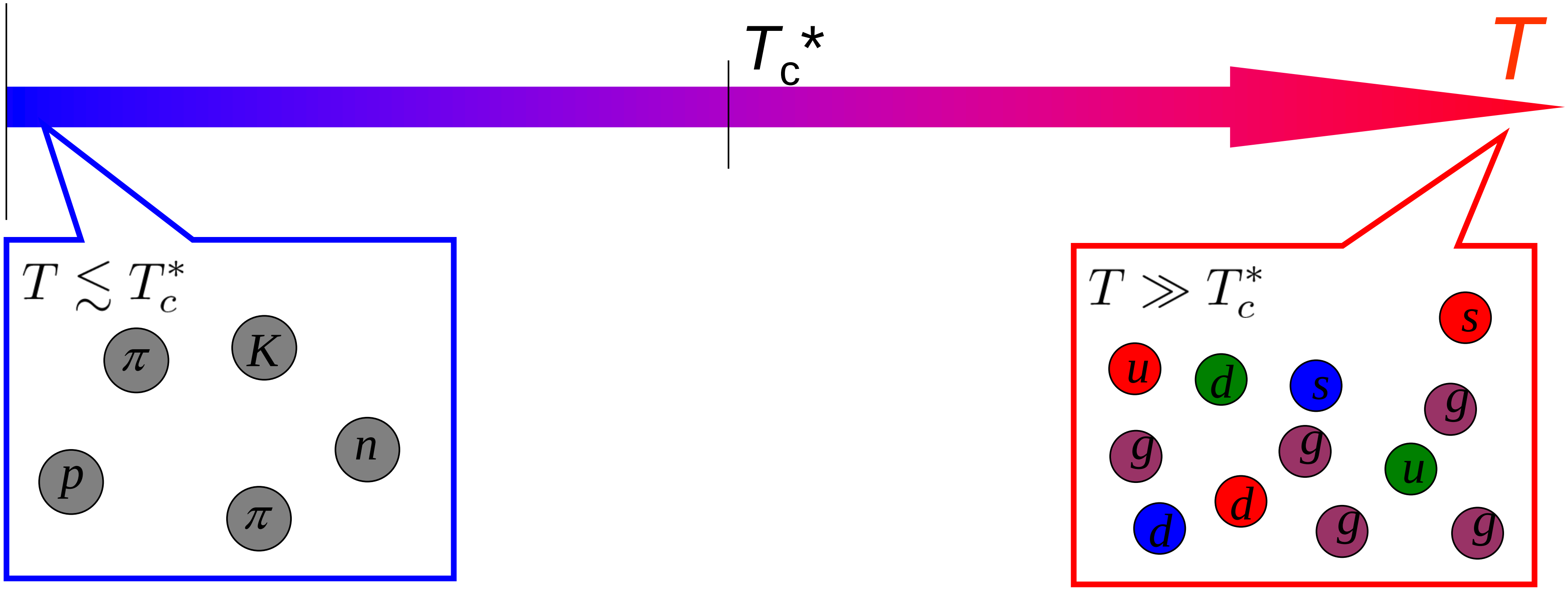
非閉じ込め相における クォーク準粒子描像と レプトン対生成率

北沢正清
（阪大）

Karsch, MK, PL**B658**, 49 (2007); PRD80, 056001 (2009);
Kaczmarek, Karsch, MK, Soeldner, PRD86, 036006 (2012);
Kim, Asakawa, MK, in preparation.

筑波大学セミナー、２０１４年１０月３０日

有限温度(ゼロ密度)QCD



QCDの低温および高温極限は単純な描像が適用でき、
かつそれらは全く異なるのに連続的につながっている。

中間領域では何が起こっているのだろうか？
低温のハドロンの運命は？高温のクォークの運命は？

Are There Quark Quasi-Particles in sQGP?

In **real** experiments:

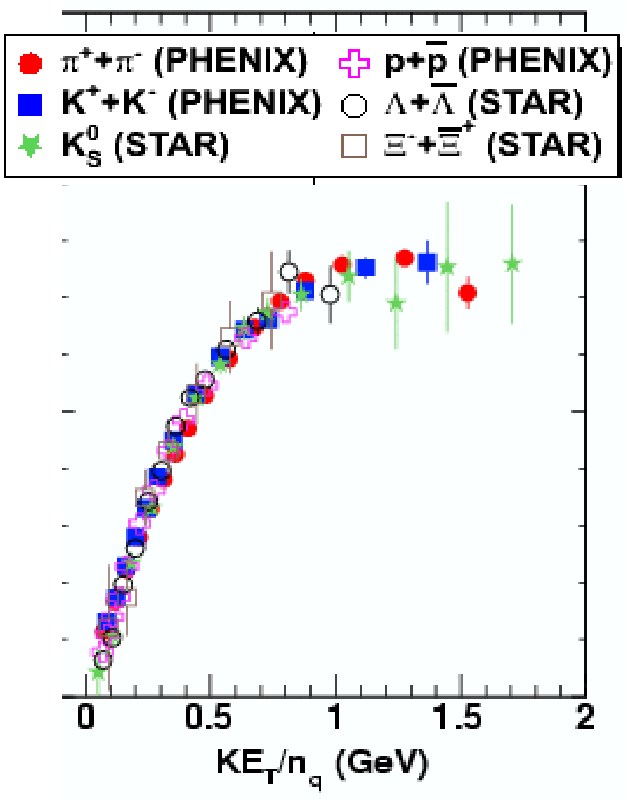
In **numerical** experiments:

Are There Quark Quasi-Particles in sQGP?

In **real** experiments:

In **numerical** experiments:

quark # scaling of v_2

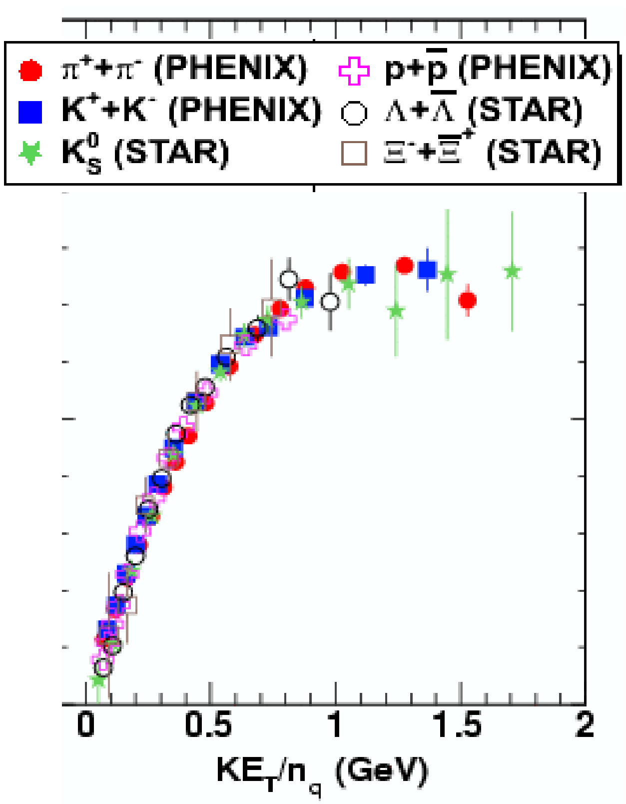


Fries, *et al*., '03

Are There Quark Quasi-Particles in sQGP?

In real experiments:

quark # scaling of v_2



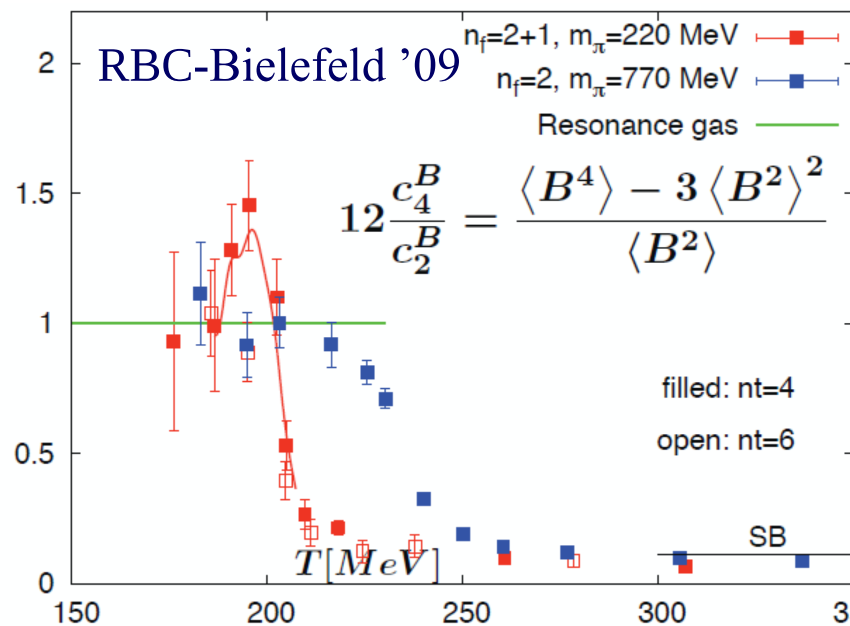
Fries, *et al.*, '03

In numerical experiments:

ratio c_4/c_2 $\begin{cases} c_2 = -\partial^2\Omega/\partial\mu_B^2 \\ c_4 = -\partial^4\Omega/\partial\mu_B^4 \end{cases}$

with particles (mass m & charge q)

$m \ll T \Rightarrow c_4/c_2 \sim q^2$



Ejiri, Karsch, Redlich, '05

ボルツマンの苦悩

１900年頃

Boltzmann

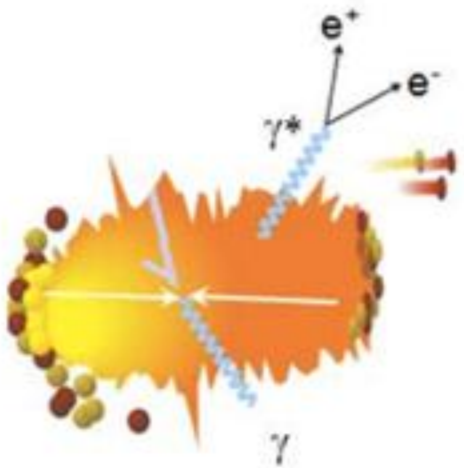
物質は原子から
構成されている

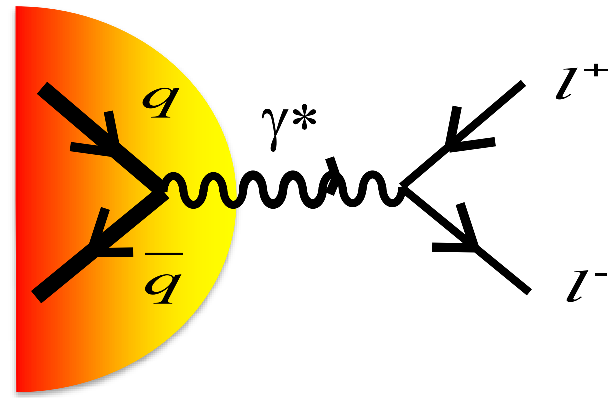
原子など考えても無
意味だ。観測しよう
がないのだから。

Mach

photos from Wikipedia

レプトン対生成率





レプトン・光子は強い
相互作用をしない

↓

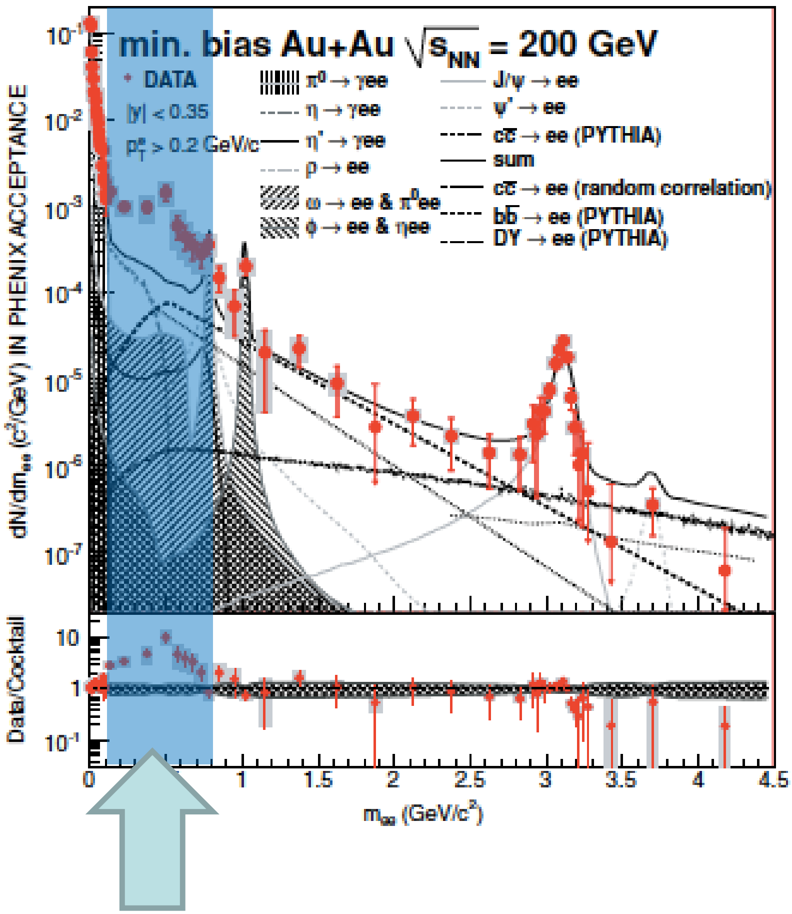
レプトン対・光子は、高温
物質からの直接的信号

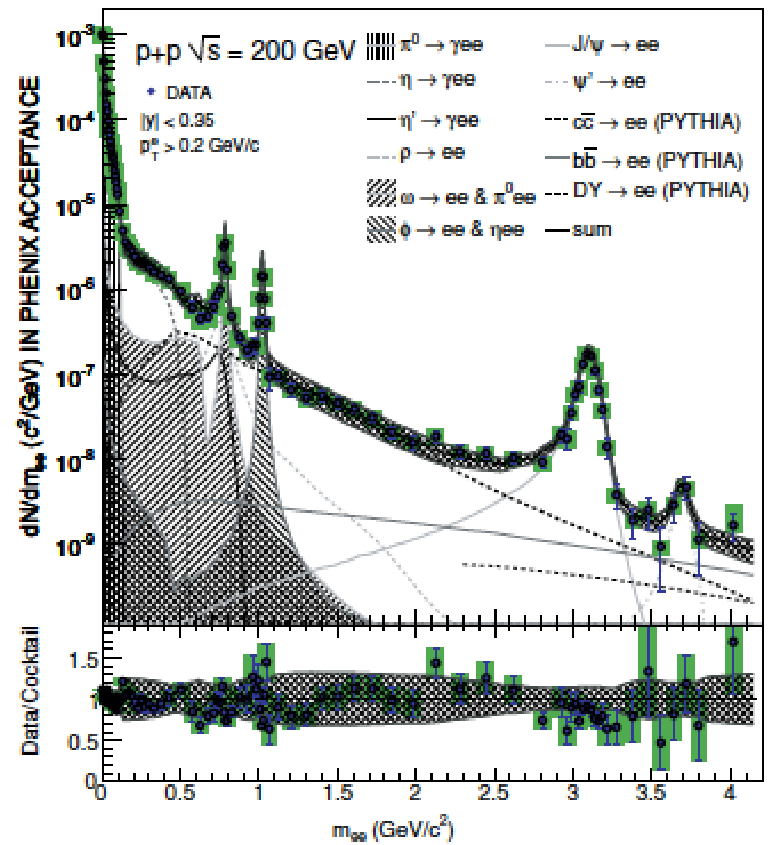
QGPでの仮想光子生成過程
||
クォーク・反クォーク対消滅

↓

クォークの準粒子描像を反映

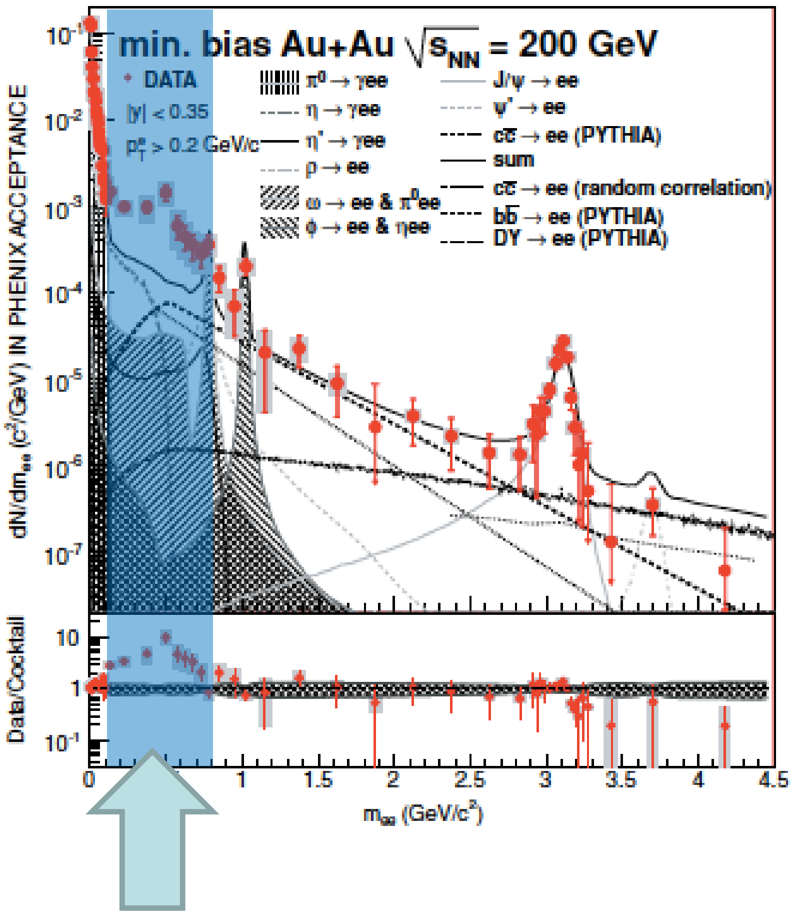
Dilepton Production Rate @ RHIC

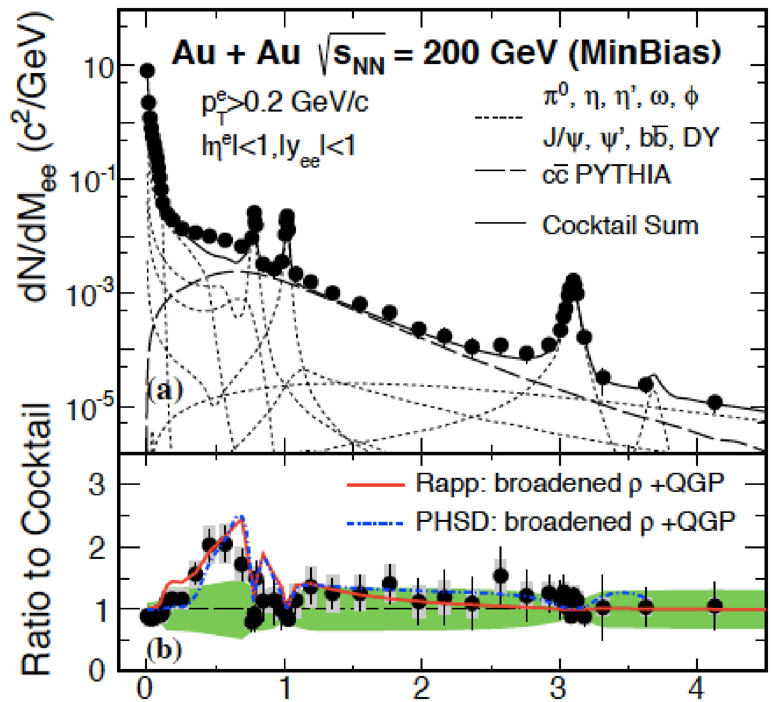




低不変質量で顕著な増大
QGP由来のレプトン対生成？

Dilepton Production Rate @ RHIC





qualitatively inconsistent result @ STAR

低不変質量で顕著な増大
QGP由来のレプトン対生成？

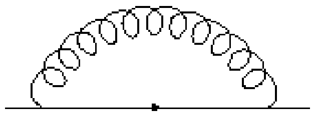
クォークスペクトル関数と準粒子描像

Quarks at Extremely High T

Klimov '82, Weldon '83
Braaten, Pisarski '89

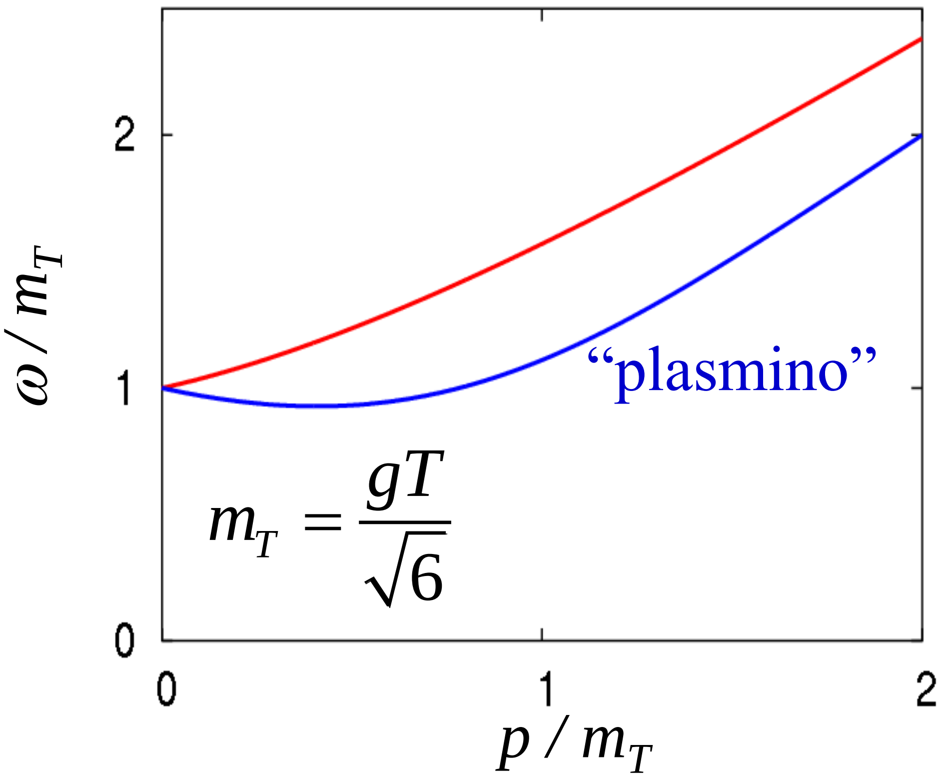
- Hard Thermal Loop approx. ($\boldsymbol{p}$, ω, $m_q \ll T$)
- 1-loop ($g \ll 1$)

$$\Sigma(\omega, \mathbf{p}) =$$



$$S(\omega, \mathbf{p}) = \frac{1}{\omega\gamma_0 - \mathbf{p}\cdot\boldsymbol{\gamma} - \Sigma(\omega, \mathbf{p})}$$

- Gauge independent spectrum
- 2 collective excitations having a "thermal mass" ~ gT
- width ~g^2T
- The plasmino mode has a minimum at finite $\boldsymbol{p}$.



Decomposition of Quark Propagator

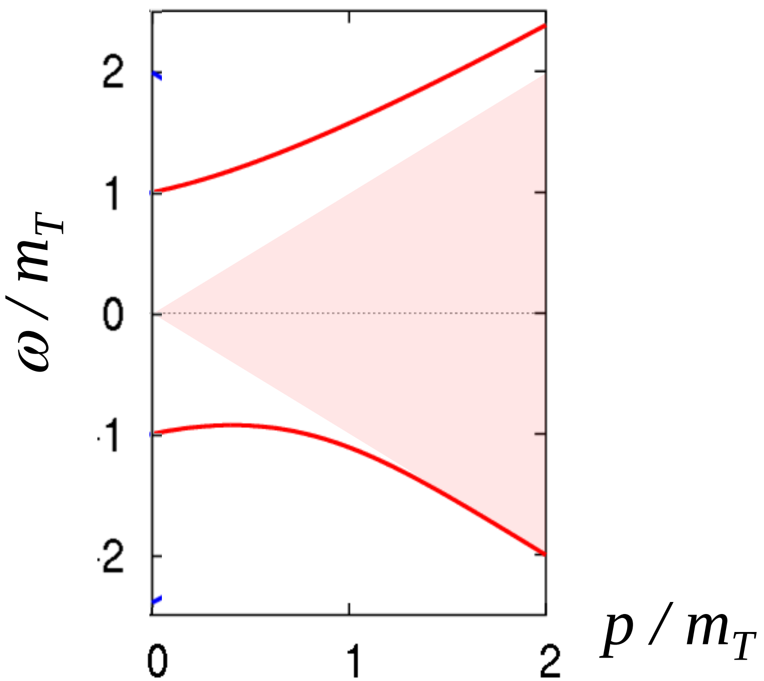
$$S(\omega,\mathbf{p}) = S_+(\omega,\mathbf{p})\Lambda_+(\vec{\mathbf{p}})\gamma^0 + S_-(\omega,\mathbf{p})\Lambda_-(\vec{\mathbf{p}})\gamma^0$$

$$\Lambda_\pm(\mathbf{p}) = \frac{E_\mathbf{p} \pm \gamma_0(\mathbf{p}\cdot\vec{\gamma} + m)}{2E_\mathbf{p}}$$

$$E_\mathbf{p} = \sqrt{\mathbf{p}^2 + m^2}$$

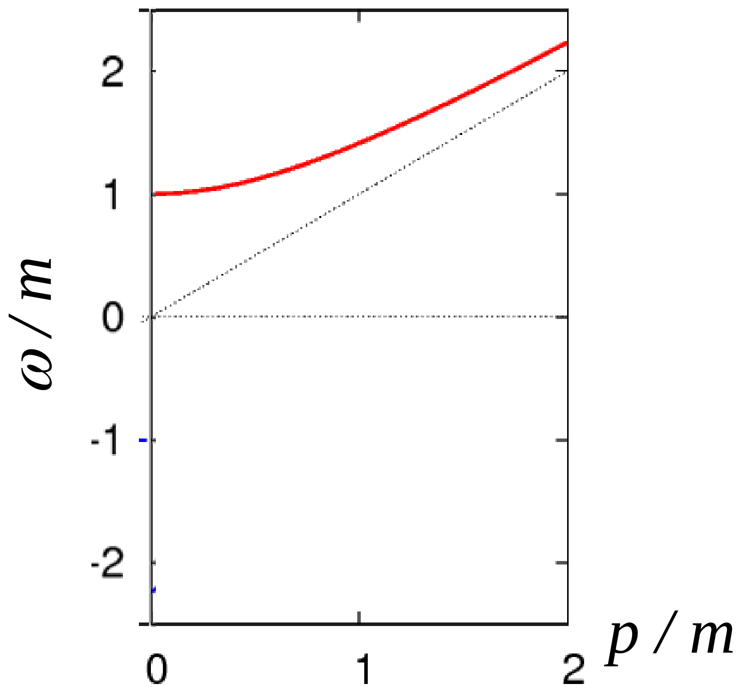
HTL (high T limit)

$$S_{\rm HTL}(\omega,\mathbf{p}) = \frac{L_+(\mathbf{p})\gamma^0}{\omega - p - \Sigma_+} + \frac{L_-(\mathbf{p})\gamma^0}{\omega + p - \Sigma_-}$$



Free quark with mass m

$$S_{\rm free}(\omega,\mathbf{p}) = \frac{\Lambda_+(\mathbf{p})\gamma^0}{\omega - E_\mathbf{p}} + \frac{\Lambda_-(\mathbf{p})\gamma^0}{\omega + E_\mathbf{p}}$$



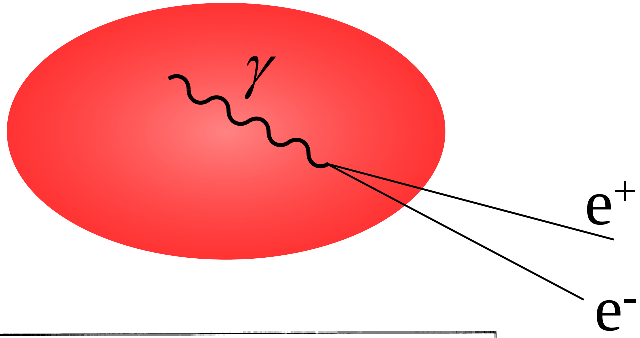
Effects of Plasmino Minimum

minimum @ $p>0$ ➡ divergence of DoS

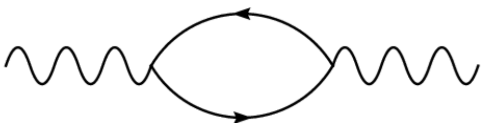
➡ **van Hove singularity**

- Example: dilepton production rate

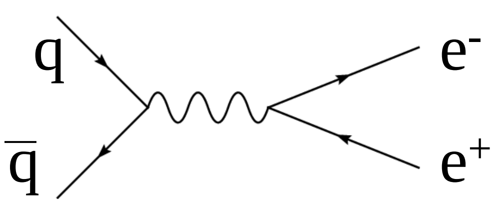
  observable in heavy-ion collisions

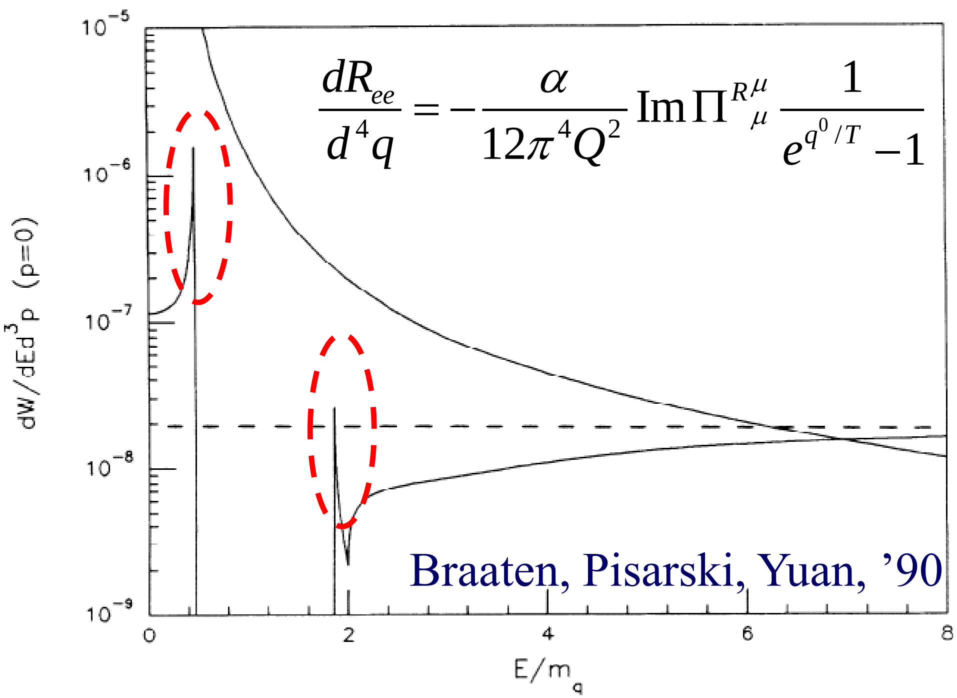


•photon self-energy:



•decay process:



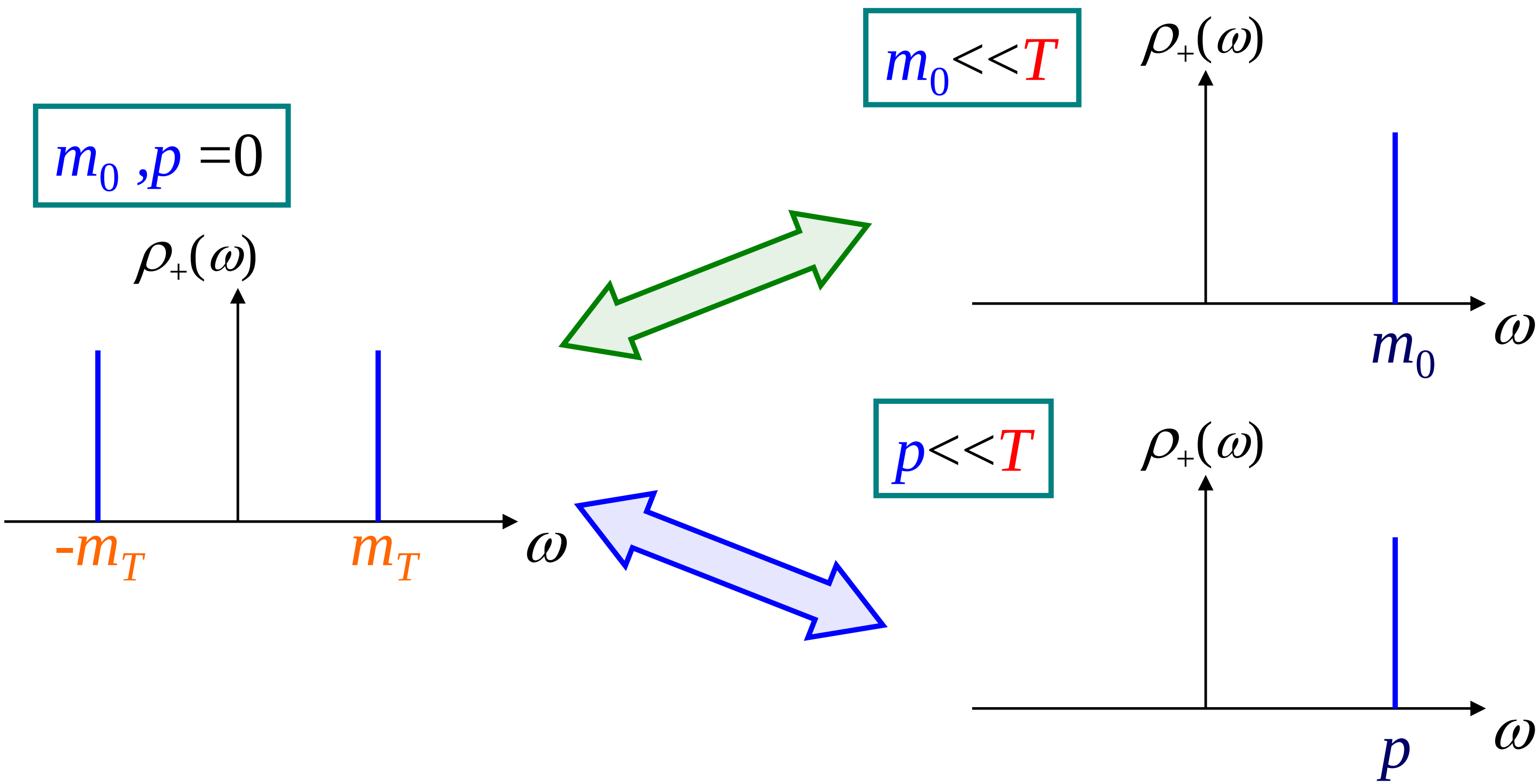


Lattice Study of Quarks above T_c

Karsch, MK, '07, '09

✓dispersion relation (p dependence)
✓bare quark mass (m_0) dependence

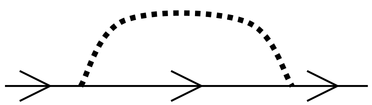
$$\rho(\omega) = \rho_+(\omega)\Lambda_+\gamma^0 + \rho_-(\omega)\Lambda_-\gamma^0$$



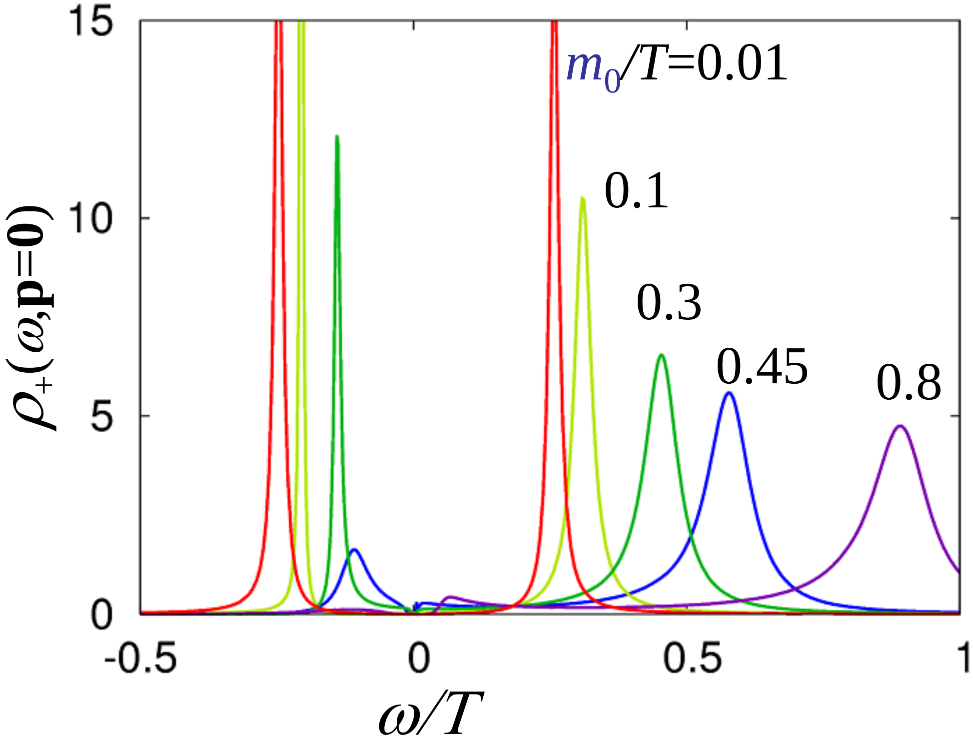
Fermion Spectrum in QED & Yukawa Model

Baym, Blaizot, Svetisky, '92

- Yukawa model: $L = i\bar{\psi}(i\partial\!\!\!/ - m_0 - g\sigma)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma$
- 1-loop approx.:



Spectral Function for g =1 , T =1



- m_0/T <<1
  thermal mass m_T=gT/4
- m_0/T >>1
  single peak at m_0

Plasmino peak disappears as m_0 /T becomes larger.

cf.) massless fermion + massive boson
MK, Kunihiro, Nemoto,'06

Extracting Spectral Functions

$$D(\tau) = \int_{-\infty}^{+\infty} d\omega K(\omega,\tau)\rho(\omega)$$

$$K(\omega,\tau) = \frac{e^{(\beta/2-\tau)\omega}}{e^{\beta\omega/2} \pm e^{-\beta\omega/2}}$$

$D(\tau) = \langle T_\tau O(\tau)O(0)\rangle$

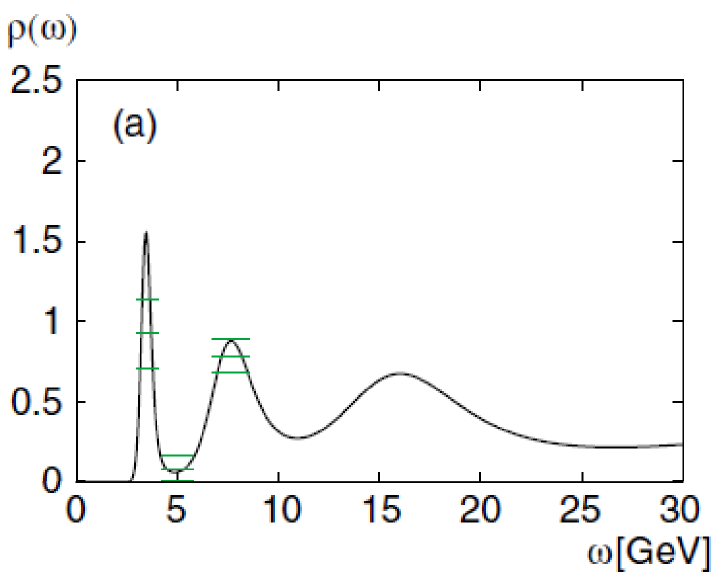
lattice observable

discrete and noisy

spectral function

continuous

Ill-posed problem



MEM analysis of $\rho(\omega)$

most probable image estimated by
lattice data + prior knowledge
Asakawa, Hatsuda, Nakahara, 1999

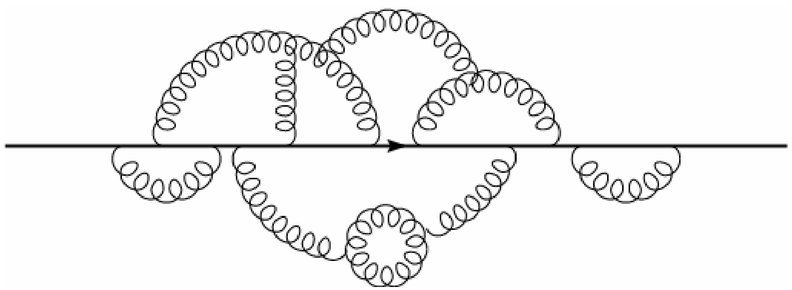
qualitative structure of $\rho(\omega)$.
errors only for average for finite range

Lattice Study of Quarks above T_c

Karsch, MK, '07, '09;
Kaczmarek, MK+,'12

- quenched approximation
- clover improved Wilson
- Landau gauge fixing

T/T_c	β	$N_x^3 \text{x} N_t$
3	7.45	128^3x16
		64^3x16
		48^3x16
	7.19	48^3x12
1.5	6.87	128^3x16
		64^3x16
		48^3x16
	6.64	48^3x12
1.25	6.72	64^3x16
		48^3x16



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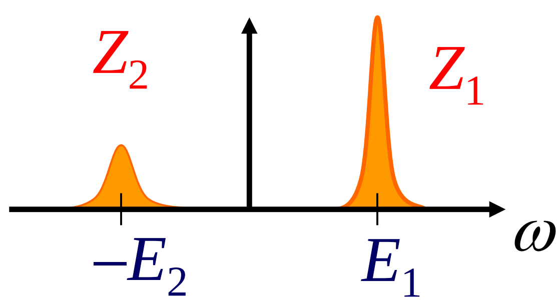
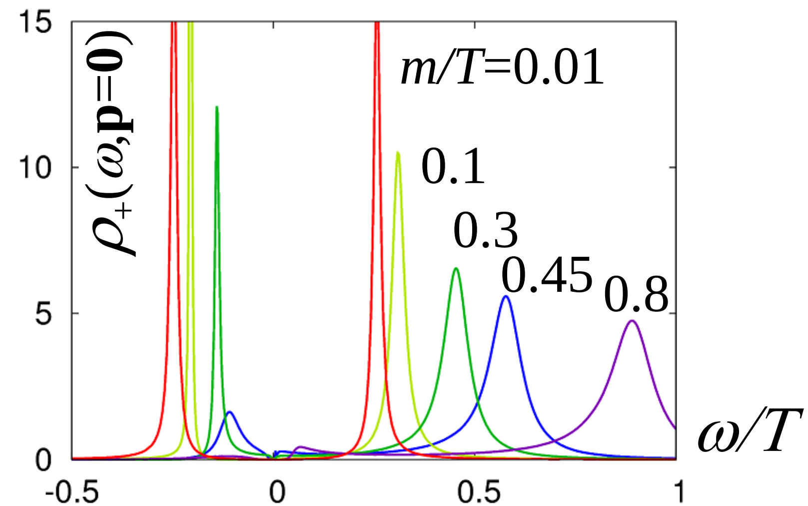
- 2-pole ansatz for $\rho_+(\omega)$.

$$\rho_+(\omega) = Z_1\delta(\omega - E_1) + Z_2\delta(\omega + E_2)$$

4-parameter fit E_1, E_2, Z_1, Z_2





Dirac Structure of Quark Spectrum

$$\rho(\mathbf{p},\omega)=\rho_0(\mathbf{p},\omega)\gamma^0-\rho_V(\mathbf{p},\omega)\vec{p}\cdot\vec{\gamma}+\rho_S(\mathbf{p},\omega)$$

zero momentum

$$\rho(\mathbf{p},\omega)=\rho_0(\mathbf{p},\omega)\gamma^0+\rho_S(\mathbf{p},\omega)$$
$$=\rho_+^M L_+\gamma^0+\rho_-^M L_-\gamma^0$$

$$\rho_\pm^M=\rho_0\pm\rho_S$$

$$L_\pm=\frac{1\pm\gamma^0}{2}$$

chirally symmetric

$$\rho(\mathbf{p},\omega)=\rho_0(\mathbf{p},\omega)\gamma^0-\rho_V(\mathbf{p},\omega)\vec{p}\cdot\vec{\gamma}$$
$$=\rho_+^P P_+\gamma^0+\rho_-^P P_-\gamma^0$$

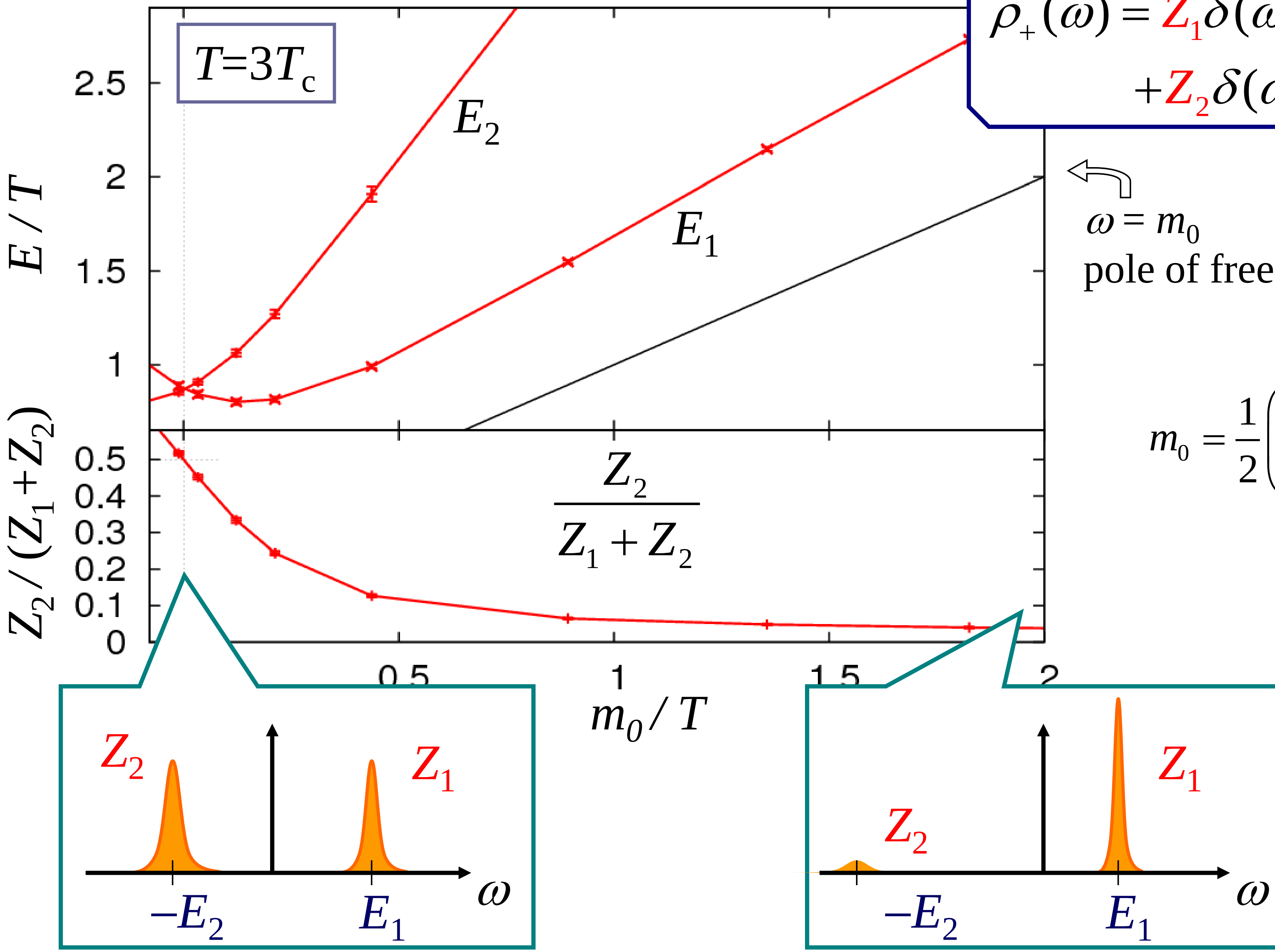
$$\rho_\pm^P=\rho_0\pm\rho_V$$

$$P_\pm=\frac{1\pm\gamma^0\vec{p}\cdot\vec{\gamma}}{2}$$

Spectral Function

$T = 3T_c$ 64^3x16 ($\beta = 7.459$)

$$\rho_+(\omega) = Z_1\delta(\omega - E_1) + Z_2\delta(\omega + E_2)$$



$\omega = m_0$
pole of free quark

$$m_0 = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

Spectral Function

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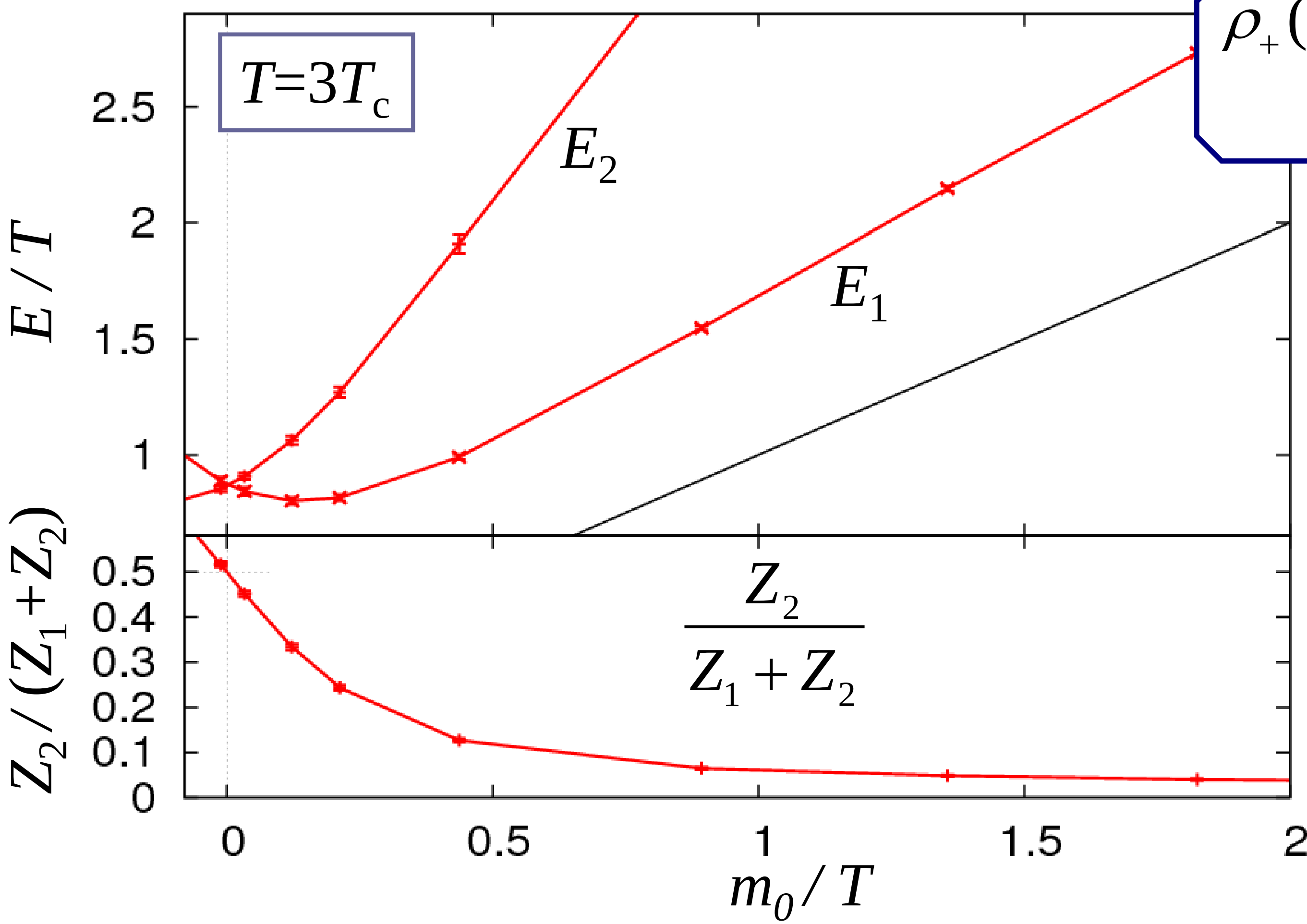


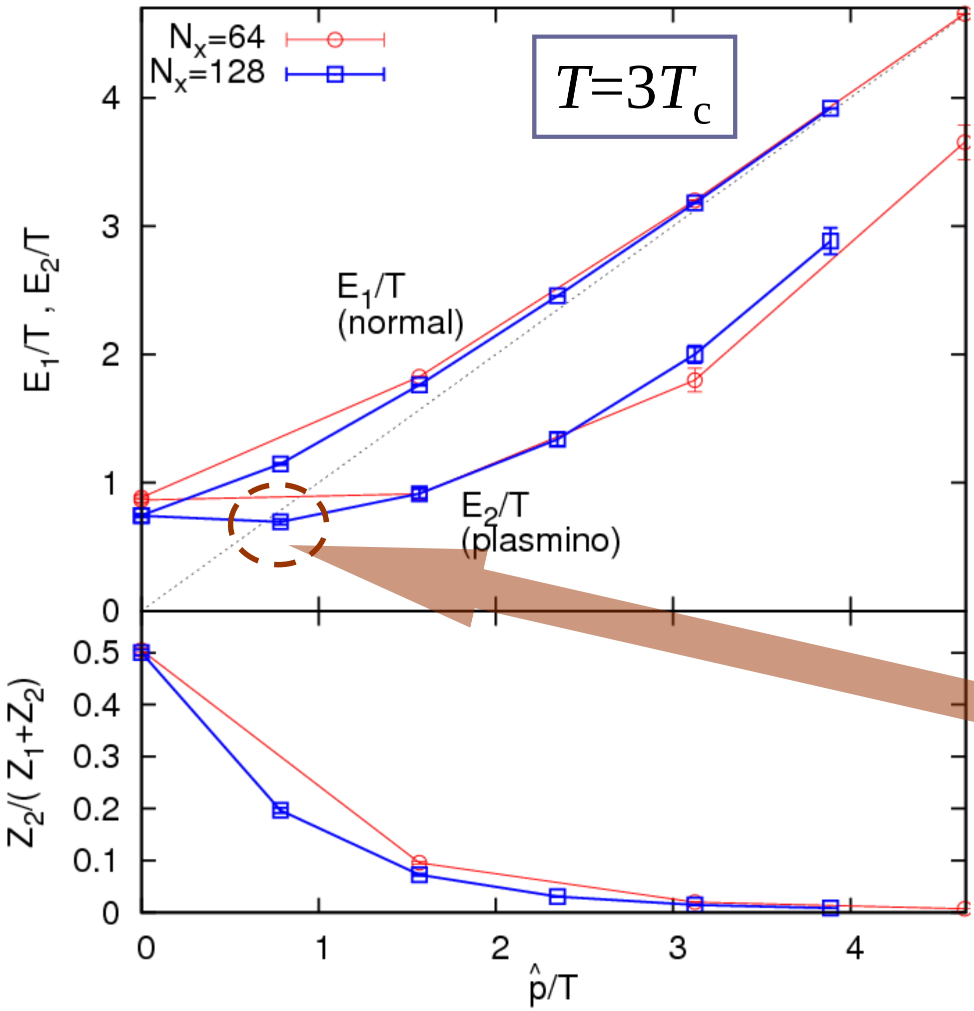
$\omega = m_0$
pole of free quark

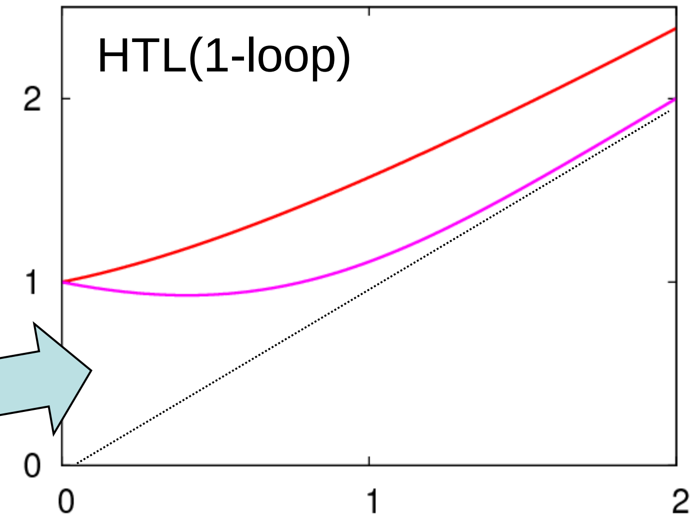
$$m_0 = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

- Limiting behaviors for $m_0 \to 0, m_0 \to \infty$ are as expected.
- Quark propagator approaches the chiral symmetric one near m_0=0.
- E_2>E_1 : qualitatively different from the 1-loop result.

Quark Dispersion on 128^3x16 Lattice

Kaczmare,MK+ 2012



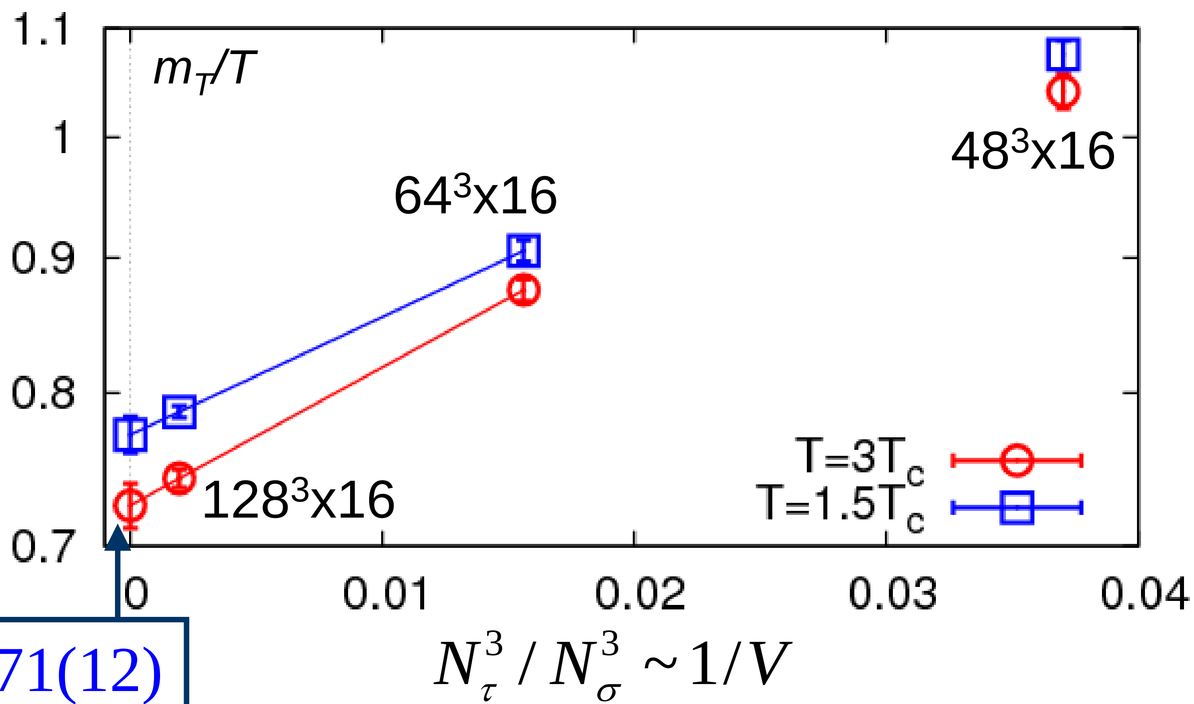


E_2 has a minimum at p>0

- Existence of the plasmino minimum is indicated.
- E_2, however, is not the position of plasmino pole.

Spatial Volume Dependence of m_T

Kaczmare,MK+ 2012



m_T/T=0.771(12)
m_T/T=0.725(14)

- Note: lowest p

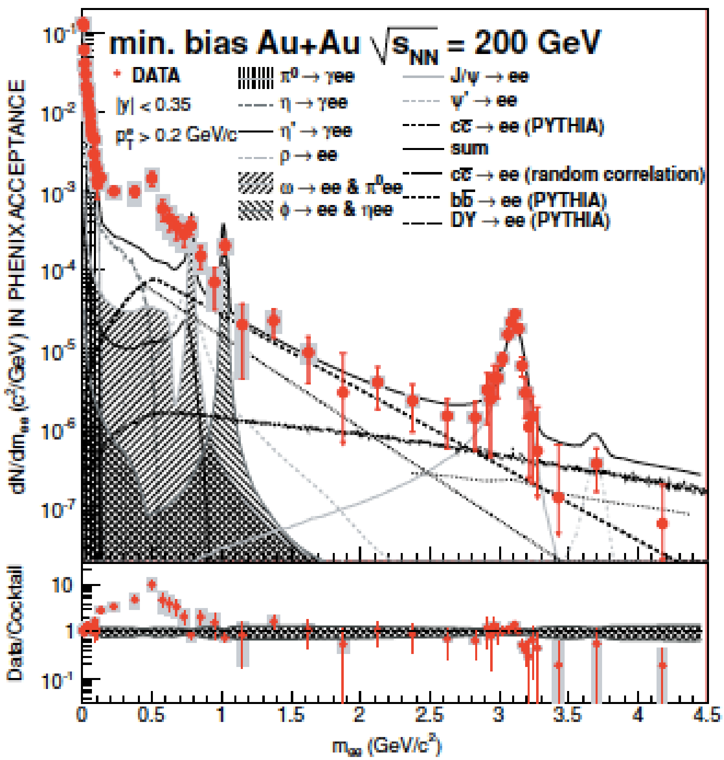
$$p_{\min} = \frac{2\pi}{L_x} = 2\pi T \frac{N_\tau}{N_x}$$

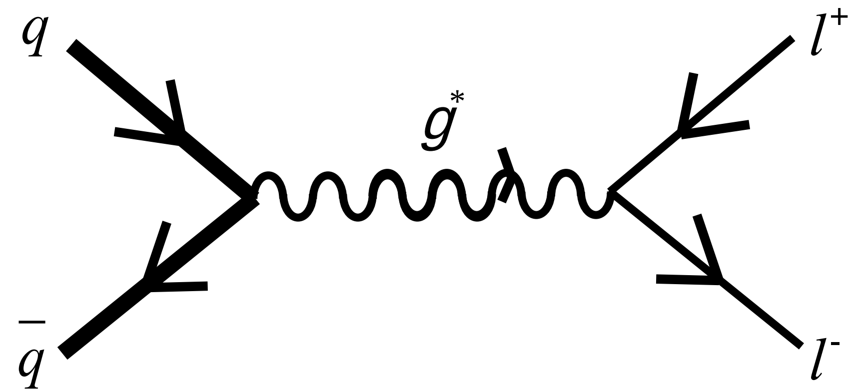
- N_x/N_t=4 → $p_{\min}$~1.57T
- N_x/N_t=8 → $p_{\min}$~0.79T

- Result on 128^3x16 seems to converge.
- Much larger lattice is desirable.
- No lattice spacing dependence.

レプトン対生成率の解析

光子・レプトン対（仮想光子）生成率





媒質からの光子･レプトン対生成率

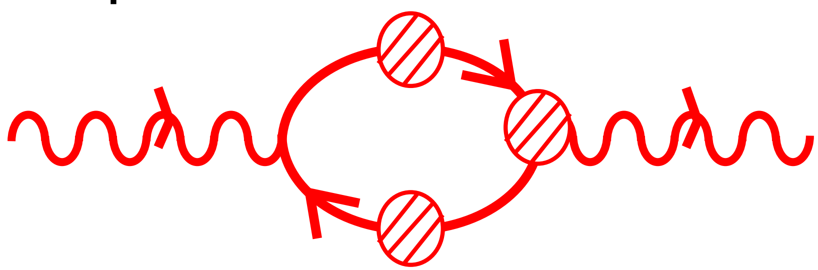
$$\frac{d^4G}{dq_0 d^3q} = \frac{a}{12p^4}\frac{1}{e^{bq_0}-1}\frac{1}{q^2}\mathrm{Im}\,P^{m}_{m}$$

McLerran, Toimela (1985);
Weldon(1990);
Gale, Kapusta (1991)

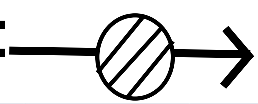
Virtual Photon Self Energy

Strongly interacting ⇒ Non-perturbative analysis is desirable

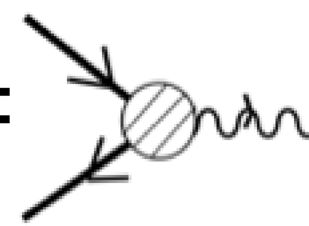
Schwinger-Dyson equation

$P_{mn} =$ 

exact quark propagator

$S =$ 

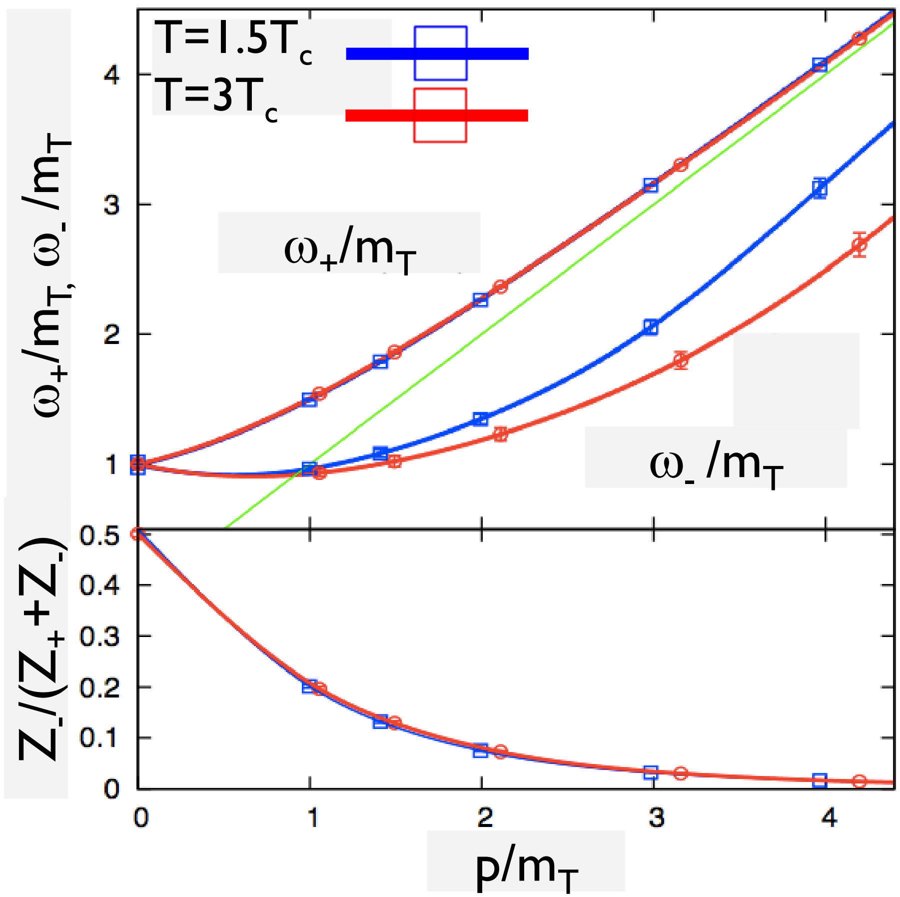
exact vertex

$G_m =$ 

using a lattice quark propagator

W-T identity

準粒子の分散関係

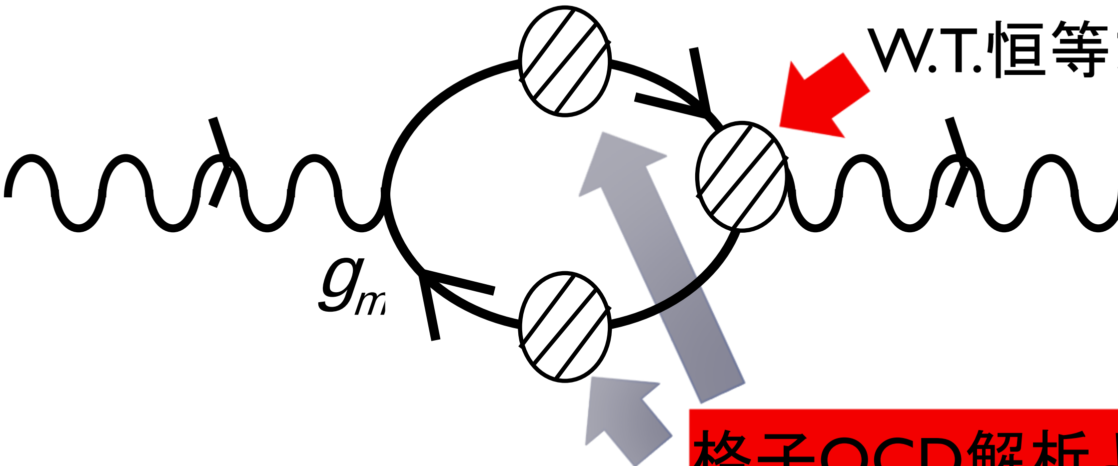


格子QCDで計算した分散関係のデータ点を3次スプライン補完でフィット

$\omega_+(p)$: 準粒子のノーマル状態の分散関係

$\omega_-(p)$: 準粒子のプラズミーノ状態(媒質中でのみ存在する状態)の分散関係

バーテックス補正



W.T.恒等式を満たす頂点関数G_n

格子QCD解析より得られた $S(p)$

$$k^m \mathrm{G}_m(p+k, p) = S^{-1}(p+k) - S^{-1}(p)$$

からΓ_μを決定したい

Γ_μ4成分に対して拘束条件が1つしか無い

バーテックス補正2

$$k^0\Gamma_0 + k^1\Gamma_1 + k^2\Gamma_2 + k^3\Gamma_3 = S^{-1}(p+k) - S^{-1}(p)$$

***k*=0**のとき空間の対称性から0になる (滑らかさを仮定)

$k^i \to 0 \ (i = 1, 2, 3)$ の極限で

$$k_0\Gamma_0(p+k,p)\big|_{\boldsymbol{k}=0} \quad = \quad S^{-1}(p_0+k_0,\boldsymbol{p}) - S^{-1}(p_0,\boldsymbol{p})$$

バーテックス補正2

$$k^0\Gamma_0 + k^1\Gamma_1 + k^2\Gamma_2 + k^3\Gamma_3 = S^{-1}(p+k) - S^{-1}(p)$$

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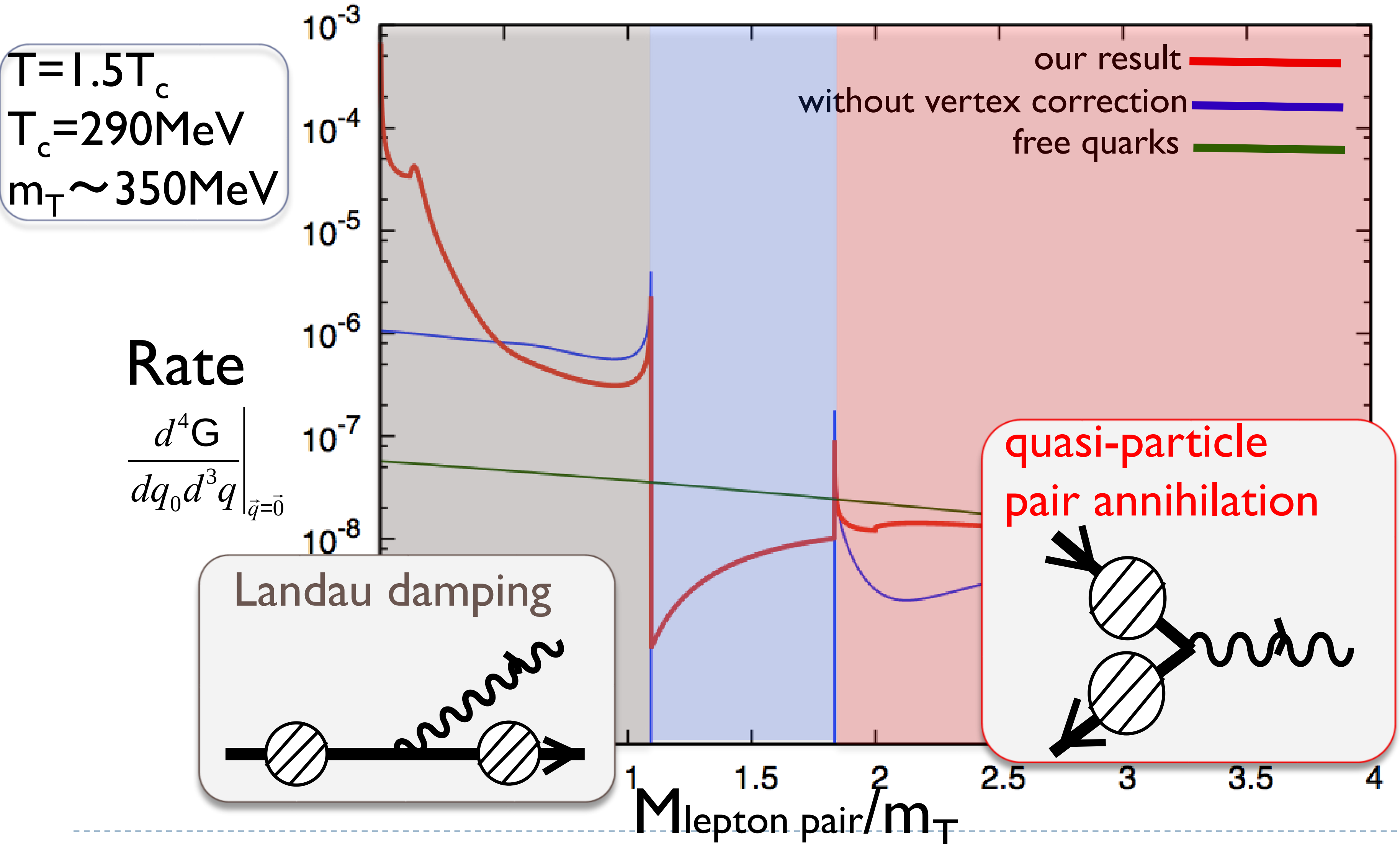
$$k_0\Gamma_0(p+k,p)\big|_{\boldsymbol{k}=0} = S^{-1}(p_0+k_0,\boldsymbol{p}) - S^{-1}(p_0,\boldsymbol{p})$$

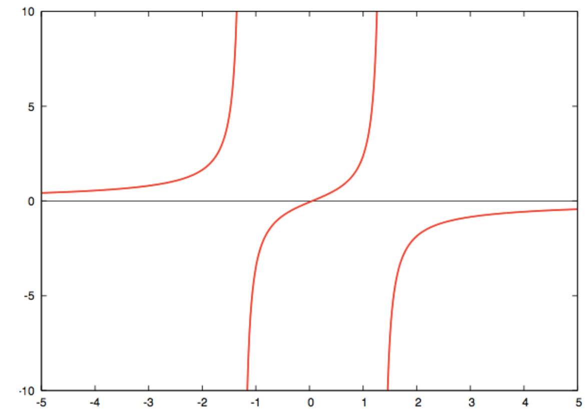
Γ_0 のk依存性がないと仮定 ⇒ Γ_i が決まる

Numerical results



Additional pole due to vertex correction

behavior of propagator



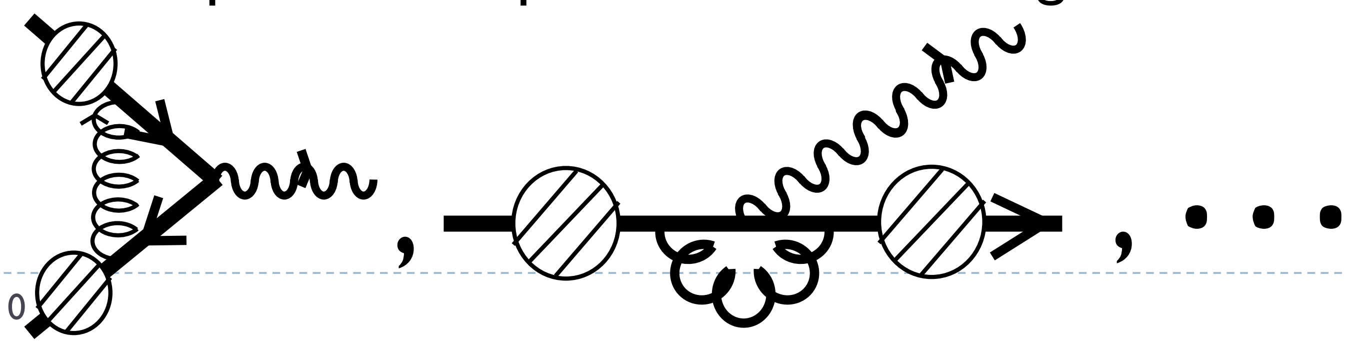
“Two pole ansatz” → S has a zero point

↓

emergence of a pole in Γ_μ

↓

Additional pole means production from diagrams like,

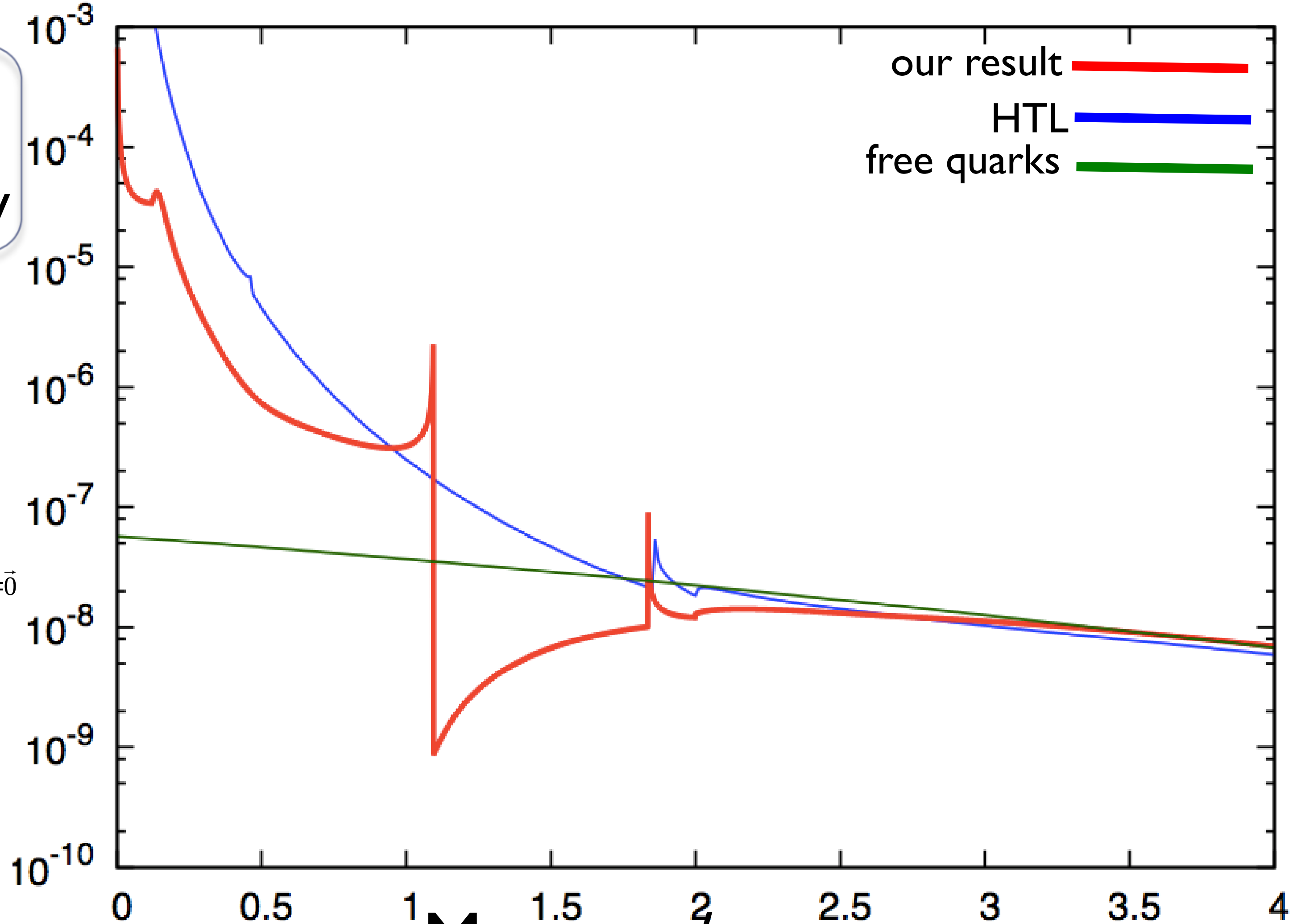


Compared to HTL

$T=1.5T_c$
$T_c=290\text{MeV}$
$m_T \sim 350\text{MeV}$

Rate

$$\left.\frac{d^4G}{dq_0 d^3q}\right|_{\vec{q}=\vec{0}}$$

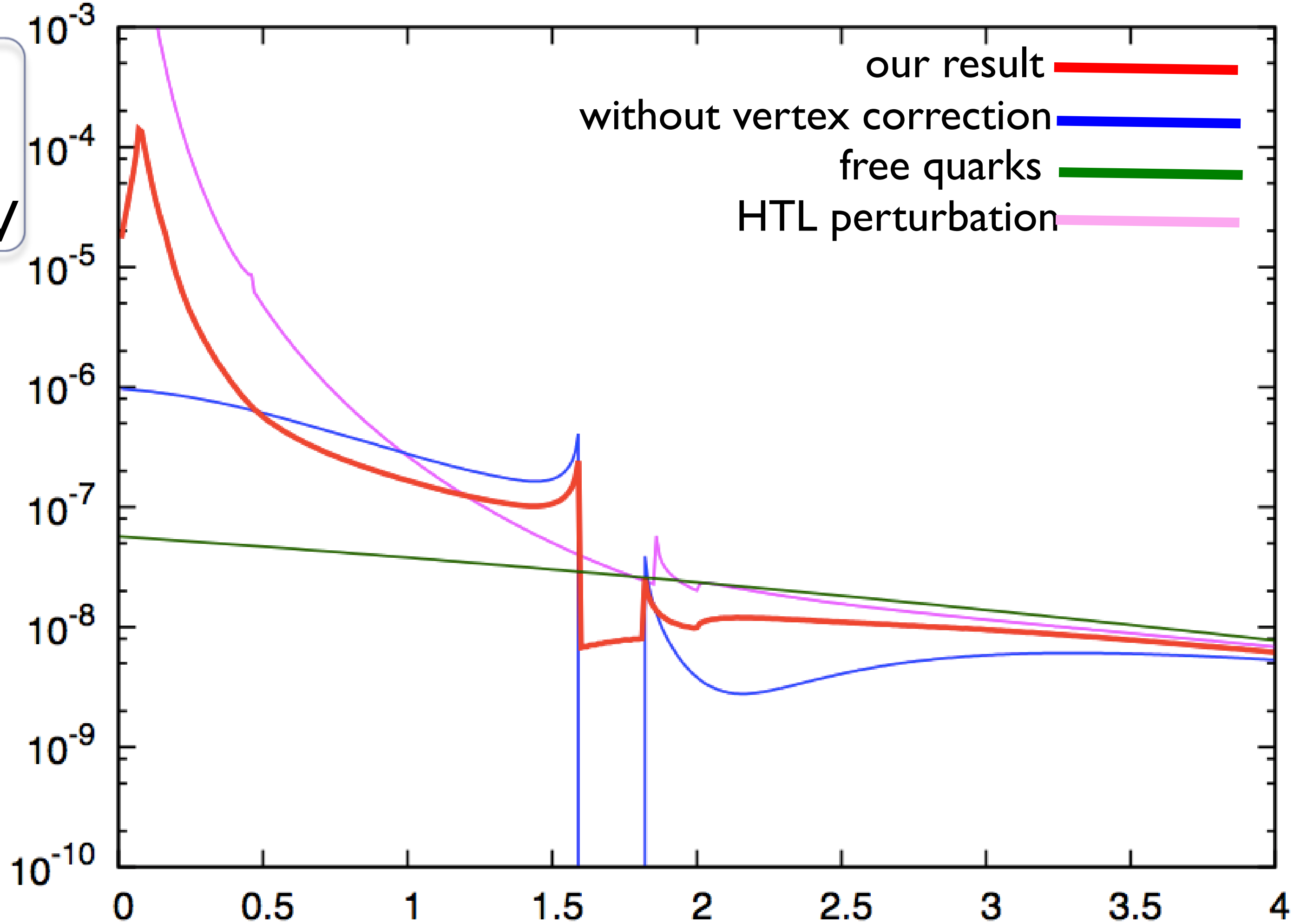


$M_{\text{lepton pair}}/m_T$

$3T_c$

T=3T$_c$
T$_c$=290MeV
$m_T \sim 650$MeV

Rate
$$\left.\frac{d^4G}{dq_0 d^3q}\right|_{\vec{q}=\vec{0}}$$



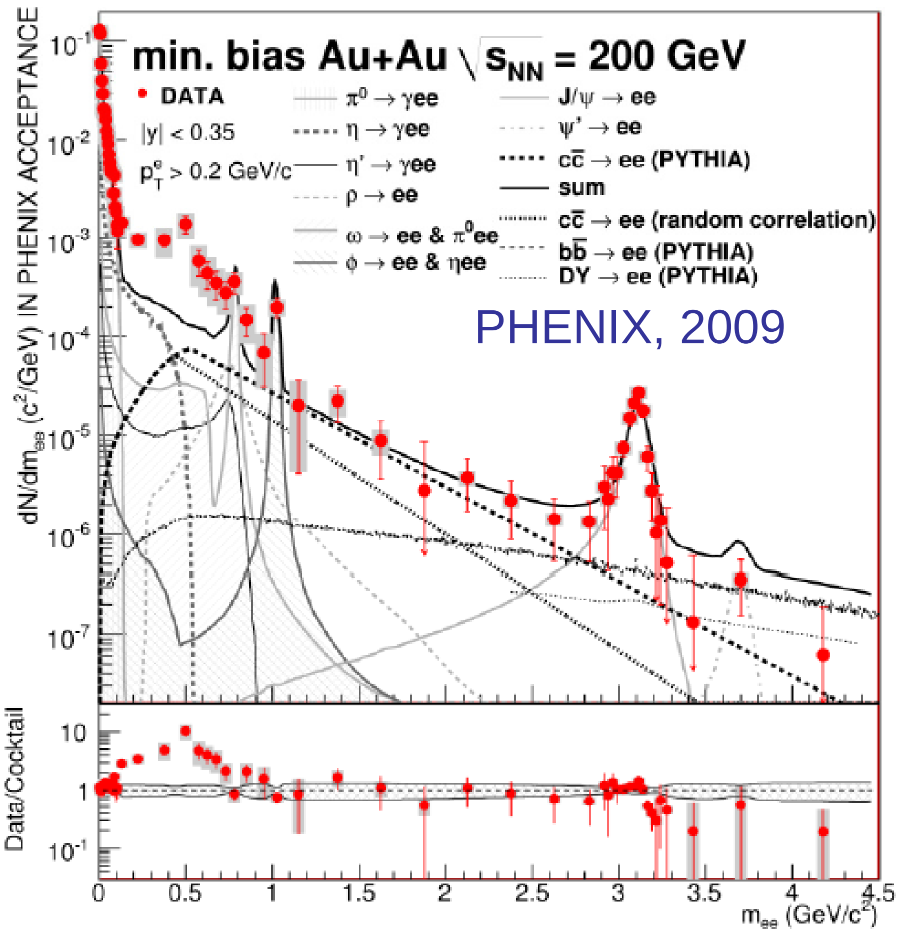
$M_{\text{lepton pair}}/m_T$

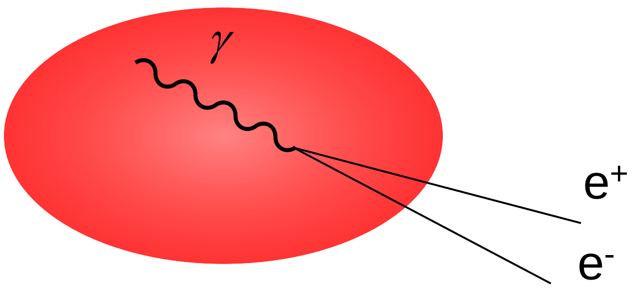
Summary

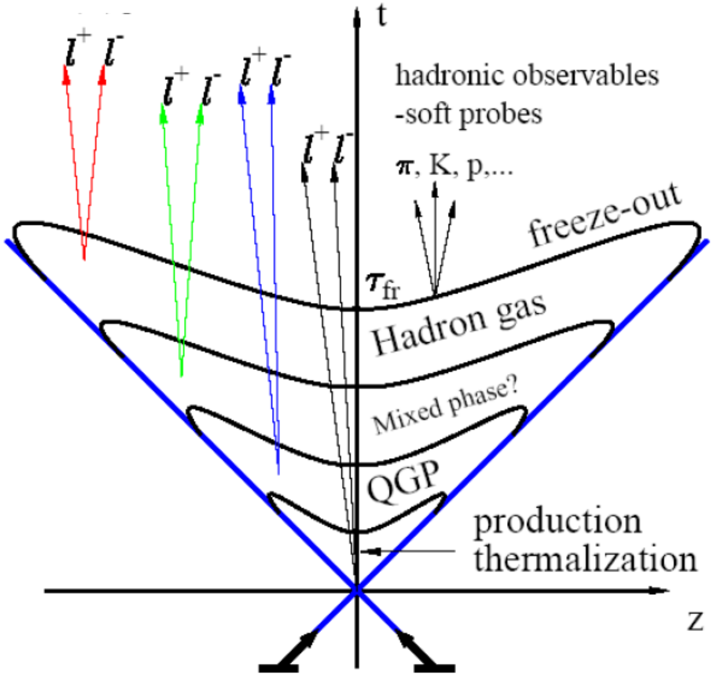
- 格子QCDによるクォーク伝搬関数の解析は、臨界温度より高温であれば非摂動領域においてもクォーク場が準粒子励起を持ち、かつ熱質量を持つことを示唆している。

- 格子QCDの相関関数から求めたクォーク伝搬関数と、WT関係式を満たす頂点関数を使って、レプトン対生成率の解析を行った。
- 臨界温度付近の非閉じ込め物質のレプトン対生成率は、低エネルギー領域で、自由クォークのレートより10倍程度大きくなることが示唆された。

Dilepton Production Rate







- Most direct probes of the QGP.
- They are produced in all stages of time evolution.

Choice of Source

- Wall source, instead of point source

$$\begin{cases} \text{point: } S(\mathbf{p}=\mathbf{0},\tau) = \sum_{\mathbf{x}} \langle \psi(\mathbf{x},\tau)\bar{\psi}(\mathbf{0},0)\rangle \\ \text{wall: } S(\mathbf{p}=\mathbf{0},\tau) = \frac{1}{V}\sum_{\mathbf{x},\mathbf{y}} \langle \psi(\mathbf{x},\tau)\bar{\psi}(\mathbf{y},0)\rangle \end{cases}$$

- same (or, less) numerical cost
- quite effective to reduce error!!

Quality of data on **128^3x16** lattice is about **3** times better than on **64^3x16**.

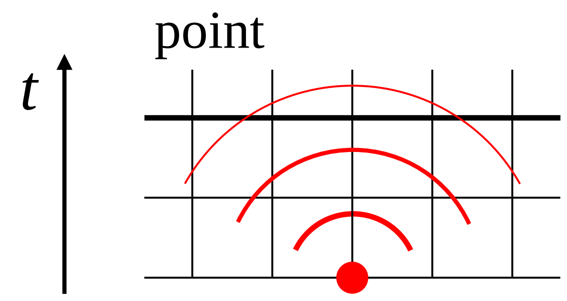
$\sim \sqrt{8}$

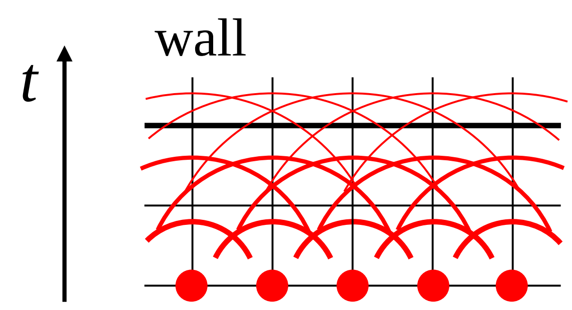
What's the source?

$$K = \not{D} - m_0$$

$$K\phi_{\text{result}} = \phi_{\text{source}}$$

$$\phi_{\text{result}} = K^{-1}\phi_{\text{source}}$$



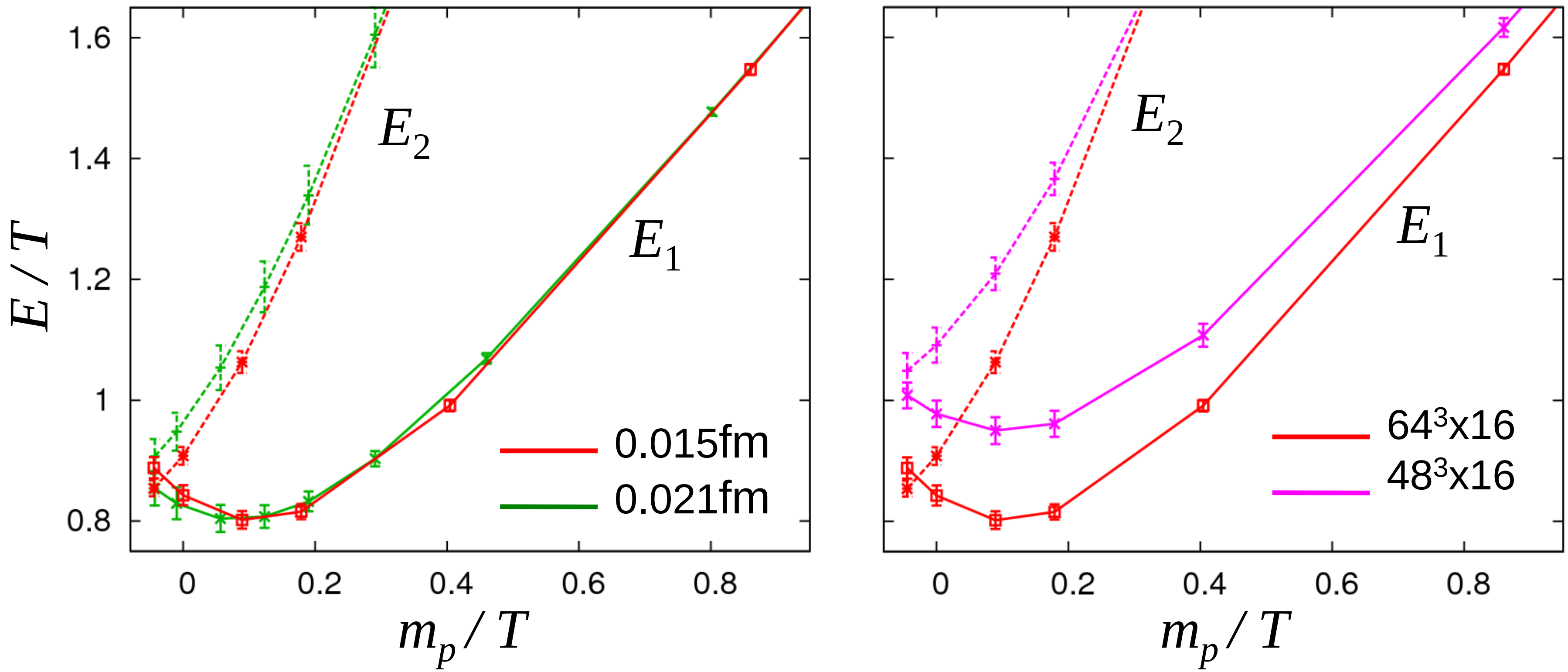


Lattice Spacing and Volume Dependences

for T=3T_c

Lattice spacing dependence

Spatial volume dependence



•No lattice spacing dependence within statistical error.

•strong volume dependence even for N_σ/N_τ=4.