rangle = 8\langle (\delta N_p^{\rm (net)})^3 \rangle - 12\langle \delta N_p^{\rm (net)} \delta N_p^{\rm (tot)} \rangle + 6\langle N_p^{\rm (net)} \rangle,$$

$$\langle (\delta N_{\rm B}^{\rm (net)})^4 \rangle_c = 16\langle (\delta N_p^{\rm (net)})^4 \rangle_c - 48\langle (\delta N_p^{\rm (net)})^2 \delta N_p^{\rm (tot)} \rangle + 48\langle (\delta N_p^{\rm (net)})^2 \rangle + 12\langle (\delta N_p^{\rm (tot)})^2 \rangle - 26\langle N_p^{\rm (tot)} \rangle,$$

Difference btw Baryon and Proton Numbers

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

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(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

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$$\begin{cases} 2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\ 2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\ 2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots \end{cases}$$

genuine info. noise

For free gas

$$2\langle(\delta N_p^{(\mathrm{net})})^n\rangle_c = \langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^n\rangle_c$$

Strange Baryons

Decay Rates:

Λ $\quad m_\Lambda \simeq 1116[\text{MeV}]$

$$\Rightarrow p : n \simeq 1.6 : 1$$

Σ $\quad m_\Sigma \simeq 1190[\text{MeV}]$

$$\Rightarrow p : n \simeq 1 : 1.8$$

Decay modes:

- $\Lambda \rightarrow p + \pi^-$ 64%
- $\Lambda \rightarrow n + \pi^0$ 36%
- $\Sigma^+ \rightarrow p + \pi^0$ 52%
- $\Sigma^+ \rightarrow n + \pi^+$ 48%
- $\Sigma^0 \rightarrow \Lambda \rightarrow p + \pi^-$ 64%
- $\Sigma^0 \rightarrow \Lambda \rightarrow n + \pi^0$ 36%
- $\Sigma^- \rightarrow n + \pi^-$

Regarding these ratios even, protons from these decays is incorporated into the binomial distribution. Then, $N_N \rightarrow N_B$

Time Evolution of Higher Order Cumulants

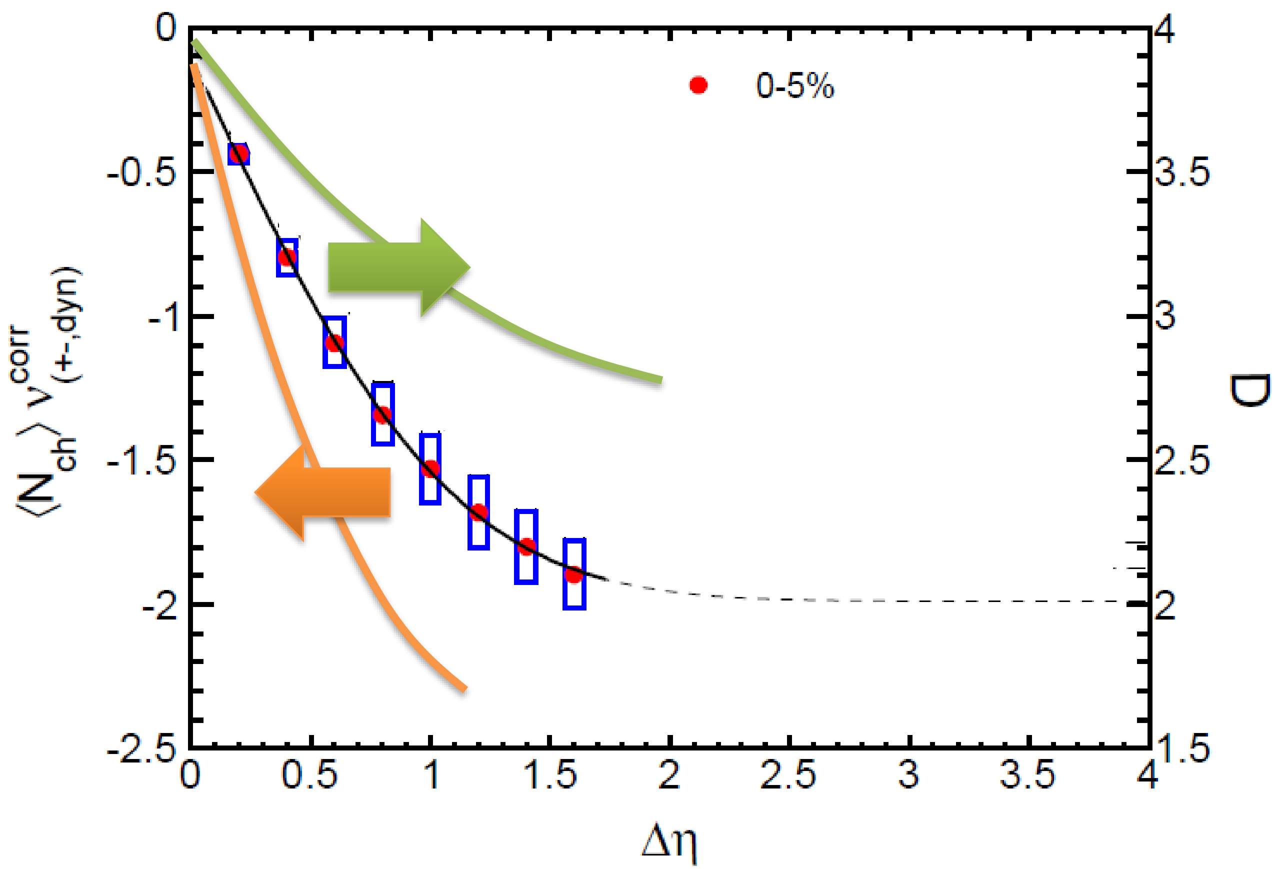
MK, Asakawa, Ono, arXiv:1307.xxxx

30min

$<\delta N_Q^4>$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

Left (suppression) or Right (hadronic)



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n$$

⬇

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

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Conservation Law

$$\partial_\tau n = -\partial_\eta j$$

Fick's Law

$$j = -D\partial_\eta n + \xi$$

Fluctuation-Dissipation Relation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

Stochastic force

- Local correlation (hydrodynamics) $\langle \xi(\eta_1, \tau_1)\xi(\eta_2, \tau_2)\rangle \sim \delta(\eta_1 - \eta_2)\delta(\tau_1 - \tau_2)$
- Equilibrium fluc. $\langle \delta Q(t)^2 \rangle \xrightarrow[t \to \infty]{} \chi_2 \Delta\eta$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta, \tau)$$

χ_2 : susceptibility

$$\langle \xi(k_1, \tau_1)\xi(k_2, \tau_2)\rangle = \frac{2\chi_2}{D}\delta(k_1 + k_2)\delta(\tau_1 - \tau_2)$$

$\Delta\eta$ Dependence

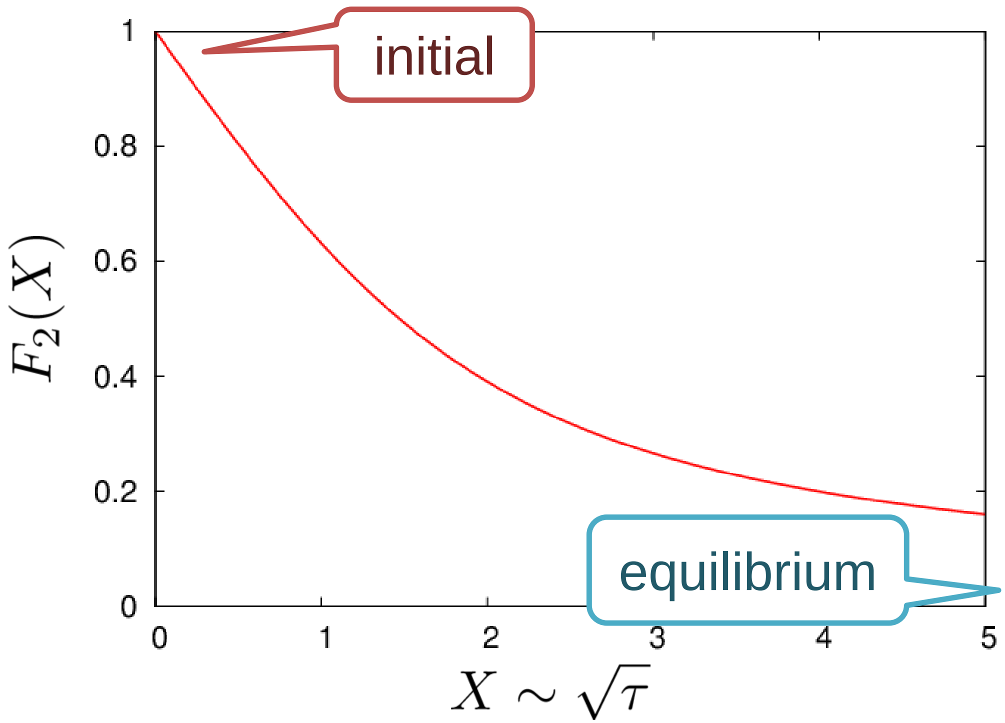
Shuryak, Stephanov, 2001

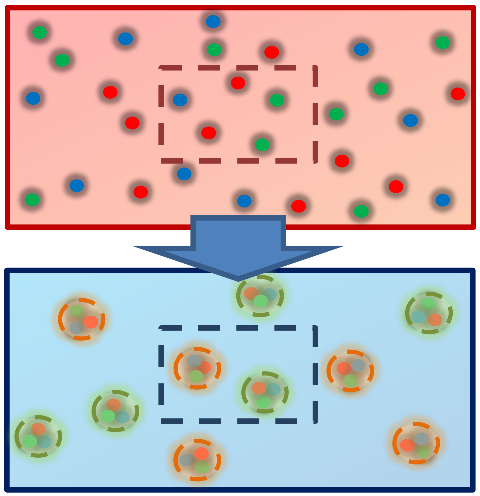
- Initial condition: $\langle \delta n(\eta_1, 0)\delta n(\eta_2, 0)\rangle = \sigma_2 \delta(\eta_1 - \eta_2)$
- Translational invariance

$$\langle \delta Q(\tau)^2 \rangle = \underbrace{\sigma_2 F_2(X)}_{\text{initial}} + \underbrace{\chi_2 (1 - F_2(X))}_{\text{equilibrium}}$$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta\, n(\eta, \tau)$$

$$X = \frac{2\sqrt{D\tau}}{\Delta\eta}$$





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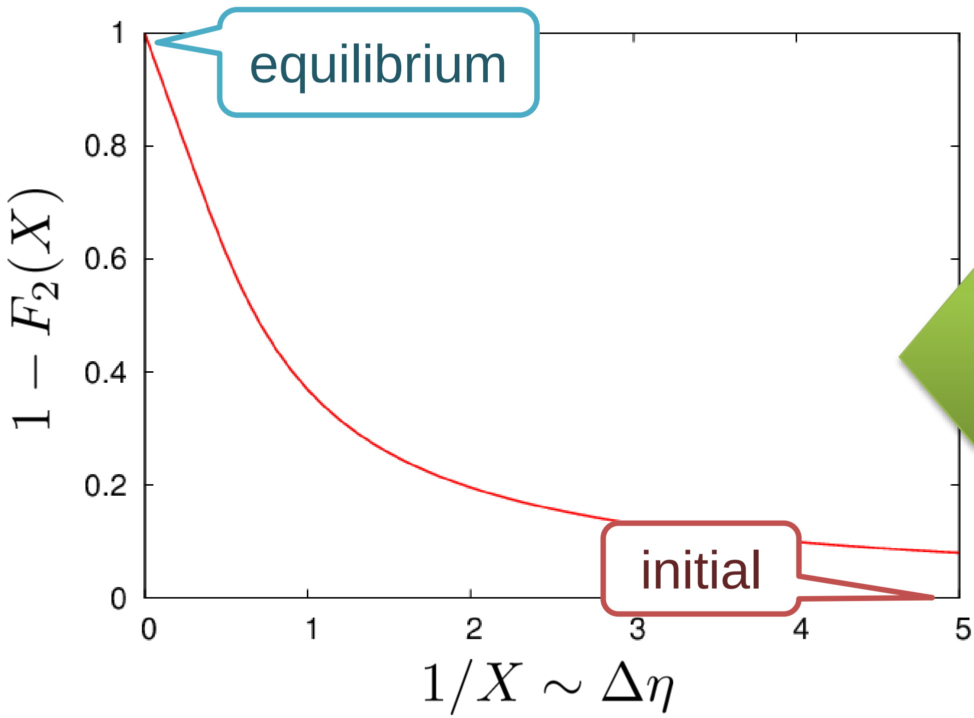

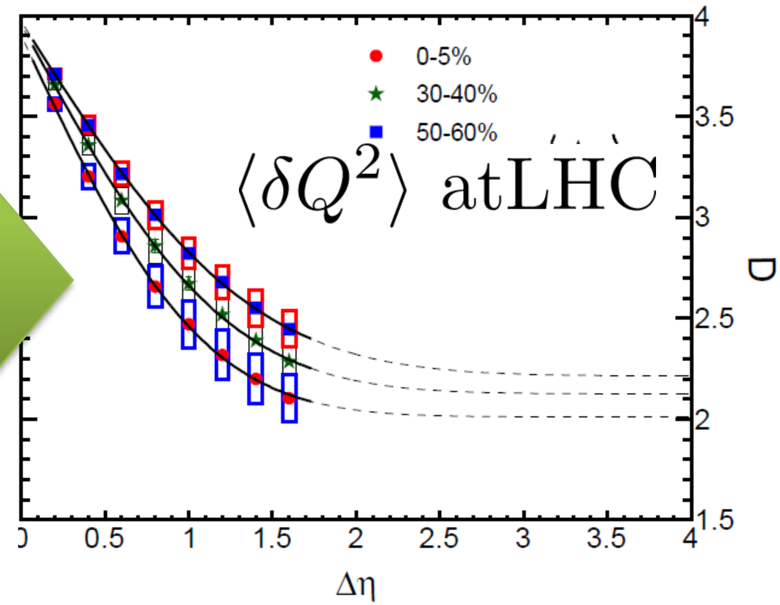

Non-Gaussian Stochastic Force ??

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Stochastif Force : 3rd order

- Local correlation (hydrodynamics) $\langle \xi(\eta_1,\tau_1)\xi(\eta_2,\tau_2)\xi(\eta_3,\tau_3)\rangle$
$$\sim \delta(\eta_1-\eta_2)\delta(\eta_2-\eta_3)\delta(\tau_1-\tau_2)\delta(\tau_2-\tau_3)$$
- Equilibrium fluc. $\langle \delta Q(t)^3\rangle \underset{t\to\infty}{\longrightarrow} \chi_3 \Delta\eta$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta,\tau)$$

χ_3 : third − moment

Caution!

- $$\langle \xi(k_1,\tau_1)\xi(k_2,\tau_2)\xi(k_3,\tau_3)\rangle = \frac{\chi_3}{\gamma}\frac{k_1^2+k_2^2+k_3^2}{k_1k_2k_3}\delta(k_1+k_2+k_3)$$
$$\times\delta(\tau_1-\tau_2)\delta(\tau_2-\tau_3)$$

  diverge in long wavelength

- No a priori extension of FD relation to higher orders

Caution!

- $$\langle \xi(k_1,\tau_1)\xi(k_2,\tau_2)\xi(k_3,\tau_3)\rangle = \frac{\chi_3}{\gamma}\frac{k_1^2+k_2^2+k_3^2}{k_1k_2k_3}\delta(k_1+k_2+k_3) \times\delta(\tau_1-\tau_2)\delta(\tau_2-\tau_3)$$

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- No a priori extension of FD relation to higher orders

- Theorem

  Markov process + continuous variable

  → Gaussian random force

  cf) Gardiner, "Stochastic Methods"

- Hydrodynamics ⇒ Local equilibrium with many particles

  ⇒ Gaussian due to central limit theorem

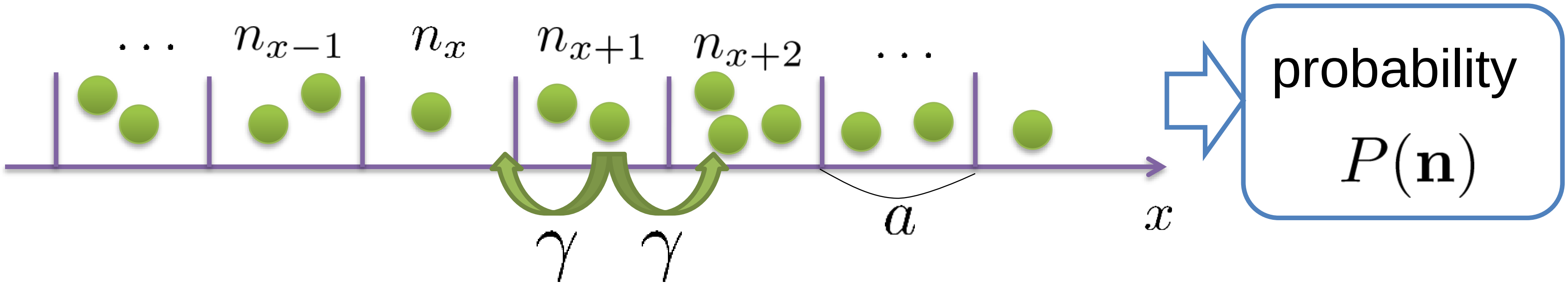
Three “NON”s

Physics of non-Gaussianity in heavy-ion collisions is a particular problem!

- **Non-Gaussian** — Non-Gaussianitiy is irrelevant in large systems
- **Non-critical** — critical enhancement is not observed in HIC so far
- **Non-equilibrium** — Fluctuations are not equilibrated in HIC

Diffusion Master Equation

Divide spatial coordinate into discrete cells



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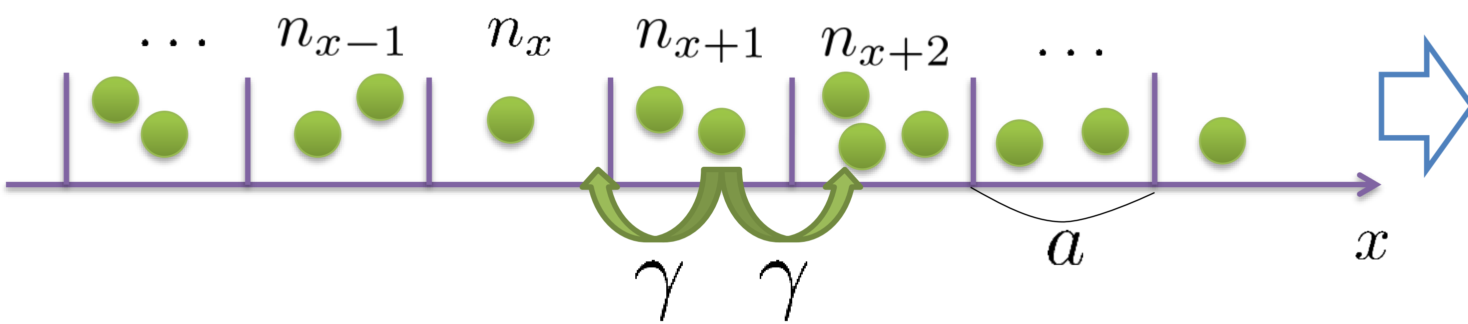

probability

$P(\mathbf{n})$

Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x \Big[(n_x+1)\left\{P(\mathbf{n}+\hat{x}-\widehat{x+1}) + P(\mathbf{n}+\hat{x}-\widehat{x-1})\right\} - 2n_x P(\mathbf{n})\Big]$$

x-hat: lattice-QCD notation

Solve the DME **exactly**, and take *a*→0 limit

No approx., ex. van Kampen’s system size expansion

Solution of DME

1st $\langle \tilde{n}_k \rangle(t) = e^{-\omega_k t} \langle \tilde{n}_k \rangle_0$ $\qquad \omega_k \simeq \gamma a^2 k^2$

initial

- Deterministic part ←→ diffusion equation at long wave length (1/a<<k)

$$\partial_t \langle n_x(t) \rangle = \gamma a^2 \partial_x^2 \langle n_x(t) \rangle$$

- Appropriate continuum limit with $\gamma a^2 = D$

Solution of DME

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- Appropriate continuum limit with $\gamma a^2 = D$

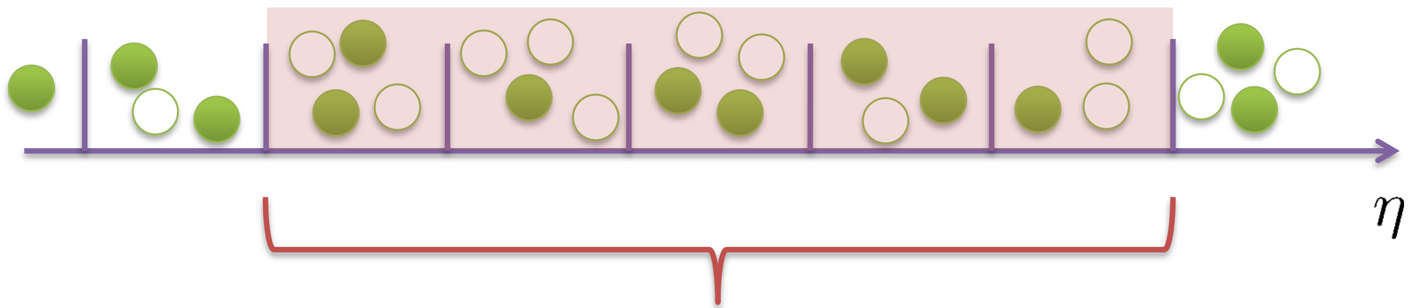
2nd

$$\langle \delta\tilde{n}_{k_1} \delta\tilde{n}_{k_2} \rangle (t) = \langle \tilde{n}_{k_1+k_2} \rangle_0 \left(e^{-\omega_{k_1+k_2} t} - e^{-(\omega_{k_1} + \omega_{k_2})t} \right) + \langle \delta\tilde{n}_{k_1} \delta\tilde{n}_{k_2} \rangle_0 e^{-(\omega_{k_1} + \omega_{k_2})t}$$

- Consistent with stochastic diffusion eq. (for sufficiently smooth initial condition)

Net Charge Number

Prepare 2 species of (non-interacting) particles



$$\bar{Q}(\tau) = \int_0^{\Delta\eta} d\eta \left(n_1(\eta,\tau) - n_2(\eta,\tau) \right)$$

Let us investigate

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ at freezeout time t

Initial Condition at Hadronization

- Boost invariance / infinitely long system
- Local equilibration / local correlation
- Initial fluctuations

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ $\quad \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ $\quad \langle Q_{(\mathrm{tot})}^2 \rangle_c$

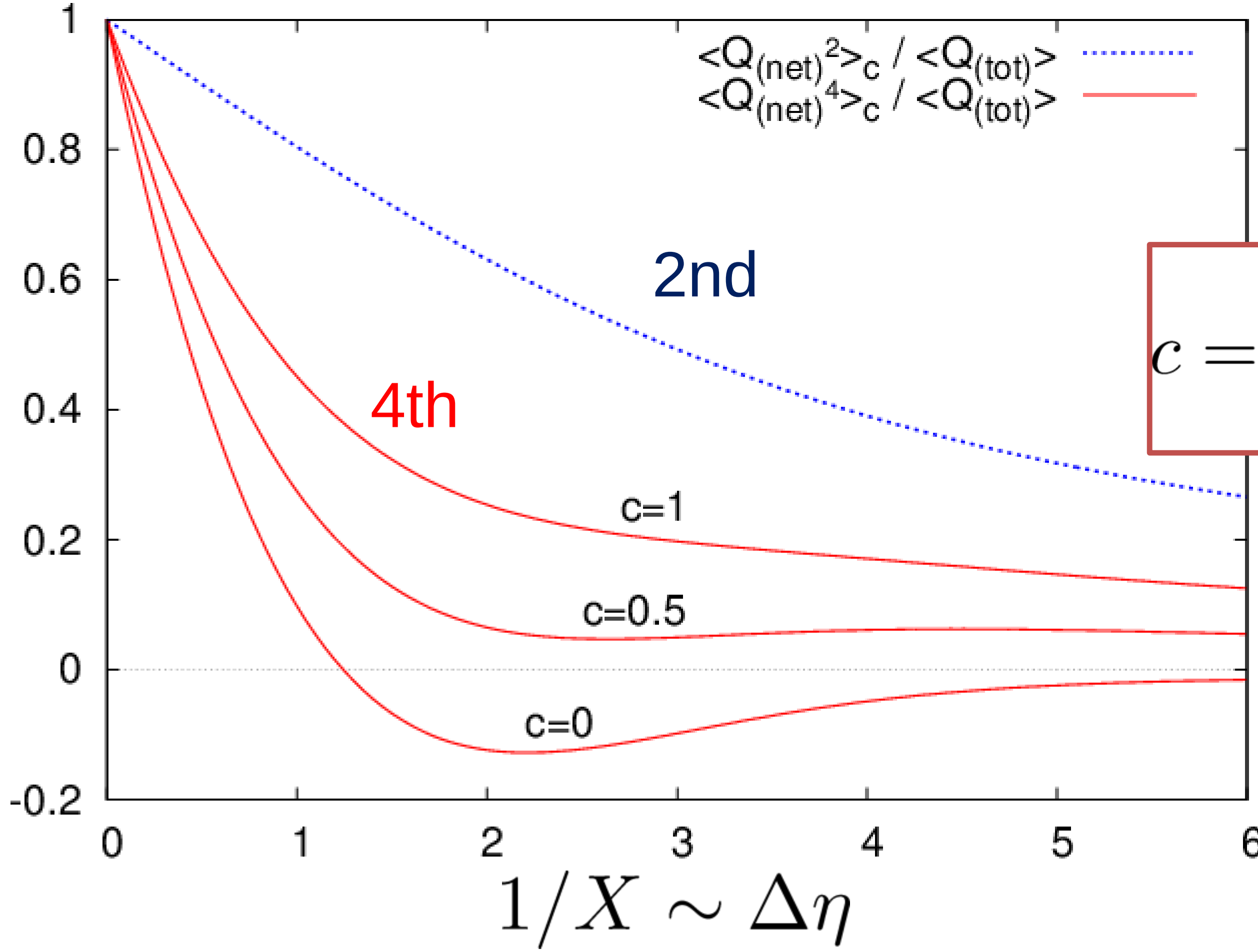
suppression owing to local charge conservation

strongly dependent on hadronization mechanism

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

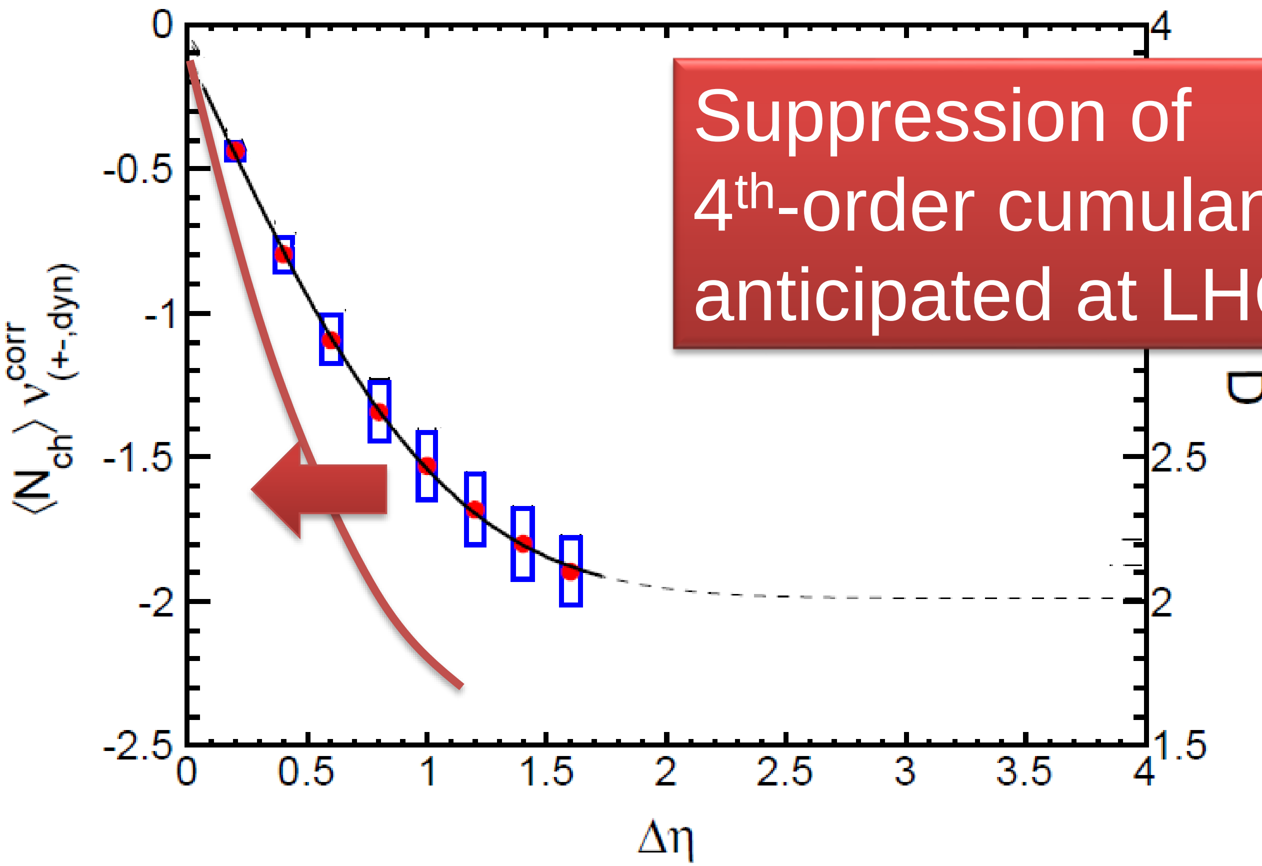
$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$



$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$\langle \delta N_Q^4 \rangle$ @ LHC

Assumptions
- boost invariant system
- small fluctuations of CC at hadronization
- short correlation in hadronic stage

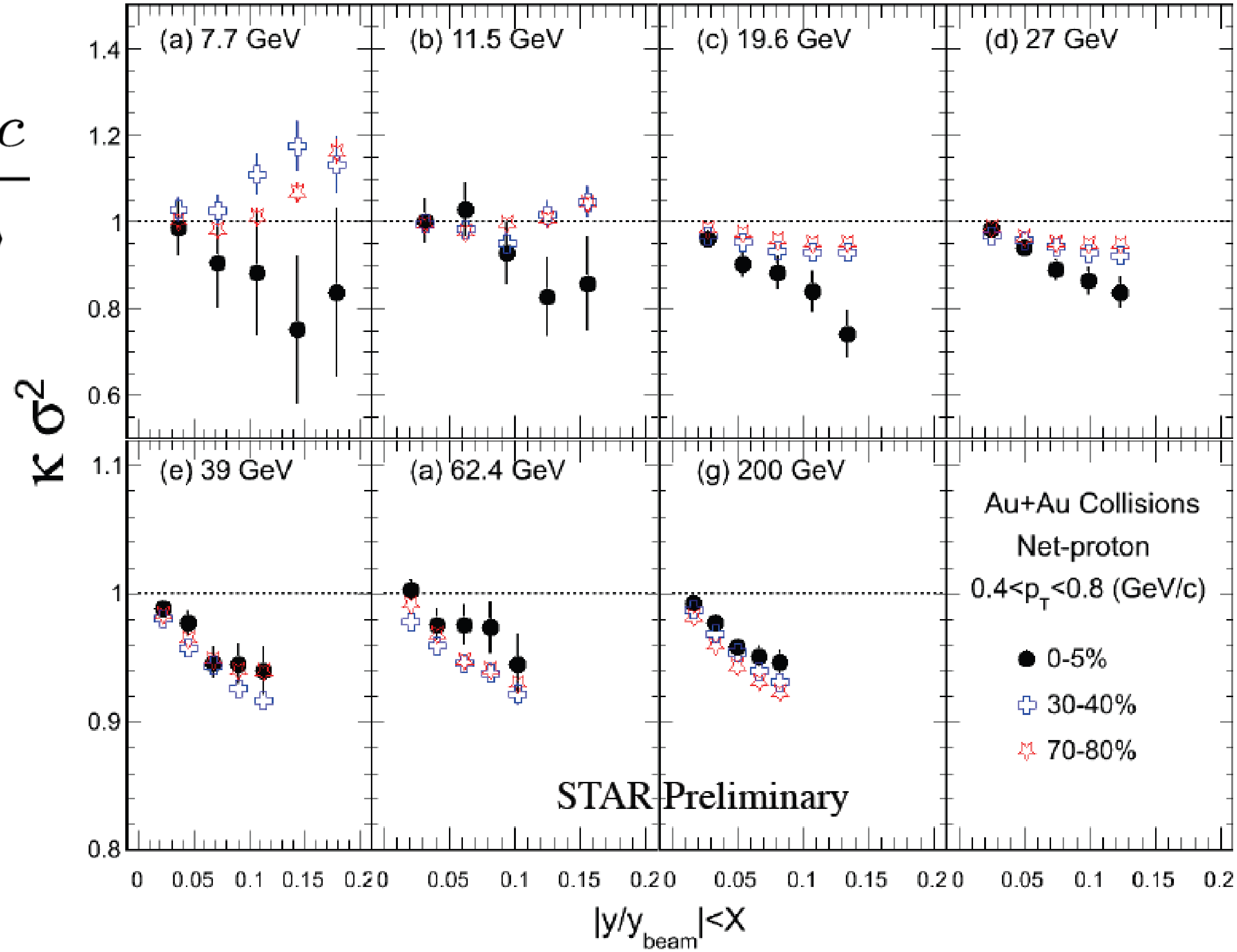


Suppression of 4^{th}-order cumulant is anticipated at LHC energy!

$\Delta\eta$ Dependence at STAR

STAR, QM2012

$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$

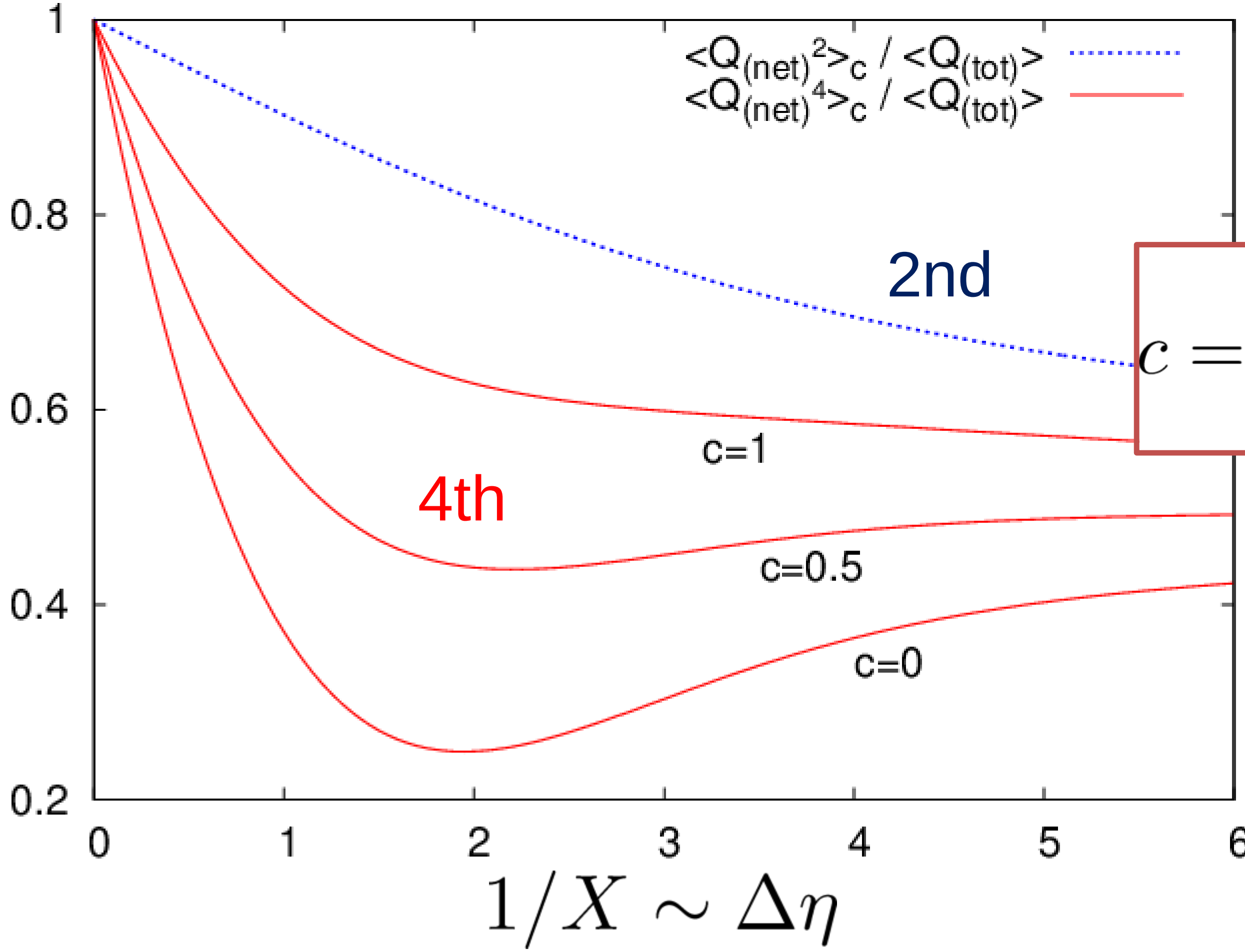


$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$ decreases as $\Delta\eta$ becomes larger at RHIC.

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0.5 \langle Q_{(\mathrm{tot})} \rangle$$



$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

Summary

Plenty of physics in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2 \rangle_c, \ \langle N_B^2 \rangle_c, \ \langle N_Q^4 \rangle_c, \ \langle N_B^4 \rangle_c, \ \langle N_{ch}^2 \rangle_c, \cdots$$

Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC

- history of hot medium
- mechanism of hadronization
- diffusion constant

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$$\langle N_Q^2 \rangle_c,\ \langle N_B^2 \rangle_c,\ \langle N_Q^4 \rangle_c,\ \langle N_B^4 \rangle_c,\ \langle N_{ch}^2 \rangle_c, \cdots$$

Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC

- history of hot medium
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- diffusion constant

Search of QCD Phase Structure

Open Questions & Future Work

- Why the primordial fluctuations are observed only at LHC, and not RHIC ?
- Extract more information on each stage of fireballs using fluctuations

- Model refinement
  - Including the effects of
    nonzero correlation length / relaxation time
    global charge conservation

  - Non Poissonian system ← interaction of particles

Chemical Reaction 1

$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

x: # of X
a: # of A (**fixed**)

Master eq.: $$\frac{\partial}{\partial t}P(x,t) = k_2 a P(x-1,t) + k_1(x+1)P(x+1,t) - (k_1 x + k_2 a)P(x,t)$$

Cumulants with fixed initial condition $P(x,0) = \delta_{x,N_0}$

$$\langle x(t)\rangle = N_0 e^{-k_1 t} + N_{eq}(1 - e^{-k_1 t})$$

$$\langle \delta x(t)^2\rangle = N_0(e^{-k_1 t} - e^{-2k_1 t}) + N_{eq}(1 - e^{-k_1 t})$$

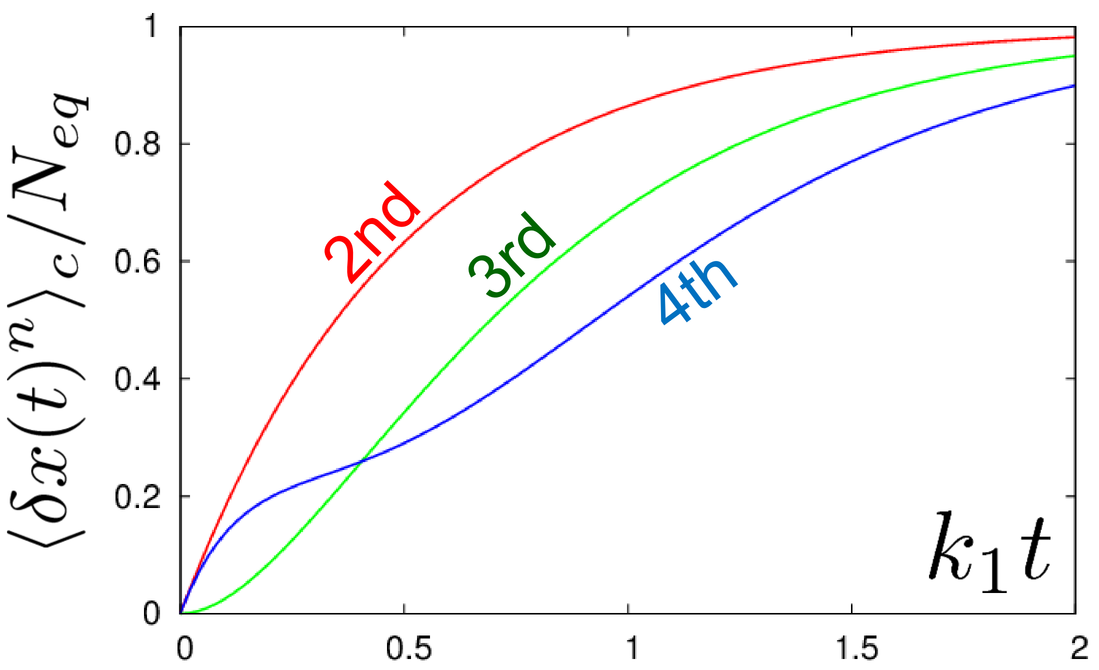
$$\langle \delta x(t)^3\rangle = N_0(e^{-k_1 t} - 3e^{-2k_1 t} + 2e^{-3k_1 t}) + N_{eq}(1 - e^{-k_1 t})$$

initial   equilibrium

Chemical Reaction 2

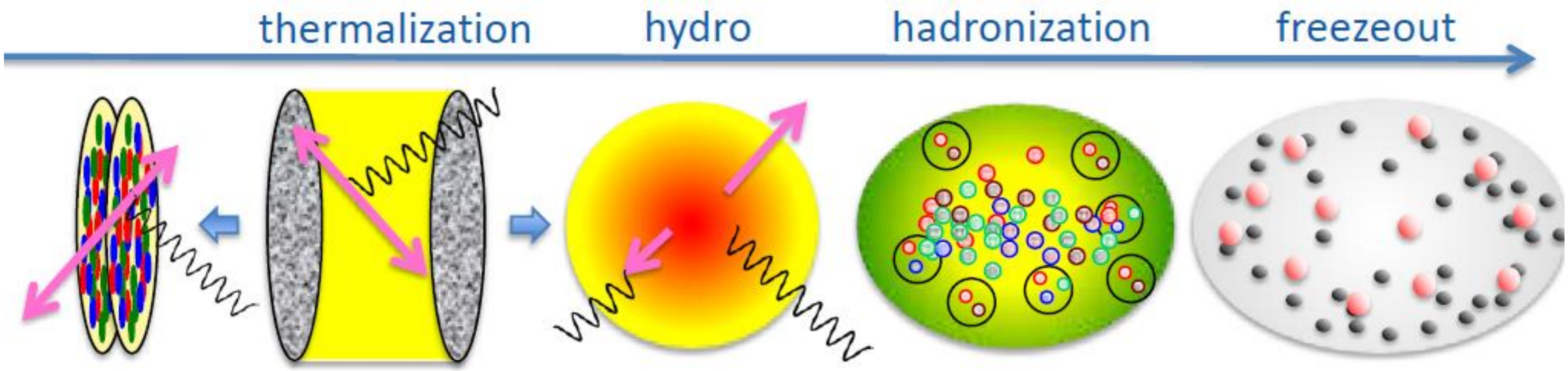
$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

$$N_0 = N_{eq} \Rightarrow \begin{cases} \langle x(t) \rangle = N_{eq} \\ \langle \delta x(t)^2 \rangle = N_{eq}(1 - e^{-2k_1 t}) \\ \langle \delta x(t)^3 \rangle = N_{eq}(1 - 3e^{-2k_1 t} + 2e^{-3k_1 t}) \end{cases}$$



Higher-order cumulants grow slower.

Evolution of Fluctuations



Fluctuation in initial state

Time evolution in the QGP

approach to HRG by diffusion

volume fluctuation

experimental effects
particle missID, etc.

Central Limit Theorem

CLT

Large system → Gaussian

↓

Higher-order cumulants suppressed
as system volume becomes larger?

Central Limit Theorem

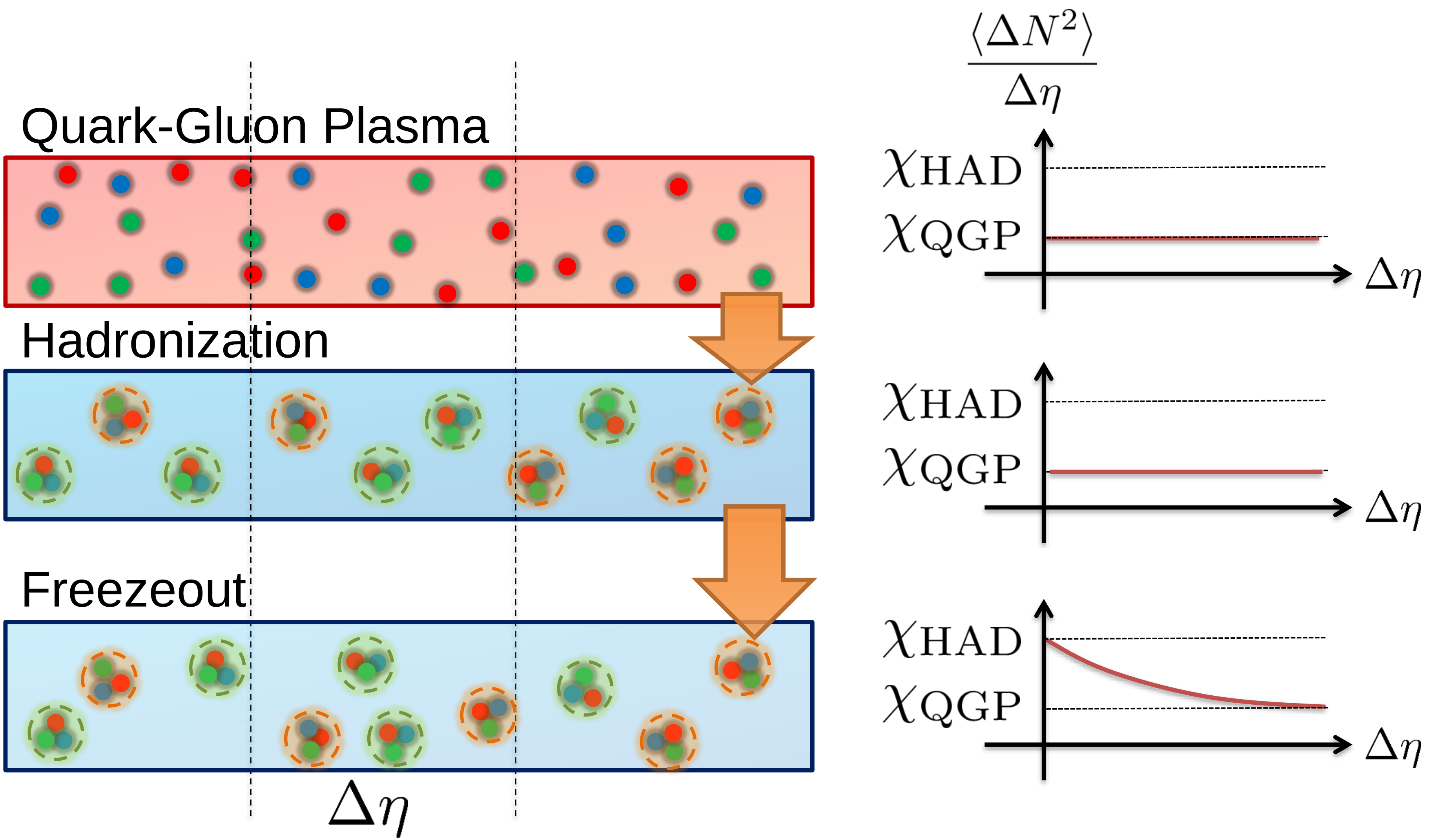
CLT

$$\bar{n} = \frac{N}{\sqrt{V}} \quad \text{:Gaussian}$$

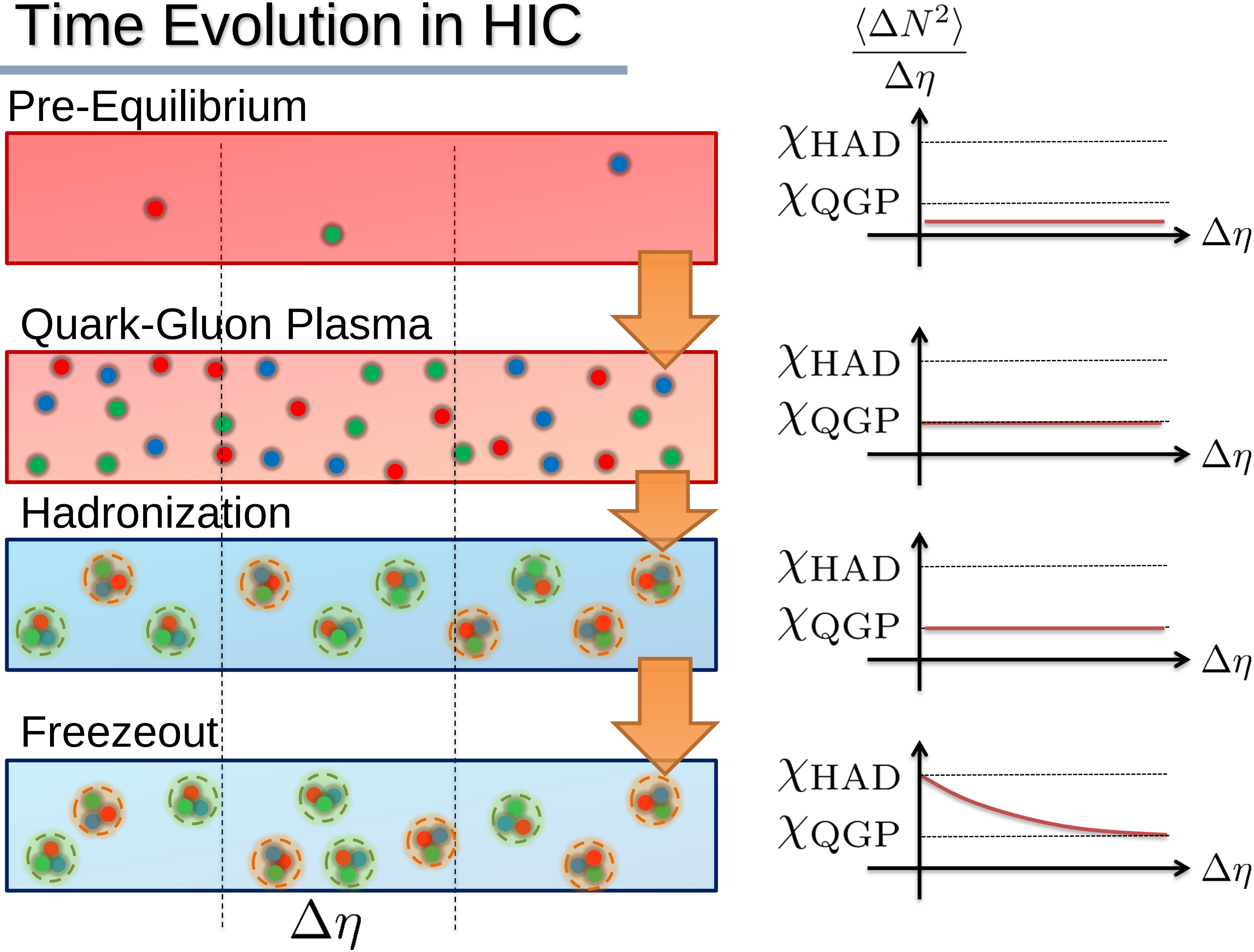
$$\frac{\langle \bar{n}^k \rangle_c}{\langle \bar{n}^2 \rangle_c} = \frac{\langle N^k \rangle_c}{\langle N^2 \rangle_c} V^{(2-k)/2} \underset{k \geq 3}{\rightarrow} 0$$

- In a large system,
  - Cumulants $\langle N^k \rangle_c$ are nonzero.
  - Their experimental measurements are difficult.

Time Evolution in HIC



Time Evolution in HIC



Time Evolution in HIC