重イオン衝突における 保存電荷ゆらぎの ダイナミクス

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Beam-Energy Scan





Observables in equilibrium are fluctuating.







Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.



Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.



Non-Gaussianity in CMB

fluctuations (correlations)

$\langle \delta n_1 \delta n_2 \rangle, \langle \delta n_1 \delta n_2 \delta n_3 \rangle, \langle \delta n_1 \delta n_2 \delta n_3 \delta n_4 \rangle_c, \cdots$ Non-Gaussianity



PLANCK : statistics insufficient to see non-Gaussianity...(2013)

 Fluctuations reflect properties of matter.
Enhancement near the critical point Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);...
Ratios between cumulants of conserved charges Asakawa,Heintz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)
Signs of higher order cumulants Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)



Conserved Charges : Theoretical Advantage



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Simple thermodynamic relations

$$\left< \delta N_c^n \right> = \frac{1}{V T^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}$$

 Intuitive interpretation for the behaviors of cumulants

ex:
$$\langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B}$$



観測にかかるゆらぎは、いつ形成されたのか?

ゆらぎのダイナミクス(動的振る舞い)の議論が必要

保存電荷の場合



境界を通過する電荷 のみが変化に寄与



非保存電荷の場合



体積内の任意の場所で 電荷が変化できる $\tau \rightarrow \text{const.}$

for $V \to \infty$

観測にかかるゆらぎは、いつ形成されたのか?

*∆η*内の保存電荷量は、初期段階の ものが終状態まで生き残ることが期 待できる。

Asakawa, Heinz, Muller, '00 Jeon, Koch, '00 Shuryak, Stephanov, '02



Note: STAR - $\begin{bmatrix} -0.5 < \eta < 0.5 \\ 0.4 < p < 0.8[GeV] \end{bmatrix}$



Free Boltzmann → Poisson
$$\langle \delta N^n \rangle_c = \langle N \rangle$$



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$
$$\Longrightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$



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$$3N_B = N_q$$



Free Boltzmann → Poisson $\langle \delta N^n \rangle_c = \langle N \rangle$



Proton # Fluctuations @ STAR-BES



$$S\sigma = \frac{\langle (\delta N_p^{(\text{net})})^3 \rangle}{\langle (\delta N_p^{(\text{net})})^2 \rangle}, \quad \kappa \sigma^2 = \frac{\langle (\delta N_p^{(\text{net})})^4 \rangle_c}{\langle (\delta N_p^{(\text{net})})^2 \rangle}$$

Proton # Fluctuations @ STAR-BES



Proton # Fluctuations @ STAR-BES



Proton # Cumulants @ STAR-BES



Proton # Cumulants @ STAR-BES



Charge Fluctuations @ STAR-BES



Charge Fluctuation @ LHC



 $\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

$\Delta\eta$ Dependence @ ALICE



Time Evolution of CC



Variation of a conserved charge in Δy is achieved only through diffusion.

The larger Δy , the slower diffusion

Dissipation of a Conserved Charge



Dissipation of a Conserved Charge





Time Evolution in HIC







$\Delta\eta$ Dependence @ ALICE



ゆらぎのΔη 依存性には、高温物質の 時間発展の歴史が刻まれている!

Cumulants : HIC vs Lattice



Cumulants as Probes of the Medium





T<150MeVでの格子データは、HRG描像と矛盾しない

Time Evolution of Higher Order Cumulants

MK, Asakawa, Ono, arXiv:1307.2978 Sakaida, Asakawa, MK, in progress

$\Delta\eta$ Dependence @ ALICE



ゆらぎのΔη 依存性には、高温物質の 時間発展の歴史が刻まれている!
$<\delta N_{\rm B}^2>$ and $<\delta N_{\rm p}^2>$ @ LHC ?

 $\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$ should have different $\Delta \eta$ dependence.

0-5% 3.5 $\frac{\langle \delta N^2 \rangle(\eta)}{\langle \delta N^2 \rangle(0)}$ δN_O^2 2.5 0.5**-**1.5 2.5 3.5 0.5 1.5 2 3 0 Δŋ $\langle (\delta N_p^{(\text{net})})^2 \rangle = \frac{1}{4} \langle (\delta N_{\text{B}}^{(\text{net})})^2 \rangle + \frac{1}{4} \langle N_{\text{B}}^{(\text{tot})} \rangle$

 $<\delta N_{0}^{4} > @ LHC ?$



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II Kapusta, Muller, Stephanov, 2012

Diffusion equation

$$\partial_{\tau} n = D \partial_{\eta}^2 n$$

Stochastic diffusion equation

$$\partial_{\tau} n = D \partial_{\eta}^2 n + \partial_{\eta} \xi(\eta, \tau)$$

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Conservation Law Fick's Law
$$\partial_{\tau}n=-\partial_{\eta}j$$
 $j=-D\partial_{\eta}n+\xi$

Fluctuation-Dissipation Relation

$$\partial_{\tau} n = D \partial_{\eta}^2 n + \partial_{\eta} \xi(\eta, \tau)$$

Stochastic force

 \Box Local correlation $\langle \xi(\eta_1, \tau_1) \xi(\eta_2, \tau_2) \rangle \sim \delta(\eta_1 - \eta_2) \delta(\tau_1 - \tau_2)$ (hydrodynamics)

■ Equilibrium fluc.
$$\langle \delta Q(t)^2 \rangle \xrightarrow[t \to \infty]{} \chi_2 \Delta \eta$$

 $Q(\tau) = \int_0^{\Delta \eta} d\eta n(\eta, \tau)$



 $\langle \xi(k_1, \tau_1) \xi(k_2, \tau_2) \rangle = \frac{2\chi_2}{D} \delta(k_1 + k_2) \delta(\tau_1 - \tau_2)$

· susceptionity

$\Delta\eta$ Dependence

 $\square \text{ Initial condition: } \langle \delta n(\eta_1, 0) \delta n(\eta_2, 0) \rangle = \sigma_2 \delta(\eta_1 - \eta_2)$

Translational invariance



$\Delta\eta$ Dependence

Shuryak, Stephanov, 2001

□ Initial condition: $\langle \delta n(\eta_1, 0) \delta n(\eta_2, 0) \rangle = \sigma_2 \delta(\eta_1 - \eta_2)$

Translational invariance



Non-Gaussian Stochastic Force ??

$$\partial_{\tau} n = D \partial_{\eta}^2 n + \partial_{\eta} \xi(\eta, \tau)$$

Stochastif Force : 3rd order

 $\square \text{ Local correlation } \langle \xi(\eta_1, \tau_1) \xi(\eta_2, \tau_2) \xi(\eta_3, \tau_3) \rangle$ (hydrodynamics) $\sim \delta(\eta_1 - \eta_2) \delta(\eta_2 - \eta_3) \delta(\tau_1 - \tau_2) \delta(\tau_2 - \tau_3)$

 $\square \text{ Equilibrium fluc. } \langle \delta Q(t)^3 \rangle \xrightarrow[t \to \infty]{} \chi_3 \Delta \eta$

$$Q(\tau) = \int_0^{\Delta \eta} d\eta n(\eta, \tau)$$

 χ_3 : third – moment

Caution!

$$\begin{array}{c|c} \Box \ \langle \xi(k_1, \tau_1) \xi(k_2, \tau_2) \xi(k_3, \tau_3) \rangle = \frac{\chi_3}{\gamma} \frac{k_1^2 + k_2^2 + k_3^2}{k_1 k_2 k_3} \delta(k_1 + k_2 + k_3) \\ \\ \hline \text{diverge in long} \\ \text{wavelength} \end{array} \\ \begin{array}{c} \times \delta(\tau_1 - \tau_2) \delta(\tau_2 - \tau_3) \end{array} \end{array}$$

No a priori extension of FD relation to higher orders

Caution!

$$\begin{array}{c|c} \Box \ \langle \xi(k_1, \tau_1) \xi(k_2, \tau_2) \xi(k_3, \tau_3) \rangle = \frac{\chi_3}{\gamma} \frac{k_1^2 + k_2^2 + k_3^2}{k_1 k_2 k_3} \delta(k_1 + k_2 + k_3) \\ \\ \hline \text{diverge in long} \\ \text{wavelength} \end{array} \\ \begin{array}{c} \times \delta(\tau_1 - \tau_2) \delta(\tau_2 - \tau_3) \end{array} \end{array}$$

No a priori extension of FD relation to higher orders

Theorem
 Markov process + continuous variable
 Gaussian random force

cf) Gardiner, "Stochastic Methods"

Hydrodynamics Local equilibrium with many particles Gaussian due to central limit theorem

Thee "NON"s

重イオン衝突での高次ゆらぎの観測・解析は、 物理学として相当に特殊な問題である。

Non-Gaussian

通常、高次ゆらぎは観測困難。 適度に小さい系

Thee "NON"s

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■ Non-Gaussian 適度に小さい系

□ Non-critical 韻

観測されたゆらぎの値は、 自由ガスとたかだか2倍のずれ

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Non-critical
 観測されたゆらぎの値は、
 自由ガスとたかだか2倍のずれ

Non-equilibrium

平衡に至る非定常過程を記述する必要性。

Diffusion Master Equation



Diffusion Master Equation



Solve the DME **exactly**, and take $a \rightarrow 0$ limit

No approx., ex. van Kampen's system size expansion

Solution of DME



Solution of DME

1st
$$\langle \tilde{n}_k \rangle(t) = e^{-\omega_k t} \langle \tilde{n}_k \rangle_0$$
 $\omega_k \simeq \gamma a^2 k^2$
initial
Deterministic part $\leftarrow \rightarrow$ diffusion equation
at long wave length (1/a<\partial_t \langle n_x(t) \rangle = \gamma a^2 \partial_x^2 \langle n_x(t) \rangle
Appropriate continuum limit with $\gamma a^2 = D$

2nd
$$\langle \delta \tilde{n}_{k_1} \delta \tilde{n}_{k_2} \rangle(t) = \langle \tilde{n}_{k_1+k_2} \rangle_0 (e^{-\omega_{k_1+k_2}t} - e^{-(\omega_{k_1}+\omega_{k_2})t})$$

 $+ \langle \delta \tilde{n}_{k_1} \delta \tilde{n}_{k_2} \rangle_0 e^{-(\omega_{k_1}+\omega_{k_2})t}$



Consistent with stochastic diffusion eq. (for smooth initial condition)

Net Charge Number

Prepare 2 species of (non-interacting) particles



Let us investigate

 $\langle \bar{Q}^2
angle_c ~~ \langle \bar{Q}^4
angle_c$ at freezeout time t

Time Evolution in Hadronic Phase

Hadronization (initial condition)



Boost invariance / infinitely long system
 Local equilibration / local correlation



Time Evolution in Hadronic Phase

Hadronization (initial condition)





Freezeout



Total Charge Number

In recombination model,



 \square $N_B^{(\text{tot})}$ can fluctuate, while $N_B^{(\text{net})}$ does not.



$\Delta \eta$ Dependence at Freezeout



⁴> @ LHC

boost invariant system

Assumptions -

- small fluctuations of CC at hadronization
- short correlation in hadronic stage



$\Delta\eta$ Dependence at STAR

STAR, QM2012



decreases as $\Delta\eta$ becomes larger at RHIC.

坂井田、et al., in prep.

HICで作られたQGPは有限系 全系を観測すれば、保存電荷はゆらがない



Diffusion Equation w/ Boundaries

$$\partial_{\tau} n = D \partial_{\eta}^2 n + \partial_{\eta} \xi$$



拡散方程式を、境界条件付きで解く



3 Free Parameters

 $y_{
m total}$







有限体積効果は、Δη依存性から読み取ることができる

ALICEの結果には、有限体積効果はほとんど寄与しない



有限体積効果は、∆η依存性から読み取ることができる

- ALICEの結果には、有限体積効果はほとんど寄与しない
- STARでは?!?

Summary

Plenty of physics in $\Delta \eta$ dependences of various cumulants

 $\langle N_Q^2 \rangle_c, \ \langle N_B^2 \rangle_c, \ \langle N_Q^4 \rangle_c, \ \langle N_B^4 \rangle_c,$ $\langle N_{ch}^2 \rangle_c, \cdots$

Physical meanings of fluctuation obs. in experiments. Diagnosing dynamics of HIC
history of hot medium
mechanism of hadronization
diffusion constant

Summary

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Search of QCD Phase Structure

Summary

Conserved charge fluctuations are observable both in lattice simulations and heavy ion collisions. The comparison of the results in these two "experiments" will provide us many information to understand the QCD at nonzero T/m.

A lot of efforts are required both sides:
 Lattice: Higher statistics
 HIC: reconstructing baryon #, acceptance, etc.

Rapidity window dependences of cumulants in HIC are valuable tools to understand the non-thermal nature of fluctuations.

高温物質の時間発展



まとめ




Secondary Protons



Secondary Protons

