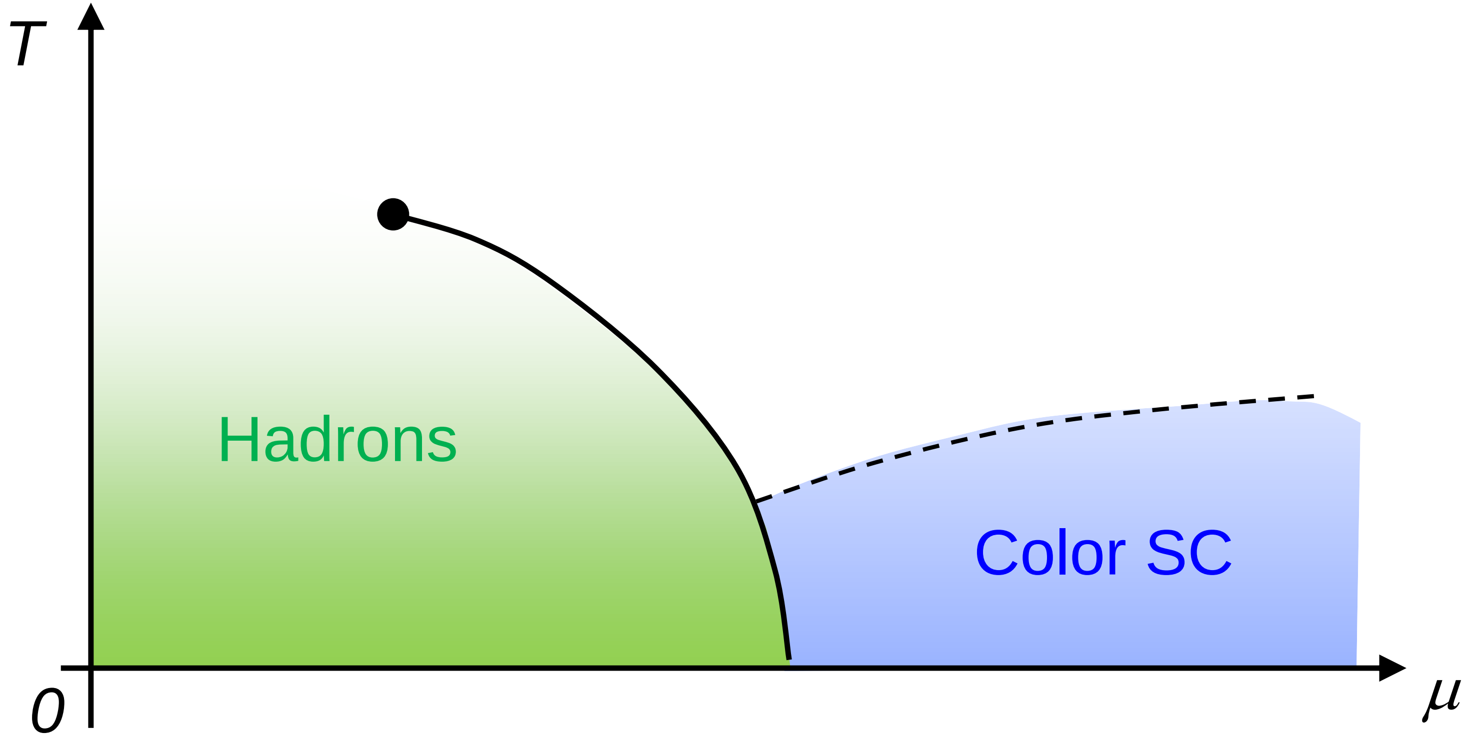
Non-Gaussian Fluctuations in Relativistic Heavy Ion Collisions

Masakiyo Kitazawa
(Osaka U.)

Asakawa, MK, arXiv:1512.05038 [Review]
MK, Asakawa, Ono, Phys. Lett. B728, 386-392 (2014)
Sakaida, Asakawa, MK, PRC90, 064911 (2014)
MK, Nucl. Phys. A942, 65 (2015)

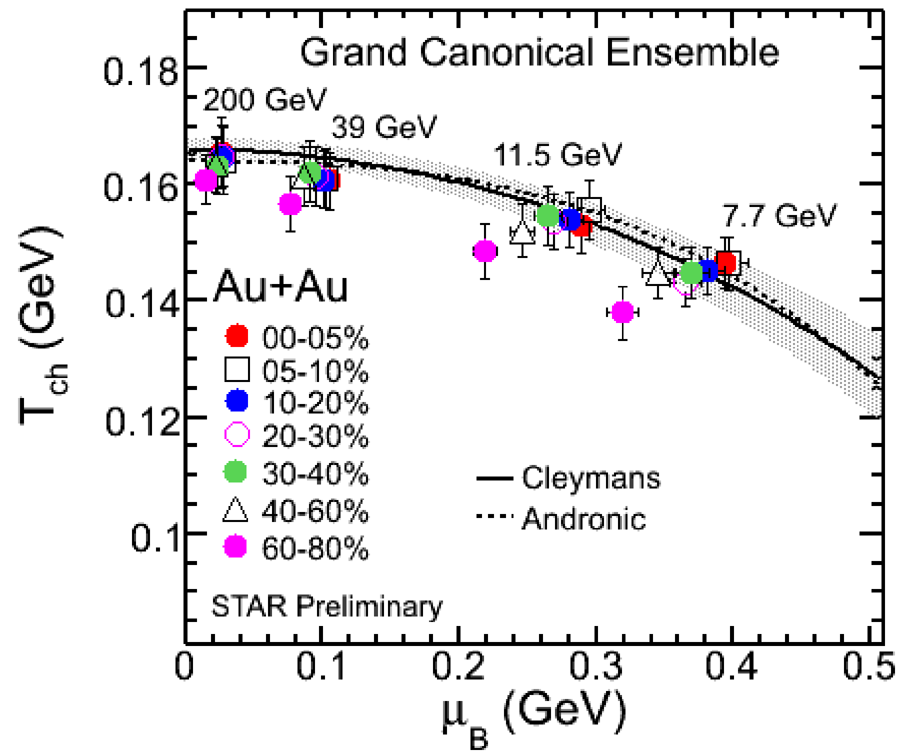
CiRfSE Workshop, Tsukuba U.,19/Jan./2016

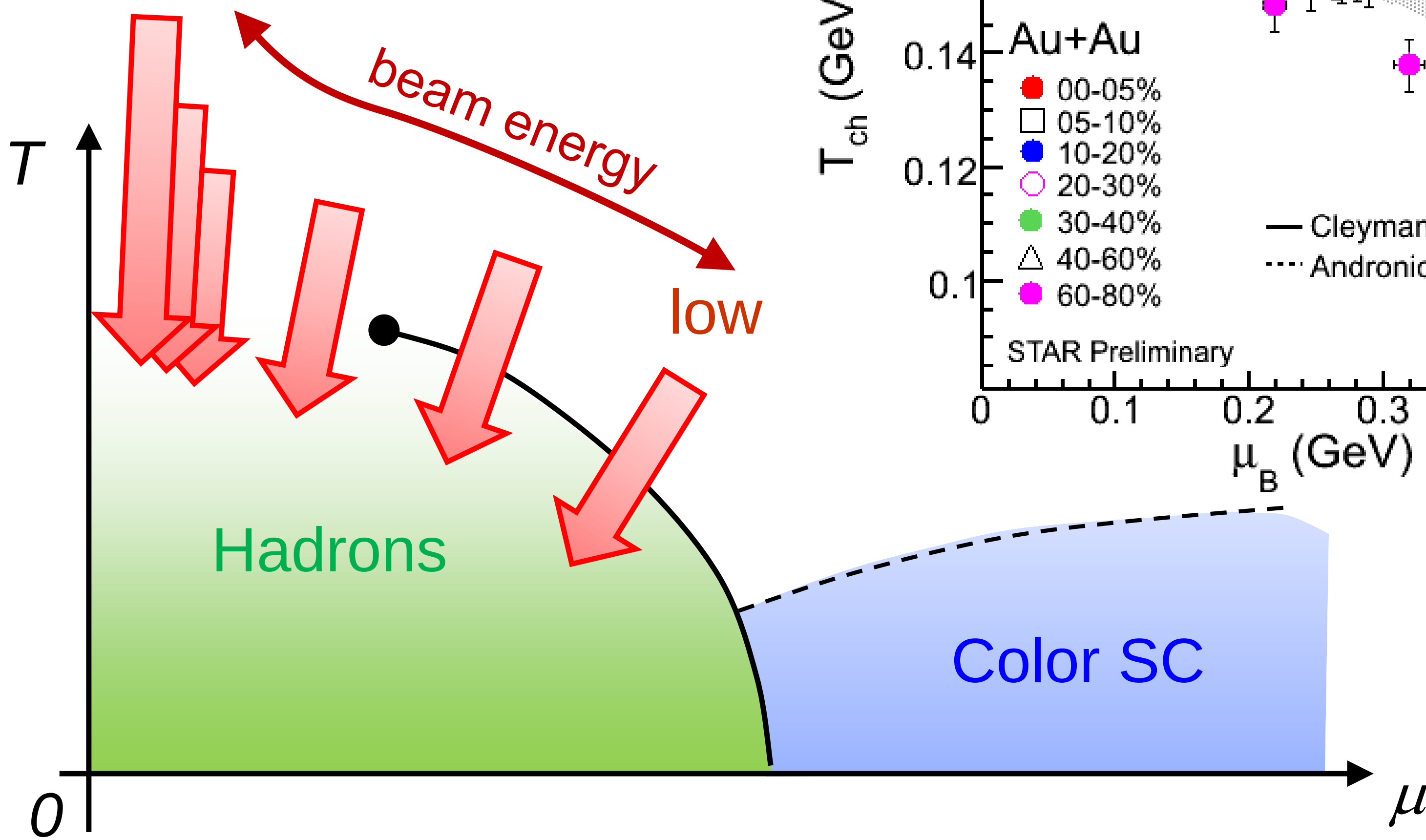
Beam-Energy Scan



Beam-Energy Scan

STAR 2012

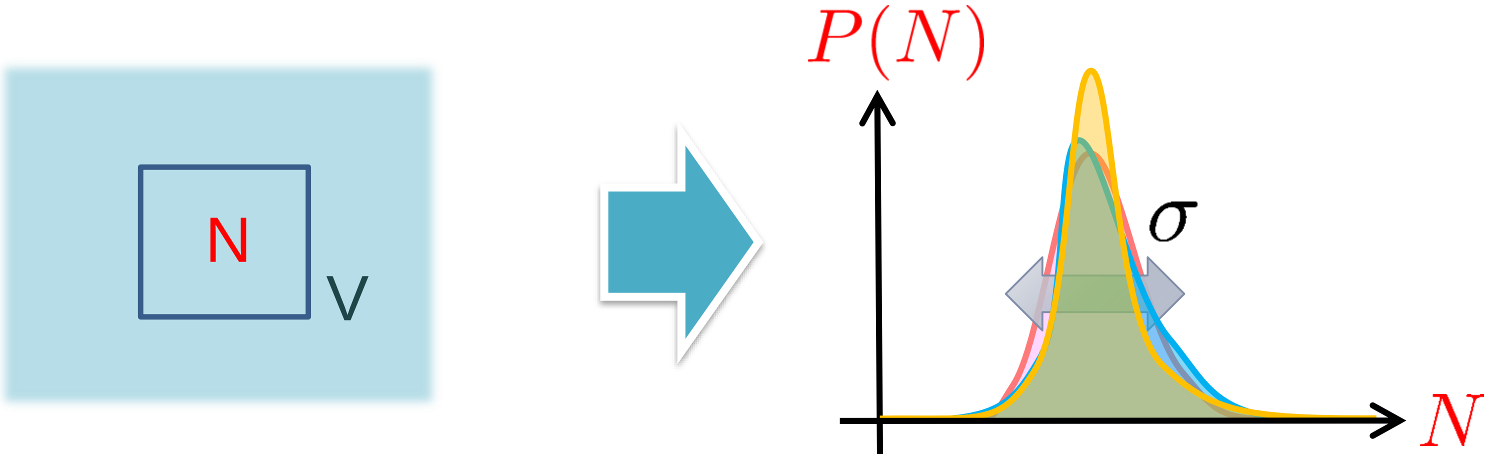




Fluctuations

Fluctuations

Observables in equilibrium are fluctuating.



$$\delta N = N - \langle N \rangle$$

- Variance: $\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \dfrac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

Gaussian

non-Gaussianity

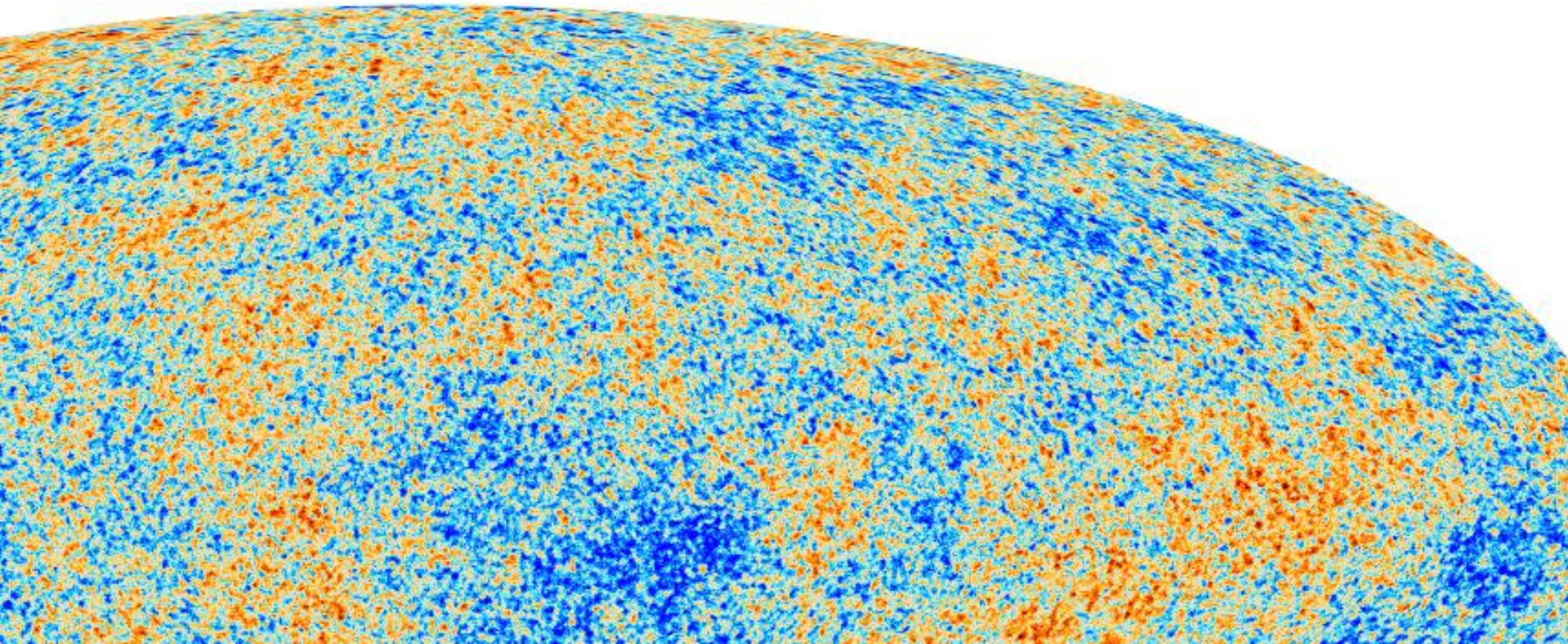
In “haiku”, a Japanese short style poem, a poet wrote…

Even on one blade of grass
the cool wind lives

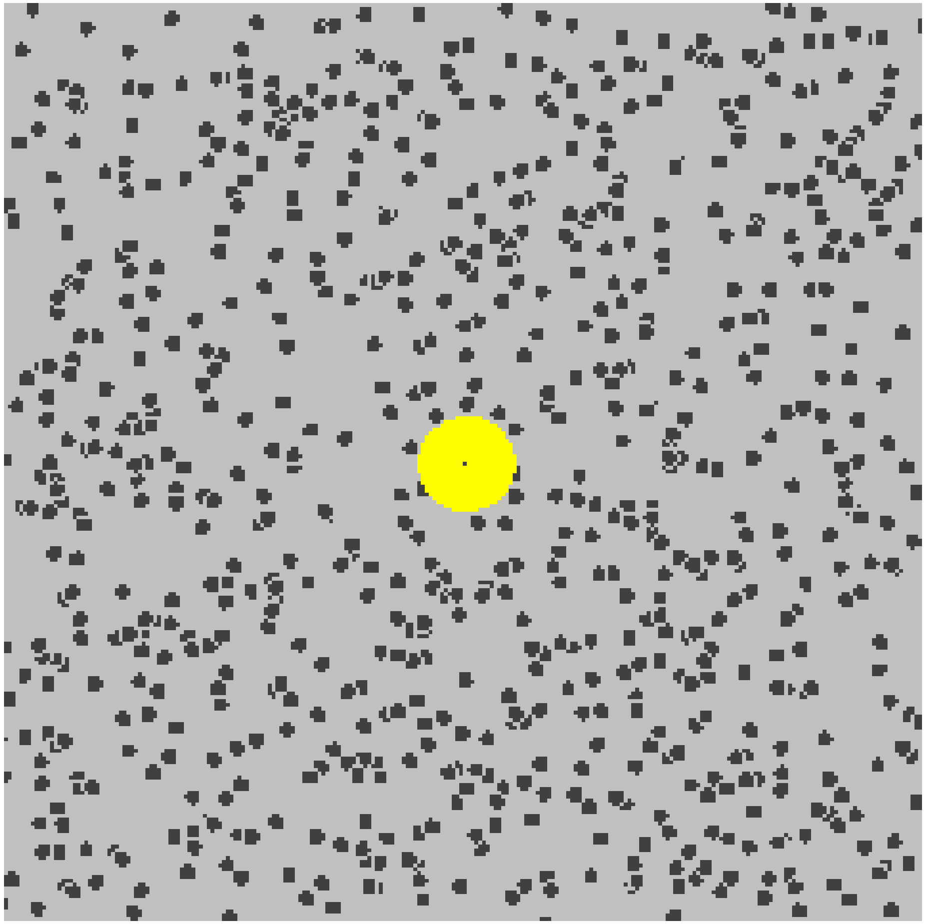
Issa Kobayashi
1814

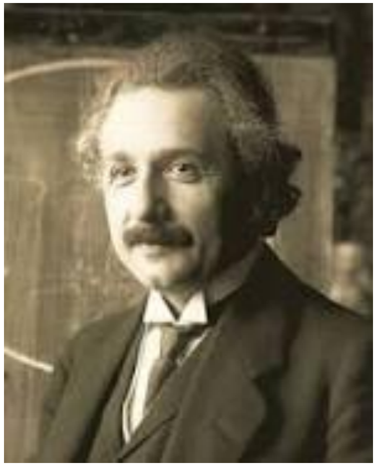
一本の草も涼風宿りけり
小林一茶

Physicists can feel hot early Universe
13 800 000 000 years ago
in tiny fluctuations of
cosmic microwave



Physicists can feel the existence of microscopic atoms behind random fluctuations of Brownian pollens

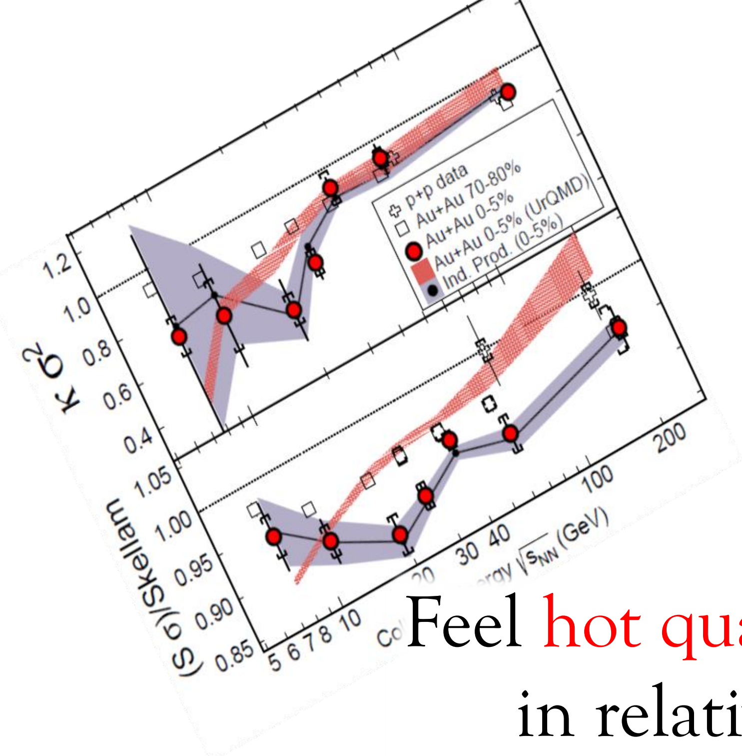


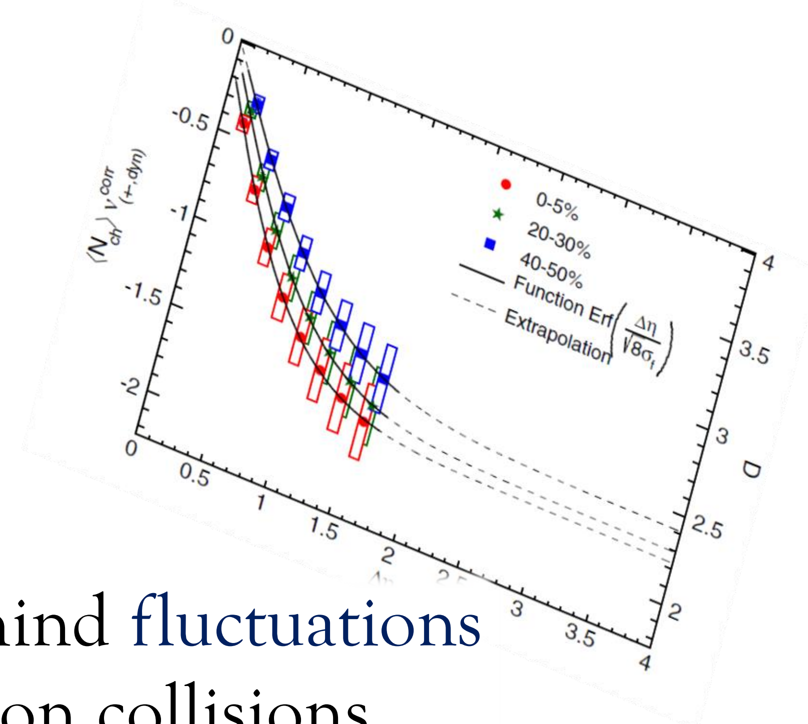


A. Einstein
1905

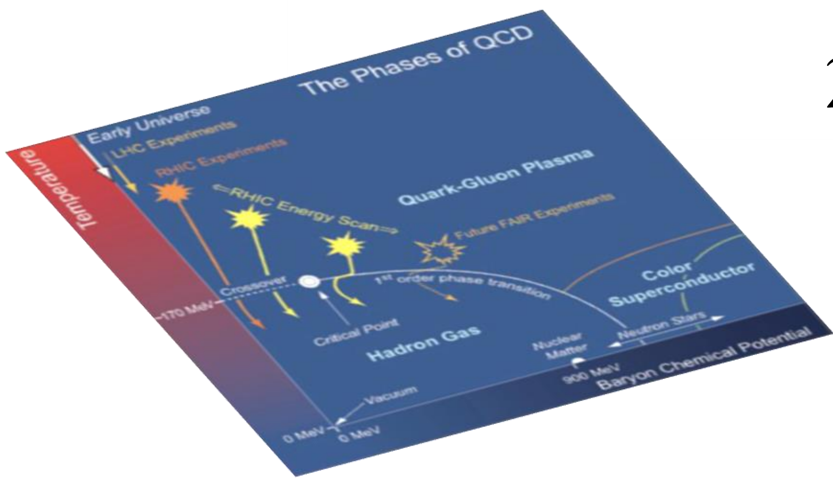
Feel hot quark wind behind fluctuations
in relativistic heavy ion collisions

2010-

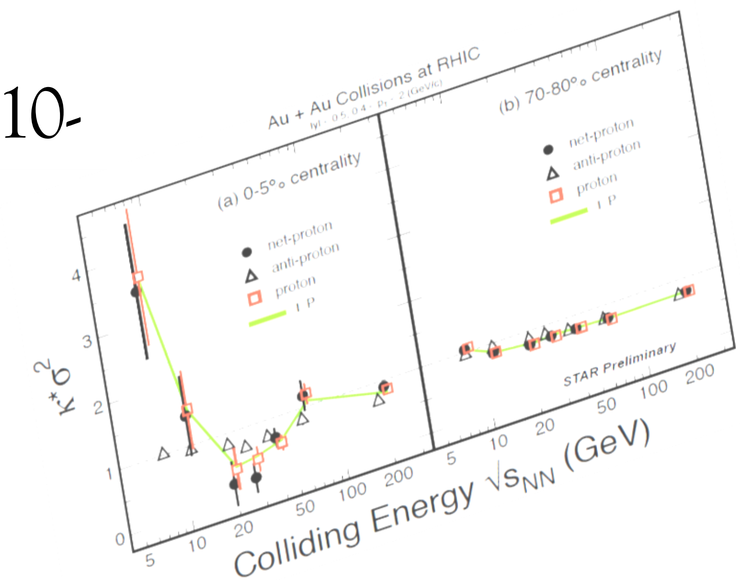




Feel hot quark wind behind fluctuations in relativistic heavy ion collisions



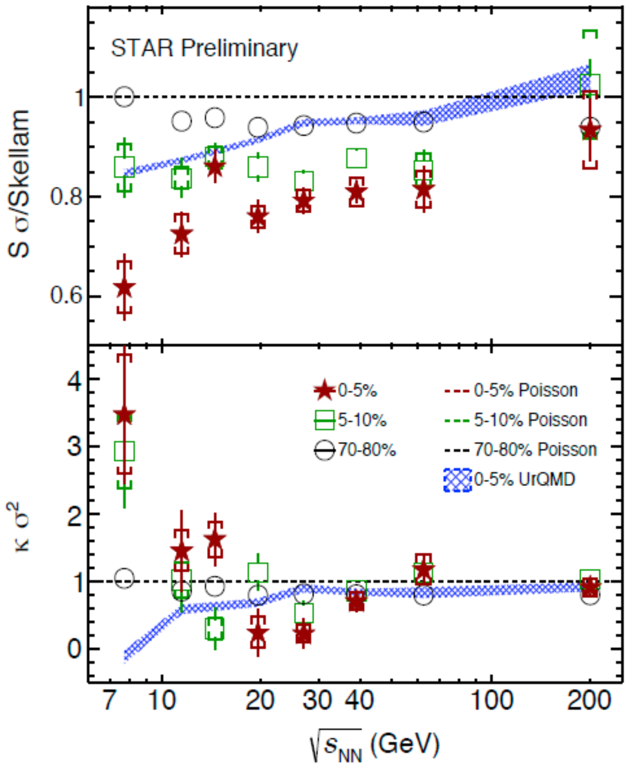
2010-

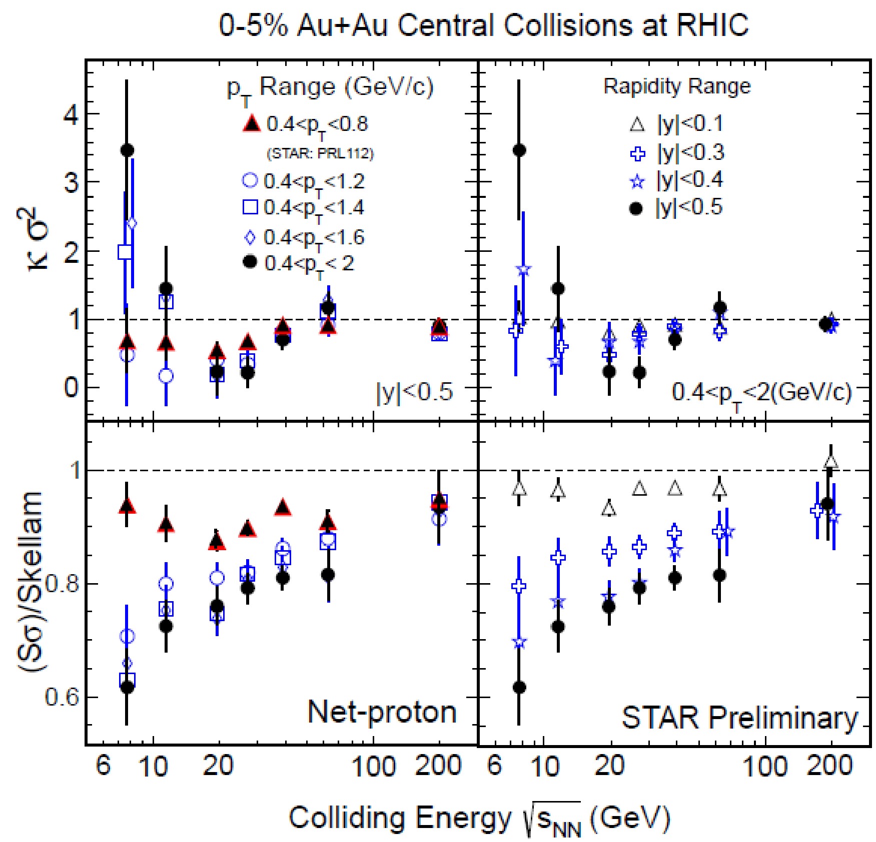


Non-Gaussian Fluctuations in Heavy-Ion Collisions

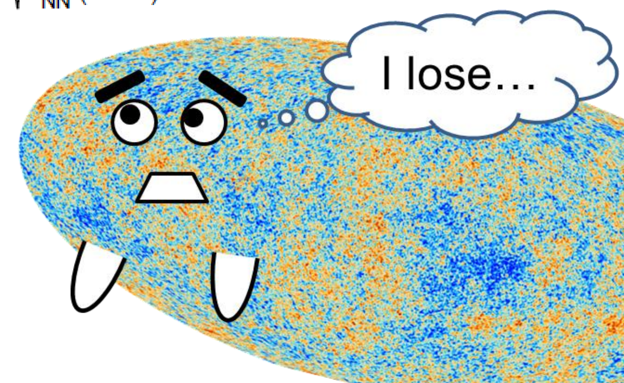
Non-Gaussianity in Exp.

X. Luo+, STAR Collab. 2010~



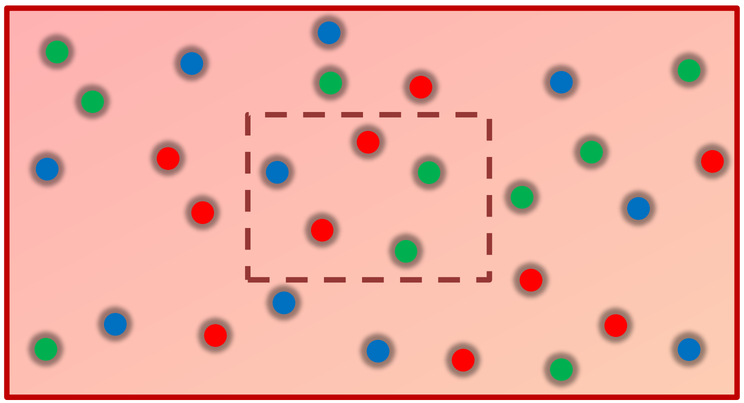


Non-zero **non**-Gaussian cumulants have been established!



Fluctuations and Elemental Charge

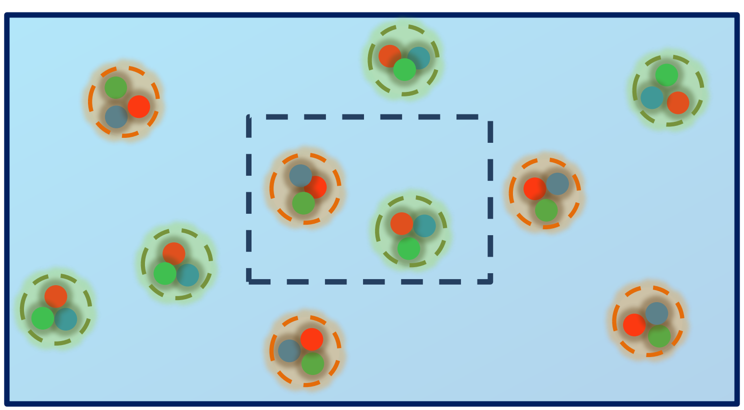
Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000
Ejiri, Karsch, Redlich, 2005



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$



$$\langle \delta N_B^n \rangle_c = \langle N_B \rangle$$

Free Boltzmann → Poisson

$$\langle \delta N^n \rangle_c = \langle N \rangle$$

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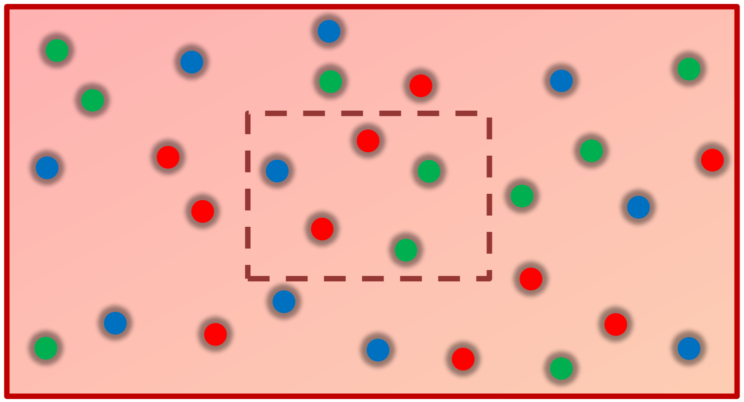

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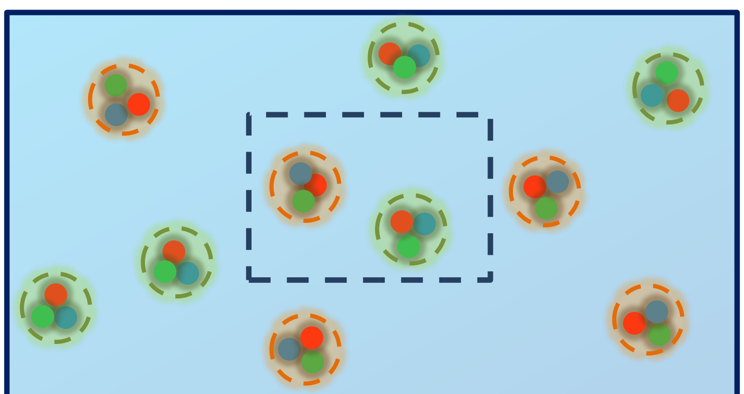

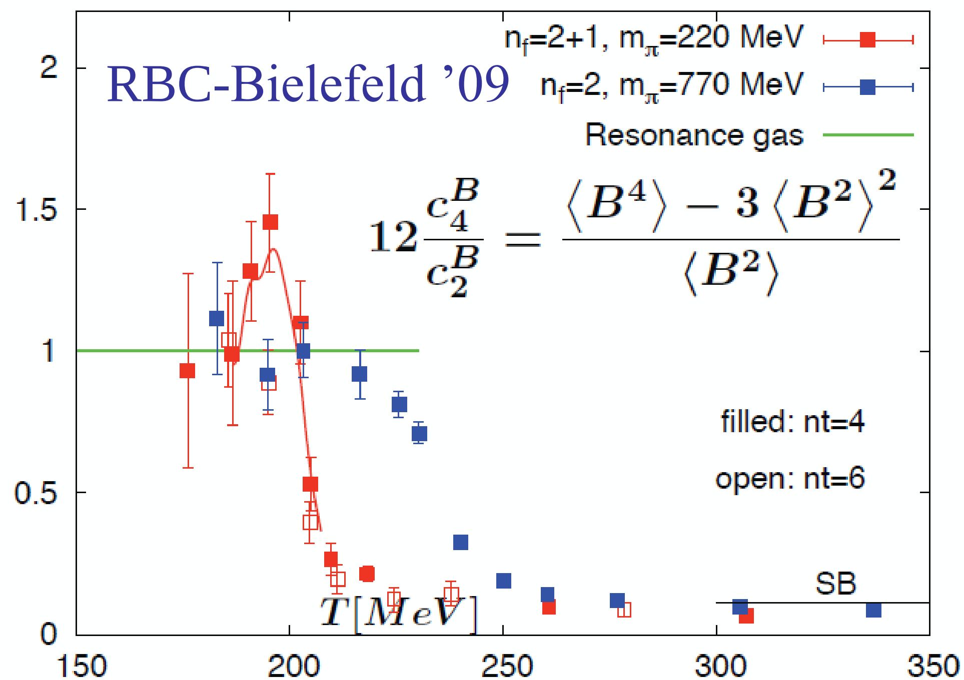

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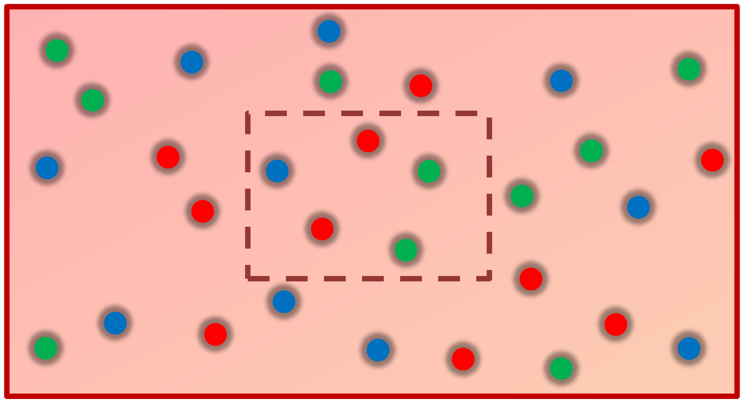

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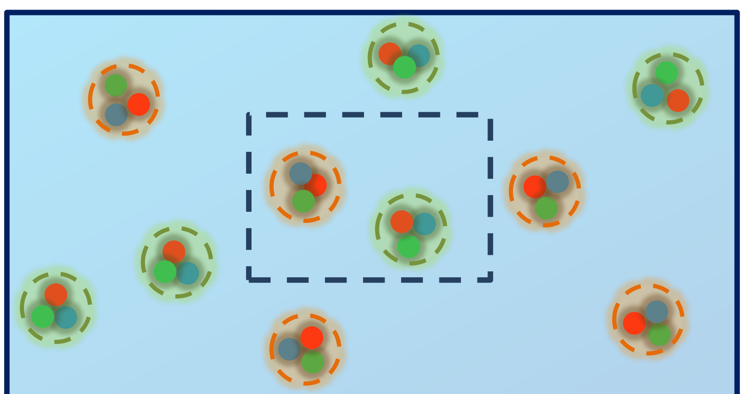
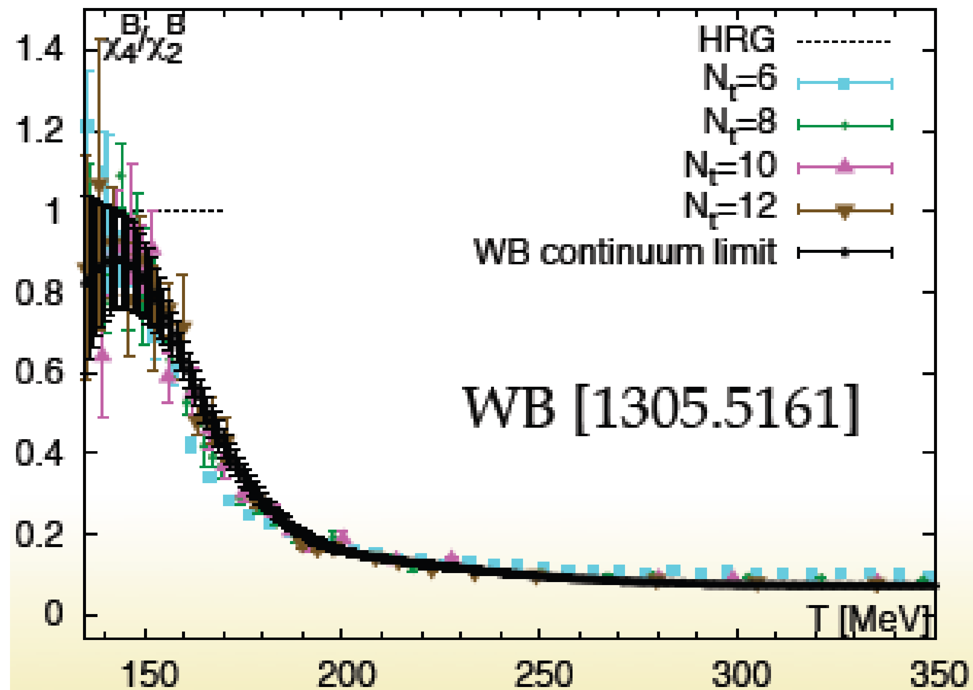
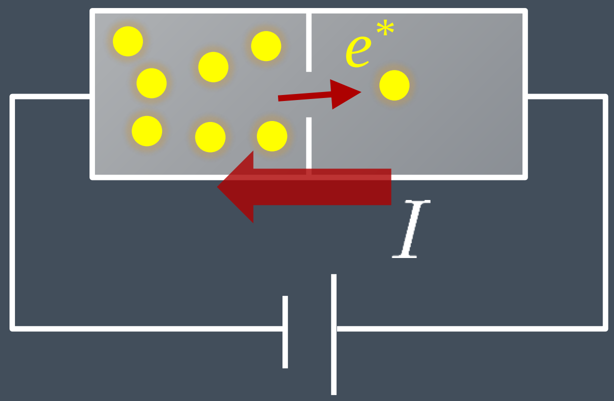




Shot Noise



$$S_{\text{shot}} \sim \langle \delta I^2 \rangle$$

$$S_{\text{shot}} = 2e^*\langle I \rangle$$

charge of quasi-particles

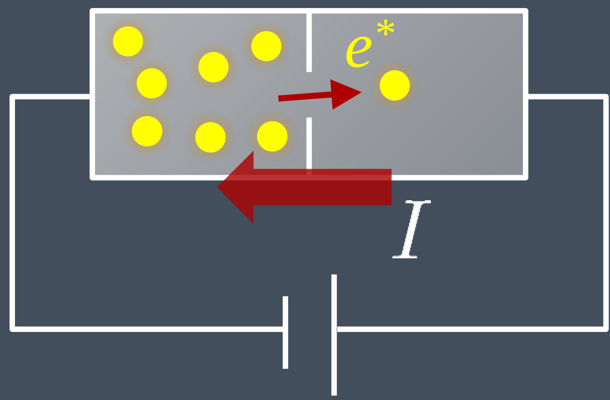
Total charge Q:

$$Q = e\langle N \rangle$$

$$\langle \delta Q^2 \rangle = e^2 \langle \delta N^2 \rangle = e^2 \langle N \rangle = eQ$$

$$\frac{\langle \delta Q^2 \rangle}{Q} = e$$

Shot Noise



$$S_{\rm shot} \sim \langle \delta I^2 \rangle$$

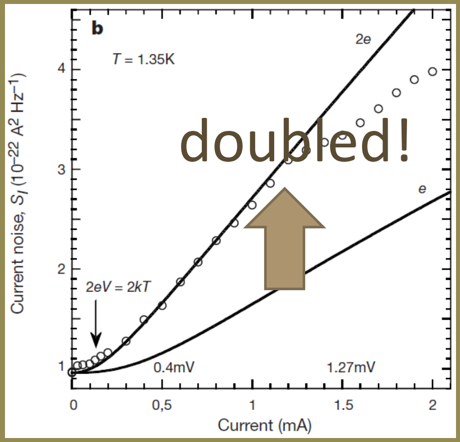
$$S_{\rm shot} = 2e^* \langle I \rangle$$

charge of quasi-particles

Superconductors with Cooper Pairs

$$e^* = 2e$$

Jehl+, Nature **405**,50 (2000)



Fractional Quantum Hall Systems

$$e^* = \frac{q}{p} e$$

Saminadayar+, PRL**79**,2526 (1997)

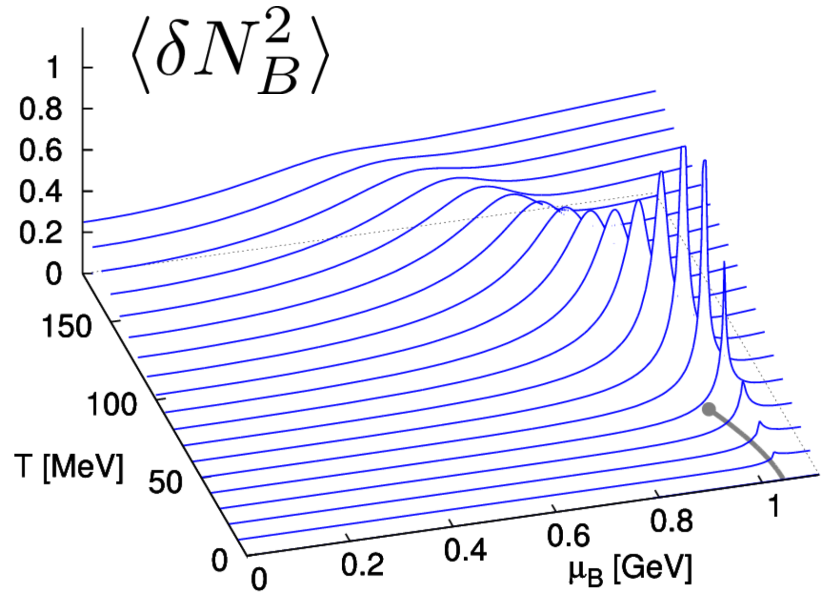
Higher order cumulants:

3rd order: ex. Beenakker+, PRL90,176802(2003)

up to 5th order: Gustavsson+, Surf.Sci.Rep.**64**,191(2009)

Fluctuation and QCD Critical Point

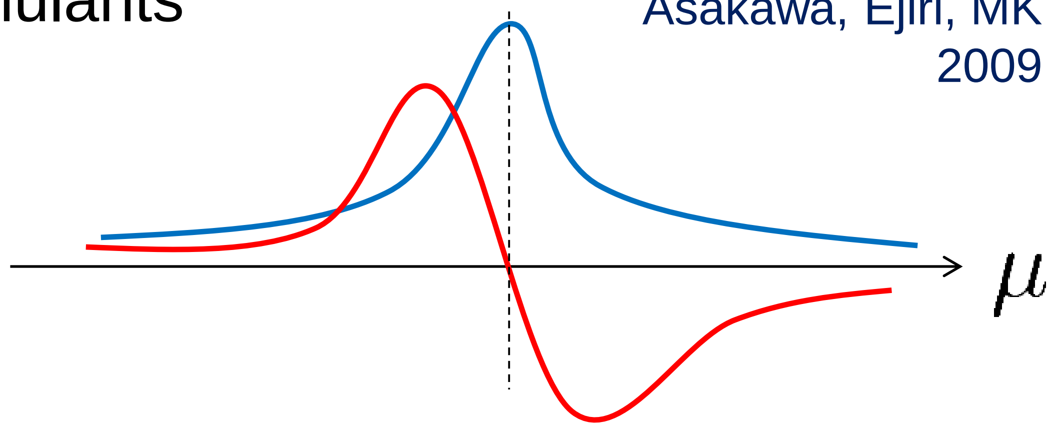
Fluctuations diverge at the QCD critical point



Asakawa, Ejiri, MK 2009

- Geometric interpretation to signs of higher order cumulants

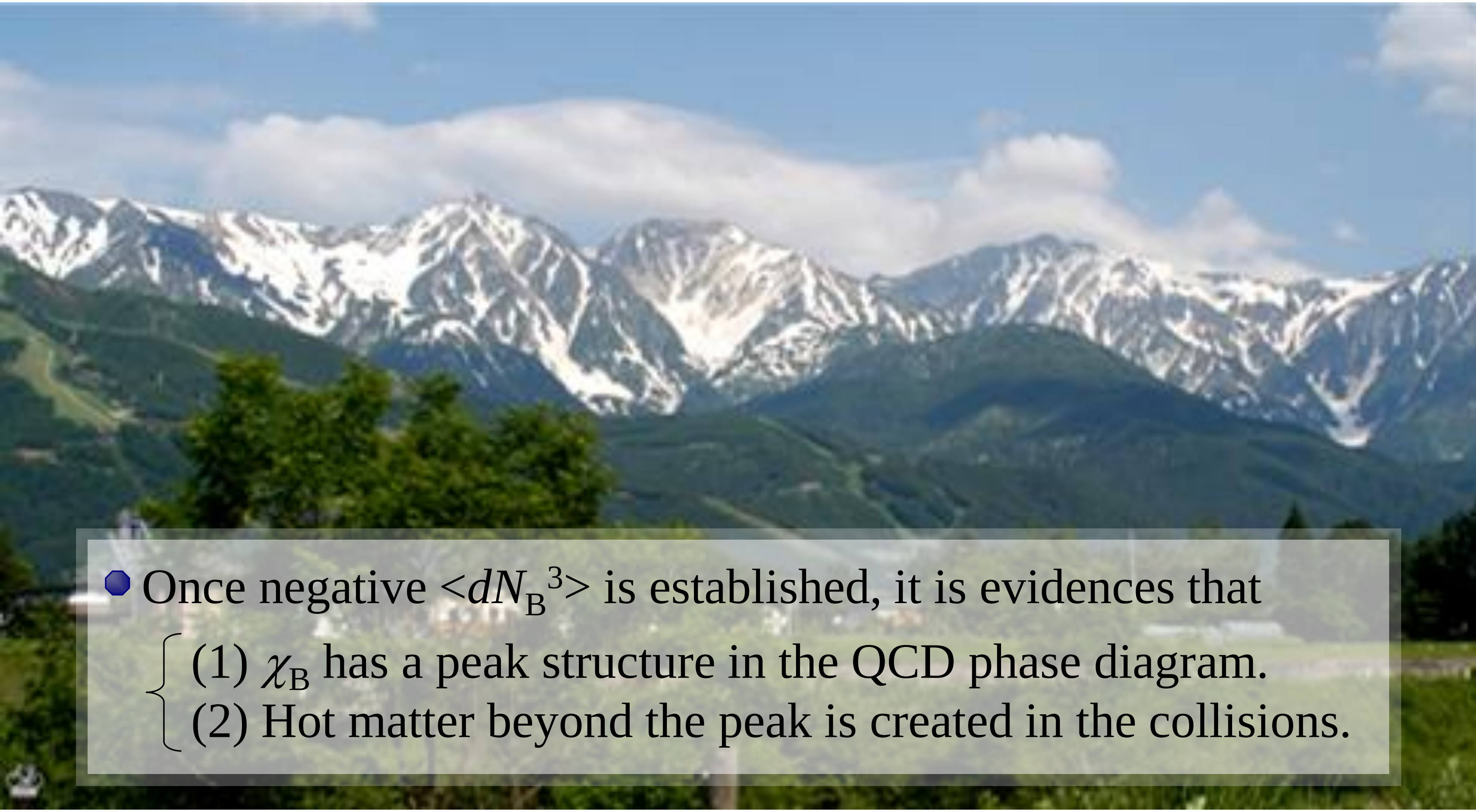
$$\langle \delta N^3 \rangle = T \frac{\partial \langle \delta N^2 \rangle}{\partial \mu}$$



- More severe divergence for higher-order cumulants

Stephanov, 2009

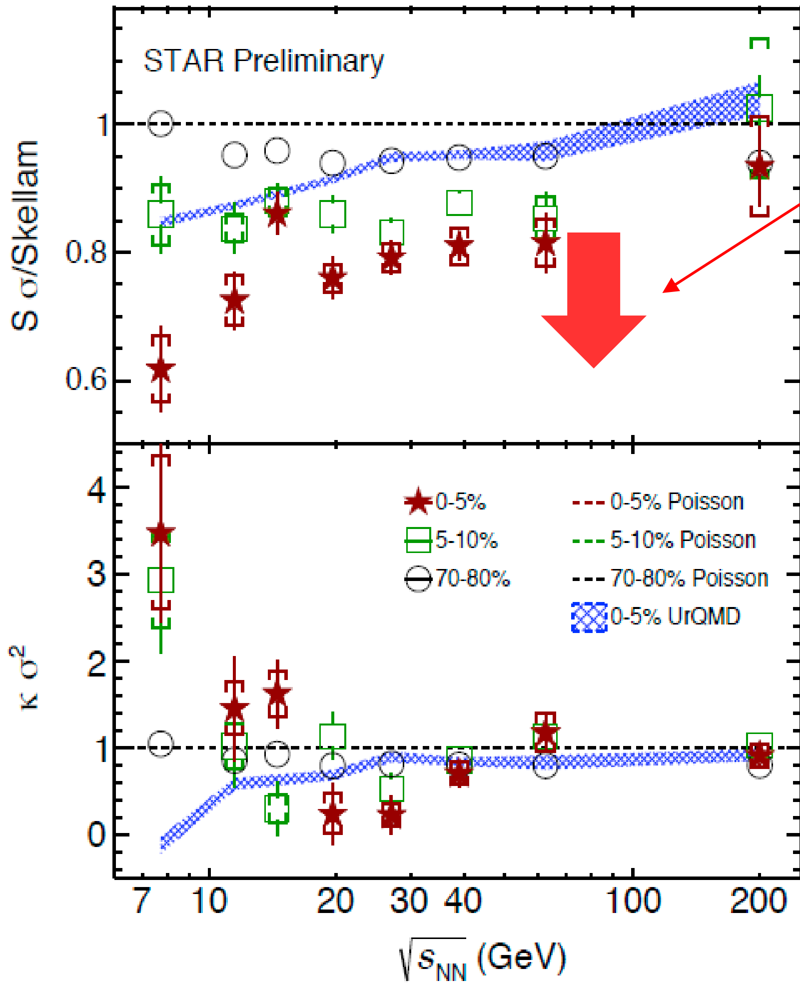
Impact of Negative Third Moments



- Once negative $<dN_B^3>$ is established, it is evidences that
  - (1) χ_B has a peak structure in the QCD phase diagram.
  - (2) Hot matter beyond the peak is created in the collisions.

-
  - **No** dependence on any specific models.
  - Just the sign! **No** normalization (such as by N_{ch}).

X. Luo, STAR, QM2015



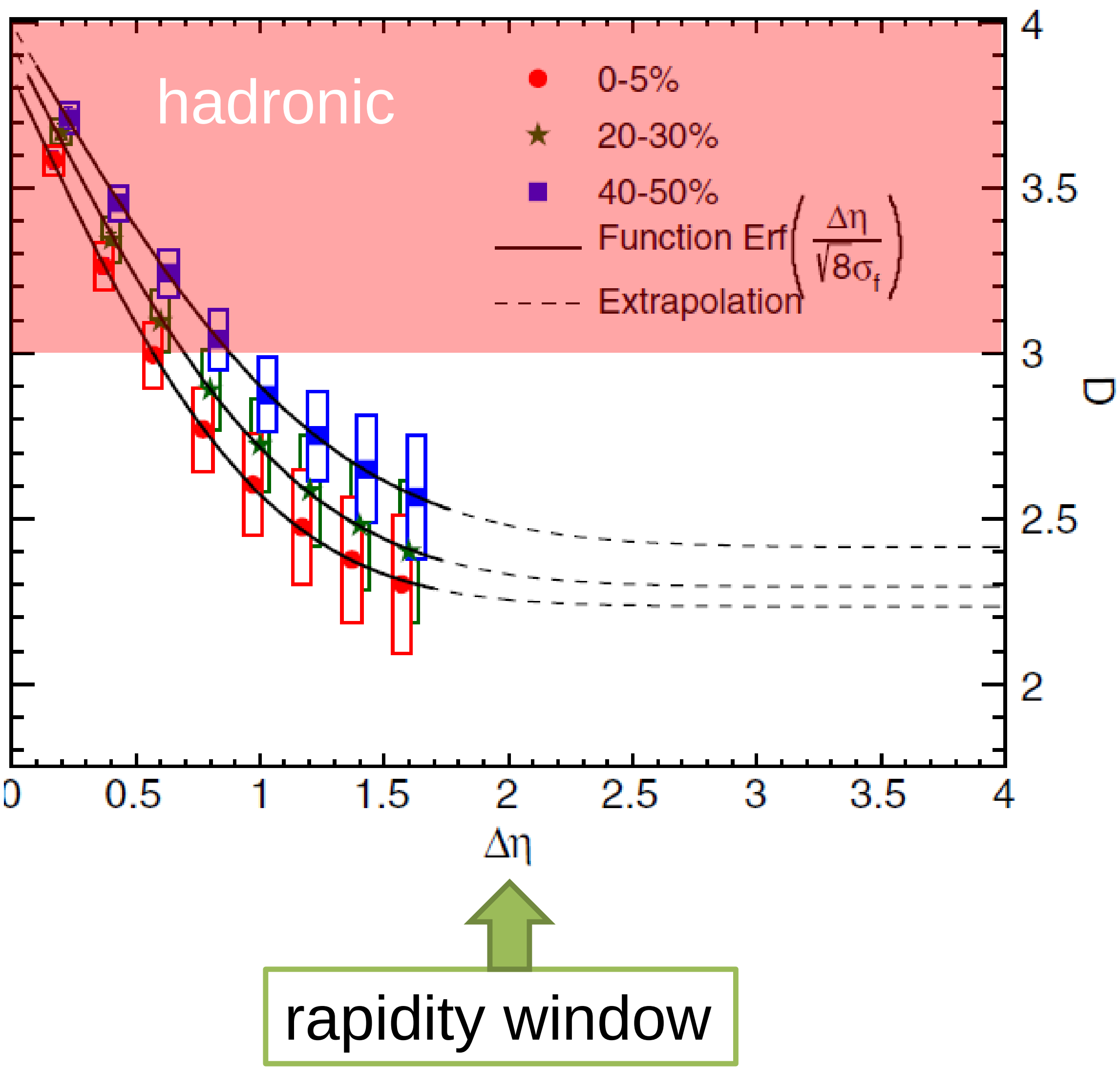
Clear suppression!

ex. Asakawa, Ejiri, MK, 2009

Rapidity Window Dependences of Gaussian Fluctuations

Δη Dependence @ ALICE

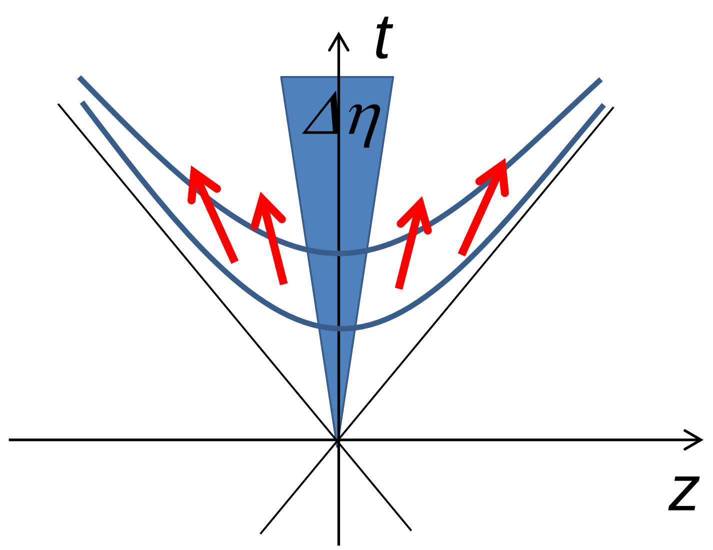
ALICE
PRL 2013



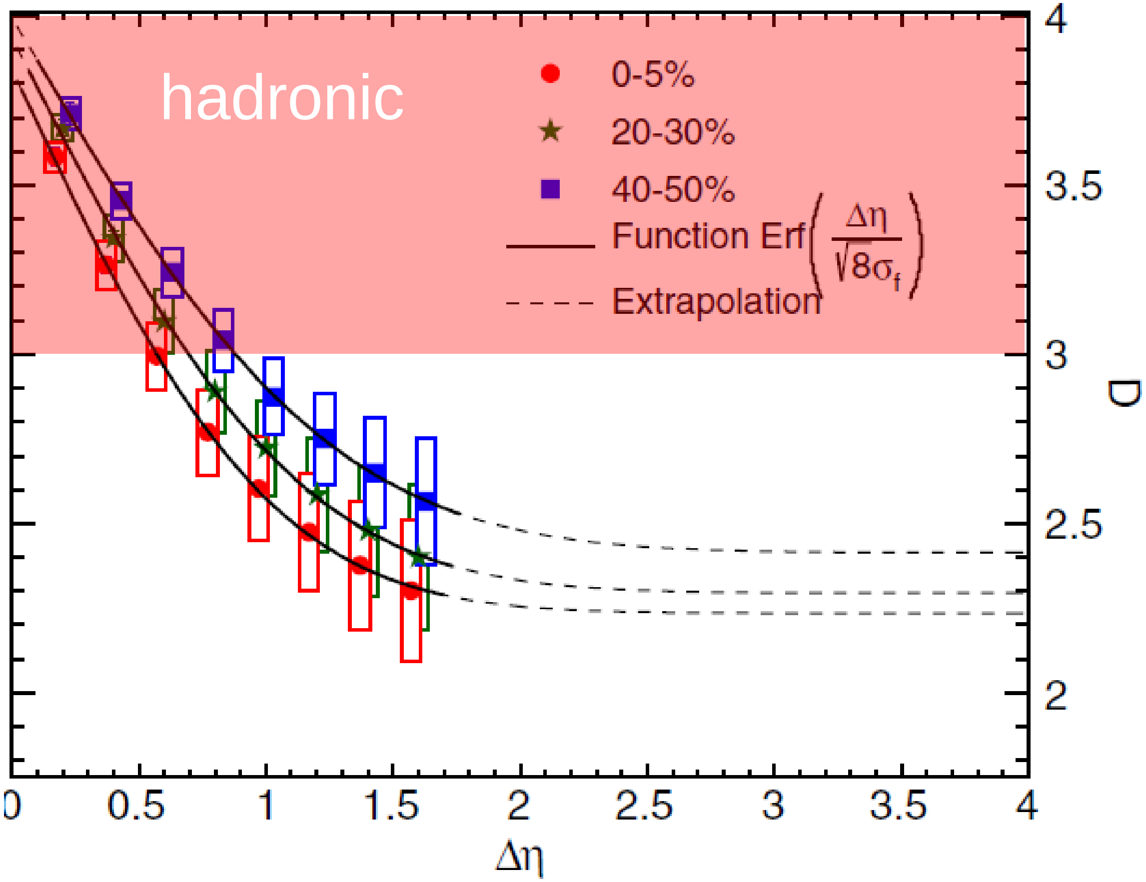
D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- D ~ 3-4 Hadronic
- D ~ 1-1.5 Quark



Δη Dependence @ ALICE



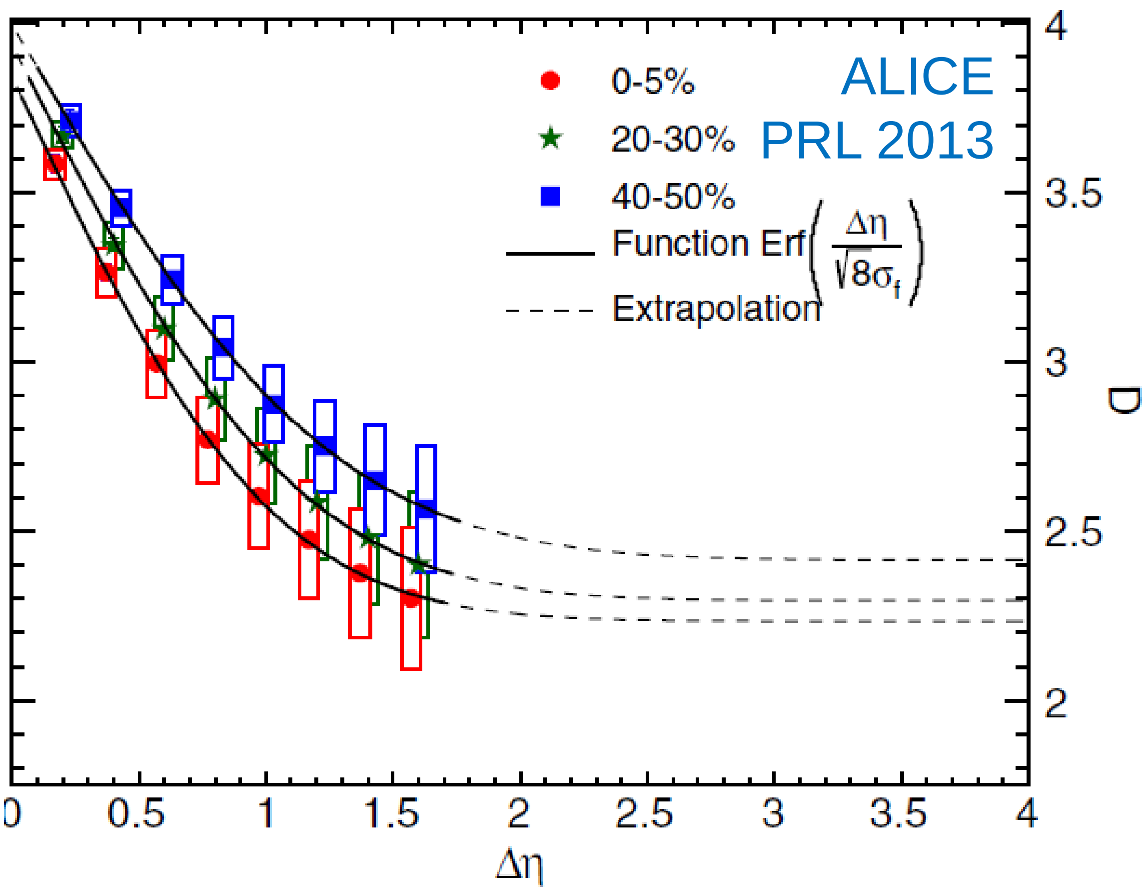
rapidity window

$$D \sim \frac{\langle \delta N_Q \rangle^2}{\Delta \eta}$$

has to be a constant in equil. medium

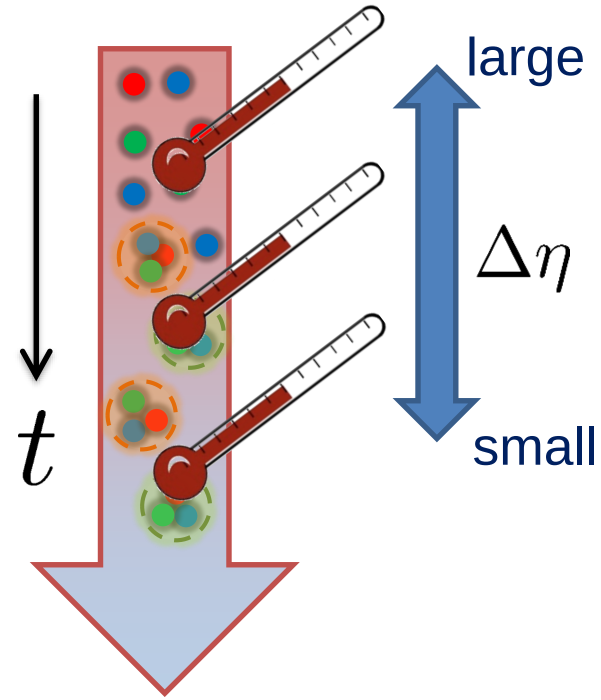
Fluctuation of N_Q at ALICE is not the equilibrated one.

Δη Dependence @ ALICE



rapidity window

Δη dependent thermometer?



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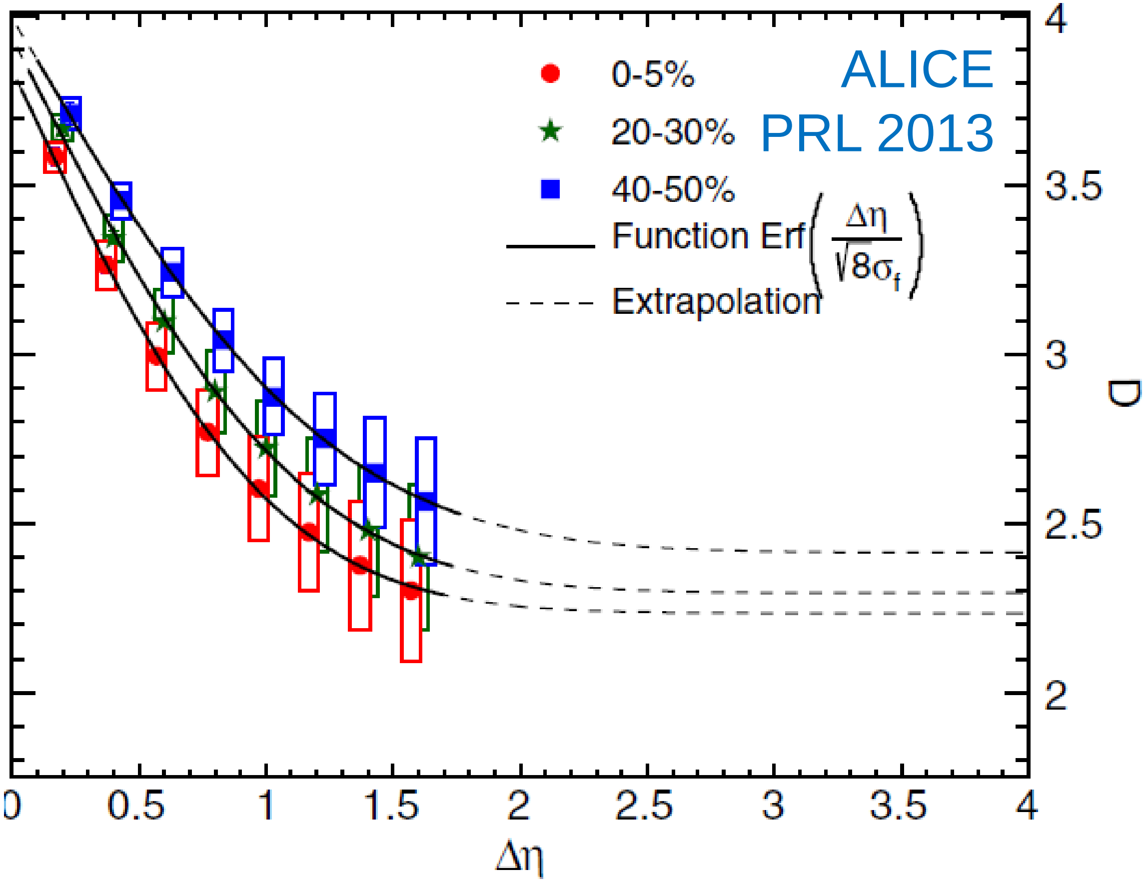
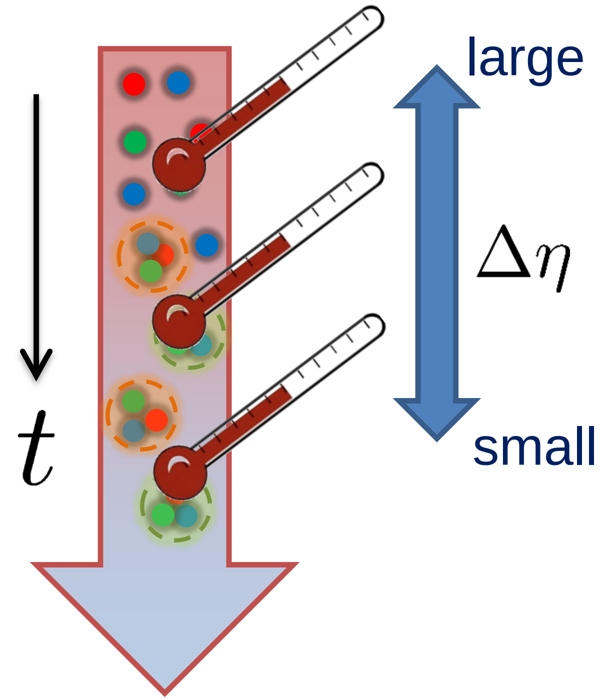
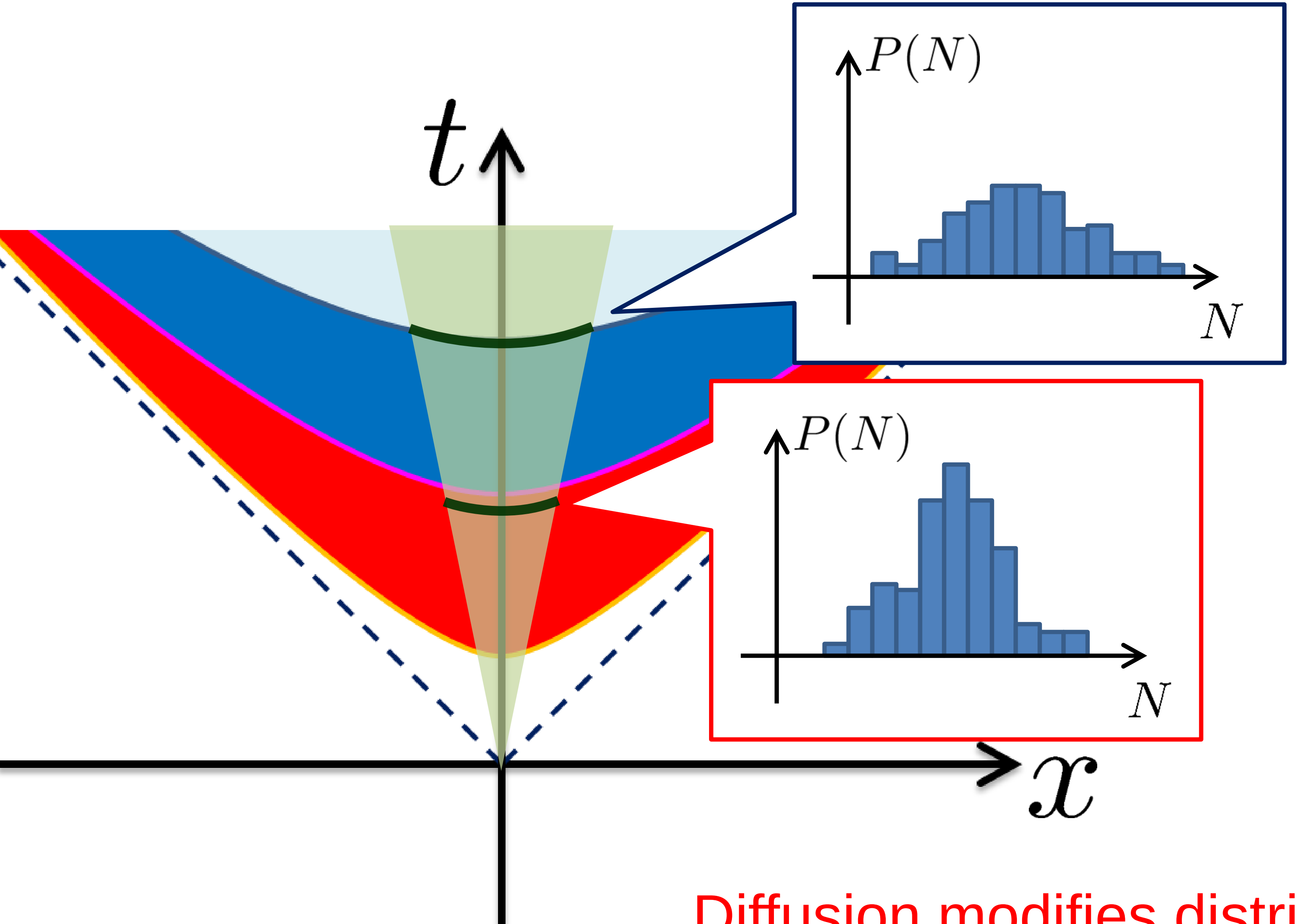


Δη dependent thermometer?



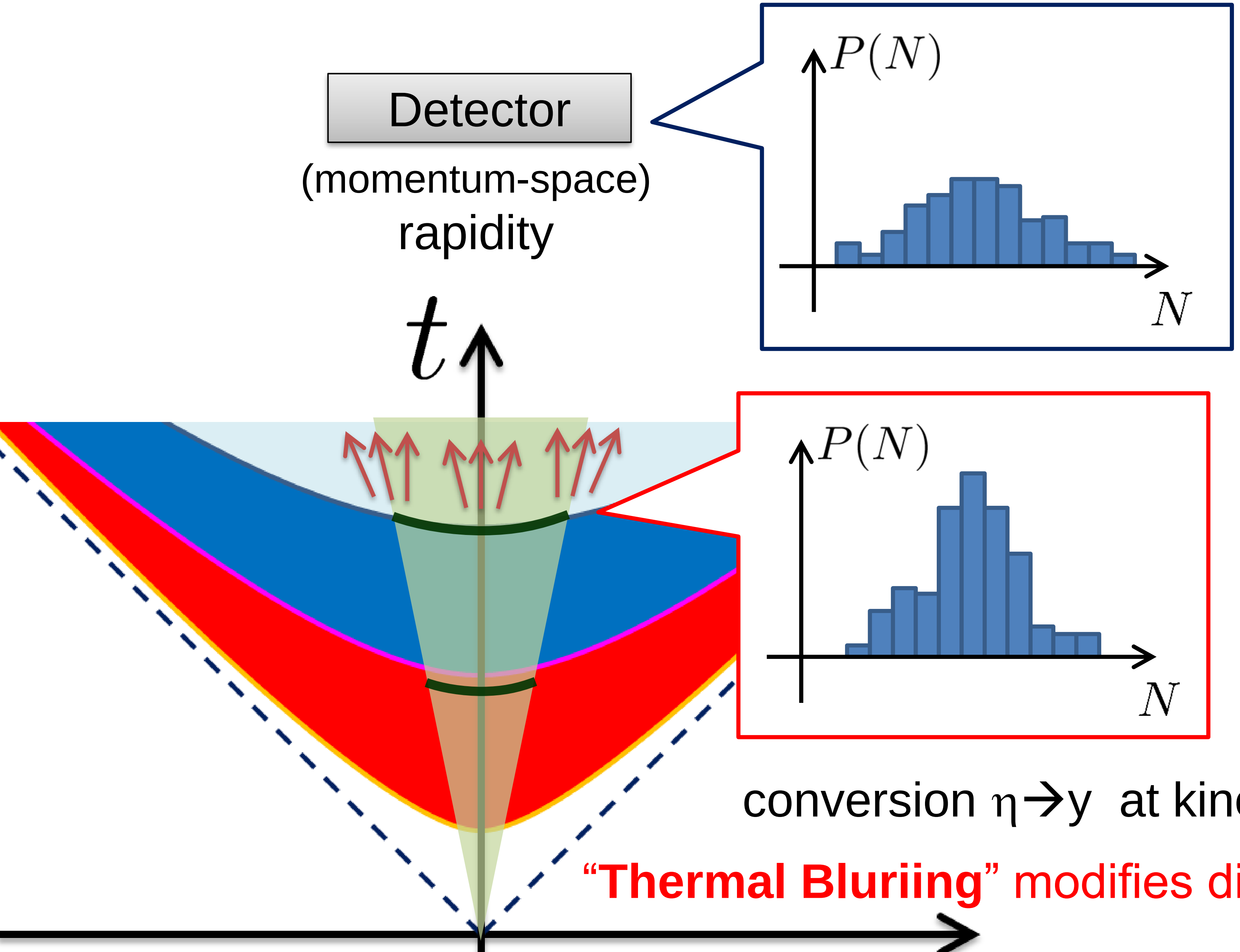
Δη dependences of fluctuation observables encode history of the hot medium!

Time Evolution of Fluctuations



Diffusion modifies distribution

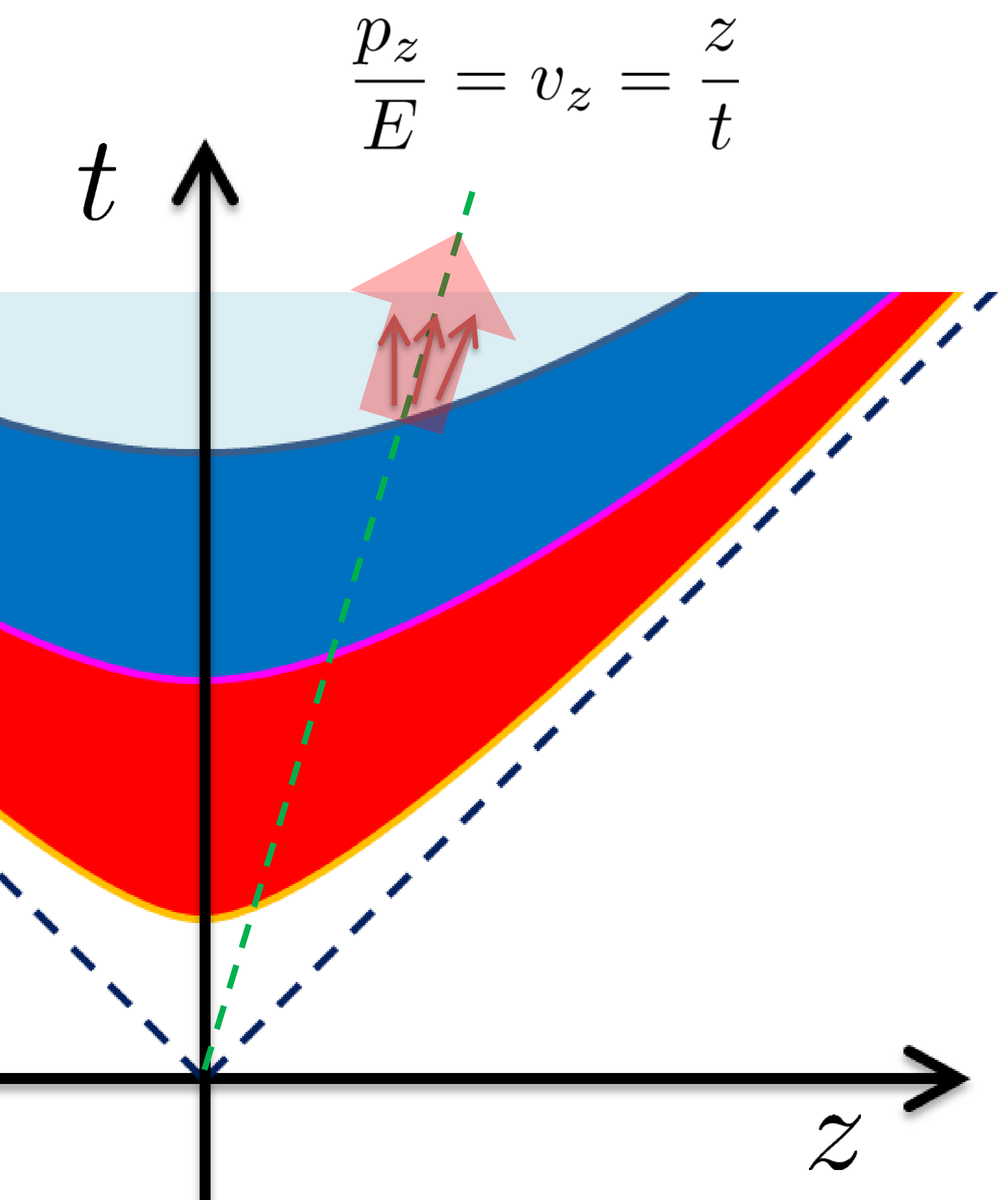
Time Evolution of Fluctuations



conversion η→y at kinetic f.o.

"**Thermal Bluriing**" modifies distribution

Thermal Blurring



Under Bjorken picture,

coordinate-space rapidity of medium

||

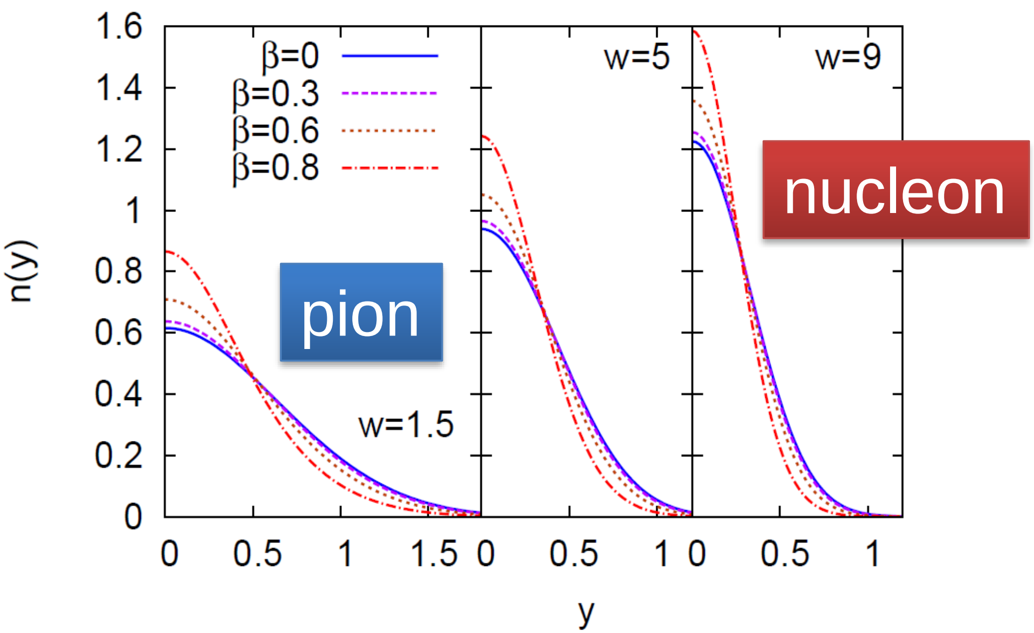
momentum-space rapidity of medium

$\nparallel$

momentum-space rapidity of individual particles

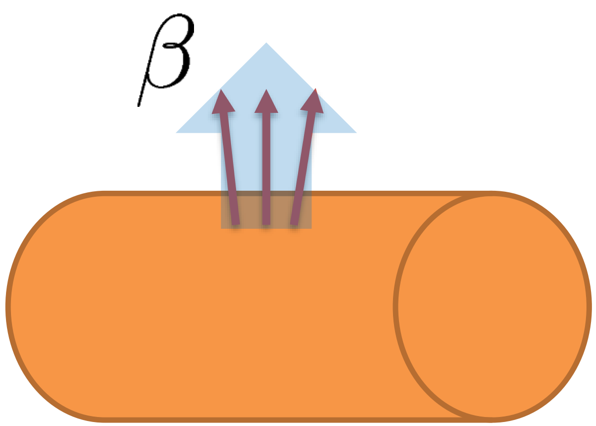
Thermal distribution in η space

Y. Ohnishi+
in preparation

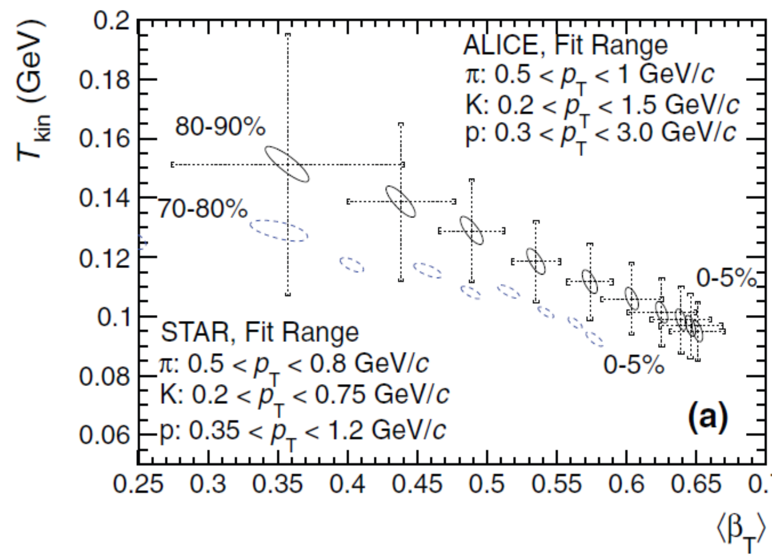


$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$



Blast wave squeezes the distribution in rapidity space



- blast wave
- flat freezeout surface

Thermal distribution in η space

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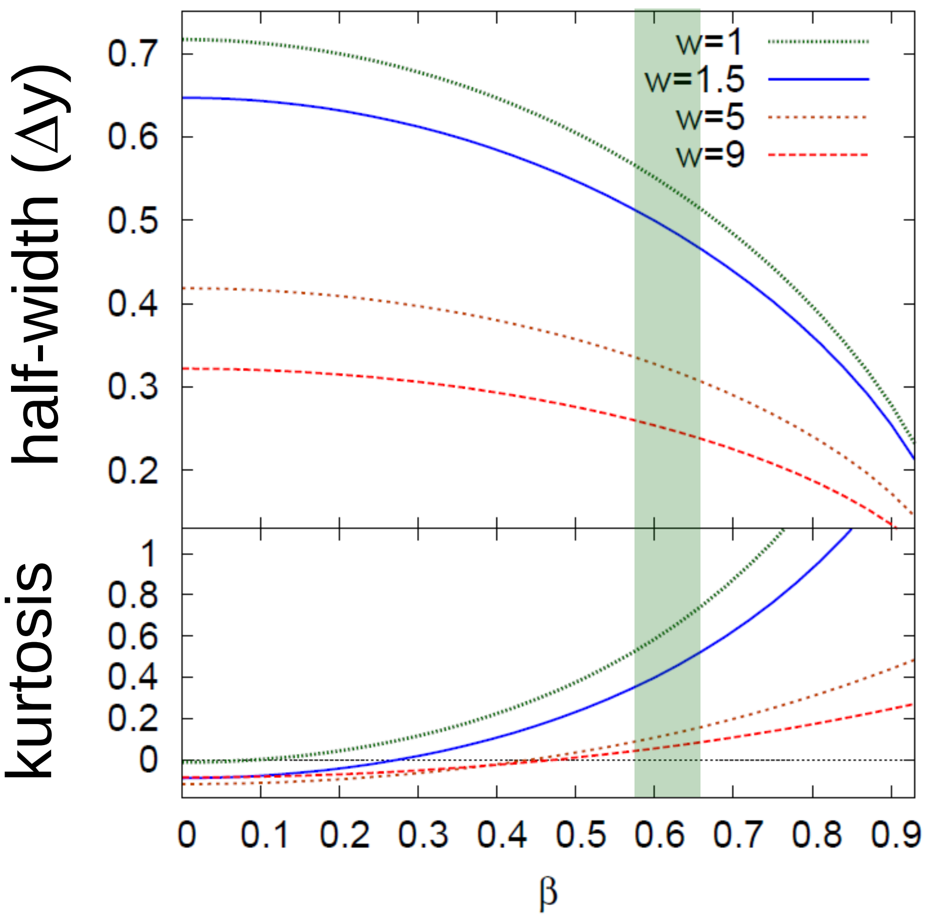

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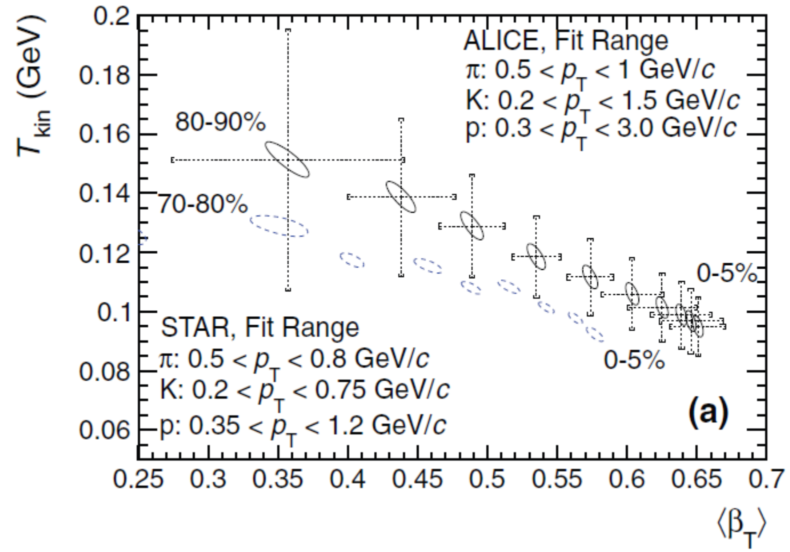
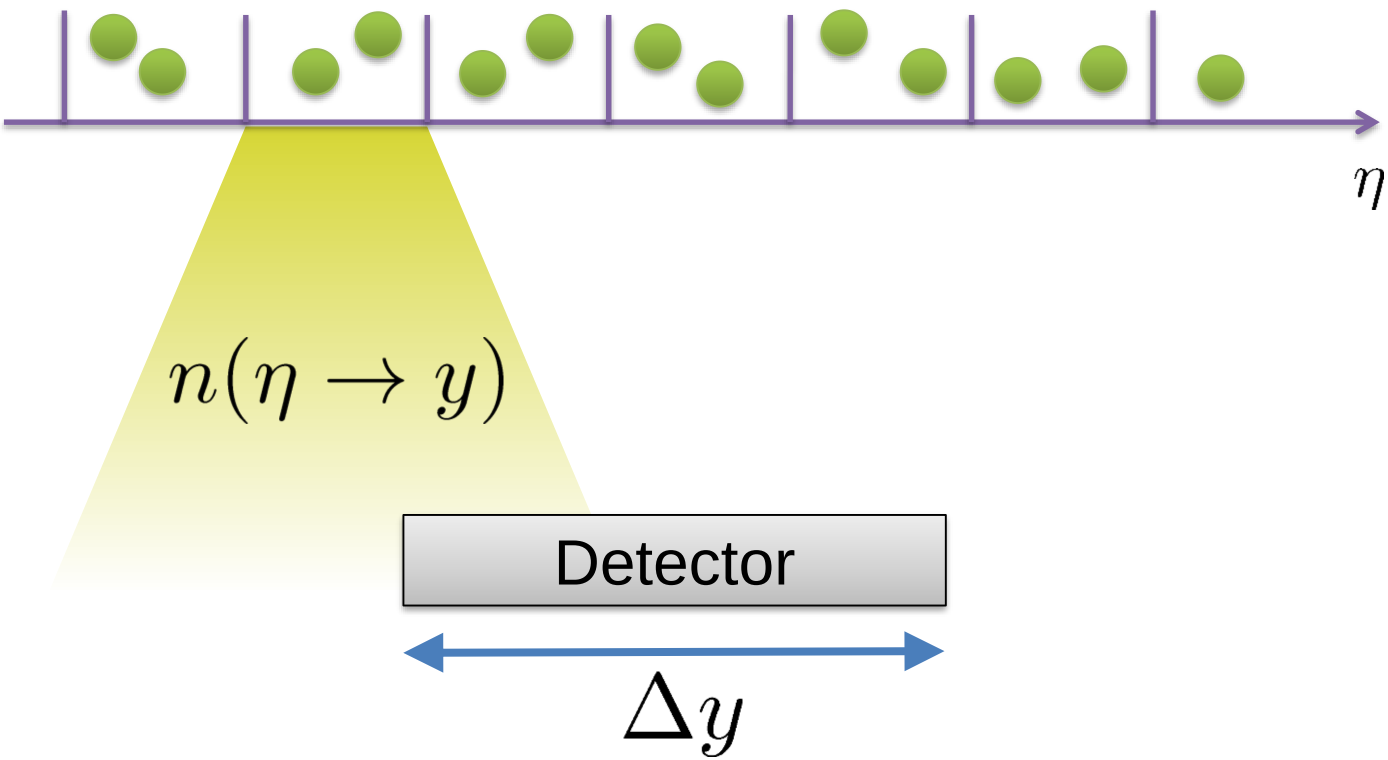


Rapidity distribution is not far away from Gaussian.

- blast wave
- flat freezeout surface

Formalism



- Particles arrive at the detector with some probability.
- Sum all of them up. Make the distribution.
- Take the continuum limit.

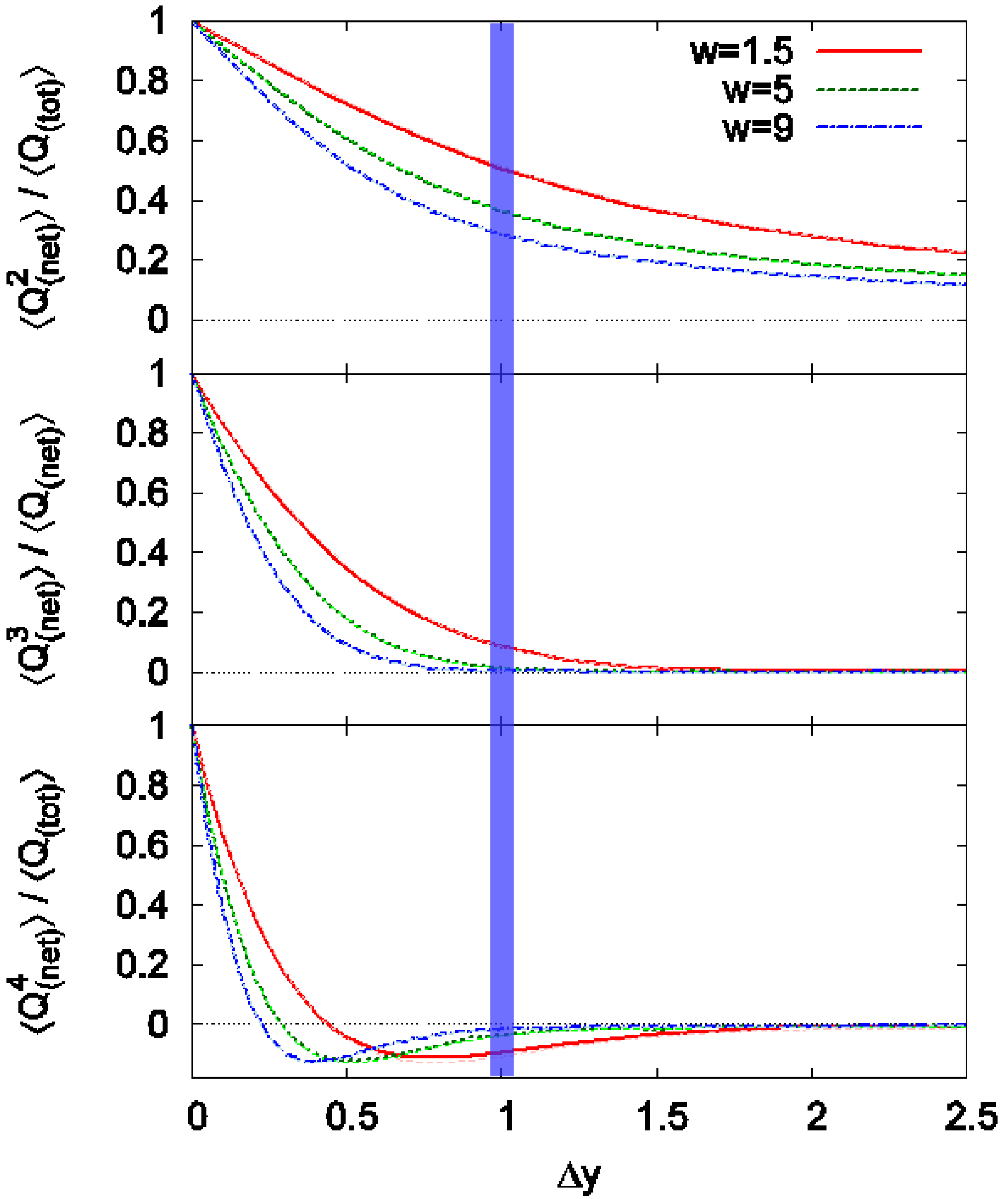
Δη Dependence

With **vanishing fluctuations** before thermal blurring

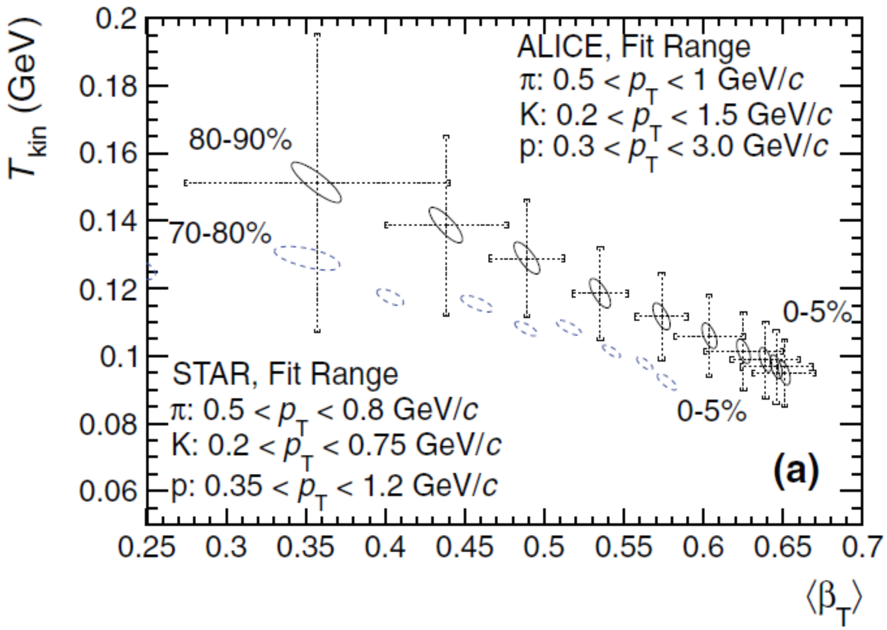
Cumulants after blurring take nonzero values

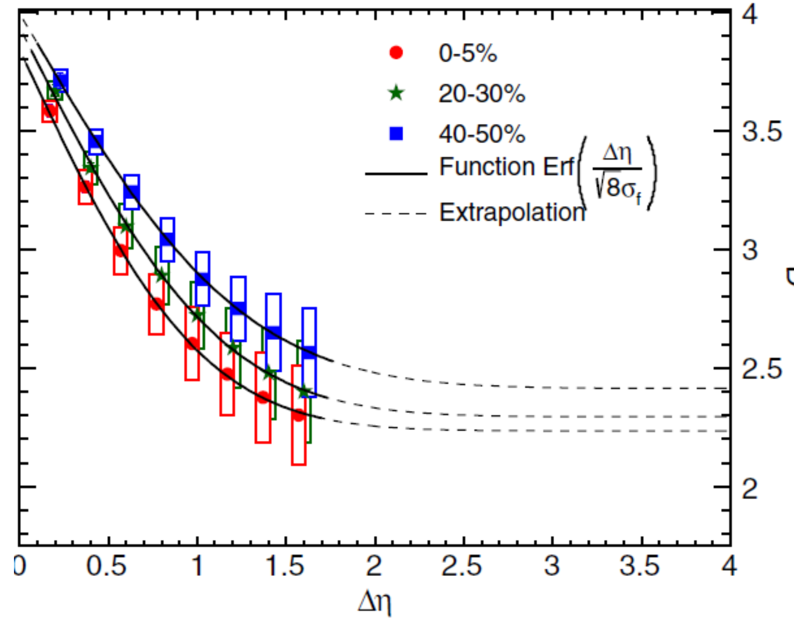
With Δy=1, the effect is **not** well suppressed

Cumulants after blurring



Centrality Dependence



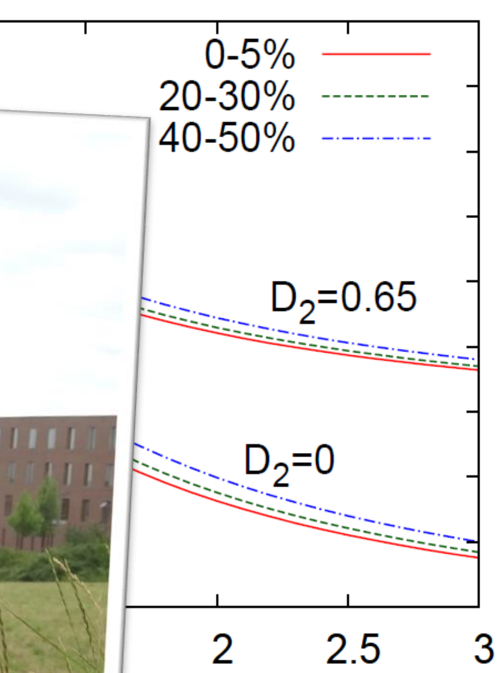


More central ⇒ { lower T, larger β } ⇒ Weaker blurring

Is the centrality dependence understood solely by the thermal blurring at kinetic f.o.?

Centrality Dependence

$$D_2 = \frac{\langle \delta N_{\rm Q}^2 \rangle}{\langle \delta N_{\rm Q}^2 \rangle_{\rm eq.}}$$



Assumptions:

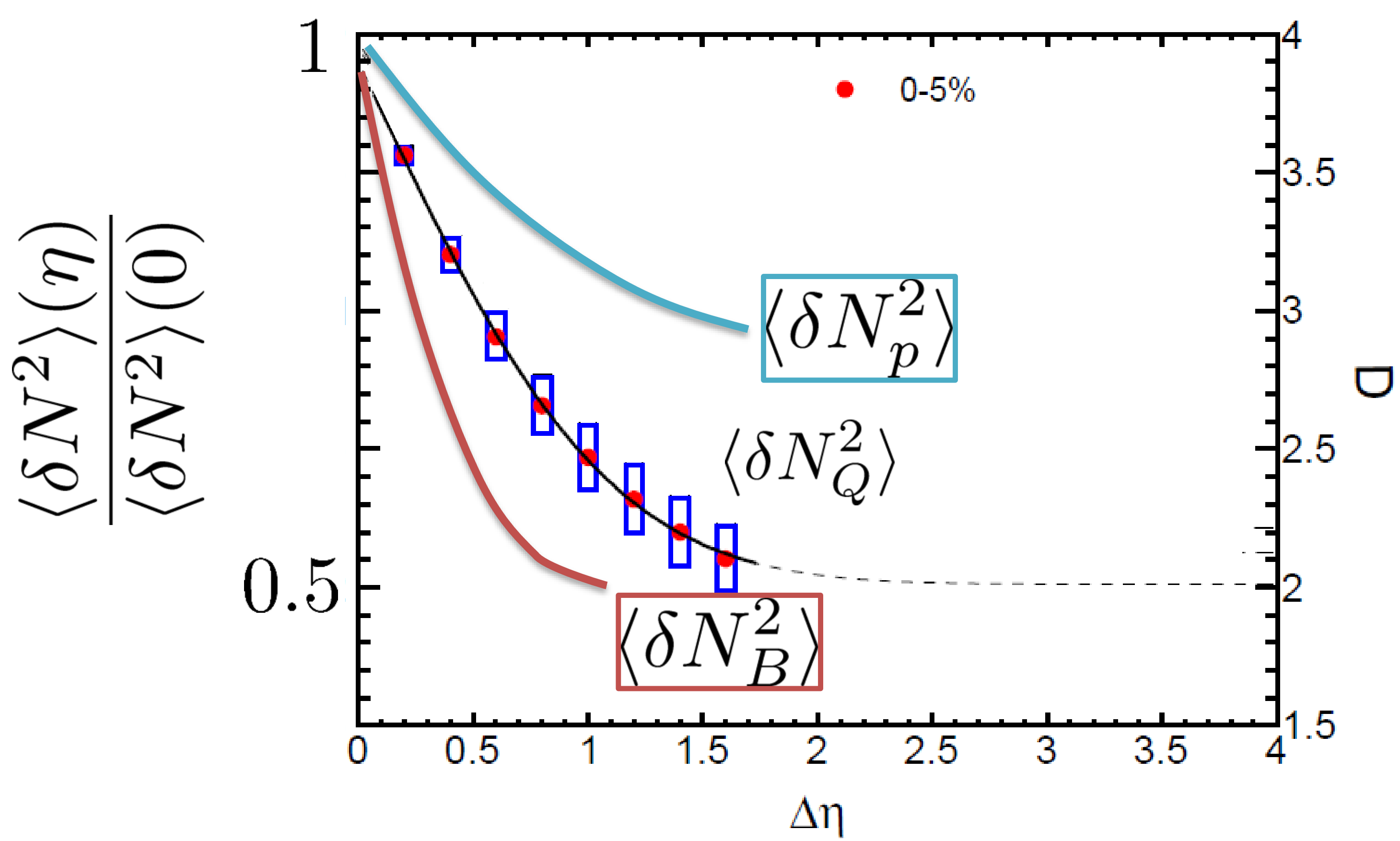
- Centrality independent cumulant at kinetic f.o.
- Thermal blurring at kinetic f.o.

☐ Centrality dep. c… an
qualitatively des…

Rapidity Window Dependences of **Non-**Gaussian Fluctuations

$<\delta N_B{}^2>$ and $< \delta N_p{}^2 >$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
should have different Δη dependence.

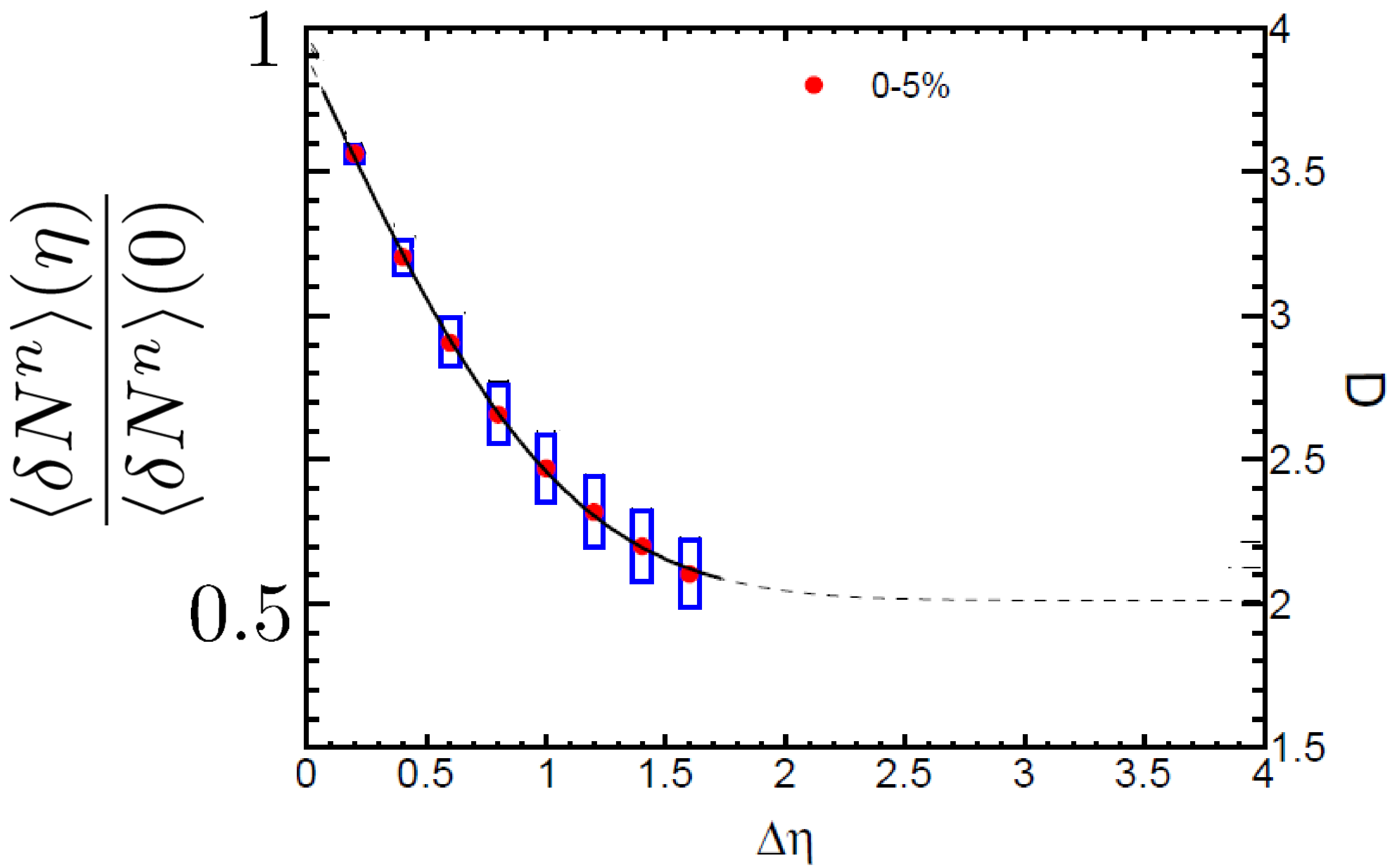


Baryon # cumulants are experimentally observable! MK, Asakawa, 2012

$<\delta N_Q{}^4>$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

suppression or enhancement



$\langle \delta N_Q^4 \rangle$ @ LHC ?

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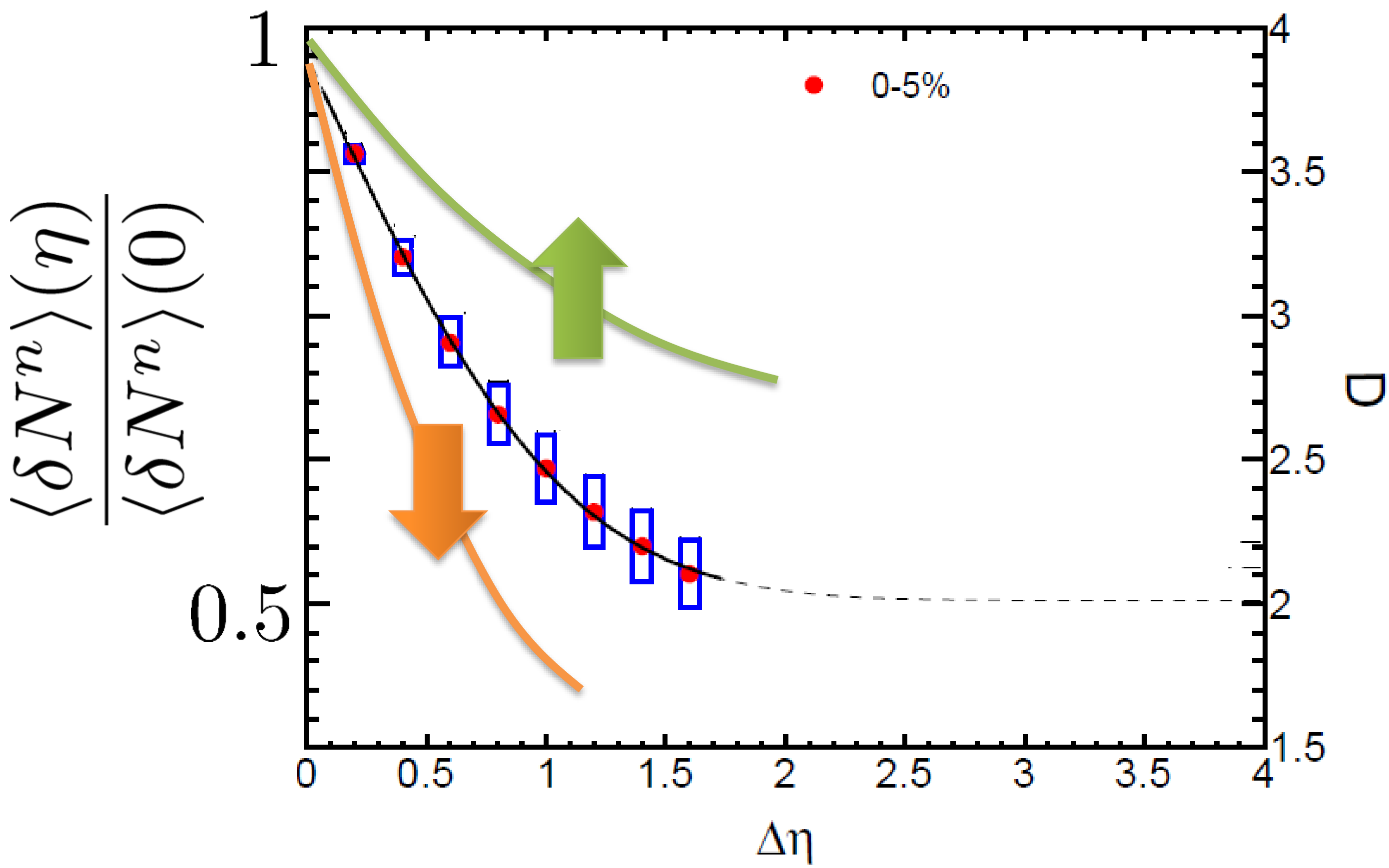


Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

Shuryak, Stephanov, 2001

Markov (white noise)
+
continuity

↓

Gaussian noise

cf) Gardiner, "Stochastic Methods"

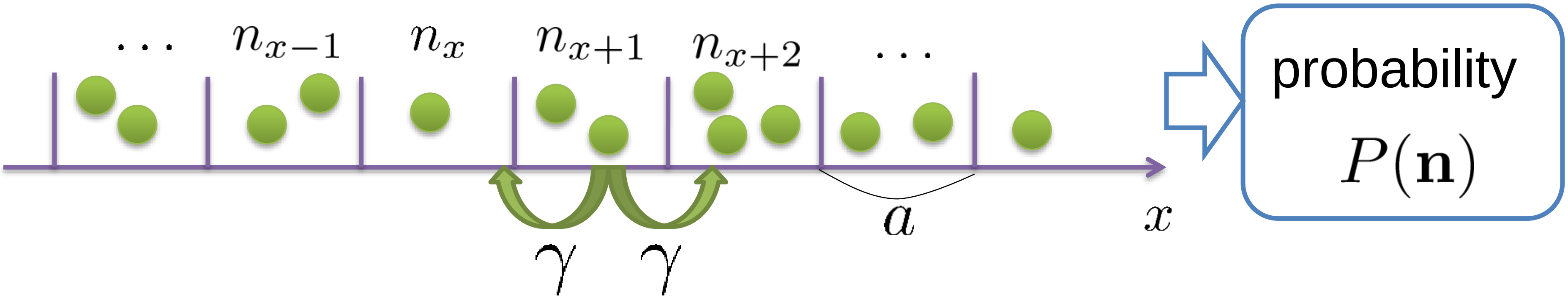
↓

Fluctuation of *n* is
Gaussian in equilibrium

Diffusion Master Equation

MK, Asakawa, Ono, 2014
MK, 2015

Divide spatial coordinate into discrete cells



Diffusion Master Equation

MK, Asakawa, Ono, 2014
MK, 2015

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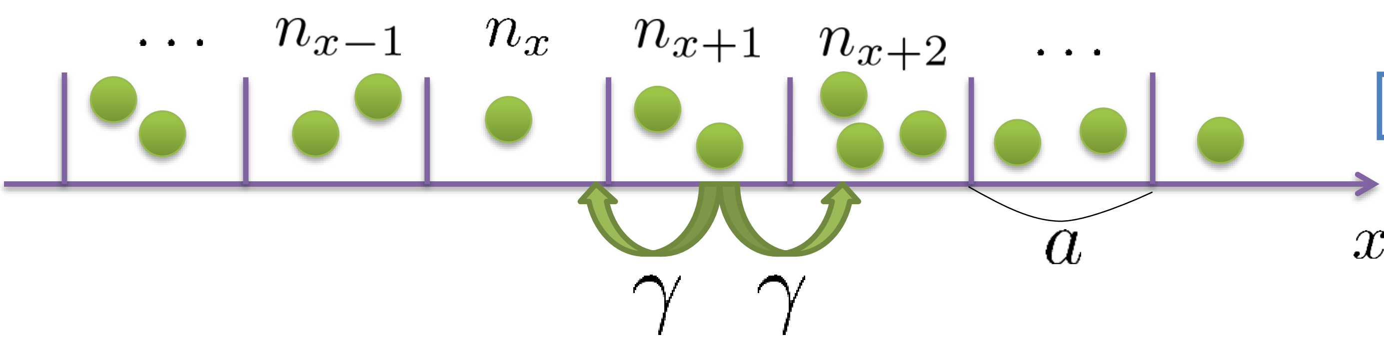


probability

$P(\mathbf{n})$

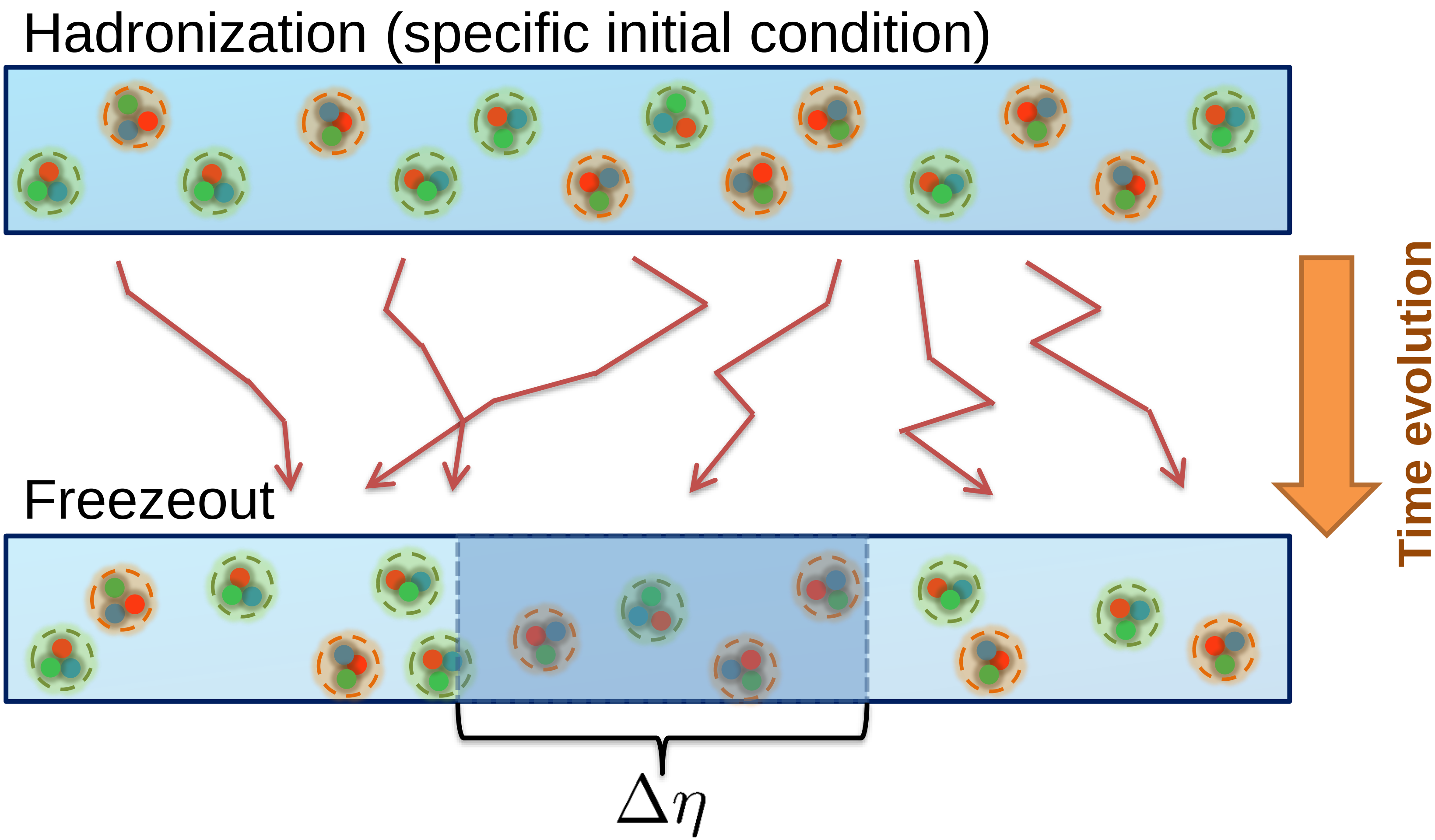
Master Equation for $P(n)$

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x+1)\{P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x+1})+P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x-1})\} - 2n_x P(\mathbf{n})]$$

Solve the DME **exactly**, and take *a*→0 limit

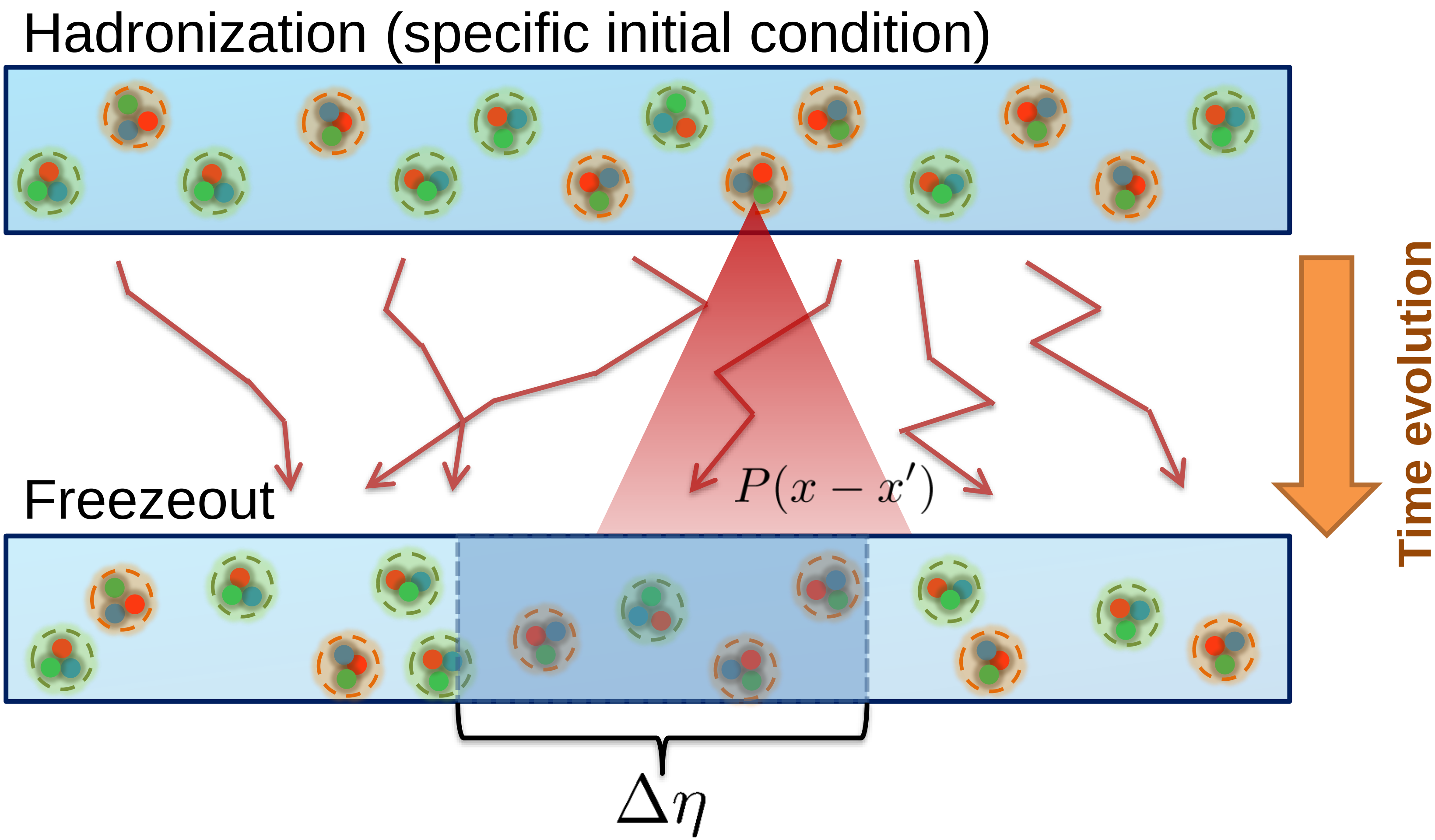
No approx., ex. van Kampen’s system size expansion

A Brownian Particle's Model



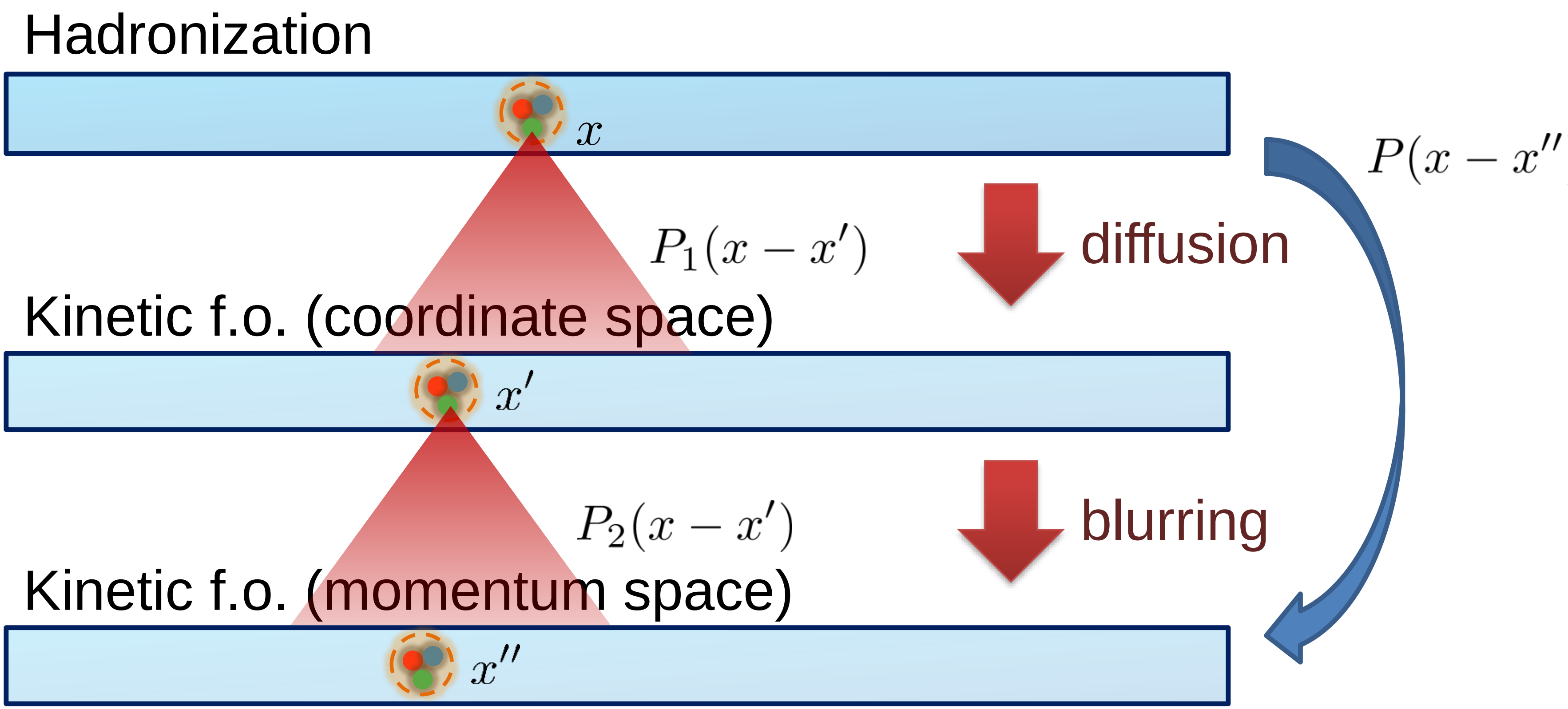
Initial distribution + motion of each particle
→ cumulants of particle # in $\Delta\eta$

A Brownian Particle's Model



Initial distribution + motion of each particle
→ cumulants of particle # in Δη

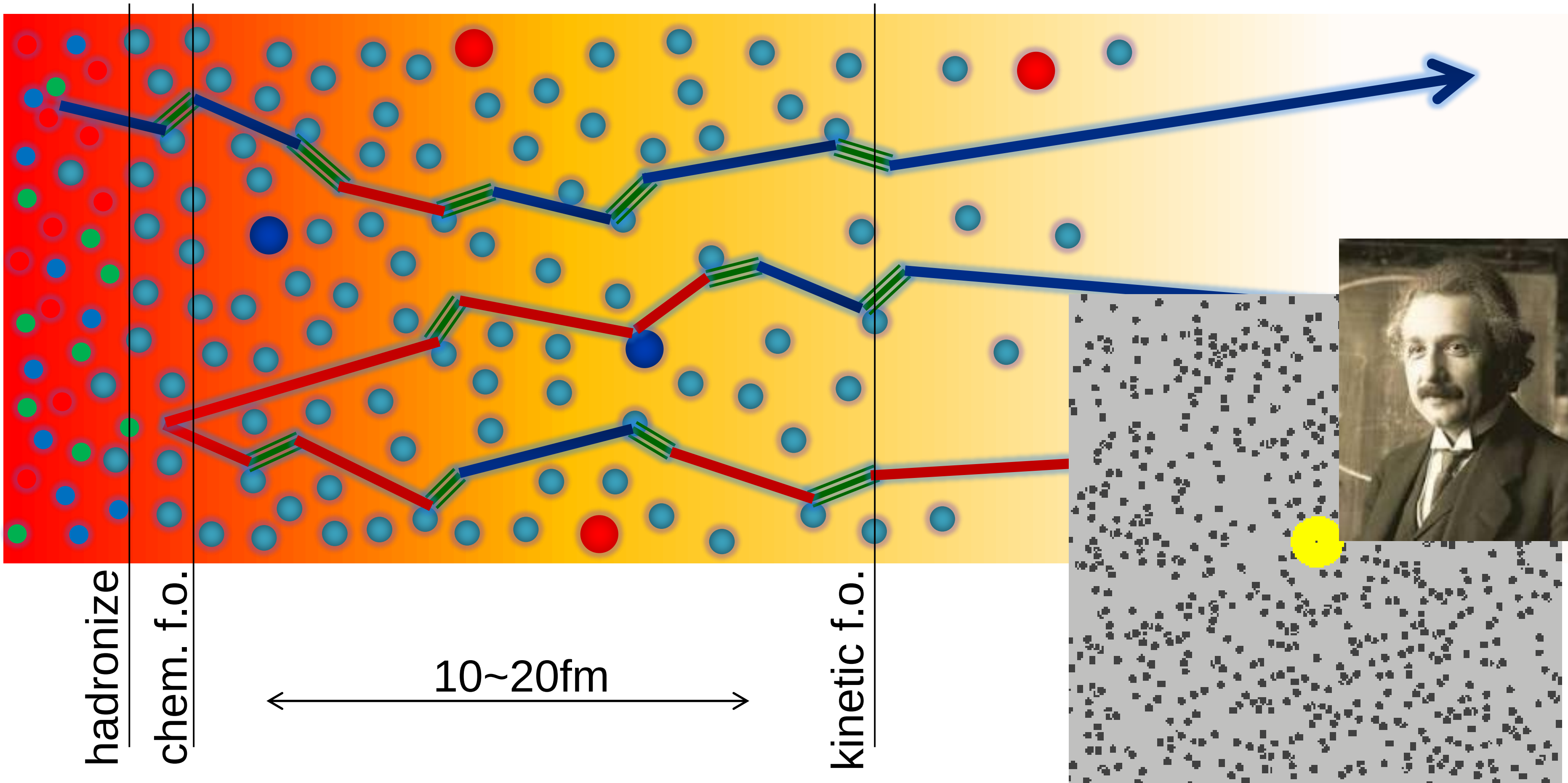
Diffusion + Thermal Blurring



Total diffusion: $P(x - x'') = \int dx' P_1(x - x') P_2(x' - x'')$

- Diffusion + thermal blurring = described by a single P(x)
- Both are consistent with Gaussian → Single Gaussian

Baryons in Hadronic Phase



$p, \bar{p}$ — mesons

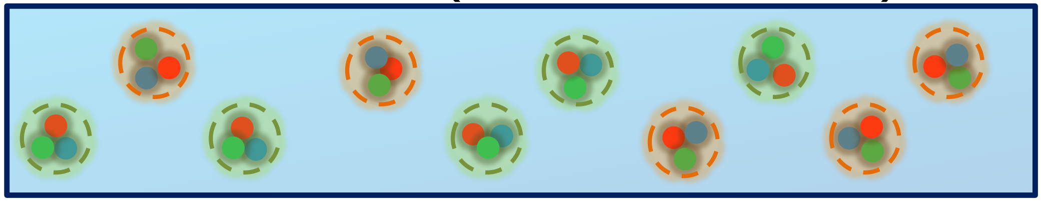
$n, \bar{n}$ — baryons

$\Delta(1232)$

Baryons behave like Brownian pollens in water

Time Evolution in Hadronic Phase

Hadronization (initial condition)



- Boost invariance / infinitely long system
- Local equilibration / local correlation

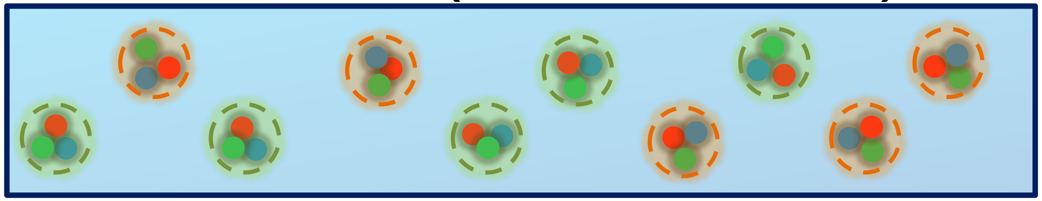
$\langle \bar{Q}^2 \rangle_c, \ \langle \bar{Q}^3 \rangle_c, \ \langle \bar{Q}^4 \rangle_c$ $\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ $\langle Q^2_{(\mathrm{tot})} \rangle_c, \langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c$

suppression owing to local charge conservation

strongly dependent on hadronization mechanism

Time Evolution in Hadronic Phase

Hadronization (initial condition)



Diffusion + Blurring

- Boost invariance / infinitely long system
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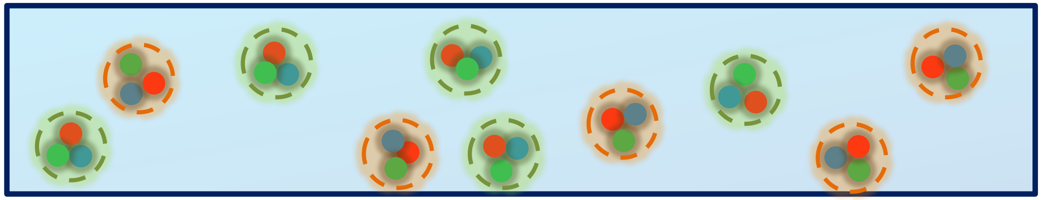
suppression owing to local charge conservation

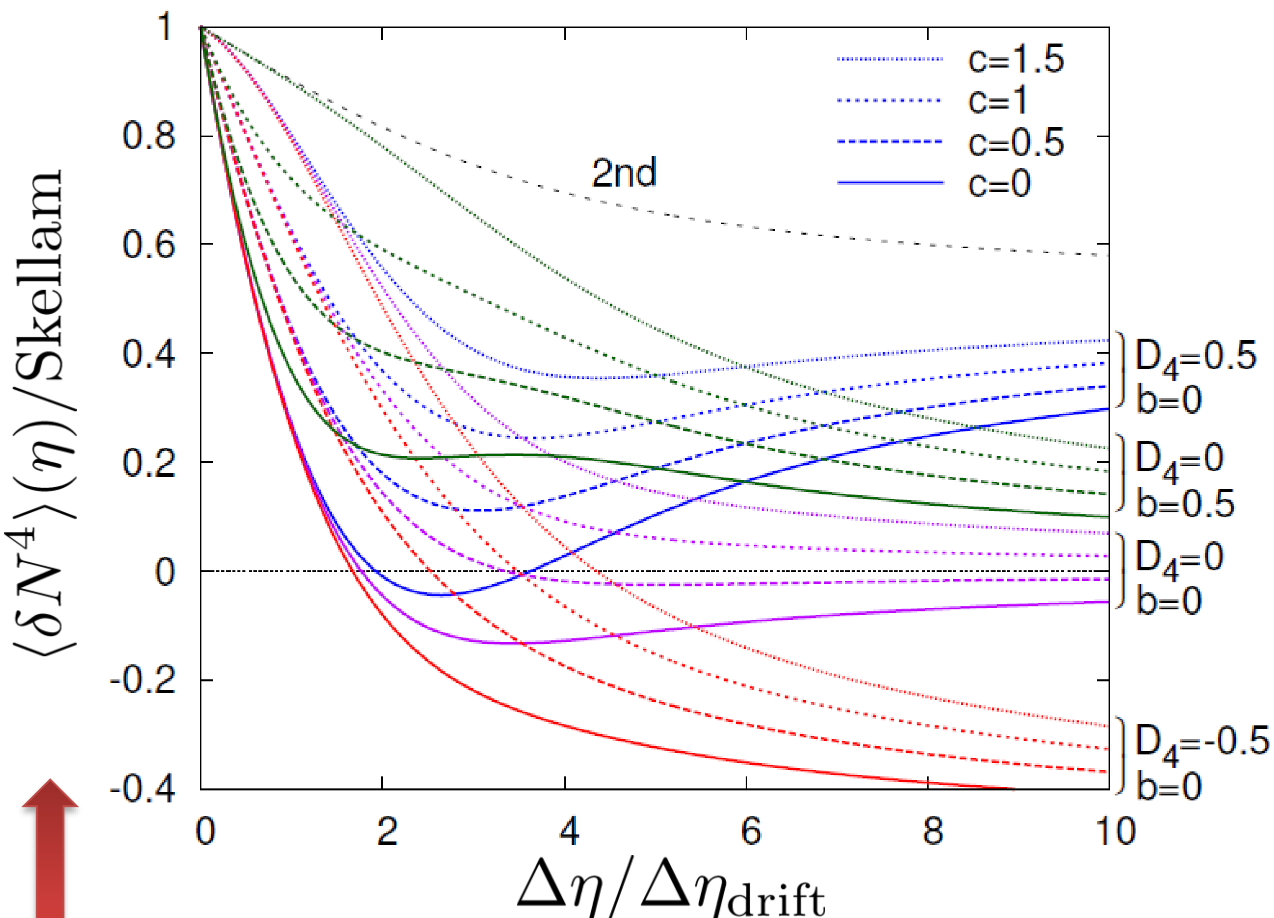
strongly dependent on hadronization mechanism

Detector



Δη Dependence: 4th order

MK, NPA (2015)



new normalization

Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

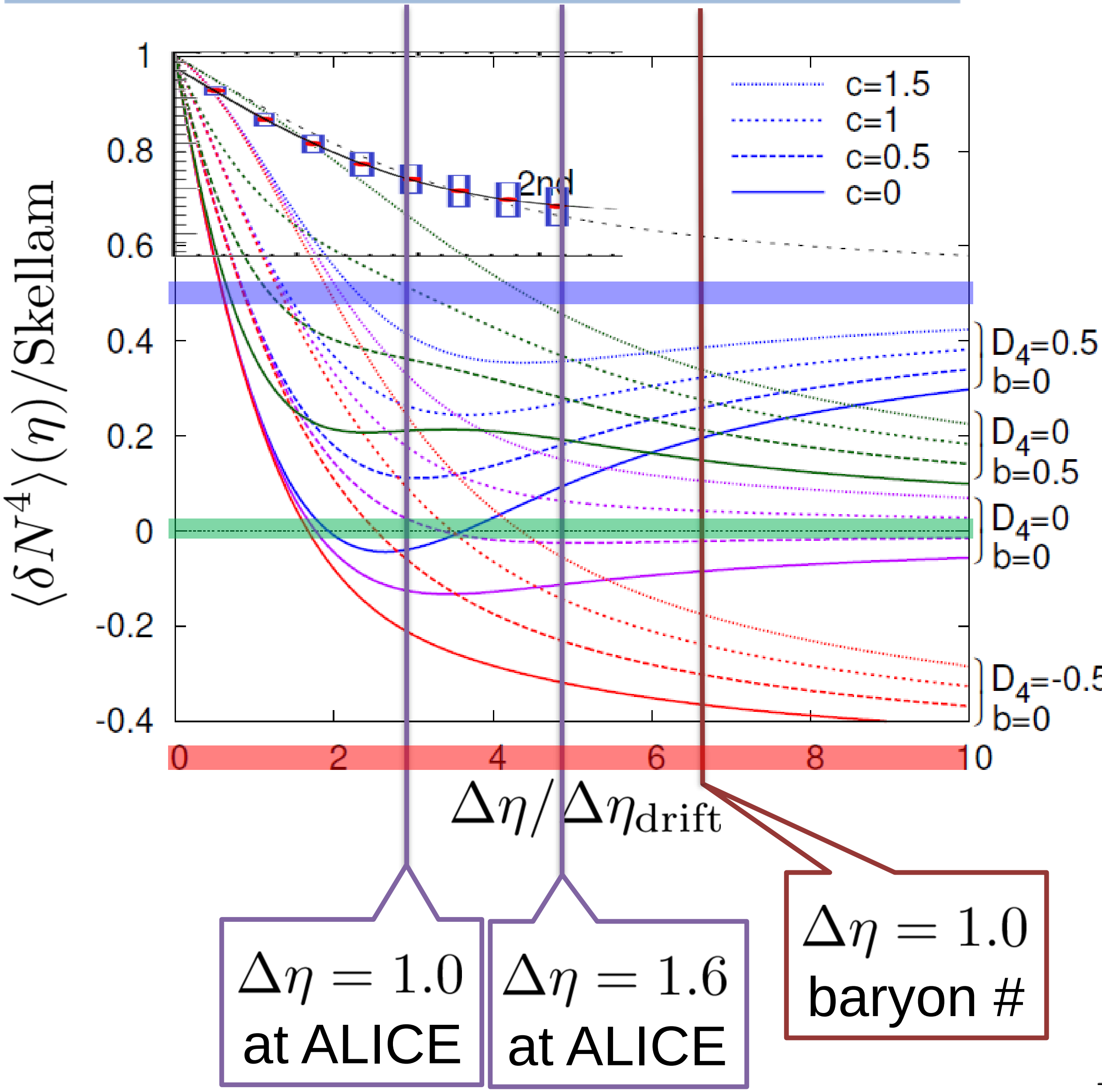
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

Characteristic Δη dependences!

Δη Dependence: 4[th] order

MK, NPA (2015)



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

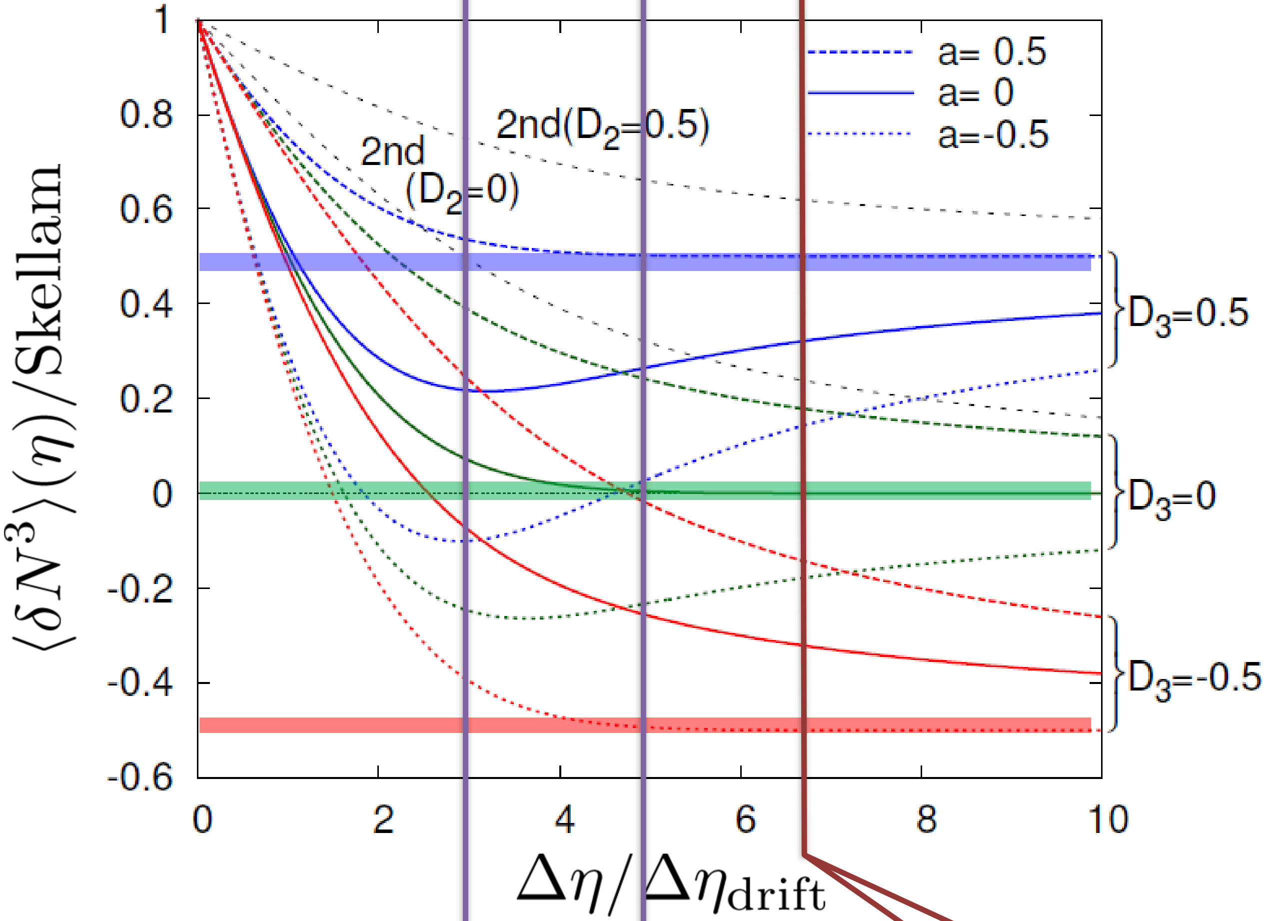
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

$$D \sim M^{-1}$$

Δη Dependence: 3rd order

MK, NPA (2015)



$\Delta\eta = 1.0$ at ALICE

$\Delta\eta = 1.6$ at ALICE

$\Delta\eta = 1.0$ baryon #

Initial Condition

$$D_3 = \frac{\langle Q^3_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

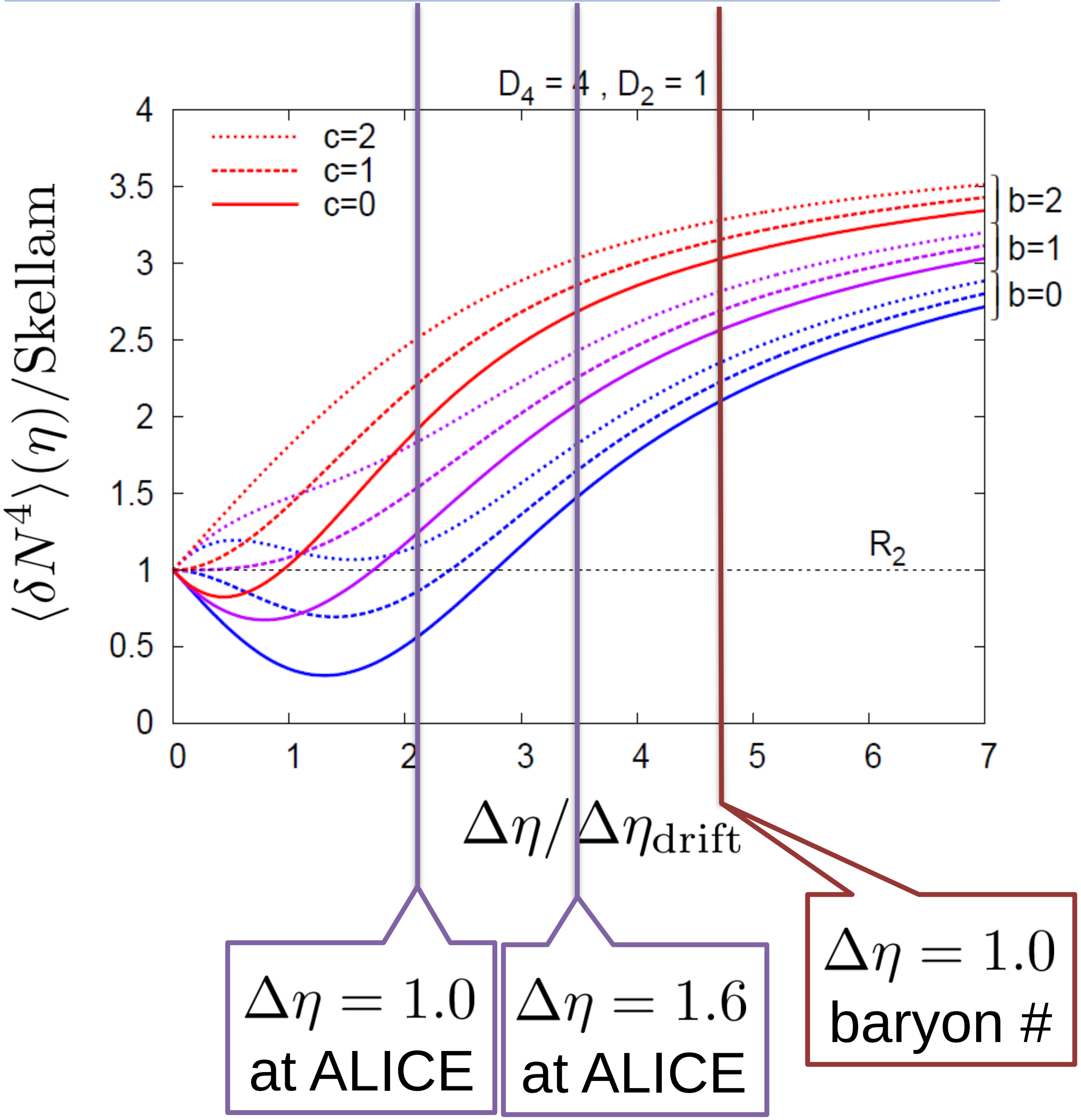
$$a = \frac{\langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} = 0.5$$

$$D \sim M^{-1}$$

4^{th} order : Large Initial Fluc.

MK, NPA (2015)



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\text{net})} \rangle_c}{\langle Q_{(\text{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\text{net})} Q_{(\text{tot})} \rangle_c}{\langle Q_{(\text{net})} \rangle}$$

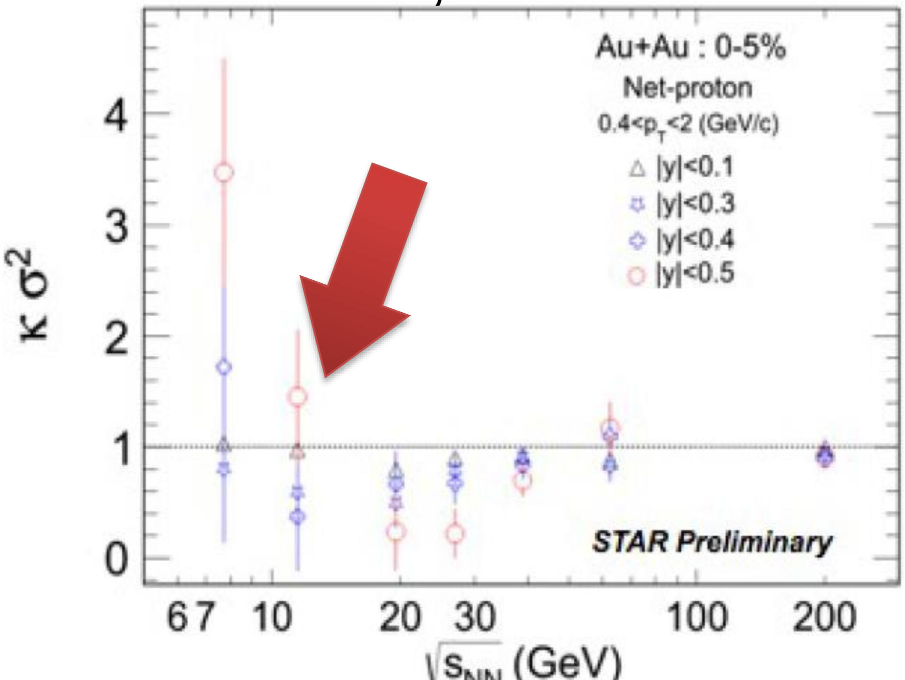
$$c = \frac{\langle Q^2_{(\text{tot})} \rangle_c}{\langle Q_{(\text{tot})} \rangle}$$

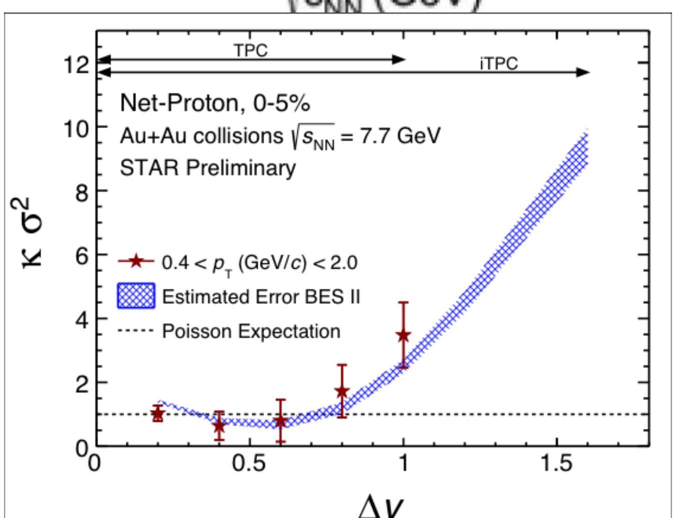
$$D_2 = \frac{\langle Q^2_{(\text{net})} \rangle_c}{\langle Q_{(\text{tot})} \rangle} = 1$$

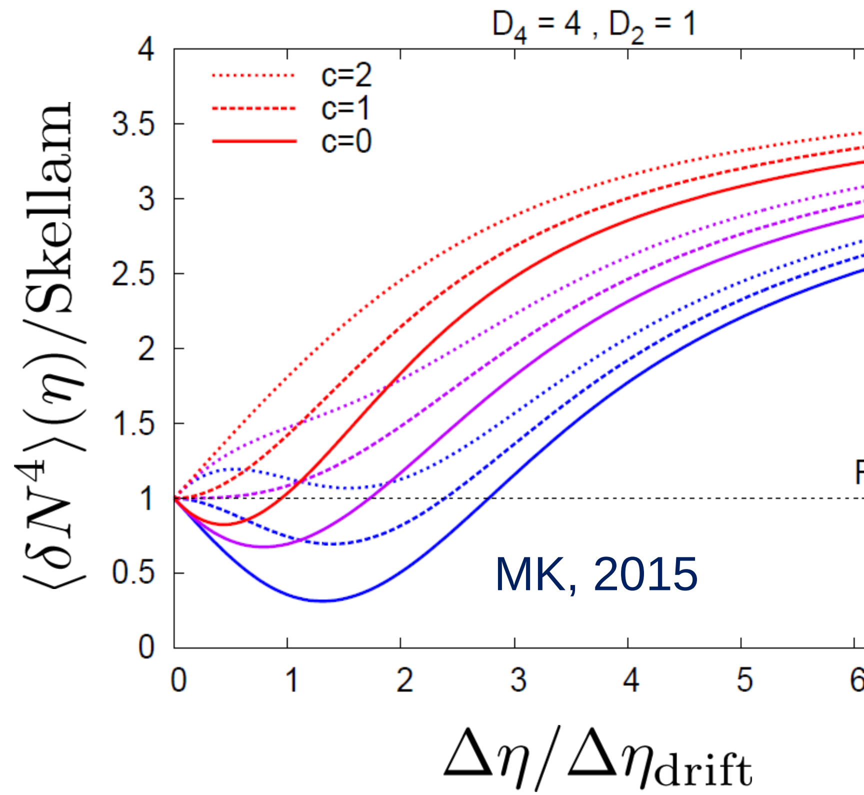
$$D \sim M^{-1}$$

Δη Dependence @ STAR

X. Luo, CPOD2014







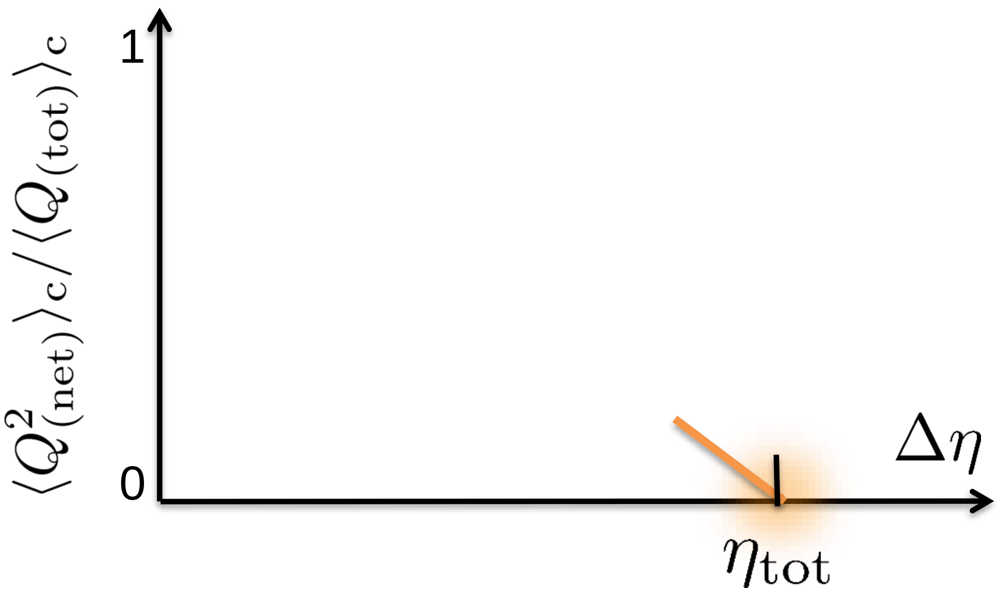
Non-monotonic dependence on Δy ?

Effect of Global Charge Conservation (Finite Volume Effect)

Sakaida, Asakawa, MK, PRC, 2014

Global Charge Conservation

Conserved charges in the total system do no fluctuate!



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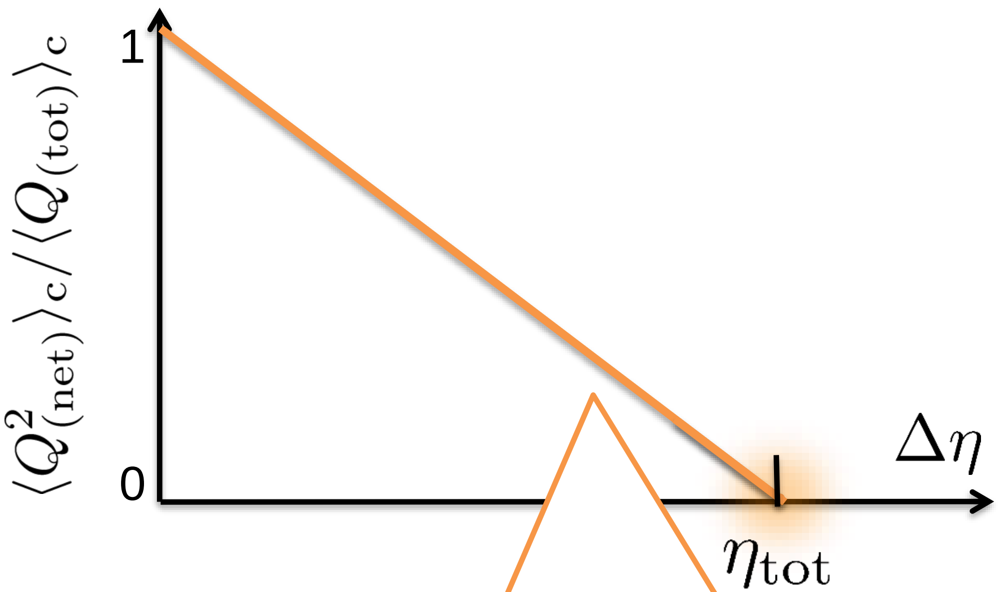


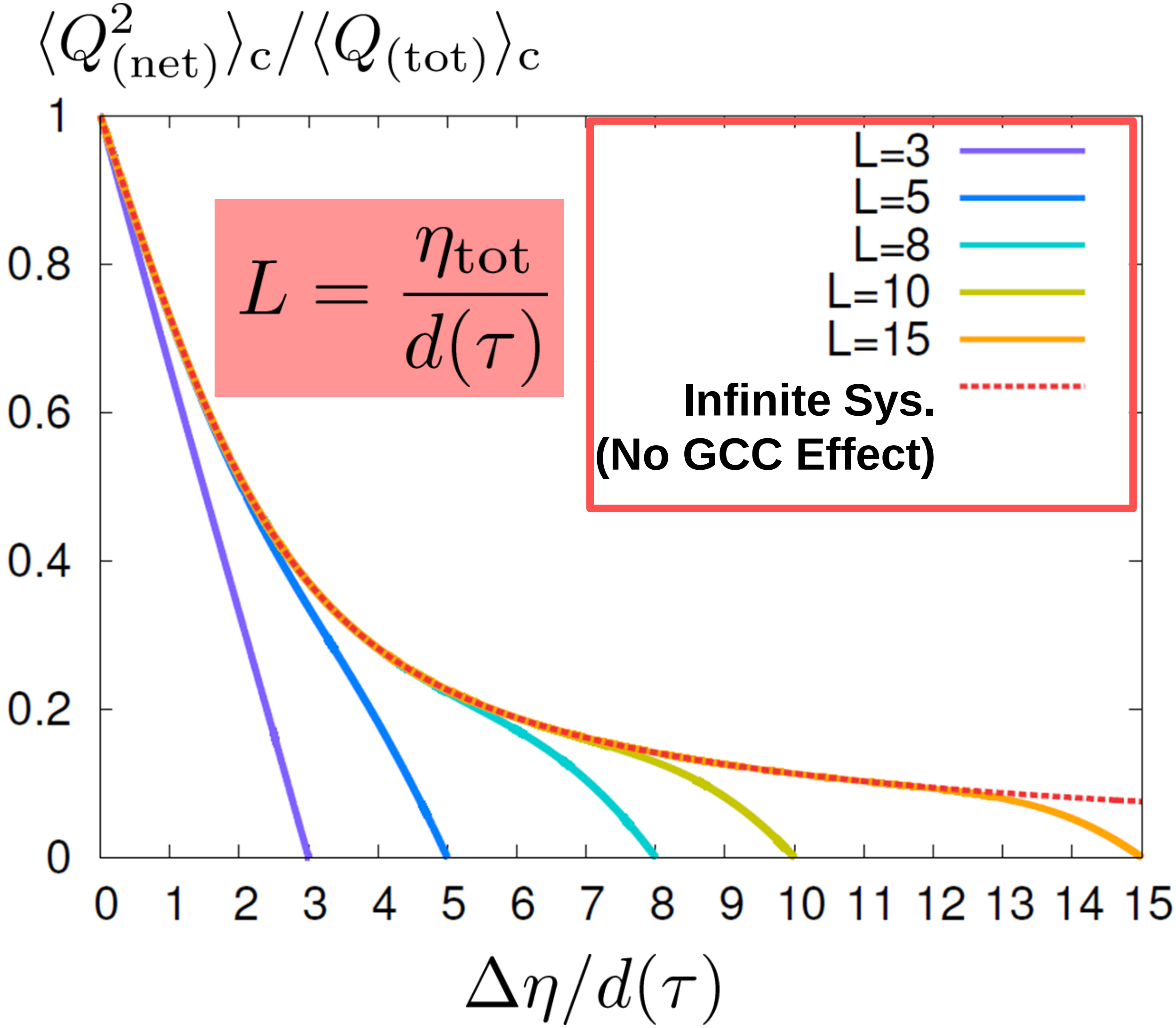
An Estimate of GCC Effect

$$\langle \delta N^2 \rangle_{\mathrm{GCC}} = \langle \delta N^2 \rangle_{\mathrm{inf}} \times \left(1 - \frac{\Delta\eta}{\eta_{\mathrm{tot}}} \right)$$

Jeon, Koch, PRL2000; Bleicher, Jeon, Koch (2000)

Diffusion in Finite Volume

Solve the diffusion master equation in finite volume



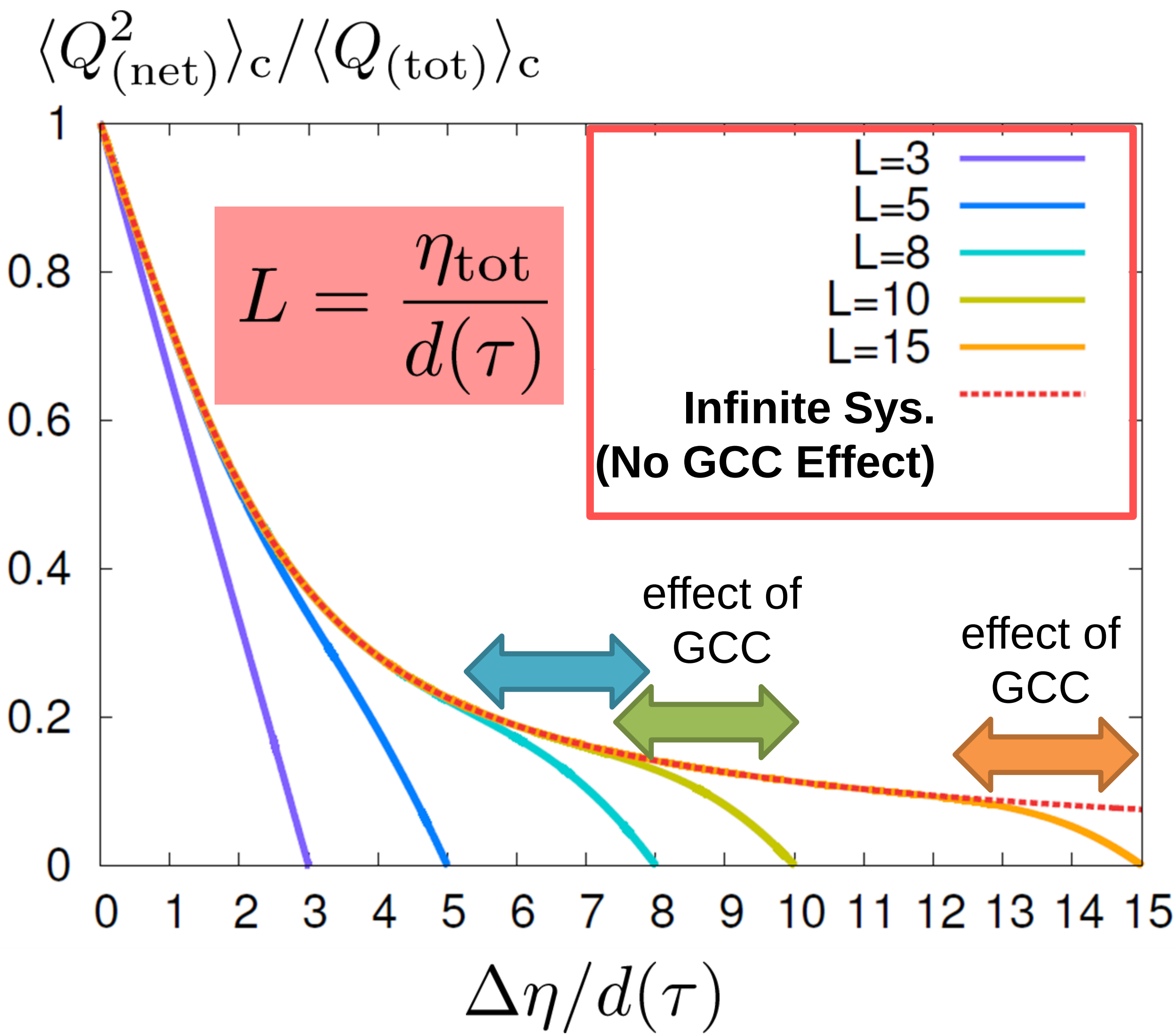
$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

: Average Diffusion Length

$D(\tau)$: Diffusion Coefficient

Diffusion in Finite Volume

Solve the diffusion master equation in finite volume



$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

: Average Diffusion Length

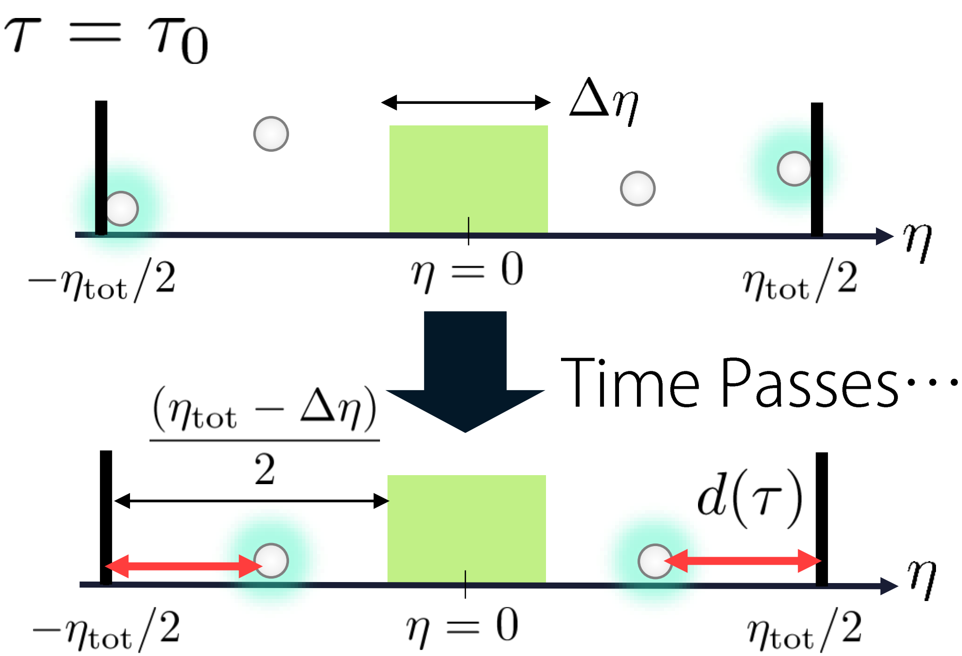
$D(\tau)$: Diffusion Coefficient

suppression only for

$$\Delta\eta/d \geq L-2$$

Physical Interpretation

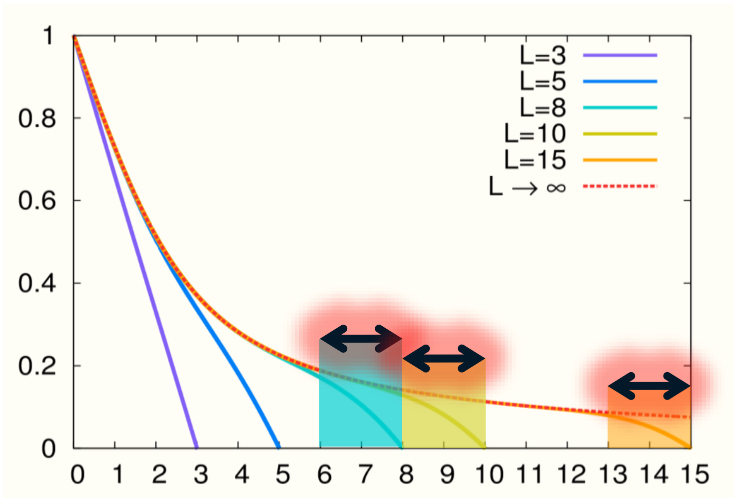
slide by M. Sakaida



$d(\tau)$: Averaged Diffusion Distance

$D(\tau)$: Diffusion Coefficient

η_{tot} : Total Length of Matter

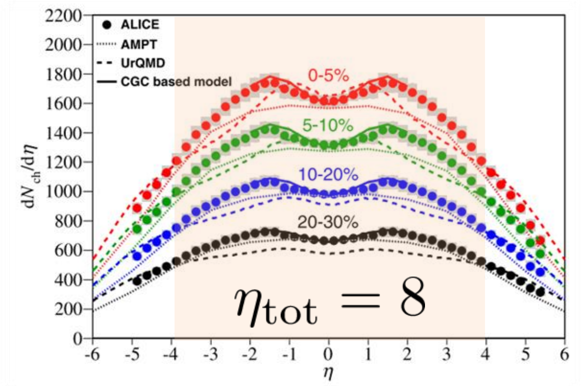


Condition for effects of the GCC

$$\Delta\eta/d \geq L - 2 \Leftrightarrow \frac{\eta_{tot} - \Delta\eta}{2} \leq d$$

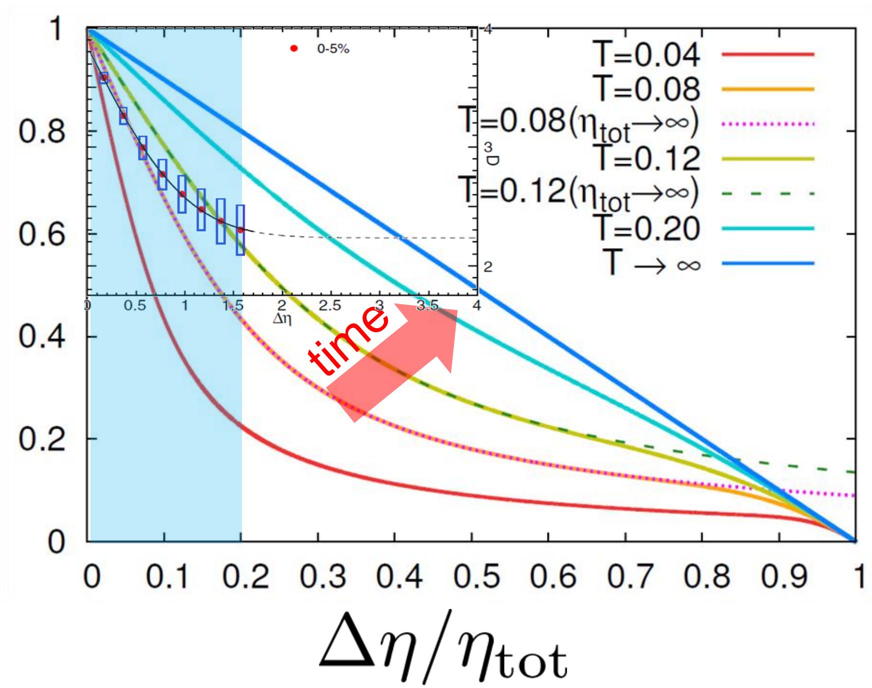
Effects of the GCC appear only near the boundaries.

Comparison with ALICE Result

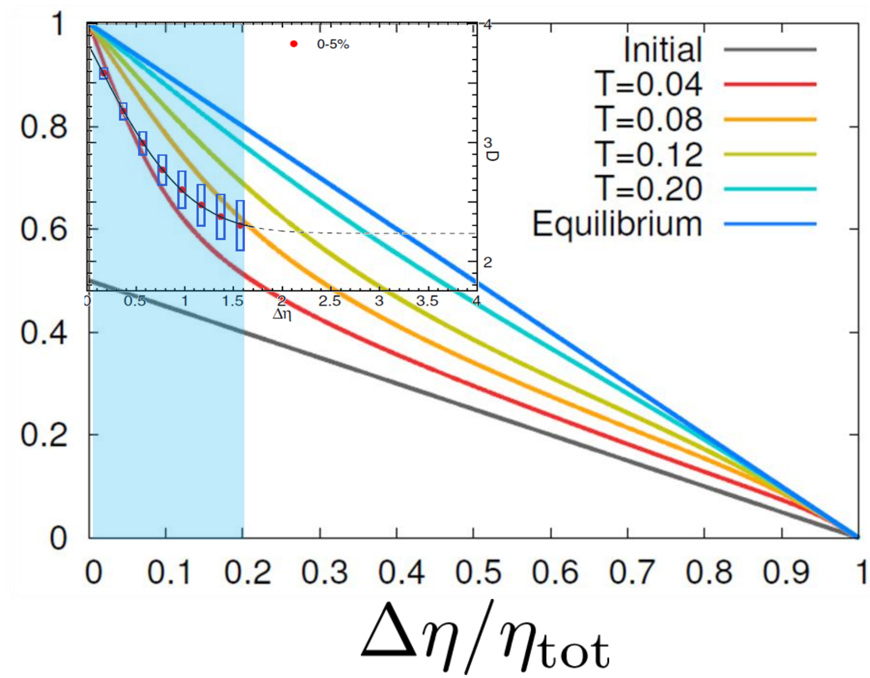


$$\langle Q^2_{(\mathrm{net})}\rangle_c / \langle Q_{(\mathrm{tot})}\rangle_c$$

without initial fluc.



with initial fluc.



- No GCC effect in ALICE experiments!
- Same conclusion for higher order cumulants

$$T = \frac{d(\tau)}{\eta_{tot}}$$

Summary

Plenty of information in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2\rangle_c,\ \langle N_Q^3\rangle_c,\ \langle N_Q^4\rangle_c,\ \langle N_B^2\rangle_c,\ \langle N_B^3\rangle_c,\ \langle N_B^4\rangle_c,\ \langle N_S^2\rangle_c,\ \cdots$$

and those of non-conserved charges, mixed cumulants…

⇒ With $\Delta\eta$ dep. we can explore

- primordial thermodynamics
- non-thermal and transport property
- effect of thermal blurring

Future Studies

□ Experimental side:

- ➢ rapidity window dependences
- ➢ baryon number cumulants
- ➢ BES for SPS- to LHC-energies

□ Theoretical side:

- ➢ rapidity window dependences in dynamical models
- ➢ description of non-equilibrium non-Gaussianity
- ➢ accurate measurements on the lattice

□ Both sides:

- ➢ Compare theory and experiment carefully
- ➢ Let's accelerate our understanding on fluctuations!