Understanding Fluctuations
using rapidity window and collision energy dependences

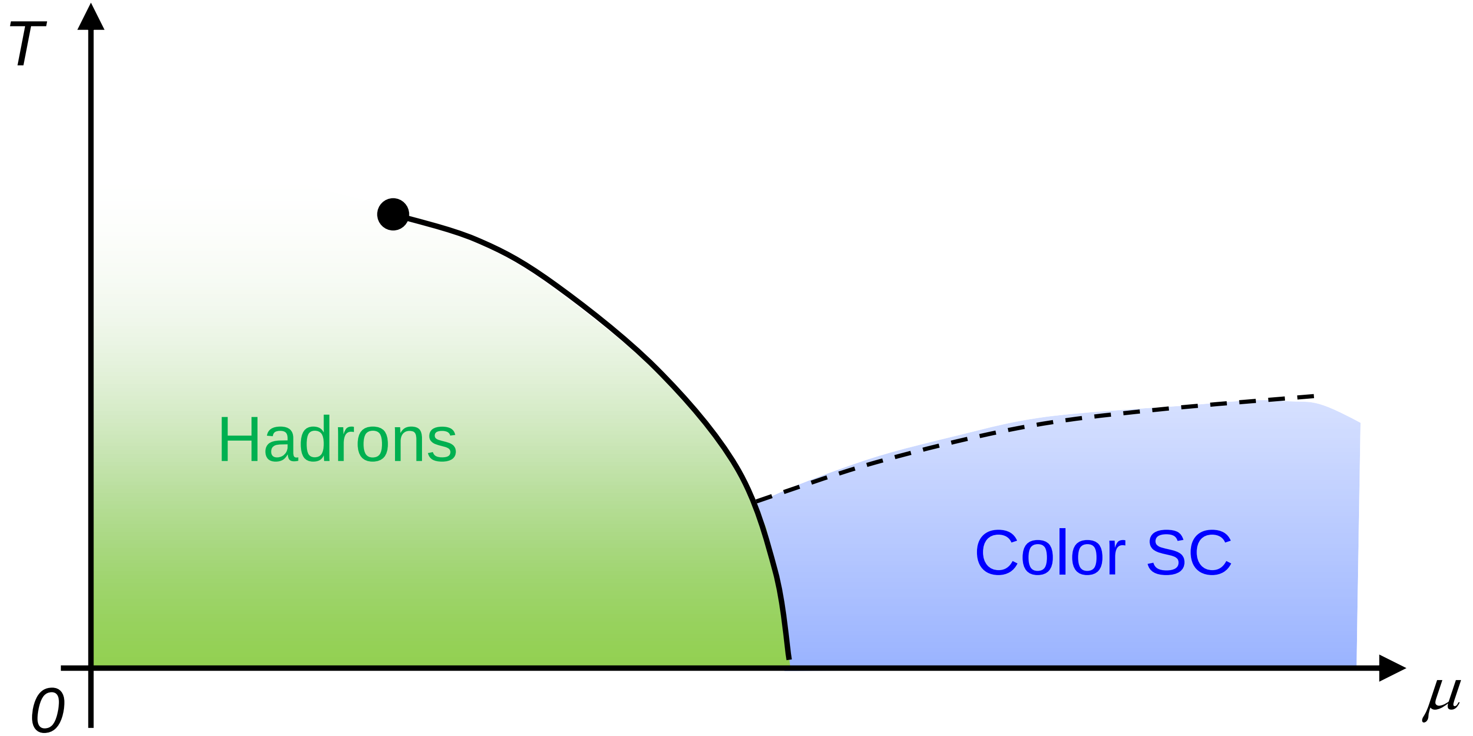
Masakiyo Kitazawa

(Osaka U.)

MK, Asakawa, Ono, Phys. Lett. B728 (2014) 386-392
Sakaida, Asakawa, MK, arXiv:1409.6866
MK, to appear soon!

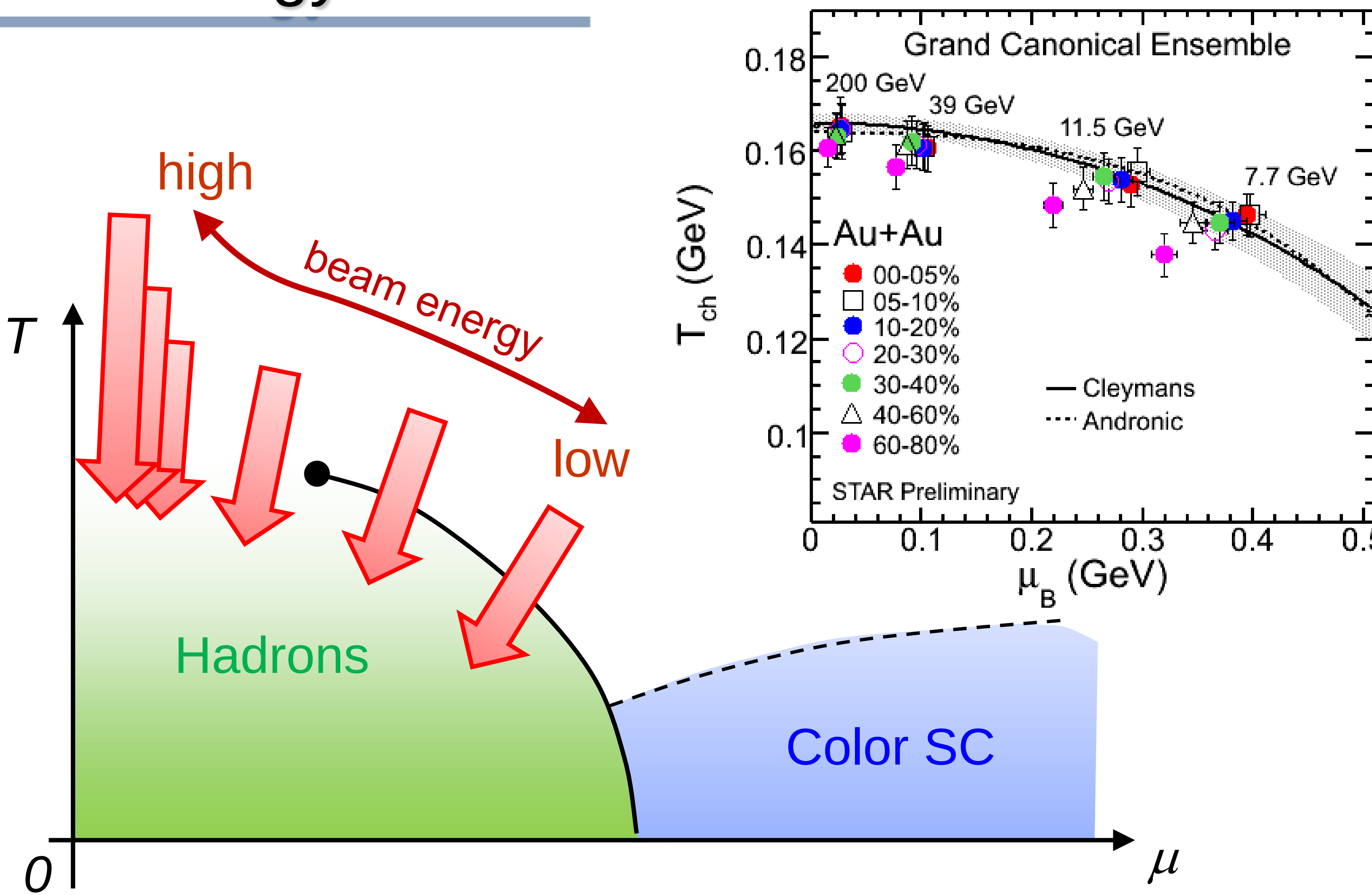
BES-II workshop, Berkeley, 28/Sep./2014

Beam-Energy Scan



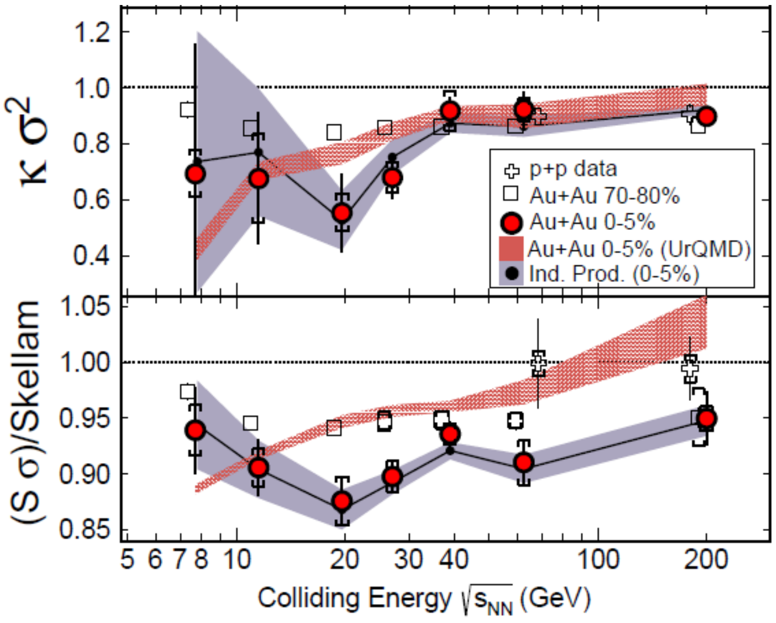
Beam-Energy Scan

STAR 2012



Cumulants up to 4th Order in 2014

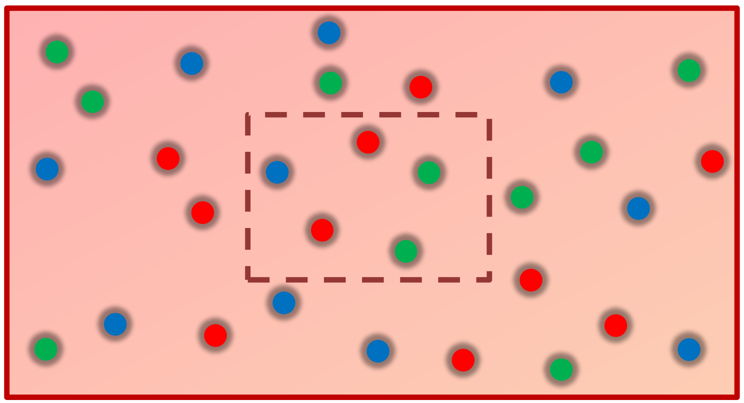
STAR, PRL 2014



This is a great achievement in physics!

Fluctuations and Elemental Charge

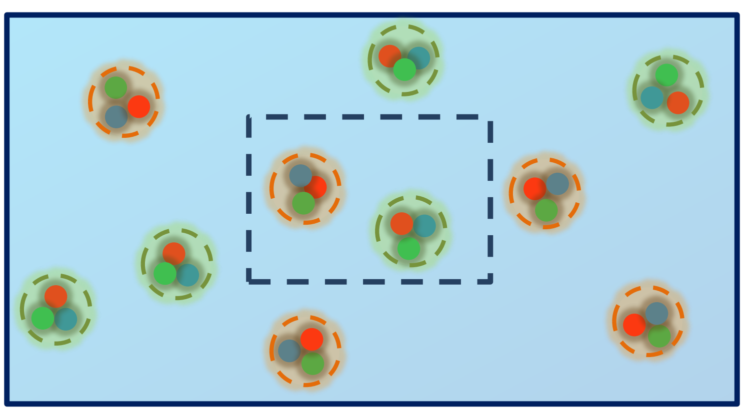
Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000
Ejiri, Karsch, Redlich, 2005



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$

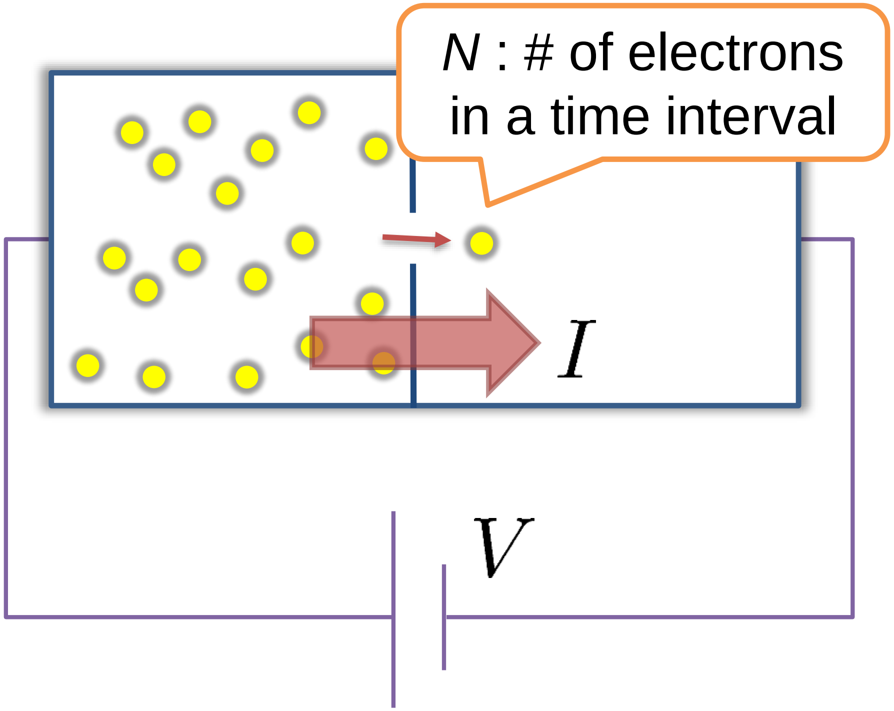


$$\langle \delta N_B^n \rangle_c = \langle N_B \rangle$$

Free Boltzmann → Poisson

$$\langle \delta N^n \rangle_c = \langle N \rangle$$

Shot Noise



Schottky, 1918

Total charge in the interval

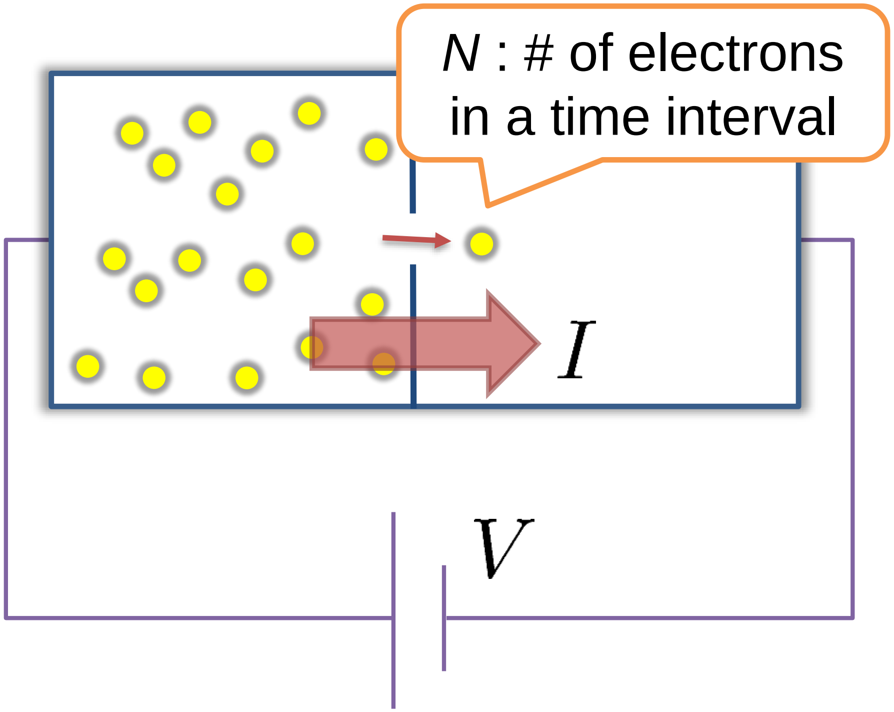
$$Q = e\langle N\rangle$$

$$\langle \delta Q^2\rangle = e^2\langle \delta N^2\rangle = e^2\langle N\rangle = eQ$$

$$\frac{\langle \delta Q^2\rangle}{Q} = e$$

- ✓ Quantum Hall effect
  Saminadayar+, PRL**79**,2526(1997)
- ✓ Superconductor
  X. Jehl+, Nature405,50 (2000)

Shot Noise



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- ✓ Quantum Hall effect
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- ✓ Superconductor
  X. Jehl+, Nature405,50 (2000)

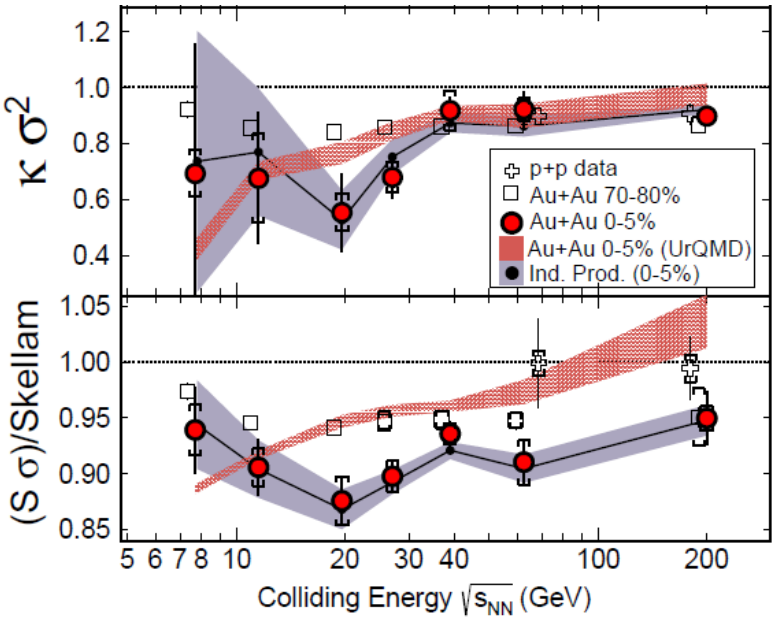
Higher order cumulants:

$$\langle \delta Q^3 \rangle = e^3 \langle N \rangle = e^2 Q$$

3rd order
ex. Beenakker+, PRL90,176802(2003)

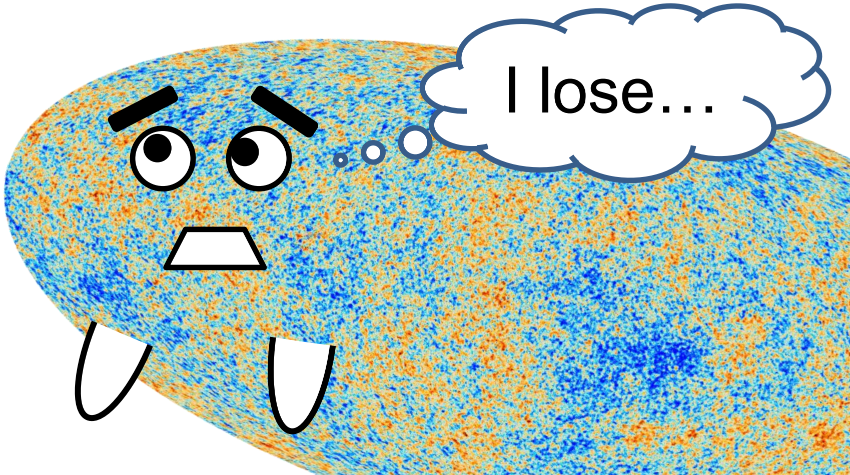
up to 5th order
Gustavsson+, Surf.Sci.Rep.**64**,191(2009)

Cumulants up to 4th Order in 2014



This is a great achievement in physics!

However,
still a lot of things to do…



Many Things to Do

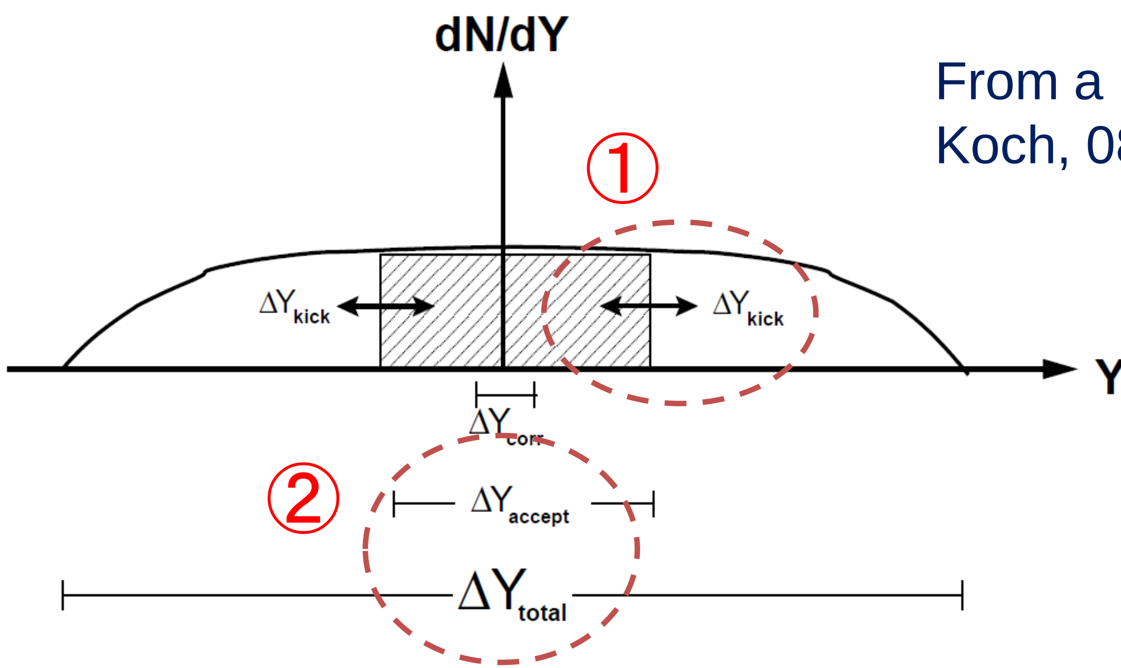
- Message to Experimentalists:
  - Measure rapidity window dependences
  - Determine baryon number cumulants
- Message to Theorists:
  - Do not directly compare your thermal results with exp.
  - Let's pursue descriptions of non-eq. non-Gaussianity.
- Message to Latticians:
  - Do not directly compare your results with exp.
  - Measure more cumulants more accurately

Caveats

Experimental results on
- Non-equilibrium state
- Non-Gaussian fluctuations

Caveats

Experimental results on { Non-equilibrium state / Non-Gaussian fluctuations

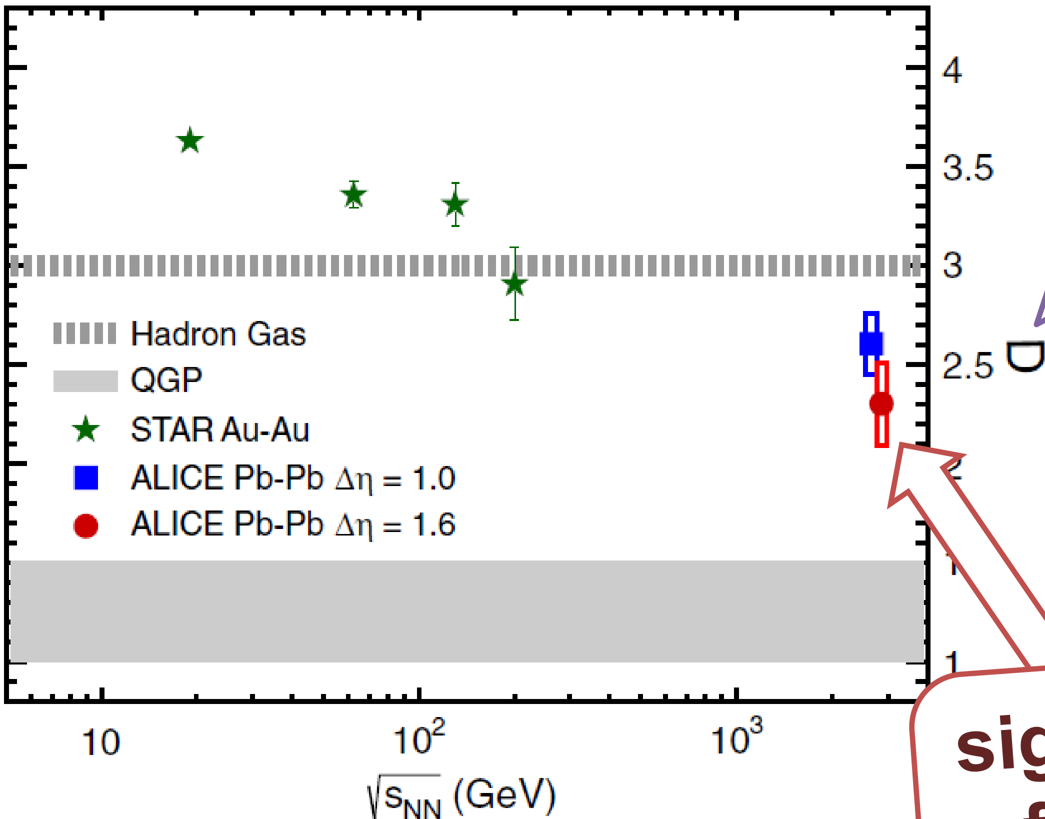


From a Review
Koch, 0810.2520

Rapidity Window Dependences of Higher Order Cumulants

MK, Asakawa, Ono, Phys. Lett. B728 (2014) 386-392;
MK, to appear soon!

Charge Fluctuation @ LHC



ALICE, PRL110,152301(2013)

D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

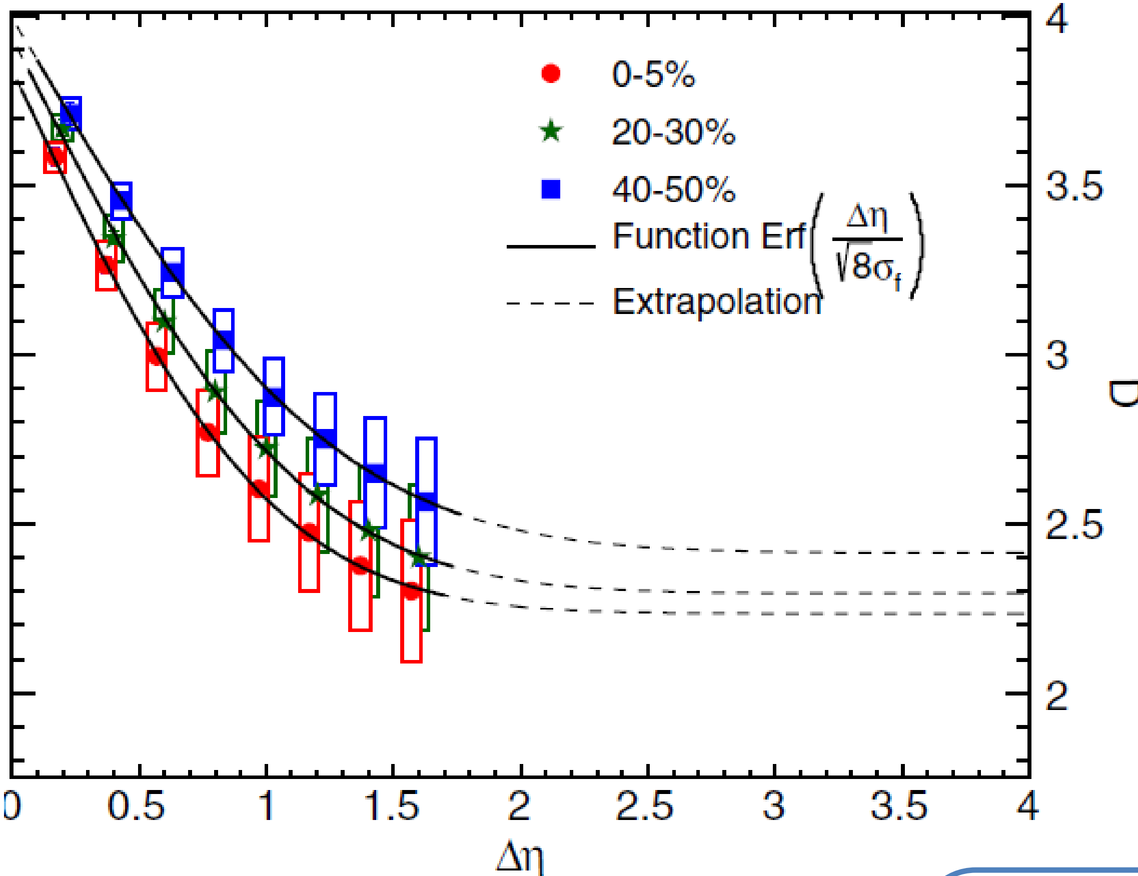
- D ~ 3-4 Hadronic
- D ~ 1-1.5 Quark

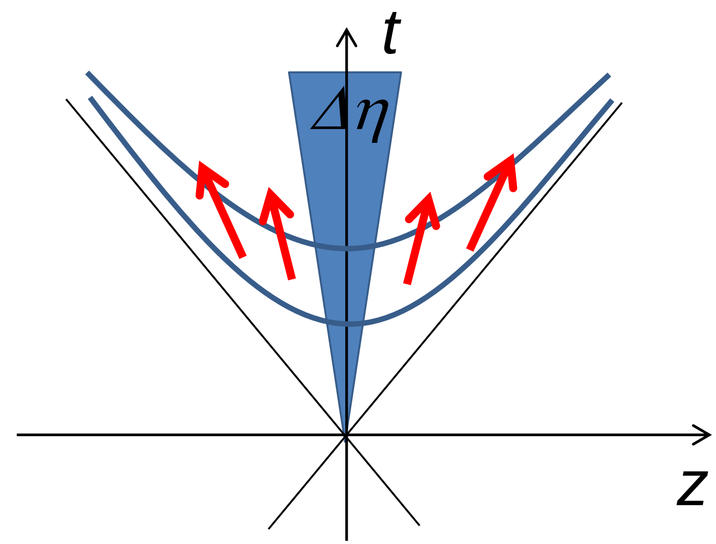
significant suppression from hadronic value at LHC energy!

$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

Δη Dependence @ ALICE

ALICE
PRL 2013



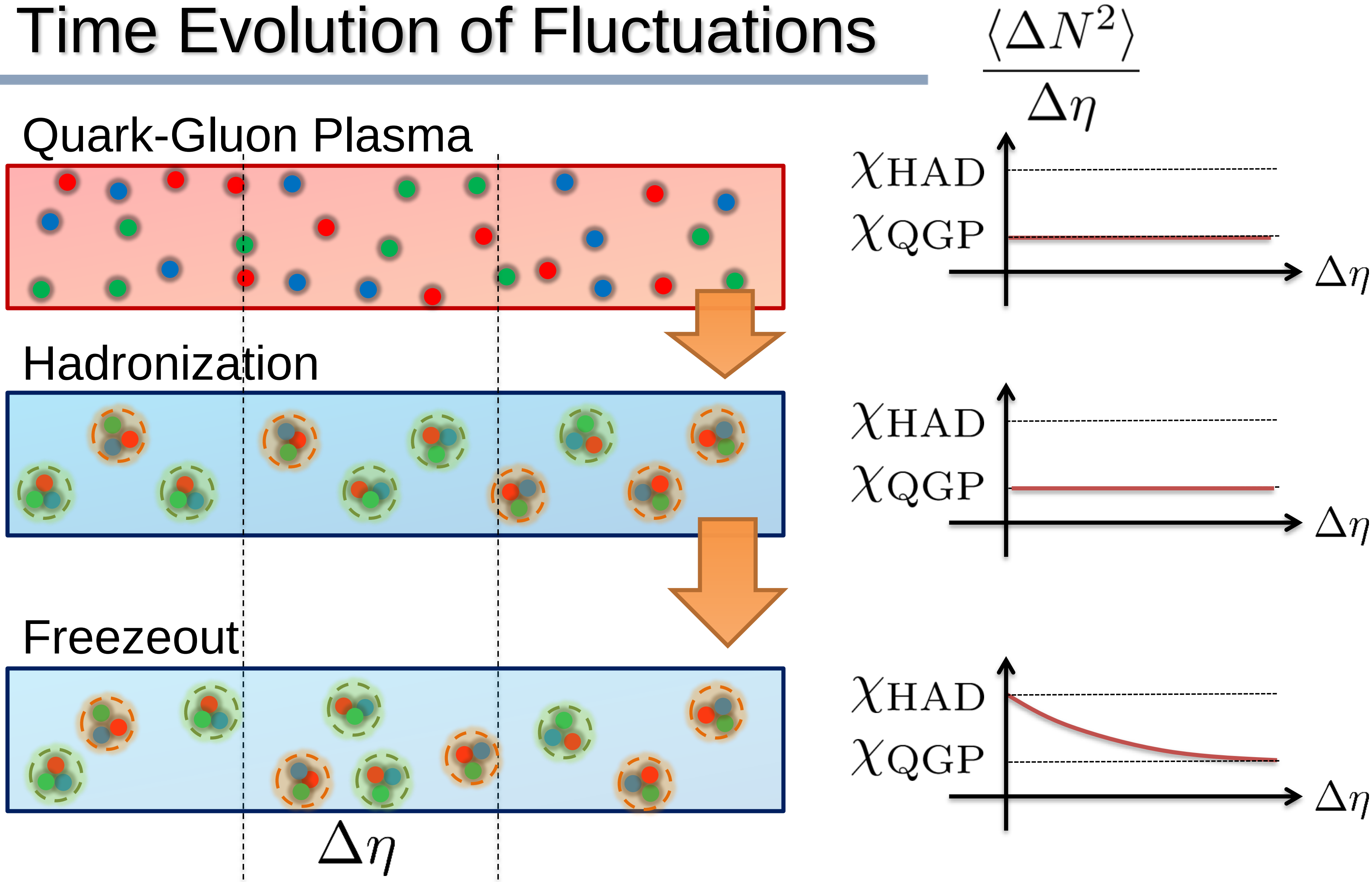


rapidity window

Same information as

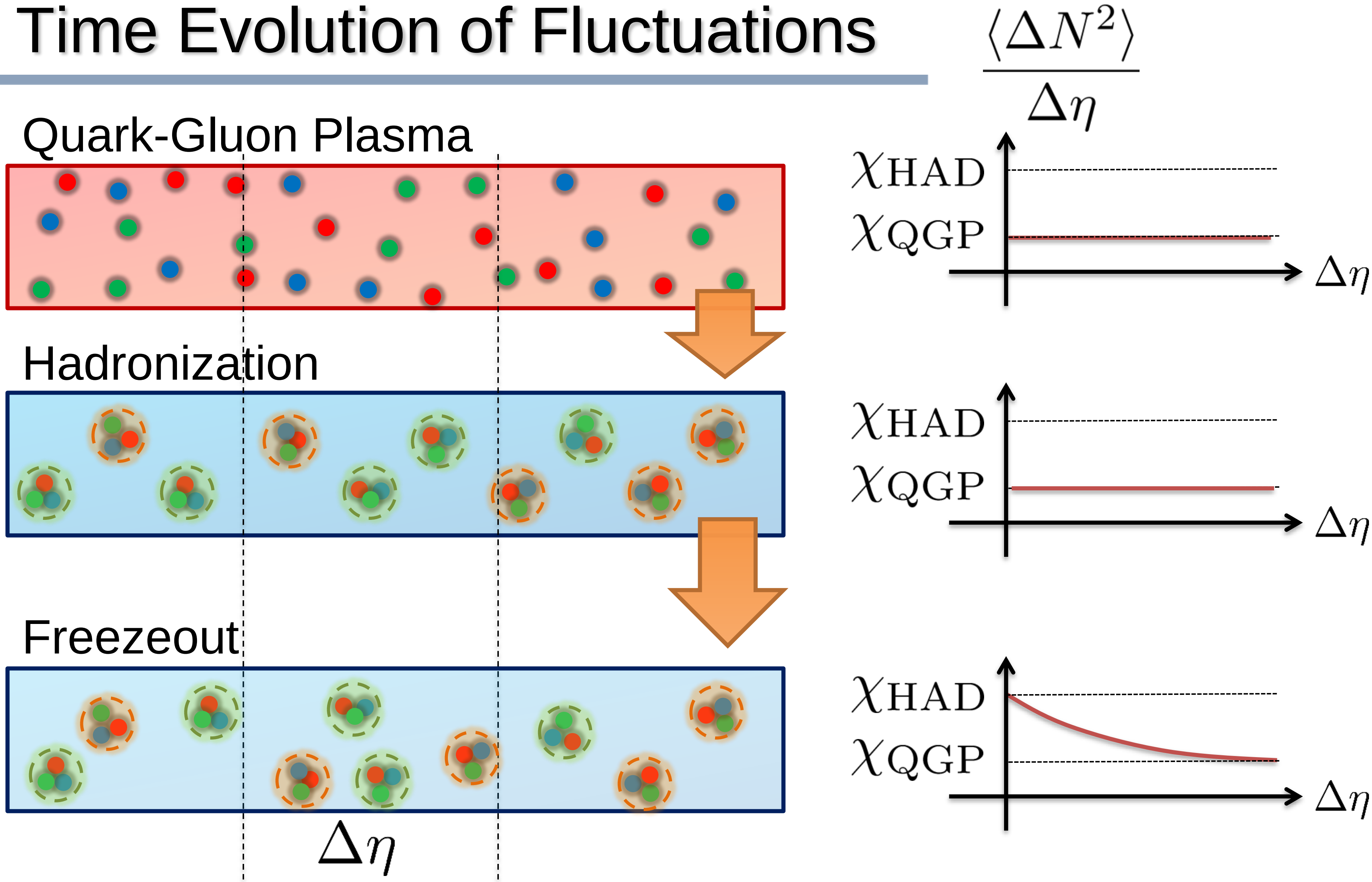
- ◻ 2 particle corr.: $\langle n(\eta) n(0) \rangle$
- ◻ Balance function

Time Evolution of Fluctuations



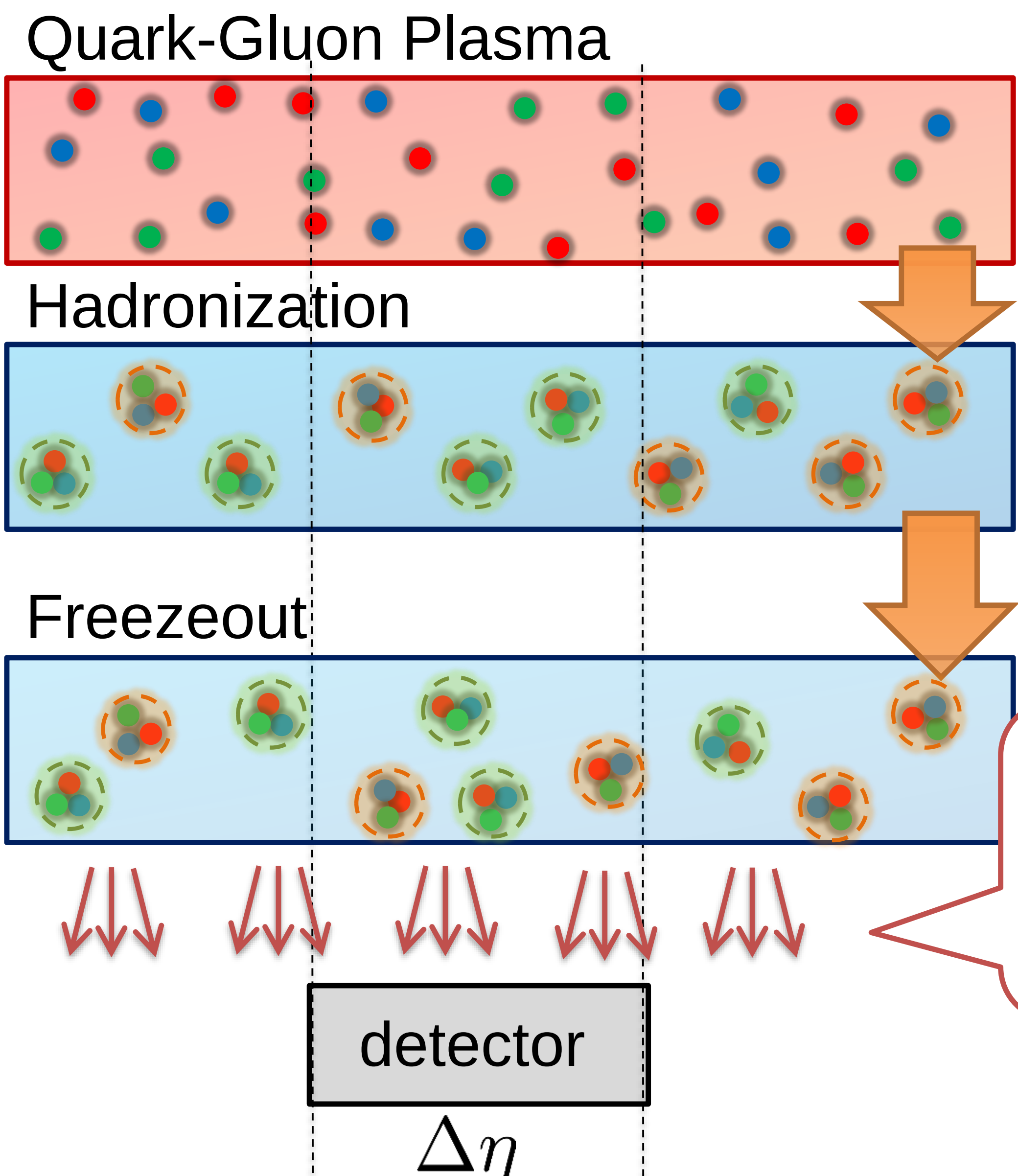
Fluctuations continue to change until kinetic freezeout!!

Time Evolution of Fluctuations



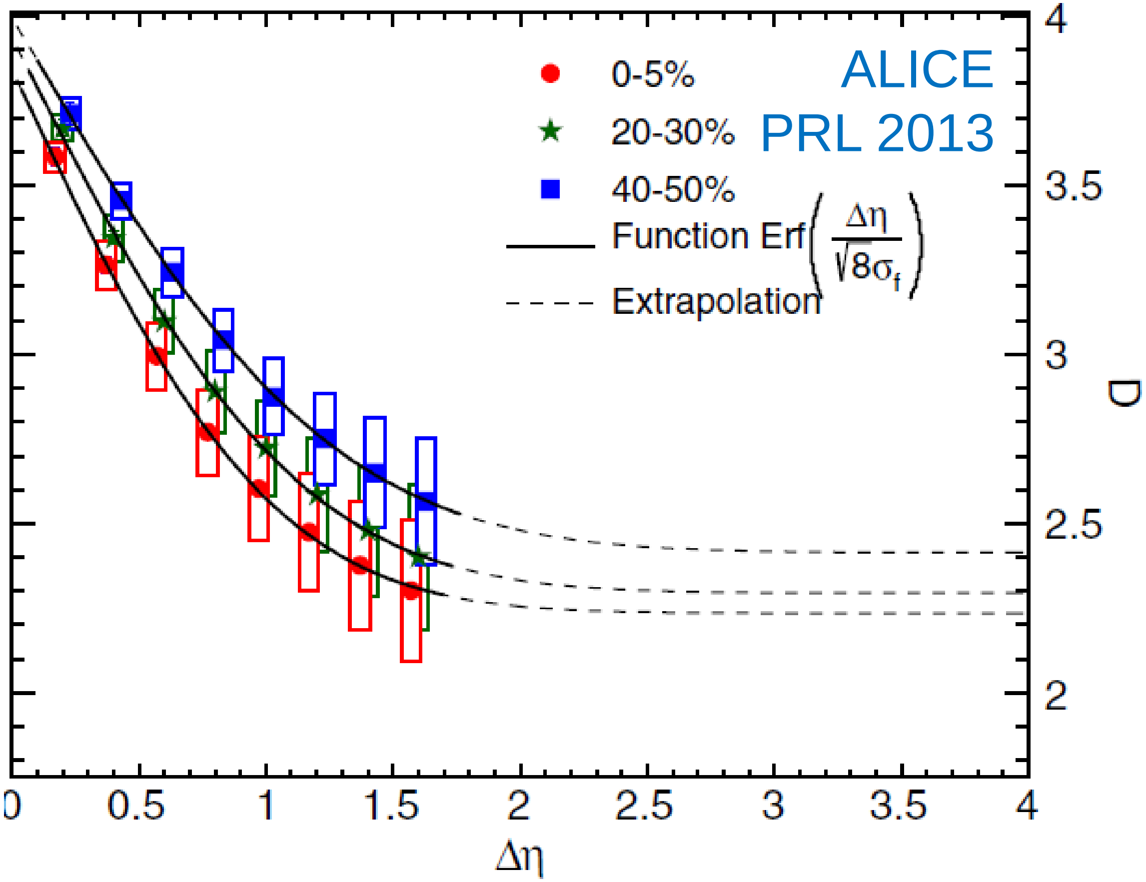
Variation of a conserved charge is achieved only through diffusion. ⇨ The larger $\Delta\eta$, the slower diffusion

Conversion of Rapidities

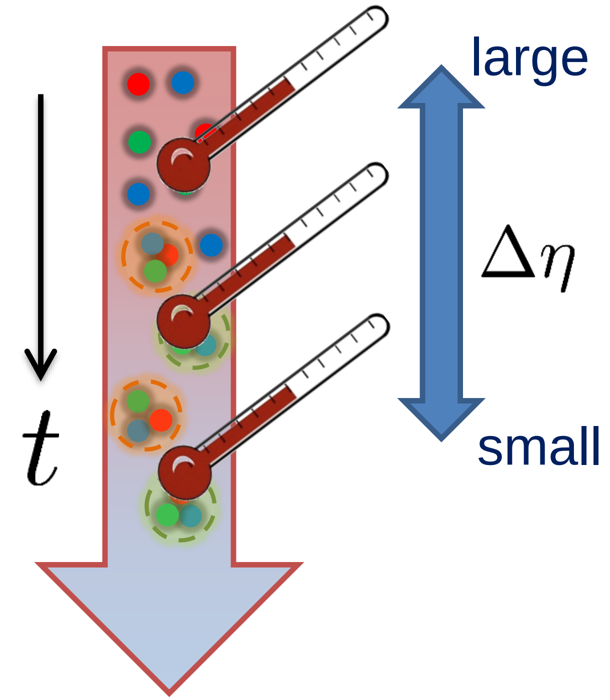


conversion
from coordinate-space
to momentum-space
rapidities

Δη Dependence @ ALICE



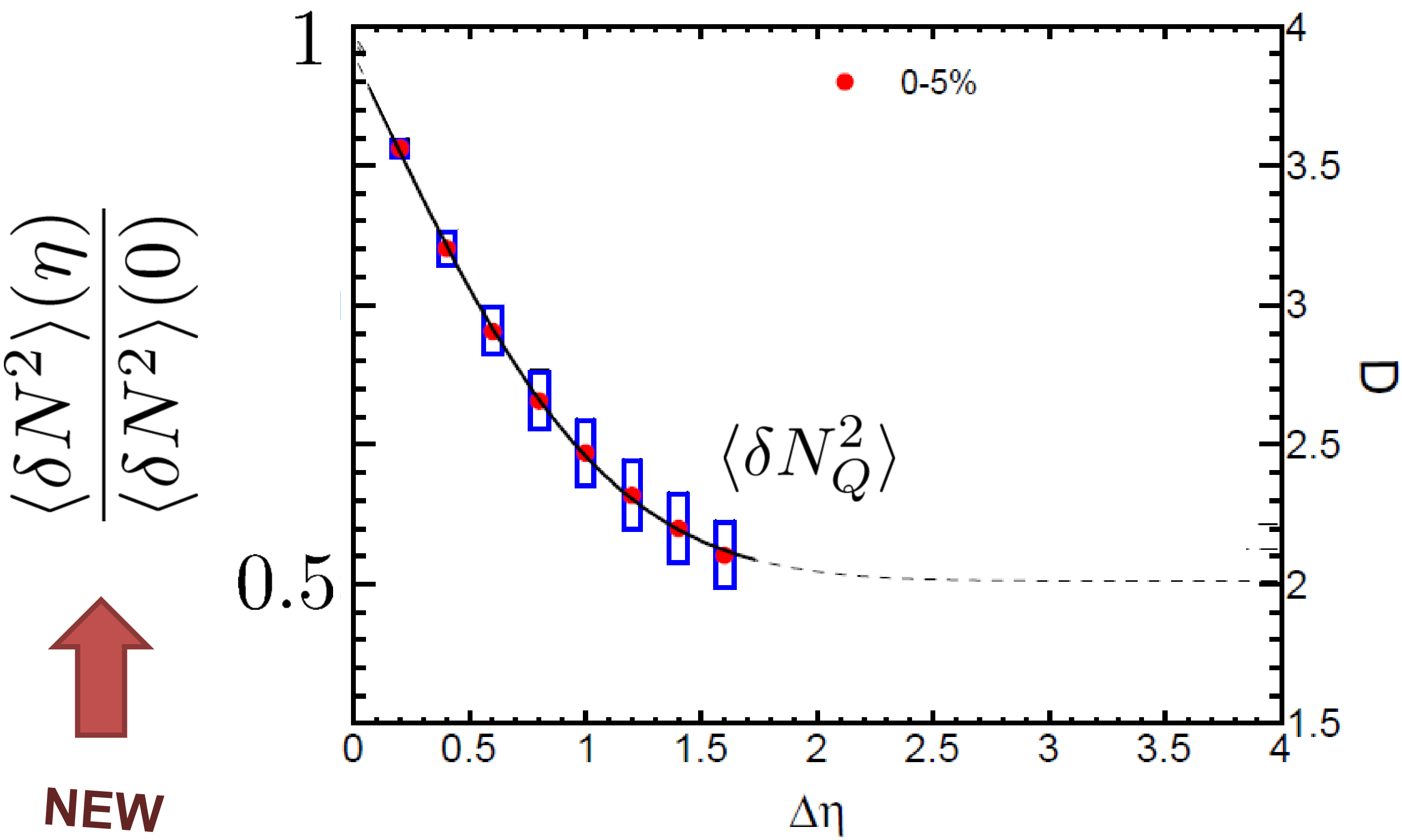
Δη dependent thermometer?



Δη dependences of fluctuation observables encode history of the hot medium!

$<\delta N_B^2>$ and $< \delta N_p^2 >$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
should have different $\Delta\eta$ dependence.



NEW normalization!

$<\delta N_B^2>$ and $< \delta N_p^2 >$ @ LHC ?

$$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$$

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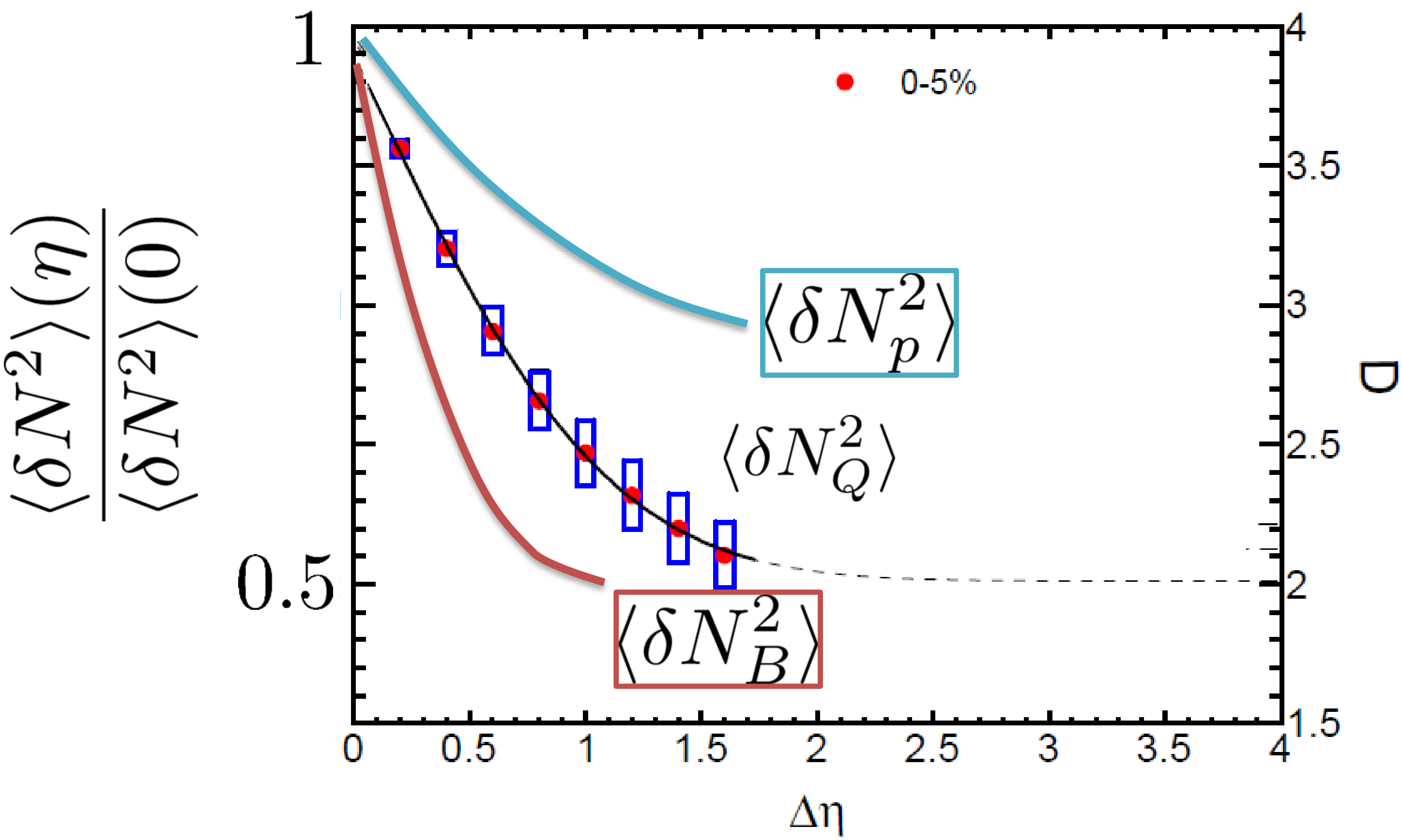
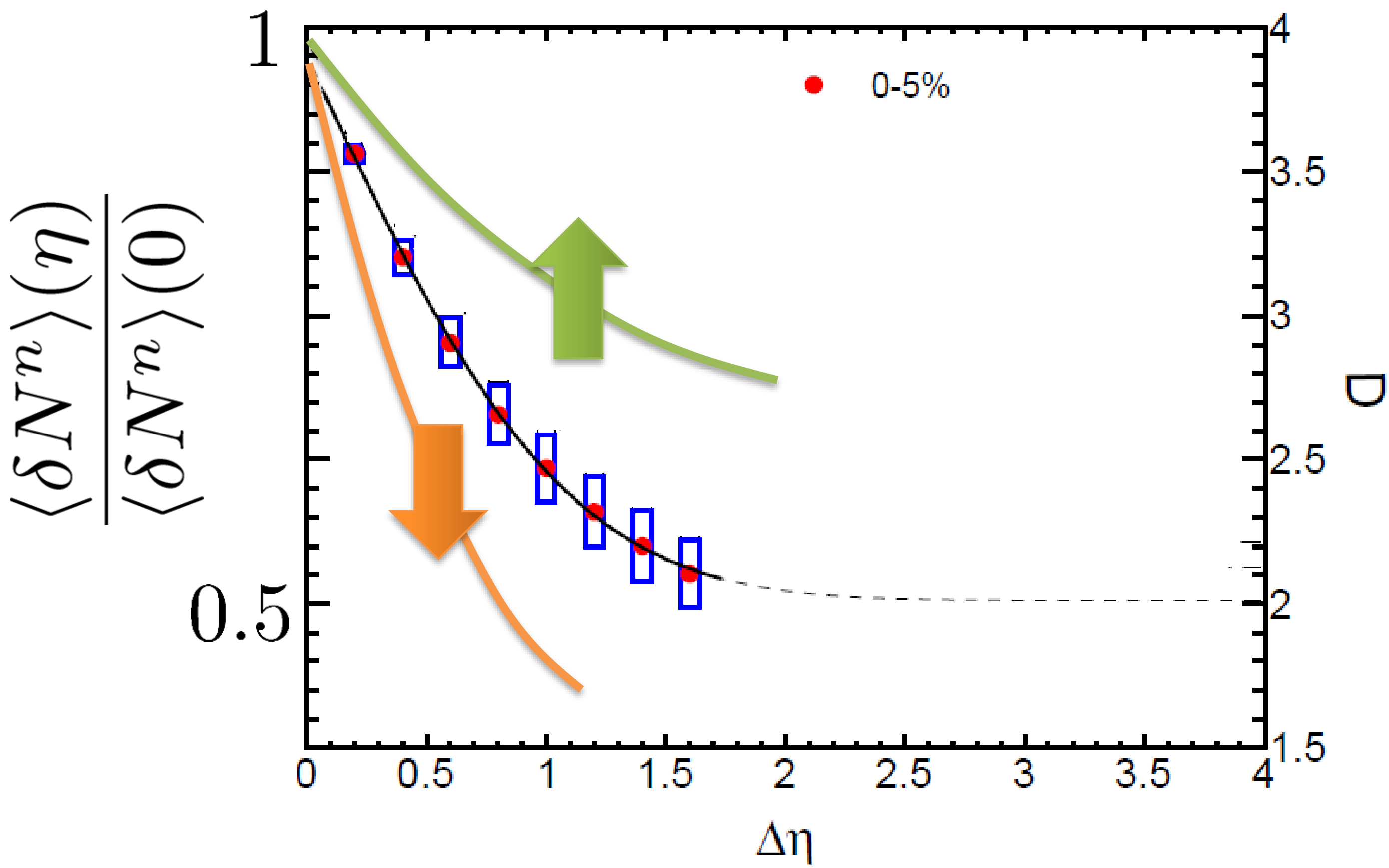


Baryon # cumulants are experimentally observable! MK, Asakawa, 2011;2012

$\langle \delta N_Q^4 \rangle$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

suspression or enhancement



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Stephanov, Shuryak, 2001

Markov (white noise)
+
continuity

↓

Gaussian noise

↓

Fluctuation of *n* is Gaussian in equilibrium

cf) Gardiner, "Stochastic Methods"

How to Introduce Non-Gaussianity?

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant *D*(*n*)
  - colored noise
  - discretization of *n*

How to Introduce Non-Gaussianity?

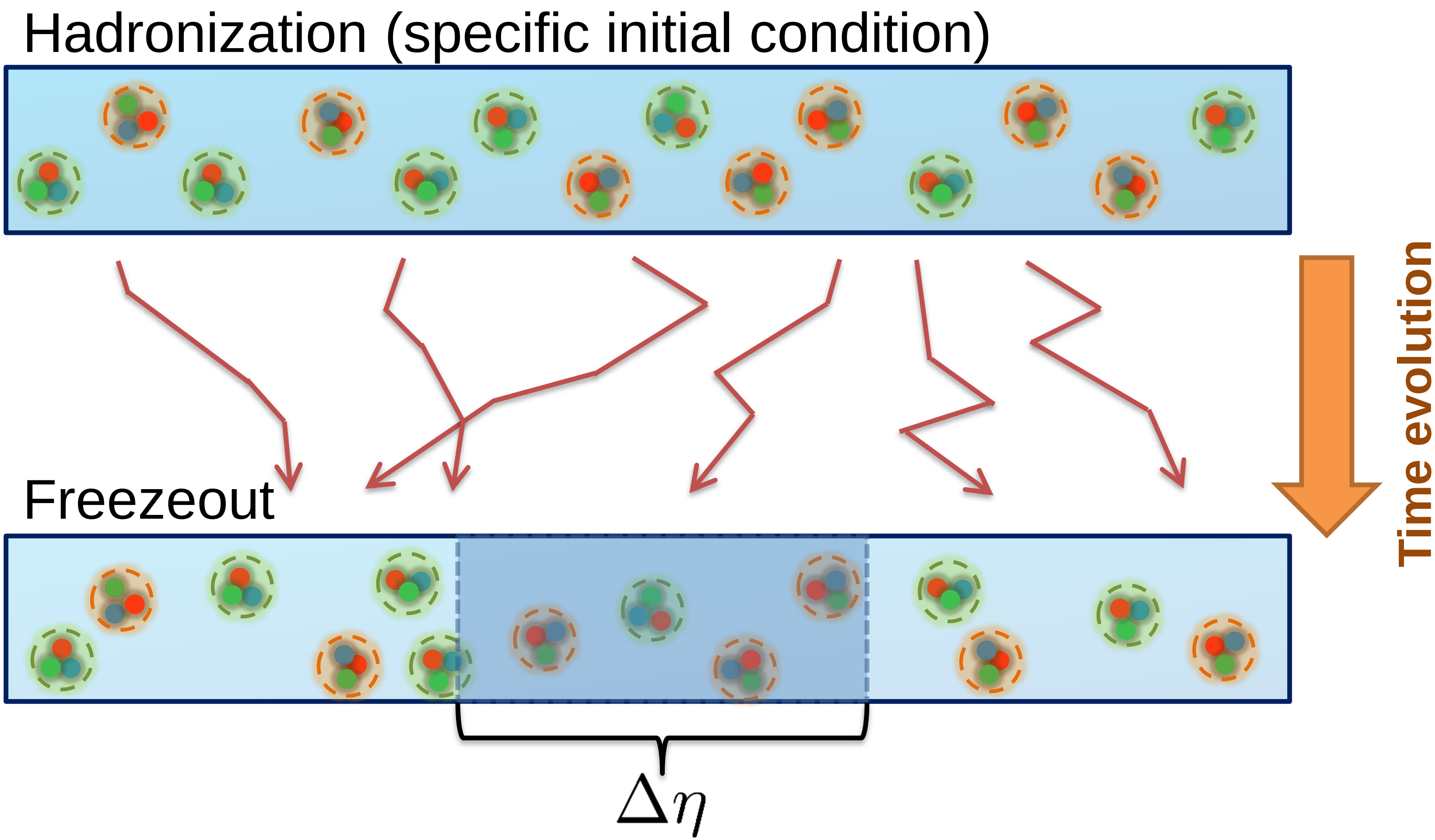
Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant *D*(*n*)
  - colored noise
  - discretization of *n* ⟸ **our choice**

REMARK: Fluctuations measured in HIC are almost Poissonian.

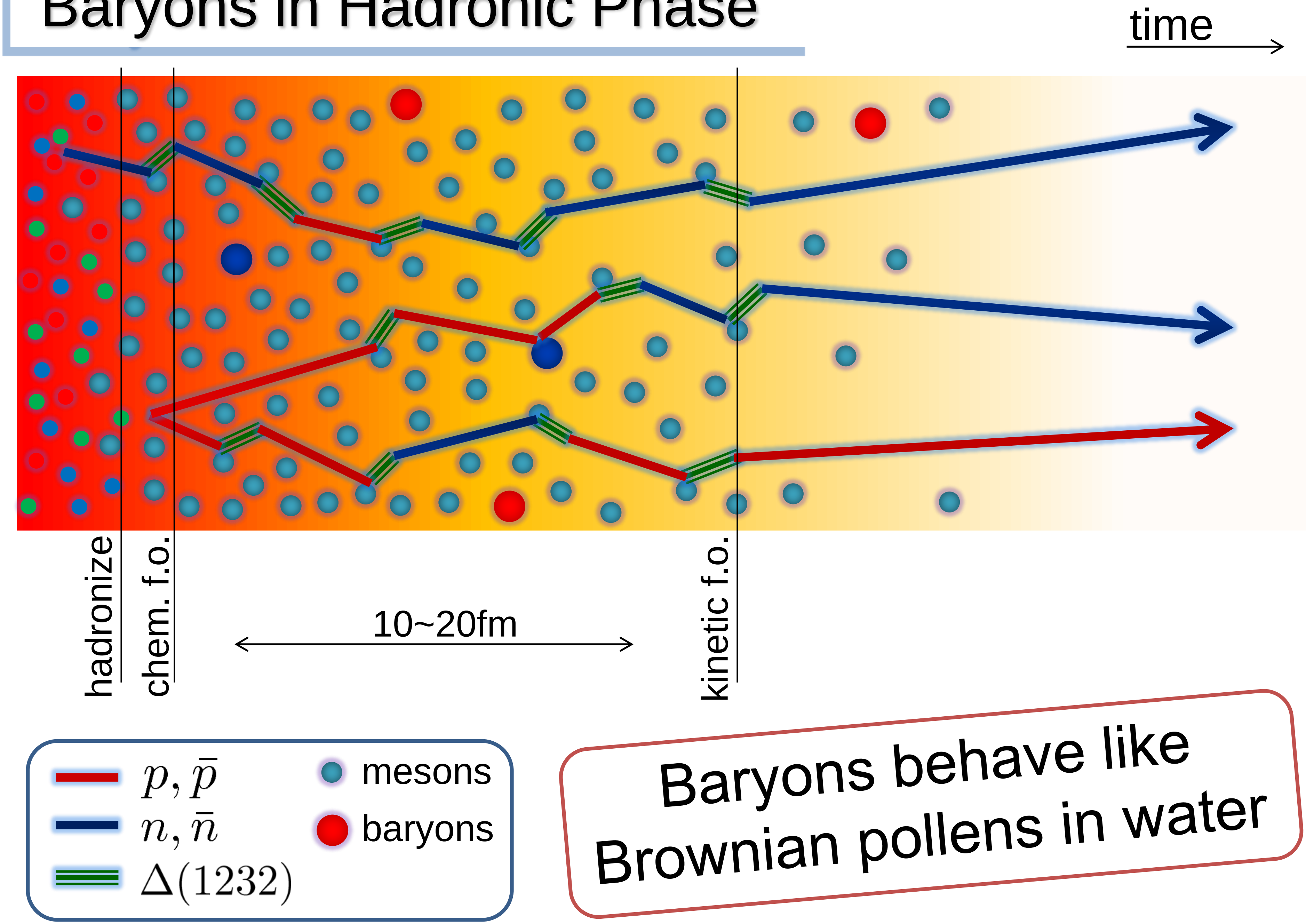
A Brownian Particle's Model



① Describe time evolution of Brownian particles exactly

② Obtain cumulants of particle # in $\Delta\eta$

Baryons in Hadronic Phase

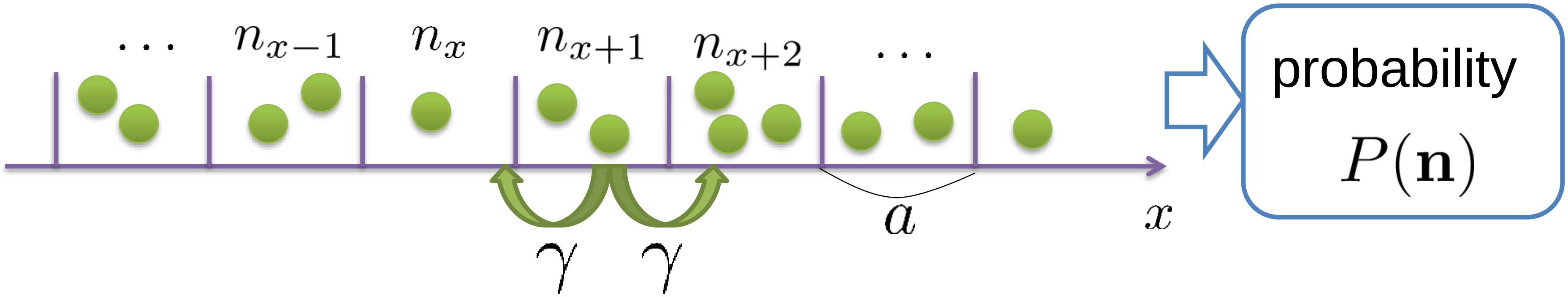


Baryons behave like
Brownian pollens in water

Diffusion Master Equation

MK, Asakawa, Ono, 2014

Divide spatial coordinate into discrete cells



Diffusion Master Equation

MK, Asakawa, Ono, 2014

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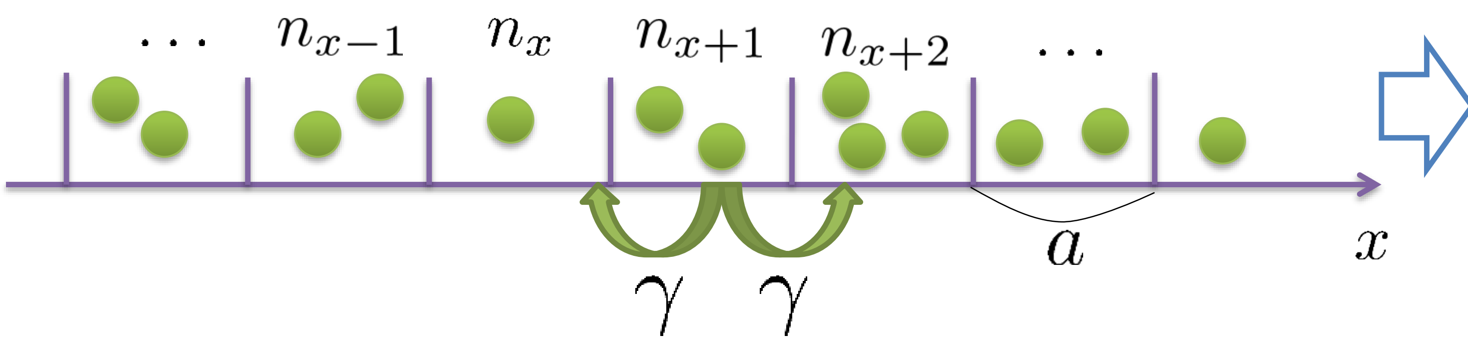


probability

$P(\mathbf{n})$

Master Equation for $P(n)$

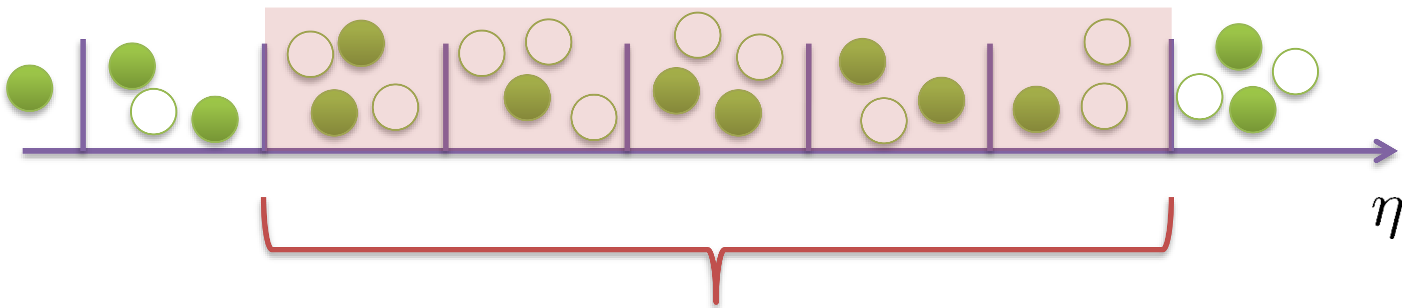
$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x+1)\{P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x+1}) + P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x-1})\} - 2n_x P(\mathbf{n})]$$

Solve the DME **exactly**, and take *a*→0 limit

No approx., ex. van Kampen’s system size expansion

Net Charge Number

Prepare 2 species of (non-interacting) particles

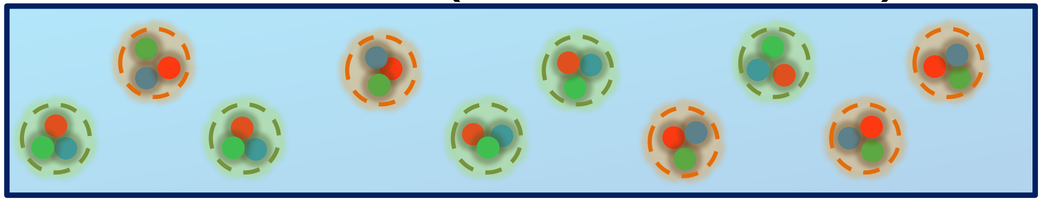


$$\bar{Q}(\tau) = \int_0^{\Delta\eta} d\eta \, (n_1(\eta,\tau) - n_2(\eta,\tau))$$

Time evolution of $\overline{\mathrm{Q}}$ up to Gauusianity is consistent with the stochastic diffusion equation

Time Evolution in Hadronic Phase

Hadronization (initial condition)



- Boost invariance / infinitely long system
- Local equilibration / local correlation

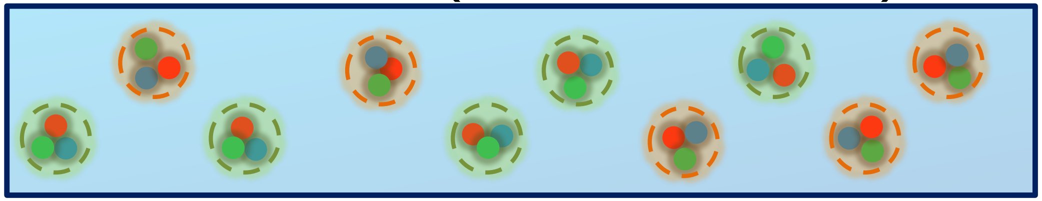
$\langle \bar{Q}^2 \rangle_c, \ \langle \bar{Q}^3 \rangle_c, \ \langle \bar{Q}^4 \rangle_c$ $\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ $\langle Q^2_{(\mathrm{tot})} \rangle_c, \langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c$

suppression owing to local charge conservation

strongly dependent on hadronization mechanism

Time Evolution in Hadronic Phase

Hadronization (initial condition)



Time evolution via DME

- Boost invariance / infinitely long system
- Local equilibration / local correlation

$\langle \bar{Q}^2 \rangle_c, \ \langle \bar{Q}^3 \rangle_c, \ \langle \bar{Q}^4 \rangle_c$ $\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ $\langle Q^2_{(\mathrm{tot})} \rangle_c, \langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c$

suppression owing to local charge conservation

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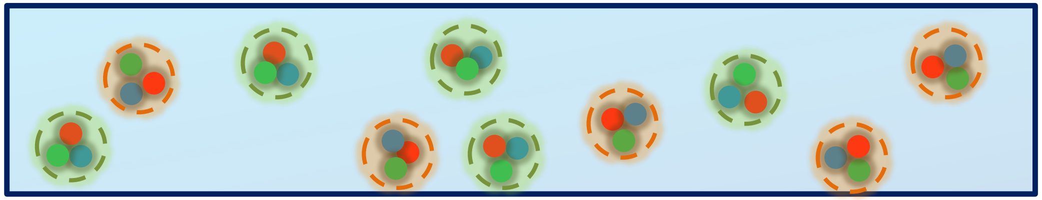
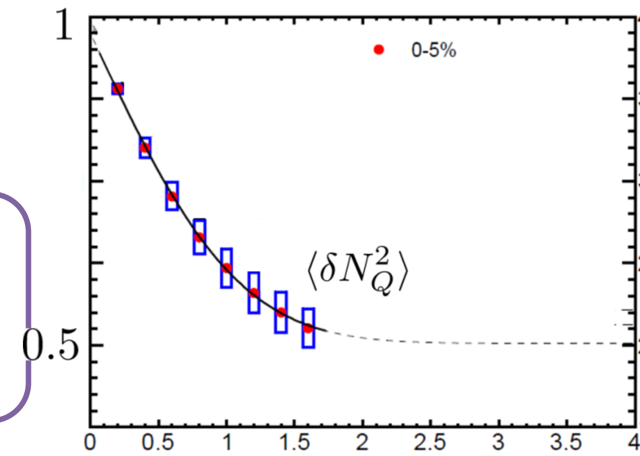
Freezeout

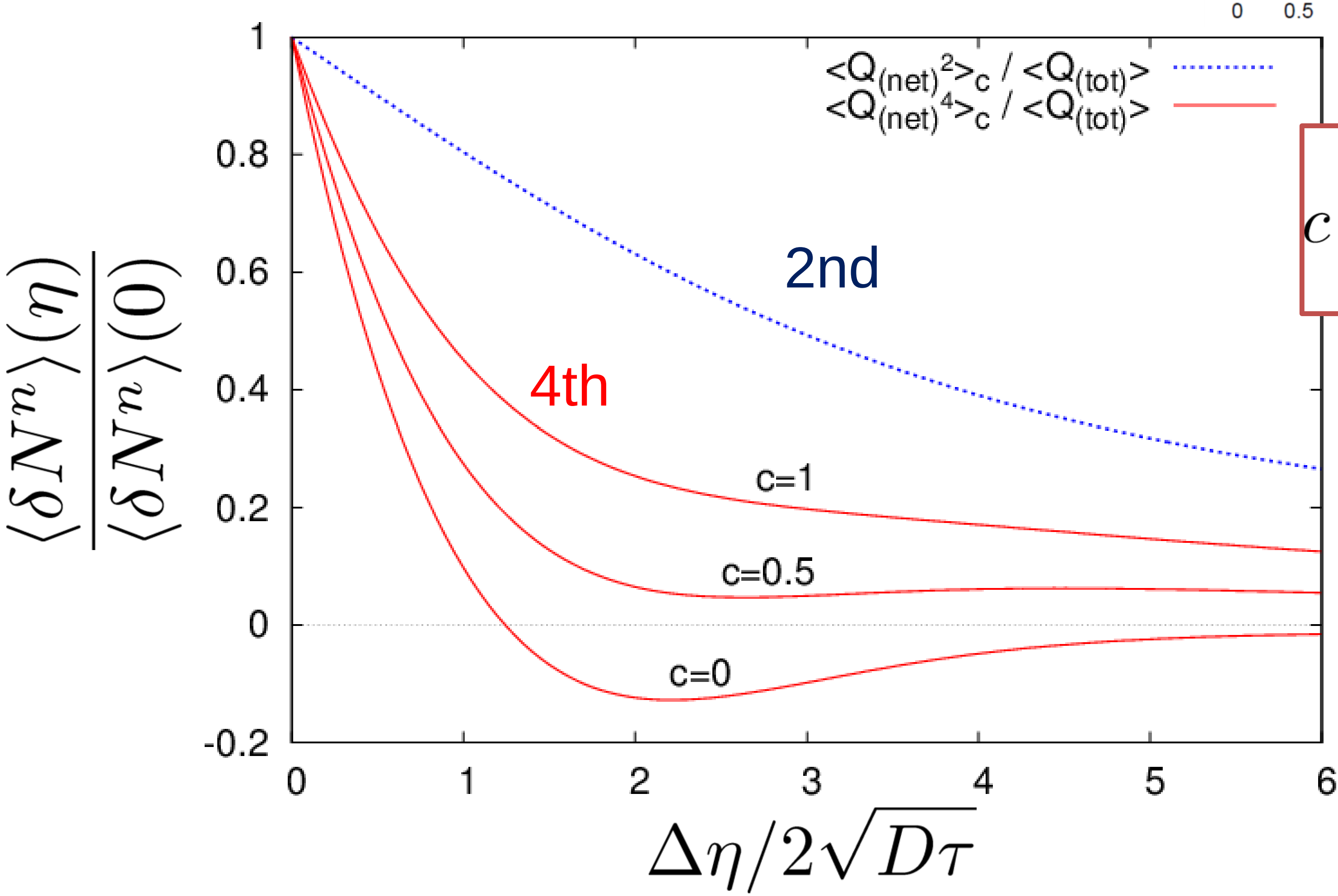


$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$



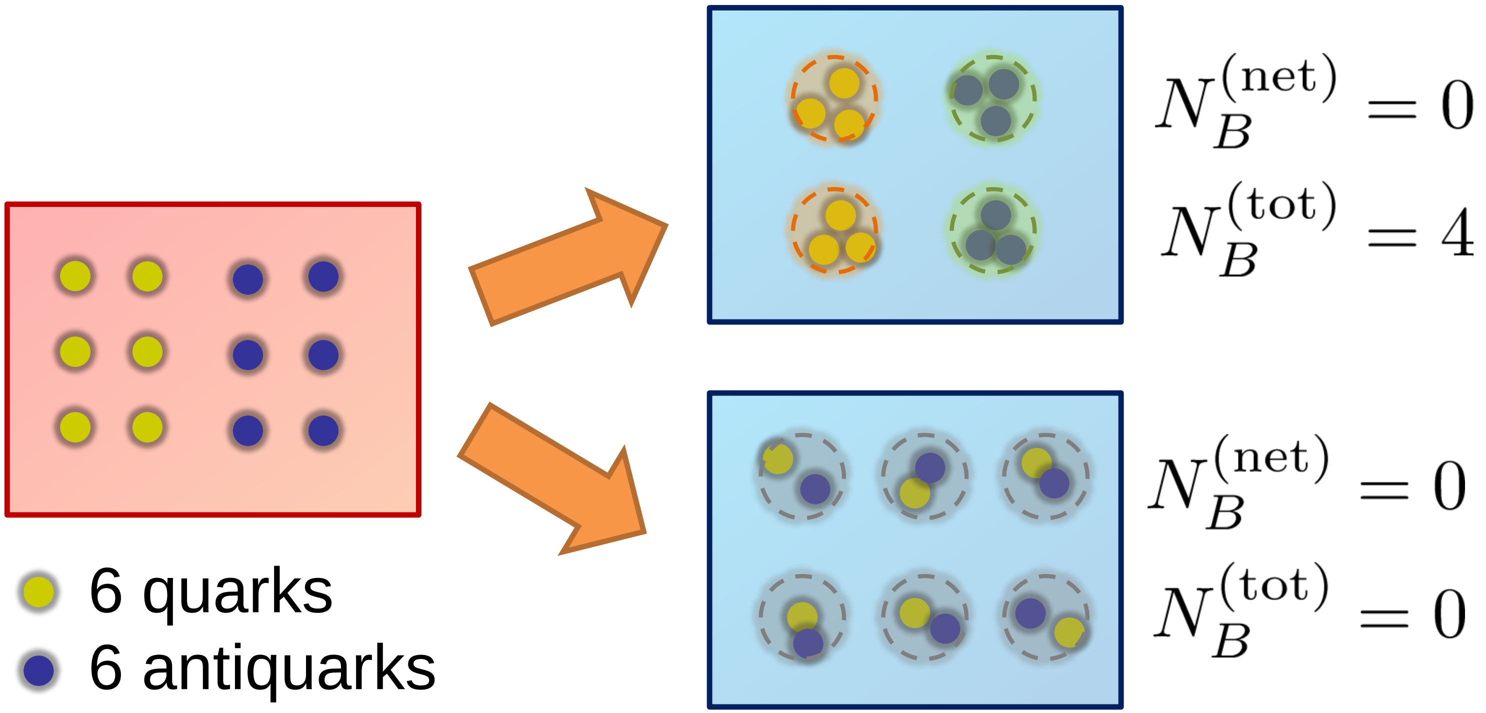


$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

Total Charge Number

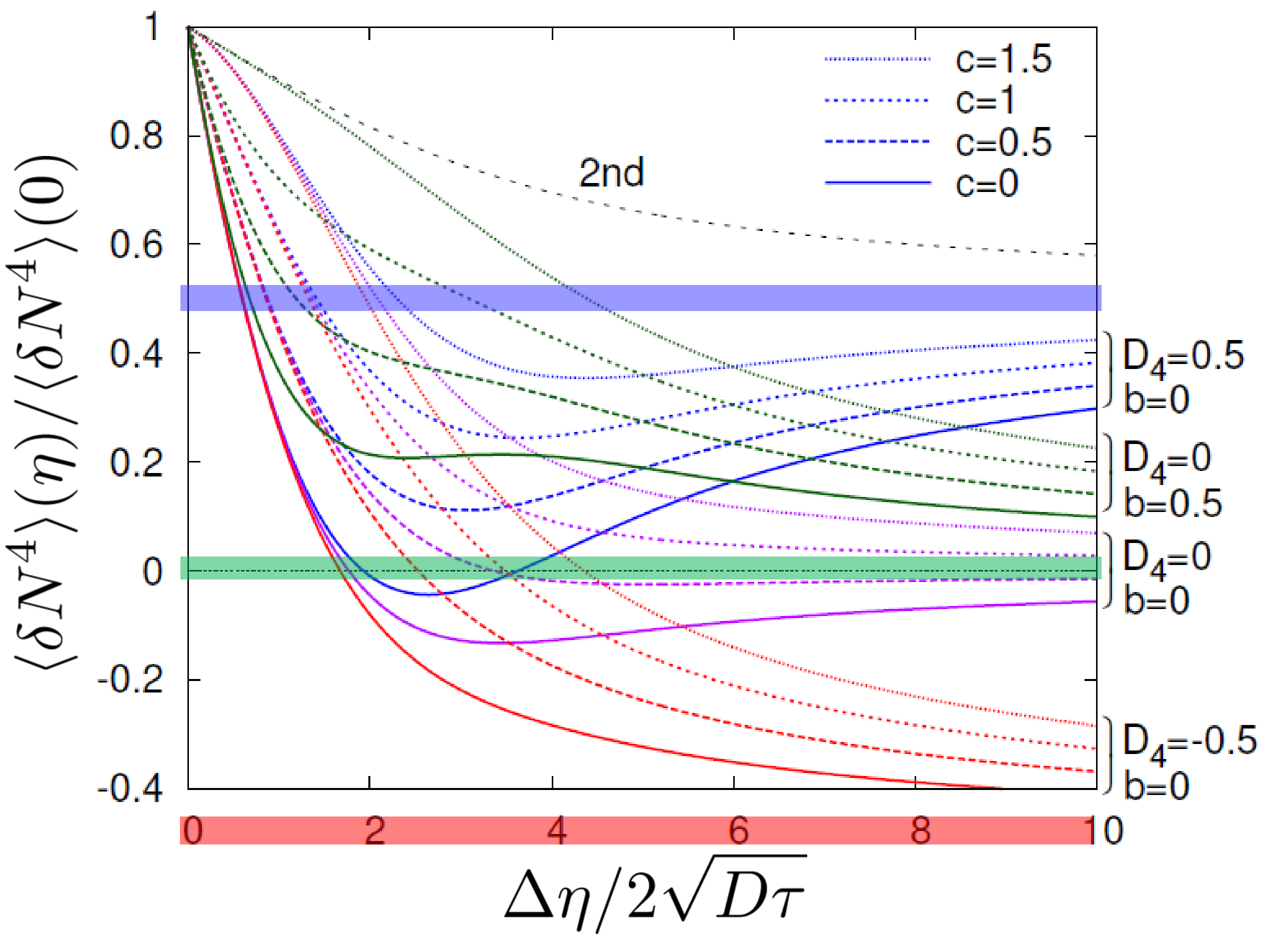
In recombination model,



- $N_B^{(\mathrm{tot})}$ can fluctuate, while $N_B^{(\mathrm{net})}$ does not.

Δη Dependence: 4th order

MK, to appear soon



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

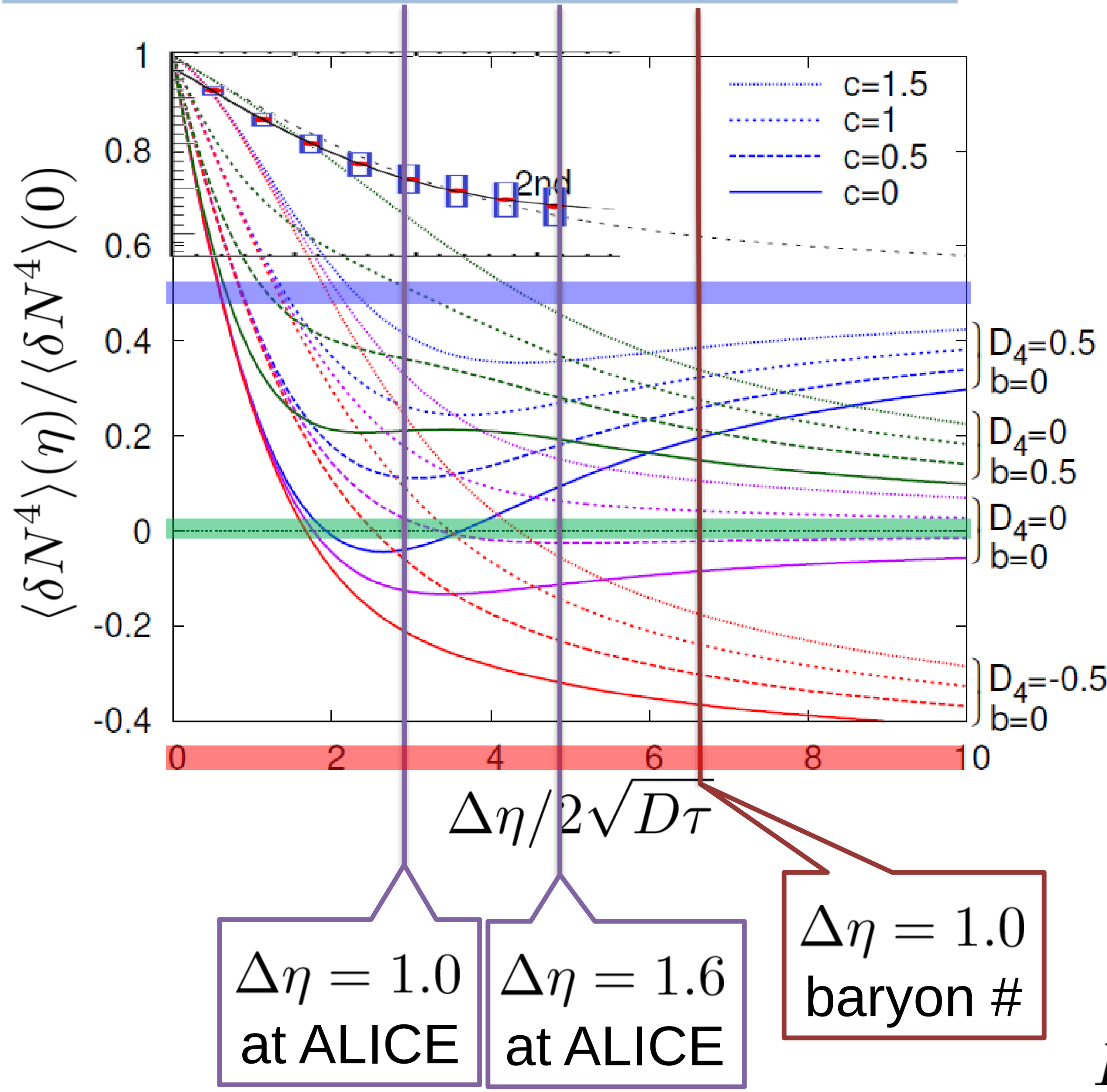
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

Charcteristic Δη dependences!

Δη Dependence: 4th order

MK, to appear soon



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$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

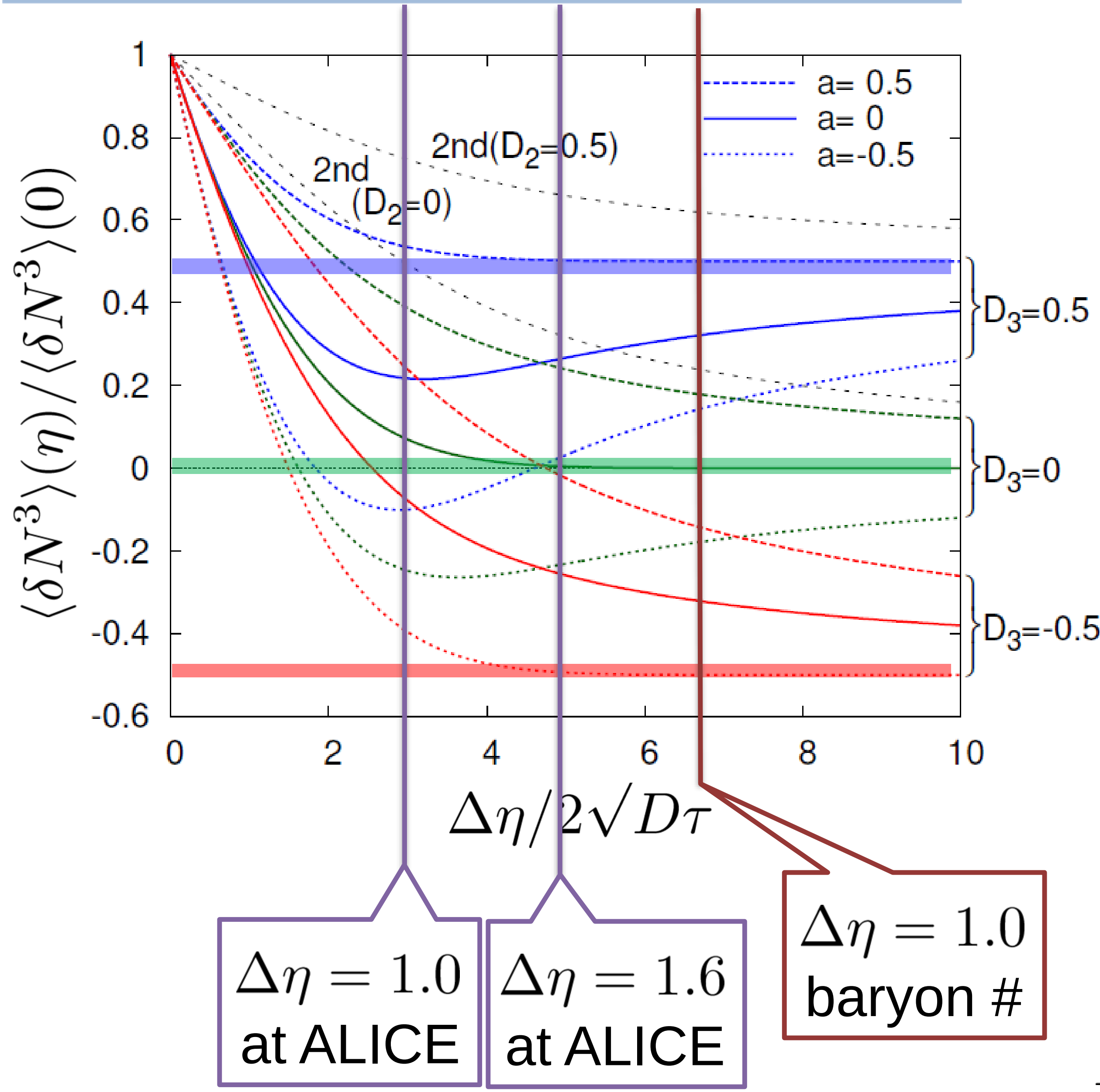
$$c = \frac{\langle Q^2_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} = 0.5$$

$$D \sim M^{-1}$$

Δη Dependence: 3rd order

MK, to appear soon



Initial Condition

$$D_3 = \frac{\langle Q^3_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$a = \frac{\langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

$$D \sim M^{-1}$$

Problems

- Relaxation from a specific initial condition to Skellam value
  - We assume perfect equilibration at T_c
  - Cumulants near T_c can be studied by Lattice.

- No pair creation/annihilation below T_{chem}
  - For baryons, this would be justified below T_c
  - But, may not for electric charges.

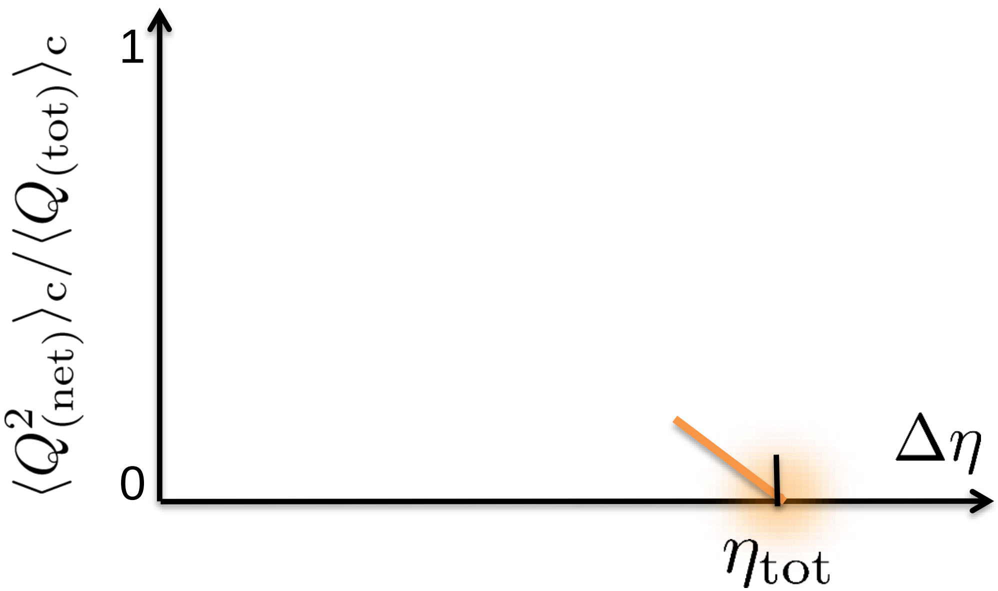
- No finite volume effect

Effect of Global Charge Conservation (Finite Volume Effect)

Sakaida, Asakawa, MK, arXiv:1409.6866

Global Charge Conservation

Conserved charges in the total system do no fluctuate!



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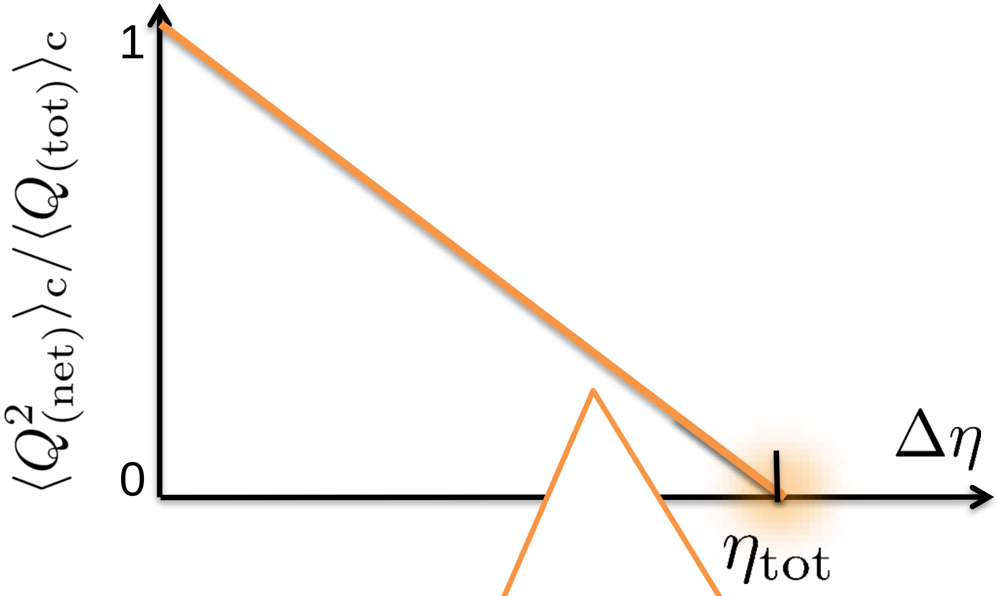


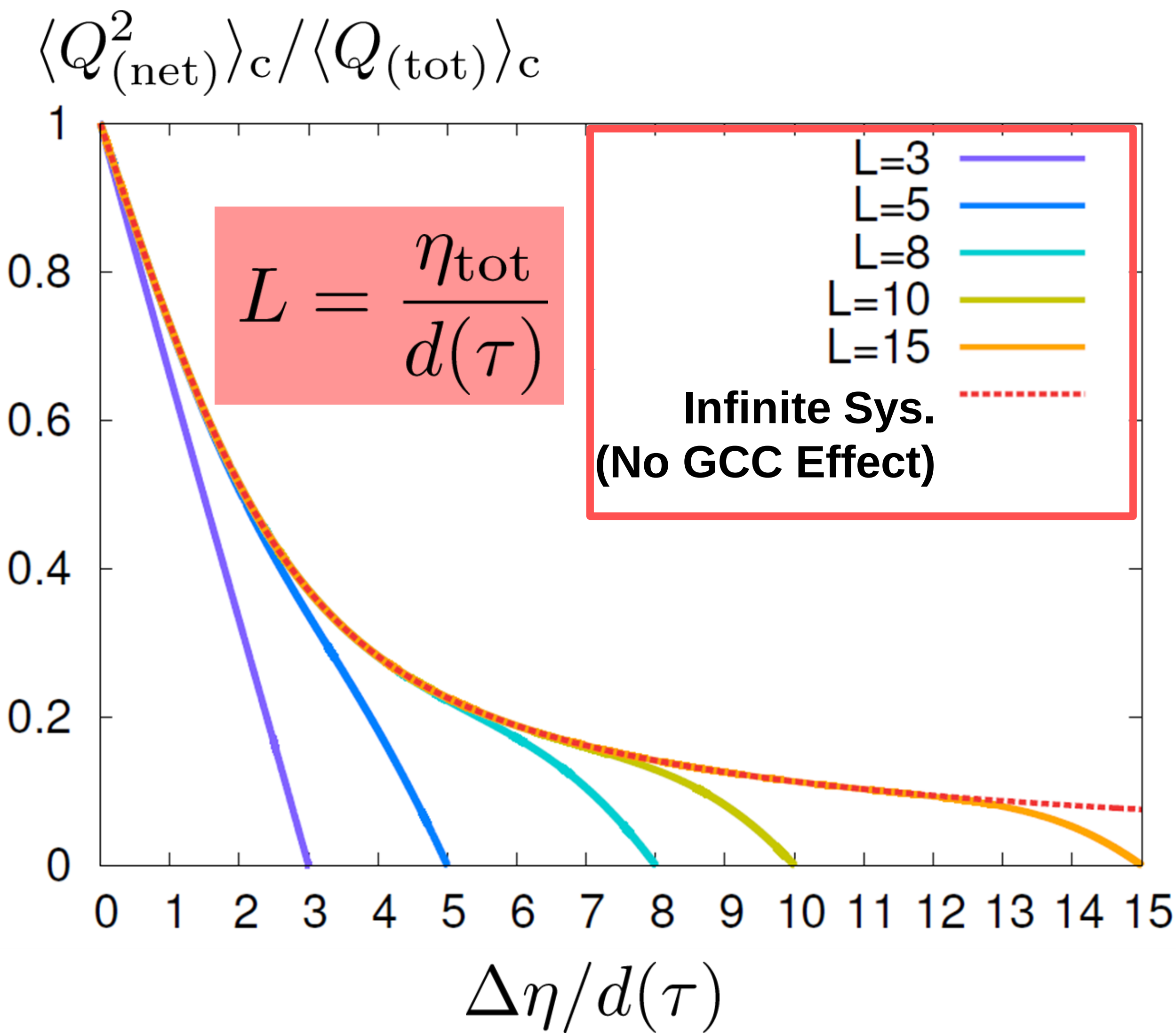
An Estimate of GCC Effect

$$\langle \delta N^2 \rangle_{\mathrm{GCC}} = \langle \delta N^2 \rangle_{\mathrm{inf}} \times \left(1 - \frac{\Delta\eta}{\eta_{\mathrm{tot}}} \right)$$

Jeon, Koch, PRL2000; Bleicher, Jeon, Koch (2000)

Diffusion in Finite Volume

Solve the diffusion master equation in finite volume



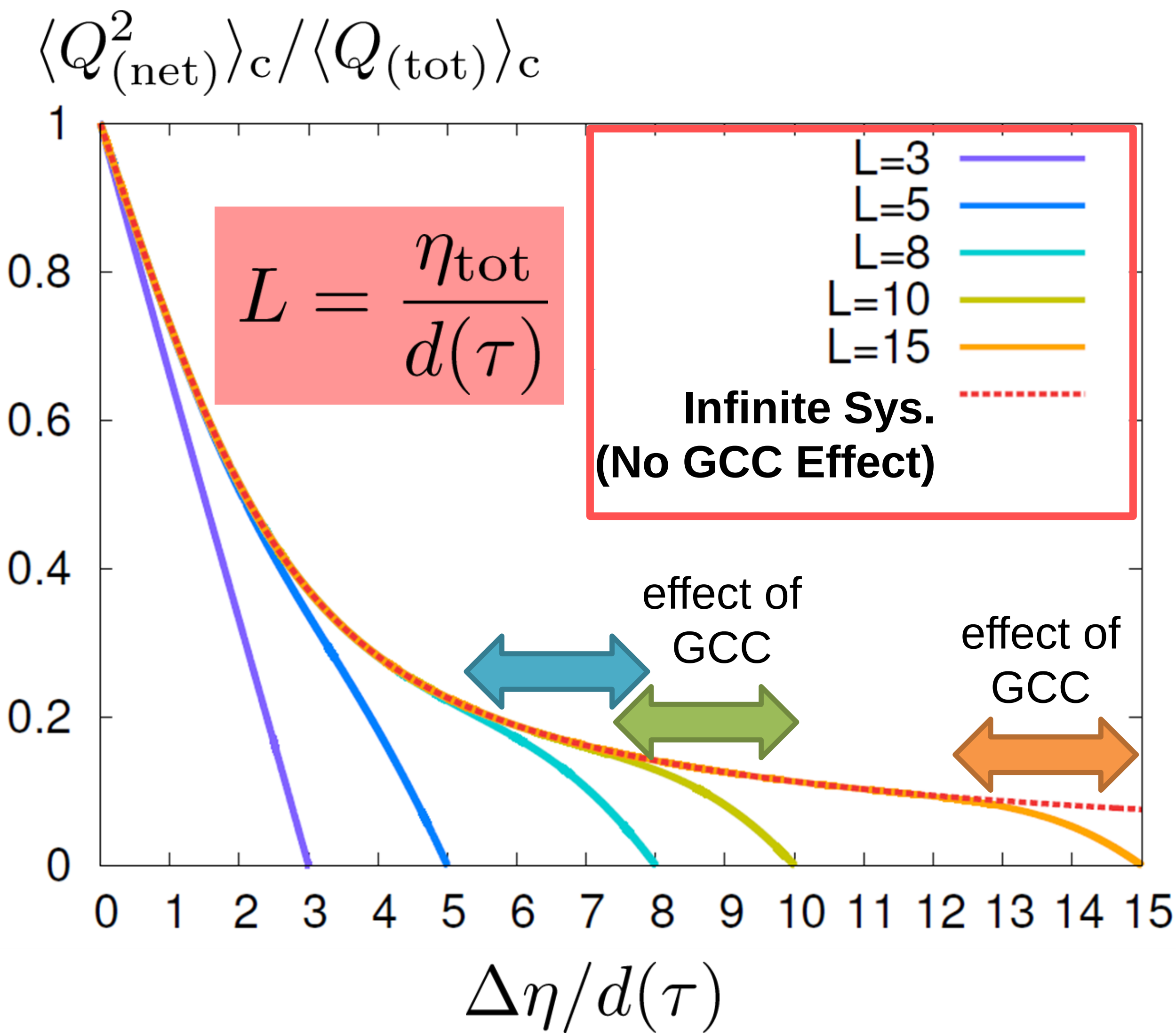
$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

: Average Diffusion Length

$D(\tau)$: Diffusion Coefficient

Diffusion in Finite Volume

Solve the diffusion master equation in finite volume



$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

: Average Diffusion Length

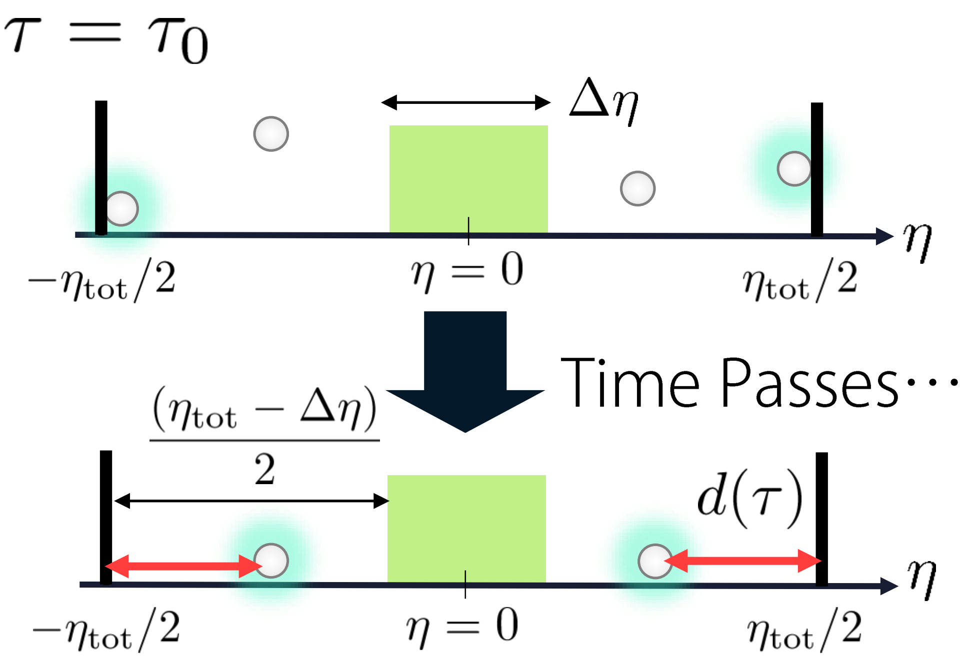
$D(\tau)$: Diffusion Coefficient

suppression only for

$$\Delta\eta/d \geq L - 2$$

Physical Interpretation

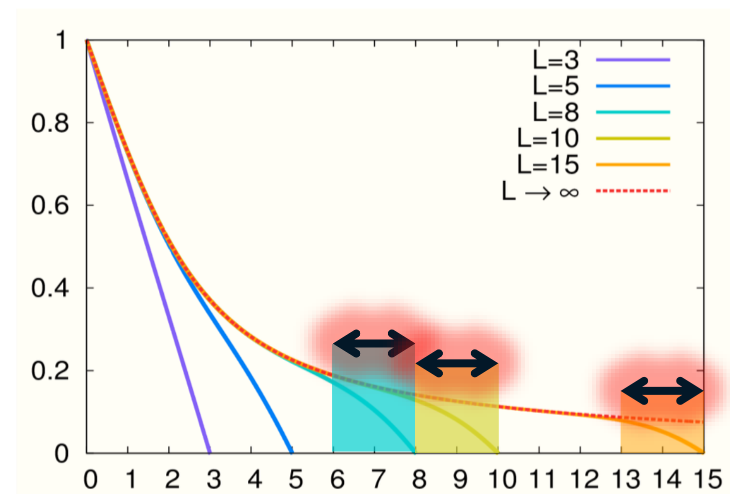
slide by M. Sakaida



$d(\tau)$: Averaged Diffusion Distance

$D(\tau)$: Diffusion Coefficient

η_{tot} : Total Length of Matter

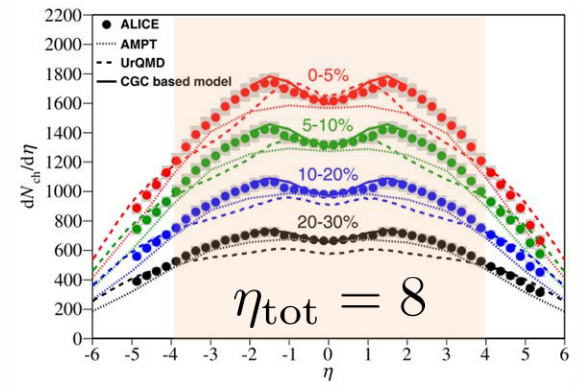


Condition for effects of the GCC

$$\Delta\eta/d \geq L - 2 \Leftrightarrow \frac{\eta_{tot} - \Delta\eta}{2} \leq d$$

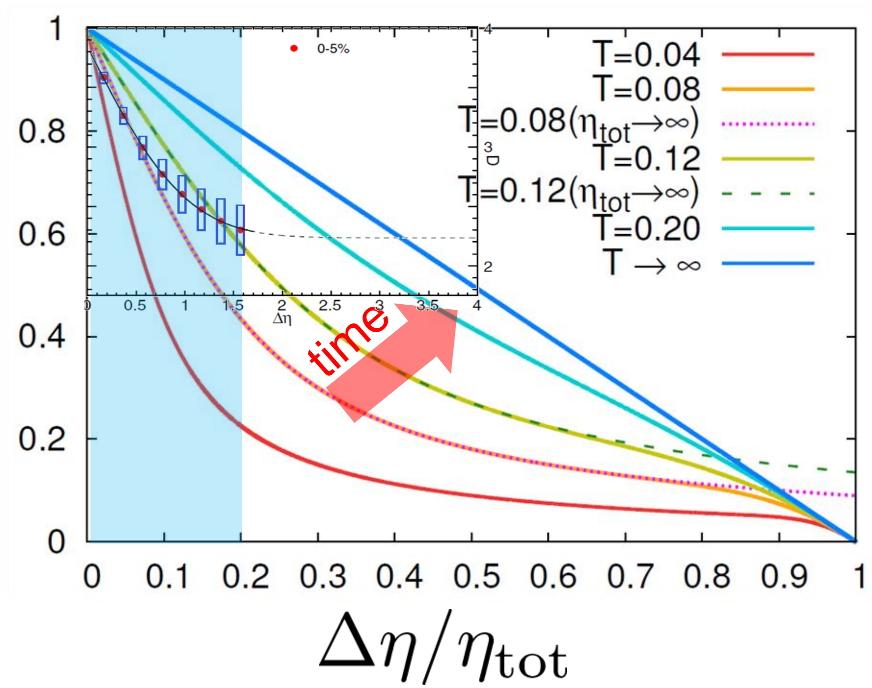
Effects of the GCC appear only near the boundaries.

Comparison with ALICE Result

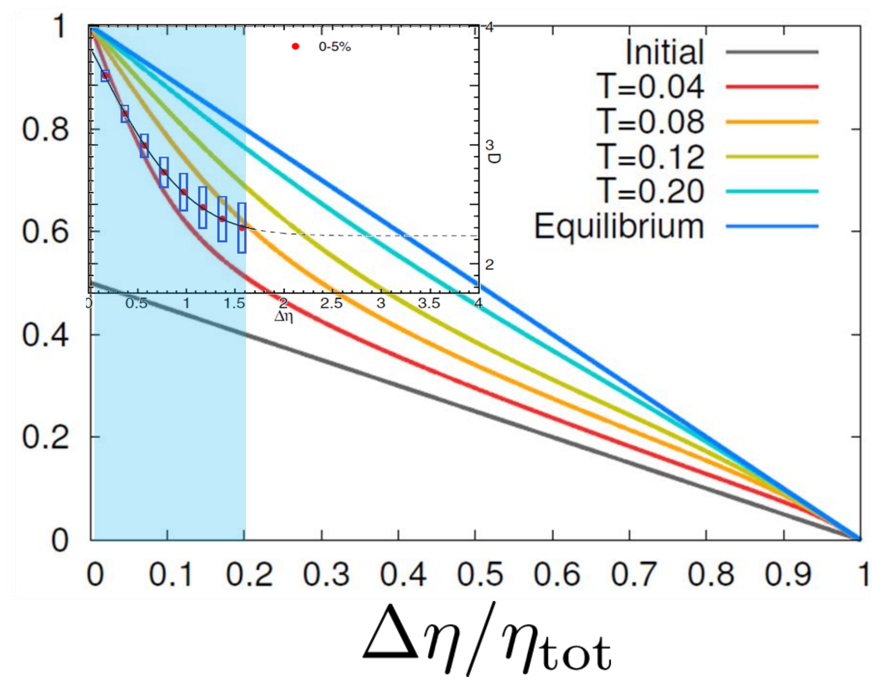


$$\langle Q^2_{(net)} \rangle_c / \langle Q_{(tot)} \rangle_c$$

without initial fluc.



with initial fluc.

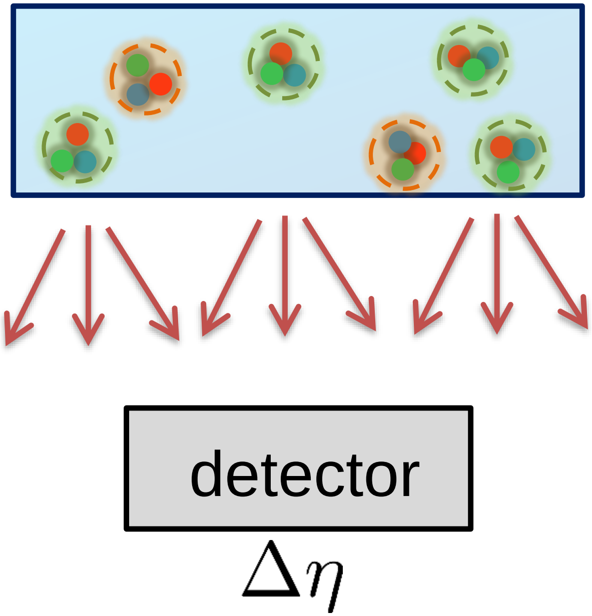


- No GCC effect in ALICE experiments!
- Same conclusion for higher order cumulants

$$T = \frac{d(\tau)}{\eta_{tot}}$$

Very Low Energy Collisions

- Large contribution of global charge conservation
- Violation of Bjorken scaling



Careful treatment is required to interpret fluctuations at low beam energies!

Many information should be encoded in $\Delta\eta$ dep.

Summary

- Measurement of non-Gaussianity at STAR
- Non-equilibrium behavior at ALICE

➡ Theoretical description to treat both "non"s is needed!

Plenty of information in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2 \rangle_c,\ \langle N_Q^3 \rangle_c,\ \langle N_Q^4 \rangle_c,\ \langle N_B^2 \rangle_c,\ \langle N_B^3 \rangle_c,\ \langle N_B^4 \rangle_c$$

and those of non-conserved charges, mixed cumulants…

Many Things to Do

- Message to Experimentalists:
  - Measure rapidity window dependences
  - Determine baryon number cumulants
- Message to Theorists:
  - Do not directly compare your thermal results with exp.
  - Let's pursue descriptions of non-eq. non-Gaussianity.
- Message to Latticians:
  - Do not directly compare your results with exp.
  - Measure more cumulants more accurately

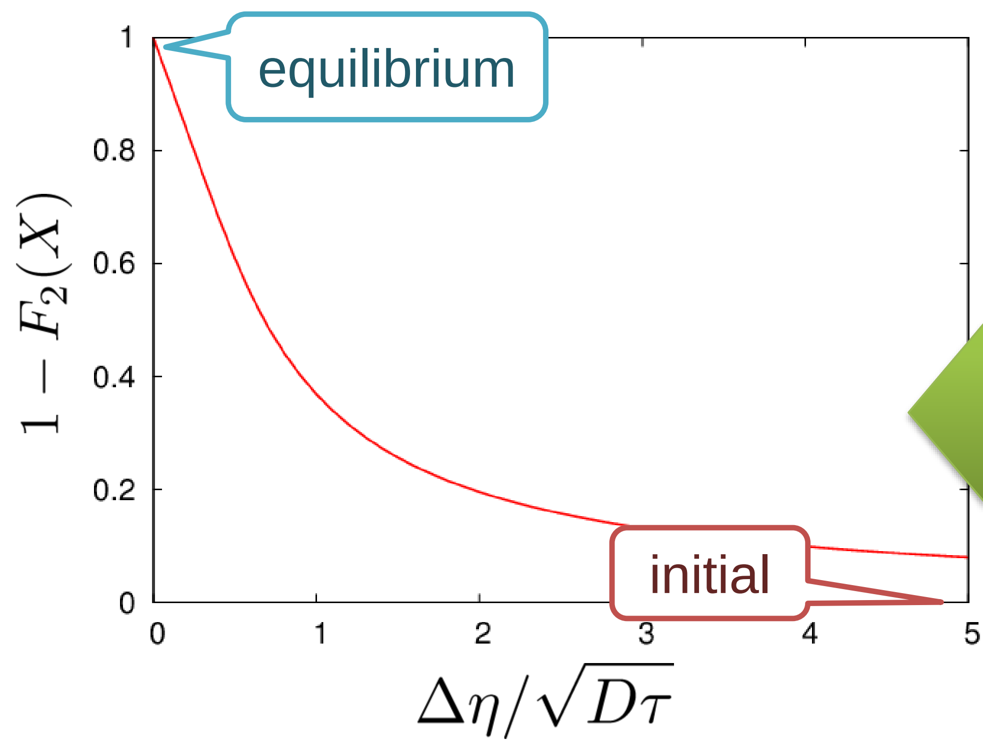
$\Delta\eta$ Dependence

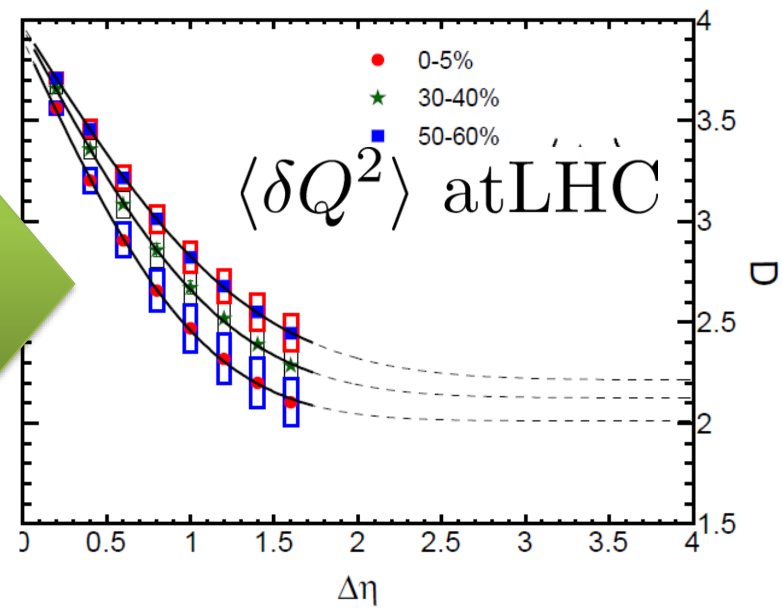
Shuryak, Stephanov, 2001

- Initial condition: $\langle \delta n(\eta_1, 0)\delta n(\eta_2, 0)\rangle = \sigma_2 \delta(\eta_1 - \eta_2)$
- Translational invariance

$$Q(\tau) = \int_0^{\Delta\eta} d\eta\, n(\eta, \tau)$$

$$\langle \delta Q(\tau)^2 \rangle = \underbrace{\sigma_2 F_2(X)}_{\text{initial}} + \underbrace{\chi_2 (1 - F_2(X))}_{\text{equilibrium}}$$





Open Questions & Future Work

- Why the primordial fluctuations are observed only at LHC, and not RHIC ?
- Extract more information on each stage of fireballs using fluctuations

- Model refinement
  - Including the effects of
    nonzero correlation length / relaxation time
    global charge conservation

  - Non Poissonian system ← interaction of particles