Diffusion of Conserved-Charge Fluctuations

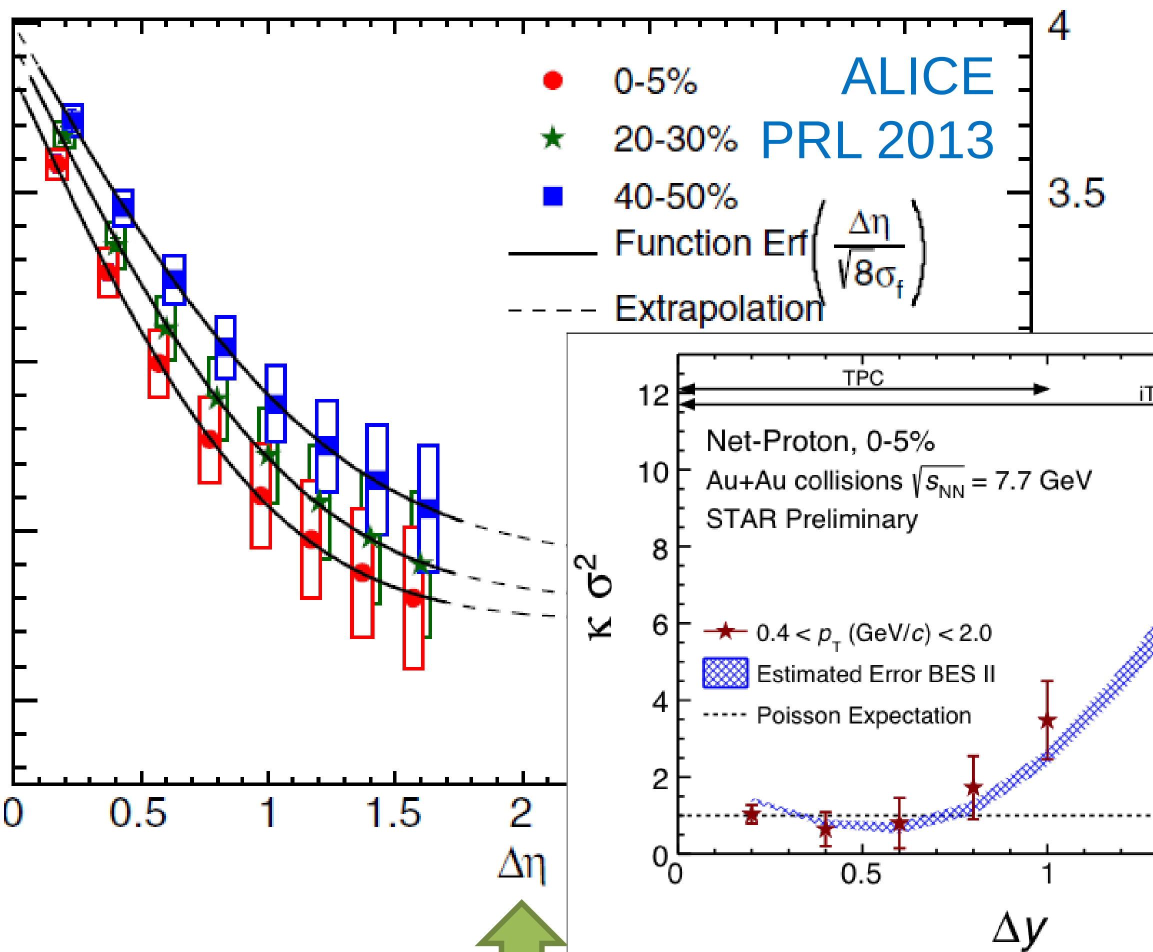
Masakiyo Kitazawa

(Osaka U.)

MK, Asakawa, Ono, Phys. Lett. B728, 386-392 (2014)
Sakaida, Asakawa, MK, PRC90, 064911 (2014)
MK, arXiv:1505.04349, Nucl. Phys. A942, 65 (2015)

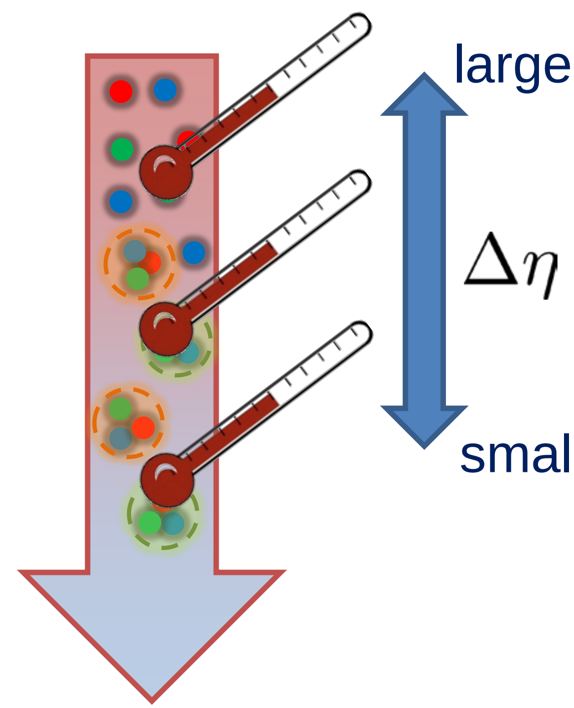
Workshop on Fluctuation, GSI, 4/Nov./2015

Objective 1: $\Delta\eta$ Dependence @ ALICE



rapidity window

$\Delta\eta$ dependent thermometer?

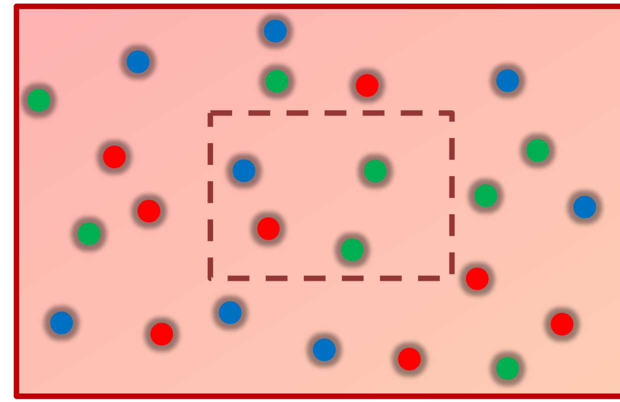


Objective 2

☐ Theories

on the basis of statistical mechanics

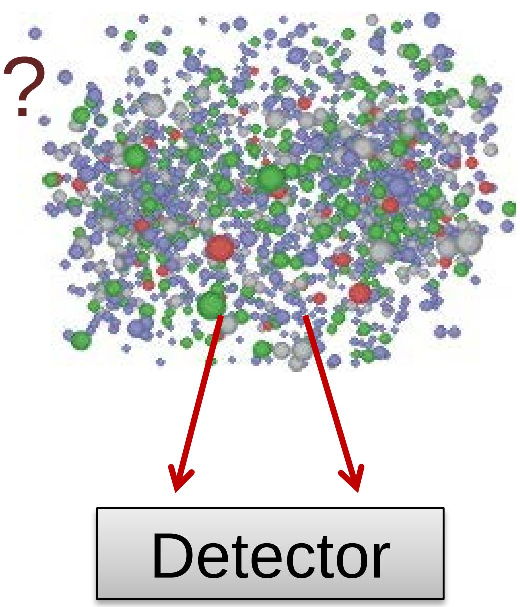
- fluctuation in a spatial volume
- equilibration (in an early time)



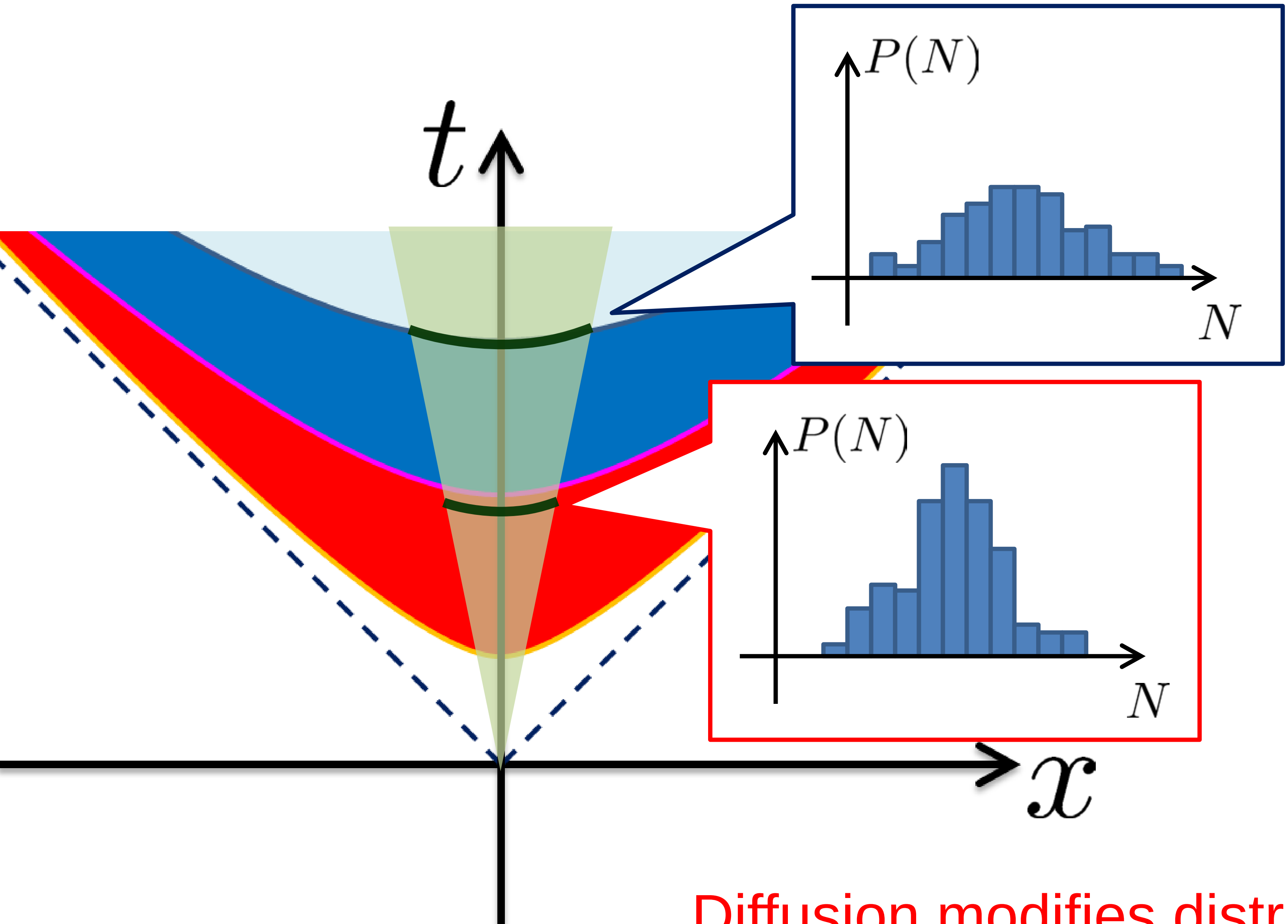
What are their relation?

☐ Experiments

- final state fluctuation
- fluctuation in a pseudo rapidity

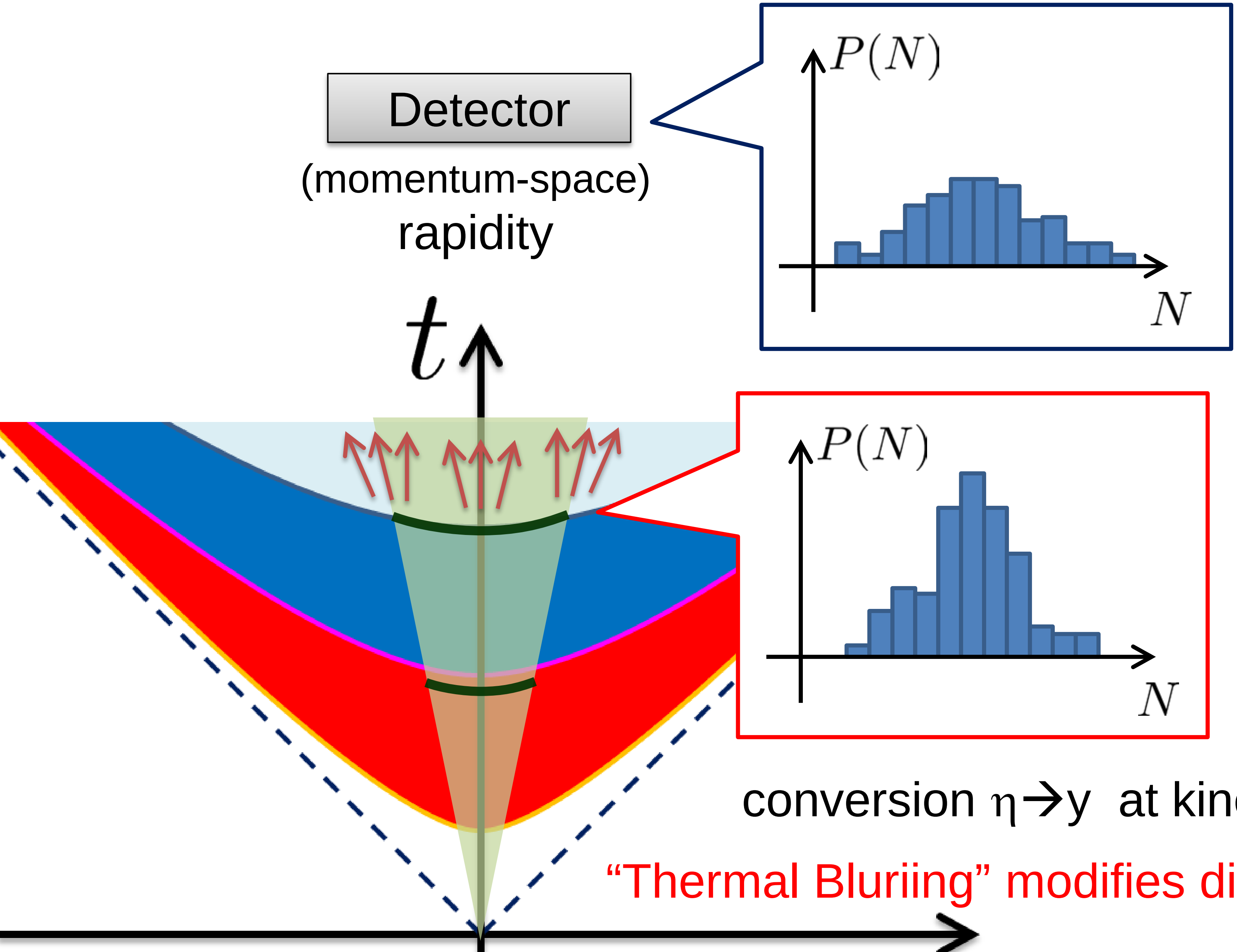


Time Evolution of Fluctuations



Diffusion modifies distribution

Time Evolution of Fluctuations

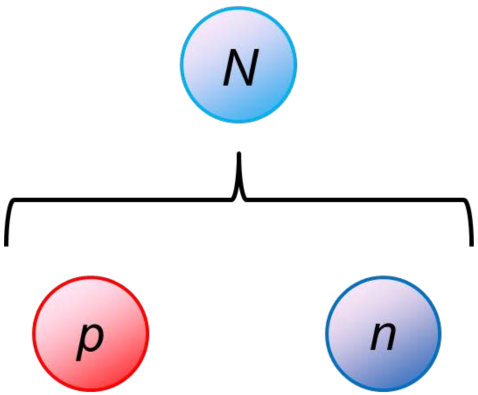


conversion η→y at kinetic f.o.

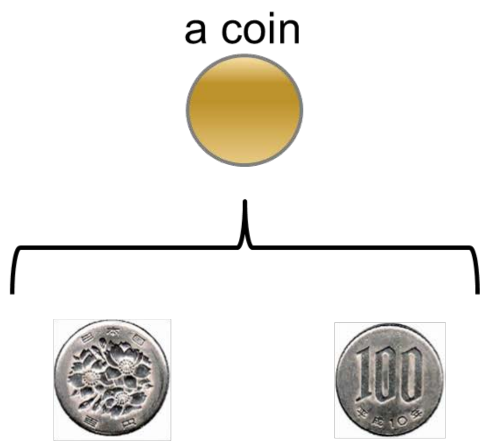
"Thermal Bluriing" modifies distribution revisiting

Slot Machine Analogy

An Exercise (Old Ver.).
Rconstructing Baryon Number Cumulants

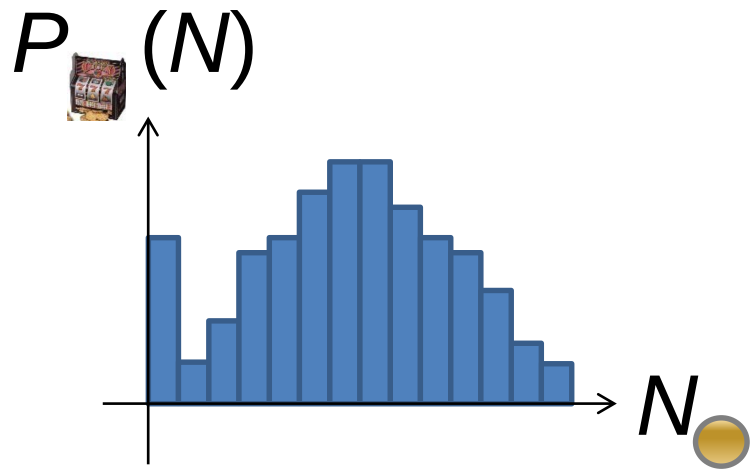


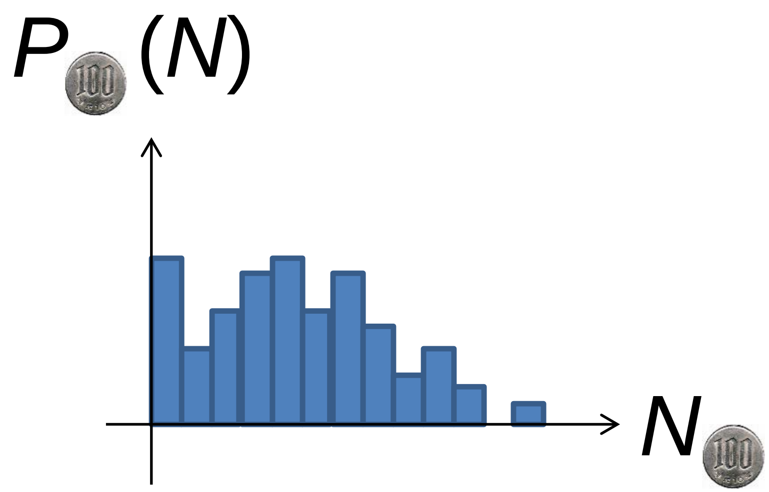
Nucleons have
two isospin states.



Coins have two sides.

Slot Machine Analogy

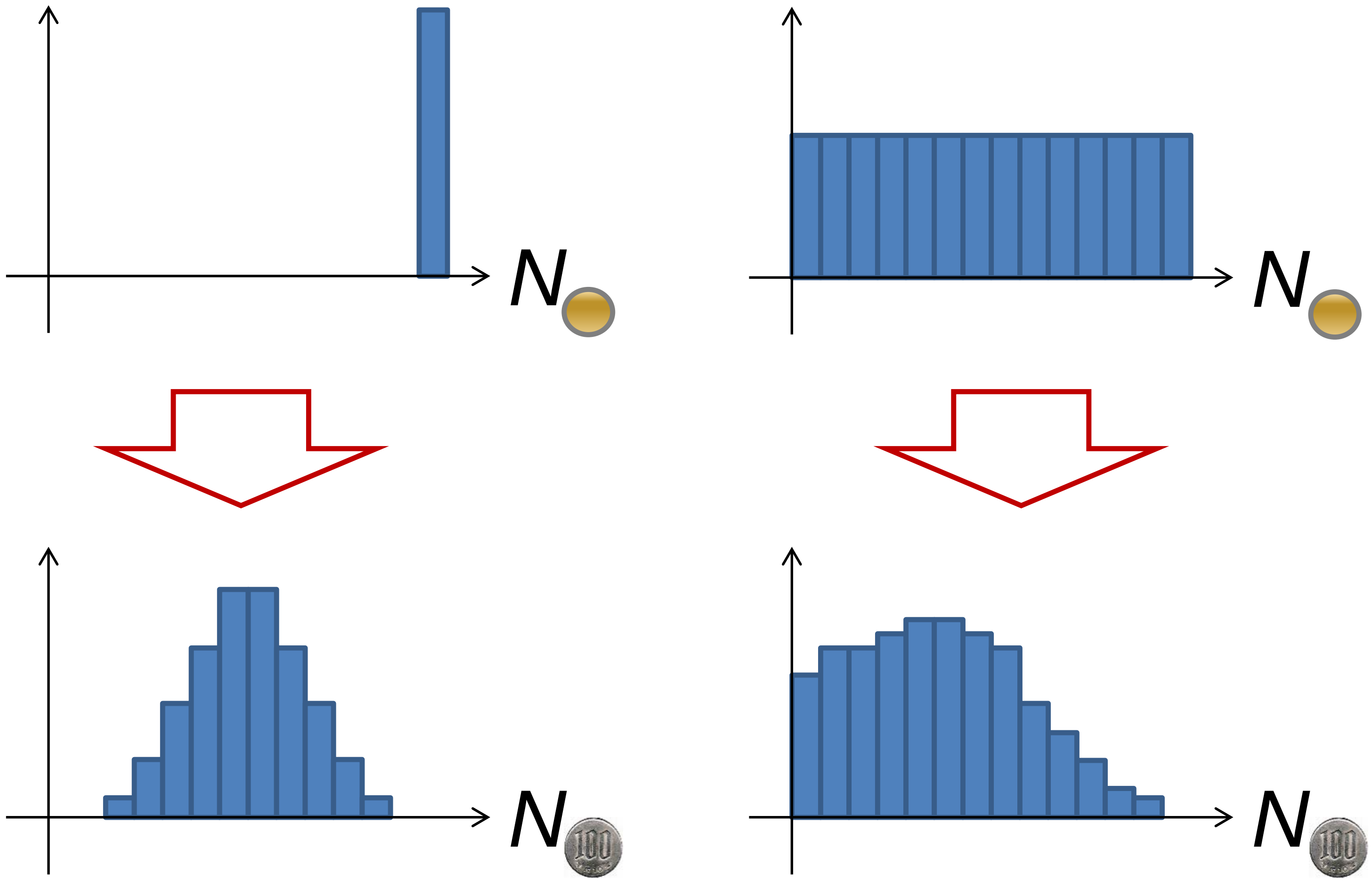




Extreme Examples

Fixed # of coins

Constant probabilities



Reconstructing Total Coin Number

MK, Asakawa, 2012

$$P_{100}(N_{100}) = \sum_{\bullet} P_{\text{slot}}(N_{\bullet})\, B_{1/2}(N_{100}; N_{\bullet})$$

binomial distribution

Application to efficiency correction:

- MK, Asakawa, 2012
- Bzdak, Koch, 2012, 2015
- Luo, 2012, 2015, …

Caveat
We still have 50% efficiency loss in proton #

Difference btw Baryon and Proton Numbers

MK, Asakawa, 2012

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

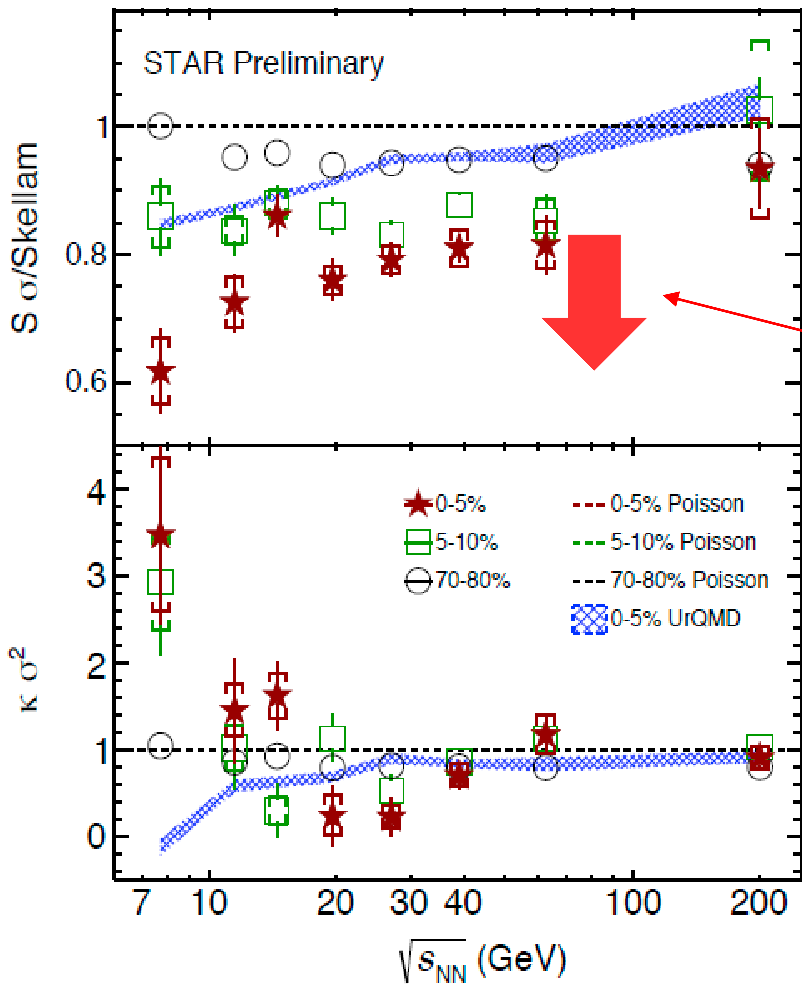
$$\Rightarrow \begin{cases} 2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\rangle_\mathrm{free} \\ 2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle_\mathrm{free} \\ 2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^4\rangle_c + \cdots \end{cases}$$

genuine info. noise (Poisson)

For free gas

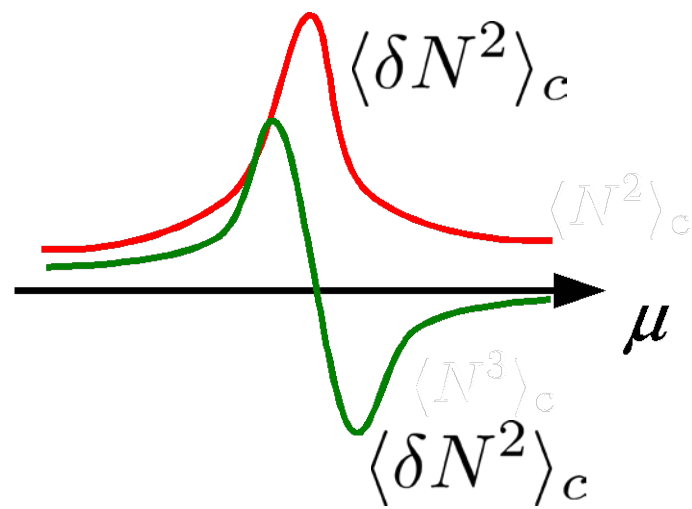
$$2\langle(\delta N_p^{(\mathrm{net})})^n\rangle_c = \langle(\delta N_\mathrm{N}^{(\mathrm{net})})^n\rangle_c$$

STAR, QM2015



Clear suppression!

ex. Asakawa, Ejiri, MK, 2009



Main Part

Diffusion of conserved-charge fluctuations and rapidity blurring

2 Slot Machines

Can we get the cumulants of single slot machine?

$\langle N^n \rangle_c$  $\langle N^n \rangle_c$ 

2 Slot Machines

Can we get the cumulants of single slot machine?

$$\langle N^n \rangle_c$$  $$= \frac{1}{2} \langle N^n \rangle_c$$ 

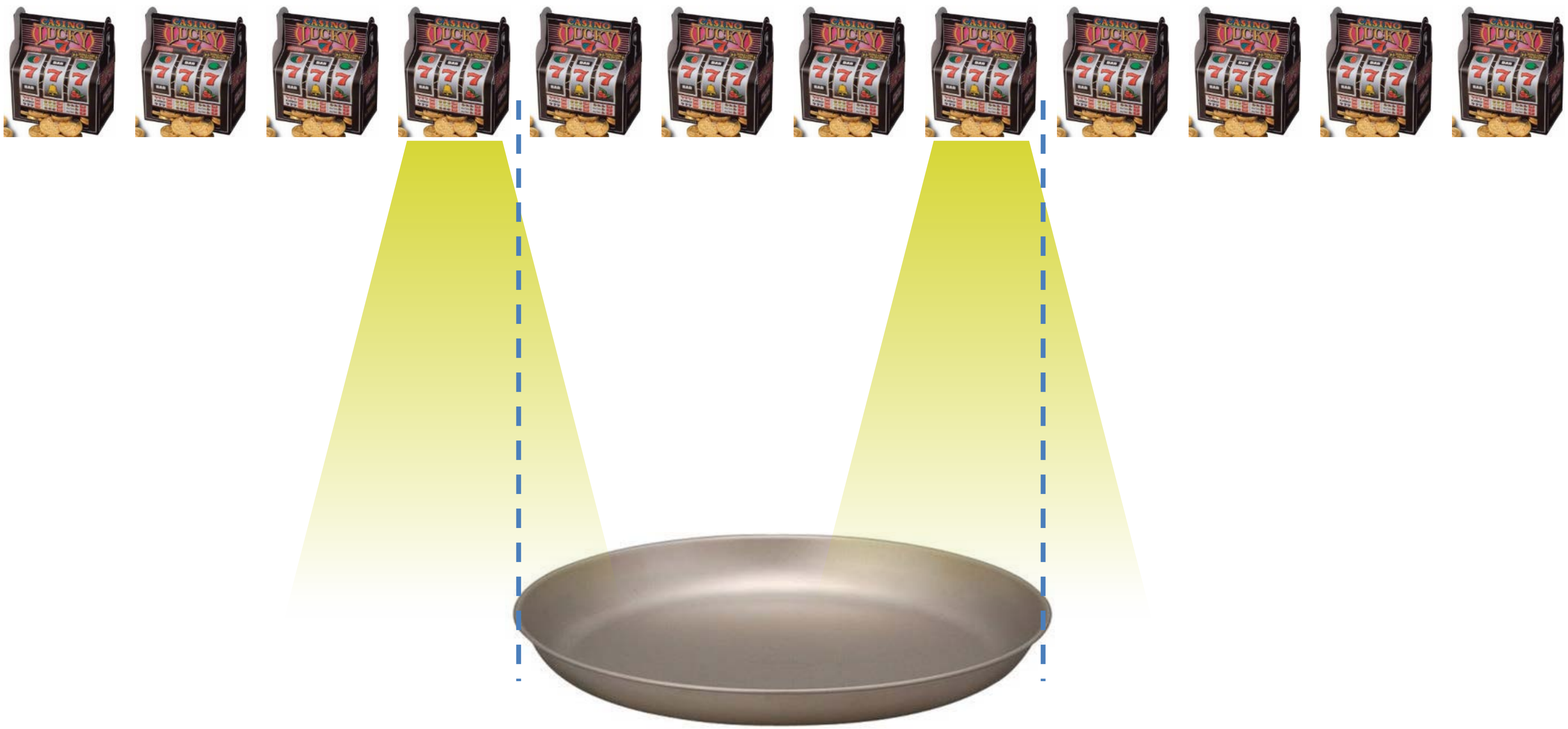
Many Slot Machines

$$\langle N^n \rangle_c$$  $$= \frac{1}{4} \langle N^n \rangle_c$$ 

Many Slot Machines

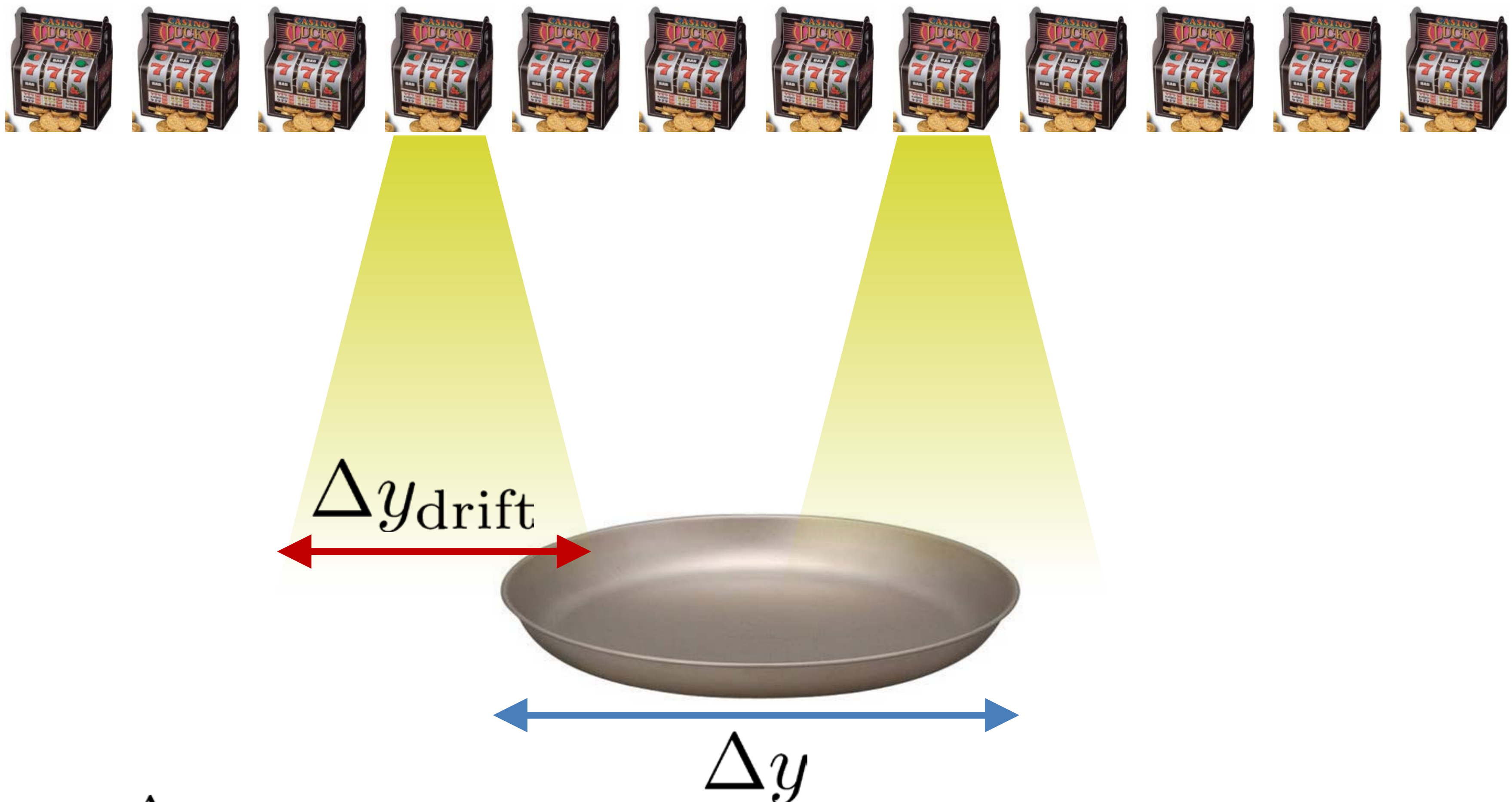
$\langle N^n \rangle_c$  $= \frac{1}{4} \langle N^n \rangle_c$ 

Many Slot Machines



$\langle N^n \rangle_c$  $\neq \frac{1}{4} \langle N^n \rangle_c$ 

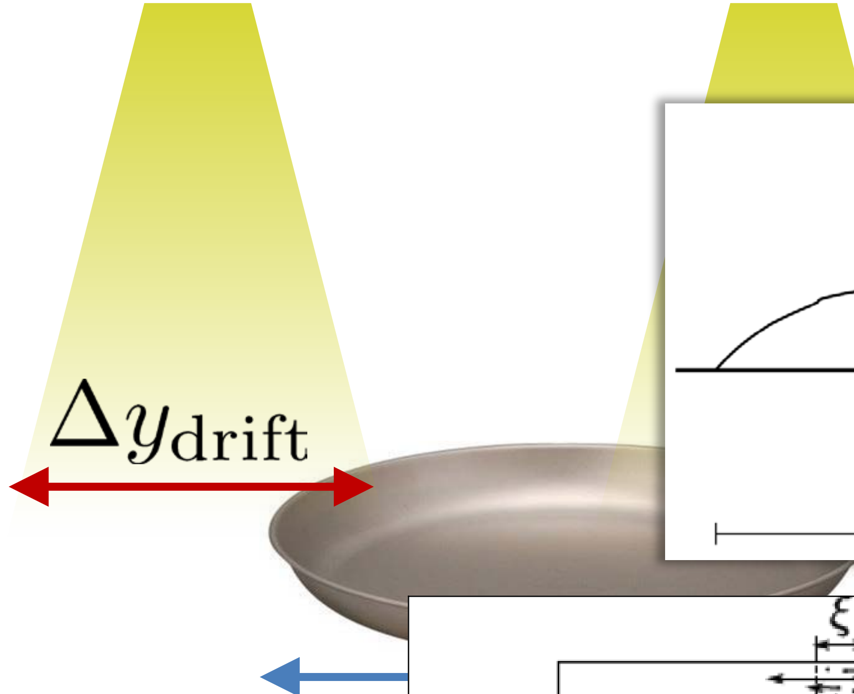
Many Slot Machines

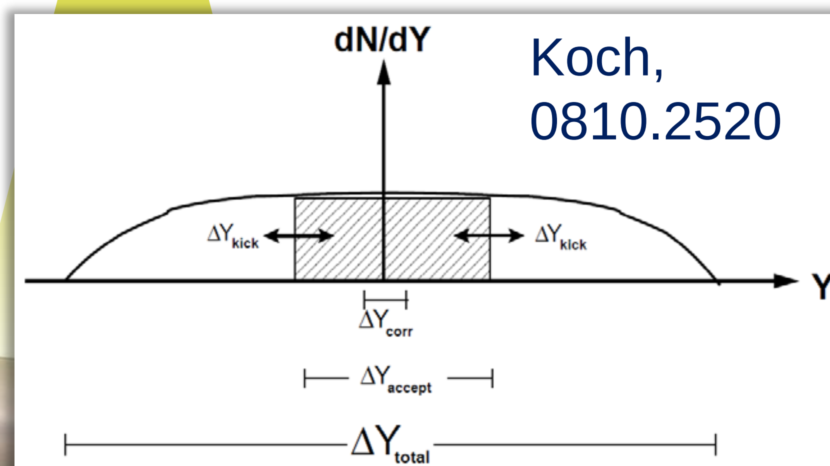


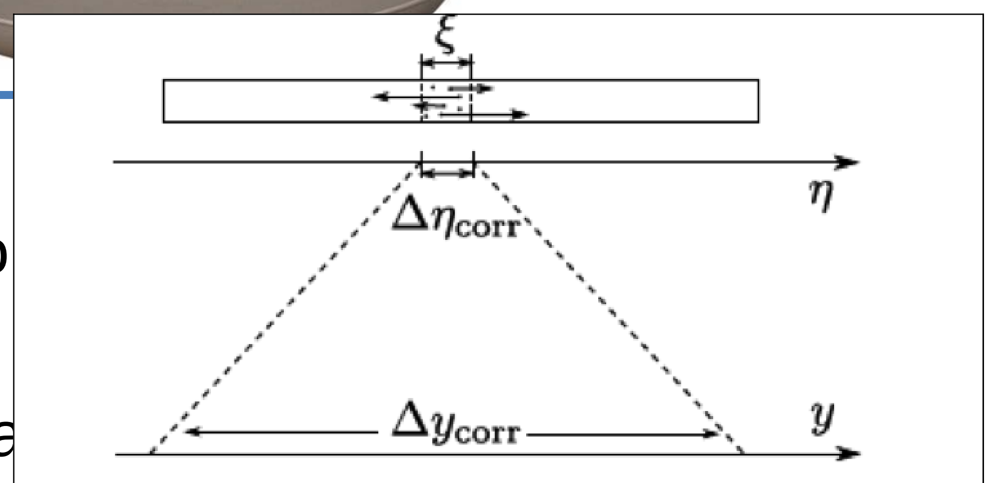
$$\frac{\Delta y}{\Delta y_{\mathrm{drift}}} \to 0$$: Poisson

$$\frac{\Delta y}{\Delta y_{\mathrm{drift}}} \to \infty$$: cumulants of 

Many Slot Machines







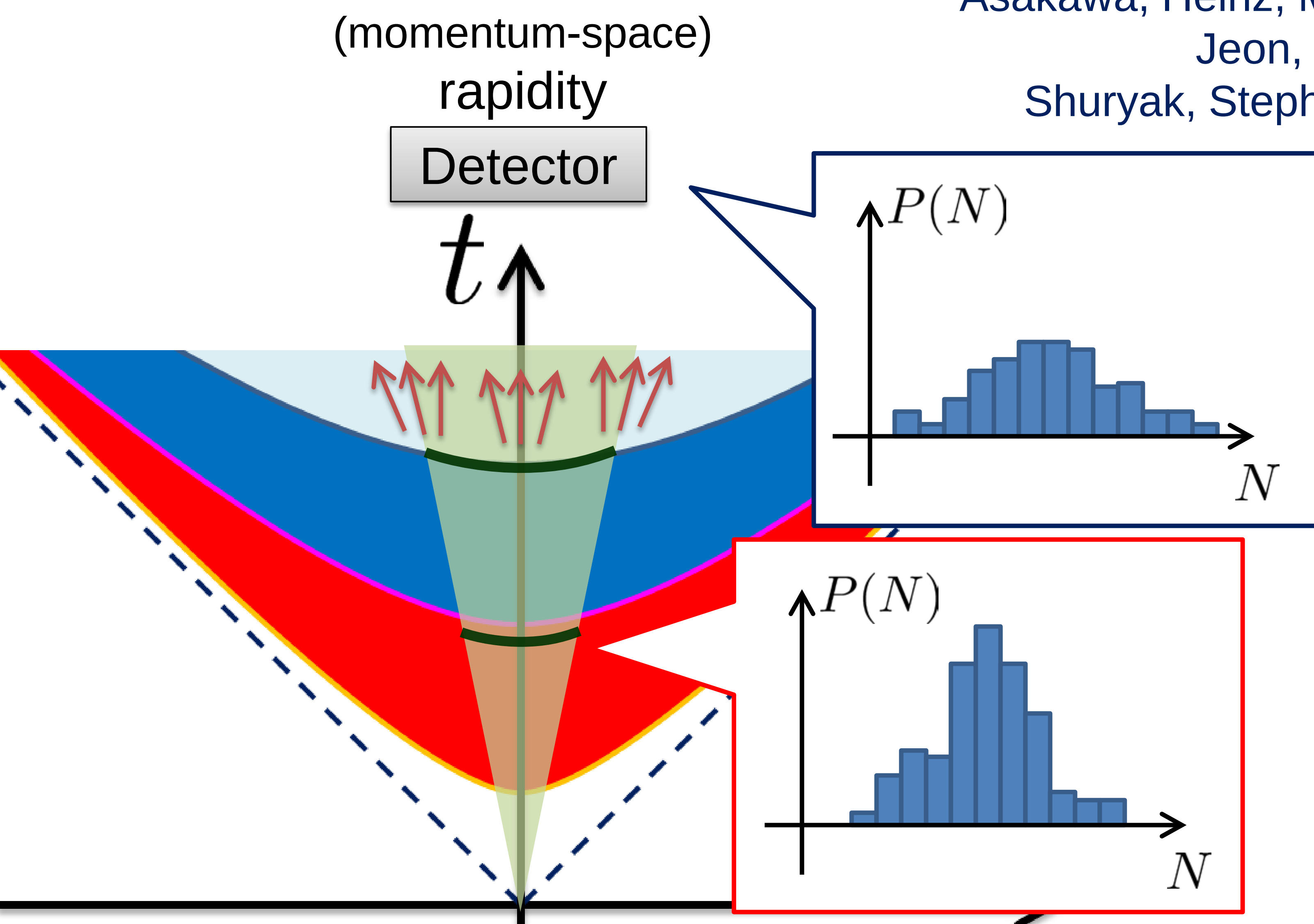
$$\frac{\Delta y}{\Delta y_{\mathrm{drift}}} \to 0$$: Poisso

$$\frac{\Delta y}{\Delta y_{\mathrm{drift}}} \to \infty$$: cumula

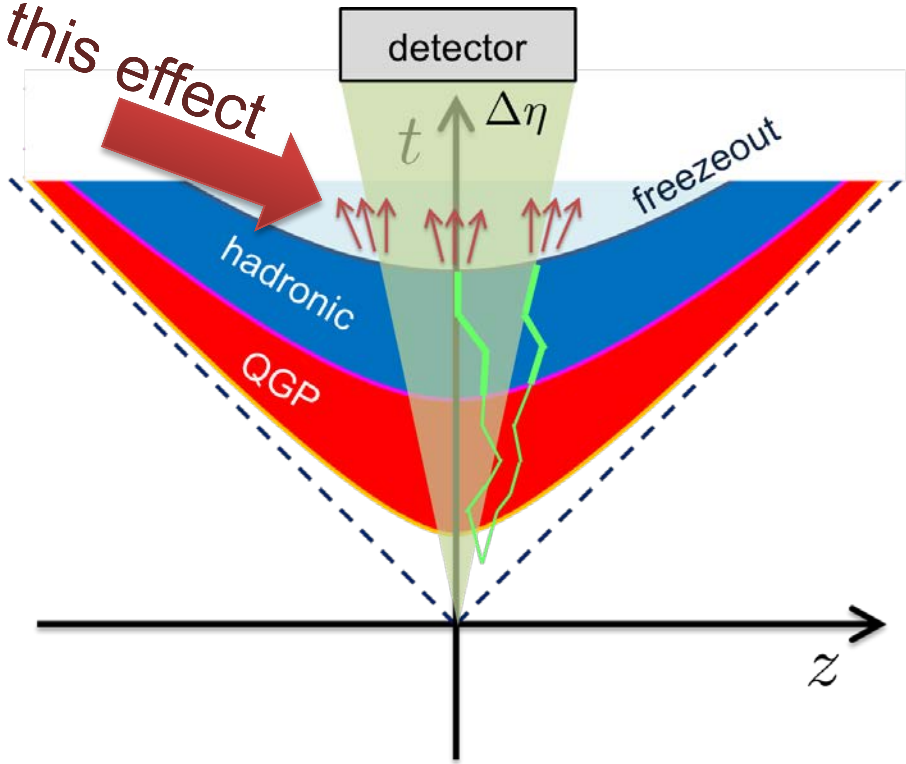
Time Evolution of Fluctuations

Diffusion modifies distribution

Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000
Shuryak, Stephanov, 2001

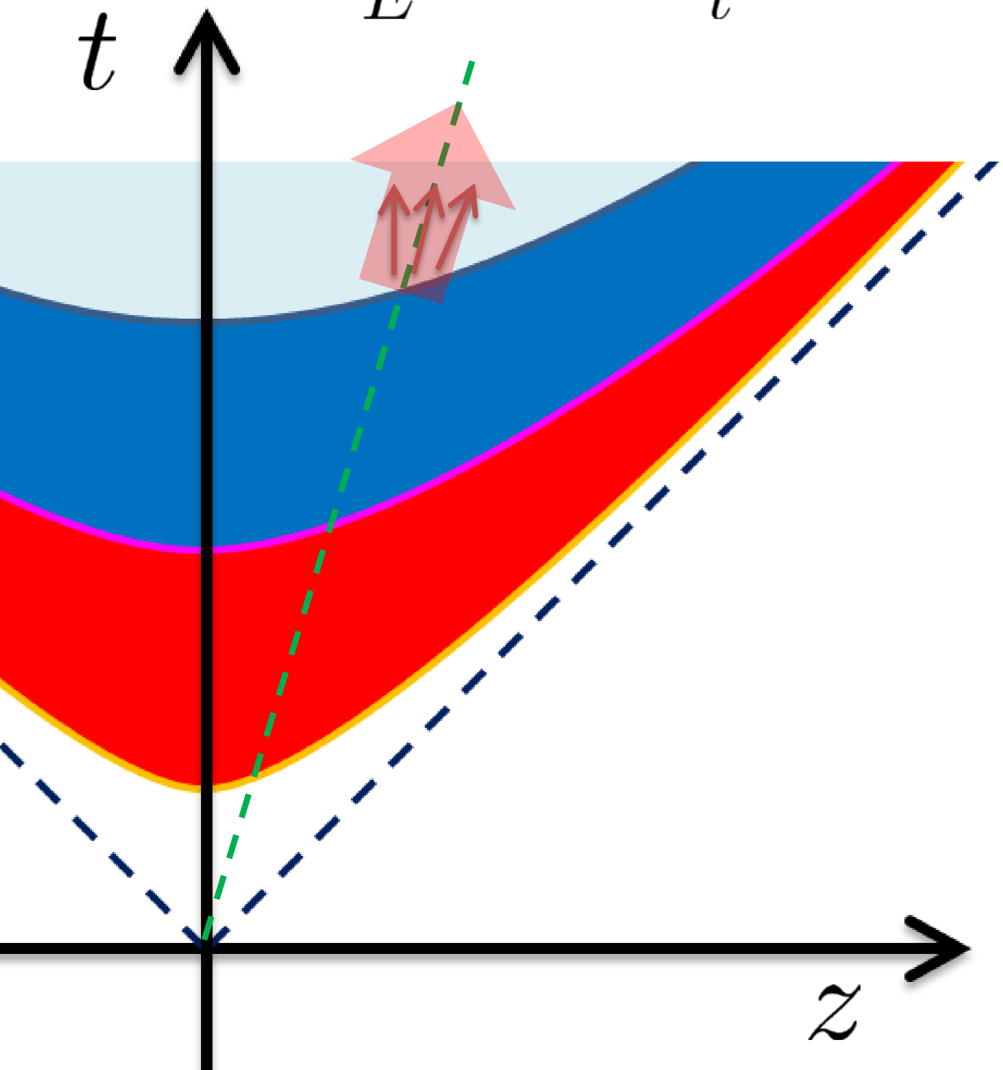


Thermal Blurring



Thermal Blurring

$$\frac{p_z}{E} = v_z = \frac{z}{t}$$



Under Bjorken picture,

coordinate-space rapidity of medium

$=$

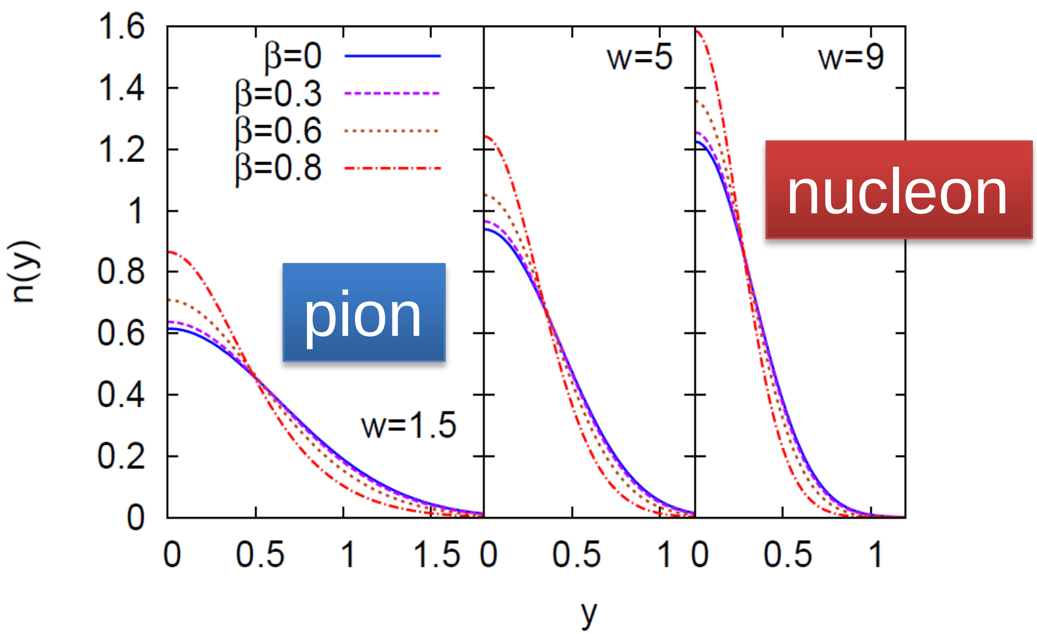
momentum-space rapidity of medium

$\neq$

coordinate-space rapidity of individual particles

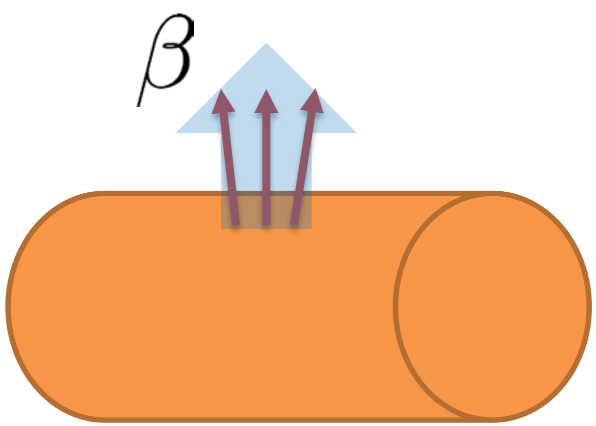
Thermal distribution in η space

Y. Ohnishi+
in preparation

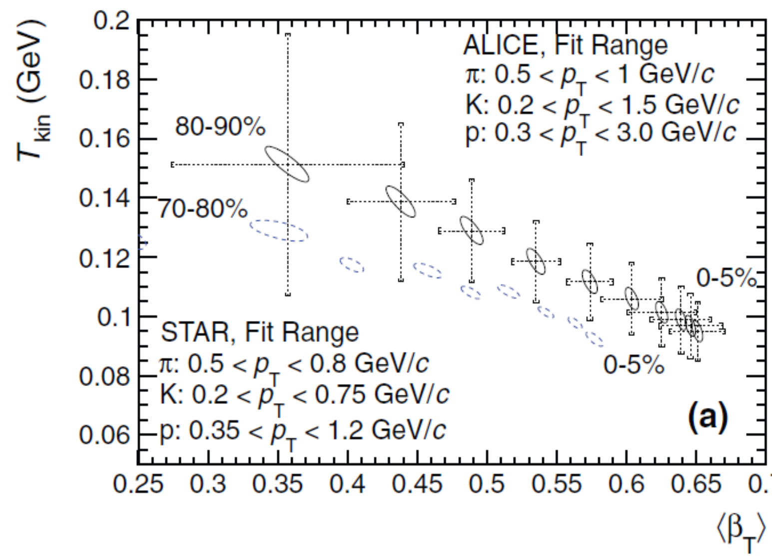


$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$



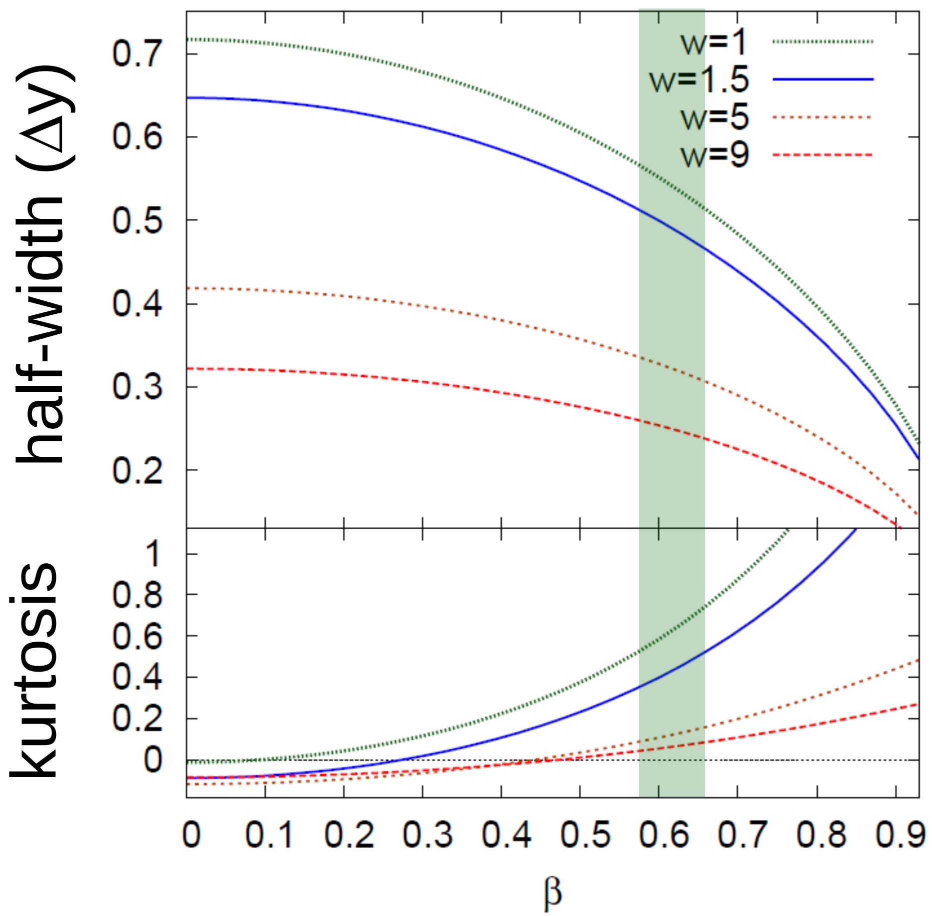
Blast wave squeezes the distribution in rapidity space



- blast wave
- flat freezeout surface

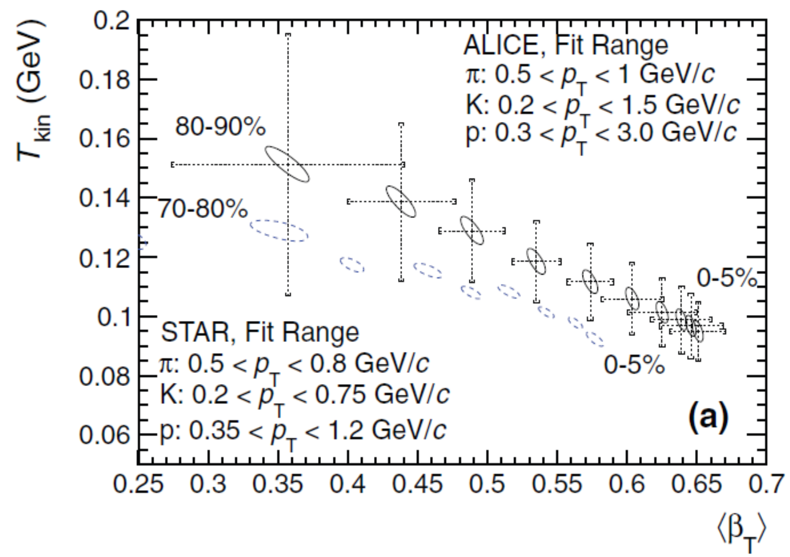
Thermal distribution in η space

Y. Ohnishi+
in preparation



$$w = \frac{m}{T}$$

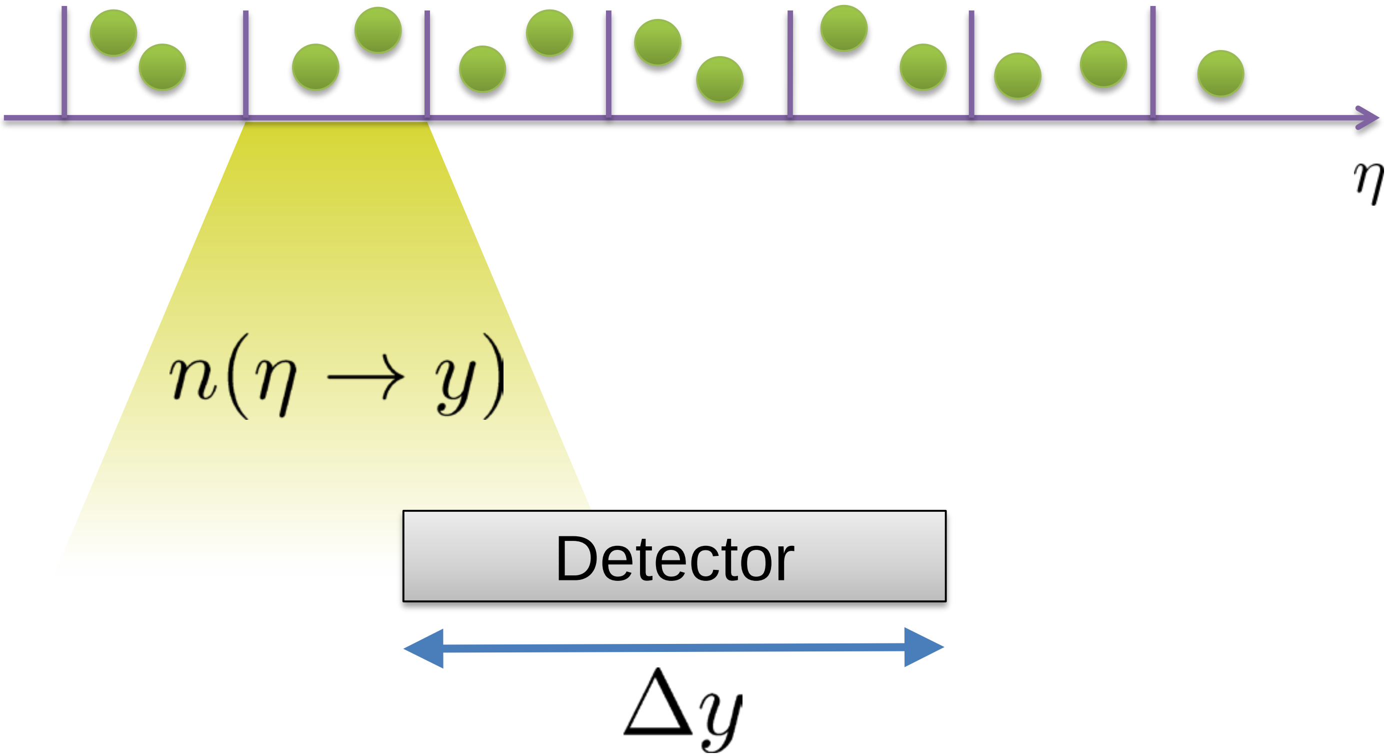
- pions $w \simeq 1.5$
- nucleons $w \simeq 9$



Rapidity distribution is not far away from Gaussian.

- blast wave
- flat freezeout surface

Formalism



- Particles arrive at the detector with some probability.
- Sum all of them up. Make the distribution.
- Take the continuum limit.

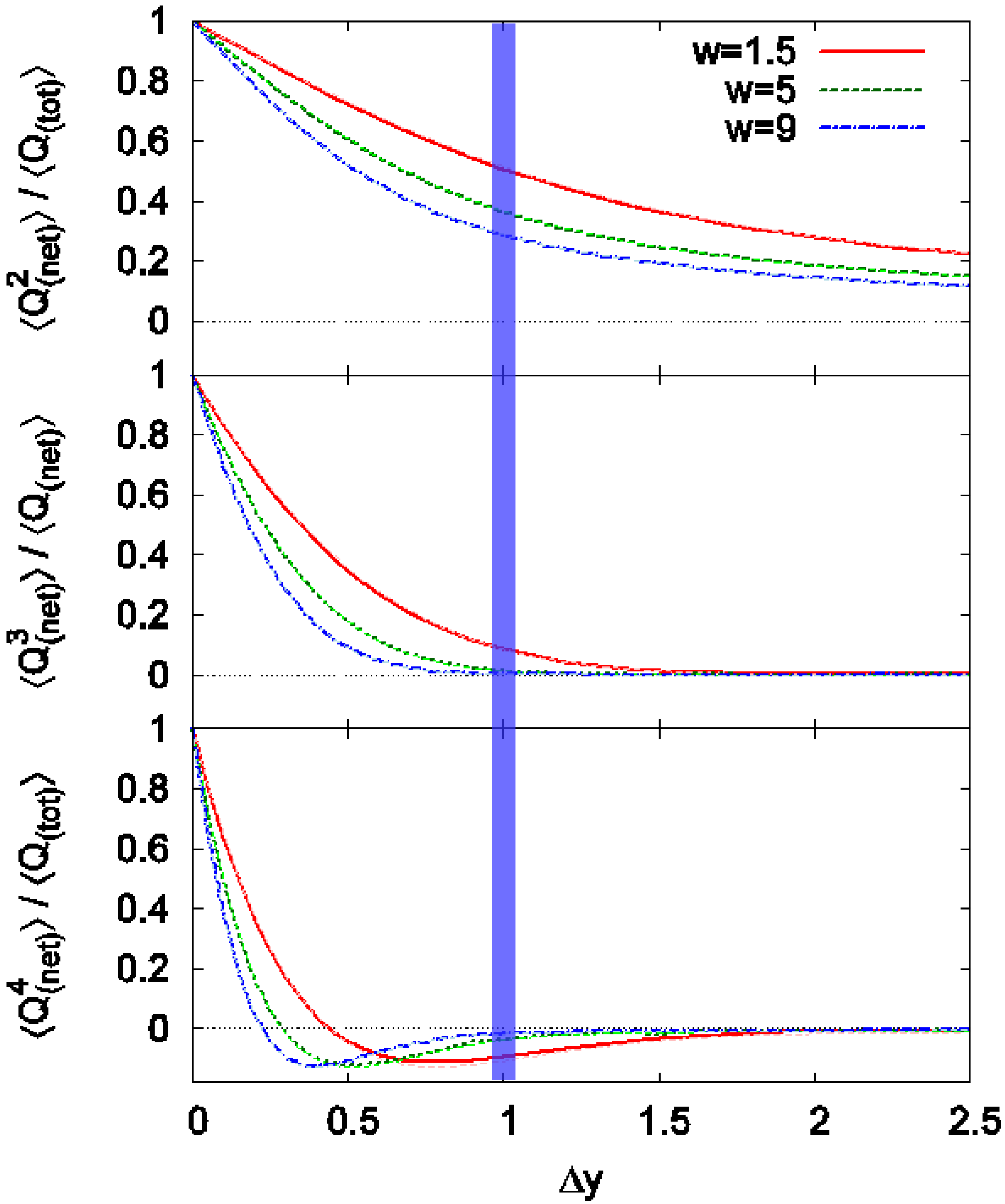
Δη Dependence

With vanishing fluctuations **before** thermal blurring

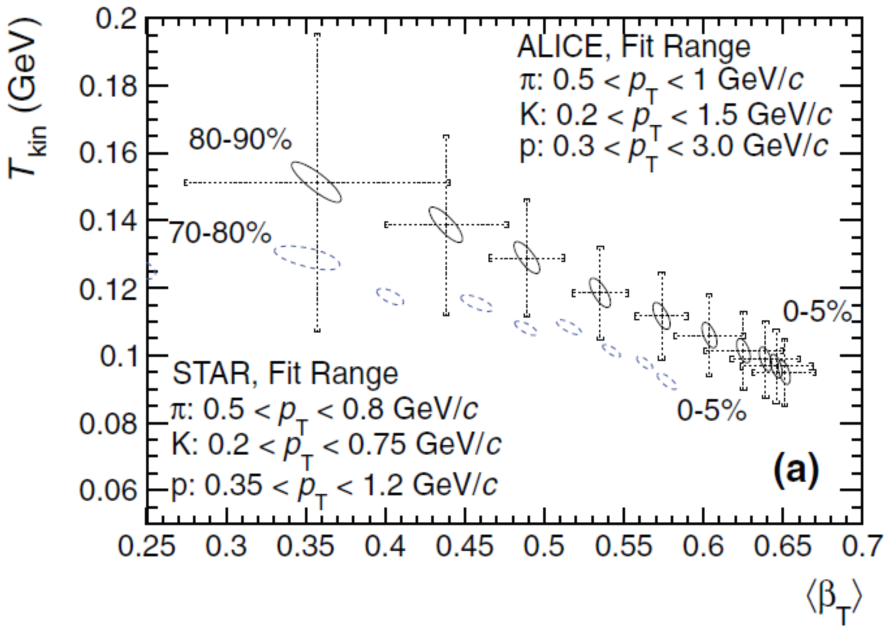
Cumulants **after** blurring take nonzero values

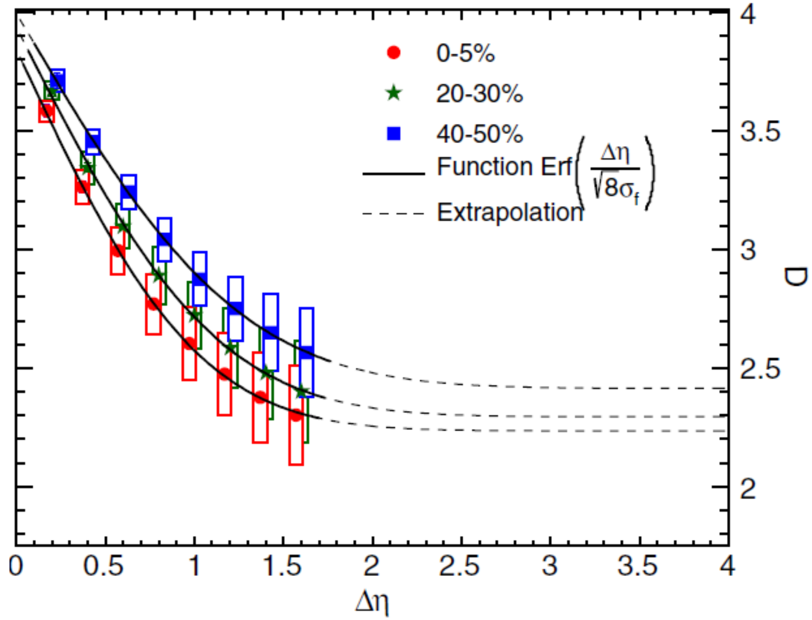
With Δy=1, the effect is **not** well suppressed

Cumulants after blurring



Centrality Dependence





More central → { lower T, larger β } → Weaker blurring

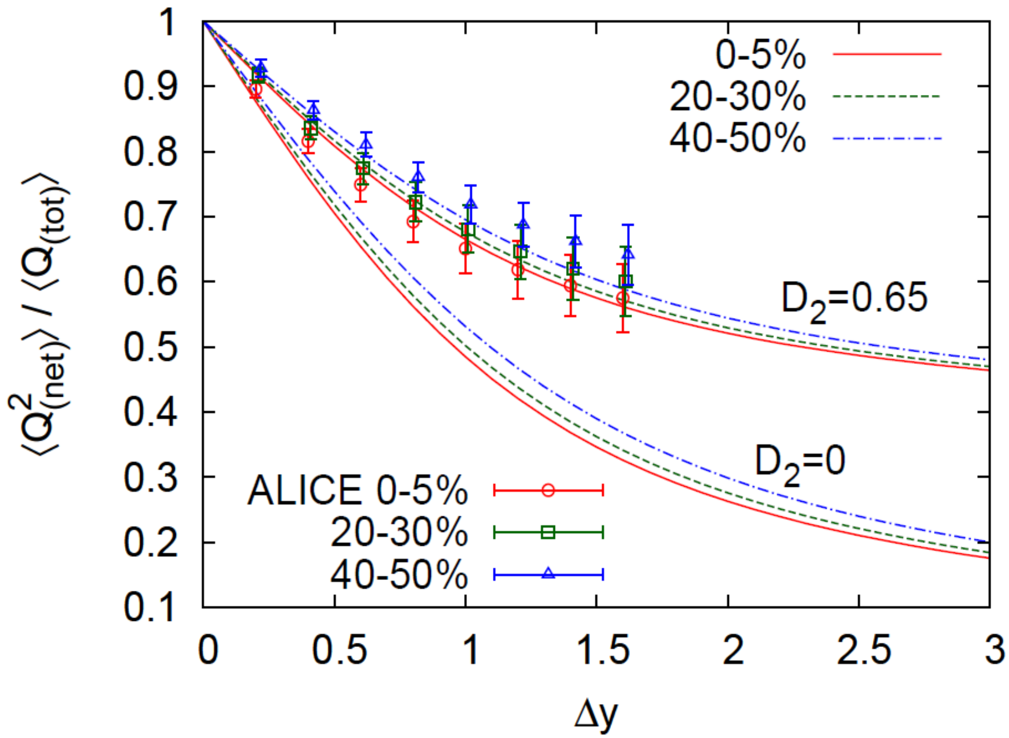
Is the centrality dependence understood solely by the thermal blurring at kinetic f.o.?

Centrality Dependence

$$D_2 = \frac{\langle \delta N_Q^2 \rangle}{\langle \delta N_Q^2 \rangle_{\mathrm{eq.}}}$$

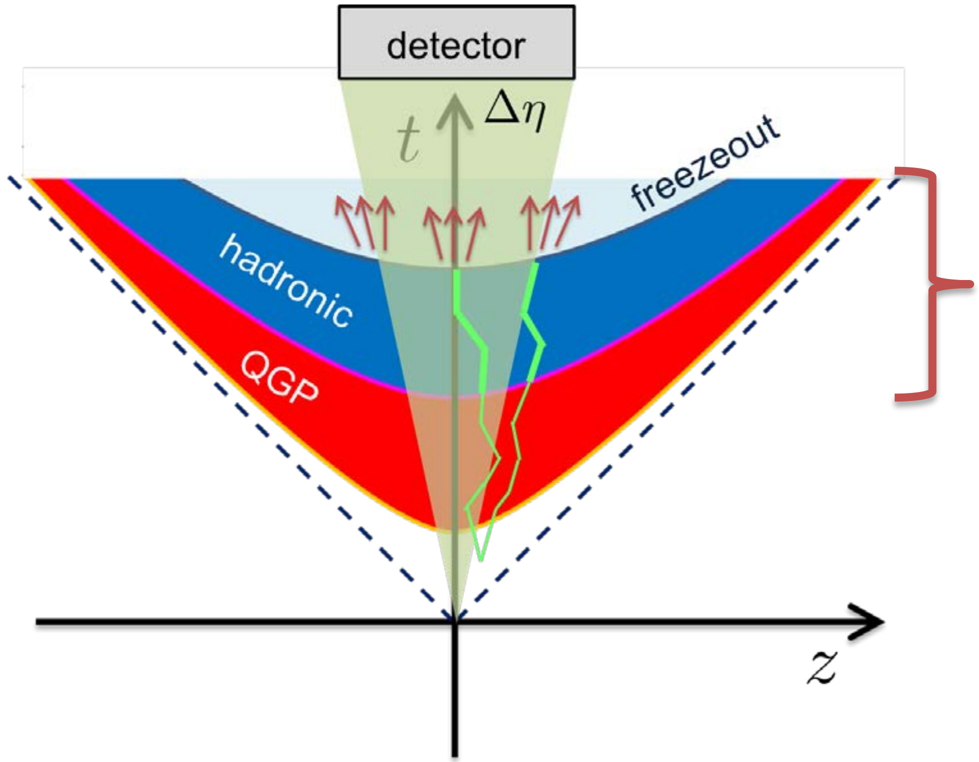
Assumptions:

- Centrality independent cumulant at kinetic f.o.
- Thermal blurring at kinetic f.o.



□ Centrality dep. of blast wave parameters can qualitatively describe the one of $\langle \delta N_Q^2 \rangle$

Diffusion (+ Thermal Blurring), of Non-Gaussian Cumulants

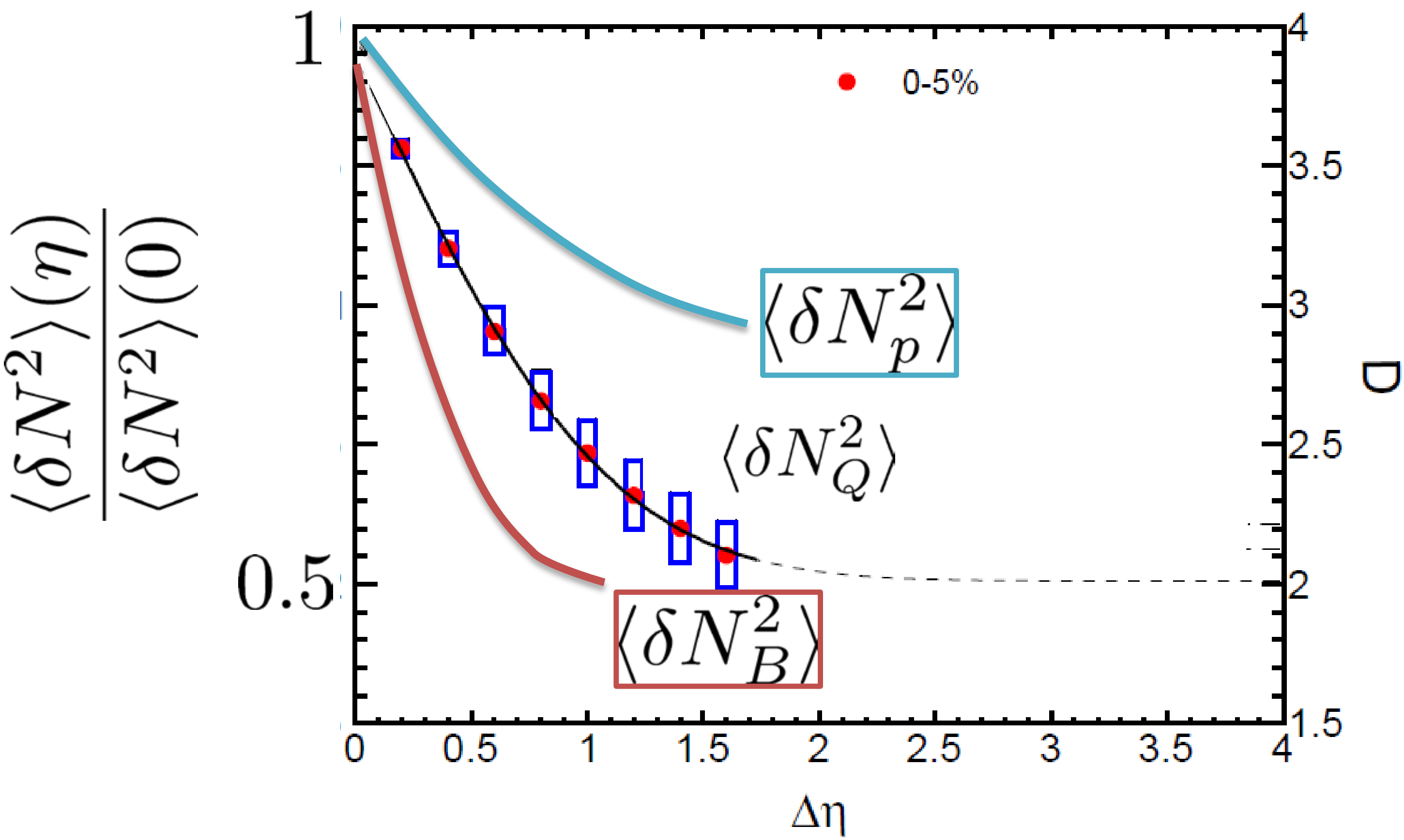


these processes

$<\delta N_B{}^2>$ and $<\delta N_p{}^2>$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$

should have different Δη dependence.

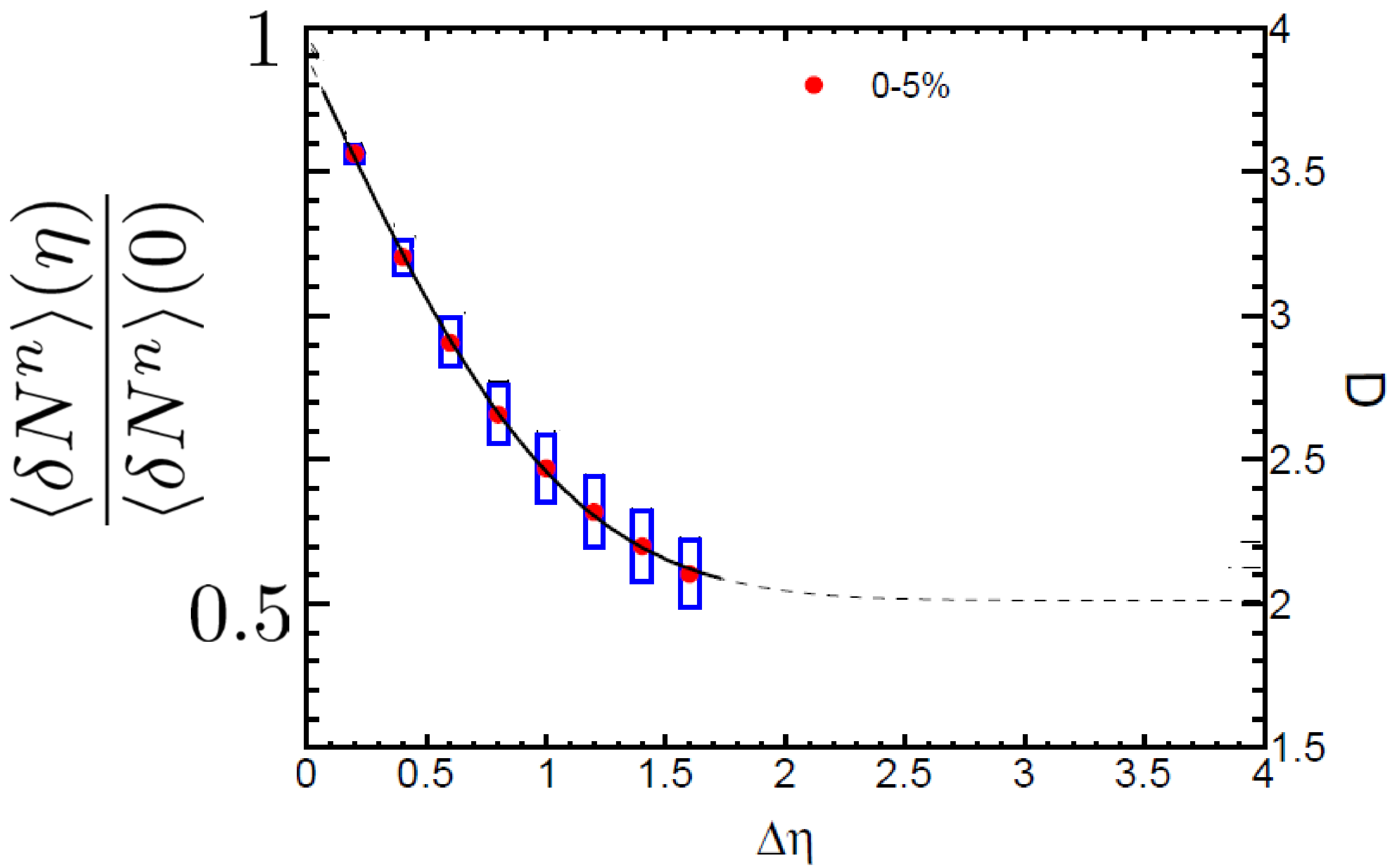


Baryon # cumulants are experimentally observable! MK, Asakawa, 2012

$<\delta N_Q^4>$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

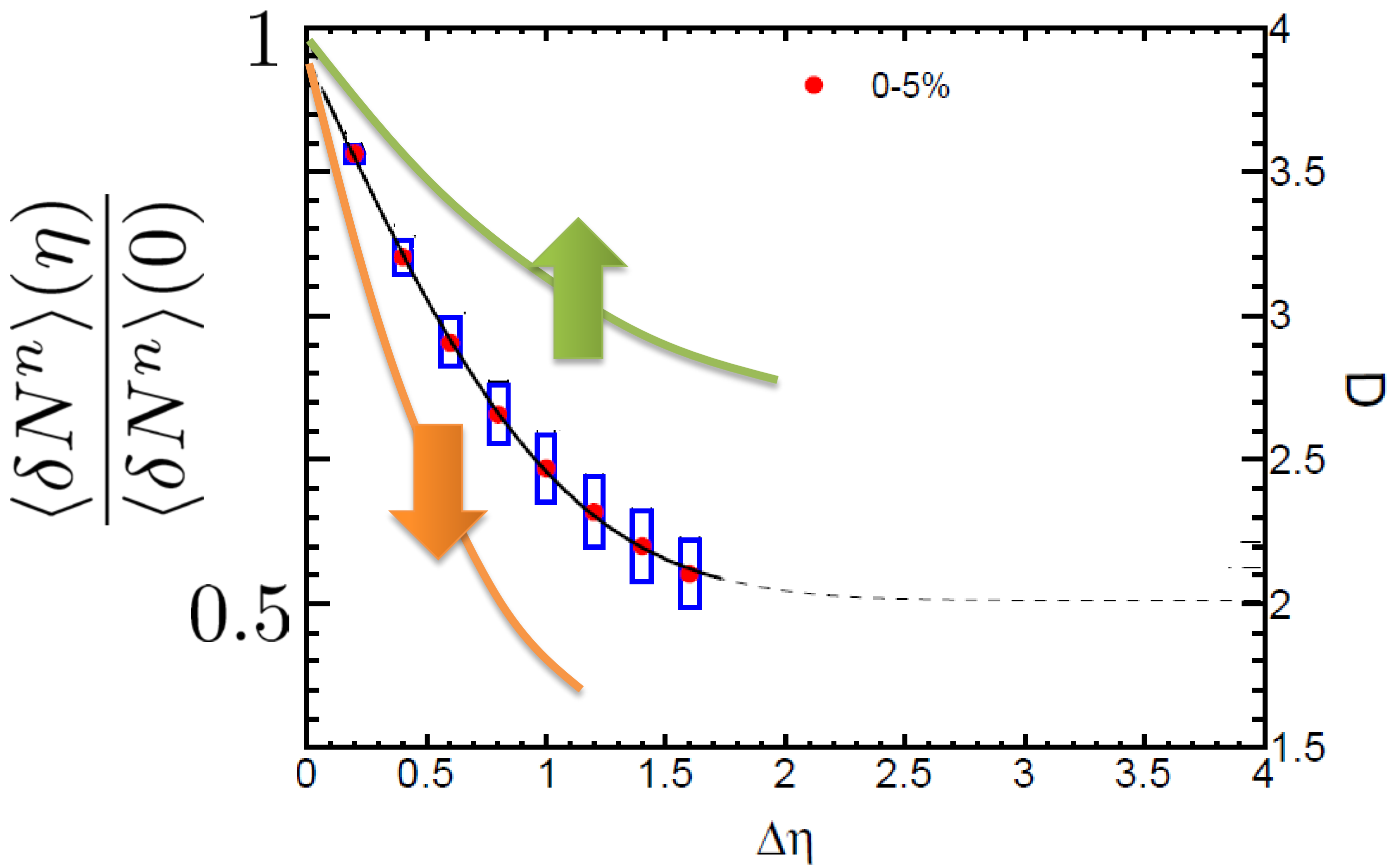
suppression or enhancement



$<\delta N_Q{}^4>$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

suppression or enhancement



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Shuryak, Stephanov, 2001

Markov (white noise)
+
continuity

Gaussian noise

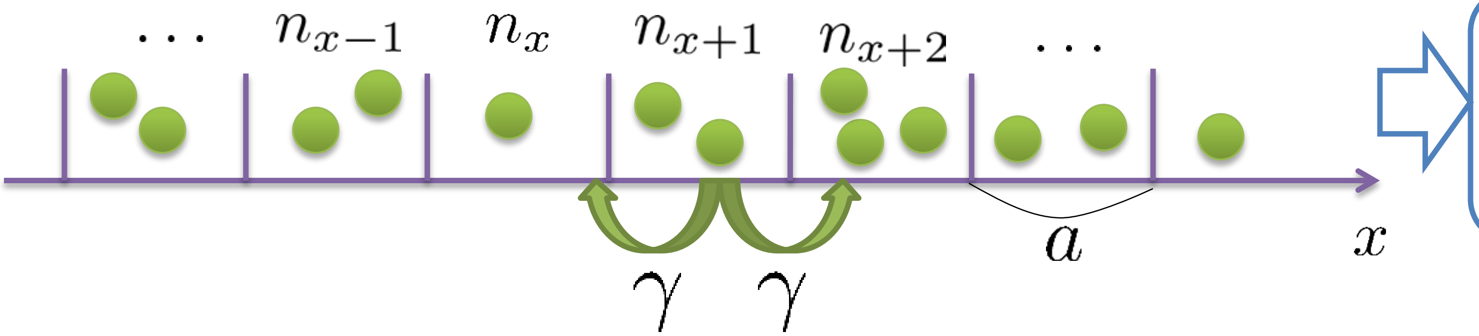
Fluctuation of *n* is
Gaussian in equilibrium

cf) Gardiner, “Stochastic Methods”

Diffusion Master Equation

MK, Asakawa, Ono, 2014
MK, 2015

Divide spatial coordinate into discrete cells



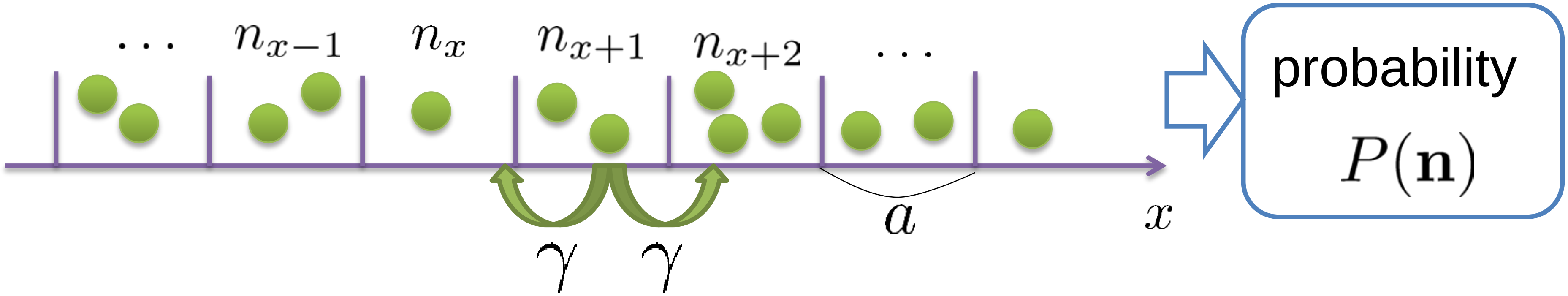
probability

$P(\mathbf{n})$

Diffusion Master Equation

MK, Asakawa, Ono, 2014
MK, 2015

Divide spatial coordinate into discrete cells



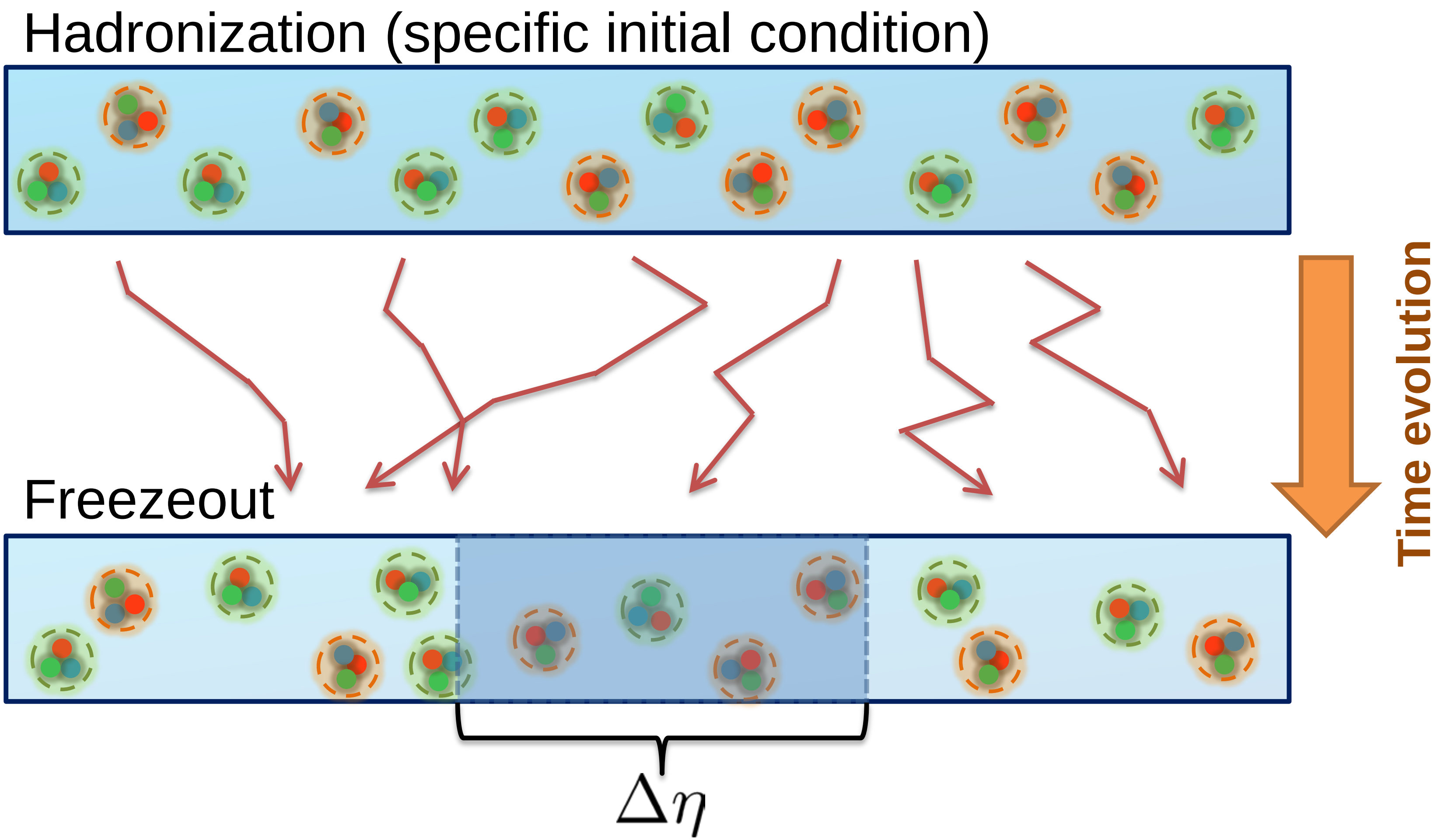
Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma\sum_x[(n_x+1)\{P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x+1})+P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x-1})\} - 2n_xP(\mathbf{n})]$$

Solve the DME **exactly**, and take *a*→0 limit

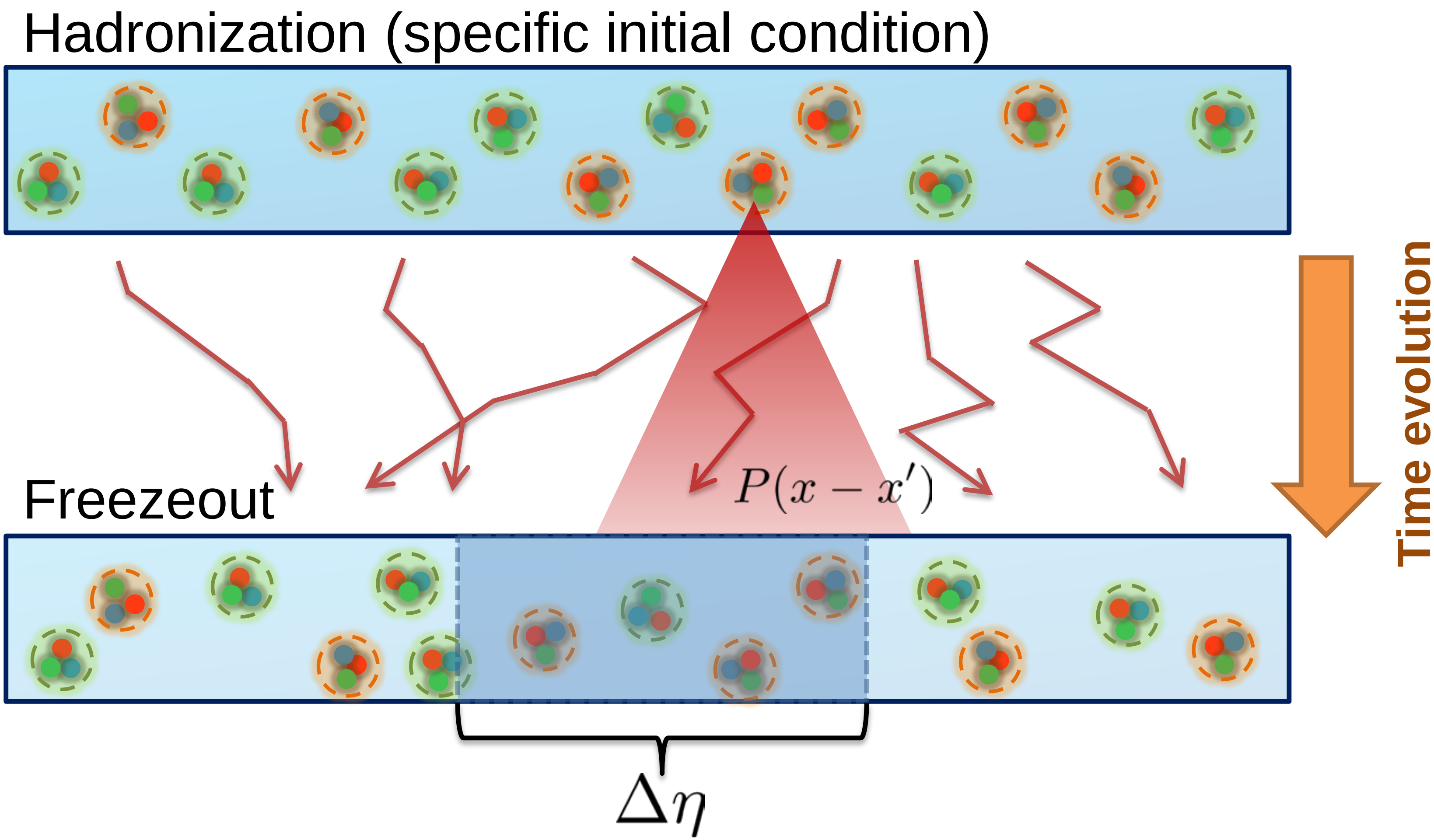
No approx., ex. van Kampen's system size expansion

A Brownian Particle's Model



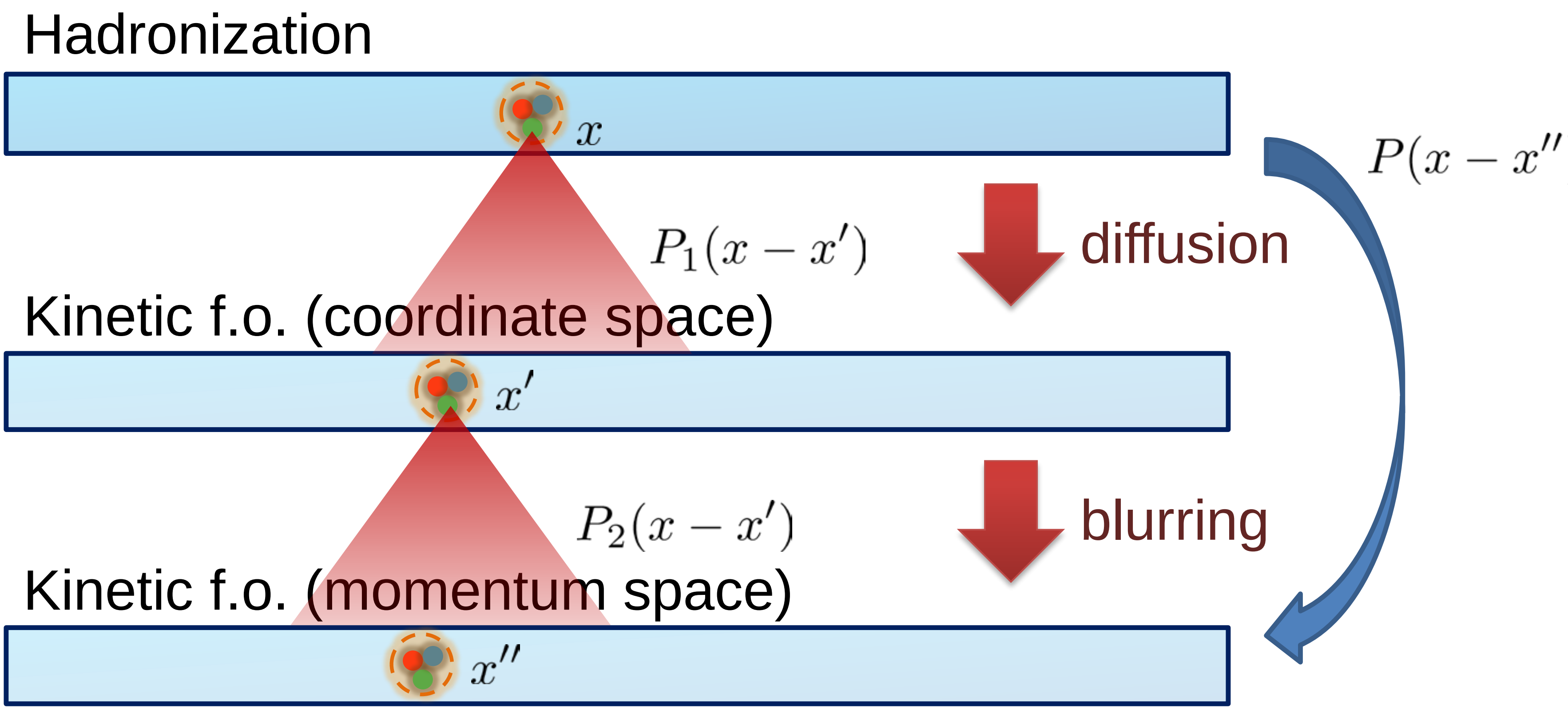
Initial distribution + motion of each particle
→ cumulants of particle # in $\Delta\eta$

A Brownian Particle's Model



Initial distribution + motion of each particle
→ cumulants of particle # in $\Delta\eta$

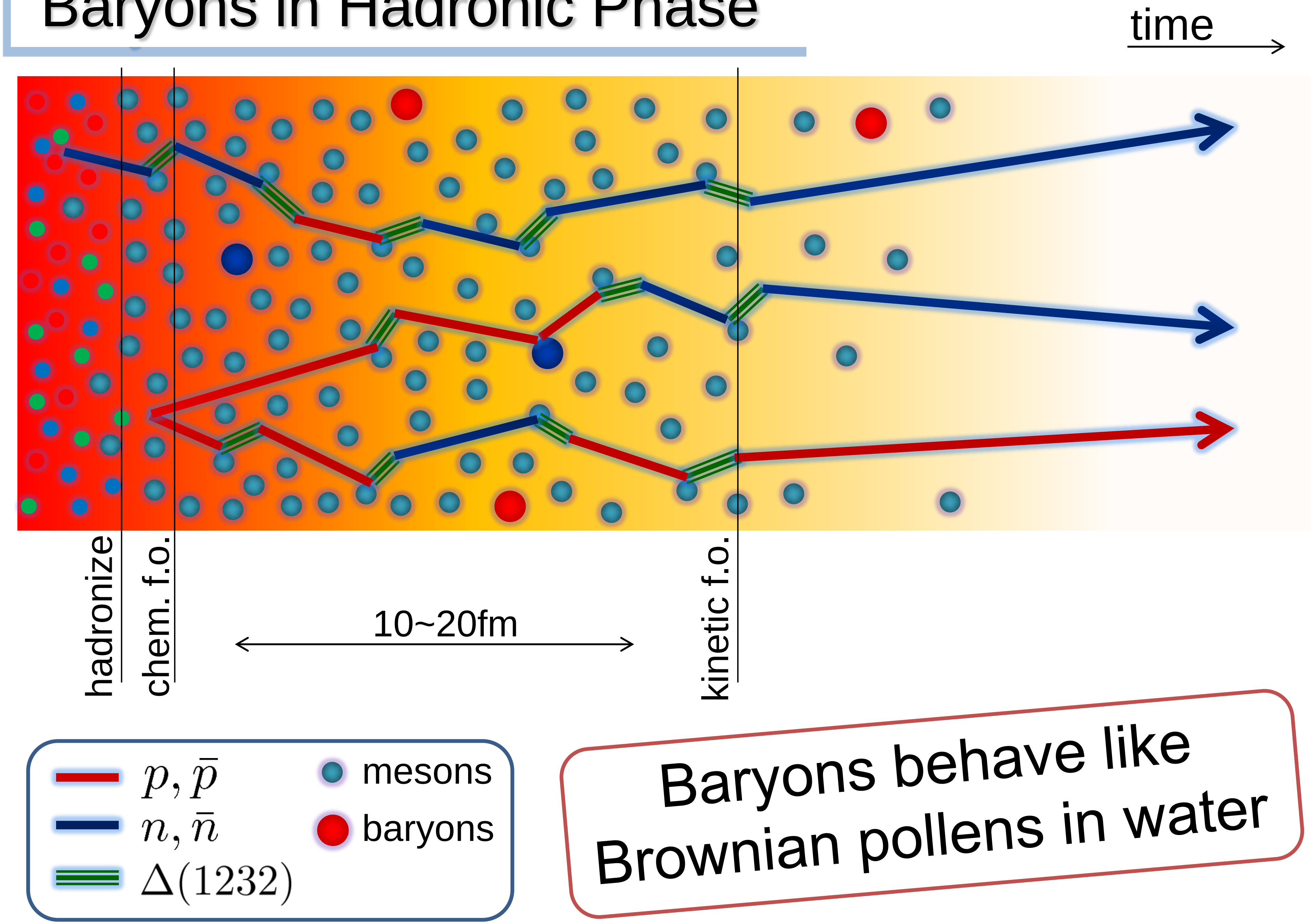
Diffusion + Thermal Blurring



Total diffusion: $P(x - x'') = \int dx' P_1(x - x') P_2(x' - x'')$

- ☐ Diffusion + thermal blurring = described by a single P(x)
- ☐ Both are consistent with Gaussian → Single Gaussian

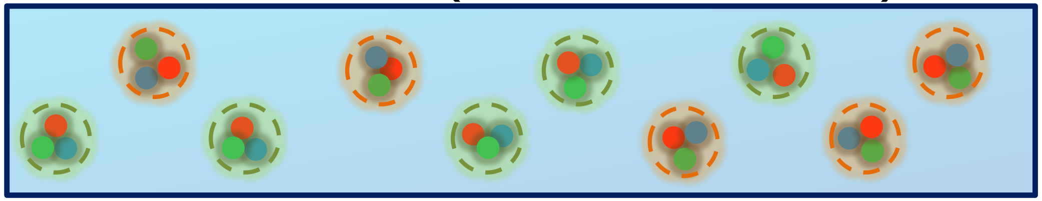
Baryons in Hadronic Phase



Baryons behave like Brownian pollens in water

Time Evolution in Hadronic Phase

Hadronization (initial condition)



- Boost invariance / infinitely long system
- Local equilibration / local correlation

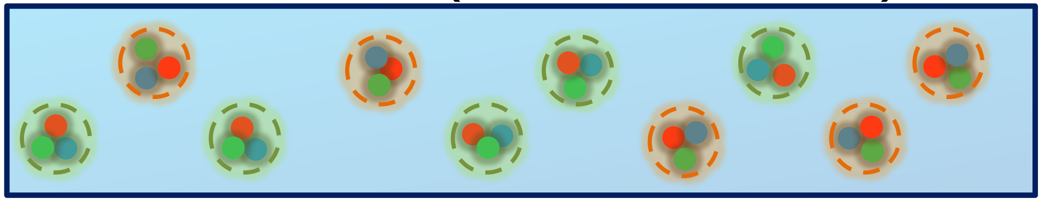
$\langle \bar{Q}^2 \rangle_c, \ \langle \bar{Q}^3 \rangle_c, \ \langle \bar{Q}^4 \rangle_c$ $\quad \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ $\quad \langle Q^2_{(\mathrm{tot})} \rangle_c, \langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c$

suppression owing to local charge conservation

strongly dependent on hadronization mechanism

Time Evolution in Hadronic Phase

Hadronization (initial condition)



Diffusion + Blurring

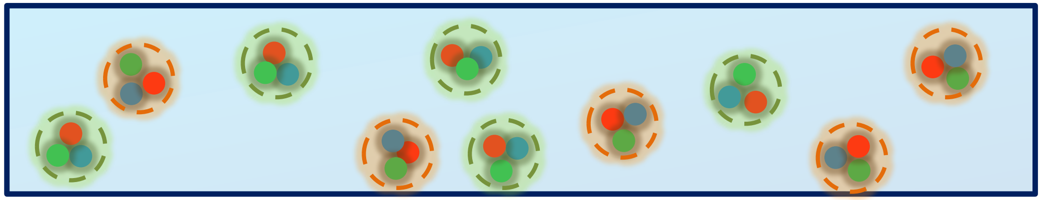
- Boost invariance / infinitely long system
- Local equilibration / local correlation

$\langle \bar{Q}^2 \rangle_c, \ \langle \bar{Q}^3 \rangle_c, \ \langle \bar{Q}^4 \rangle_c$ $\quad \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ $\quad \langle Q^2_{(\mathrm{tot})} \rangle_c, \langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c$

suppression owing to local charge conservation

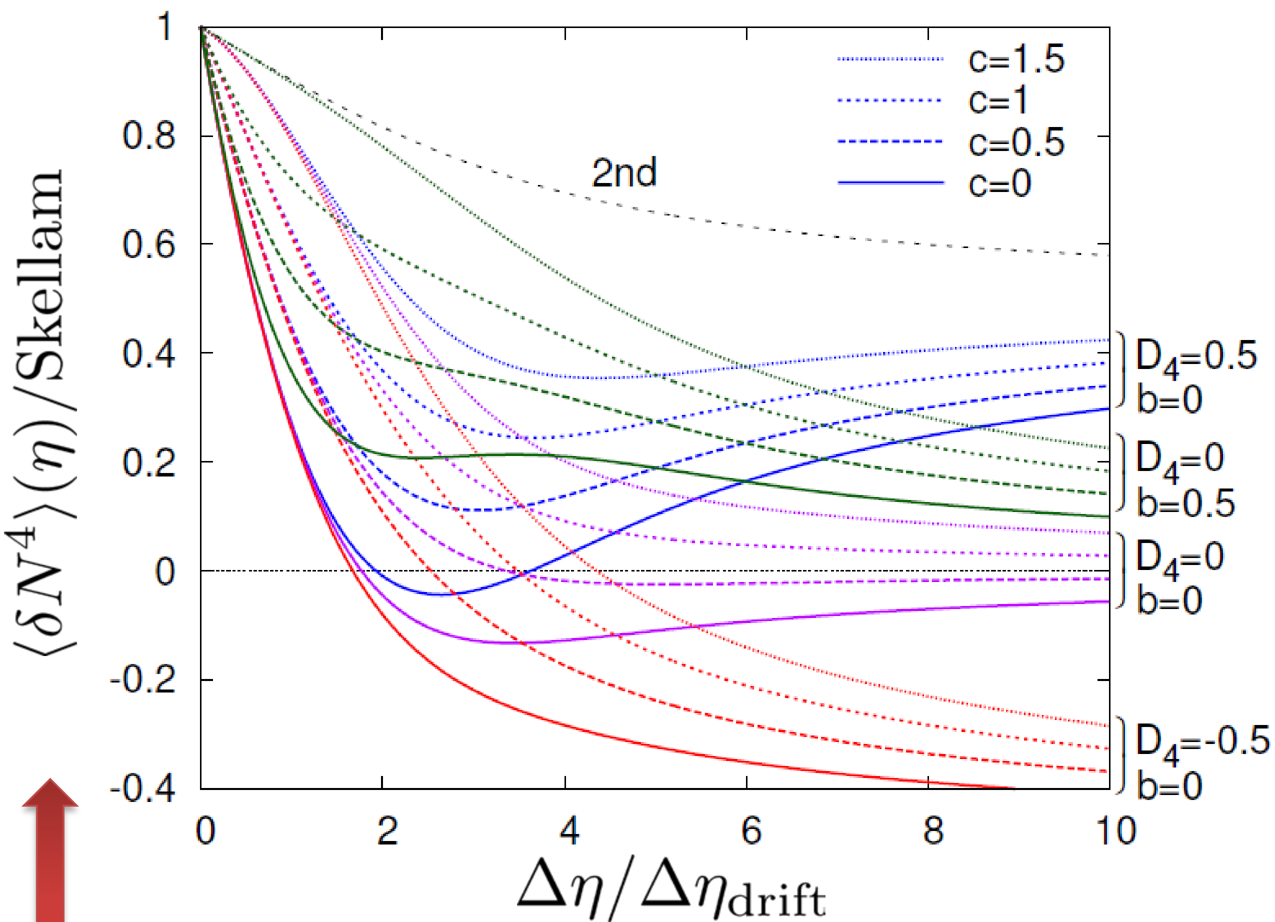
strongly dependent on hadronization mechanism

Detector



Δη Dependence: 4th order

MK, NPA (2015)



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

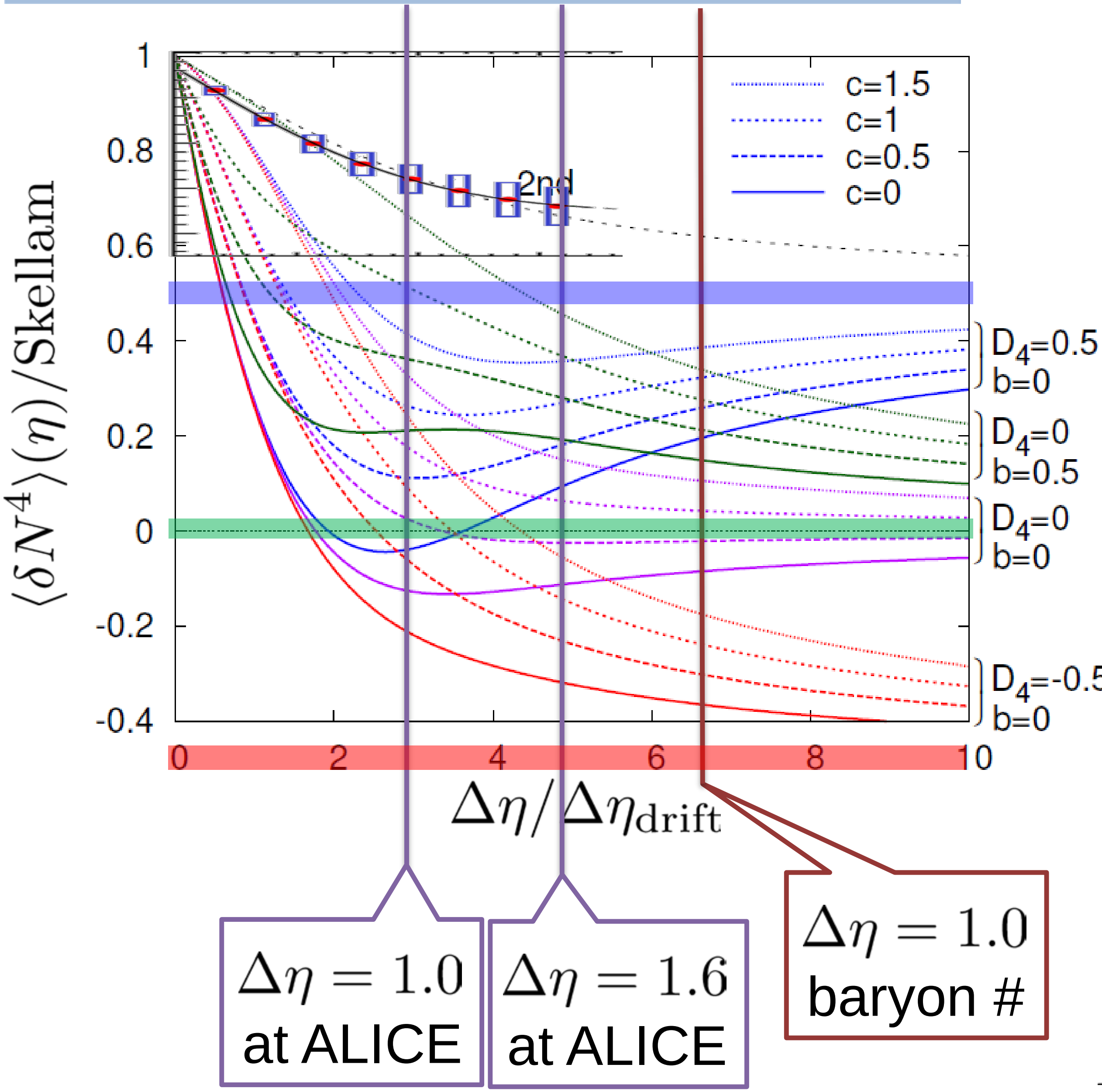
$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

new normalization

Characteristic Δη dependences!

Δη Dependence: 4th order

MK, NPA (2015)



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

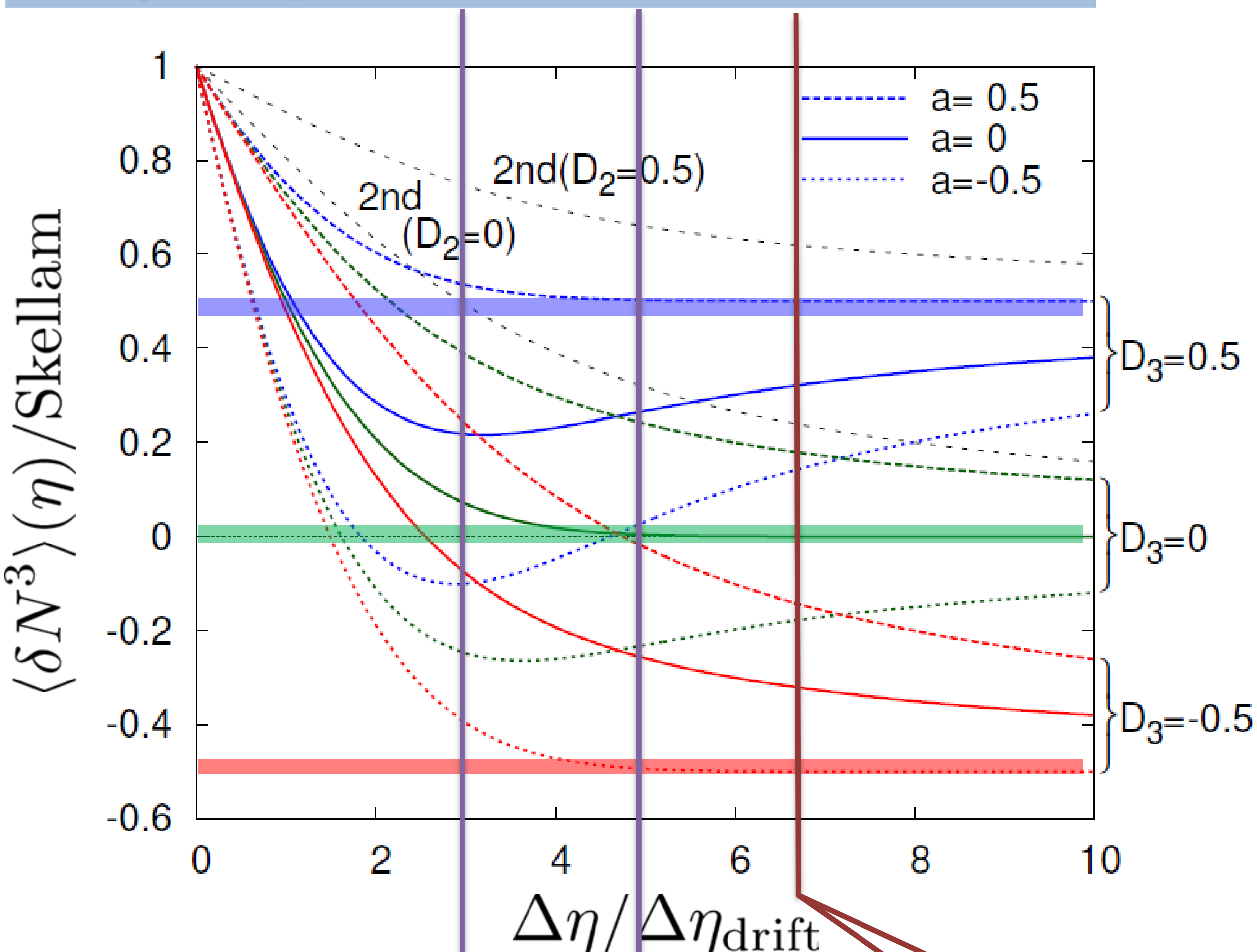
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

$$D \sim M^{-1}$$

Δη Dependence: 3rd order

MK, NPA (2015)



$\Delta\eta = 1.0$ at ALICE

$\Delta\eta = 1.6$ at ALICE

$\Delta\eta = 1.0$ baryon #

Initial Condition

$$D_3 = \frac{\langle Q^3_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

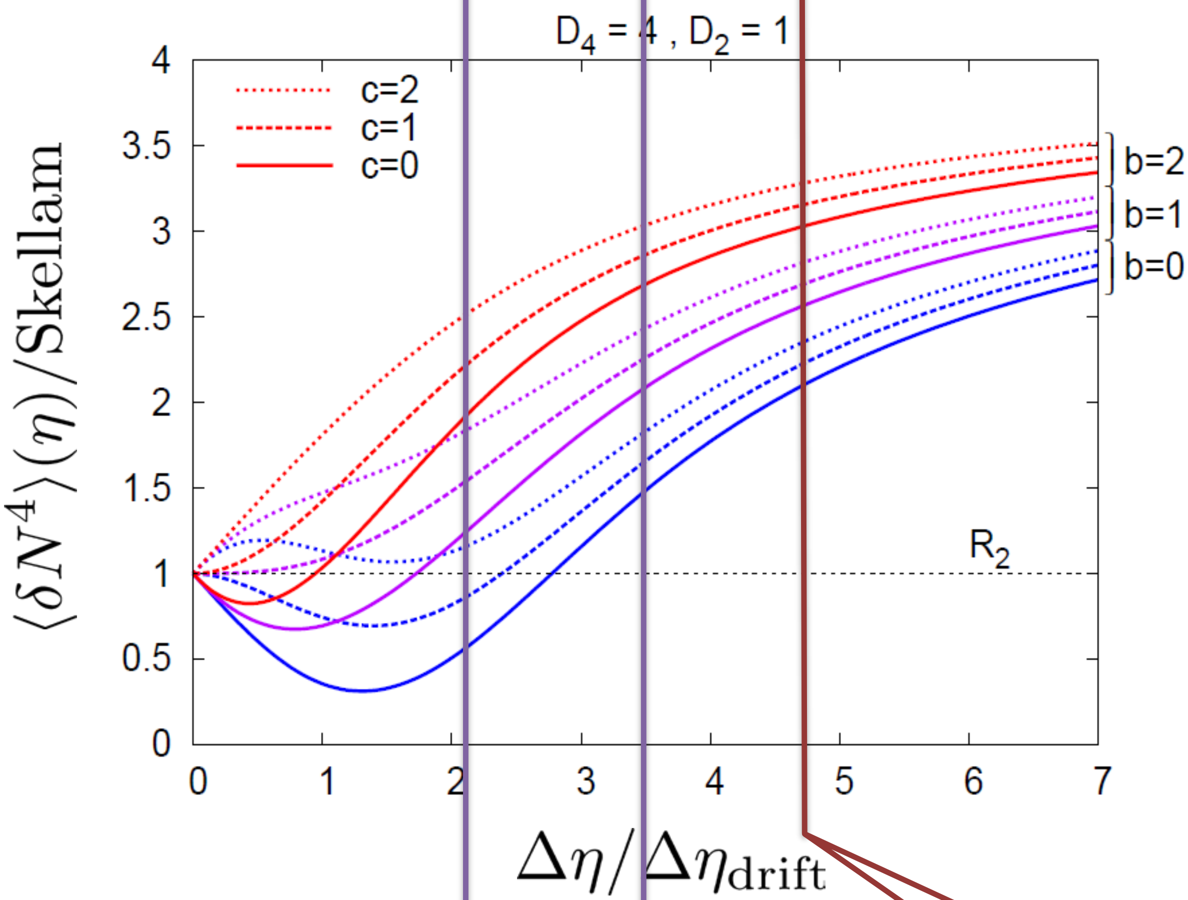
$$a = \frac{\langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

$$D \sim M^{-1}$$

4^{th} order : Large Initial Fluc.

MK, NPA (2015)



$\Delta\eta = 1.0$
at ALICE

$\Delta\eta = 1.6$
at ALICE

$\Delta\eta = 1.0$
baryon #

Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

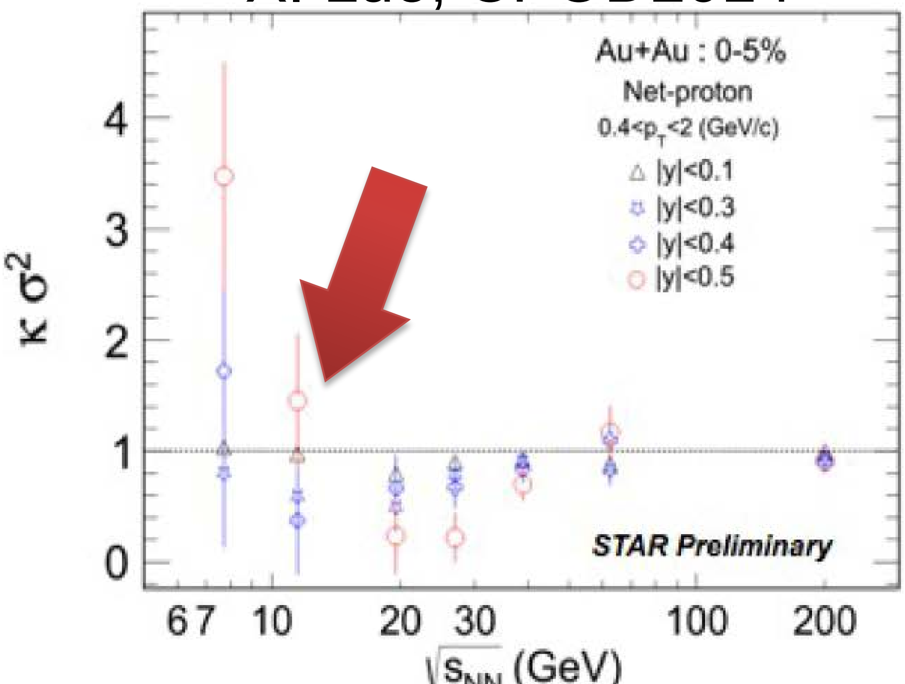
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

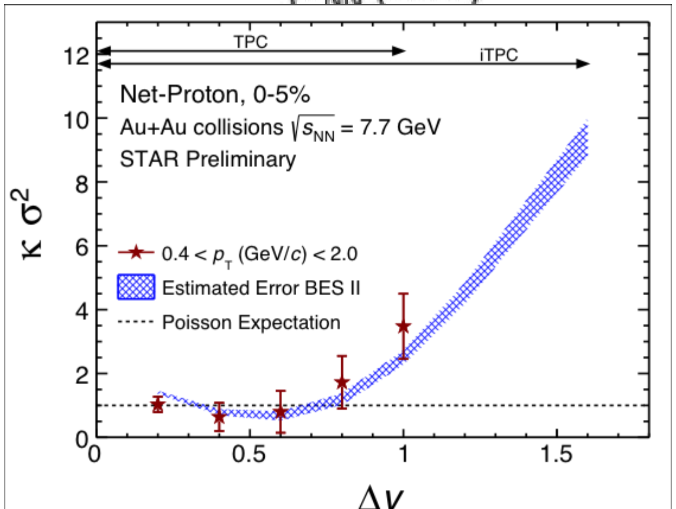
$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

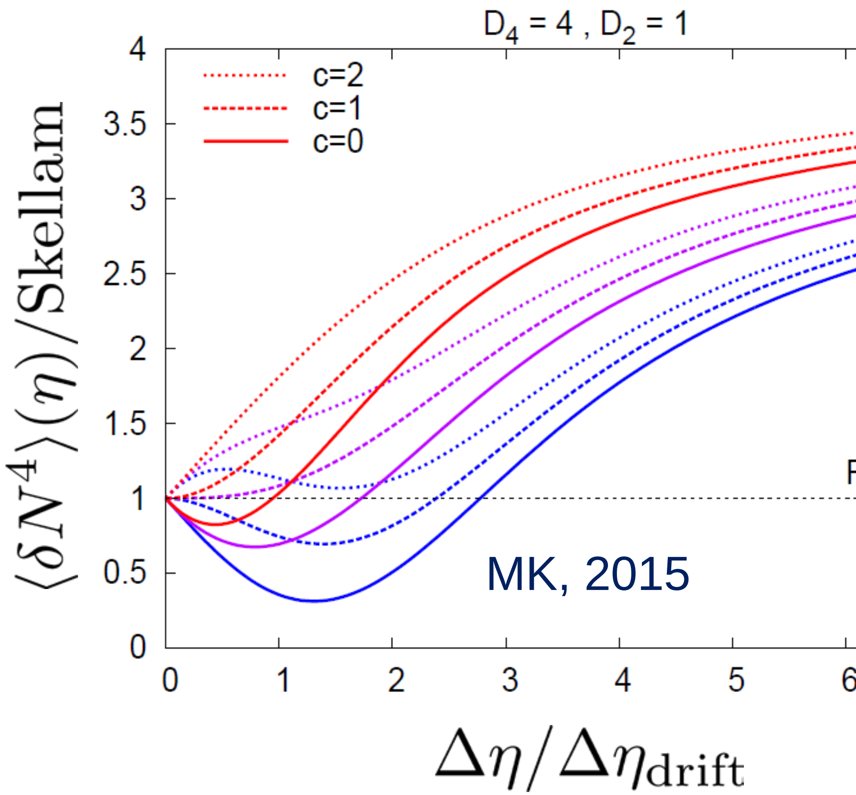
$$D \sim M^{-1}$$

Δη Dependence @ STAR

X. Luo, CPOD2014







Non-monotonic dependence on Δy ?

Summary

Plenty of information in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2 \rangle_c, \ \langle N_Q^3 \rangle_c, \ \langle N_Q^4 \rangle_c, \ \langle N_B^2 \rangle_c, \ \langle N_B^3 \rangle_c, \ \langle N_B^4 \rangle_c, \ \langle N_S^2 \rangle_c, \ \cdots$$

and those of non-conserved charges, mixed cumulants…

With $\Delta\eta$ dep. we can explore

- primordial thermodynamics
- non-thermal and transport property
- effect of thermal blurring

Future Studies

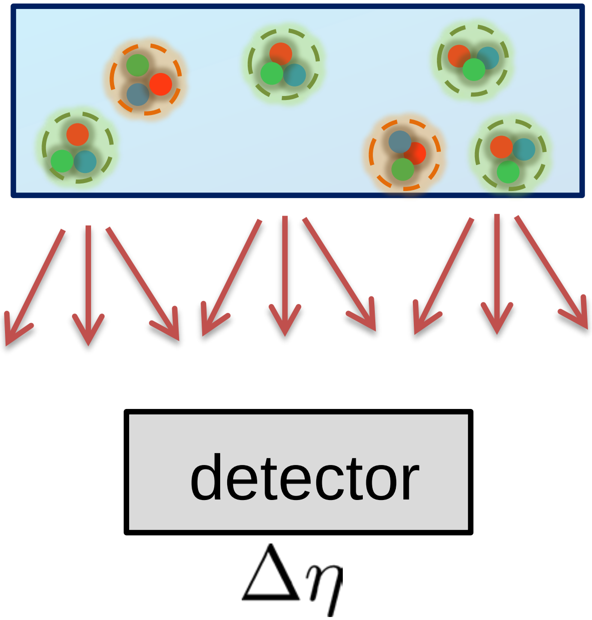
- **Experimental side:**
  - rapidity window dependences
  - baryon number cumulants
  - BES for SPS- to LHC-energies

- **Theoretical side:**
  - rapidity window dependences in dynamical models
  - description of non-equilibrium non-Gaussianity
  - accurate measurements on the lattice

- **Both sides:**
  - Compare theory and experiment carefully
  - Let's accelerate our understanding on fluctuations!

Very Low Energy Collisions

- Large contribution of global charge conservation
- Violation of Bjorken scaling



Fluctuations at low $\sqrt{s}$ should be interpreted carefully!
Comparison with statistical mechanics would not make sense…

How to Introduce Non-Gaussianity?

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant *D*(*n*)
  - colored noise
  - discretization of *n*

How to Introduce Non-Gaussianity?

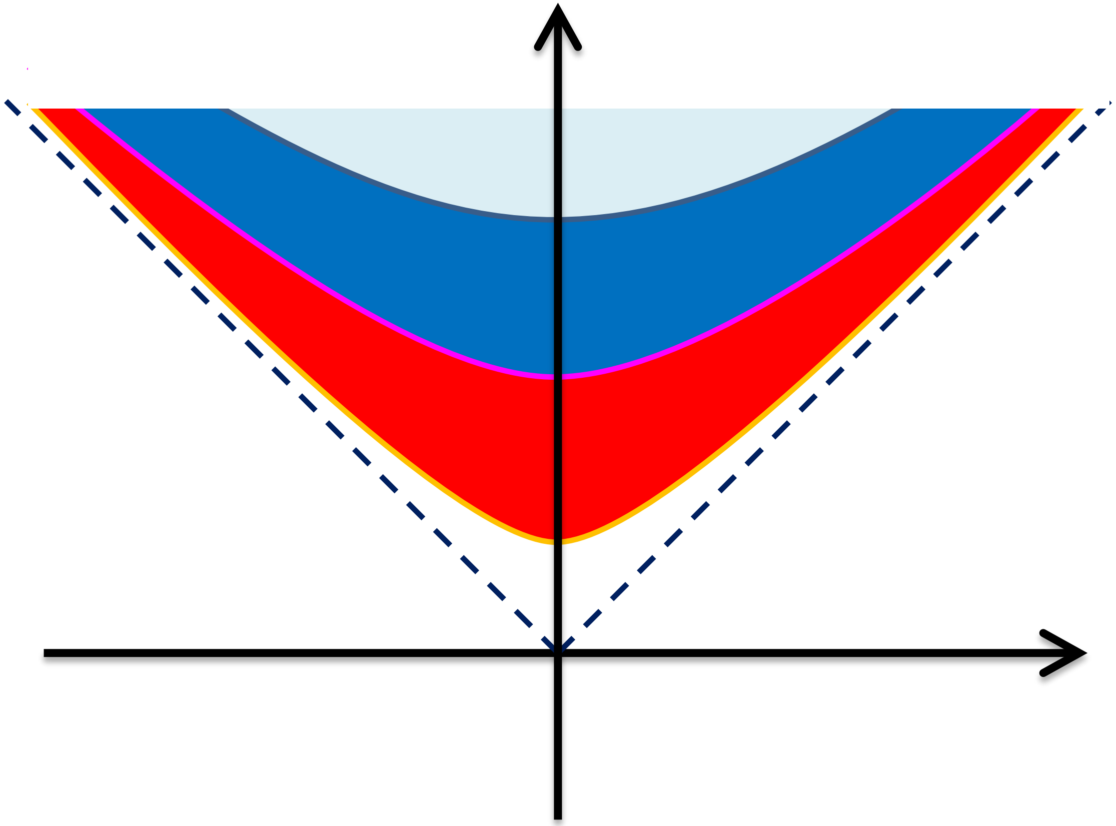
Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

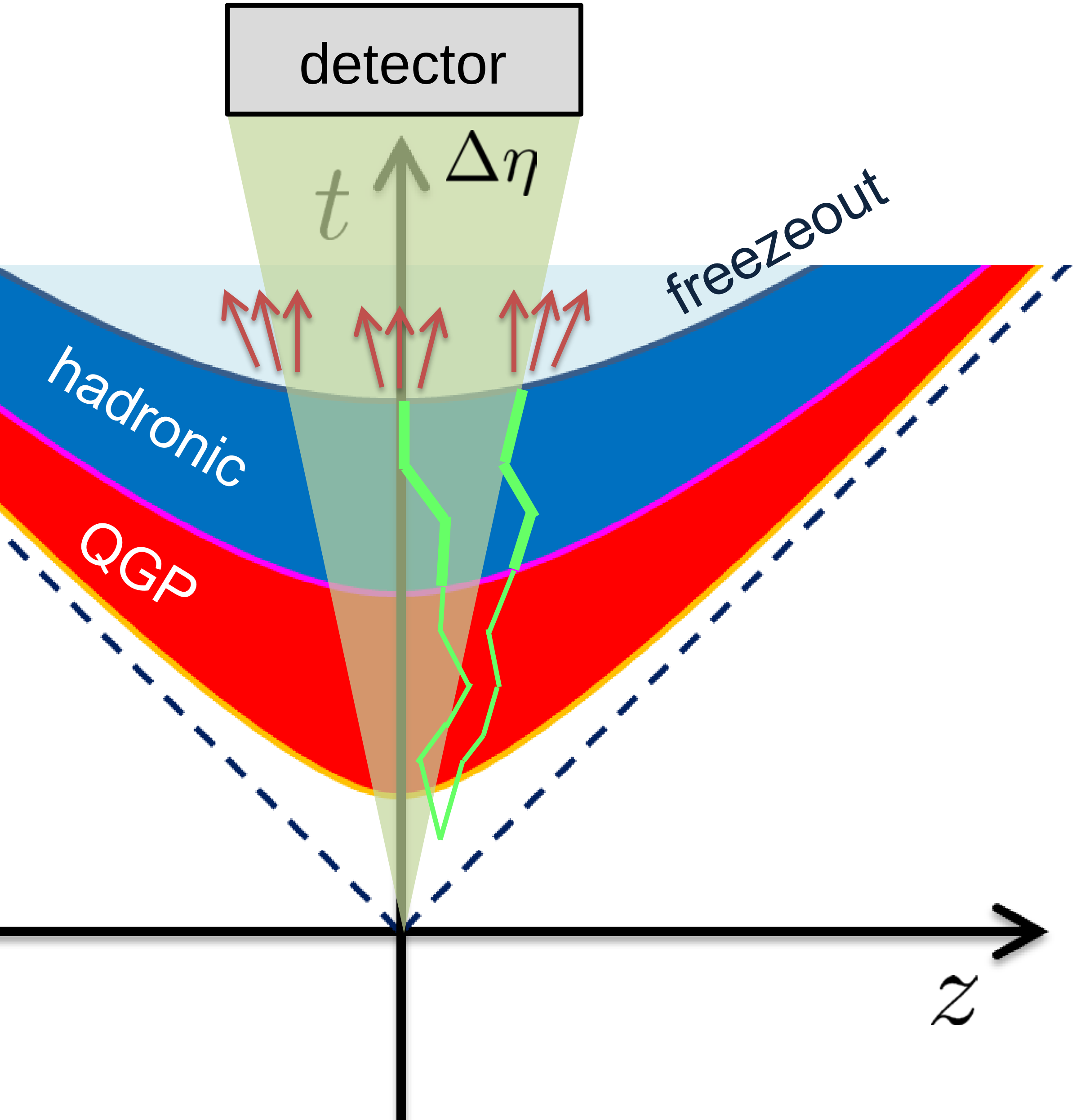
- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant *D*(*n*)
  - colored noise
  - discretization of *n* ← **our choice**

REMARK: Fluctuations measured in HIC are almost Poissonian.

Time Evolution of Fluctuations



Time Evolution of Fluctuations



Particle # in Δη

① continues to change until kinetic freezeout due to diffusion.

② changes due to a conversion y → η at kinetic freezeout

"Thermal Blurring"