重イオン衝突実験における熱ゆらぎ
～非平衡性と非ガウス性を中心に～

Masakiyo Kitazawa
(Osaka U.)
Beam-Energy Scan

Quark-Gluon Plasma

Hadrons

Color SC
Beam-Energy Scan

**STAR 2012**

Grand Canonical Ensemble

- 200 GeV
- 39 GeV
- 11.5 GeV
- 7.7 GeV

**Au+Au**

- 00-05%
- 05-10%
- 10-20%
- 20-30%
- 30-40%
- 40-60%
- 60-80%

**STAR Preliminary**

- Cleymans
- Andronic

**Hadrons**

**Color SC**
Bulk (Thermal) Fluctuations

Observables in equilibrium are fluctuating!

\[ \langle \delta N^2 \rangle = V \chi_2 = \sigma^2 \]

\[ S = \frac{\langle \delta N^3 \rangle}{\sigma^3} \]

\[ \kappa = \frac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2} \]

Gaussian

non-Gaussianity
Event-by-Event Analysis

Pioneering studies on $\sigma$:

- Search of QCD critical point  Stephanov, Rajagopal, Shuryak, PRL (1998)
- Quark deconfinement  Asakawa, Heinz, Muller PRL; Jeon, Koch PRL (2000)
My Messages

- Fluctuations are invaluable observables in HIC
- But, we must understand them in more detail
- It’s possible, interesting, and important
Why Fluctuations?
Brownian Motion

Fluctuations opened atomic physics

from Wikipedia

A. Einstein 1905
Shot Noise at Normal-Superconductor Junction


Similar experiments for fractional QHE ex. Saminadayar+, PRL 79, 2526 (1997)
Conserved Charges: Theoretical Advantage

- Definite definition for operators
  - as a Noether current
  - calculable on any theory
  ex: on the lattice
Conserved Charges: Theoretical Advantage

- Definite definition for operators
  - as a Noether current
  - calculable on any theory
    ex: on the lattice

- Simple thermodynamic relations
  \[
  \left\langle \delta N_c^n \right\rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}
  \]
  - Intuitive interpretation for the behaviors of cumulants
    ex: \[
    \left\langle \delta N_B^3 \right\rangle = \frac{1}{VT^2} \frac{\partial \left\langle \delta N_B^2 \right\rangle}{\partial \mu_B}
    \]

Asakawa, Ejiri, MK, 2009
Conserved-Charge Fluctuations

Fluctuations of CC: rigorously defined in a theory
- operators as the Noether current
- as derivatives of the partition function

They are lattice observables

Fluctuations of CC
II
LAT-HIC crossover

QCD phase diagram 3, Wed. 11:00-13:30
Recent Progress in Lattice Community

From LATTICE2013 presentations
Fluctuations

Free Boltzmann $\rightarrow$ Poisson

$$\langle \delta N^n \rangle_c = \langle N \rangle$$

$$\langle \delta N^q_q \rangle_c = \langle N_q \rangle$$

$$\langle \delta N^n_B \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$

$$\langle \delta N^n_B \rangle_c = \langle N_B \rangle$$
Fluctuations

Free Boltzmann $\rightarrow$ Poisson

$\langle \delta N^n \rangle_c = \langle N \rangle$

$\langle \delta N^q \rangle_c = \langle N_q \rangle$

$\langle \delta N^B_B \rangle_c = \frac{1}{3n-1} \langle N_B \rangle$

$3 N_B = N_q$

RBC-Bielefeld ’09

$12 \frac{c^B_4}{c^B_2} = \frac{\langle B^4 \rangle - 3 \langle B^2 \rangle^2}{\langle B^2 \rangle}$

filled: nt=4
open: nt=6
Electric Charge Fluctuation

Asakawa, Heinz, Muller; Jeon, Koch, 2000

$$|q_q| = 1/3, 2/3$$

$$|q_B| = 1$$

**D-measure**

$$D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- $D \sim 3-4$ Hadronic
- $D \sim 1-1.5$ QGP
Electric Charge Fluctuation


$D$ measure

$$D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- $D \sim 3-4$ Hadronic
- $D \sim 1-1.5$ QGP

- No suppression at RHIC energy
- Fluctuations @ LHC cannot be described by hadronic d.o.f.
Rapidity Window Dependence

**ALICE, PRL 110, 152301 (2013)**

- Smaller $\Delta \eta$ more hadronic
- Larger $\Delta \eta$ more QGP like

Same information in
- $\langle \delta N_Q^{\text{(net)}}(\eta_1) \delta N_Q^{\text{(net)}}(\eta_2) \rangle$
- $\simeq$ balance function
to be studied by fluctuating hydro.

Stochastic Diffusion eq.
Shuryak,Stephanov,2001
Time Evolution in HIC

Quark-Gluon Plasma

Hadronization

Freezeout

\[ \langle \Delta N^2 \rangle / \Delta \eta \]

\[ \chi_{\text{HAD}} \]

\[ \chi_{\text{QGP}} \]

\[ \Delta \eta \]

\[ \Delta \eta \]
Various Contributions

- Initial fluctuations
- Effect of jets
- Negative binomial (?)
- Final state rescattering
- Coordinate vs pseudo rapidities
- Particle missID
- Efficiency correction
- Global charge conservation
Global Charge Conservation

For equilibrated medium

\[ \langle \delta N^2 \rangle_{\text{exp.}} = \langle \delta N^2 \rangle_{\text{GC}} \times \left( 1 - \frac{\Delta y}{y_{\text{tot}}} \right) \]

Jeon, Koch, 2000
Bleicher, Jeon, Koch, 2001
Global Charge Conservation

For equilibrated medium

$$\langle \delta N^2 \rangle_{\text{exp.}} = \langle \delta N^2 \rangle_{\text{GC}} \times \left(1 - \frac{\Delta y}{y_{\text{tot}}} \right)$$

Solving the time evolution...

GCC effect on the hadronic diffusion is negligible in the ALICE result!

Sakaida, Poster I-35
Electric-Charge Fluctuations

- Electric charge fluctuations is suppressed at LHC!
- The suppression is most probably attributed to primordial physics
- Qualitative difference b/w RHIC and LHC

... but why?
Non Gaussianity
Non-Gaussianity

CMB
Cosmic Microwave Background

- No statistically-significant signals
  Planck, 2013

Mesoscopic Systems

- Full counting statistics
- Cumulants up to 5th order
Non-Gaussianity in HIC

- Ratio of conserved charges
  Ejiri, Karsch, Redlich (2005)
- Critical enhancement
  Stephanov (2009)
- Sign change
  Asakawa, Ejiri, MK (2009); Friman+ (2011); Stephanov (2011)
- Strange confinement
  BNL-Bielefeld (2013)
- Distribution funcs themselves
  Morita+ (2013); Nakamura (Wed.)
Ratio of Cumulants

$\langle \delta N^n_q \rangle_c = \langle N_q \rangle$

$\Rightarrow \langle \delta N^n_B \rangle_c = \frac{1}{3n-1} \langle N_B \rangle$

$\frac{\langle \delta N^4_B \rangle_c}{\langle \delta N^2_B \rangle} = \begin{cases} 1 & \text{hadronic} \\ 1/9 & \text{quark-gluon} \end{cases}$
Strange Confinement

**Baryonic**

\[ \langle \delta N_B^2 \rangle = \langle \delta N_B^4 \rangle \]

\[ \langle \delta N_B^2 \rangle - \langle \delta N_B^4 \rangle_c \]

\[ \begin{cases} = 0 & \text{baryons confined} \\ \neq 0 & \text{something else} \end{cases} \]

**Strangeness**

\[ \langle \delta N_s \delta N_B^3 \rangle_c - \langle \delta N_s \delta N_B \rangle \]

\[ \begin{cases} = 0 & \text{strangeness confined} \\ \neq 0 & \text{something else} \end{cases} \]

**Many lattice studies** (LAT-HIC crossover):

Budapest-Wuppertal, 2013; BW,1403.4578; BNL-Bi.,1404.4043; Gupta+,1405.2206; Ratti, Wed.; Schmidt, Wed.; Nakamura, Wed.; Sharma, J-13
Cumulants: HIC@RHIC vs Lattice

fluctuations “exp + lattice”

μ/T discrepancy

particle abundance (chem. freezeout $T$)

parameter window constrained by lattice

BNL-Bielefeld, LATTICE2013
Exp. results are close to and less than Poissonian values.

Something interesting around $\sqrt{s_{NN}} \approx 20$GeV
Effects of Various Contributions

- Initial fluctuations: \textcolor{red}{Enhance}
- Effect of jets: \textcolor{red}{Enhance}
- Negative binomial (?): \textcolor{red}{Enhance}
- Final state rescattering: \textcolor{yellow}{Enhance to Poisson}
- Coordinate vs pseudo rapidities: \textcolor{yellow}{Enhance to Poisson}
- Particle missID: \textcolor{yellow}{Enhance to Poisson}
- Efficiency correction: \textcolor{yellow}{Enhance to Poisson}
- Global charge conservation: \textcolor{blue}{Suppress}
Caution!!

Let’s clarify their relation!

MK, Asakawa (2012;2012)
Nucleon has two isospin states.

A coin has two sides.
Slot Machine Analogy

$P_{\text{tot}}(N)$

$P_{\text{head}}(N)$

$N_{\text{tot}}$

$N_{\text{head}}$
Reconstructing Total Coin Number

\[ P_{\text{head}}(N_{\text{head}}) = \sum_{N_{\text{tot}}} B_{1/2}(N_{\text{head}}; N_{\text{tot}}) P_{\text{tot}}(N_{\text{tot}}) \]
Charge Exchange Reaction

\[ p, n \rightarrow \Delta(1232) \rightarrow p, n \]
\[ \pi \rightarrow \pi \]

Isospin of N is not frozen at chemical freezeout!

\[ p + \pi^+ \rightarrow \Delta^{++} \rightarrow p + \pi^+ \]
\[ 3 \]
\[ p + \pi^0 \rightarrow \Delta^+ \rightarrow p + \pi^0 \]
\[ n + \pi^+ \rightarrow \Delta^+ \rightarrow n + \pi^+ \]
\[ 1 \]
\[ p + \pi^- \rightarrow \Delta^0 \rightarrow p + \pi^- \]
\[ n + \pi^0 \rightarrow \Delta^0 \rightarrow n + \pi^0 \]
\[ 2 \]
\[ n + \pi^- \rightarrow \Delta^- \rightarrow n + \pi^- \]
\[ \text{cross sections of } p \]

\[ p + \pi \rightarrow \Delta^{+,0} \rightarrow p : n \]
\[ = 5 : 4 \]

mean reaction time \(< 5\text{fm/c}\)
Nucleons in Hadronic Medium

- so many pions
- rare NN collisions
- no quantum corr.

**NOTE:**

- hadronize
- chem. f.o.
- kinetic f.o.

10 \sim 20\text{fm}

- $p, \bar{p}$
- $n, \bar{n}$
- mesons
- baryons
- $\Delta(1232)$
Difference b/w $N_B$ and $N_p$

Assumptions:

- Net-cumulants deviate from thermal value.
- But, $N_B$, $N_B^{-1}$ are Poissionian.

\[
\begin{align*}
2\langle (\delta N_{p}^{(\text{net})})^2 \rangle &= \frac{1}{2} \langle (\delta N_{B}^{(\text{net})})^2 \rangle + \frac{1}{2} \langle (\delta N_{B}^{(\text{net})})^2 \rangle_{\text{free}} \quad \text{genuine info.} \\
2\langle (\delta N_{p}^{(\text{net})})^3 \rangle &= \frac{1}{4} \langle (\delta N_{B}^{(\text{net})})^3 \rangle + \frac{3}{4} \langle (\delta N_{B}^{(\text{net})})^3 \rangle_{\text{free}} \\
2\langle (\delta N_{p}^{(\text{net})})^4 \rangle_{c} &= \frac{1}{8} \langle (\delta N_{B}^{(\text{net})})^4 \rangle_{c} + \cdots \\
\end{align*}
\]

Proton number cumulants are dominated by Poissonian noise.

\[\text{cf.) Nahrgang+, arXiv:1402.1238}\]
Efficiency Correction

If efficiency for each particle is uncorrelated

\[ P_{\text{exp.}}(N) = \sum_{N'} B_{\epsilon}(N; N') P(N') \]

for Particle missID: Ono, Asakawa, MK, PRC, 2013
More Information on/from Fluctuations

$\Delta \eta$ dependence

MK, Asakawa, Ono, PLB728, 386 (2014)
How does the 4-th order cumulant behave as a function of $\Delta \eta$?
Fluctuating Hydrodynamics?

- Distributions in experiments are close to Poissonian
- Cumulants are expected to increase in the hadronic medium

These behaviors cannot be described by the theory of hydrodynamic fluctuations.
Hydrodynamic Fluctuations

Stochastic diffusion equation

\[ \partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau) \]

- Fluctuation of \( n \) is Gaussian in equilibrium
- Gaussian noise
- Markov (white noise) + continuity

Landau, Lifshitz, Statistical Mechanics II
Kapusta, Muller, Stephanov, 2012
Stephanov, Shuryak, 2001

cf) Gardiner, “Stochastic Methods”
How to Introduce Non-Gaussianity?

Stochastic diffusion equation

$$\partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

- Choices to introduce non-Gaussianity in equil.:
  - $n$ dependence of diffusion constant $D(n)$
  - colored noise
  - discretization of $n$
How to Introduce Non-Gaussianity?

**Stochastic diffusion equation**

\[
\partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)
\]

- Choices to introduce non-Gaussianity in equil.:
  - \( n \) dependence of diffusion constant \( D(n) \)
  - colored noise
  - **discretization of \( n \)** \( \rightarrow \) our choice

**REMARK:** Fluctuations measured in HIC are almost Poissonian.
Nucleons in Hadronic Medium

Baryons in hadronic medium behave like Brownian pollen

hadronize
chem. f.o.
kinetic f.o.

10 ~ 20 fm

- $p, \bar{p}$
- $n, \bar{n}$
- mesons
- baryons
- $\Delta(1232)$
Diffusion Master Equation

Mark Kawai, Fumiaki Asakawa, and Masahiro Ono, PLB **728**, 386 (2014)

Divide spatial coordinate into discrete cells

\[ \frac{\partial}{\partial t} P(n) = \gamma \sum_x [(n_x + 1) \{ P(n + e_x - e_{x+1}) + P(n + e_x - e_{x-1}) \} - 2n_x P(n)] \]

Solve the DME exactly, and take \( a \to 0 \) limit.
hadronization
chemical freezeout

Initial condition
• boost invariance
• locality of fluctuations
• small cumulants

Brownian diffusion

kinetic freezeout

Comments:
• agreement with stochastic diffusion eq. up to Gaussian fluctuation
• Poisson (Skellam) distribution in equilibrium: consistent with HRG
Solution of DME in a $a \rightarrow 0$ Limit

1st order (deterministic) $\langle n \rangle$
- consistent with diffusion equation with $D = \gamma a^2$

Continuum limit with fixed $D = \gamma a^2$

2nd order $\langle \delta n^2 \rangle$
- consistent with stochastic diffusion eq.
  (for sufficiently smooth initial conditions)

Shuryak, Stephanov, 2001

Nontrivial results for non-Gaussian fluctuations
Net Charge Number

Prepare 2 species of (non-interacting) particles

\[
\bar{Q}(\tau) = \int_0^{\Delta \eta} \, d\eta \left( n_1(\eta, \tau) - n_2(\eta, \tau) \right)
\]

Let us investigate
\[
\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c
\] at freezeout time \( t \)
Time Evolution in Hadronic Phase

Hadronization (initial condition)

- Boost invariance / infinitely long system
- Local equilibration / local correlation

\[
\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c \quad \langle \bar{Q}^2 \bar{Q}_{(tot)} \rangle_c \quad \langle \bar{Q}^2_{(tot)} \rangle_c
\]

- suppression owing to local charge conservation
- strongly dependent on hadronization mechanism
Time Evolution in Hadronic Phase

Hadronization (initial condition)

- Boost invariance / infinitely long system
- Local equilibration / local correlation

\[ \langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c \quad \langle \bar{Q}^2 Q_{(tot)} \rangle_c \quad \langle Q_{(tot)}^2 \rangle_c \]

suppression owing to local charge conservation

strongly dependent on hadronization mechanism

Freezeout
**Δη Dependence at Freezeout**

Initial fluctuations:

\[
\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(tot)} \rangle_c = 0
\]

\[
c = \frac{\langle Q_{(net)}^2 \rangle_c / \langle Q_{(tot)} \rangle_c}{\langle Q_{(net)}^4 \rangle_c / \langle Q_{(tot)} \rangle_c}
\]

Parameter sensitive to hadronization.
In recombination model,

$N_B^{(\text{net})} = 0$
$N_B^{(\text{tot})} = 4$

$N_B^{(\text{net})} = 0$
$N_B^{(\text{tot})} = 0$

$N_B^{(\text{tot})}$ can fluctuate, while $N_B^{(\text{net})}$ does not.
Initial fluctuations:
\[ \langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{\text{tot}} \rangle_c = 0.5 \langle Q_{\text{tot}} \rangle \]

\[ c = \frac{\langle Q_{\text{net}}^2 \rangle_c / \langle Q_{\text{tot}} \rangle}{\langle Q_{\text{net}}^4 \rangle_c / \langle Q_{\text{tot}} \rangle} \]

parameter sensitive to hadronization
4th order Cumulant at ALICE

MK, Asakawa, Ono (2014)
Sakaida+, poster I-35

rapidity coverage at ALICE ($\eta_{tot} = 8$)

4th order cumulant is sensitive to initial fluctuation / transport property / confinement

It can be non-monotonic and negative!
How does the 4-th order cumulant behave as a function of $\Delta \eta$?
Suggestions to Experimentalists

- many conserved charges
  - electric charge, baryon number, (and strangeness?)
  - with different diffusion constants

- various cumulants
  - second, third, fourth, mixed, (and much higher?)

- $\Delta \eta$ window dependences
  - primordial thermodynamics, transport property, confinement
  - no normalization

- Beam Energy Scan
  - LHC, RHIC-BES, FAIR, NICA, J-PARC, ...
Fluctuations are invaluable observables in HIC.

But, we must understand them in more detail.

It’s possible, interesting, and important.

We are just arriving at the starting point to explore QCD phase structure with fluctuations!
Summary

Fluctuations in HIC are nonthermal!

Plenty of physics in $\Delta \eta$ dependences of various cumulants

Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC
- history of hot medium
- mechanism of hadronization
- diffusion constant

$\langle N_Q^2 \rangle_c, \langle N_B^2 \rangle_c, \langle N_Q^4 \rangle_c, \langle N_B^4 \rangle_c, \langle N_{ch}^2 \rangle_c, \cdots$
backup
$\frac{\langle \delta N_{p}^{4}\rangle_{c}}{\langle \delta N_{p}^{2}\rangle}$ decreases as $\Delta \eta$ becomes larger at RHIC energy.
Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$

- $N_N$ nucleons
- $N_{\bar{N}}$ anti-nucleons

$\Rightarrow F(N_N, N_{\bar{N}})$

- $N_p$ protons
- $N_n$ neutrons

$\Rightarrow B(N_p; N_N) B(N_{\bar{p}}; N_{\bar{N}})$

for any phase space in the final state.
$3^{rd} \ & 4^{th} \ Order \ Fluctuations$

$N_B \rightarrow N_p$

\[
\langle (\delta N_p^{(net)})^3 \rangle = \frac{1}{8} \langle (\delta N_B^{(net)})^3 \rangle + \frac{3}{8} \langle \delta N_B^{(net)} \delta N_B^{(tot)} \rangle,
\]

\[
\langle (\delta N_p^{(net)})^4 \rangle_c = \frac{1}{16} \langle (\delta N_B^{(net)})^4 \rangle_c + \frac{3}{8} \langle (\delta N_B^{(net)})^2 \delta N_B^{(tot)} \rangle
\]

\[
+ \frac{3}{16} \langle (\delta N_B^{(tot)})^2 \rangle - \frac{1}{8} \langle N_B^{(tot)} \rangle,
\]

$N_p \rightarrow N_B$

\[
\langle (\delta N_B^{(net)})^3 \rangle = 8 \langle (\delta N_p^{(net)})^3 \rangle - 12 \langle \delta N_p^{(net)} \delta N_p^{(tot)} \rangle
\]

\[
+ 6 \langle N_p^{(net)} \rangle,
\]

\[
\langle (\delta N_B^{(net)})^4 \rangle_c = 16 \langle (\delta N_p^{(net)})^4 \rangle_c - 48 \langle (\delta N_p^{(net)})^2 \delta N_p^{(tot)} \rangle
\]

\[
+ 48 \langle (\delta N_p^{(net)})^2 \rangle + 12 \langle (\delta N_p^{(tot)})^2 \rangle - 26 \langle N_p^{(tot)} \rangle,
\]
Strange Baryons

Decay Rates:

\[ \Lambda \quad m_\Lambda \simeq 1116 \text{[MeV]} \]

\[ \Sigma \quad m_\Sigma \simeq 1190 \text{[MeV]} \]

\[ p : n \simeq 1.6 : 1 \]

\[ p : n \simeq 1 : 1.8 \]

Decay modes:

\[ \Lambda \rightarrow p + \pi^- \quad 64\% \]

\[ \Sigma^+ \rightarrow p + \pi^0 \quad 52\% \]

\[ \Sigma^0 \rightarrow \Lambda + \pi^- \quad 64\% \]

\[ \Sigma^- \rightarrow n + \pi^- \quad 64\% \]

\[ \Sigma^0 \rightarrow n + \pi^0 \quad 36\% \]

Regarding these ratios even, protons from these decays is incorporated into the binomial distribution. Then, \( N_N \rightarrow N_B \)
$\Delta \eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle Q^2 \rangle_c = \langle Q^4 \rangle_c = \langle Q^2 Q_{(\text{tot})} \rangle_c = 0$$

Parameter sensitive to hadronization
In recombination model, 6 quarks and 6 antiquarks can fluctuate, while $N_B^{(net)}$ does not.
Exp. results are close to Poissonian values.
Proton number cumulants are lower than the Poissonian values.
2nd Order Cumulant

consistent with stochastic diffusion equation
Search of QCD Phase Structure

Stronger correlation length dep.

\[ \langle \delta N^2 \rangle \sim \xi^2, \quad \langle \delta N^3 \rangle \sim \xi^{4.5}, \quad \langle \delta N^4 \rangle_c \sim \xi^7 \]

Sign of cumulants

\[ \langle \delta N^n \rangle = T \frac{\partial^n}{\partial \hat{\mu}^n} \ln Z \]
\[ \langle \delta N^3 \rangle = \frac{\partial}{\partial \hat{\mu}} \langle \delta N^2 \rangle \]

Stephanov, 2009
Asakawa, Ejiri, MK, 2009
Friman+, 2011
Stephanov, 2011
Fluctuations

- Fluctuations reflect properties of matter.
- Enhancement near the critical point
  - Stephanov, Rajagopal, Shuryak ('98); Hatta, Stephanov ('02); Stephanov ('09);...
- Ratios between cumulants of conserved charges
  - Asakawa, Heinz, Muller ('00); Jeon, Koch ('00); Ejiri, Karsch, Redlich ('06)
- Signs of higher order cumulants
  - Asakawa, Ejiri, MK ('09); Friman, et al. ('11); Stephanov ('11)