Dynamics of Non-Gaussianity in Heavy Ion Collisions

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MK, Asakawa, PRC85, 021901C(2012); PRC86, 024904(2012)
MK, Asakawa, Ono, arXiv:1307.2978

NFQCD, YITP, Kyoto, 27/Nov./2013
Beam-Energy Scan

Hadrons

Color SC
Beam-Energy Scan

**STAR 2012**

The diagram illustrates the phase diagram of hadrons and color-superconducting (Color SC) matter as a function of beam energy and temperature. The region to the left of the black line represents high energy, while the region to the right is low energy. The phase transition from hadrons to Color SC is indicated by the arrows. The graph shows different energy levels, such as 200 GeV, 39 GeV, 11.5 GeV, and 7.7 GeV, with corresponding data points indicating the phase transition.
Fluctuations

- Fluctuations reflect properties of matter.
- Enhancement near the critical point
  Stephanov, Rajagopal, Shuryak ('98); Hatta, Stephanov ('02); Stephanov ('09);…
- Ratios between cumulants of conserved charges
  Asakawa, Heinz, Muller ('00); Jeon, Koch ('00); Ejiri, Karsch, Redlich ('06)
- Signs of higher order cumulants
  Asakawa, Ejiri, MK ('09); Friman, et al. ('11); Stephanov ('11)
Definite definition for operators

- as a Noether current
- calculable on any theory

ex: on the lattice
Conserved Charges: Theoretical Advantage

- Definite definition for operators
  - as a Noether current
  - calculable on any theory
  
  ex: on the lattice

- Simple thermodynamic relations

\[
\langle \delta N_c^n \rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}
\]

- Intuitive interpretation for the behaviors of cumulants

ex: \[
\langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B}
\]
Conserved Charge Fluctuations

\[ \frac{\langle \delta N_B^4 \rangle_c}{\langle \delta N_B^2 \rangle} \]

Cumulants of \( N_B \) and \( N_Q \) are **suppressed** at high \( T \).

Asakawa, Heinz, Muller, 2000; Jeon, Koch, 2000; Ejiri, Karsch, Redlich, 2006; Asakawa, Ejiri, MK, 2009; Friman, et al., 2011; Stephanov, 2011
Proton # Cumulants @ STAR-BES

\[ \frac{C_4}{C_2} \]

\[ \frac{C_3}{C_1} = \frac{C_3}{C_2} \text{ Poissonian} \]

CAUTION!
proton number ≠ baryon number

MK, Asakawa, 2011;2012
Charge Fluctuation @ LHC

ALICE, PRL110, 152301 (2013)

D-measure

\[ D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle} \]

- \( D \sim 3-4 \) Hadronic
- \( D \sim 1-1.5 \) Quark

significant suppression from hadronic value at LHC energy!

\( \langle \delta N_Q^2 \rangle \) is not equilibrated at freeze-out at LHC energy!
$\Delta \eta$ Dependence @ ALICE

[Graph showing the dependence of $\Delta \eta$ on rapidity windows and functions like $\text{Erf}(\frac{\Delta \eta}{\sqrt{8\sigma_f}})$]

ALICE PRL 2013

rapidity window
Time Evolution of Fluctuations

Variation of a conserved charge is achieved only through diffusion.

The larger $\Delta \eta$, the slower diffusion.
**Δη Dependence @ ALICE**

Δη dependences of fluctuation observables encode history of the hot medium!
Cumulants: HIC@RHIC vs Lattice

Parameter window constrained by lattice

$\mu / T$ discrepancy

fluctuations “exp + lattice”

$\sqrt{s}$ [GeV]: 39

62.4

200

HotQCD, LATTICE2013
$<\delta N_B^2>$ and $<\delta N_\rho^2>$ @ LHC?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_\rho^2 \rangle$
should have different $\Delta \eta$ dependence.

Baryon # cumulants are experimentally observable! MK, Asakawa, 2011;2012
$\langle \delta N_B^2 \rangle$ and $\langle \delta N_p^2 \rangle @ LHC$?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$

should have different $\Delta \eta$ dependence.

Baryon # cumulants are experimentally observable! MK, Asakawa, 2011;2012
How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta \eta$?

- **suppression**
- **enhancement**
Baryon vs Proton Number Fluctuations

MK, Asakawa, PRC85,021901C(2012); PRC86, 024904(2012)

\[
\frac{\langle \delta N_B^m \rangle_c}{\langle \delta N_B^n \rangle_c} \neq \frac{\langle \delta N_p^n \rangle_c}{\langle \delta N_p^m \rangle_c}
\]

- \( \langle \delta N_B^m \rangle_c \) are experimentally observable
Nucleons have two isospin states.

MK, Asakawa, 2012
Nucleon Isospin as Two Sides of a Coin

Nucleons have two isospin states.

Coins have two sides.

MK, Asakawa, 2012
Slot Machine Analogy

\[ P_x(N) = 100 + \]
Extreme Examples

Fixed # of coins

Constant probabilities

N

N

N

N
Reconstructing Total Coin Number

\[ P(N) = \sum P(N) B_{1/2}(N; N) \]

\[ B_p(k; N) = p^k (1 - p)^{N-k} \binom{N}{k} \quad \text{binomial distr. func.} \]
Nucleons in Hadronic Phase

- Hadronization
- Chemical freeze-out
- Kinetic freeze-out

10~20fm

\[ m_\pi \simeq T \ll m_N - \mu_N \]

- \( n_N \ll 1 \)
  - rare NN collisions
- No quantum corr.

\( n_N \ll n_\pi \)

- Many pions
Isospin of baryons can vary after chemical freezeout via charge exchange reactions mediated by $\Delta(1232)$:

$\pi \rightarrow \Delta(1232) \rightarrow \pi$

$p, n \rightarrow \Delta(1232) \rightarrow p, n$

$I = 3/2$

$\Gamma \simeq 1.8$ [fm]

$200$ mb $= 20$ fm$^2$
\( \Delta(1232) \)

Cross sections of \( p \):

\[
\begin{align*}
    p + \pi^+ &\rightarrow \Delta^{++} \rightarrow p + \pi^+ \\
    p + \pi^0 &\rightarrow \Delta^+ \rightarrow p + \pi^0 \\
    n + \pi^+ &\rightarrow \Delta^+ \rightarrow n + \pi^+ \\
    p + \pi^- &\rightarrow \Delta^0 \rightarrow p + \pi^- \\
    n + \pi^0 &\rightarrow \Delta^0 \rightarrow n + \pi^0 \\
    n + \pi^- &\rightarrow \Delta^- \rightarrow n + \pi^- 
\end{align*}
\]

Decay rates of \( \Delta \):

\[
\begin{align*}
    3 &\quad 1:2 \\
    1 &\quad 2:1 \\
    2 &\quad 1:2
\end{align*}
\]
$\Delta(1232)$

Cross sections of $\rho$

$p + \pi^+ \rightarrow \Delta^{++} \rightarrow p + \pi^+$

$p + \pi^0 \rightarrow \Delta^+ \rightarrow p + \pi^0$

$n + \pi^+ \rightarrow \Delta^+ \rightarrow n + \pi^+$

$p + \pi^- \rightarrow \Delta^0 \rightarrow p + \pi^-$

$n + \pi^0 \rightarrow \Delta^0 \rightarrow n + \pi^0$

$n + \pi^- \rightarrow \Delta^- \rightarrow n + \pi^-$

$p + \pi \rightarrow \Delta^{+,0} \rightarrow p : n$

$= 5 : 4$
\[ \Delta(1232) \]

**Cross sections of \( p \):**

\[
\begin{align*}
p + \pi^+ & \rightarrow \Delta^{++} \rightarrow p + \pi^+ \\
p + \pi^0 & \rightarrow \Delta^+ \rightarrow p + \pi^0 \\
n + \pi^+ & \rightarrow \Delta^+ \rightarrow n + \pi^+ \\
p + \pi^- & \rightarrow \Delta^0 \rightarrow p + \pi^- \\
n + \pi^0 & \rightarrow \Delta^0 \rightarrow n + \pi^0 \\
n + \pi^- & \rightarrow \Delta^- \rightarrow n + \pi^- \\
p + \pi & \rightarrow \Delta^{+,0} \\
\rightarrow p : n \\
= 5 : 4
\end{align*}
\]

**Decay rates of \( \Delta \):**

**Lifetime to create \( \Delta^+ \) or \( \Delta^0 \):**

\[
\tau^{-1} = \int \frac{d^3k_\pi}{(2\pi)^3} \sigma(E_{\text{cm}}) \nu_\pi n(E_\pi)
\]

(freezeout time) \( \approx 20 \) [fm]

**BW for \( \sigma \) & thermal pion:**

\[ T [\text{MeV}] \]

\[ \tau [\text{fm}] \]
(1) \( N_B^{(\text{net})} = N_B - N_{\bar{B}} \) deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for \( N_B, N_{\bar{B}} \).

\[
\begin{align*}
2\langle (\delta N_p^{(\text{net})})^2 \rangle & = \frac{1}{2} \langle (\delta N_B^{(\text{net})})^2 \rangle + \frac{1}{2} \langle (\delta N_B^{(\text{net})})^2 \rangle_{\text{free}} \\
2\langle (\delta N_p^{(\text{net})})^3 \rangle & = \frac{1}{4} \langle (\delta N_B^{(\text{net})})^3 \rangle + \frac{3}{4} \langle (\delta N_B^{(\text{net})})^3 \rangle_{\text{free}} \\
2\langle (\delta N_p^{(\text{net})})^4 \rangle_{c} & = \frac{1}{8} \langle (\delta N_B^{(\text{net})})^4 \rangle_{c} + \cdots
\end{align*}
\]

genuine info. Poissonian noise

Difference from Poisson (thermal) distribution is suppressed in proton number fluctuations.
Difference btw Baryon and Proton Numbers

(1) \( N_{B}^{\text{net}} = N_{B} - N_{\bar{B}} \) deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for \( N_{B}, N_{\bar{B}} \).

\[
2\langle (\delta N_{p}^{\text{net}})^2 \rangle = \frac{1}{2} \langle (\delta N_{B}^{\text{net}})^2 \rangle + \frac{1}{2} \langle (\delta N_{\bar{B}}^{\text{net}})^2 \rangle_{\text{free}}
\]

\[
2\langle (\delta N_{p}^{\text{net}})^3 \rangle = \frac{1}{4} \langle (\delta N_{B}^{\text{net}})^3 \rangle + \frac{3}{4} \langle (\delta N_{\bar{B}}^{\text{net}})^3 \rangle_{\text{free}}
\]

The observed \( \kappa \sigma^2 \) is a measure of the deviation from the Poisson (thermal) distribution, which is affected by genuine info. and Poissonian noise.

from Poisson (thermal) distribution possessed in proton number fluctuations.
Time Evolution of Higher Order Cumulants

MK, Asakawa, Ono, arXiv:1307.2978
How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta \eta$?

suppression or enhancement
Hydrodynamic Fluctuations

Stochastic diffusion equation

\[ \partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau) \]

Fluctuation of \( n \) is Gaussian in equilibrium

Markov (white noise) + continuity

Gaussian noise

cf) Gardiner, “Stochastic Methods”

Landau, Lifshitz, Statistical Mechanics II
Kapusta, Muller, Stephanov, 2012
Stephanov, Shuryak, 2001
How to Introduce Non-Gaussianity?

**Stochastic** diffusion equation

\[ \partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau) \]

- Choices to introduce non-Gaussianity in equil.:
  - \( n \) dependence of diffusion constant \( D(n) \)
  - colored noise
  - discretization of \( n \)
How to Introduce Non-Gaussianity?

**Stochastic diffusion equation**

\[ \partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau) \]

- Choices to introduce non-Gaussianity in equil.:
  - \( n \) dependence of diffusion constant \( D(n) \)
  - colored noise
  - **discretization of \( n \)**

**Remark:** Fluctuations measured in HIC are almost Poissonian.
Diffusion Master Equation

Divide spatial coordinate into discrete cells

\[ \cdots n_{x-1} \quad n_x \quad n_{x+1} \quad n_{x+2} \quad \cdots \]

\[\gamma \quad \gamma \quad \alpha\]

probability

\[P(n)\]
Diffusion Master Equation

Divide spatial coordinate into discrete cells

\[ \gamma \quad \gamma \quad \alpha \]

Master Equation for \( P(n) \)

\[ \frac{\partial}{\partial t} P(n) = \gamma \sum_x [(n_x + 1) \{ P(n + e_x - e_{x+1}) + P(n + e_x - e_{x-1}) \} - 2n_x P(n)] \]

Solve the DME **exactly**, and take \( a \rightarrow 0 \) limit

No approx., ex. van Kampen’s system size expansion
Baryons in Hadronic Phase

Baryons behave like Brownian pollens in water

- $p, \bar{p}$
- $n, \bar{n}$
- $\Delta(1232)$

hadronize
chem. f.o.

10~20 fm

kinetic f.o.
Net Charge Number

Prepare 2 species of (non-interacting) particles

\[ \bar{Q}(\tau) = \int_{0}^{\Delta \eta} d\eta \left( n_1(\eta, \tau) - n_2(\eta, \tau) \right) \]

Let us investigate

\[ \langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c \] at freezeout time t
Solution of DME in a $\rightarrow 0$ Limit

1st order (deterministic) $\langle n \rangle$
- consistent with diffusion equation with $D=\gamma a^2$

Continuum limit with fixed $D=\gamma a^2$

2nd order $\langle \delta n^2 \rangle$
- consistent with stochastic diffusion eq. (for sufficiently smooth initial conditions)

Nontrivial results for non-Gaussian fluctuations

Shuryak, Stephanov, 2001
Time Evolution in Hadronic Phase

Hadronization (initial condition)

- Boost invariance / infinitely long system
- Local equilibration / local correlation

\[
\left\langle \bar{Q}^2 \right\rangle_c \quad \left\langle \bar{Q}^4 \right\rangle_c \quad \left\langle \bar{Q}^2 Q_{(tot)} \right\rangle_c \quad \left\langle Q_{(tot)}^2 \right\rangle_c
\]

suppression owing to local charge conservation

strongly dependent on hadronization mechanism
**Time Evolution in Hadronic Phase**

**Hadronization (initial condition)**

- Boost invariance / infinitely long system
- Local equilibration / local correlation

- $\langle \bar{Q}^2 \rangle_c$, $\langle \bar{Q}^4 \rangle_c$
- $\langle \bar{Q}^2 Q_{(tot)} \rangle_c$
- $\langle Q_{(tot)}^2 \rangle_c$

**Suppression owing to local charge conservation**

**Freezeout**

**Time evolution via DME**

**Strongly dependent on hadronization mechanism**
In recombination model,

\[ N_B^{(net)} = 0 \]
\[ N_B^{(tot)} = 4 \]

\[ N_B^{(net)} = 0 \]
\[ N_B^{(tot)} = 0 \]

- \( N_B^{(tot)} \) can fluctuate, while \( N_B^{(net)} \) does not.
Initial fluctuations:

\[ \langle Q^2 \rangle_c = \langle Q^4 \rangle_c = \langle Q^2 Q_{(tot)} \rangle_c = 0 \]
Δη Dependence at Freezeout

Initial fluctuations:

\[ \langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(tot)} \rangle_c = 0.5 \langle Q_{(tot)} \rangle \]

\[ c = \frac{\langle Q_{(net)}^2 \rangle_c}{\langle Q_{(tot)} \rangle_c} \]

parameter sensitive to hadronization
Assumptions

- boost invariant system
- small fluctuations of CC at hadronization
- short correlation in hadronic stage

$\langle \delta N_Q^4 \rangle$ \@ LHC

$\Delta \eta$ dependences encode various information on the dynamics of HIC!

4th-order cumulant will be suppressed at LHC energy!
$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$ decreases as $\Delta \eta$ becomes larger at RHIC energy.
OUR SUGGESTIONS:

- Plot $\langle \delta N^2 \rangle$ and $\langle \delta N^4 \rangle$ separately
- Plot baryon number cumulants
Global Charge Conservation

Solve SDE or DME in a finite volume

Effect of a boundary appears only in the range of diffusion length

- Effect of GCC can be read off from $\Delta \eta$ dependence.
- No GCC effect in ALICE experiments!
Summary

Fluctuations in HIC are nonthermal!

Plenty of physics in $\Delta \eta$ dependences of various cumulants

$\langle N_Q^2 \rangle_c$, $\langle N_B^2 \rangle_c$, $\langle N_Q^4 \rangle_c$, $\langle N_B^4 \rangle_c$, $\langle N_{ch}^2 \rangle_c$, \ldots

Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC
- history of hot medium
- mechanism of hadronization
- diffusion constant

LATTICE inputs
Summary

Fluctuations in HIC are nonthermal!

Plenty of physics in $\Delta \eta$ dependences of various cumulants

$\langle N_Q^2 \rangle_c, \langle N_B^2 \rangle_c, \langle N_Q^4 \rangle_c, \langle N_B^4 \rangle_c, \langle N_{ch}^2 \rangle_c, \cdots$

Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC
- history of hot medium
- mechanism of hadronization
- diffusion constant

Search of QCD Phase Structure in HIC

LATTICE inputs
Open Questions & Future Work

- Why the primordial fluctuations are observed only at LHC, and not RHIC?
- Extract more information on each stage of fireballs using fluctuations

- Model refinement
  - Including the effects of nonzero correlation length / relaxation time
    global charge conservation

- Non Poissonian system ↔ interaction of particles
Chemical Reaction 1

\[ \text{Master eq.:} \quad \frac{\partial}{\partial t} P(x, t) = k_2 a P(x - 1, t) + k_1 (x + 1) P(x + 1, t) 
- (k_1 x + k_2 a) P(x, t) \]

Cumulants with fixed initial condition \( P(x, 0) = \delta_{x,N_0} \)

\[
\begin{align*}
\langle x(t) \rangle &= N_0 e^{-k_1 t} + N_{eq}(1 - e^{-k_1 t}) \\
\langle \delta x(t)^2 \rangle &= N_0 (e^{-k_1 t} - e^{-2k_1 t}) + N_{eq}(1 - e^{-k_1 t}) \\
\langle \delta x(t)^3 \rangle &= N_0 (e^{-k_1 t} - 3e^{-2k_1 t} + 2e^{-3k_1 t}) + N_{eq}(1 - e^{-k_1 t})
\end{align*}
\]
Chemical Reaction 2

\[ X \xrightarrow[k_1]{k_2} A \]

\[ N_0 = N_{eq} \]

\[
\begin{align*}
\langle x(t) \rangle &= N_{eq} \\
\langle \delta x(t)^2 \rangle &= N_{eq}(1 - e^{-2k_1 t}) \\
\langle \delta x(t)^3 \rangle &= N_{eq}(1 - 3e^{-2k_1 t} + 2e^{-3k_1 t})
\end{align*}
\]

Higher-order cumulants grow slower.
Time Evolution in HIC

Quark-Gluon Plasma

Hadronization

Freezeout

\[ \langle \Delta N^2 \rangle \]

\[ \Delta \eta \]

\[ \chi_{\text{HAD}} \]

\[ \chi_{\text{QGP}} \]

\[ \Delta \eta \]
Hydrodynamic Fluctuations

Diffusion equation

\[ \partial_\tau n = D \partial^2_\eta n \]

Stochastic diffusion equation

\[ \partial_\tau n = D \partial^2_\eta n + \partial_\eta \xi(\eta, \tau) \]

Stochastic Force determined by fluctuation-dissipation relation
Δη Dependence

- Initial condition: \( \langle \delta n(\eta_1, 0)\delta n(\eta_2, 0) \rangle = \sigma_2 \delta(\eta_1 - \eta_2) \)
- Translational invariance

\[
Q(\tau) = \int_0^{\Delta \eta} d\eta n(\eta, \tau)
\]

\[
\langle \delta Q(\tau)^2 \rangle = \sigma_2 F_2(X) + \chi_2(1 - F_2(X))
\]

Shuryak, Stephanov, 2001
Non-Gaussianity in Fluctuating Hydro?

It is **impossible** to directly extend the theory of hydro fluctuations to treat higher orders.

- No a priori extension of FD relations to higher orders
- **Theorem**
  - Markov process + continuous variable
  - Gaussian random force

*cf* Gardiner, “Stochastic Methods”
Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in HIC.

\[
\langle \delta N^2 \rangle, \langle \delta N^3 \rangle, \langle \delta N^4 \rangle_c, \cdots
\]
Non-Gaussianity

fluctuations (correlations)

\[ \langle \delta n_1 \delta n_2 \rangle, \langle \delta n_1 \delta n_2 \delta n_3 \rangle, \langle \delta n_1 \delta n_2 \delta n_3 \delta n_4 \rangle_c, \ldots \]

Non-Gaussianity

PLANCK: statistics insufficient to see non-Gaussianity…(2013)