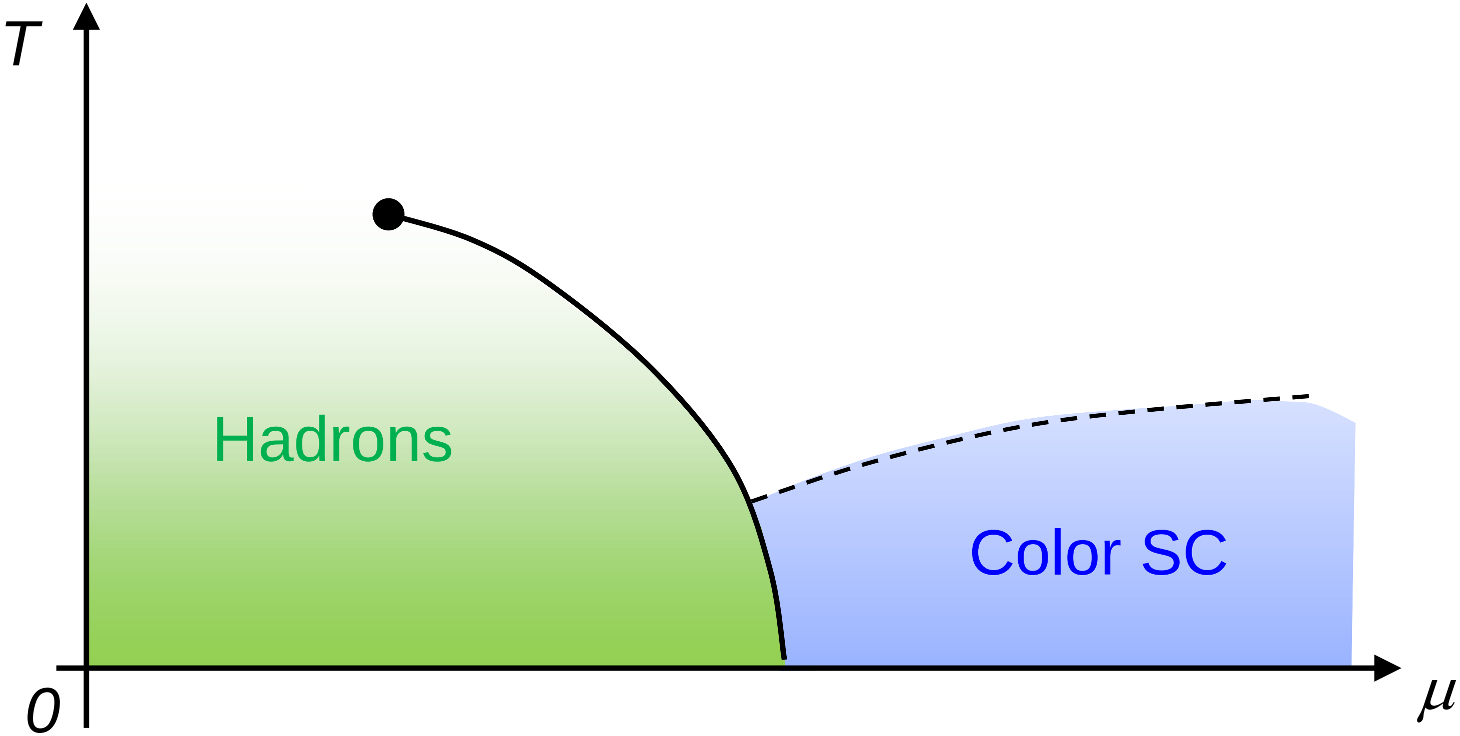
重イオン衝突における ゆらぎの話

北沢 正清
（阪大）

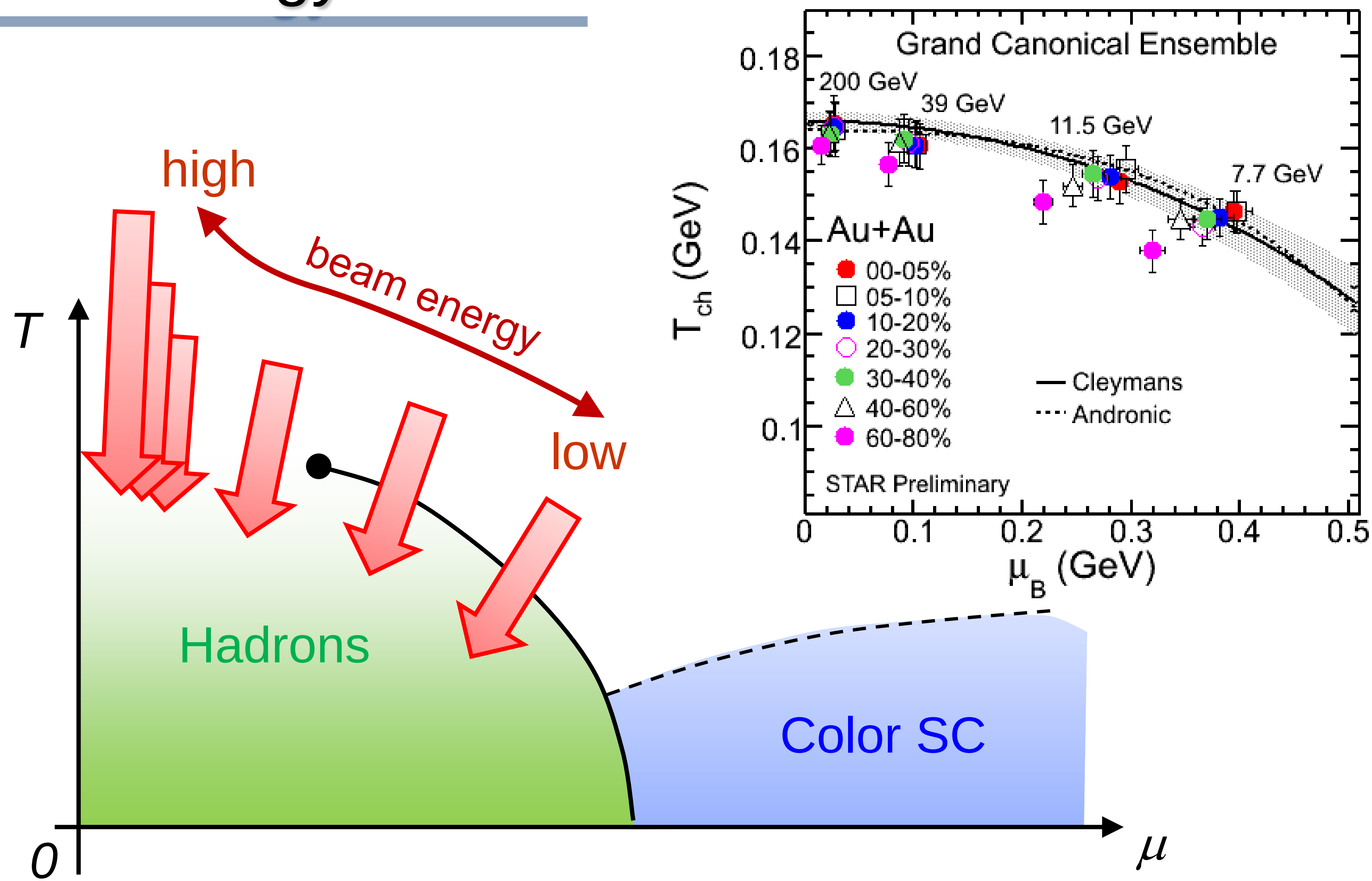
筑波大学, 2013/Jul./9

Beam-Energy Scan



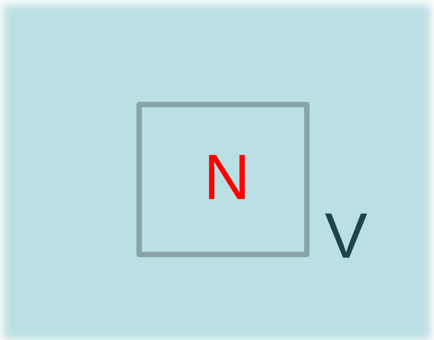
Beam-Energy Scan

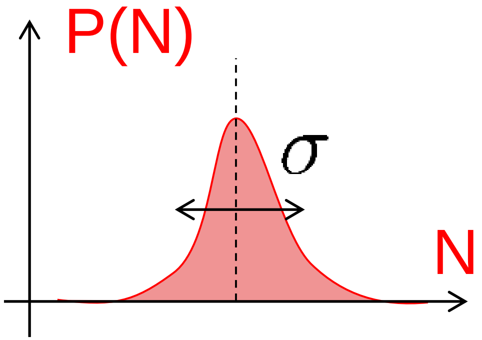
STAR 2012



Fluctuations

Observables in equilibrium are fluctuating.





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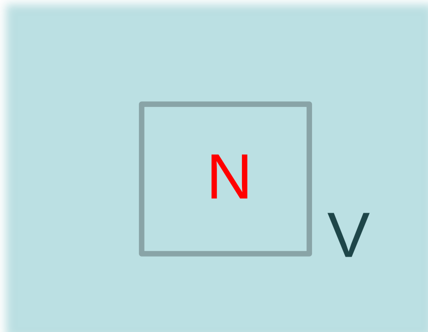
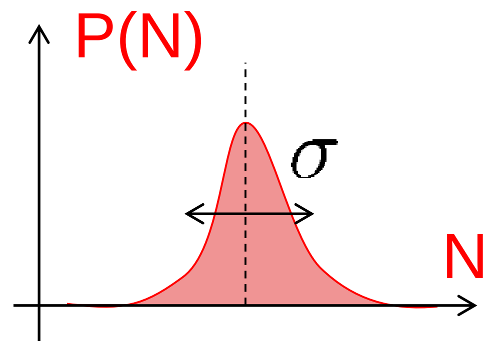


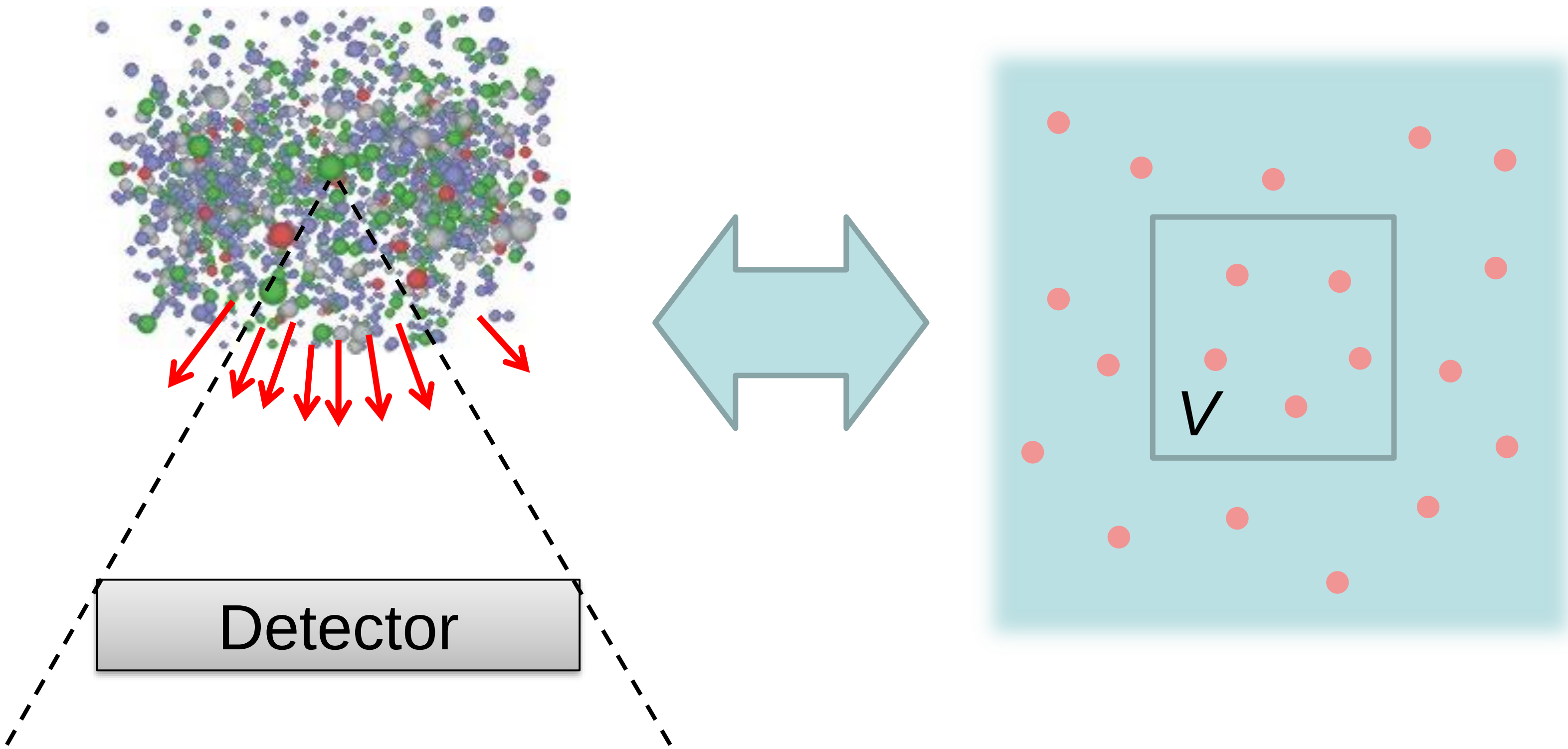


$$\delta N = N - \langle N \rangle$$

- Variance: $\langle \delta N^2 \rangle = V \chi_2 = \sigma^2$
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \dfrac{\langle \delta N^4 \rangle - 3 \langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.



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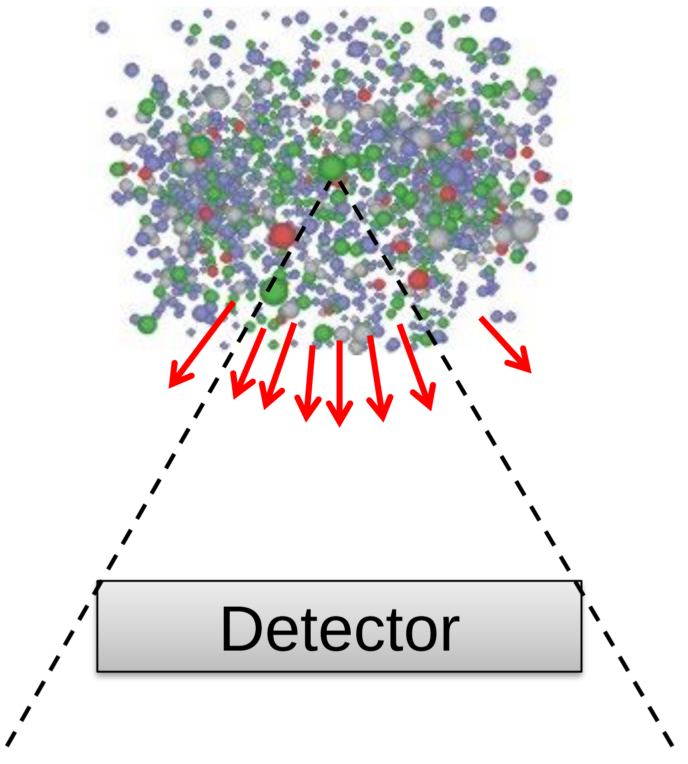
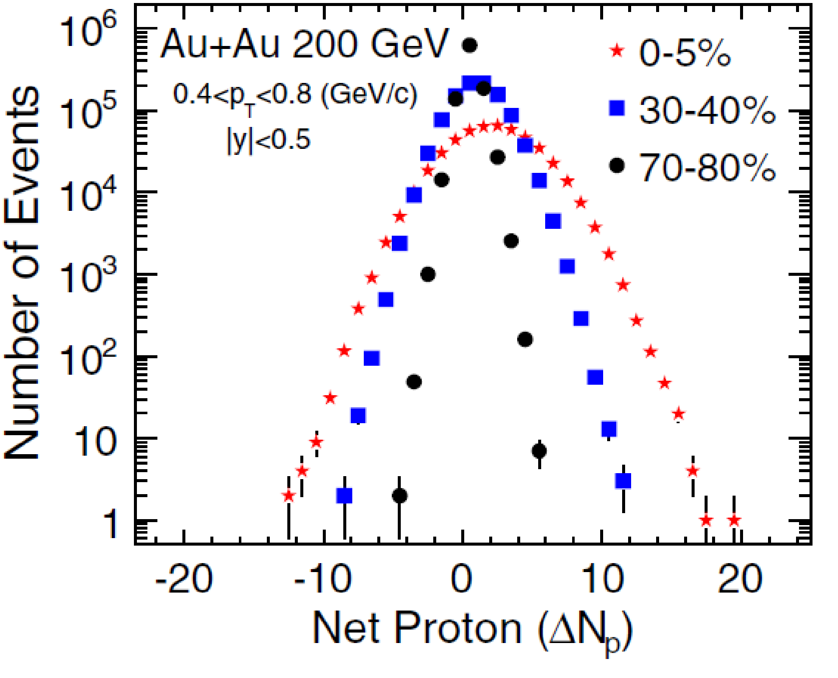
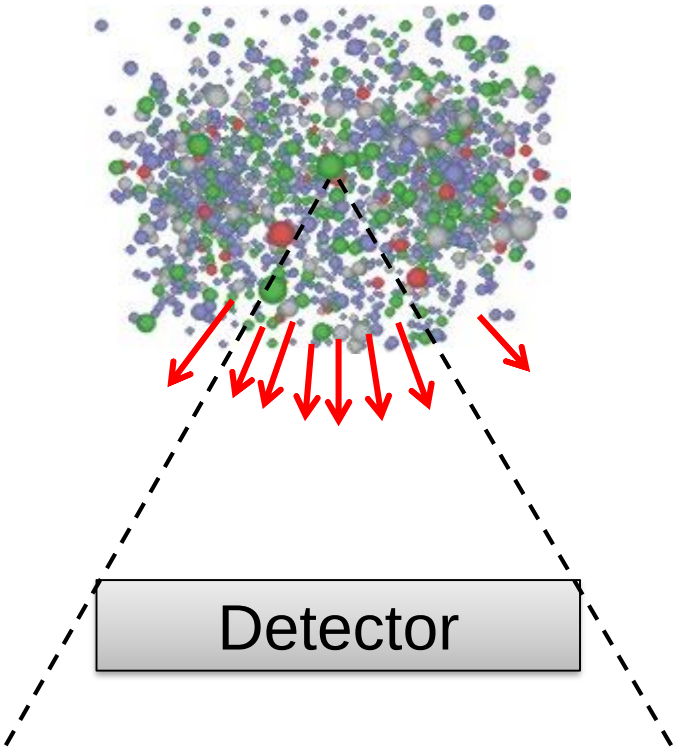


STAR, PRL**105** (2010)



観測にかかるゆらぎは、いつ形成されたのか？

ゆらぎのダイナミクス（動的振る舞い）の議論が必要



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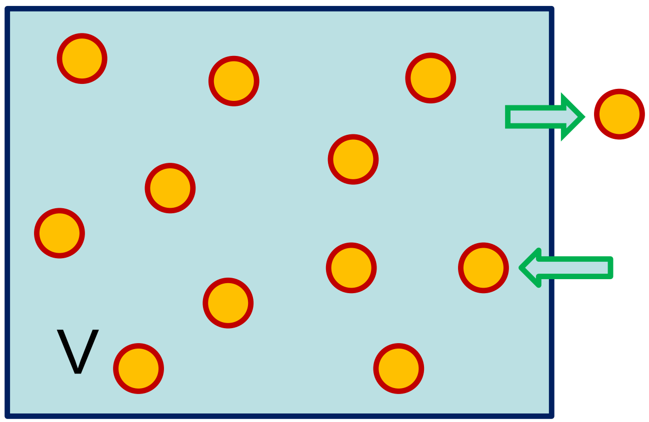
保存電荷の場合

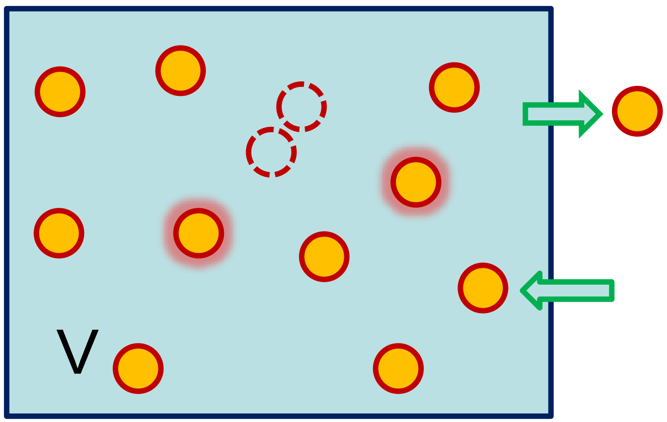


境界を通過する電荷のみが変化に寄与

$$\tau \to \infty$$

for $V \to \infty$

非保存電荷の場合



体積内の任意の場所で電荷が変化できる

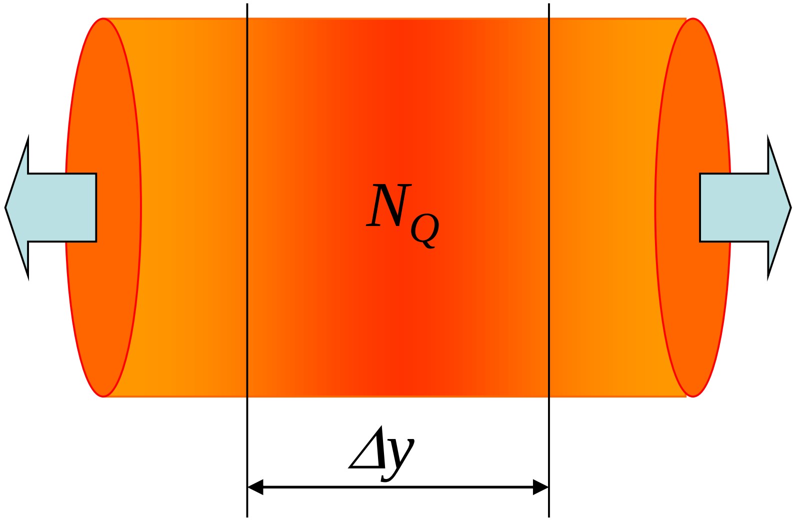
$$\tau \to \mathrm{const.}$$

for $V \to \infty$

観測にかかるゆらぎは、いつ形成されたのか？

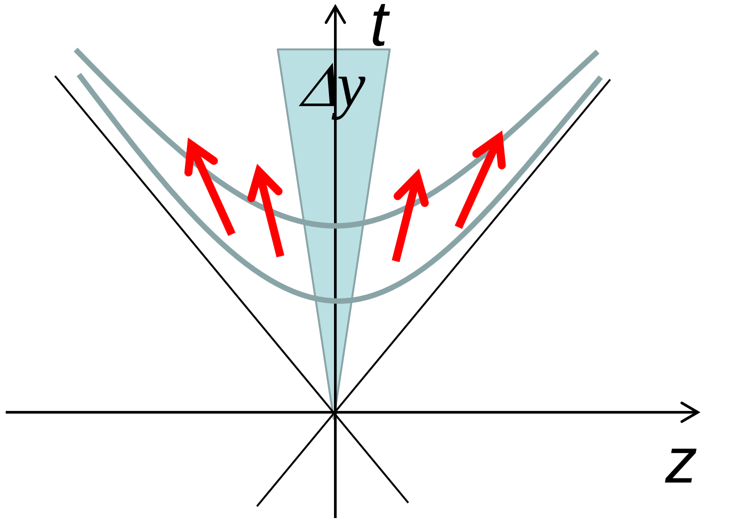
$\Delta\eta$内の保存電荷量は、初期段階のものが終状態まで生き残ることが期待できる。

Asakawa, Heinz, Muller, '00
Jeon, Koch, '00
Shuryak, Stephanov, '02



Note:

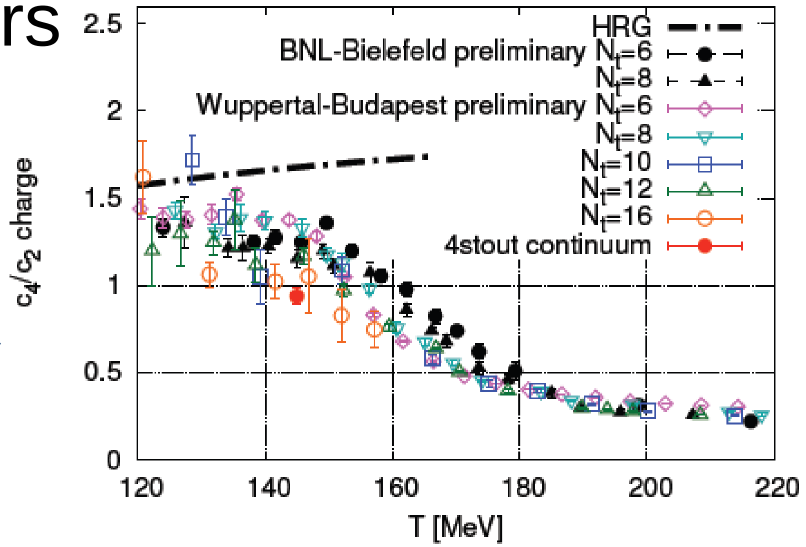
STAR $\begin{cases} -0.5 < \eta < 0.5 \\ 0.4 < p < 0.8[GeV] \end{cases}$



Conserved Charges : Theoretical Advantage

- Definite definition for operators
  - as a Noether current
  - calculable on any theory

ex: on the lattice



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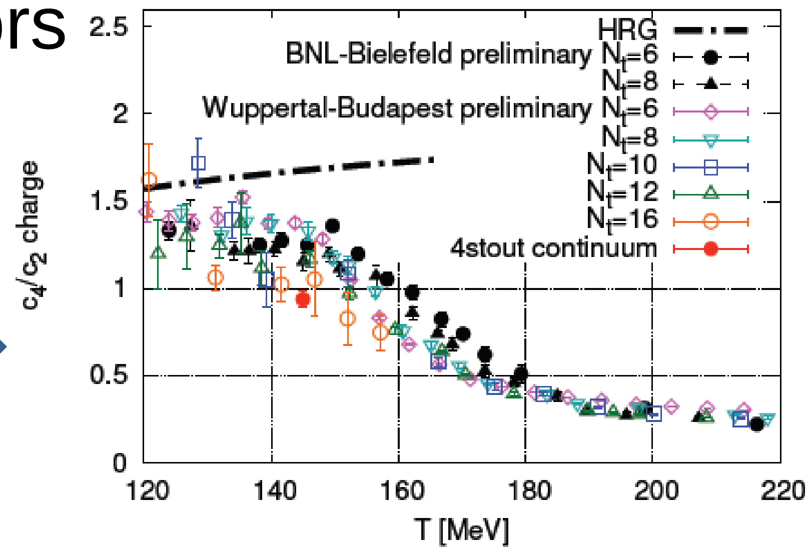


- Simple thermodynamic relations

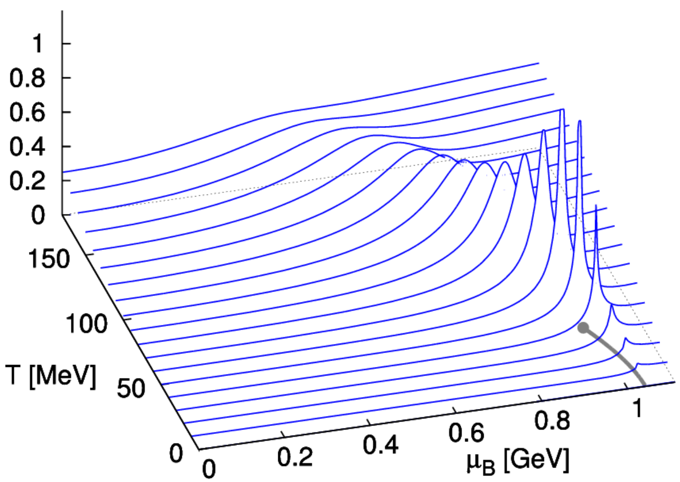
$$\langle \delta N_c^n \rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}$$

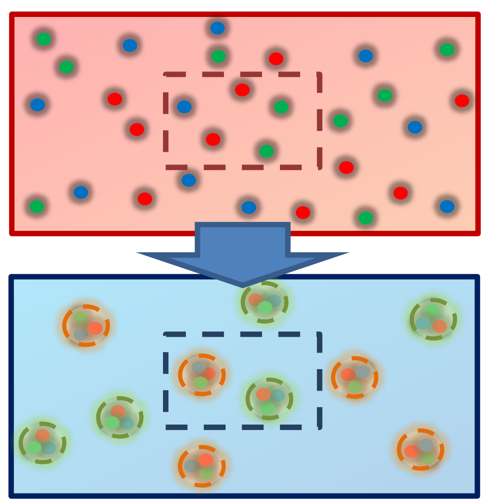
  - Intuitive interpretation for the behaviors of cumulants

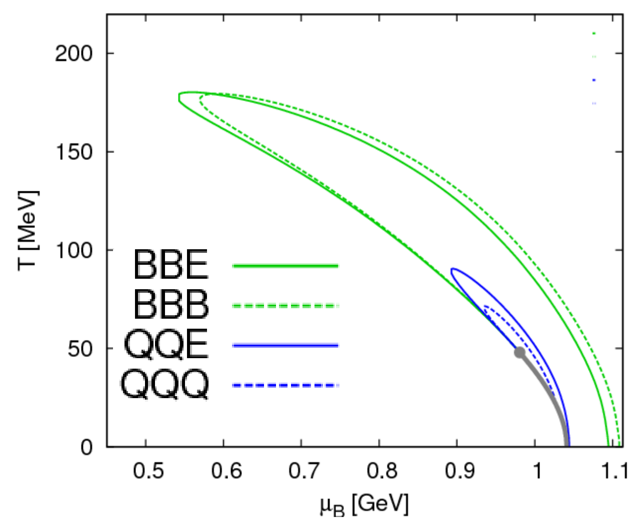
ex: $$\langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B}$$

Fluctuations

- Fluctuations reflect properties of matter.
  - Enhancement near the critical point
    Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);…
  - Ratios between cumulants of conserved charges
    Asakawa,Heintz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)
  - Signs of higher order cumulants
    Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)



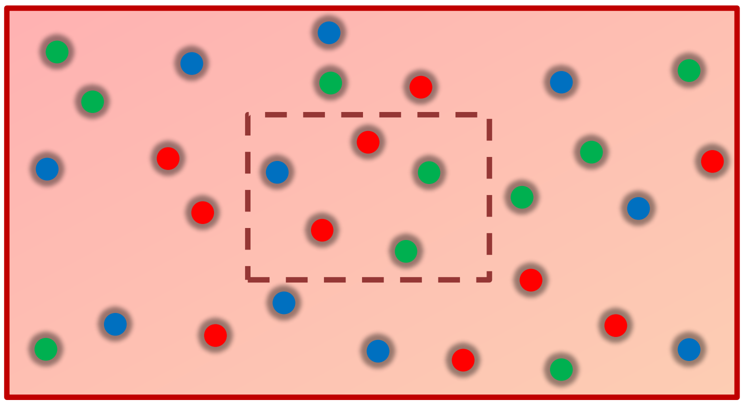




Fluctuations

Free Boltzmann → Poisson

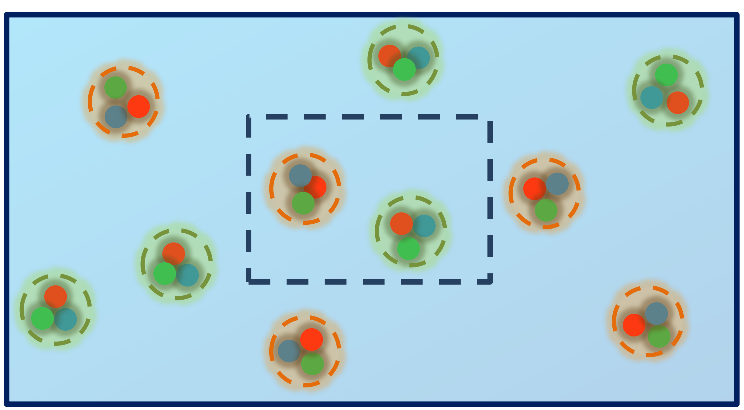
$$\langle \delta N^n \rangle_c = \langle N \rangle$$



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$



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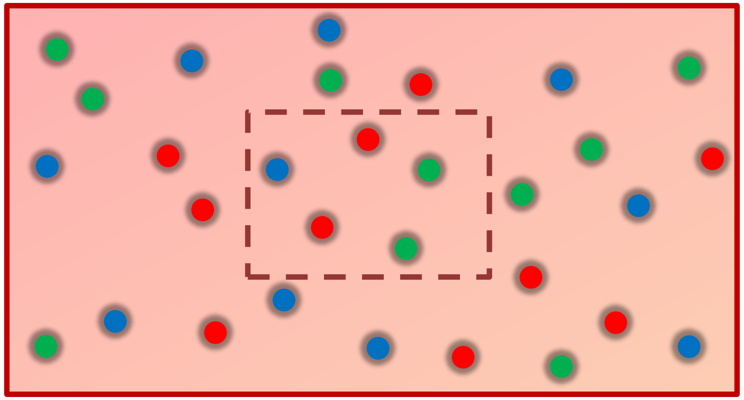

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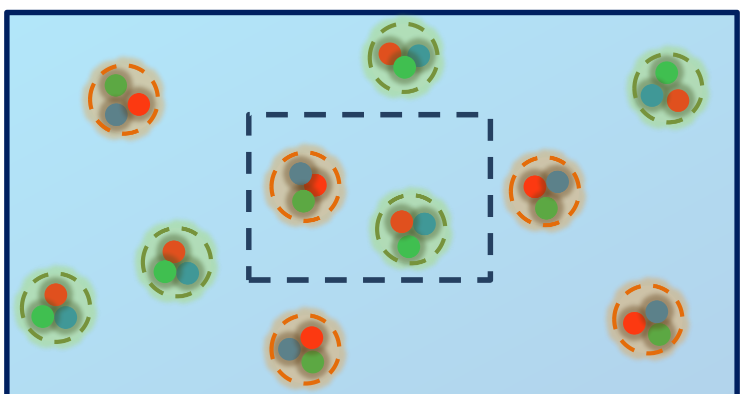
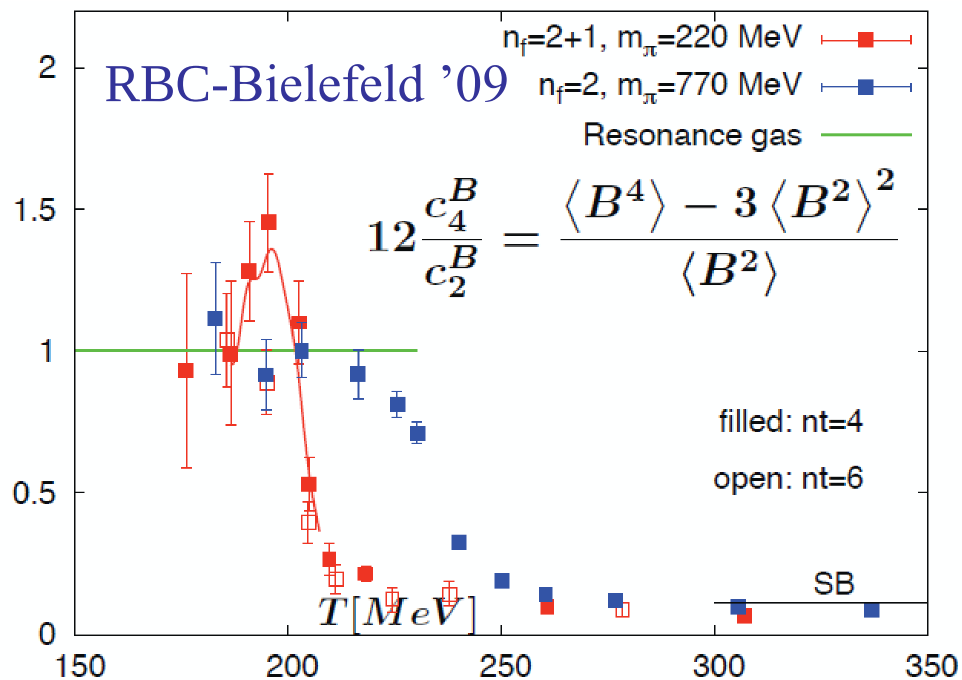
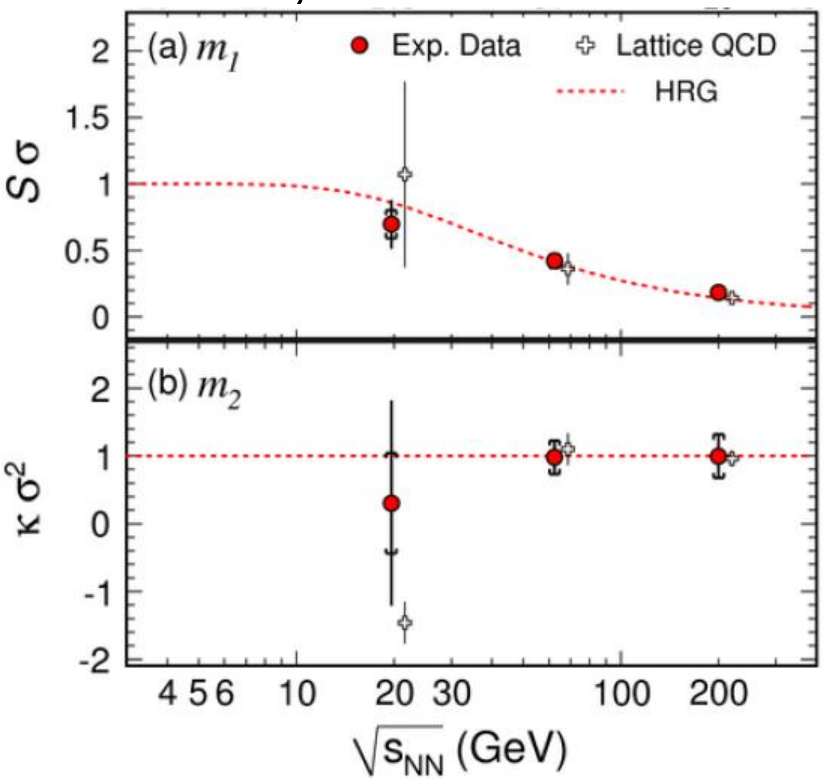




Proton # Fluctuations @ STAR-BES

STAR, PRL2010



$$S\sigma = \frac{\langle(\delta N_p^{(\mathrm{net})})^3\rangle}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}, \quad \kappa\sigma^2 = \frac{\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}$$

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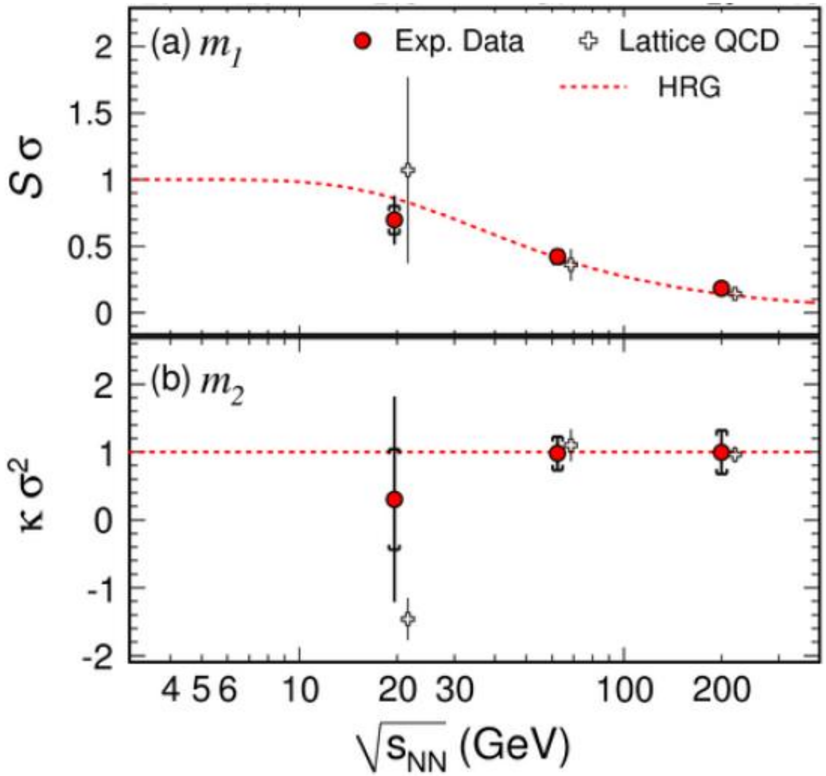
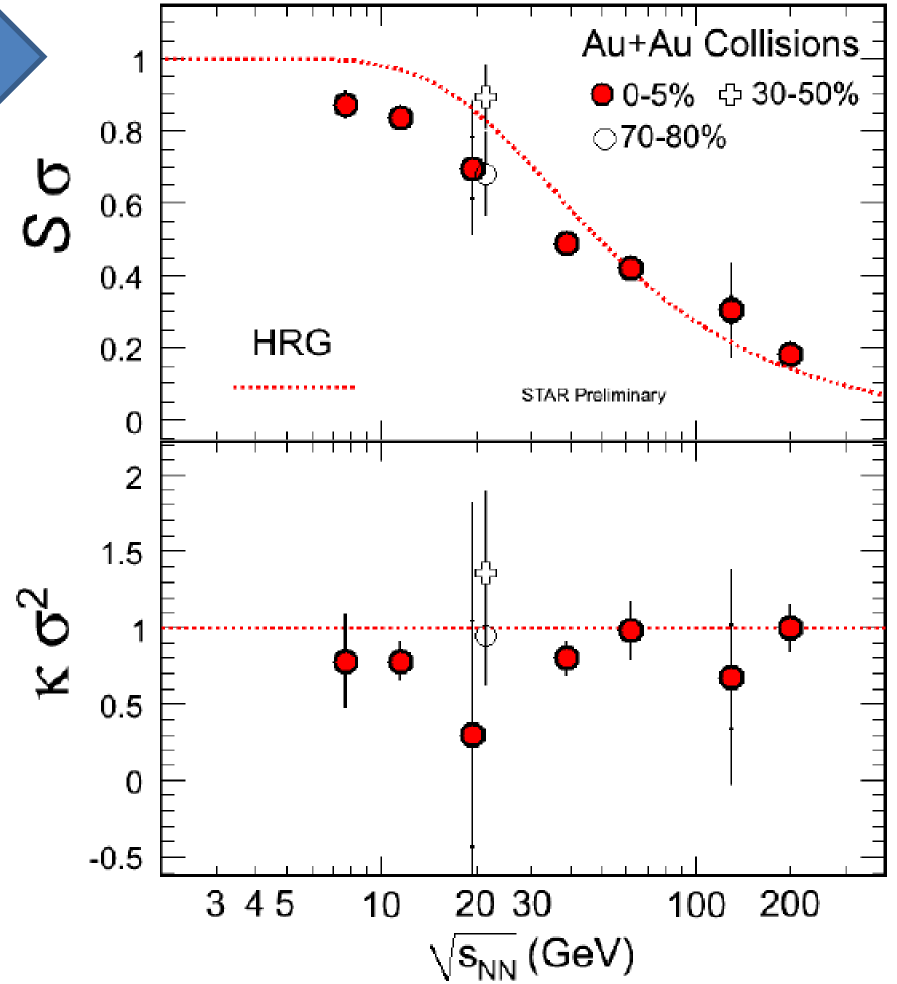


STAR, 2011



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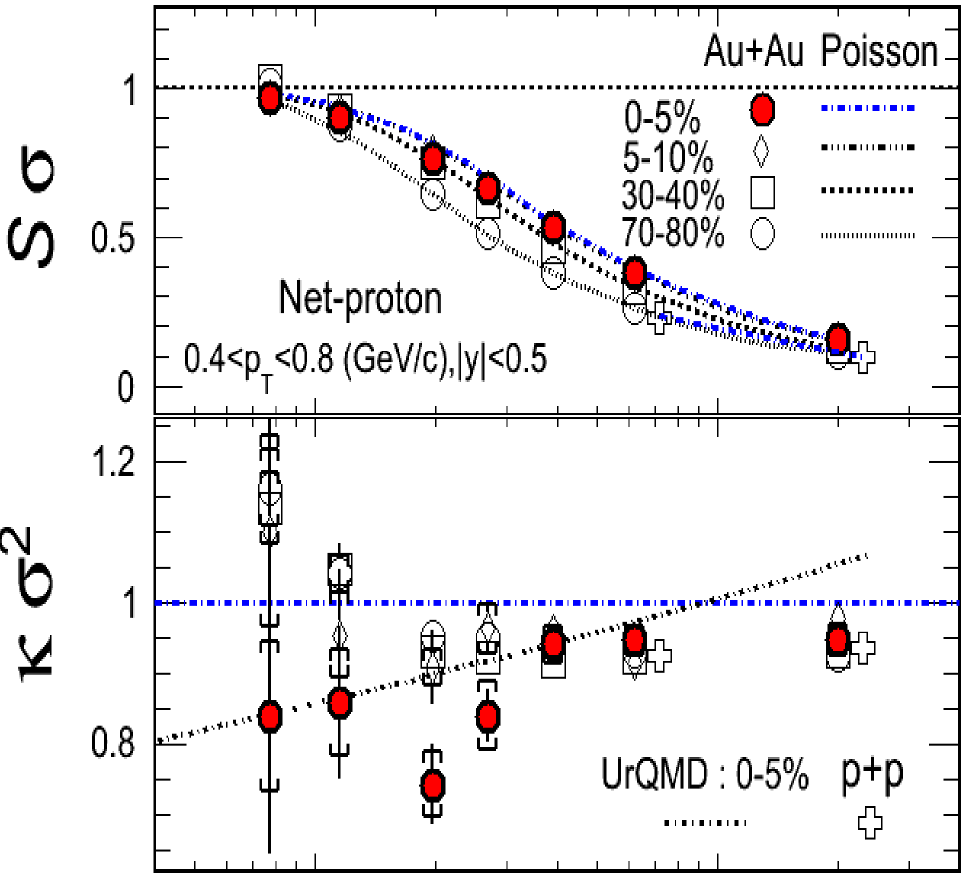
high μ

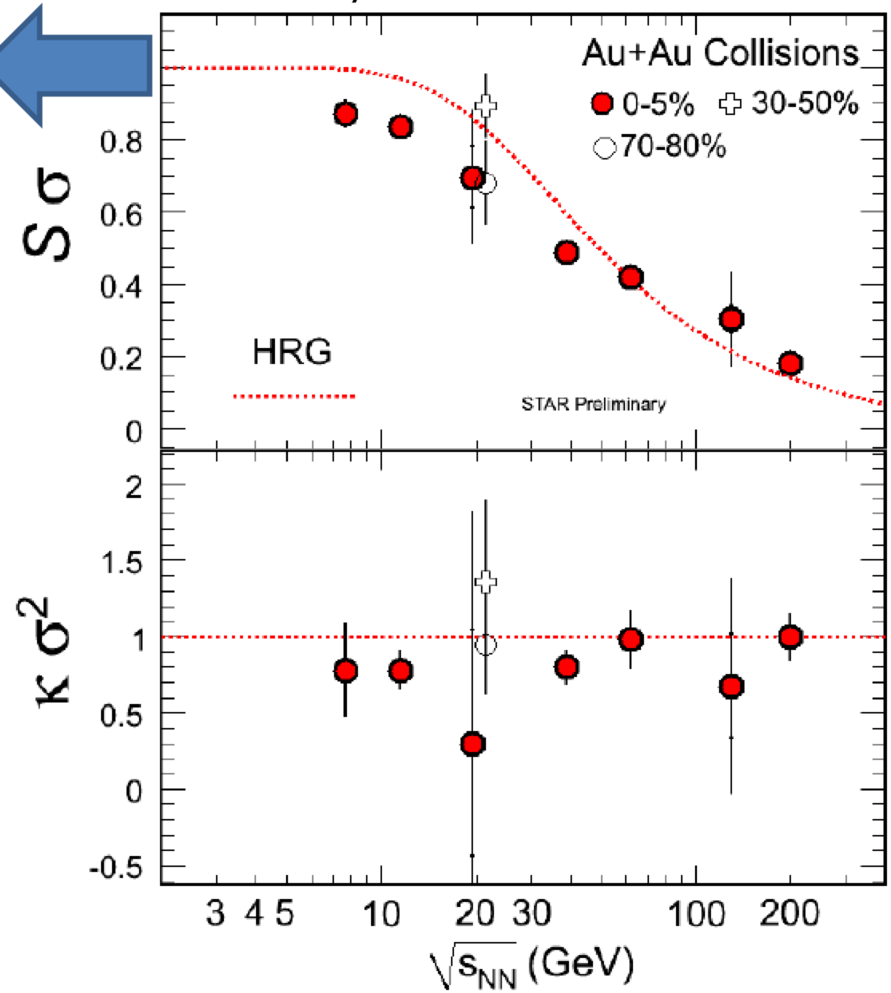
low μ

Proton # Fluctuations @ STAR-BES

STAR, 2012 (Quark Matter)



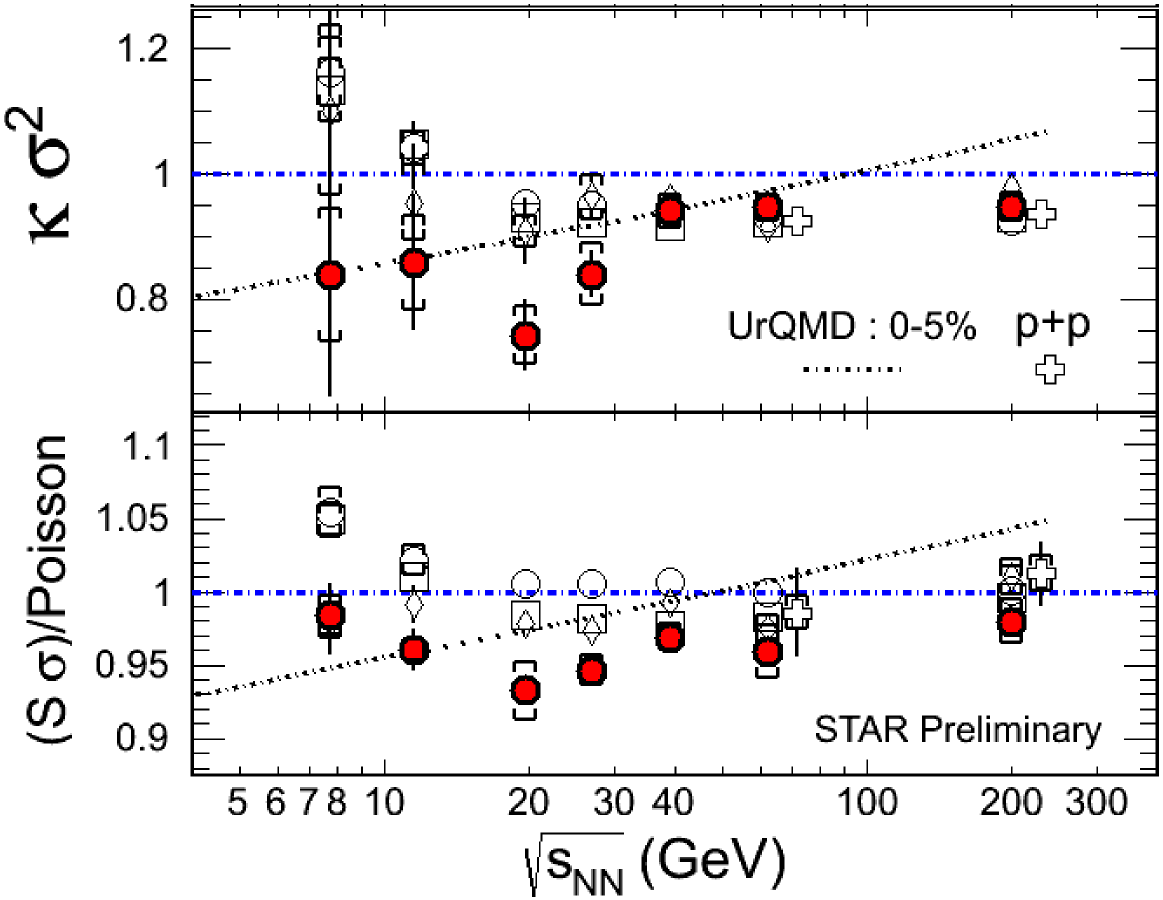
STAR, 2011



high μ

low μ

Proton # Cumulants @ STAR-BES



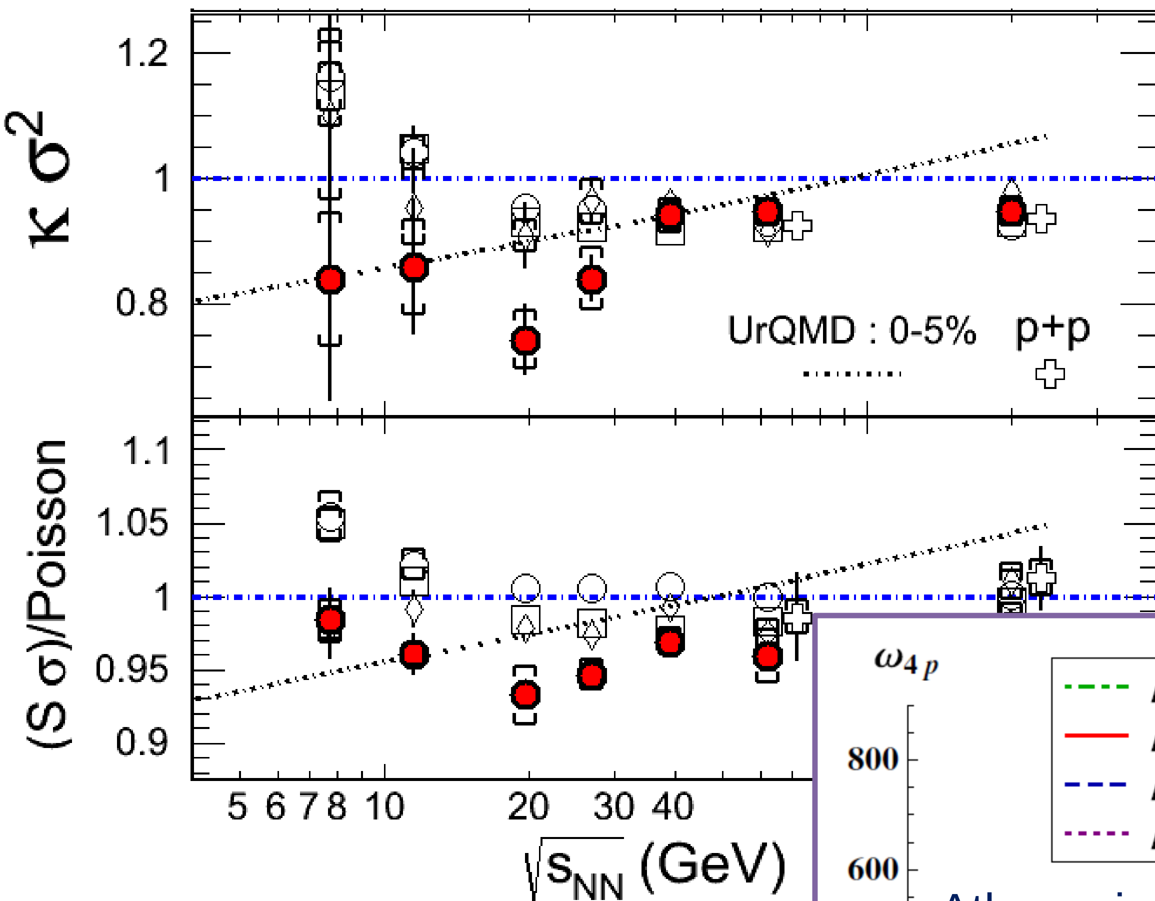
STAR,QM2012

$$\frac{C_4}{C_2}$$

$$\frac{C_3}{C_1} = \frac{C_3/C_2}{\text{Poissonian}}$$

No characteristic signals on
phase transition to QGP nor QCD CP

Proton # Cumulants @ STAR-BES



STAR,QM2012

$$\frac{C_4}{C_2}$$

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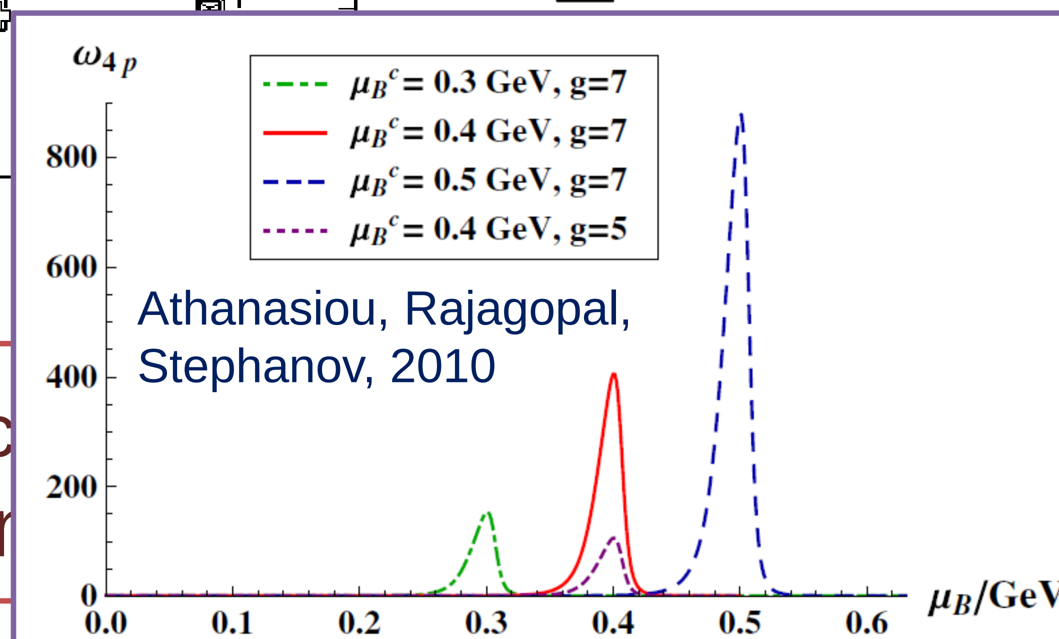
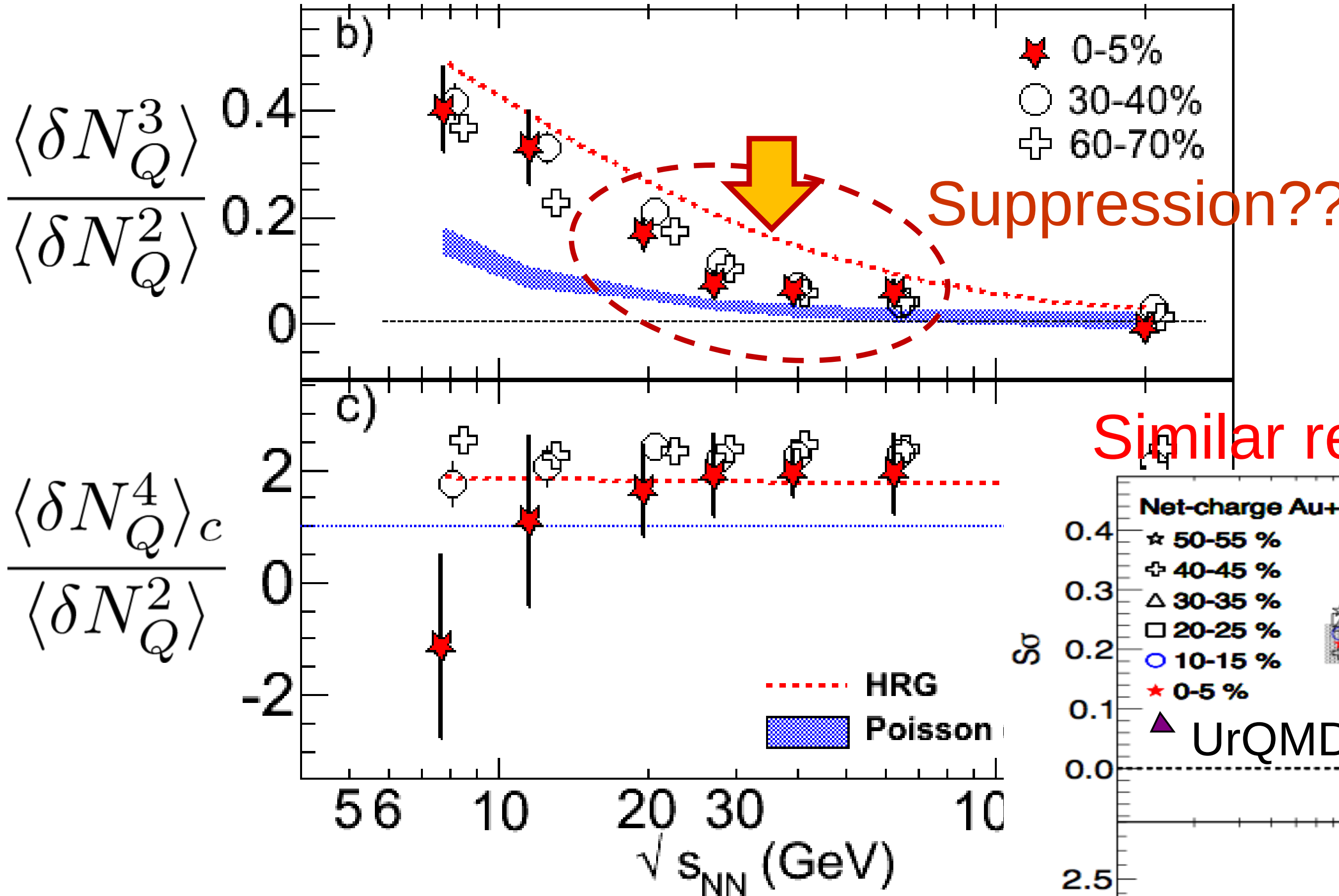
No charac

phase transitio

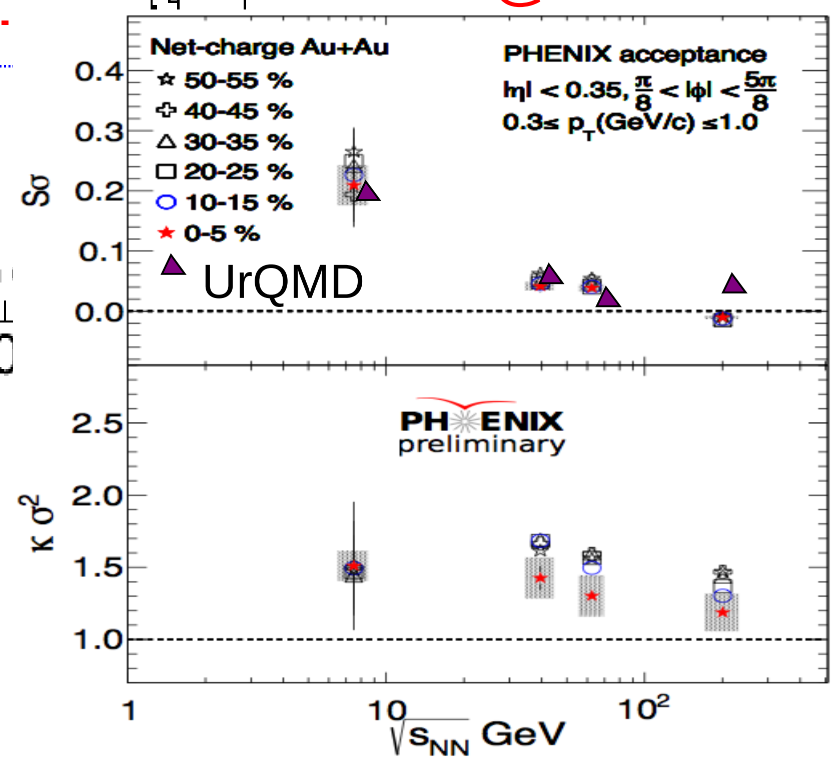


Charge Fluctuations @ STAR-BES

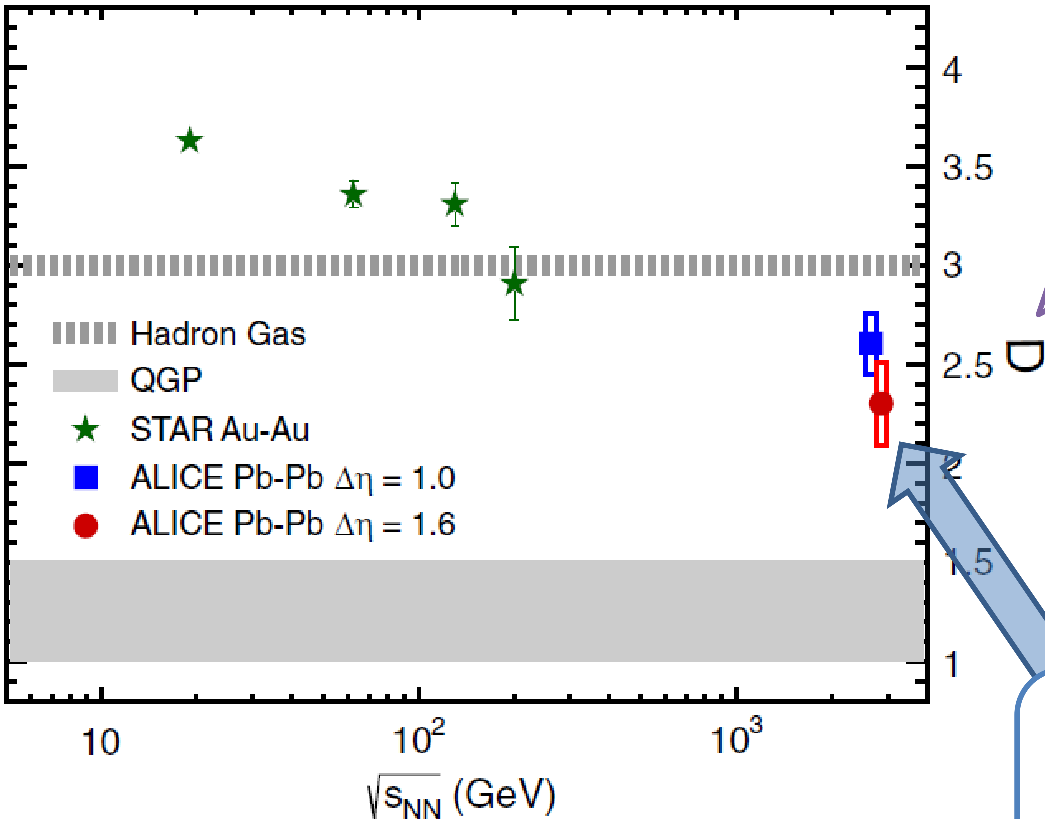
STAR, QM2012



Similar result @ PHENIX



Charge Fluctuation @ LHC



ALICE, PRL110,152301(2013)

D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

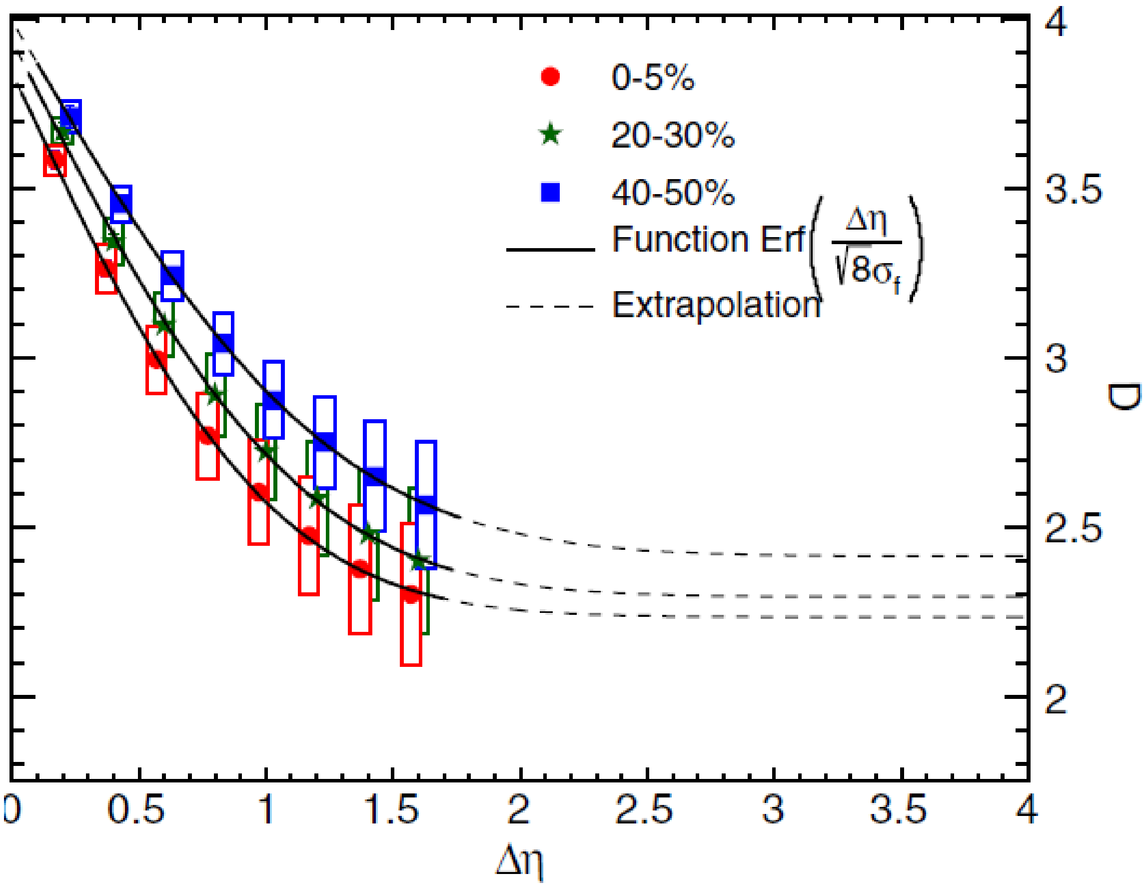
- D ~ 3-4 Hadronic
- D ~ 1 Quark

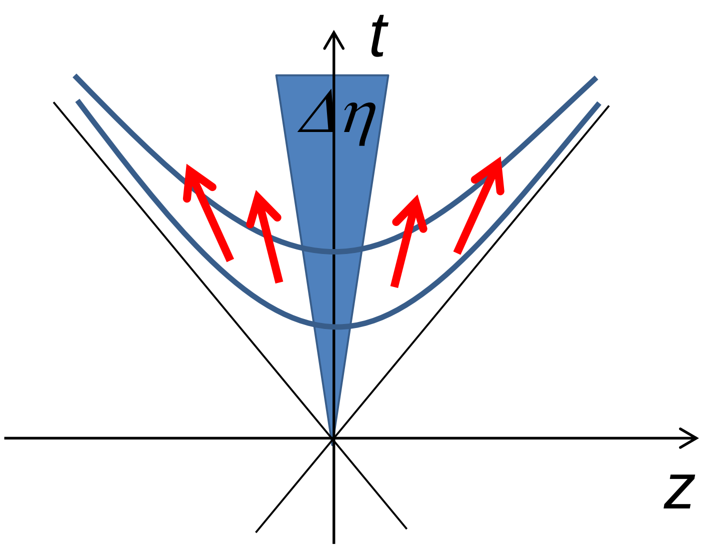
LHC:
significant suppression from hadronic value

⇒ LHC終状態の電荷ゆらぎは、ハドロン化以前に生成されたものを強く反映している！

Δη Dependence @ ALICE

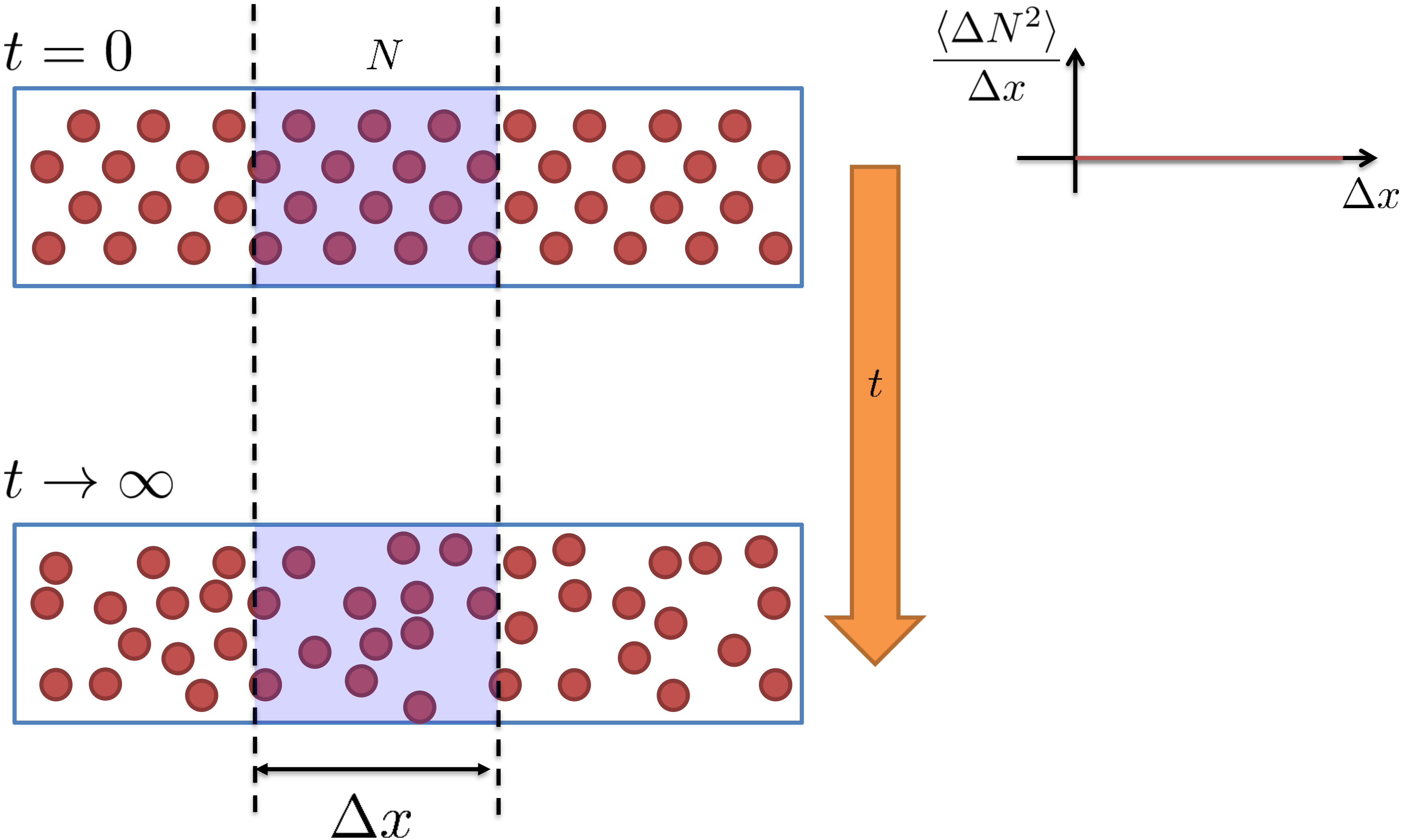
ALICE
PRL 2013





rapidity window

Dissipation of a Conserved Charge



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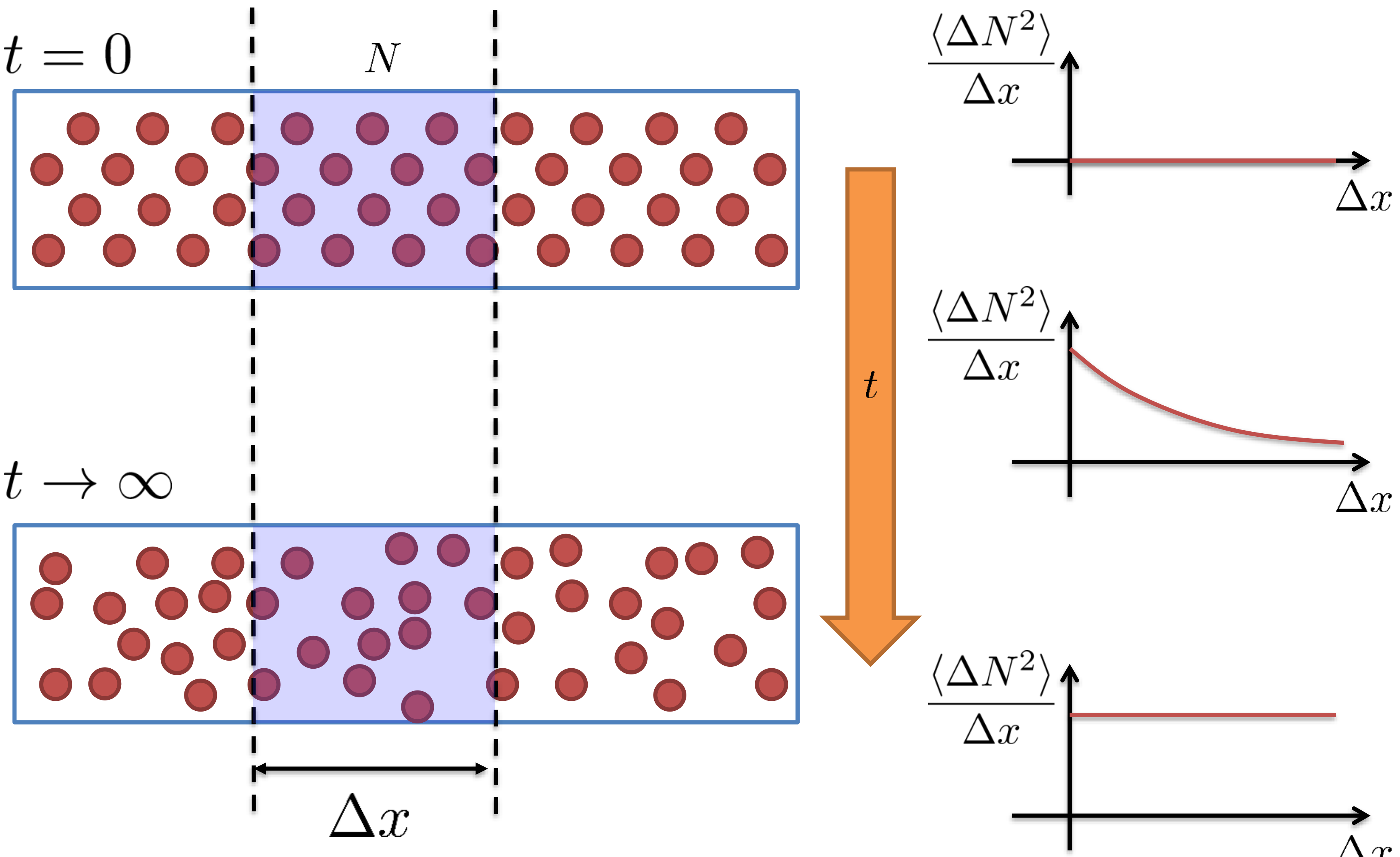
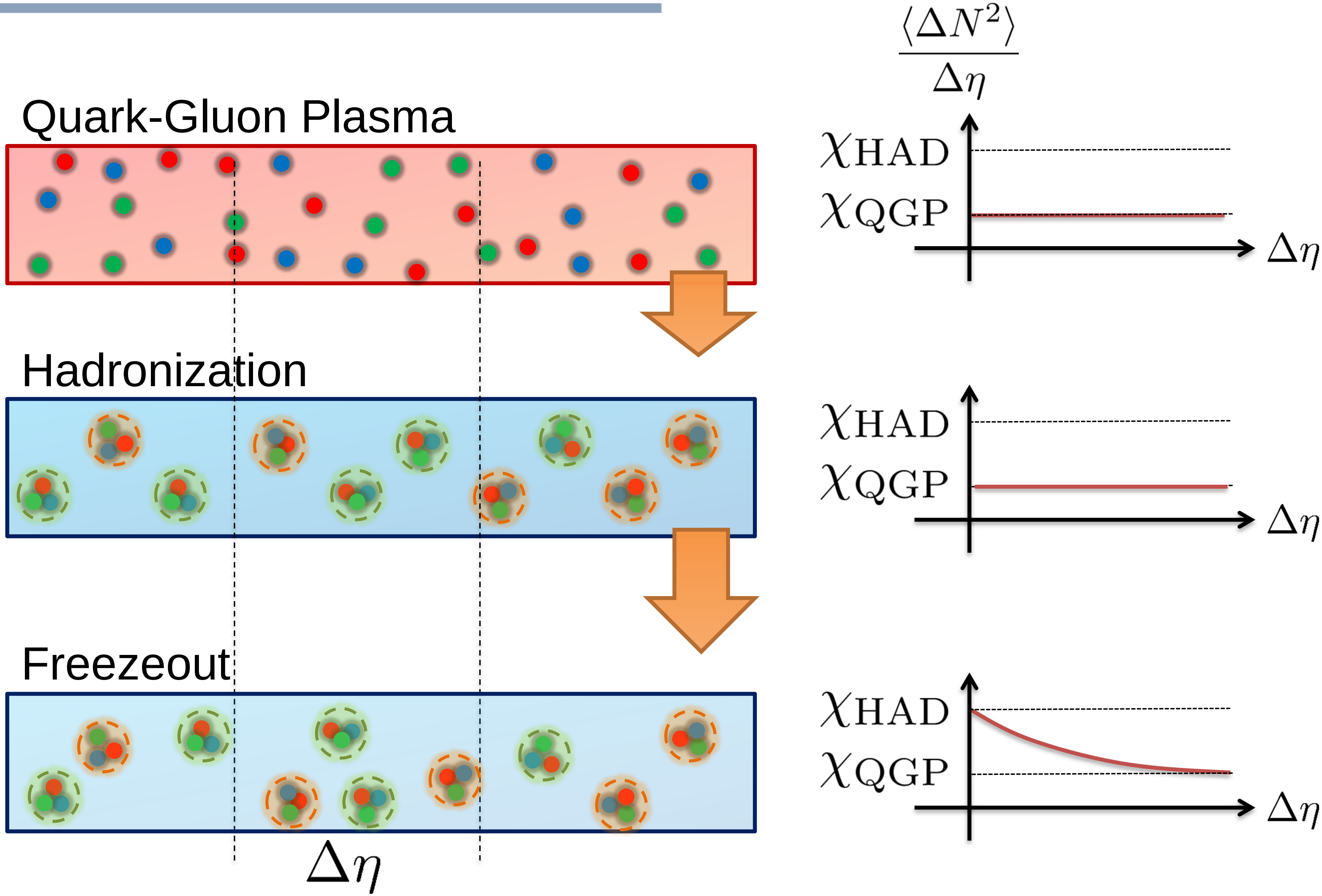
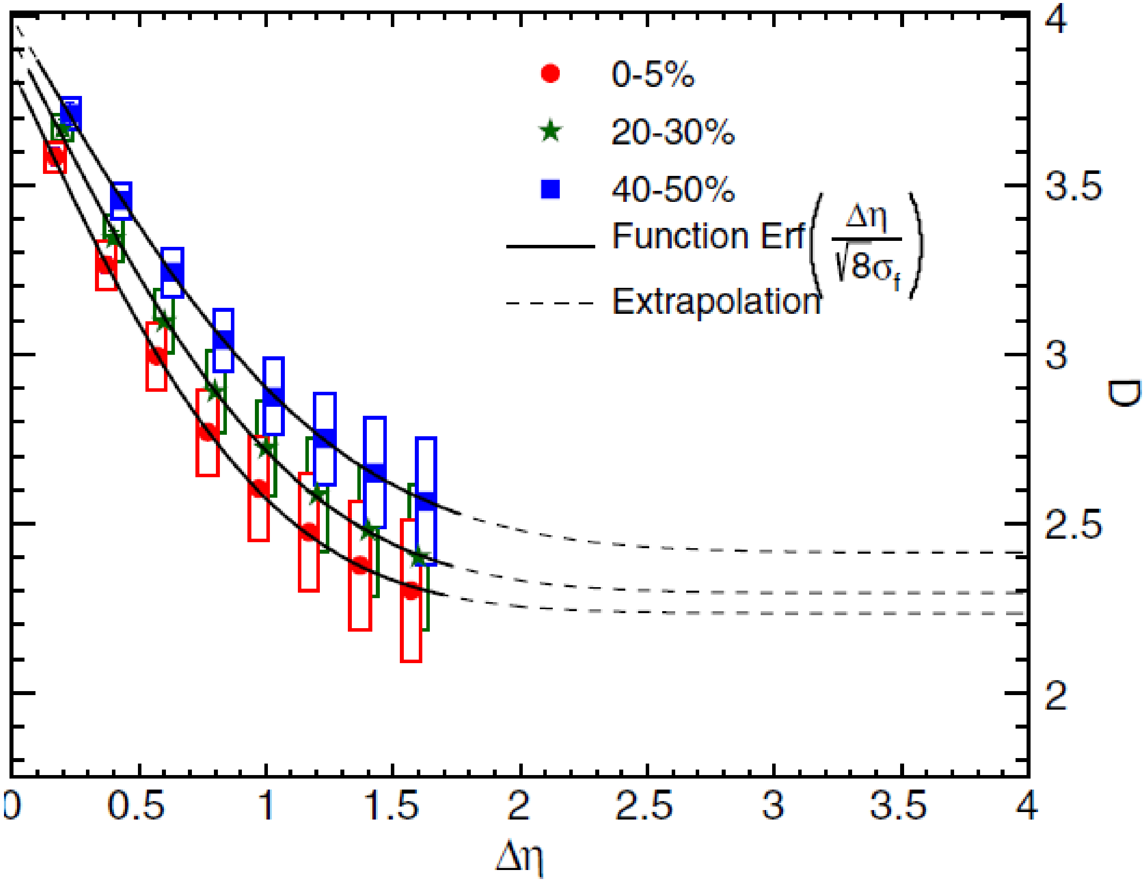


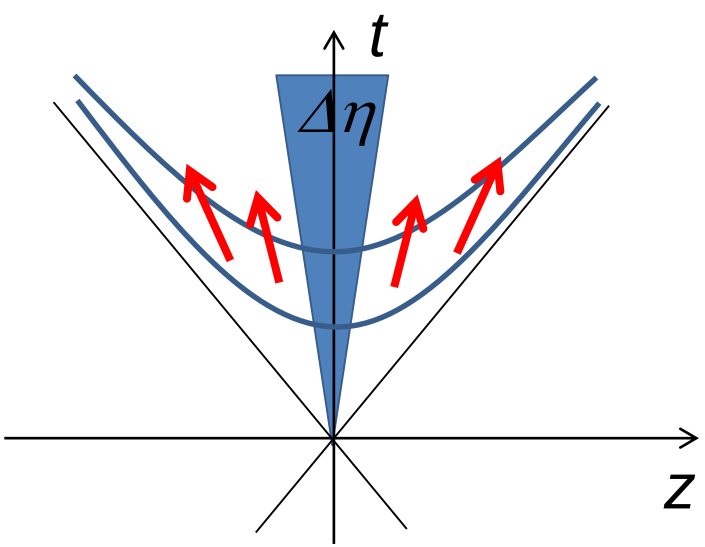
Time Evolution in HIC



Δη Dependence @ ALICE



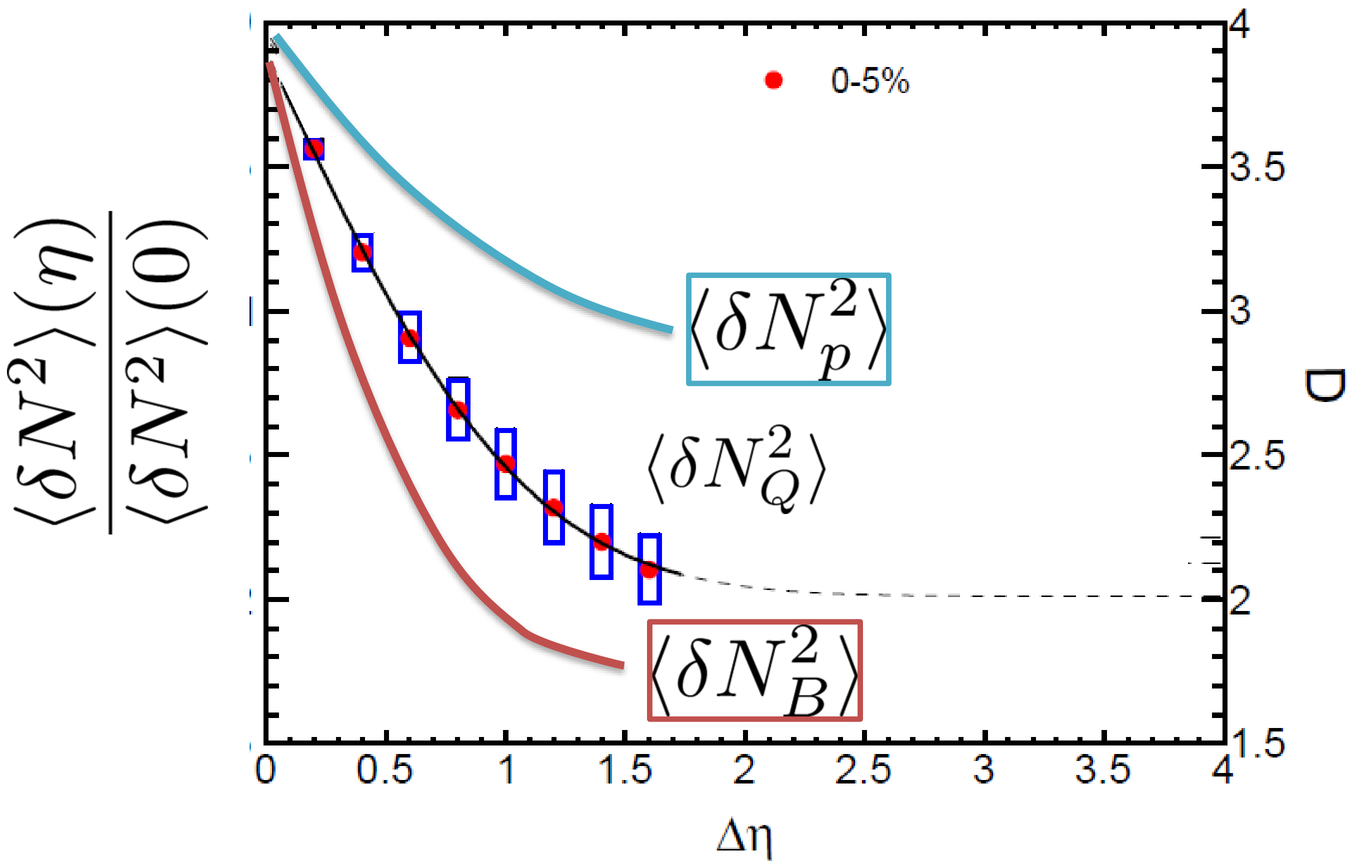
ALICE
PRL 2013



ゆらぎのΔη 依存性には、高温物質の
時間発展の情報が刻まれている！

$<\delta N_B{}^2>$ and $<\delta N_p{}^2>$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
should have different Δη dependence.

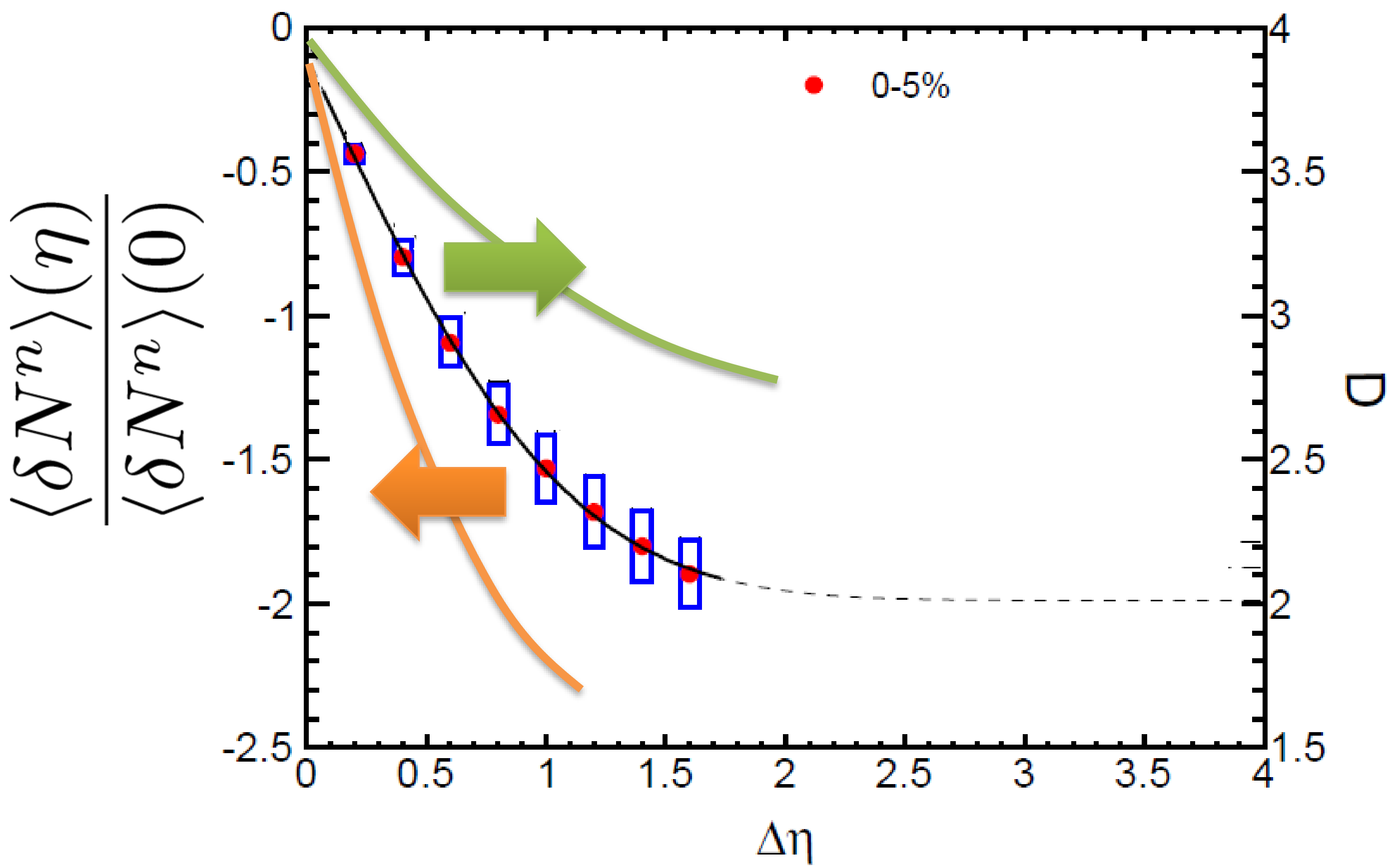


$$\langle (\delta N_p^{(\mathrm{net})})^2 \rangle = \frac{1}{4} \langle (\delta N_B^{(\mathrm{net})})^2 \rangle + \frac{1}{4} \langle N_B^{(\mathrm{tot})} \rangle$$

$\langle \delta N_Q^4 \rangle$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

Left (suppression) or Right (hadronic)

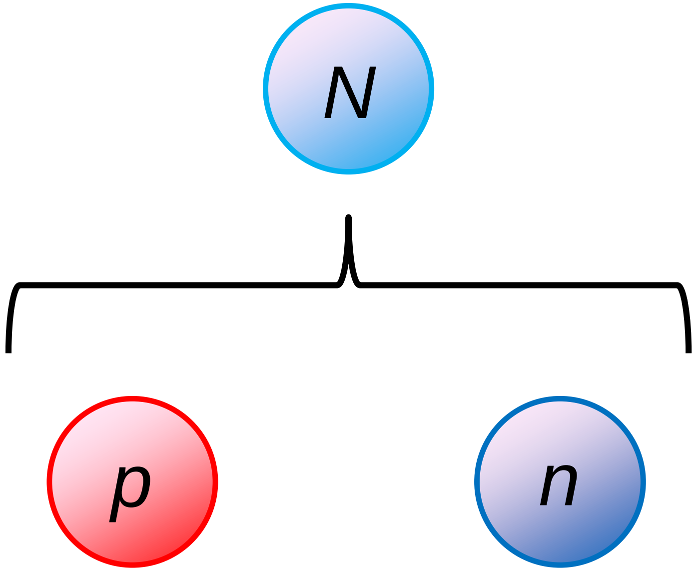


Baryon vs Proton Number Fluctuations

MK, Asakawa, PRC85,021901C(2012); PRC86, 024904(2012)

- $$\frac{\langle \delta N_B^n \rangle_c}{\langle \delta N_B^m \rangle_c} \neq \frac{\langle \delta N_p^n \rangle_c}{\langle \delta N_p^m \rangle_c}$$
- $\langle \delta N_B^n \rangle_c$ are experimentally observable

Nucleon Isospin as Two Sides of a Coin



Nucleons have
two isospin states.

MK, Asakawa,2012

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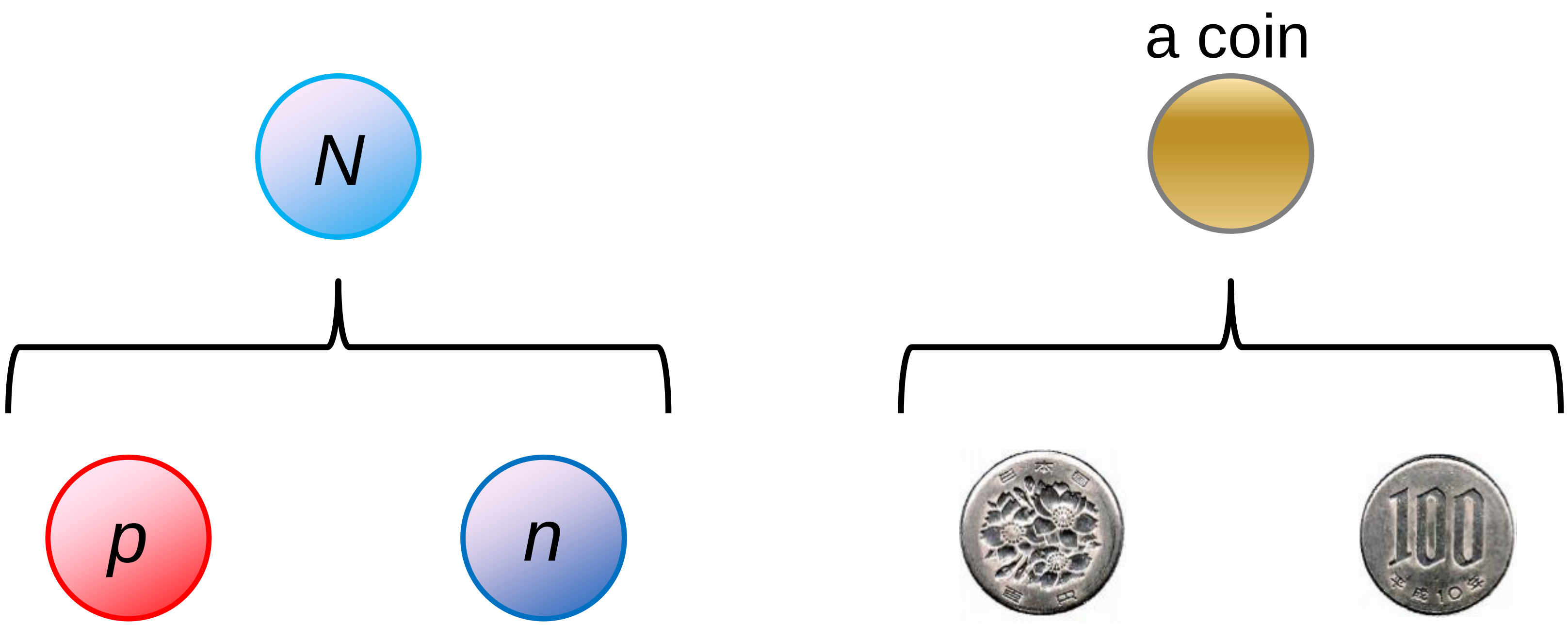

Nucleons have
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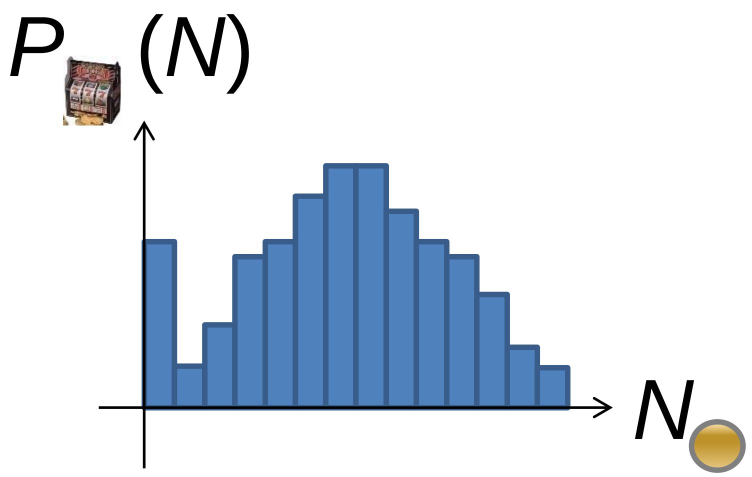
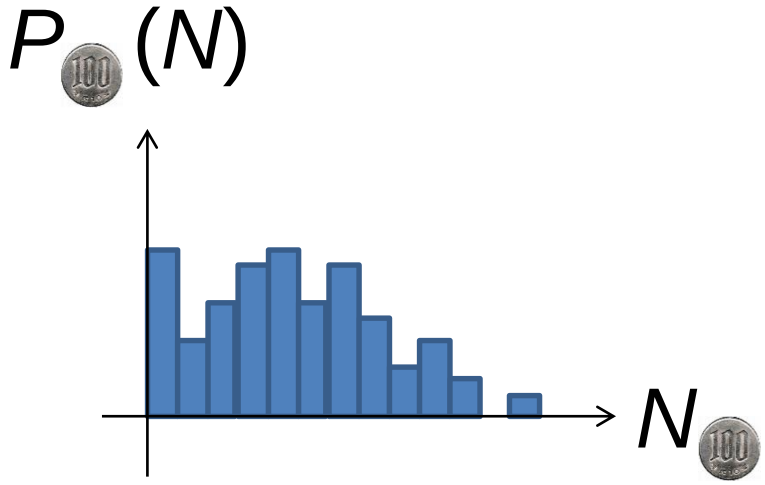
Coins have two sides.

MK, Asakawa,2012

Slot Machine Analogy

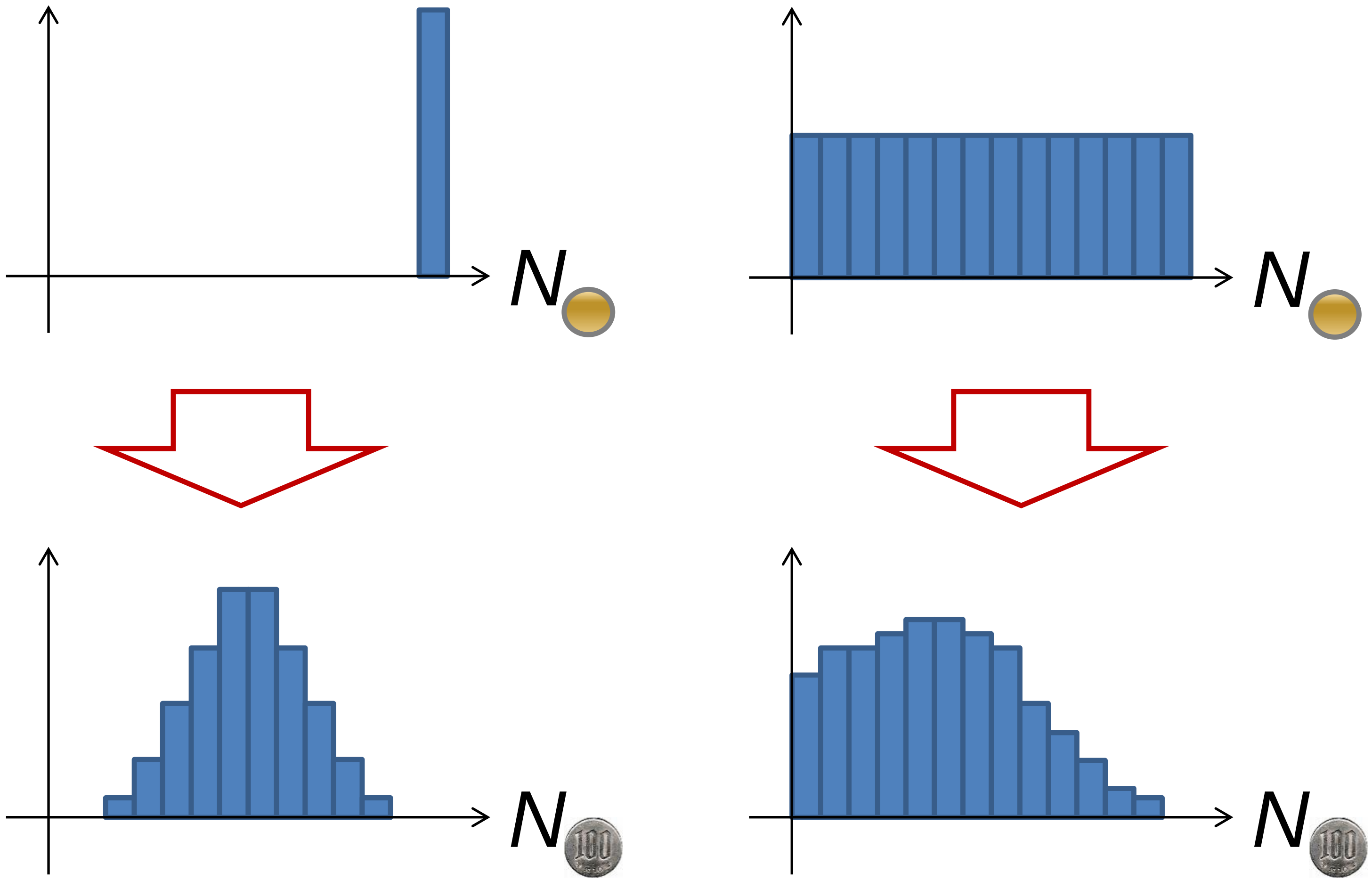




Extreme Examples

Fixed # of coins

Constant probabilities



Reconstructing Total Coin Number

$$P_{(100)}(N_{(100)}) = \sum_{\bullet} P_{\text{slot}}(N_{\bullet})\, B_{1/2}(N_{(100)}; N_{\bullet})$$

$$B_p(k;N) = p^k (1-p)^{N-k}\ {}_kC_N$$:binomial distr. func.

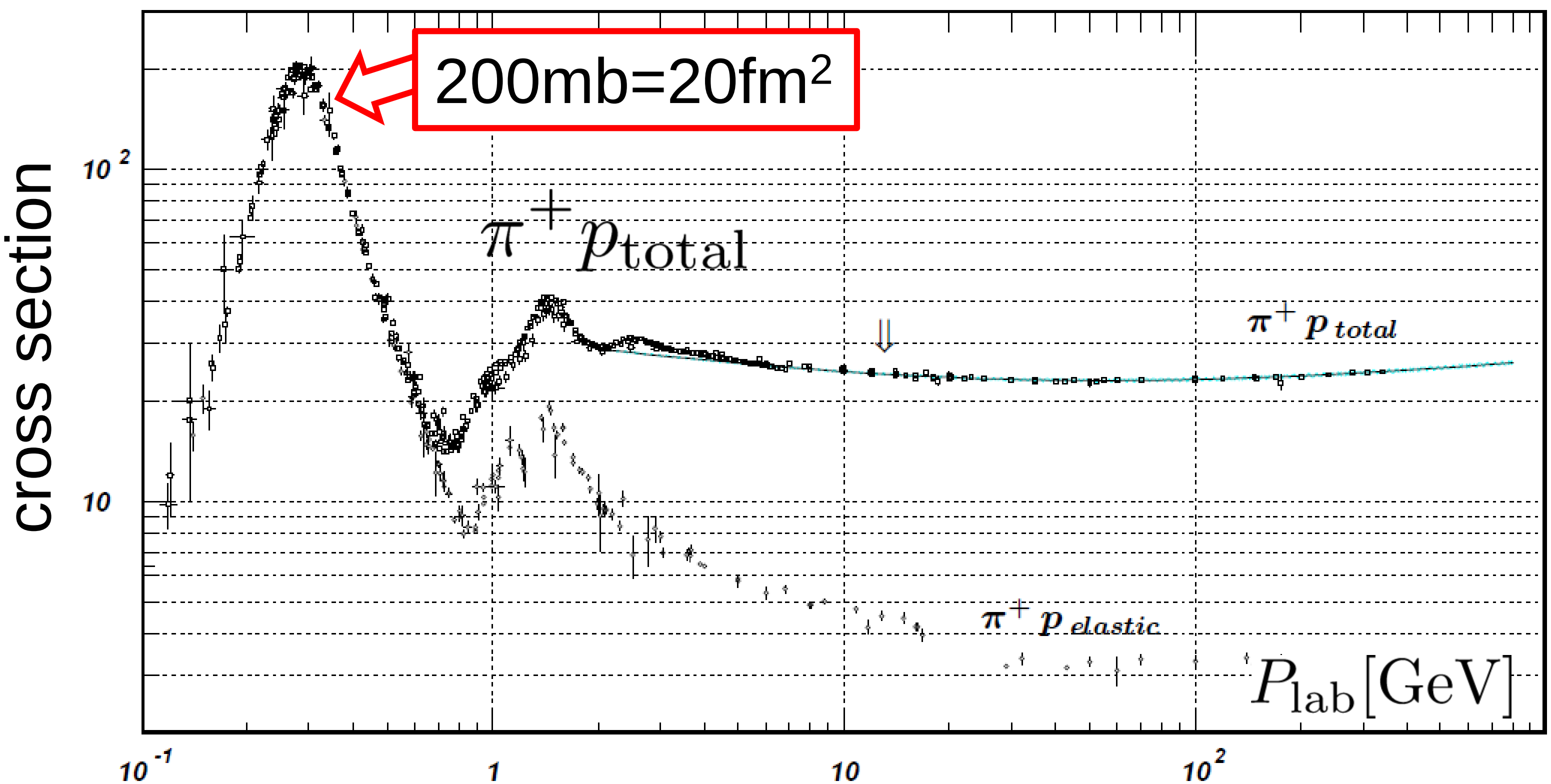
Nucleon Isospin in Hadronic Medium

- Isospin of baryons can vary after chemical freezeout via charge exchange reactions mediated by Δ(1232):

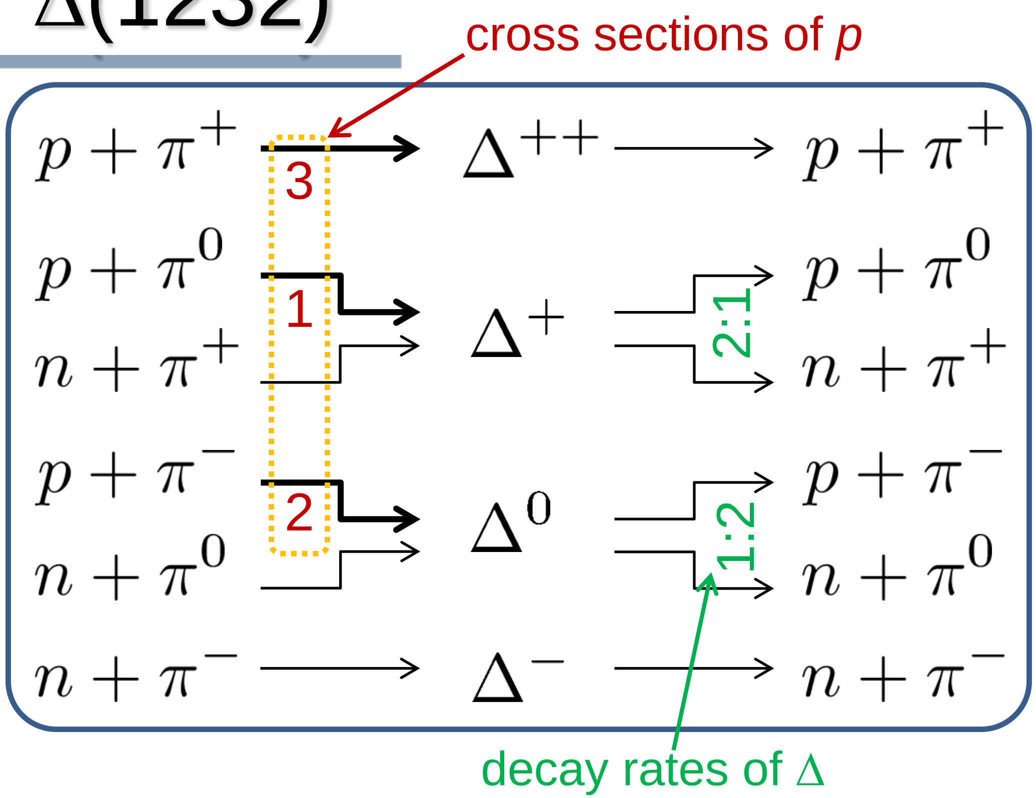
$$\begin{matrix} p, n \searrow & & \nearrow p, n \\ & \Delta(1232) & \\ \pi \nearrow & & \searrow \pi \end{matrix}$$

$$I = 3/2$$

$$\Gamma \simeq 1.8\ [\mathrm{fm}]$$



Δ(1232)



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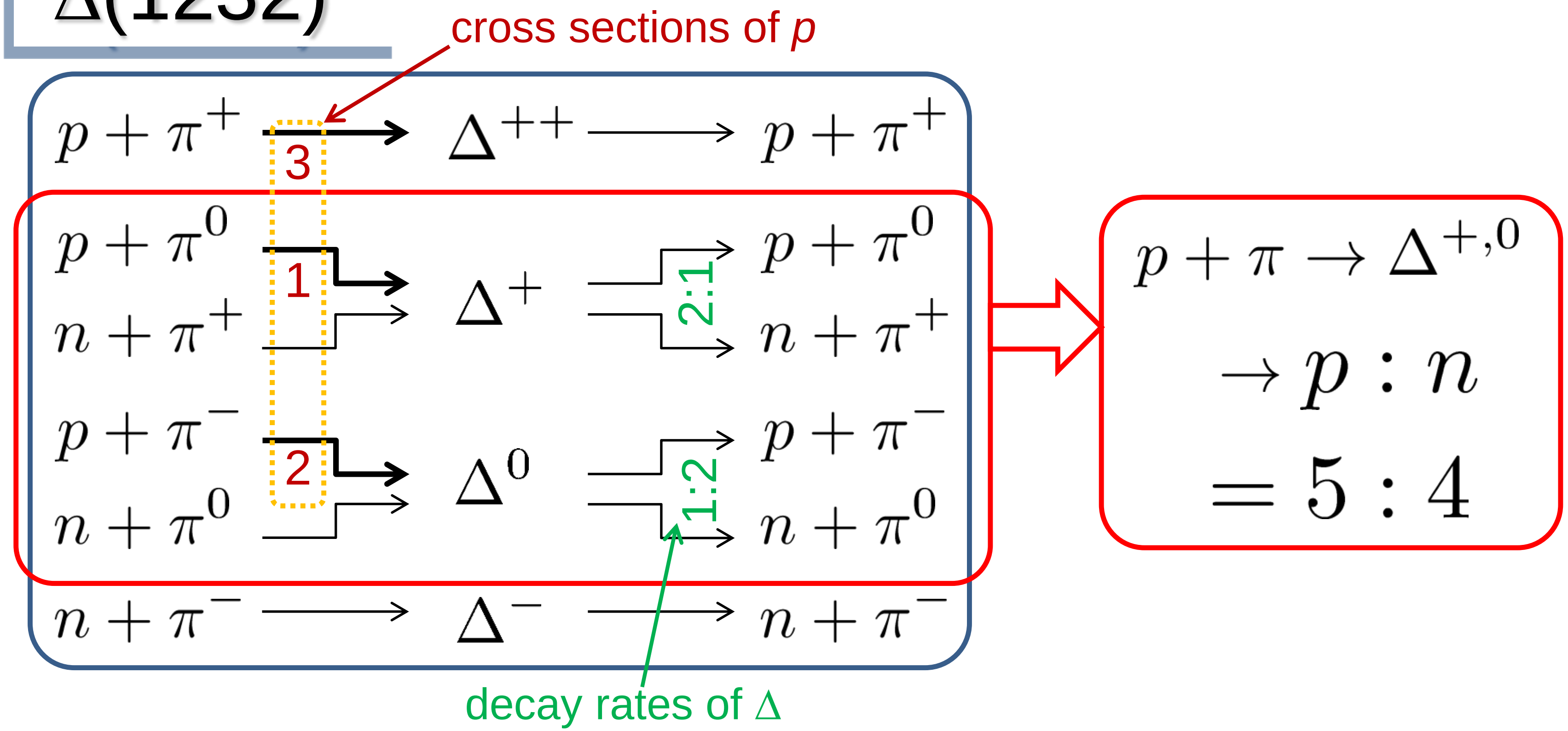
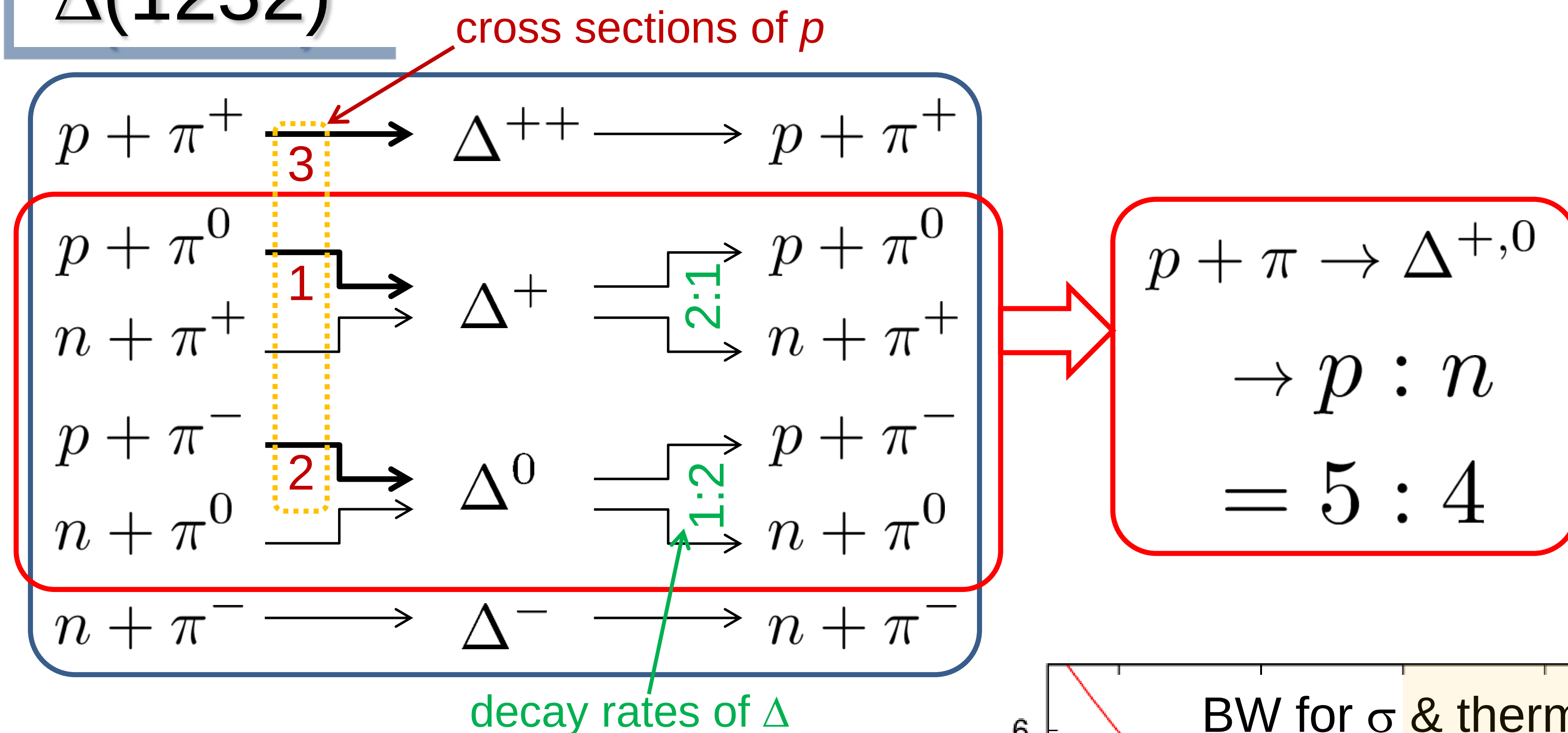

cross sections of p
p + π+ → Δ++ → p + π+
3
p + π0
n + π+
1
Δ+
2:1
p + π0
n + π+
p + π−
n + π0
2
Δ0
1:2
p + π−
n + π0
n + π− → Δ− → n + π−
decay rates of Δ
p + π → Δ+,0
→ p : n
= 5 : 4

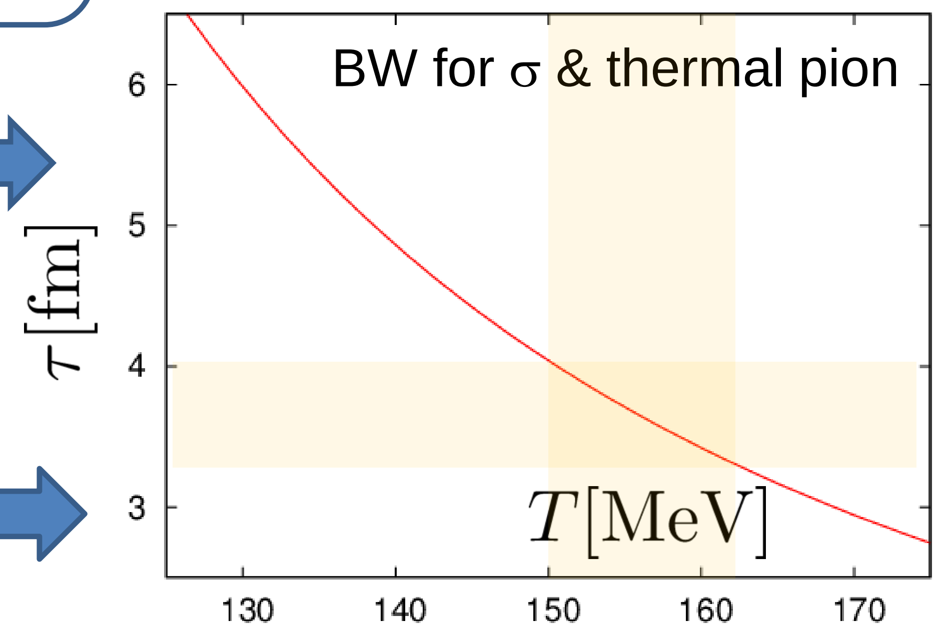
Δ(1232)



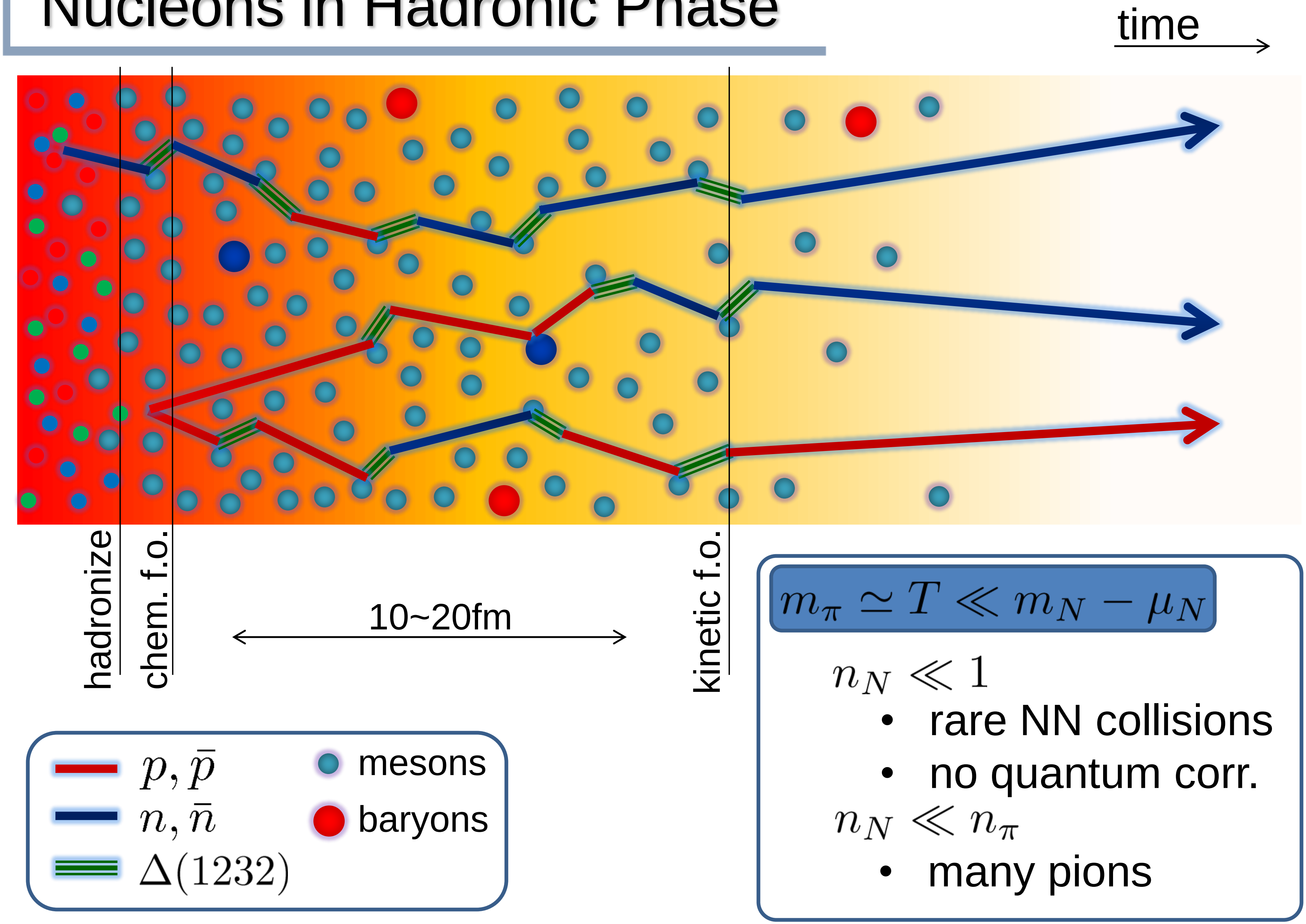
Lifetime to create Δ^+ or Δ^0

$$\tau^{-1} = \int \frac{d^3k_\pi}{(2\pi)^3}\sigma(E_{\rm cm})v_\pi n(E_\pi)$$

$$(\text{freezeout time}) \simeq 20[\text{fm}]$$



Nucleons in Hadronic Phase



$m_\pi \simeq T \ll m_N - \mu_N$

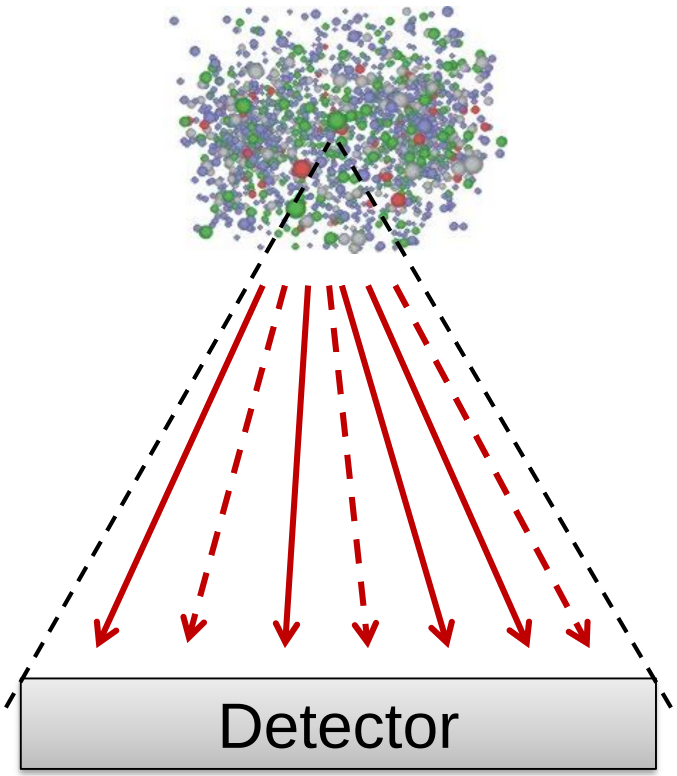
$n_N \ll 1$

- rare NN collisions
- no quantum corr.

$n_N \ll n_\pi$

- many pions

Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$



- $\left\{ \begin{array}{l} \longrightarrow N_N \text{ nucleons} \\ \dashrightarrow N_{\bar{N}} \text{ anti-nucleons} \end{array} \right.$

  ➡ $F(N_N, N_{\bar{N}})$

- $N_N \left\{ \begin{array}{l} N_p \text{ protons} \\ N_n \text{ neutrons} \end{array} \right.$

  ➡ $B(N_p; N_N)$

binomial distribution func.

$$\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$
$$= F(N_N, N_{\bar{N}}) B(N_p; N_N) B(N_{\bar{p}}; N_{\bar{N}})$$

- for any phase space in the final state.

Difference btw Baryon and Proton Numbers

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

$$
\begin{cases}
2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots
\end{cases}
$$

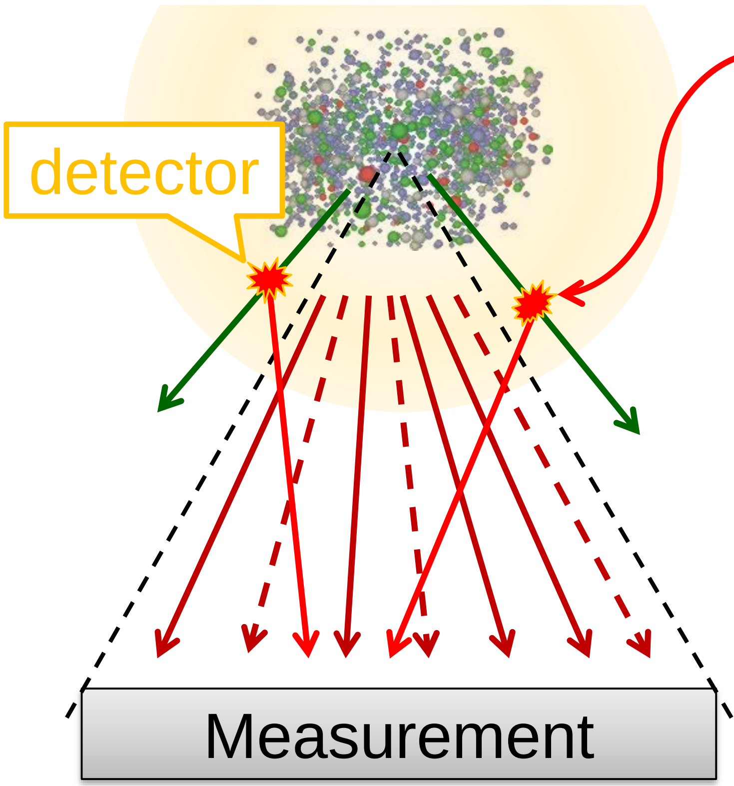
genuine info. noise

For free gas

$$2\langle(\delta N_p^{(\mathrm{net})})^n\rangle_c = \langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^n\rangle_c$$

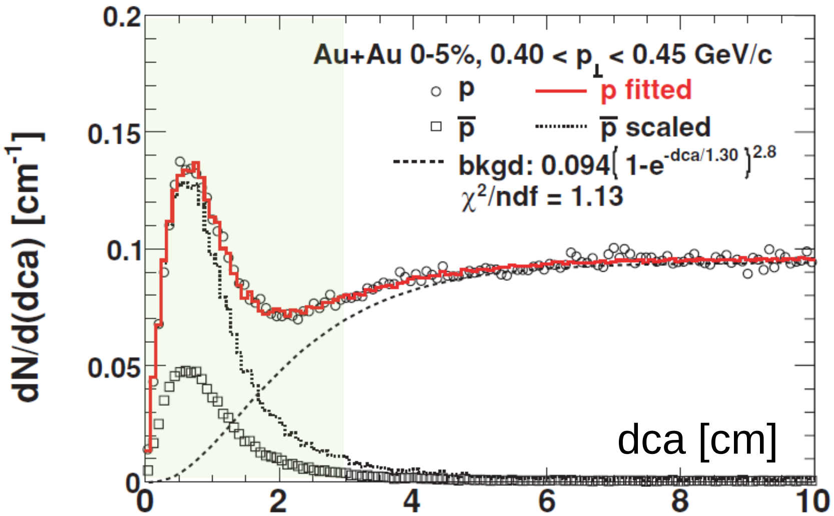
Secondary Protons

MK+, in preparation



Secondary (knockout) protons

20% of observed protons @ STAR



STAR, PRC79,034909(2009)

Secondary Protons

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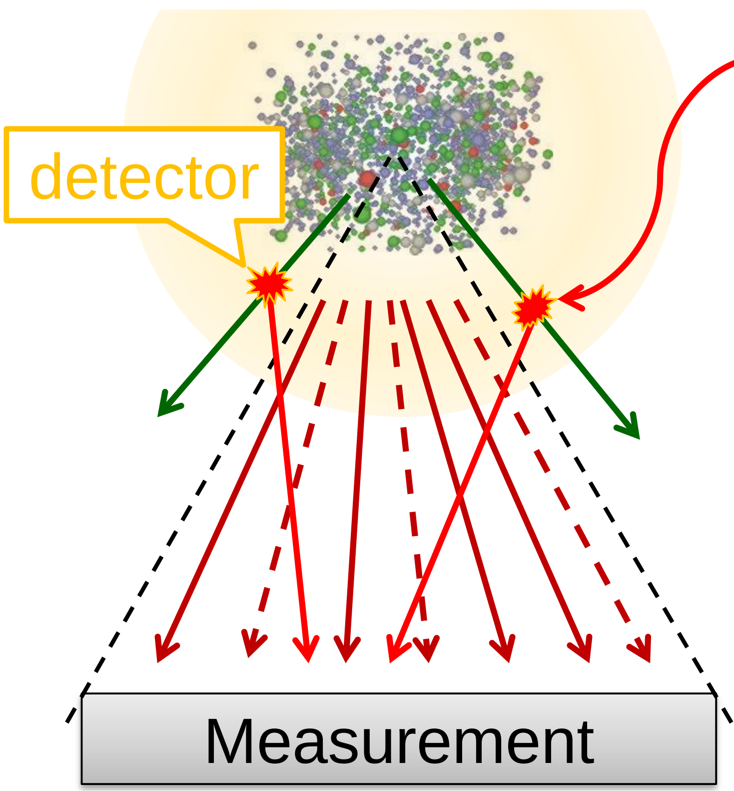

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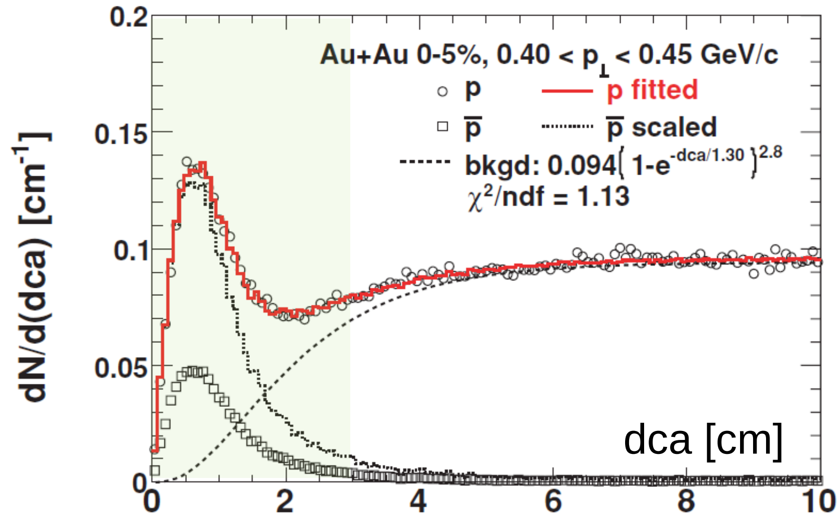
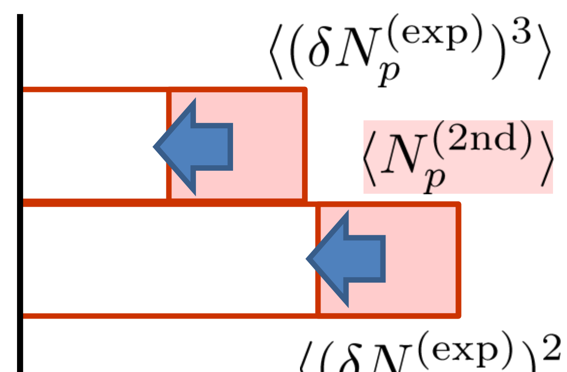


STAR, PRC79,034909(2009)

Their contribution can be eliminated!

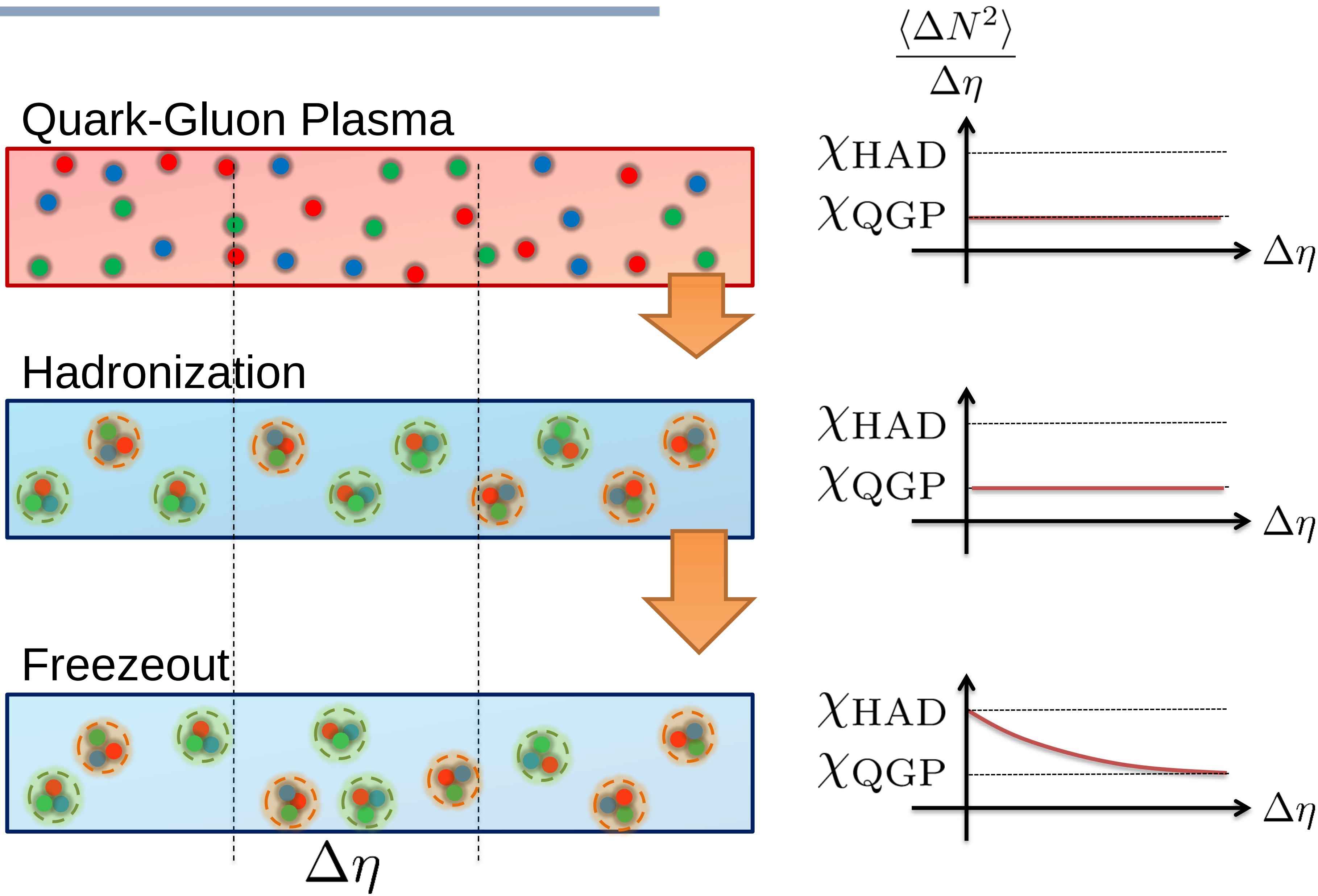
$$\langle(\delta N_p^{(\mathrm{QGP})})^n\rangle_c = \langle(\delta N_p^{(\exp)})^n\rangle_c - \langle N_p^{(\mathrm{2nd})}\rangle$$



Time Evolution of Higher Order Cumulants

MK, Asakawa, Ono, arXiv:1307.xxxx

Time Evolution in HIC



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n$$

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

$\Delta\eta$ Dependence

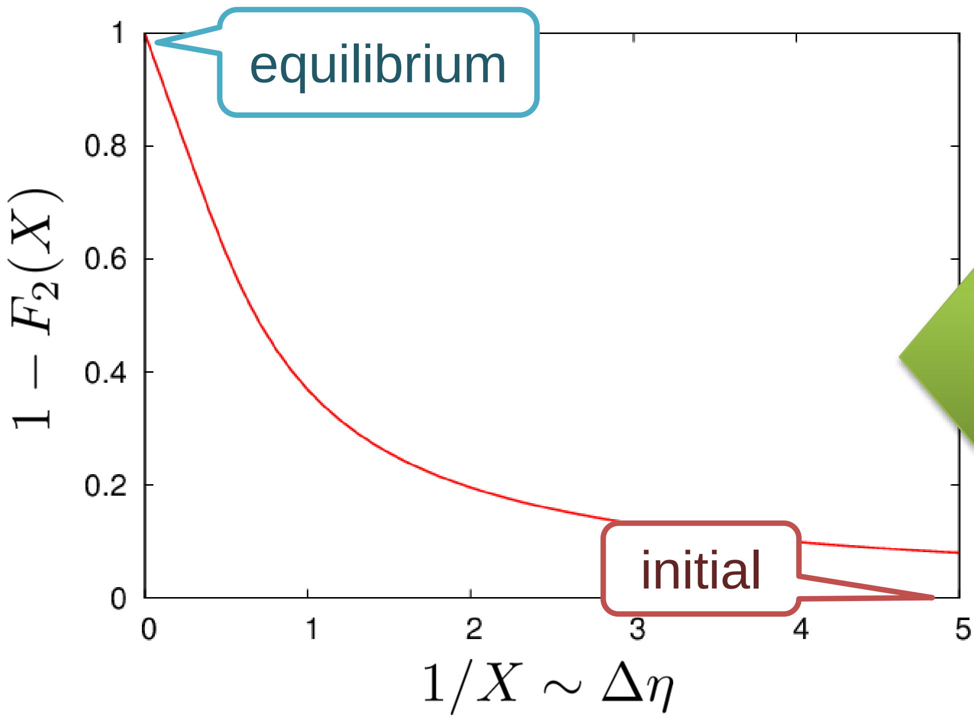
Shuryak, Stephanov, 2001

- Initial condition: $\langle \delta n(\eta_1, 0)\delta n(\eta_2, 0)\rangle = \sigma_2 \delta(\eta_1 - \eta_2)$
- Translational invariance

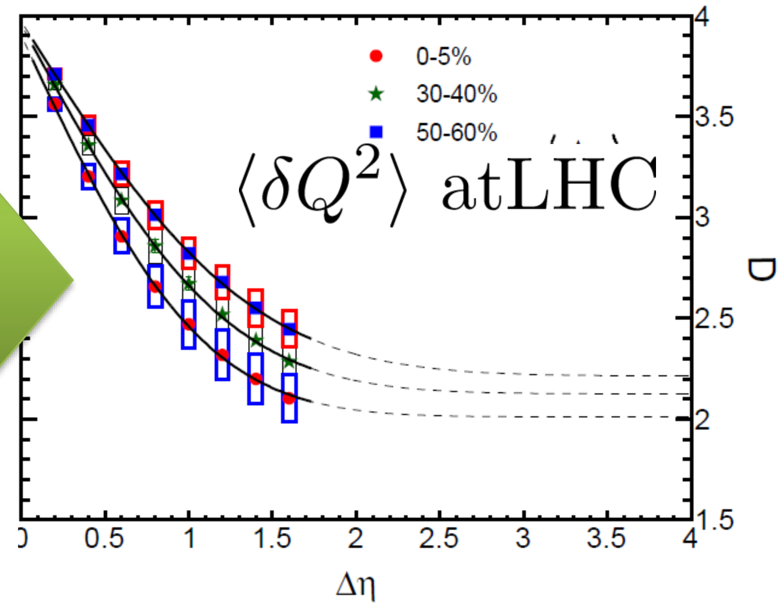
$$\langle \delta Q(\tau)^2 \rangle = \underbrace{\sigma_2 F_2(X)}_{\text{initial}} + \underbrace{\chi_2(1 - F_2(X))}_{\text{equilibrium}}$$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta\, n(\eta, \tau)$$

$2\sqrt{D\tau}$



$\langle \delta Q^2 \rangle$ at LHC



Thee “NON”s

重イオン衝突での高次ゆらぎの観測・解析は、
物理学として**相当に**特殊な問題である。

- **Non-Gaussian**

通常、高次ゆらぎは観測困難。
適度に小さい系

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  観測されたゆらぎの値は、
  自由ガスとたかだか2倍のずれ

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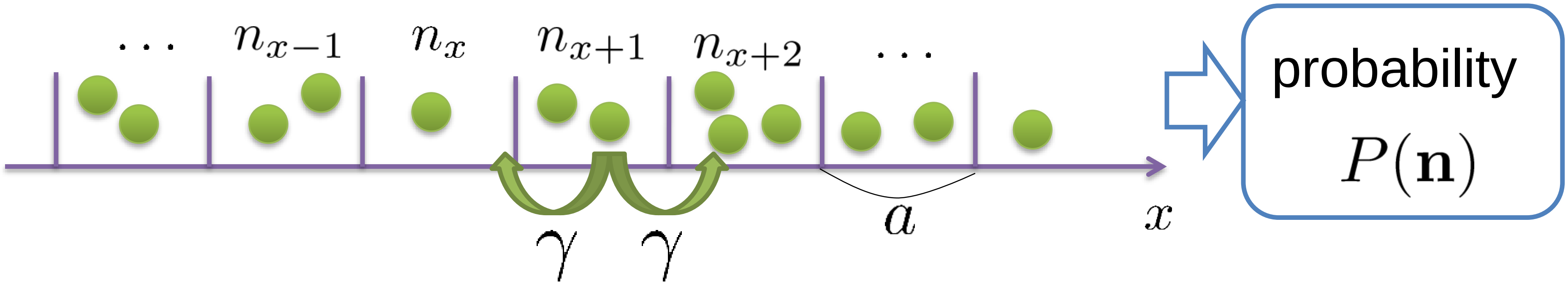
  通常、高次ゆらぎは観測困難。
  適度に小さい系
- **Non-critical**
  観測されたゆらぎの値は、
  自由ガスとたかだか２倍のずれ
- **Non-equilibrium**
  平衡に至る非定常過程を
  記述する必要性。

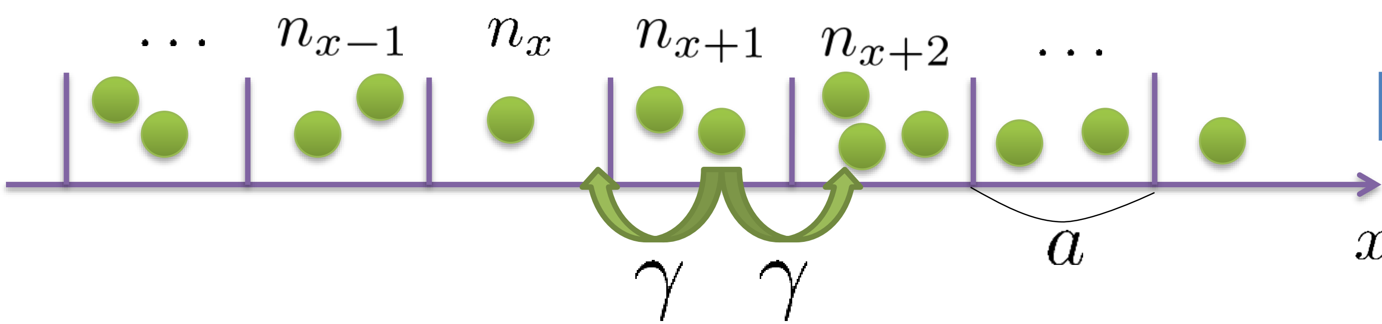
Diffusion Master Equation

Divide spatial coordinate into discrete cells



Diffusion Master Equation

Divide spatial coordinate into discrete cells



probability

$P(\mathbf{n})$

Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x+1)\left\{P(\mathbf{n}+\hat{x}-\widehat{x+1}) + P(\mathbf{n}+\hat{x}-\widehat{x-1})\right\} - 2n_x P(\mathbf{n})]$$

x-hat: lattice-QCD notation

Solve the DME **exactly**, and take *a*→0 limit

No approx., ex. van Kampen's system size expansion

Solution of DME

1st $\langle \tilde{n}_k \rangle (t) = e^{-\omega_k t} \langle \tilde{n}_k \rangle_0$ $\qquad \omega_k \simeq \gamma a^2 k^2$

initial

- Deterministic part ←→ diffusion equation at long wave length (1/a<<k)

$$\partial_t \langle n_x(t) \rangle = \gamma a^2 \partial_x^2 \langle n_x(t) \rangle$$

- Appropriate continuum limit with $\gamma a^2 = D$

Solution of DME

1st $\langle \tilde{n}_k \rangle (t) = e^{-\omega_k t} \langle \tilde{n}_k \rangle_0$ $\qquad \omega_k \simeq \gamma a^2 k^2$

↑ initial

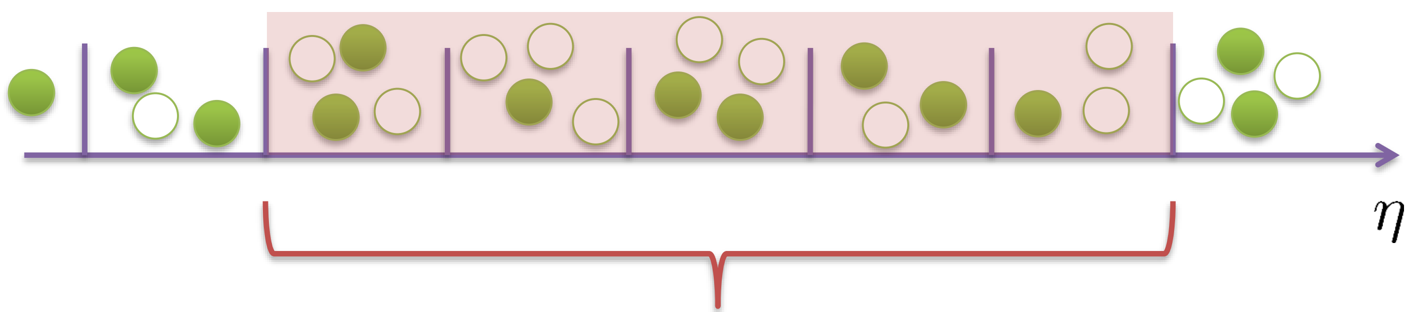
- Deterministic part ←→ diffusion equation at long wave length (1/a<<k)
- Appropriate continuum limit with $\gamma a^2 = D$

2nd
$$\langle \delta\tilde{n}_{k_1} \delta\tilde{n}_{k_2} \rangle (t) = \langle \tilde{n}_{k_1+k_2} \rangle_0 \left(e^{-\omega_{k_1+k_2} t} - e^{-(\omega_{k_1}+\omega_{k_2})t} \right) + \langle \delta\tilde{n}_{k_1} \delta\tilde{n}_{k_2} \rangle_0 e^{-(\omega_{k_1}+\omega_{k_2})t}$$

- Consistent with stochastic diffusion eq. (for sufficiently smooth initial condition)

Net Charge Number

Prepare 2 species of (non-interacting) particles



$$\bar{Q}(\tau) = \int_0^{\Delta\eta} d\eta \, (n_1(\eta,\tau) - n_2(\eta,\tau))$$

Let us investigate

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ at freezeout time t

Initial Condition at Hadronization

- Boost invariance / infinitely long system
- Local equilibration / local correlation
- Initial fluctuations

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ — suppression owing to local charge conservation

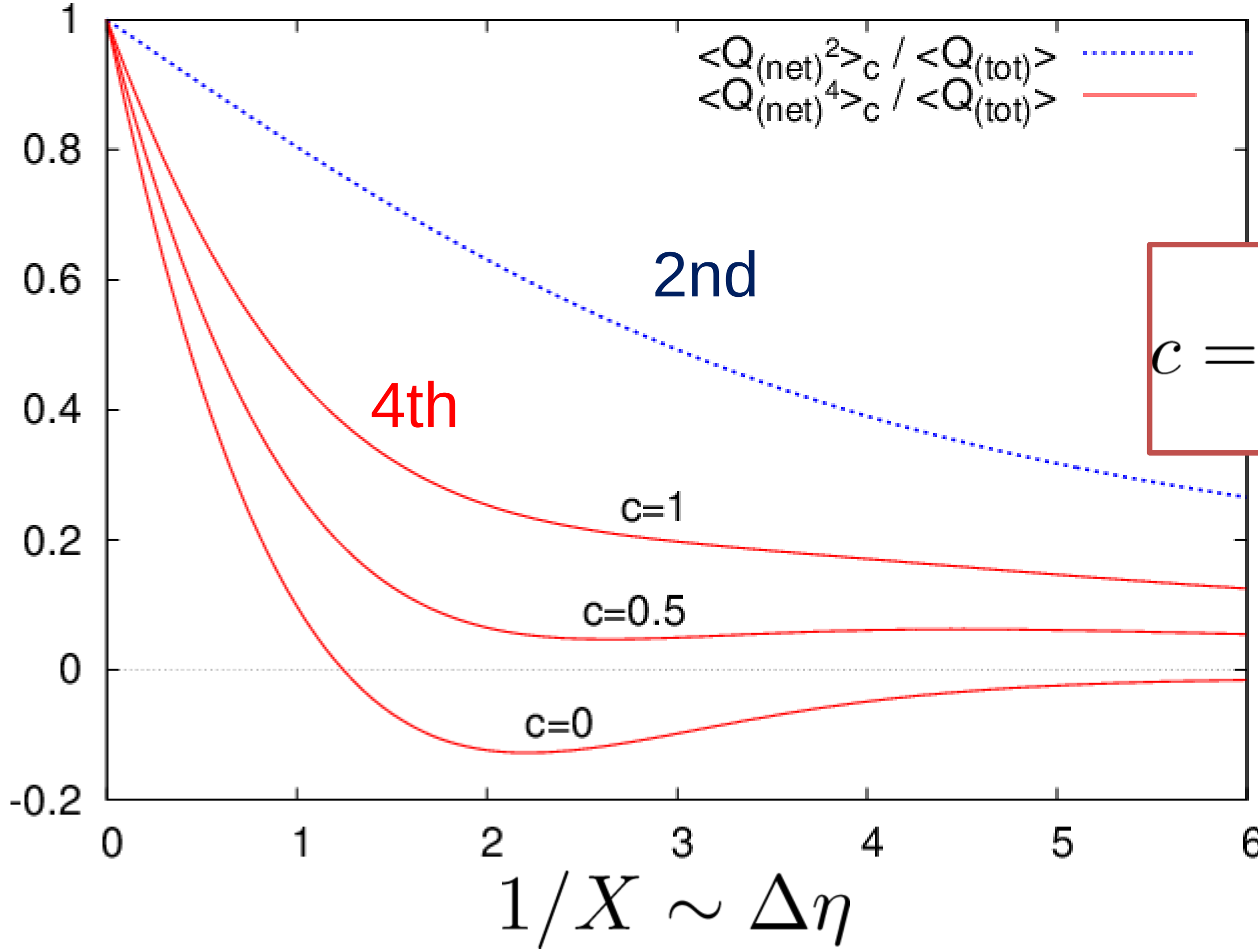
$\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$

$\langle Q^2_{(\mathrm{tot})} \rangle_c$ — strongly dependent on hadronization mechanism

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

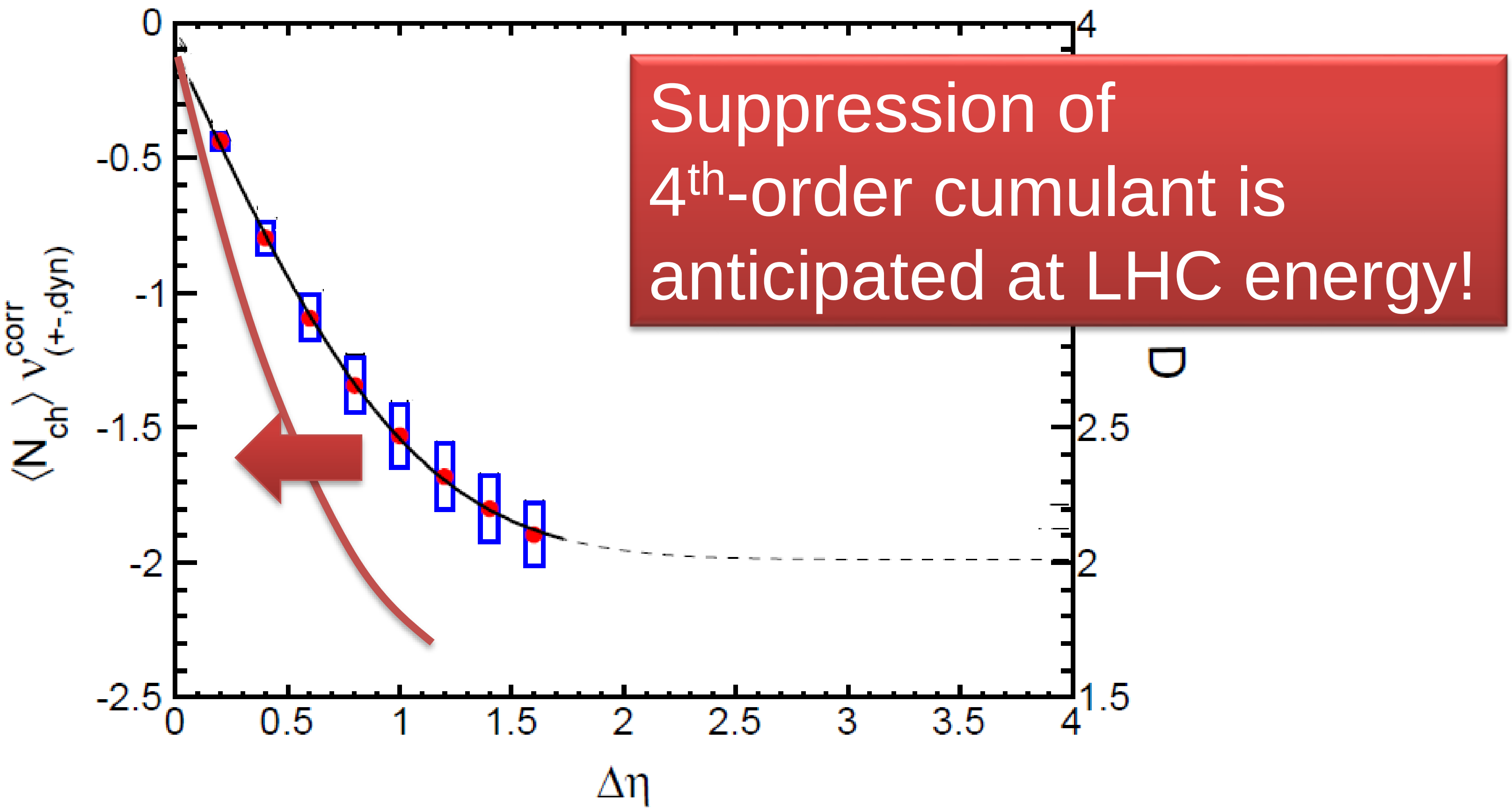
$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$



$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$<\delta N_Q^4>$ @ LHC

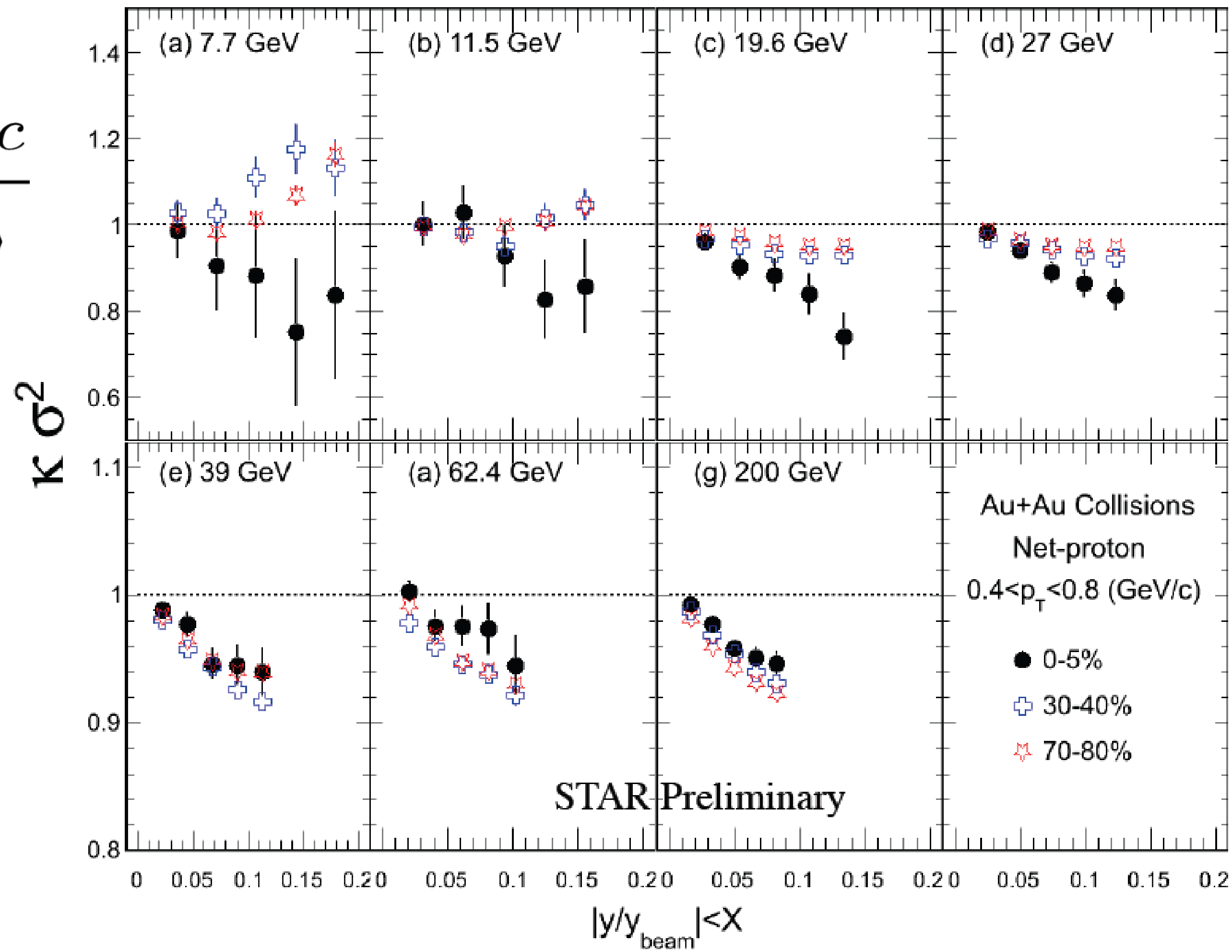
Assumptions
- boost invariant system
- small fluctuations of CC at hadronization
- short correlation in hadronic stage



Δη Dependence at STAR

STAR, QM2012

$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$

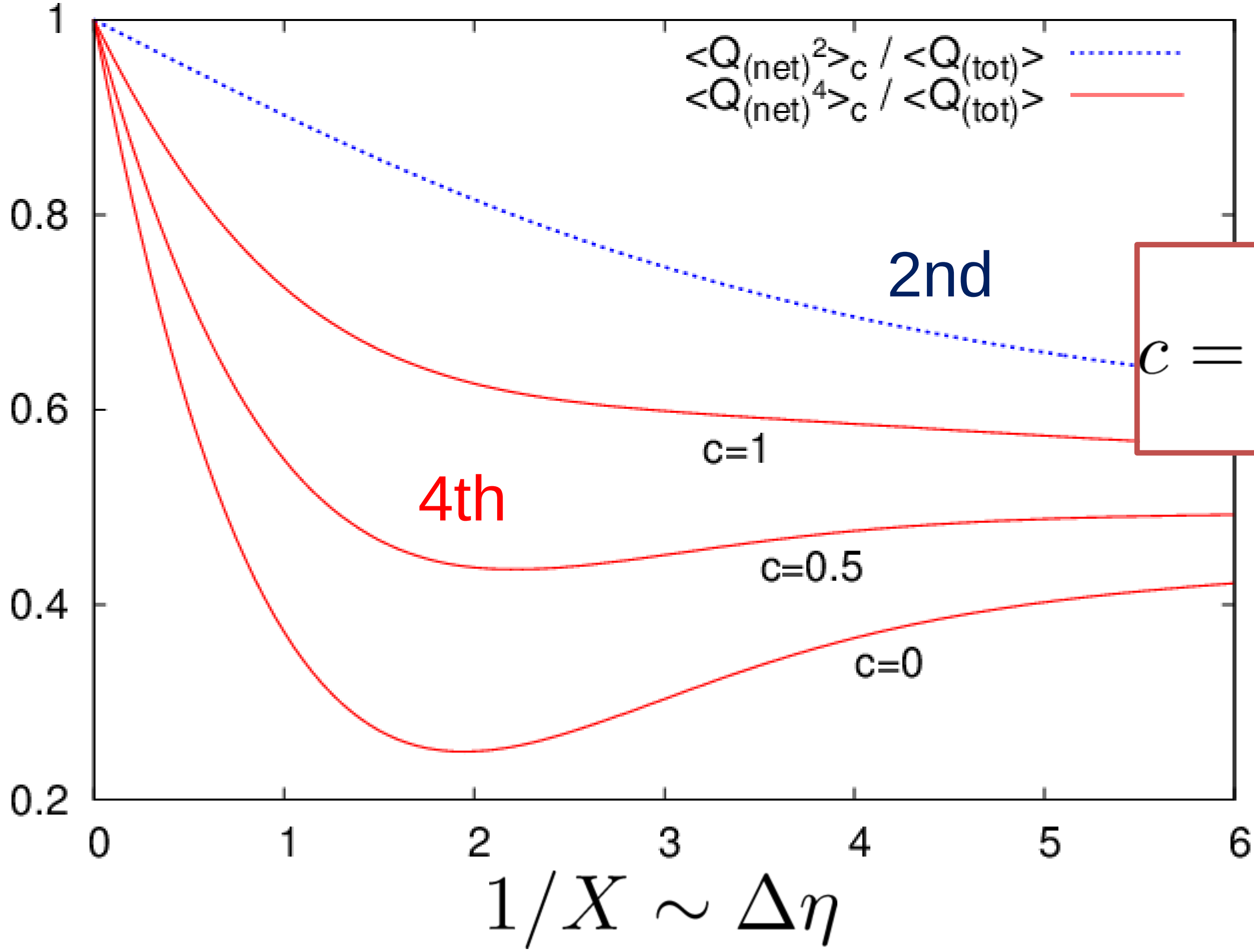


$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$ decreases as Δη becomes larger at RHIC.

$\Delta\eta$ Dependence at Freezeout

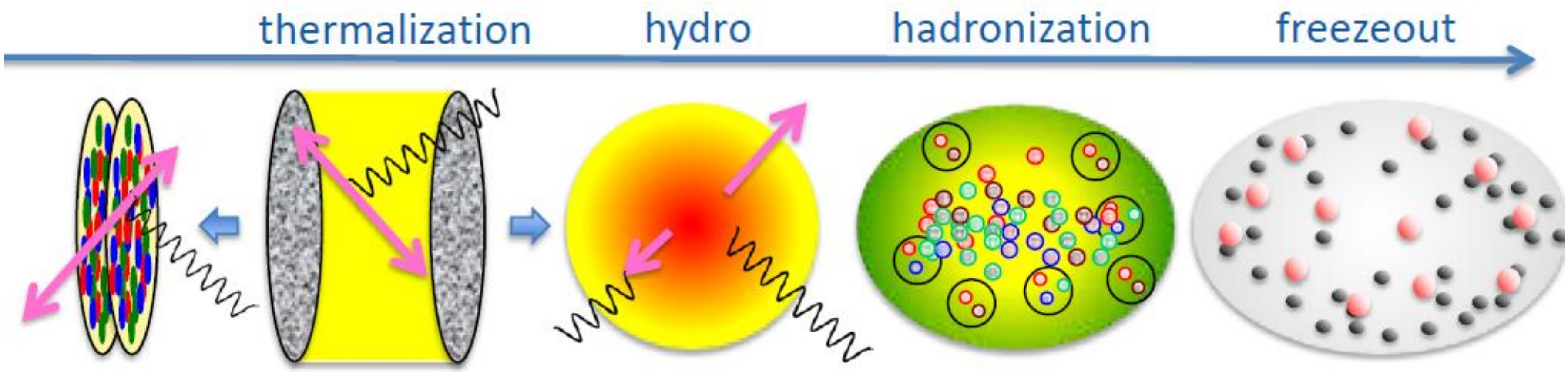
Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\rm tot)} \rangle_c = 0.5 \langle Q_{(\rm tot)} \rangle$$



$$c = \frac{\langle Q^2_{(\rm tot)} \rangle_c}{\langle Q_{(\rm tot)} \rangle}$$

高温物質の時間発展



熱平衡化段階でのゆらぎ

QGP相内での時間発展

拡散によるHRGへの接近

検出器

初期エネルギー密度 ⟶ 体積ゆらぎ

まとめ

保存量初期ゆらぎの痕跡は終状態に残されており
LHC、RHICで観測にかかり始めている

ただし、RHIC-BES領域では即座にQCD相構造が
直ちに見えるほど明瞭ではないので、

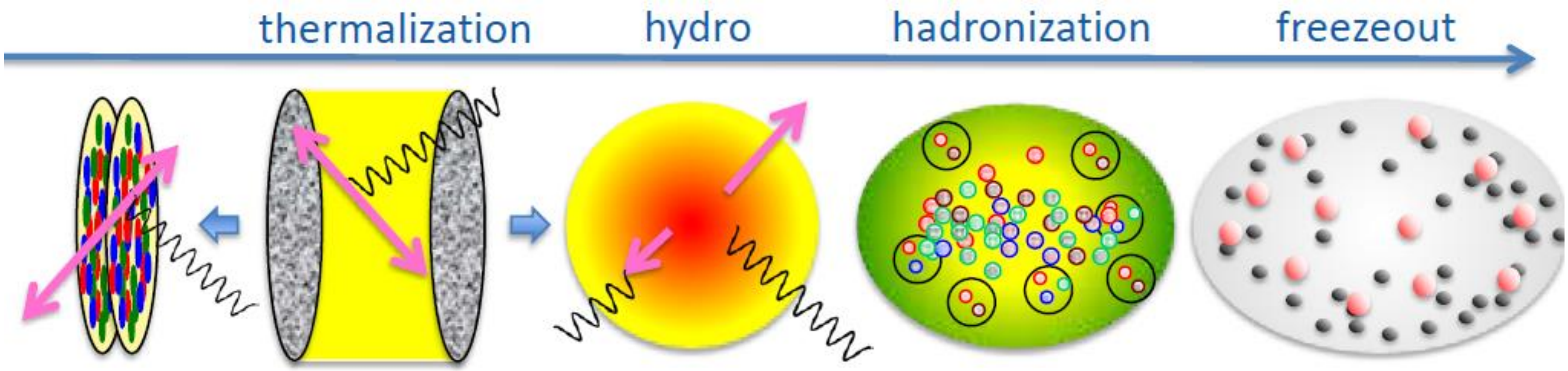
実験的課題

- ➢ バリオン数ゆらぎの構成
- ➢ Q_4/Q_2@ALICE
- ➢ efficiency, acceptance, …
- ➢ エネルギーゆらぎ
- ➢ BES from RHIC to LHC

理論的課題

- ➢ RHIC D-puzzleの解決
- ➢ RHIC-BESの結果の解釈
- ➢ ゆらぎの動的時間発展
- ➢ 2点相関関数等との比較
- ➢ 新しい観測量の提案

Evolution of Fluctuations



Fluctuation in initial state

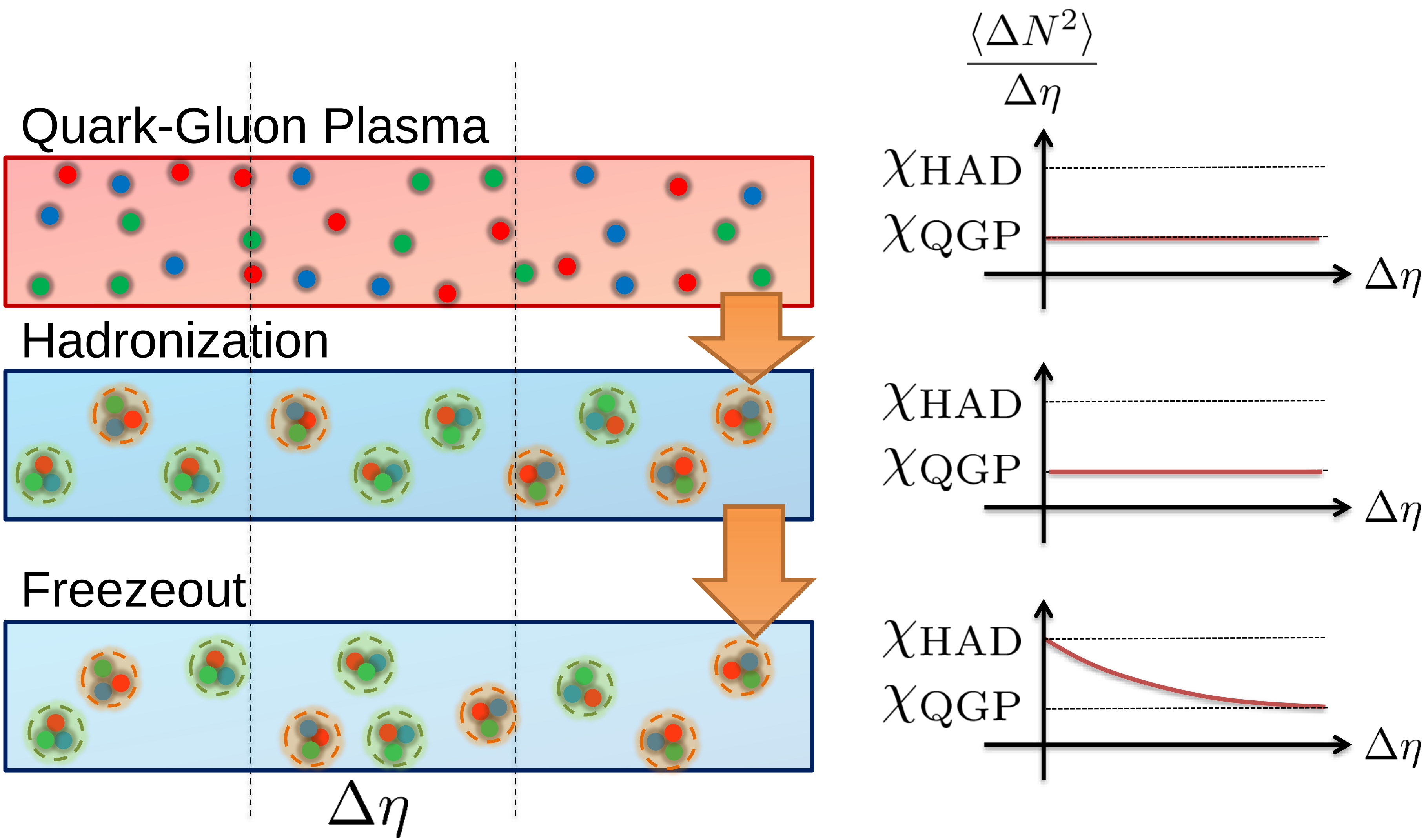
Time evolution in the QGP

approach to HRG by diffusion

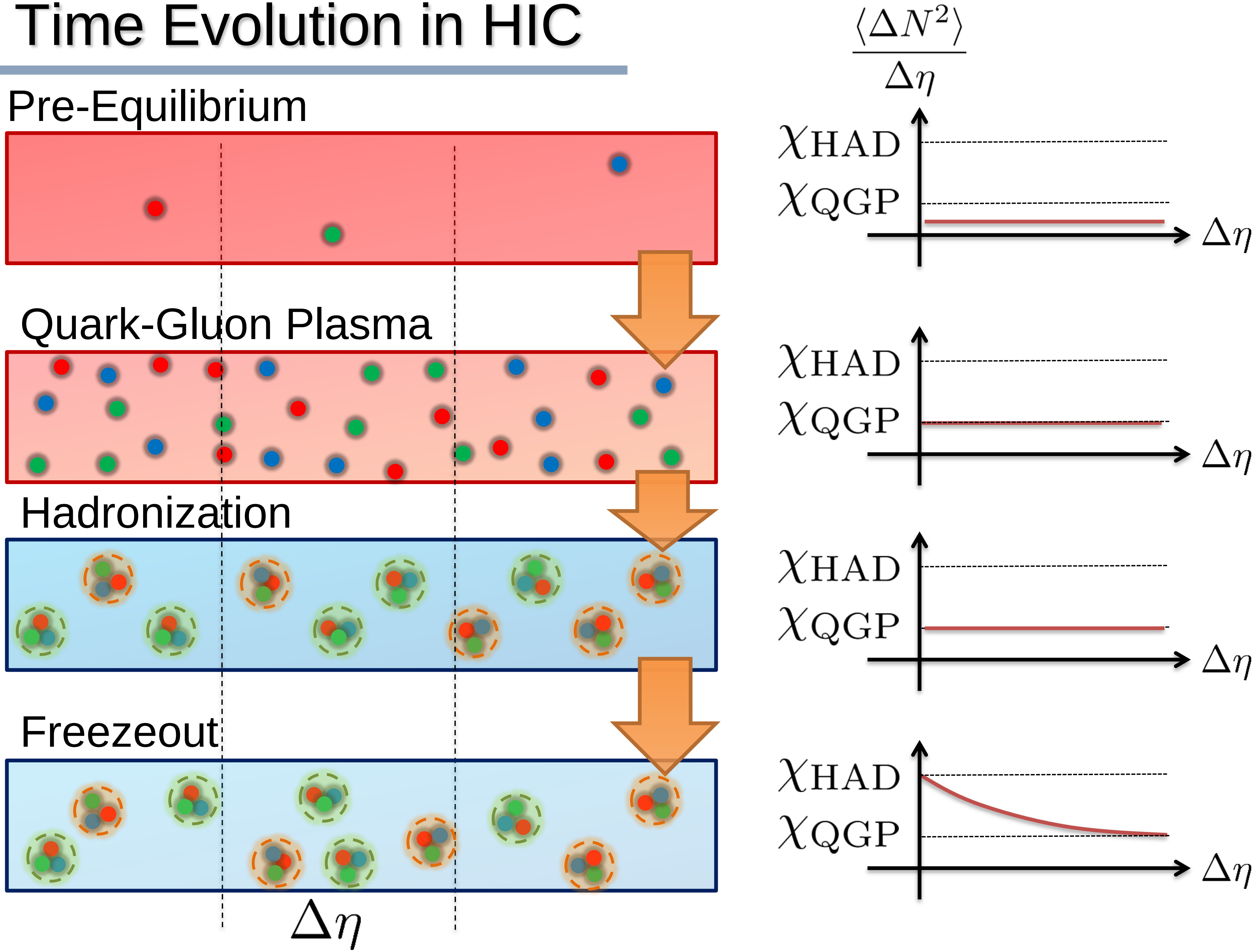
experimental effects
particle missID, etc.

volume fluctuation

Time Evolution in HIC



Time Evolution in HIC



Time Evolution in HIC