Thermal Fluctuations in Heavy Ion Collisions

Masakiyo Kitazawa
(Osaka U.)
Beam-Energy Scan

Quark-Gluon Plasma

Hadrons

Color SC
Beam-Energy Scan

Grand Canonical Ensemble

- Au+Au
  - 00-05%
  - 05-10%
  - 10-20%
  - 20-30%
  - 30-40%
  - 40-60%
  - 60-80%

STAR Preliminary

Muons
- Cleymans
- Andronic

Color SC

Hadrons
Bulk (Thermal) Fluctuations

Observables in equilibrium are fluctuating!

\[ \langle \delta N^2 \rangle = V \chi_2 = \sigma^2 \]

\[ S = \frac{\langle \delta N^3 \rangle}{\sigma^3} \]

\[ \kappa = \frac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2} \]

Gaussian

non-Gaussianity
Event-by-Event Analysis

Pioneering studies on $\sigma$:

- Search of QCD critical point  Stephanov, Rajagopal, Shuryak, PRL (1998)
- Quark deconfinement  Asakawa, Heinz, Muller PRL; Jeon, Koch PRL (2000)
My Messages

- Fluctuations are invaluable observables in HIC

- But, we must understand them in more detail

- It’s possible, interesting, and important
Why Fluctuations?
Brownian Motion

A. Einstein
1905

Fluctuations opened atomic physics
Shot Noise at Normal-Superconductor Junction


Similar experiments for fractional QHE ex. Saminadayar+, PRL 79, 2526 (1997)
Definite definition for operators
- as a Noether current
- calculable on any theory

ex: on the lattice
Conserved Charges: Theoretical Advantage

- Definite definition for operators
  - as a Noether current
  - calculable on any theory
  ex: on the lattice

- Simple thermodynamic relations
  \[ \langle \delta N_c^n \rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n} \]
  - Intuitive interpretation for the behaviors of cumulants
  ex: \[ \langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B} \]

Asakawa, Ejiri, MK, 2009
Conserved-Charge Fluctuations

**Fluctuations of CC**: rigorously defined in a theory
- operators as the Noether current
- as derivatives of the partition function

They are lattice observables

**Fluctuations of CC**

II

LAT-HIC crossover

QCD phase diagram 3, Wed. 11:00-13:30
Recent Progress in Lattice Community
From LATTICE2013 presentations
Fluctuations

Free Boltzmann $\rightarrow$ Poisson

\[
\langle \delta N_n \rangle_c = \langle N \rangle
\]

\[
\langle \delta N_q^n \rangle_c = \langle N_q \rangle
\]

\[
\langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle
\]

\[
3N_B = N_q
\]

\[
\langle \delta N_B^n \rangle_c = \langle N_B \rangle
\]
Fluctuations

Free Boltzmann $\rightarrow$ Poisson

$$\langle \delta N^n \rangle_c = \langle N \rangle$$

$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\langle \delta N_B^n \rangle_c = \frac{1}{3n-1} \langle N_B \rangle$$

$$3N_B = N_q$$

RBC-Bielefeld '09

$$12 \frac{c_B^4}{c_B^2} = \frac{\langle B^4 \rangle - 3 \langle B^2 \rangle^2}{\langle B^2 \rangle}$$

filled: nt=4

open: nt=6

$T [MeV]$
Electric Charge Fluctuation

Asakawa, Heinz, Muller; Jeon, Koch, 2000

\[ |q_q| = 1/3, 2/3 \]

\[ |q_B| = 1 \]

**D-measure**

\[ D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle} \]

- \( D \sim 3-4 \) Hadronic
- \( D \sim 1-1.5 \) QGP
Electric Charge Fluctuation

PHENIX (2002); STAR (2003)
ALICE, PRL 110, 152301 (2013)

D measure

\[ D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle} \]

- \( D \sim 3-4 \) Hadronic
- \( D \sim 1-1.5 \) QGP

- No suppression at RHIC energy
- Fluctuations @ LHC cannot be described by hadronic d.o.f.
Rapidity Window Dependence

ALICE, PRL 110, 152301 (2013)

hadronic

• Smaller $\Delta \eta$
  more hadronic

• Larger $\Delta \eta$
  more QGP like

Same information in

• $\langle \delta N^{(\text{net})}_Q(\eta_1) \delta N^{(\text{net})}_Q(\eta_2) \rangle$

• $\simeq$ balance function

to be studied by fluctuating hydro.

Stochastic Diffusion eq.
Shuryak, Stephanov, 2001

$1/X \sim \Delta \eta$
Time Evolution in HIC

Quark-Gluon Plasma

Hadronization

Freezeout

\[\langle \Delta N^2 \rangle / \Delta \eta\]

\[\chi_{\text{HAD}}\]

\[\chi_{\text{QGP}}\]
Various Contributions

- Initial fluctuations  
  - Enhance
- Effect of jets  
  - Enhance
- Negative binomial (?)  
  - Enhance
- Final state rescattering  
  - Enhance to Poisson
- Coordinate vs pseudo rapidities  
  - Enhance to Poisson
- Particle missID  
  - Enhance to Poisson
- Efficiency correction  
  - Enhance to Poisson
- Global charge conservation  
  - Suppress
Global Charge Conservation

For equilibrated medium

\[
\langle \delta N^2 \rangle_{\text{exp.}} = \langle \delta N^2 \rangle_{\text{GC}} \times \left( 1 - \frac{\Delta y}{\eta_{\text{tot}}} \right)
\]
Global Charge Conservation

For equilibrated medium

$$\langle \delta N^2 \rangle_{\text{exp.}} = \langle \delta N^2 \rangle_{\text{GC}} \times \left( 1 - \frac{\Delta y}{y_{\text{tot}}} \right)$$

Solving the time evolution...

GCC effect on the hadronic diffusion is negligible in the ALICE result!

Sakaida, Poster I-35
Electric-Charge Fluctuations

- Electric charge fluctuations is suppressed at LHC!
- The suppression is most probably attributed to primordial physics
- Qualitative difference b/w RHIC and LHC

... but why?
Non Gaussianity
Non-Gaussianity

CMB
Cosmic Microwave Background

• No statistically-significant signals
  Planck, 2013

Mesoscopic Systems

• Full counting statistics
• Cumulants up to 5th order
Non-Gaussianity in HIC

- Ratio of conserved charges
  Ejiri, Karsch, Redlich (2005)
- Critical enhancement
  Stephanov (2009)
- Sign change
  Asakawa, Ejiri, MK (2009); Friman+ (2011); Stephanov (2011)
- Strange confinement
  BNL-Bielefeld (2013)
- Distribution funcs themselves
  Morita+ (2013); Nakamura (Wed.)
Ratio of Cumulants

Ejiri, Karsch, Redlich, 2005

\[ \langle \delta N_q^n \rangle_c = \langle N_q \rangle \]

\[ \Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3n-1} \langle N_B \rangle \]

\[ \frac{\langle \delta N_B^4 \rangle_c}{\langle \delta N_B^2 \rangle_c} = \begin{cases} 1 & \text{hadronic} \\ 1/9 & \text{quark-gluon} \end{cases} \]
Strange Confinement

Baryonic
\[ \langle \delta N_B^2 \rangle = \langle \delta N_B^4 \rangle \]

\[ \langle \delta N_B^2 \rangle - \langle \delta N_B^4 \rangle_c \begin{cases} = 0 & \text{baryons confined} \\ \neq 0 & \text{something else} \end{cases} \]

Strangeness

\[ \langle \delta N_s \delta N_B^3 \rangle_c - \langle \delta N_s \delta N_B \rangle \begin{cases} = 0 & \text{strangeness confined} \\ \neq 0 & \text{something else} \end{cases} \]

Many lattice studies (LAT-HIC crossover):
Budapest-Wuppertal, 2013; BW,1403.4578; BNL-Bi.,1404.4043; Gupta+,1405.2206; Ratti, Wed.; Schmidt, Wed.; Nakamura, Wed.; Sharma, J-13
Cumulants: HIC@RHIC vs Lattice

Parameter window constrained by lattice

BNL-Bielefeld, LATTICE2013

Fluctuations “exp + lattice”

Particle abundance (chem. freezeout $T$)
Proton Number Cumulants at RHIC-BES

Exp. results are close to and less than Poissonian values.

omething interesting around $\sqrt{s_{NN}} \approx 20$GeV
Effects of Various Contributions

- Initial fluctuations
- Effect of jets
- Negative binomial (?)

- Final state rescattering
- Coordinate vs pseudo rapidities
- Particle missID
- Efficiency correction
- Global charge conservation
Caution!!

proton number cumulants $\neq$ baryon number cumulants

Let’s clarify their relation!
MK, Asakawa (2012;2012)
Nucleon has two isospin states.

A coin has two sides.

MK, Asakawa, 2012
Slot Machine Analogy

\[ P_{\text{tot}}(N) \]
\[ N_{\text{tot}} \]

\[ P_{\text{head}}(N) \]
\[ N_{\text{head}} \]
Reconstructing Total Coin Number

\[ P_{\text{head}}(N_{\text{head}}) = \sum_{N_{\text{tot}}} B_{1/2}(N_{\text{head}}; N_{\text{tot}}) P_{\text{tot}}(N_{\text{tot}}) \]
Isospin of N is not frozen at chemical freezeout!

**Cross sections of $p$**

\[
p + \pi^+ \rightarrow \Delta^{++} \rightarrow p + \pi^+ \]

\[
p + \pi^0 \rightarrow \Delta^+ \rightarrow p + \pi^0 \]

\[
n + \pi^+ \rightarrow \Delta^+ \rightarrow n + \pi^+ \]

\[
p + \pi^- \rightarrow \Delta^0 \rightarrow p + \pi^- \]

\[
n + \pi^0 \rightarrow \Delta^0 \rightarrow n + \pi^0 \]

\[
n + \pi^- \rightarrow \Delta^- \rightarrow n + \pi^- \]

**Decay rates of $\Delta$**

\[
p + \pi \rightarrow \Delta^{+,0} \rightarrow p : n = 5 : 4 \]

**Mean reaction time**

\[< 5\text{fm}/c\]
Nucleons in Hadronic Medium

- so many pions
- rare NN collisions
- no quantum corr.
Difference b/w \( N_B \) and \( N_p \)

Assumptions:
- Net-cumulants deviate from thermal value
- But, \( N_B, N_B^- \) are Poissonian

\[
\begin{align*}
2\langle (\delta N_p^{(net)})^2 \rangle &= \frac{1}{2} \langle (\delta N_B^{(net)})^2 \rangle + \frac{1}{2} \langle (\delta N_B^{(net)})^2 \rangle_{\text{free}} \\
2\langle (\delta N_p^{(net)})^3 \rangle &= \frac{1}{4} \langle (\delta N_B^{(net)})^3 \rangle + \frac{3}{4} \langle (\delta N_B^{(net)})^3 \rangle_{\text{free}} \\
2\langle (\delta N_p^{(net)})^4 \rangle_c &= \frac{1}{8} \langle (\delta N_B^{(net)})^4 \rangle_c + \ldots
\end{align*}
\]

genuine info. noise

Proton number cumulants are dominated by Poissonian noise

cf.) Nahrgang+, arXiv:1402.1238
Efficiency Correction

If efficiency for each particle is uncorrelated, the binomial correction to the distribution function is given by:

\[ P_{\text{exp.}}(N) = \sum_{N'} B_c(N; N') P(N') \]

Electric charge

for Particle missID: Ono, Asakawa, MK, PRC, 2013

STAR, arXiv:1402.1558

MK, Asakawa, 2012
Bdzak, Koch, 2012
STAR, 2013
More Information on/from Fluctuations

$\Delta \eta$ dependence

MK, Asakawa, Ono, PLB 728, 386 (2014)
$\Delta \eta$ Dep. of Non-Gaussianity

How does the 4-th order cumulant behave as a function of $\Delta \eta$?
Fluctuating Hydrodynamics?

- Distributions in experiments are close to Poissonian.
- Cumulants are expected to increase in the hadronic medium.

These behaviors **cannot** be described by the theory of hydrodynamic fluctuations.
Hydrodynamic Fluctuations

Stochastic diffusion equation

\[ \partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau) \]

Fluctuation of \( n \) is Gaussian in equilibrium

Markov (white noise) + continuity

Gaussian noise

cf) Gardiner, "Stochastic Methods"

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012
Stephanov, Shuryak, 2001
How to Introduce Non-Gaussianity?

**Stochastic diffusion equation**

\[ \partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau) \]

- Choices to introduce non-Gaussianity in equil.:
  - \( n \) dependence of diffusion constant \( D(n) \)
  - colored noise
  - discretization of \( n \)
How to Introduce Non-Gaussianity?

**Stochastic diffusion equation**

\[ \partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau) \]

- Choices to introduce non-Gaussianity in equil.:
  - \( n \) dependence of diffusion constant \( D(n) \)
  - colored noise
  - discretization of \( n \) **our choice**

**REMARK:** Fluctuations measured in HIC are almost Poissonian.
Nucleons in Hadronic Medium

Baryons in hadronic medium behave like Brownian pollen

hadronize
chem. f.o.

$10 \sim 20 \text{fm}$

$p, \bar{p}$
$n, \bar{n}$
$\Delta(1232)$

mesons
baryons
Divide spatial coordinate into discrete cells

Master Equation

\[
\frac{\partial}{\partial t} P(n) = \gamma \sum_x [(n_x + 1) \{ P(n + e_x - e_{x+1}) + P(n + e_x - e_{x-1}) \} - 2n_x P(n)]
\]

Solve the DME exactly, and take \( \alpha \rightarrow 0 \) limit
hadronization
chemical freezeout

Initial condition
- boost invariance
- locality of fluctuations
- small cumulants

Brownian diffusion

kinetic freezeout

Comments:
- agreement with stochastic diffusion eq. up to Gaussian fluctuation
- Poisson (Skellam) distribution in equilibrium: consistent with HRG
Solution of DME in a $a \to 0$ Limit

1st order (deterministic) $\langle n \rangle$
- consistent with diffusion equation with $D = \gamma a^2$
- Continuum limit with fixed $D = \gamma a^2$

2nd order $\langle \delta n^2 \rangle$
- consistent with stochastic diffusion eq.
  (for sufficiently smooth initial conditions)

Shuryak, Stephanov, 2001

Nontrivial results for non-Gaussian fluctuations
Prepare 2 species of (non-interacting) particles

\[
\bar{Q}(\tau) = \int_0^{\Delta \eta} d\eta \left( n_1(\eta, \tau) - n_2(\eta, \tau) \right)
\]

Let us investigate

\[\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c\]

at freezeout time \(t\)
Time Evolution in Hadronic Phase

Hadronization (initial condition)

- Boost invariance / infinitely long system
- Local equilibration / local correlation

\[
\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c \quad \langle \bar{Q}^2 Q_{(tot)} \rangle_c \quad \langle Q_{(tot)}^2 \rangle_c
\]

- suppression owing to local charge conservation
- strongly dependent on hadronization mechanism
**Time Evolution in Hadronic Phase**

Hadronization (initial condition)

- Boost invariance / infinitely long system
- Local equilibration / local correlation

Suppression owing to local charge conservation

Strongly dependent on hadronization mechanism

Freezeout
$\Delta \eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(tot)} \rangle_c = 0$$

Parameter sensitive to hadronization:

$$c = \frac{\langle Q_{(net)}^2 \rangle_c / \langle Q_{(tot)} \rangle_c}{\langle Q_{(net)}^4 \rangle_c / \langle Q_{(tot)} \rangle_c}$$
In recombination model,

\[ N_B^{(\text{net})} = 0 \]
\[ N_B^{(\text{tot})} = 4 \]
\[ N_B^{(\text{net})} = 0 \]
\[ N_B^{(\text{tot})} = 0 \]

\[ N_B^{(\text{tot})} \] can fluctuate, while \[ N_B^{(\text{net})} \] does not.
**Initial fluctuations:**

\[
\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q^{(\text{tot})} \rangle_c = 0.5 \langle Q^{(\text{tot})} \rangle_c
\]
$4^{\text{th}}$ order Cumulant at ALICE

MK, Asakawa, Ono (2014)
Sakaida+, poster I-35

rapidity coverage at ALICE ($\eta_{\text{tot}} = 8$)

$4^{\text{th}}$ order cumulant is sensitive to initial fluctuation / transport property / confinement

It can be non-monotonic and negative!
How does the 4-th order cumulant behave as a function of $\Delta \eta$?
Suggestions to Experimentalists

- many conserved charges
  electric charge, baryon number, (and strangeness?)
  with different diffusion constants

- various cumulants
  second, third, fourth, mixed, (and much higher?)

- \( \Delta \eta \) window dependences
  primordial thermodynamics, transport property, confinement
  no normalization

- Beam Energy Scan
  LHC, RHIC-BES, FAIR, NICA, J-PARC, ...
My Messages

- Fluctuations are invaluable observables in HIC
- But, we must understand them in more detail
- It’s possible, interesting, and important

We are just arriving at the starting point to explore QCD phase structure with fluctuations!
Summary

Fluctuations in HIC are nonthermal!

Plenty of physics in $\Delta \eta$ dependences of various cumulants

$\langle N_Q^2 \rangle_c, \langle N_B^2 \rangle_c, \langle N_Q^4 \rangle_c, \langle N_B^4 \rangle_c,$

$\langle N_{ch}^2 \rangle_c, \ldots$

Diagnosing dynamics of HIC
- history of hot medium
- mechanism of hadronization
- diffusion constant

Physical meanings of fluctuation obs. in experiments.
\( \frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle} \) decreases as \( \Delta \eta \) becomes larger at RHIC energy.
Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$

- $N_N$ nucleons
- $N_{\bar{N}}$ anti-nucleons

$\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) = F(N_N, N_{\bar{N}})B(N_p; N_N)B(N_{\bar{p}}; N_{\bar{N}})$

- for any phase space in the final state.
3rd & 4th Order Fluctuations

\[ N_B \rightarrow N_p \]

\[ \langle (\delta N_p^{(\text{net})})^3 \rangle = \frac{1}{8} \langle (\delta N_B^{(\text{net})})^3 \rangle + \frac{3}{8} \langle \delta N_B^{(\text{net})} \delta N_B^{(\text{tot})} \rangle, \]

\[ \langle (\delta N_p^{(\text{net})})^4 \rangle_c = \frac{1}{16} \langle (\delta N_B^{(\text{net})})^4 \rangle_c + \frac{3}{8} \langle (\delta N_B^{(\text{net})})^2 \delta N_B^{(\text{tot})} \rangle \]

\[ + \frac{3}{16} \langle (\delta N_B^{(\text{tot})})^2 \rangle - \frac{1}{8} \langle N_B^{(\text{tot})} \rangle, \]

\[ N_p \rightarrow N_B \]

\[ \langle (\delta N_B^{(\text{net})})^3 \rangle = 8 \langle (\delta N_p^{(\text{net})})^3 \rangle - 12 \langle \delta N_p^{(\text{net})} \delta N_p^{(\text{tot})} \rangle \]

\[ + 6 \langle N_p^{(\text{net})} \rangle, \]

\[ \langle (\delta N_B^{(\text{net})})^4 \rangle_c = 16 \langle (\delta N_p^{(\text{net})})^4 \rangle_c - 48 \langle (\delta N_p^{(\text{net})})^2 \delta N_p^{(\text{tot})} \rangle \]

\[ + 48 \langle (\delta N_p^{(\text{net})})^2 \rangle + 12 \langle (\delta N_p^{(\text{tot})})^2 \rangle - 26 \langle N_p^{(\text{tot})} \rangle, \]
Strange Baryons

Decay Rates:

\[ \Lambda \quad m_\Lambda \simeq 1116 \text{[MeV]} \]

\[ \sum \quad m_\sum \simeq 1190 \text{[MeV]} \]

\[ p : n \simeq 1.6 : 1 \]

\[ p : n \simeq 1 : 1.8 \]

Decay modes:

\[ \Lambda \rightarrow p + \pi^- \quad 64\% \]

\[ \sum^+ \rightarrow p + \pi^0 \quad 52\% \]

\[ \sum^0 \rightarrow \Lambda \rightarrow p + \pi^- \quad 64\% \]

\[ \sum^- \rightarrow n + \pi^- \quad 36\% \]

\[ \sum^0 \rightarrow \Lambda \rightarrow n + \pi^0 \quad 36\% \]

Regarding these ratios even, protons from these decays is incorporated into the binomial distribution. Then, \( N_\sum \rightarrow N_B \)
$\Delta \eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\text{tot})} \rangle_c = 0$$

Parameter sensitive to hadronization
Total Charge Number

In recombination model,

- $N_B^{(\text{net})} = 0$
- $N_B^{(\text{tot})} = 4$

- $N_B^{(\text{net})} = 0$
- $N_B^{(\text{tot})} = 0$

- $N_B^{(\text{tot})}$ can fluctuate, while $N_B^{(\text{net})}$ does not.
Higher Order Cumulants @ STAR

(Net-) Proton Number

(Net-) Electric Charge

Exp. results are close to Poissonian values.
Proton number cumulants are lower than the Poissonian values.

STAR, PRL 112, 032302 (2014)
STAR, arXiv:1402.1558
2\textsuperscript{nd} Order Cumulant

consistent with stochastic diffusion equation
Search of QCD Phase Structure

Stronger correlation length dep.

\[ \langle \delta N^2 \rangle \sim \xi^2, \quad \langle \delta N^3 \rangle \sim \xi^{4.5}, \quad \langle \delta N^4 \rangle_c \sim \xi^7 \]

Sign of cumulants

\[ \langle \delta N^n \rangle = T \frac{\partial^n}{\partial \hat{\mu}^n} \ln Z \]

\[ \langle \delta N^3 \rangle = \frac{\partial}{\partial \hat{\mu}} \langle \delta N^2 \rangle \]

Stephanov, 2009

Asakawa, Ejiri, MK, 2009

Friman+, 2011

Stephanov, 2011
Fluctuations

- Fluctuations reflect properties of matter.
- Enhancement near the critical point: Stephanov, Rajagopal, Shuryak ('98); Hatta, Stephanov ('02); Stephanov ('09); …
- Ratios between cumulants of conserved charges: Asakawa, Heinz, Muller ('00); Jeon, Koch ('00); Ejiri, Karsch, Redlich ('06)
- Signs of higher order cumulants: Asakawa, Ejiri, MK ('09); Friman, et al. ('11); Stephanov ('11)