文学に学ぶクォークの物理

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物理院生春の学校 2015年3月9日

話題その一

小林一茶と重イオン衝突実験

非ガウスゆらぎで探るクォーク・グルオン・プラズマ

話題その二

論語と格子QCD数値シミュレーション

「勾配フロー」がもたらす格子QCDの新展開

話題その一

小林一茶と重イオン衝突実験

非ガウスゆらぎで探るクォーク・グルオン・プラズマ

一本の草も涼風宿りけり

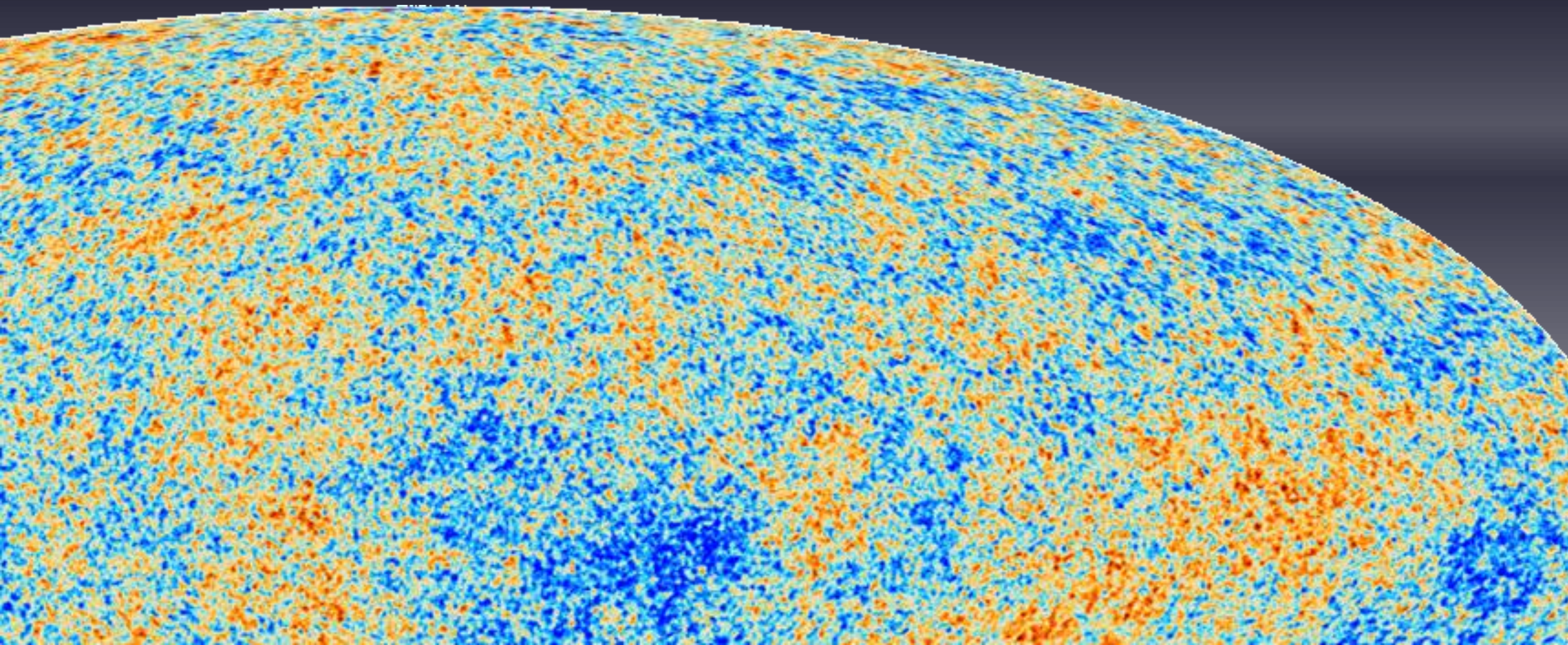
even on one blade of grass the cool wind lives

小林一茶

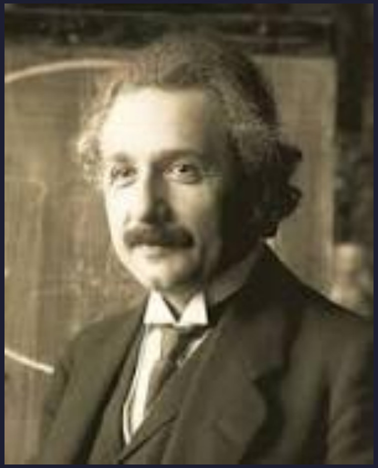
Issa Kobayashi

1814

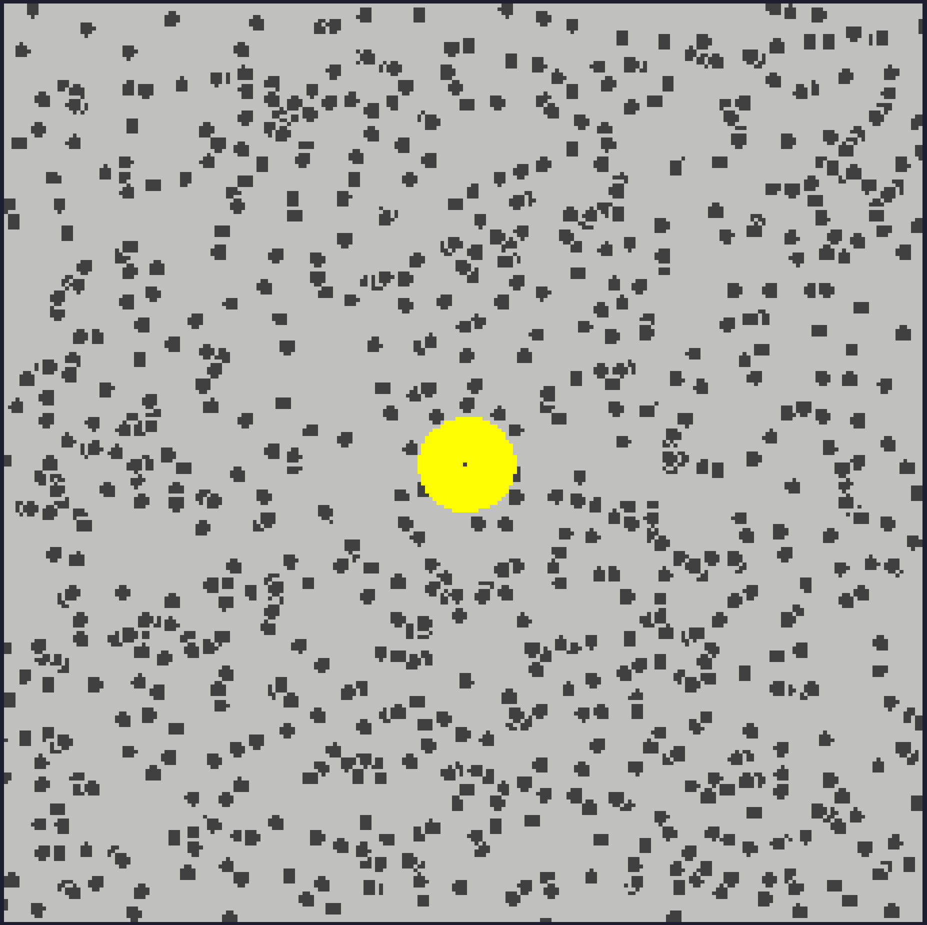
Physicists can feel hot early Universe
13 800 000 000 years ago
in tiny fluctuations of
cosmic microwave



Physicists can feel the existence of microscopic atoms behind random fluctuations of Brownian pollens



A. Einstein
1905



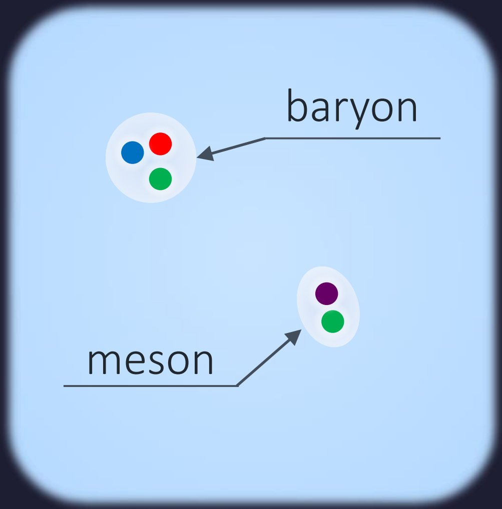
quarks

Feel quarks behind fluctuations
in relativistic heavy ion collisions

2012~

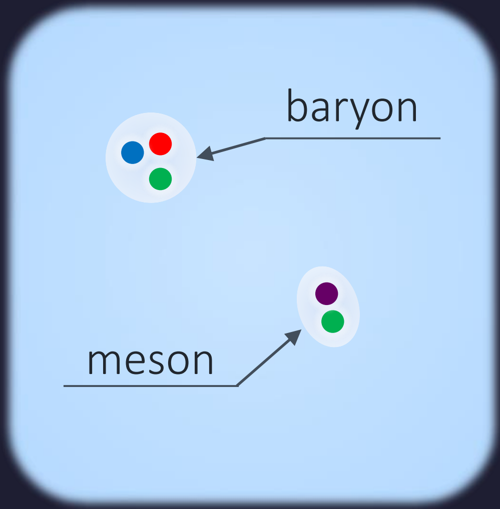
Quark-Gluon Plasma (QGP)

vacuum



Quark-Gluon Plasma (QGP)

vacuum



As T increases …

Early Universe

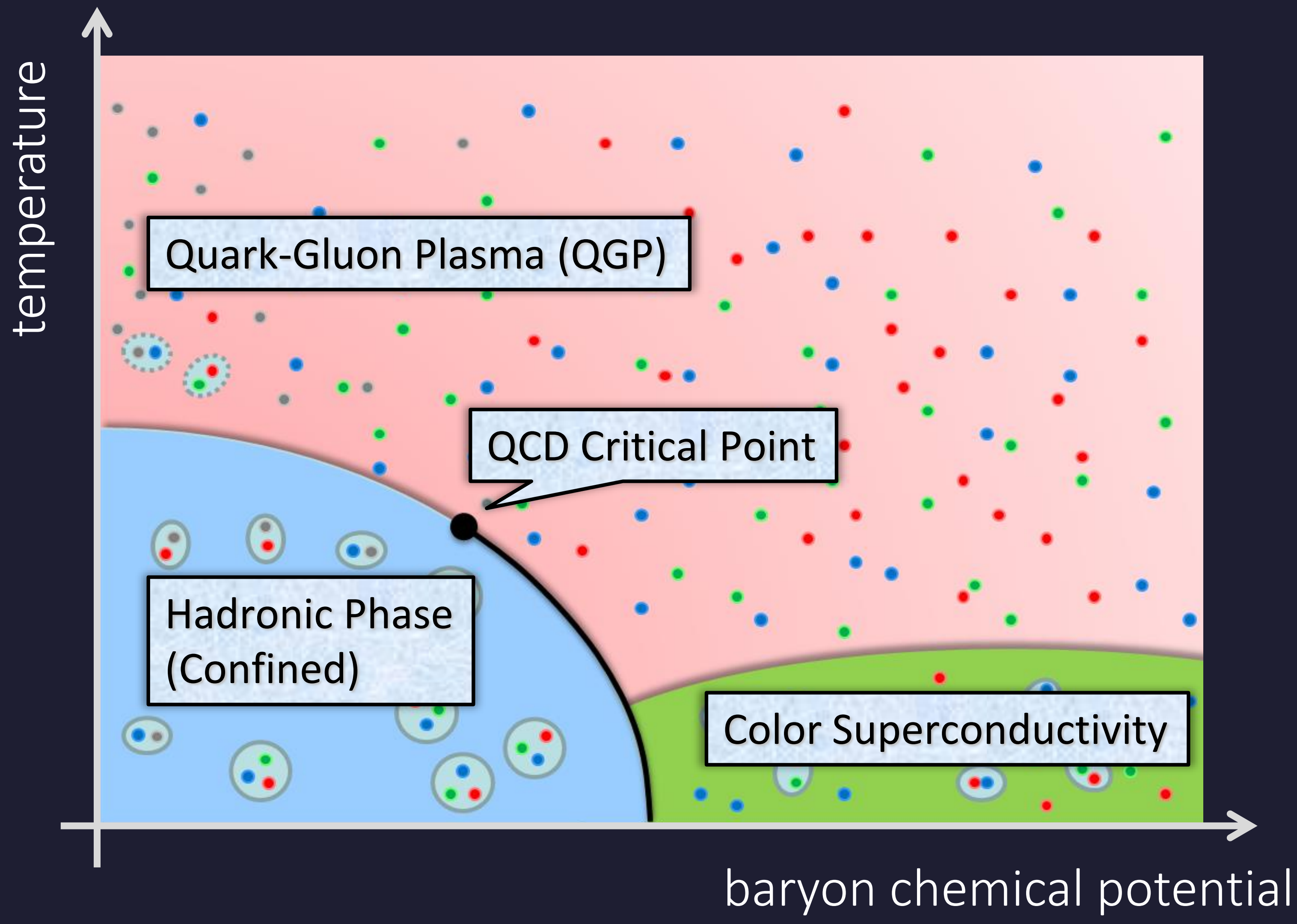
Quark-Gluon Plasma (QGP)

vacuum

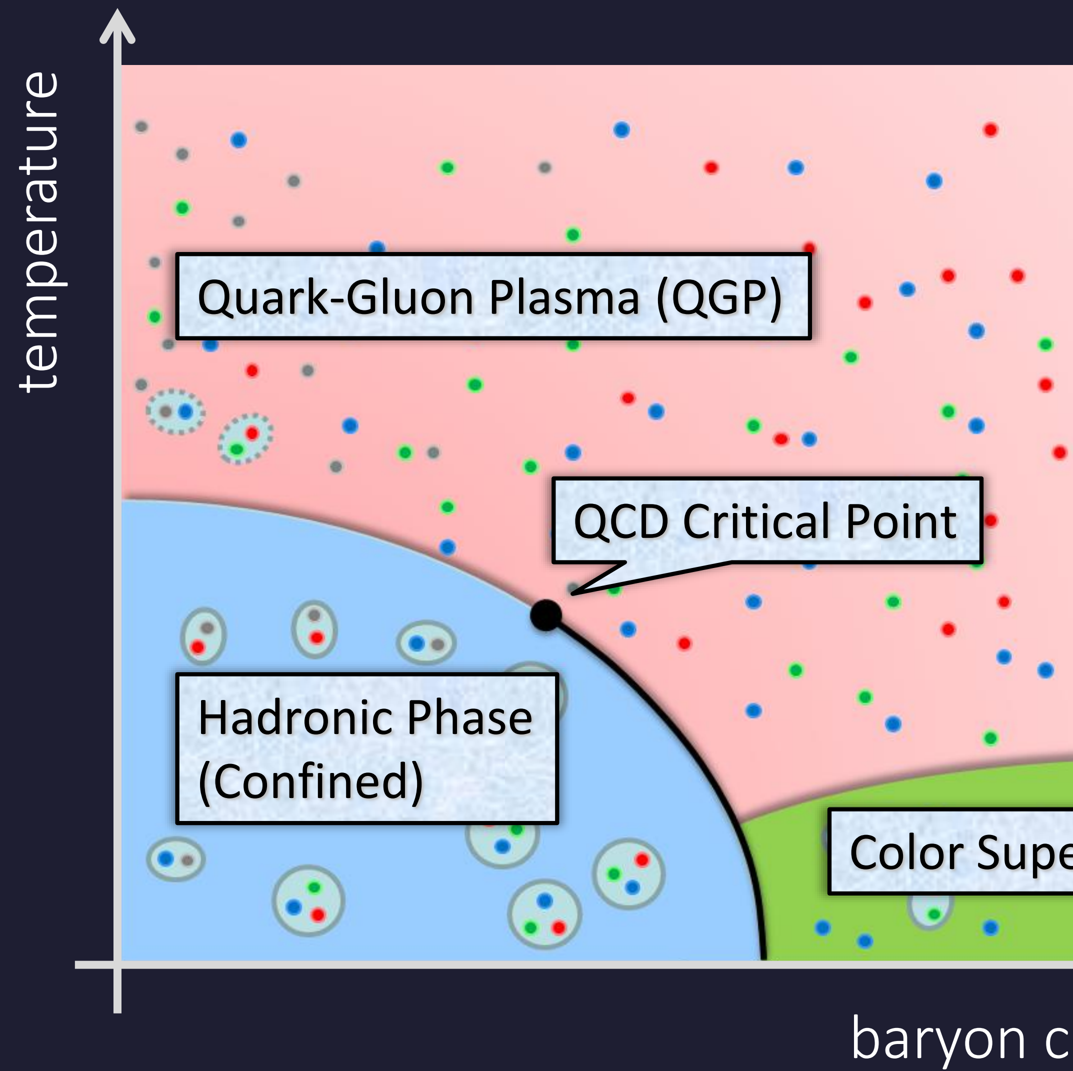
As *T* increases …

Early Universe

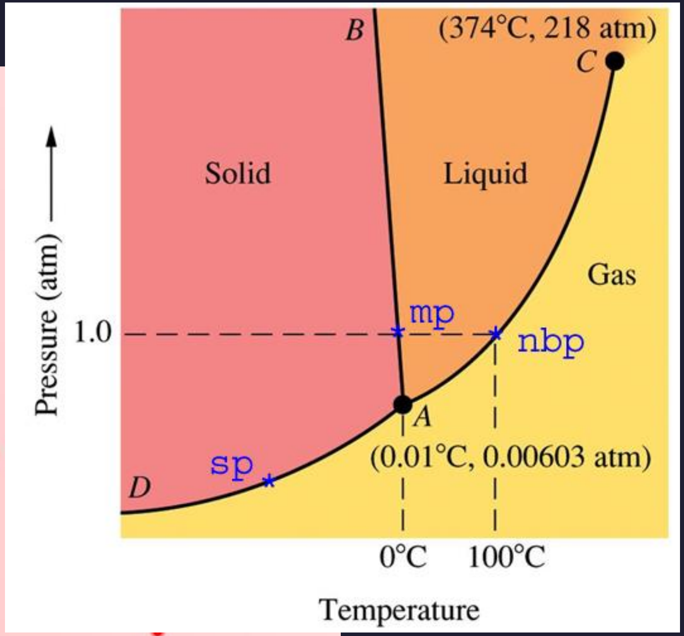
QCD Phase Diagram



QCD Phase Diagram



Phase diagram of water



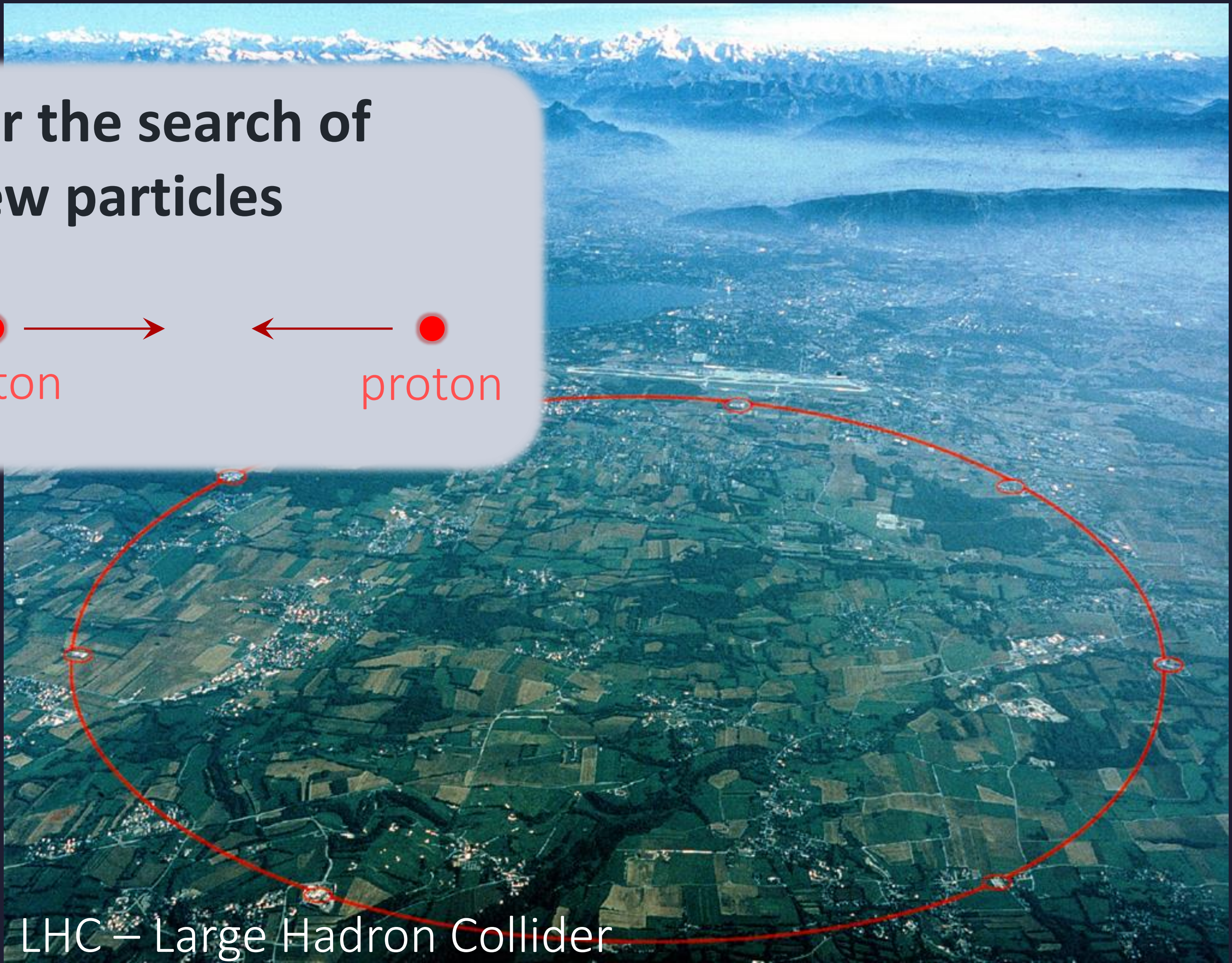
Relativistic Heavy Ion Collisions

LHC – Large Hadron Collider

Relativistic Heavy Ion Collisions

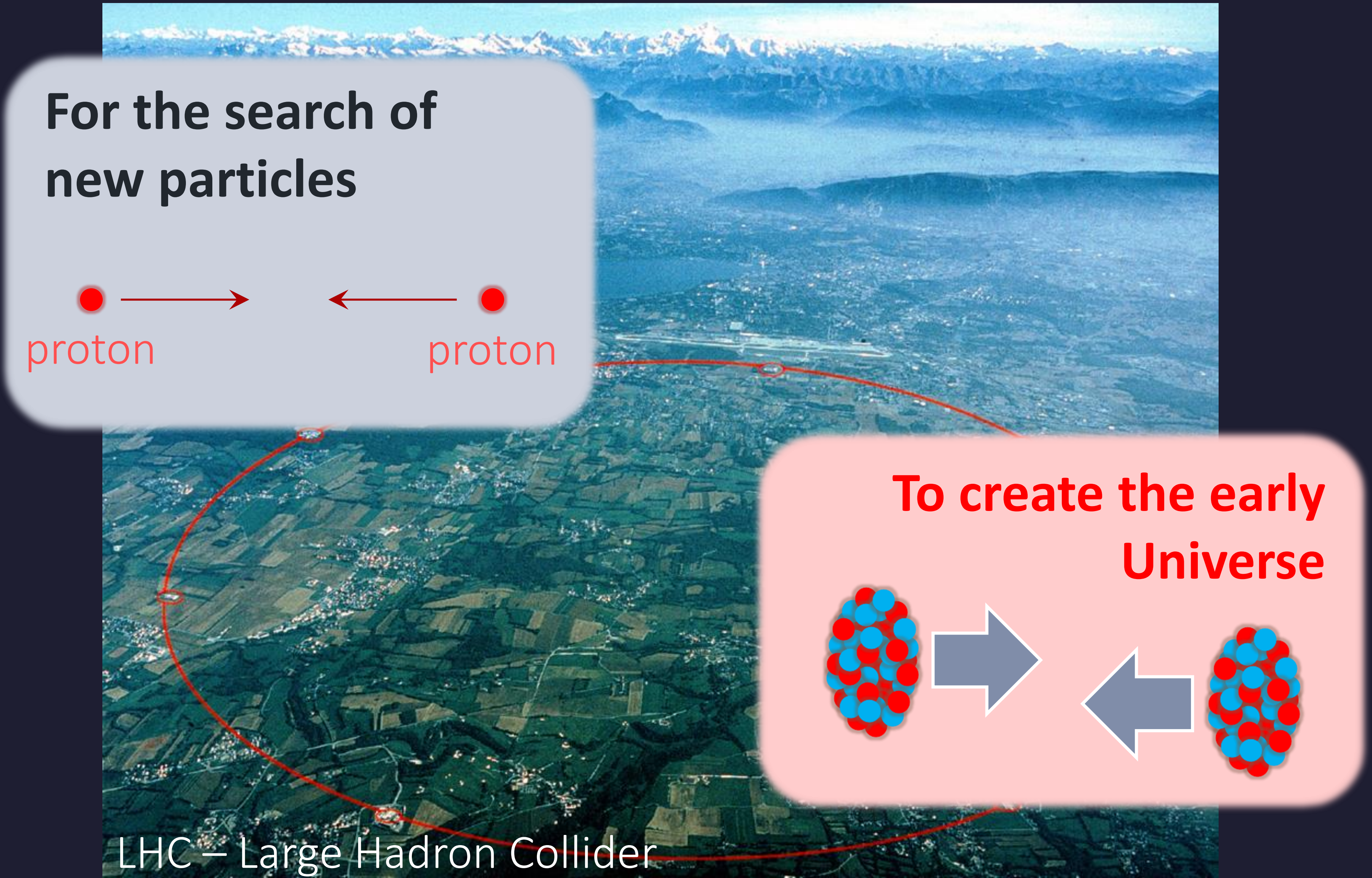
For the search of new particles

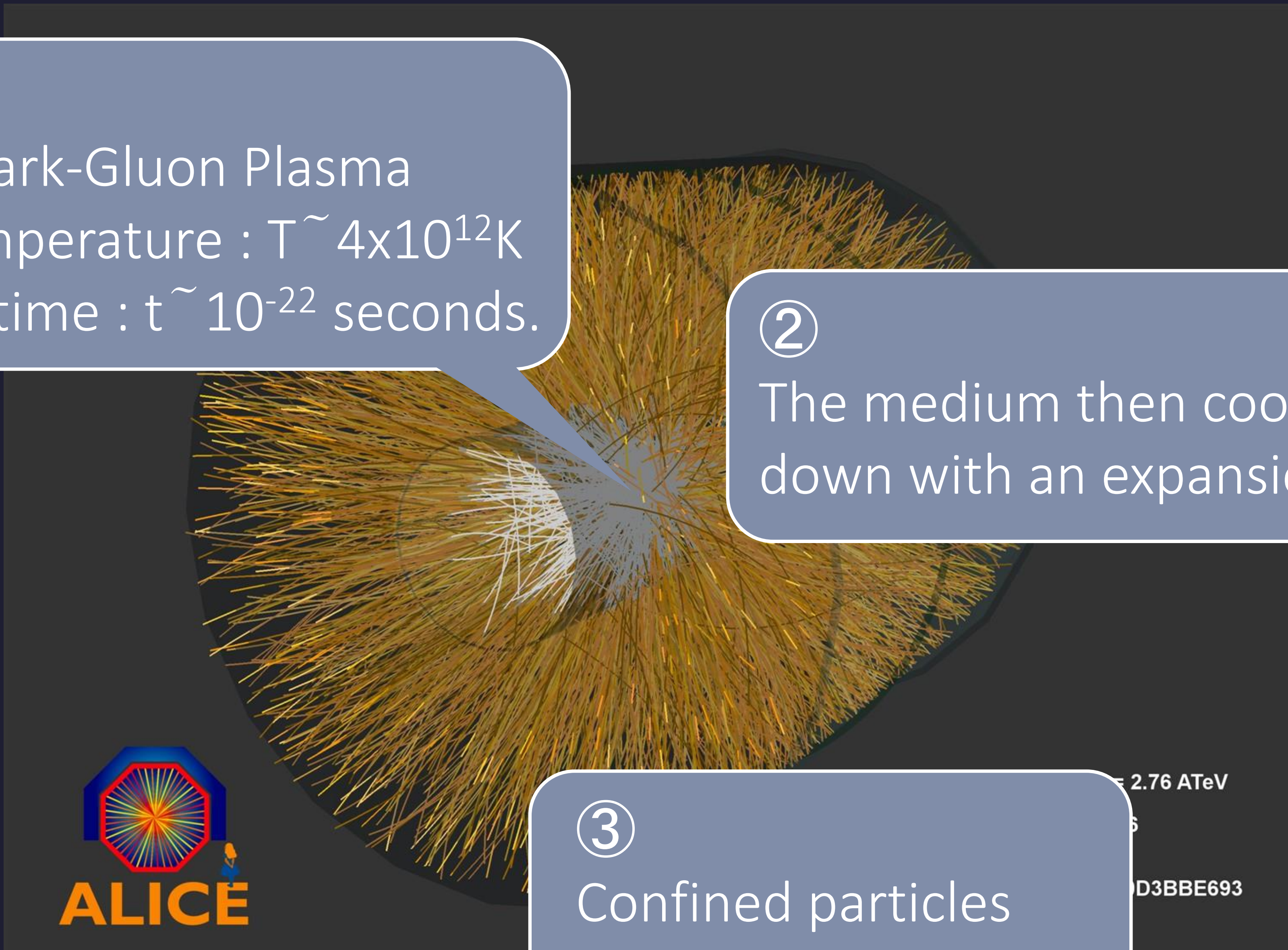
proton → ← proton



LHC – Large Hadron Collider

Relativistic Heavy Ion Collisions





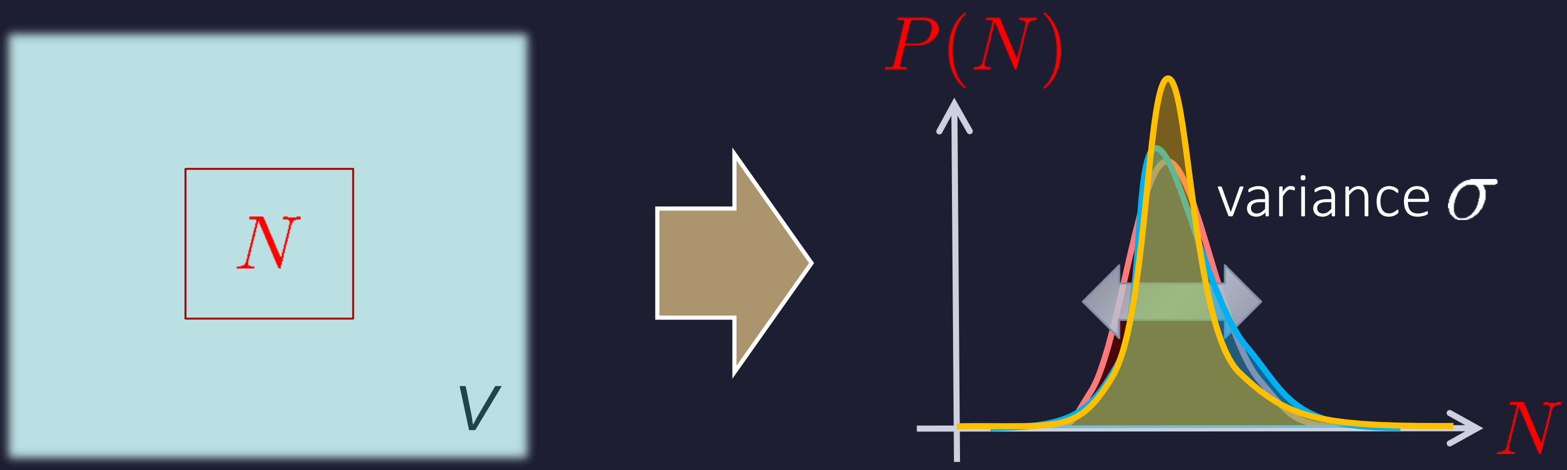
① Quark-Gluon Plasma
temperature : T ~ 4×10^{12} K
lifetime : t ~ 10^{-22} seconds.

② The medium then cools down with an expansion.

③ Confined particles arrive at the detector.

Thermal Fluctuations

Observables in equilibrium are fluctuating!



$$\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$$

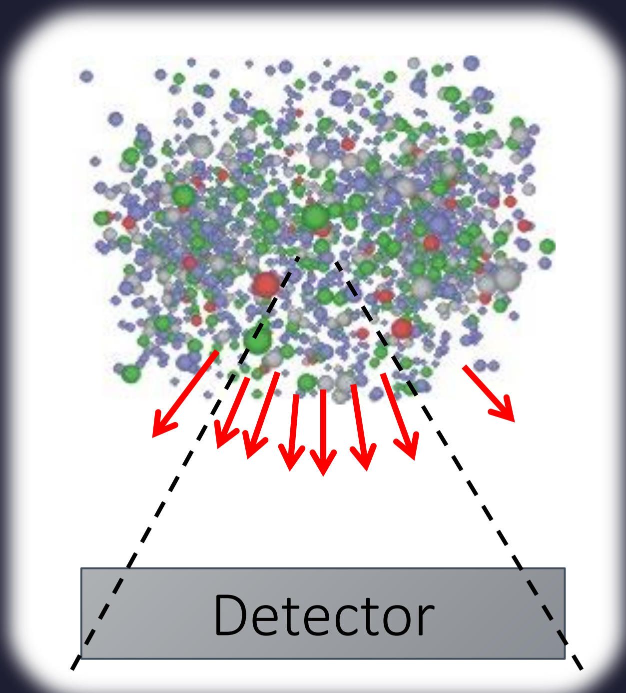
$$S = \frac{\langle \delta N^3 \rangle}{\sigma^3}$$

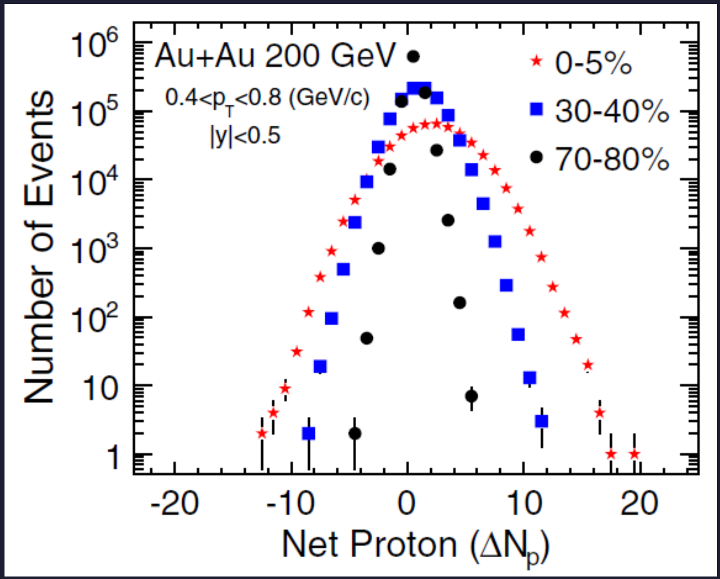
$$\kappa = \frac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$$

Gaussian

non-Gaussianity

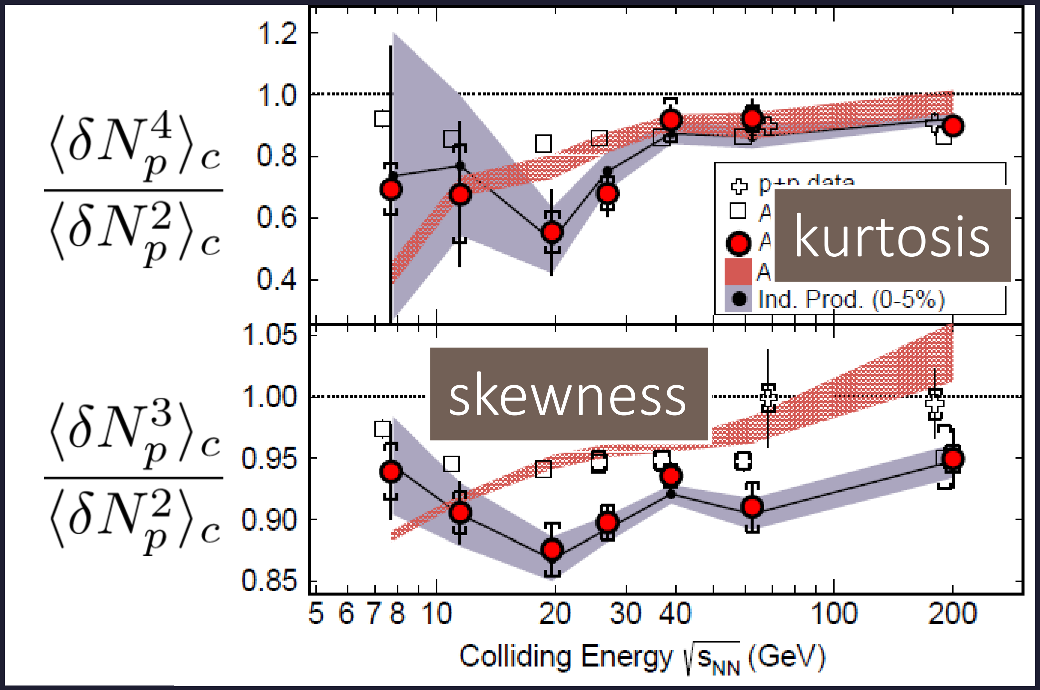
Event-by-Event Measurement



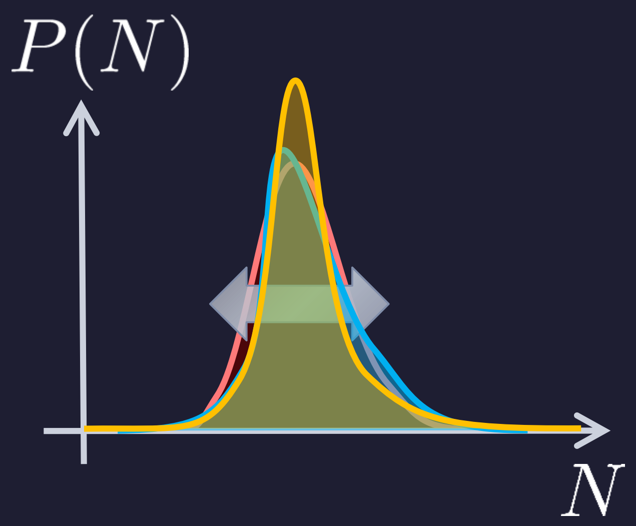


STAR Collaboration, PRL 2010

Non-Gaussianity @ RHIC

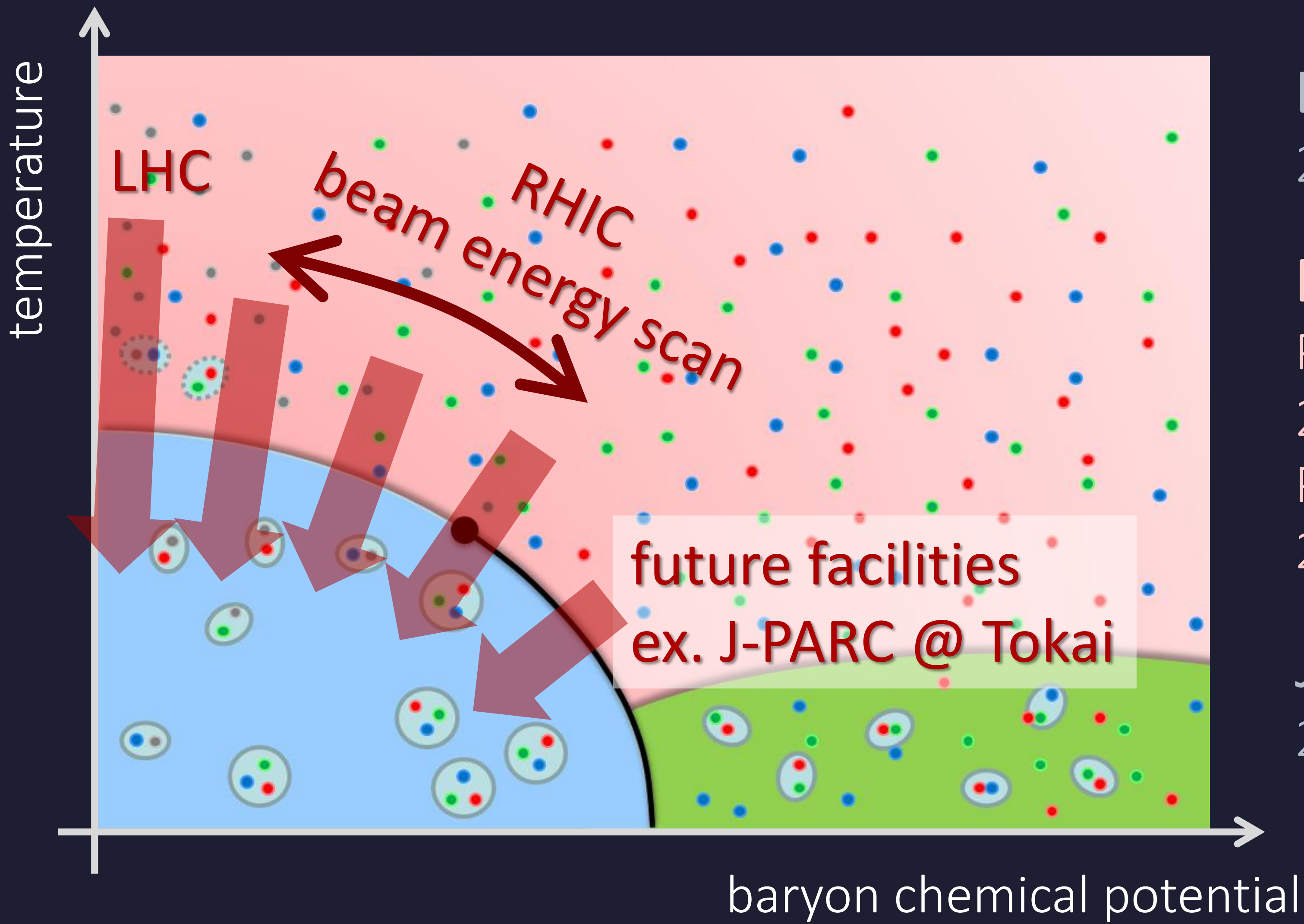


STAR Collaboration, PRL 2014



- Nonzero higher-order cumulants of conserved charges (skewness and kurtosis)
- They are not far from Poissonian values.

Search for QCD Phase Structure



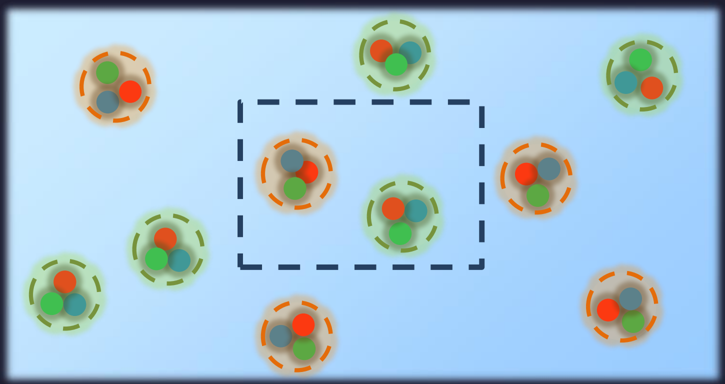
LHC
2010~

RHIC-BES
Phase I
2010~2014
Phase II
2015~

J-PARC
2018~ ⁇

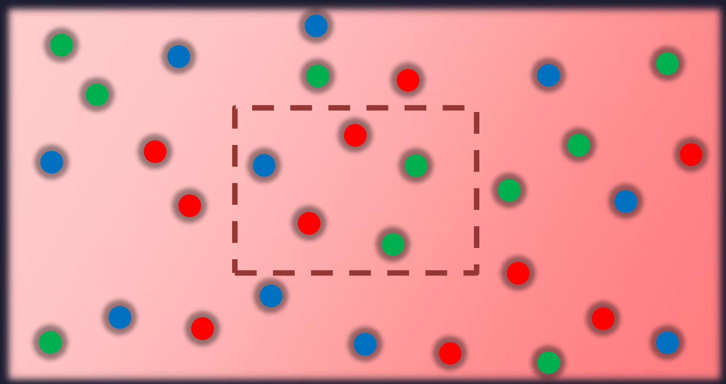
Signal of Quark Deconfinement

Hadronic



$$|q_B| = 0, 1, \ \ |q_Q| = 0, 1$$

Quark-Gluon



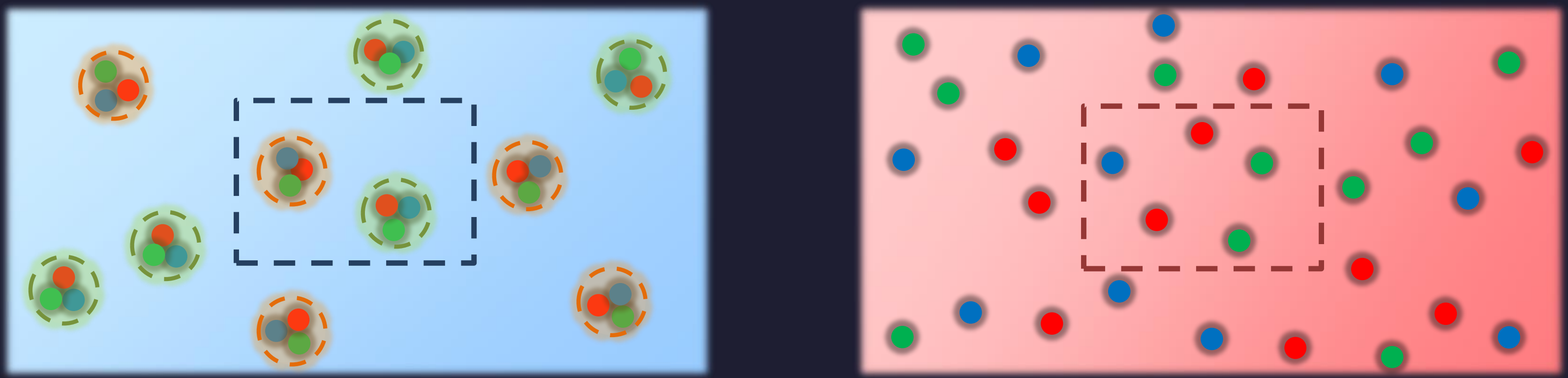
$$|q_B| = 1/3, \ \ |q_Q| = 1/3, 2/3$$

Elemental charge carried by quasi-particles decreases in QGP

Asakawa, Heinz, Muller PRL (2000); Jeon, Koch, ibid.

Signal of Quark Deconfinement

Hadronic | Quark-Gluon



$|q_B| = 0, 1,\ \ |q_Q| = 0, 1$ | $|q_B| = 1/3,\ \ |q_Q| = 1/3, 2/3$

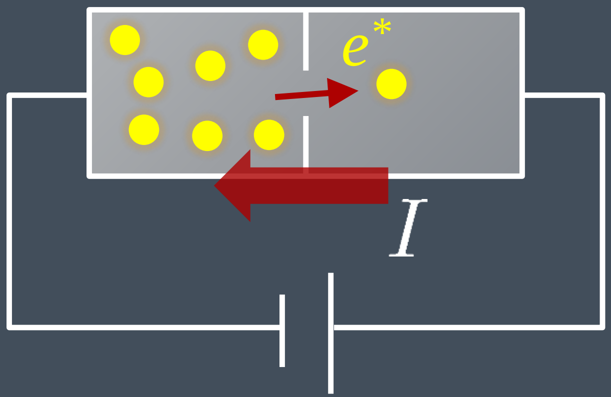
Elemental charge carried by quasi-particles decreases in QGP

⇓

Corresponding thermal fluctuations decrease in QGP

Asakawa, Heinz, Muller PRL (2000); Jeon, Koch, ibid.

Shot Noise

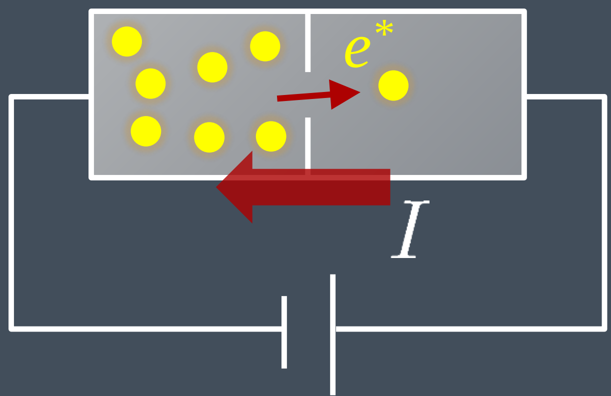


$$S_{\text{shot}} \sim \langle \delta I^2 \rangle$$

$$S_{\text{shot}} = 2e^* \langle I \rangle$$

charge of quasi-particles

Shot Noise



$$S_{\mathrm{shot}} \sim \langle \delta I^2 \rangle$$

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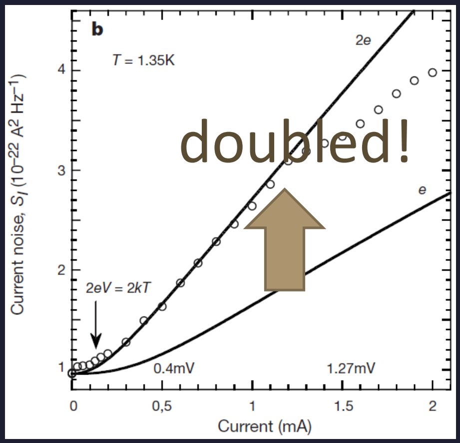
charge of quasi-particles

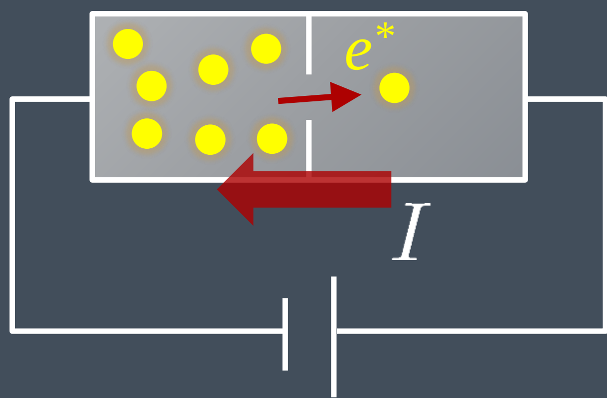
Superconductors with Cooper Pairs

$$e^* = 2e$$

Jehl+, Nature **405**,50 (2000)



Shot Noise



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charge of quasi-particles

Superconductors with Cooper Pairs

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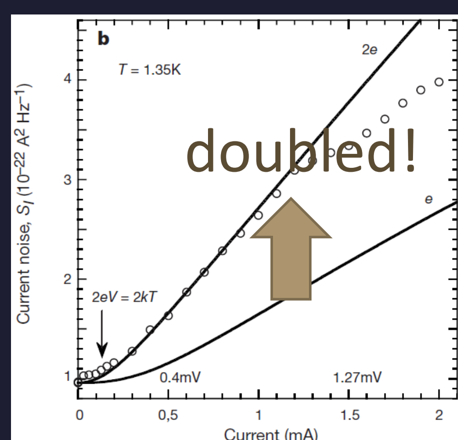


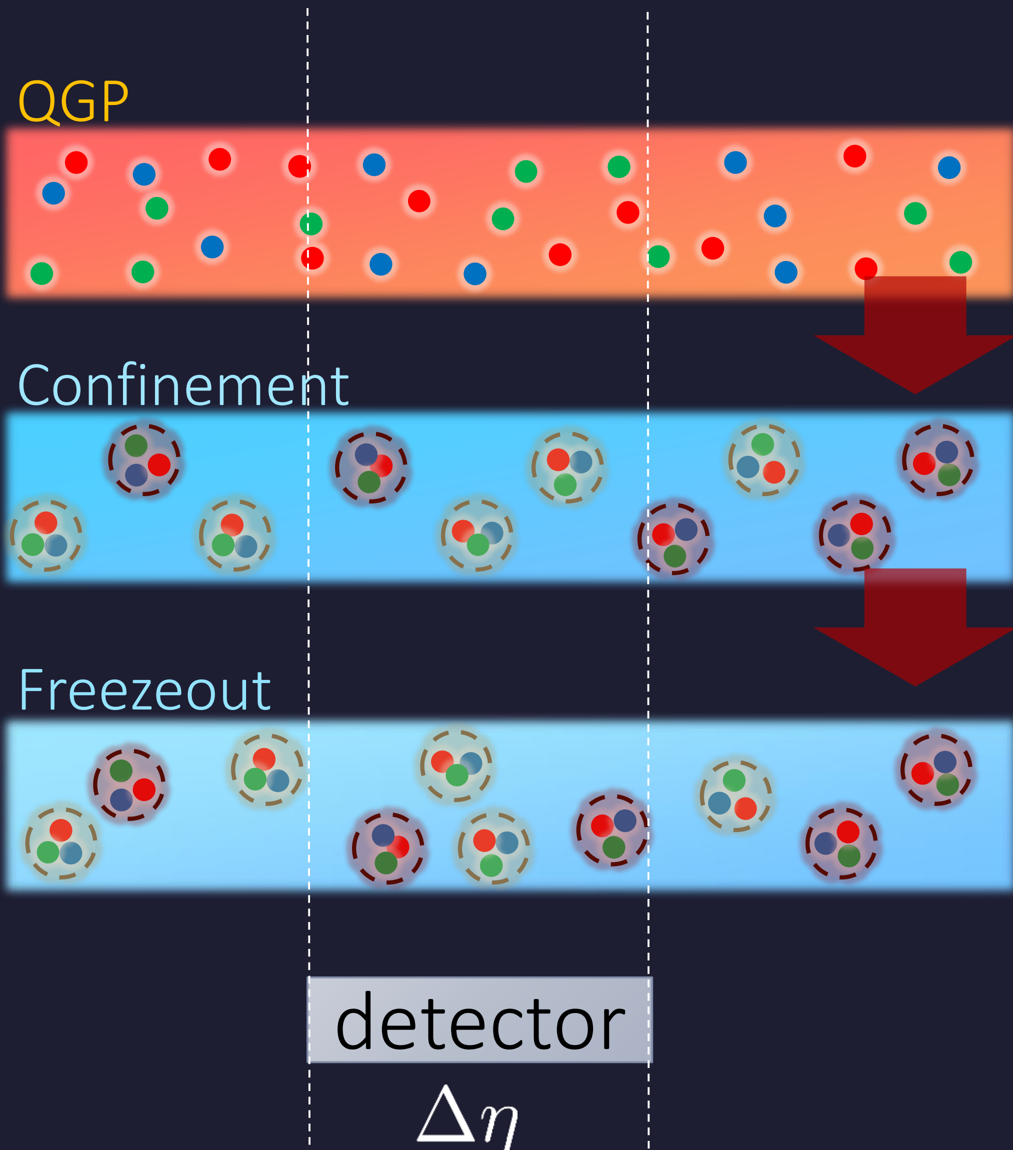
Jehl+, Nature **405**,50 (2000)

Fractional Quantum Hall Systems

$$e^* = \frac{q}{p} e$$

Saminadayar+, PRL**79**,2526 (1997)

Diffusion of Fluctuations



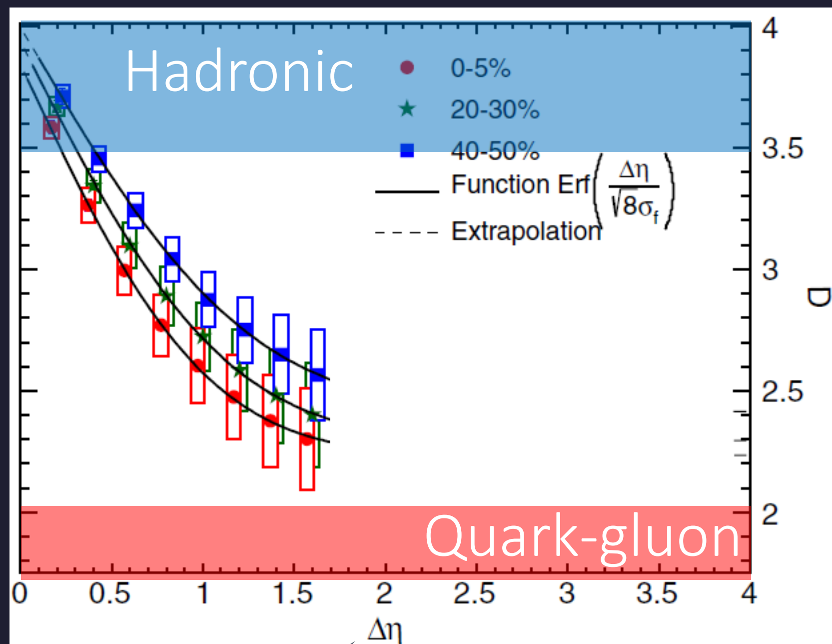
Experiments can vary spatial volume to measure fluctuations

The larger $\Delta\eta$, the earlier fluctuations

Electric Charge Fluctuations @ LHC

ALICE Collaboration,
PRL **110**, 152301 2013

一本の草も涼風宿りけり　一茶



$\sim \dfrac{\langle \delta N_Q^2 \rangle}{V}$

$\sim V$

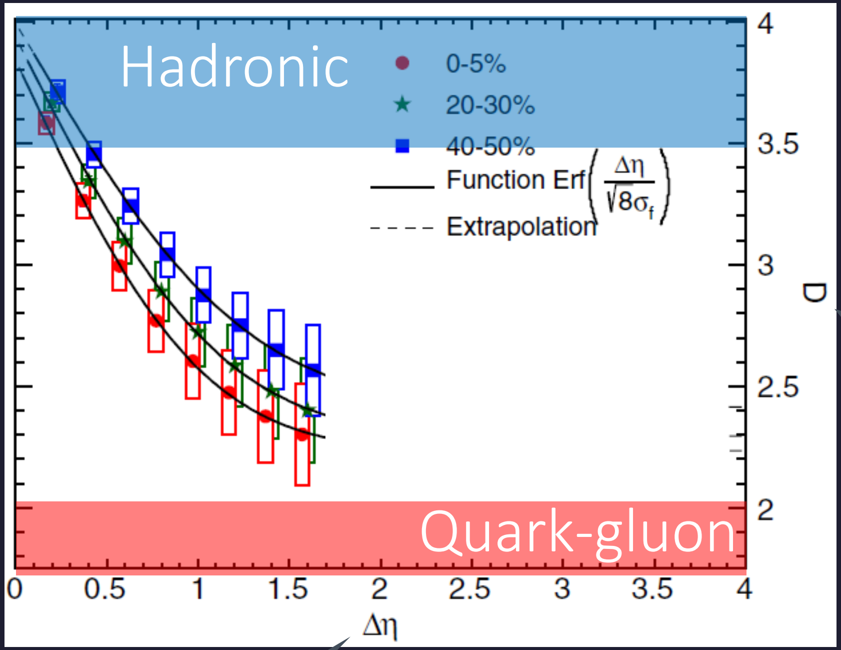
Fluctuation is more QGP-like as *V* becomes larger.
The Δη dependence encodes history of the medium!

Diffusion of non-Gaussianity

MK,Asakawa,Ono,PLB**728**,386(2014)
Sakaida, Asakawa, MK, PRC**90**,064911 (2014)

Electric Charge Fluctuations @ LHC

ALICE Collaboration,
PRL **110**, 152301 2013



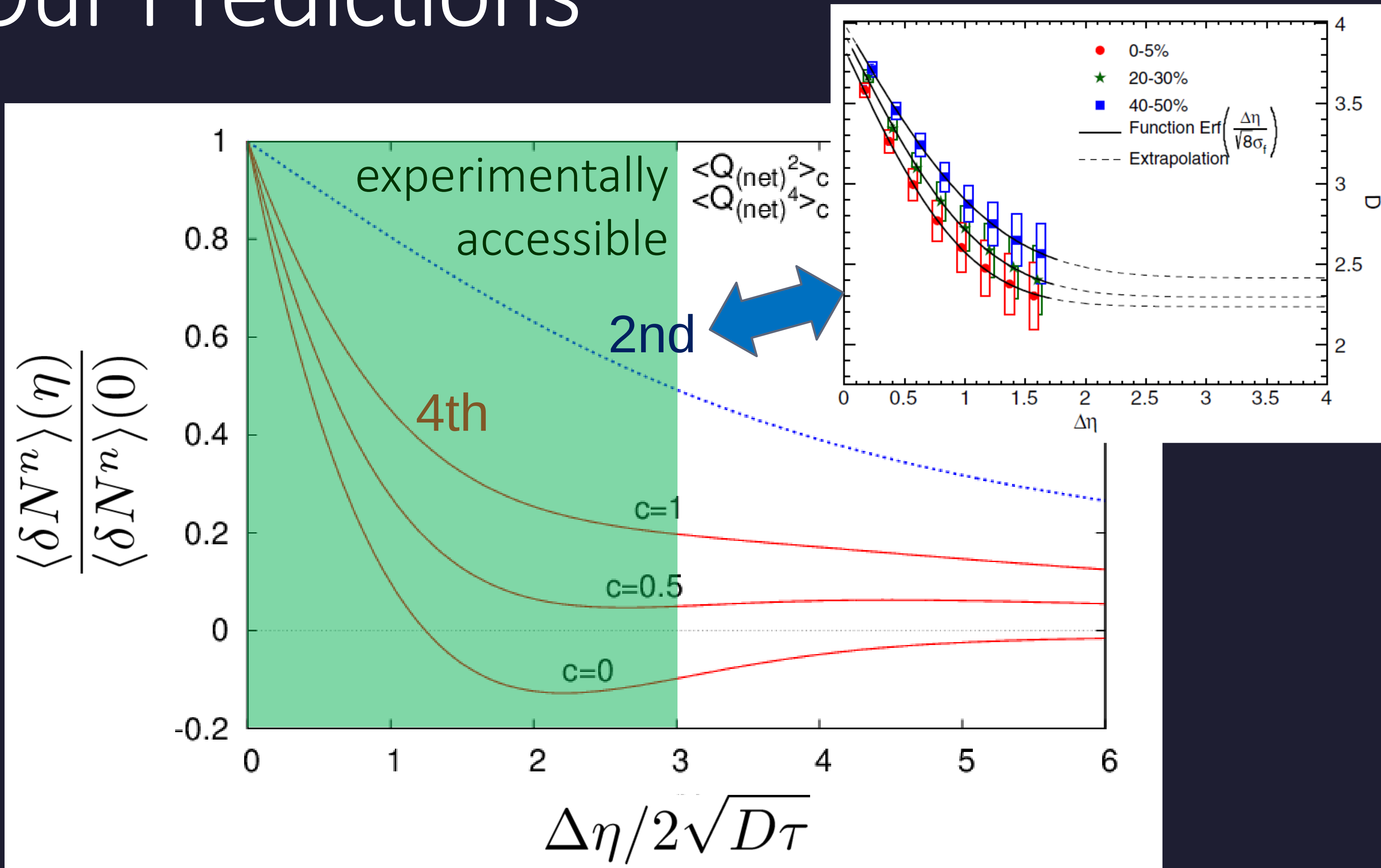
$\sim \frac{\langle \delta N_Q^2 \rangle}{V}$

$\sim V$

- ☐ Experimental results only for 2nd order fluctuation
- ☐ No results on $\Delta\eta$ dependence of higher-order cumulants

Our Predictions

ALICE Collaboration, 2013



Volume dep. of non-Gaussianity encodes more information!

Relation b/w baryon and proton number cumulants

MK, Asakawa, PRC85,021901C(2012);
MK, Asakawa, PRC86, 024904(2012)

バリオン数 ≃ 陽子数？？

理論的予言

バリオン数
≃
陽子数＋中性子数

実験的観測量

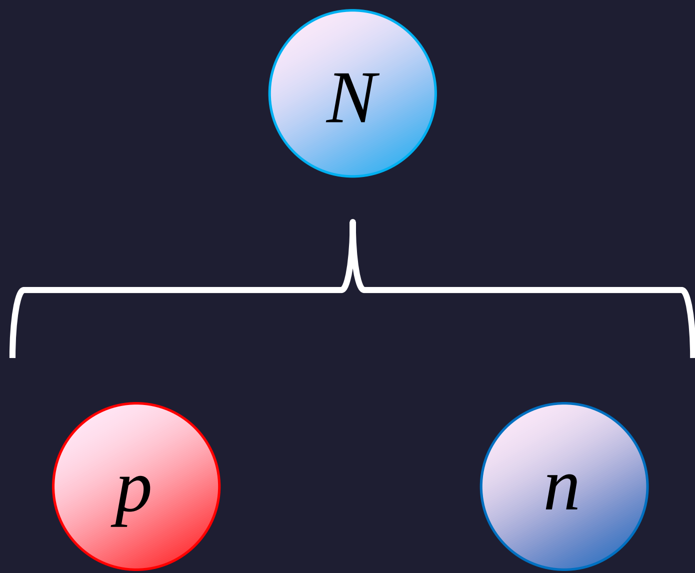
陽子数

※保存電荷であるバリオン数は理論的取り扱いが容易

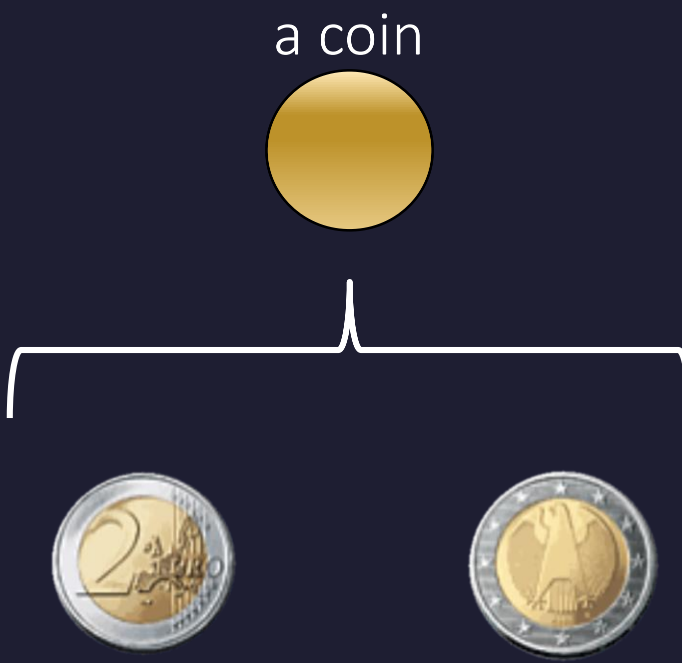
※検出器は、電荷を持たない中性子を観測できない

両者を同一視してよいのか？

Nucleon isospin and a coin



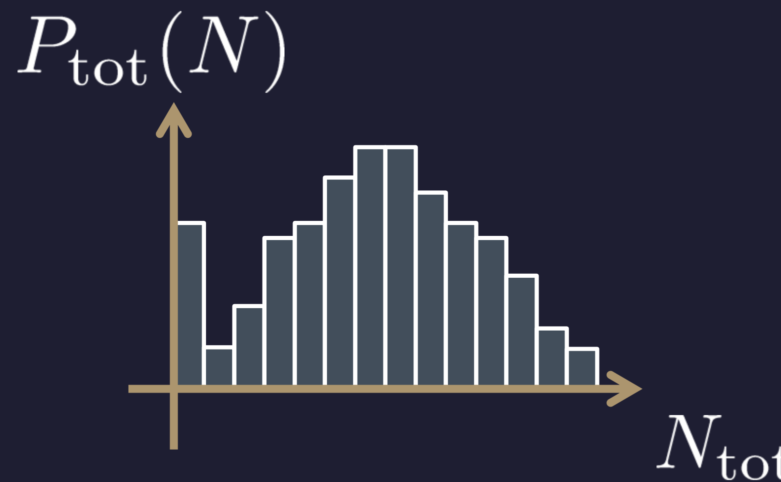
Nucleon has
two isospin states.

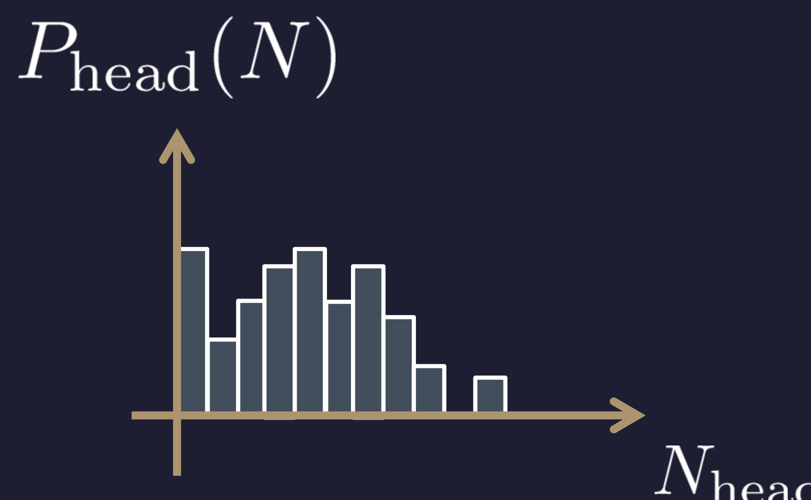


A coin has two sides.

MK, Asakawa,2012

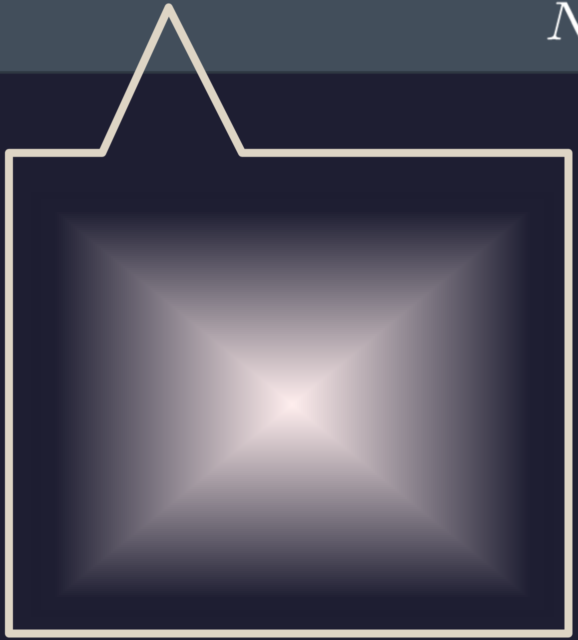
Slot Machine Analogy

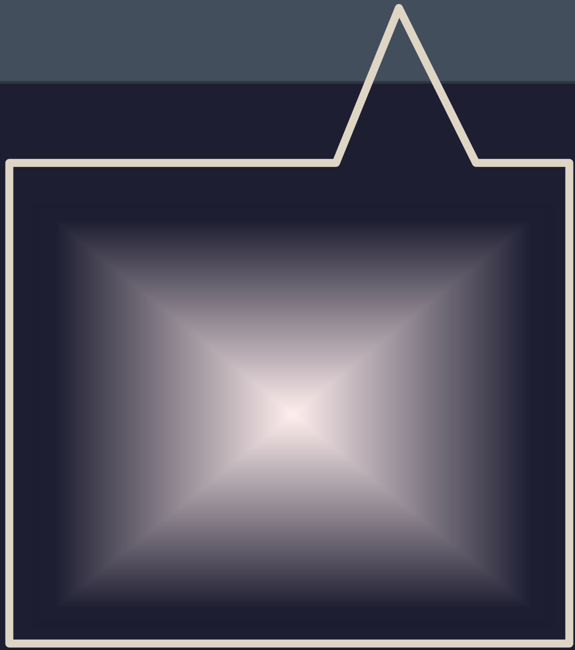




Reconstructing Total Coin Number

$$P_{\text{head}}(N_{\text{head}}) = \sum_{N_{\text{tot}}} B_{1/2}(N_{\text{head}}; N_{\text{tot}}) P_{\text{tot}}(N_{\text{tot}})$$





Relation b/w baryon/proton

$$\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{4}\langle N_{\mathrm{B}}^{(\mathrm{tot})}\rangle$$

$$\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle = 4\langle(\delta N_p^{(\mathrm{net})})^2\rangle - 2\langle N_p^{(\mathrm{tot})}\rangle$$

And, similar formulae for higher order cumulants

Summary

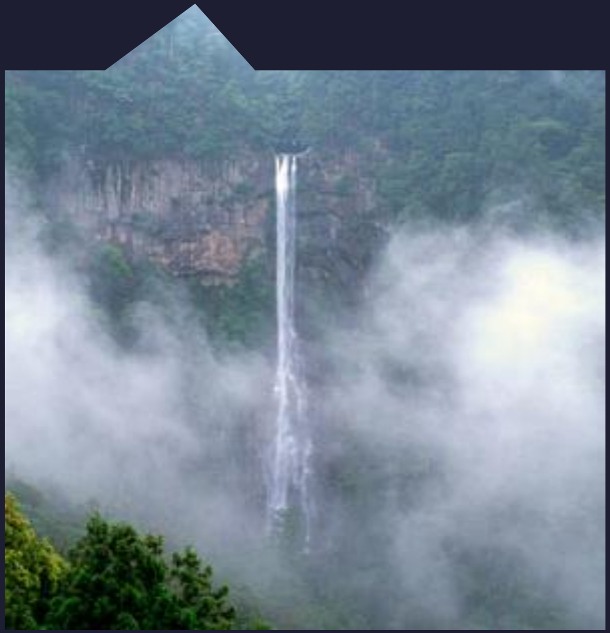
- Fluctuations are invaluable tools in physics, as well as in our daily life.

- Fluctuations acquire much attention in relativistic heavy-ion collisions. In particular, their non-Gaussianity is one of the latest topics in this realm.

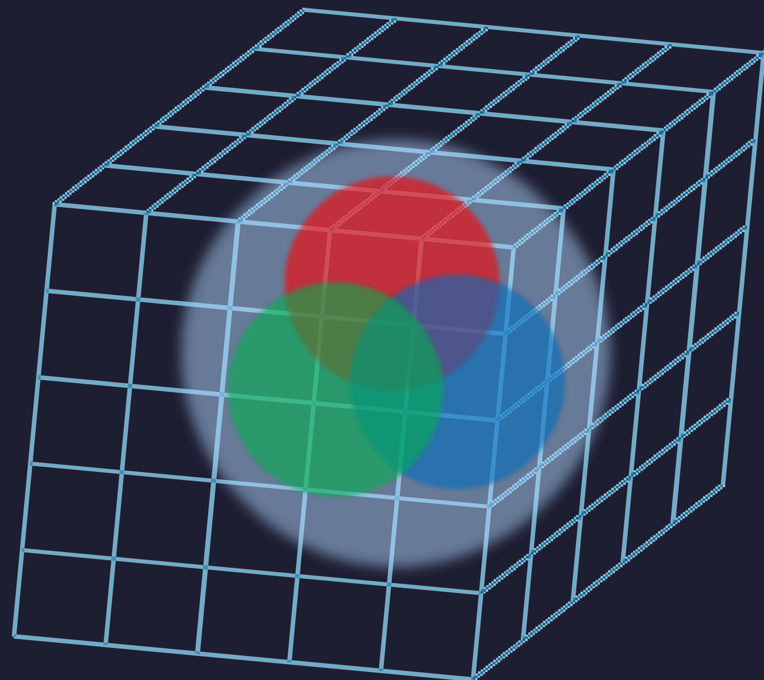
話題その二

論語と格子QCD数値シミュレーション

「勾配フロー変換」がもたらす格子QCDの新展開



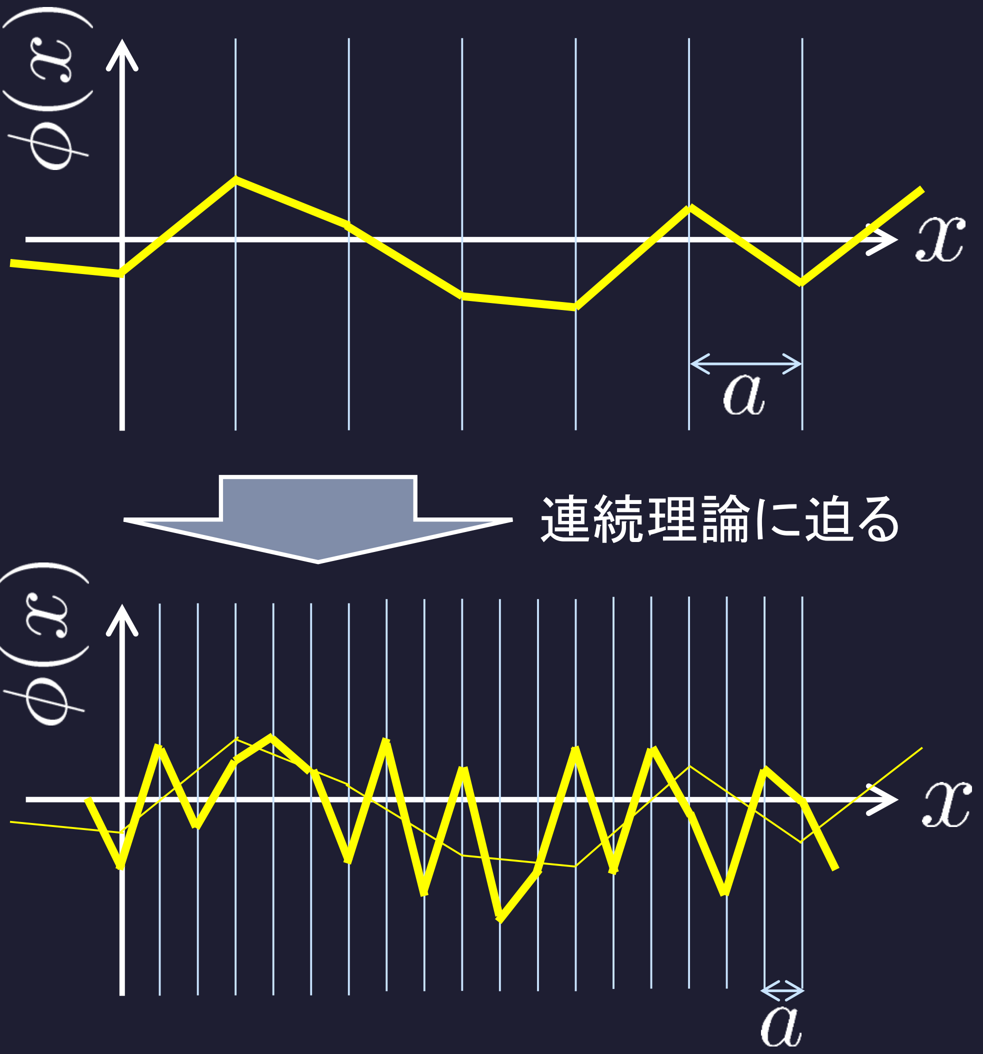
格子QCD数値シミュレーション



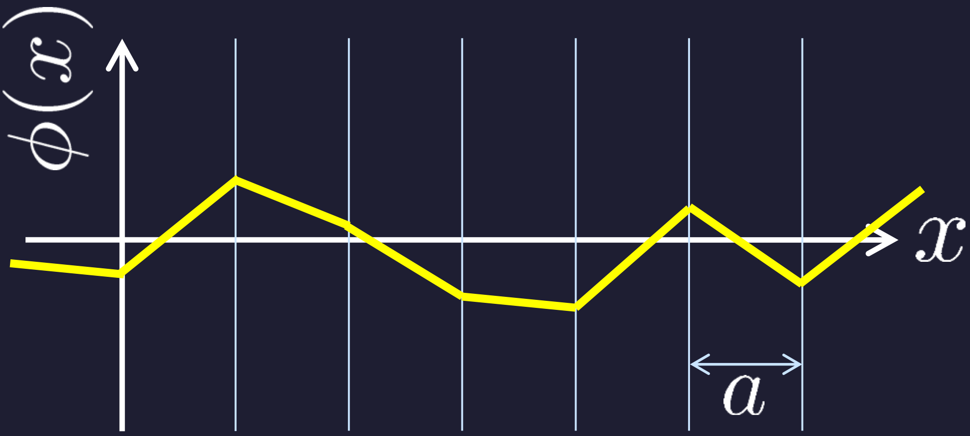
量子色力学（QCD）を
第一原理的に取り扱う現状唯一の手段

ただし、正確な予言を行うためには「連続極限」への外挿が必要

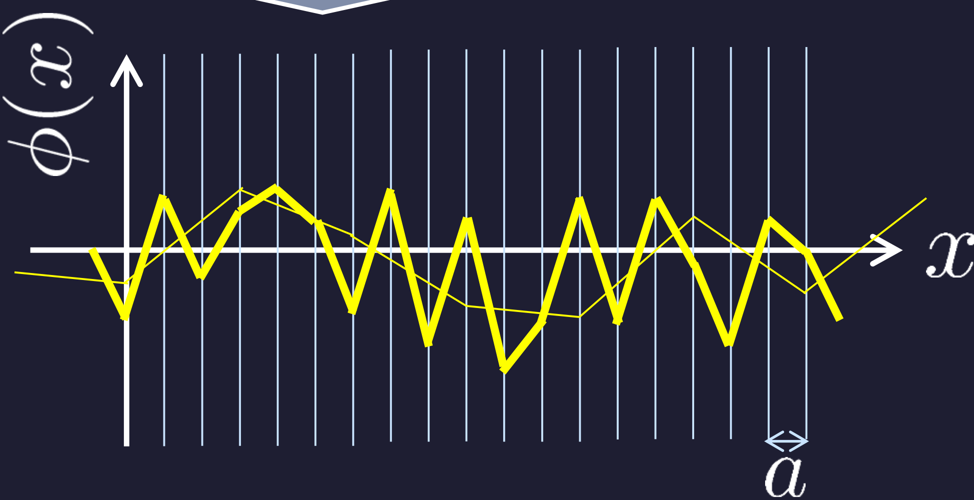
格子間隔と計算コスト



格子間隔と計算コスト



連続理論に迫る



ϕのノイズは$a^{1/2}$に比例

$d\phi(x)/dx$ のノイズは、
aの減少と共に増大

数値コストが爆発的に増大

量子場の理論の
紫外発散に対応した
原理的困難

見小利則大事不成

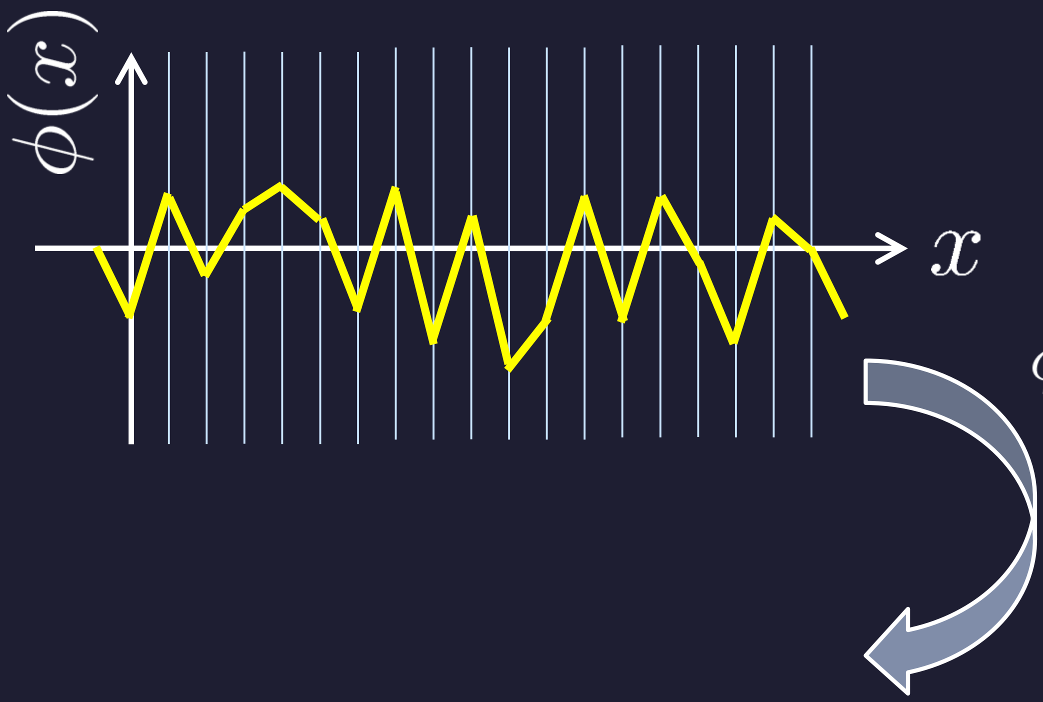
小利を見ればすなわち大事成らず

Miss the wood for the trees

孔子

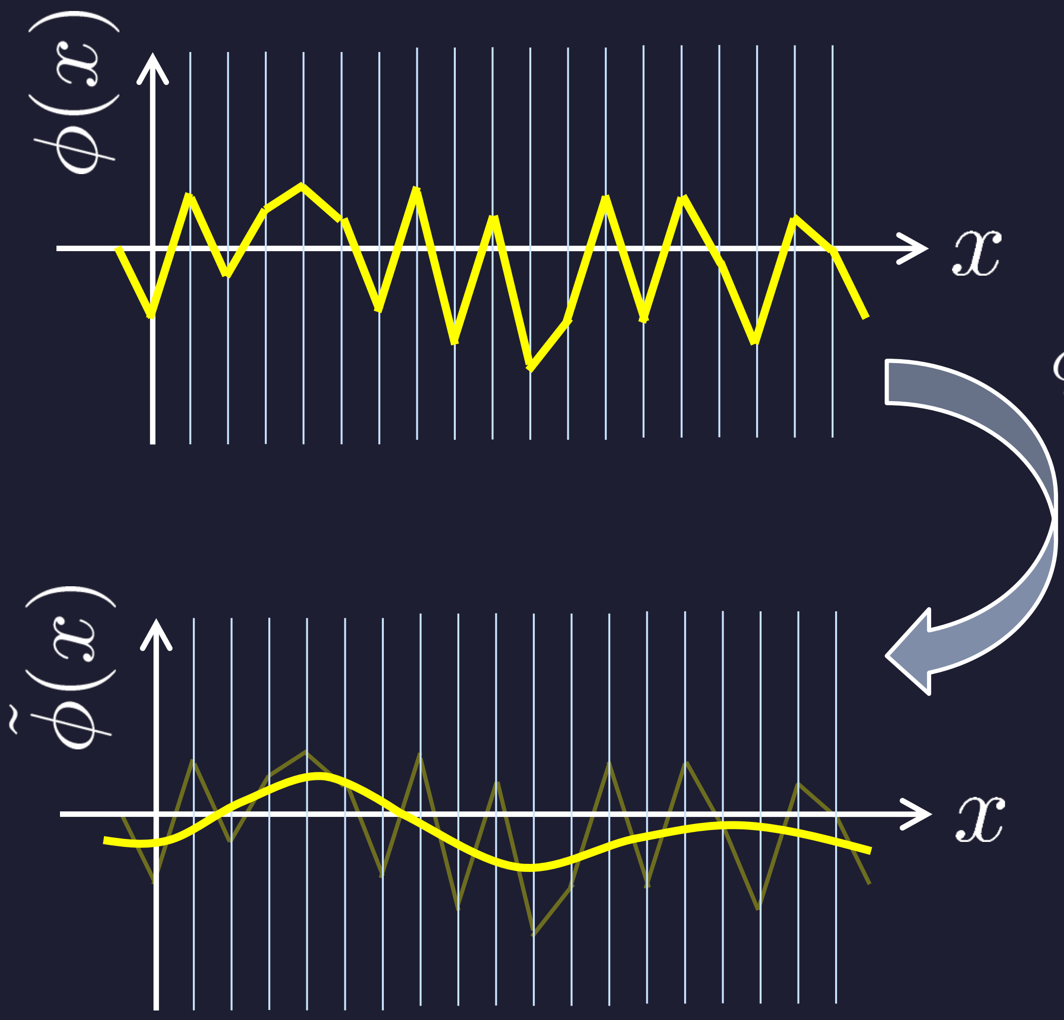
（論語、子路13）

解決策:にじませてみる



$$\tilde{\phi}(x) \sim \int dx' e^{-x'^2/r^2} \phi(x')$$

解決策:にじませてみる



$$\tilde{\phi}(x) \sim \int dx' e^{-x'^2/r^2} \phi(x')$$

にじませた場上では、
ノイズ激減！！

問題点

- 場をにじませた時点で、もはや別の理論になっているのでは？

- にじませるんだったら、粗い格子でシミュレーションしているのと同じなのではないか？

最近の発展
「もとの理論」と「にじませた理論」との間の理論的関係づけ
「にじませた理論」を測定に使う方法論の確立
Luscher, 2010; Luscher, Weiss, 2011; Suzuki, 2014; ...

「勾配フロー」方程式 ＝「にじませた場」生成法

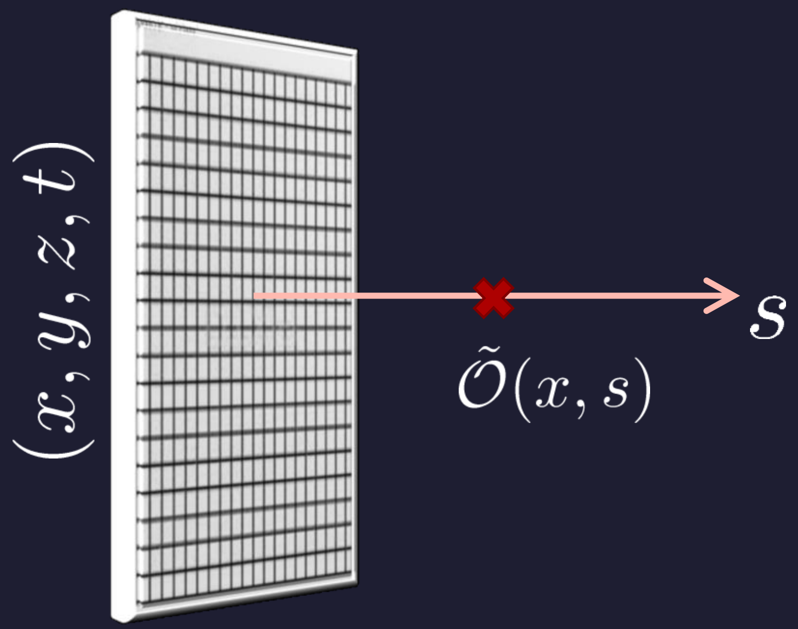
$$\frac{\partial}{\partial s}\phi(x,s) = \sum_{\mu} \frac{\partial^2}{\partial x_\mu^2}\phi(x,s)$$

↑ 連続変換のパラメータ

↑ 元の4次元理論の空間変数

- 空間4次元世界の拡散方程式
- sの増加とともに、場の「にじみ」が進行

(x,y,z,t,s)5次元理論



$$O(x) \Longleftrightarrow O(x, s)$$

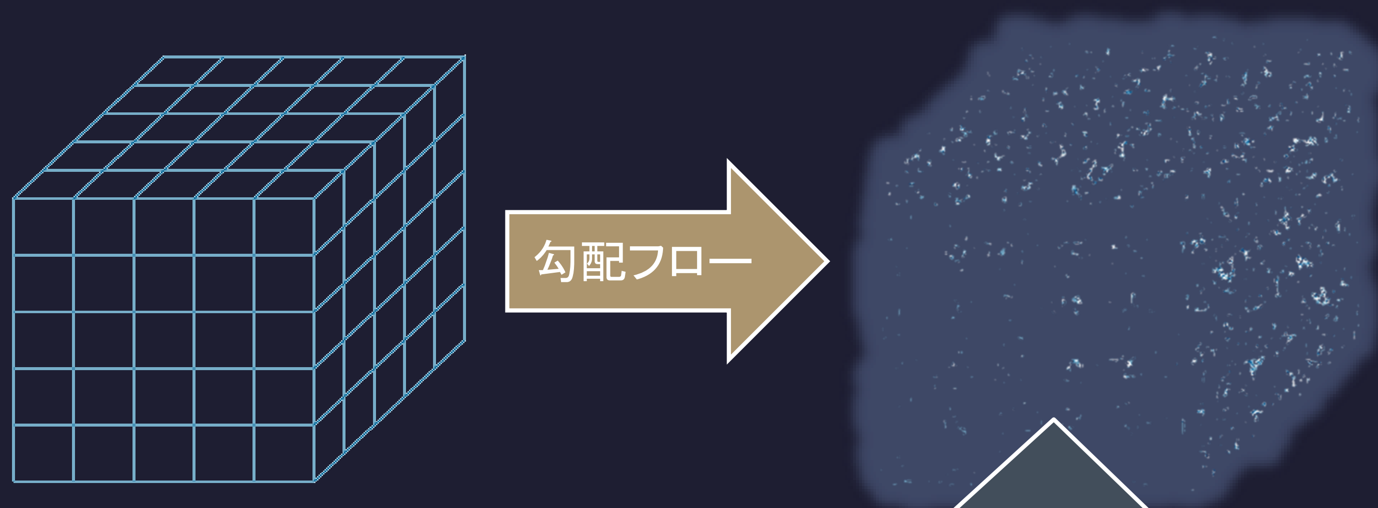
4次元理論と、有限sの関係が確立

Luscher, Weiss, 2011; Suzuki, 2014

エネルギー運動量テンソル

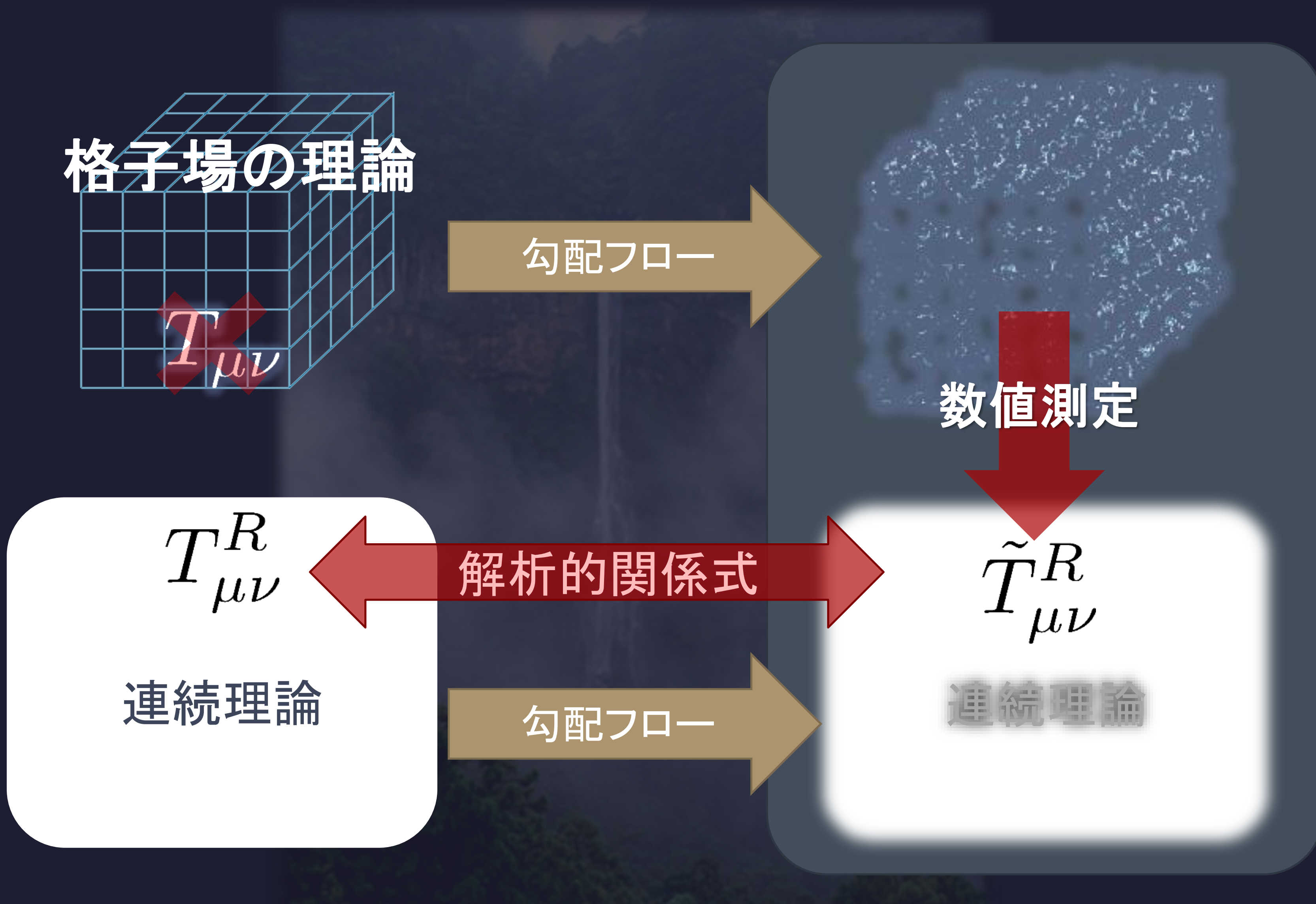
$$T_{\mu\nu}$$

- 時空の並進対称性と深く関係
- 格子上ではうまく定義できない
- 仮に測定できても、ノイズがでかい



にじんで格子が見えなくなった場の上でなら、問題なく定義可能！

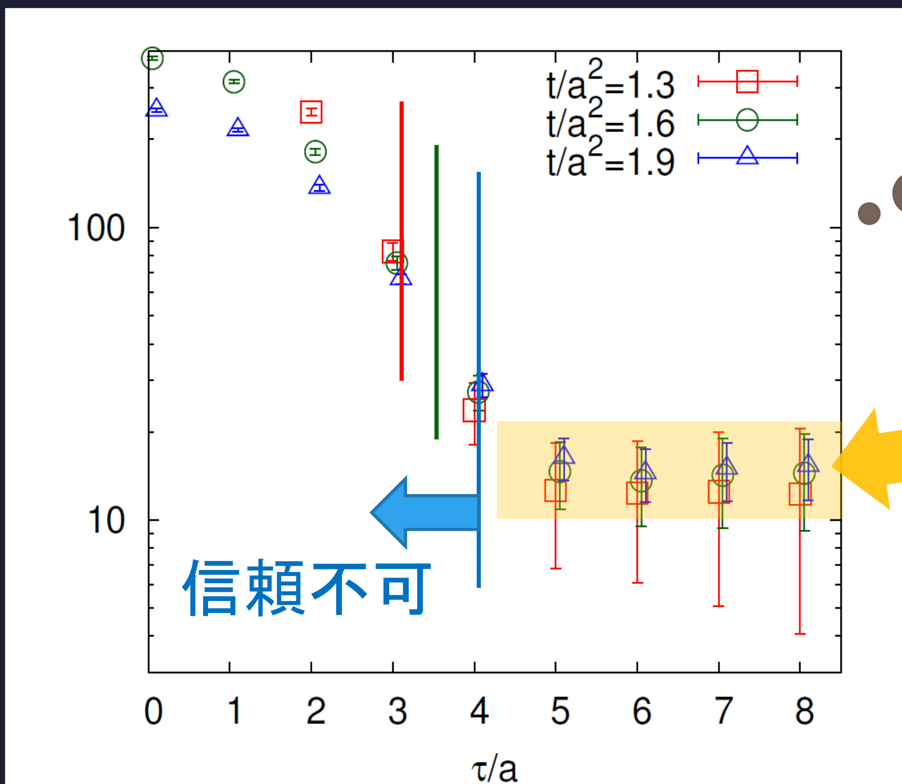
勾配フロー法


格子場の理論
$T_{\mu\nu}$
勾配フロー
数値測定
$T^R_{\mu\nu}$
解析的関係式
$\tilde{T}^R_{\mu\nu}$
連続理論
勾配フロー
連続理論

エネルギー時間依存性の測定

Kitazawa+, PoS(LATTICE2014),014

$$\langle \delta T_{00}(\tau)\delta T_{00}(0)\rangle / T^5$$



虚時間

系の全エネルギーの
時間依存性みたいな量

- □ まともに測定したこと自体が世界初
- □ （虚）時間τ に依存しない

✓ エネルギー保存則

✓ 線形応答関係式の直接的確認

まとめ

- 場の理論の数値解析は、連続極限に迫る際に数値コストが爆発的に増大する。これは紫外発散に伴う、原理的問題。
- 最近、格子上の場を系統的に「にじませる」手法＝「勾配フロー方程式」が提案された。
- 我々が行った幾つかの数値解析は、この方法が正しく機能し、かつ実用上著しく有用であることを示唆している。

見小利則大事不成

を克服せよ！