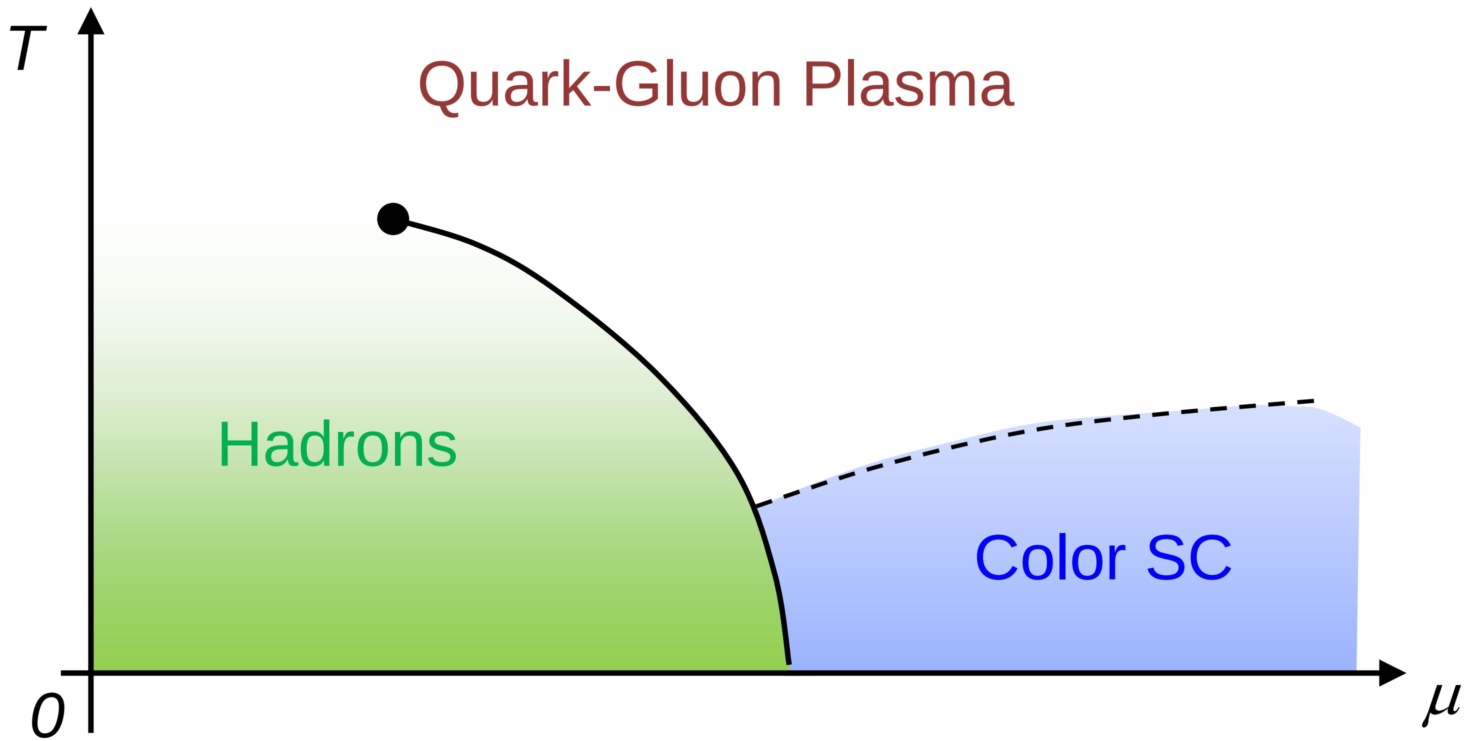
Thermal Fluctuations in Heavy Ion Collisions

Masakiyo Kitazawa

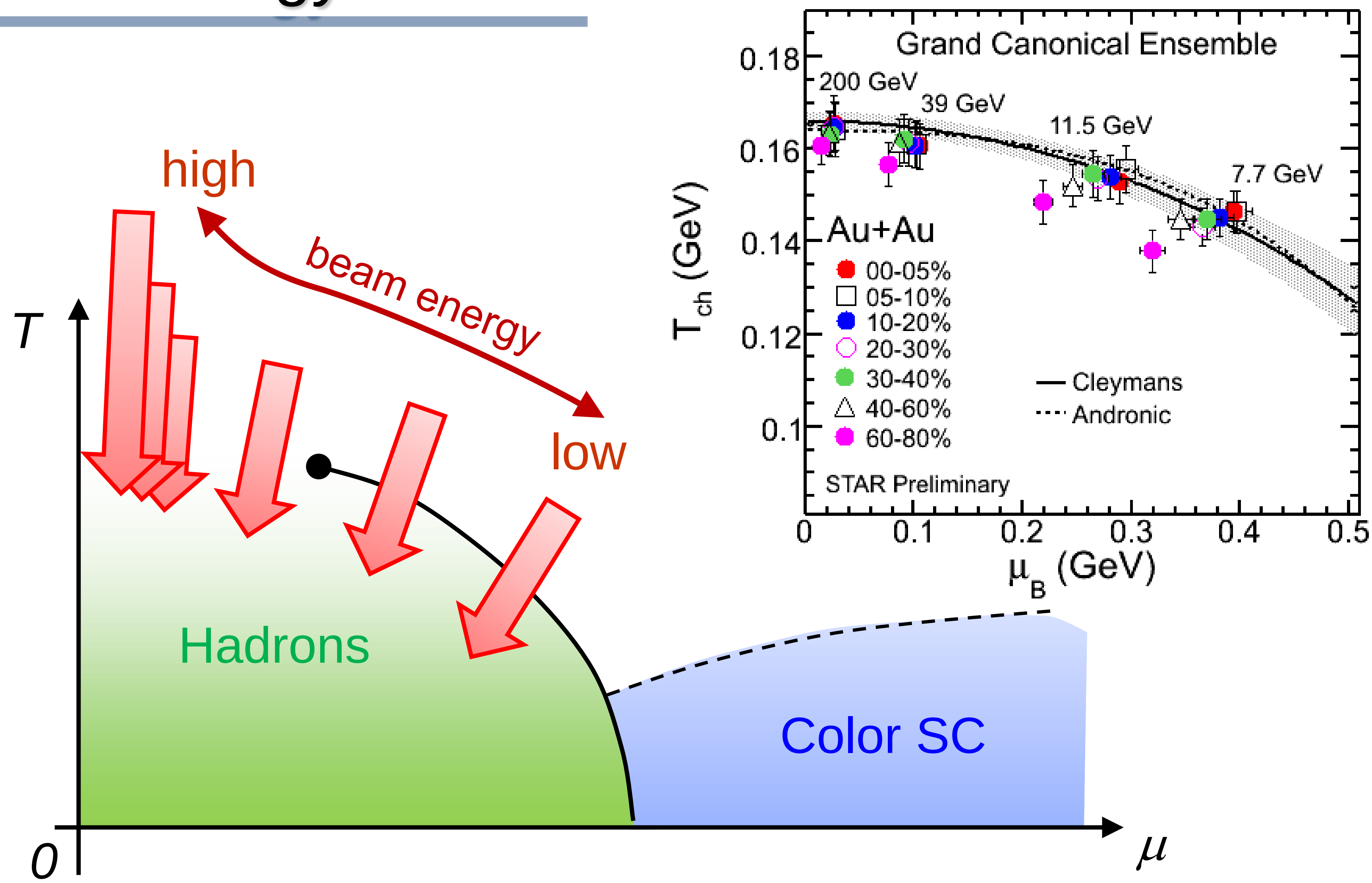
(Osaka U.)

Beam-Energy Scan



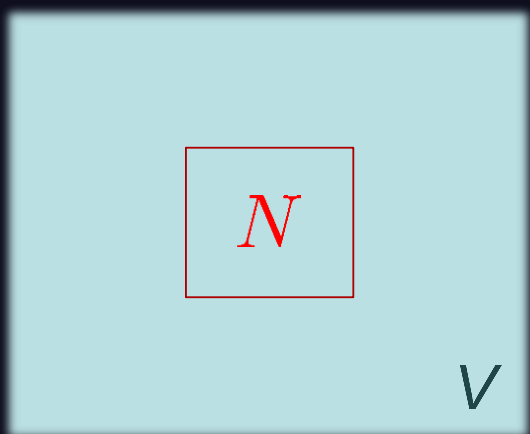
Beam-Energy Scan

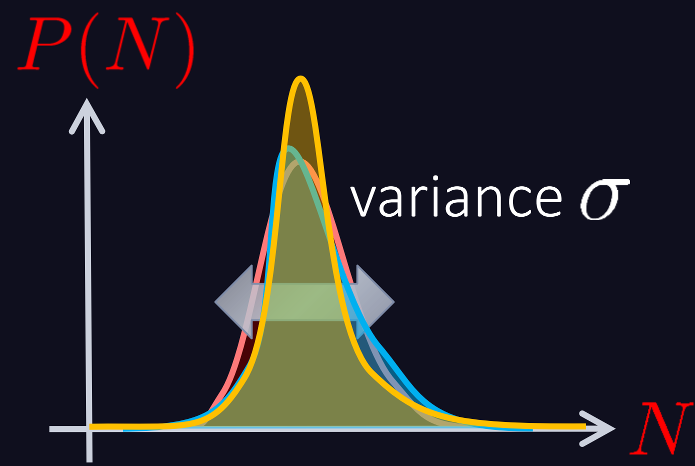
STAR 2012



Bulk (Thermal) Fluctuations

Observables in equilibrium are fluctuating!





$$\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$$

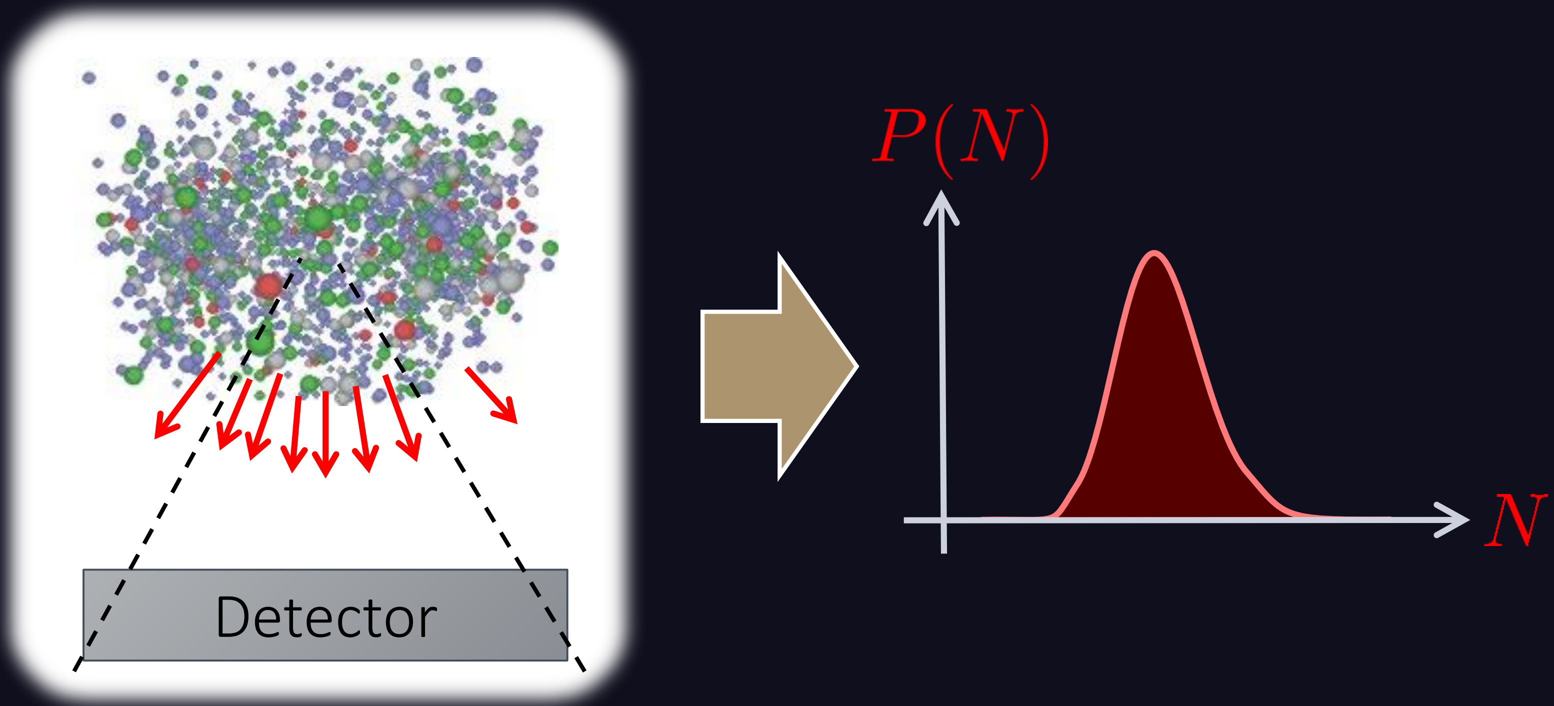
Gaussian

$$S = \frac{\langle \delta N^3 \rangle}{\sigma^3}$$

$$\kappa = \frac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$$

non-Gaussianity

Event-by-Event Analysis



Pioneering studies on σ:

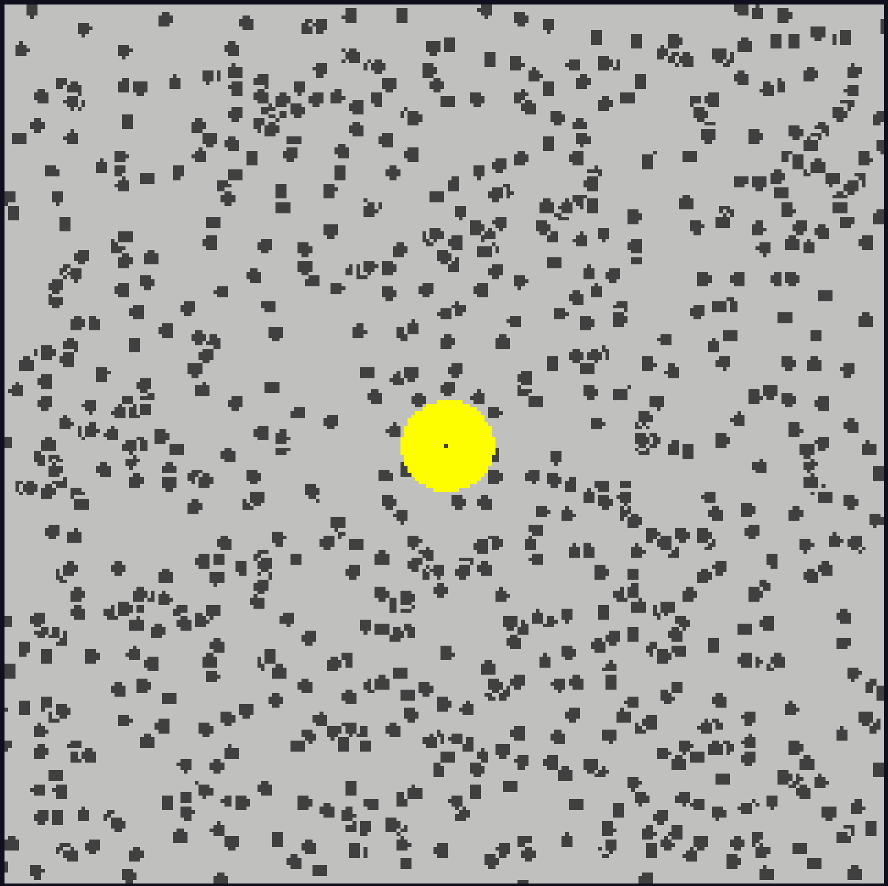
- Search of QCD critical point Stephanov,Rajagopal,Shuryak,PRL(1998)
- Quark deconfinement Asakawa,Heinz,Muller PRL; Jeon,Koch PRL(2000)

My Messages

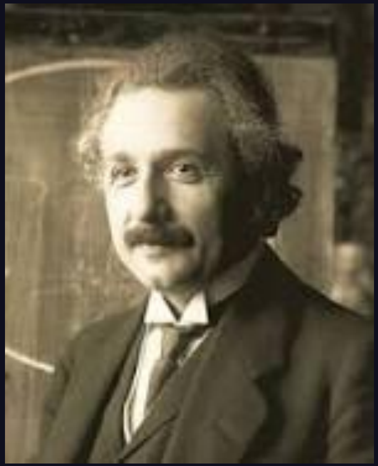
- ☐ Fluctuations are invaluable observables in HIC
- ☐ But, we must understand them in more detail
- ☐ It’s possible, interesting, and important

Why Fluctuations?

Brownian Motion



from Wikipedia

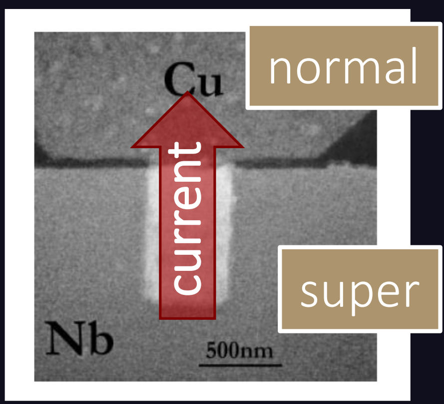


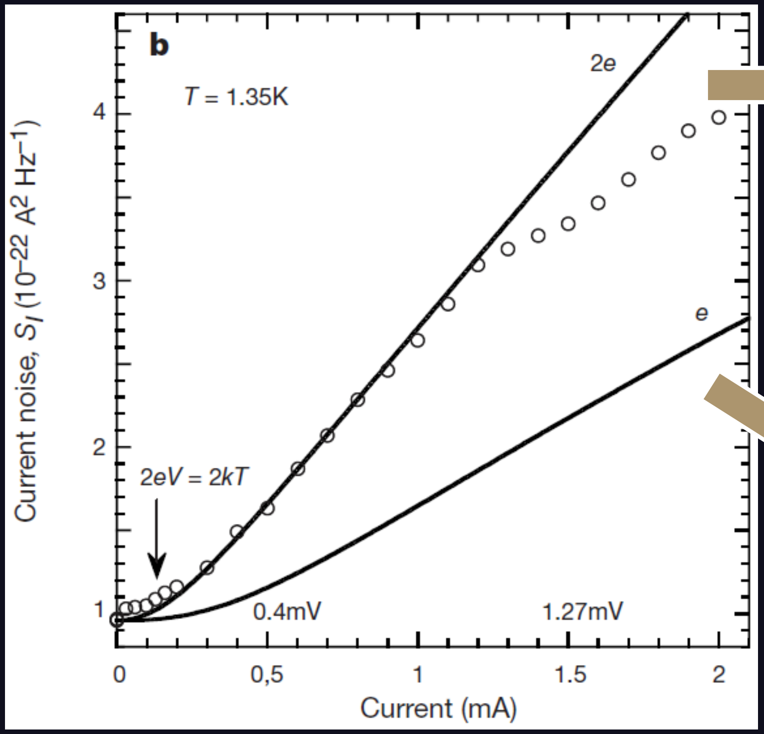
A. Einstein
1905

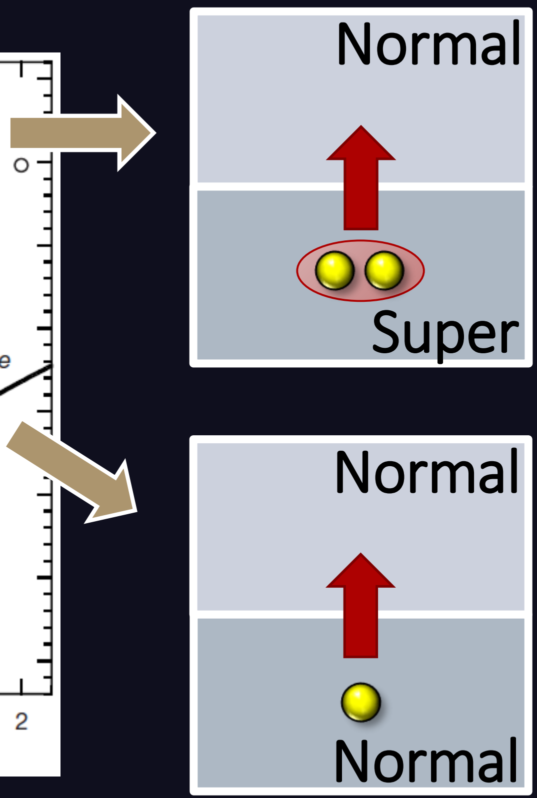
Fluctuations opened atomic physics

Shot Noise at Normal-Superconductor Junction

X. Jehl+, Nature **405**,50 (2000)





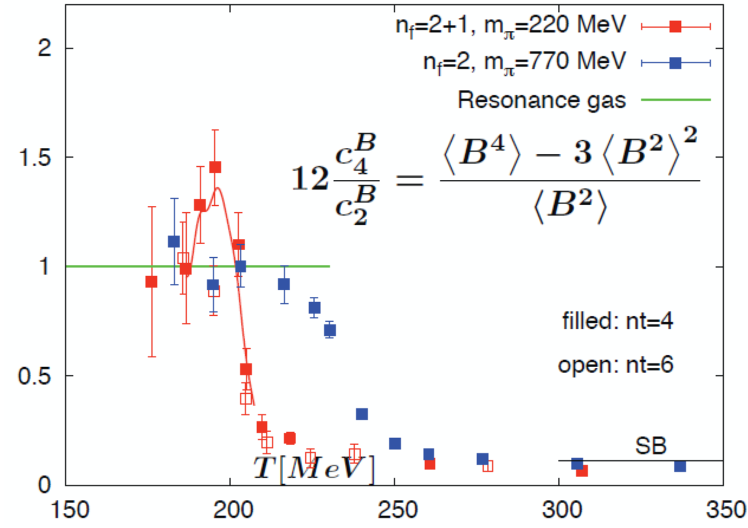


Similar experiments for fractional QHE ex. Saminadayar+, PRL**79**,2526(1997)

Conserved Charges : Theoretical Advantage

- Definite definition for operators
  - as a Noether current
  - calculable on any theory

ex: on the lattice



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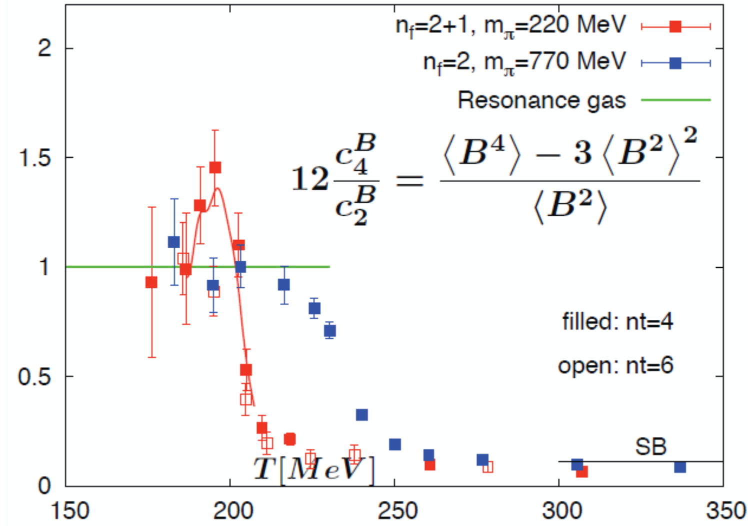


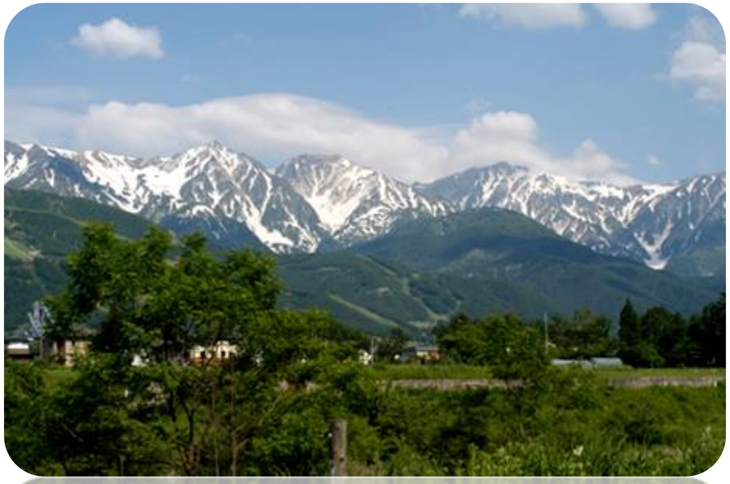
Asakawa, Ejiri, MK, 2009

- Simple thermodynamic relations

$$\langle \delta N_c^n \rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}$$

  - Intuitive interpretation for the behaviors of cumulants

ex: $\langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B}$



Conserved-Charge Fluctuations

Fluctuations of CC : rigorously defined in a theory

- operators as the Noether current
- as derivatives of the partition function

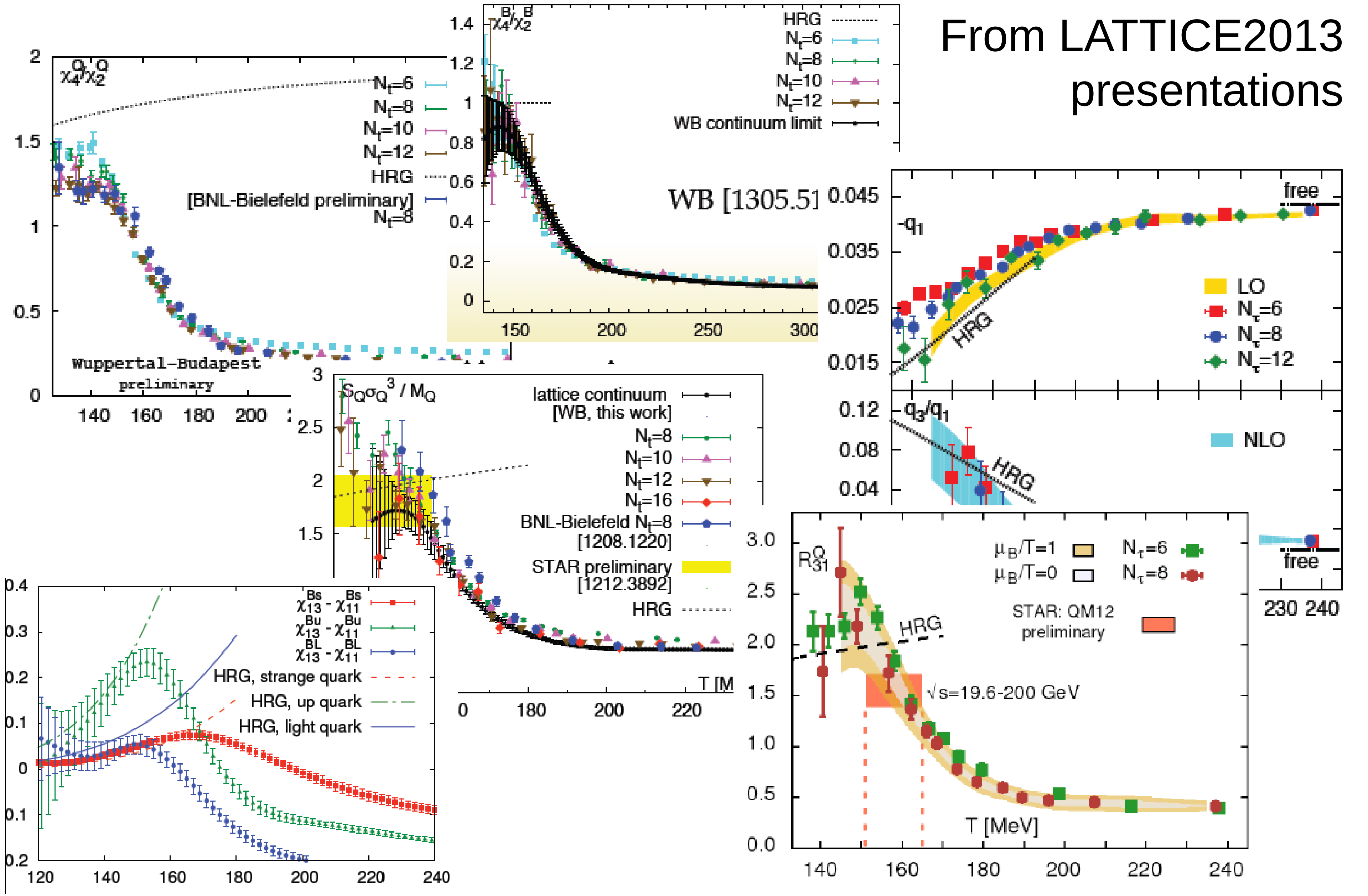
They are lattice observables

Fluctuations of CC
II
LAT-HIC crossover

QCD phase diagram 3, Wed. 11:00-13:30

Recent Progress in Lattice Community

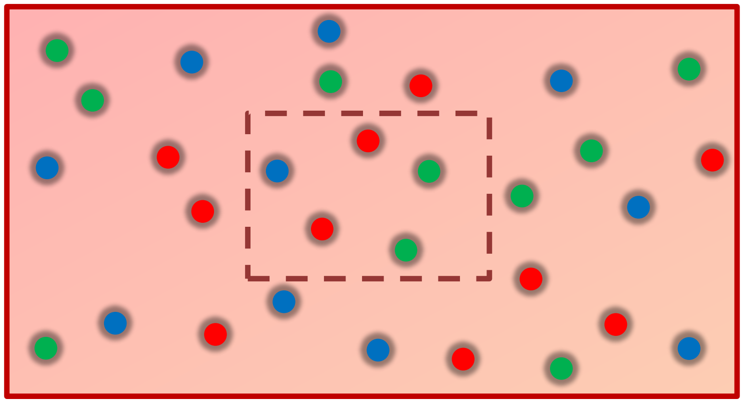
From LATTICE2013 presentations



Fluctuations

Free Boltzmann → Poisson

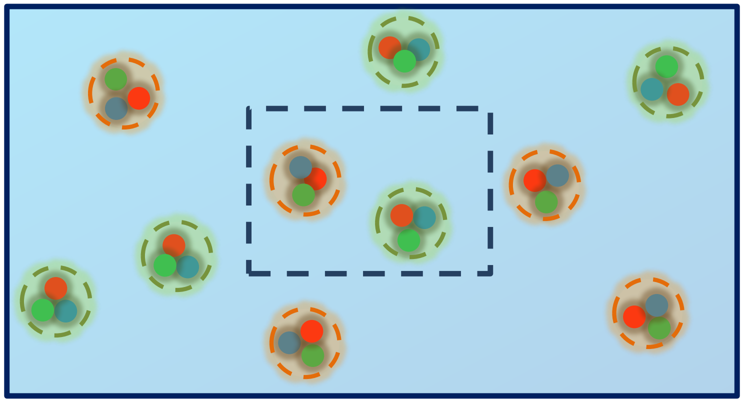
$$\langle \delta N^n \rangle_c = \langle N \rangle$$



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$



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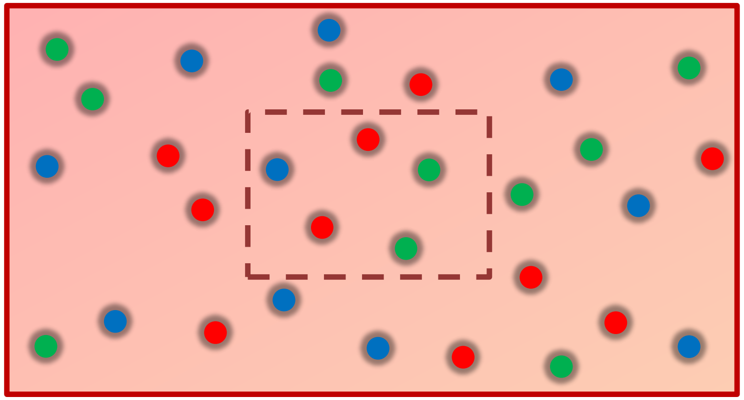

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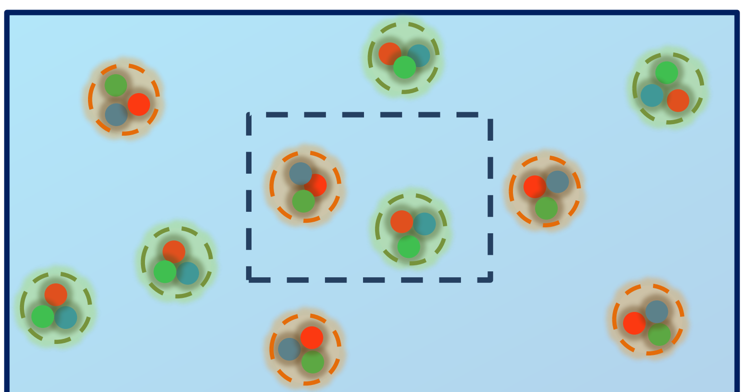
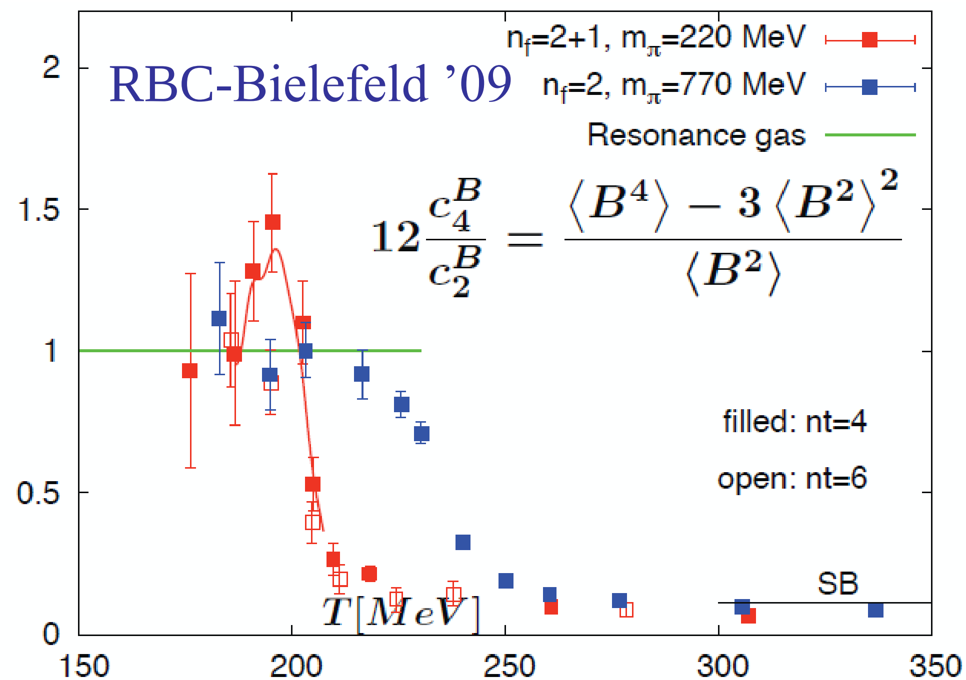
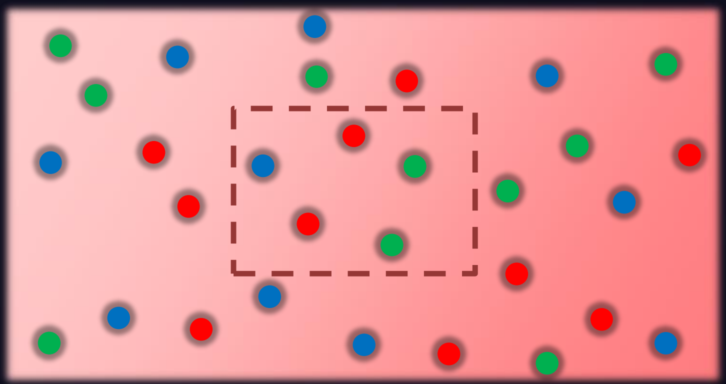


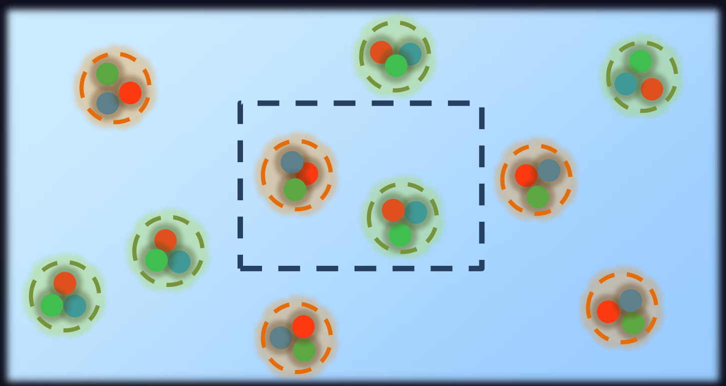


Electric Charge Fluctuation

Asakawa,Heinz,Muller; Jeon,Koch,2000



$$|q_q| = 1/3, 2/3$$



$$|q_B| = 1$$

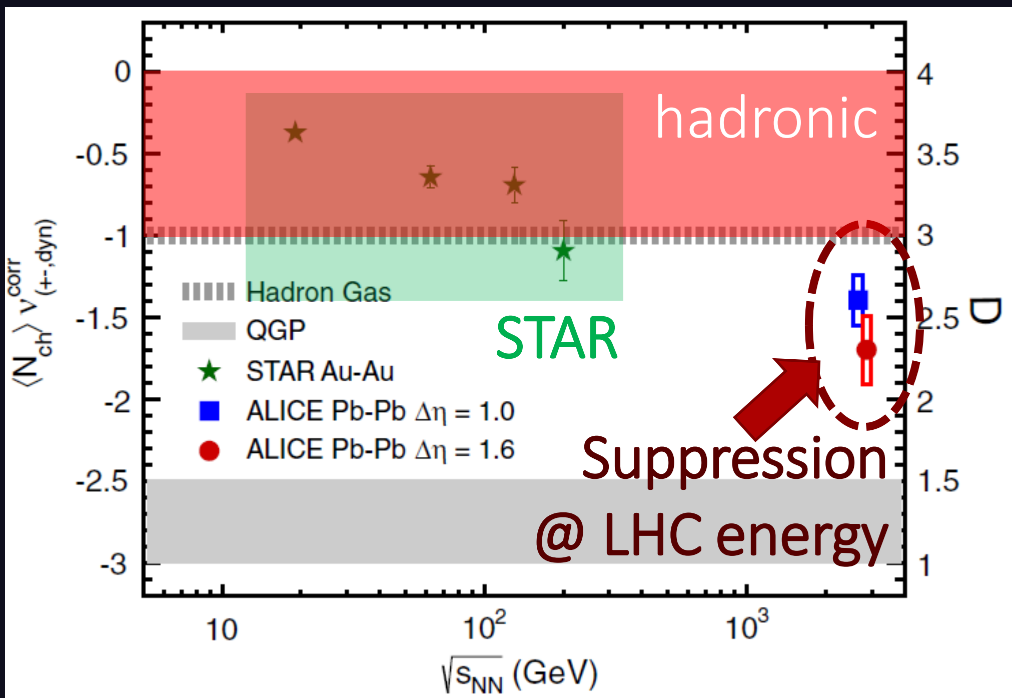
D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- D ~ 3-4 Hadronic
- D ~ 1-1.5 QGP

Electric Charge Fluctuation

PHENIX (2002); STAR (2003)
ALICE, PRL **110**, 152301 (2013)



D measure

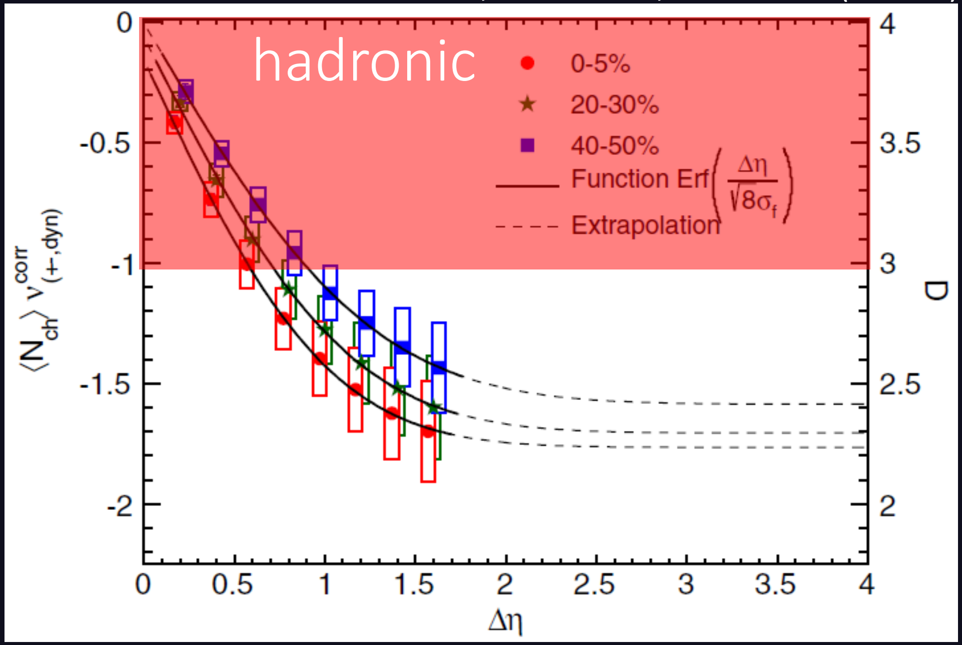
$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- *D* ~ 3-4 Hadronic
- *D* ~ 1-1.5 QGP

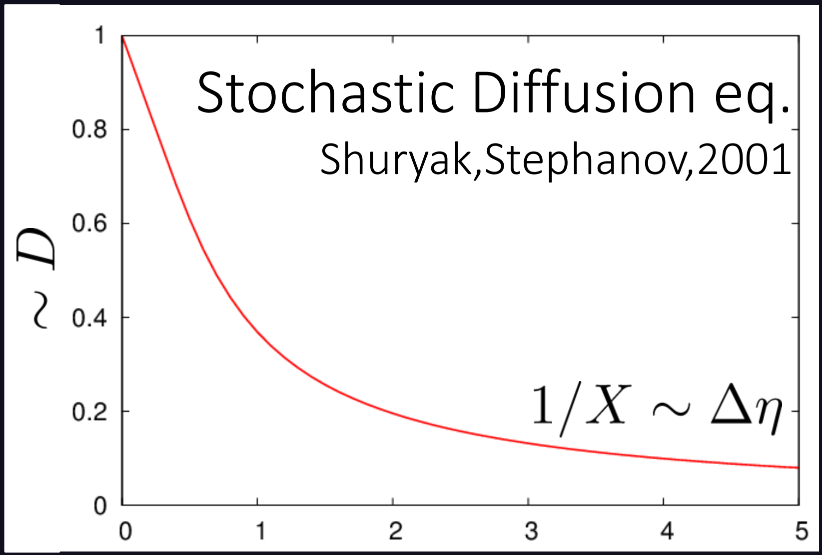
- No suppression at RHIC energy
- Fluctuations @ LHC **cannot** be described by hadronic d.o.f.

Rapidity Window Dependence

ALICE, PRL **110**, 152301 (2013)



- Smaller $\Delta\eta$
  more hadronic
- Larger $\Delta\eta$
  more QGP like

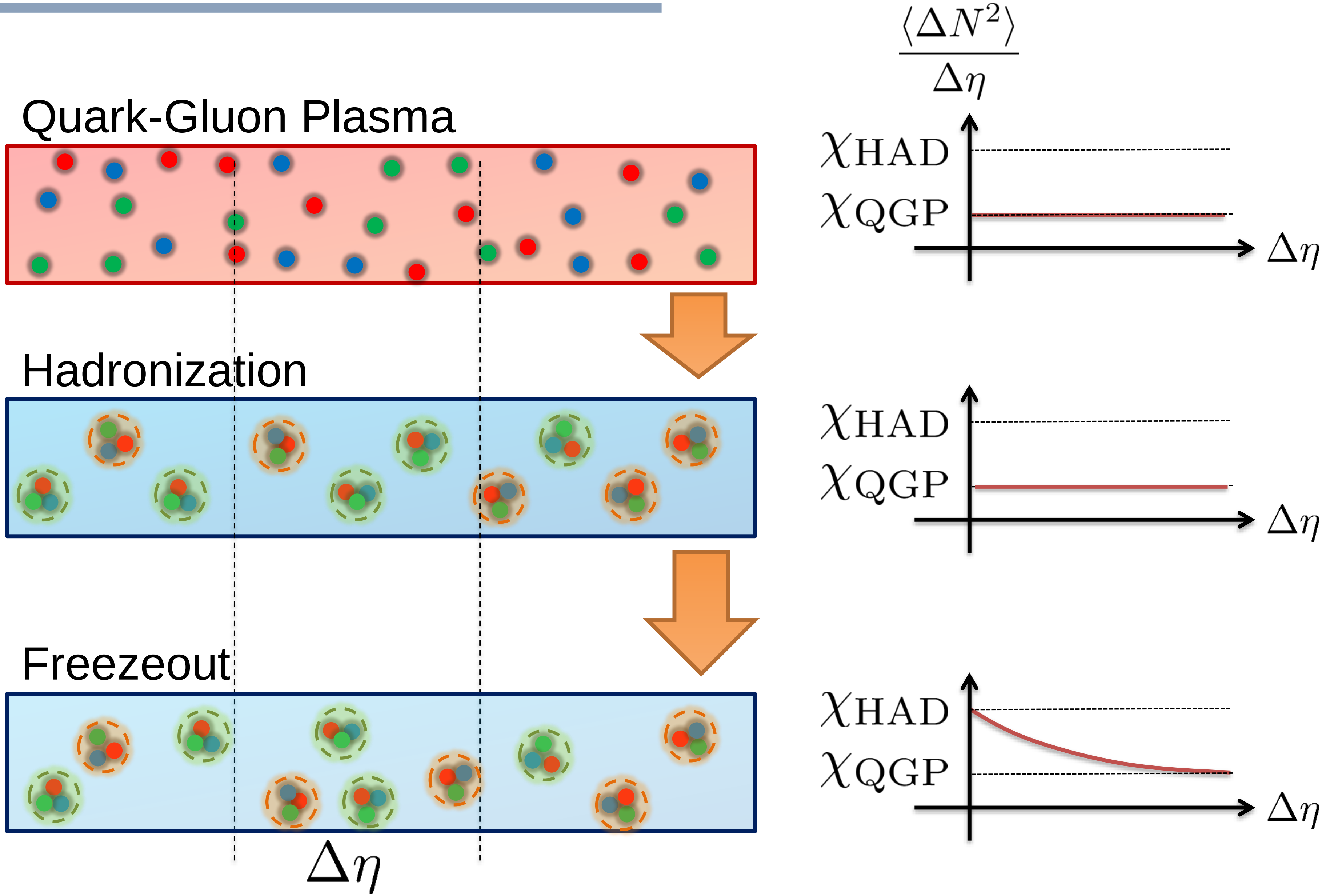


Same information in

- $\langle \delta N_Q^{(net)}(\eta_1)\delta N_Q^{(net)}(\eta_2)\rangle$
- $\simeq$ balance function

to be studied by fluctuating hydro.

Time Evolution in HIC



Various Contributions

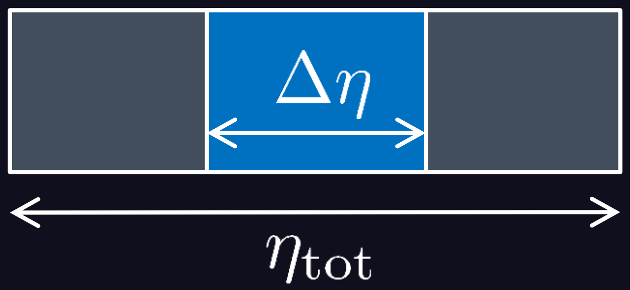
- Initial fluctuations ⬆ Enhance
- Effect of jets ⬆ Enhance
- Negative binomial (?) ⬆ Enhance
- Final state rescattering ↗ **Enhance** to Poisson
- Coordinate vs pseudo rapidities ↗ **Enhance** to Poisson
- Particle missID ↗ **Enhance** to Poisson
- Efficiency correction ↗ **Enhance** to Poisson
- Global charge conservation ⬇ **Suppress**

Global Charge Conservation

Jeon,Koch,2000
Bleicher,Jeon,Koch,2001

For equilibrated medium

$$\langle \delta N^2 \rangle_{\mathrm{exp.}} = \langle \delta N^2 \rangle_{\mathrm{GC}} \times \left(1 - \frac{\Delta y}{y_{\mathrm{tot}}} \right)$$



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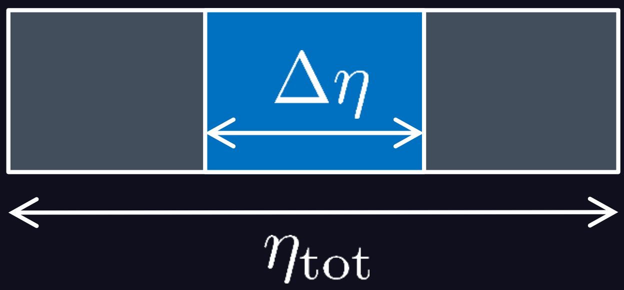
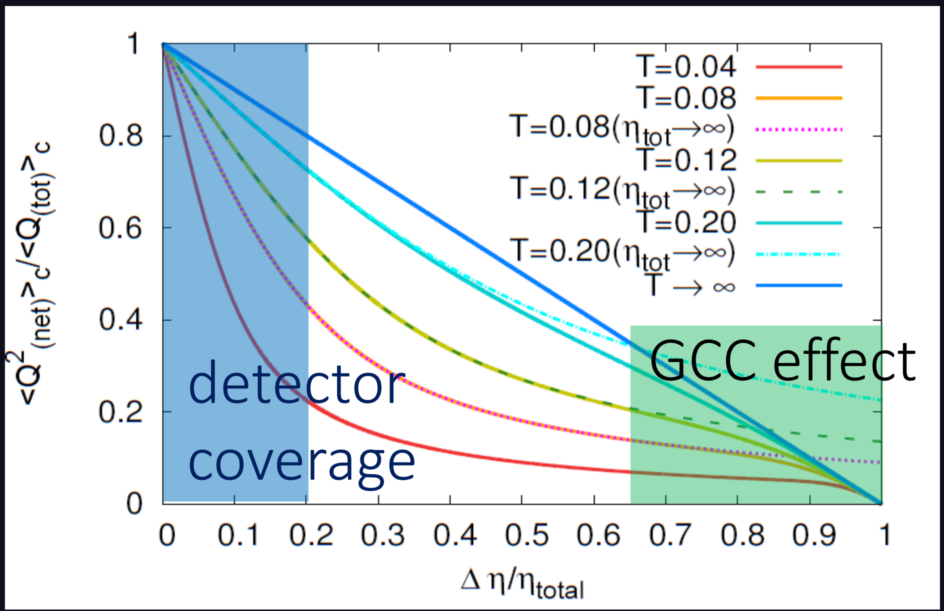


Solving the time evolution…



GCC effect on the hadronic diffusion **is negligible** in the ALICE result!

Sakaida, Poster I-35

Electric-Charge Fluctuations

- Electric charge fluctuations is suppressed at LHC!
- The suppression is most probably attributed to primordial physics
- Qualitative difference b/w RHIC and LHC

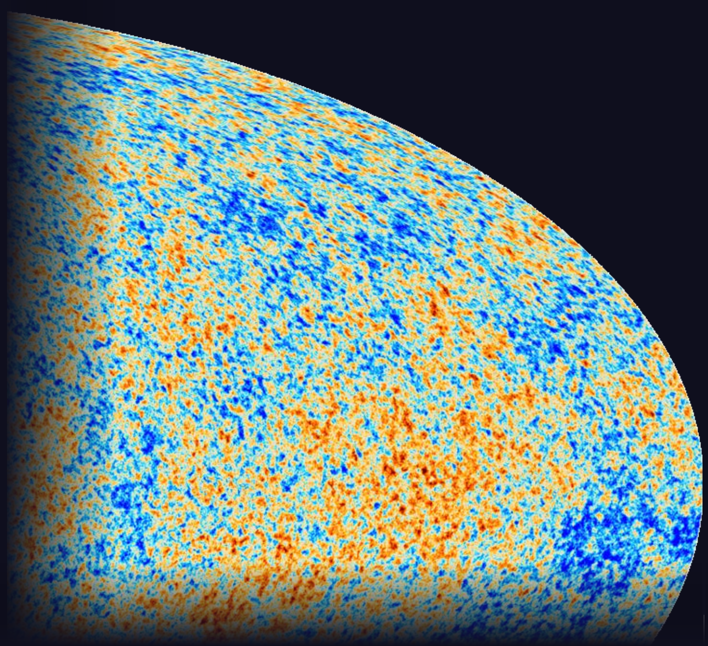
… but why?

Non Gaussianity

Non-Gaussianity

CMB

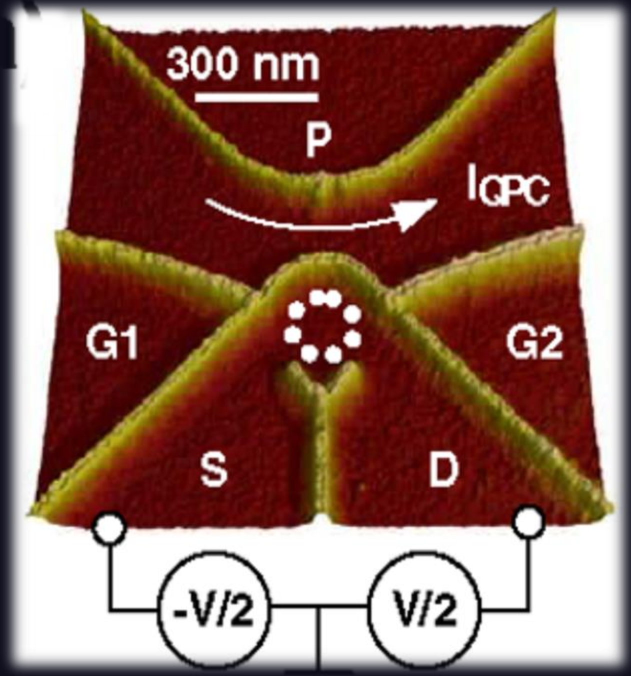
Cosmic Microwave Background



- No statistically-significant signals

Planck, 2013

Mesoscopic Systems



- Full counting statistics
- Cumulants up to 5th order

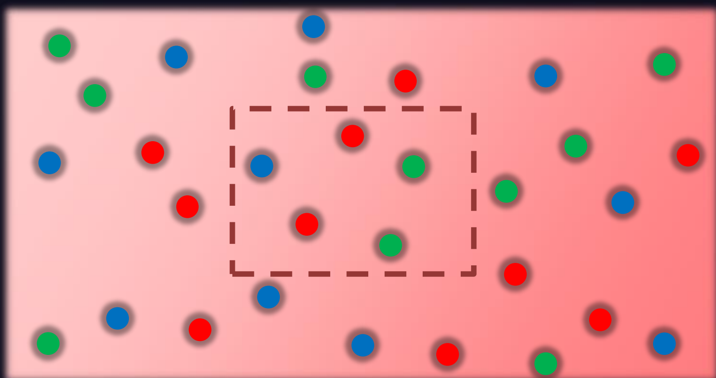
Gustavsson+, Surf.Sci.Rep.**64**,191(2009)

Non-Gaussianity in HIC

- Ratio of conserved charges
  Ejiri,Karsch,Redlich(2005)
- Critical enhancement
  Stephanov(2009)
- Sign change
  Asakawa,Ejiri,MK (2009); Friman+(2011); Stephanov(2011)
- Strange confinement
  BNL-Bielefeld(2013)
- Distribution funcs themselves
  Morita+(2013); Nakamura (Wed.)

Ratio of Cumulants

Ejiri, Karsch, Redlich, 2005



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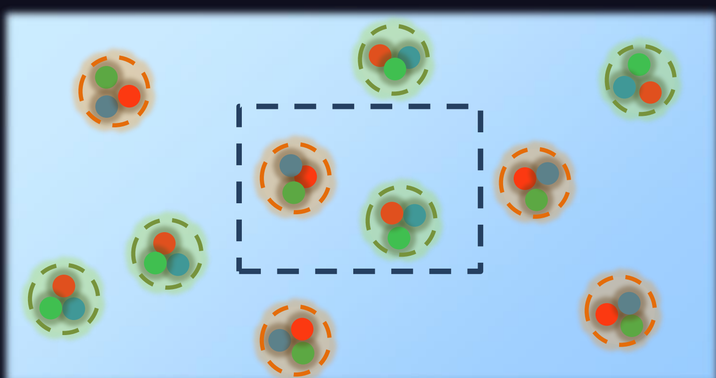
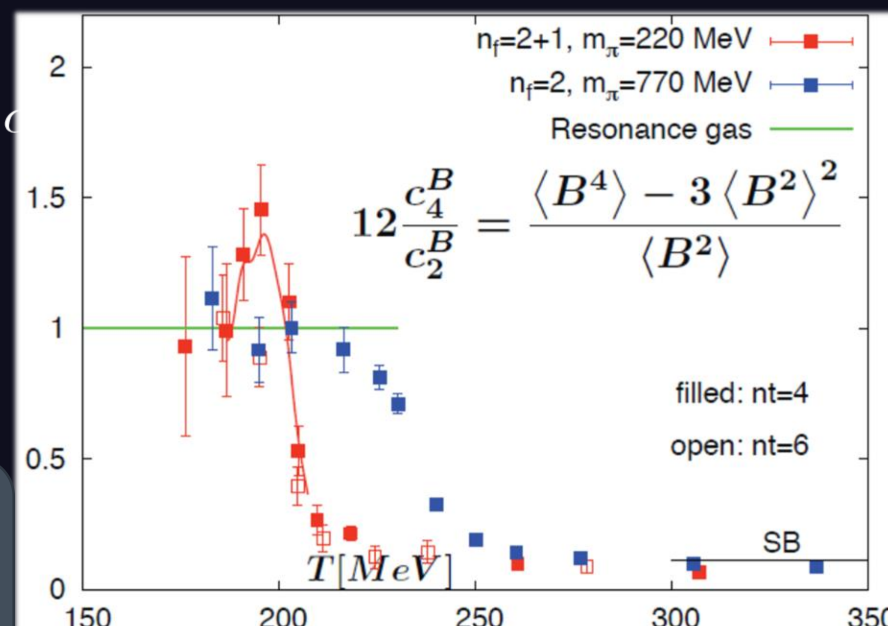


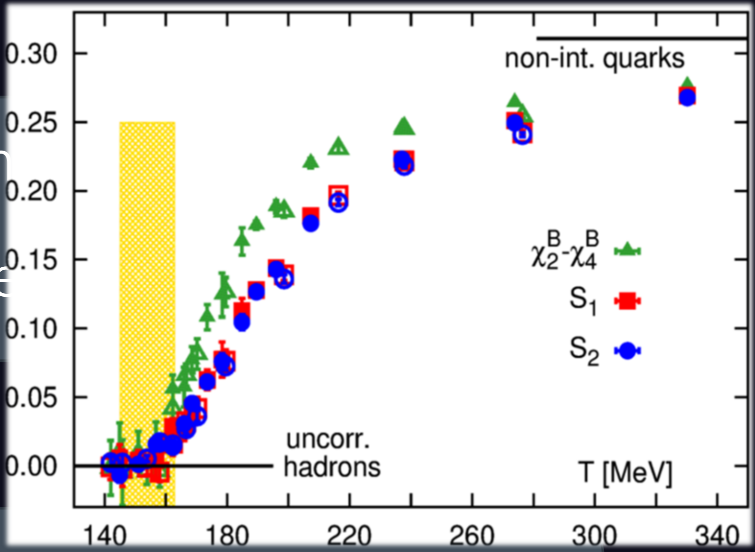
$\langle \delta N_B^n \rangle_c$



$$\frac{\langle \delta N_B^4 \rangle_c}{\langle \delta N_B^2 \rangle} = \begin{cases} 1 & \text{hadronic} \\ 1/9 & \text{quark-gluon} \end{cases}$$

Strange Confinement

BNL-Bielefeld,PRL(2013)
H.T. Ding, This Morning



Baryonic $\langle \delta N_B^2 \rangle = \langle \delta N_B^4 \rangle$

$$\langle \delta N_B^2 \rangle - \langle \delta N_B^4 \rangle_c \begin{cases} = 0 & \text{baryons con} \\ \neq 0 & \text{something e} \end{cases}$$

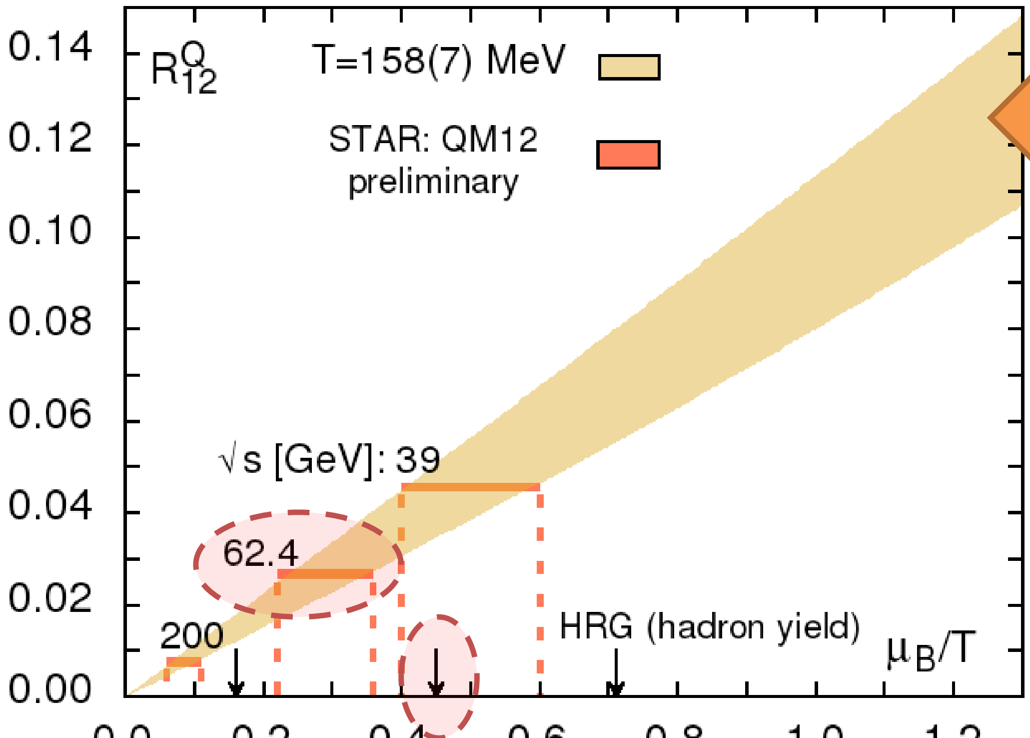
Strangeness

$$\langle \delta N_s \delta N_B^3 \rangle_c - \langle \delta N_s \delta N_B \rangle \begin{cases} = 0 & \text{strangeness confined} \\ \neq 0 & \text{something else} \end{cases}$$

Many lattice studies (LAT-HIC crossover):

Budapest-Wuppertal, 2013; BW,1403.4578; BNL-Bi.,1404.4043; Gupta+,1405.2206; Ratti, Wed.; Schmidt, Wed.; Nakamura, Wed.; Sharma, J-13

Cumulants : HIC@RHIC vs Lattice



parameter window constrained by lattice

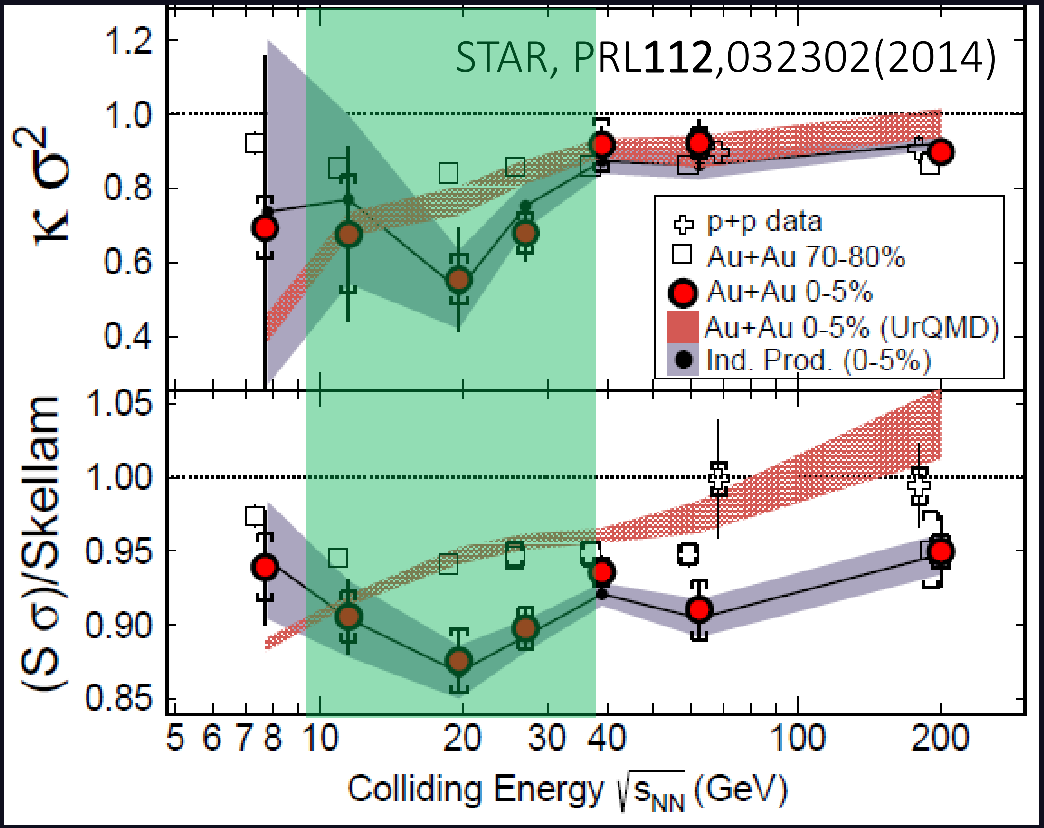
BNL-Bielefeld, LATTICE2013

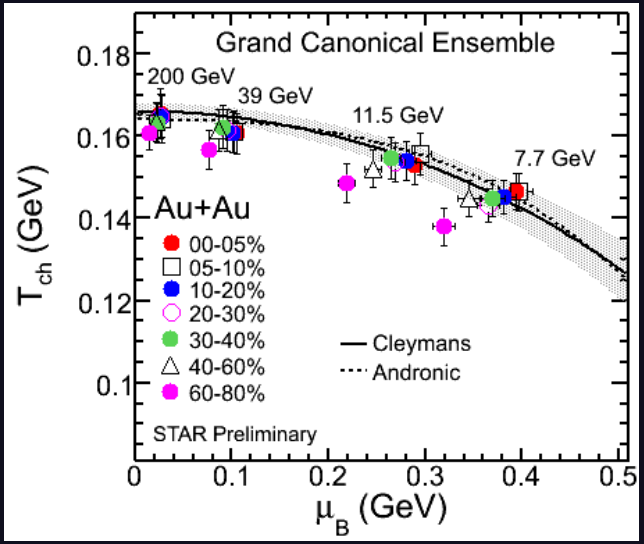
fluctuations "exp + lattice" ⟷ μ/T ⟷ particle abundance (chem. freezeout T)

discrepancy

Proton Number Cumulants at RHIC-BES

STAR 2012





- Exp. results are close to and less than Poissonian values.
- Something interesting around $\sqrt{s_{\rm NN}} \simeq 20{\rm GeV}$

Effects of Various Contributions

- Initial fluctuations **Enhance**
- Effect of jets **Enhance**
- Negative binomial (?) **Enhance**
- Final state rescattering **Enhance** to Poisson
- Coordinate vs pseudo rapidities **Enhance** to Poisson
- Particle missID **Enhance** to Poisson
- Efficiency correction **Enhance** to Poisson
- Global charge conservation **Suppress**

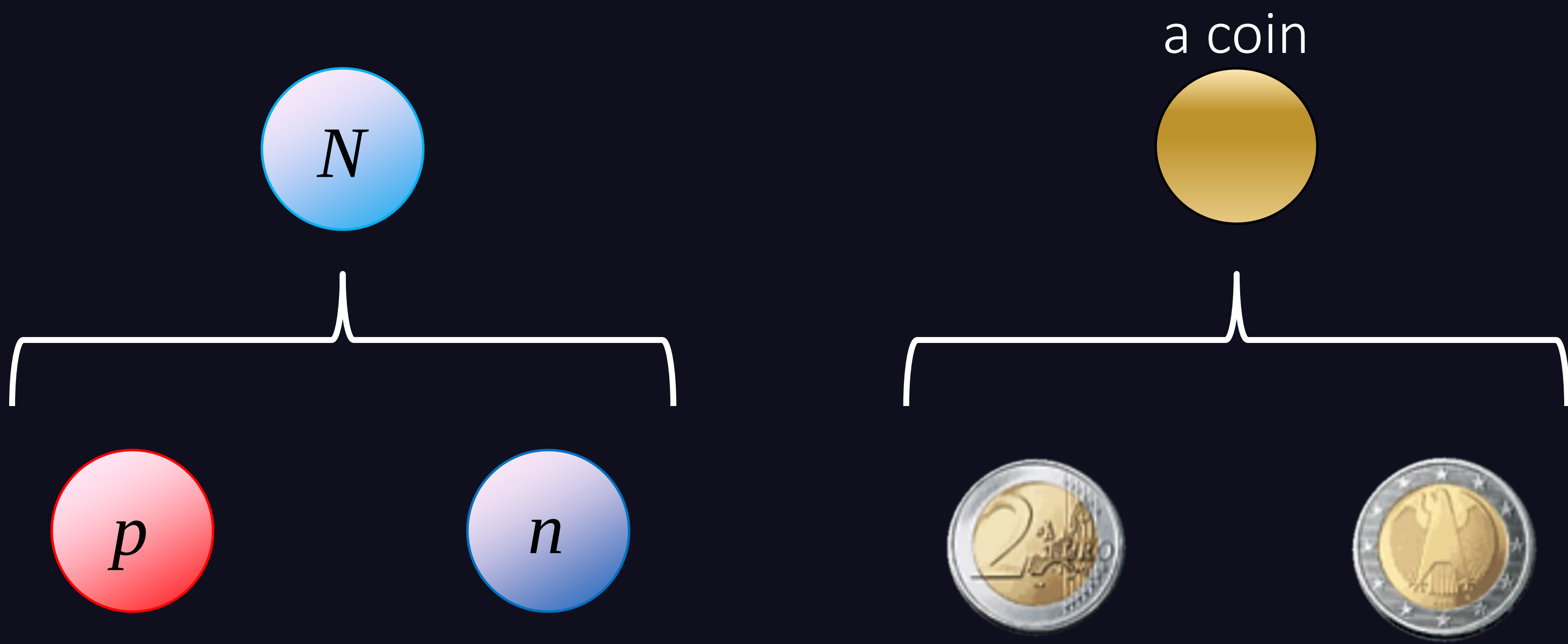
Caution!!

proton number cumulants $\neq$ baryon number cumulants

Let's clarify their relation!

MK, Asakawa (2012;2012)

Nucleon isospin and a coin

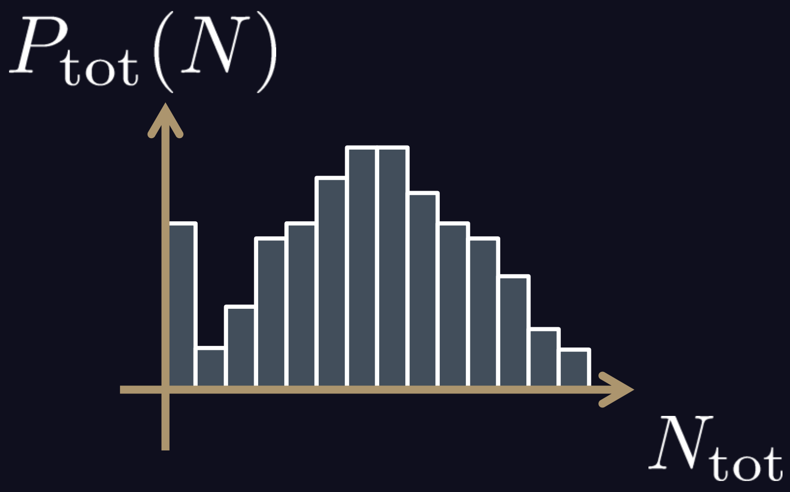


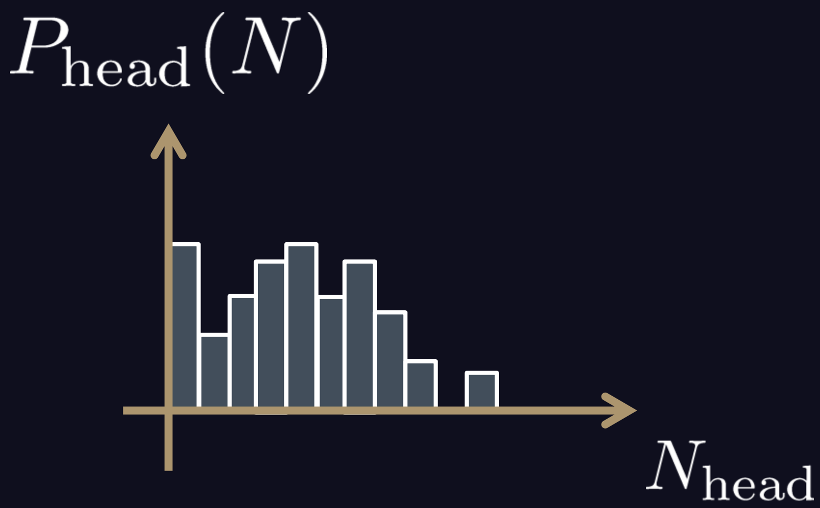
Nucleon has
two isospin states.

A coin has two sides.

MK, Asakawa,2012

Slot Machine Analogy



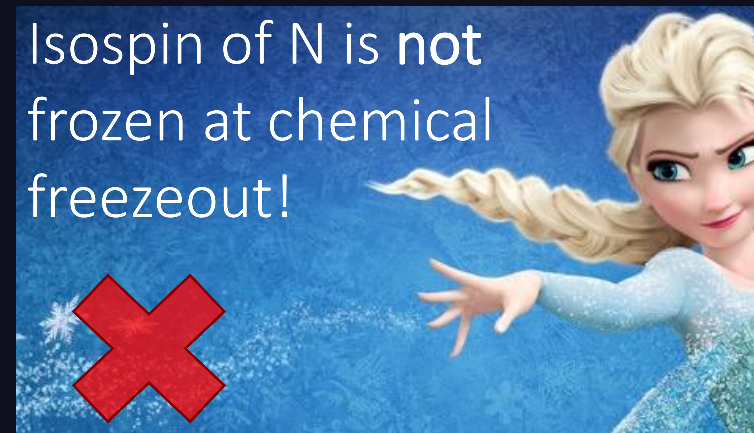


Reconstructing Total Coin Number

$$P_{\text{head}}(N_{\text{head}}) = \sum_{N_{\text{tot}}} B_{1/2}(N_{\text{head}}; N_{\text{tot}}) P_{\text{tot}}(N_{\text{tot}})$$

Charge Exchange Reaction

$$p, n + \pi \rightarrow \Delta(1232) \rightarrow p, n + \pi$$



cross sections of p

$$p+\pi^+ \xrightarrow{3} \Delta^{++} \longrightarrow p+\pi^+$$

$$p+\pi^0 \xrightarrow{1},\ n+\pi^+ \longrightarrow \Delta^+ \longrightarrow p+\pi^0,\ n+\pi^+ \quad (2:1)$$

$$p+\pi^- \xrightarrow{2},\ n+\pi^0 \longrightarrow \Delta^0 \longrightarrow p+\pi^-,\ n+\pi^0 \quad (1:2)$$

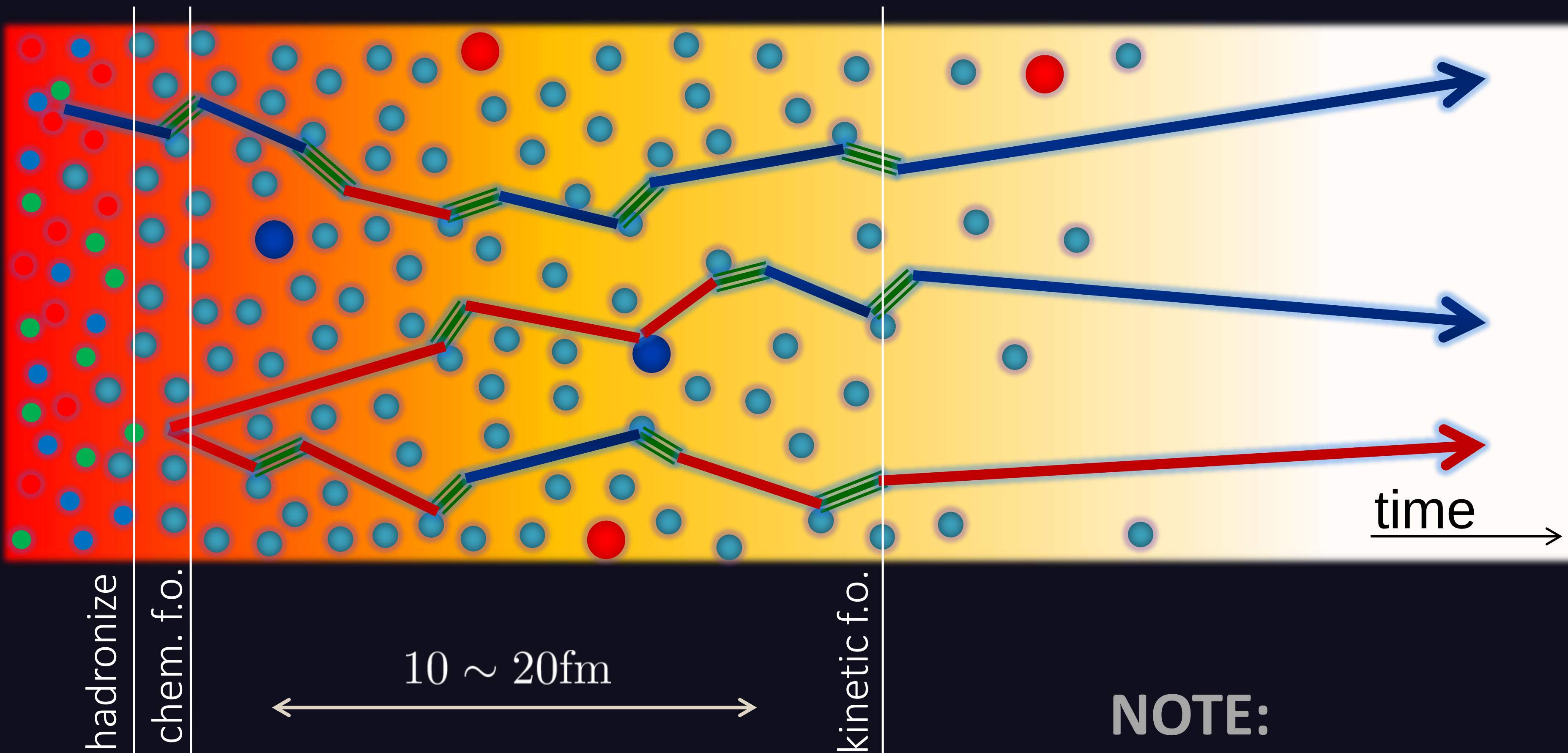
$$n+\pi^- \longrightarrow \Delta^- \longrightarrow n+\pi^-$$

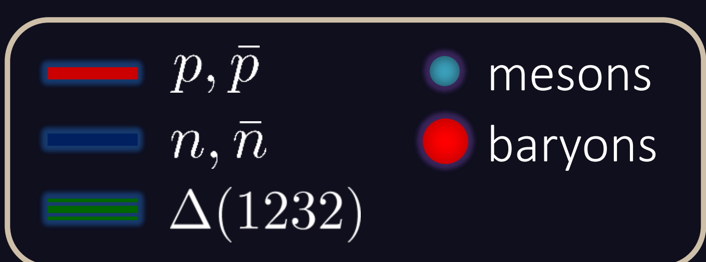
decay rates of Δ

$$p+\pi \rightarrow \Delta^{+,0}$$
$$\rightarrow p:n$$
$$=5:4$$

mean reaction time
< 5fm/c

Nucleons in Hadronic Medium





NOTE:

- so many pions
- rare NN collisions
- no quantum corr.

Difference b/w N_B and N_p

Assumptions: net-cumulants deviate from thermal value
But, $N_\mathrm{B}, N_{\bar{\mathrm{B}}}$ are Poissoian

$$\begin{cases} 2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\rangle_\mathrm{free} \\ 2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle_\mathrm{free} \\ 2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^4\rangle_c + \cdots \end{cases}$$

genuine info. noise

Proton number cumulants are dominated by Poissonian noise

cf.) Nahrgang+, arXiv:1402.1238

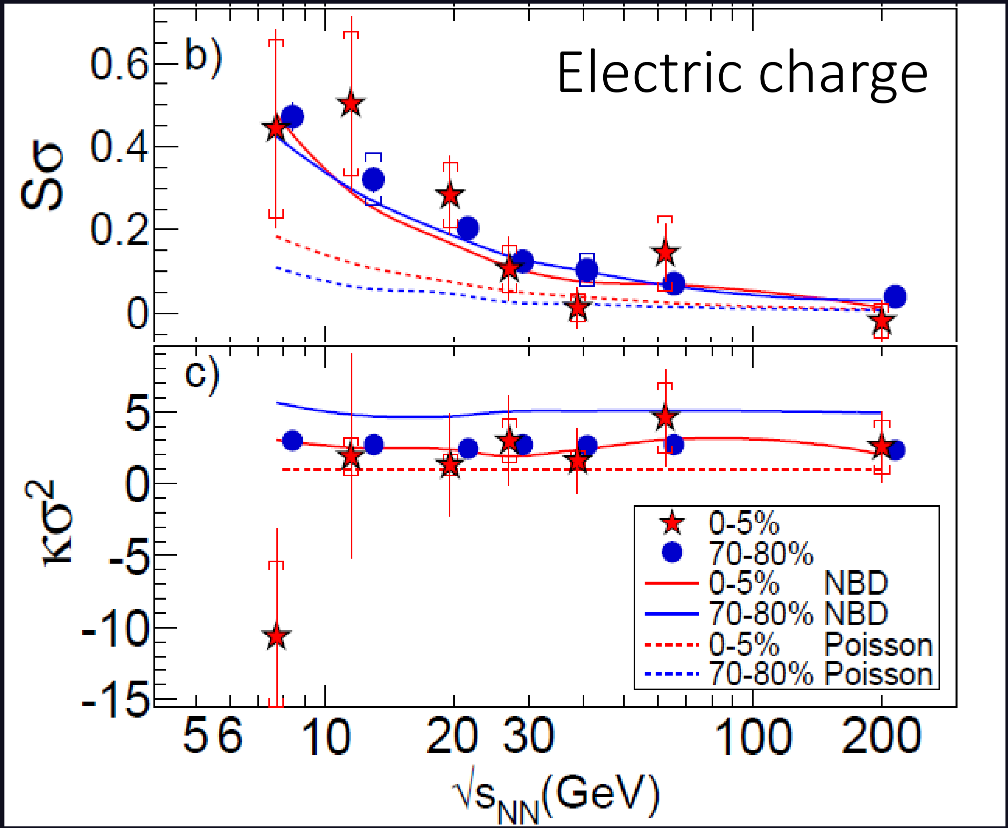
Efficiency Correction

MK, Asakawa, 2012
Bdzak, Koch, 2012
STAR, 2013

If efficiency for each particle is uncorrelated

binomial correction to distribution function

$$P_{\mathrm{exp.}}(N) = \sum_{N'} B_\epsilon(N;N')P(N')$$



STAR, arXiv:1402.1558

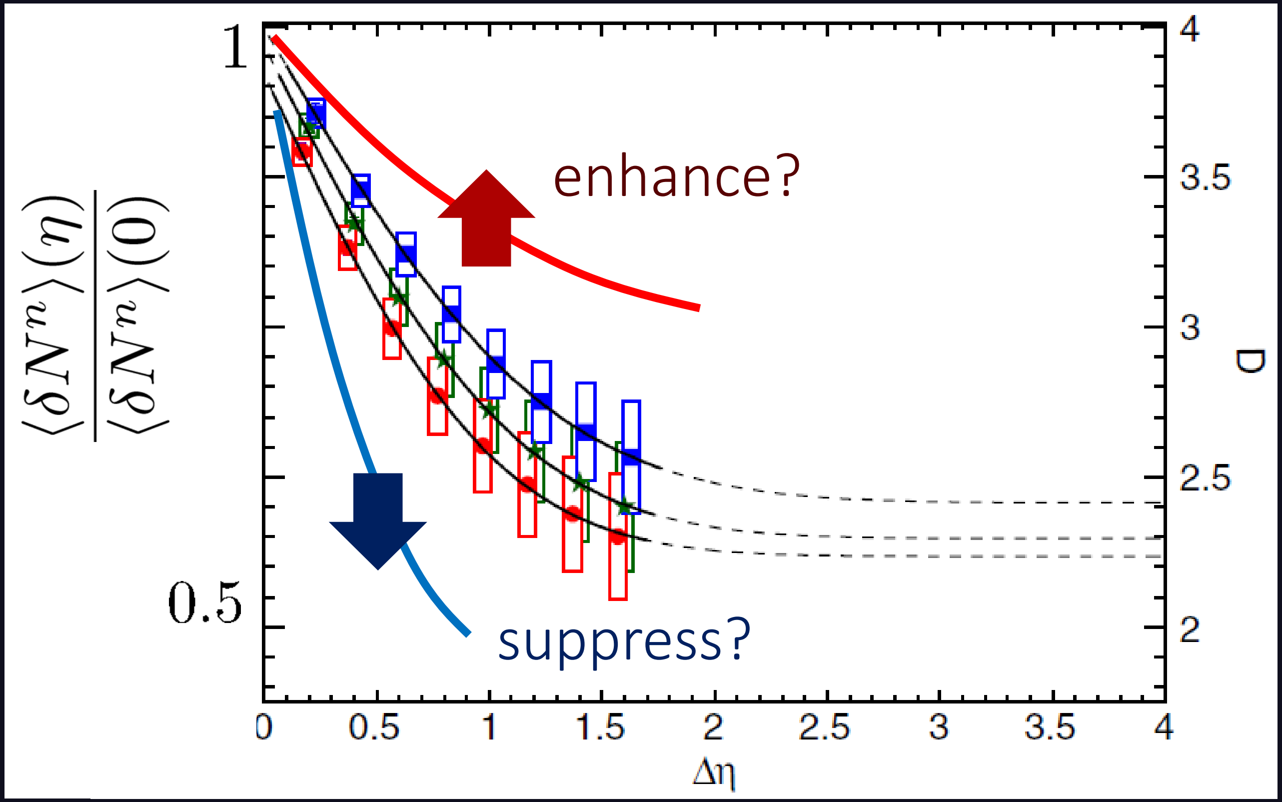
for Particle missID: Ono,Asakawa,MK, PRC,2013

More Information on/from Fluctuations

$\Delta\eta$ dependence

MK, Asakawa, Ono,PLB**728**, 386 (2014)

$\Delta\eta$ Dep. of Non-Gaussianity



How does the 4-th order cumulant behave as a function of Δη?

Fluctuating Hydrodynamics?

- Distributions in experiments are close to Poissonian
- Cumulants are expected to **increase** in the hadronic medium

These behaviors **cannot** be described by the theory of hydrodynamic fluctuations

Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012
Stephanov, Shuryak, 2001

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Markov (white noise)
+
continuity

⇓

Gaussian noise

⇓

Fluctuation of *n* is Gaussian in equilibrium

cf) Gardiner, "Stochastic Methods"

How to Introduce Non-Gaussianity?

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant $D(n)$
  - colored noise
  - discretization of *n*

How to Introduce Non-Gaussianity?

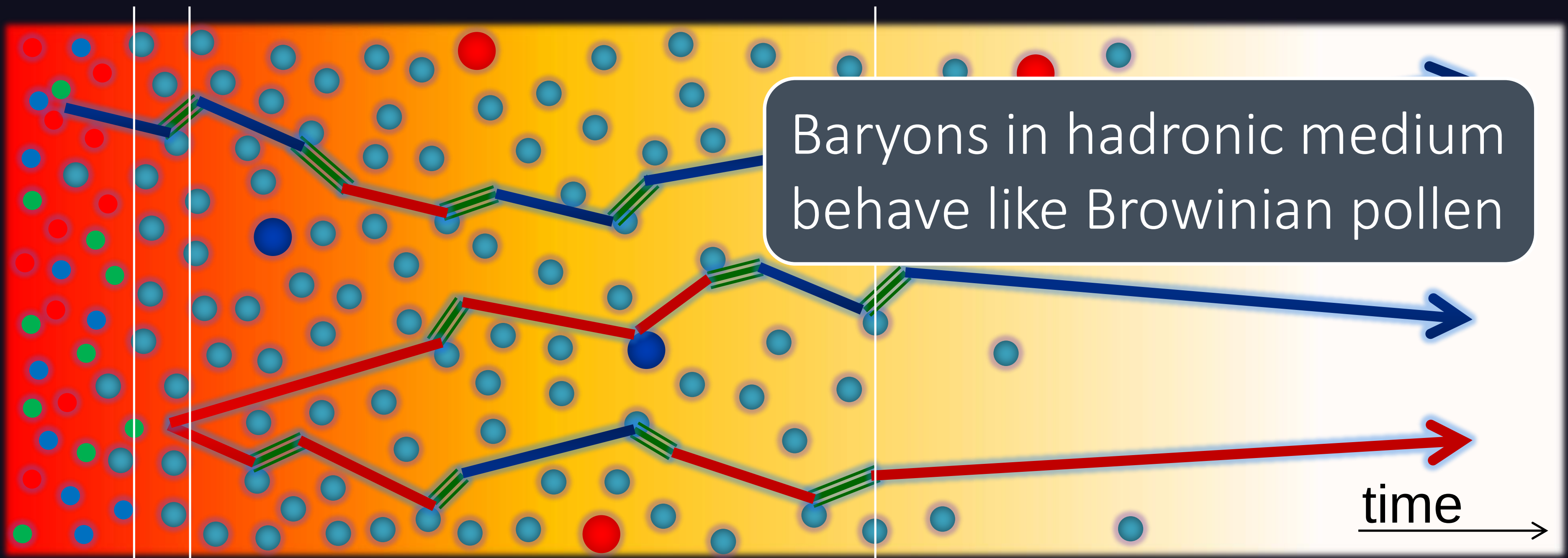
Stochastic diffusion equation

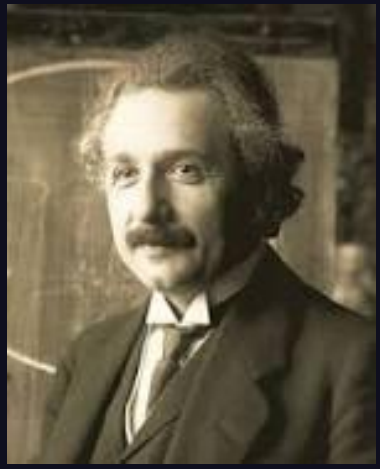
$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant *D*(*n*)
  - colored noise
  - discretization of *n* ⇐ **our choice**

REMARK: Fluctuations measured in HIC are almost Poissonian.

Nucleons in Hadronic Medium

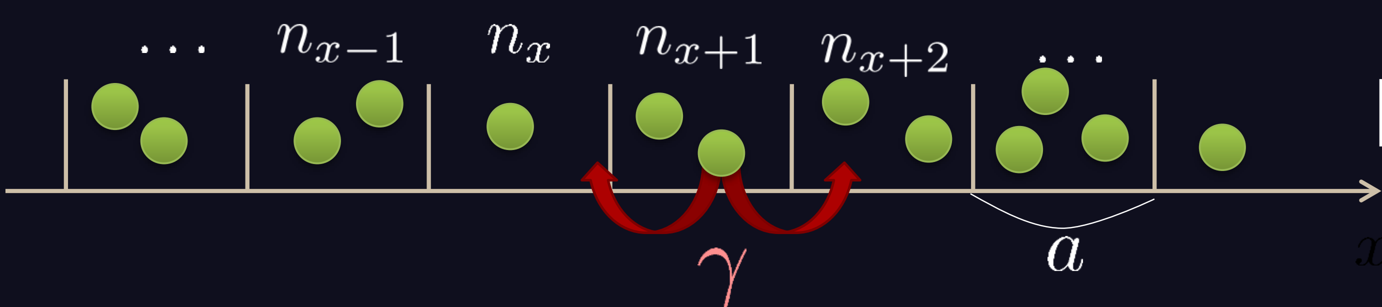




Diffusion Master Equation

MK,Asakawa,Ono,PLB**728**,386(2014)

Divide spatial coordinate into discrete cells



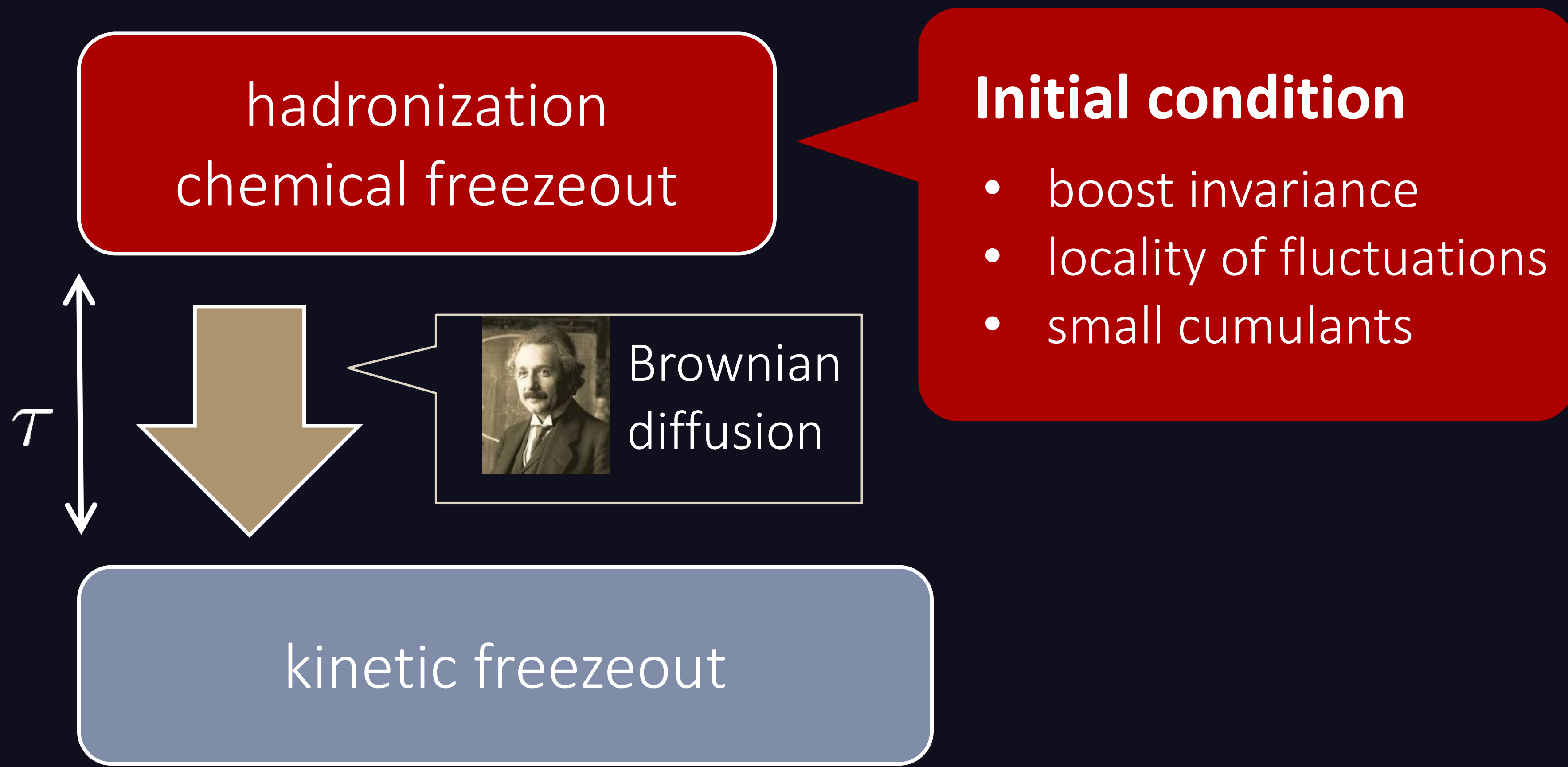
probability

$P(\mathbf{n})$

Master Equation

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x+1)\{P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x+1})+P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x-1})\} -2n_x P(\mathbf{n})]$$

Solve the DME exactly, and take $a \rightarrow 0$ limit



Comments:

- agreement with stochastic diffusion eq. up to Gaussian fluctuaion
- Poisson (Skellam) distribution in equilibrium: consistent with HRG

Solution of DME in a→0 Limit

1st order (deterministic) $\langle n \rangle$

- consistent with diffusion equation with $D=\gamma a^2$

  ⇨ Continuum limit with fixed $D=\gamma a^2$

2nd order $\langle \delta n^2 \rangle$

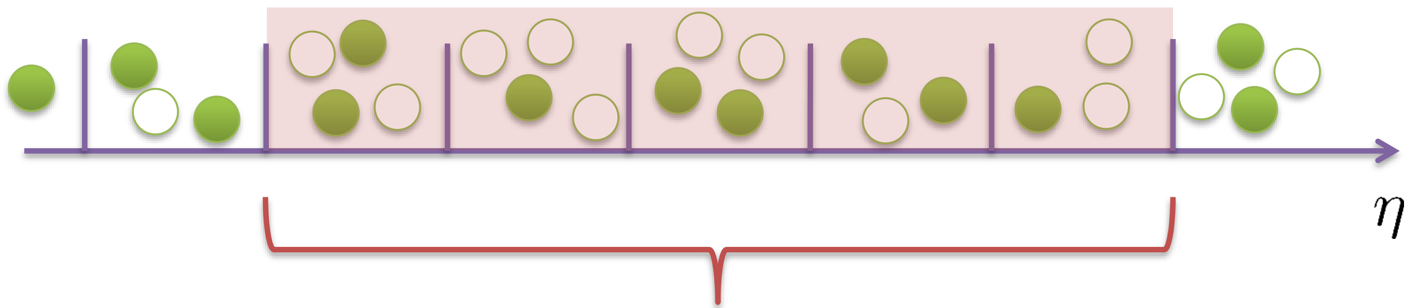
- consistent with stochastic diffusion eq.
  (for sufficiently smooth initial conditions)

Shuryak, Stephanov, 2001

Nontrivial results for non-Gaussian fluctuations

Net Charge Number

Prepare 2 species of (non-interacting) particles



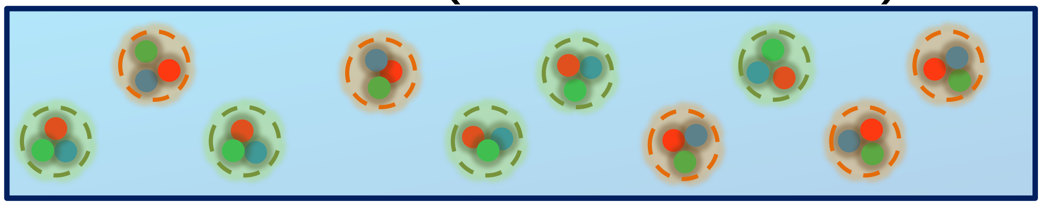
$$\bar{Q}(\tau) = \int_0^{\Delta\eta} d\eta \, (n_1(\eta,\tau) - n_2(\eta,\tau))$$

Let us investigate

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ at freezeout time t

Time Evolution in Hadronic Phase

Hadronization (initial condition)



- Boost invariance / infinitely long system
- Local equilibration / local correlation

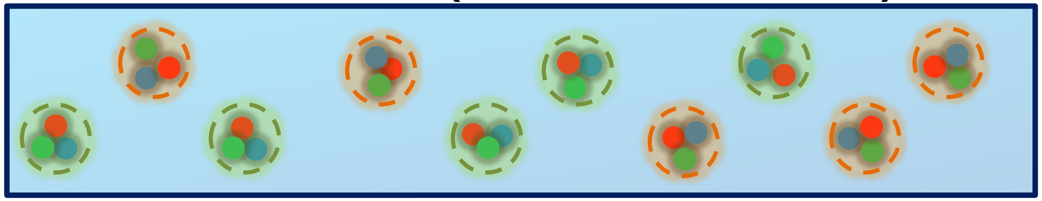
$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ ↑ suppression owing to local charge conservation

$\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$

$\langle Q_{(\mathrm{tot})}^2 \rangle_c$ ↑ strongly dependent on hadronization mechanism

Time Evolution in Hadronic Phase

Hadronization (initial condition)



Time evolution via DME

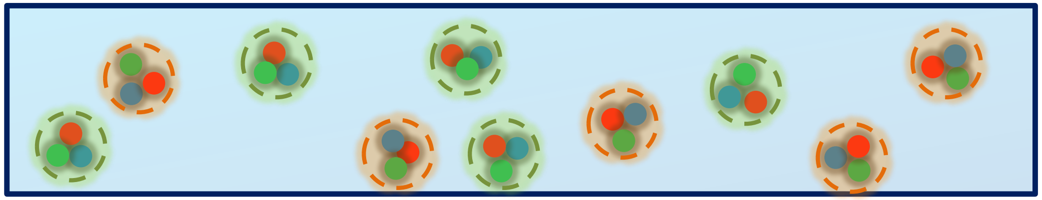
- Boost invariance / infinitely long system
- Local equilibration / local correlation

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ — suppression owing to local charge conservation

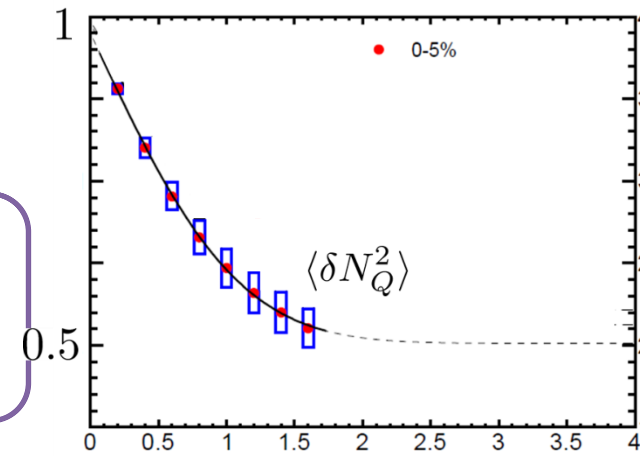
$\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$

$\langle Q^2_{(\mathrm{tot})} \rangle_c$ — strongly dependent on hadronization mechanism

Freezeout

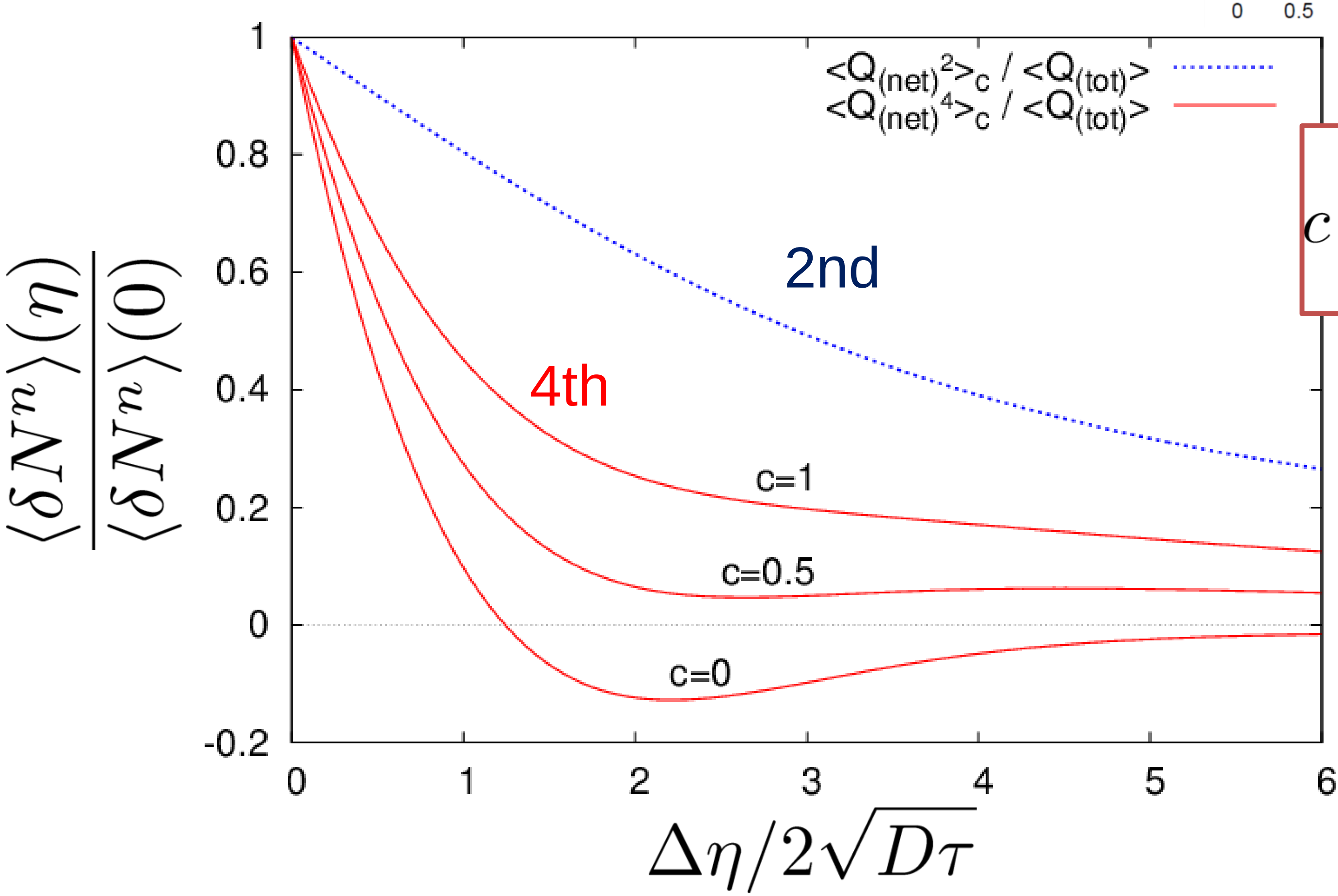


$\Delta\eta$ Dependence at Freezeout



Initial fluctuations:

$$\langle\bar{Q}^2\rangle_c = \langle\bar{Q}^4\rangle_c = \langle\bar{Q}^2 Q_{(\mathrm{tot})}\rangle_c = 0$$

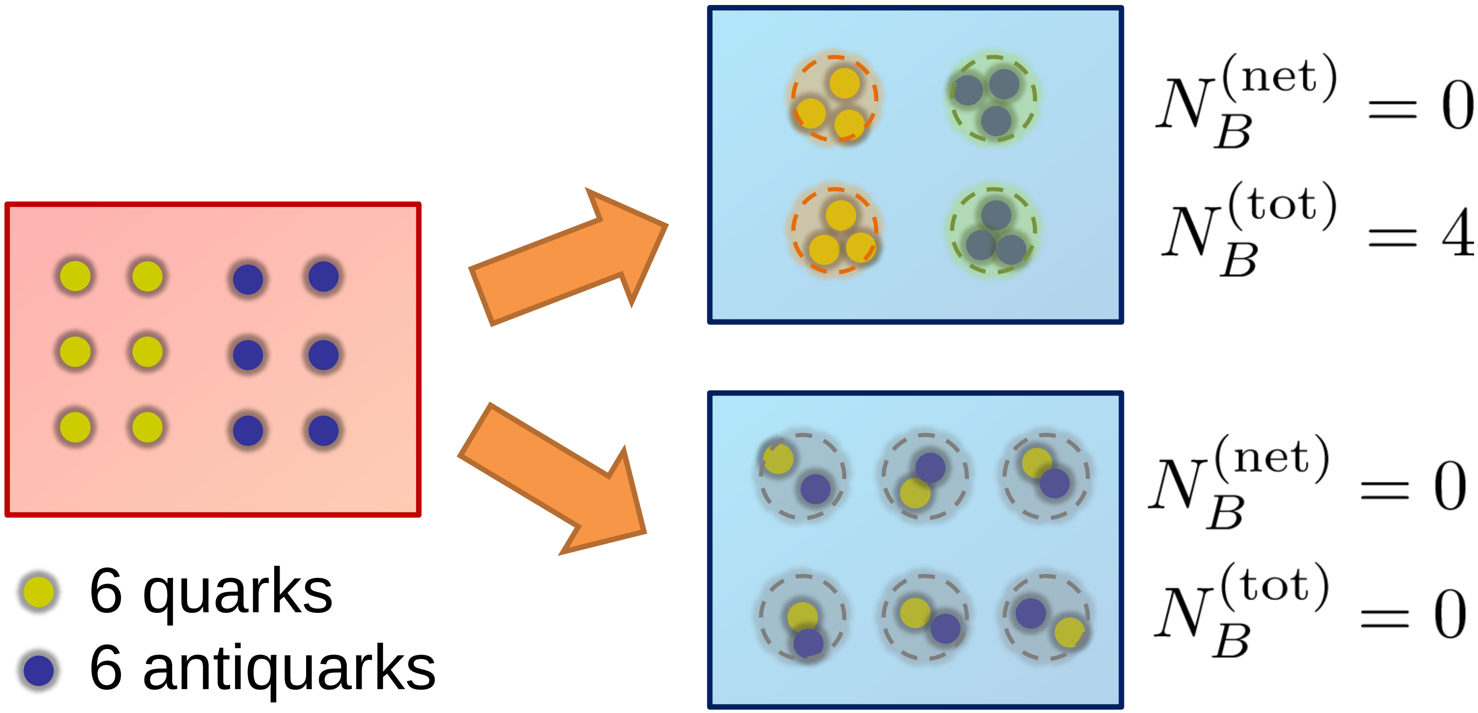


$$c = \frac{\langle Q_{(\mathrm{tot})}^2\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

parameter sensitive to hadronization

Total Charge Number

In recombination model,

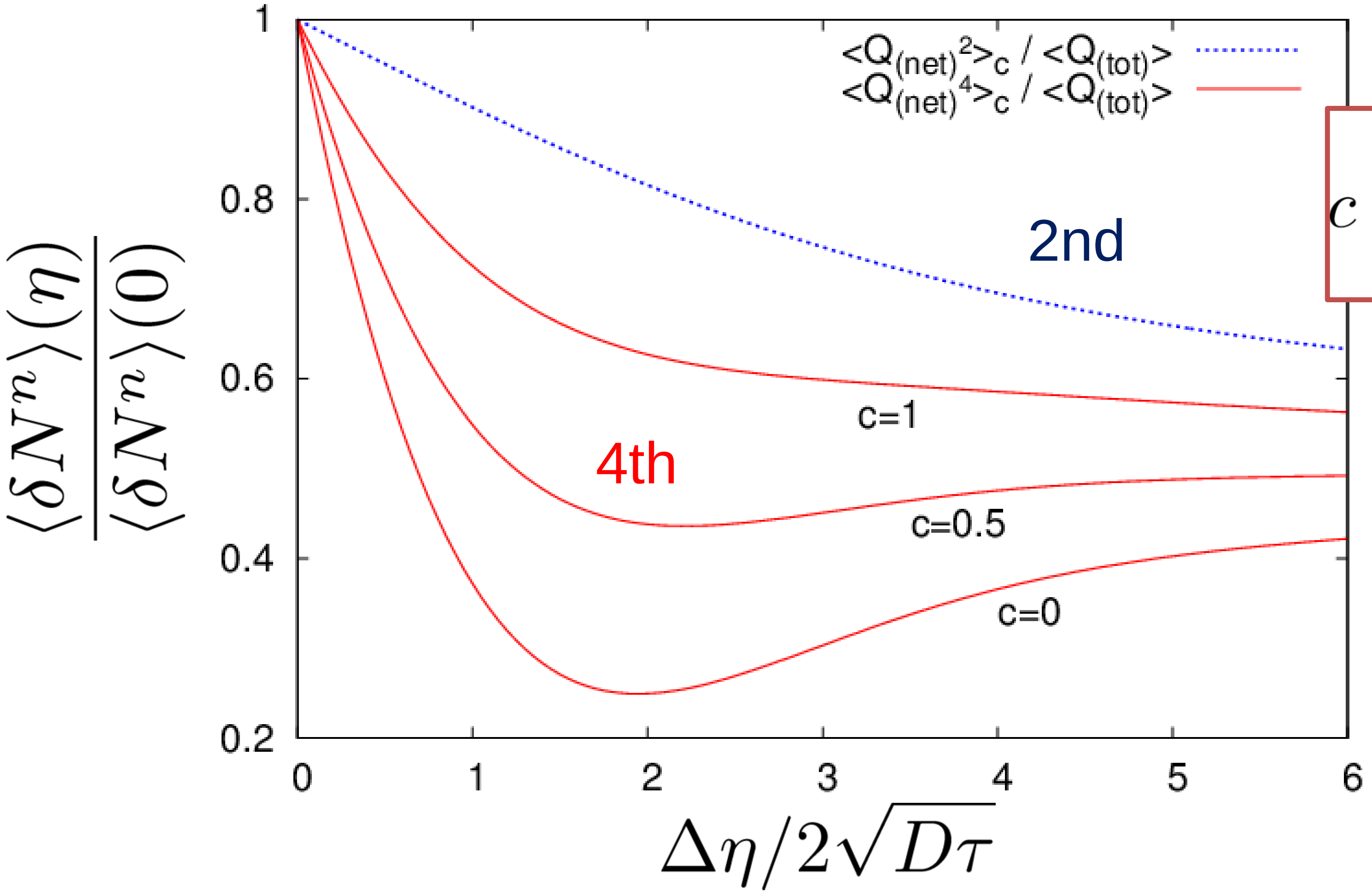


- $N_B^{(\mathrm{tot})}$ can fluctuate, while $N_B^{(\mathrm{net})}$ does not.

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0.5 \langle Q_{(\mathrm{tot})} \rangle$$



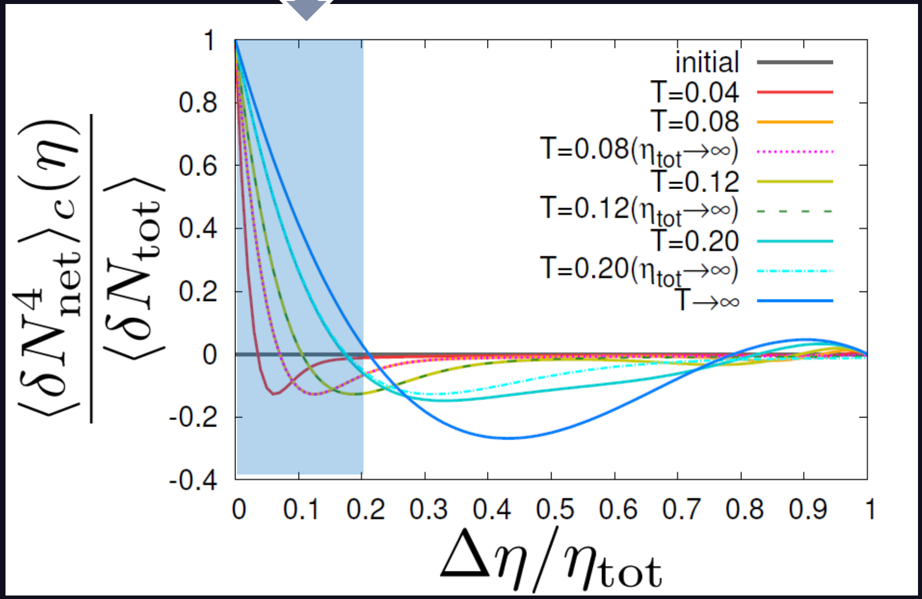
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

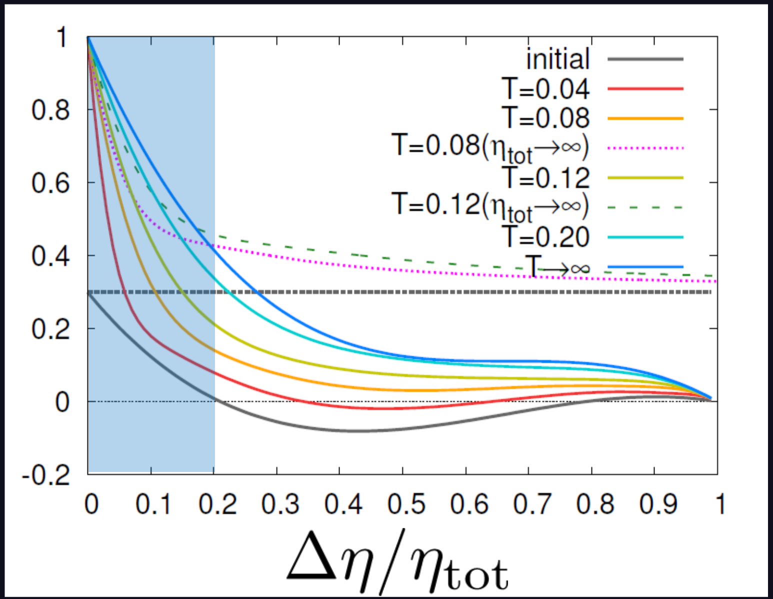
parameter sensitive to hadronization

4^{th} order Cumulant at ALICE

MK, Asakawa,Ono (2014)
Sakaida+, poster I-35

rapidity coverage at ALICE $(\eta_{\rm tot} = 8)$

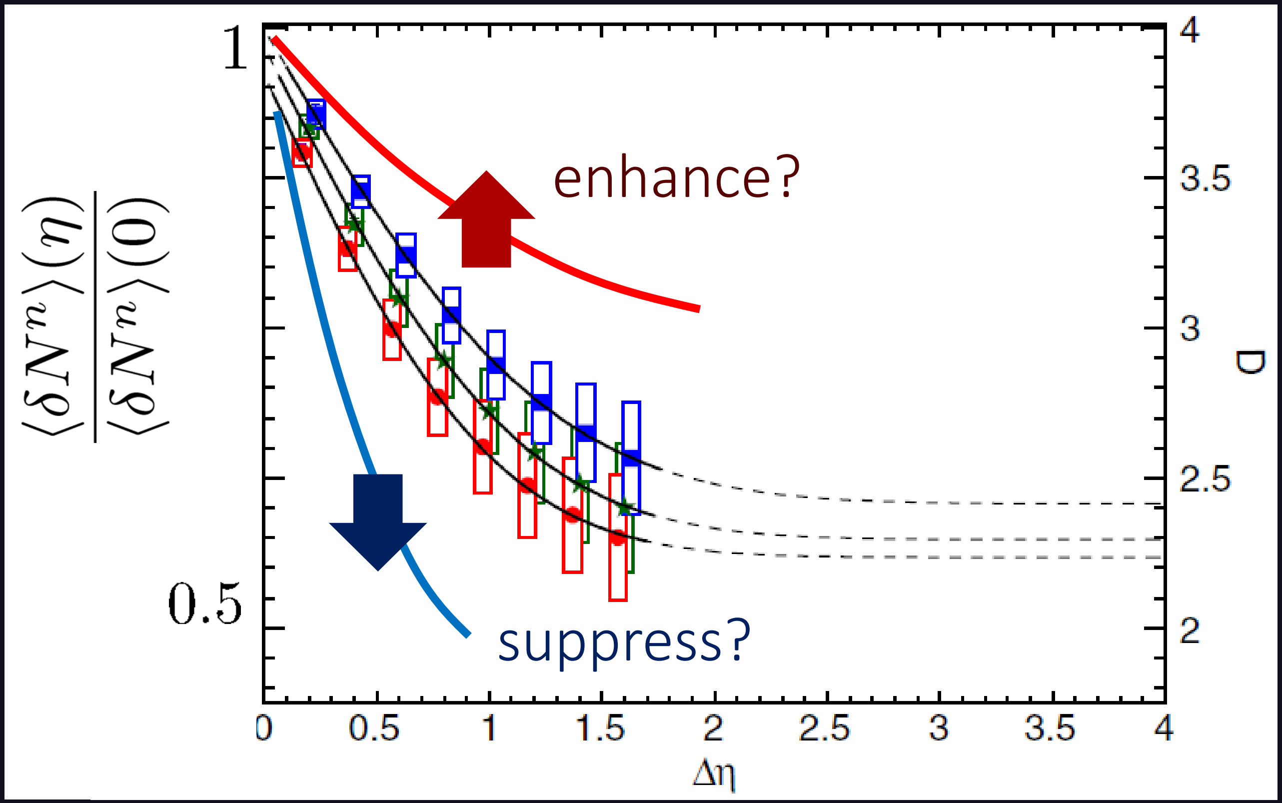




4^{th} order cumulant is sensitive to
initial fluctuation / transport property / confinement

It can be non-monotonic and negative!

$\Delta\eta$ Dep. of Non-Gaussianity



How does the 4-th order cumulant behave as a function of Δη?

Suggestions to Experimentalists

- many conserved charges

  electric charge, baryon number, (and strangeness?)
  with different diffusion constans

- various cumulants

  second, third, fourth, mixed, (and much higher?)

- $\Delta\eta$ window dependences

  primordial thermodynamics, transport property, confinement
  no normalization

- Beam Energy Scan

  LHC, RHIC-BES, FAIR, NICA, J-PARC, …

My Messages

- ☐ Fluctuations are invaluable observables in HIC
- ☐ But, we must understand them in more detail
- ☐ It's possible, interesting, and important

We are just arriving at the starting point to explore QCD phase structure with fluctuations!

Summary

Fluctuations in HIC are nonthermal!

Plenty of physics in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2 \rangle_c,\ \langle N_B^2 \rangle_c,\ \langle N_Q^4 \rangle_c,\ \langle N_B^4 \rangle_c,\ \langle N_{ch}^2 \rangle_c, \cdots$$

Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC

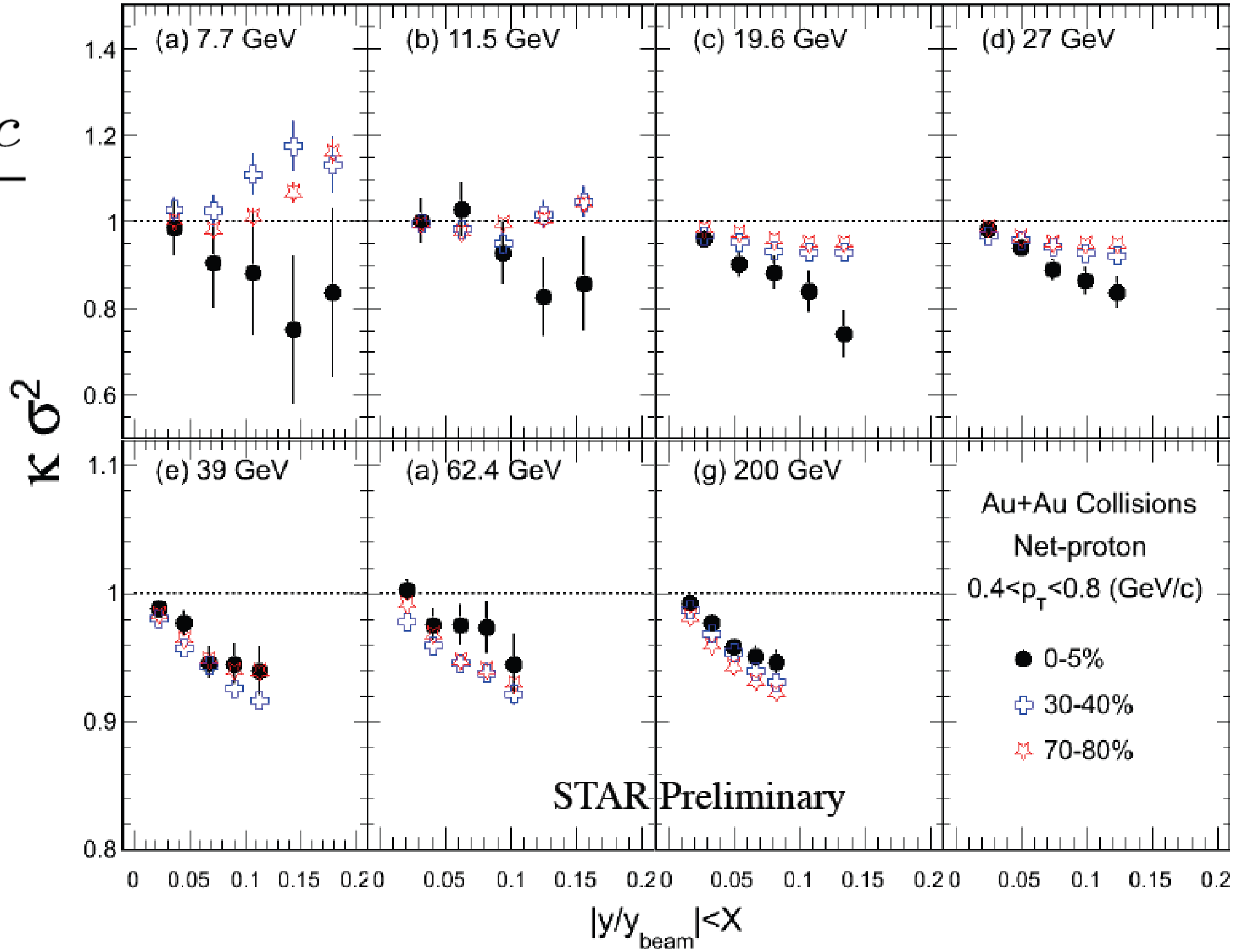
- history of hot medium
- mechanism of hadronization
- diffusion constant

backup

Δη Dependence at STAR

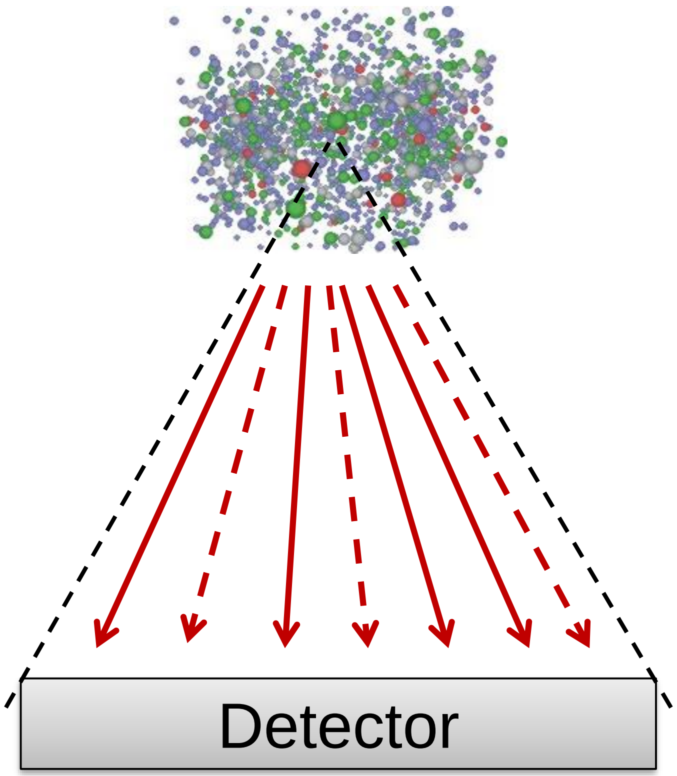
STAR, QM2012

$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$



$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$ decreases as Δη becomes larger at RHIC energy.

Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$



- $\begin{cases} N_N \text{ nucleons} \\ N_{\bar{N}} \text{ anti-nucleons} \end{cases}$

  ➡ $F(N_N, N_{\bar{N}})$

- $N_N \begin{cases} N_p \text{ protons} \\ N_n \text{ neutrons} \end{cases}$

  ➡ $B(N_p; N_N)$

  binomial distribution func.

$$\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$
$$= F(N_N, N_{\bar{N}})B(N_p; N_N)B(N_{\bar{p}}; N_{\bar{N}})$$

- for any phase space in the final state.

3rd & 4th Order Fluctuations

$N_\mathrm{B} \to N_p$

$$\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{8}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle + \frac{3}{8}\langle\delta N_\mathrm{B}^{(\mathrm{net})}\delta N_\mathrm{B}^{(\mathrm{tot})}\rangle,$$

$$\begin{aligned}\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c =& \frac{1}{16}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^4\rangle_c + \frac{3}{8}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\delta N_\mathrm{B}^{(\mathrm{tot})}\rangle \\ &+ \frac{3}{16}\langle(\delta N_\mathrm{B}^{(\mathrm{tot})})^2\rangle - \frac{1}{8}\langle N_\mathrm{B}^{(\mathrm{tot})}\rangle,\end{aligned}$$

$N_p \to N_\mathrm{B}$

$$\begin{aligned}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle =& 8\langle(\delta N_p^{(\mathrm{net})})^3\rangle - 12\langle\delta N_p^{(\mathrm{net})}\delta N_p^{(\mathrm{tot})}\rangle \\ &+ 6\langle N_p^{(\mathrm{net})}\rangle,\end{aligned}$$

$$\begin{aligned}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^4\rangle_c =& 16\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c - 48\langle(\delta N_p^{(\mathrm{net})})^2\delta N_p^{(\mathrm{tot})}\rangle \\ &+ 48\langle(\delta N_p^{(\mathrm{net})})^2\rangle + 12\langle(\delta N_p^{(\mathrm{tot})})^2\rangle - 26\langle N_p^{(\mathrm{tot})}\rangle,\end{aligned}$$

Strange Baryons

Decay Rates:

Λ $\quad m_\Lambda \simeq 1116[\mathrm{MeV}]$

$$\Rightarrow p : n \simeq 1.6 : 1$$

Σ $\quad m_\Sigma \simeq 1190[\mathrm{MeV}]$

$$\Rightarrow p : n \simeq 1 : 1.8$$

Decay modes:

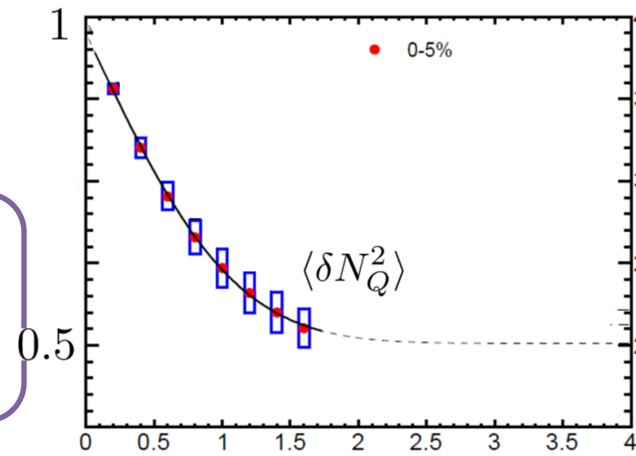
$\Lambda \to$	$p + \pi^-$	64%
	$n + \pi^0$	36%
$\Sigma^+ \to$	$p + \pi^0$	52%
	$n + \pi^+$	48%
$\Sigma^0 \to \Lambda \to$	$p + \pi^-$	64%
	$n + \pi^0$	36%
$\Sigma^- \to$	$n + \pi^-$	

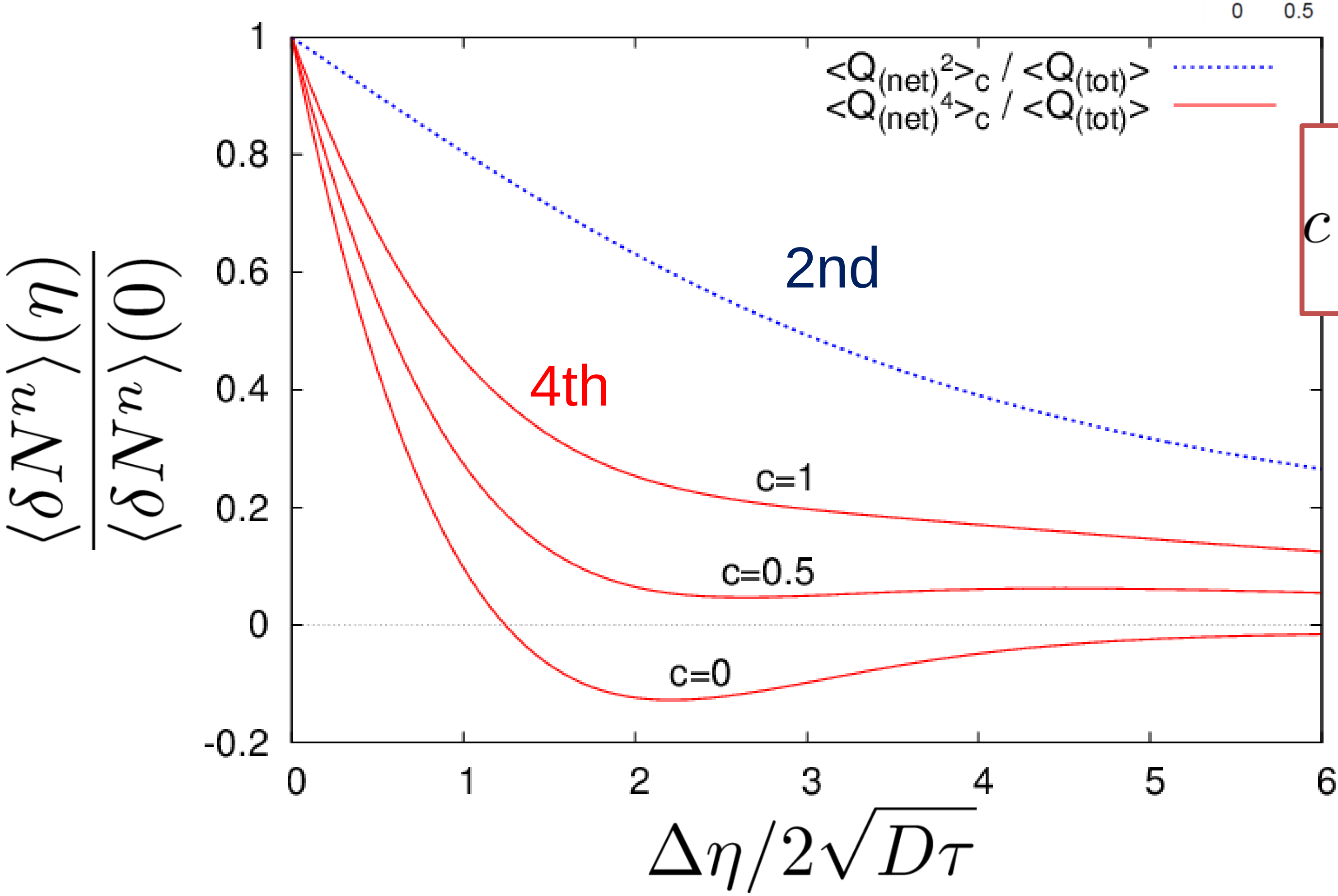
Regarding these ratios even, protons from these decays is incorporated into the binomial distribution. Then, $N_N \to N_B$

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$



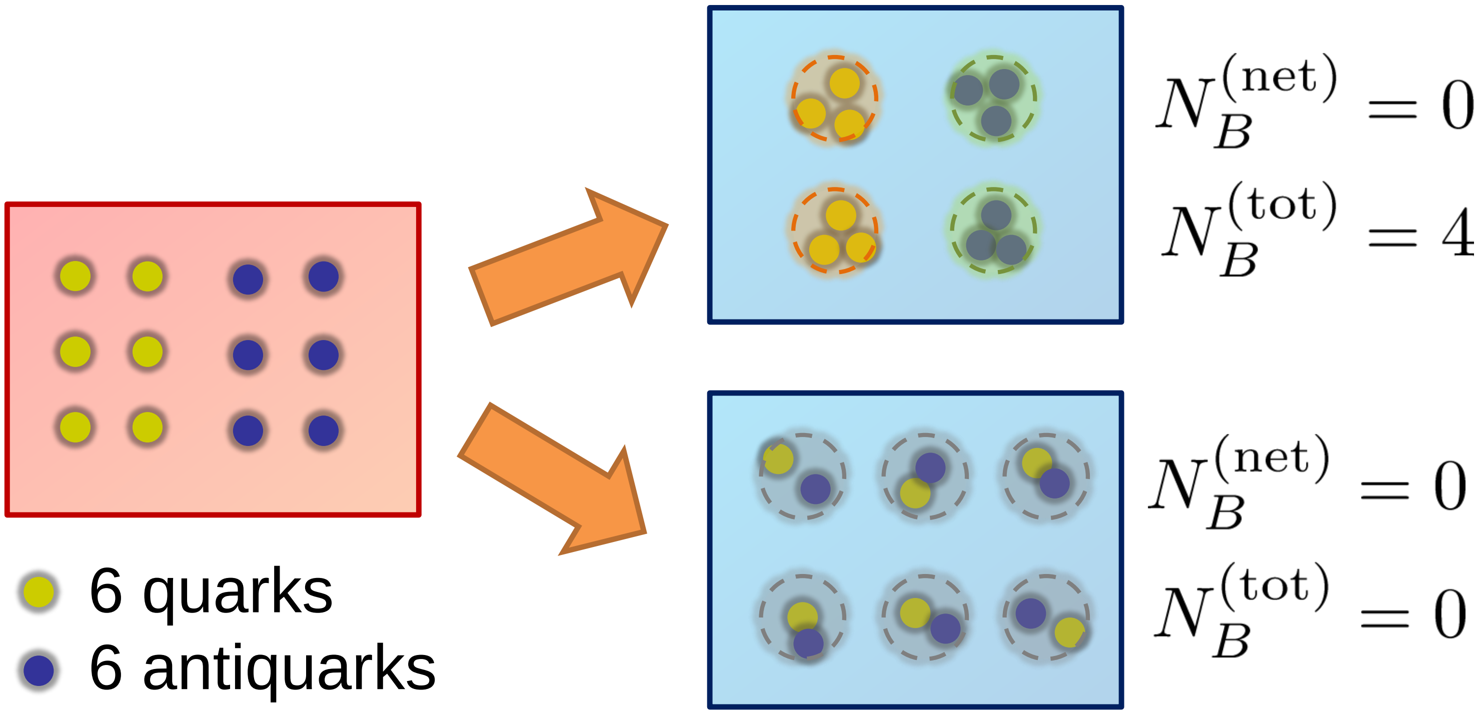


$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

Total Charge Number

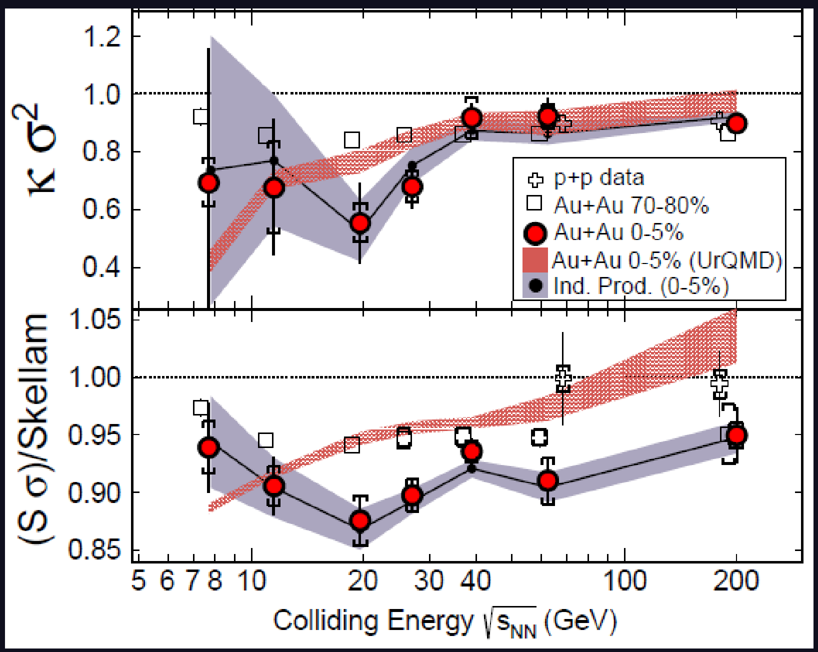
In recombination model,



- $N_B^{(\mathrm{tot})}$ can fluctuate, while $N_B^{(\mathrm{net})}$ does not.

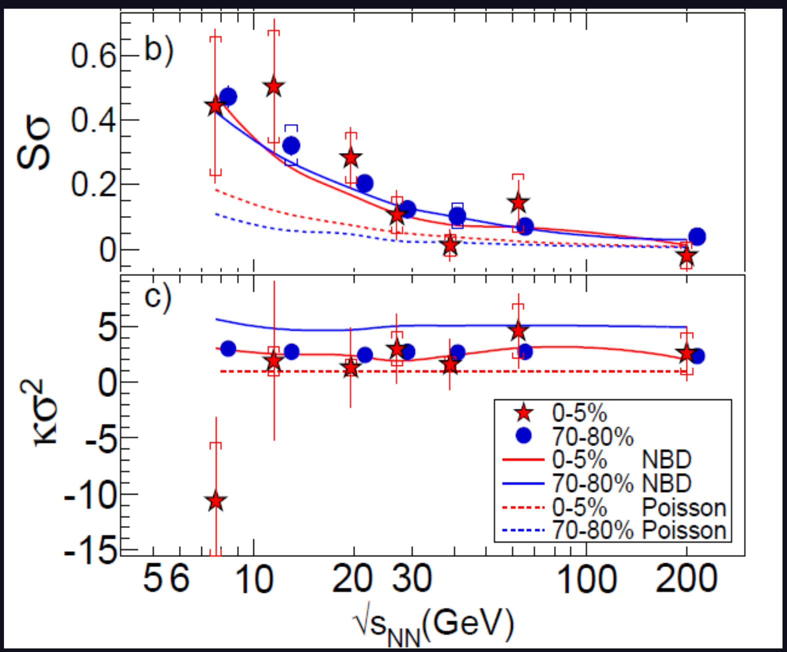
Higher Order Cumulants @ STAR

(Net-) Proton Number



STAR, PRL**112**,032302(2014)

(Net-) Electric Charge

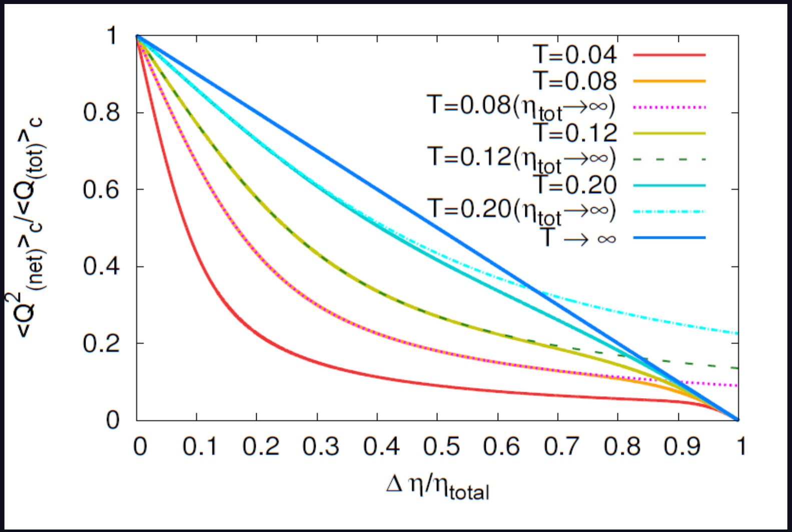


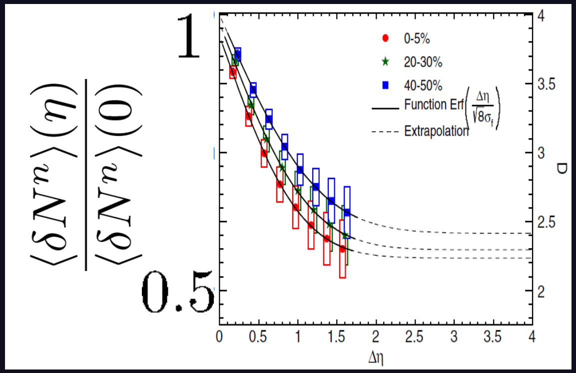
STAR, arXiv:1402.1558

- Exp. results are close to Poissonian values.
- Proton number cumulants are lower than the Poissonian values.

2^{nd} Order Cumulant

consistent with stochastic diffusion equation





Search of QCD Phase Structure

Stronger correlation length dep.

Stephanov, 2009

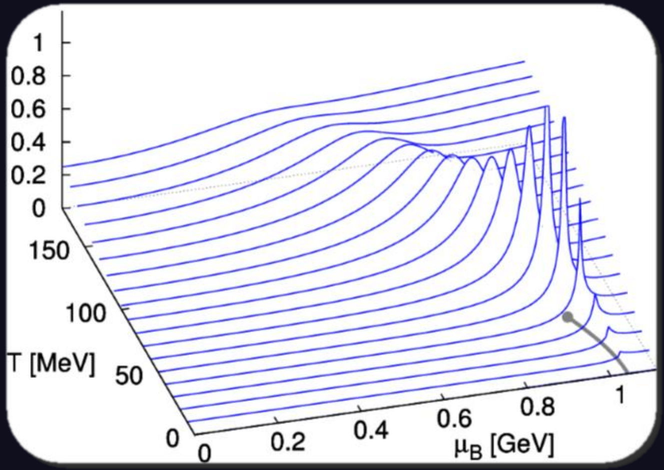
$$\langle \delta N^2 \rangle \sim \xi^2,\ \langle \delta N^3 \rangle \sim \xi^{4.5},\ \langle \delta N^4 \rangle_c \sim \xi^7$$

Sign of cumulants

Asakawa, Ejiri, MK, 2009
Friman+, 2011
Stephanov, 2011

$$\langle \delta N^n \rangle = T \frac{\partial^n}{\partial \hat{\mu}^n} \ln Z$$

$$\langle \delta N^3 \rangle = \frac{\partial}{\partial \hat{\mu}} \langle \delta N^2 \rangle$$



Fluctuations

- Fluctuations reflect properties of matter.
  - Enhancement near the critical point

    Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);…
  - Ratios between cumulants of conserved charges

    Asakawa,Heinz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)
  - Signs of higher order cumulants

    Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)