Energy-Momentum Tensor and Thermodynamics of Lattice Gauge Theory from Gradient Flow

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for FlowQCD Collaboration
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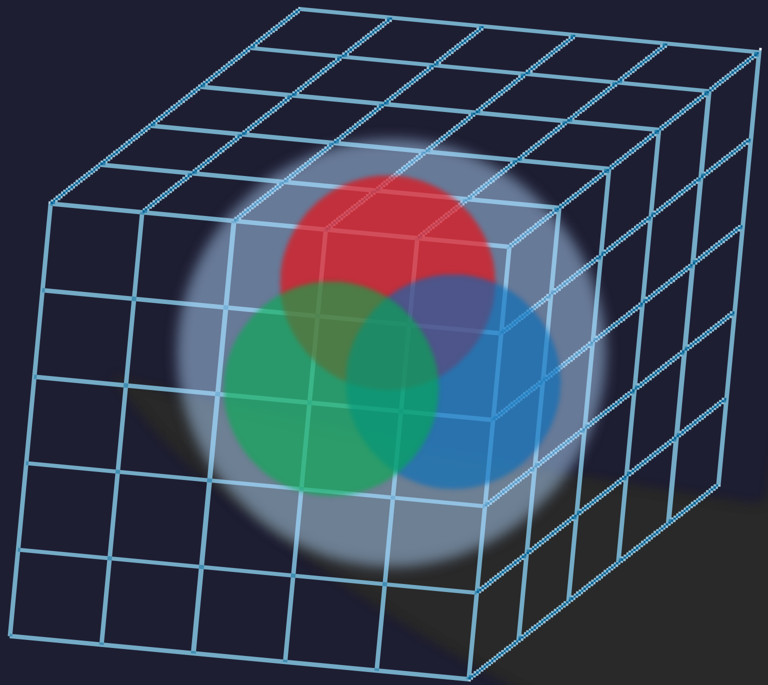
FlowQCD, PR**D90**,011501R (2014)

ATHIC2014, 7 Aug. 2014, Osaka U.

Lattice QCD



First principle calculation of QCD
Monte Carlo for path integral

hadron spectra, chiral symmetry, phase transition, etc.

Gradient Flow

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

Luscher, 2010

A powerful tool for various analyses on the lattice

$$T_{\mu\nu}$$

Poincare symmetry

$$T_{\mu\nu}$$

energy

momentum

$$\begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

stress

pressure

Einstein Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

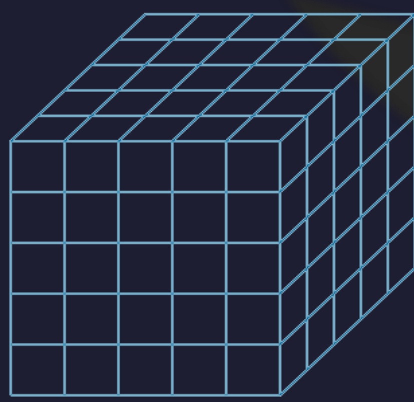
Hydrodynamic Eq.

$$\partial_\mu T_{\mu\nu} = 0$$

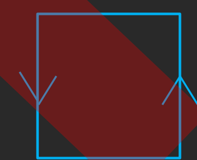
$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial

because of the explicit breaking of Lorentz symmetry



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

② Its measurement is extremely noisy

due to high dimensionality and etc.

the more severe on the finer lattices

stem from	small	lose	big
因	小	失	大

stem from	small	lose	big
因	小	失	大

- miss the wood for the trees
- 見小利則大事不成

孔子(論語、子路13)

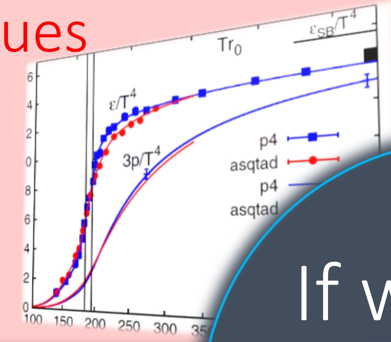
If we have

$$T_{\mu\nu}$$

Thermodynamics

direct measurement of expectation values

$$\langle T_{00} \rangle, \langle T_{ii} \rangle$$



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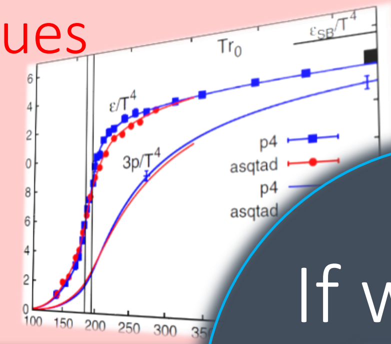

Fluctuations and Correlations

viscosity, specific heat, ...

$$\eta = \int_0^\infty dt \langle T_{12}; T_{12} \rangle$$

$$c_V \sim \langle \delta T_{00}^2 \rangle$$

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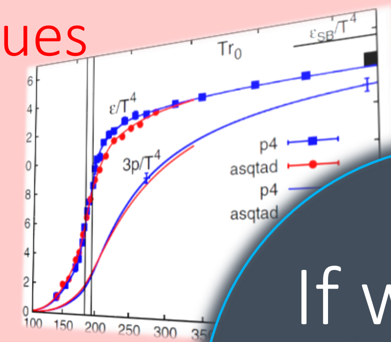

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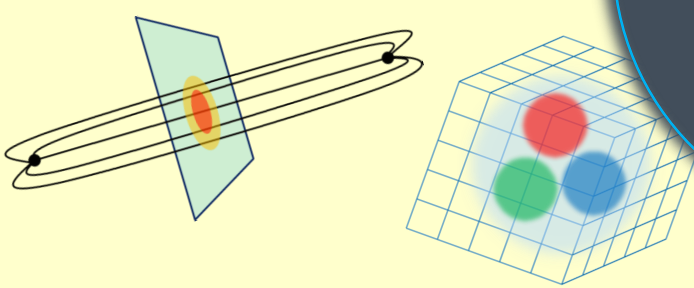
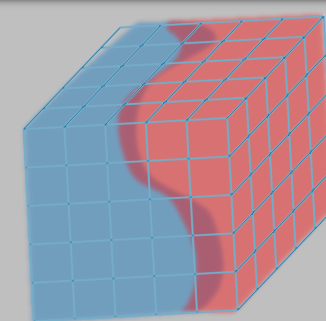


- confinement string
- EM distribution in hadrons

Hadron Structure



- vacuum configuration
- mixed state on 1st transition

Vacuum Structure

Themodynamics: Integral Method

ε : energy density
p : pressure

↔

$\varepsilon - 3p$
directly observable

Themodynamics: Integral Method

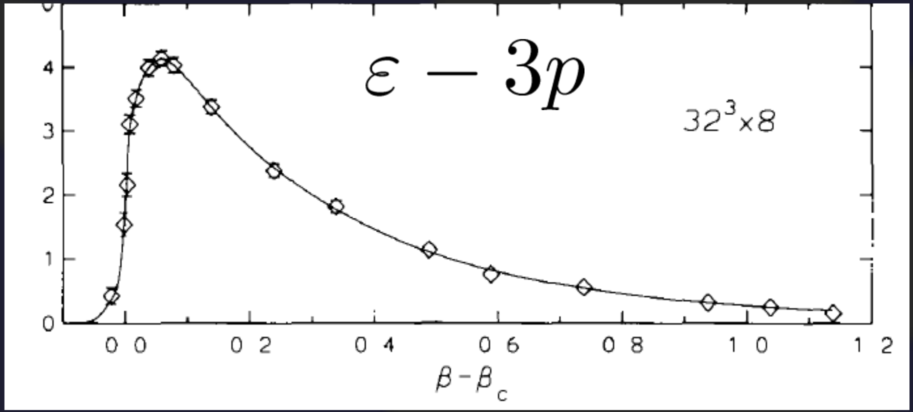
ε : energy density
p : pressure

$\varepsilon - 3p$
directly observable

$$T\frac{\partial(p/T^4)}{\partial T} = \frac{\varepsilon - 3p}{T^4}$$

$$\frac{p}{T^4} = \int_{T_0}^{T} dT \frac{\varepsilon - 3p}{T^5}$$

Boyd+ 1996



- measurements of e-3p for many T
- vacuum subtraction for each T
- information on beta function

Gradient Flow

YM Gradient Flow

Luscher, 2010

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

t: “flow time”
dim:[length^2]

$$A_\mu(0,x) = A_\mu(x)$$

YM Gradient Flow

Luscher, 2010

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length^2]

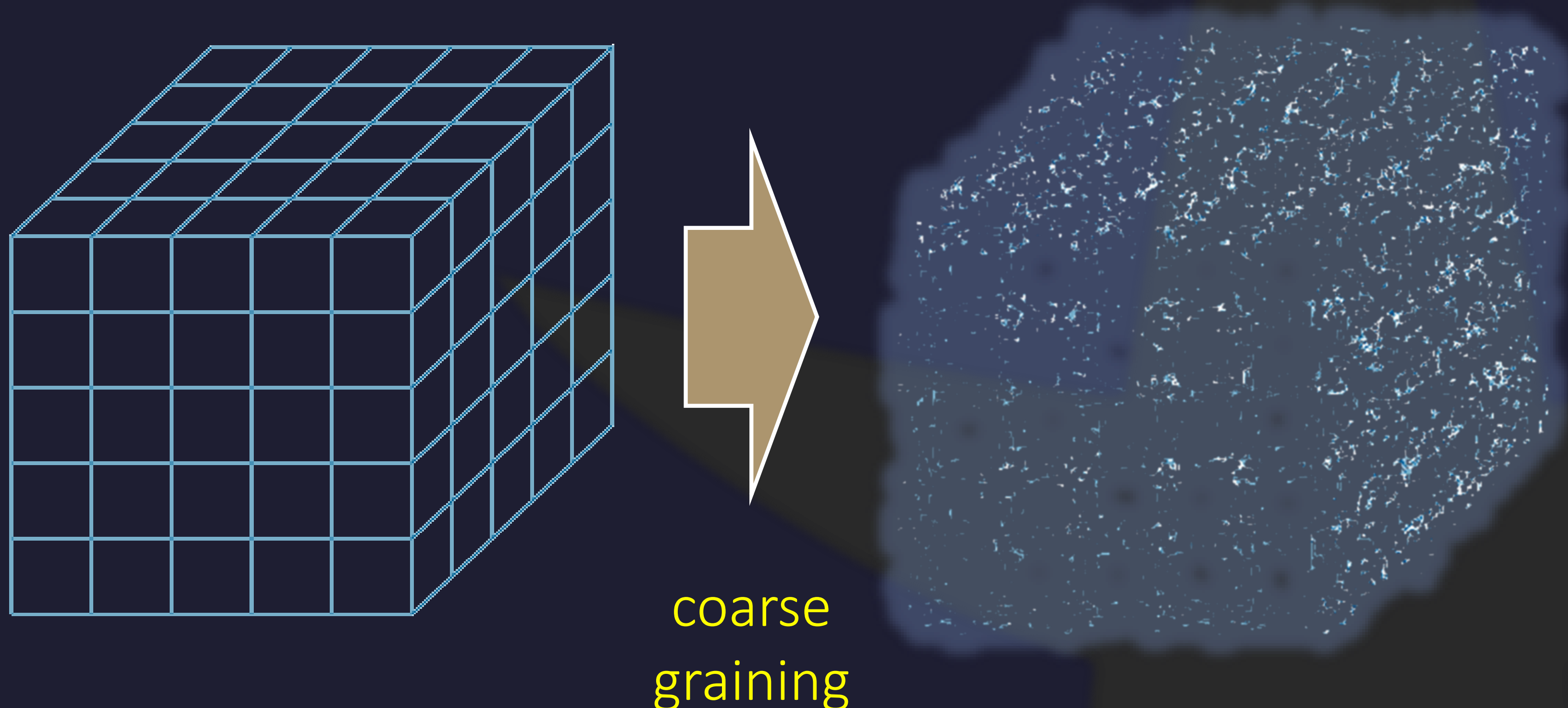
- transform gauge field like diffusion equation

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion length $d \sim \sqrt{8t}$

- This is NOT the standard cooling/smearing
- All composite operators at t>0 are UV finite Luescher,Weisz,2011

失小得大



- This is NOT the standard cooling/smearing

Small Flow Time Expansion of Operators and EMT

$$T_{\mu\nu}$$

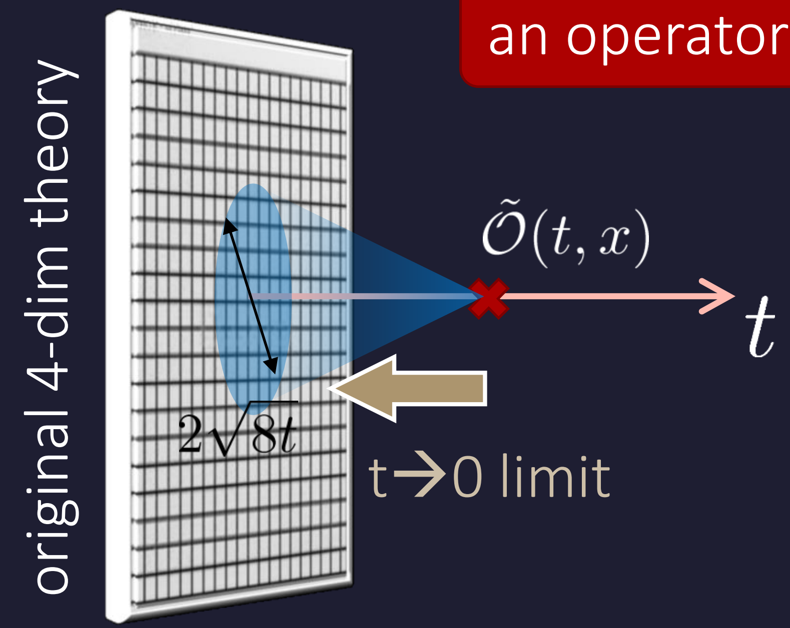
Small t Expansion

Luescher, Weisz, 2011

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at t>0

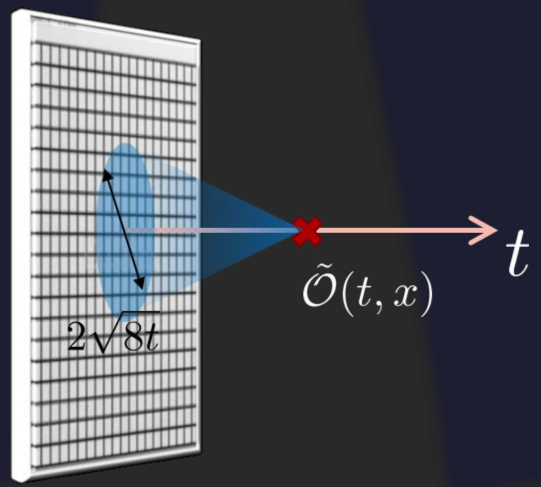
remormalized operators of original theory



Constructing EMT

Suzuki, 2013
DelDebbio,Patella,Rago,2013

$$\tilde{\mathcal{O}}(t,x) \underset{t\to 0}{\longrightarrow} \sum_i c_i(t)\mathcal{O}_i^R(x)$$



- **gauge-invariant dimension 4 operators**

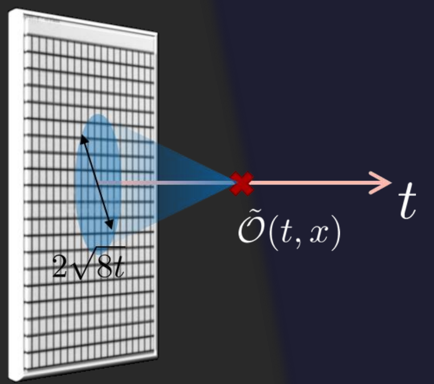
$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

Constructing EMT 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$



Suzuki coeffs.

$$\begin{cases} \alpha_U(t) = g^2\left[1 + 2b_0 s_1 g^2 + O(g^4)\right] \\ \alpha_E(t) = \frac{1}{2b_0}\left[1 + 2b_0 s_2 g^2 + O(g^4)\right] \end{cases}$$

$g = g(1/\sqrt{8t})$

$s_1 = 0.03296\ldots$

$s_2 = 0.19783\ldots$

Constructing EMT 2

Suzuki, 2013

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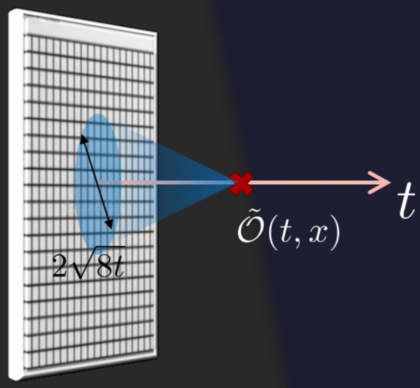

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$g = g(1/\sqrt{8t})$

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Remormalized EMT

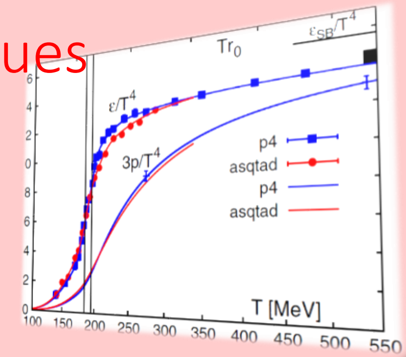
$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\text{subt.}}\right]$$

Numerical Analysis: Thermodynamics

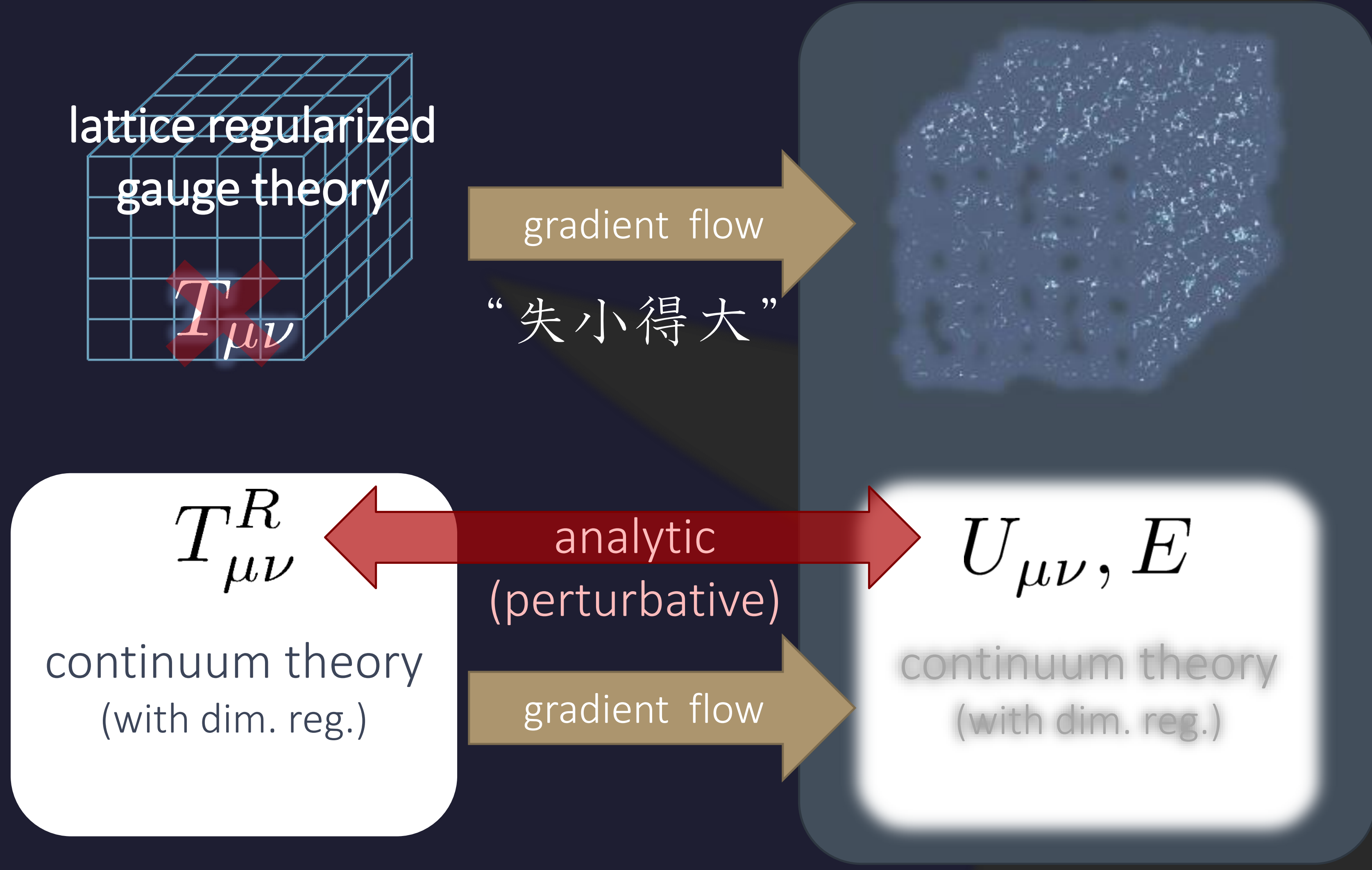
Thermodynamics

direct measurement of expectation values

$$\langle T_{00} \rangle, \langle T_{ii} \rangle$$



Gradient Flow Method



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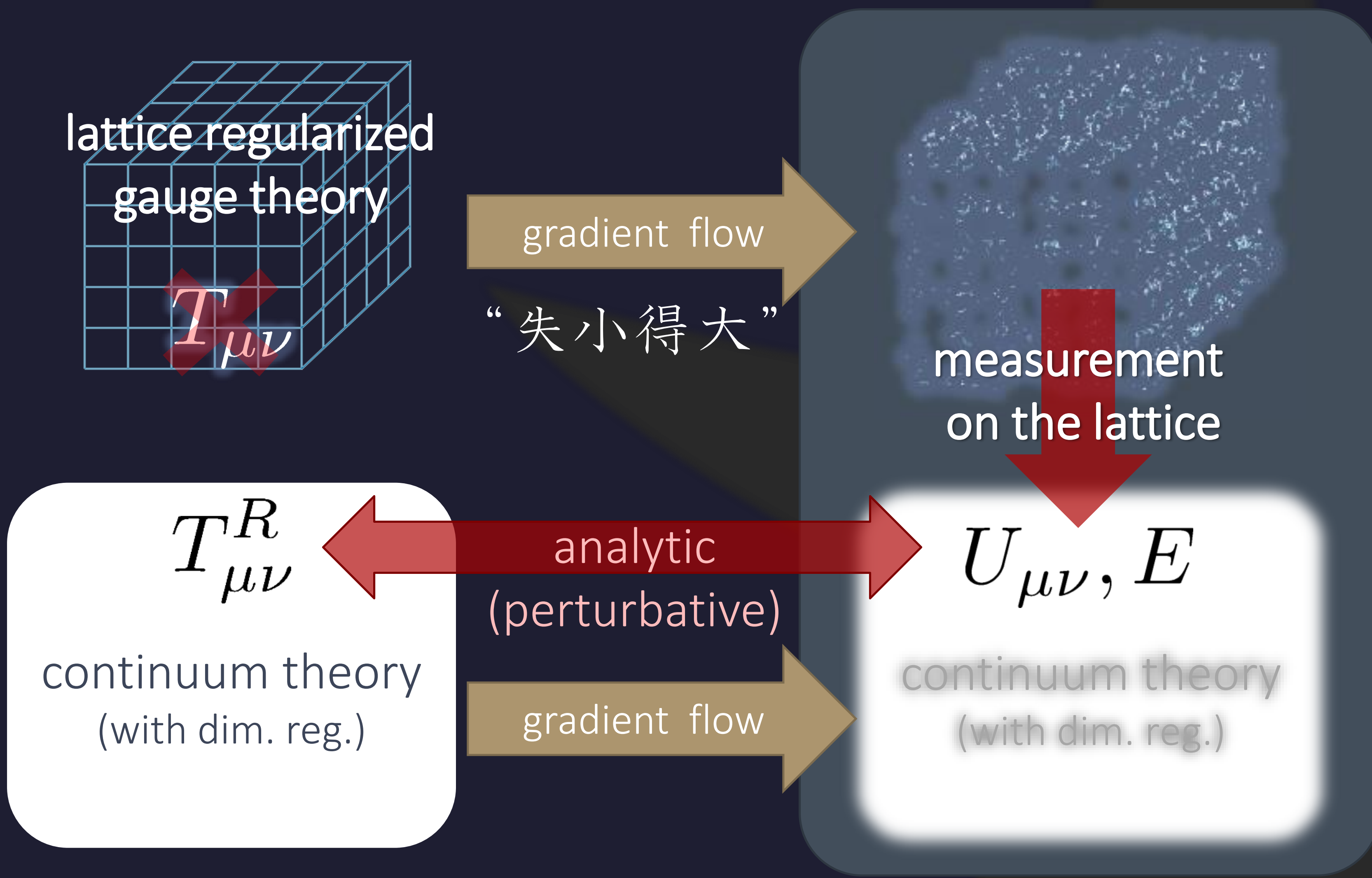
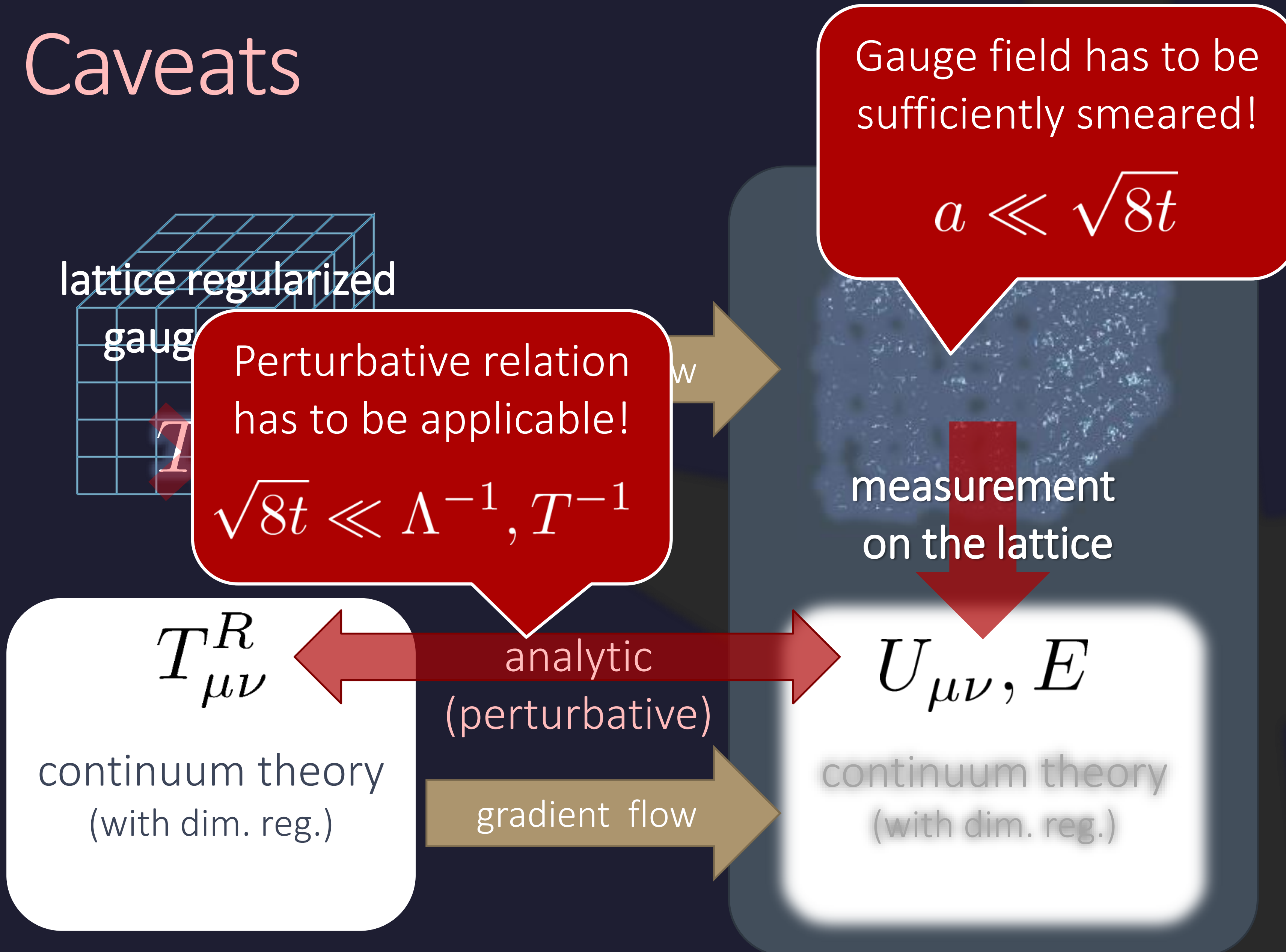
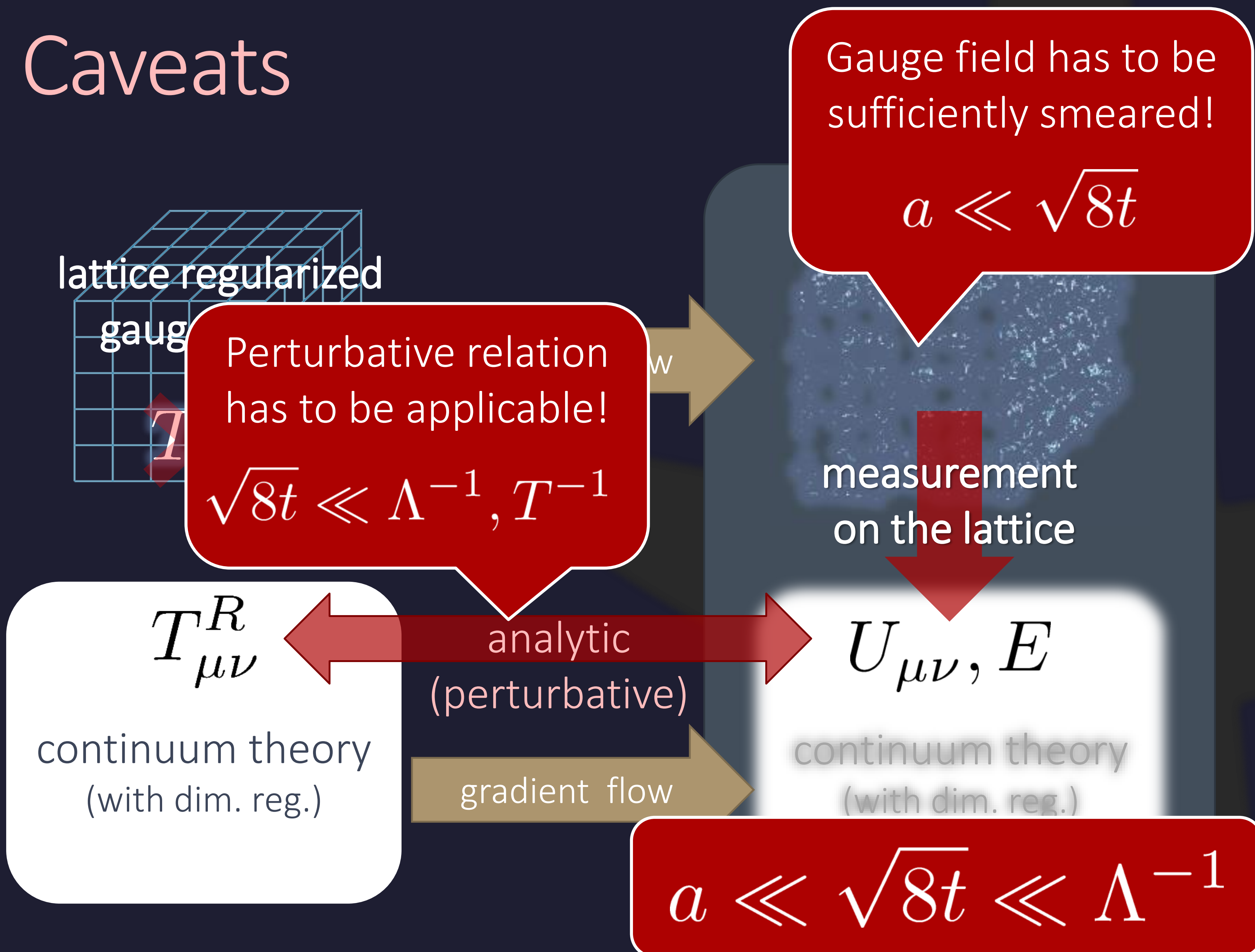


Caveats


Gauge field has to be sufficiently smeared!
$a \ll \sqrt{8t}$
lattice regularized
Perturbative relation has to be applicable!
$\sqrt{8t} \ll \Lambda^{-1}, T^{-1}$
measurement on the lattice
$T^R_{\mu\nu}$
continuum theory
(with dim. reg.)
analytic
(perturbative)
$U_{\mu\nu}, E$
continuum theory
(with dim. reg.)
gradient flow

Caveats



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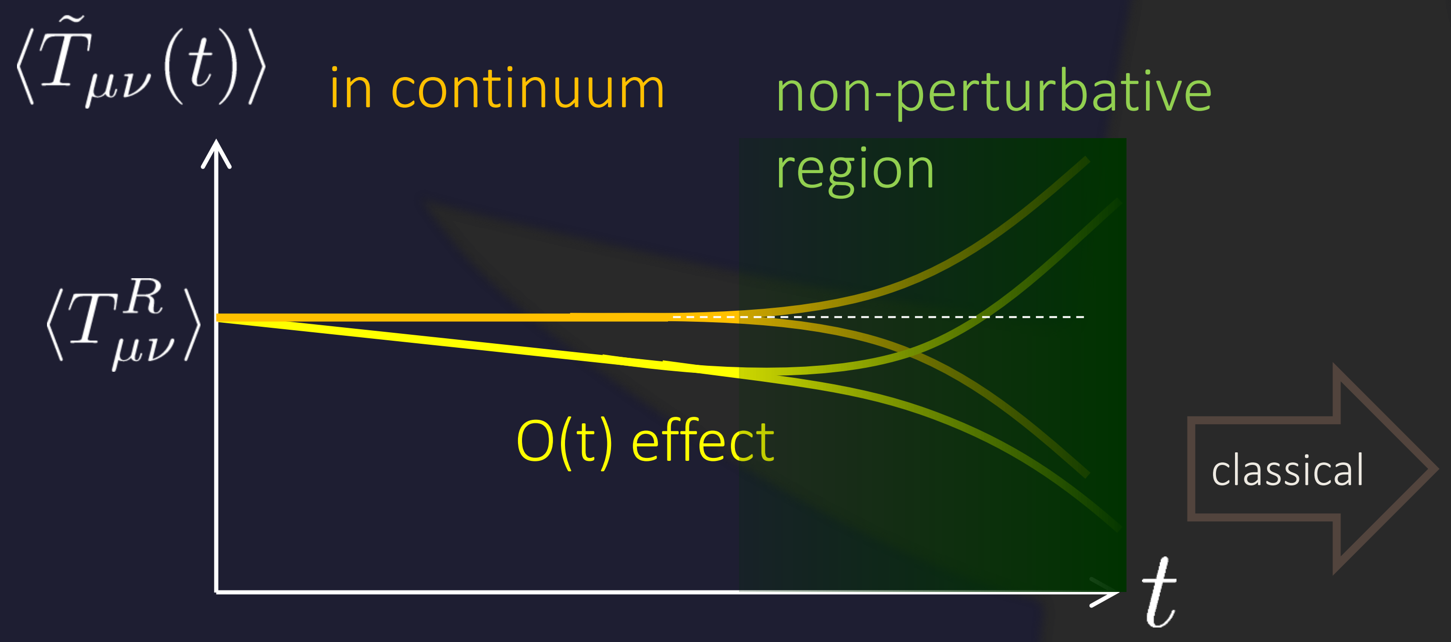
$a \ll \sqrt{8t} \ll \Lambda^{-1}$

$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\text{subt.}} \qquad T^R_{\mu\nu} = \lim_{t \to 0} \tilde{T}_{\mu\nu}(t)$$



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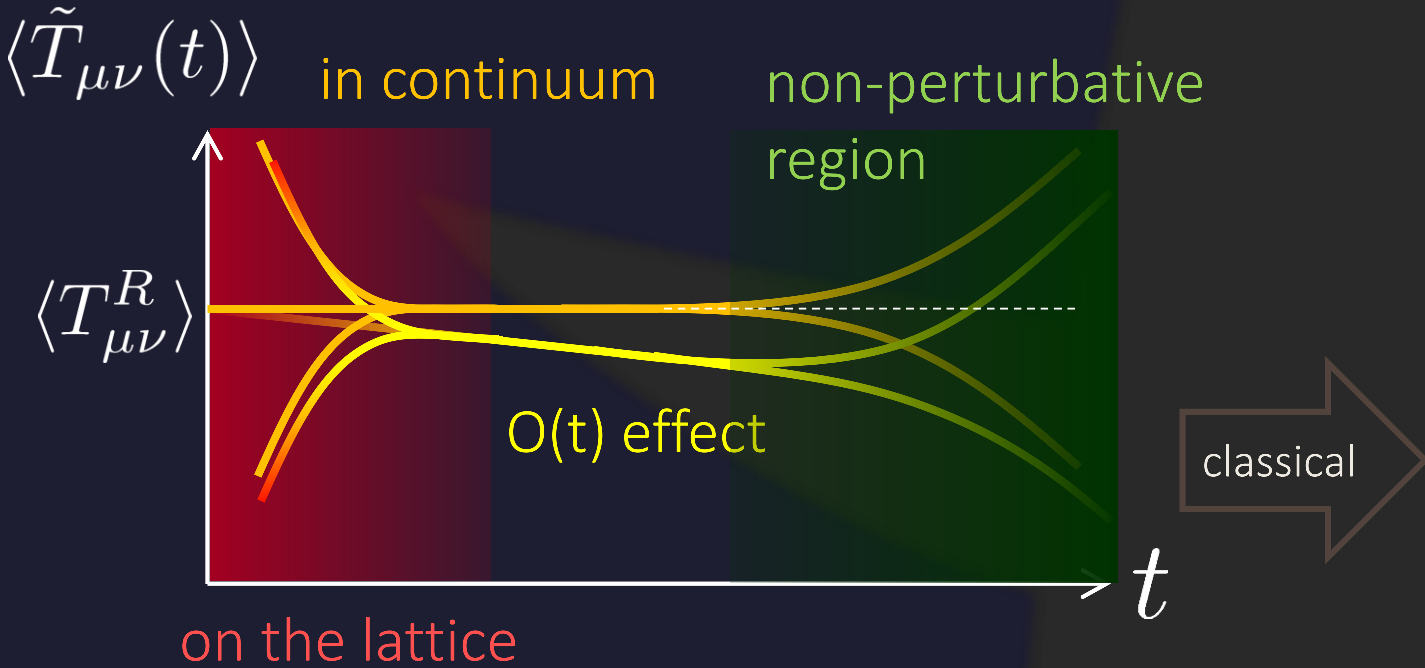
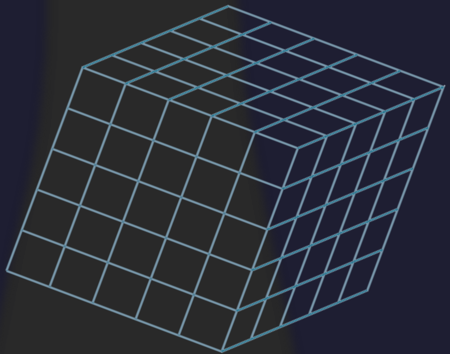


- $t \to 0$ limit with keeping $t \gg a^2$

Numerical Simulation



- SU(3) YM theory
- Wilson gauge action

twice finer lattice!

Simulation 1

(arXiv:1312.7492)

- lattice size: $32^3 \times N_t$
- Nt = 6, 8, 10
- β = 5.89 – 6.56
- ~300 configurations

using SX8 @ RCNP
SR16000 @ KEK

Simulation 2

(new, preliminary)

- lattice size: $64^3 \times N_t$
- Nt = 10, 12, 14, 16
- β = 6.4 – 7.4
- ~2000 configurations

using BlueGeneQ @ KEK
efficiency ~40%

ε-3p at T=1.65T$_c$

$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\mathrm{subt.}}$$

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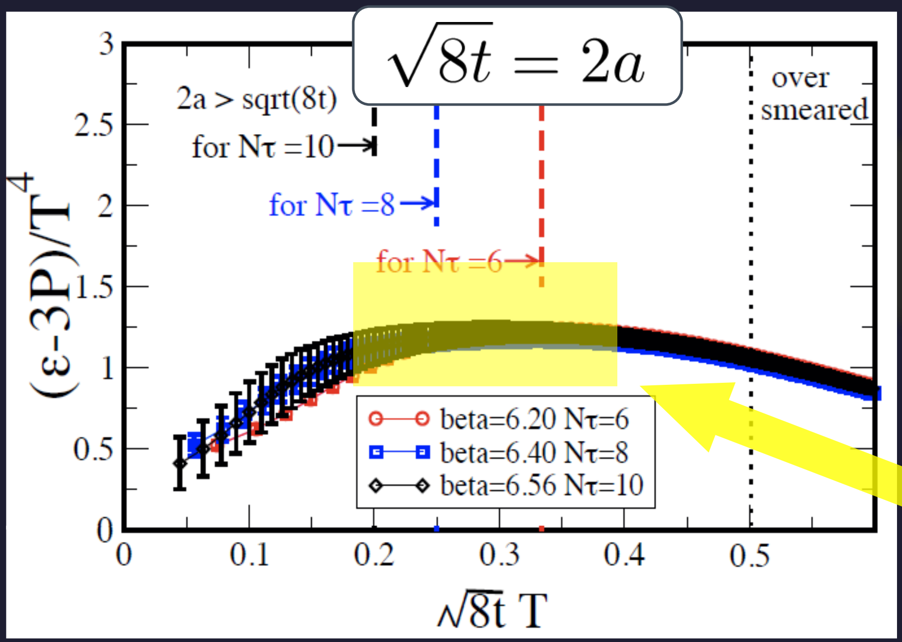

Emergent plateau!

$$2a \lesssim \sqrt{8t} \lesssim 0.4T^{-1}$$

Nt=6,8,10
~300 confs.

the range of t where the EMT formula is successfully used!

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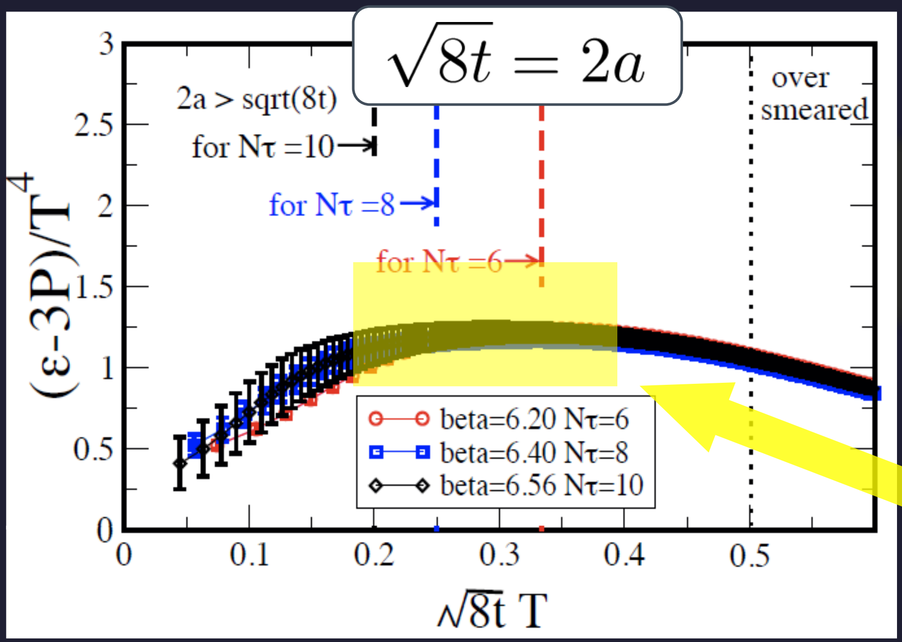

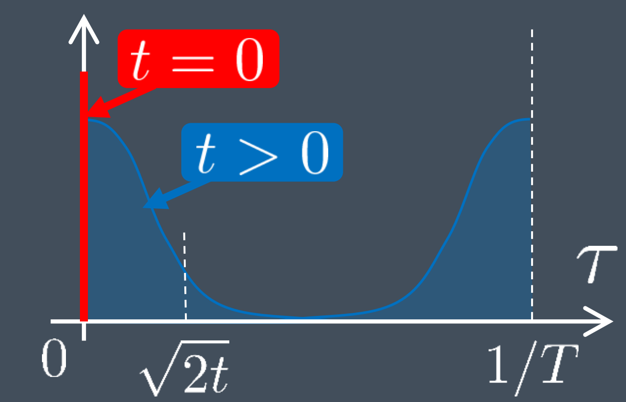

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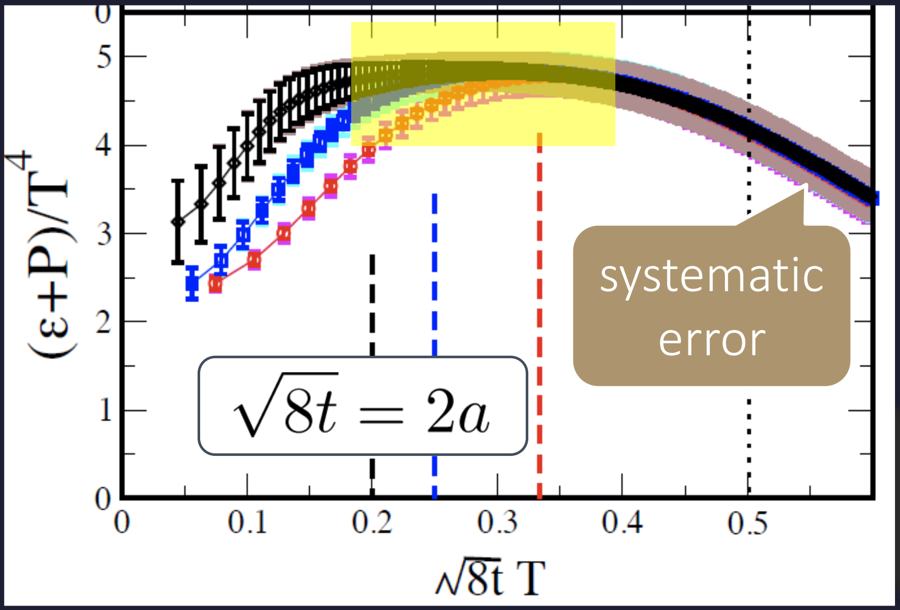
Entropy Density at T=1.65Tc



Emergent plateau!

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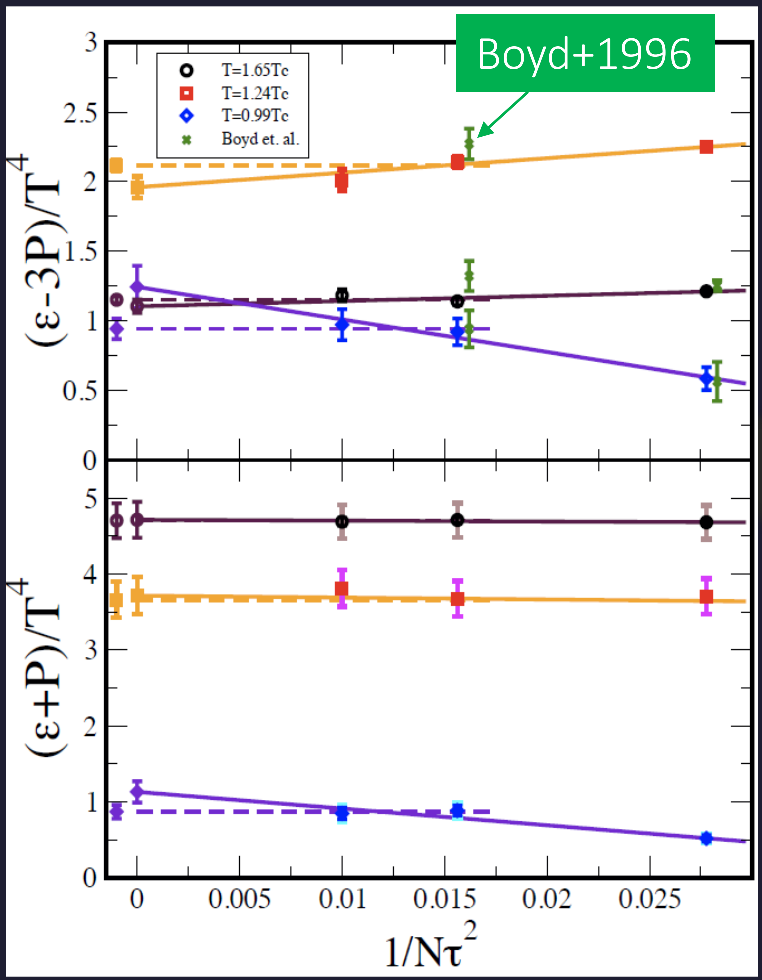
Nt=6,8,10
~300 confs.

Direct measurement of e+p on a given T!

NO integral / **NO** vacuum subtraction

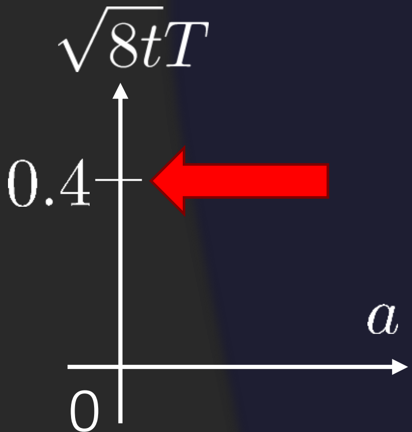
Continuum Limit



32^3xNt

Nt = 6, 8, 10

T/Tc=0.99, 1.24, 1.65



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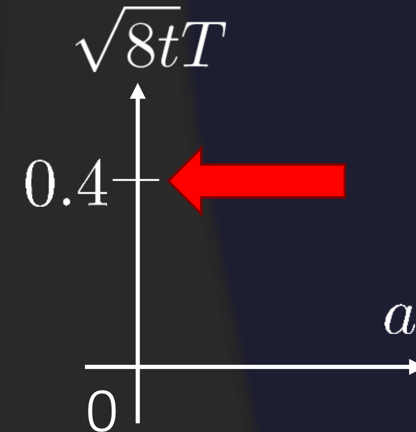

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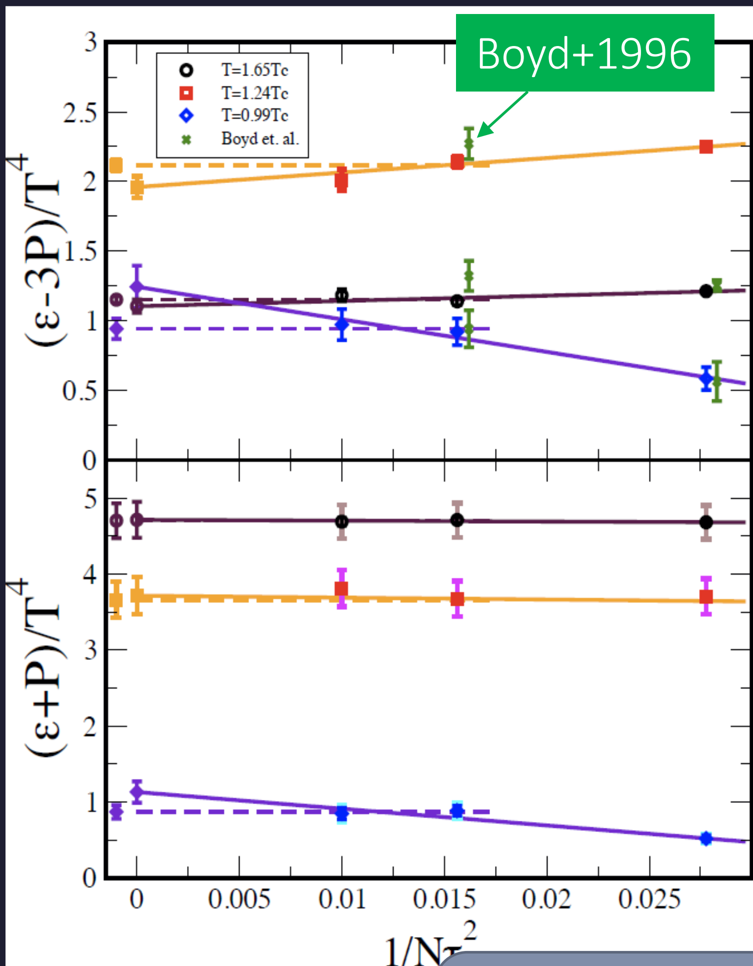
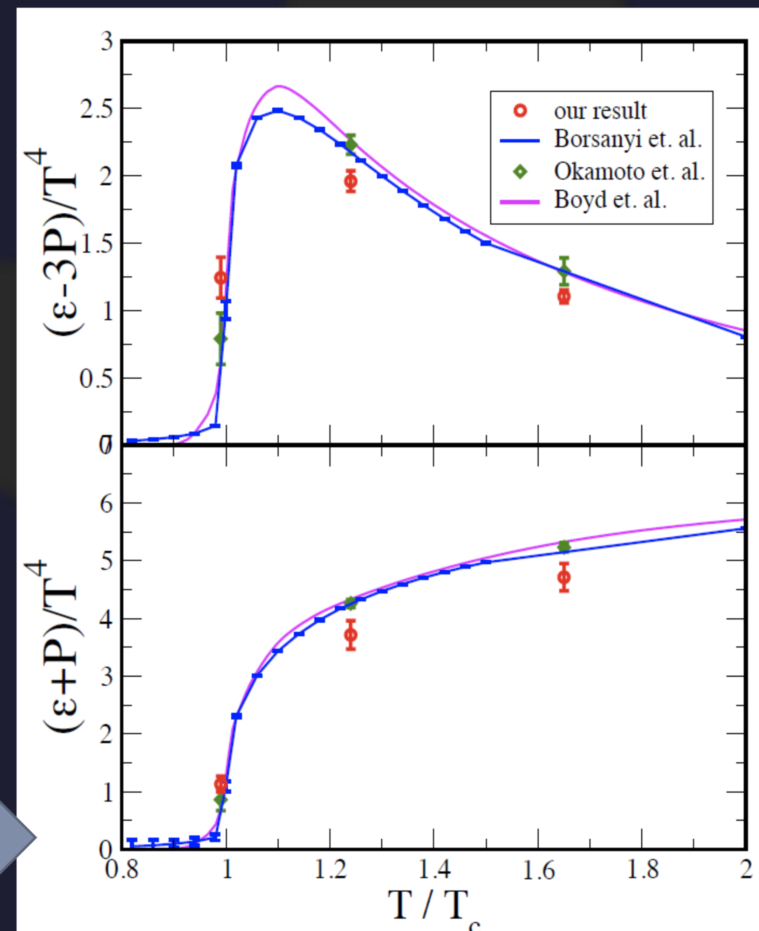


Comparison with previous studies



Numerical Simulation

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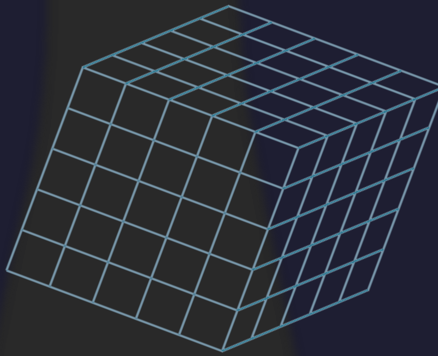

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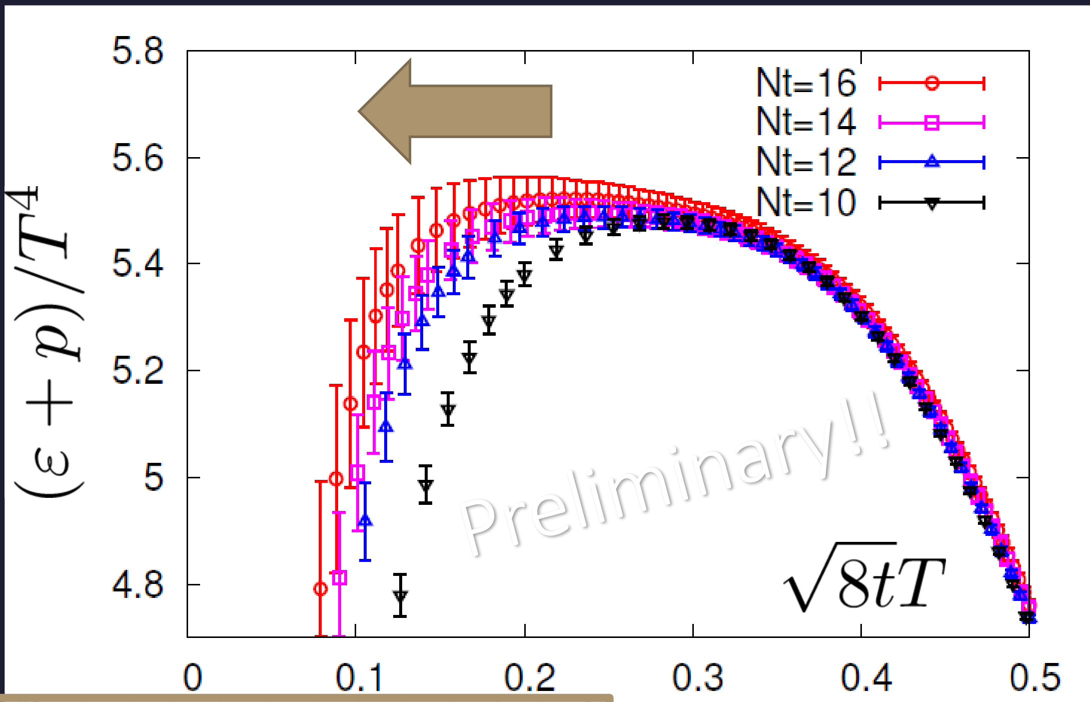
efficiency ~40%

Entropy Density on Finer Lattices

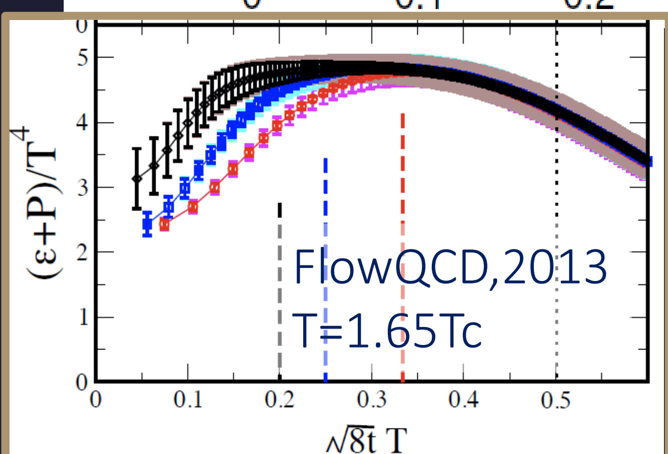


T = 2.31Tc

64^3xNt

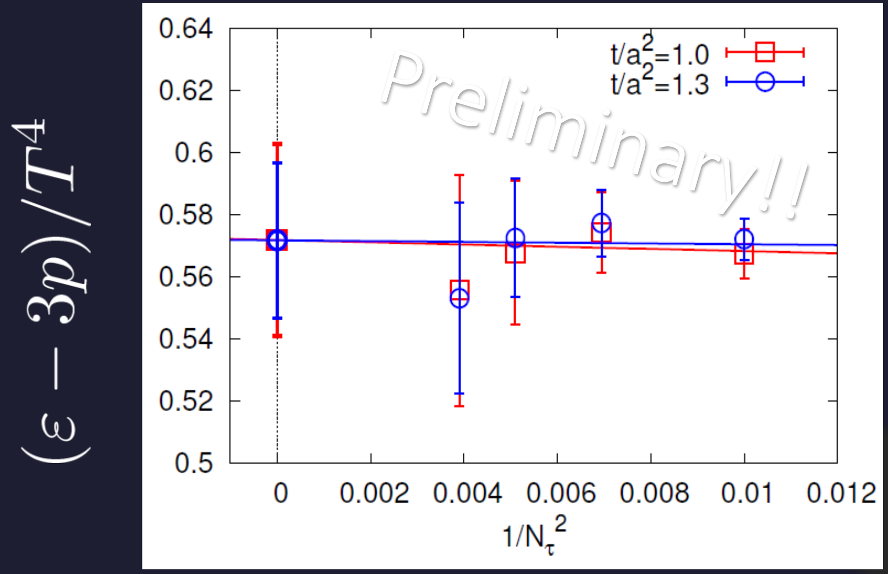
Nt = 10, 12, 14, 16

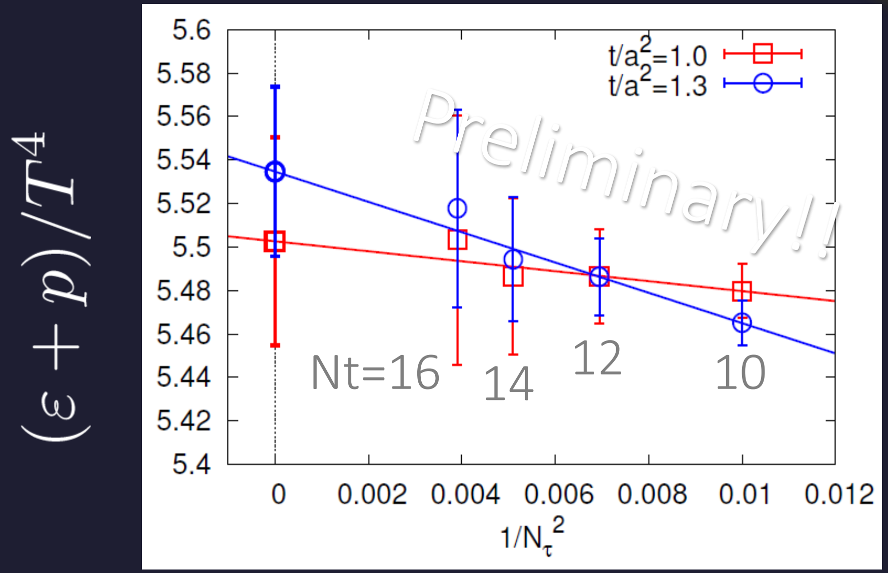
2000 confs.



- The wider plateau on the finer lattices
- Plateau may have a nonzero slope

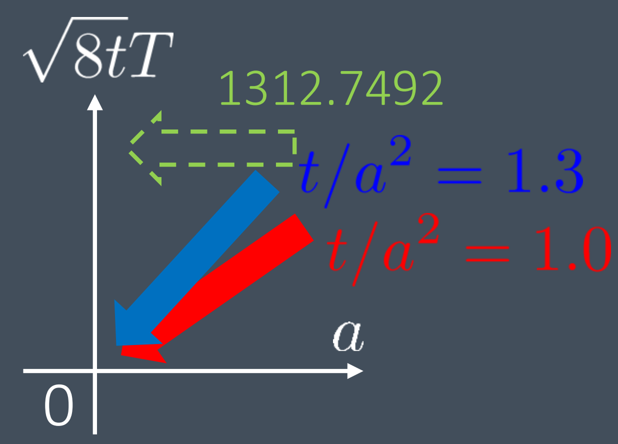
Continuum Extrapolation





- T=2.31Tc
- 2000 confs
- Nt = 10 ~ 16

a→0 limit with fixed t/a^2



Continuum extrapolation is stable

Summary

$$T^R_{\mu\nu}(x)$$

Summary

EMT formula from gradient flow

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\mathrm{subt.}}\right]$$

This formula can successfully define and calculate the EMT on the lattice. It’s direct and intuitive

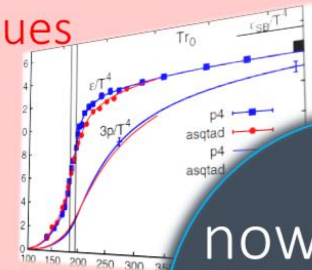
Gradient flow provides us novel methods of various analyses on the lattice avoiding 因小失大 problem

Many Future Studies!!

Thermodynamics

direct measurement of expectation values

$\langle T_{00} \rangle, \langle T_{ii} \rangle$



Fluctuations and Correlations

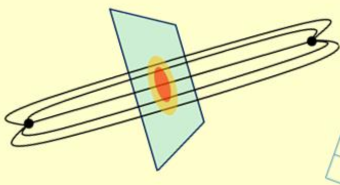
viscosity, specific heat, ...

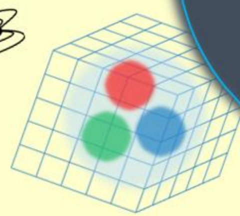
$c_V \sim \langle \delta T_{00}^2 \rangle$

$\eta = \langle T_{12} ; T_{12} \rangle$

now we have

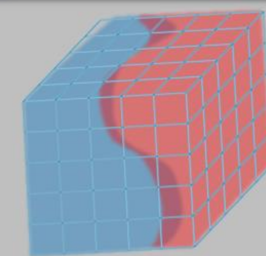
$T_{\mu\nu}$





- confinement string
- EM distribution in hadrons

Hadron Structure



- vacuum configuration
- mixed state on 1st transition

Vacuum Structure

Other observables

full QCD Makino,Suzuki,2014

non-pert. improvement Patella 7E(Thu)

O(a) improvement

Nogradi, 7E(Thu); Sint, 7E(Thu)

Monahan, 7E(Thu)

and etc.

Numerical Analysis: EMT Correlators

Fluctuations and Correlations

viscosity, specific heat, ...

$$\eta = \int_0^\infty dt \langle T_{12}; T_{12} \rangle$$

$$c_V \sim \langle \delta T_{00}^2 \rangle$$

EMT Correlator

- Kubo Formula: T_{12} correlator ←→ shear viscosity

$$\eta = \int_0^\infty dt \int_0^{1/T} d\tau \int d^3x \langle T_{12}(x, -i\tau) T_{12}(0, t) \rangle$$

  - Hydrodynamics describes long range behavior of $T_{\mu\nu}$

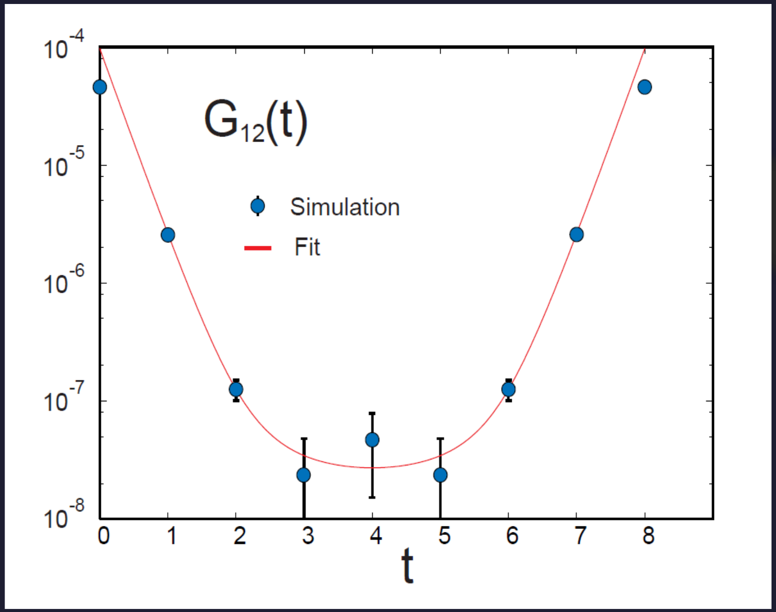
- Energy fluctuation ←→ specific heat

$$c_V = \frac{\langle \delta E^2 \rangle}{V T^2}$$

EMT Correlator : Noisy...

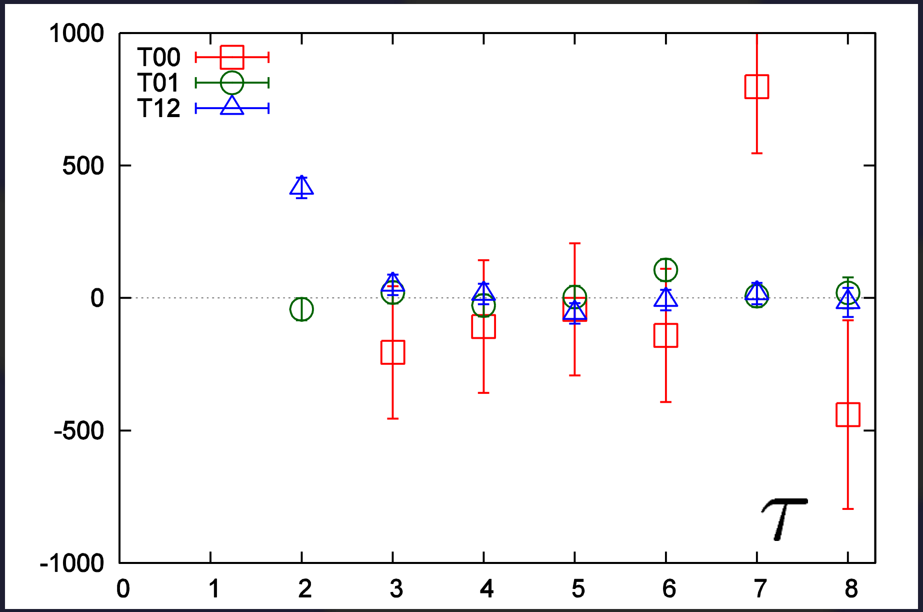
With naïve EMT operators

$\langle T_{12}(\tau)T_{12}(0)\rangle$



Nakamura, Sakai, PRL,2005
$N_t=8$
improved action
$\sim10^6$ configurations

$\langle T_{\mu\nu}(\tau)T_{\mu\nu}(0)\rangle$



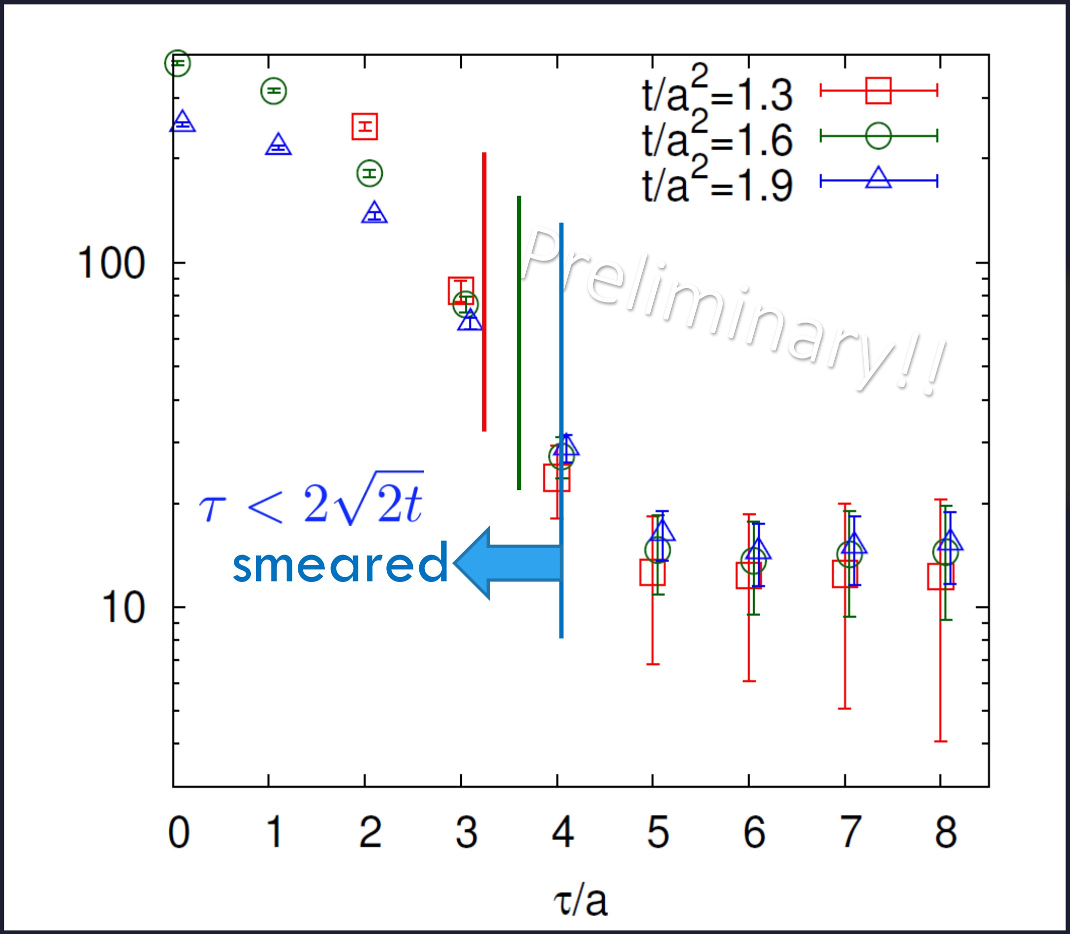
Nt=16
standard action
5×10^4 configurations

... no signal

Energy Correlation Function

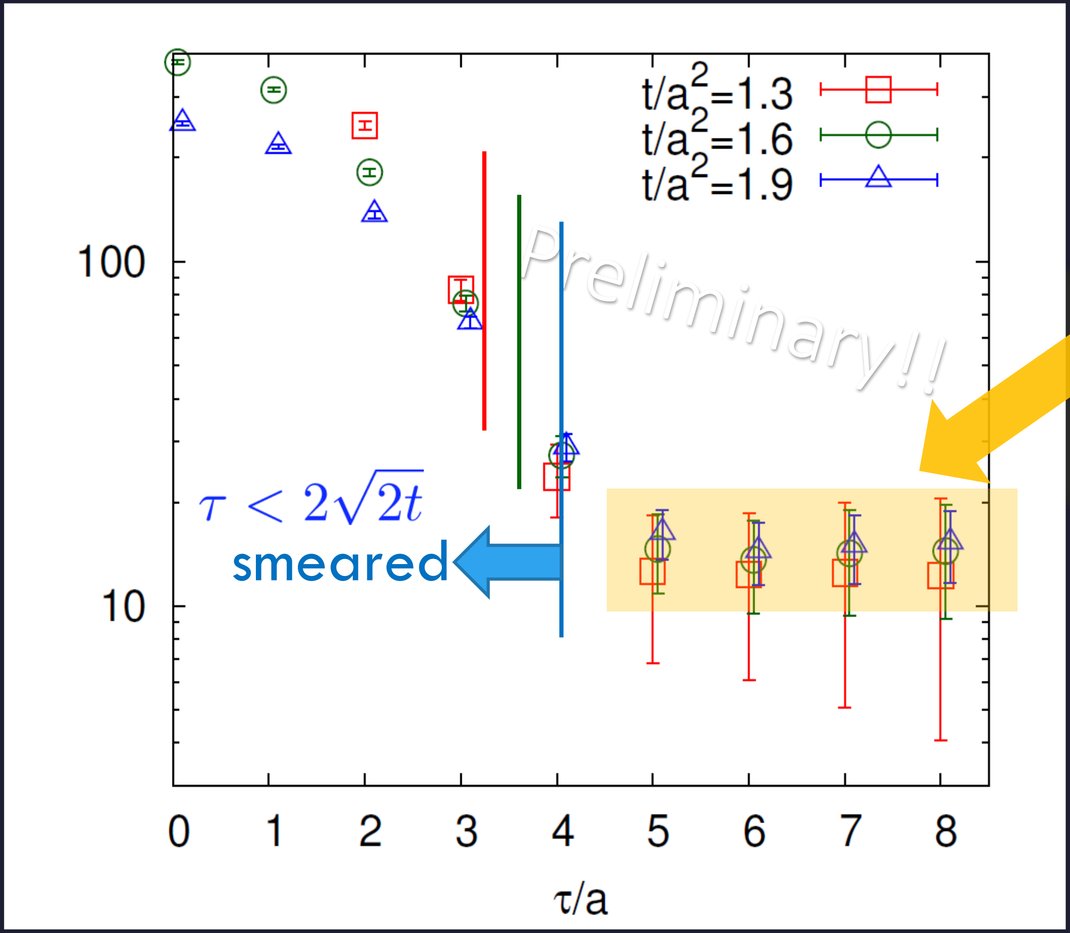
$$\langle T_{00}(\tau) T_{00}(0) \rangle / T^5$$

T=2.31Tc
b=7.2, Nt=16
2000 confs
p=0 correlator



Energy Correlation Function

$$\langle T_{00}(\tau)T_{00}(0)\rangle/T^5$$

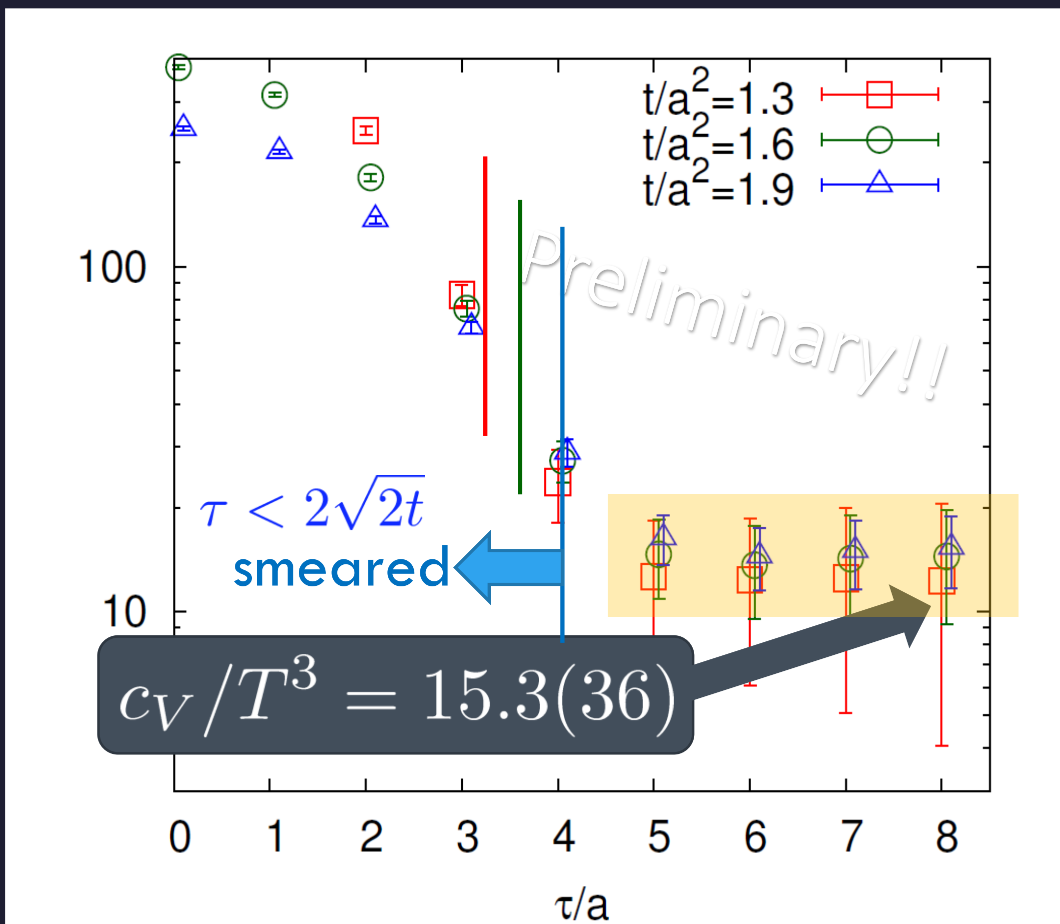


T=2.31Tc
b=7.2, Nt=16
2000 confs
p=0 correlator

- τ independent const.
  → energy conservation

Energy Correlation Function

$$\langle T_{00}(\tau)T_{00}(0)\rangle/T^5$$



T=2.31Tc

b=7.2, Nt=16

2000 confs

p=0 correlator

- specific heat

$$c_V = \frac{\langle \delta E^2 \rangle}{VT^2}$$

→ Novel approach to measure specific heat!

Gavai, Gupta, Mukherjee, 2005

$$c_V/T^3 = 15(1) \quad T/T_c = 2$$

$$= 18(2) \quad T/T_c = 3$$

differential method / cont lim.