

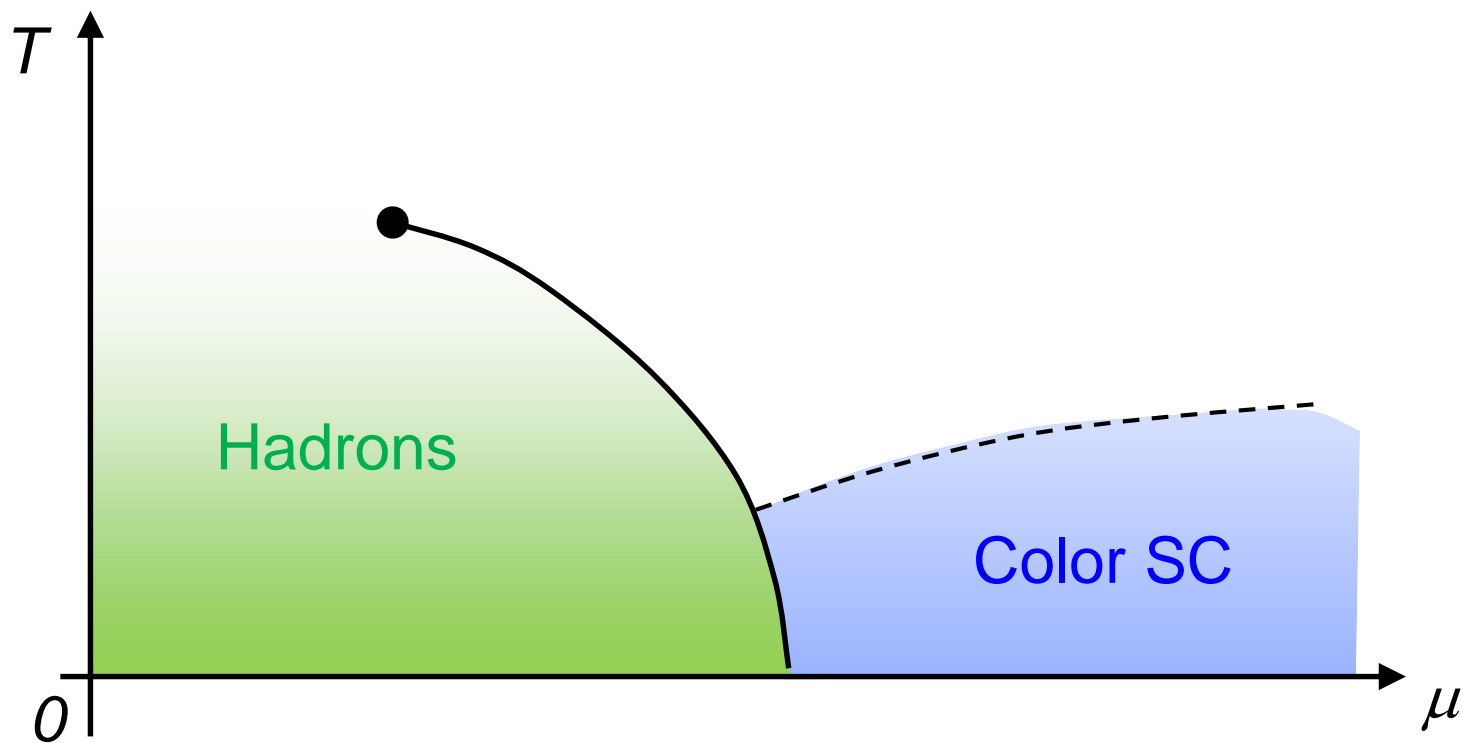
重イオン衝突における 非ガウスゆらぎ

北沢 正清
(阪大)

MK, Asakawa, Ono, arXiv:1307.2978

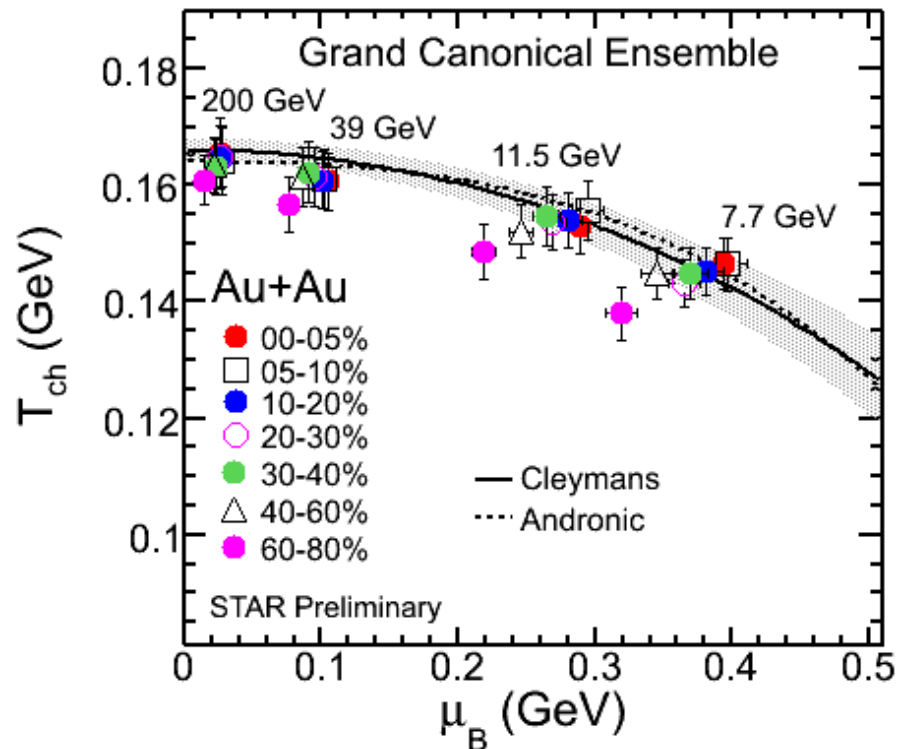
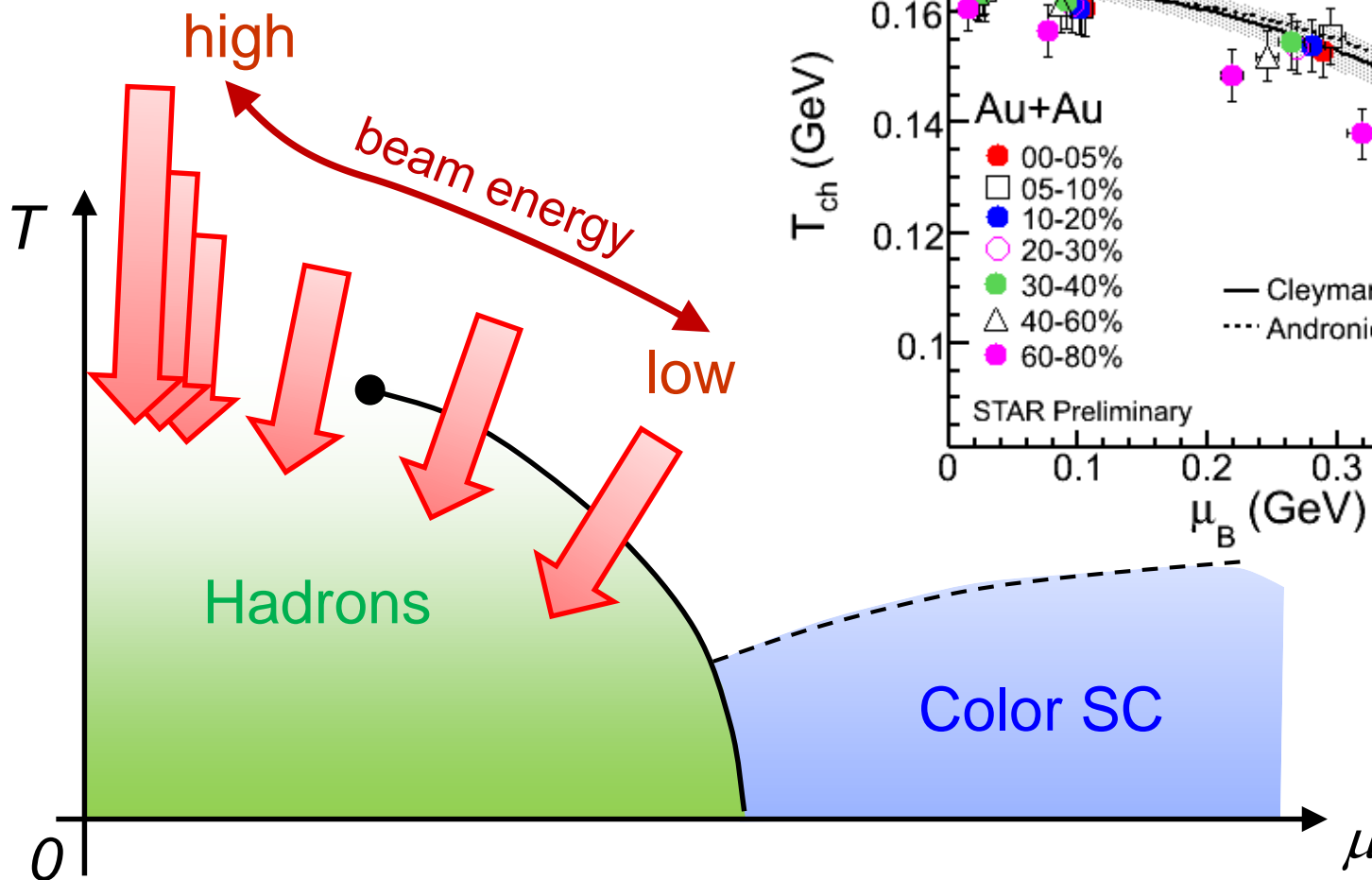
「熱場の量子論とその応用」、京大基研、2013年8月28日

Beam-Energy Scan



Beam-Energy Scan

STAR 2012



Fluctuations

Fluctuations reflect properties of matter.

Enhancement near the critical point

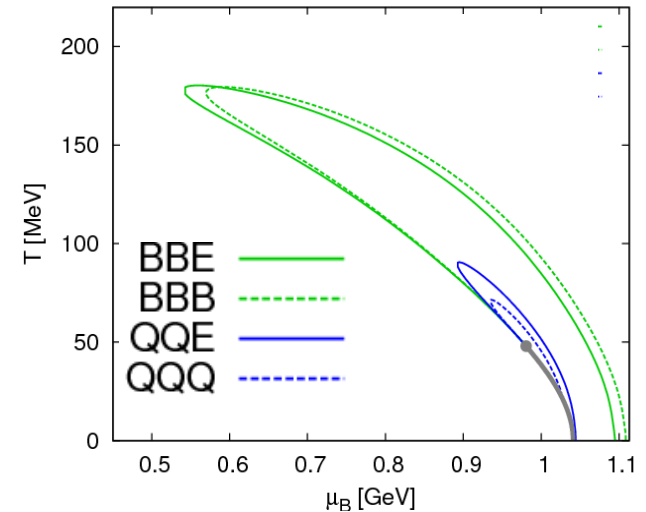
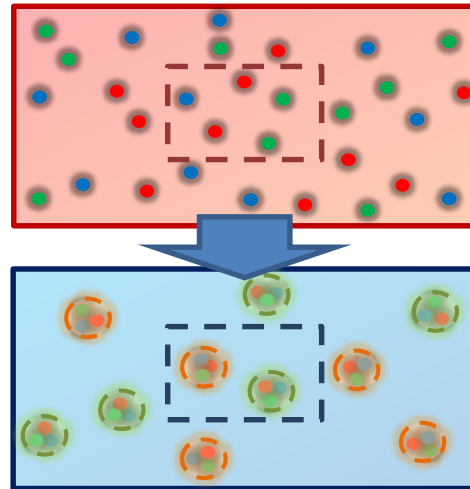
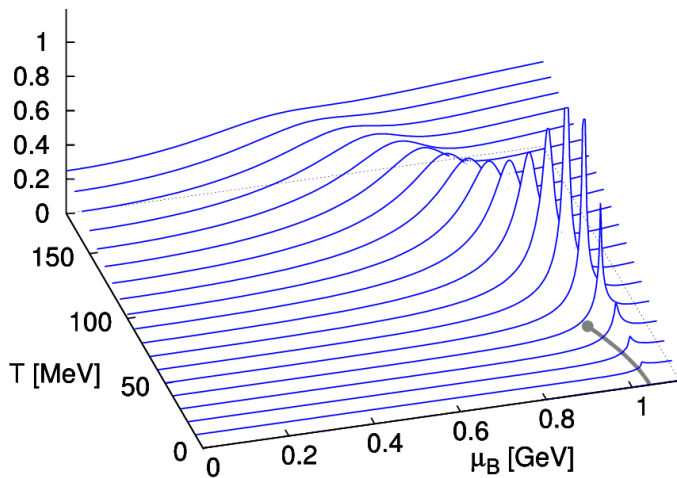
Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);...

Ratios between cumulants of conserved charges

Asakawa,Heinz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)

Signs of higher order cumulants

Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)

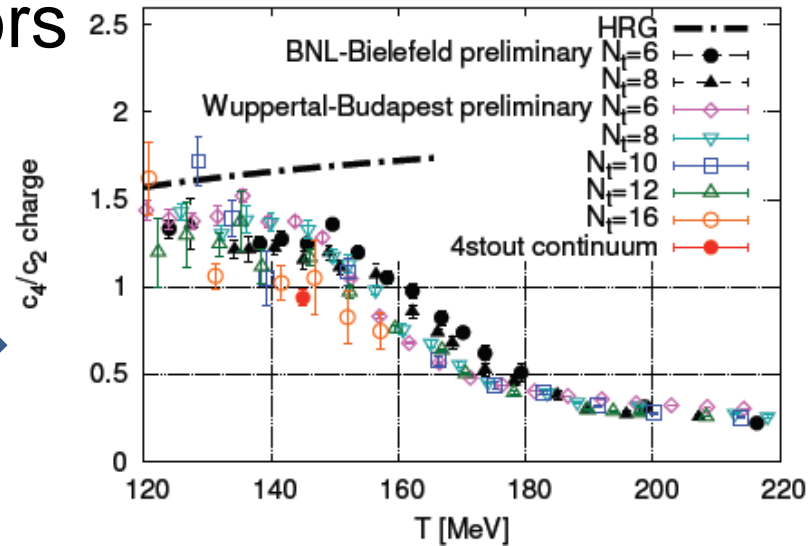


Conserved Charges : Theoretical Advantage

□ Definite definition for operators

- as a Noether current
- calculable on any theory

ex: on the lattice

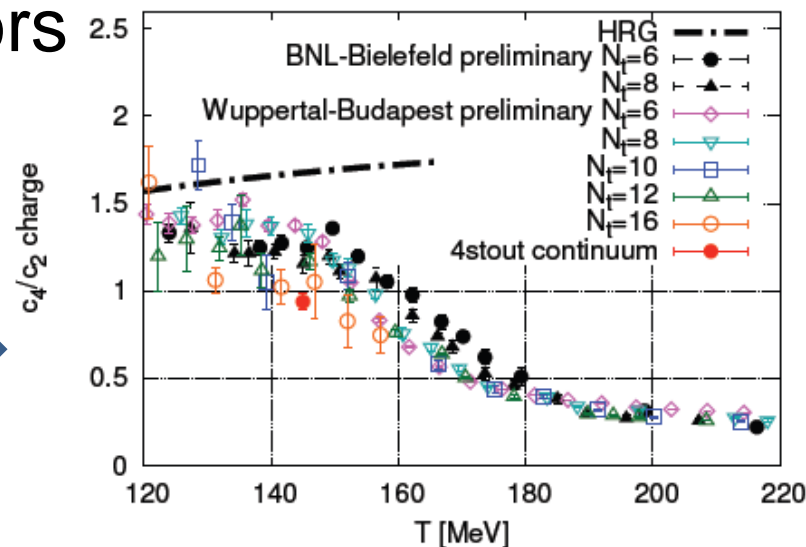


Conserved Charges : Theoretical Advantage

□ Definite definition for operators

- as a Noether current
- calculable on any theory

ex: on the lattice →



□ Simple thermodynamic relations

$$\langle \delta N_c^n \rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}$$

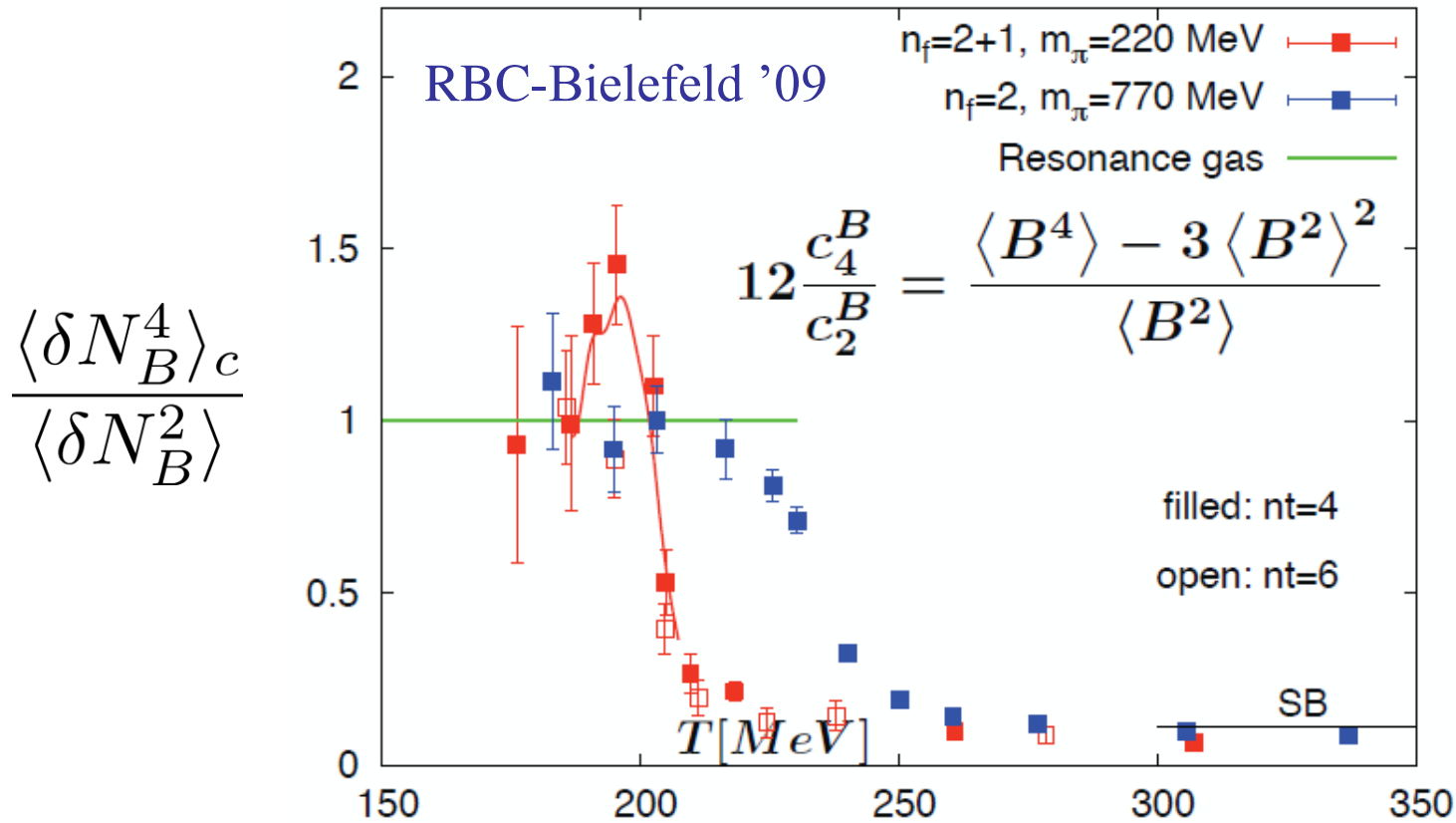
- Intuitive interpretation for the behaviors of cumulants

ex: $\langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B}$



Asakawa, Ejiri, MK, 2009

Conserved Charge Fluctuations

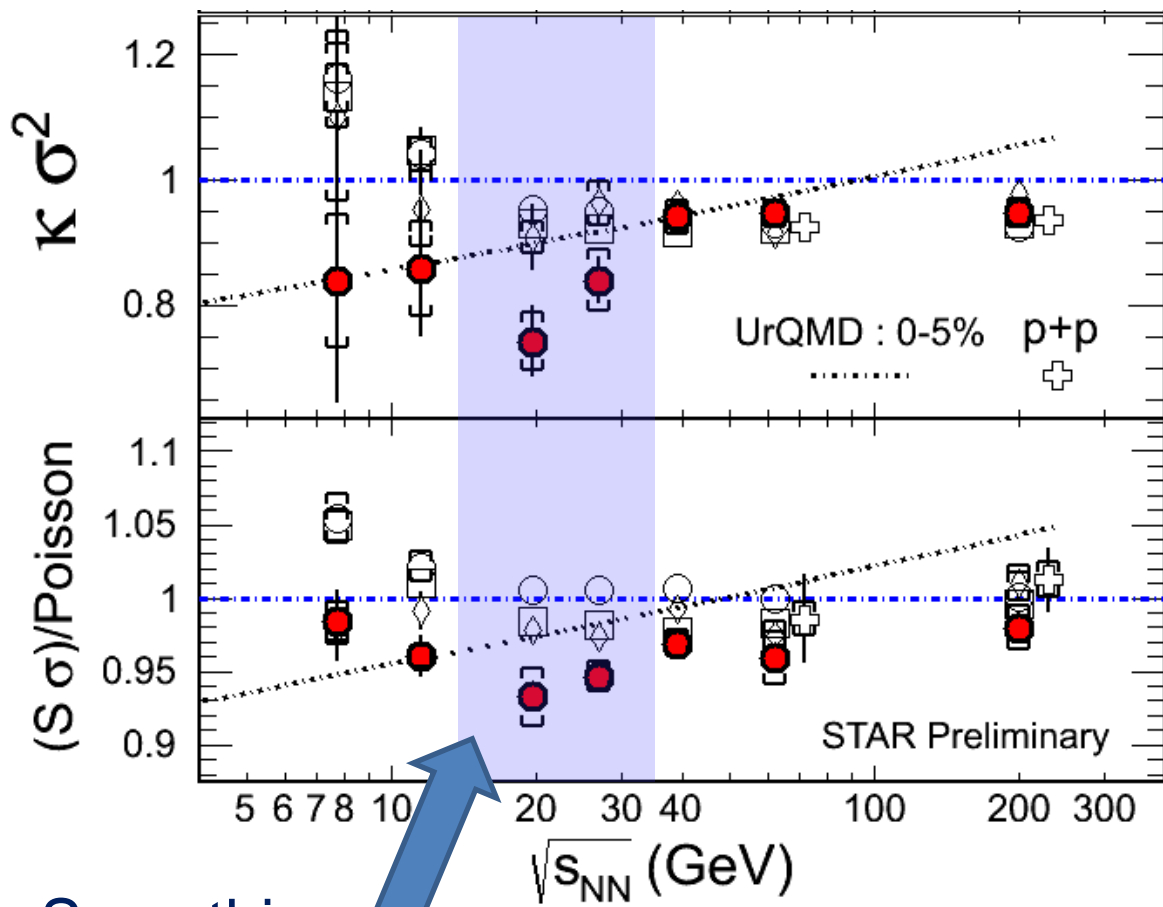


Cumulants of N_B and N_Q are **suppressed** at high T .

Asakawa, Heinz, Muller, 2000; Jeon, Koch, 2000;
 Ejiri, Karsch, Redlich, 2006; Asakawa, Ejiri, MK, 2009;
 Friman, et al., 2011; Stephanov, 2011

Proton # Cumulants @ STAR-BES

STAR, QM2012



$$\frac{C_4}{C_2}$$

$$\frac{C_3}{C_1} = \frac{C_3/C_2}{\text{Poissonian}}$$

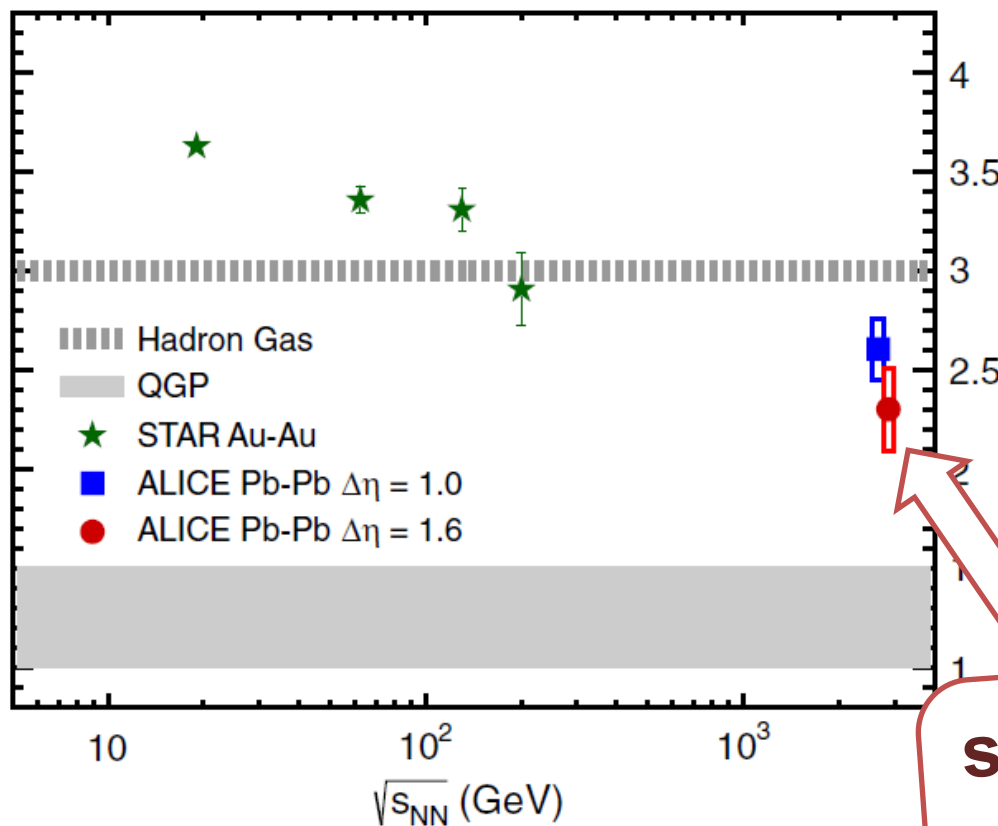
Something interesting??



CAUTION!
 proton number \neq baryon number
 MK, Asakawa, 2011;2012

Charge Fluctuation @ LHC

ALICE, PRL110,152301(2013)



D-measure

$$D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

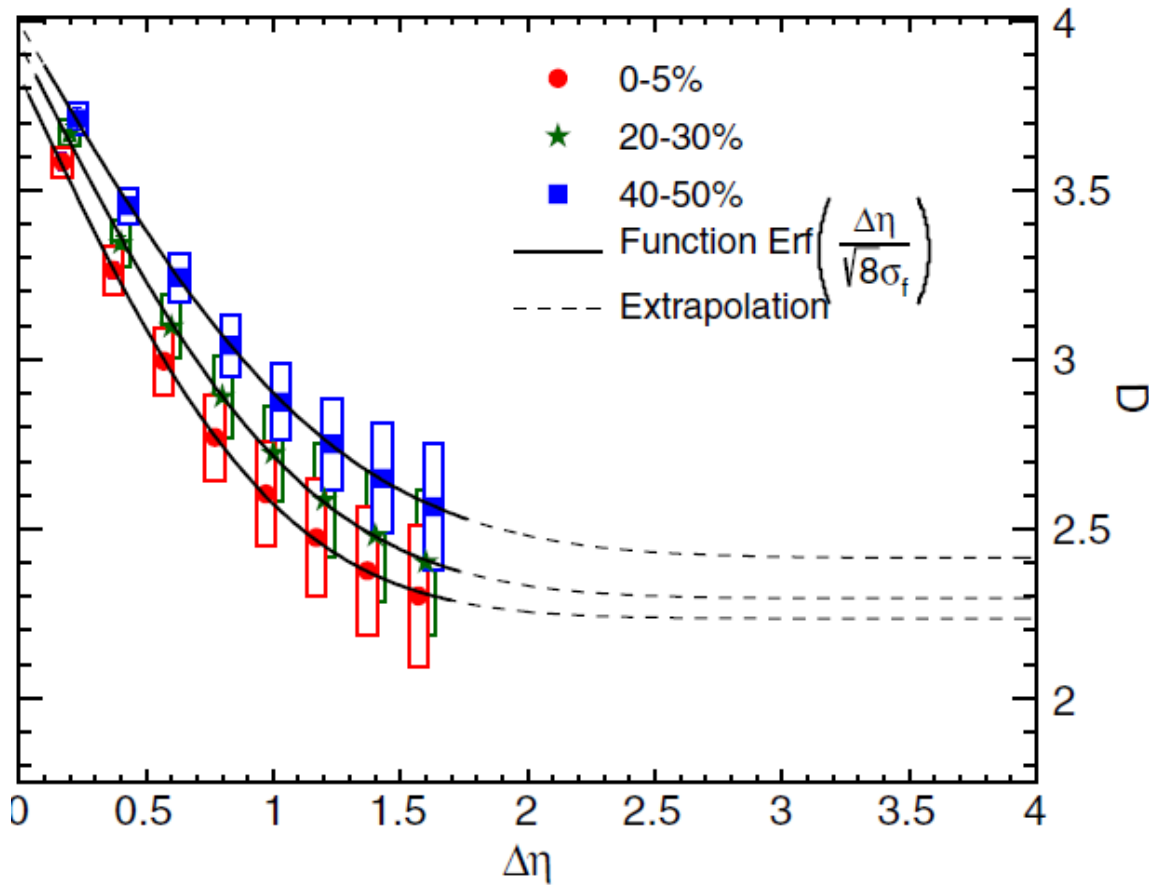
- $D \sim 3-4$ Hadronic
- $D \sim 1$ Quark

**significant suppression
from hadronic value
at LHC energy!**

$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

$\Delta\eta$ Dependence @ ALICE

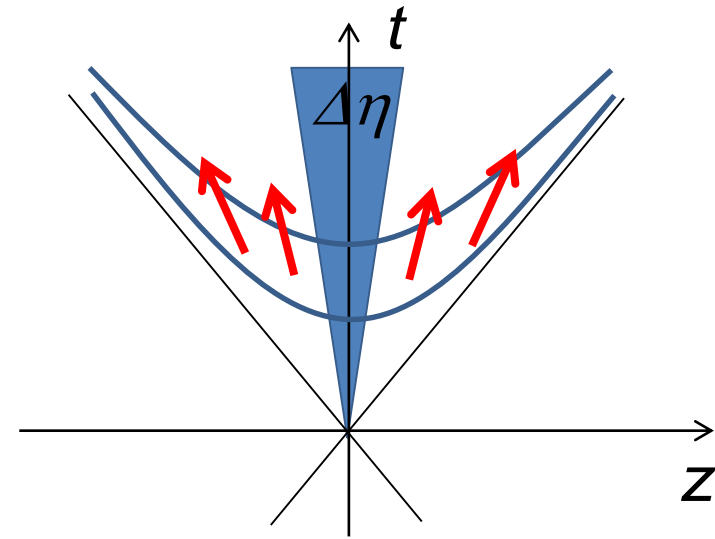
ALICE
PRL 2013



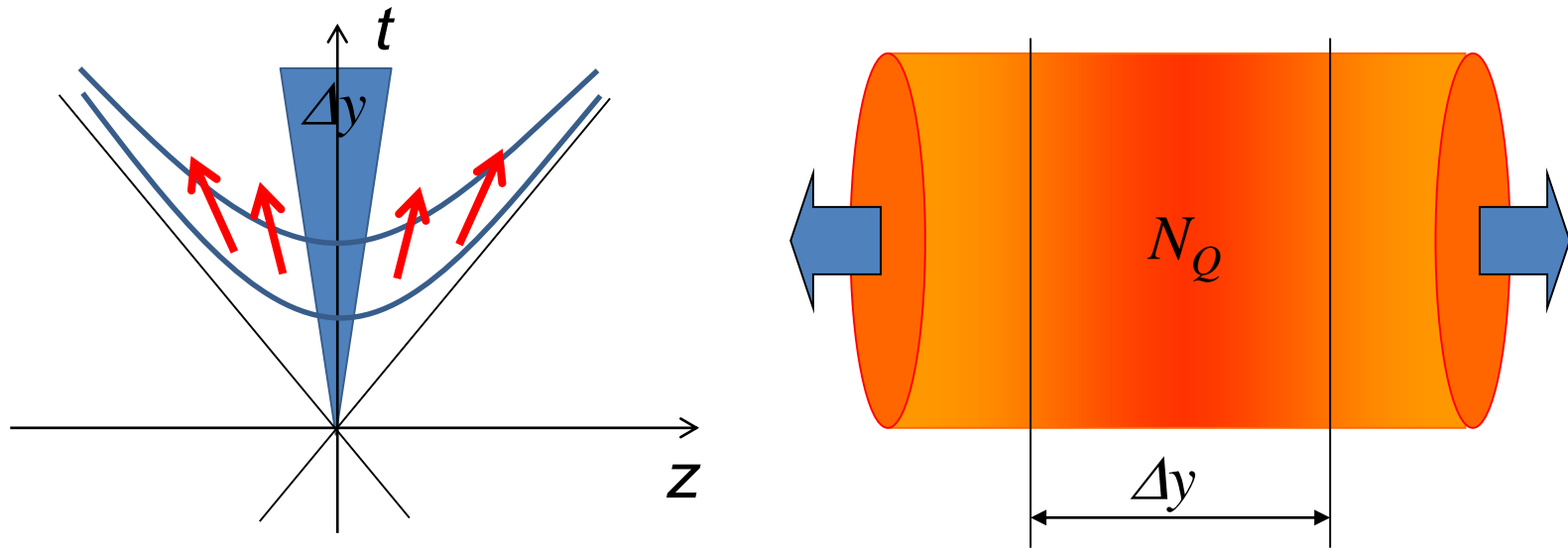
$\Delta\eta$

↑

rapidity window



Time Evolution of CC



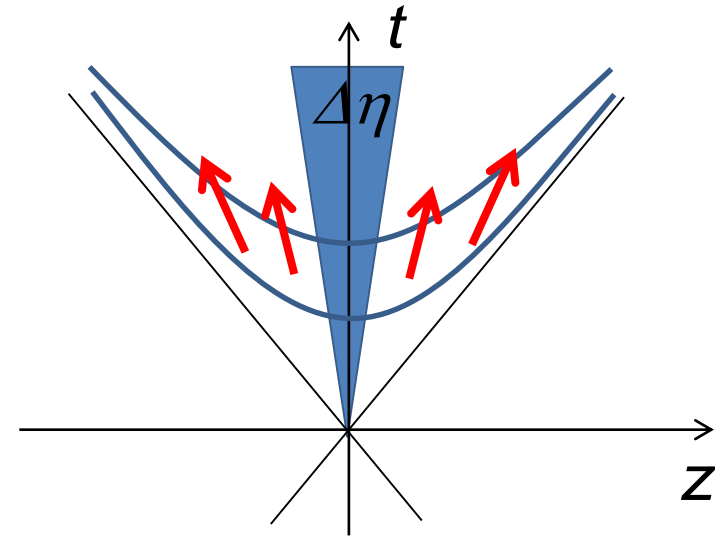
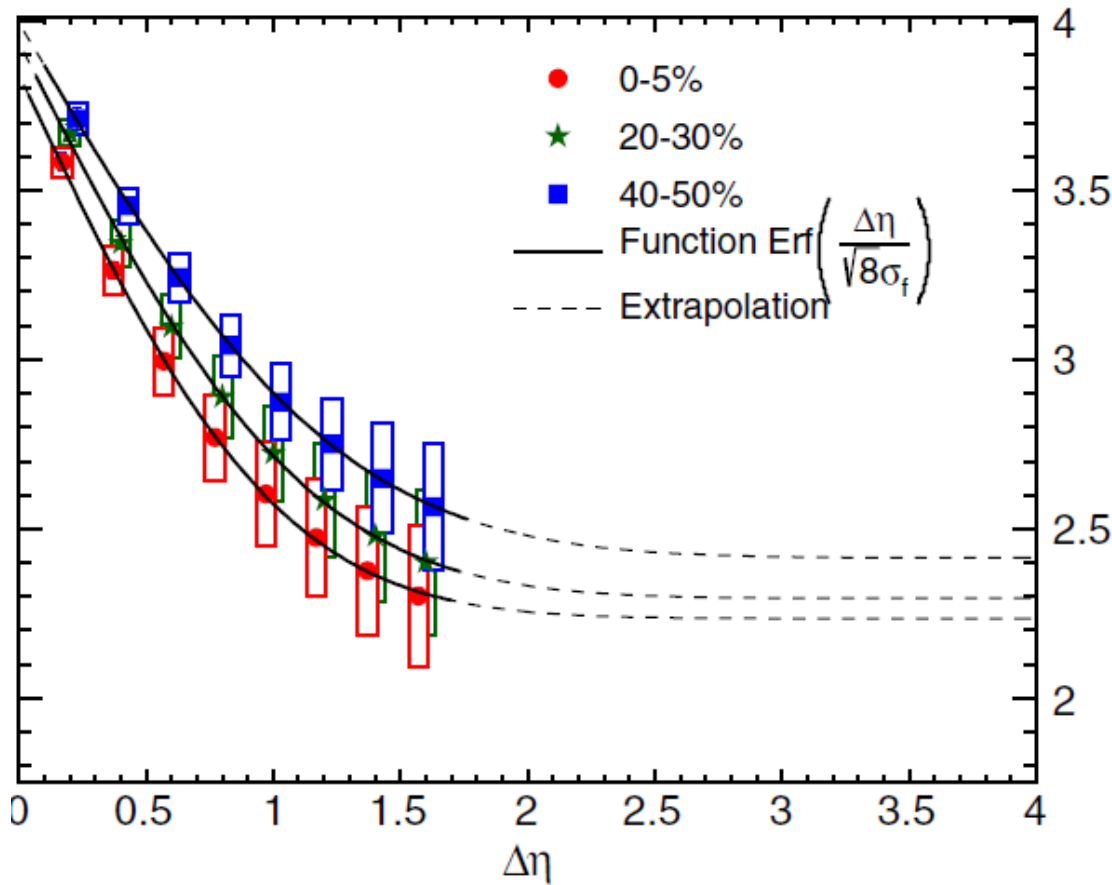
Variation of a conserved charge in Δy is achieved only through diffusion.



The larger Δy , the slower diffusion

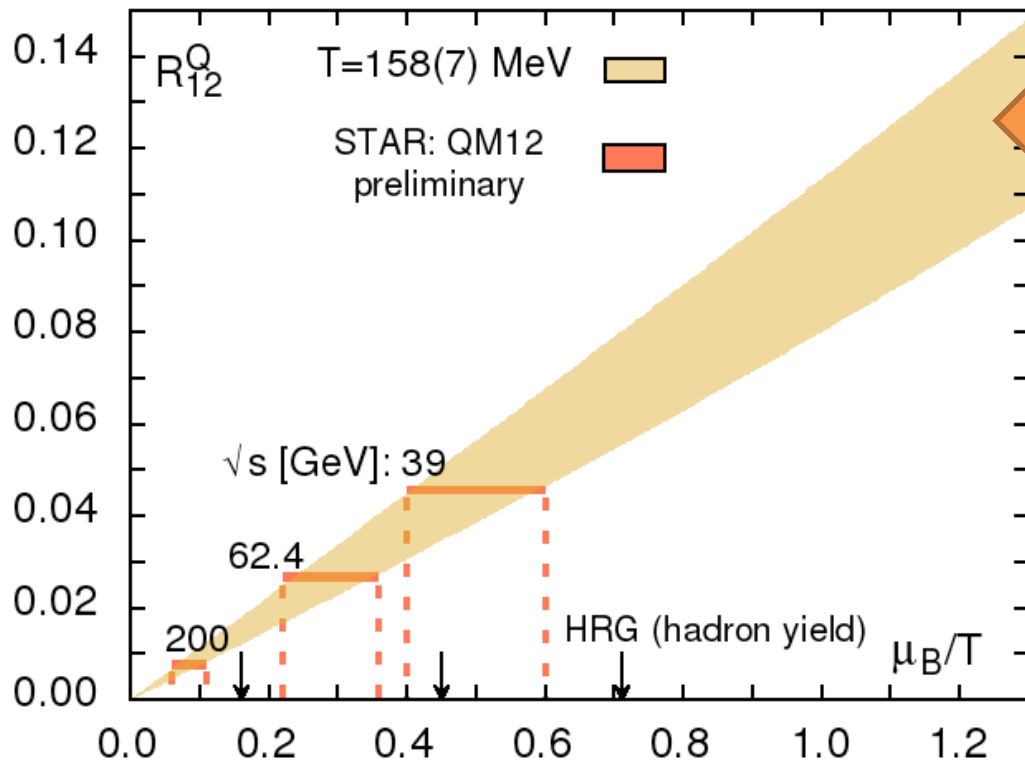
$\Delta\eta$ Dependence @ ALICE

ALICE
PRL 2013



$\Delta\eta$ dependences of fluctuation observables
encode history of the hot medium!

Cumulants : HIC vs Lattice



格子QCDで得られた
ゆらぎ- μ/T 関係線

実験の観測値 + 格子



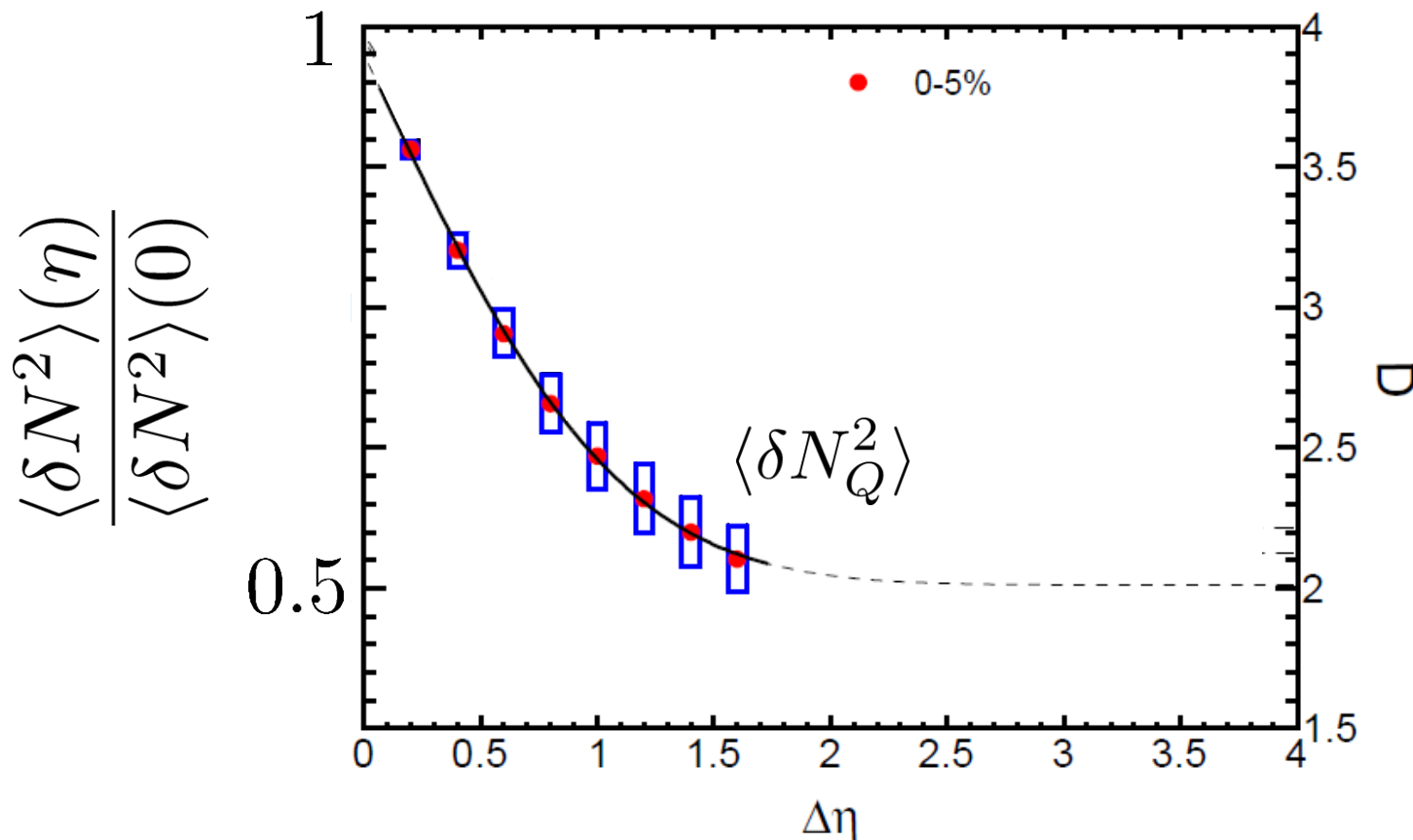
化学凍結

不一致

$\langle \delta N_B^2 \rangle$ and $\langle \delta N_p^2 \rangle$ @ LHC ?

$$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$$

should have different $\Delta\eta$ dependence.

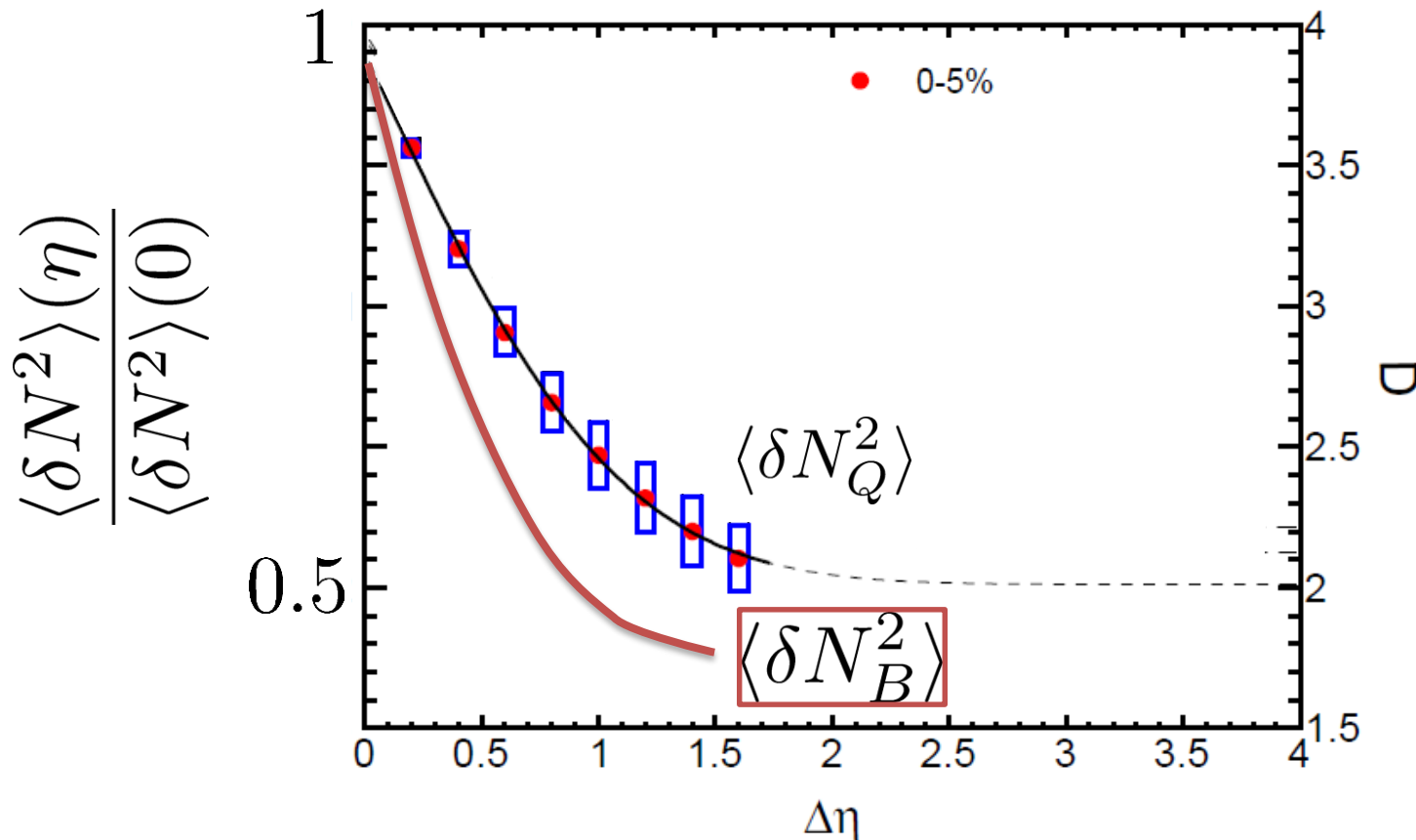


Baryon # cumulants are experimentally observable! [MK, Asakawa, 2011;2012](#)

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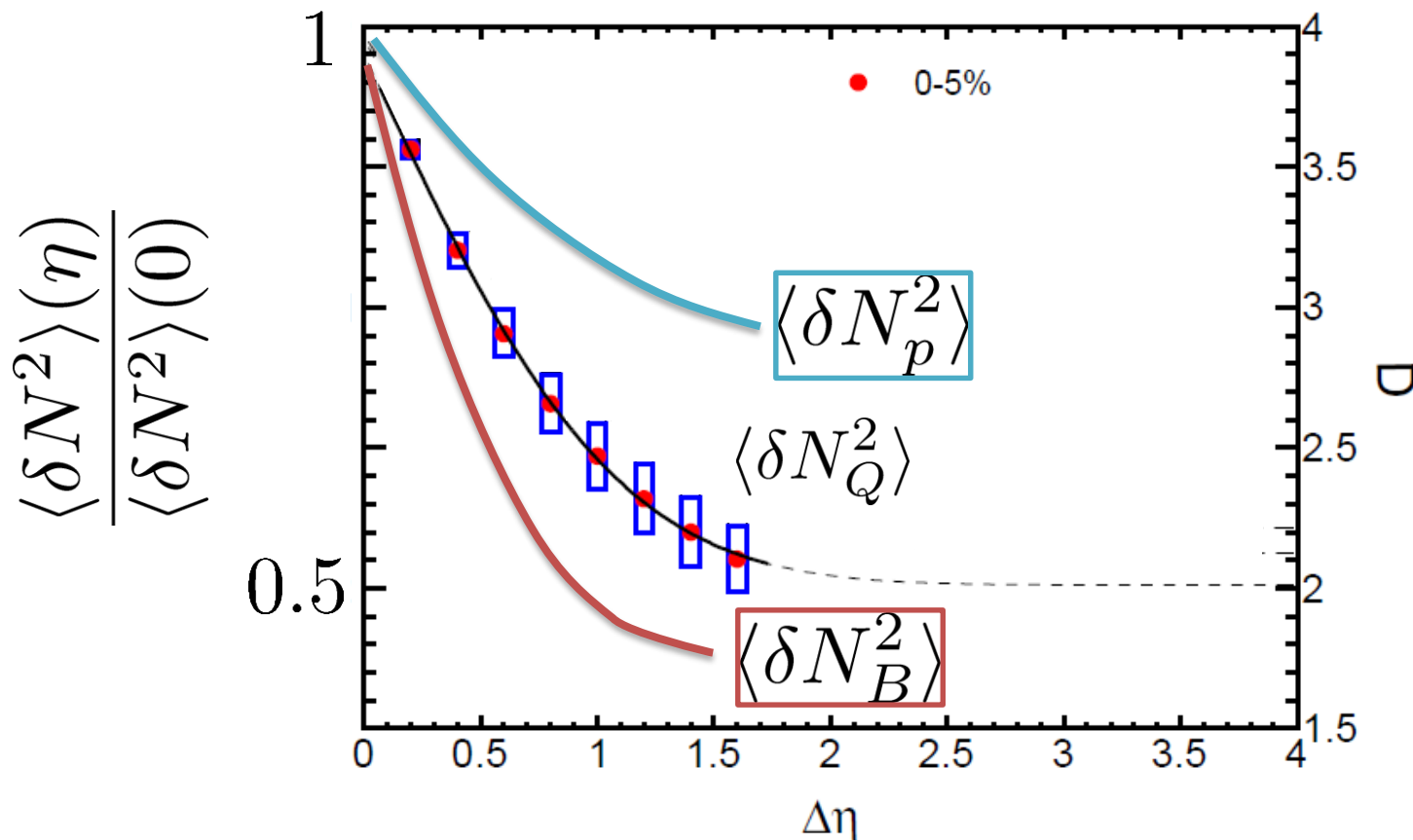


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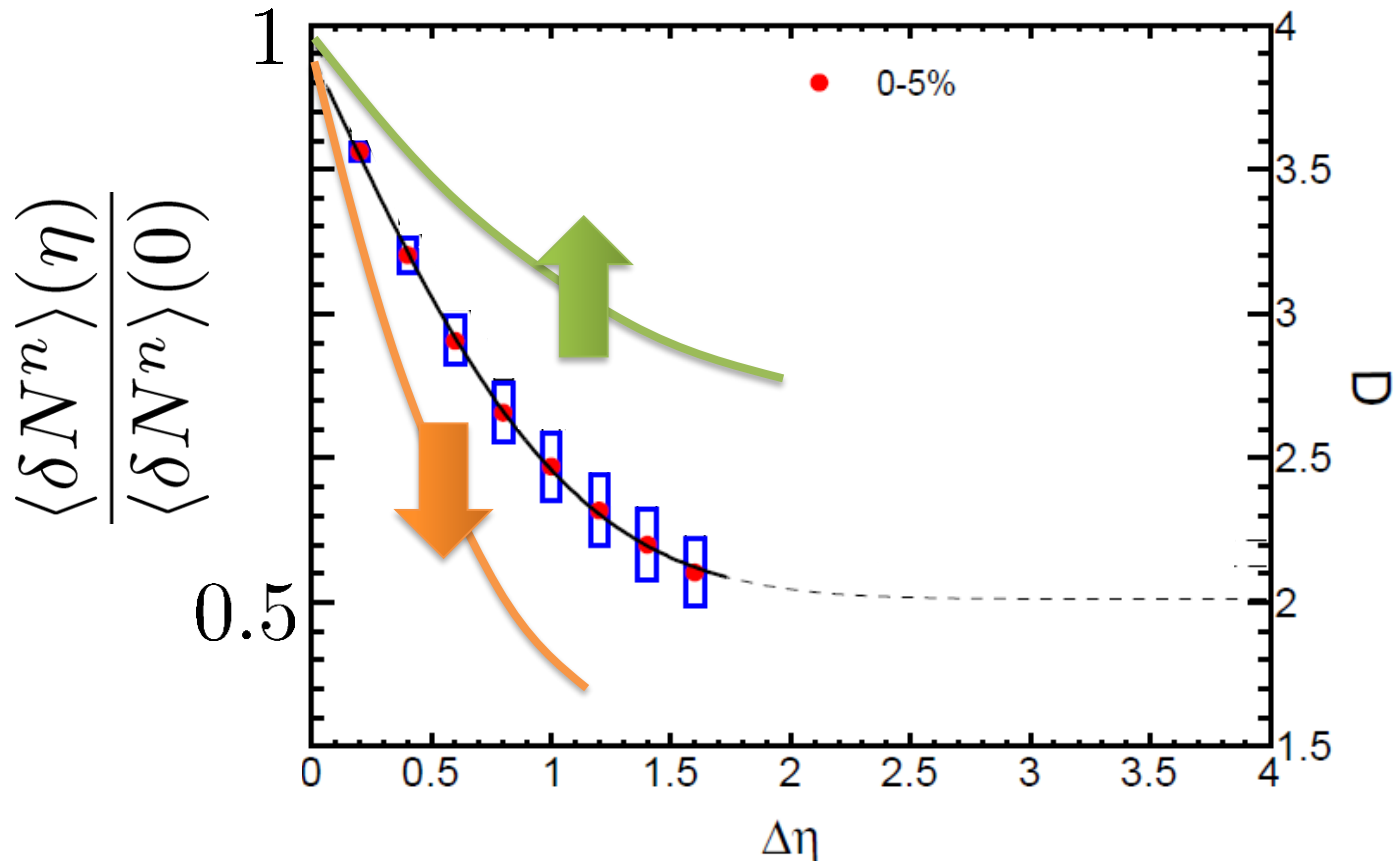
$\langle \delta N_Q^4 \rangle$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

suppression

or

enhancement



Three “NON”s

Physics of non-Gaussianity in heavy-ion collision is a **particular** problem!

□ **Non-Gaussian**

Non-Gaussianity is irrelevant in large systems

□ **Non-critical**

Critical enhancement is not observed in HIC so far

□ **Non-equilibrium**

Fluctuations are not equilibrated in HIC

Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechanics II
Kapusta, Muller, Stephanov, 2012
Stephanov, Shuryak, 2001

Stochastic diffusion equation

$$\partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$



Gaussian fluctuation
in equilibrium

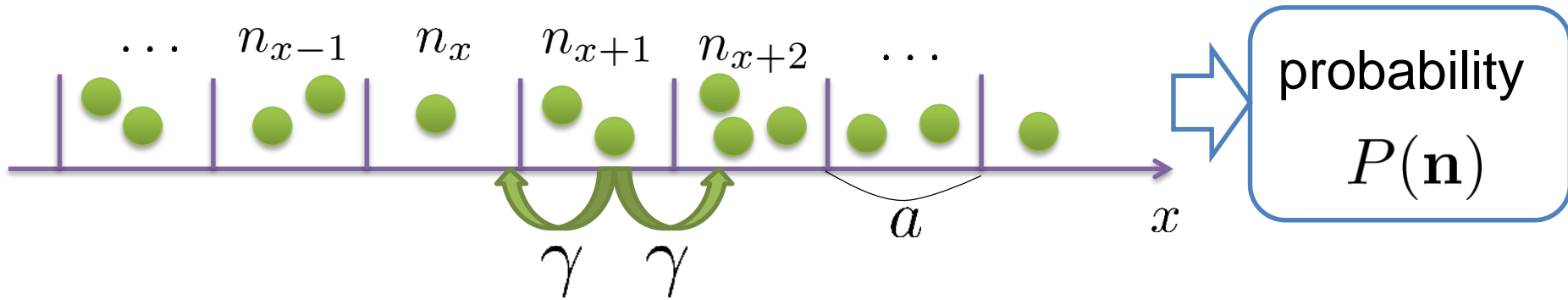
Markov (temporary local)
+
continuity



Gaussian

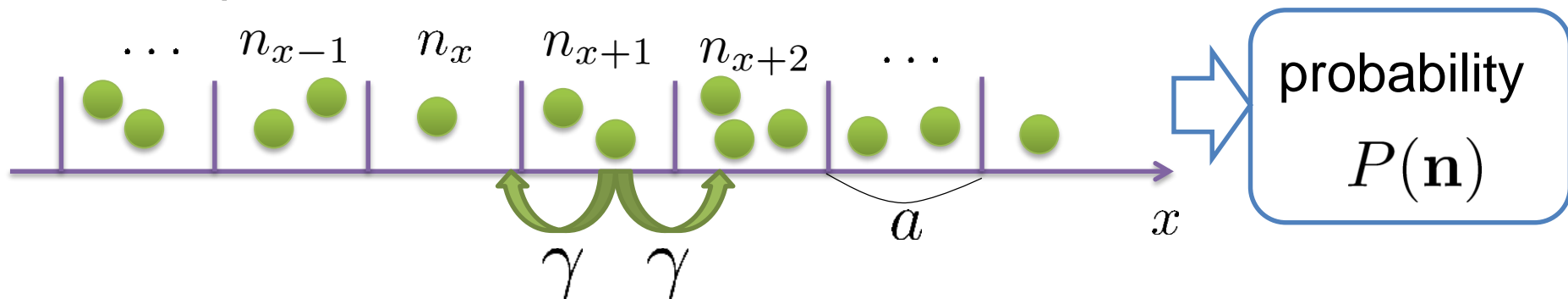
Diffusion Master Equation

Divide spatial coordinate into discrete cells



Diffusion Master Equation

Divide spatial coordinate into discrete cells



Master Equation for $P(n)$

$$\frac{\partial}{\partial t} P(\mathbf{n}) = \gamma \sum_x [(n_x + 1) \{P(\mathbf{n} + \mathbf{e}_x - \mathbf{e}_{x+1}) + P(\mathbf{n} + \mathbf{e}_x - \mathbf{e}_{x-1})\} - 2n_x P(\mathbf{n})]$$

Solve the DME **exactly**, and take $a \rightarrow 0$ limit

No approx., ex. van Kampen's system size expansion


Solution of DME

1st

$$\langle \tilde{n}_k \rangle(t) = e^{-\omega_k t} \langle \tilde{n}_k \rangle_0$$

$$\omega_k \simeq \gamma a^2 k^2$$

 initial

 Deterministic part \leftrightarrow diffusion equation at long wave length ($1/a \ll k$)

$$\partial_t \langle n_x(t) \rangle = \gamma a^2 \partial_x^2 \langle n_x(t) \rangle$$

 Appropriate continuum limit with $\gamma a^2 = D$


Solution of DME

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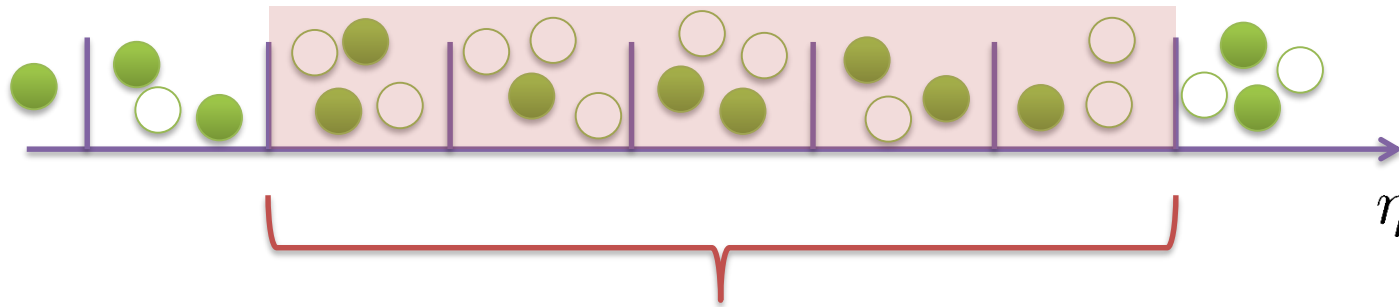
2nd

$$\langle \delta \tilde{n}_{k_1} \delta \tilde{n}_{k_2} \rangle(t) = \langle \tilde{n}_{k_1+k_2} \rangle_0 (e^{-\omega_{k_1+k_2} t} - e^{-(\omega_{k_1} + \omega_{k_2}) t}) + \langle \delta \tilde{n}_{k_1} \delta \tilde{n}_{k_2} \rangle_0 e^{-(\omega_{k_1} + \omega_{k_2}) t}$$

 Consistent with stochastic diffusion eq. (for smooth initial condition)

Net Charge Number

Prepare 2 species of (non-interacting) particles



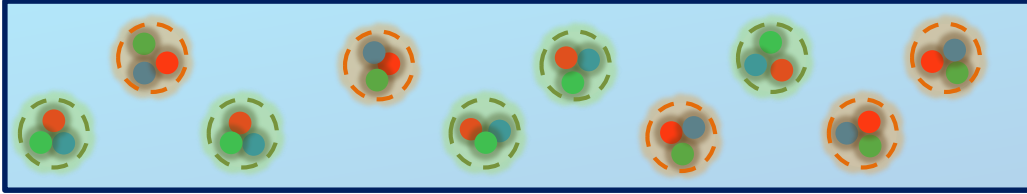
$$\bar{Q}(\tau) = \int_0^{\Delta\eta} d\eta (n_1(\eta, \tau) - n_2(\eta, \tau))$$

Let us investigate

$$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c \quad \text{at freezeout time } t$$

Time Evolution in Hadronic Phase

Hadronization (initial condition)



- Boost invariance / infinitely long system
- Local equilibration / local correlation

$$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$$

$$\langle \bar{Q}^2 Q_{(\text{tot})} \rangle_c$$

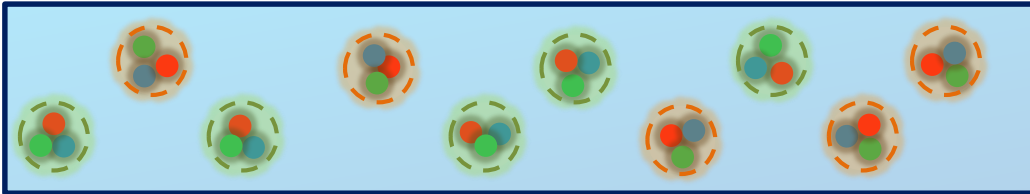
$$\langle Q_{(\text{tot})}^2 \rangle_c$$

suppression owing to
local charge conservation

strongly dependent on
hadronization mechanism

Time Evolution in Hadronic Phase

Hadronization (initial condition)



Time evolution via DME

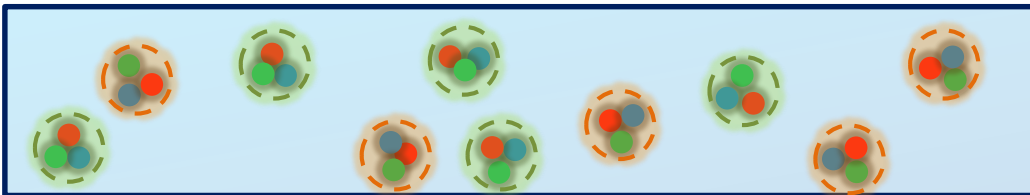
- Boost invariance / infinitely long system
- Local equilibration / local correlation

$$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c \quad \langle \bar{Q}^2 Q_{(\text{tot})} \rangle_c \quad \langle Q_{(\text{tot})}^2 \rangle_c$$

suppression owing to local charge conservation

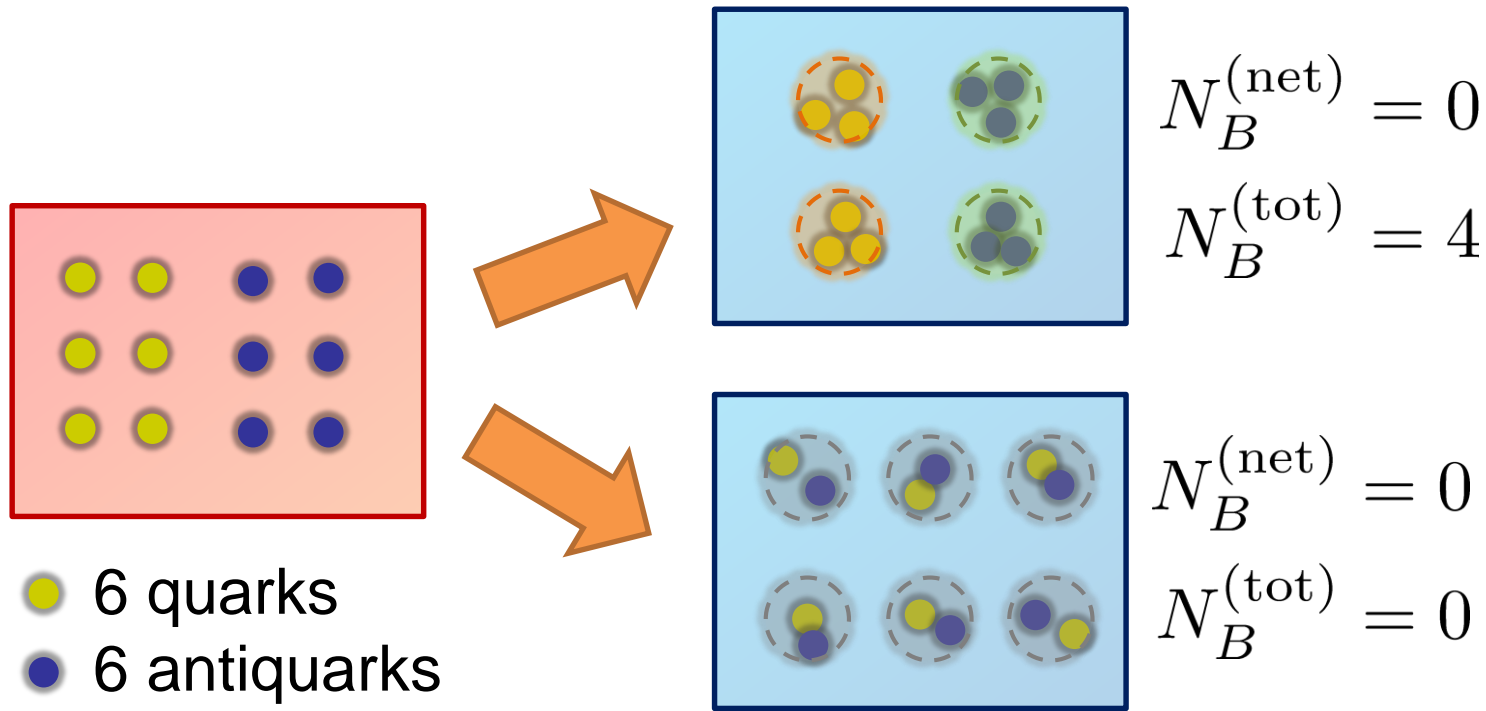
strongly dependent on hadronization mechanism

Freezeout



Total Charge Number

In recombination model,

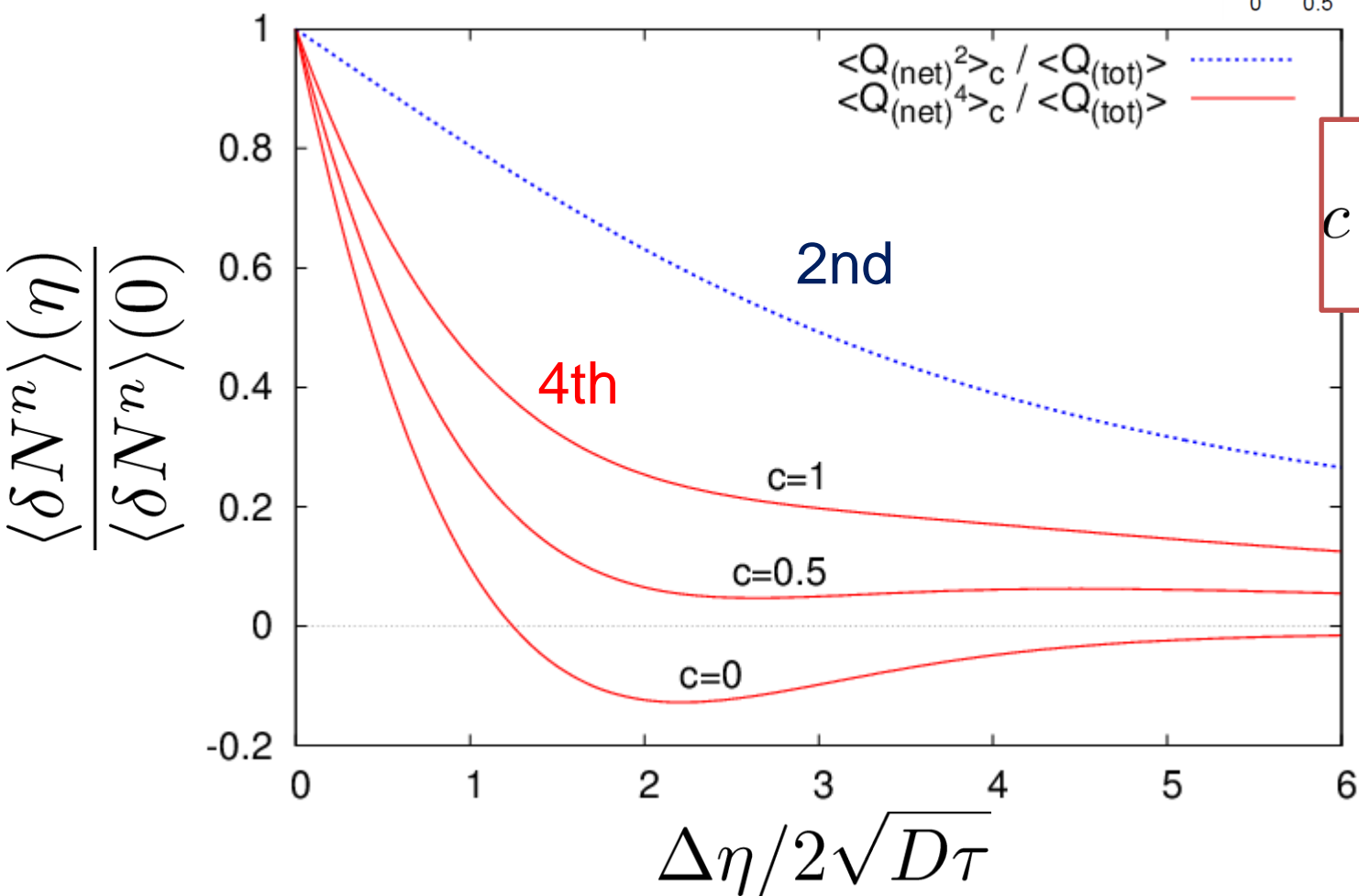
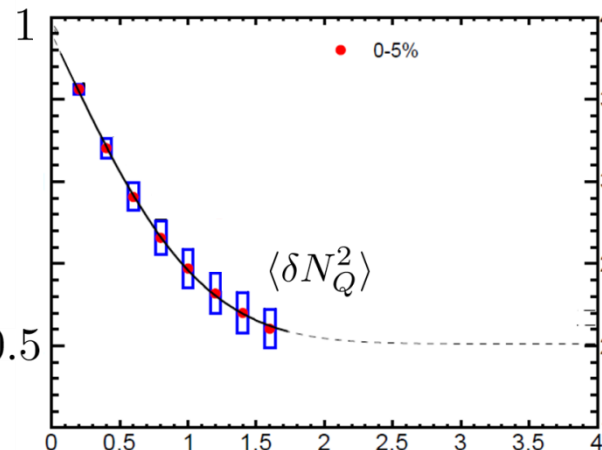


□ $N_B^{(\text{tot})}$ can fluctuate, while $N_B^{(\text{net})}$ does not.

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\text{tot})} \rangle_c = 0$$



$$c = \frac{\langle Q_{(\text{tot})}^2 \rangle_c}{\langle Q_{(\text{tot})} \rangle}$$

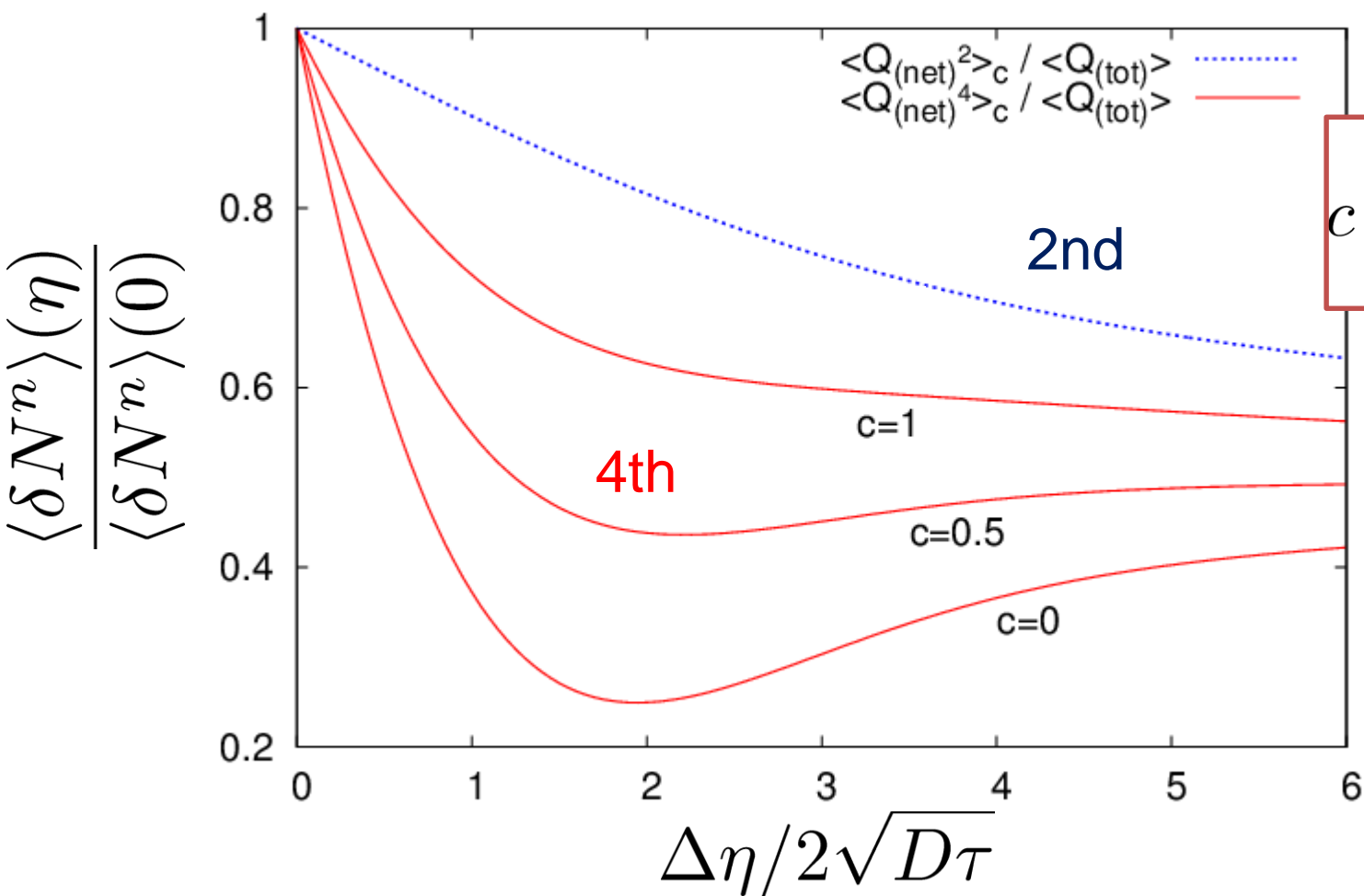


parameter
sensitive to
hadronization

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\text{tot})} \rangle_c = 0.5 \langle Q_{(\text{tot})} \rangle$$



$$c = \frac{\langle Q_{(\text{tot})}^2 \rangle_c}{\langle Q_{(\text{tot})} \rangle}$$

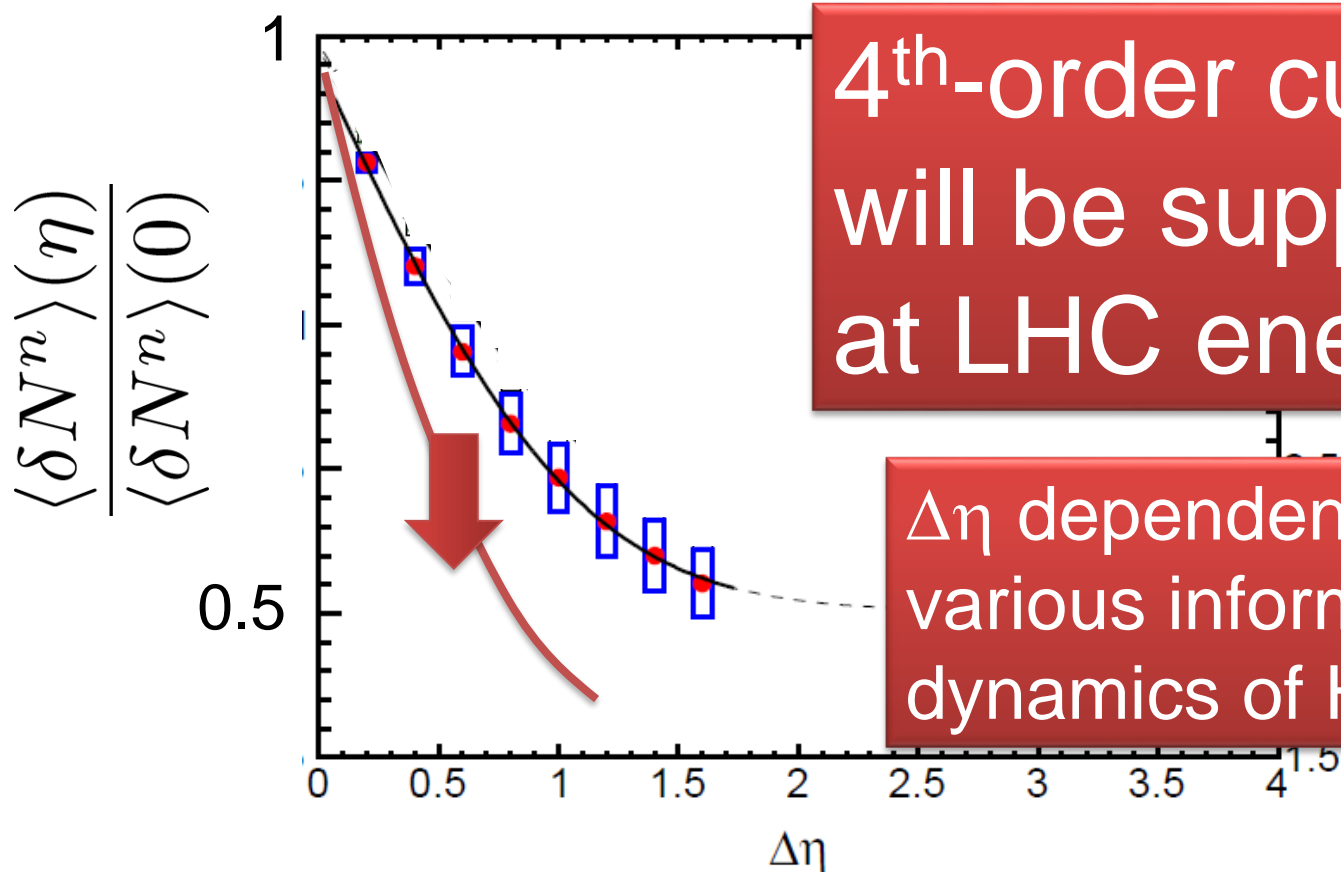


parameter sensitive to hadronization

$\langle \delta N_Q^4 \rangle @ \text{LHC}$

Assumptions

- boost invariant system
- small fluctuations of CC at hadronization
- short correlation in hadronic stage

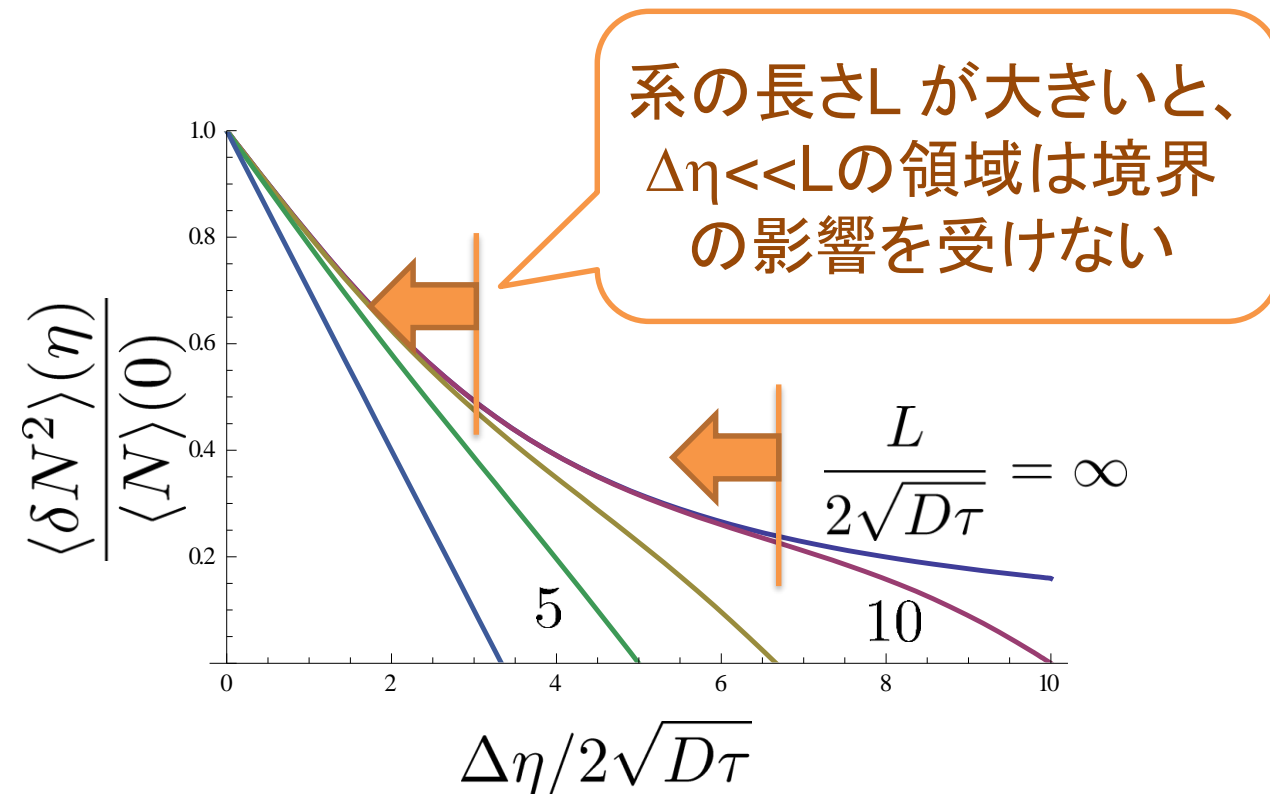


4th-order cumulant will be suppressed at LHC energy!

$\Delta\eta$ dependences encode various information on the dynamics of HIC!

HICで作られたQGPは有限系

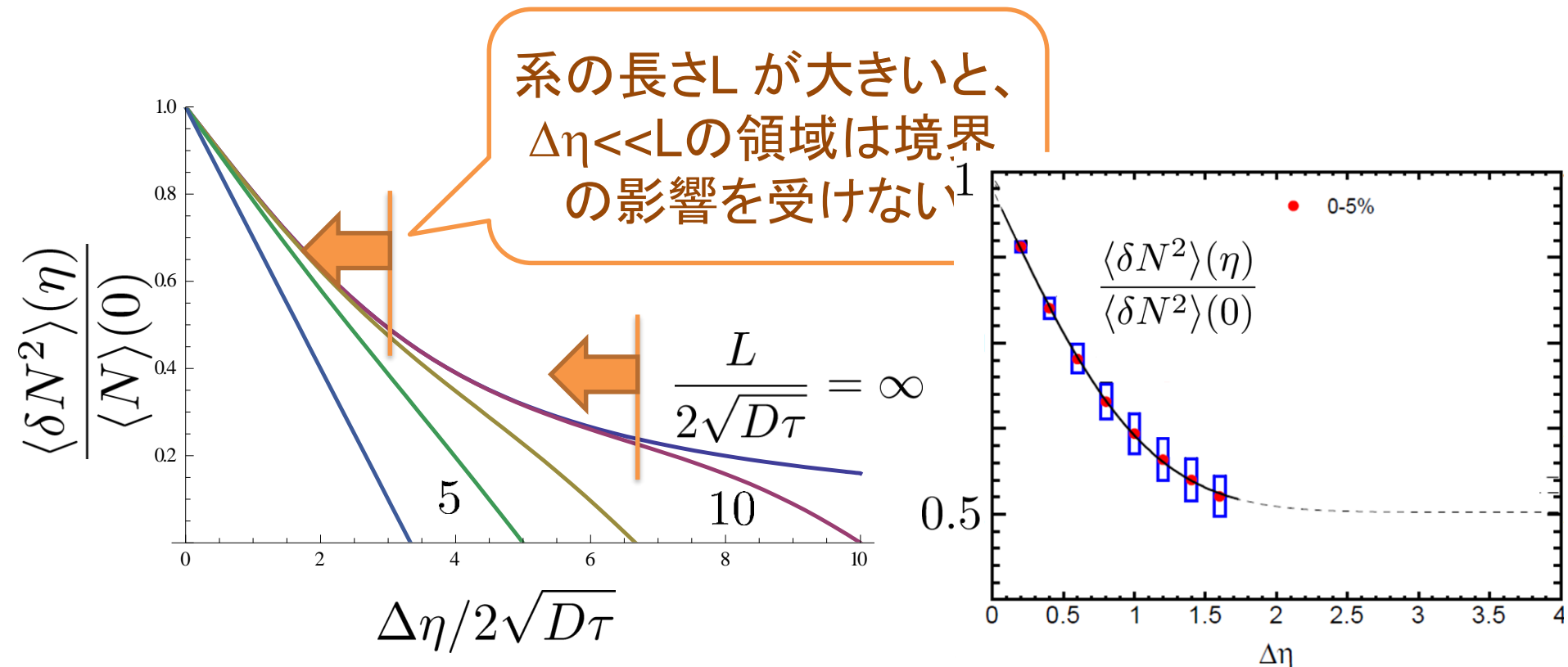
➡ 有限長さLの系でゆらぎの時間発展を解く



- 有限体積効果は、 $\Delta\eta$ 依存性から読み取ることができる
- ALICEの結果は、有限体積効果が寄与していないことを示唆

HICで作られたQGPは有限系

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Summary

Plenty of physics in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2 \rangle_c, \langle N_B^2 \rangle_c, \langle N_Q^4 \rangle_c, \langle N_B^4 \rangle_c, \\ \langle N_{ch}^2 \rangle_c, \dots$$

Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC

- history of hot medium
- mechanism of hadronization
- diffusion constant

Summary

Plenty of physics in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2 \rangle_c, \langle N_B^2 \rangle_c, \langle N_Q^4 \rangle_c, \langle N_B^4 \rangle_c, \\ \langle N_{ch}^2 \rangle_c, \dots$$

Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC

- history of hot medium
- mechanism of hadronization
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Search of QCD Phase Structure

Open Questions & Future Work

- Why the primordial fluctuations are observed only at LHC, and not RHIC ?
- Extract more information on each stage of fireballs using fluctuations

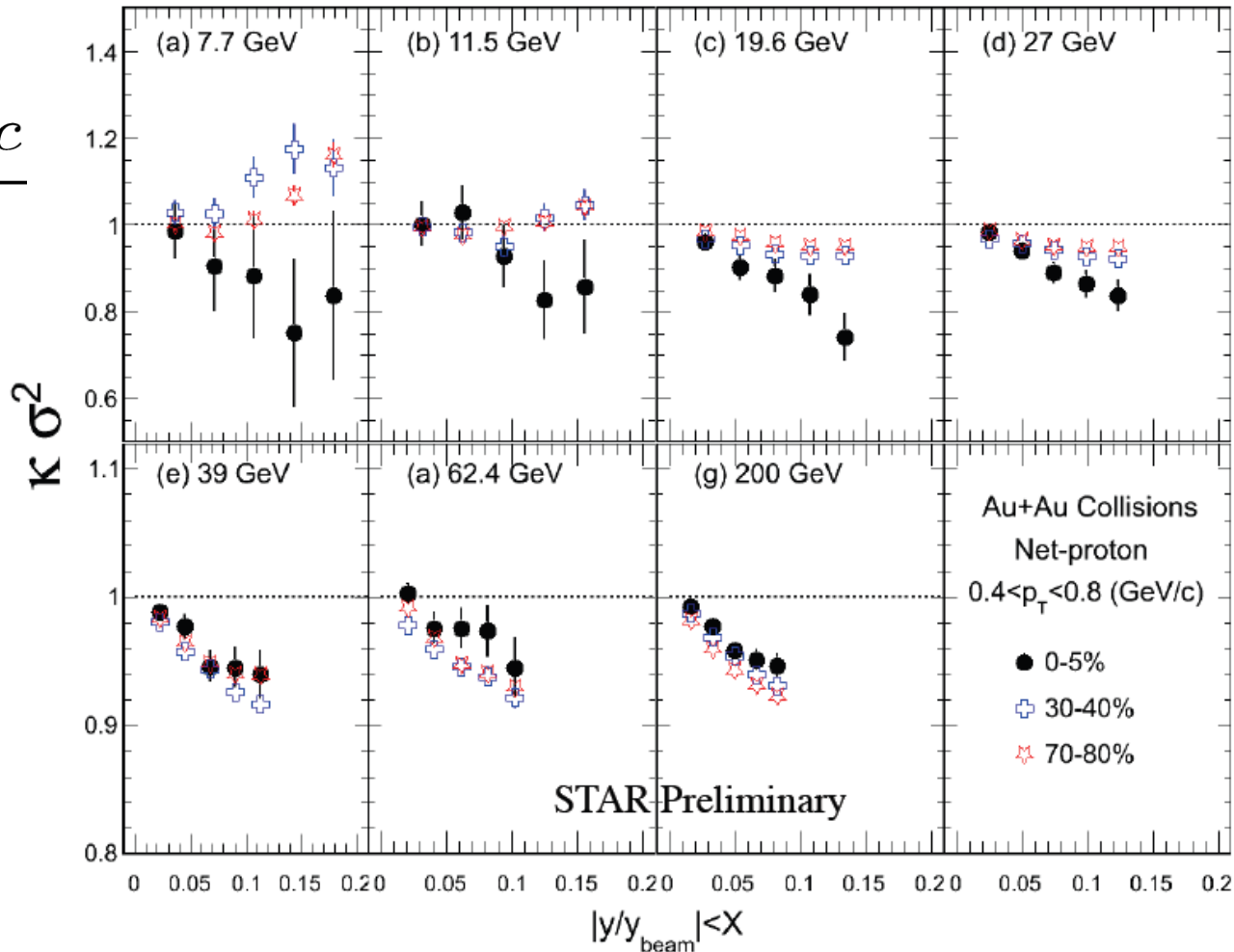
- Model refinement
 - Including the effects of
nonzero correlation length / relaxation time
global charge conservation

 - Non Poissonian system ← interaction of particles

$\Delta\eta$ Dependence at STAR

STAR, QM2012

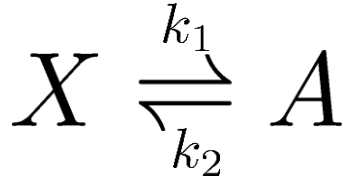
$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$



$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$

decreases as $\Delta\eta$ becomes larger at RHIC energy.

Chemical Reaction 1



x: # of X

a: # of A (**fixed**)

Master eq.:
$$\frac{\partial}{\partial t} P(x, t) = k_2 a P(x - 1, t) + k_1 (x + 1) P(x + 1, t) - (k_1 x + k_2 a) P(x, t)$$



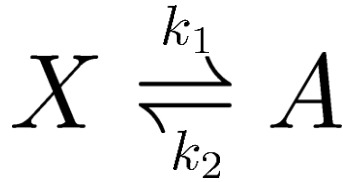
Cumulants with fixed initial condition $P(x, 0) = \delta_{x, N_0}$

$$\langle x(t) \rangle = N_0 e^{-k_1 t} + N_{eq} (1 - e^{-k_1 t})$$

$$\langle \delta x(t)^2 \rangle = N_0 (e^{-k_1 t} - e^{-2k_1 t}) + N_{eq} (1 - e^{-k_1 t})$$

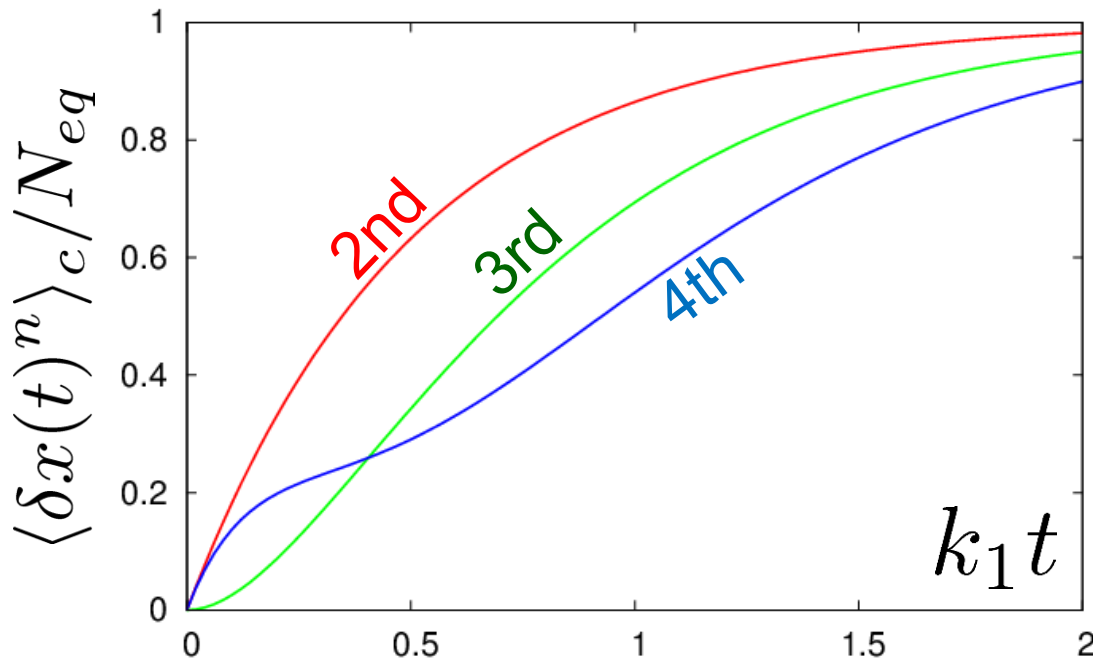
$$\langle \delta x(t)^3 \rangle = \underbrace{N_0 (e^{-k_1 t} - 3e^{-2k_1 t} + 2e^{-3k_1 t})}_{\text{initial}} + \underbrace{N_{eq} (1 - e^{-k_1 t})}_{\text{equilibrium}}$$

Chemical Reaction 2



$$N_0 = N_{eq}$$

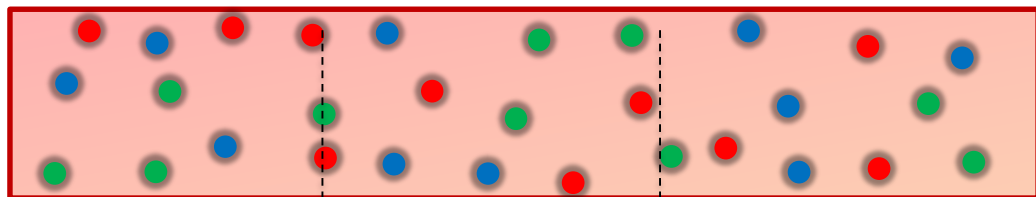
$$\begin{cases} \langle x(t) \rangle = N_{eq} \\ \langle \delta x(t)^2 \rangle = N_{eq}(1 - e^{-2k_1 t}) \\ \langle \delta x(t)^3 \rangle = N_{eq}(1 - 3e^{-2k_1 t} + 2e^{-3k_1 t}) \end{cases}$$



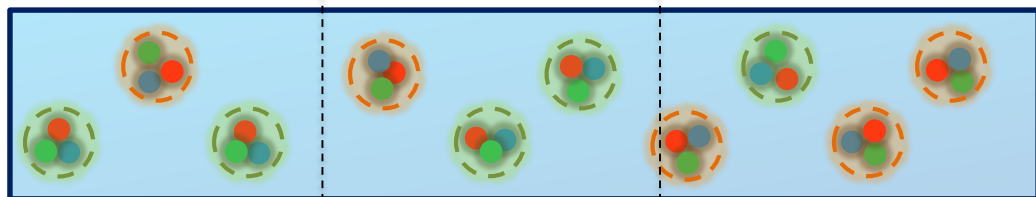
Higher-order
cumulants
grow slower.

Time Evolution in HIC

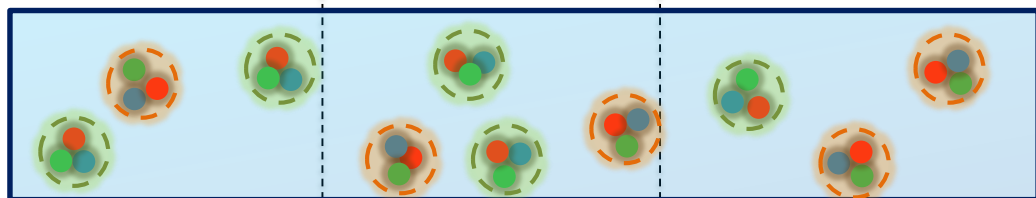
Quark-Gluon Plasma



Hadronization

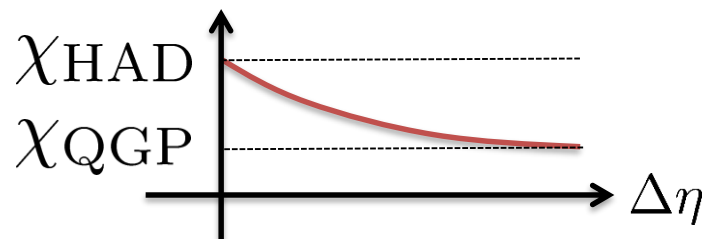
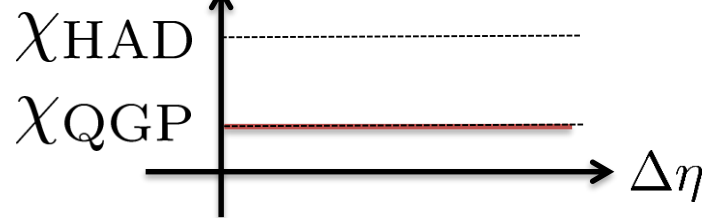


Freezeout



$\Delta\eta$

$$\frac{\langle \Delta N^2 \rangle}{\Delta\eta}$$



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechanics II
Kapusta, Muller, Stephanov, 2012

Diffusion equation

$$\partial_\tau n = D \partial_\eta^2 n$$



Stochastic diffusion equation

$$\partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$



Stochastic Force

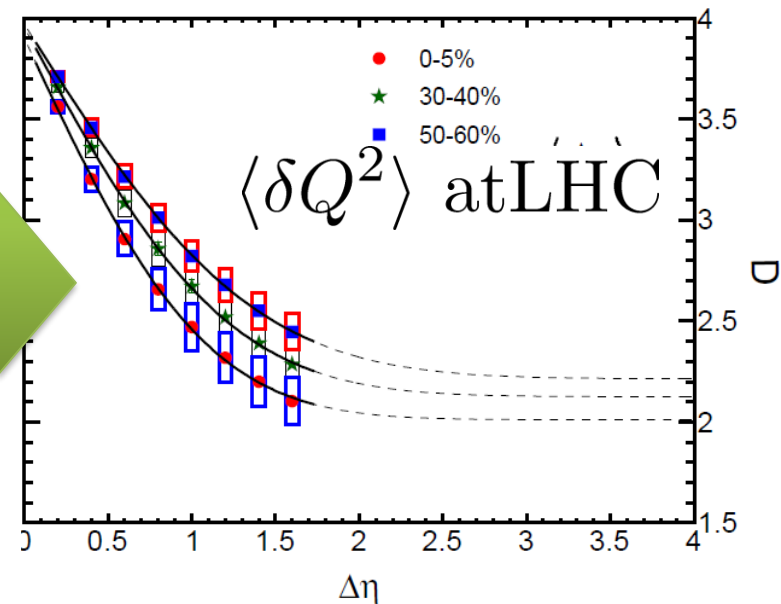
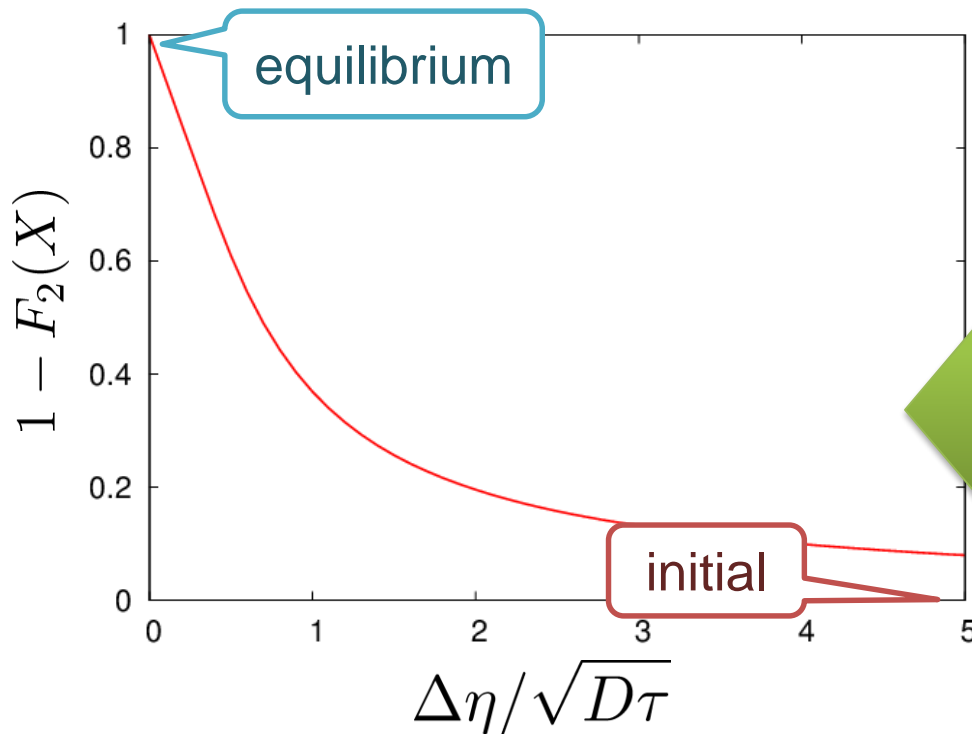
determined by fluctuation-dissipation relation

$\Delta\eta$ Dependence

Shuryak, Stephanov, 2001

- Initial condition: $\langle \delta n(\eta_1, 0) \delta n(\eta_2, 0) \rangle = \sigma_2 \delta(\eta_1 - \eta_2)$
- Translational invariance

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta, \tau) \quad \rightarrow \quad \langle \delta Q(\tau)^2 \rangle = \underbrace{\sigma_2 F_2(X)}_{\text{initial}} + \underbrace{\chi_2(1 - F_2(X))}_{\text{equilibrium}}$$



Non-Gaussianity in Fluctuating Hydro?

It is **impossible** to directly extend the theory of hydro fluctuations to treat higher orders.

□ No a priori extension of FD relations to higher orders

□ **Theorem**

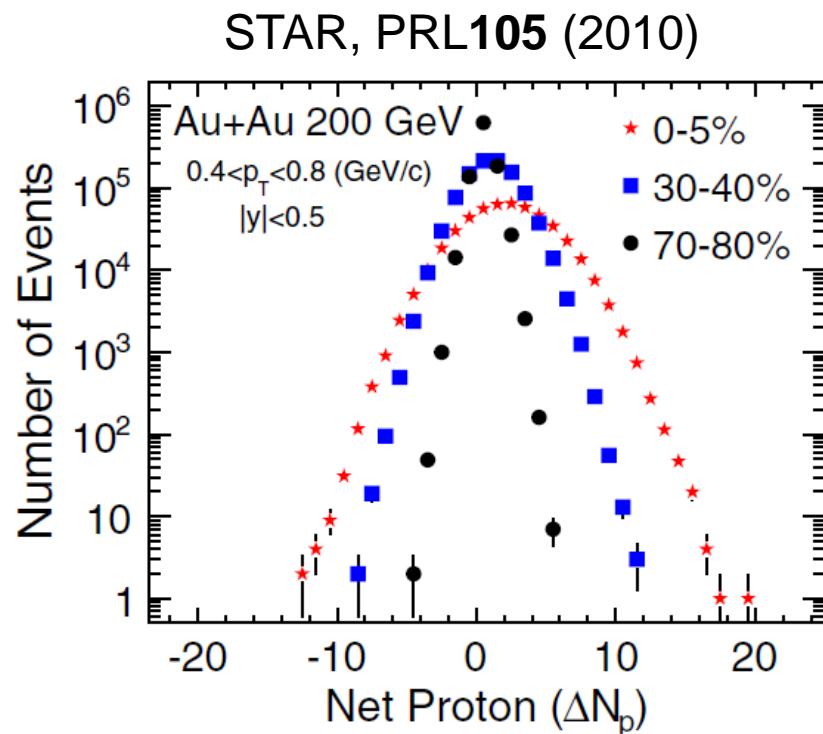
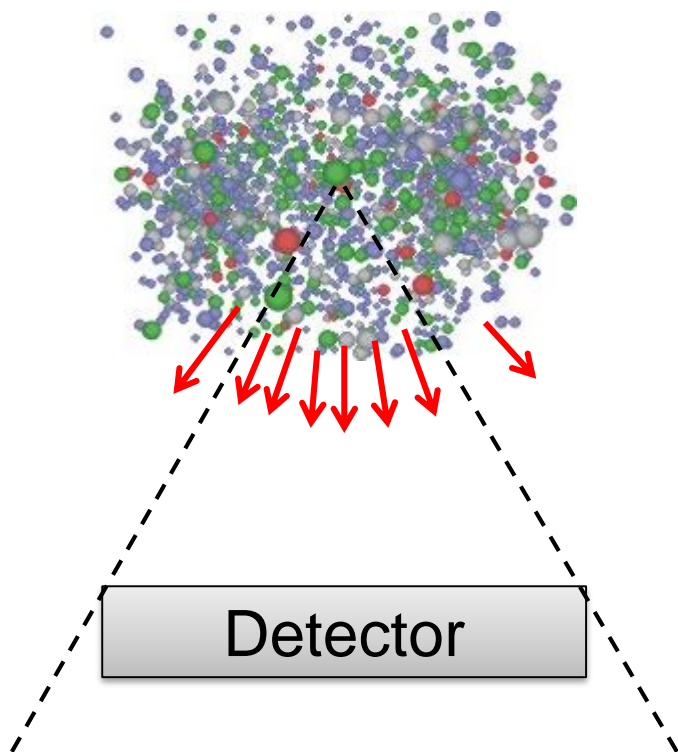
Markov process + continuous variable

→ Gaussian random force

cf) Gardiner, "Stochastic Methods"

Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in HIC.



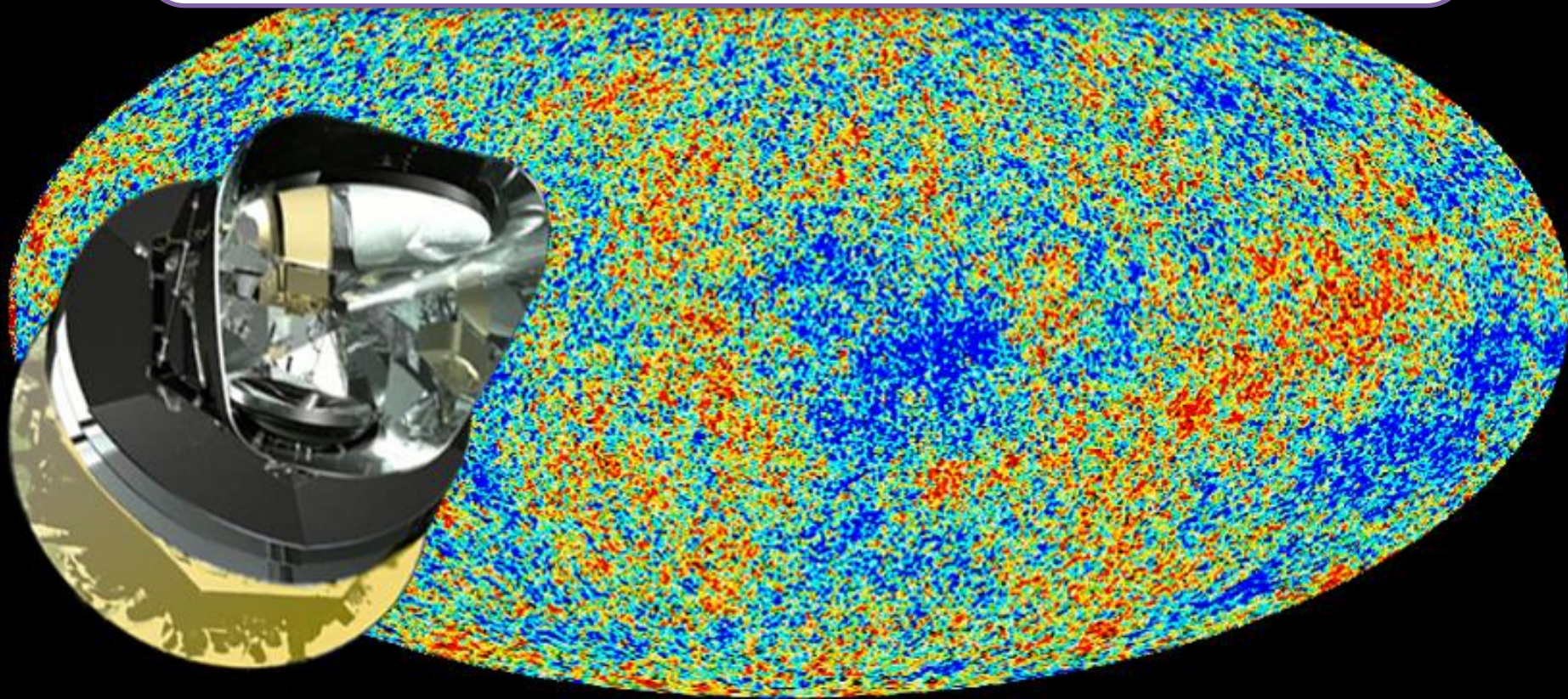
$$\langle \delta N^2 \rangle, \langle \delta N^3 \rangle, \langle \delta N^4 \rangle_c, \dots$$

Non-Gaussianity

fluctuations (correlations)

$$\langle \delta n_1 \delta n_2 \rangle, \langle \delta n_1 \delta n_2 \delta n_3 \rangle, \langle \delta n_1 \delta n_2 \delta n_3 \delta n_4 \rangle_c, \dots$$

→ Non-Gaussianity



PLANCK : statistics insufficient to see non-Gaussianity...(2013)