Gradient Flow and Energy-Momentum Tensor in Lattice Gauge Theory

Masakiyo Kitazawa (Osaka U.)

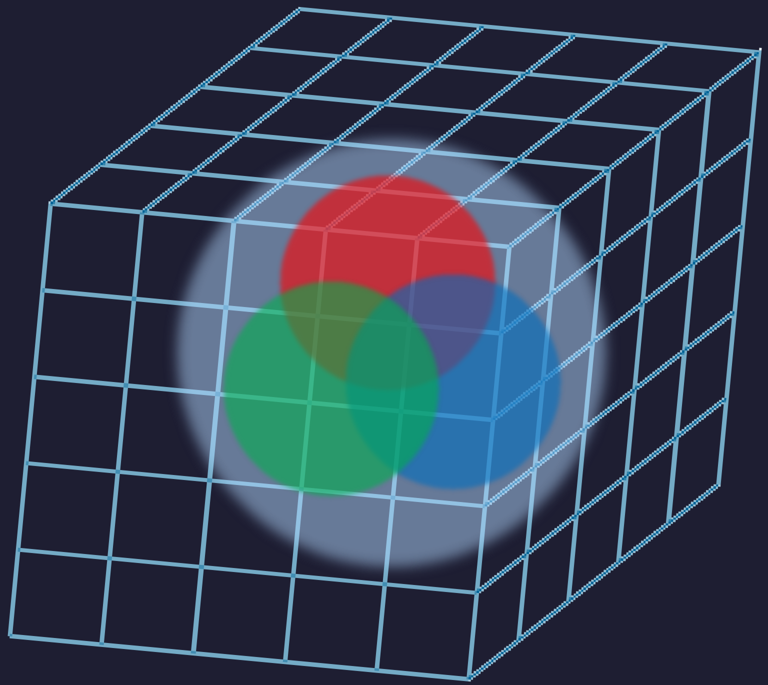
for FlowQCD Collaboration

Asakawa, Hatsuda, Iritani, Itou, MK, Suzuki

FlowQCD, arXiv:1312.7492[hep-lat]; to appear in PRD

BNL seminar, 30 June 2014

Lattice QCD



First principle calculation of QCD

Monte Carlo for path integral

hadron spectra, chiral symmetry, phase transition, etc.

Gradient Flow

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

Luscher, 2010

A powerful tool for various analyses on the lattice

Gradient Flow

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

Luscher, 2010

A powerful tool for various analyses on the lattice

D. Nogradi, LATTICE2014,7B

Why care?

- Tuesday 14:55 – Nathan Brown – Gradient Flow Analysis on MILC HISQ Ensembles
- Tuesday 14:35 – Andrea Shindler – Beyond the Standard Model Matrix Elements with the gradient flow
- Tuesday 14:35 – Liam Keegan – TEK twisted gradient flow running coupling
- Wednesday 09:00 – Anna Hasenfratz – Improved gradient flow for step scaling function and scale setting
- Wednesday 09:20 – Jarno Rantaharju – The gradient flow running coupling in SU2 with 8 flavors
- Wednesday 11:10 – Marco Ce – Testing the WittenVeneziano mechanism with the YangMills gradient flow on the lattice
- Thursday 14:55 – Agostino Patella – Energy-momentum tensor on the lattice and Wilson flow
- Thursday 15:15 – Masanori Okawa – String tension from smearing and Wilson flow methods
- Thursday 15:55 – Stefan Sint – How to reduce $O(a^2)$ effects in gradient flow observables
- Friday 10:15 – Alberto Ramos – Wilson flow and renormalization
- Saturday 09:30 – Kitazawa Masakiyo – Measurement of thermodynamics using Gradient Flow

Gradient Flow

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

Luscher, 2010

A powerful tool for various analyses on the lattice

Applications:
- ① scale setting
- ② running coupling
- ③ topology
- ④ operator relation → thermodynamics / EMT correlator
- ⑤ autocorrelation
- ⑥ etc.

Lattice Scale Setting

- ☐ a(β)
- ☐ previous references
  - string tension
  - Sommer scale

SU(3) pure YM	Edwards, Heller, Klassen, 1998	$\beta<6.56$
Wilson gauge	Alpha-Collab., 1998	$\beta<6.57$
	Necco, Sommer, 2002	$\beta<6.92$
	(Durr, Fodor, Hoelbling, 2007	$\beta<6.92$)

We perform the precision scale setting of
SU(3) YM theory up tp $\beta=7.5$ using gradient flow

$$T_{\mu\nu}$$

Poincare symmetry

$$T_{\mu\nu}$$

energy momentum

$$\begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

stress pressure

Einstein Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

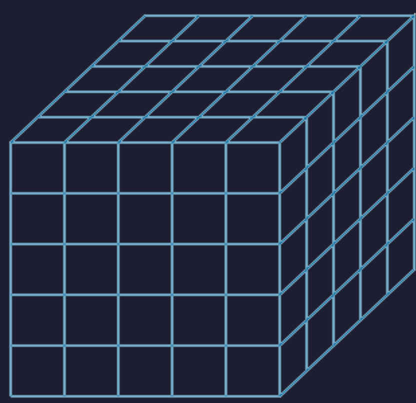
Hydrodynamic Eq.

$$\partial_\mu T_{\mu\nu} = 0$$

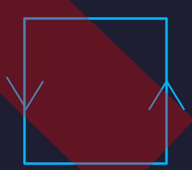
$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial

because of the explicit breaking of Lorentz symmetry



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

② Its measurement is extremely noisy

due to high dimensionality and etc.

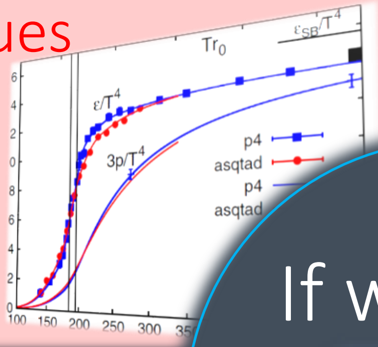
If we have

$$T_{\mu\nu}$$

Thermodynamics

direct measurement of expectation values

$$\langle T_{00} \rangle, \langle T_{ii} \rangle$$



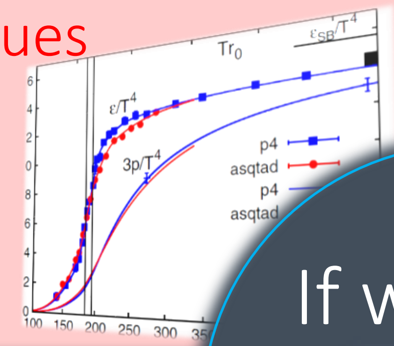
If we have

$$T_{\mu\nu}$$

Thermodynamics

direct measurement of expectation values

$$\langle T_{00} \rangle, \langle T_{ii} \rangle$$



Fluctuations and Correlations

viscosity, specific heat, ...

$$\eta = \int_0^\infty dt \langle T_{12}; T_{12} \rangle$$

$$c_V \sim \langle \delta T_{00}^2 \rangle$$

If we have

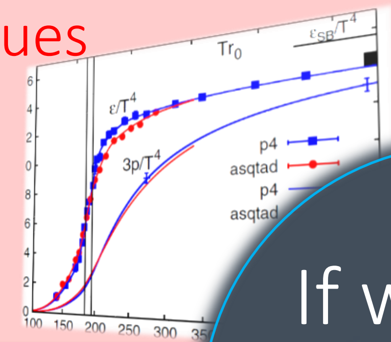
$$T_{\mu\nu}$$

We construct the EMT using gradient flow and measure these quantities

Thermodynamics

direct measurement of expectation values

$$\langle T_{00} \rangle, \langle T_{ii} \rangle$$



Fluctuations and Correlations

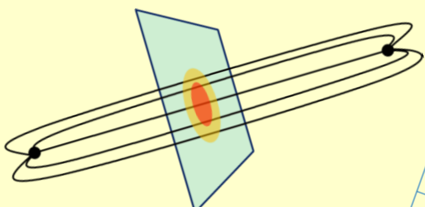
viscosity, specific heat, ...

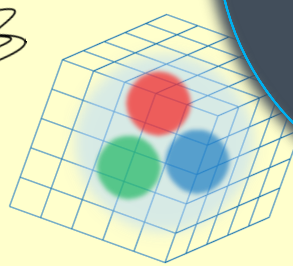
$$\eta = \int_0^\infty dt \langle T_{12}; T_{12} \rangle$$

$$c_V \sim \langle \delta T_{00}^2 \rangle$$

If we have

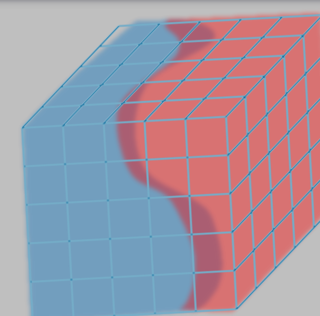
$$T_{\mu\nu}$$





- confinement string
- EM distribution in hadrons

Hadron Structure



- vacuum configuration
- mixed state on 1st transition

Vacuum Structure

Gradient Flow

YM Gradient Flow

Luscher, 2010

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

t: “flow time”
dim:[length^2]

$$A_\mu(0,x) = A_\mu(x)$$

YM Gradient Flow

Luscher, 2010

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length^2]

- transform gauge field like diffusion equation

$$\boxed{\partial_t A_\mu} = D_\nu G_{\mu\nu} = \boxed{\partial_\nu \partial_\nu A_\mu} + \cdots$$

- diffusion length $d \sim \sqrt{8t}$

- This is NOT the standard cooling/smearing
- All composite operators at t>0 are UV finite Luescher,Weisz,2011

Lattice Scale Setting

Flow Time Dep. of an Observable

Luscher, 2010

$\langle \mathcal{O}(t) \rangle$: universal function of t

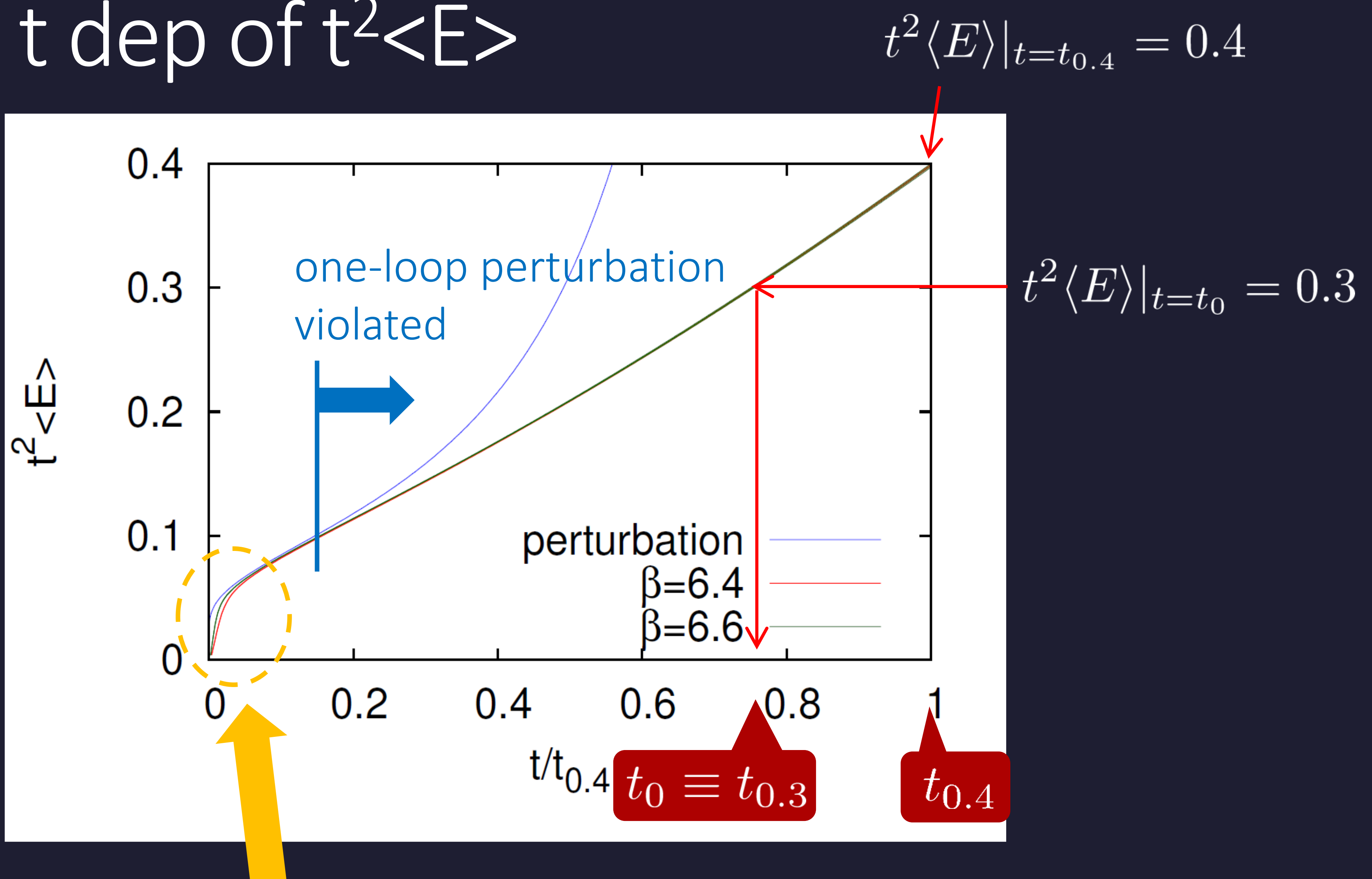
$\mathcal{O}$: an observable

use this function used to determine a(β)

$\langle \mathcal{O}(t_0) \rangle = \mathrm{const}$ ⇨ $t_0 = \hat{t} a^2$

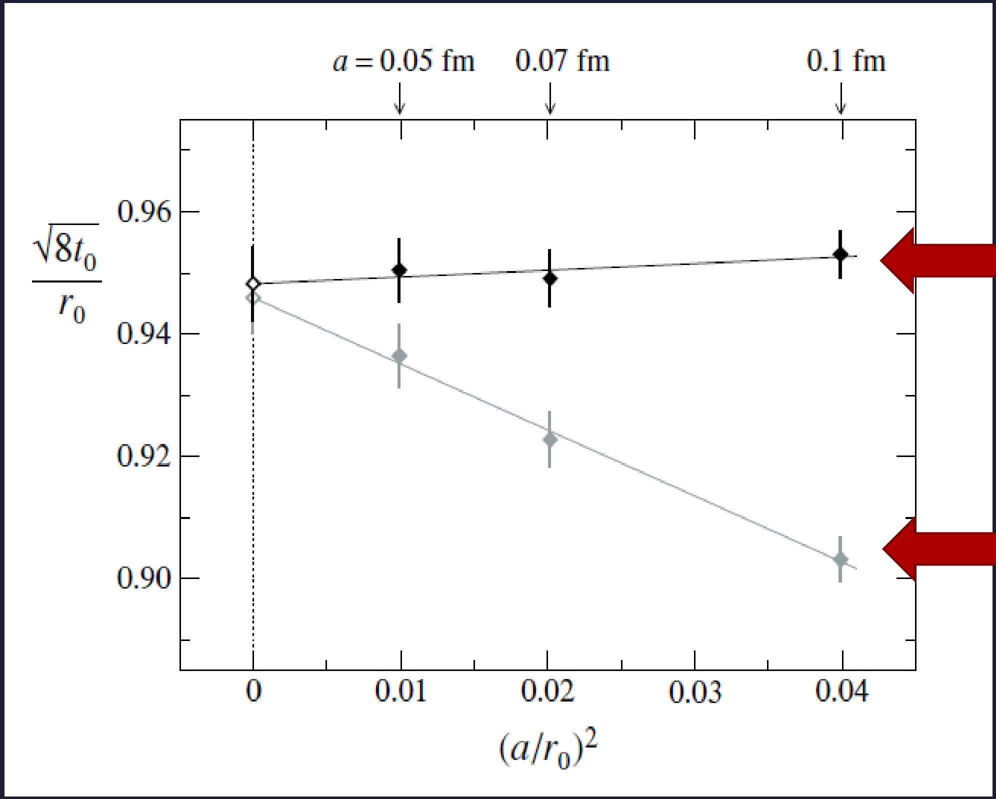
- standard choice of *O*: $\mathcal{O}(t) = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \equiv E$
- perturbative formula: $t^2 \langle E \rangle = \frac{3}{(4\pi)^2} g^2 (1 + k_1 g^2 + \cdots)$ $\quad g = g(1/\sqrt{8t})$

t dep of $t^2\langle E\rangle$



lattice discretization effect

$t_\#$ Scale Setting

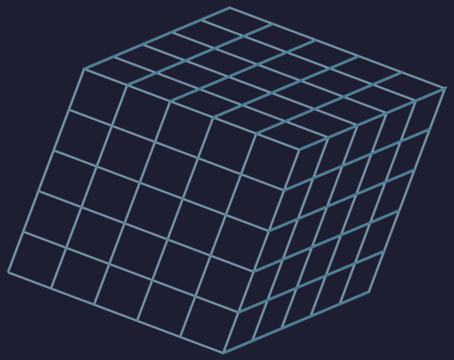


clover operator for FF

plaquette action

good agreement with previous scales

Numerical Analysis

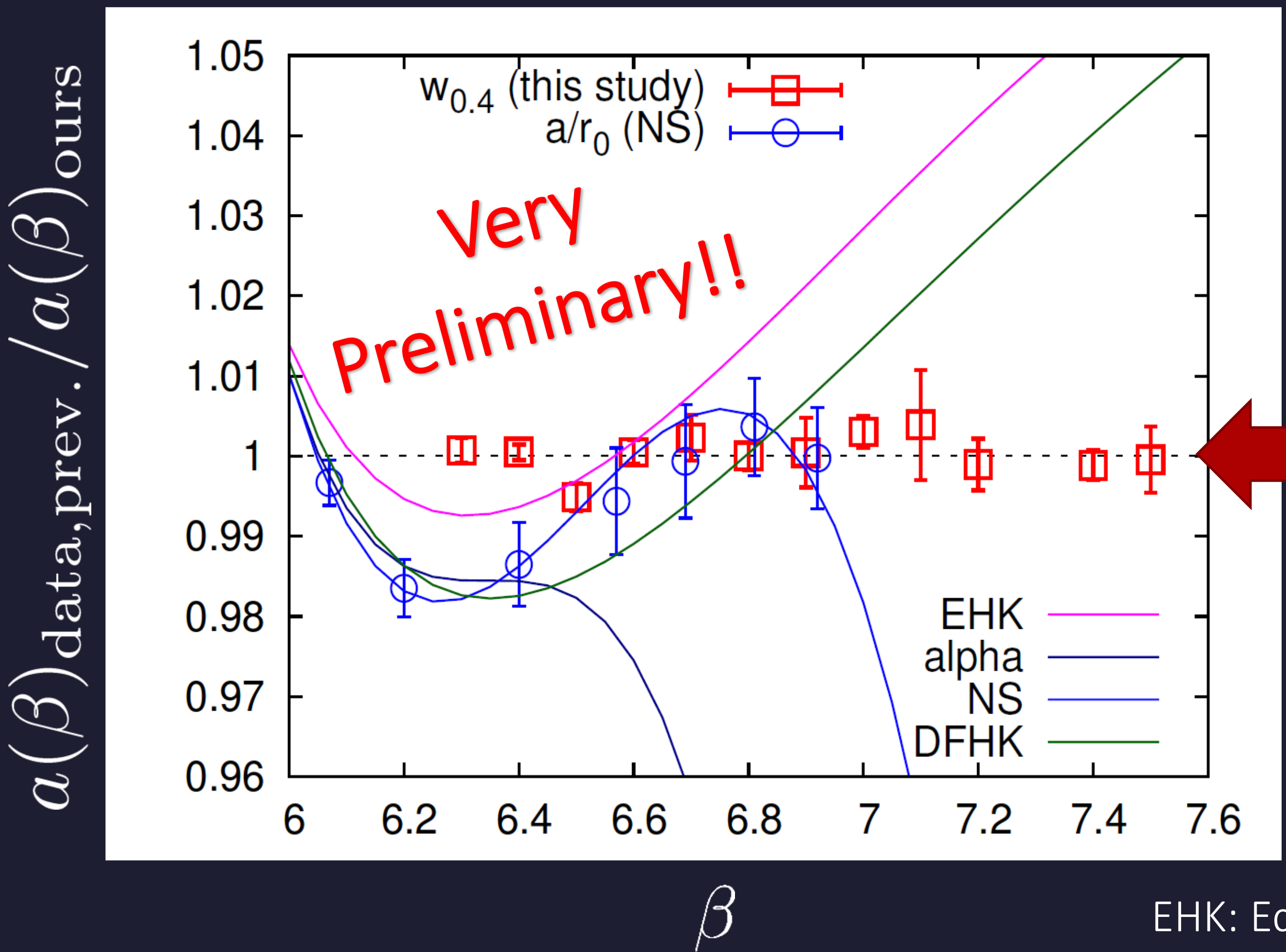


- SU(3) YM theory
- Wilson gauge action
- w_0 scaling

β	size	N_{conf}	β	size	N_{conf}
6.3	64^4	30	6.9	64^4	30
6.4	64^4	100	7.0	96^4	100
6.5	64^4	30	7.2	96^4	50
6.6	64^4	100	7.4	128^4	40
6.7	64^4	30	7.5	128^4	40
6.8	64^4	100			

each configuration is separated by 1000 gauge updates (HB+OR^5)

Comparison with Previous Studies



$w_{0.4}/r_0$ determined by 4 data points at $6.5<\beta<6.92$

scale determined by w_0 scaling

EHK: Edwards, Heller, Klassen, 1998
Alpha-Collaboration, 1998
NS: Necco, Sommer, 2002
DFHK: Durr, Fodor, Hoelbling, 2007

Small Flow Time Expansion of Operators and EMT

$$T_{\mu\nu}$$

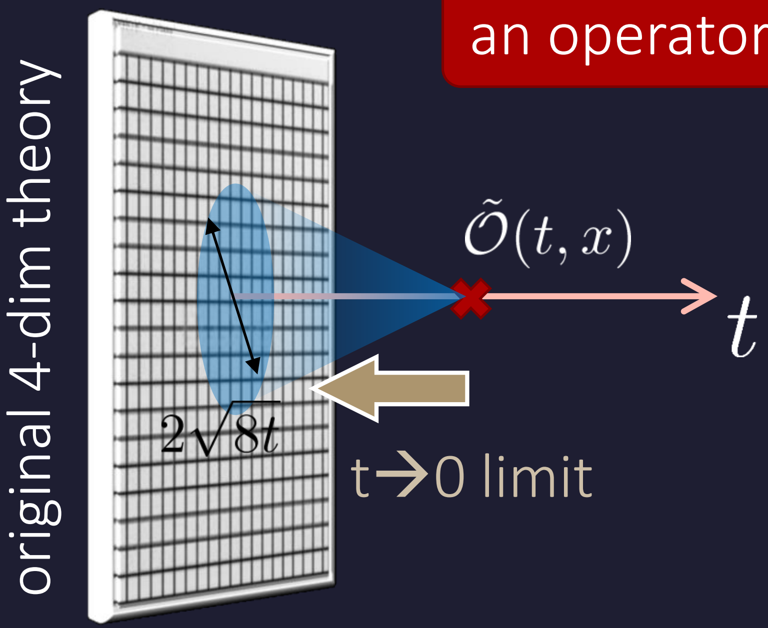
Operator Relation

Luescher, Weisz, 2011

$$\tilde{\mathcal{O}}(t,x) \underset{t\to 0}{\longrightarrow} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at t>0

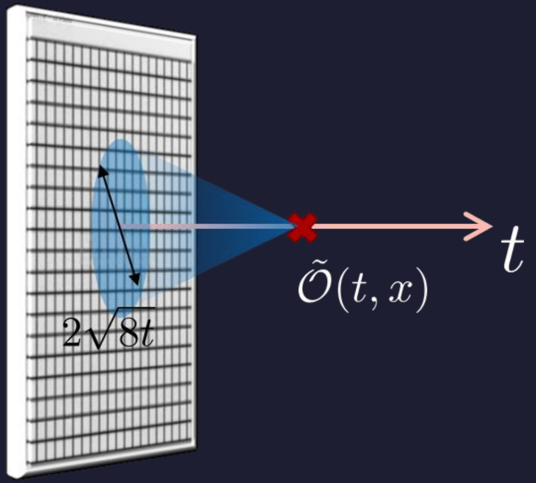
remormalized operators of original theory



Constructing EMT

Suzuki, 2013
DelDebbio,Patella,Rago,2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$



- **gauge-invariant dimension 4 operators**

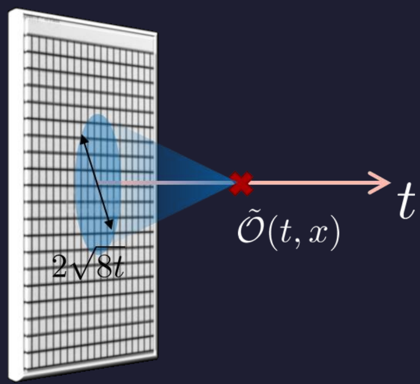
$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

Constructing EMT 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$



Suzuki coeffs.
$$\begin{cases} \alpha_U(t) = g^2\left[1 + 2b_0 s_1 g^2 + O(g^4)\right] \\ \alpha_E(t) = \dfrac{1}{2b_0}\left[1 + 2b_0 s_2 g^2 + O(g^4)\right] \end{cases}$$

$g = g(1/\sqrt{8t})$

$s_1 = 0.03296\ldots$

$s_2 = 0.19783\ldots$

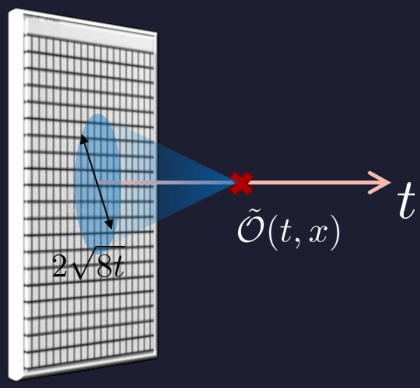
See also, Patella, Parallel7E, Thu.

Constructing EMT 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$



Suzuki coeffs. $\begin{cases} \alpha_U(t) = g^2\left[1 + 2b_0 s_1 g^2 + O(g^4)\right] \\ \alpha_E(t) = \dfrac{1}{2b_0}\left[1 + 2b_0 s_2 g^2 + O(g^4)\right] \end{cases}$

$g = g(1/\sqrt{8t})$

$s_1 = 0.03296\ldots$

$s_2 = 0.19783\ldots$

Remormalized EMT

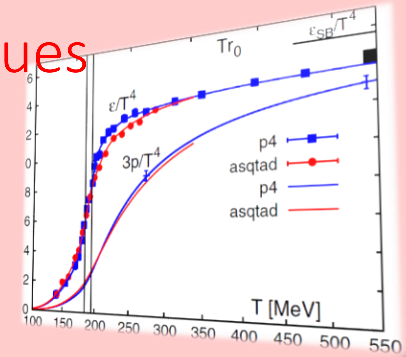
$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\text{subt.}}\right]$$

Numerical Analysis: thermodynamics

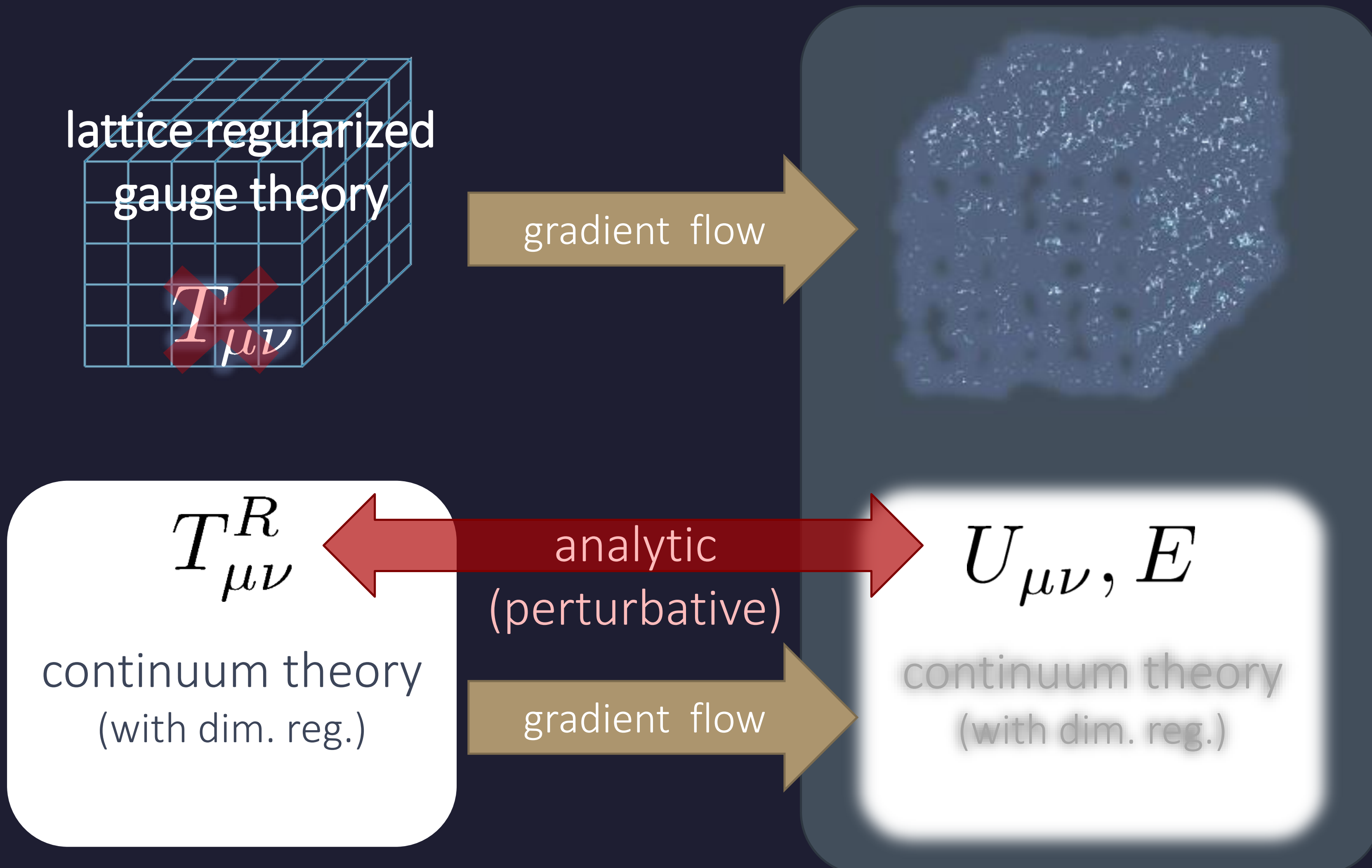
Thermodynamics

direct measurement of expectation values

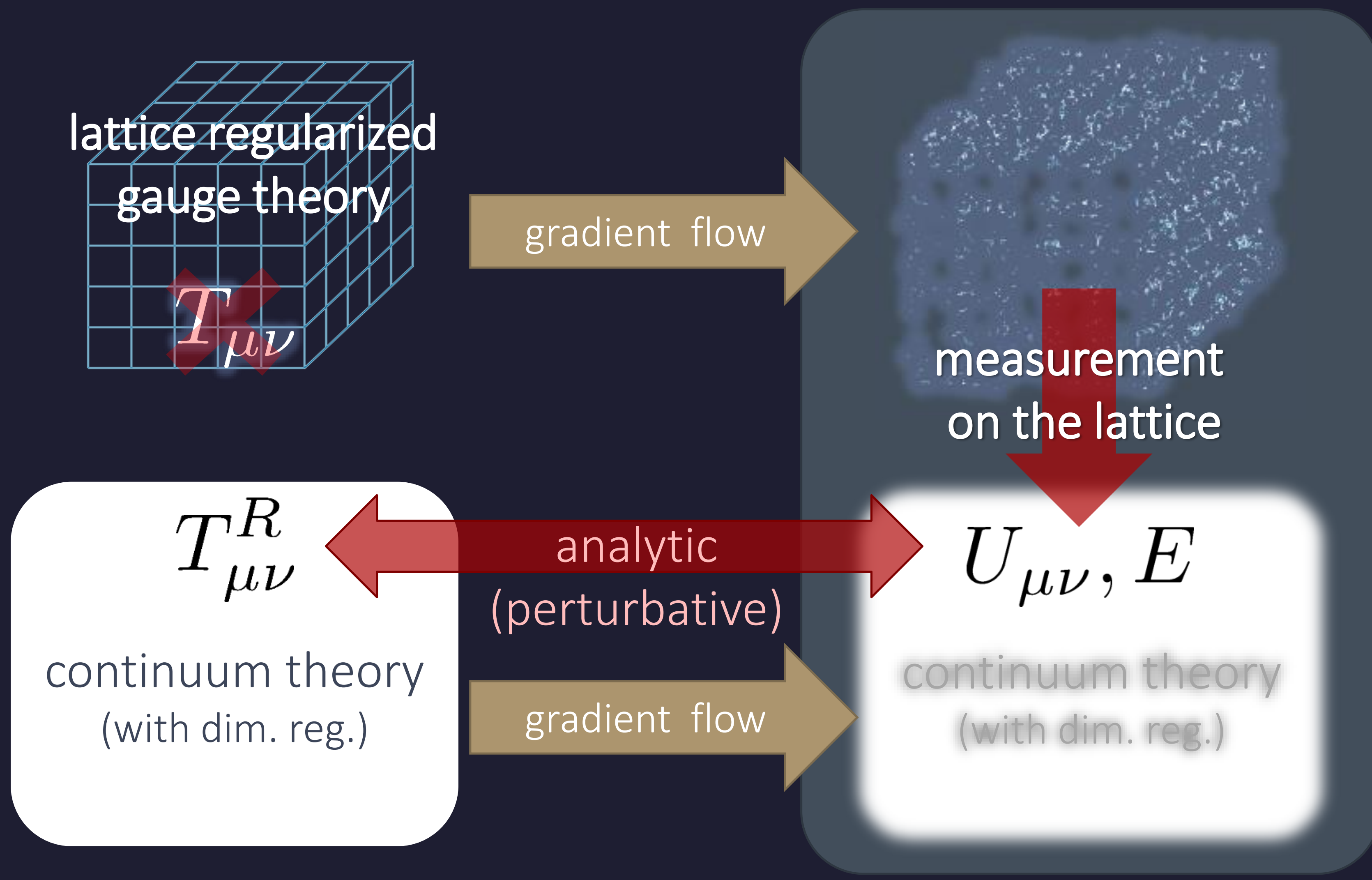
$$\langle T_{00} \rangle, \langle T_{ii} \rangle$$



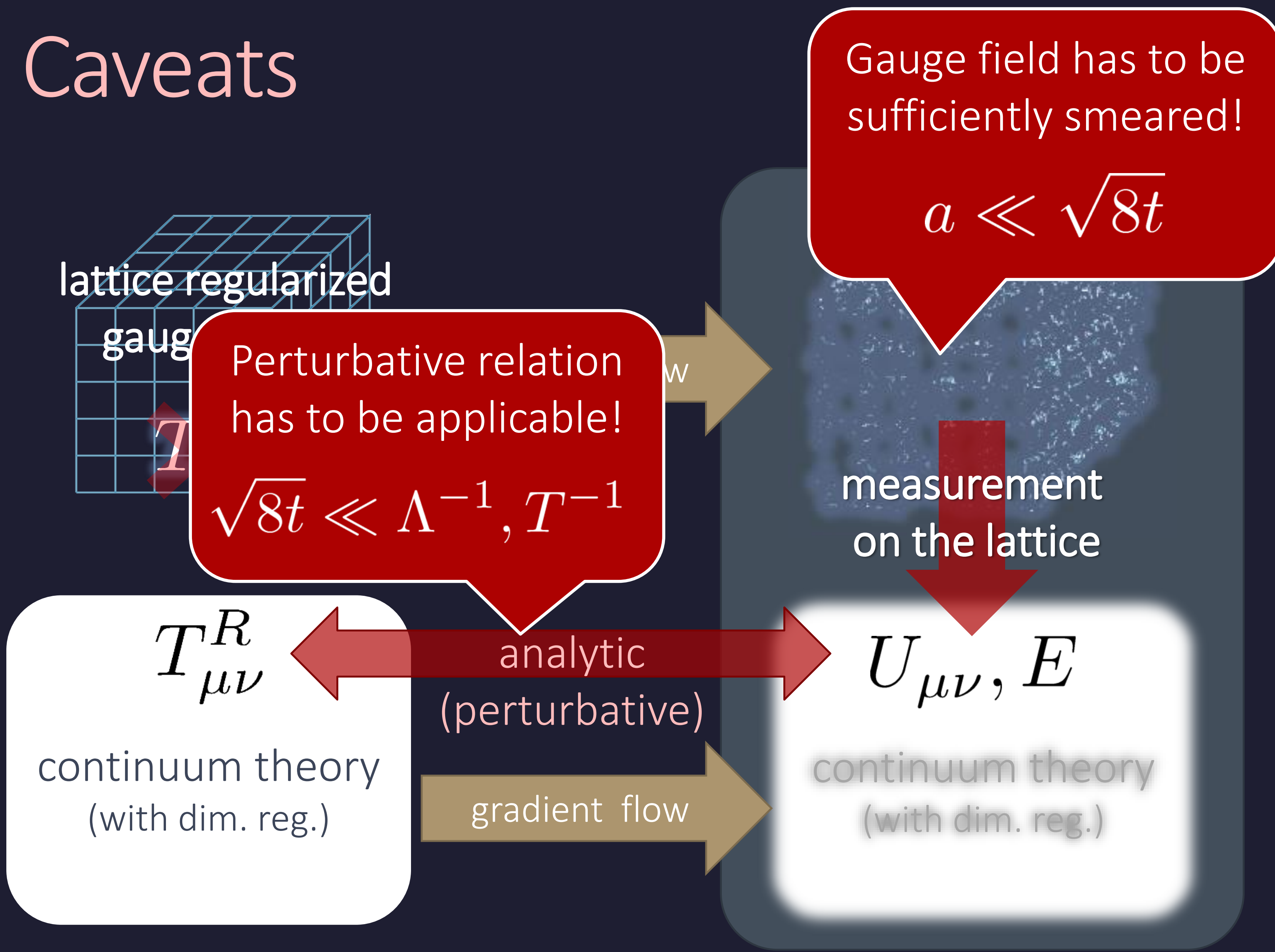
Gradient Flow Method



Gradient Flow Method

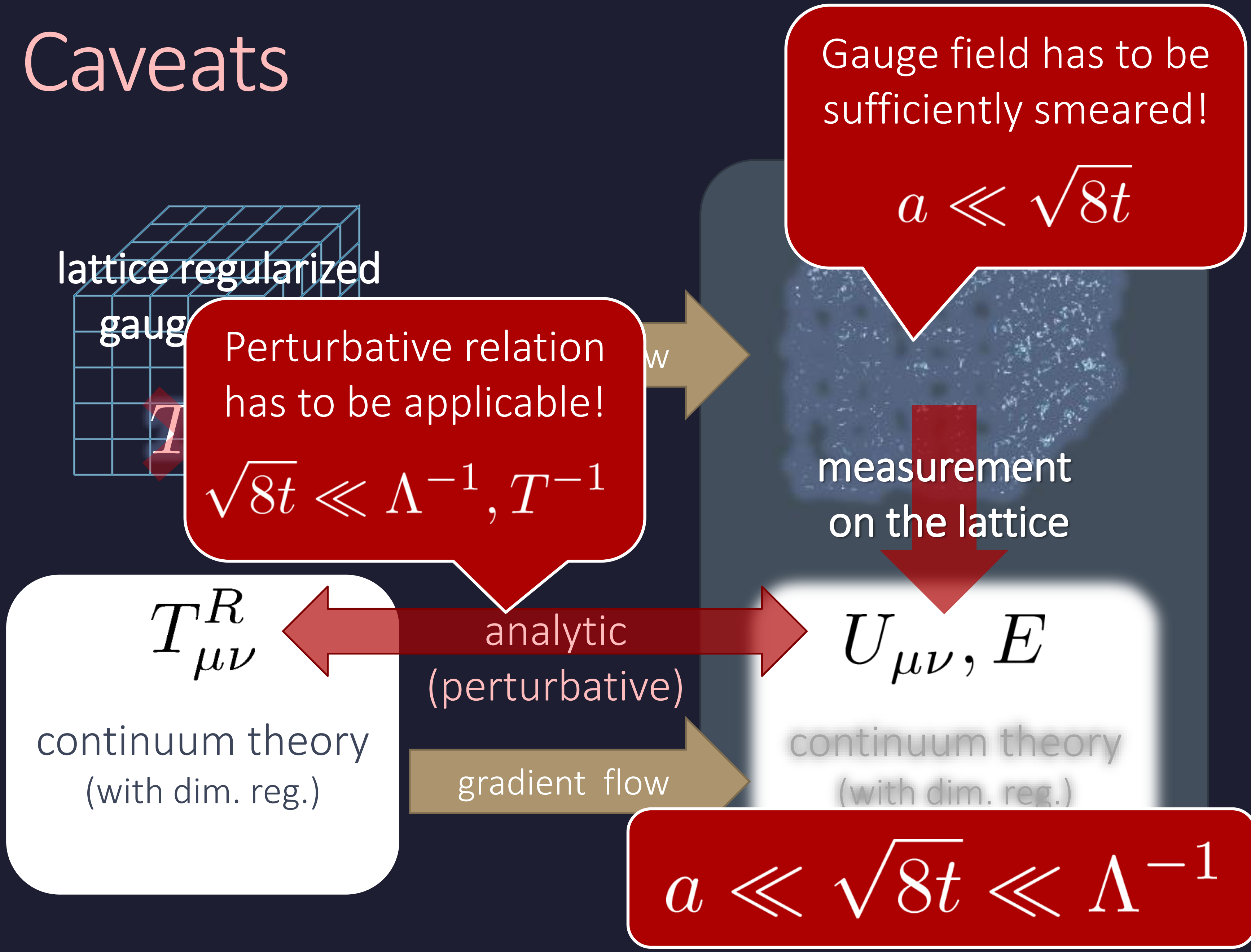


Caveats

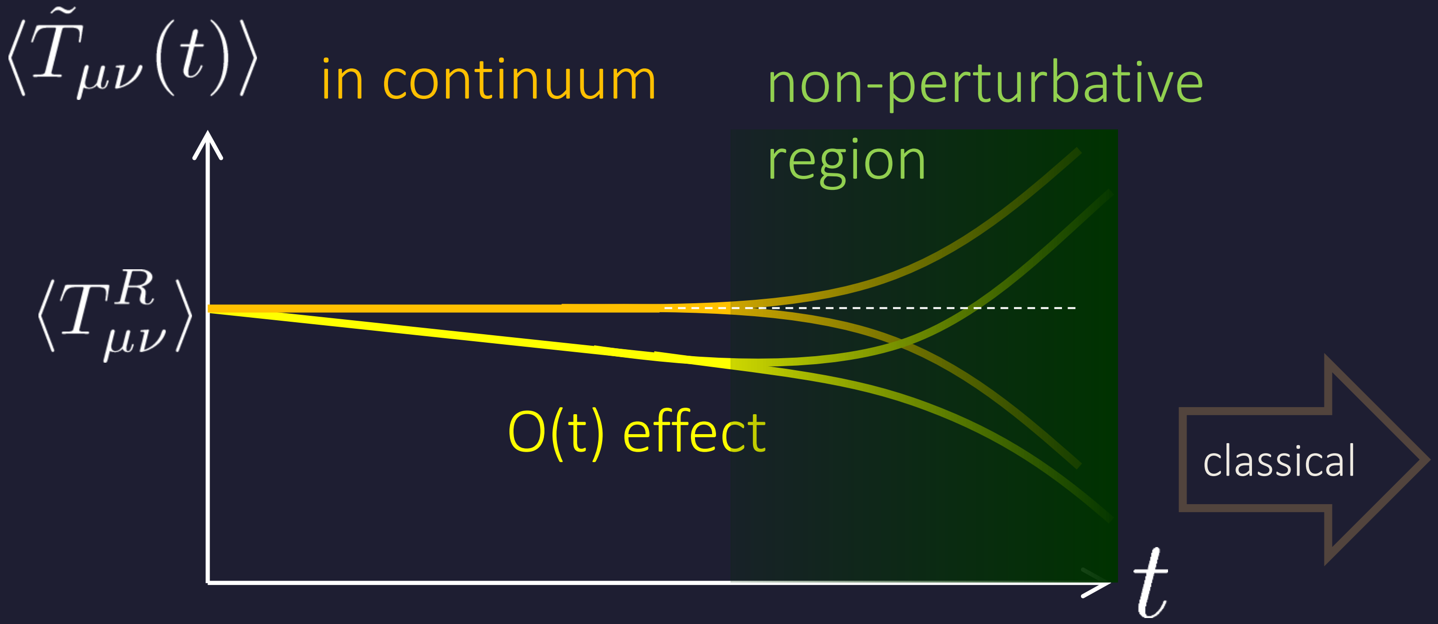


Gauge field has to be sufficiently smeared!
$a \ll \sqrt{8t}$
lattice regularized
Perturbative relation has to be applicable!
$\sqrt{8t} \ll \Lambda^{-1}, T^{-1}$
measurement on the lattice
$T^R_{\mu\nu}$
analytic (perturbative)
$U_{\mu\nu}, E$
continuum theory (with dim. reg.)
gradient flow
continuum theory (with dim. reg.)

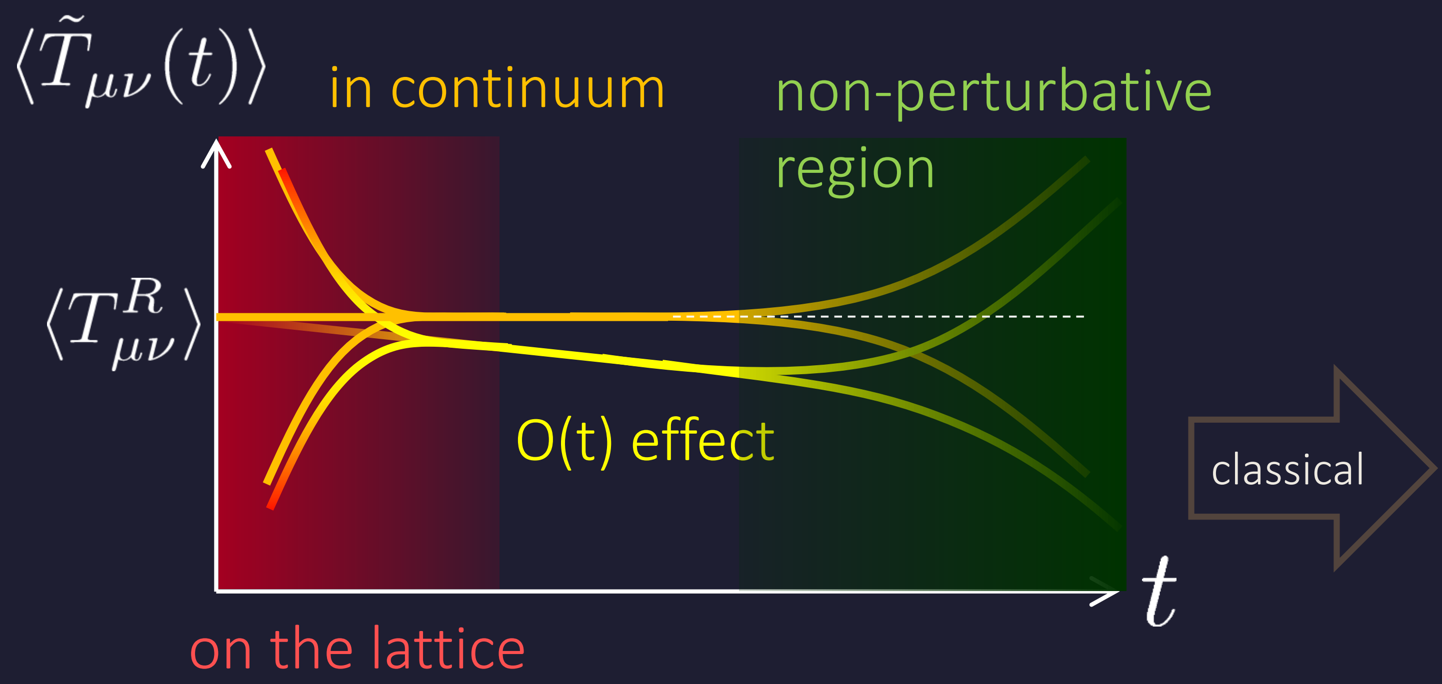
Caveats


Gauge field has to be sufficiently smeared!
$a \ll \sqrt{8t}$
lattice regularized
Perturbative relation has to be applicable!
$\sqrt{8t} \ll \Lambda^{-1}, T^{-1}$
measurement on the lattice
$T^R_{\mu\nu}$
analytic (perturbative)
$U_{\mu\nu}, E$
continuum theory (with dim. reg.)
gradient flow
continuum theory (with dim. reg.)
$a \ll \sqrt{8t} \ll \Lambda^{-1}$

$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\rm subt.} \qquad T^R_{\mu\nu} = \lim_{t\to 0} \tilde{T}_{\mu\nu}(t)$$



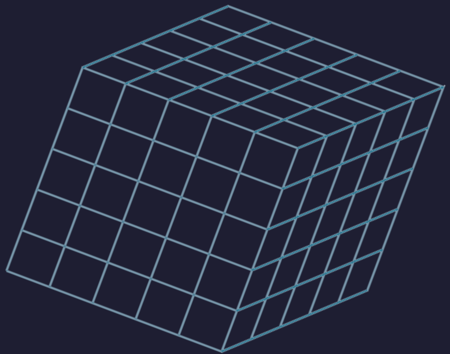
$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\text{subt.}} \qquad T^R_{\mu\nu} = \lim_{t\to 0} \tilde{T}_{\mu\nu}(t)$$



- t→0 limit with keeping t>>a^2

Numerical Simulation

- SU(3) YM theory
- Wilson gauge action



twice finer lattice!

Simulation 1

(arXiv:1312.7492)

- lattice size: $32^3 \mathrm{x} N_t$
- Nt = 6, 8, 10
- β = 5.89 – 6.56
- ~300 configurations

using SX8 @ RCNP
SR16000 @ KEK

Simulation 2

(new, preliminary)

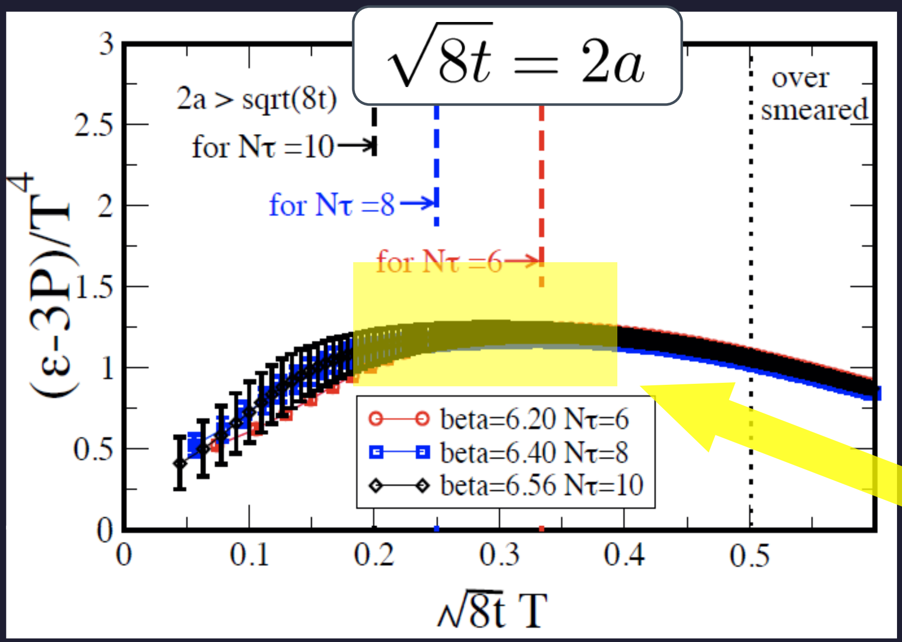
- lattice size: $64^3 \mathrm{x} N_t$
- Nt = 10, 12, 14, 16
- β = 6.4 – 7.4
- ~2000 configurations

using BlueGeneQ @ KEK
efficiency ~40%

ε-3p at T=1.65T$_c$

$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\text{subt.}}$$

$$T^R_{\mu\nu} = \lim_{t\to 0} \tilde{T}_{\mu\nu}(t)$$



Emergent plateau!

$$2a \lesssim \sqrt{8t} \lesssim 0.4T^{-1}$$

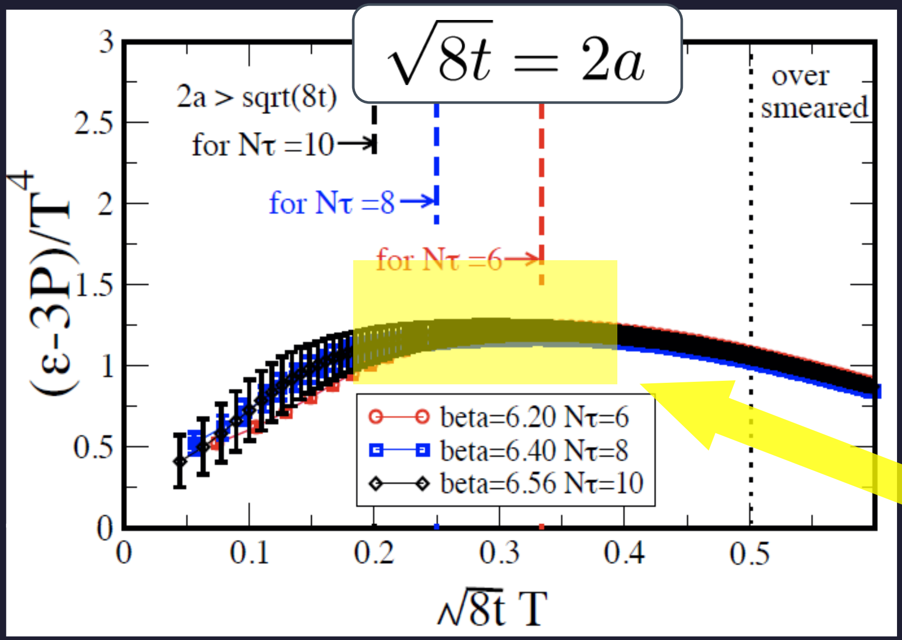
Nt=6,8,10
~300 confs.

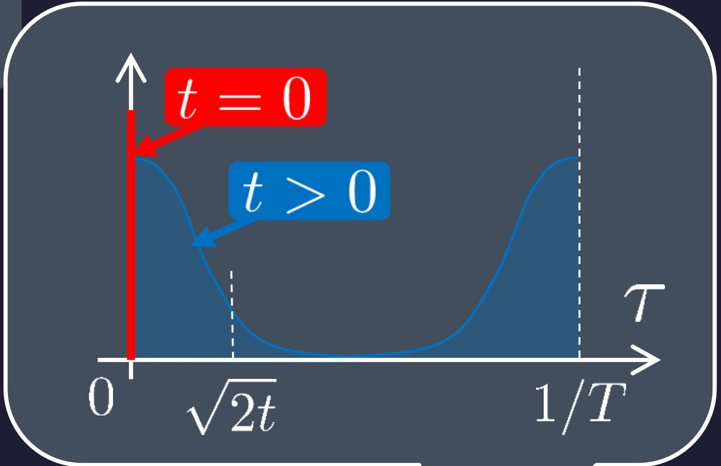
the range of t where the EMT formula is successfully used!

ε-3p at T=1.65T$_c$

$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\text{subt.}}$$

$$T^R_{\mu\nu} = \lim_{t\to 0} \tilde{T}_{\mu\nu}(t)$$





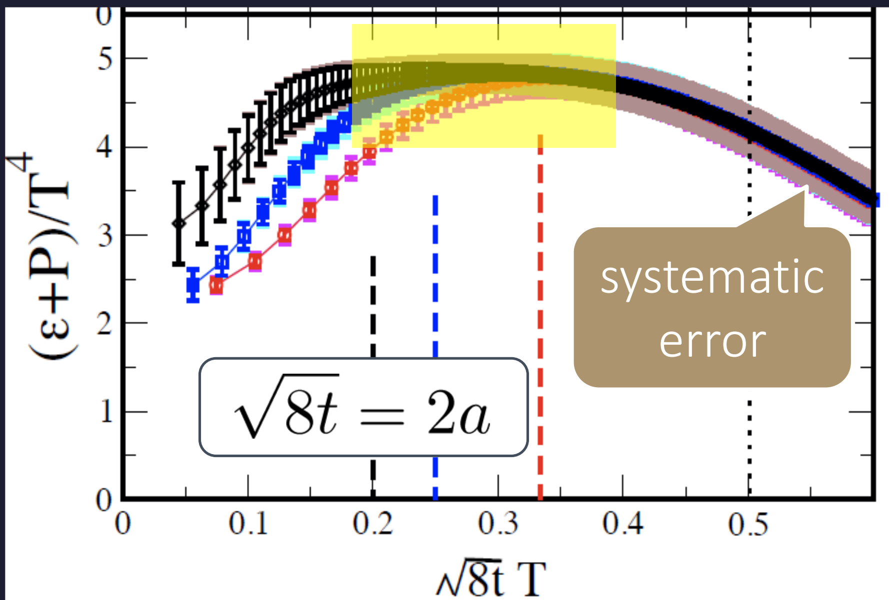
Emergent plateau!

$$2a \lesssim \sqrt{8t} \lesssim 0.4T^{-1}$$

Nt=6,8,10
~300 confs.

the range of t where the EMT formula is successfully used!

Entropy Density at T=1.65Tc



Emergent plateau!

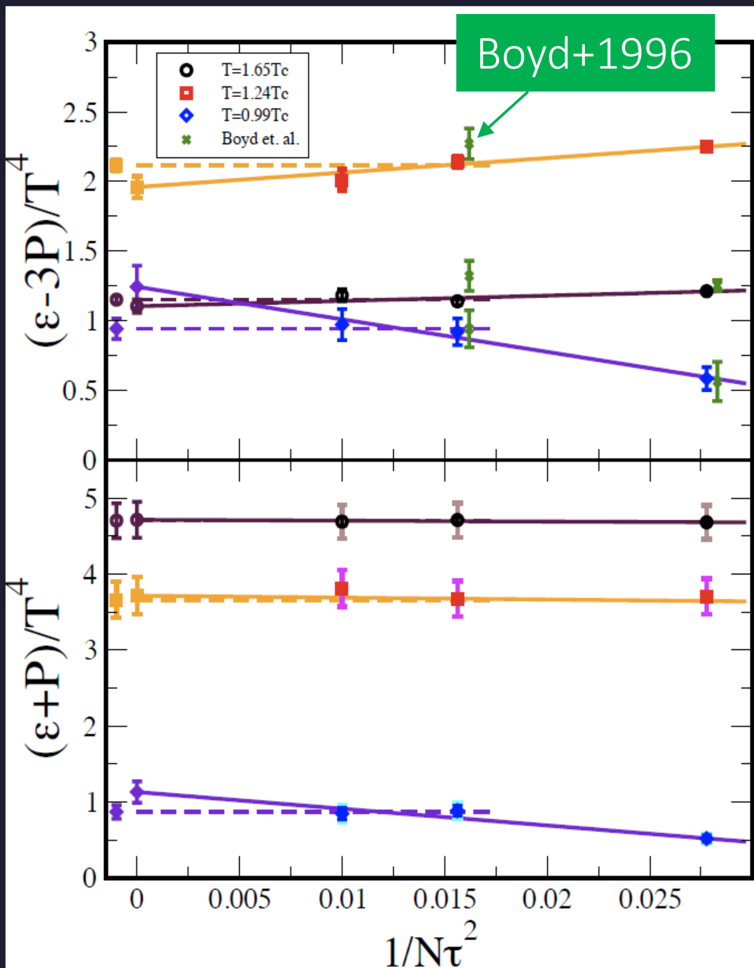
$$2a \lesssim \sqrt{8t} \lesssim 0.4T^{-1}$$

Nt=6,8,10
~300 confs.

Direct measurement of e+p on a given T!

NO integral / **NO** vacuum subtraction

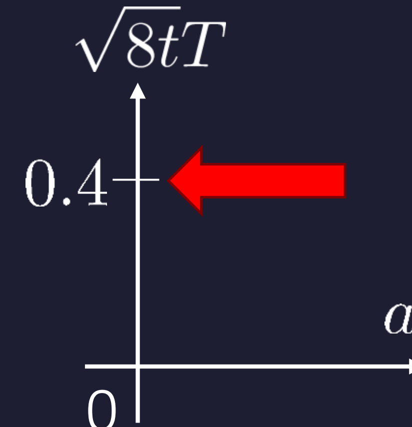
Continuum Limit



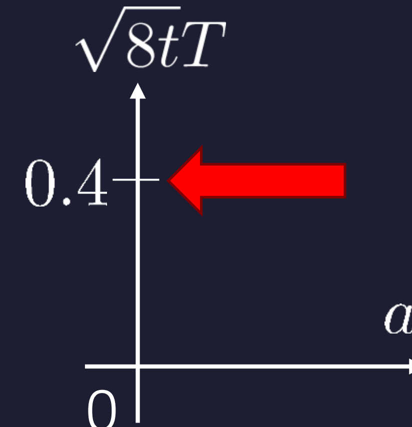
32^3xNt

Nt = 6, 8, 10

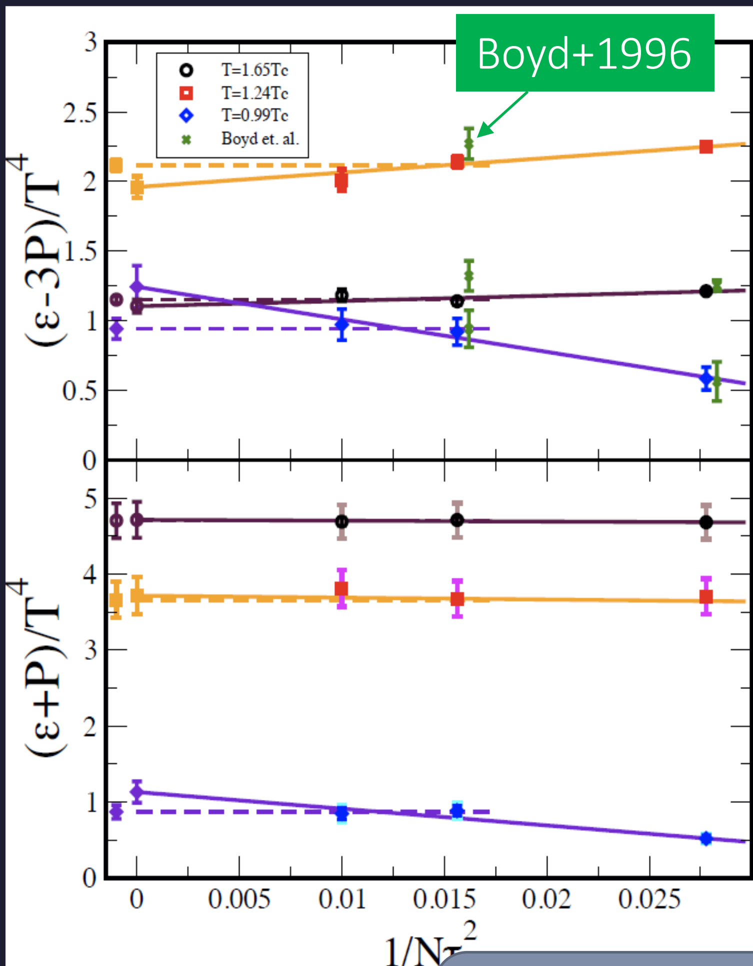
T/Tc=0.99, 1.24, 1.65

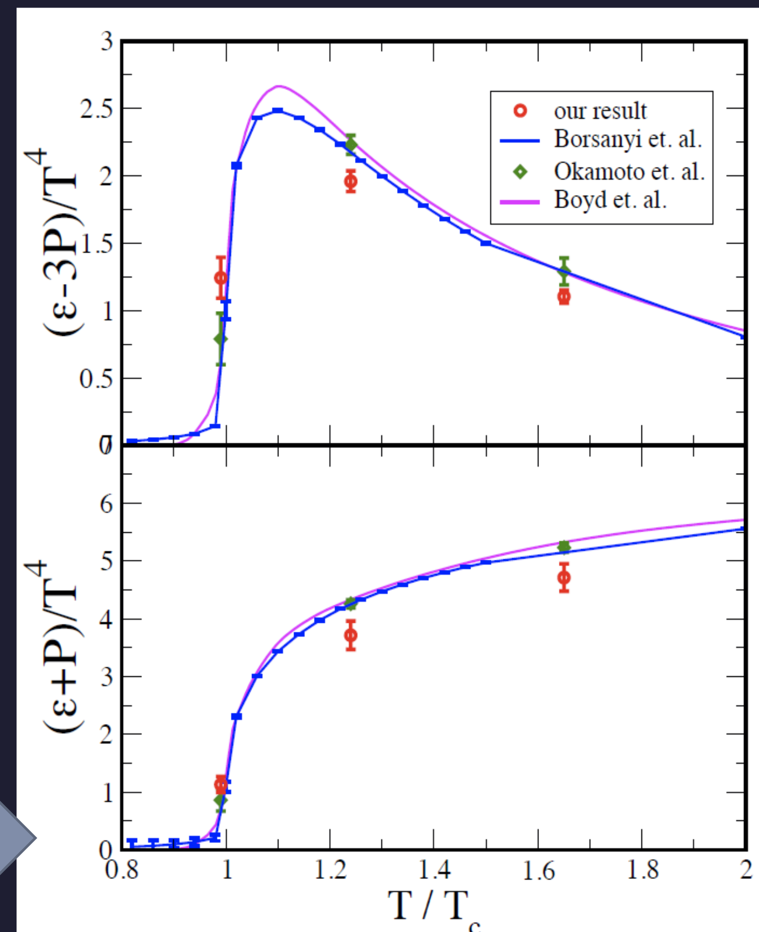


Continuum Limit



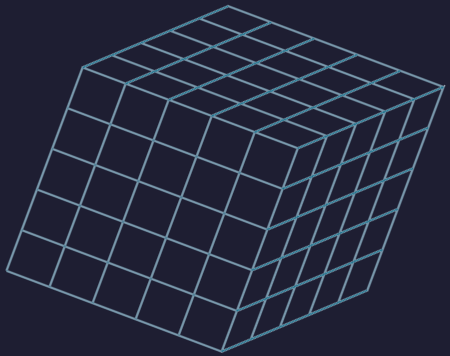
32^3xNt
Nt = 6, 8, 10
T/Tc=0.99, 1.24, 1.65





Comparison with previous studies

Numerical Simulation



- SU(3) YM theory
- Wilson gauge action

twice finer lattice!

Simulation 1
(arXiv:1312.7492)

- lattice size: $32^3 \mathrm{x} N_t$
- Nt = 6, 8, 10
- β = 5.89 – 6.56
- ~300 configurations

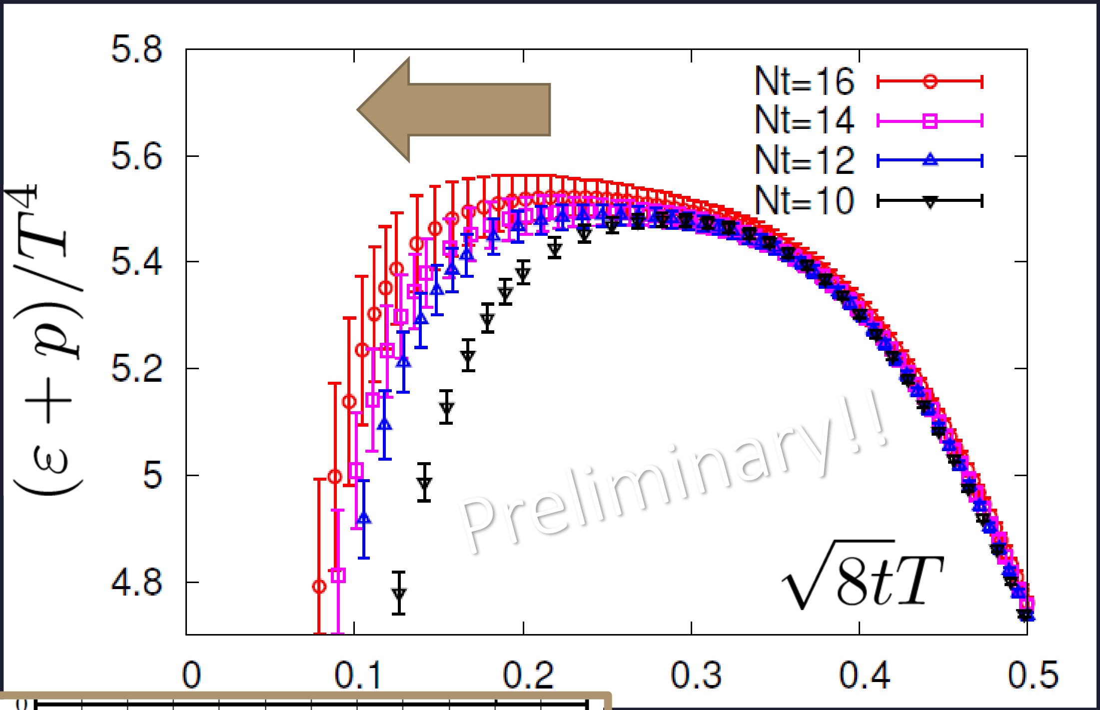
using SX8 @ RCNP
SR16000 @ KEK

Simulation 2
(new, preliminary)

- lattice size: $64^3 \mathrm{x} N_t$
- Nt = 10, 12, 14, 16
- β = 6.4 – 7.4
- ~2000 configurations

using BlueGeneQ @ KEK
efficiency ~40%

Entropy Density on Finer Lattices

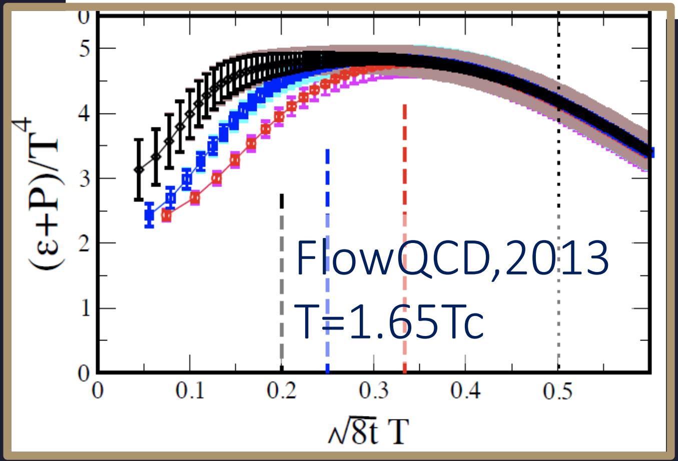


T = 2.31Tc

64^3xNt

Nt = 10, 12, 14, 16

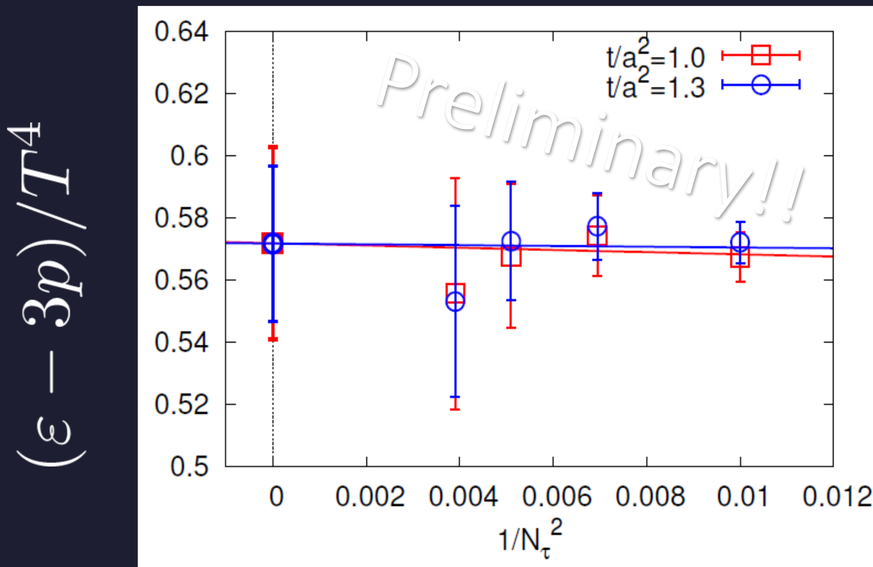
2000 confs.

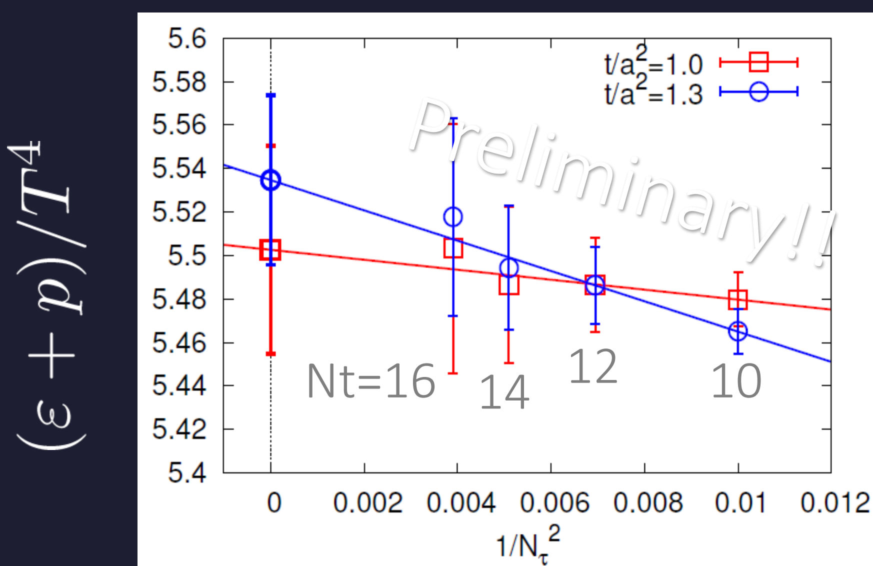


- The wider plateau on the finer lattices
- Plateau may have a nonzero slope

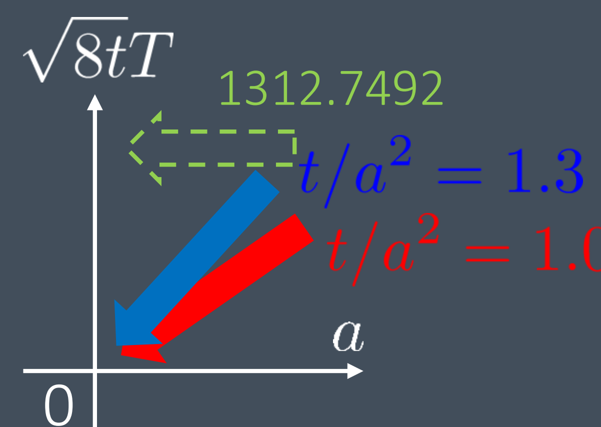
Continuum Extrapolation

- T=2.31Tc
- 2000 confs
- Nt = 10 ~ 16





a→0 limit with fixed t/a^2



Continuum extrapolation is stable

Numerical Analysis: EMT Correlators

Fluctuations and Correlations

viscosity, specific heat, ...

$$\eta = \int_0^\infty dt \langle T_{12}; T_{12} \rangle$$

$$c_V \sim \langle \delta T_{00}^2 \rangle$$

EMT Correlator

- Kubo Formula: T_{12} correlator ←→ shear viscosity

$$\eta = \int_0^\infty dt \int_0^{1/T} d\tau \int d^3x \langle T_{12}(x, -i\tau) T_{12}(0,t) \rangle$$

  - Hydrodynamics describes long range behavior of $T_{\mu\nu}$

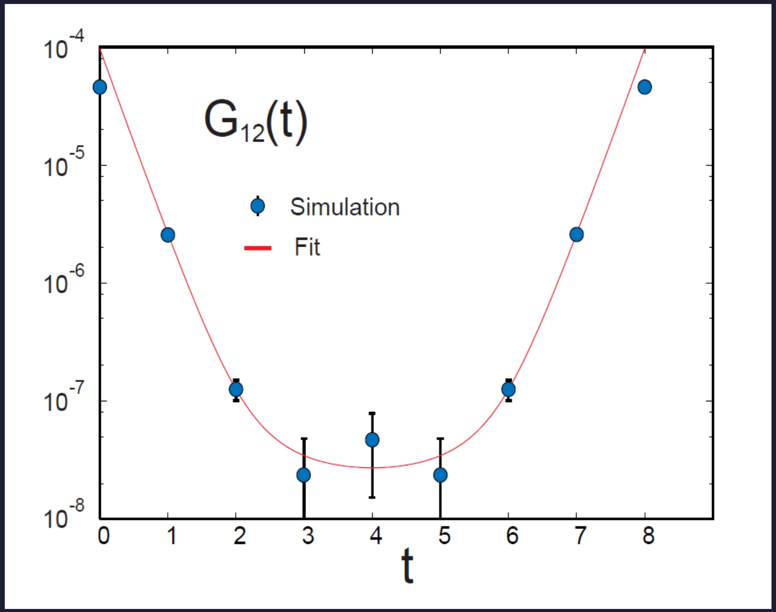
- Energy fluctuation ←→ specific heat

$$c_V = \frac{\langle \delta E^2 \rangle}{VT^2}$$

EMT Correlator : Noisy…

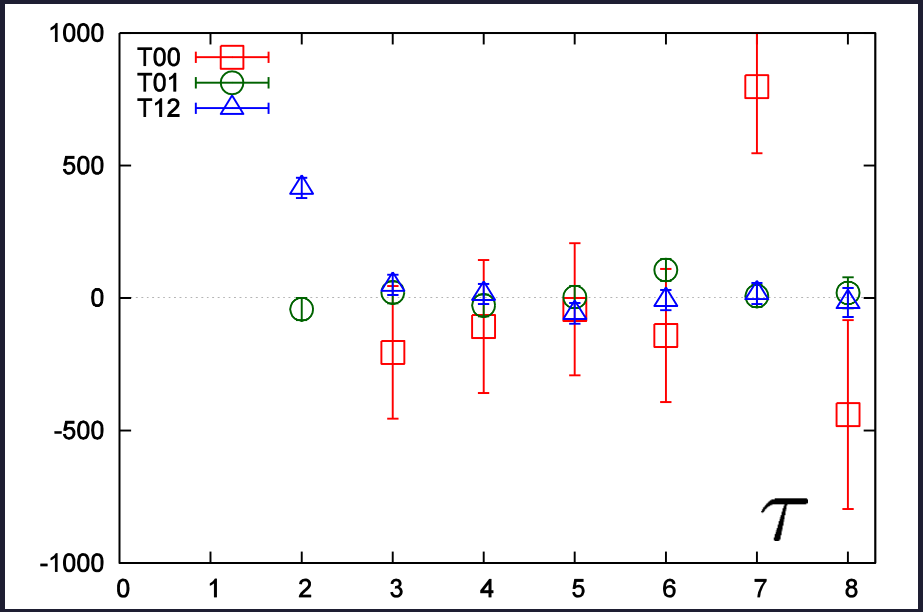
With naïve EMT operators

$$\langle T_{12}(\tau) T_{12}(0) \rangle$$



Nakamura, Sakai, PRL,2005
N_t=8
improved action
~10^6 configurations

$$\langle T_{\mu\nu}(\tau) T_{\mu\nu}(0) \rangle$$

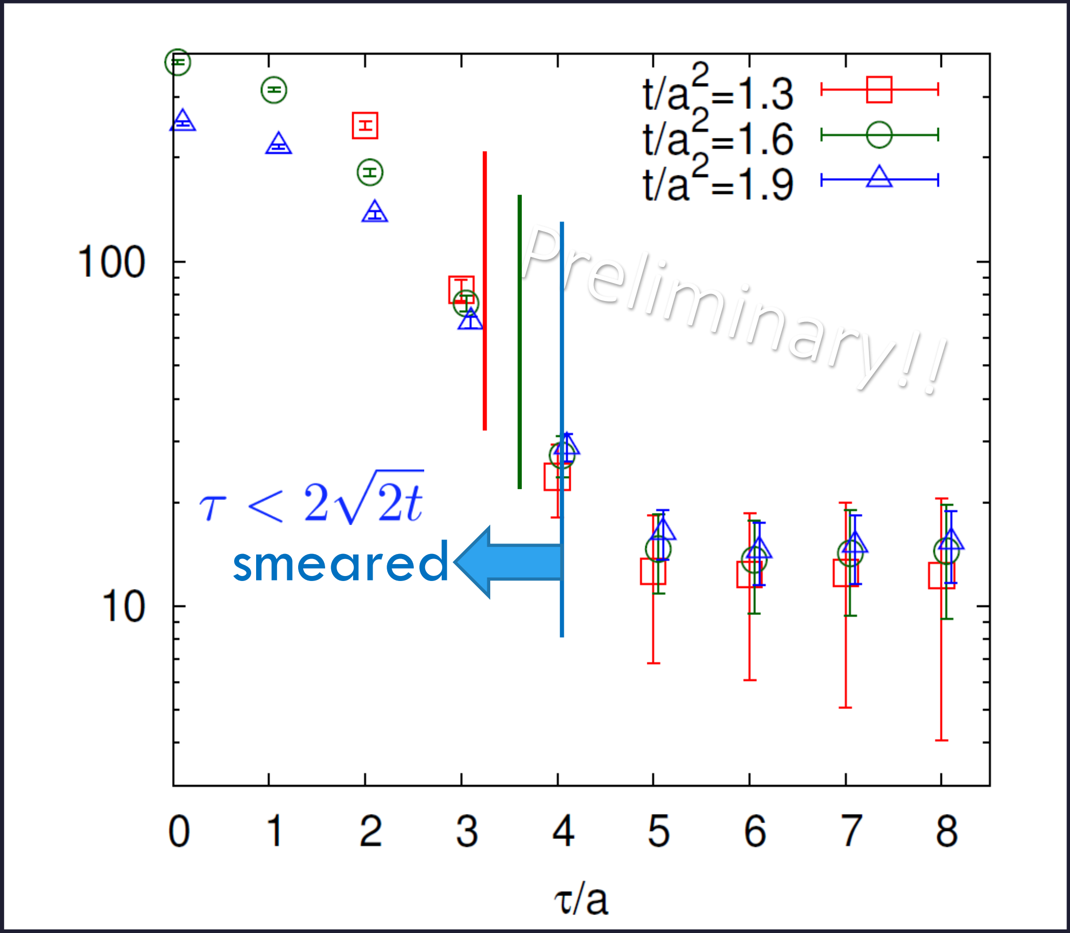


Nt=16
standard action
5x10^4 configurations

… no signal

Energy Correlation Function

$\langle T_{00}(\tau) T_{00}(0) \rangle / T^5$



T=2.31Tc

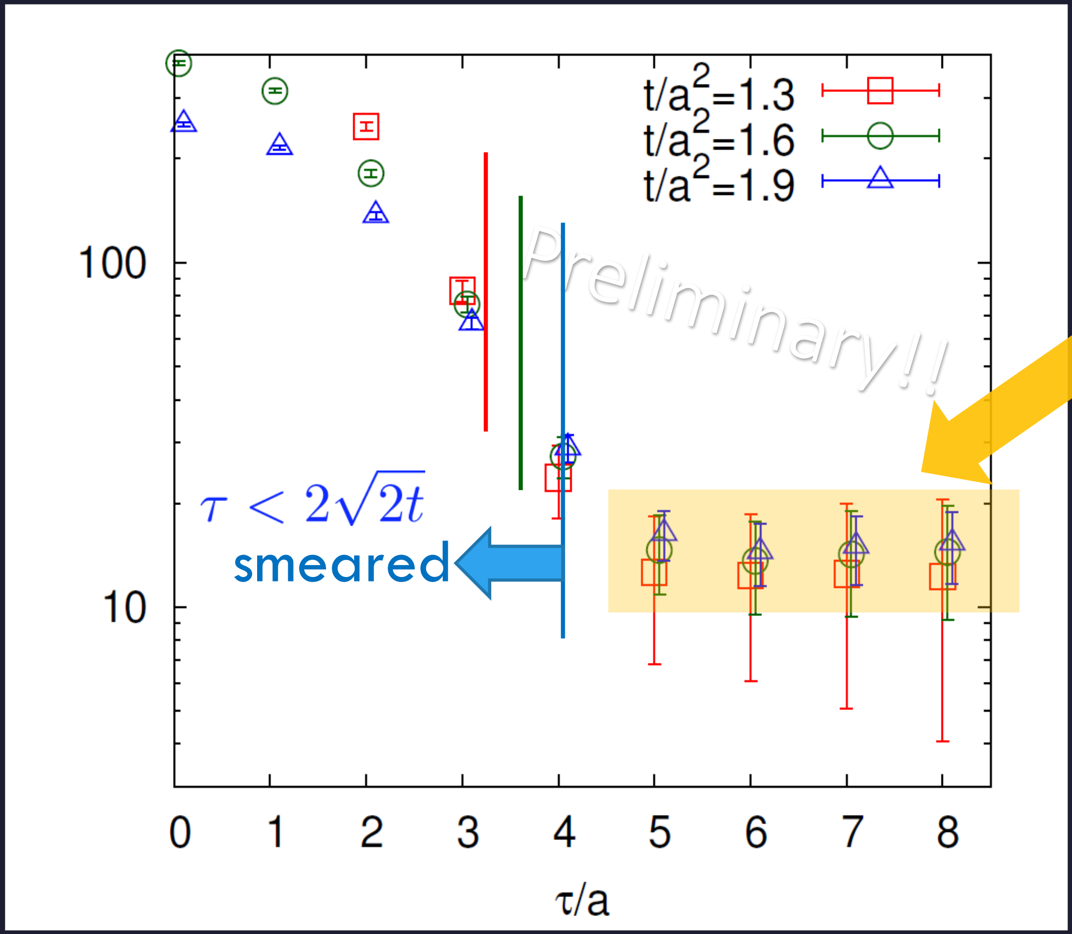
b=7.2, Nt=16

2000 confs

p=0 correlator

Energy Correlation Function

$\langle T_{00}(\tau)T_{00}(0)\rangle/T^5$



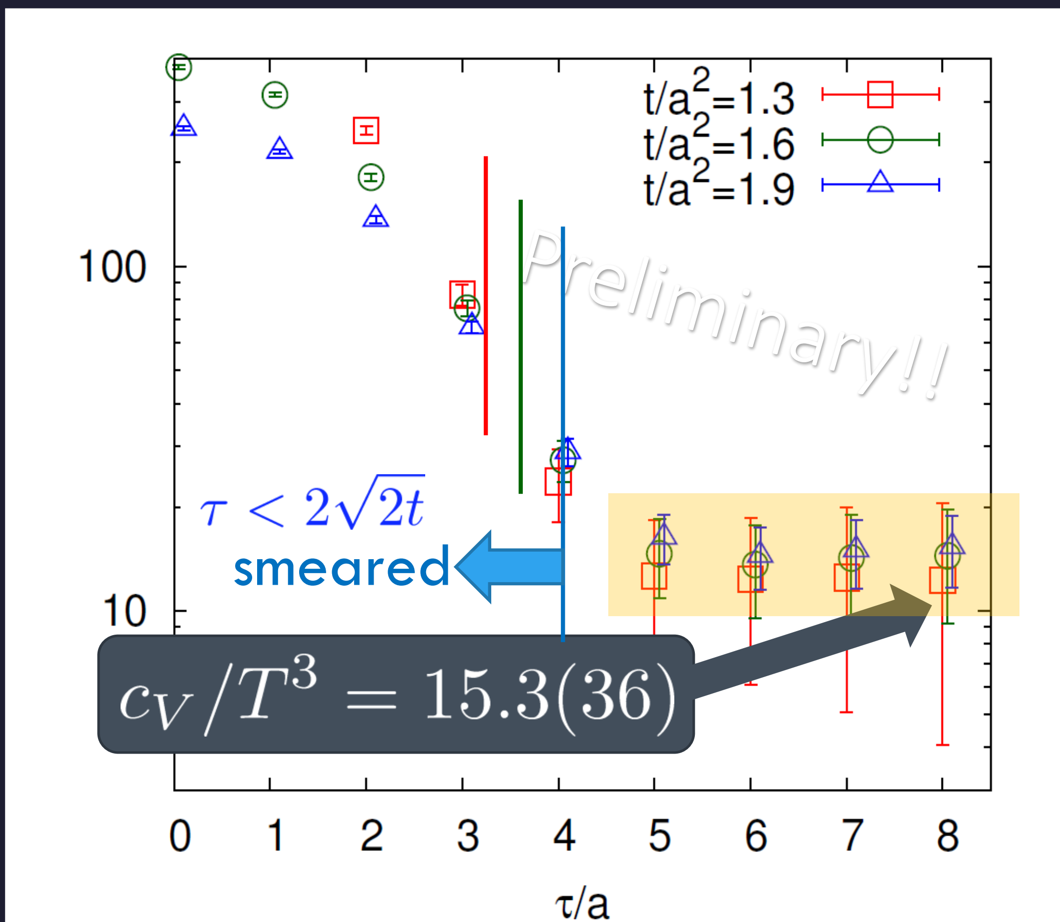
T=2.31Tc
b=7.2, Nt=16
2000 confs
p=0 correlator

- τ independent const.
  → energy conservation

Energy Correlation Function

$$\langle T_{00}(\tau) T_{00}(0) \rangle / T^5$$

T=2.31Tc
b=7.2, Nt=16
2000 confs
p=0 correlator



- specific heat

$$c_V = \frac{\langle \delta E^2 \rangle}{VT^2}$$

→ Novel approach to measure specific heat!

Gavai, Gupta, Mukherjee, 2005

$$c_V/T^3 = 15(1) \quad T/T_c = 2$$
$$= 18(2) \quad T/T_c = 3$$

differential method / cont lim.

Summary

$$T^{R}_{\mu\nu}(x)$$

Summary

EMT formula from gradient flow

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\mathrm{subt.}}\right]$$

This formula can successfully define and calculate the EMT on the lattice

It provides us with novel approaches to measure various observables on the lattice!

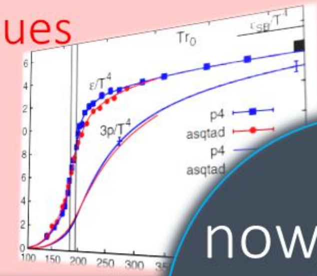
This method is direct, intuitive and less noisy

Many Future Studies!!

Thermodynamics

direct measurement of expectation values

$\langle T_{00} \rangle, \langle T_{ii} \rangle$



Fluctuations and Correlations

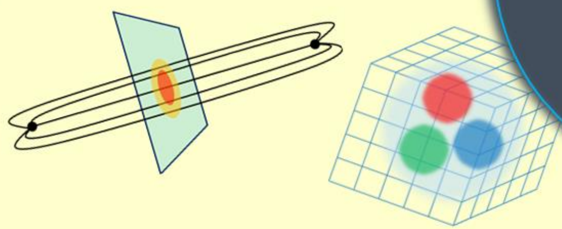
viscosity, specific heat, ...

$c_V \sim \langle \delta T_{00}^2 \rangle$

$\eta = \langle T_{12}; T_{12} \rangle$

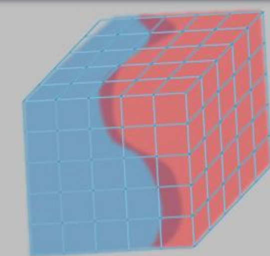
now we have

$T_{\mu\nu}$



- confinement string
- EM distribution in hadrons

Hadron Structure



- vacuum configuration
- mixed state on 1st transition

Vacuum Structure

Other observables

full QCD Makino,Suzuki,2014

non-pert. improvement Patella 7E(Thu)

O(a) improvement

Nogradi, 7E(Thu); Sint, 7E(Thu)

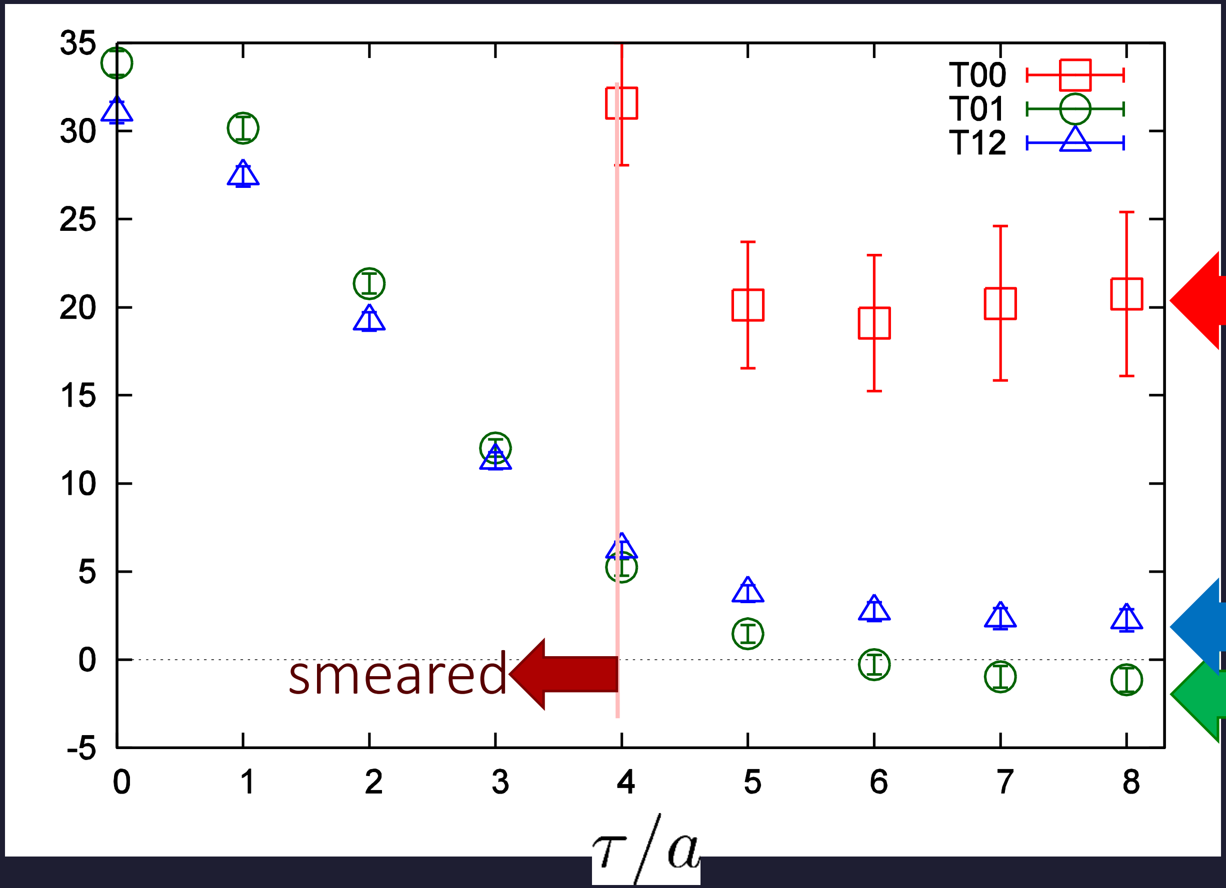
Monahan, 7E(Thu)

and etc.

Correlation Function

$$C_{\mu\nu}(\tau) = \int d^3x \langle T_{\mu\nu}(x,\tau) T_{\mu\nu}(0,0) \rangle$$

64^3x16
β=7.2 (T~2.3Tc)
1200 confs
t/a^2=1.9



$C_{44}(\tau)$:constant
←conservation law!

$$\partial_\tau \langle \delta E(\tau) \delta E(0) \rangle = 0$$
$$(\text{for } \tau \neq 0)$$

$C_{12}(\tau)$

$C_{41}(\tau)$
negative← $i^2=-1$