低エネルギー衝突での熱ゆらぎ観測に関する問題点

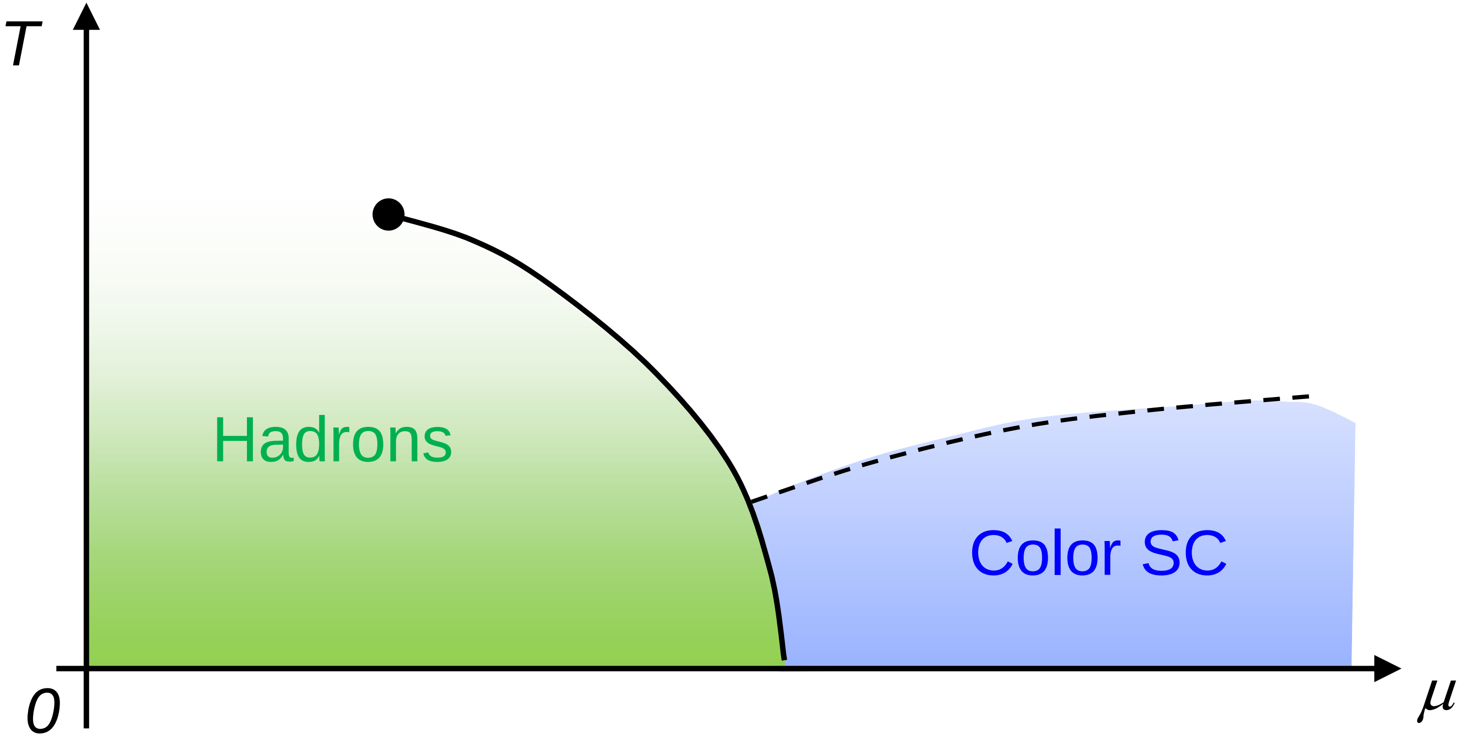
Masakiyo Kitazawa

(Osaka U.)

MK, Asakawa, Ono, Phys. Lett. B728 (2014) 386-392
Sakaida, Asakawa, MK, arXiv:1409.6866
MK, to appear soon!

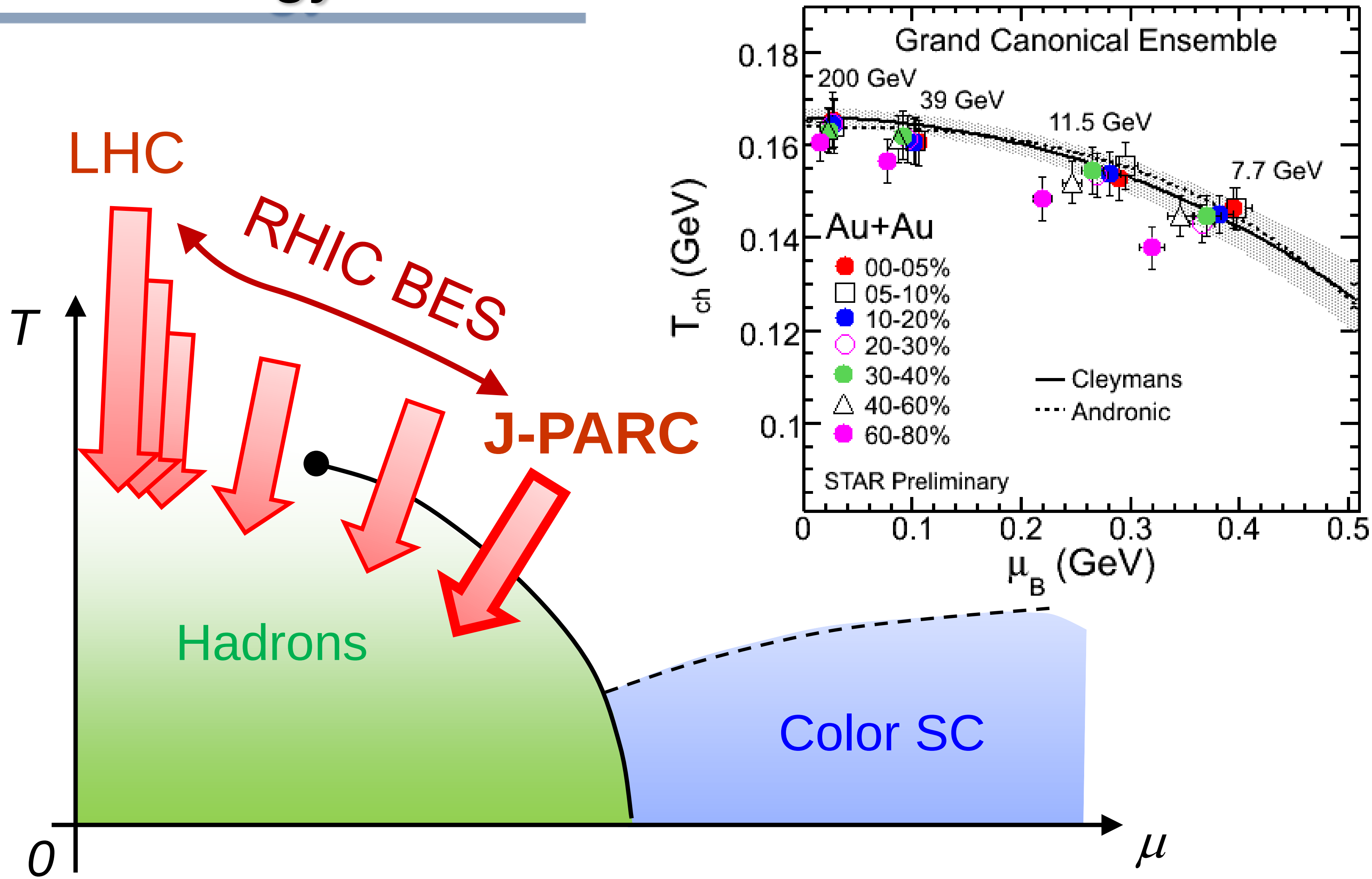
『J-PARCにおける重イオン衝突実験が拓く新しい物理』, KEK, 2014/11/26

Beam-Energy Scan



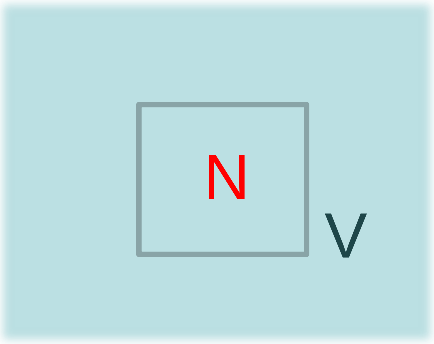
Beam-Energy Scan

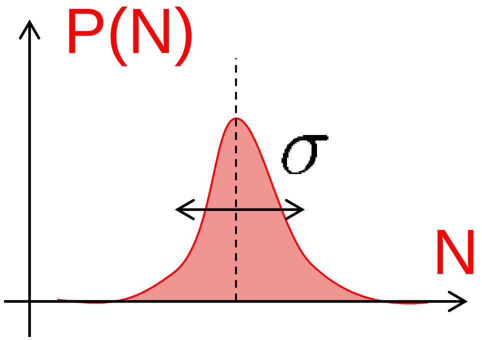
STAR 2012



Fluctuations

Observables in equilibrium are fluctuating.





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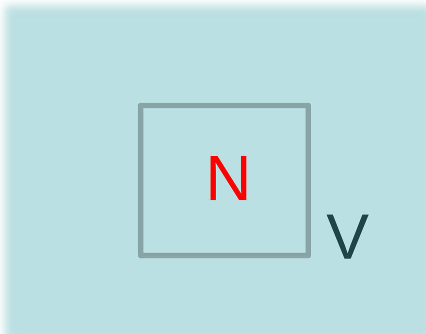
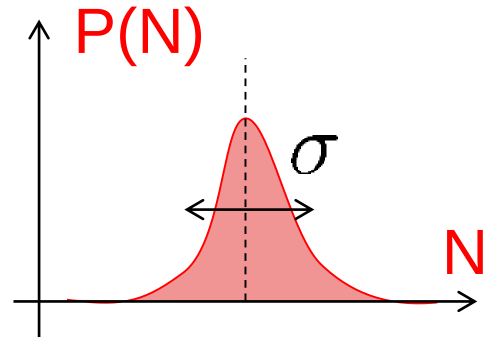


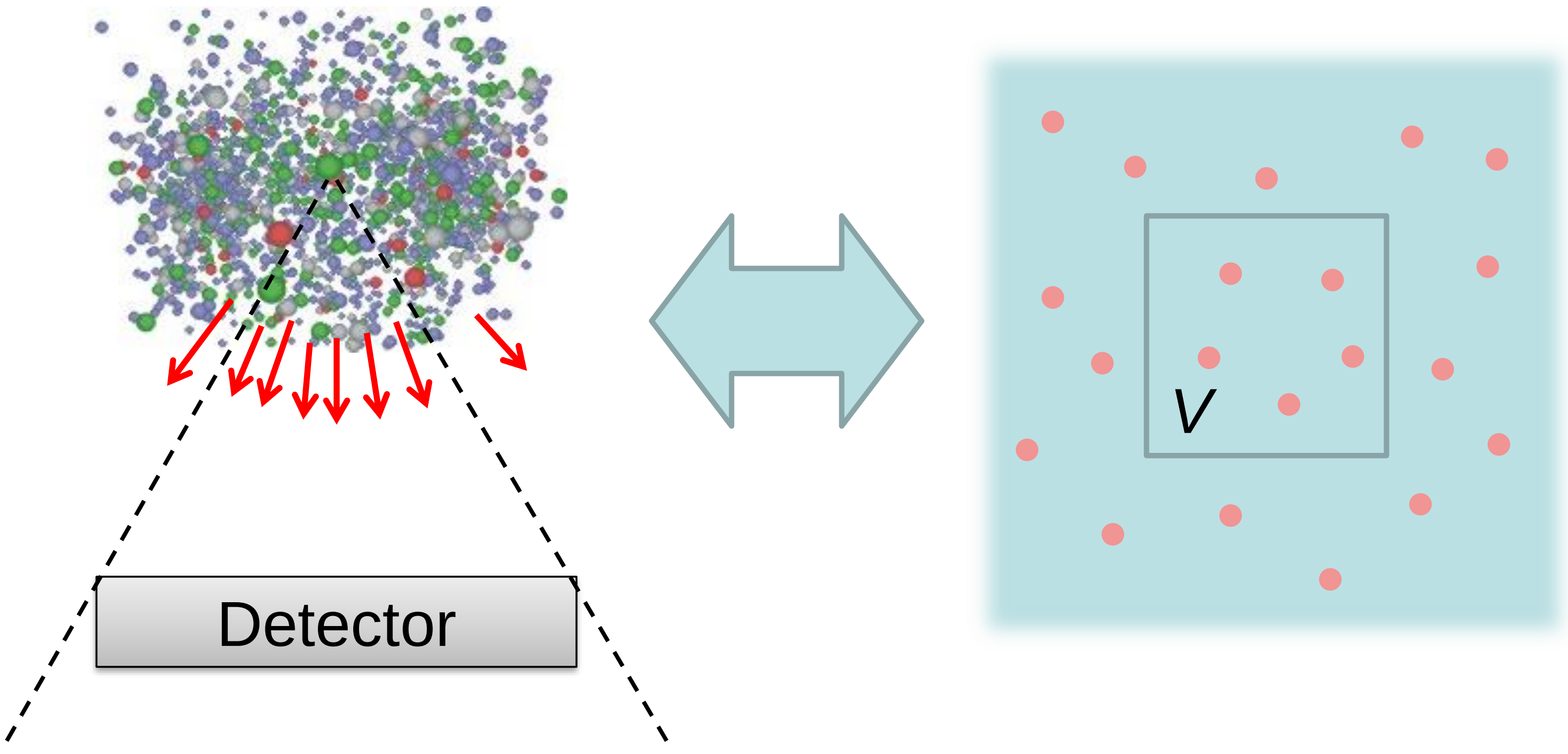


$$\delta N = N - \langle N \rangle$$

- Variance: $\langle \delta N^2 \rangle = V \chi_2 = \sigma^2$
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \dfrac{\langle \delta N^4 \rangle - 3 \langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.



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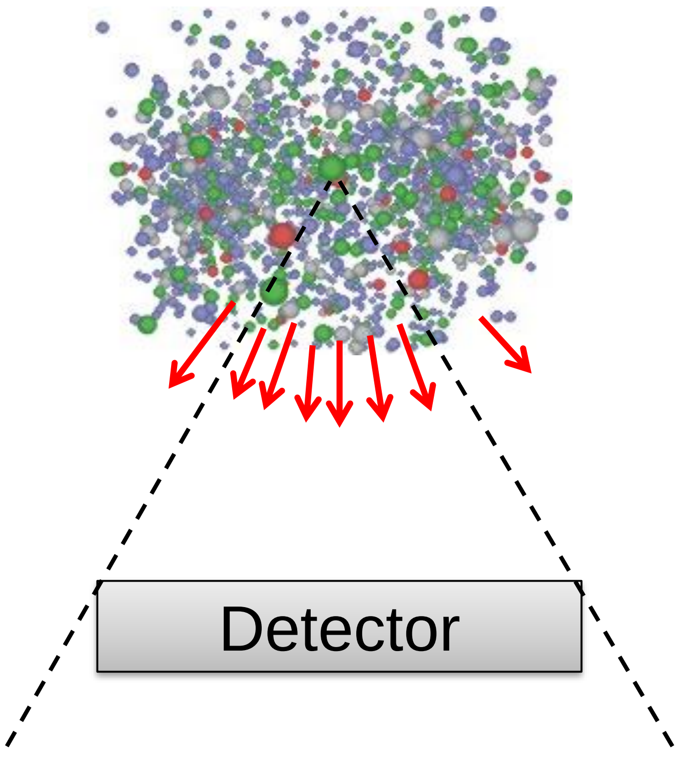
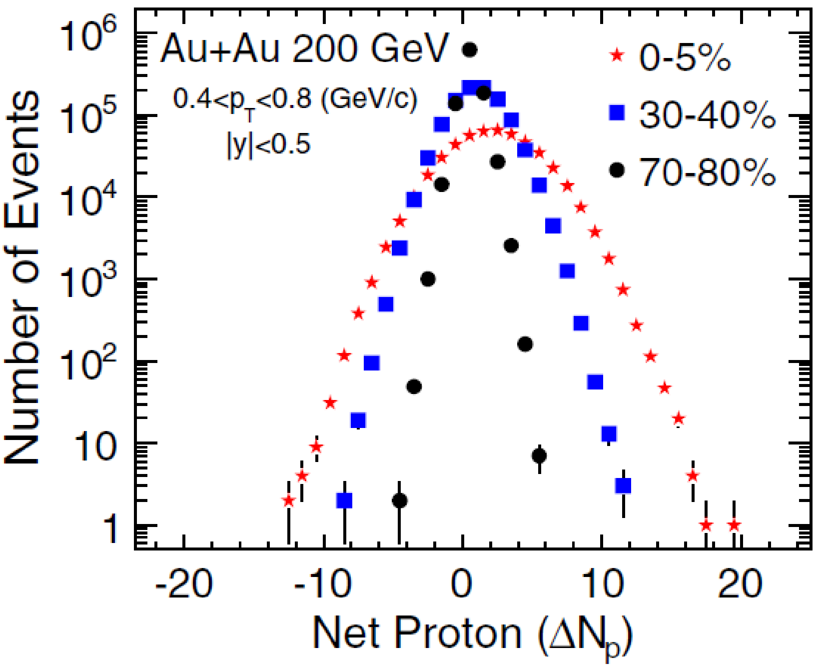
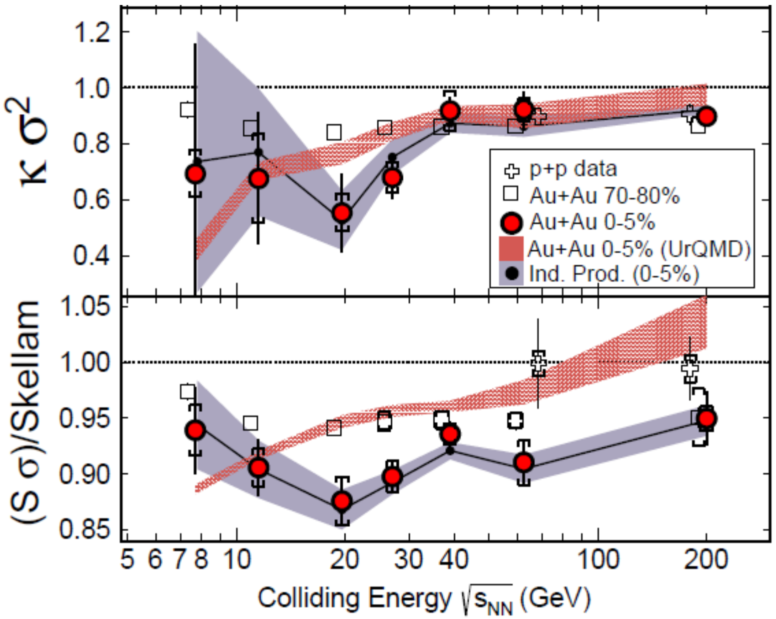


STAR, PRL**105** (2010)



Cumulants up to 4th Order in 2014

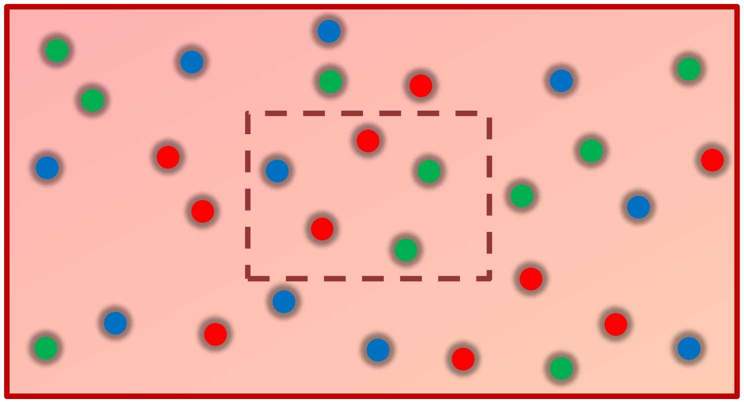
STAR, PRL 2014



This is a great achievement in physics!

Fluctuations and Elemental Charge

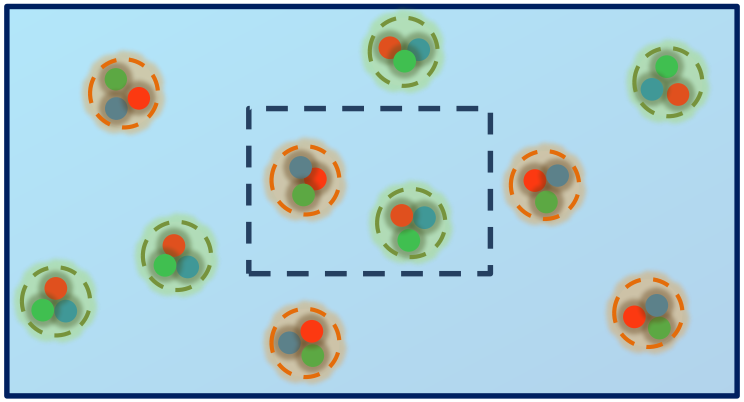
Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000
Ejiri, Karsch, Redlich, 2005



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$

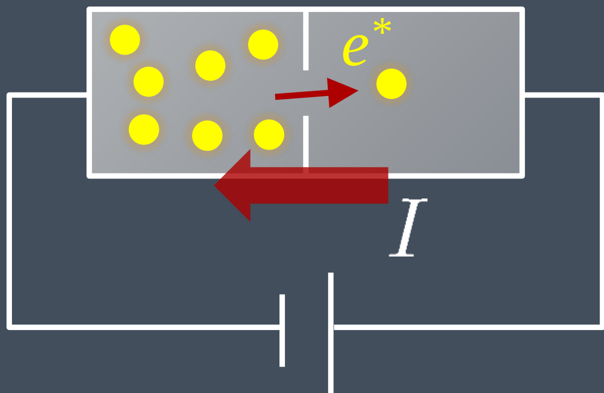


$$\langle \delta N_B^n \rangle_c = \langle N_B \rangle$$

Free Boltzmann → Poisson

$$\langle \delta N^n \rangle_c = \langle N \rangle$$

Shot Noise



$$S_{\text{shot}} \sim \langle \delta I^2 \rangle$$

$$S_{\text{shot}} = 2e^* \langle I \rangle$$

charge of quasi-particles

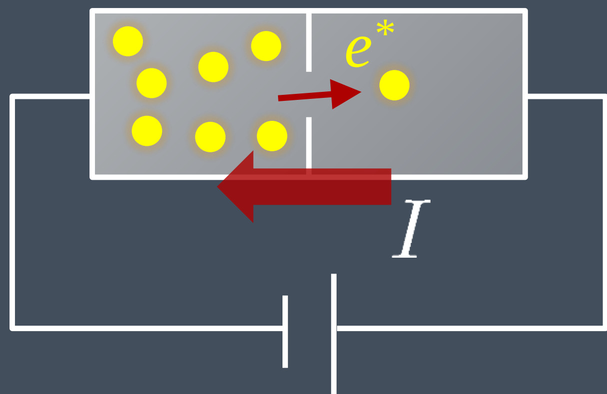
Total charge Q:

$$Q = e\langle N \rangle$$

$$\langle \delta Q^2 \rangle = e^2 \langle \delta N^2 \rangle = e^2 \langle N \rangle = eQ$$

$$\frac{\langle \delta Q^2 \rangle}{Q} = e$$

Shot Noise



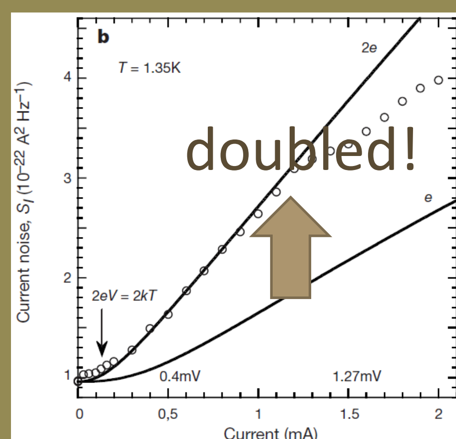
$$S_{\text{shot}} \sim \langle \delta I^2 \rangle$$
$$S_{\text{shot}} = 2e^*\langle I \rangle$$

charge of quasi-particles

Superconductors with Cooper Pairs

$$e^* = 2e$$

Jehl+, Nature **405**,50 (2000)



Fractional Quantum Hall Systems

$$e^* = \frac{q}{p}e$$

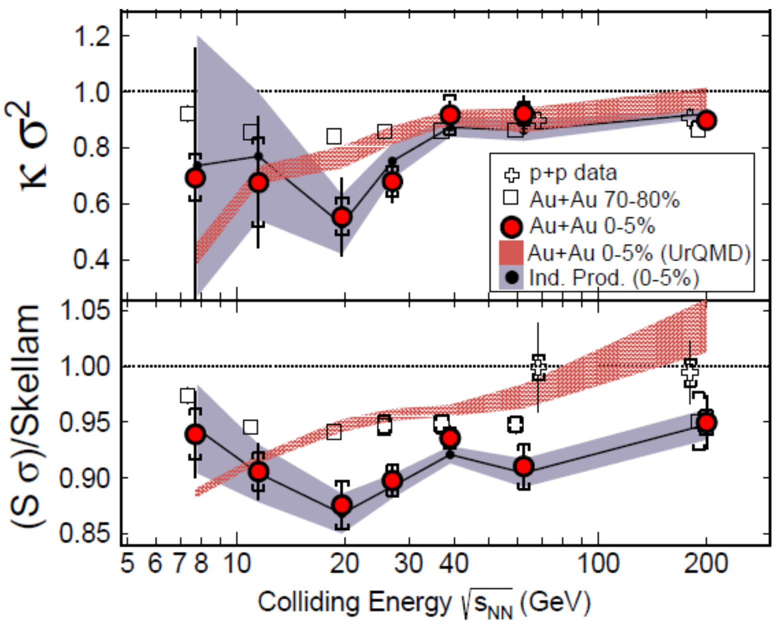
Saminadayar+, PRL**79**,2526 (1997)

Higher order cumulants:

3rd order: ex. Beenakker+, PRL90,176802(2003)

up to 5th order: Gustavsson+, Surf.Sci.Rep.**64**,191(2009)

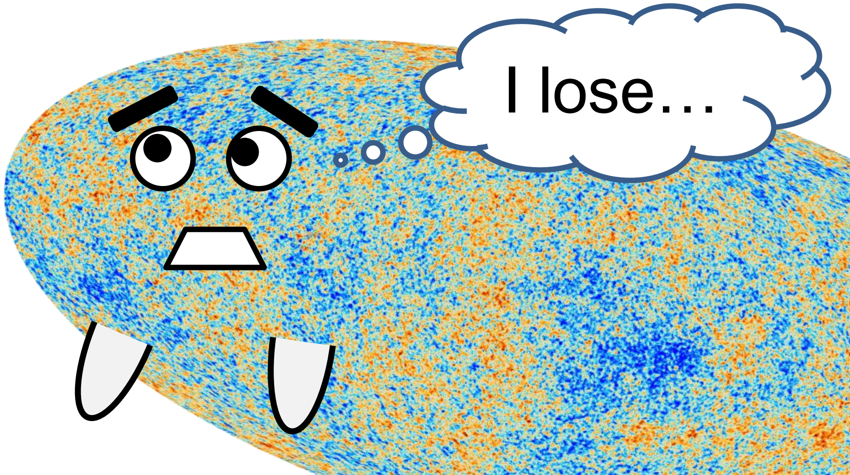
Cumulants up to 4th Order in 2014



This is a great achievement in physics!

重イオン衝突実験は
完全計数統計解析が可能である

However,
still a lot of things
to do…



Many Things to Do

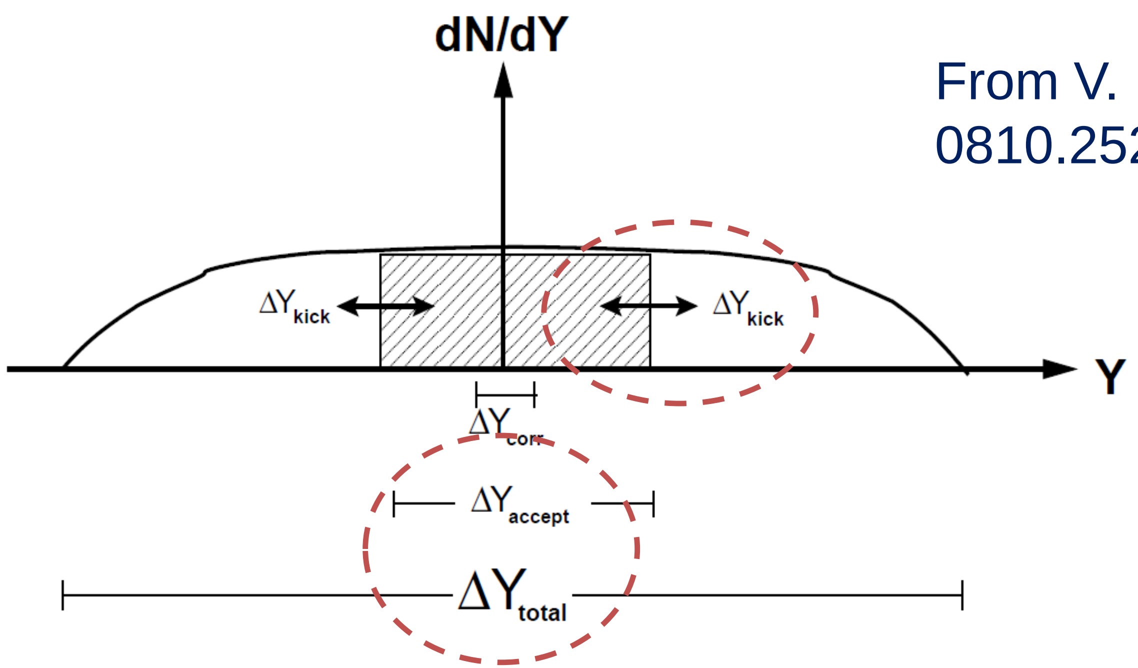
- Message to Experimentalists:
  - Measure rapidity window dependences
  - Determine baryon number cumulants

- Message to Theorists:
  - Do not directly compare your thermal results with exp.
  - Let's pursue descriptions of non-eq. non-Gaussianity.

- Message to Latticians:
  - Do not directly compare your results with exp.
  - Measure more cumulants more accurately

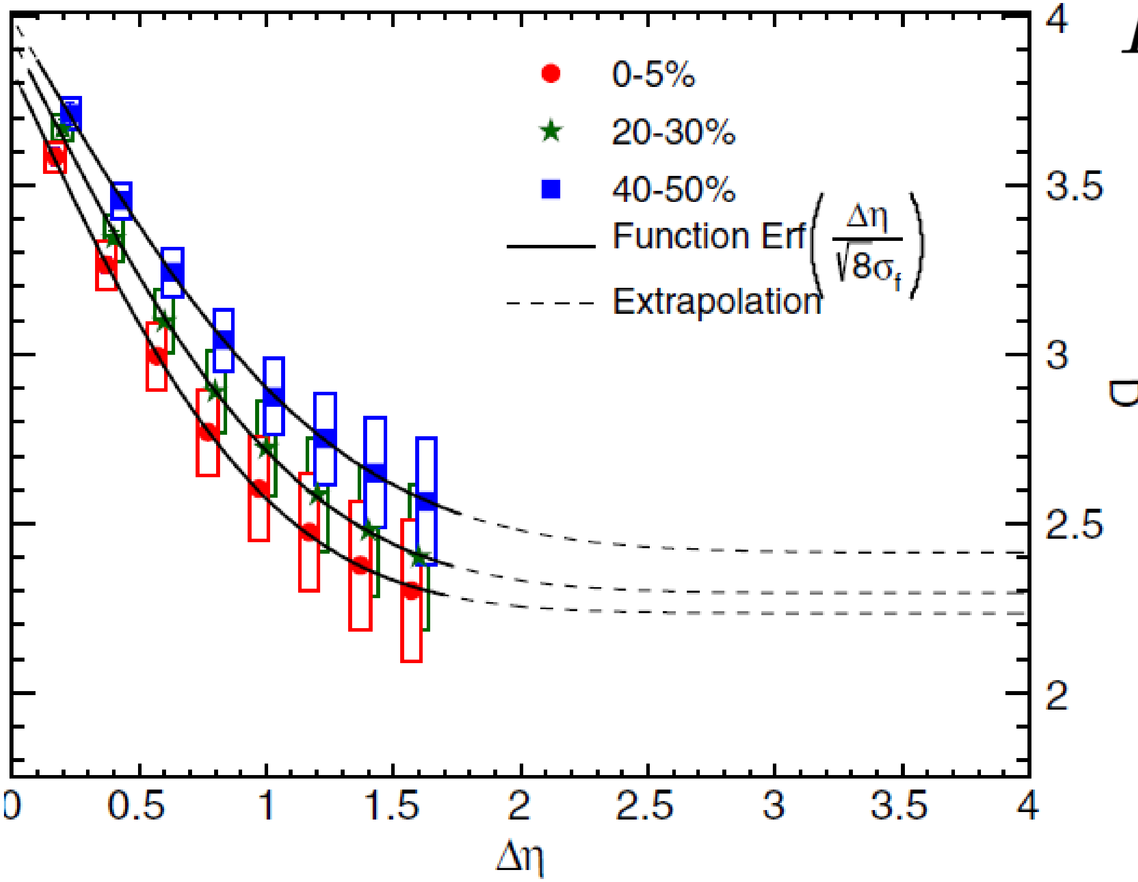
HICが観測するゆらぎは、何者か？

実験が観測するゆらぎは、化学フリーズアウト時における熱平衡状態のもの？



From V. Koch, 0810.2520

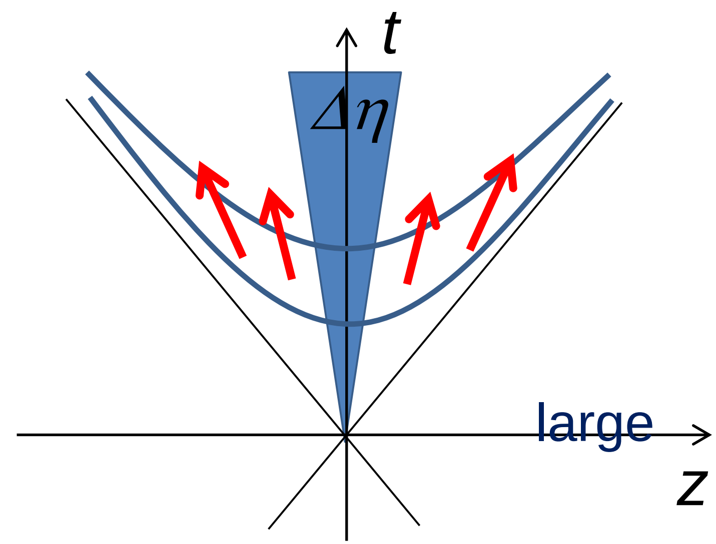
Δη Dependence @ ALICE



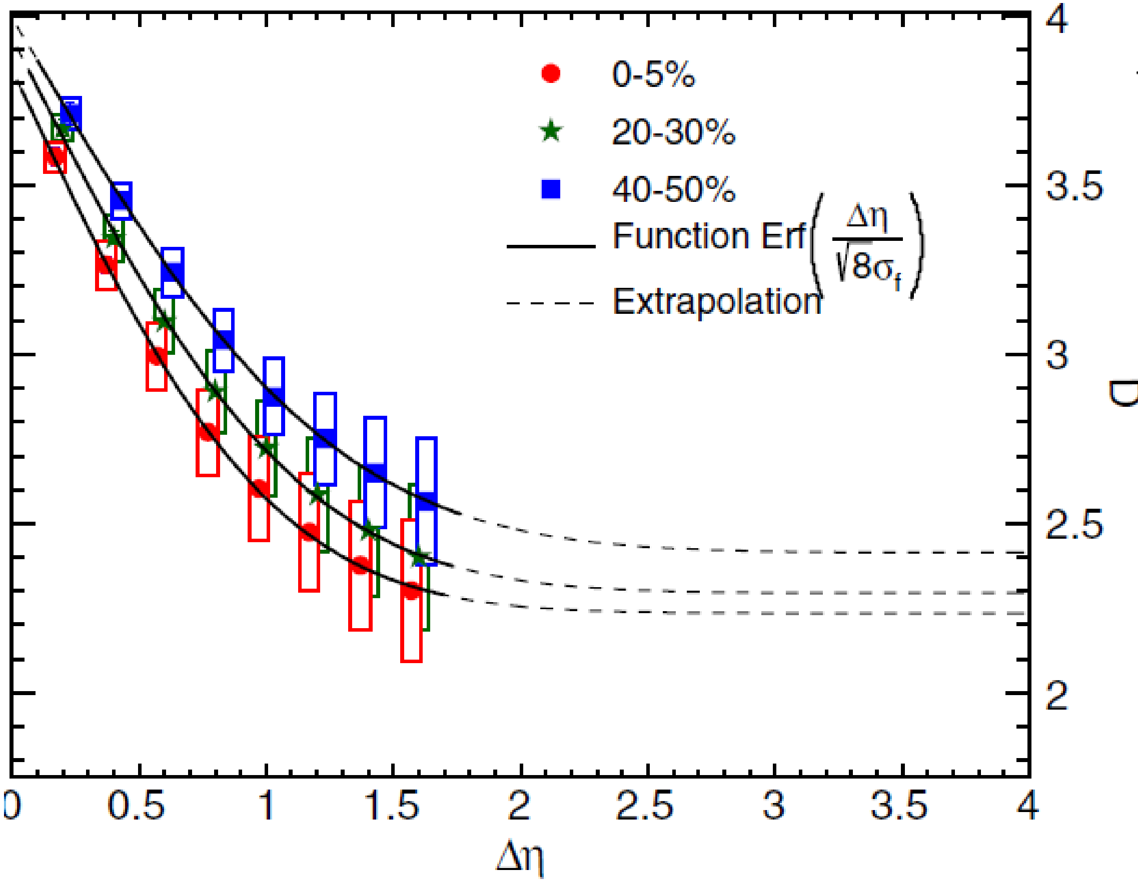
rapidity window

$$D \sim \frac{\langle \delta N_Q^2 \rangle}{V}$$

ALICE
PRL 2013



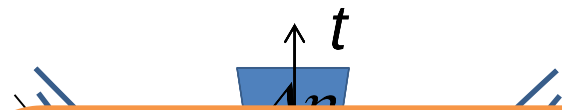
Δη Dependence @ ALICE



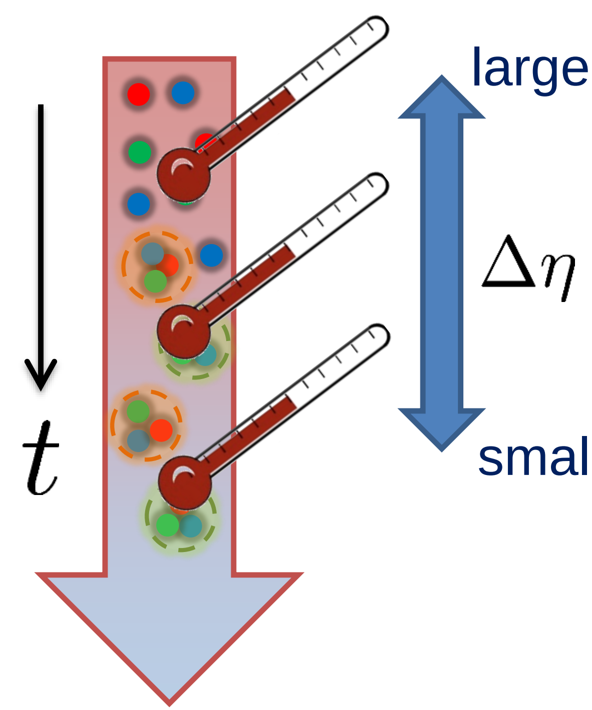
rapidity window

$$D \sim \langle \delta N_Q^2 \rangle$$

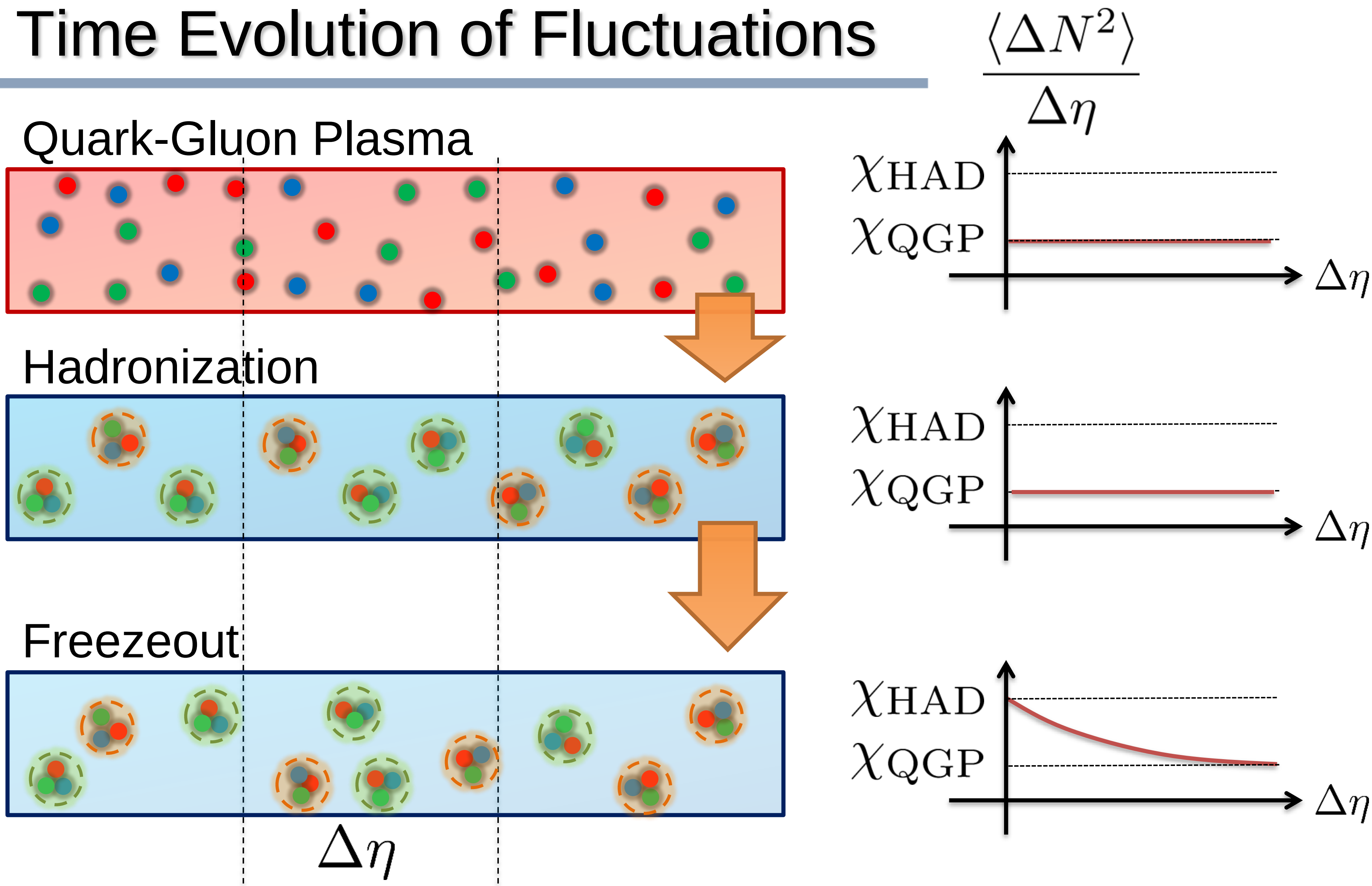
ALICE
PRL 2013



Δη dependent thermometer?

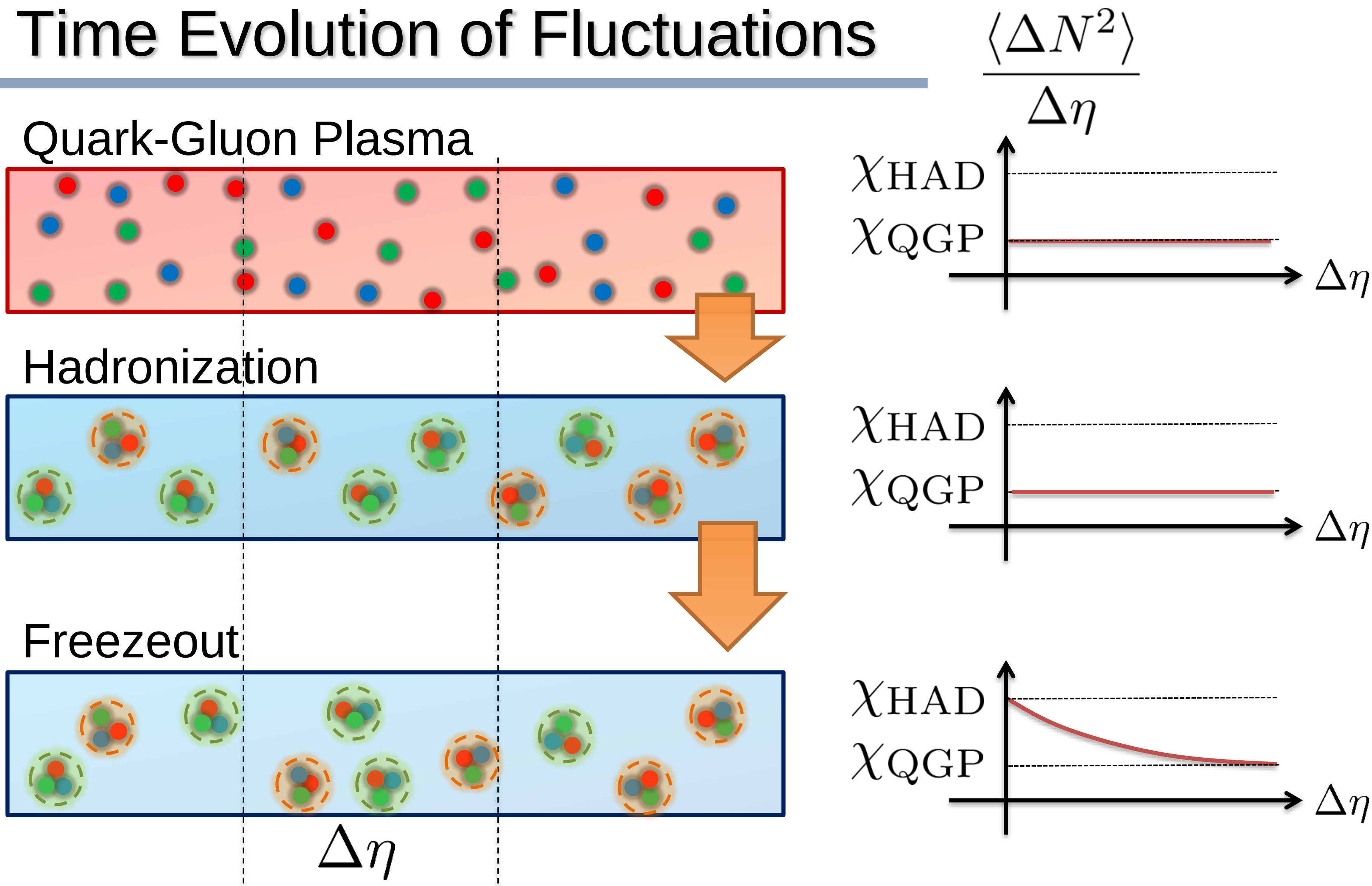


Time Evolution of Fluctuations



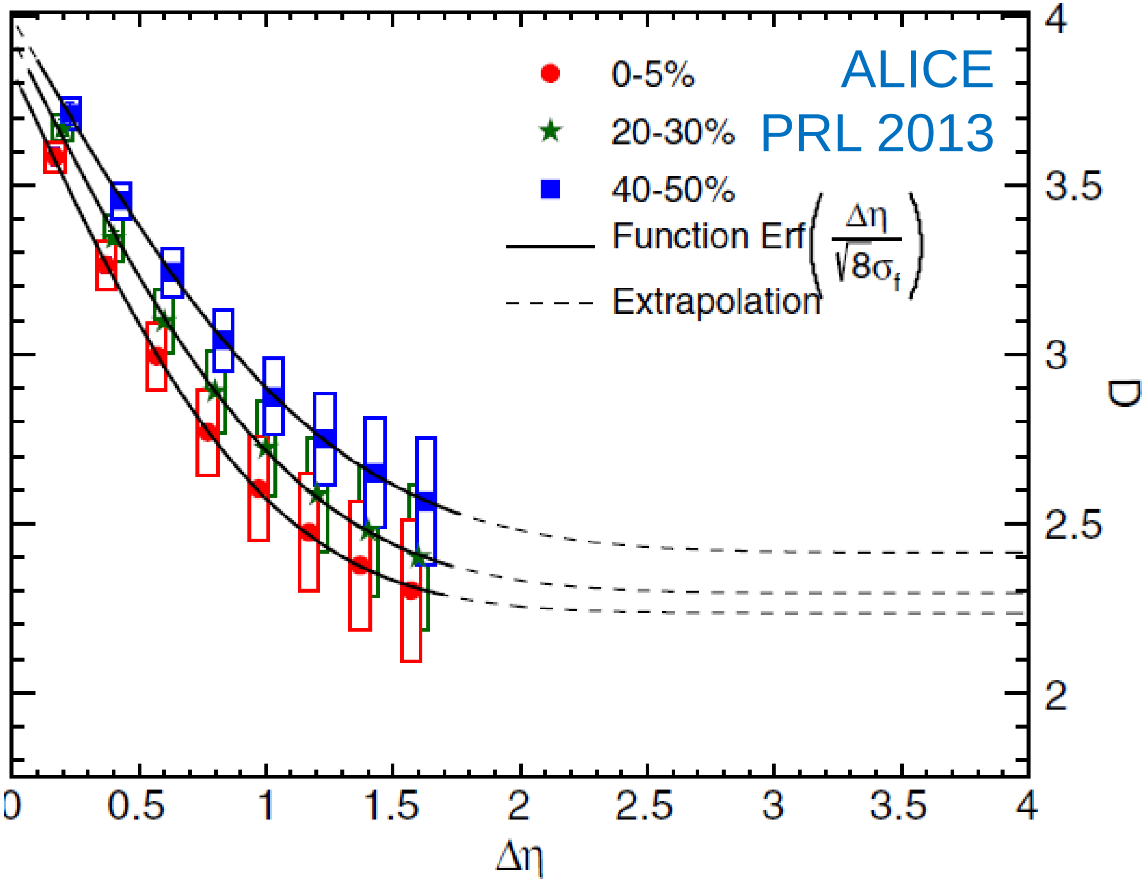
Fluctuations continue to change until kinetic freezeout!!

Time Evolution of Fluctuations

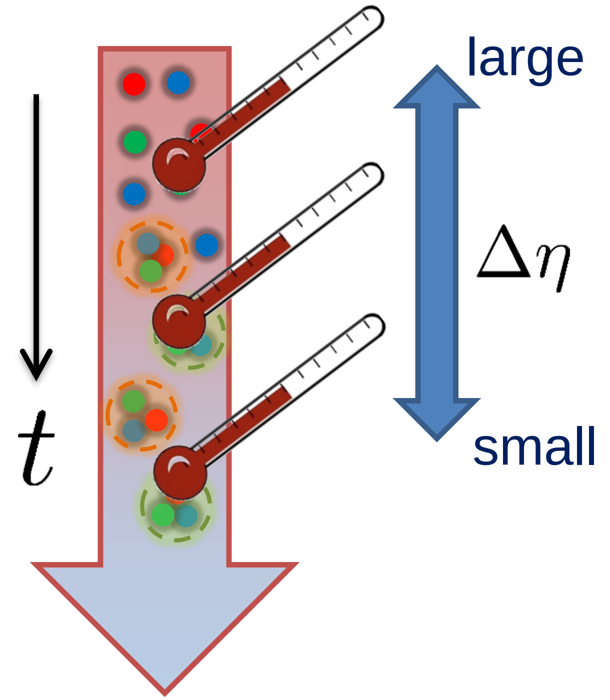


Variation of a conserved charge is achieved only through diffusion. ⇨ The larger $\Delta\eta$, the slower diffusion

Δη Dependence @ ALICE

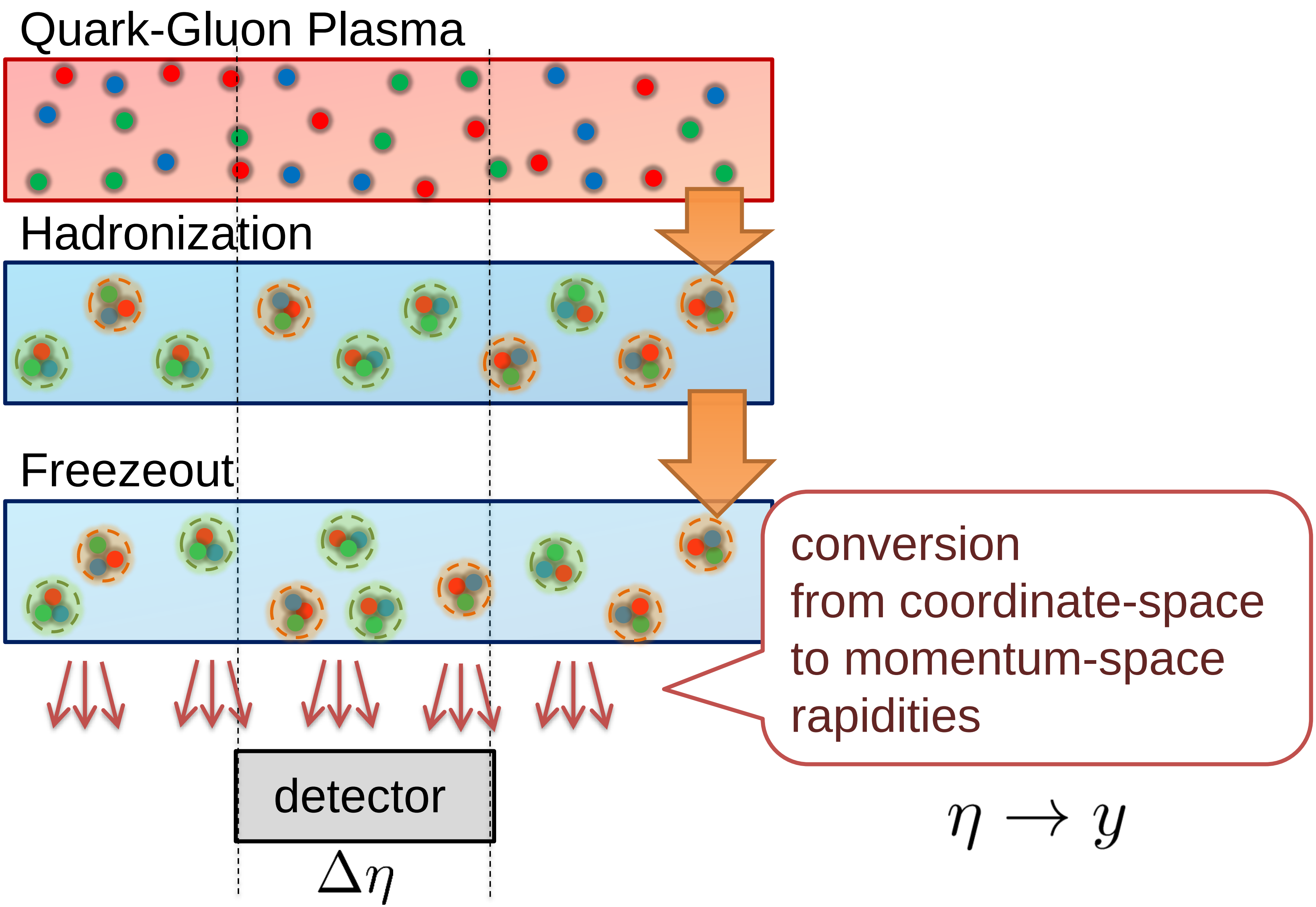


Δη dependent thermometer?



Δη dependences of fluctuation observables encode history of the hot medium!

Conversion of Rapidities

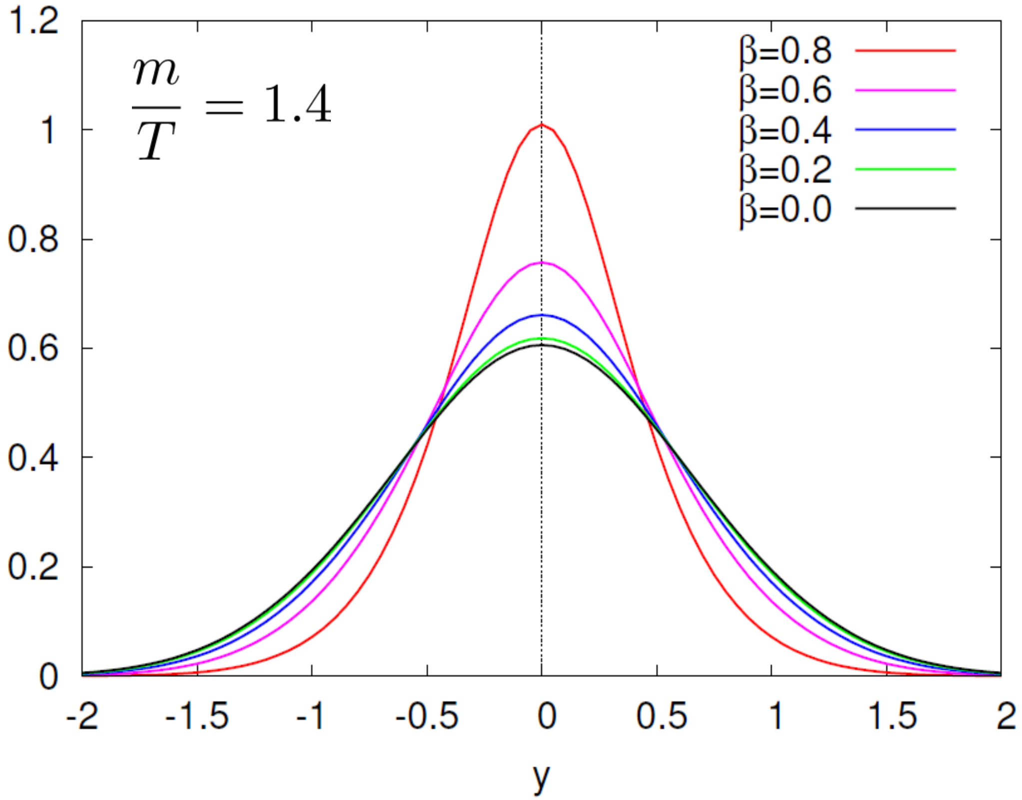


$$\eta \to y$$

Blurring with Rapidity Conversion

Y. Onishi+,
in prep
（修士論文）

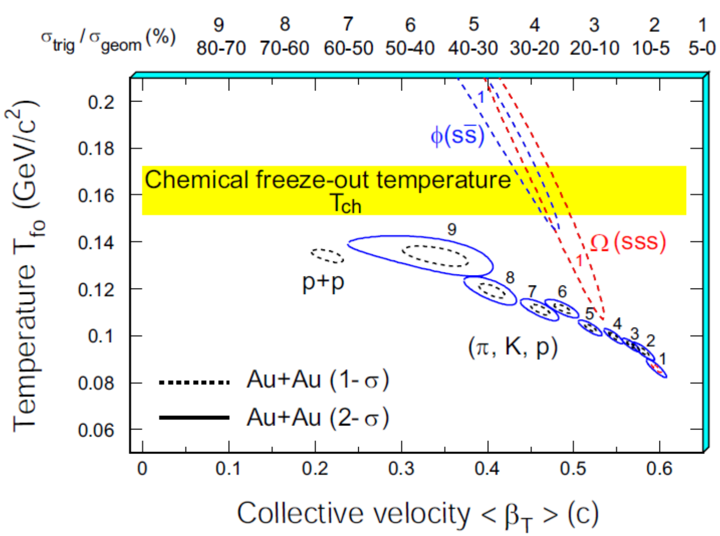
y distribution of a particle at η=0



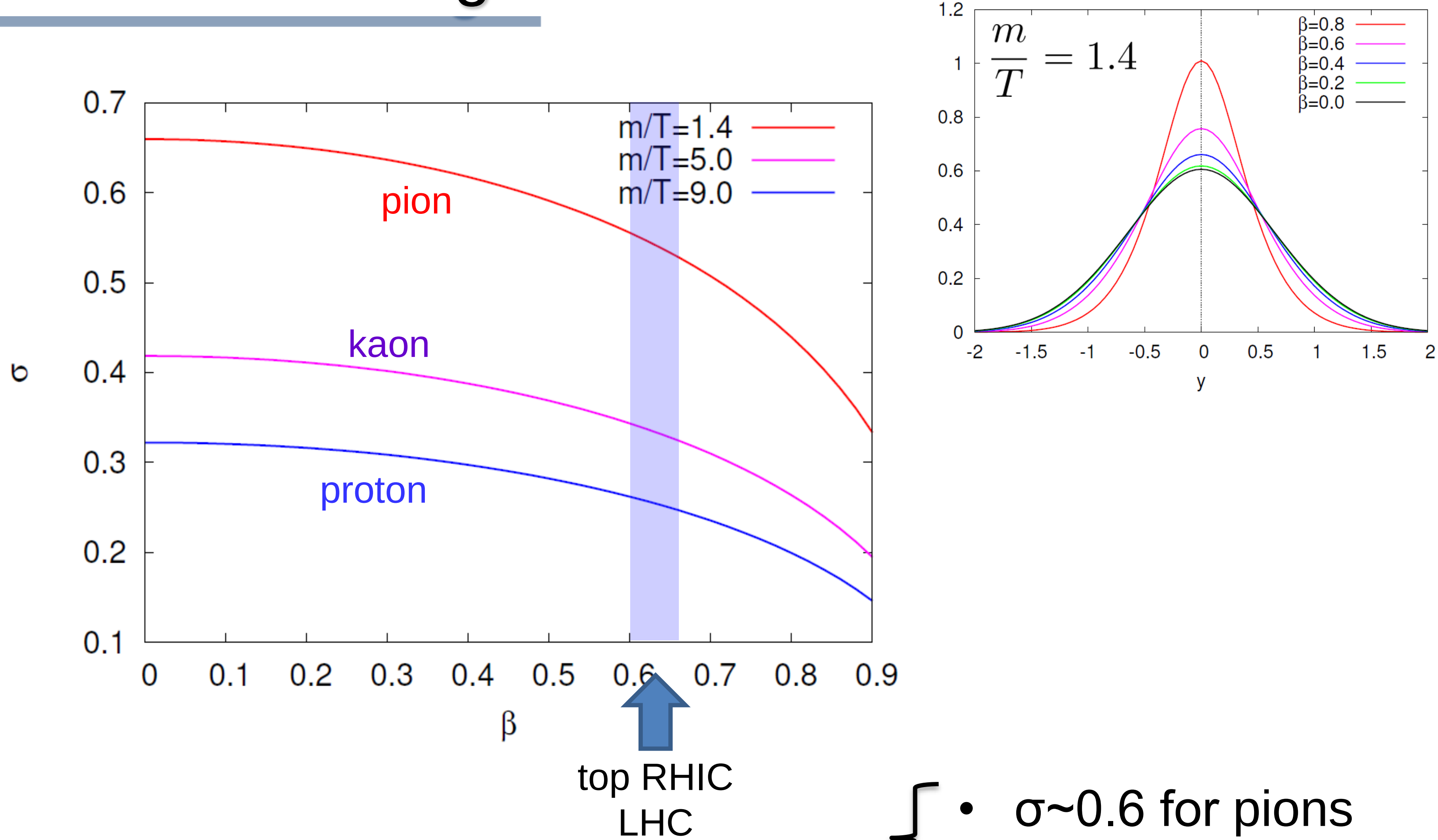
- Assume Bjorken expansion
- y: momentum rapidity
- η: coordinate-space rapidity

thermal distribution + blast wave

blast wave fit for 200GeV @STAR



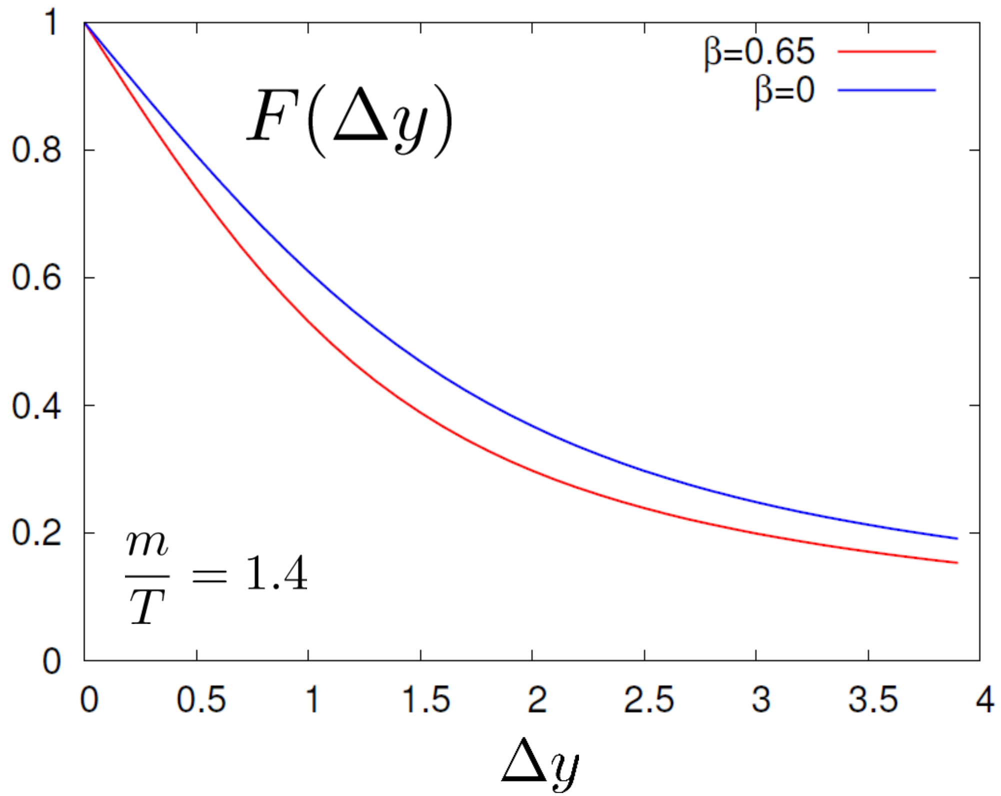
Width of Blurring



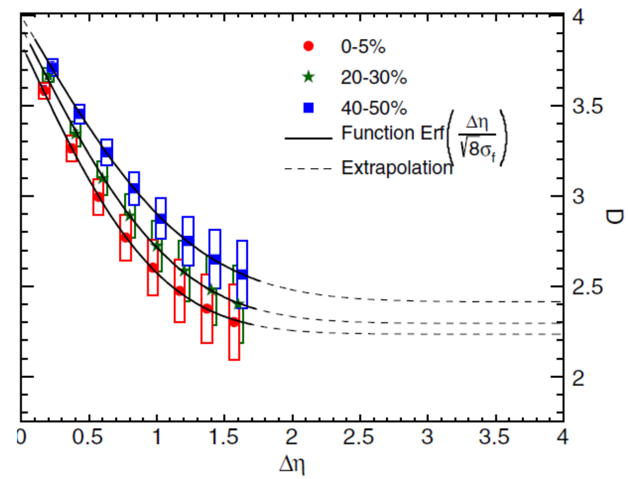
- σ~0.6 for pions
- σ~0.3 for protons

y dependence

$$\frac{\langle \delta N^2 \rangle_{\Delta y}}{\langle \delta N^2 \rangle_{\rm eq}} = F(\Delta y) + (1 - F(\Delta y)) \frac{\langle \delta N^2 \rangle}{\langle \delta N^2 \rangle_{\rm eq}}$$



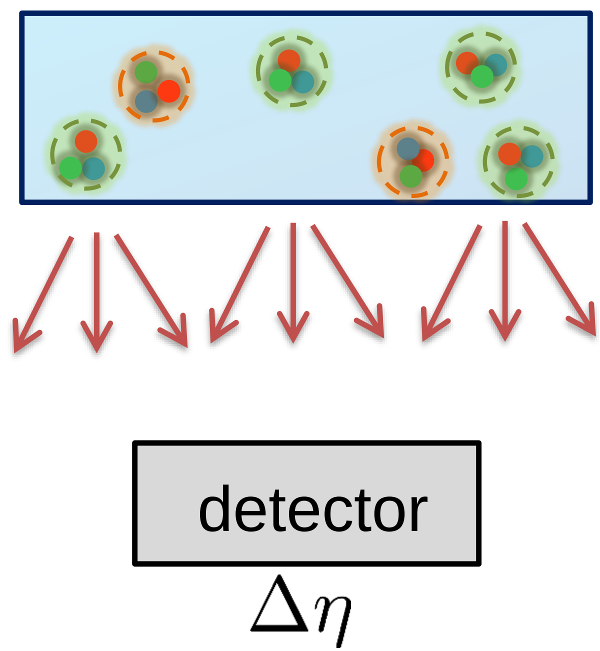
ALICE electric charge 2nd



- 電磁電荷に対し、にじみ効果はかなり大きい。
- Δy=1では、熱平衡値との平均程度。

Very Low Energy Collisions

- Large contribution of global charge conservation
- Violation of Bjorken scaling



→坂井田さんのトーク

Careful treatment is required to interpret fluctuations at low beam energies!

統計力学との比較は、おそらくほとんど意味をなさない。
多点相関関数から物理を抜き出す等の新しいアイディアを…

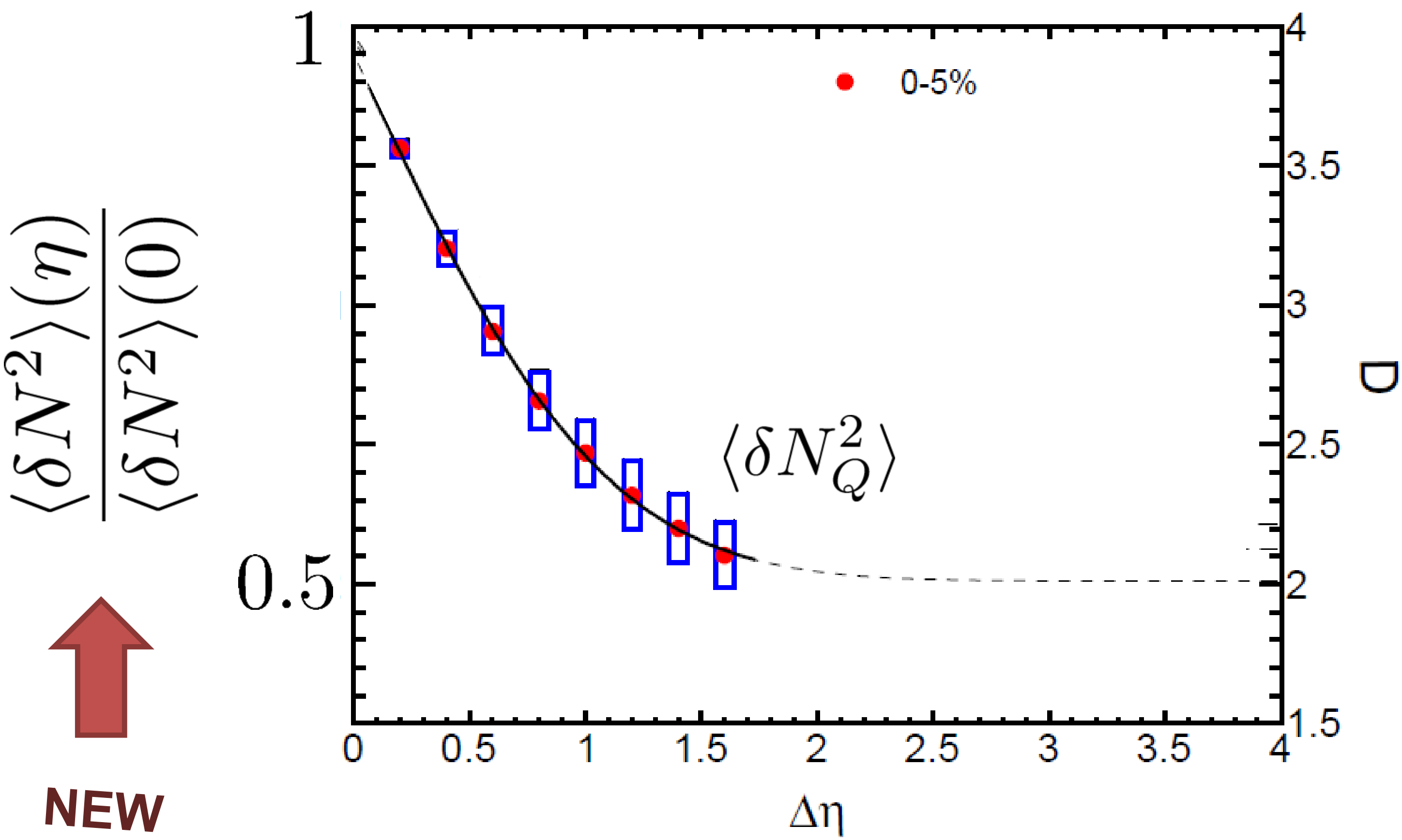
Rapidity Window Dependences of Higher Order Cumulants

MK, Asakawa, Ono, Phys. Lett. B728 (2014) 386-392;
MK, to appear soon

$\langle \delta N_B^2 \rangle$ and $\langle \delta N_p^2 \rangle$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$

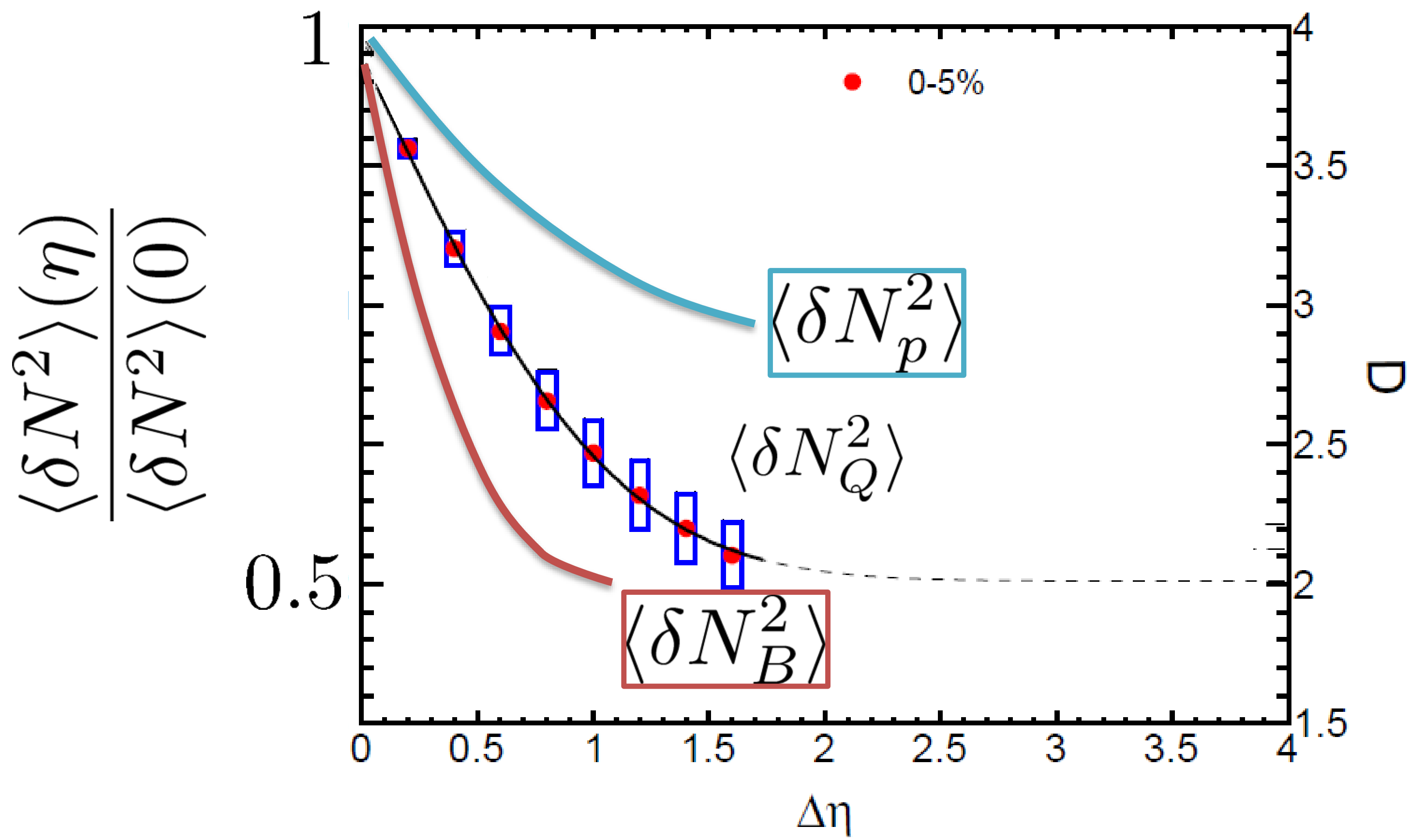
should have different $\Delta\eta$ dependence.



NEW normalization!

$<\delta N_B^2>$ and $< \delta N_p^2 >$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
should have different Δη dependence.



Baryon # cumulants are experimentally observable! MK, Asakawa, 2011;2012

$\langle \delta N_B^2 \rangle$ and $\langle \delta N_p^2 \rangle$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
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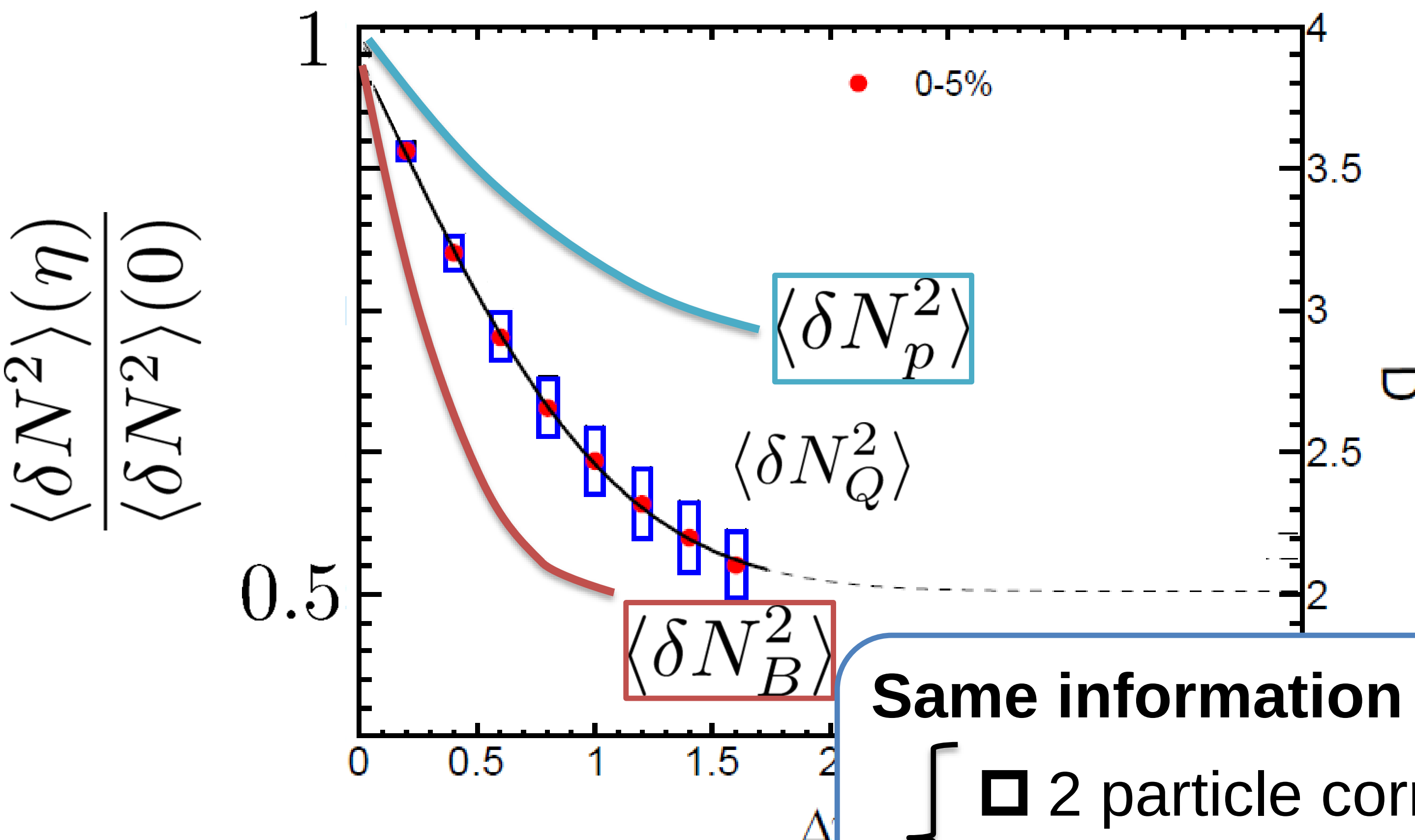


Baryon # cumulants are experimentally

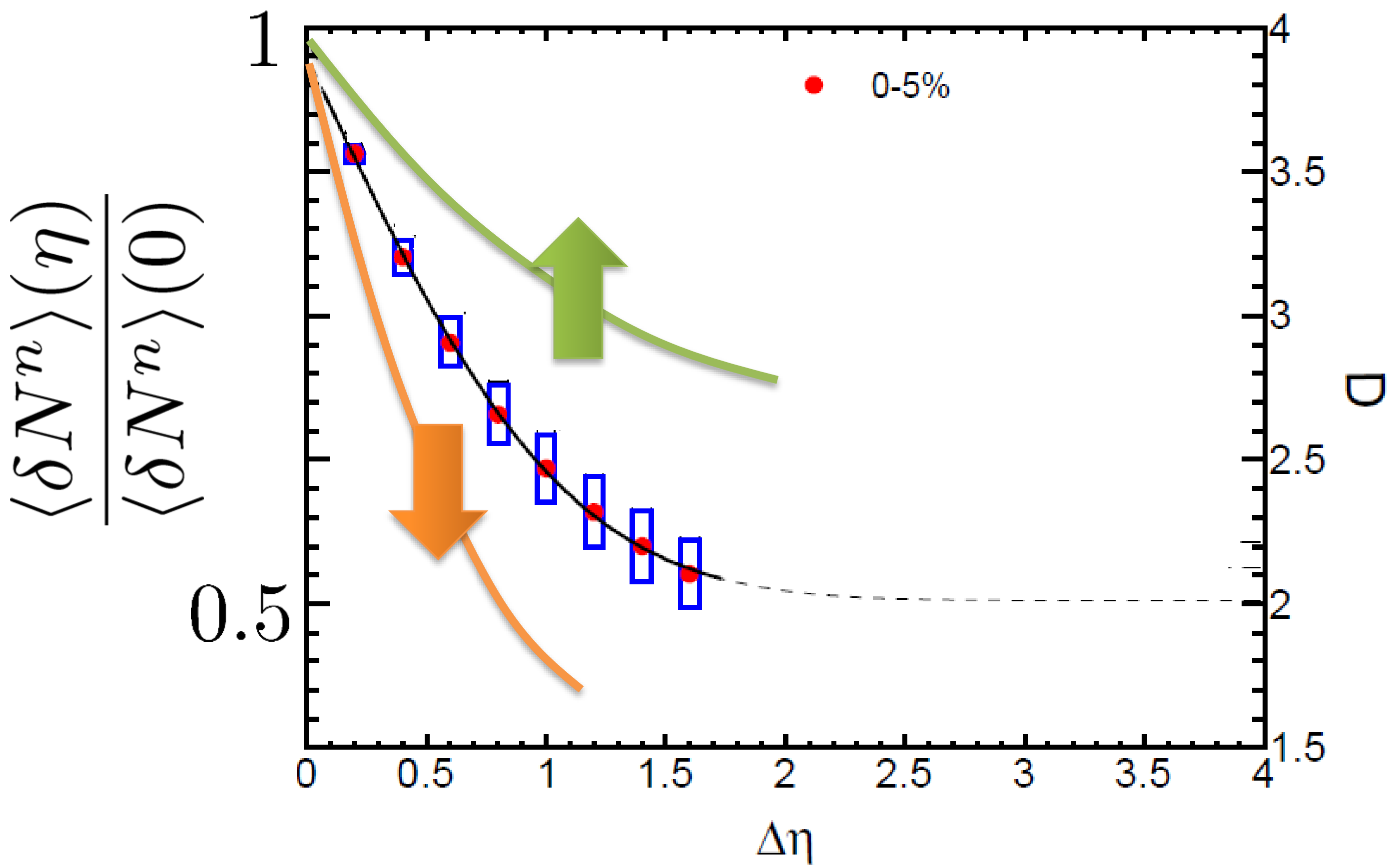
Same information as

- 2 particle corr.: $\langle n(\eta) n(0) \rangle$
- Balance function

$<\delta N_Q{}^4>$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

suppression or enhancement



How to Introduce Non-Gaussianity?

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

⇒ Fluctuation of *n* is Gaussian in equilibrium

How to Introduce Non-Gaussianity?

Stochastic diffusion equation

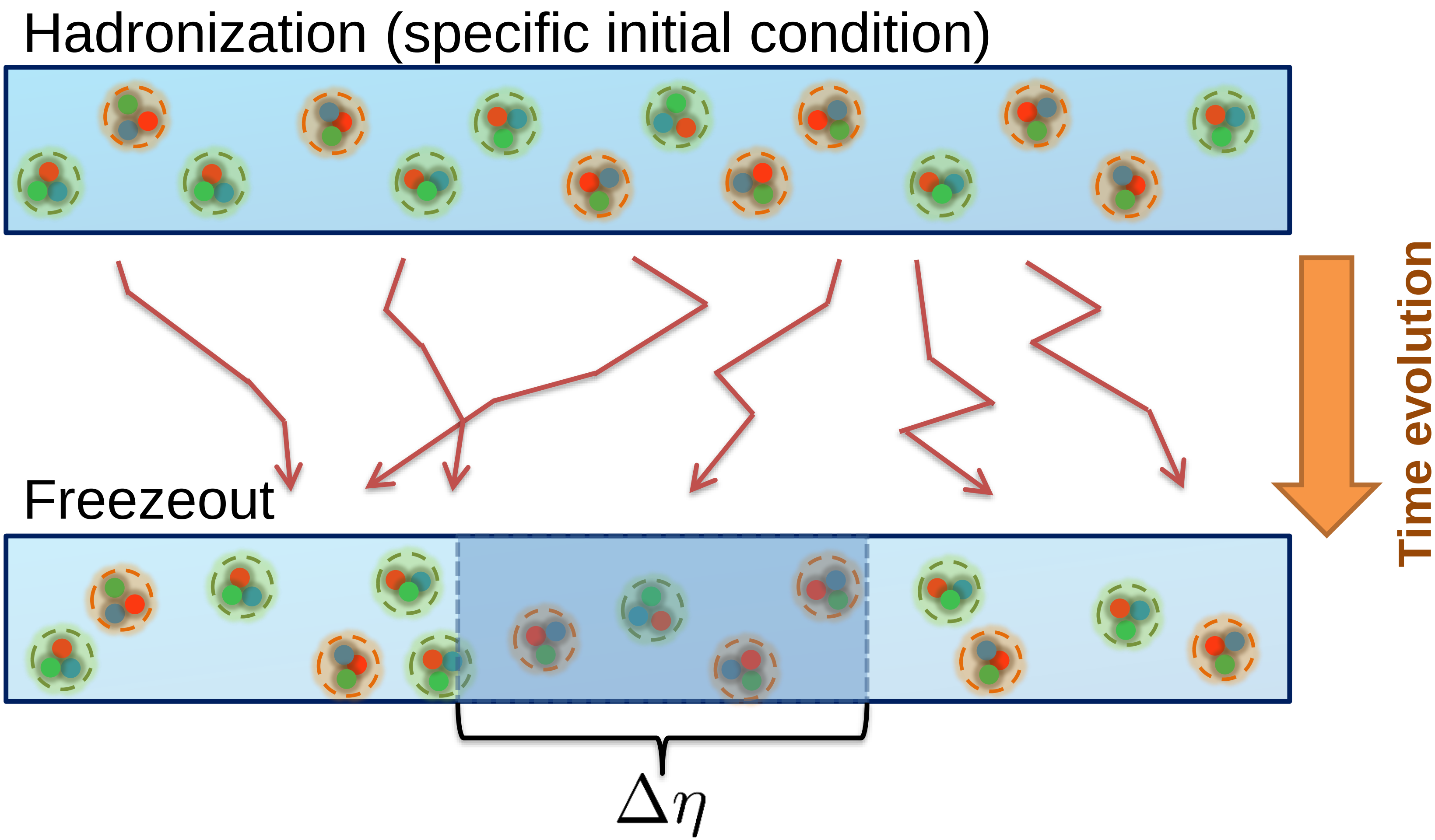
$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Fluctuation of *n* is Gaussian in equilibrium

- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant *D*(*n*)
  - colored noise
  - discretization of *n* ← **our choice**

REMARK: Fluctuations measured in HIC are almost Poissonian.

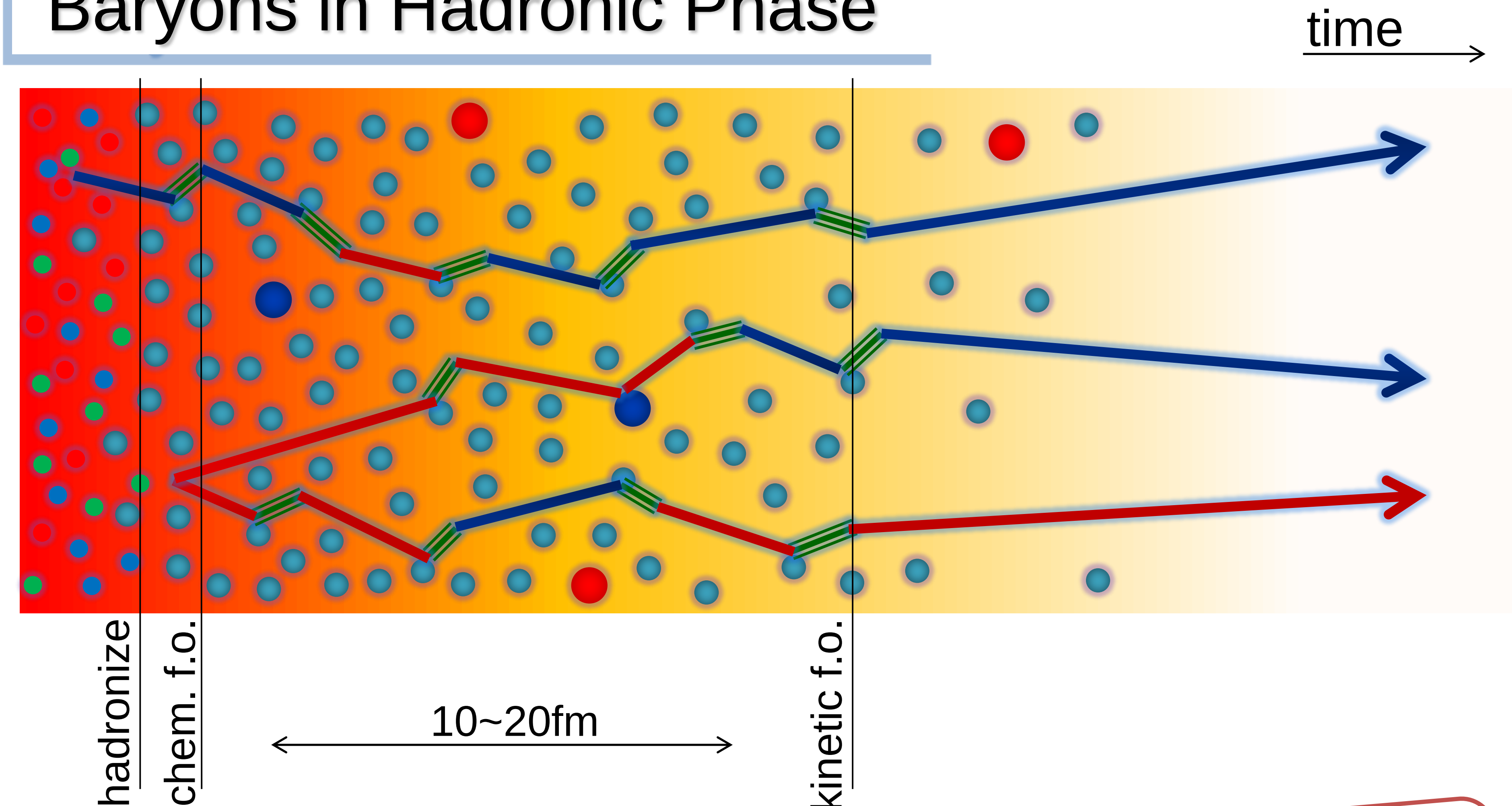
A Brownian Particle's Model



① Describe time evolution of Brownian particles exactly

② Obtain cumulants of particle # in $\Delta\eta$

Baryons in Hadronic Phase

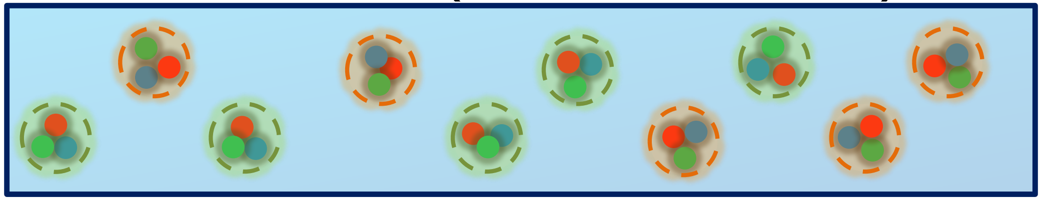


- $p, \bar{p}$
- $n, \bar{n}$
- $\Delta(1232)$
- mesons
- baryons

Baryons behave like Brownian pollens in water

Initial Condition @ Hadronization

Hadronization (initial condition)



- Boost invariance / infinitely long system
- Local equilibration / local correlation

$\langle \bar{Q}^2 \rangle_c, \ \langle \bar{Q}^3 \rangle_c, \ \langle \bar{Q}^4 \rangle_c$ — $\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ — $\langle Q^2_{(\mathrm{tot})} \rangle_c, \langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c$

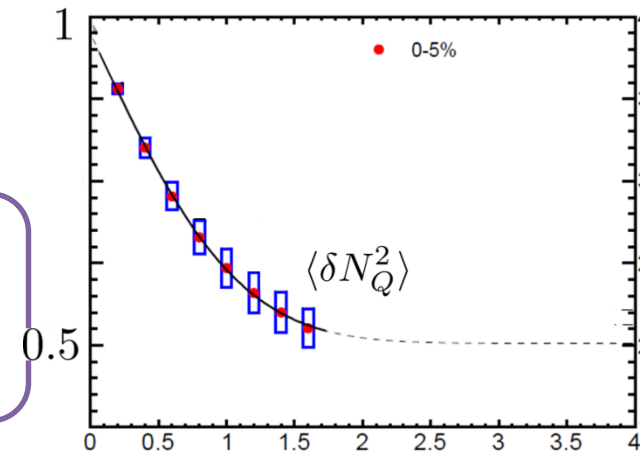
suppression owing to local charge conservation

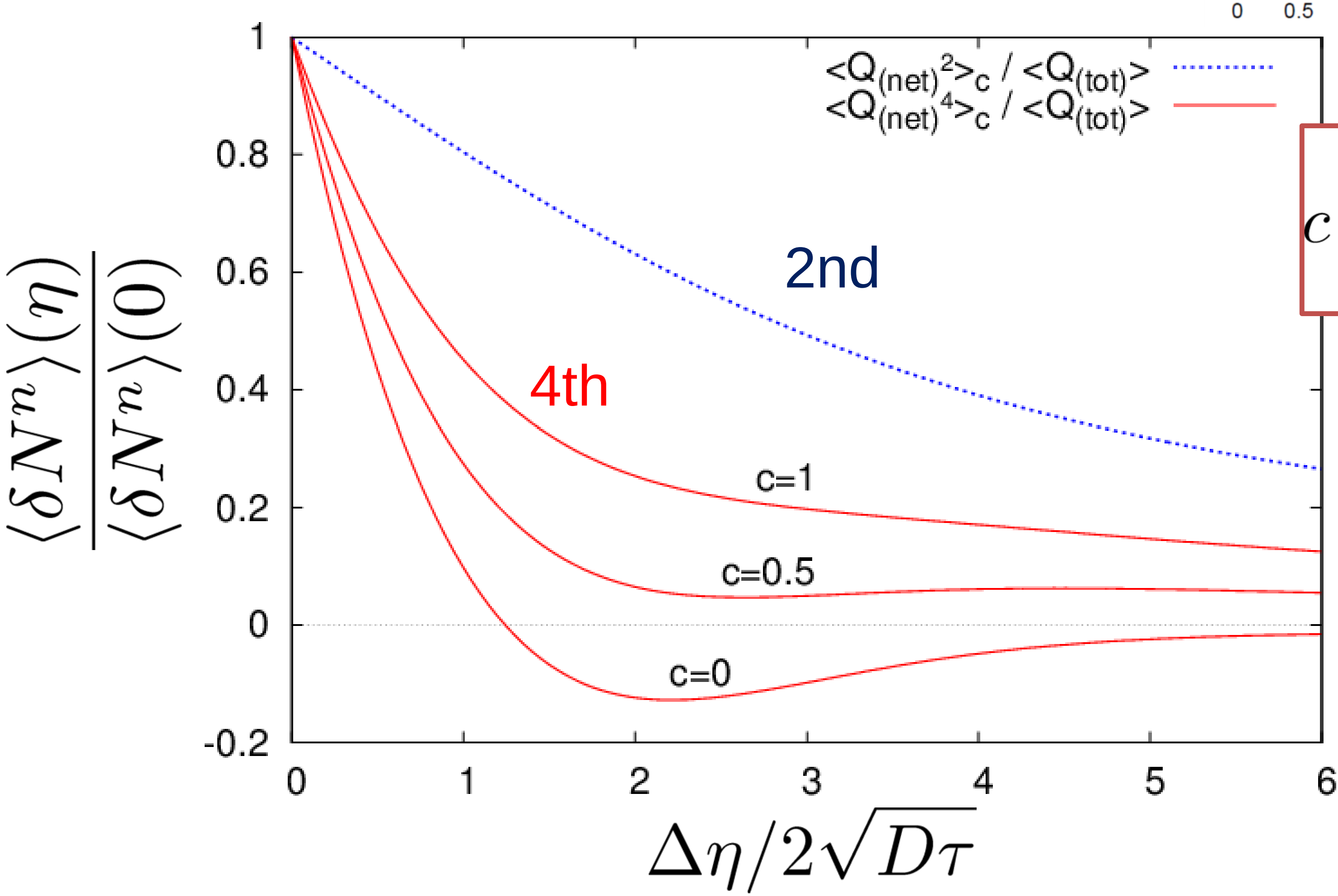
strongly dependent on hadronization mechanism

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$



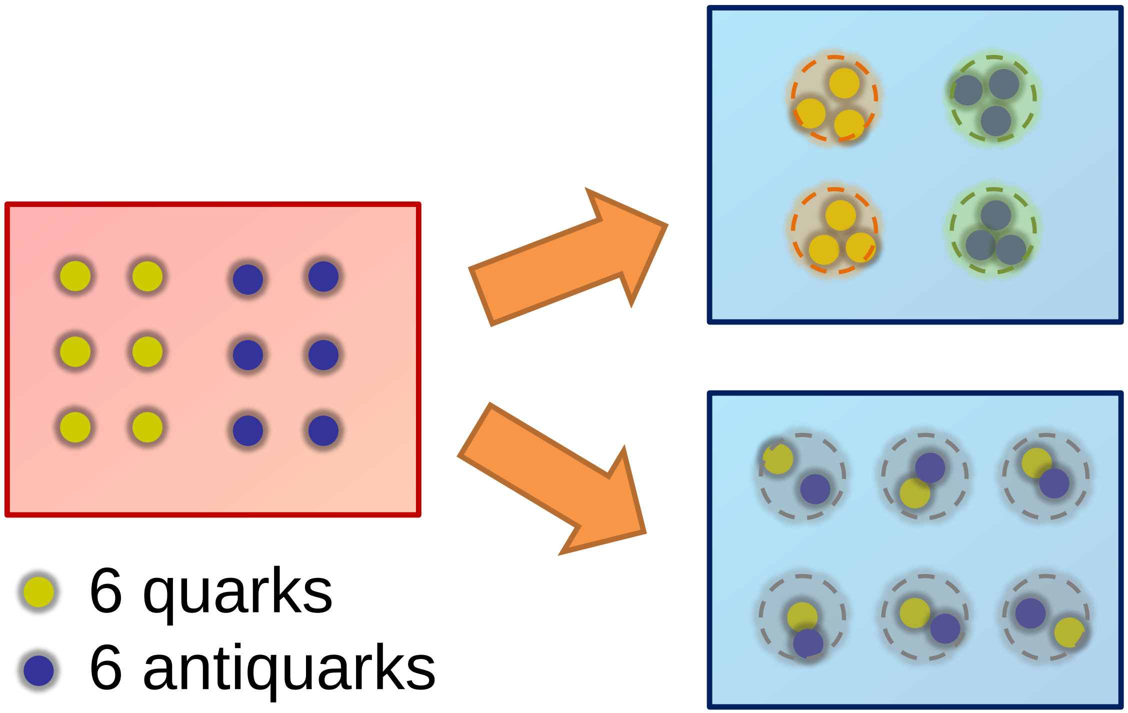


$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

Total Charge Number

In recombination model,



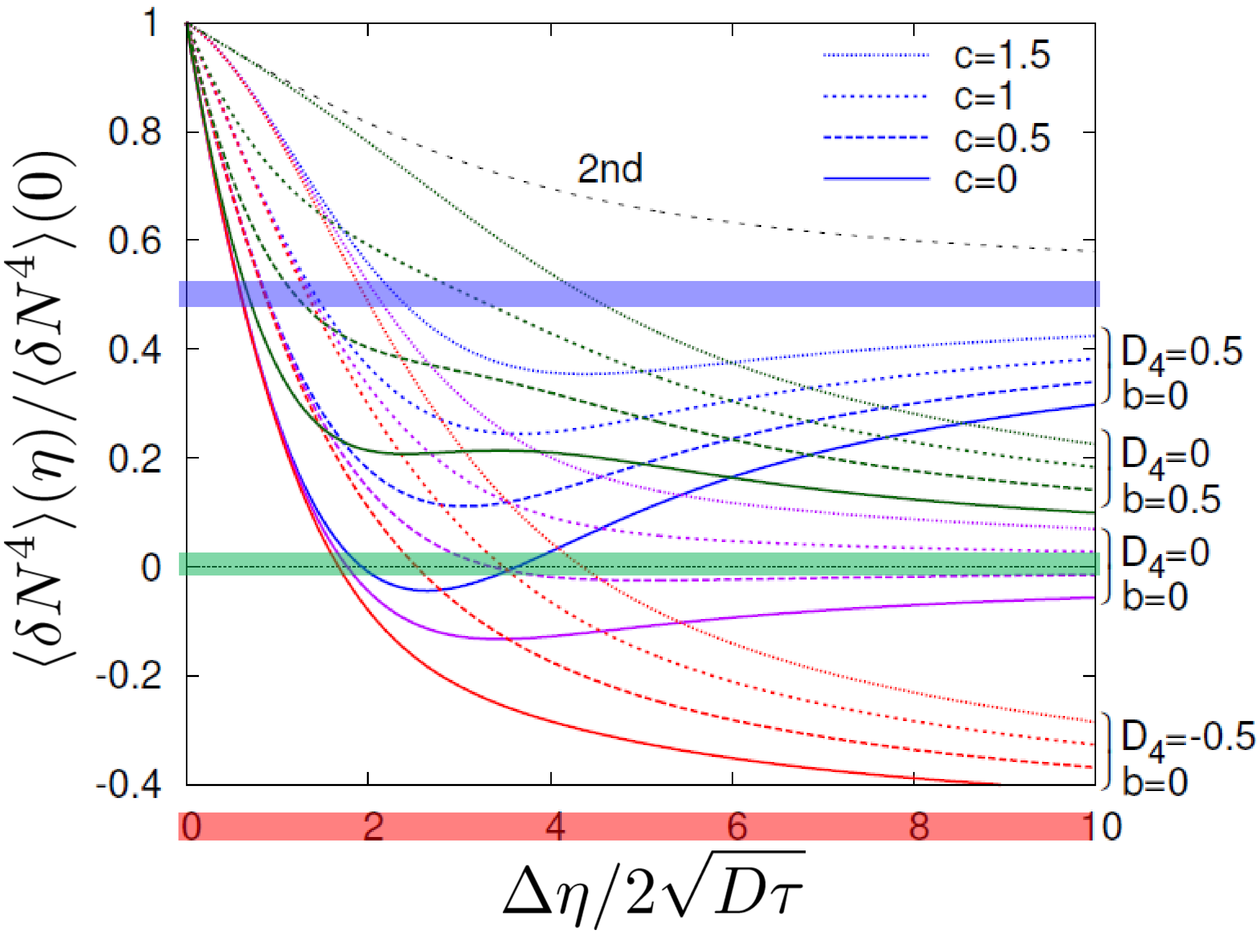
$$N_B^{(\mathrm{net})} = 0 \quad N_B^{(\mathrm{tot})} = 4$$

$$N_B^{(\mathrm{net})} = 0 \quad N_B^{(\mathrm{tot})} = 0$$

- $N_B^{(\mathrm{tot})}$ can fluctuate, while $N_B^{(\mathrm{net})}$ does not.

$\Delta\eta$ Dependence: 4th order

MK, to appear soon



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

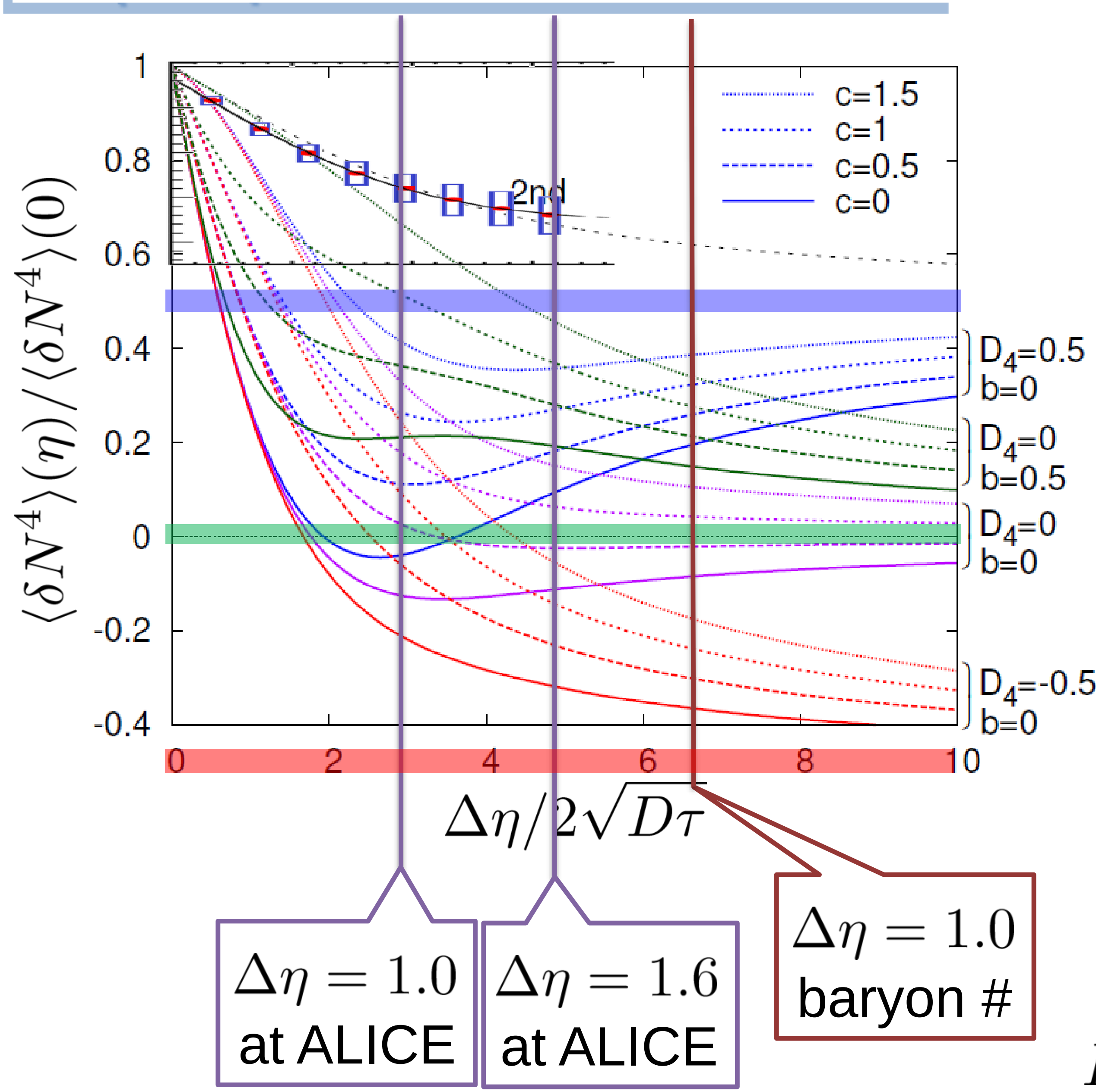
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

Charcteristic $\Delta\eta$ dependences!

Δη Dependence: 4^{th} order

MK, to appear soon



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

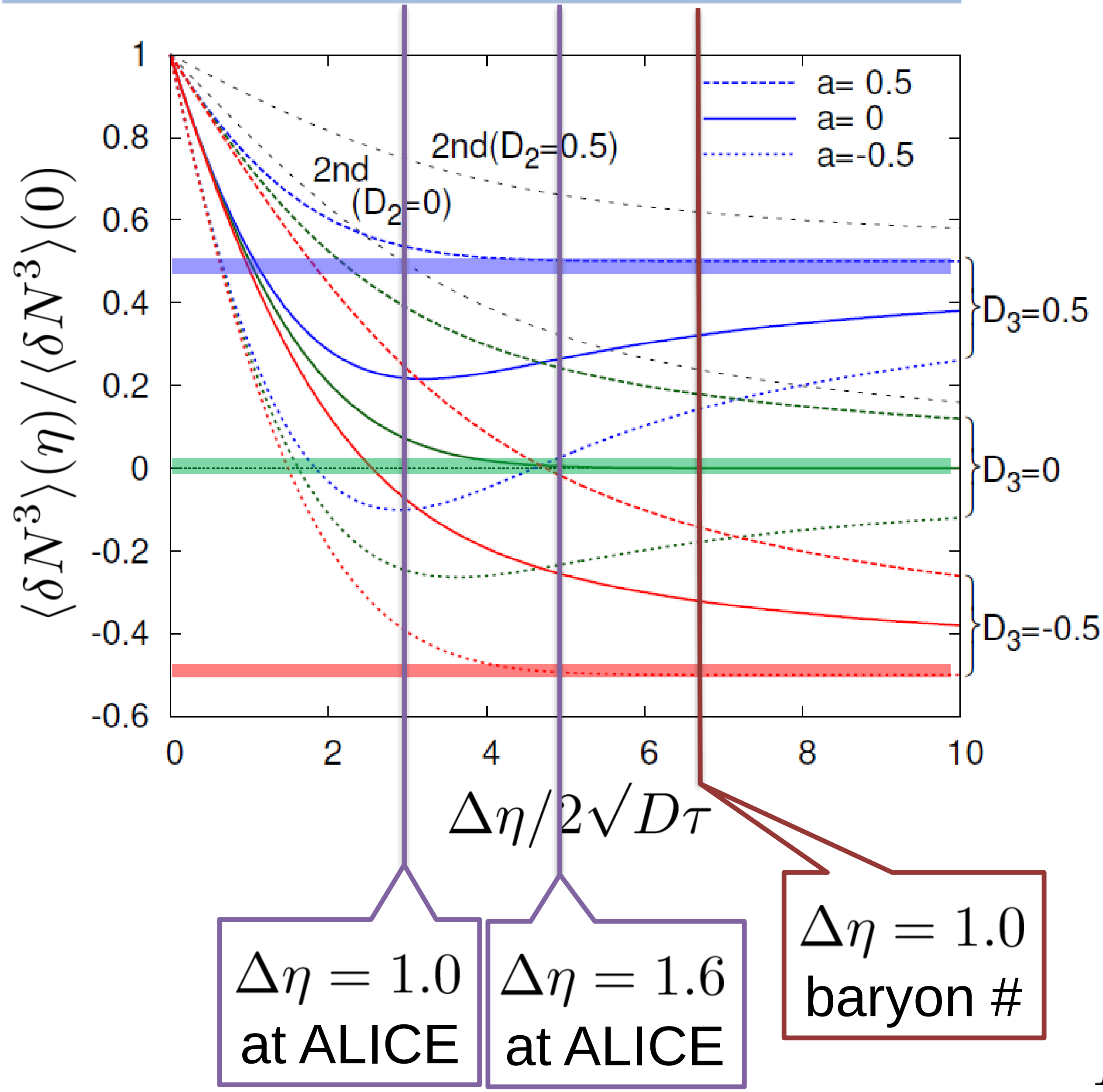
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

$$D \sim M^{-1}$$

Δη Dependence: 3rd order

MK, to appear soon



Initial Condition

$$D_3 = \frac{\langle Q^3_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$a = \frac{\langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

$$D \sim M^{-1}$$

Summary

- Fluctuations are not frozen at chemical freezeout time.
- Careful treatment is required especially for low √s.
- Multi-particle correlation should be interesting!

Plenty of information in Δη dependences of various cumulants

$$\langle N_Q^2\rangle_c,\ \langle N_Q^3\rangle_c,\ \langle N_Q^4\rangle_c,\ \langle N_B^2\rangle_c,\ \langle N_B^3\rangle_c,\ \langle N_B^4\rangle_c$$

and those of non-conserved charges, mixed cumulants…

And, multi-particle correlation functions

Many Things to Do

- Message to Experimentalists:
  - Measure rapidity window dependences
  - Determine baryon number cumulants

- Message to Theorists:
  - Do not directly compare your thermal results with exp.
  - Let’s pursue descriptions of non-eq. non-Gaussianity.

- Message to Latticians:
  - Do not directly compare your results with exp.
  - Measure more cumulants more accurately

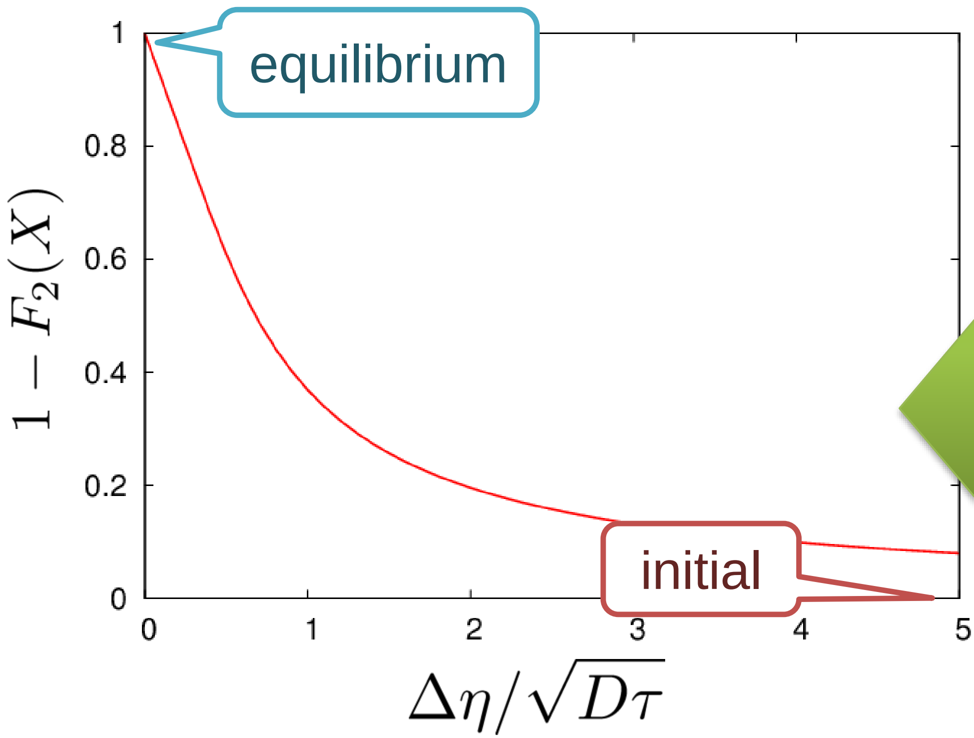
$\Delta\eta$ Dependence

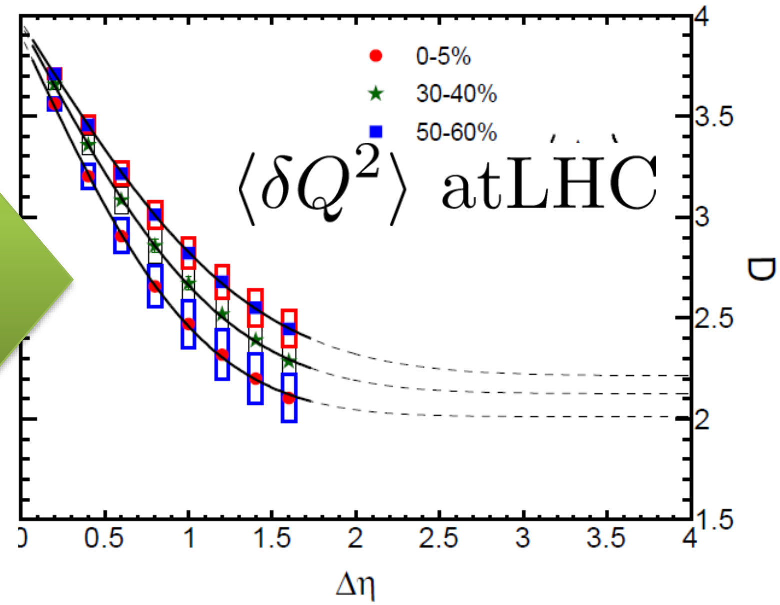
Shuryak, Stephanov, 2001

- Initial condition: $\langle \delta n(\eta_1, 0)\delta n(\eta_2, 0)\rangle = \sigma_2 \delta(\eta_1 - \eta_2)$
- Translational invariance

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta, \tau)$$

$$\langle \delta Q(\tau)^2 \rangle = \underbrace{\sigma_2 F_2(X)}_{\text{initial}} + \underbrace{\chi_2(1 - F_2(X))}_{\text{equilibrium}}$$

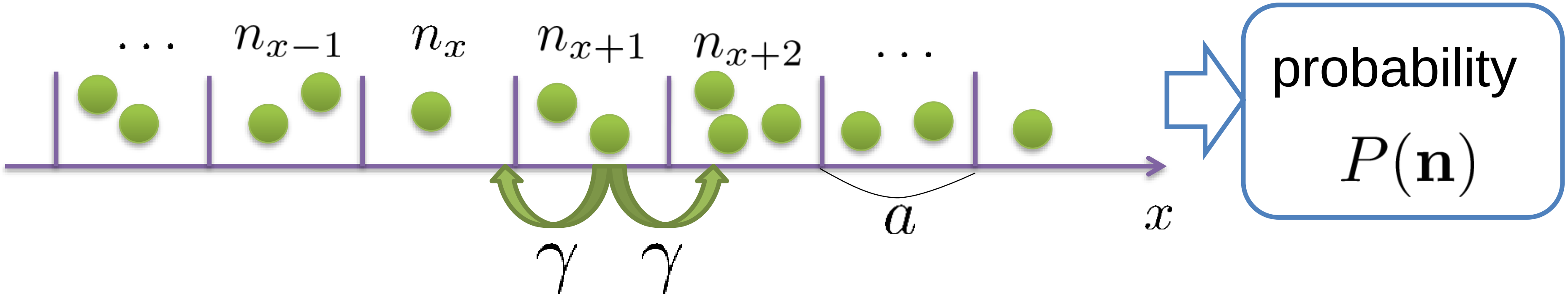




Diffusion Master Equation

MK, Asakawa, Ono, 2014

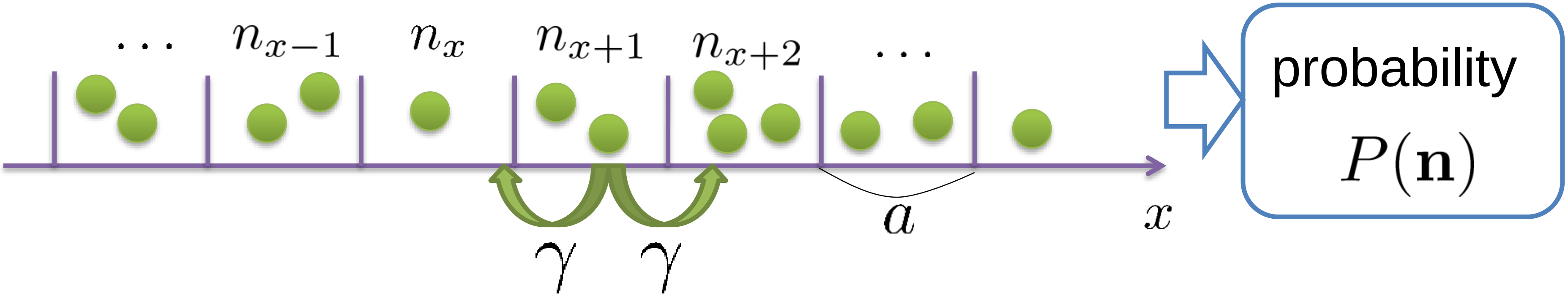
Divide spatial coordinate into discrete cells



Diffusion Master Equation

MK, Asakawa, Ono, 2014

Divide spatial coordinate into discrete cells



Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x + 1)\{P(\mathbf{n}+\mathbf{e}_x - \mathbf{e}_{x+1}) + P(\mathbf{n}+\mathbf{e}_x - \mathbf{e}_{x-1})\} - 2n_x P(\mathbf{n})]$$

Solve the DME **exactly**, and take *a*→0 limit

No approx., ex. van Kampen's system size expansion

Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

Stephanov, Shuryak, 2001

Markov (white noise)
+
continuity

Gaussian noise

Fluctuation of *n* is
Gaussian in equilibrium

cf) Gardiner, "Stochastic Methods"