Gradient Flowと エネルギー運動量テンソル

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for FlowQCD Collaboration
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FlowQCD, PR**D90**,011501R (2014)

「熱場の量子論とその応用」・理研・2014年9月4日

$$T_{\mu\nu}$$

Poincare symmetry

$$T_{\mu\nu}$$

$$\begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

energy

momentum

stress

pressure

Einstein Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

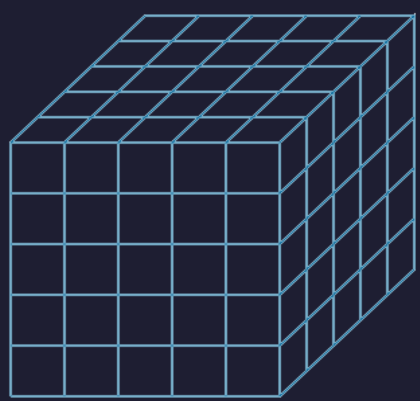
Hydrodynamic Eq.

$$\partial_\mu T_{\mu\nu} = 0$$

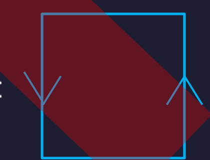
$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial

because of the explicit breaking of Lorentz symmetry



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

② Its measurement is extremely noisy

due to high dimensionality and etc.

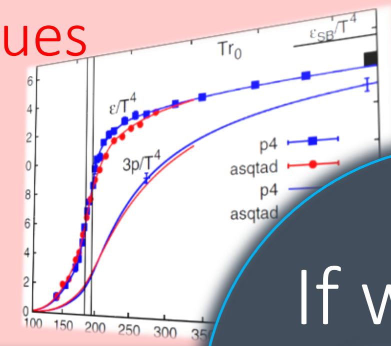
If we have

$$T_{\mu\nu}$$

Thermodynamics

direct measurement of expectation values

$\langle T_{00} \rangle, \langle T_{ii} \rangle$



Fluctuations and Correlations

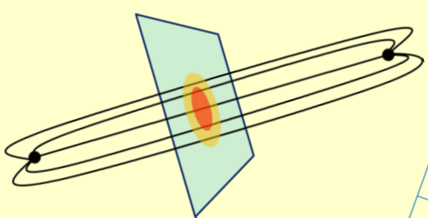
viscosity, specific heat, ...

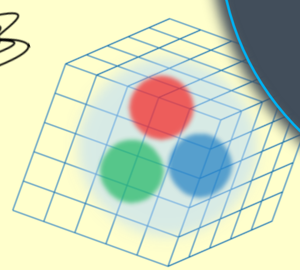
$\eta = \int_0^\infty dt \langle T_{12}; T_{12} \rangle$

$c_V \sim \langle \delta T_{00}^2 \rangle$

If we have

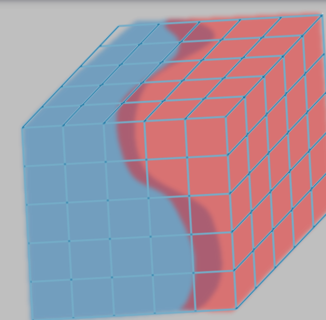
$T_{\mu\nu}$





- confinement string
- EM distribution in hadrons

Hadron Structure



- vacuum configuration
- mixed state on 1st transition

Vacuum Structure

Gradient Flow

YM Gradient Flow

Luescher, 2010

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

t: “flow time”
dim:[length^2]

$$A_\mu(0,x) = A_\mu(x)$$

YM Gradient Flow

Luescher, 2010

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

$$A_\mu(0,x) = A_\mu(x)$$

t: “flow time”
dim:[length^2]

- transform gauge field like diffusion equation

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion length $d \sim \sqrt{8t}$

- This is NOT the standard cooling/smearing
- All composite operators at t>0 are UV finite Luescher,Weisz,2011

Applications of Gradient Flow

① scale setting

② running coupling

③ topology

④ operator construction

⑤ autocorrelation, etc.

Small Flow Time Expansion of Operators and EMT

Operator Relation

Luescher, Weisz, 2011

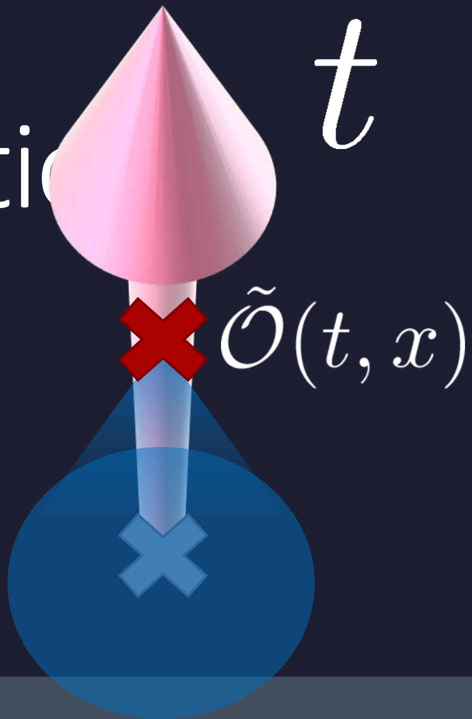
$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t \to 0]{} \sum_i c_i(t) \mathcal{O}_i^R(x)$$

an operator at t>0

remormalized operators of original theory

Operator Relati

Luescher, Weisz, 2011



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Constructing EMT

Suzuki, 2013

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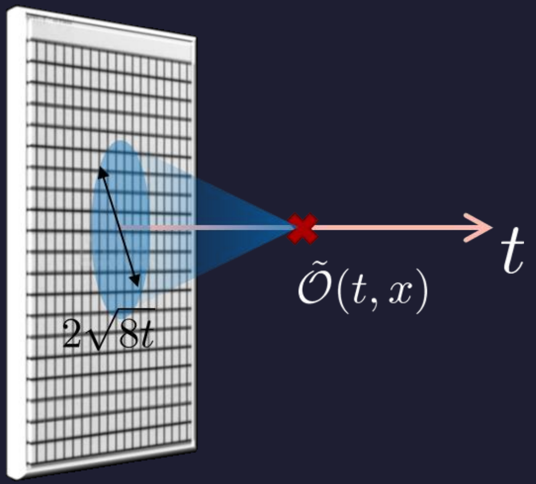


- **gauge-invariant dimension 4 operators**

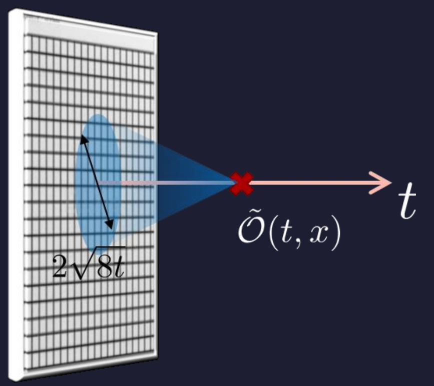
$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

Constructing EMT 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$



Suzuki coeffs.

$$\begin{cases} \alpha_U(t) = g^2\left[1 + 2b_0 s_1 g^2 + O(g^4)\right] \\ \alpha_E(t) = \dfrac{1}{2b_0}\left[1 + 2b_0 s_2 g^2 + O(g^4)\right] \end{cases}$$

$g = g(1/\sqrt{8t})$

$s_1 = 0.03296\ldots$

$s_2 = 0.19783\ldots$

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Suzuki, 2013

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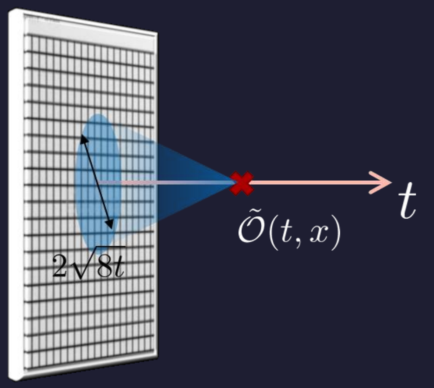

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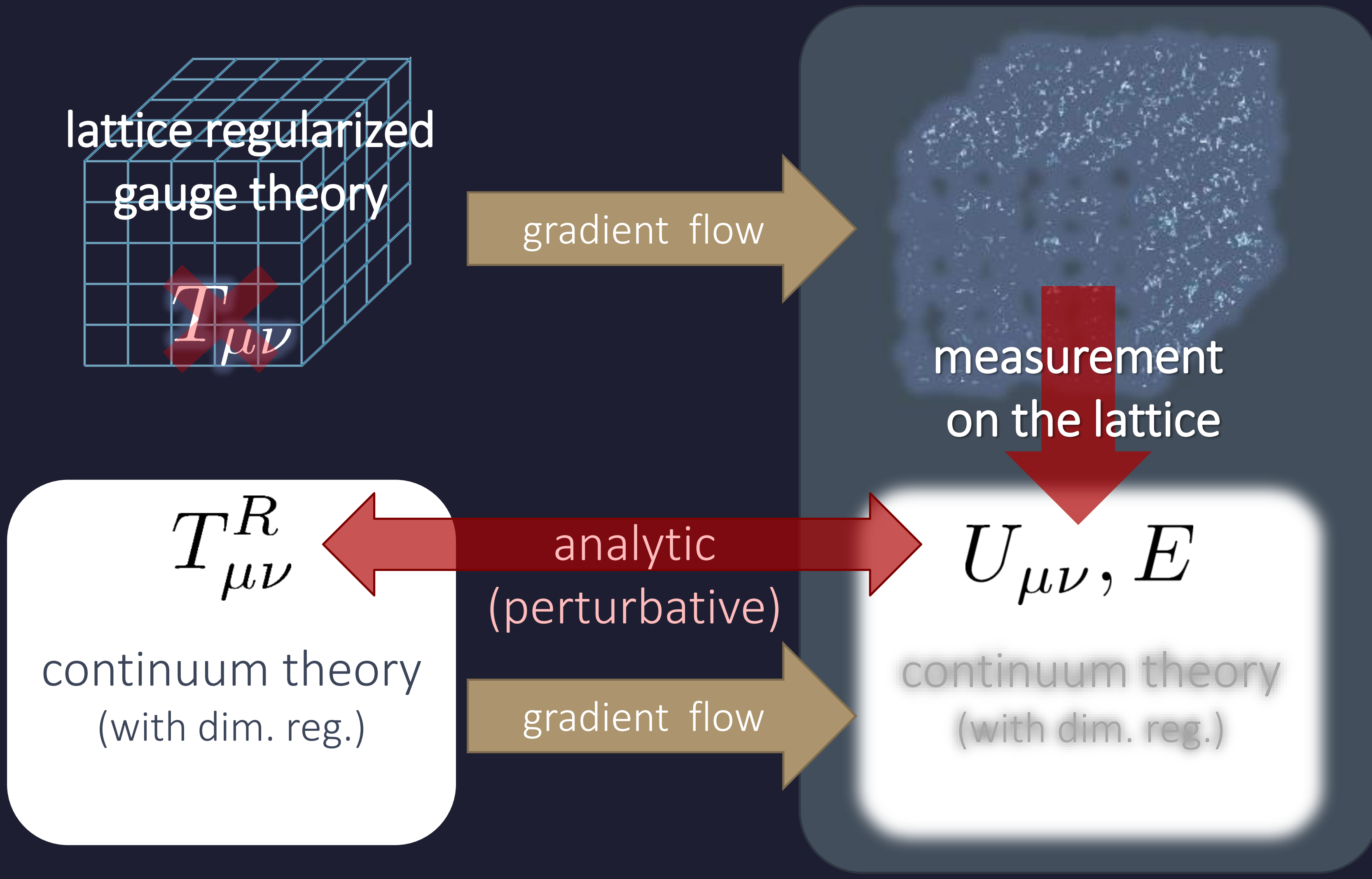
Remormalized EMT

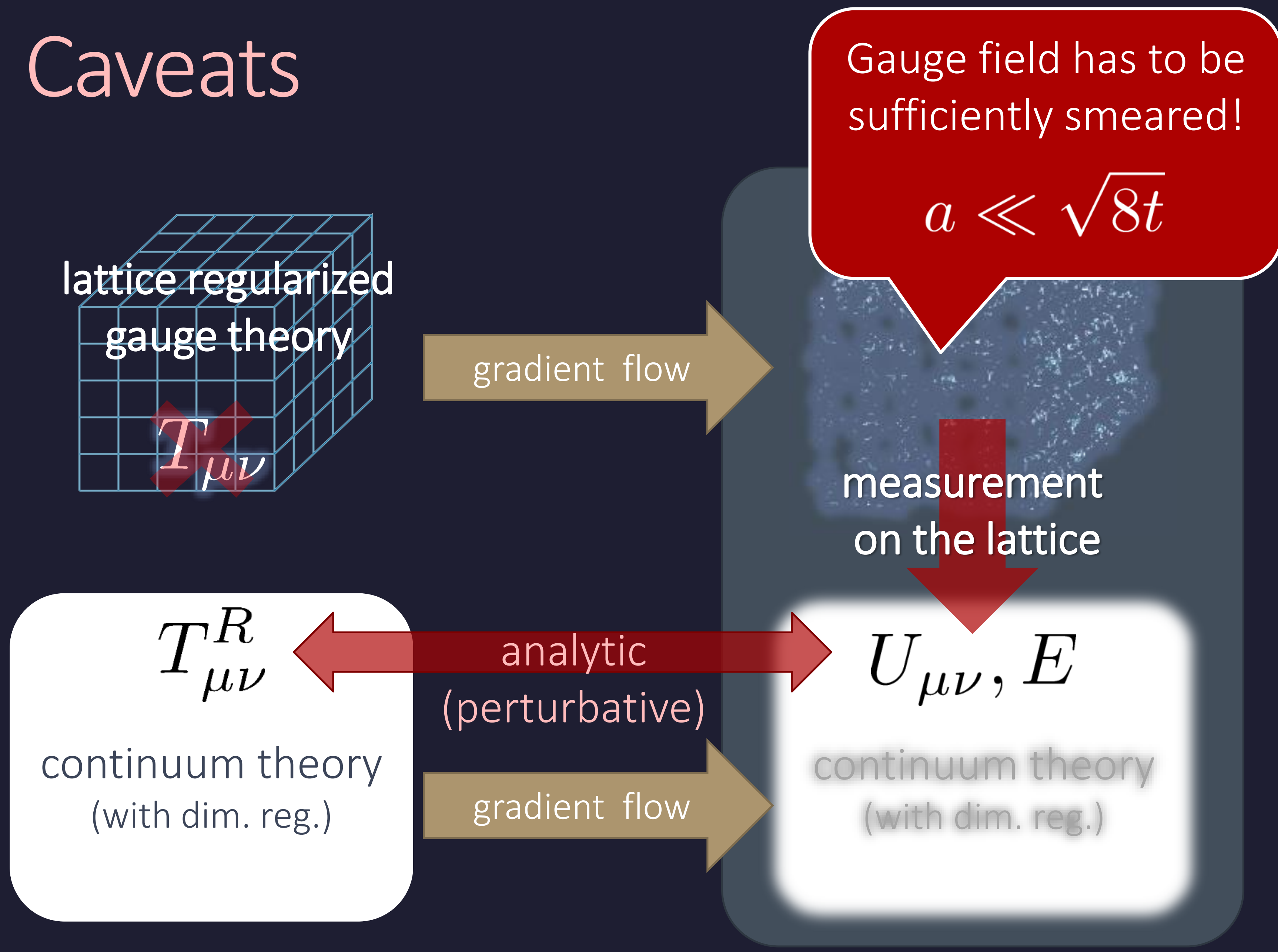
$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\text{subt.}}\right]$$

Numerical Analysis on the Lattice

Gradient Flow Method

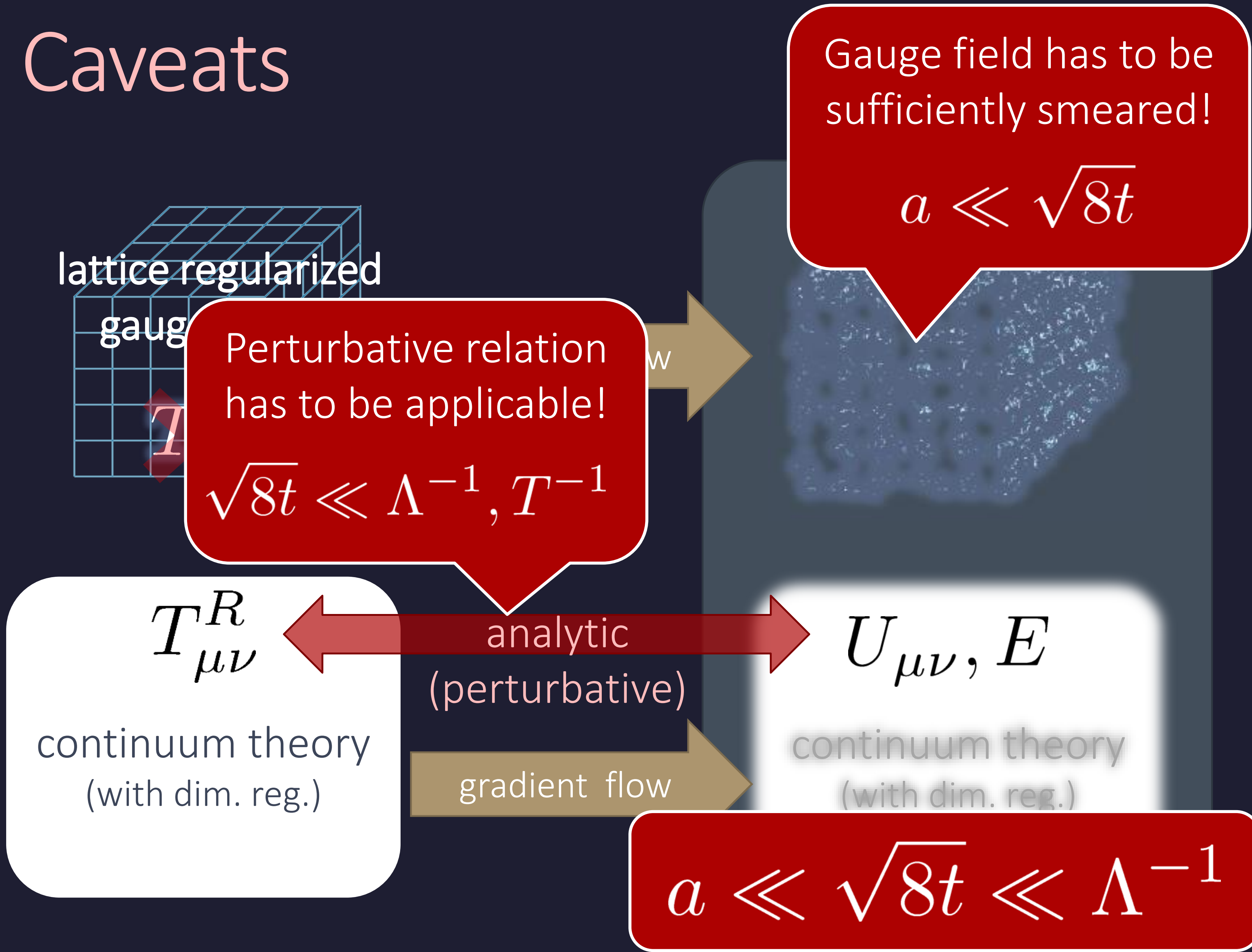


Caveats



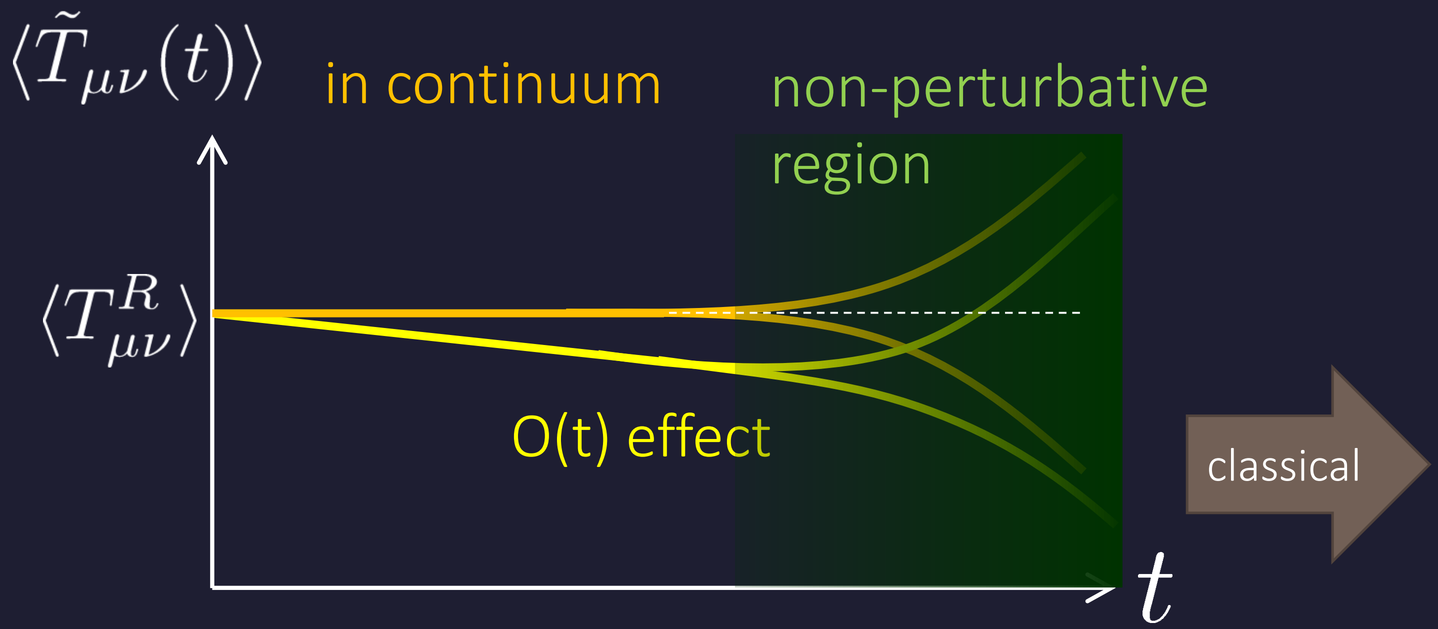
Gauge field has to be sufficiently smeared!
$a \ll \sqrt{8t}$
lattice regularized gauge theory
$T_{\mu\nu}$
gradient flow
measurement on the lattice
$T^R_{\mu\nu}$
analytic (perturbative)
$U_{\mu\nu}, E$
continuum theory (with dim. reg.)
gradient flow
continuum theory (with dim. reg.)

Caveats



Gauge field has to be sufficiently smeared!
$a \ll \sqrt{8t}$
lattice regularized
Perturbative relation has to be applicable!
$\sqrt{8t} \ll \Lambda^{-1}, T^{-1}$
$T^R_{\mu\nu}$
continuum theory
(with dim. reg.)
analytic
(perturbative)
$U_{\mu\nu}, E$
continuum theory
(with dim. reg.)
gradient flow
$a \ll \sqrt{8t} \ll \Lambda^{-1}$

$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\rm subt.} \qquad T^R_{\mu\nu} = \lim_{t\to 0} \tilde{T}_{\mu\nu}(t)$$



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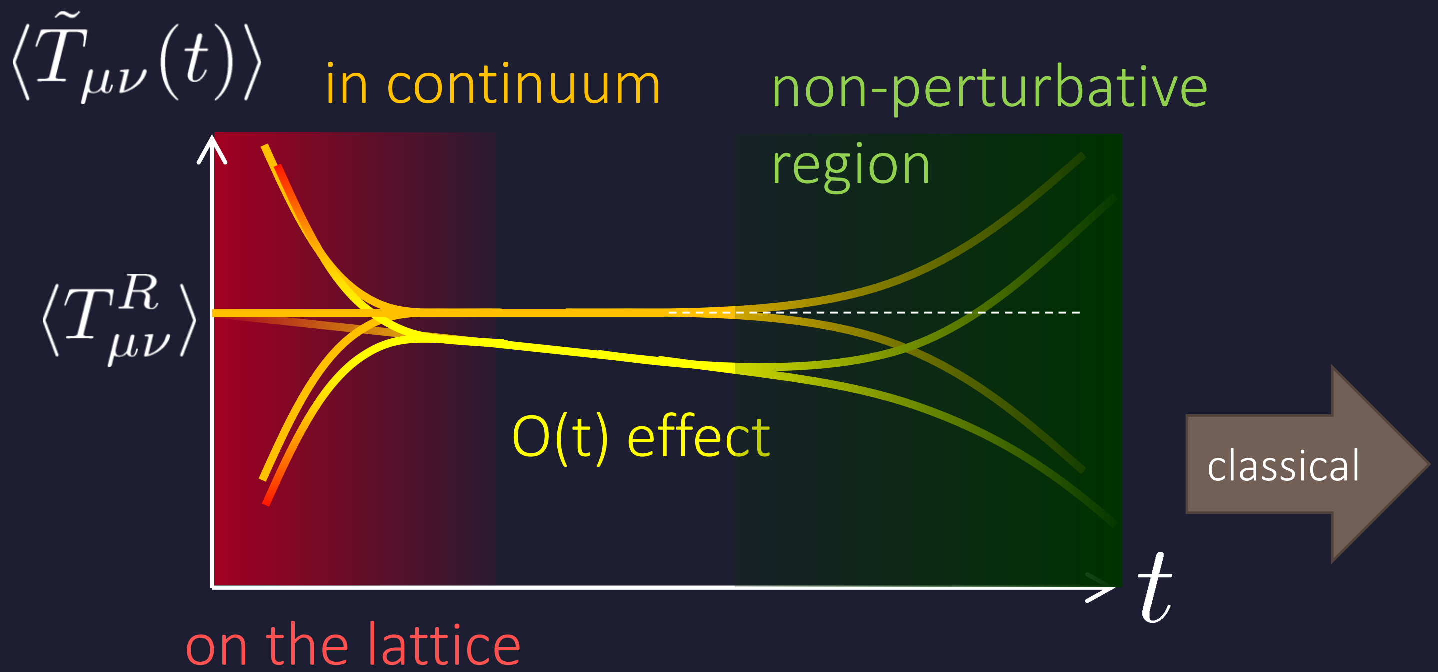
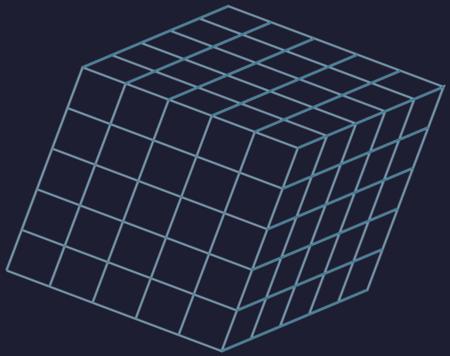


- t→0 limit with keeping t>>a^2

Numerical Simulation

- SU(3) YM theory
- Wilson gauge action



twice finer lattice!

Simulation 1

(arXiv:1312.7492)

- lattice size: $32^3 \mathrm{x} N_t$
- Nt = 6, 8, 10
- β = 5.89 – 6.56
- ~300 configurations

using SX8 @ RCNP
SR16000 @ KEK

Simulation 2

(new, preliminary)

- lattice size: $64^3 \mathrm{x} N_t$
- Nt = 10, 12, 14, 16
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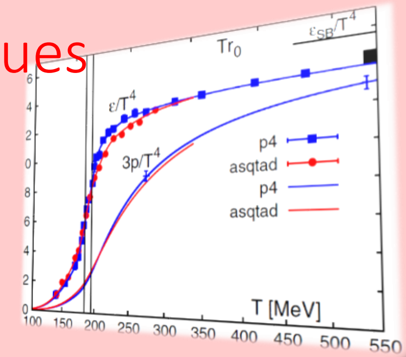
using BlueGeneQ @ KEK
efficiency ~40%

Thermodynamics

Thermodynamics

direct measurement of expectation values

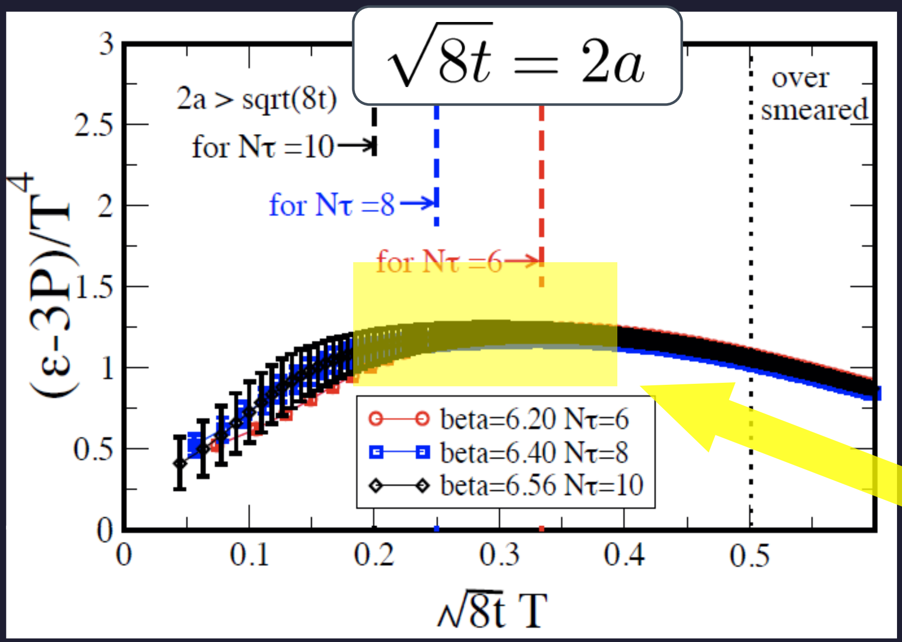
$$\langle T_{00} \rangle, \langle T_{ii} \rangle$$



“Trace Anomaly” at T=1.65T$_c$

$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\text{subt.}}$$

$$T^R_{\mu\nu} = \lim_{t\to 0} \tilde{T}_{\mu\nu}(t)$$



Emergent plateau!

$$2a \lesssim \sqrt{8t} \lesssim 0.4T^{-1}$$

Nt=6,8,10
~300 confs.

the range of t where the EMT formula is successfully used!

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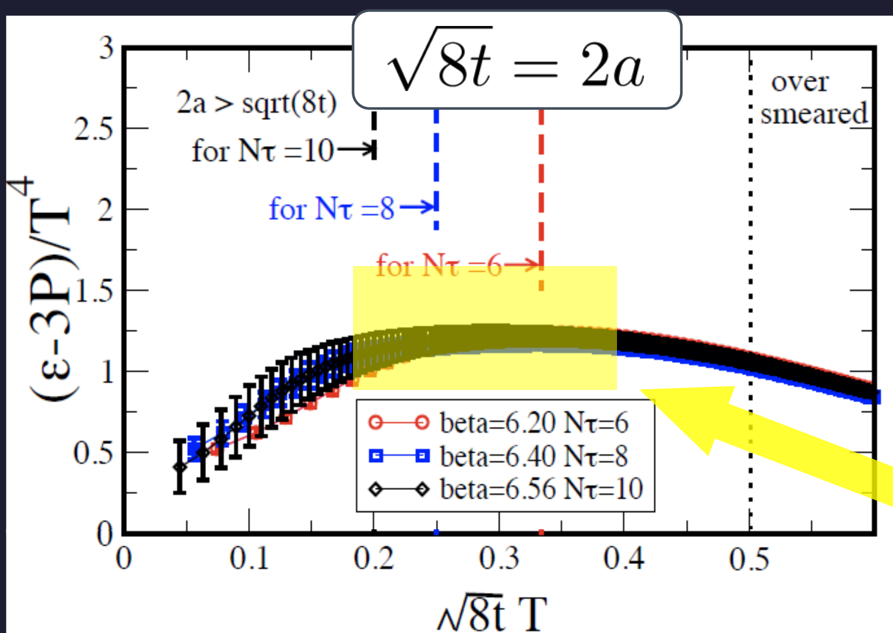

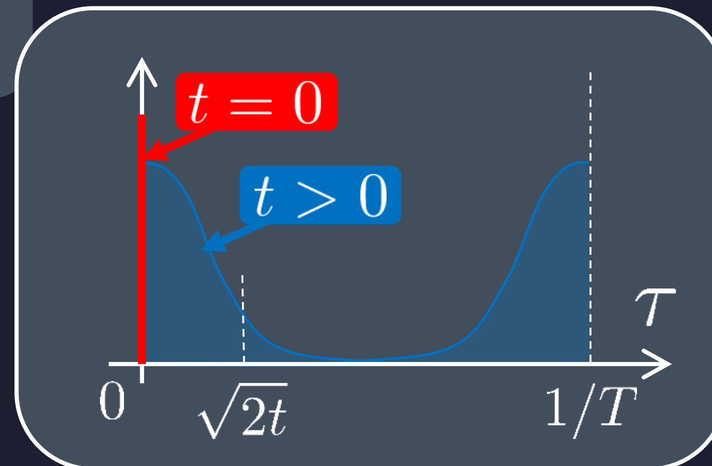

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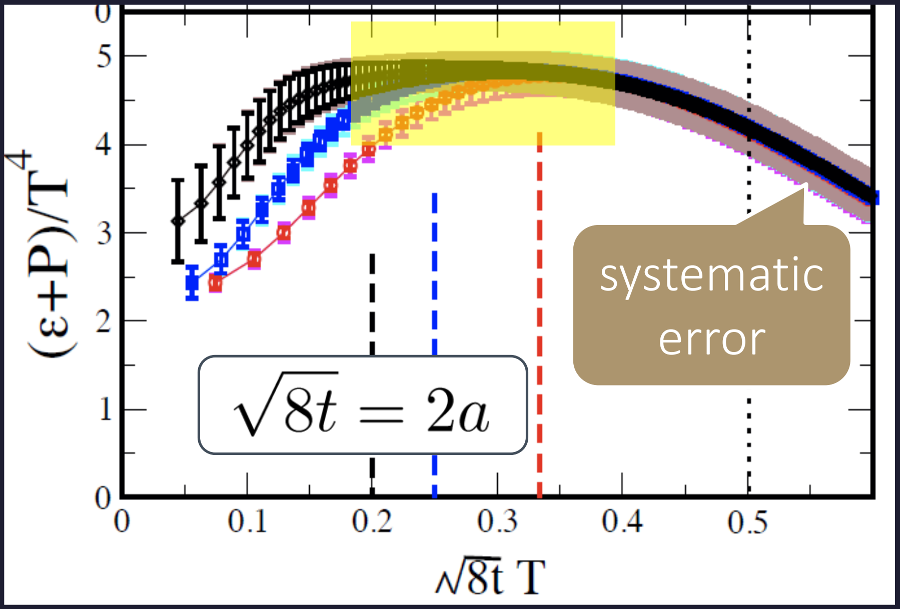
Entropy Density at T=1.65Tc



Emergent plateau!

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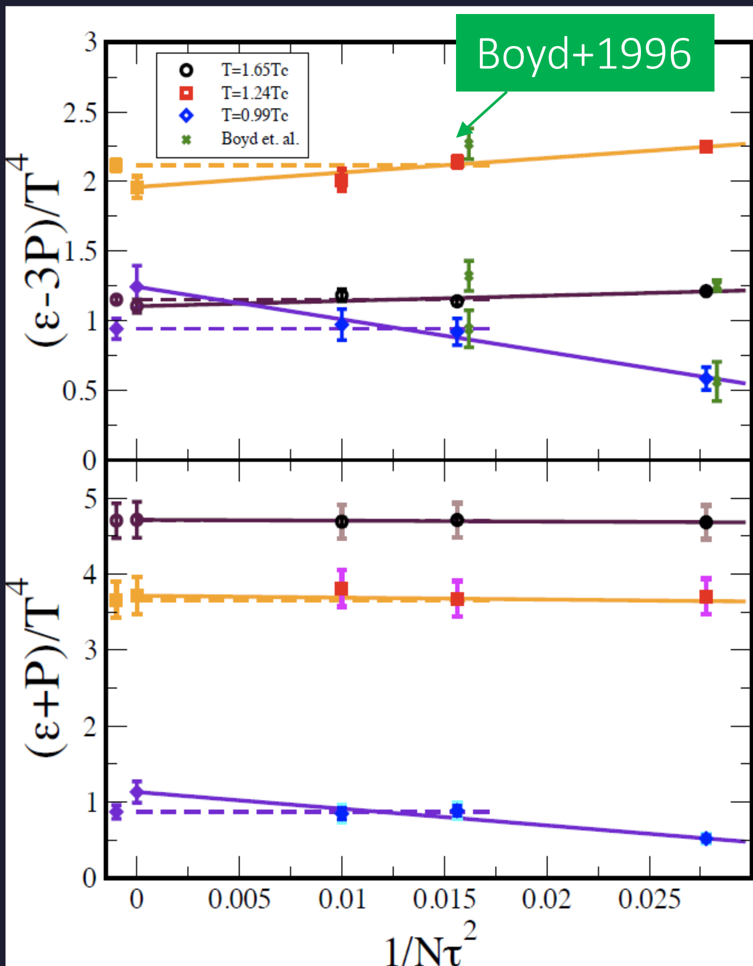
Nt=6,8,10
~300 confs.

Direct measurement of e+p on a given T!

NO integral / **NO** vacuum subtraction

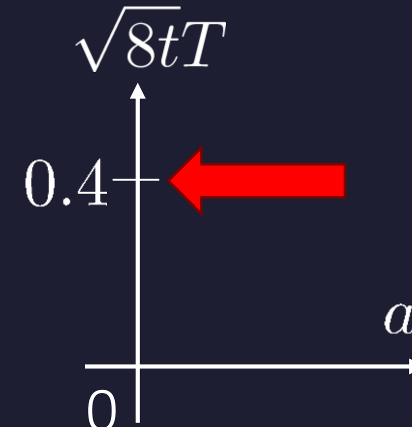
Continuum Limit



32^3xNt

Nt = 6, 8, 10

T/Tc=0.99, 1.24, 1.65



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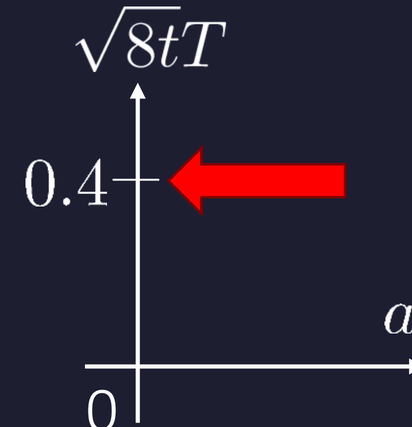

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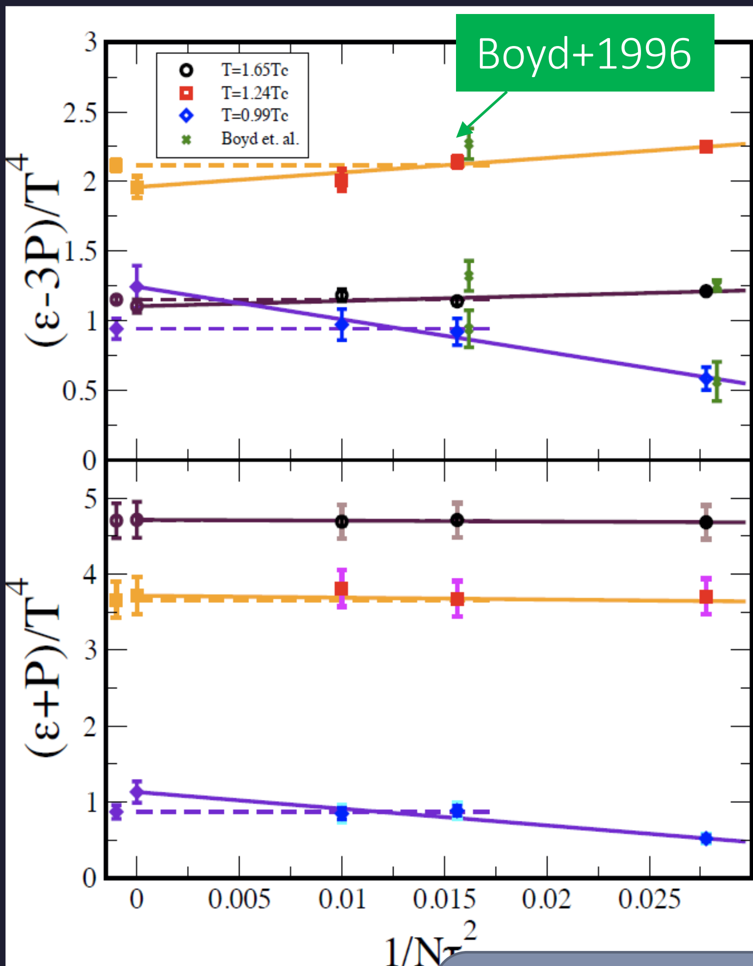
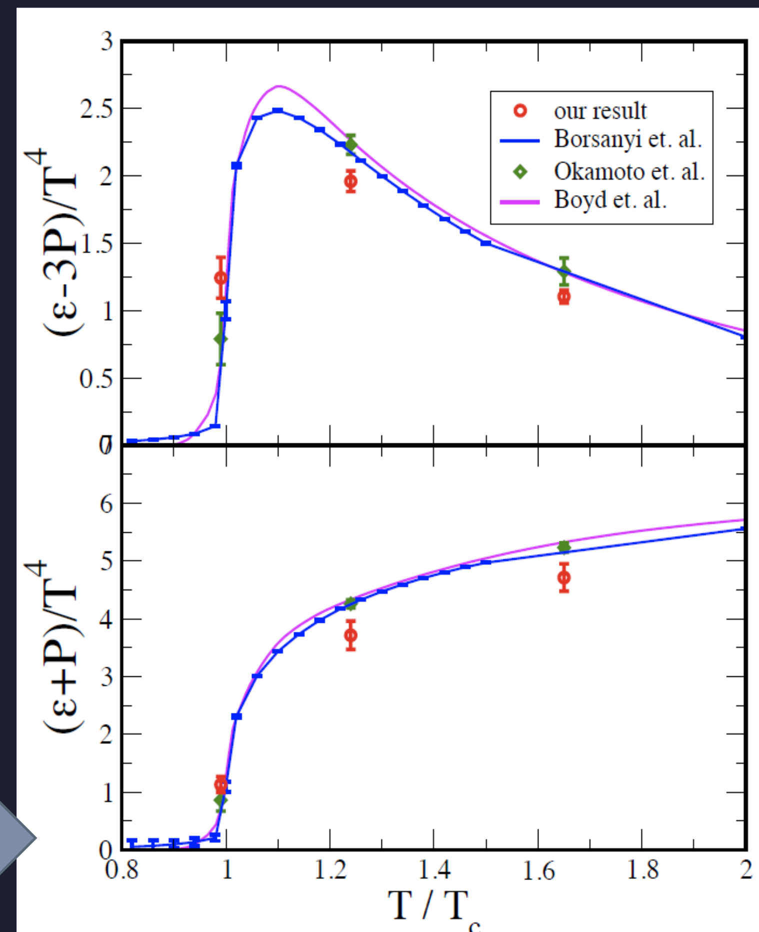
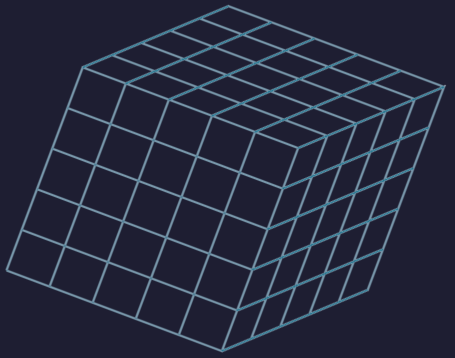


Comparison with previous studies



Numerical Simulation



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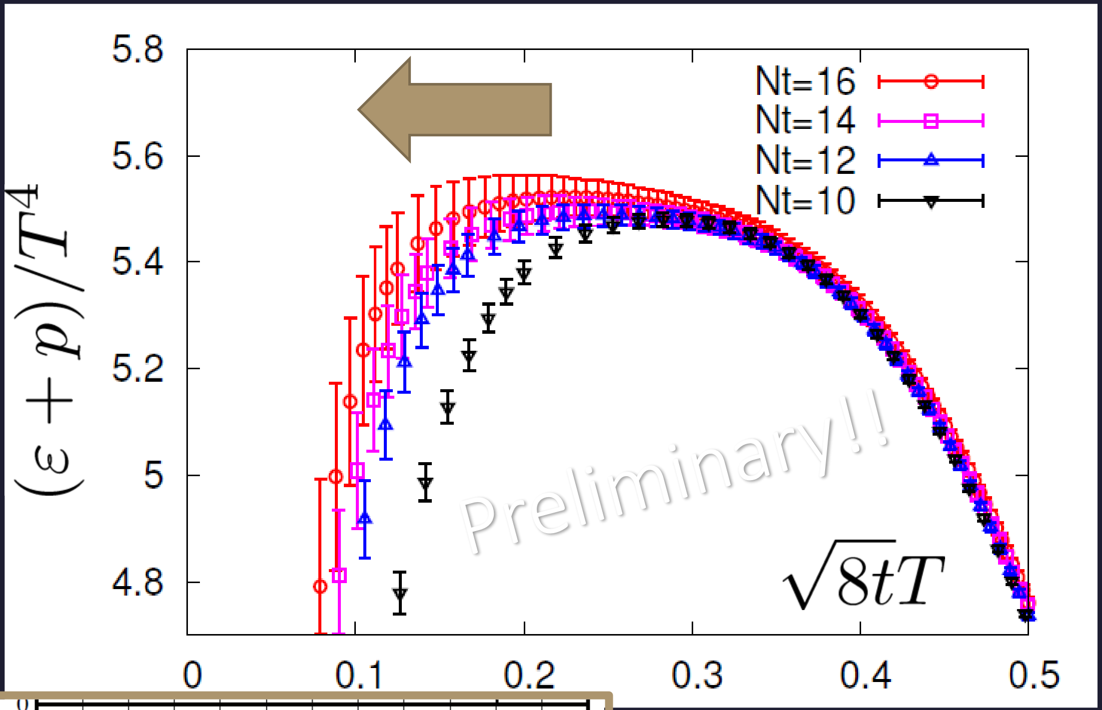
efficiency ~40%

Entropy Density on Finer Lattices

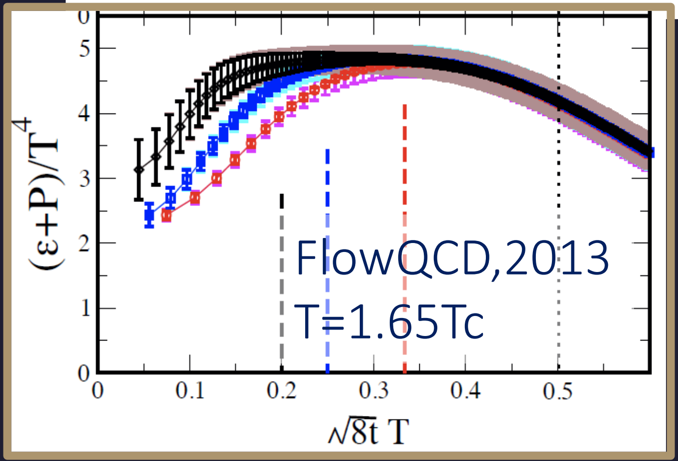


T = 2.31Tc

64^3xNt

Nt = 10, 12, 14, 16

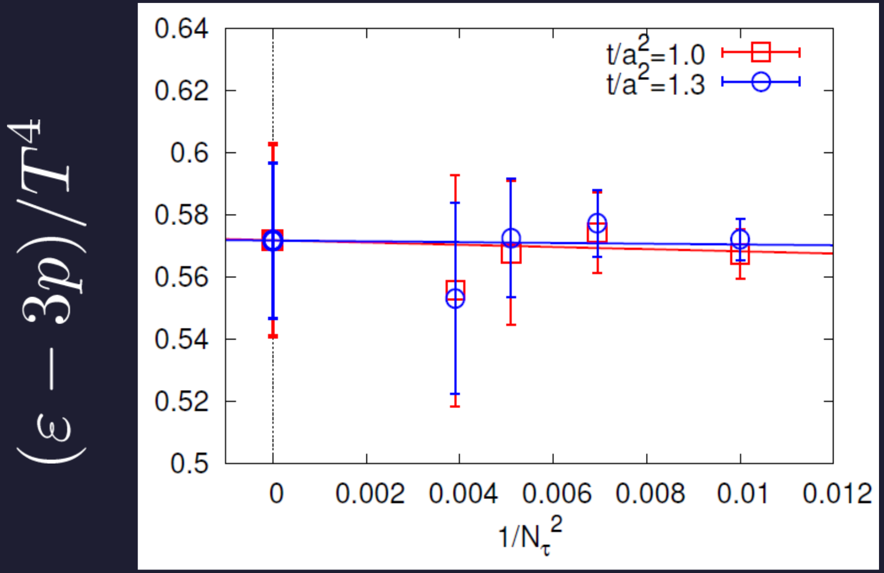
2000 confs.

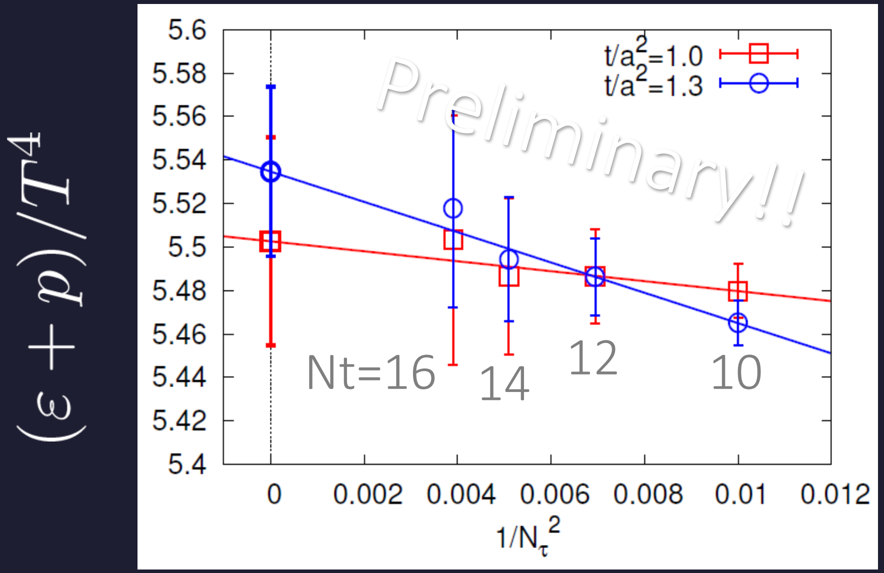


- The wider plateau on the finer lattices
- Plateau may have a nonzero slope

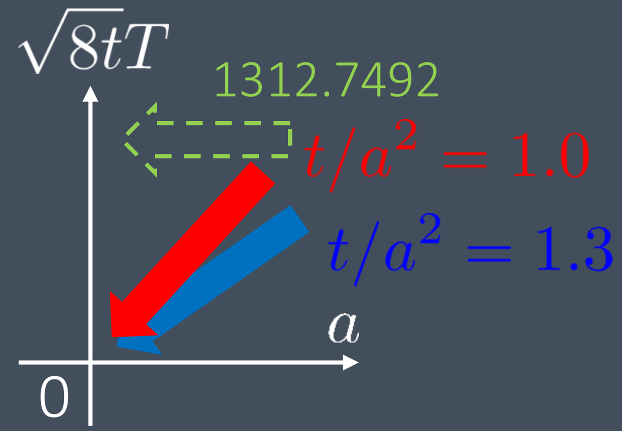
Continuum Extrapolation

- T=2.31Tc
- 2000 confs
- Nt = 10 ~ 16





a→0 limit with fixed t/a^2



Continuum extrapolation is stable

EMT Correlators

Fluctuations and Correlations

viscosity, specific heat, ...

$$\eta = \int_0^\infty dt \langle T_{12}; T_{12} \rangle$$

$$c_V \sim \langle \delta T_{00}^2 \rangle$$

EMT Correlator

- Kubo Formula: T_{12} correlator ←→ shear viscosity

$$\eta = \int_0^\infty dt \int_0^{1/T} d\tau \int d^3x \langle T_{12}(x, -i\tau) T_{12}(0,t) \rangle$$

  - Hydrodynamics describes long range behavior of $T_{\mu\nu}$

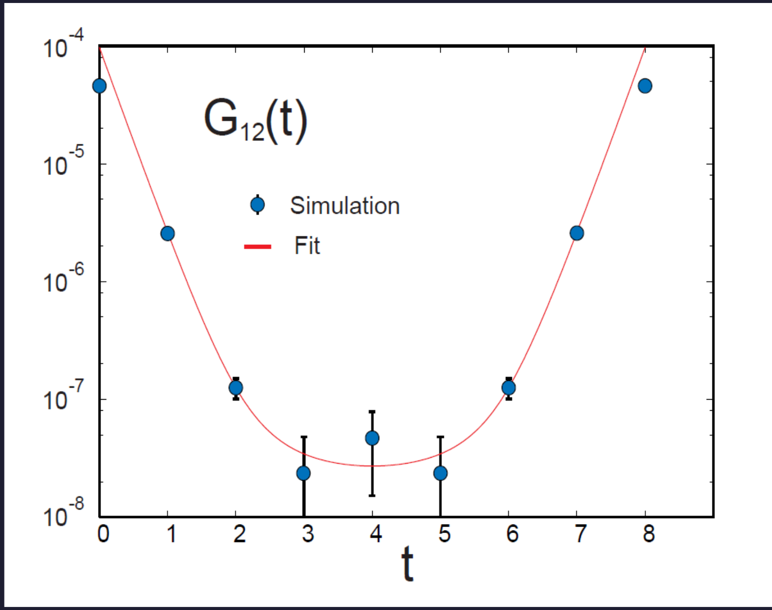
- Energy fluctuation ←→ specific heat

$$c_V = \frac{\langle \delta E^2 \rangle}{VT^2}$$

EMT Correlator : Noisy...

With naïve EMT operators

$\langle T_{12}(\tau)T_{12}(0)\rangle$



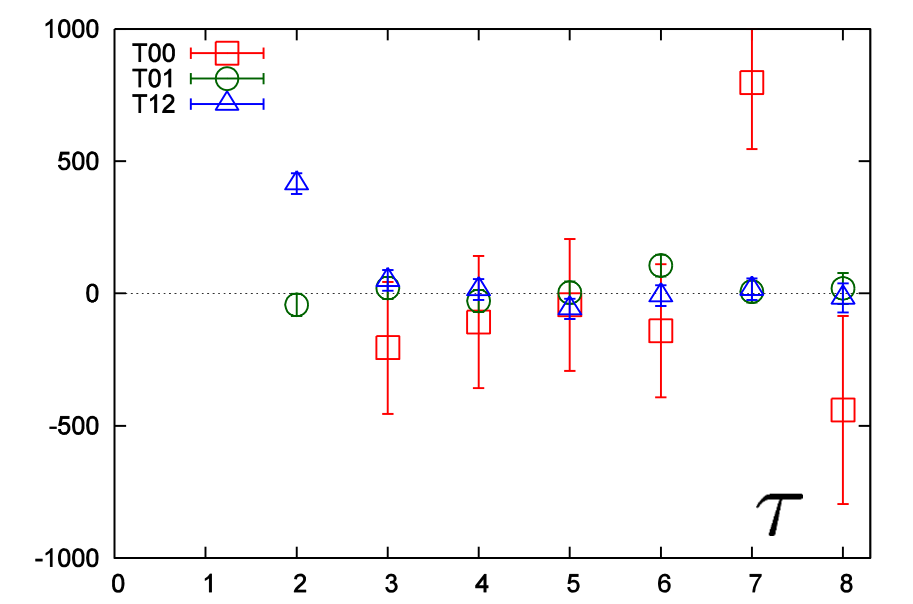
Nakamura, Sakai, PRL,2005

N_t=8

improved action

~10^6 configurations

$\langle T_{\mu\nu}(\tau)T_{\mu\nu}(0)\rangle$



Nt=16

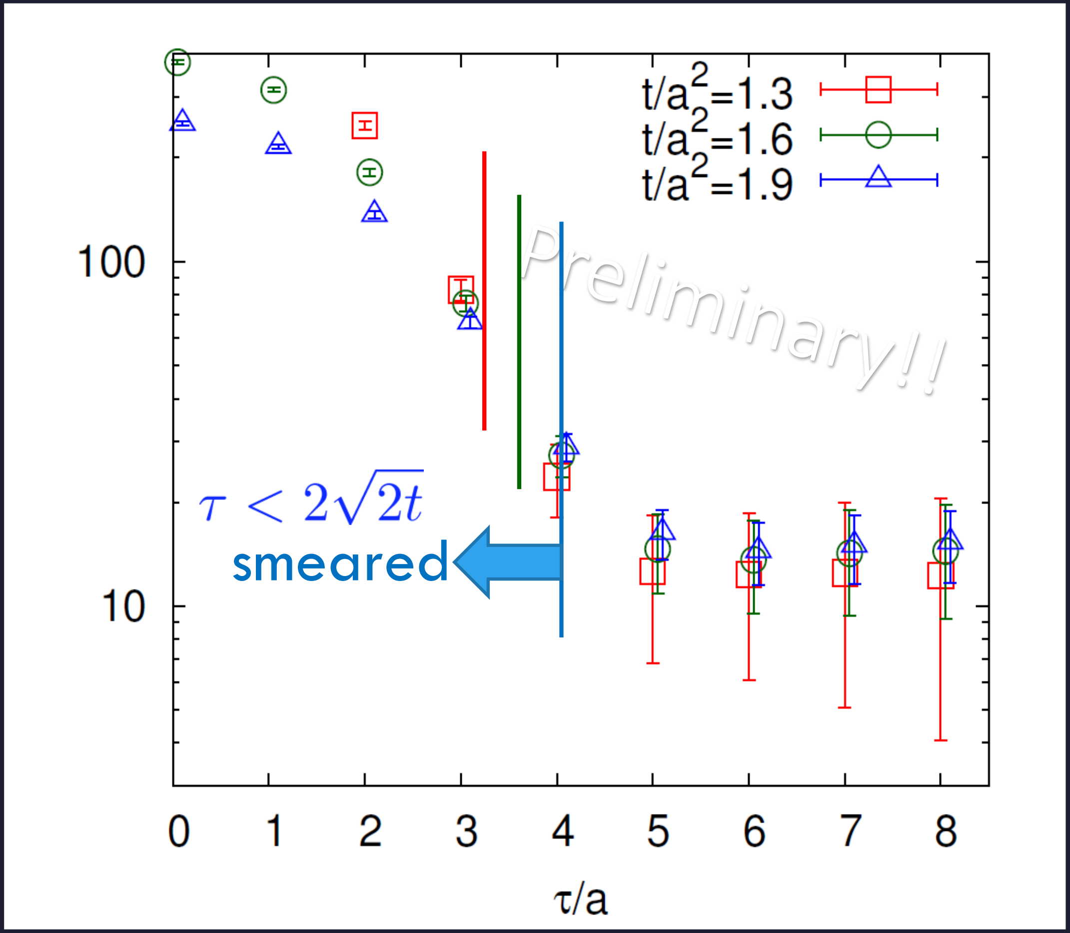
standard action

5x10^4 configurations

... no signal

Energy Correlation Function

$$\langle \delta T_{00}(\tau) \delta T_{00}(0) \rangle / T^5$$



T=2.31Tc

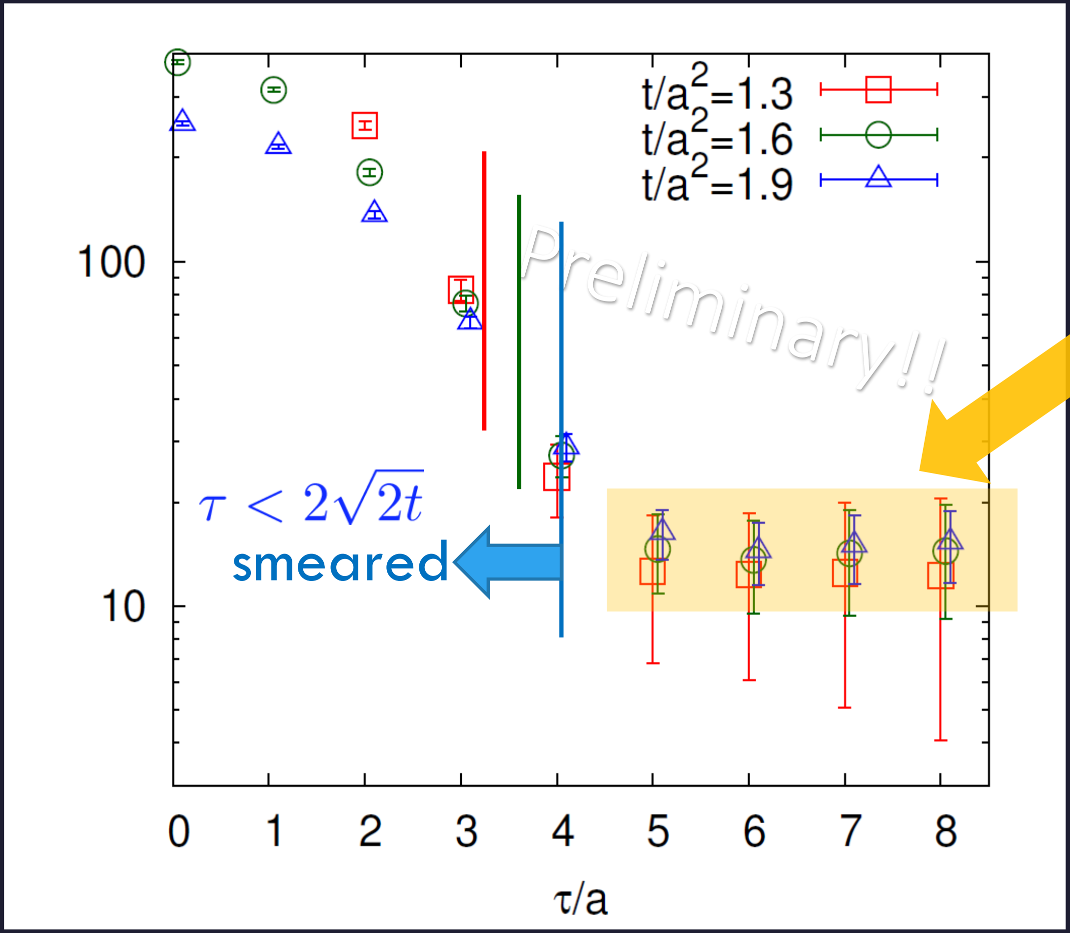
b=7.2, Nt=16

2000 confs

p=0 correlator

Energy Correlation Function

$$\langle \delta T_{00}(\tau)\delta T_{00}(0)\rangle / T^5$$

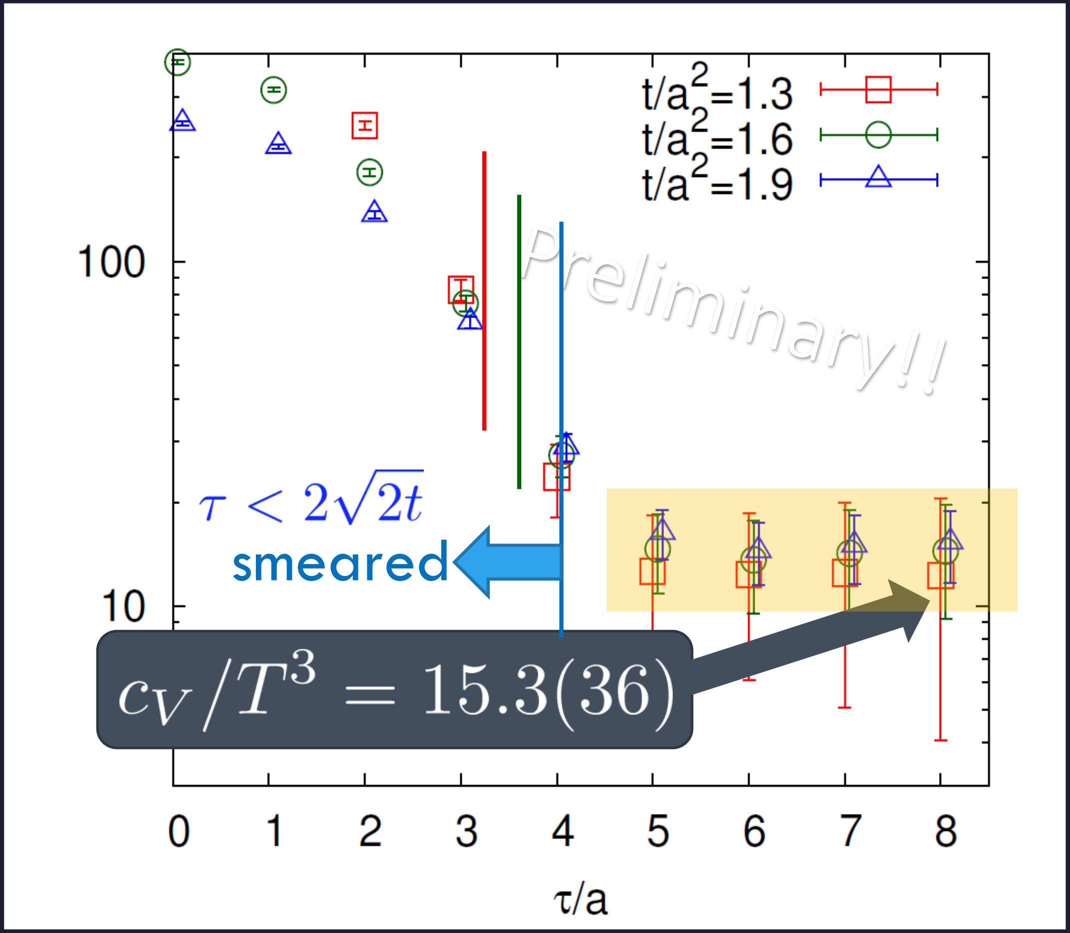


T=2.31Tc
b=7.2, Nt=16
2000 confs
p=0 correlator

□ τ independent const.
→ energy conservation

Energy Correlation Function

$$\langle \delta T_{00}(\tau) \delta T_{00}(0) \rangle / T^5$$



T=2.31Tc
b=7.2, Nt=16
2000 confs
p=0 correlator

- specific heat

$$c_V = \frac{\langle \delta E^2 \rangle}{VT^2}$$

→ Novel approach to measure specific heat!

Gavai, Gupta, Mukherjee, 2005

$$c_V/T^3 = 15(1) \quad T/T_c = 2$$
$$= 18(2) \quad T/T_c = 3$$

differential method / cont lim.

Summary

$$T^{R}_{\mu\nu}(x)$$

Summary

EMT formula from gradient flow

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\text{subt.}}\right]$$

This formula can successfully define and calculate the EMT on the lattice

This operator provides us with novel approaches to measure various observables on the lattice!

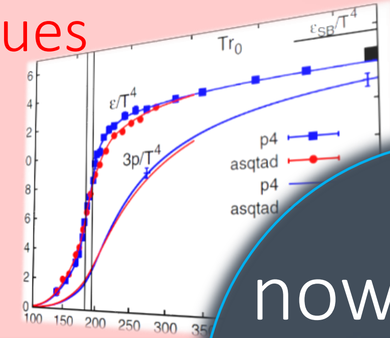
This method is direct, intuitive and less noisy

Many Future Studies!!

Thermodynamics

direct measurement of expectation values

$\langle T_{00} \rangle, \langle T_{ii} \rangle$



Fluctuations and Correlations

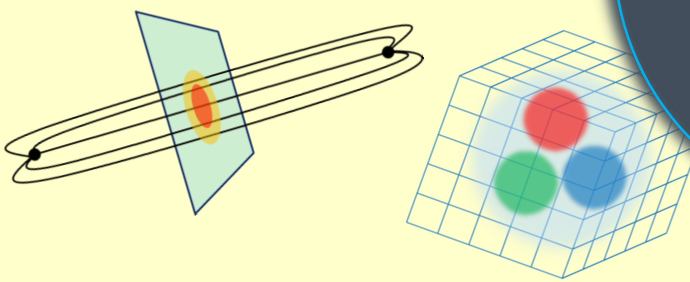
viscosity, specific heat, ...

$c_V \sim \langle \delta T_{00}^2 \rangle$

$\eta = \langle T_{12}; T_{12} \rangle$

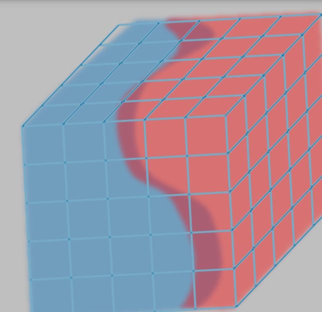
now we have

$T_{\mu\nu}$



- confinement string
- EM distribution in hadrons

Hadron Structure



- vacuum configuration
- mixed state on 1st transition

Vacuum Structure

and scale setting, full QCD Makino,Suzuki,2014, etc.