Fluctuations of Conserved Charges
- Theory, Experiment, and Lattice -

Masakiyo Kitazawa
（Osaka U.）

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QCD @ nonzero $T$

- Theory (Motivation)
- Lattice
- Heavy Ion Collisions
QCD @ nonzero $T$

Theory (Motivation)

Fluctuations of conserved charges

Lattice

Heavy Ion Collisions
QCD @ nonzero $T$

Theory (Motivation)

Lattice

Heavy Ion Collisions
Why QCD @ nonzero $T$ and $\mu$?

- Form of the matter under extreme conditions
  - QCD Phase diagram
  - New many body properties
Why QCD @ nonzero $T$ and $\mu$?

- Form of the matter under extreme conditions
  - QCD Phase diagram
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- State of the matter realized in
  - Early Universe
  - Compact stars
Why QCD @ nonzero $T$ and $\mu$?

- Form of the matter under extreme conditions
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- State of the matter realized in
  - Early Universe
  - Compact stars

- Relativistic heavy ion collisions
Relativistic Heavy Ion Collisions
Particle yields can be well described only by $T$, $\mu_B$!

canonical chemical equilibrium?
Beam-Energy Scan Program

STAR 2012

Grand Canonical Ensemble

Au+Au

00-05%
05-10%
10-20%
20-30%
30-40%
40-60%
60-80%

Cleymans
Andronic

STAR Preliminary

T_{ch} (GeV)

\mu_B (GeV)

Hadrons

Color SC

beam energy

high

low
Hadron Resonance Gas (HRG) Model

HRG model

free gas composed of known hadrons

The HRG model well describes thermodynamics calculated on the lattice.

“Trace Anomaly”  Baryon # fluctuation
Lattice and HIC : EoS

Equation of states

- Robust modelling of space-time evolution
- Small shear viscosity

Lattice

Input

Heavy Ion Collisions
Lattice and HIC: Heavy Quarkonia

Heavy quarkonia will disappear in QGP
Matsui, Satz, 1986

Charmonium SPC
Asakawa, Hatsuda, 2004

Heavy Ion Collisions

Input
Fluctuations of Conserved Charges
Observables in equilibrium are fluctuating.
Fluctuations

Observables in equilibrium are fluctuating.

- Variance: \( \langle \delta N^2 \rangle = V \chi_2 = \sigma^2 \)
- Skewness: \( S = \frac{\langle \delta N^3 \rangle}{\sigma^3} \)
- Kurtosis: \( \kappa = \frac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2} \)

Non-Gaussianity
Conserved Charge Fluctuations

- Definite definition of the operator \( \mathcal{O} \)
  - as a Noether current
  - Expectation value: \( \langle \mathcal{O} \rangle = \text{Tr}[\rho \mathcal{O}] = \int \mathcal{D} U \mathcal{O} e^{-S} \)
  - Fluctuation: \( \langle \delta \mathcal{O}^2 \rangle = \langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2 \)

Simple thermodynamic relation

\[ \langle \delta \mathcal{O}^n \rangle_c = \frac{T^n}{V} \frac{\partial^n}{\partial \mu^n} \ln Z(\mu) \quad \text{where} \quad Z(\mu) = \text{Tr} e^{-\beta (H - \mu \mathcal{O})} \]
Taylor Expansion Method & Cumulants

\[ P(T, \mu) = \frac{T}{V} \ln Z(\mu) \]

\[ = P(T, 0) + \frac{\mu}{T} \frac{\partial P(T, 0)}{\partial (\mu/T)} + \frac{1}{2} \left( \frac{\mu}{T} \right)^2 \frac{\partial^2 P(T, 0)}{\partial (\mu/T)^2} + \cdots \]

\[ \langle N \rangle \quad \langle \delta N^2 \rangle_c \]

Baryon number cumulants = Taylor expansion coeffs.
Recent Progress in Lattice Simulations

From LATTICE2013 presentations
QCD @ nonzero $T$

Theory (Motivation)

QCD @ nonzero $T$

Lattice

Heavy Ion Collisions
Fluctuations can be measured by e-by-e analysis in experiments.
Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.

\[ \langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c \]
What are Fluctuations observed in HIC?

QUESTION: When the experimentally-observed fluctuations are formed?

- at chemical freezeout?
- at kinetic freezeout?
- or, much earlier?
QCD @ nonzero $T$

Theory
(Motivation)

QCD @ nonzero $T$

Lattice

Heavy Ion Collisions
Fluctuations

- Fluctuations reflect properties of matter.
- Enhancement near the critical point
  - Stephanov, Rajagopal, Shuryak ('98); Hatta, Stephanov ('02); Stephanov ('09); …
- Ratios between cumulants of conserved charges
  - Asakawa, Heintz, Muller ('00); Jeon, Koch ('00); Ejiri, Karsch, Redlich ('06)
- Signs of higher order cumulants
  - Asakawa, Ejiri, MK ('09); Friman, et al. ('11); Stephanov ('11)
Fluctuations

Free Boltzmann $\rightarrow$ Poisson

$\langle \delta N^n \rangle_c = \langle N \rangle$

$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$

$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$

$3N_B = N_q$

$\langle \delta N_B^n \rangle_c = \langle N_B \rangle$
Fluctuations

Free Boltzmann $\rightarrow$ Poisson

$$\left\langle \delta N^n \right\rangle_c = \left\langle N \right\rangle$$

$$\left\langle \delta N^n_q \right\rangle_c = \left\langle N_q \right\rangle$$

$$\left\langle \delta N^n_B \right\rangle_c = \frac{1}{3n-1} \left\langle N_B \right\rangle$$

3$N_B = N_q$

RBC-Bielefeld '09

$$12 \frac{c_4^B}{c_2^B} = \frac{\left\langle B^4 \right\rangle - 3 \left\langle B^2 \right\rangle^2}{\left\langle B^2 \right\rangle}$$
Fluctuations

Free Boltzmann $\rightarrow$ Poisson

$$\langle \delta N^n \rangle_c = \langle N \rangle$$

$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\langle \delta N_B^n \rangle_c = \frac{1}{3n-1} \langle N_B \rangle$$

$$3N_B = N_q$$
Skellam Distribution

- Poisson + Poisson = Poisson
  \[ \langle N_1 \rangle \quad \langle N_2 \rangle \quad \langle \delta N^n \rangle_c = \langle N_1 + N_2 \rangle \]

- Poisson — Poisson = Skellam distribution
  \[ \langle N_1 \rangle \quad \langle N_2 \rangle \quad \langle \delta N^n \rangle_c = \begin{cases} 
  \langle N_1 + N_2 \rangle & \text{(n:even)} \\
  \langle N_1 - N_2 \rangle & \text{(n:odd)} 
\end{cases} \]

In the HRG model, (Net-)baryon and electric charge fluctuations are of Skellam distribution.
Fluctuations diverge at the QCD critical point.

Example: $\langle \delta N_B^2 \rangle$

Higher order cumulants are more sensitive to correlation length

\[
\begin{align*}
\langle \delta N^2 \rangle & \sim \xi^2 \\
\langle \delta N^3 \rangle & \sim \xi^{4.5} \\
\langle \delta N^4 \rangle_c & \sim \xi^7
\end{align*}
\]
Sign of Higher Order Cumulants

- $\chi_B$ has an edge along the phase boundary
  
  \[ \frac{\partial \chi_B}{\partial \mu_B} \] changes the sign at QCD phase boundary!

\[
\begin{aligned}
\chi_B &= - \frac{1}{V} \frac{\partial^2 \Omega}{\partial \mu_B^2} = \frac{\left\langle (\delta N_B)^2 \right\rangle}{VT} \\
\frac{\partial \chi_B}{\partial \mu_B} &= - \frac{1}{V} \frac{\partial^3 \Omega}{\partial \mu_B^3} = \frac{\left\langle (\delta N_B)^3 \right\rangle}{VT^2}
\end{aligned}
\]
Impact of Negative Third Moments

- Once negative $m_3(BBB)$ is established, it is evidences that
  \[ \begin{align*}
  (1) & \ \chi_B \text{ has a peak structure in the QCD phase diagram.} \\
  (2) & \ \text{Hot matter beyond the peak is created in the collisions.}
  \end{align*} \]

- \textbf{No} dependence on any specific models.
  - Just the sign! \textbf{No} normalization (such as by $N_{ch}$).
Various third moments, $\langle \delta N_B^3 \rangle$, $\langle \delta N_Q^3 \rangle$, $\langle \delta E^3 \rangle$ become negative near the phase boundary.

The behaviors can be checked by lattice and HIC!

See also, Friman, et al. ('11); Stephanov ('11).
Exploring Medium Properties

Hadronic

$B=0,1$

strangeness with baryon number

Quark-Gluon

$B=1/3$
Exploring Medium Properties

Hadronic

\[ B = 0, 1 \]

strangeness with baryon number

Quark-Gluon

\[ B = 1/3 \]

Combinations of cumulants which vanish in the HRG model

BNL-Bielefeld, PRL 2013
QCD @ nonzero $T$

Theory (Motivation)

Lattice

Heavy Ion Collisions
Proton # Fluctuations @ STAR-BES

STAR, PRL2010

\[ S\sigma = \frac{\langle (\delta N_p^{\text{net}})^3 \rangle}{\langle (\delta N_p^{\text{net}})^2 \rangle}, \quad \kappa\sigma^2 = \frac{\langle (\delta N_p^{\text{net}})^4 \rangle_c}{\langle (\delta N_p^{\text{net}})^2 \rangle} \]
Proton # Fluctuations @ STAR-BES

STAR, PRL2010

(a) $m_1$

$b)$ $m_2$

$S\sigma = \frac{\langle (\delta N_p^{(\text{net})})^3 \rangle}{\langle (\delta N_p^{(\text{net})})^2 \rangle}$, $\kappa\sigma^2 = \frac{\langle (\delta N_p^{(\text{net})})^4 \rangle_c}{\langle (\delta N_p^{(\text{net})})^2 \rangle}$

STAR, 2011

Au+Au Collisions

0-5%  30-50%  70-80%

HRG

STAR Preliminary
Proton # Fluctuations @ STAR-BES

STAR, 2012 (Quark Matter)

STAR, 2011

Au+Au Poisson

Net-proton

0.4<p_T<0.8 (GeV/c), |y|<0.5

UrQMD : 0-5% p+p

STAR Preliminary
Proton # Cumulants @ STAR-BES

\[
\frac{C_4}{C_2} \quad \frac{C_3}{C_1} = \frac{C_3}{C_2} / \text{Poissonian}
\]

CAUTION!

proton number \(\neq\) baryon number

MK, Asakawa, 2011;2012

Something interesting??
Proton Cumulants @ STAR-BES

\[ \frac{C_4}{C_2} \]

\[ \frac{C_3}{C_2} \]

Something interesting??

Athanasiou, Rajagopal, Stephanov, 2010
Electric Charge Fluctuation @ LHC

ALICE, PRL110, 152301 (2013)

D-measure

\[ D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle} \]

- \( D \approx 3-4 \) Hadronic
- \( D \approx 1-1.5 \) Quark

significant suppression from hadronic value at LHC energy!

\( \langle \delta N_Q^2 \rangle \) is not equilibrated at freeze-out at LHC energy!
$\Delta\eta$ Dependence @ ALICE

\[ D = 4 \frac{\left\langle \delta N_Q^2 \right\rangle}{\left\langle N_Q^+ + N_Q^- \right\rangle} \]
Dissipation of a Conserved Charge

\[ t = 0 \]

\[ \langle \Delta N^2 \rangle / \Delta x \]

\[ t \rightarrow \infty \]
Dissipation of a Conserved Charge

$t = 0$

$t \rightarrow \infty$
Time Evolution of Fluctuations

Variation of a conserved charge is achieved only through diffusion. The larger $\Delta \eta$, the slower diffusion.
\[ \Delta \eta \] dependences of conserved charge fluctuations encode history of dynamical evolution.
QCD @ nonzero $T$

Theory (Motivation)

QCD @ nonzero $T$

Lattice

Heavy Ion Collisions

Diagram showing connections between QCD at nonzero temperature, theory (motivation), lattice, and heavy ion collisions.
Comparison b/w Lattice & HIC

Gupta, Xu, et al., Science, 2009

- Taylor expansion method
- Chemical freezeout $T, \mu$
- Pade approx.
Cumulants : HIC@RHIC vs Lattice

Parameter window constrained by lattice

BNL-Bielefeld, LATTICE2013

fluctuations
“exp + lattice”

μ/T

discrepancy

particle abundance
(chem. freezeout T)
Many Things to Do

- Proton vs baryon number cumunants
- Are fluctuations generated with fixed \( T \)?
- Experimental environments
  - Acceptance, efficiency
  - Particle missid
- Global charge conservation
Baryon vs Proton Number Fluctuations

MK, Asakawa, PRC85, 021901C(2012); PRC86, 024904(2012)

\[
\frac{\langle \delta N_B^n \rangle_c}{\langle \delta N_B^m \rangle_c} \neq \frac{\langle \delta N_p^n \rangle_c}{\langle \delta N_p^m \rangle_c}
\]

\[
\langle \delta N_B^n \rangle_c \text{ are experimentally observable}
\]
Nucleon Isospin as Two Sides of a Coin

Nucleons have two isospin states.

MK, Asakawa, 2012
Nucleon Isospin as Two Sides of a Coin

Nucleons have two isospin states.

Coins have two sides.

MK, Asakawa, 2012
Slot Machine Analogy

\[ P(N) = 100 + \]
Extreme Examples

Fixed # of coins

Constant probabilities

N

N

N

N
Reconstructing Total Coin Number

\[ P_{\bullet} (N_{\bullet}) = \sum P_{\bullet} (N_{\bullet}) B_{1/2}(N_{\bullet}; N_{\bullet}) \]

\[ B_p(k; N) = p^k(1 - p)^{N-k} \binom{k}{N} \text{ : binomial distr. func.} \]
Nucleon Isospin in Hadronic Medium

- Isospin of baryons can vary after chemical freezeout via charge exchange reactions mediated by $\Delta(1232)$:

\[
p, n \xrightarrow{\pi} \Delta(1232) \xrightarrow{\pi} p, n \quad I = \frac{3}{2}
\]

\[
\Gamma \simeq 1.8 \text{ [fm]}
\]

![Graph showing particle interactions and cross section](image)

- Cross section $200\text{mb} = 20\text{fm}^2$
$\Delta(1232)$

cross sections of $p$

$\begin{align*}
 p + \pi^+ & \rightarrow \Delta^{++} \rightarrow p + \pi^+ \\
p + \pi^0 & \rightarrow \Delta^+ \rightarrow p + \pi^0 \\
n + \pi^+ & \rightarrow \Delta^0 \rightarrow n + \pi^+ \\
p + \pi^- & \rightarrow \Delta^- \rightarrow n + \pi^- \\
n + \pi^0 & \rightarrow \Delta^0 \rightarrow n + \pi^- \\
n + \pi^- & \rightarrow \Delta^- \rightarrow n + \pi^-
\end{align*}$

decay rates of $\Delta$
The diagram illustrates the cross sections of the decay rates of the $\Delta(1232)$, a baryon with mass 1232 MeV. The reactions shown are:

- $p + \pi^+ \rightarrow \Delta^{++} \rightarrow p + \pi^+$
- $p + \pi^0 \rightarrow \Delta^+ \rightarrow p + \pi^0$
- $n + \pi^+ \rightarrow \Delta^+ \rightarrow n + \pi^+$
- $p + \pi^- \rightarrow \Delta^0 \rightarrow p + \pi^-$
- $n + \pi^0 \rightarrow \Delta^0 \rightarrow n + \pi^0$
- $n + \pi^- \rightarrow \Delta^- \rightarrow n + \pi^-$

The decay rates are as follows:

- $p + \pi \rightarrow \Delta^{+,0}$
- $\rightarrow p : n$
- $= 5 : 4$
Nucleons in Hadronic Phase

Nucleons in Hadronic Phase

10~20 fm

- Rare NN collisions
- No quantum corr.
- Many pions

\[ m_\pi \approx T \ll m_N - \mu_N \]

- \( n_N \ll 1 \)
- Many pions
Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$

- $N_N$ nucleons
- $N_{\bar{N}}$ anti-nucleons

$F(N_N, N_{\bar{N}})$

$B(N_p; N_N)$

$B(N_{\bar{p}}; N_{\bar{N}})$

$\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) = F(N_N, N_{\bar{N}})B(N_p; N_N)B(N_{\bar{p}}; N_{\bar{N}})$

- for any phase space in the final state.
(1) $N_{B}^{(\text{net})} = N_{B} - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_{B}, N_{\bar{B}}$.

$$2\langle (\delta N_{p}^{(\text{net})})^2 \rangle = \frac{1}{2}\langle (\delta N_{B}^{(\text{net})})^2 \rangle + \frac{1}{2}\langle (\delta N_{B}^{(\text{net})})^2 \rangle_{\text{free}}$$

$$\langle (\delta N_{B}^{(\text{net})})^3 \rangle + \frac{3}{4}\langle (\delta N_{B}^{(\text{net})})^3 \rangle_{\text{free}}$$

$$\langle (\delta N_{B}^{(\text{net})})^4 \rangle_{c} + \cdots$$

For free gas

$$2\langle (\delta N_{p}^{(\text{net})})^n \rangle_{c} = \langle (\delta N_{N}^{(\text{net})})^n \rangle_{c}$$
Time Evolution of Higher Order Cumulants

MK, Asakawa, Ono, PLB728, 386, 2014
Δη Dependence @ ALICE

rapidity window
Dissipation of a Conserved Charge

$t = 0$

$t \to \infty$

$\langle \Delta N^2 \rangle / \Delta x$

$\langle \Delta N^2 \rangle / \Delta x$

$\Delta x$
How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta \eta$?

- suppression
- or
- enhancement
Hydrodynamic Fluctuations

Stochastic diffusion equation

\[ \partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau) \]

- Fluctuation of \( n \) is Gaussian in equilibrium
- Markov (white noise)
- Continuity
- Gaussian noise

cf) Gardiner, “Stochastic Methods”

Landau, Lifshitz, Statistical Mechanics II
Kapusta, Muller, Stephanov, 2012
Stephanov, Shuryak, 2001
How to Introduce Non-Gaussianity?

**Stochastic diffusion equation**

\[ \partial_{\tau} n = D \partial^2_{\eta} n + \partial_{\eta} \xi(\eta, \tau) \]

- Choices to introduce non-Gaussianity in equil.:
  - \( n \) dependence of diffusion constant \( D(n) \)
  - colored noise
  - discretization of \( n \)
How to Introduce Non-Gaussianity?

Stochastic diffusion equation

$$\partial_\tau n = D \partial^2_\eta n + \partial_\eta \xi(\eta, \tau)$$

Choices to introduce non-Gaussianity in equil.:

- $n$ dependence of diffusion constant $D(n)$
- colored noise
- discretization of $n$  

Our choice

REMARK: Fluctuations measured in HIC are almost Poissonian.
Diffusion Master Equation

Divide spatial coordinate into discrete cells

\[ n_{x-1} \quad n_x \quad n_{x+1} \quad n_{x+2} \quad \ldots \]

\[ x \]

probability

\[ P(n) \]

Hadronization

Freezeout

\[ \Delta \eta \]
Divide spatial coordinate into discrete cells

Master Equation for $P(n)$

$$\frac{\partial}{\partial t} P(n) = \gamma \sum_x [(n_x + 1) \{P(n + e_x - e_{x+1}) + P(n + e_x - e_{x-1})\} - 2n_x P(n)]$$

Solve the DME **exactly**, and take $a \to 0$ limit

No approx., ex. van Kampen’s system size expansion
Baryons in Hadronic Phase

Baryons behave like Brownian pollens in water

hadronize
chem. f.o.
kinetic f.o.

$\rho, \bar{\rho}$
$n, \bar{n}$
$\Delta(1232)$

mesons
baryons

10~20fm
Net Charge Number

Prepare 2 species of (non-interacting) particles

\[
\bar{Q}(\tau) = \int_{0}^{\Delta \eta} d\eta \left( n_1(\eta, \tau) - n_2(\eta, \tau) \right)
\]

Let us investigate \( \langle \bar{Q}^2 \rangle_c \) \( \langle \bar{Q}^4 \rangle_c \) at freezeout time \( t \)
Solution of DME in a $a \to 0$ Limit

1st order (deterministic) $\langle n \rangle$
- consistent with diffusion equation with $D = \gamma a^2$
- Continuum limit with fixed $D = \gamma a^2$

2nd order $\langle \delta n^2 \rangle$
- consistent with stochastic diffusion eq.
  (for sufficiently smooth initial conditions)

Shuryak, Stephanov, 2001

Nontrivial results for non-Gaussian fluctuations
Time Evolution in Hadronic Phase

Hadronization (initial condition)

- Boost invariance / infinitely long system
- Local equilibration / local correlation

\[
\langle \bar{Q}^2 \rangle_c, \langle \bar{Q}^4 \rangle_c, \langle \bar{Q}^2 Q_{(tot)} \rangle_c, \langle Q_{(tot)}^2 \rangle_c
\]

- Suppression owing to local charge conservation
- Strongly dependent on hadronization mechanism
Time Evolution in Hadronic Phase

Hadronization (initial condition)

- Boost invariance / infinitely long system
- Local equilibration / local correlation

\[
\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c \quad \langle \bar{Q}^2 Q_{(tot)} \rangle_c \quad \langle Q_{(tot)}^2 \rangle_c
\]

Suppression owing to local charge conservation

Strongly dependent on hadronization mechanism

Time evolution via DME

Freezeout
\[ \Delta \eta \text{ Dependence at Freezeout} \]

Initial fluctuations:
\[ \langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\text{tot})} \rangle_c = 0 \]

\[ c = \frac{\langle Q_{(\text{net})}^2 \rangle_c}{\langle Q_{(\text{tot})} \rangle_c} \]

Parameter sensitive to hadronization
Assumptions

- boost invariant system
- small fluctuations of CC at hadronization
- short correlation in hadronic stage

$\langle \delta N_Q^4 \rangle @ LHC$

4\textsuperscript{th}-order cumulant will be suppressed at LHC energy!

$\Delta \eta$ dependences encode various information on the dynamics of HIC!
Dependence at STAR

\[
\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}
\]

\(\Delta \eta\) Dependence at STAR

STAR, QM2012

\[
\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}
\]
decreases as \(\Delta \eta\) becomes larger at RHIC energy.
Many Things to do …

• Better understanding on non-thermal nature
• Critical phenomena
• Other ideas?

Theory
(Motivation)

• $\Delta \eta$ dependence of 4th order cumulant
• Baryon number cumulants
• Acceptance effect, etc.

Lattice

• More accurate data
• Various channels
• Nonzero $\mu$

Heavy Ion Collisions
Conserved charge fluctuations are observable both in lattice simulations and heavy ion collisions. The comparison of the results in these two “experiments” will provide us many information to understand the QCD at nonzero $T/\mu$.

A lot of efforts are required both sides:
- Lattice: Higher statistics
- HIC: reconstructing baryon #, acceptance, etc.

Rapidity window dependences of cumulants in HIC are valuable tools to understand the non-thermal nature of fluctuations.
In recombination model,

\[ N_B^{(\text{net})} = 0 \]
\[ N_B^{(\text{tot})} = 4 \]

\[ N_B^{(\text{net})} = 0 \]
\[ N_B^{(\text{tot})} = 0 \]

\( N_B^{(\text{tot})} \) can fluctuate, while \( N_B^{(\text{net})} \) does not.
Evolution of Fluctuations

Fluctuation in initial state

Time evolution in the QGP

Volume fluctuation

Approach to HRG by diffusion

Experimental effects, particle misID, etc.
Time Evolution in HIC

Quark-Gluon Plasma

Hadronization

Freezeout

\[ \langle \Delta N^2 \rangle \]

\[ \Delta \eta \]

\( \chi_{\text{HAD}} \)

\( \chi_{\text{QGP}} \)

\( \Delta \eta \)

\( \chi_{\text{HAD}} \)

\( \chi_{\text{QGP}} \)

\( \Delta \eta \)

\( \chi_{\text{HAD}} \)

\( \chi_{\text{QGP}} \)

\( \Delta \eta \)
Time Evolution in HIC

- Pre-Equilibrium
- Quark-Gluon Plasma
- Hadronization
- Freezeout

\[ \langle \Delta N^2 \rangle \]
\[ \frac{\Delta \eta}{\Delta \eta} \]

\( \chi_{\text{HAD}} \)
\( \chi_{\text{QGP}} \)

\( \chi_{\text{HAD}} \)
\( \chi_{\text{QGP}} \)

\( \chi_{\text{HAD}} \)
\( \chi_{\text{QGP}} \)

\( \chi_{\text{HAD}} \)
\( \chi_{\text{QGP}} \)
Time Evolution in HIC

Pre-Equilibrium

Quark-Gluon Plasma

Hadronization

Freezeout

$\langle \Delta N^2 \rangle$

$\Delta \eta$

$\chi_{HAD}$

$\chi_{QGP}$

$\Delta \eta$

$\chi_{HAD}$

$\chi_{QGP}$

$\Delta \eta$

$\chi_{HAD}$

$\chi_{QGP}$

$\Delta \eta$