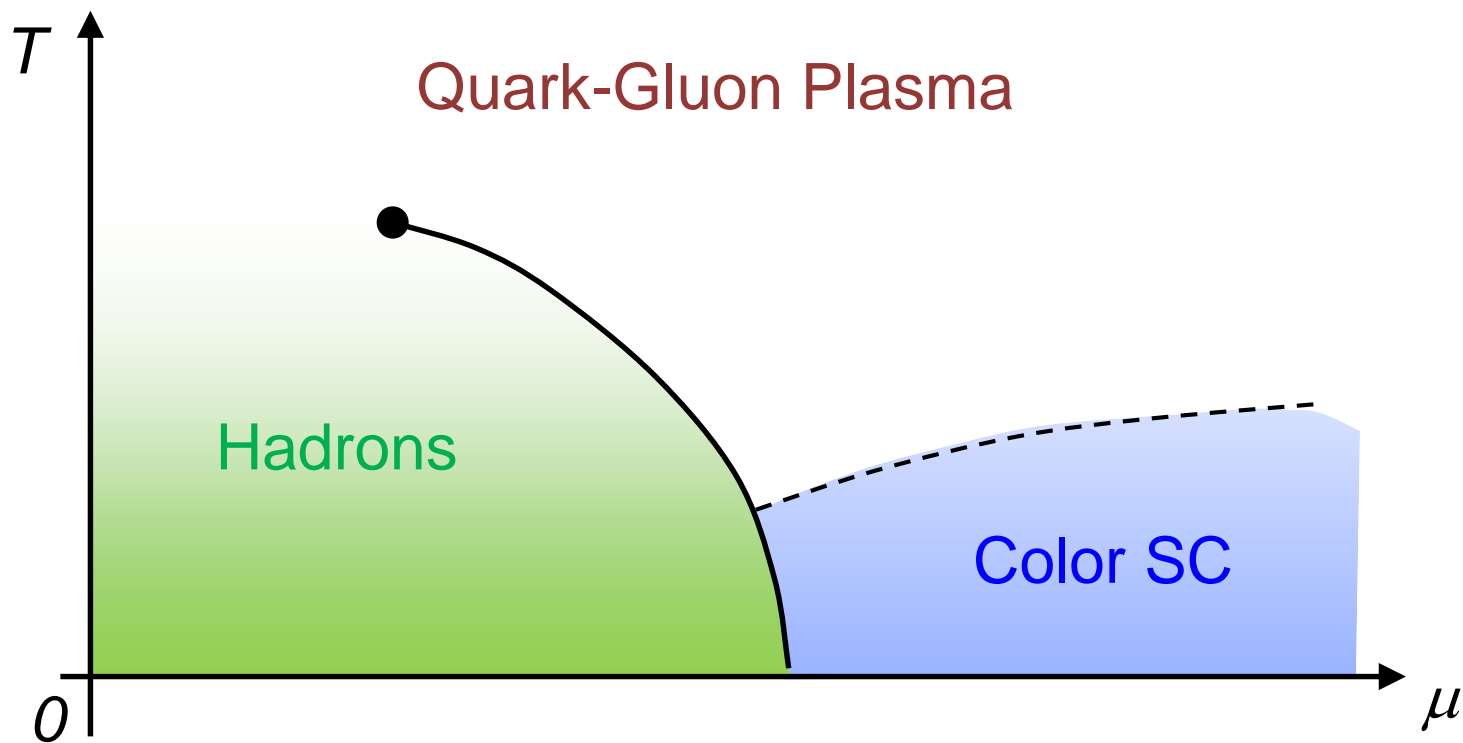


重イオン衝突実験に おける熱ゆらぎ ～非平衡性と非ガウス性を中心に～

Masakiyo Kitazawa

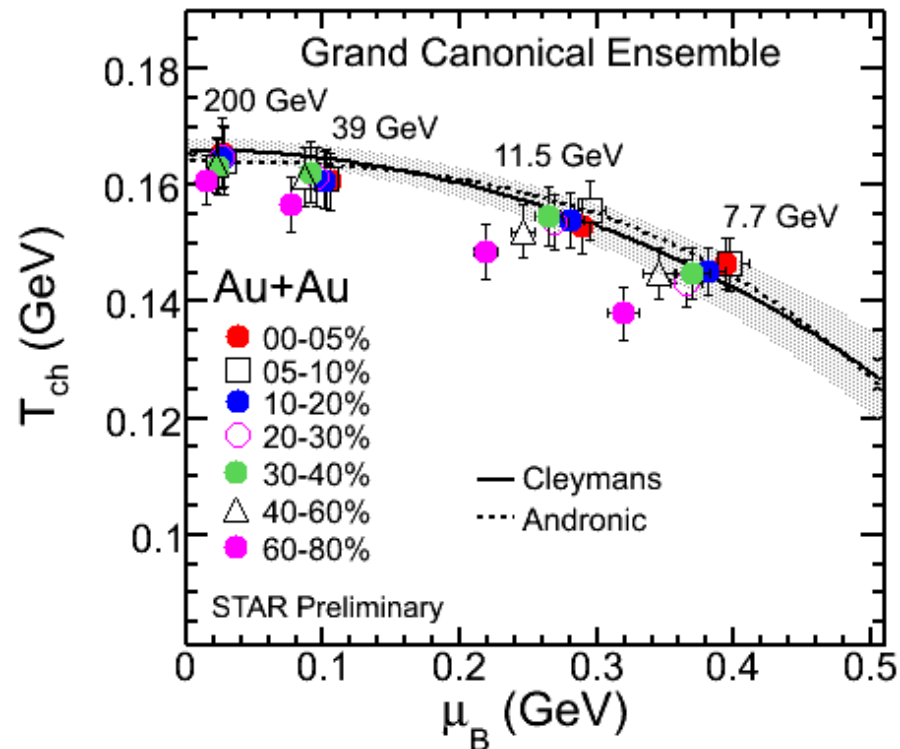
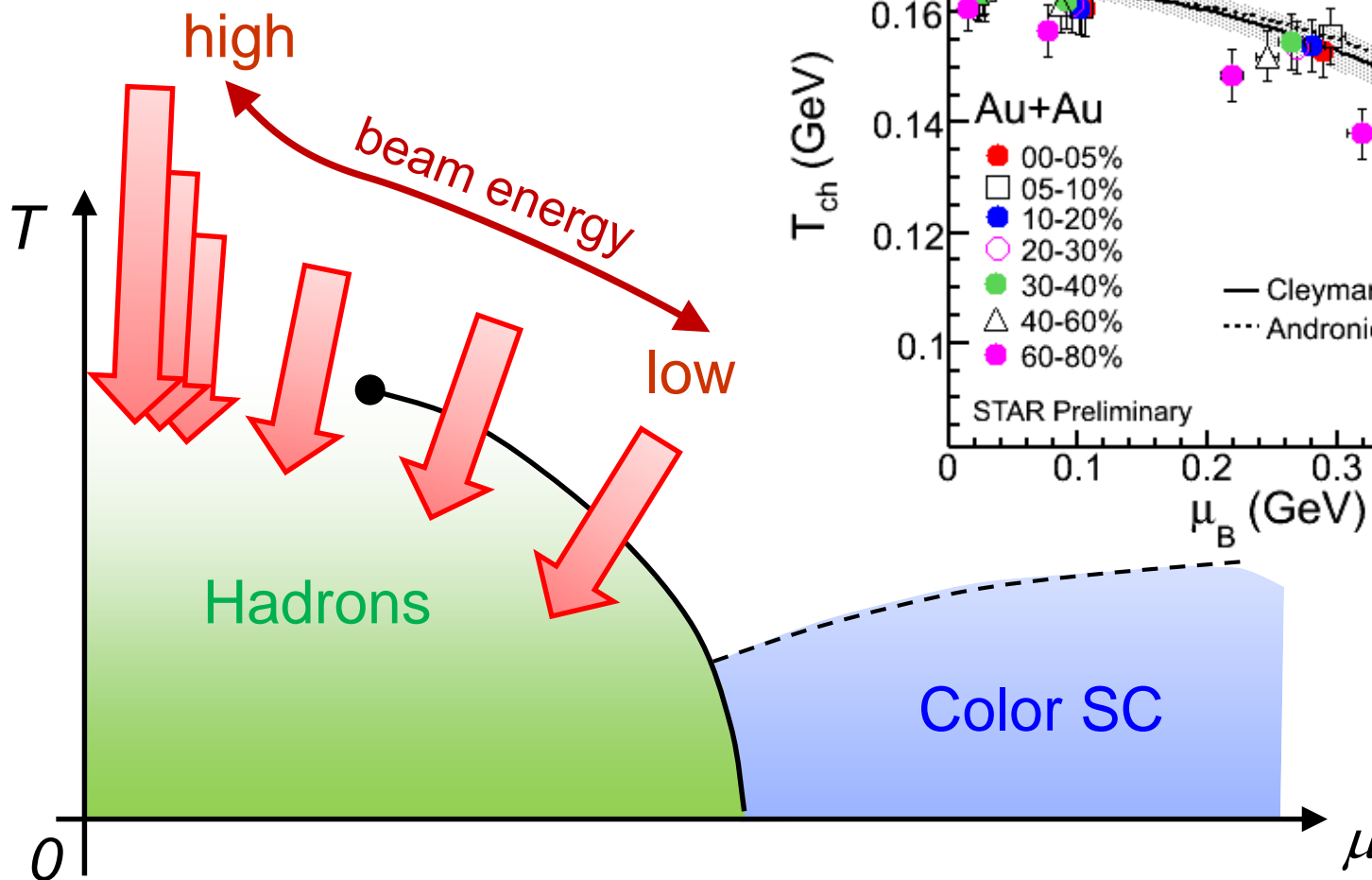
(Osaka U.)

Beam-Energy Scan



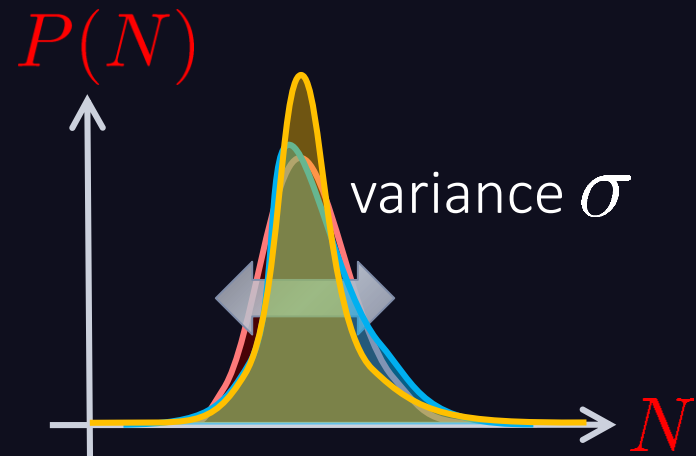
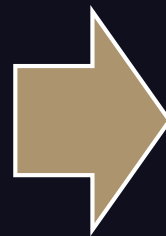
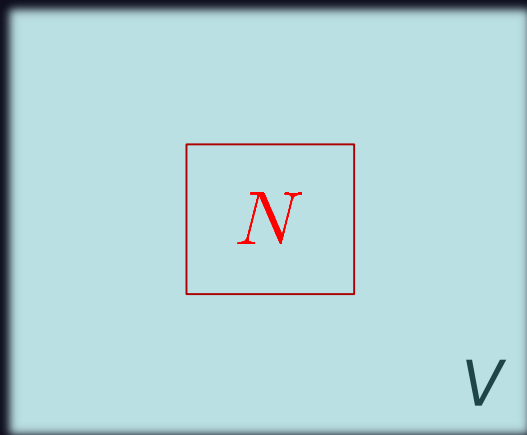
Beam-Energy Scan

STAR 2012



Bulk (Thermal) Fluctuations

Observables in equilibrium are fluctuating!



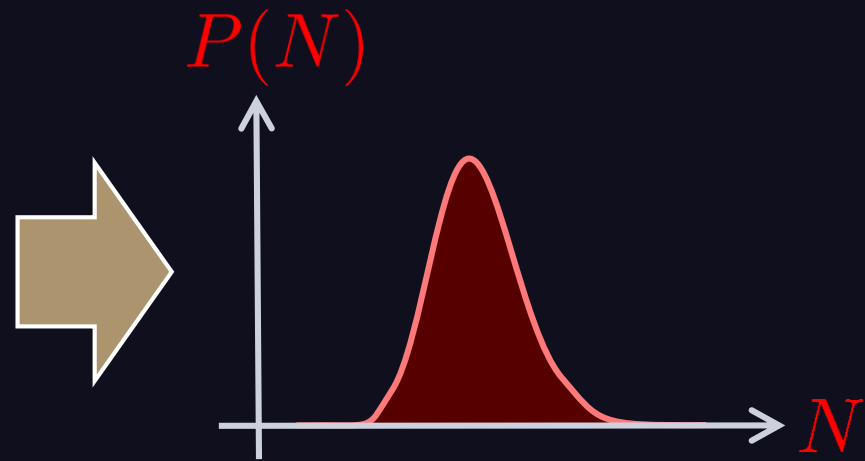
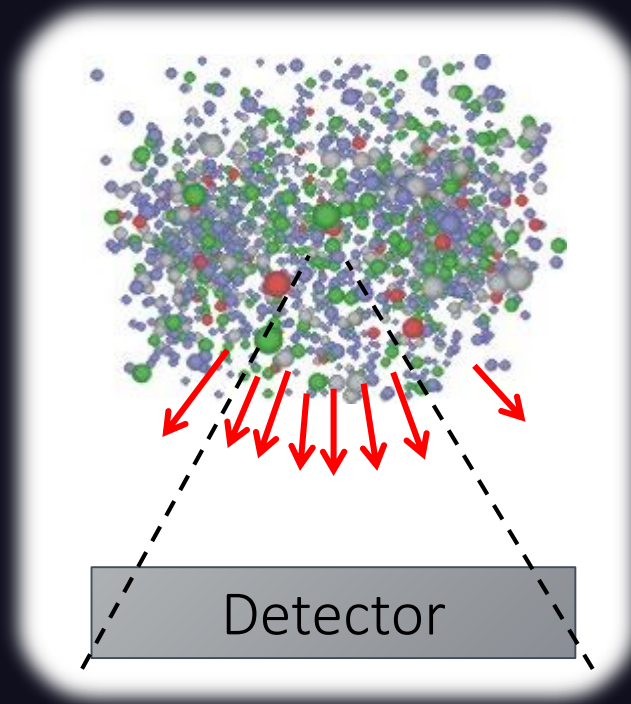
A large brown arrow pointing to the equations.

$$\left\{ \begin{array}{l} \langle \delta N^2 \rangle = V \chi_2 = \sigma^2 \\ S = \frac{\langle \delta N^3 \rangle}{\sigma^3} \\ \kappa = \frac{\langle \delta N^4 \rangle - 3 \langle \delta N^2 \rangle^2}{\chi_2 \sigma^2} \end{array} \right.$$

Gaussian

non-Gaussianity

Event-by-Event Analysis



Pioneering studies on σ :

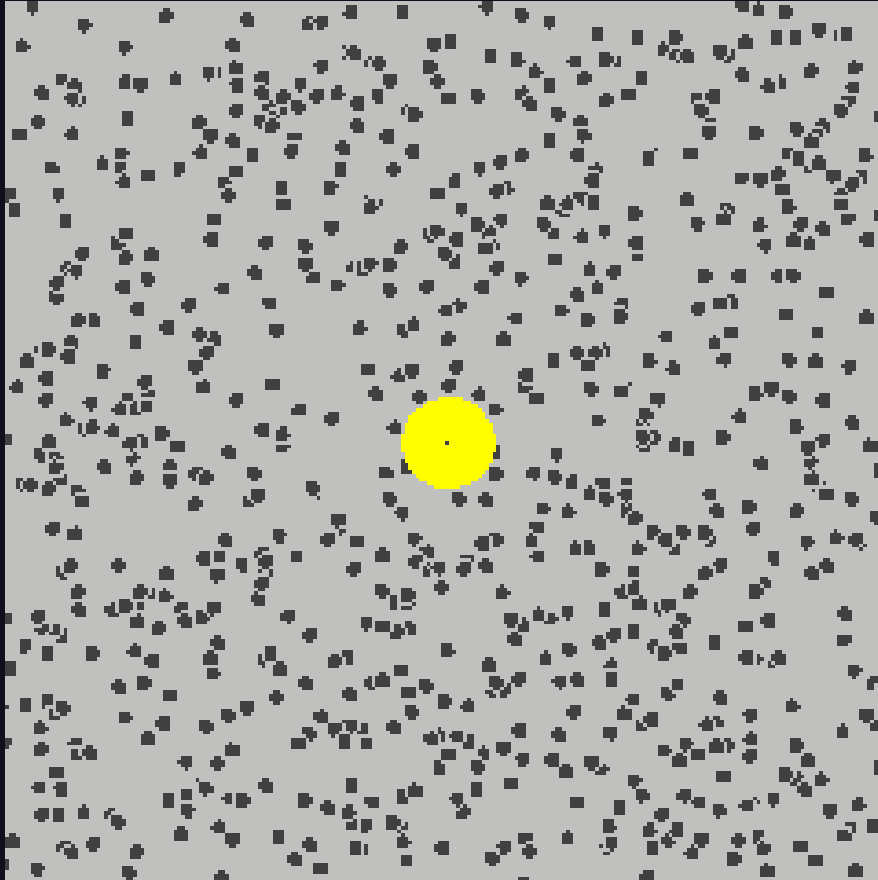
- Search of QCD critical point Stephanov,Rajagopal,Shuryak,PRL(1998)
- Quark deconfinement Asakawa,Heinz,Muller PRL; Jeon,Koch PRL(2000)

My Messages

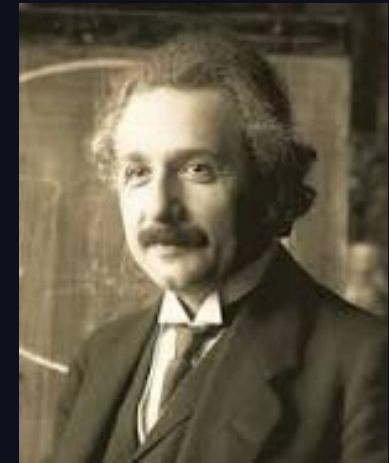
- Fluctuations are invaluable observables in HIC
- But, we must understand them in more detail
- **It's possible**, interesting, and important

Why Fluctuations?

Brownian Motion



from Wikipedia

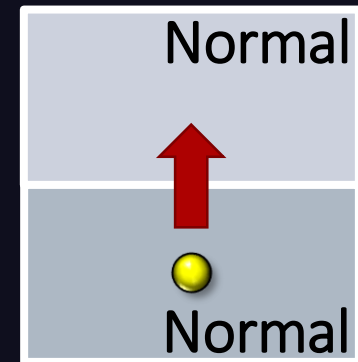
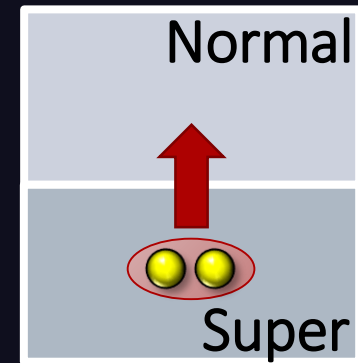
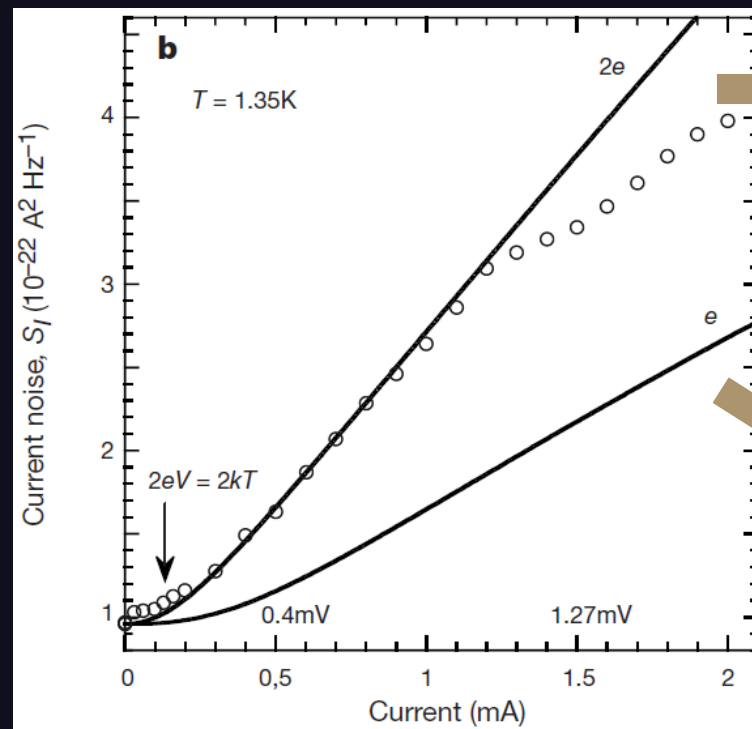
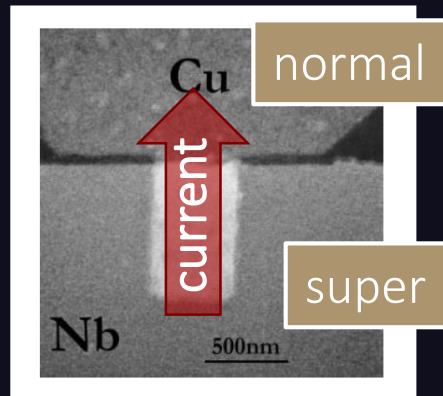


A. Einstein
1905

Fluctuations opened atomic physics

Shot Noise at Normal-Superconductor Junction

X. Jehl+, Nature 405, 50 (2000)



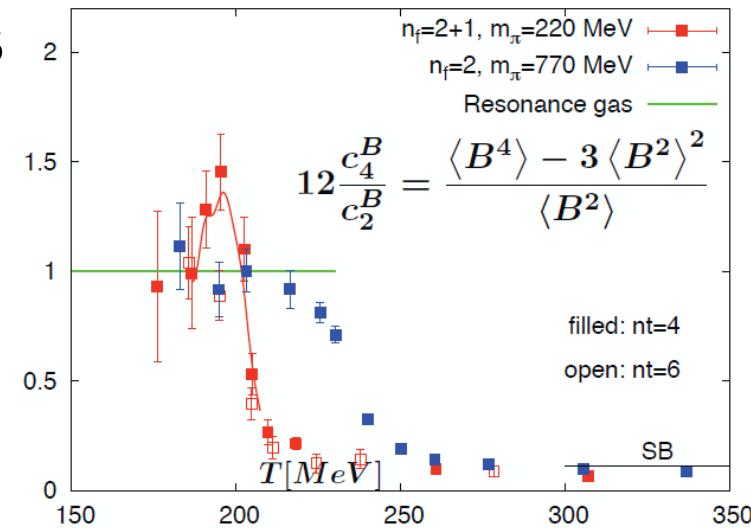
Similar experiments for fractional QHE ex. Saminadayar+, PRL 79, 2526 (1997)

Conserved Charges : Theoretical Advantage

□ Definite definition for operators

- as a Noether current
- calculable on any theory

ex: on the lattice

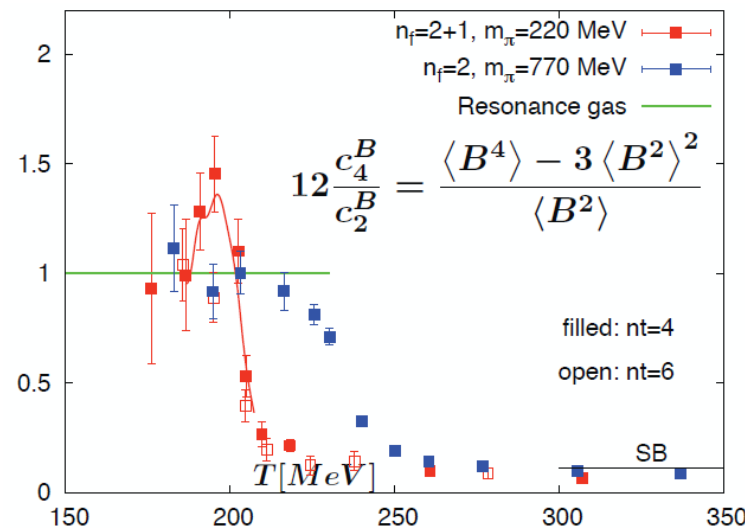


Conserved Charges : Theoretical Advantage

□ Definite definition for operators

- as a Noether current
- calculable on any theory

ex: on the lattice 

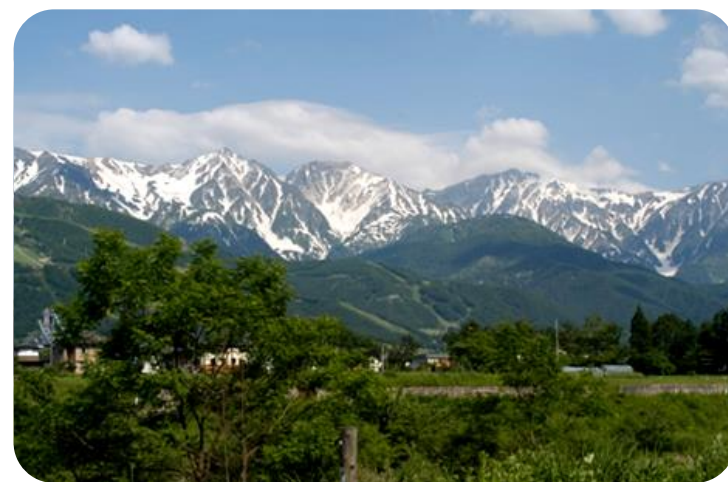


□ Simple thermodynamic relations

$$\langle \delta N_c^n \rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}$$

- Intuitive interpretation for the behaviors of cumulants

ex: $\langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B}$



Asakawa, Ejiri, MK, 2009

Conserved-Charge Fluctuations

Fluctuations of CC : rigorously defined in a theory

- operators as the Noether current
- as derivatives of the partition function

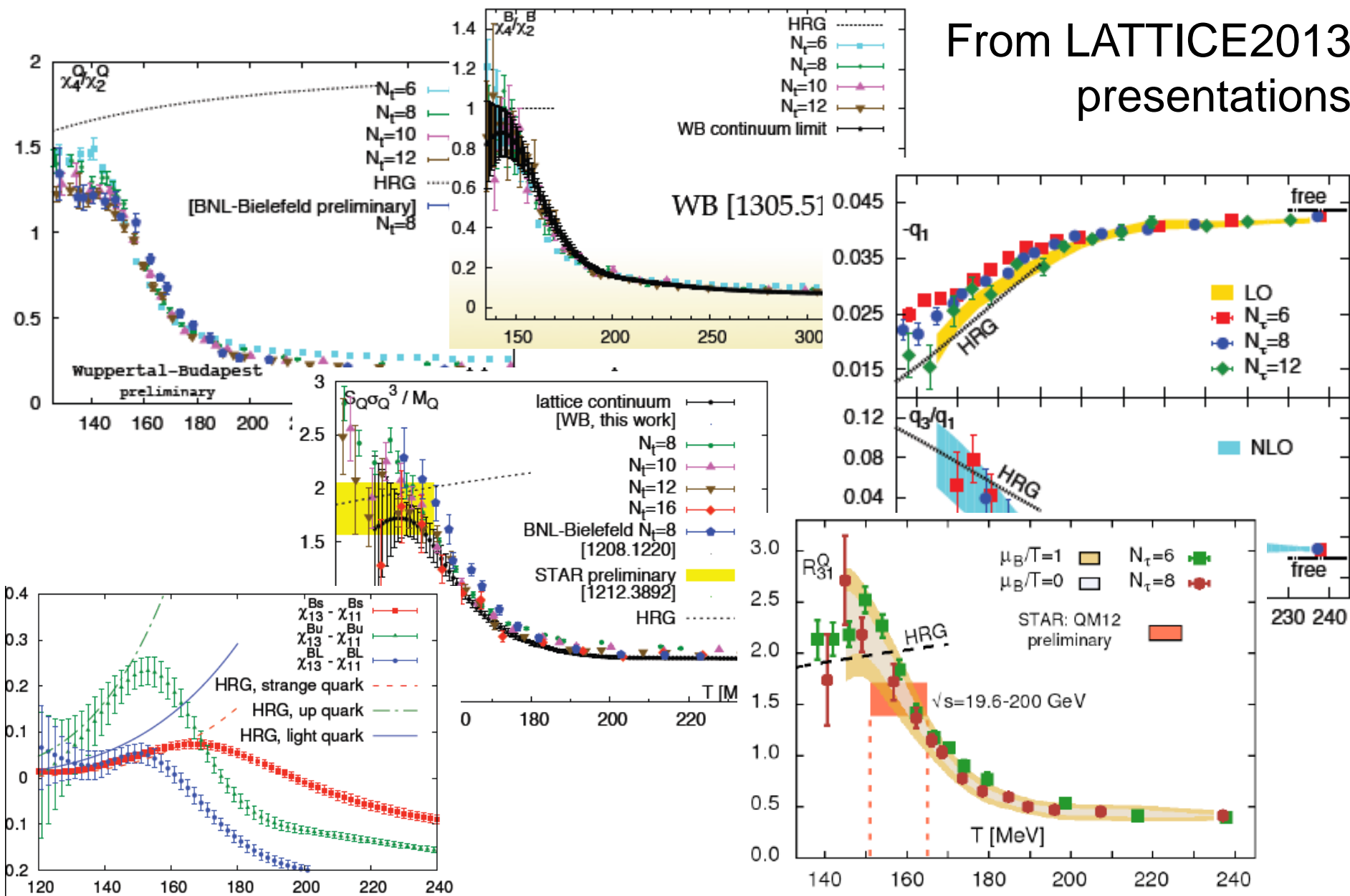
They are lattice observables

Fluctuations of CC
||
LAT-HIC crossover

QCD phase diagram 3, Wed. 11:00-13:30

Recent Progress in Lattice Community

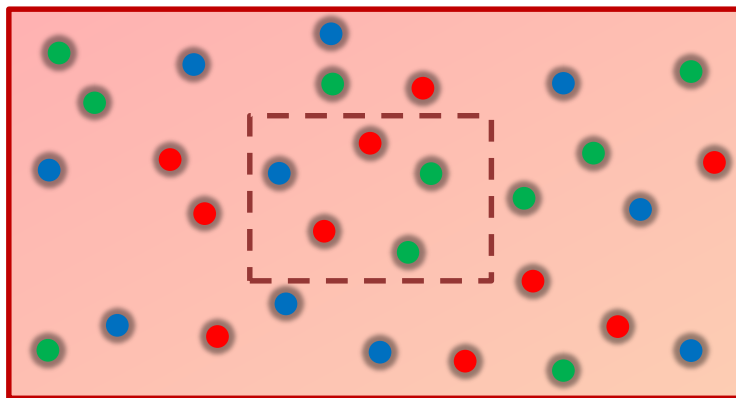
From LATTICE2013 presentations



Fluctuations

Free Boltzmann \rightarrow Poisson

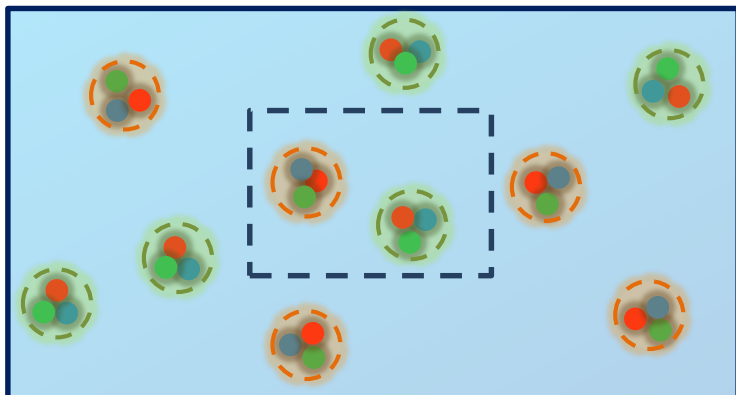
$$\langle \delta N^n \rangle_c = \langle N \rangle$$



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$

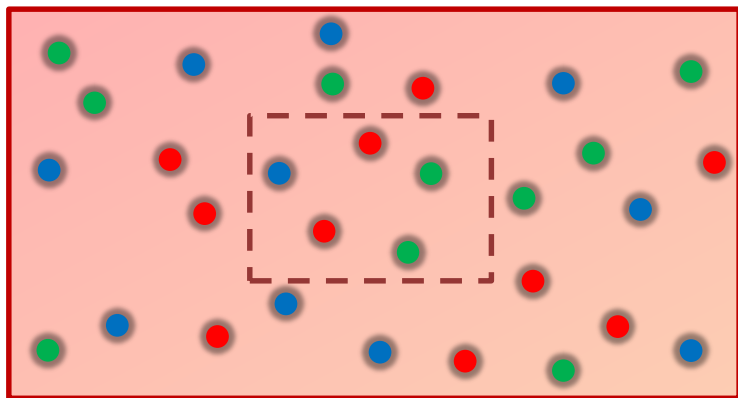


$$\langle \delta N_B^n \rangle_c = \langle N_B \rangle$$

Fluctuations

Free Boltzmann \rightarrow Poisson

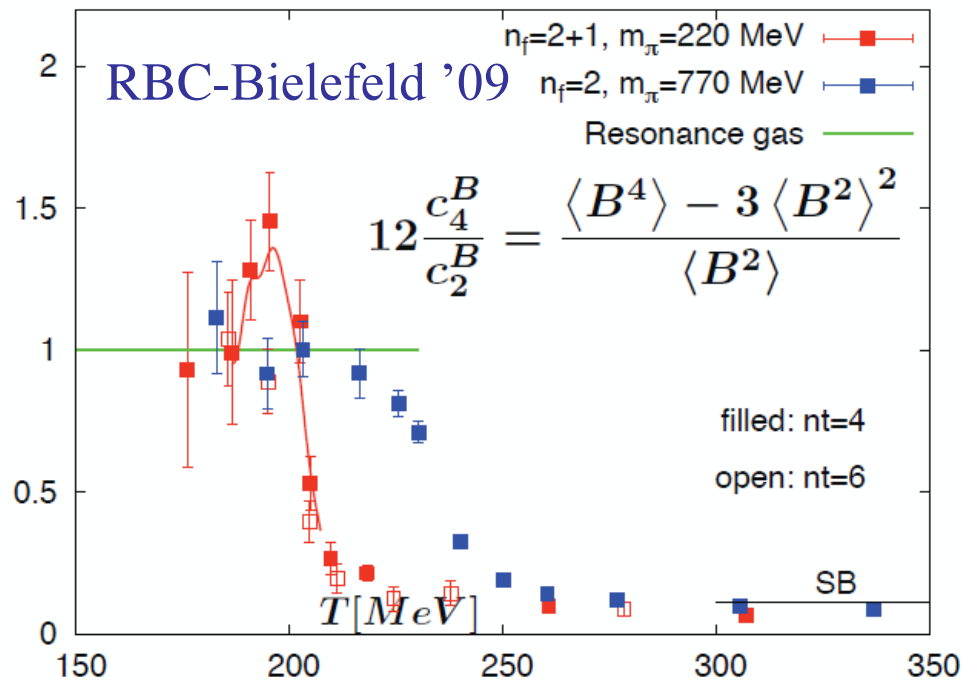
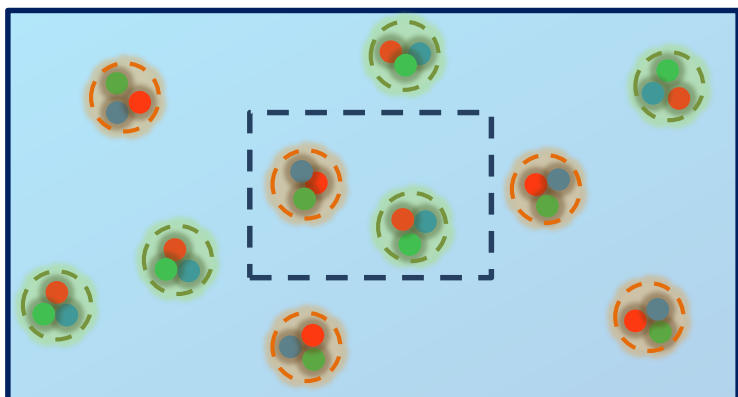
$$\langle \delta N^n \rangle_c = \langle N \rangle$$



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

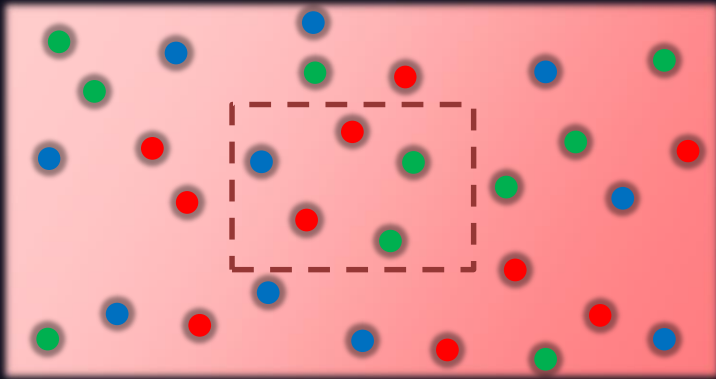
$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$

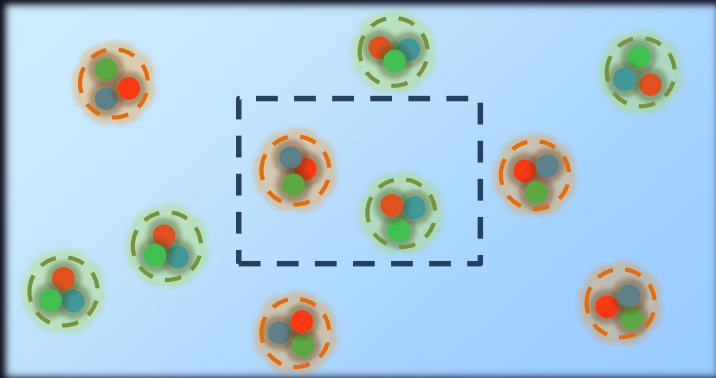


Electric Charge Fluctuation

Asakawa, Heinz, Muller; Jeon, Koch, 2000



$$|q_q| = 1/3, 2/3$$



$$|q_B| = 1$$

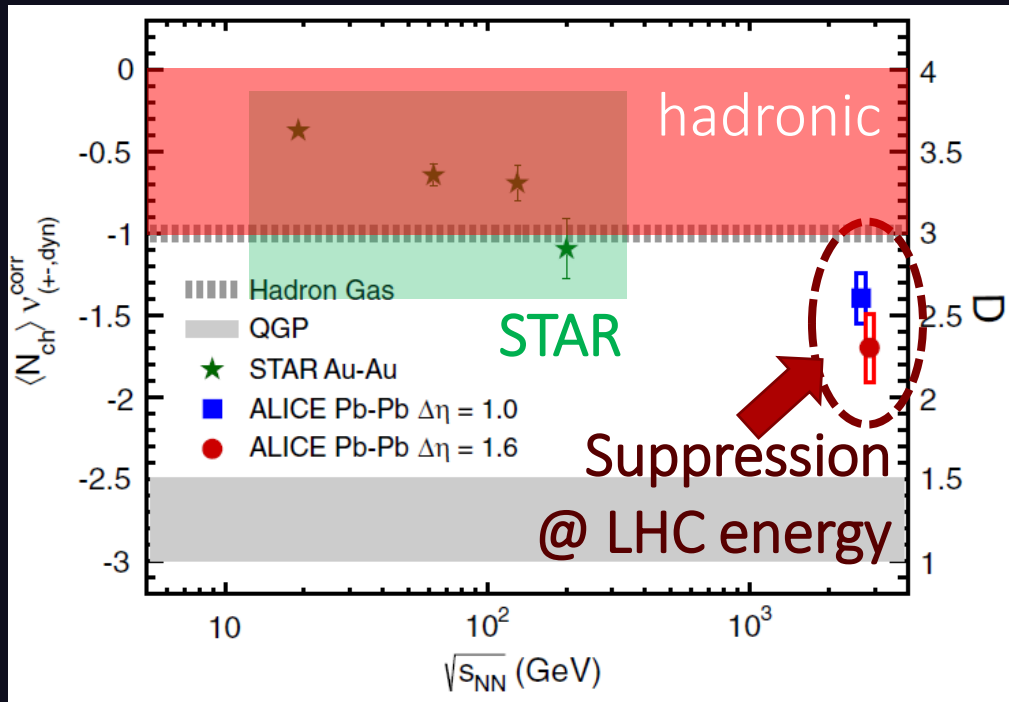
D-measure

$$D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- $D \sim 3-4$ Hadronic
- $D \sim 1-1.5$ QGP

Electric Charge Fluctuation

PHENIX (2002); STAR (2003)
ALICE, PRL 110, 152301 (2013)



D measure

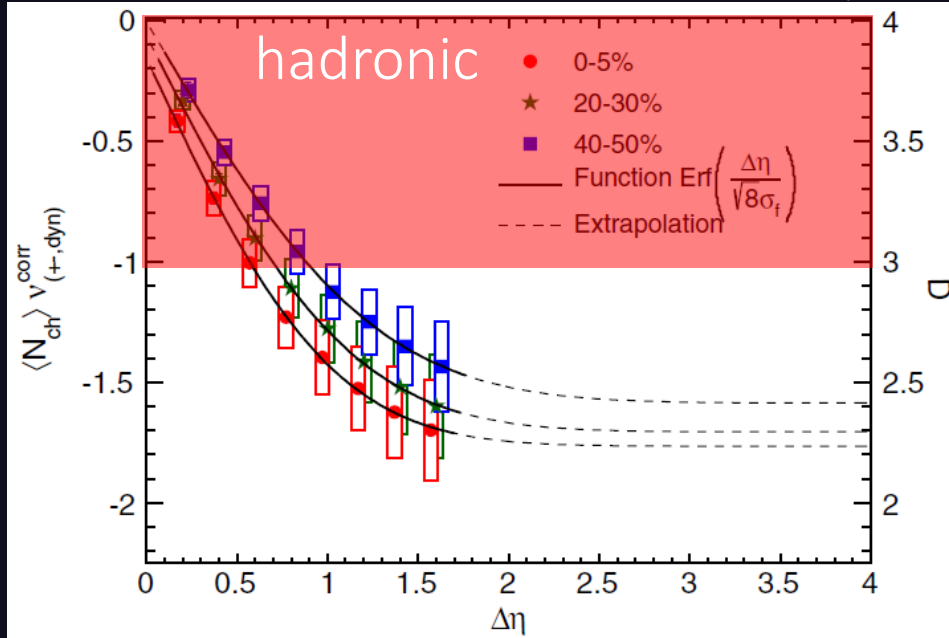
$$D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- $D \sim 3-4$ Hadronic
- $D \sim 1-1.5$ QGP

- No suppression at RHIC energy
- Fluctuations @ LHC **cannot** be described by hadronic d.o.f.

Rapidity Window Dependence

ALICE, PRL 110, 152301 (2013)

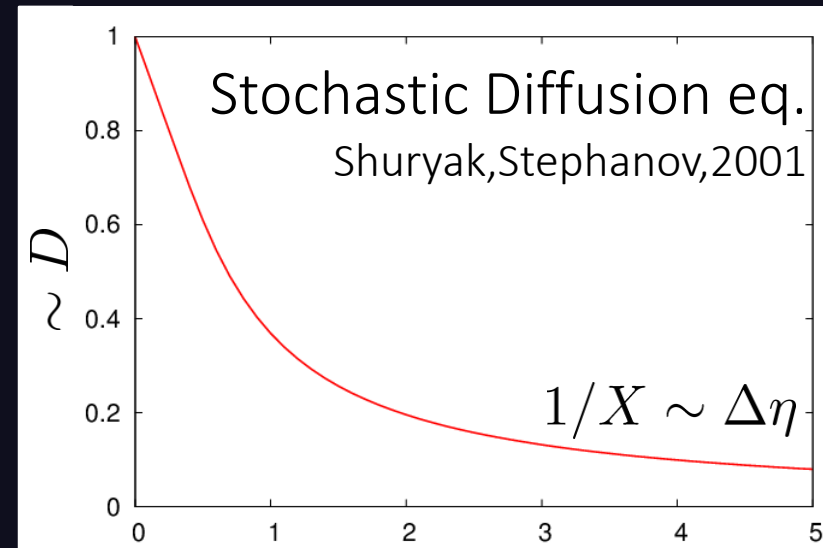


- Smaller $\Delta\eta$
more hadronic
- Larger $\Delta\eta$
more QGP like

Same information in

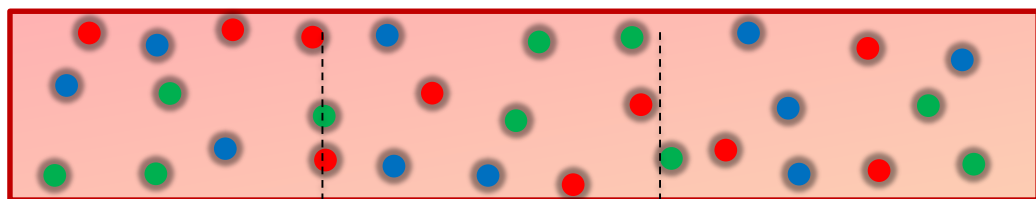
- $\langle \delta N_Q^{(\text{net})}(\eta_1) \delta N_Q^{(\text{net})}(\eta_2) \rangle$
- \simeq balance function

to be studied by fluctuating hydro.

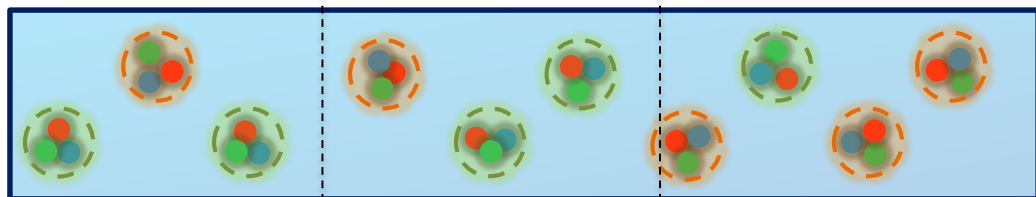


Time Evolution in HIC

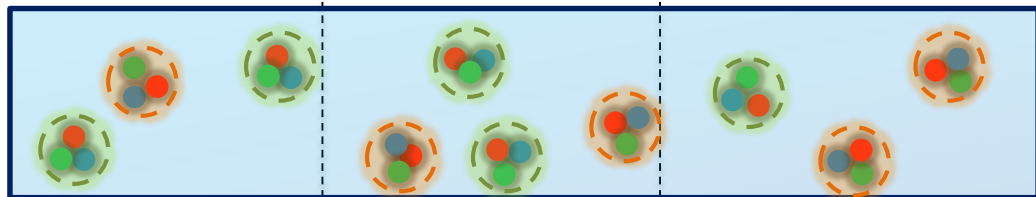
Quark-Gluon Plasma



Hadronization



Freezeout

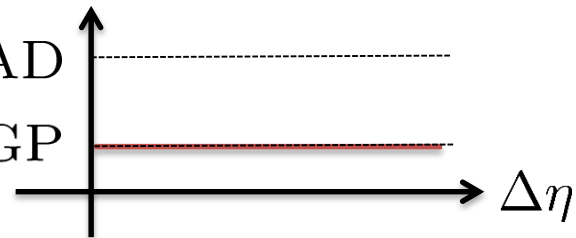


$\Delta\eta$

$$\frac{\langle \Delta N^2 \rangle}{\Delta\eta}$$

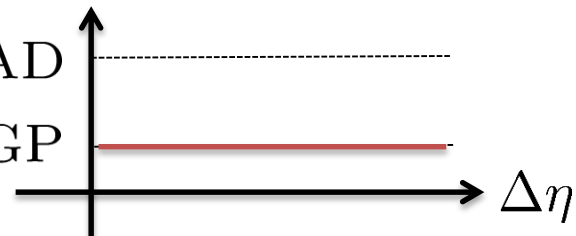
χ_{HAD}

χ_{QGP}



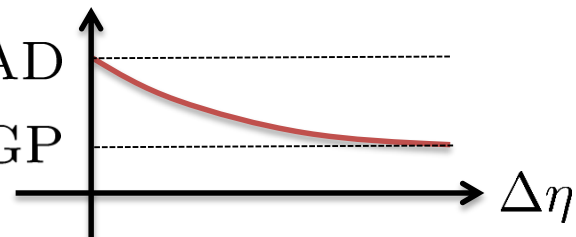
χ_{HAD}

χ_{QGP}











χ_{HAD}

χ_{QGP}



Various Contributions

- Initial fluctuations  Enhance
- Effect of jets  Enhance
- Negative binomial (?)  Enhance
- Final state rescattering  Enhance to Poisson
- Coordinate vs pseudo rapidities  Enhance to Poisson
- Particle missID  Enhance to Poisson
- Efficiency correction  Enhance to Poisson
- Global charge conservation  Suppress

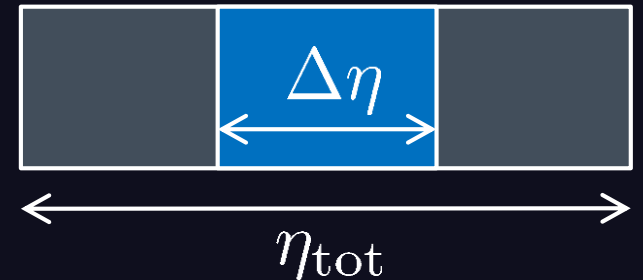
Global Charge Conservation

For equilibrated medium

Jeon, Koch, 2000

Bleicher, Jeon, Koch, 2001

$$\langle \delta N^2 \rangle_{\text{exp.}} = \langle \delta N^2 \rangle_{\text{GC}} \times \left(1 - \frac{\Delta y}{y_{\text{tot}}} \right)$$



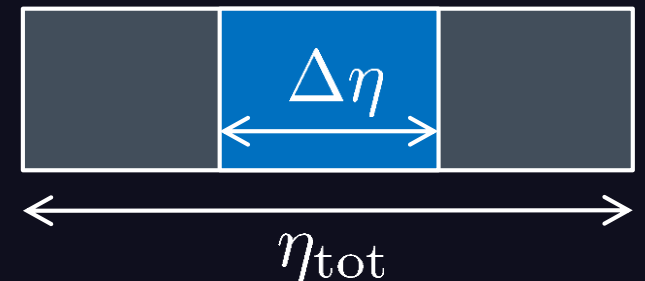
Global Charge Conservation

For equilibrated medium

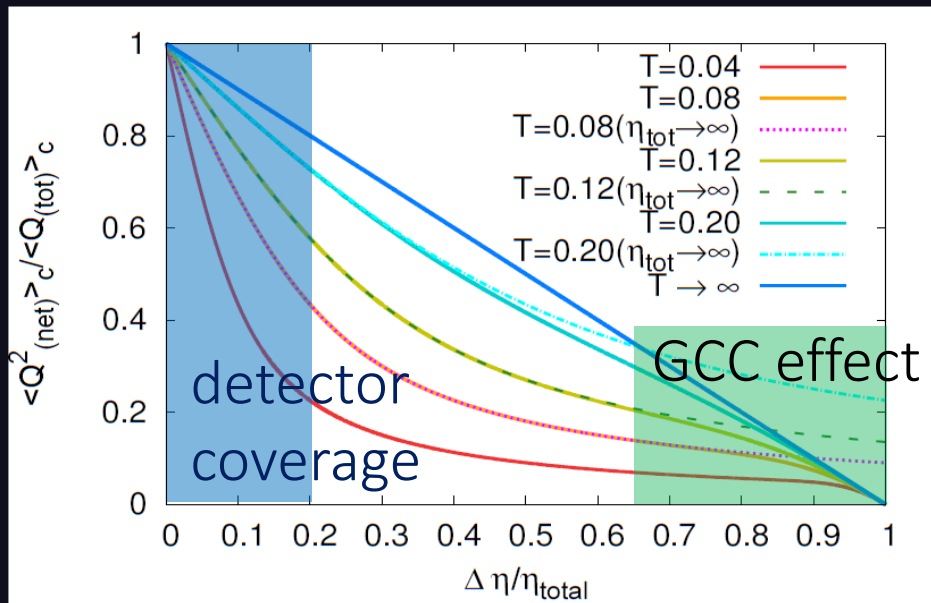
Jeon, Koch, 2000

Bleicher, Jeon, Koch, 2001

$$\langle \delta N^2 \rangle_{\text{exp.}} = \langle \delta N^2 \rangle_{\text{GC}} \times \left(1 - \frac{\Delta y}{y_{\text{tot}}} \right)$$



Solving the time evolution...



GCC effect on the hadronic diffusion **is negligible** in the ALICE result!

Electric-Charge Fluctuations

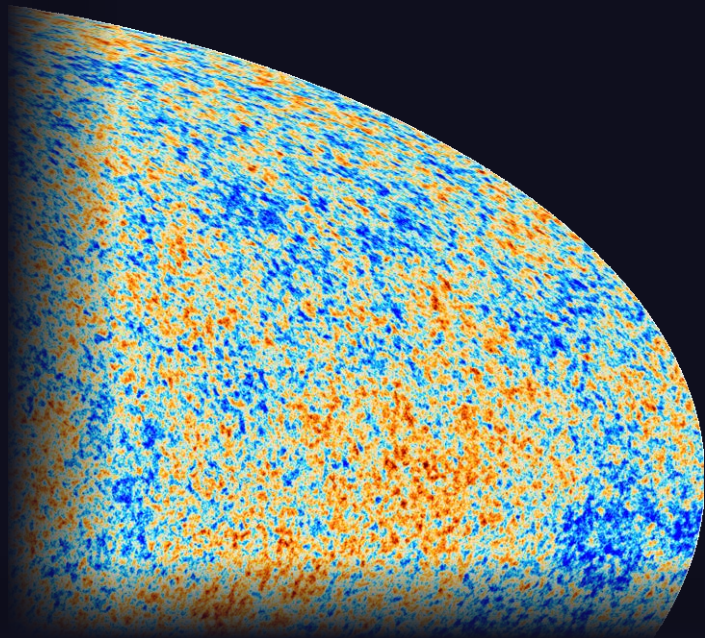
- Electric charge fluctuations is suppressed at LHC!
- The suppression is most probably attributed to primordial physics
- Qualitative difference b/w RHIC and LHC
... but why?

Non Gaussianity

Non-Gaussianity

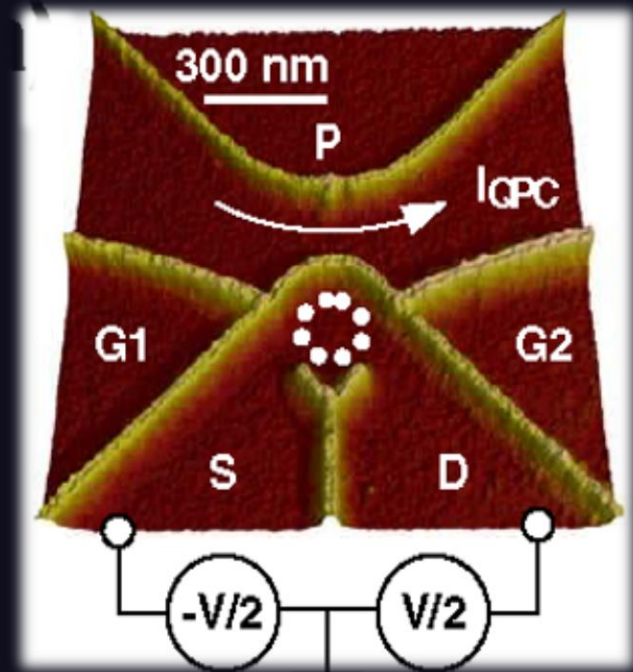
CMB

Cosmic Microwave Background



- No statistically-significant signals
Planck, 2013

Mesoscopic Systems



- Full counting statistics
- Cumulants up to 5th order

Gustavsson+, Surf.Sci.Rep.64,191(2009)

Non-Gaussianity in HIC

- Ratio of conserved charges

Ejiri,Karsch,Redlich(2005)

- Critical enhancement

Stephanov(2009)

- Sign change

Asakawa,Ejiri,MK (2009); Friman+(2011); Stephanov(2011)

- Strange confinement

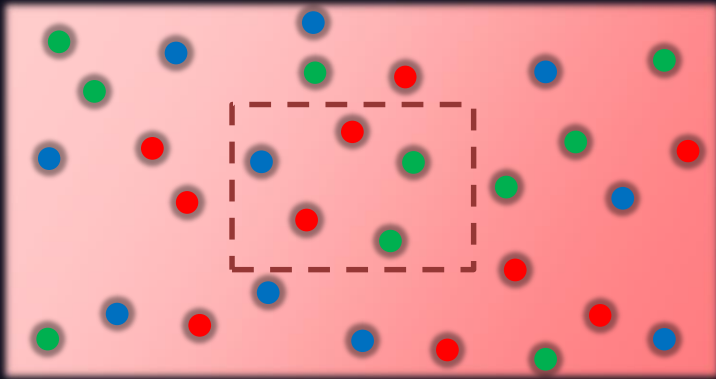
BNL-Bielefeld(2013)

- Distribution funcs themselves

Morita+(2013); Nakamura (Wed.)

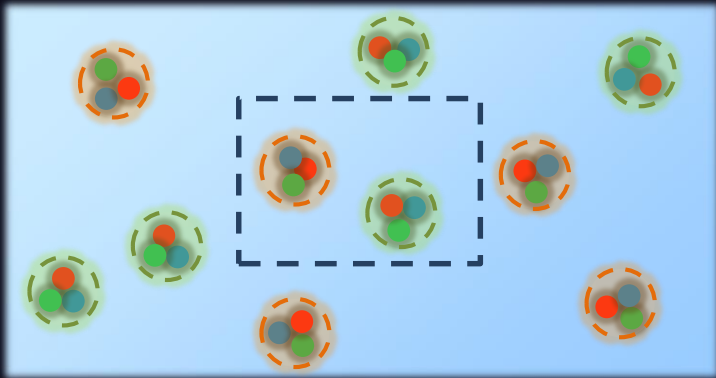
Ratio of Cumulants

Ejiri, Karsch, Redlich, 2005



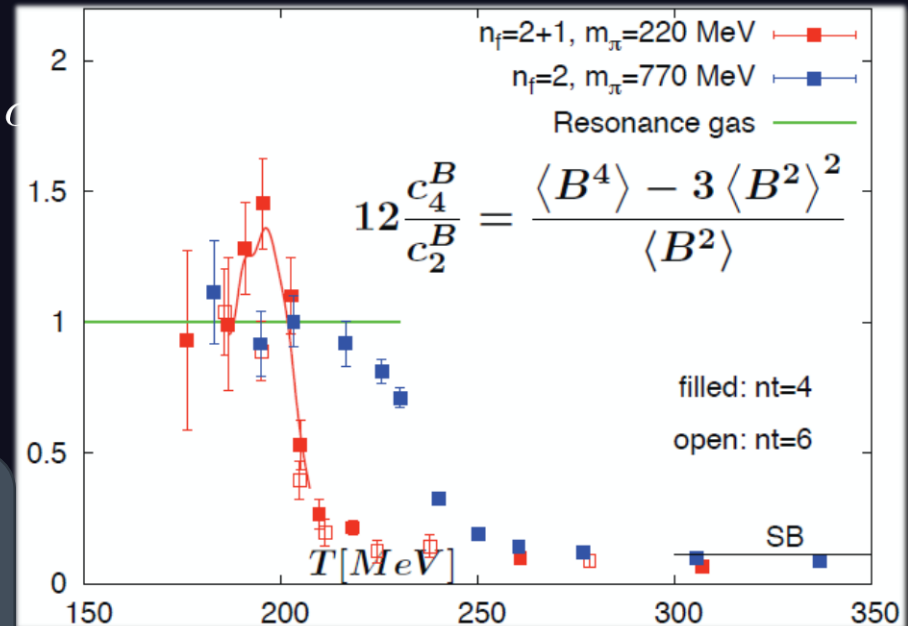
$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle^n$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle^n$$



$$\langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle^n$$

$$\frac{\langle \delta N_B^4 \rangle_c}{\langle \delta N_B^2 \rangle_c} = \begin{cases} 1 & \text{hadronic} \\ 1/9 & \text{quark-gluon} \end{cases}$$



Strange Confinement

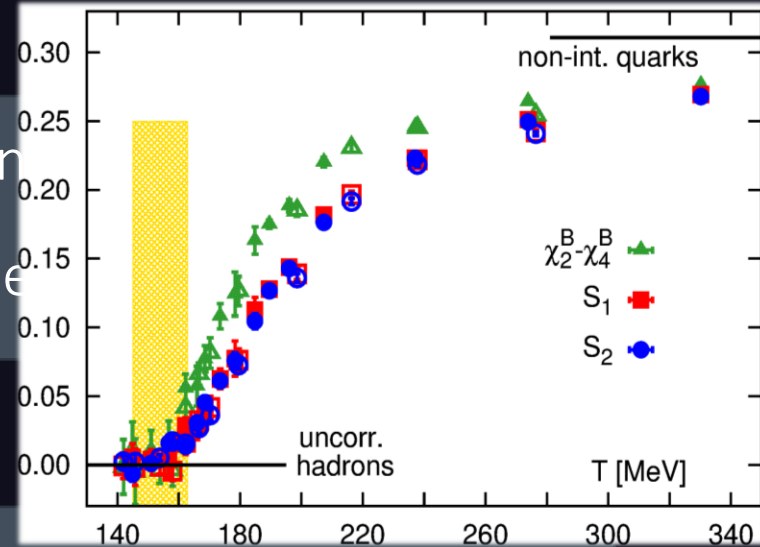
BNL-Bielefeld, PRL(2013)
H.T. Ding, This Morning

Baryonic $\langle \delta N_B^2 \rangle = \langle \delta N_B^4 \rangle$

$$\langle \delta N_B^2 \rangle - \langle \delta N_B^4 \rangle_c \begin{cases} = 0 & \text{baryons confined} \\ \neq 0 & \text{something else} \end{cases}$$

Strangeness

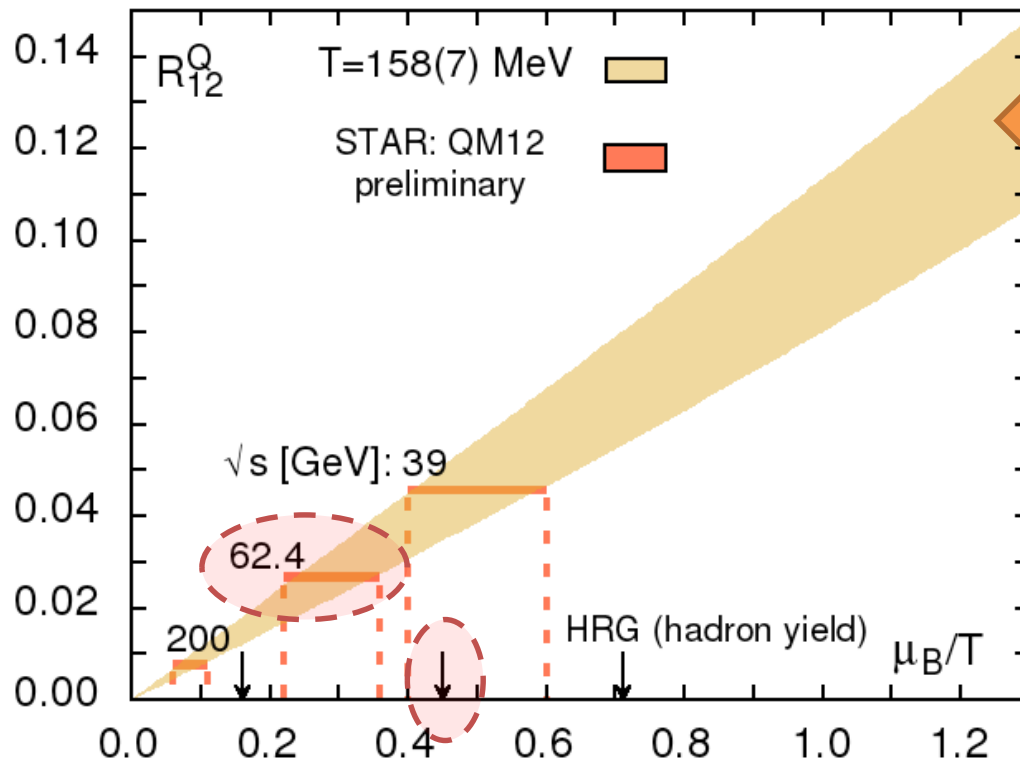
$$\langle \delta N_s \delta N_B^3 \rangle_c - \langle \delta N_s \delta N_B \rangle \begin{cases} = 0 & \text{strangeness confined} \\ \neq 0 & \text{something else} \end{cases}$$



Many lattice studies (LAT-HIC crossover):

Budapest-Wuppertal, 2013; BW,1403.4578; BNL-Bi.,1404.4043; Gupta+,1405.2206;
Ratti, Wed.; Schmidt, Wed.; Nakamura, Wed.; Sharma, J-13

Cumulants : HIC@RHIC vs Lattice



parameter window
constrained by lattice

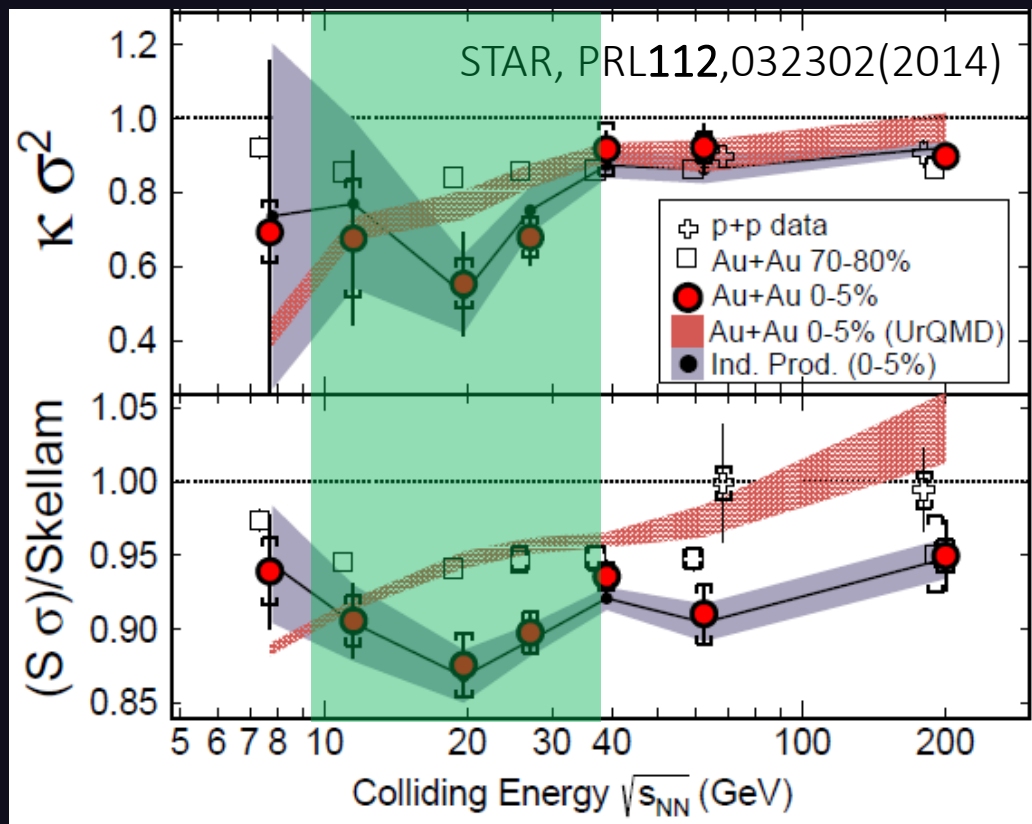
BNL-Bielefeld,
LATTICE2013

fluctuations
“exp + lattice”

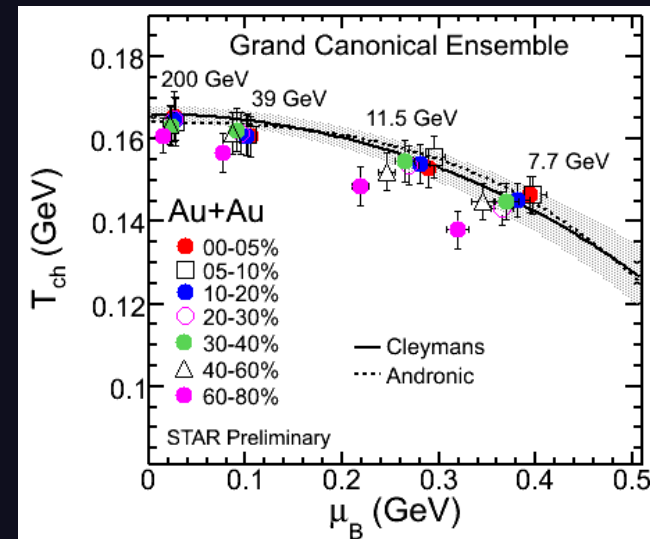
μ/T
discrepancy

particle abundance
(chem. freezeout T)

Proton Number Cumulants at RHIC-BES











STAR 2012



- Exp. results are close to and less than Poissonian values.
- Something interesting around $\sqrt{s_{NN}} \simeq 20 \text{ GeV}$

Effects of Various Contributions

- Initial fluctuations  Enhance
- Effect of jets  Enhance
- Negative binomial (?)  Enhance
- Final state rescattering  Enhance to Poisson
- Coordinate vs pseudo rapidities  Enhance to Poisson
- Particle missID  Enhance to Poisson
- Efficiency correction  Enhance to Poisson
- Global charge conservation  Suppress

Caution!!

proton number
cumulants

\neq

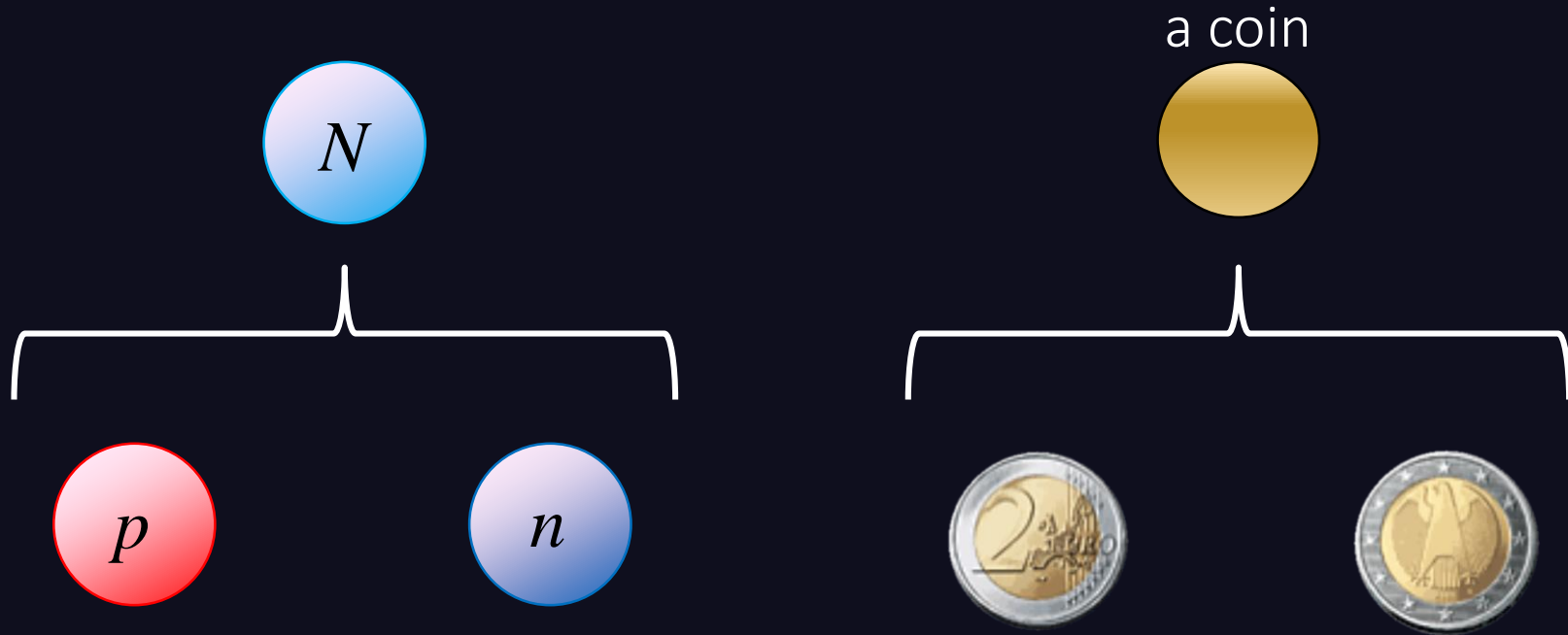
baryon number
cumulants



Let's clarify their relation!

MK, Asakawa (2012;2012)

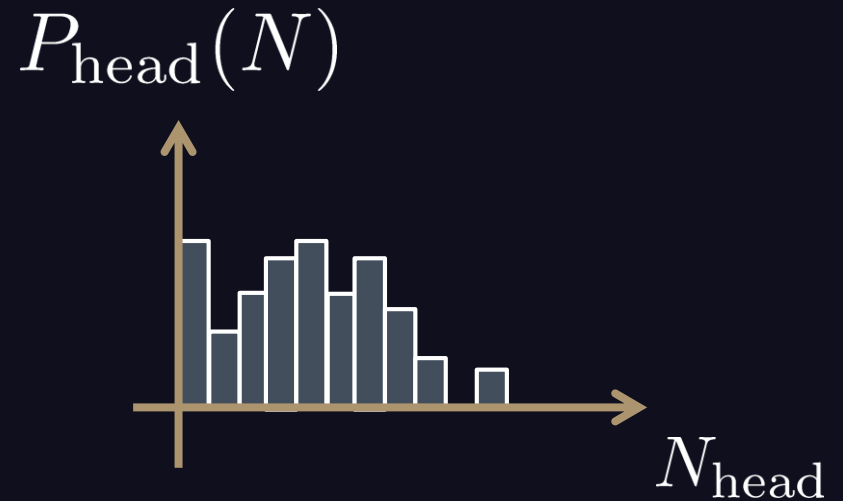
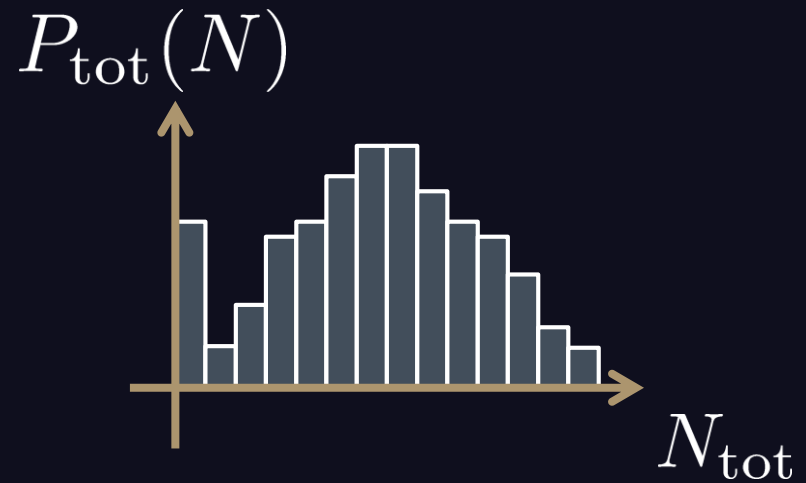
Nucleon isospin and a coin



Nucleon has
two isospin states.

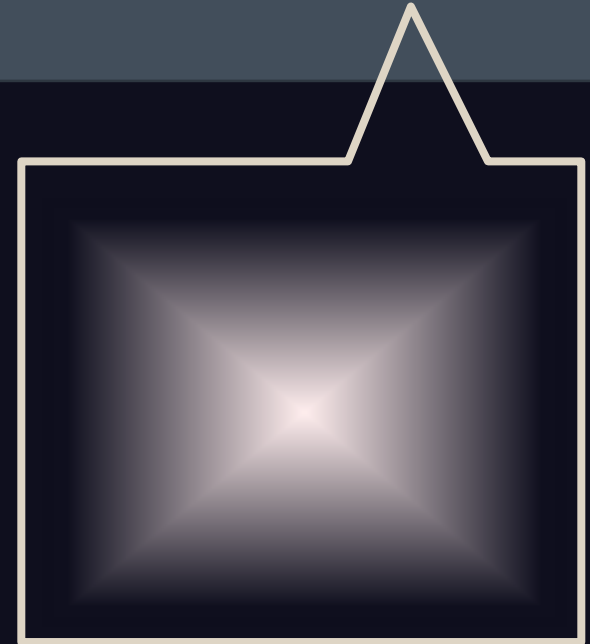
A coin has two sides.

Slot Machine Analogy

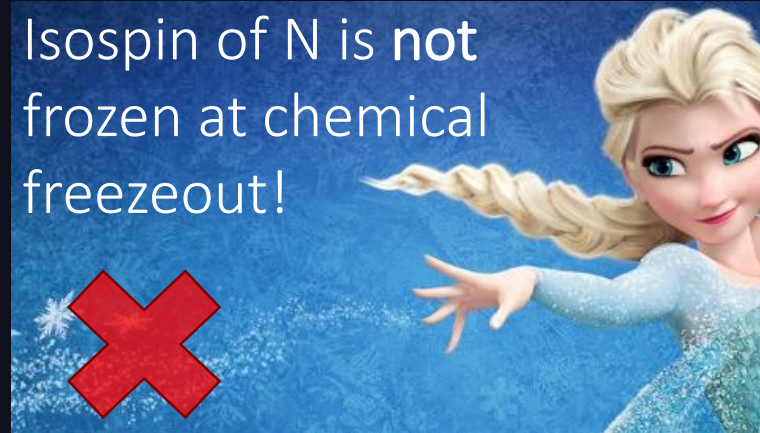


Reconstructing Total Coin Number

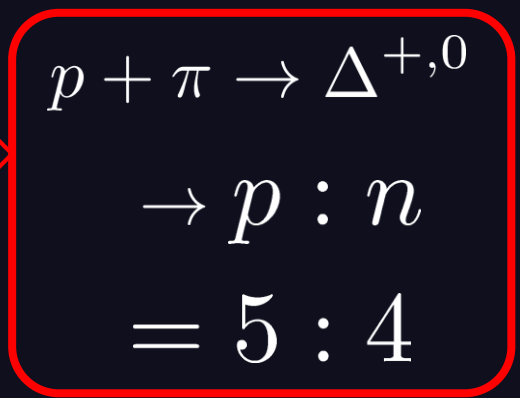
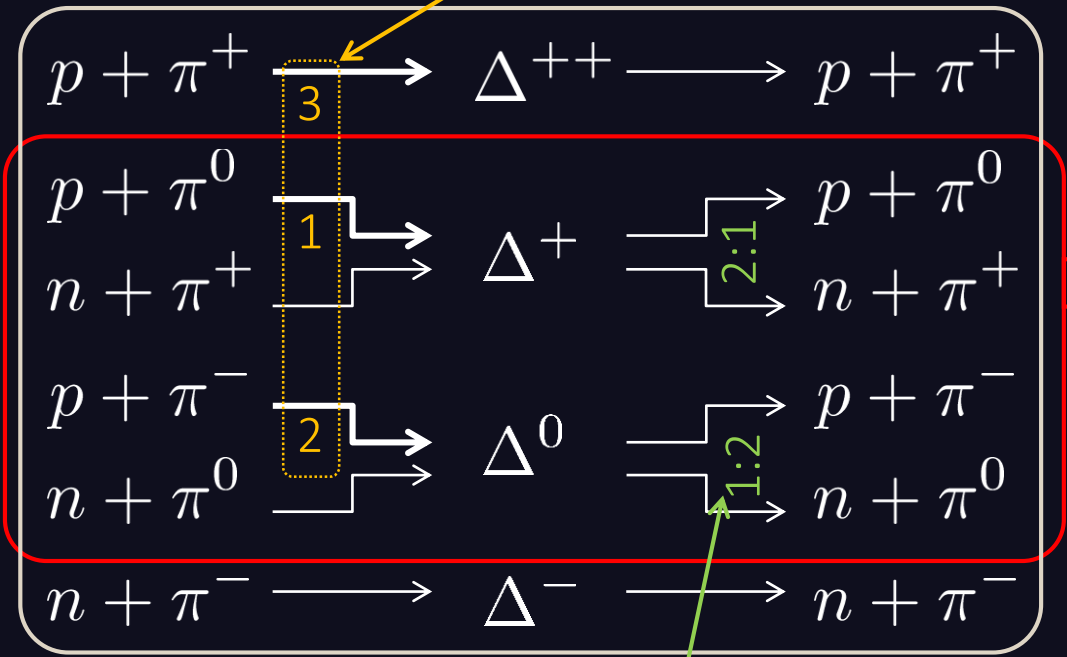
$$P_{\text{head}}(N_{\text{head}}) = \sum_{N_{\text{tot}}} B_{1/2}(N_{\text{head}}; N_{\text{tot}}) P_{\text{tot}}(N_{\text{tot}})$$



Charge Exchange Reaction



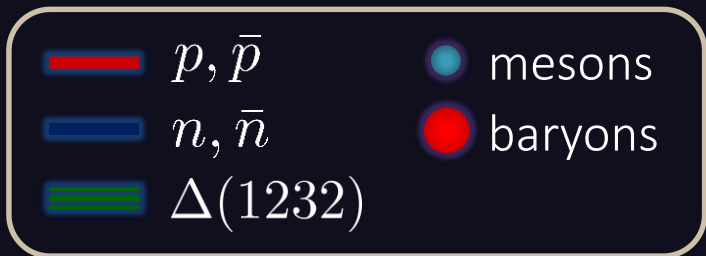
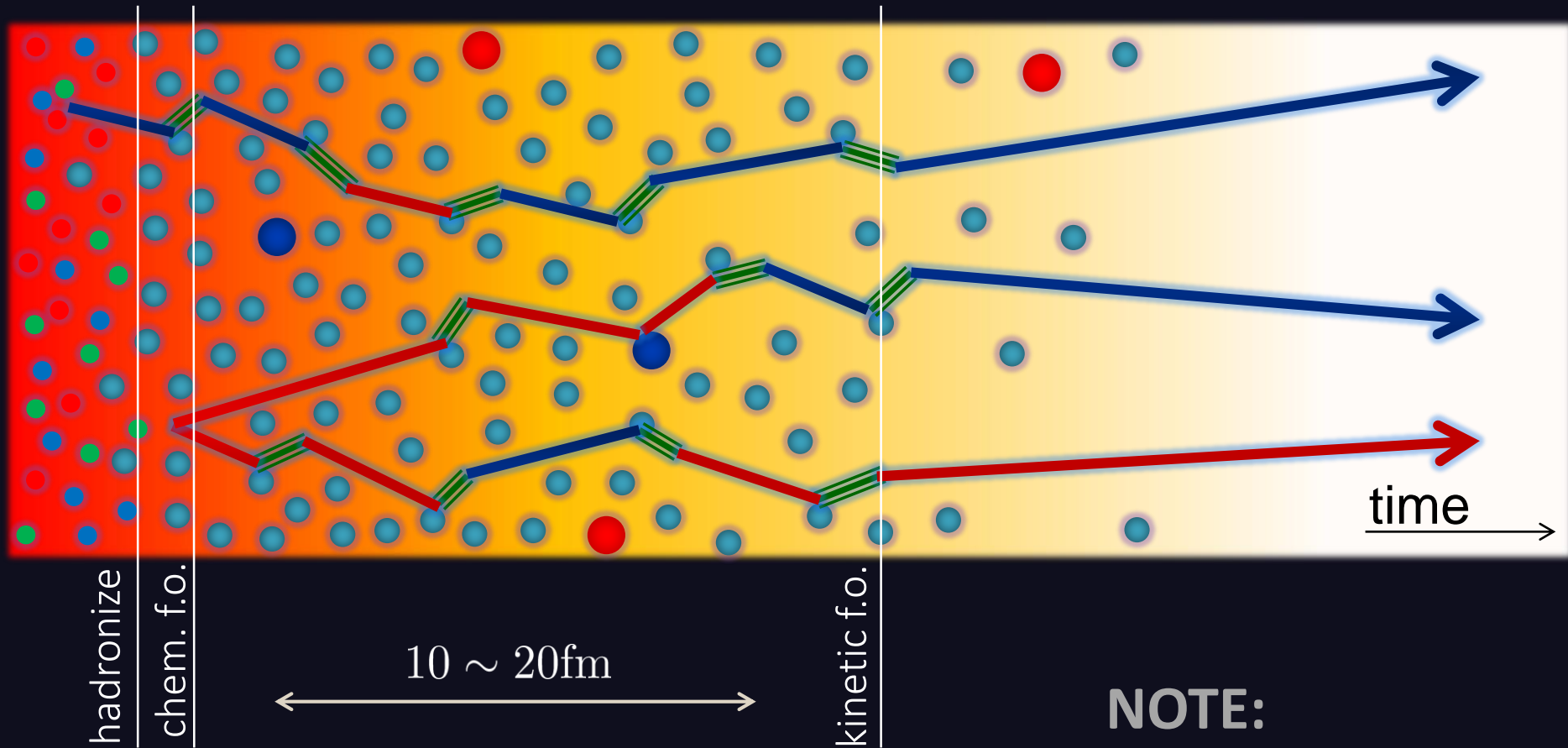
cross sections of p



decay rates of Δ

mean reaction time
 $< 5\text{fm}/c$

Nucleons in Hadronic Medium



NOTE:

- so many pions
- rare NN collisions
- no quantum corr.

Difference b/w N_B and N_p

Assumptions: net-cumulants deviate from thermal value
But, $N_B, N_{\bar{B}}$ are Poissonian

$$\left\{ \begin{array}{l} 2\langle(\delta N_p^{(\text{net})})^2\rangle = \frac{1}{2}\langle(\delta N_B^{(\text{net})})^2\rangle + \frac{1}{2}\langle(\delta N_B^{(\text{net})})^2\rangle_{\text{free}} \\ 2\langle(\delta N_p^{(\text{net})})^3\rangle = \frac{1}{4}\langle(\delta N_B^{(\text{net})})^3\rangle + \frac{3}{4}\langle(\delta N_B^{(\text{net})})^3\rangle_{\text{free}} \\ 2\langle(\delta N_p^{(\text{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_B^{(\text{net})})^4\rangle_c + \dots \end{array} \right.$$

genuine info. noise

Proton number cumulants are dominated by Poissonian noise

Efficiency Correction

MK, Asakawa, 2012

Bdzak, Koch, 2012

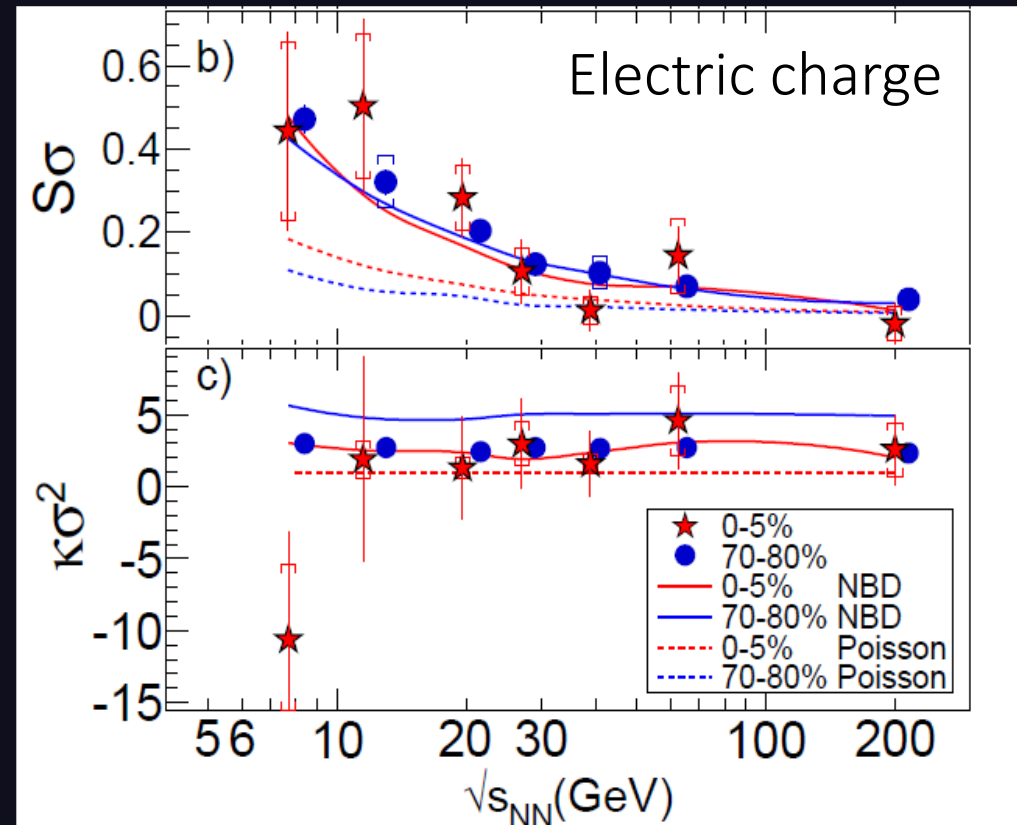
STAR, 2013

If efficiency for each
particle is uncorrelated



binomial correction to
distribution function

$$P_{\text{exp.}}(N) = \sum_{N'} B_{\epsilon}(N; N') P(N')$$



STAR, arXiv:1402.1558

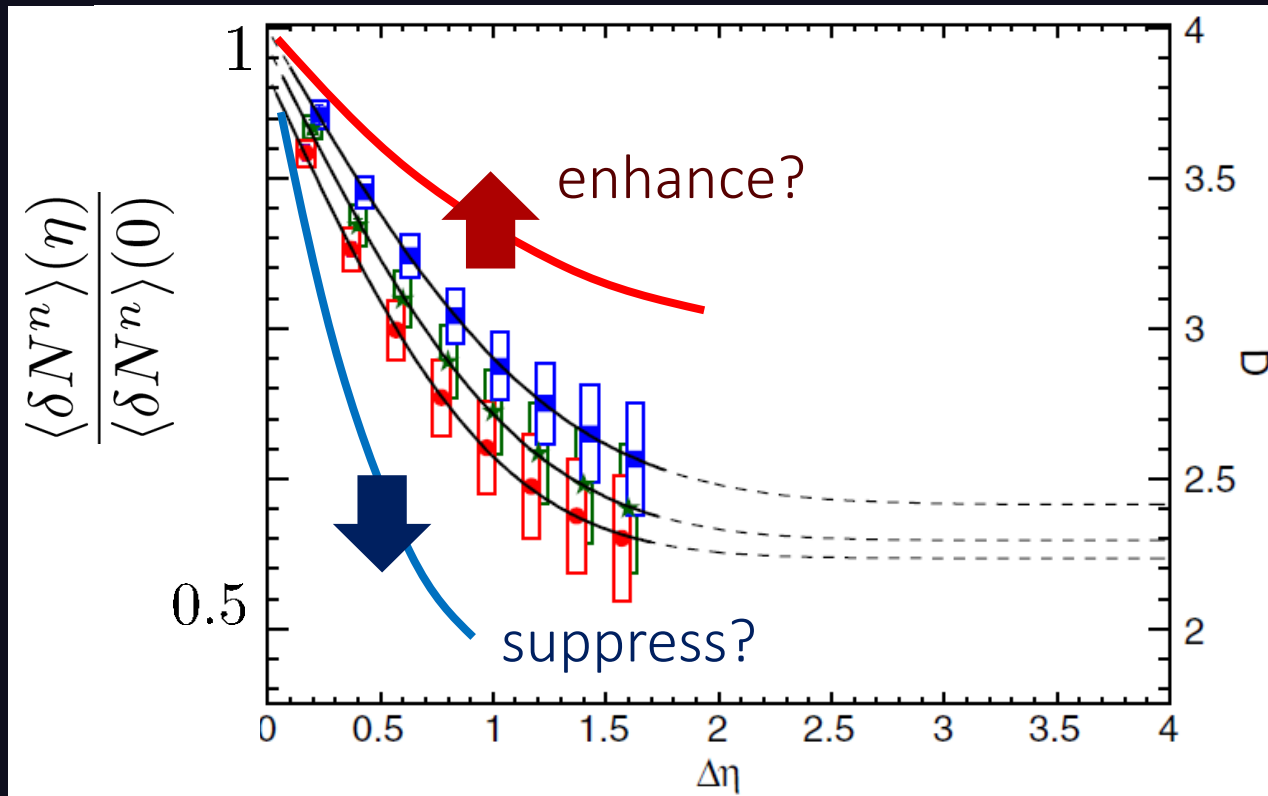
for Particle missID: Ono,Asakawa,MK, PRC,2013

More Information on/from Fluctuations

$\Delta\eta$ dependence

MK, Asakawa, Ono, PLB728, 386 (2014)

$\Delta\eta$ Dep. of Non-Gaussianity



How does the 4-th order cumulant behave as a function of $\Delta\eta$?

Fluctuating Hydrodynamics?

- Distributions in experiments are close to Poissonian
- Cumulants are expected to **increase** in the hadronic medium



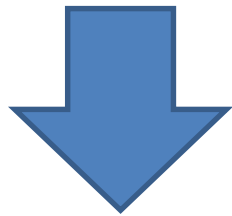
These behaviors **cannot** be described by the theory of hydrodynamic fluctuations

Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechanics II
Kapusta, Muller, Stephanov, 2012
Stephanov, Shuryak, 2001

Stochastic diffusion equation

$$\partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$



Fluctuation of n is
Gaussian in equilibrium

Markov (white noise)
+
continuity



Gaussian noise

cf) Gardiner, "Stochastic Methods"

How to Introduce Non-Gaussianity?

Stochastic diffusion equation

$$\partial_{\tau} n = D \partial_{\eta}^2 n + \partial_{\eta} \xi(\eta, \tau)$$

- Choices to introduce non-Gaussianity in equil.:
 - n dependence of diffusion constant $D(n)$
 - colored noise
 - discretization of n

How to Introduce Non-Gaussianity?

Stochastic diffusion equation

$$\partial_{\tau} n = D \partial_{\eta}^2 n + \partial_{\eta} \xi(\eta, \tau)$$

▣ Choices to introduce non-Gaussianity in equil.:

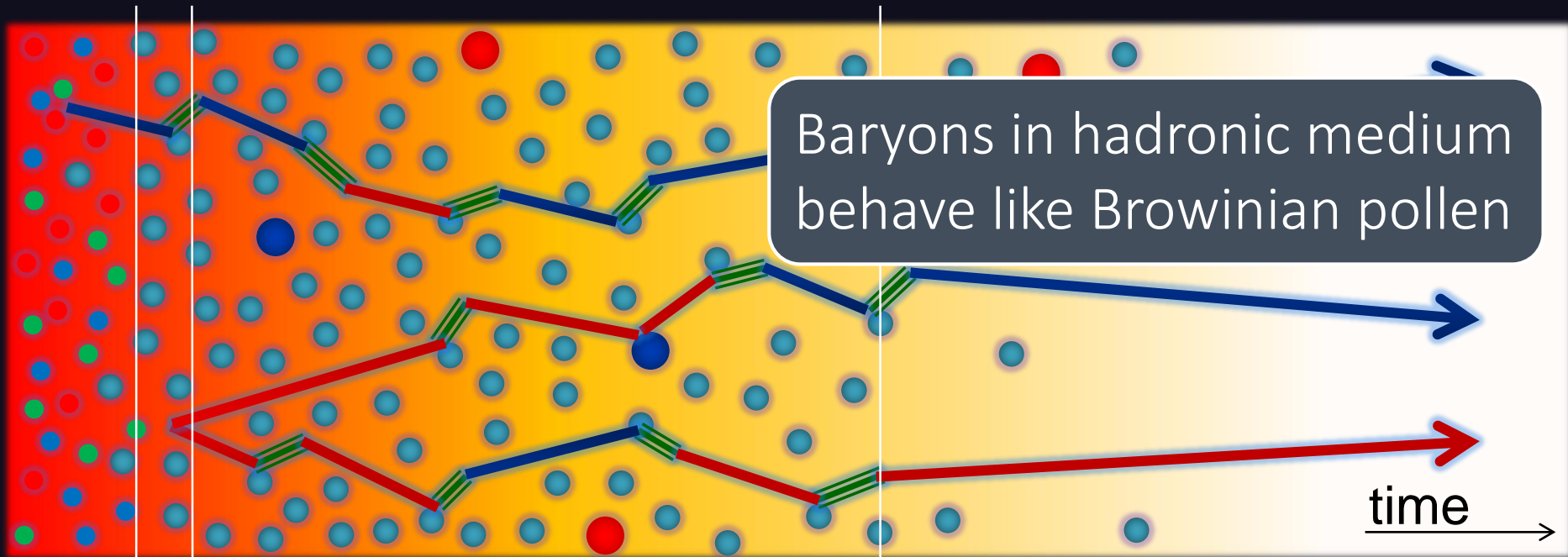
▣ n dependence of diffusion constant $D(n)$

▣ colored noise

▣ discretization of n ← **our choice**

REMARK: Fluctuations measured in HIC are almost Poissonian.

Nucleons in Hadronic Medium



Baryons in hadronic medium behave like Brownian pollen

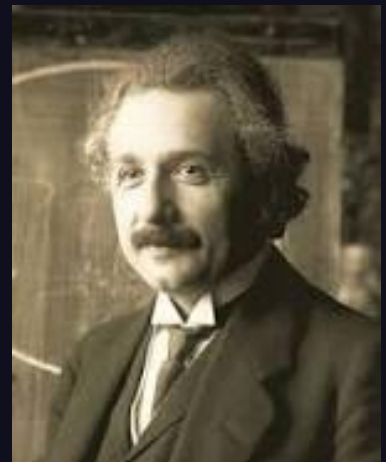
time →

hadronize
chem. f.o.

10 ~ 20fm

kinetic f.o.

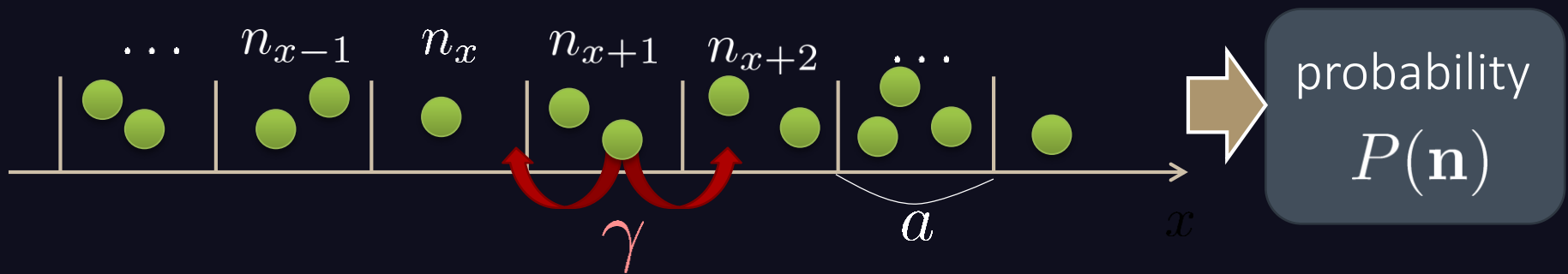
	p, \bar{p}		mesons
	n, \bar{n}		baryons
	$\Delta(1232)$		



Diffusion Master Equation

MK,Asakawa,Ono,PLB728,386(2014)

Divide spatial coordinate into discrete cells



Master Equation

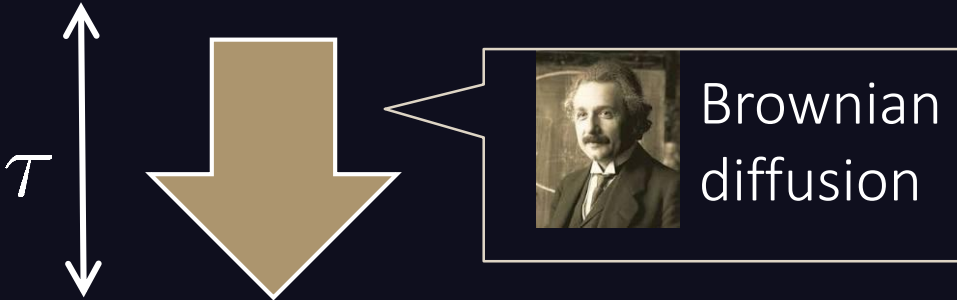
$$\frac{\partial}{\partial t} P(\mathbf{n}) = \gamma \sum_x [(n_x + 1) \{P(\mathbf{n} + \mathbf{e}_x - \mathbf{e}_{x+1}) + P(\mathbf{n} + \mathbf{e}_x - \mathbf{e}_{x-1})\} - 2n_x P(\mathbf{n})]$$

Solve the DME **exactly**, and take $a \rightarrow 0$ limit

hadronization
chemical freezeout

Initial condition

- boost invariance
- locality of fluctuations
- small cumulants



kinetic freezeout

Comments:

- agreement with stochastic diffusion eq. up to Gaussian fluctuation
- Poisson (Skellam) distribution in equilibrium: consistent with HRG

Solution of DME in $a \rightarrow 0$ Limit

1st order (deterministic) $\langle n \rangle$

- consistent with diffusion equation with $D = \gamma a^2$

➔ Continuum limit with fixed $D = \gamma a^2$

2nd order $\langle \delta n^2 \rangle$

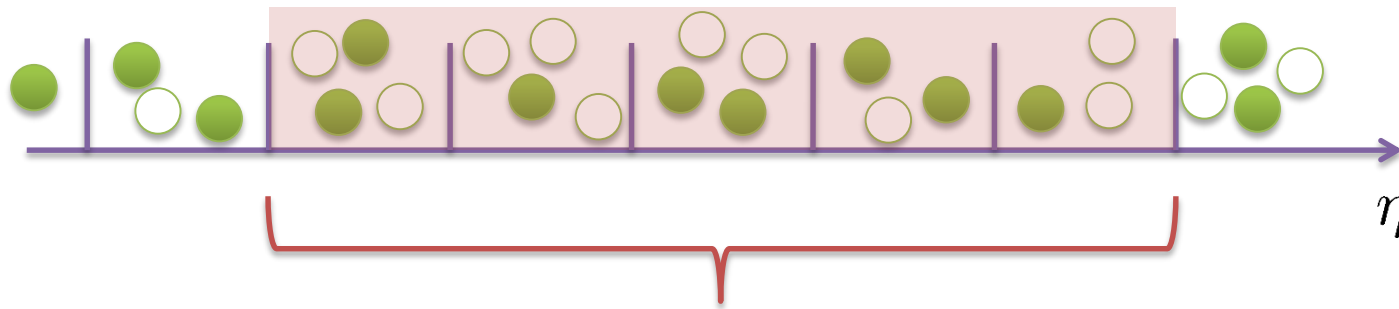
- consistent with stochastic diffusion eq.
(for sufficiently smooth initial conditions)

Shuryak, Stephanov, 2001

Nontrivial results for non-Gaussian fluctuations

Net Charge Number

Prepare 2 species of (non-interacting) particles



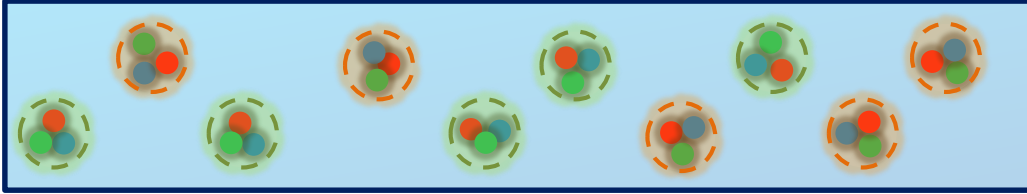
$$\bar{Q}(\tau) = \int_0^{\Delta\eta} d\eta (n_1(\eta, \tau) - n_2(\eta, \tau))$$

Let us investigate

$$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c \quad \text{at freezeout time } t$$

Time Evolution in Hadronic Phase

Hadronization (initial condition)



- Boost invariance / infinitely long system
- Local equilibration / local correlation

$$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$$

$$\langle \bar{Q}^2 Q_{(\text{tot})} \rangle_c$$

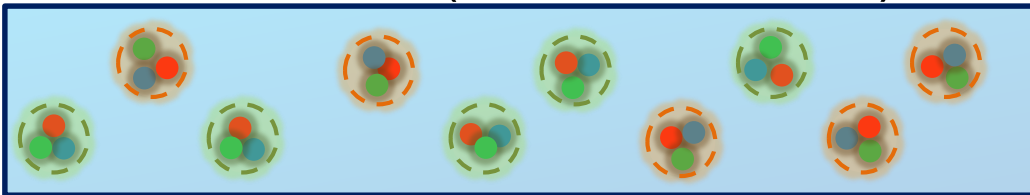
$$\langle Q_{(\text{tot})}^2 \rangle_c$$

suppression owing to
local charge conservation

strongly dependent on
hadronization mechanism

Time Evolution in Hadronic Phase

Hadronization (initial condition)



Time evolution via DME

- Boost invariance / infinitely long system
- Local equilibration / local correlation

$$\langle \bar{Q}^2 \rangle_c$$

$$\langle \bar{Q}^4 \rangle_c$$

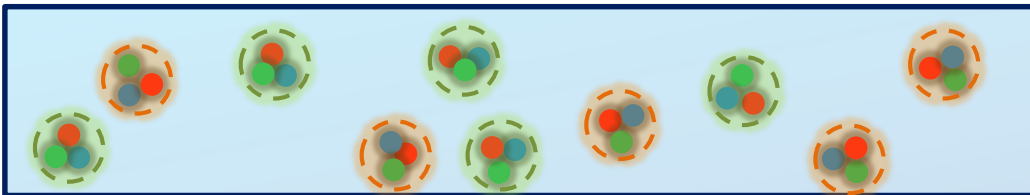
$$\langle \bar{Q}^2 Q_{(\text{tot})} \rangle_c$$

$$\langle Q_{(\text{tot})}^2 \rangle_c$$

suppression owing to local charge conservation

strongly dependent on hadronization mechanism

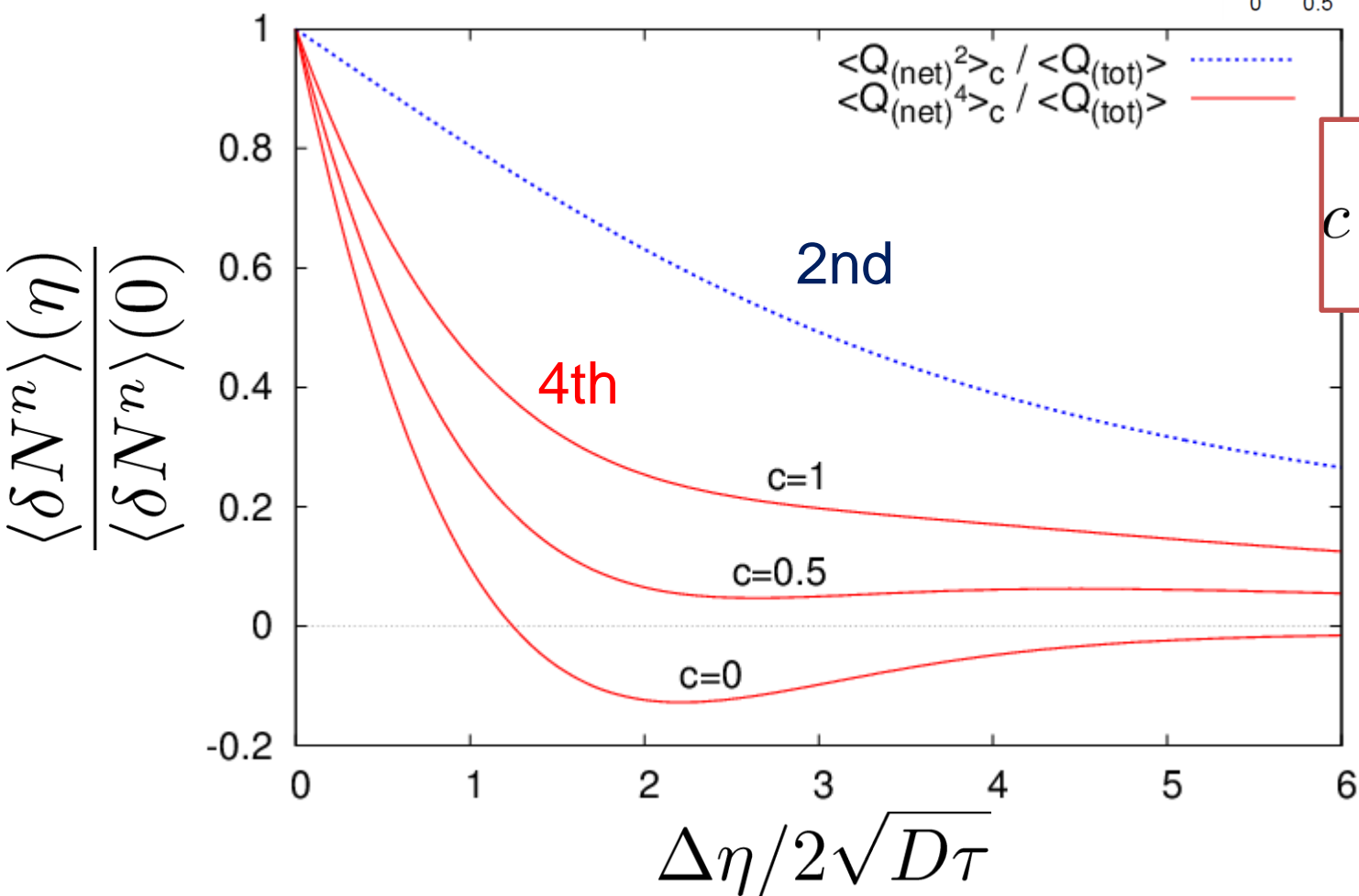
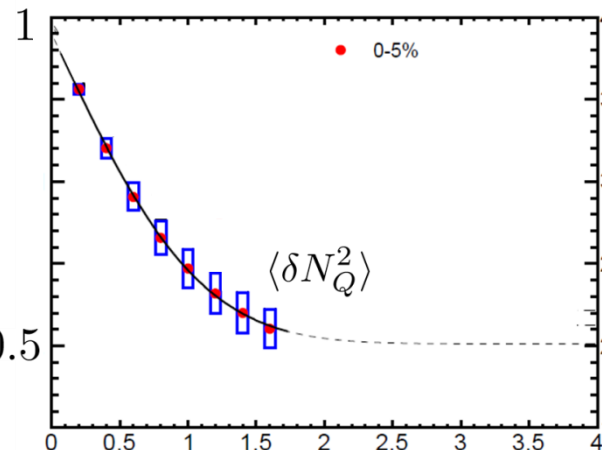
Freezeout



$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\text{tot})} \rangle_c = 0$$



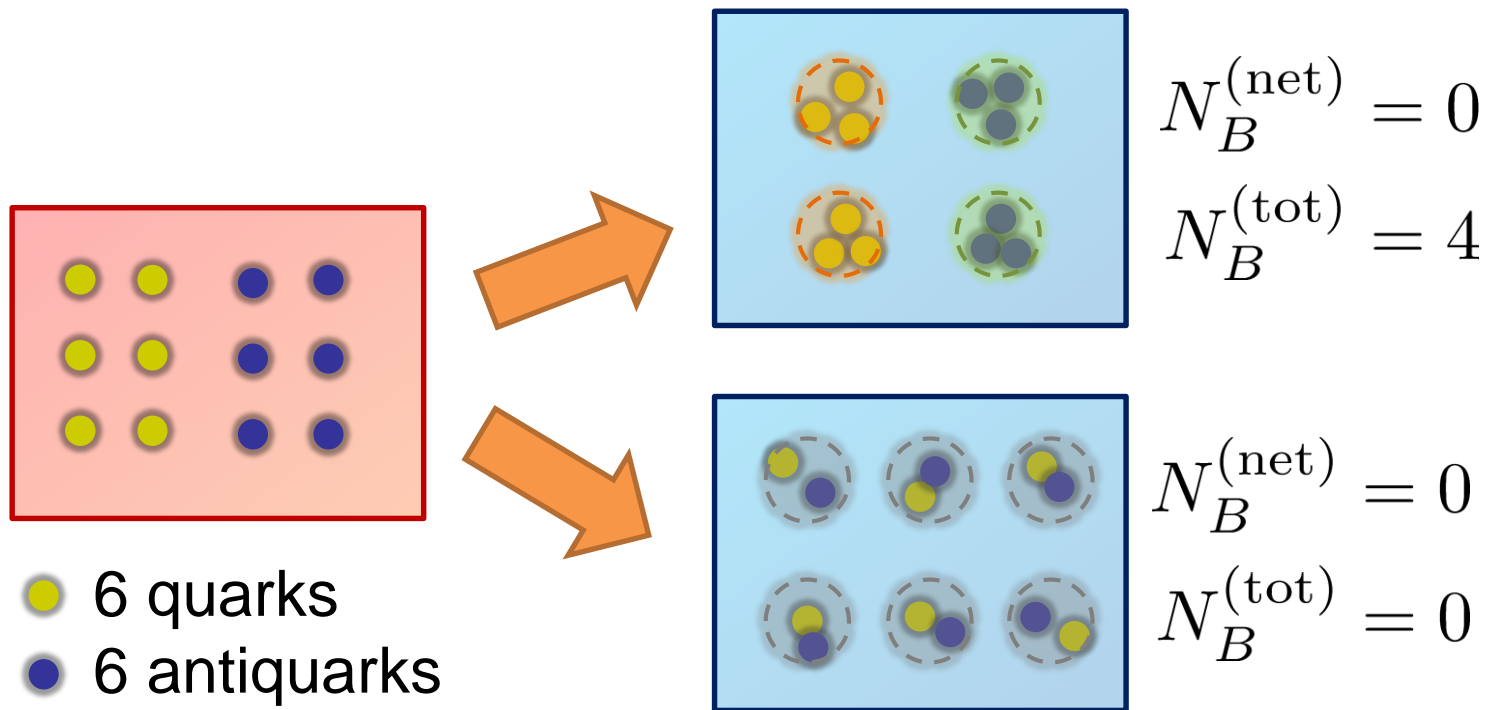
$$c = \frac{\langle Q_{(\text{tot})}^2 \rangle_c}{\langle Q_{(\text{tot})} \rangle}$$



parameter
sensitive to
hadronization

Total Charge Number

In recombination model,

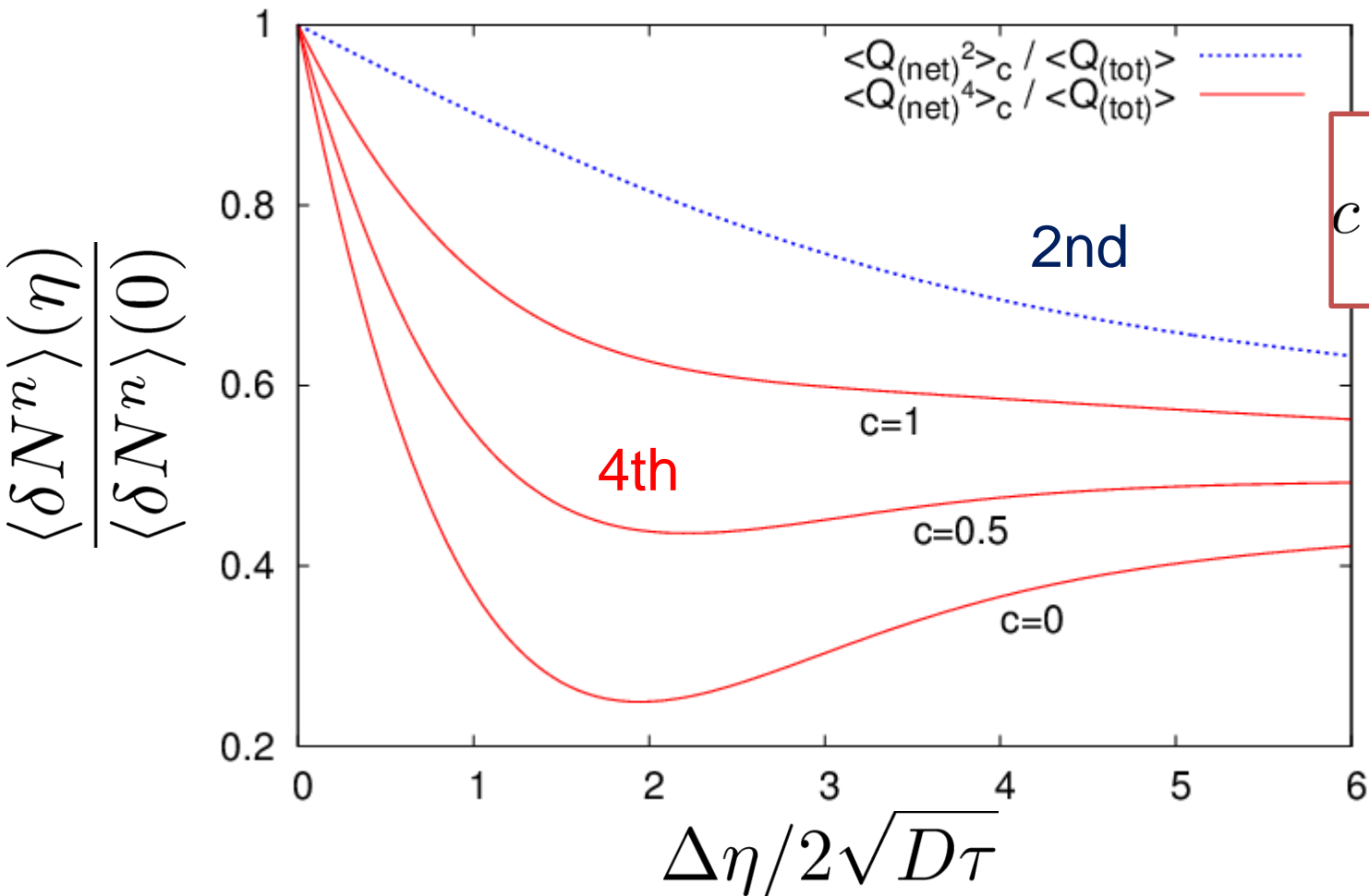


□ $N_B^{(\text{tot})}$ can fluctuate, while $N_B^{(\text{net})}$ does not.

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\text{tot})} \rangle_c = 0.5 \langle Q_{(\text{tot})} \rangle$$



$$c = \frac{\langle Q_{(\text{tot})}^2 \rangle_c}{\langle Q_{(\text{tot})} \rangle}$$



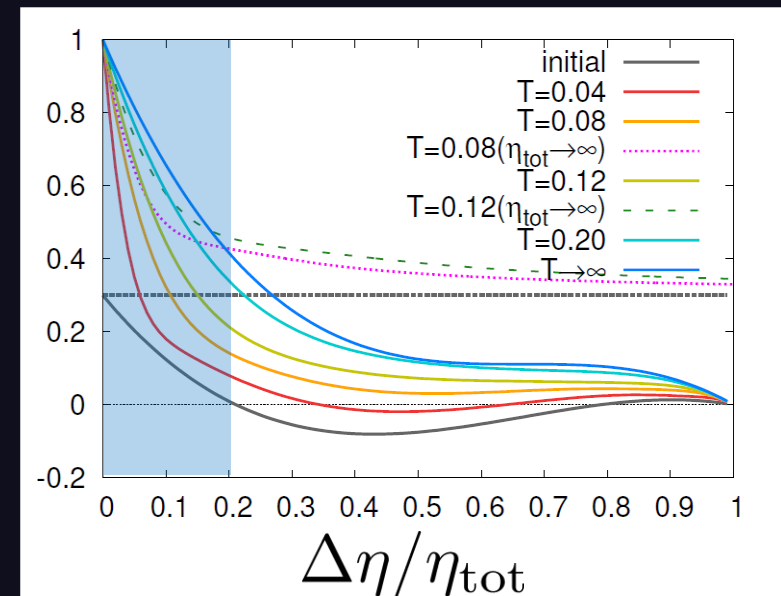
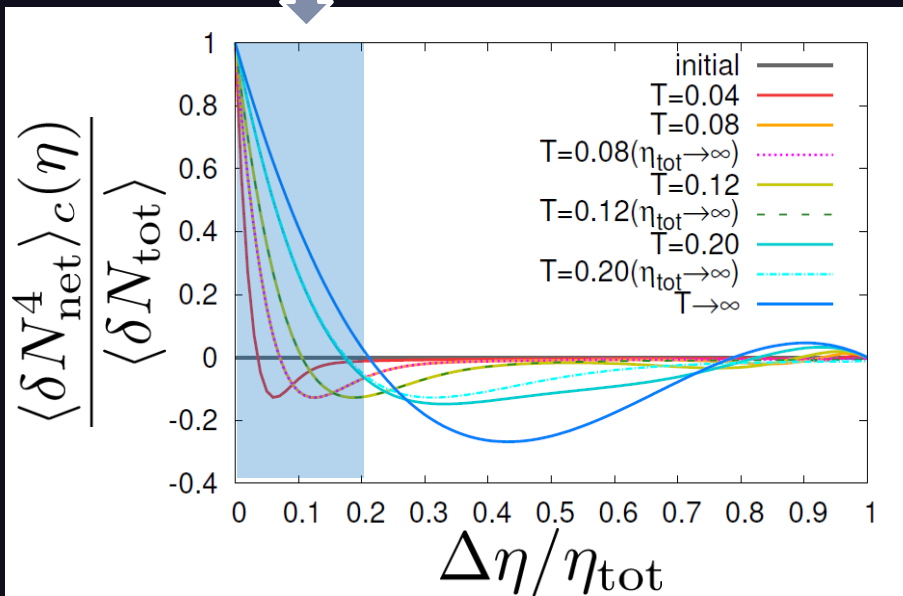
parameter sensitive to hadronization

4th order Cumulant at ALICE

MK, Asakawa, Ono (2014)

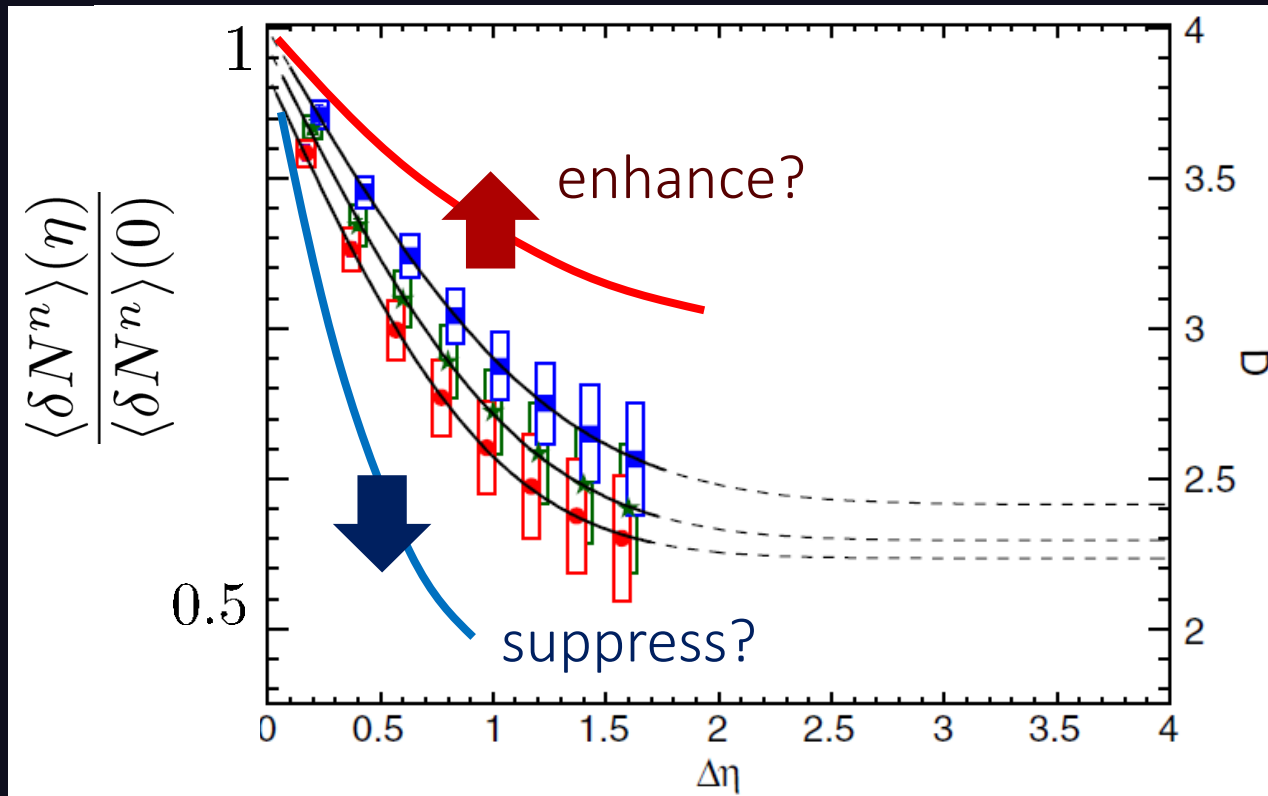
Sakaida+, poster I-35

rapidity coverage at ALICE ($\eta_{\text{tot}} = 8$)



4th order cumulant is sensitive to
initial fluctuation / transport property / confinement
It can be non-monotonic and negative!

$\Delta\eta$ Dep. of Non-Gaussianity



How does the 4-th order cumulant
behave as a function of $\Delta\eta$?

Suggestions to Experimentalists

□ many conserved charges

electric charge, baryon number, (and strangeness?)
with different diffusion constants

□ various cumulants

second, third, fourth, mixed, (and much higher?)

□ $\Delta\eta$ window dependences

primordial thermodynamics, transport property, confinement
no normalization

□ Beam Energy Scan

LHC, RHIC-BES, FAIR, NICA, J-PARC, ...

My Messages

- Fluctuations are invaluable observables in HIC
- But, we must understand them in more detail
- **It's possible**, interesting, and important

We are just arriving at the starting point to explore QCD phase structure with fluctuations!

Summary

Fluctuations in HIC are nonthermal!

Plenty of physics in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2 \rangle_c, \langle N_B^2 \rangle_c, \langle N_Q^4 \rangle_c, \langle N_B^4 \rangle_c, \\ \langle N_{ch}^2 \rangle_c, \dots$$

Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC

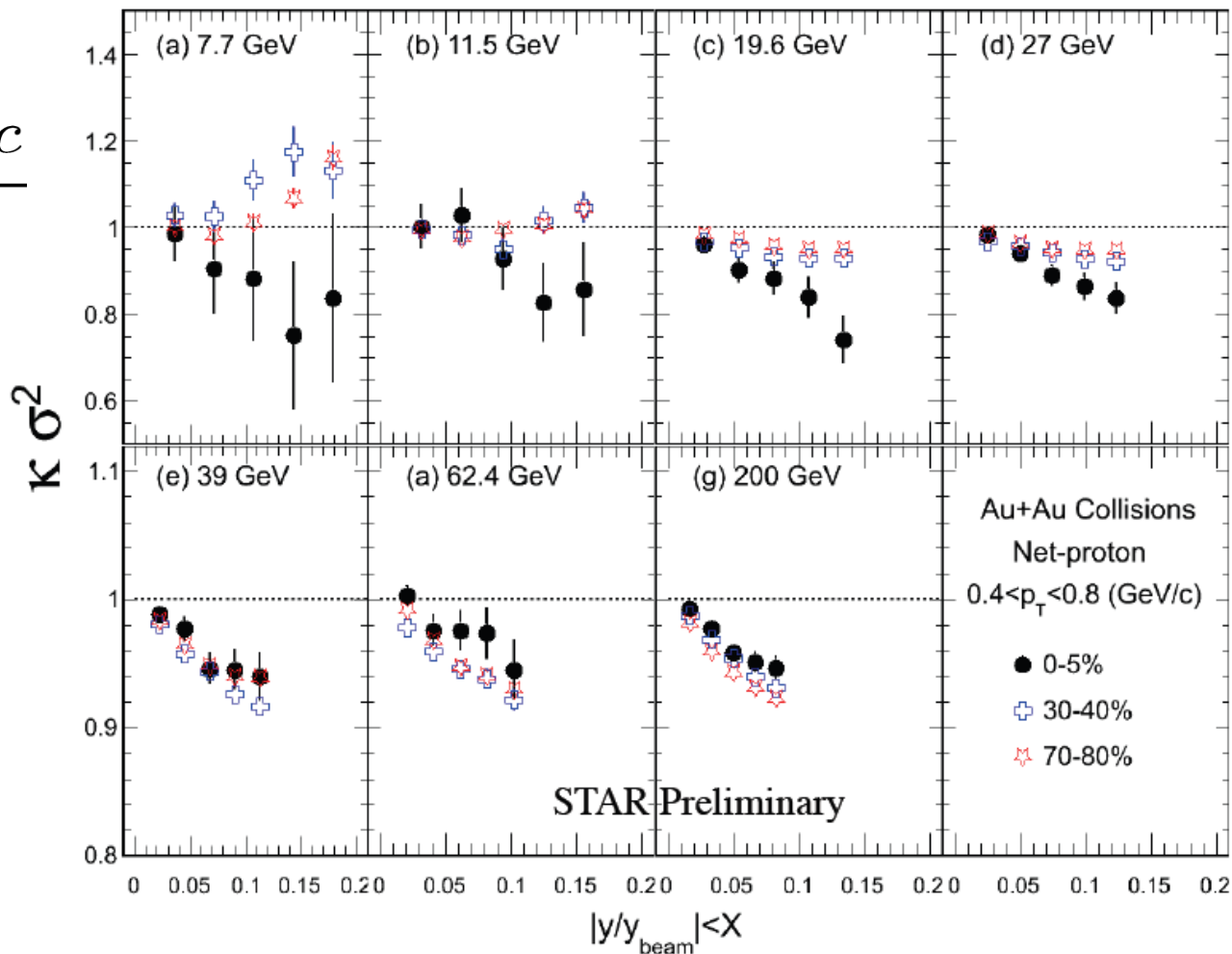
- history of hot medium
- mechanism of hadronization
- diffusion constant

backup

$\Delta\eta$ Dependence at STAR

STAR, QM2012

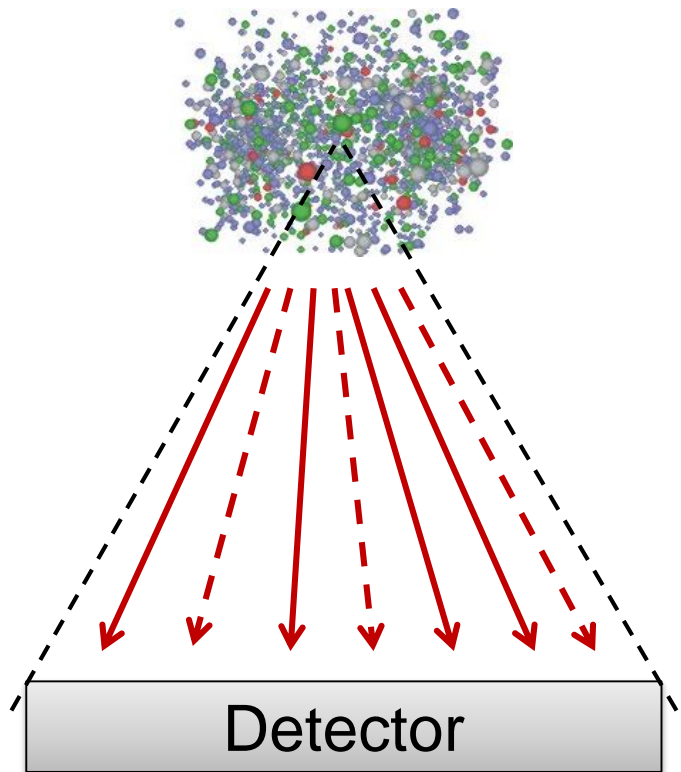
$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$



$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$

decreases as $\Delta\eta$ becomes larger at RHIC energy.

Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$



\square $\left\{ \begin{array}{l} \longrightarrow N_N \text{ nucleons} \\ \dashrightarrow N_{\bar{N}} \text{ anti-nucleons} \end{array} \right.$

$\longrightarrow F(N_N, N_{\bar{N}})$

$\square N_N \left\{ \begin{array}{l} N_p \text{ protons} \\ N_n \text{ neutrons} \end{array} \right.$

$\longrightarrow B(N_p; N_N)$

binomial distribution func.

$$\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$

$$= F(N_N, N_{\bar{N}}) B(N_p; N_N) B(N_{\bar{p}}; N_{\bar{N}})$$

➤ for any phase space in the final state.

3rd & 4th Order Fluctuations

$$N_B \rightarrow N_p$$

$$\langle (\delta N_p^{(\text{net})})^3 \rangle = \frac{1}{8} \langle (\delta N_B^{(\text{net})})^3 \rangle + \frac{3}{8} \langle \delta N_B^{(\text{net})} \delta N_B^{(\text{tot})} \rangle,$$

$$\begin{aligned} \langle (\delta N_p^{(\text{net})})^4 \rangle_c &= \frac{1}{16} \langle (\delta N_B^{(\text{net})})^4 \rangle_c + \frac{3}{8} \langle (\delta N_B^{(\text{net})})^2 \delta N_B^{(\text{tot})} \rangle \\ &\quad + \frac{3}{16} \langle (\delta N_B^{(\text{tot})})^2 \rangle - \frac{1}{8} \langle N_B^{(\text{tot})} \rangle, \end{aligned}$$

$$N_p \rightarrow N_B$$

$$\begin{aligned} \langle (\delta N_B^{(\text{net})})^3 \rangle &= 8 \langle (\delta N_p^{(\text{net})})^3 \rangle - 12 \langle \delta N_p^{(\text{net})} \delta N_p^{(\text{tot})} \rangle \\ &\quad + 6 \langle N_p^{(\text{net})} \rangle, \end{aligned}$$

$$\begin{aligned} \langle (\delta N_B^{(\text{net})})^4 \rangle_c &= 16 \langle (\delta N_p^{(\text{net})})^4 \rangle_c - 48 \langle (\delta N_p^{(\text{net})})^2 \delta N_p^{(\text{tot})} \rangle \\ &\quad + 48 \langle (\delta N_p^{(\text{net})})^2 \rangle + 12 \langle (\delta N_p^{(\text{tot})})^2 \rangle - 26 \langle N_p^{(\text{tot})} \rangle, \end{aligned}$$

Strange Baryons

Decay Rates:

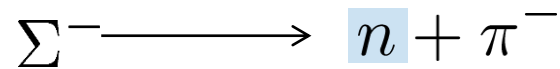
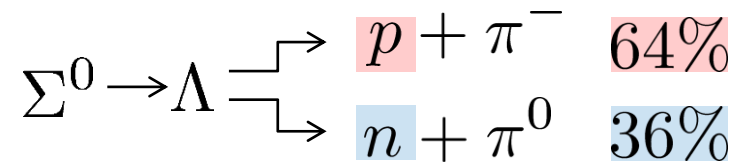
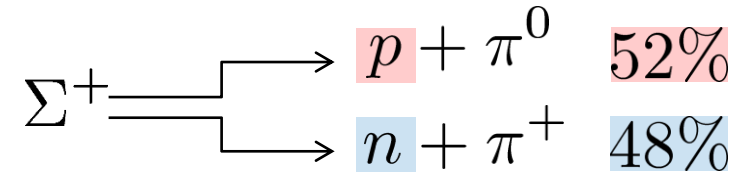
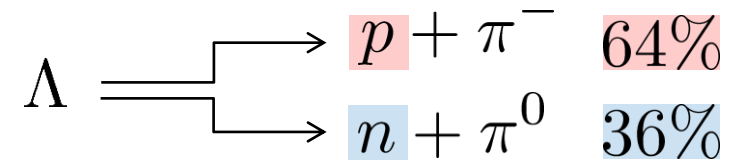
$$\Lambda \quad m_\Lambda \simeq 1116[\text{MeV}]$$

$$\Rightarrow p : n \simeq 1.6 : 1$$

$$\Sigma \quad m_\Sigma \simeq 1190[\text{MeV}]$$

$$\Rightarrow p : n \simeq 1 : 1.8$$

Decay modes:

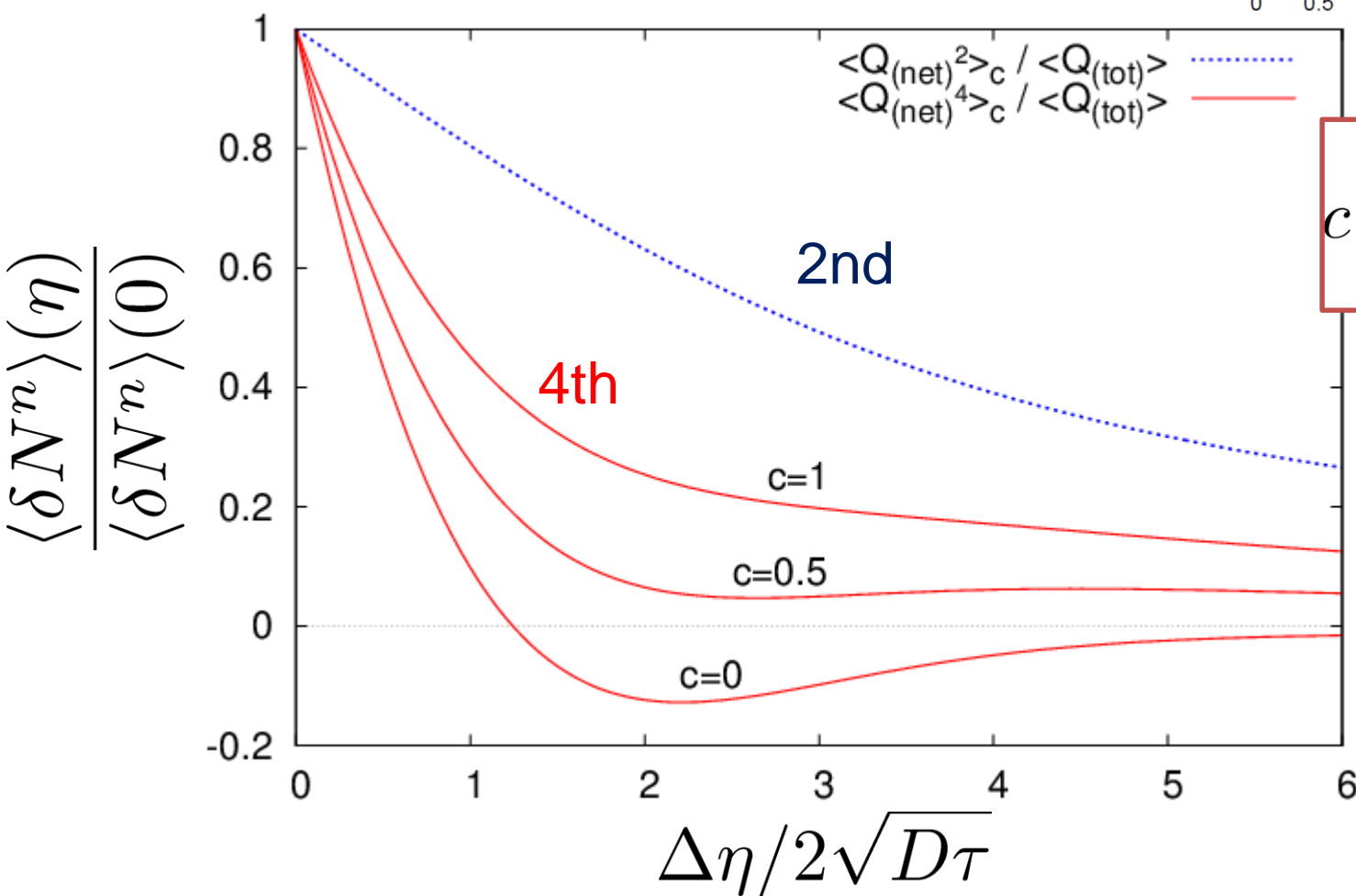
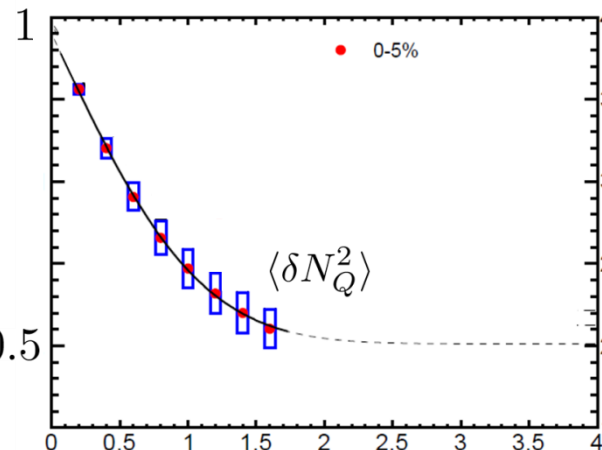


Regarding these ratios even, protons from these decays is incorporated into the binomial distribution. Then, $N_N \rightarrow N_B$

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\text{tot})} \rangle_c = 0$$



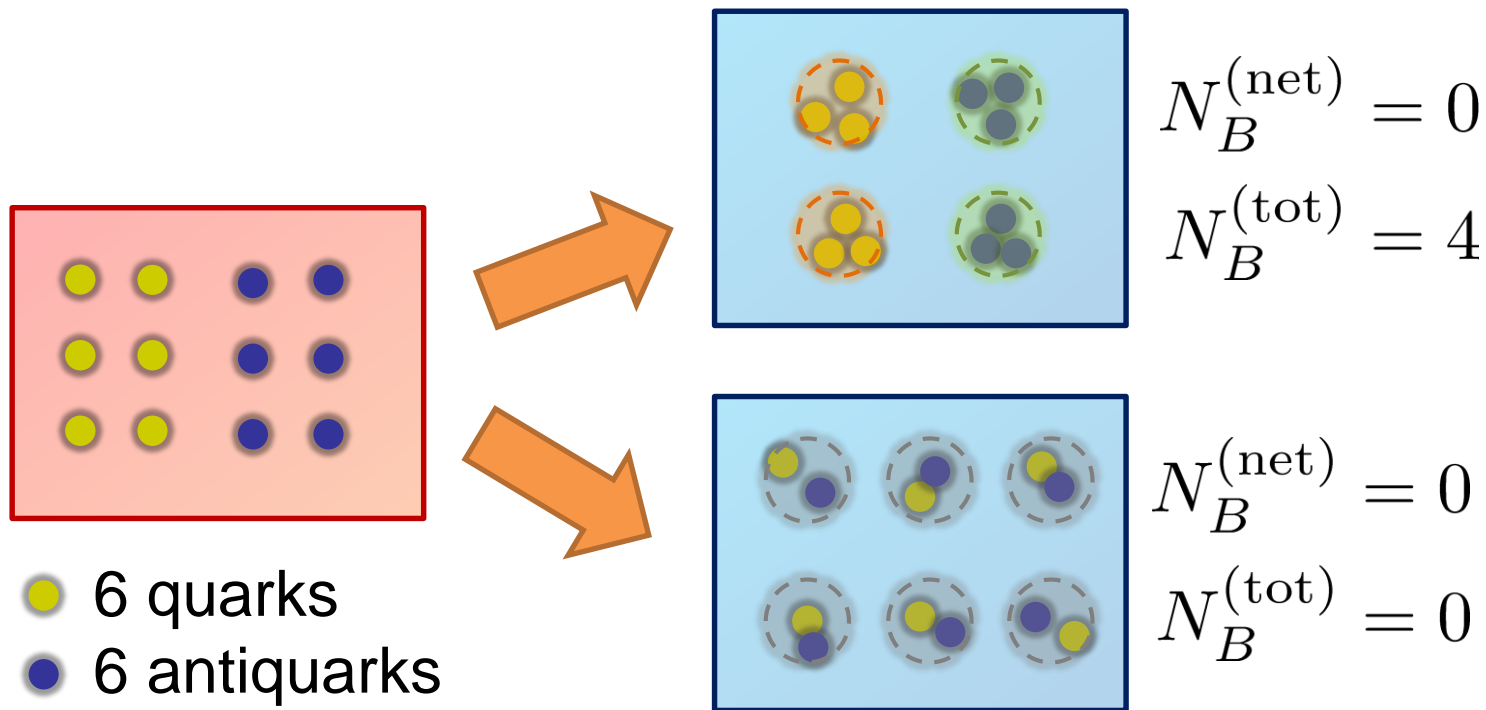
$$c = \frac{\langle Q_{(\text{tot})}^2 \rangle_c}{\langle Q_{(\text{tot})} \rangle}$$



parameter
sensitive to
hadronization

Total Charge Number

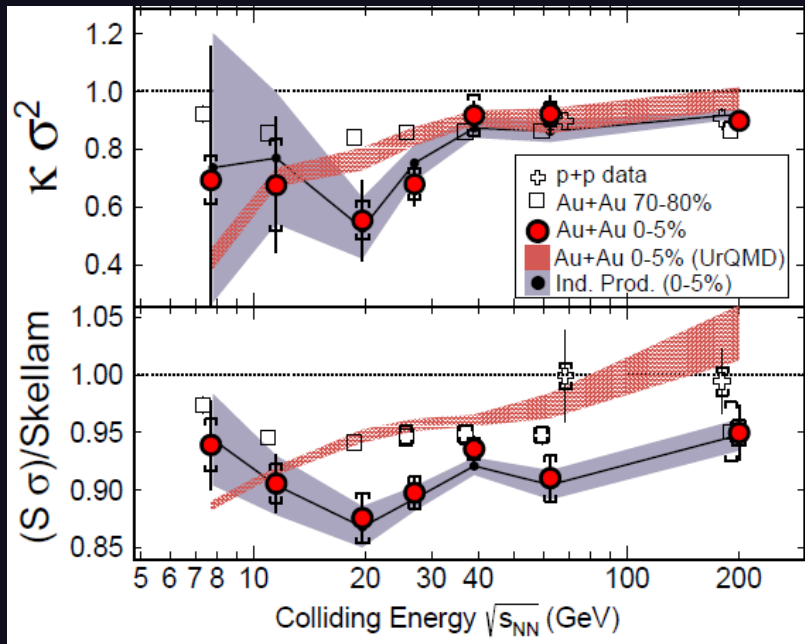
In recombination model,



□ $N_B^{(\text{tot})}$ can fluctuate, while $N_B^{(\text{net})}$ does not.

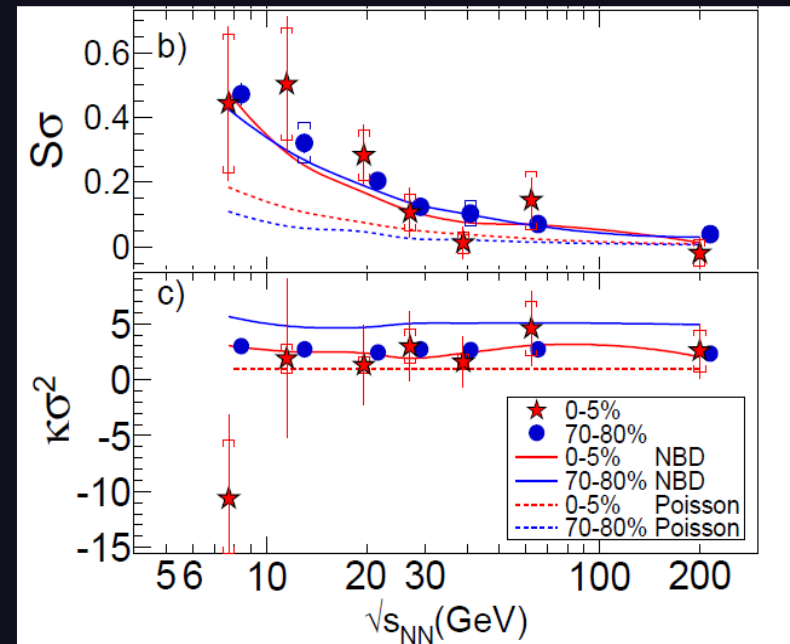
Higher Order Cumulants @ STAR

(Net-) Proton Number



STAR, PRL112,032302(2014)

(Net-) Electric Charge

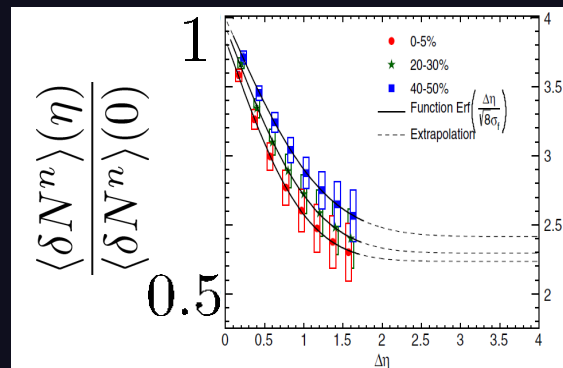
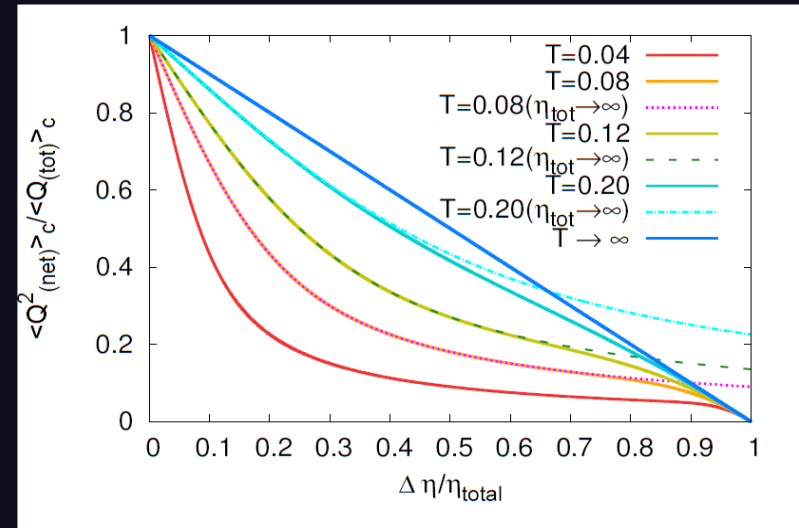


STAR, arXiv:1402.1558

- Exp. results are close to Poissonian values.
- Proton number cumulants are lower than the Poissonian values.

2nd Order Cumulant

consistent with stochastic diffusion equation



Search of QCD Phase Structure

Stronger correlation length dep.

Stephanov, 2009

$$\langle \delta N^2 \rangle \sim \xi^2, \quad \langle \delta N^3 \rangle \sim \xi^{4.5}, \quad \langle \delta N^4 \rangle_c \sim \xi^7$$

Sign of cumulants

Asakawa, Ejiri, MK, 2009

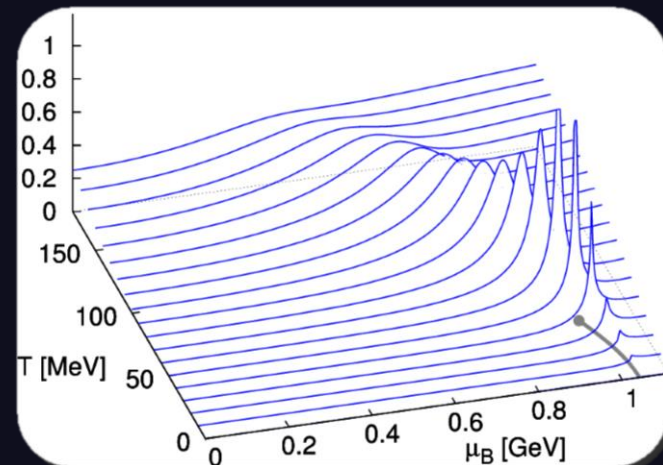
Friman+, 2011

Stephanov, 2011

$$\langle \delta N^n \rangle = T \frac{\partial^n}{\partial \hat{\mu}^n} \ln Z$$



$$\langle \delta N^3 \rangle = \frac{\partial}{\partial \hat{\mu}} \langle \delta N^2 \rangle$$



Fluctuations

Fluctuations reflect properties of matter.

Enhancement near the critical point

Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);...

Ratios between cumulants of conserved charges

Asakawa,Heinz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)

Signs of higher order cumulants

Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)

