Energy-momentum tensor and thermodynamics (of SU(3) gauge theory) from gradient flow

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FOR FLOWQCD COLLABORATION / WHOT-QCD COLLABORATION

$T_{\mu\nu}$ Energy-momentum tensor (EMT)

One of the most fundamental quantities in physics

Contain various thermal properties

1-point expectation value

$$\langle T_{00} \rangle = \varepsilon \qquad \langle T_{11} \rangle = p$$

2-point correlator (fluctuations)

$$\frac{\langle (\delta \bar{T}_{00})^2 \rangle}{VT^2} = c_V \qquad \frac{\langle (\delta \bar{T}_{01})^2 \rangle}{VT^2} = \frac{\varepsilon + p}{T^4}$$

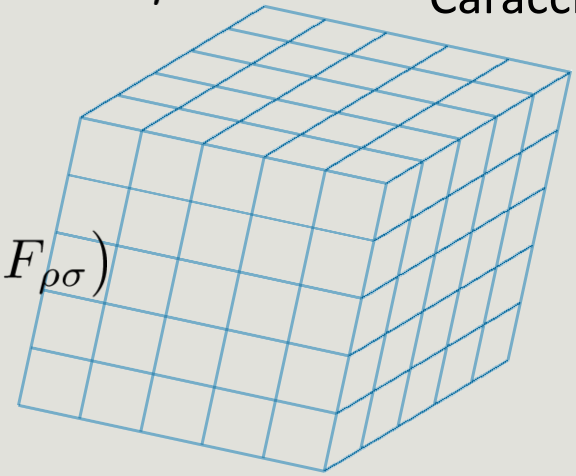
EMT on the Lattice

a standard construction

$$T_{\mu\nu} = Z_6 T^{[6]}_{\mu\nu} + Z_3 T^{[3]}_{\mu\nu} + Z_1 T^{[1]}_{\mu\nu}$$

Caracciolo+, 1990

$$\begin{cases} T^{[6]}_{\mu\nu} = \dfrac{1}{g_0^2} F_{\mu\sigma} F_{\nu\sigma} \\ T^{[3]}_{\mu\nu} = \delta_{\mu\nu} \dfrac{1}{g_0^2} \left(F_{\mu\rho} F_{\nu\rho} - \dfrac{1}{4} F_{\rho\sigma} F_{\rho\sigma} \right) \\ T^{[1]}_{\mu\nu} = \delta_{\mu\nu} \dfrac{1}{g_0^2} F_{\rho\sigma} F_{\rho\sigma} \end{cases}$$



- Non-perturbative determination of Z_6, Z_3, Z_1
  - shifted boundary condition
  - accurate nonperturbative Z_6 available

Giusti, Meyer, 2011-13
Giusti, Pepe, 2014-

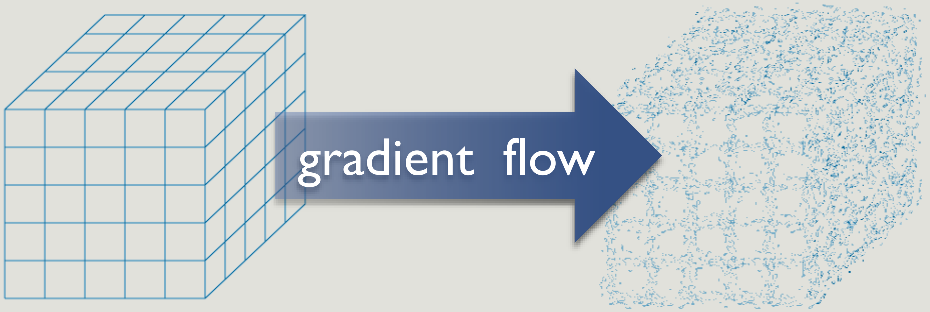
Gradient Flow

◻ Gradient Flow

Lüscher(2009–)
Narayanan, Neuberger(2006)

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- Diffusion-like equation
- smearing radius $\sqrt{8t}$



Gradient Flow

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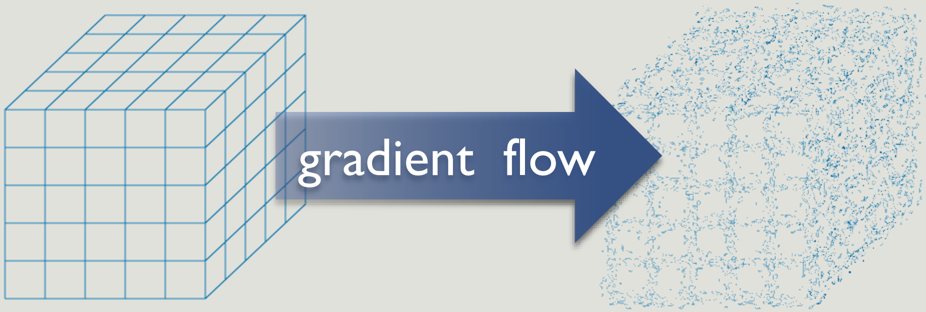
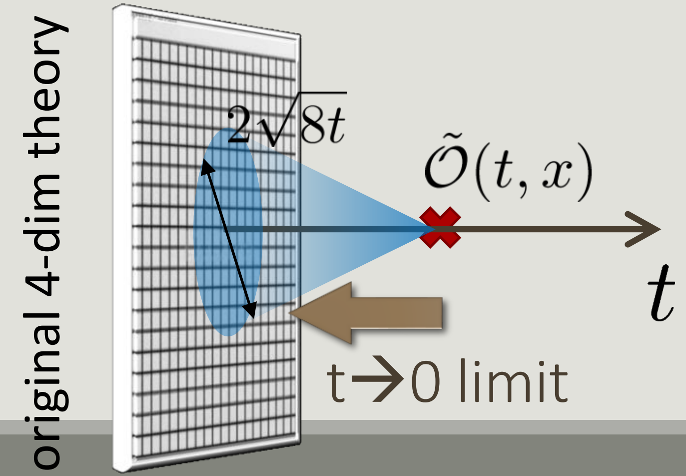


☐ Small Flow-time expansion

$$\tilde{\mathcal{O}}(t,x) \longrightarrow \sum_i c_i(t) \mathcal{O}_i^R(x)$$

Lüscher, Weisz (2011)



EMT from Gradient Flow

Suzuki, 2013

$$U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x)$$

$$E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x)$$

small flow-time expansion

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$

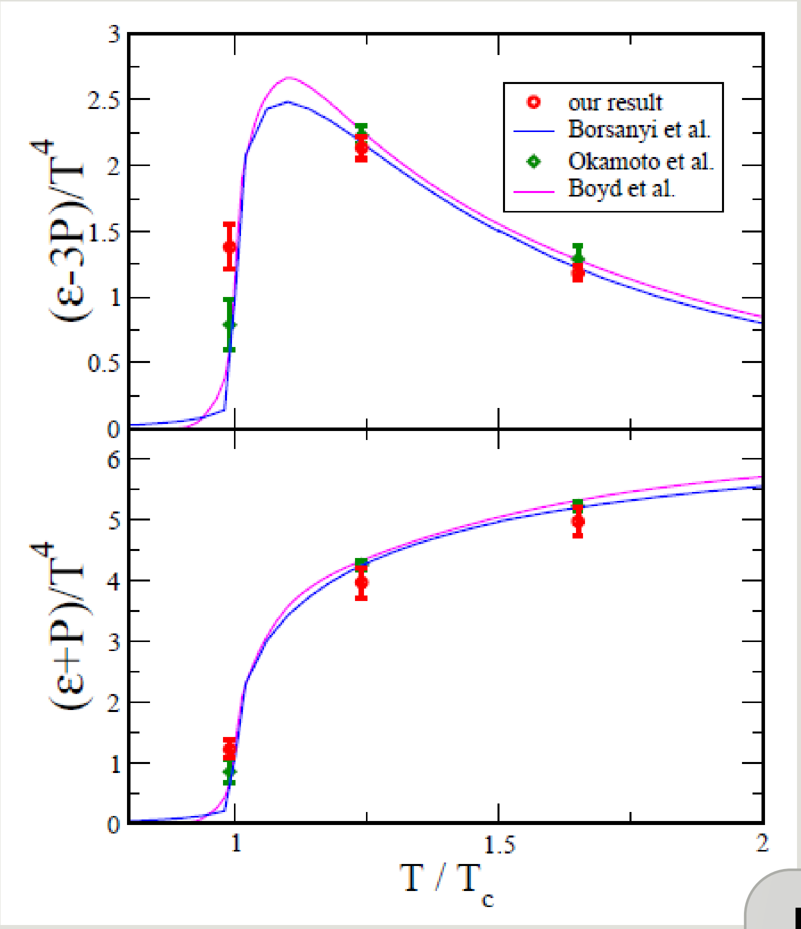
$\alpha_U,\ \alpha_E$: perturbative formula by Suzuki, 2013

Correctly Renormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\mathrm{subt.}}\right] + O(t)$$

Thermodynamics

FlowQCD, PRD90 (2014)



$$\varepsilon = \langle T_{00} \rangle \quad p = \langle T_{11} \rangle$$

- SU(3) Wilson gauge
- $32^3 \mathrm{x} N_t$
- N_t=6,8,10
- ~300 confs.
- continuum extrapolated

- ☐ Good agreement with integral method result
- ☐ Statistical error well suppressed

Applications

1. Thermodynamics

FlowQCD (2014); in preparation

2. Latent heat at 1^{st} order phase transition

Shirogane+, PRD (2016); WHOT-QCD, in preparation

3. Correlation functions

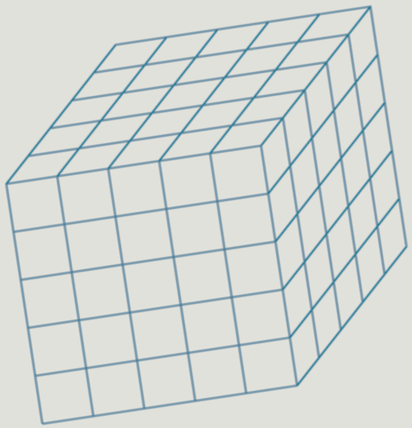
FlowQCD, in preparation

Application to full QCD: WHOT-QCD, 1609.01471 (Next talk by Kanaya)

New Result on
Thermodynamics

FlowQCD, in preparation

Numerical Simulation



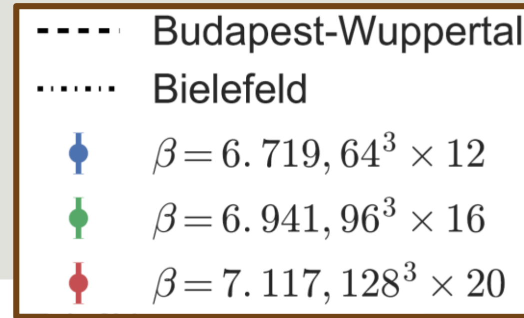
- SU(3) YM theory
- Wilson gauge action
- Parameters:
  - N_t = 12, 16, 20-24
  - aspect ratio $5.3<N_s/N_t<8$
  - 1500~2000 configurations

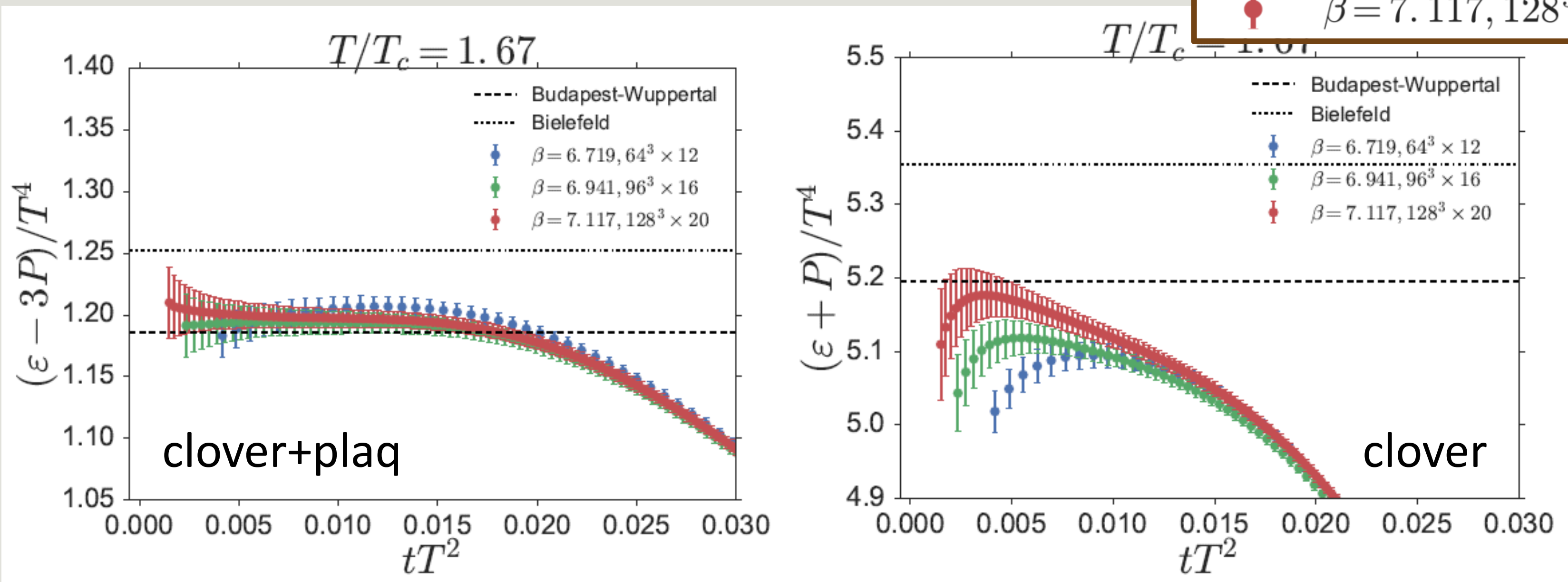
- Scale setting from gradient flow

$$\to aT_c \text{ and } a\Lambda_{\mathrm{MS}}$$

FlowQCD
1503.06516

t Dependence

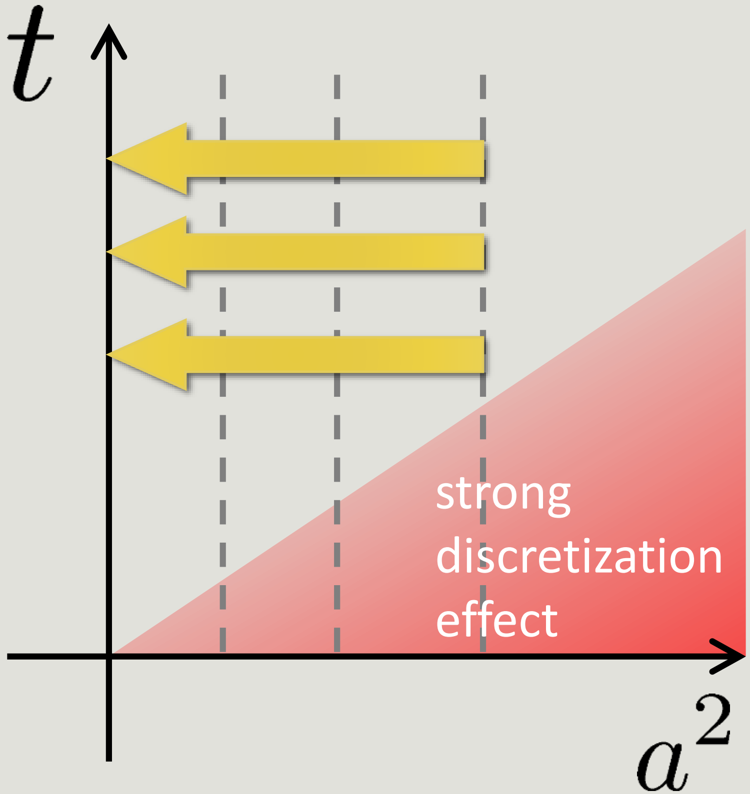




$$\begin{cases} \sqrt{8t} < a & \text{: strong discretization effect} \\ \sqrt{8t} > 1/(2T) & \text{: oversmeared} \end{cases}$$

$a < \sqrt{8t} < 1/(2T)$: Linear t dependence

Double Extrapolation

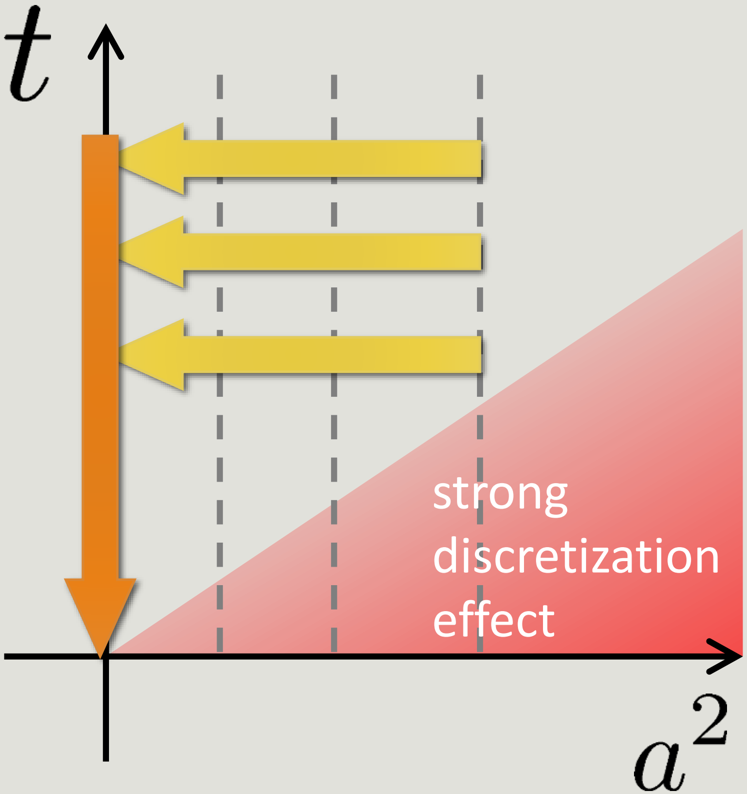


Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

Note: FlowQCD, 2014: continuum extrapolation only
WHOT-QCD, 2016: small t limit only

Double Extrapolation



Continuum extrapolation

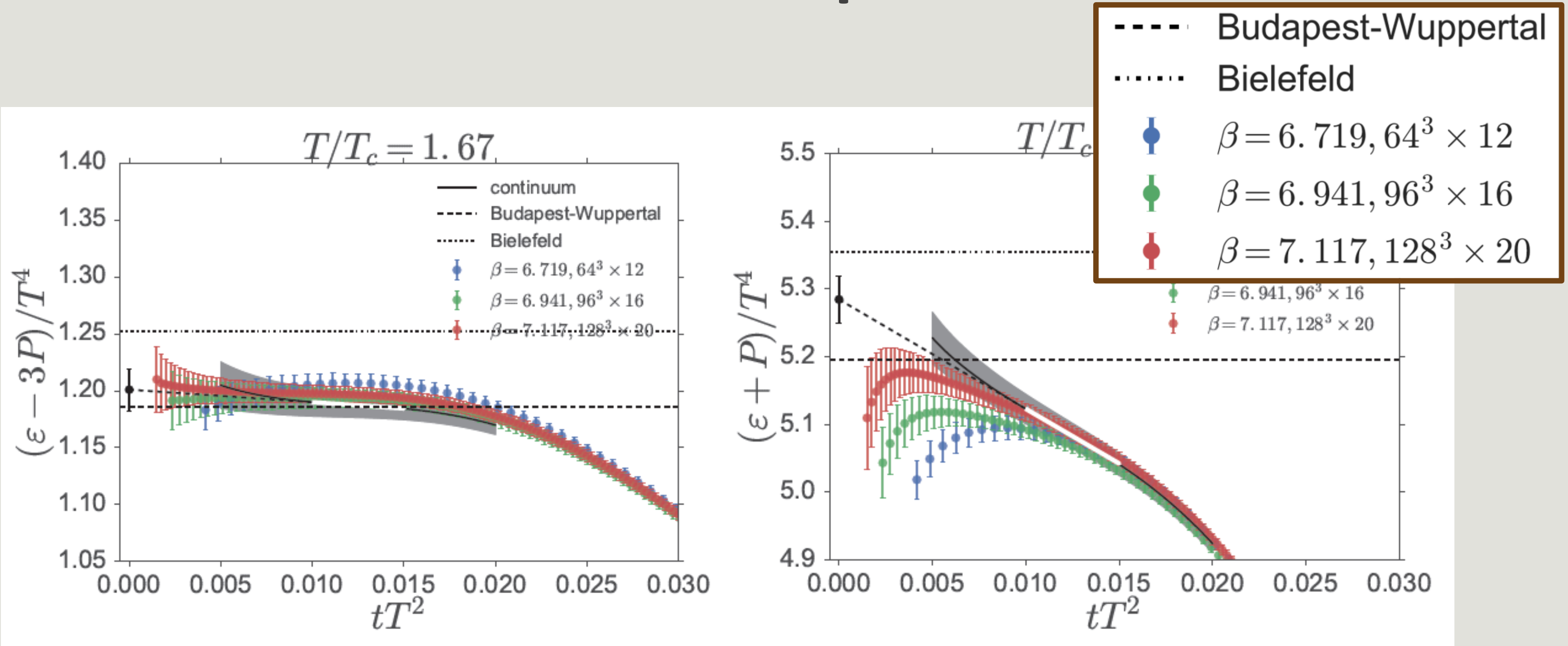
$$\langle T_{\mu\nu}(t)\rangle_{\mathrm{cont}} = \langle T_{\mu\nu}(t)\rangle_{\mathrm{lat}} + C(t)a^2$$

Small t extrapolation

$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C'(t)t$$

Note: FlowQCD, 2014: continuum extrapolation only
WHOT-QCD, 2016: small t limit only

Double Extrapolation



Black band: continuum extrapolated

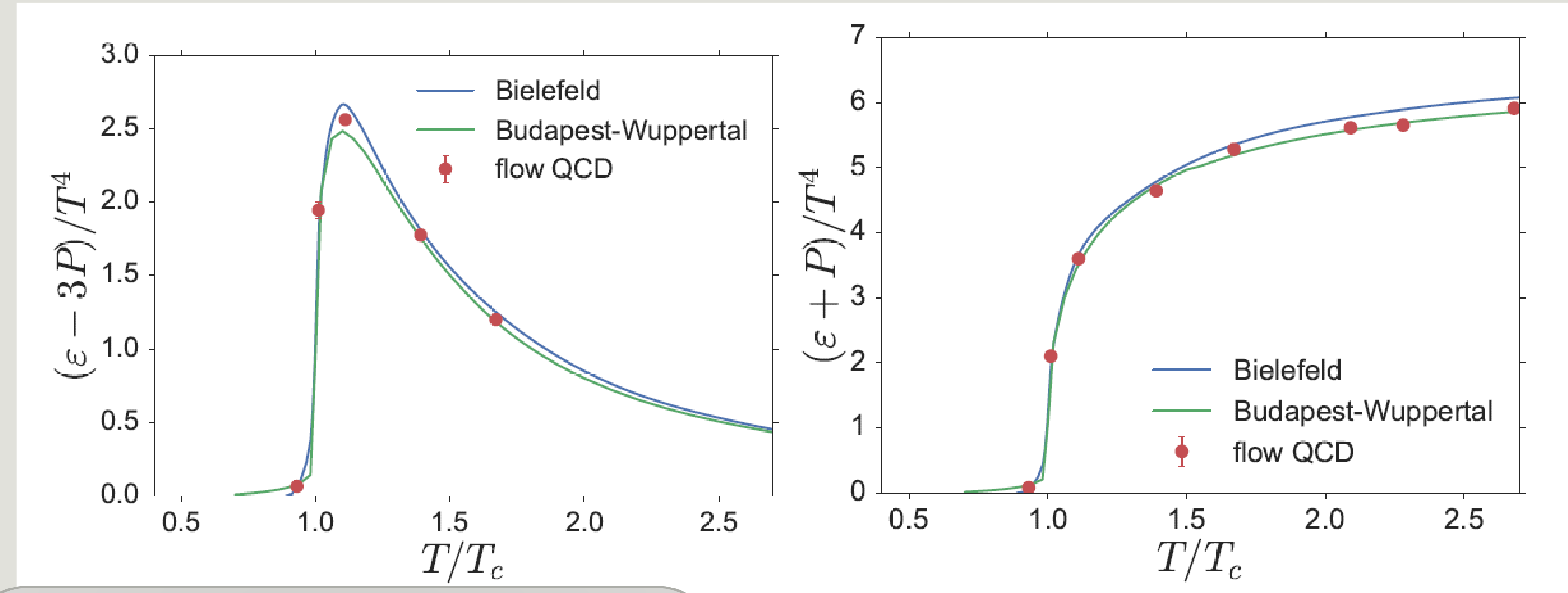
- range of t for fitting:

$$0.005 < tT^2 < 0.015$$

$$0.01 < tT^2 < 0.015$$

$$0.01 < tT^2 < 0.02$$

T Dependence



Error includes

- statistical error
- choice of t range for t→0 limit
- uncertainty in $a\Lambda_{MS}$

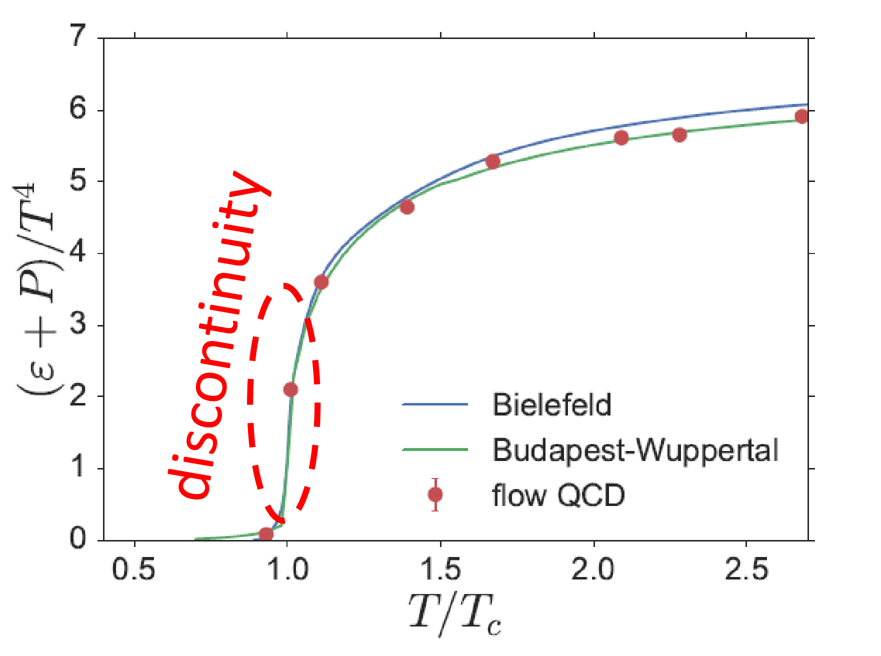
total error <1.5% for $T>1.1T_c$

- Excellent agreement with integral method
- High accuracy only with ~2000 confs.

Latent Heat
at 1^{st} order transition

WHOT-QCD Collaboration
Shirogane+, PRD (2016); in preparation
(see, poster by Shirogane)

1^{st} Order Transition



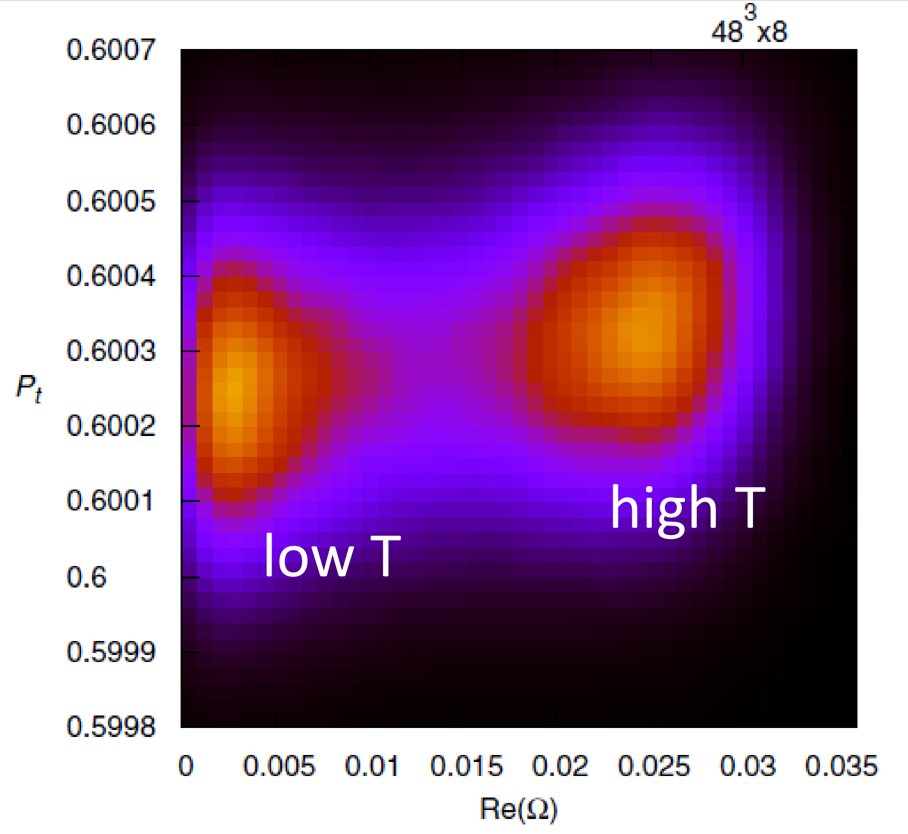
SU(3) gauge theory has a 1^{st} order phase transition

- Latent heat $\Delta\varepsilon$
  A universal constant
- Pressure gap $\Delta p=0$

- $\Delta\varepsilon$: Universal dimension-full quantity
- Δp: check of the measurement
- technics for 1^{st} tr. → future study for $\mu>0$??

Phase Coexistence at T_c

Shirogane+, PRD (2016)



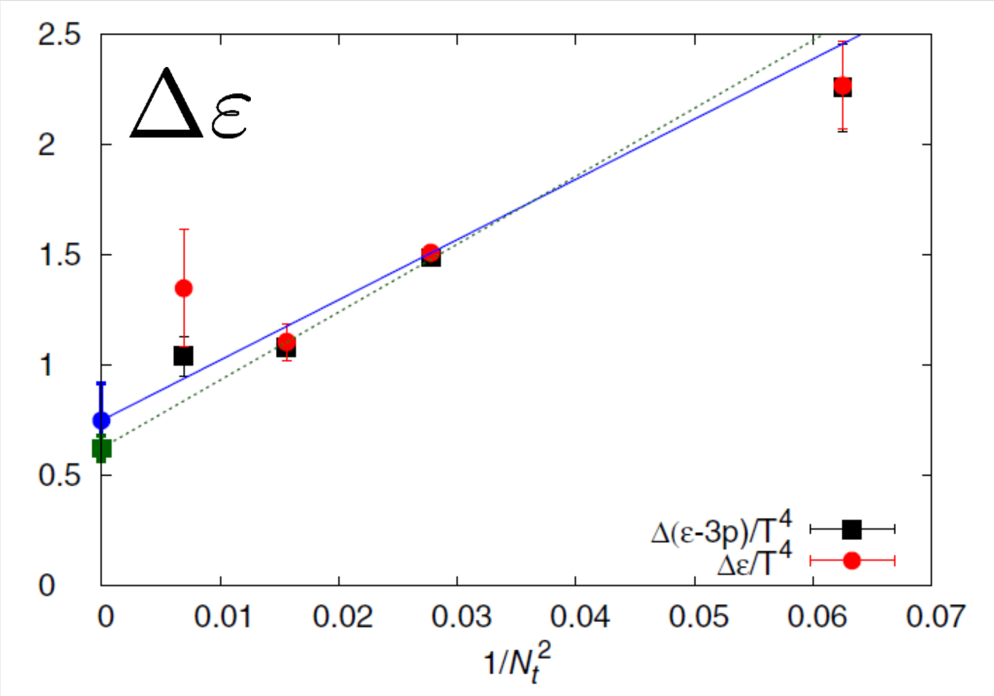
histogram in plaquette - Polyakov loop plane at $T=T_c$

- 48^3x8
- multi-point reweighting

Classification of configuration into low/high phases is possible

Result in Differential Method

Shirogane+, PRD (2016)

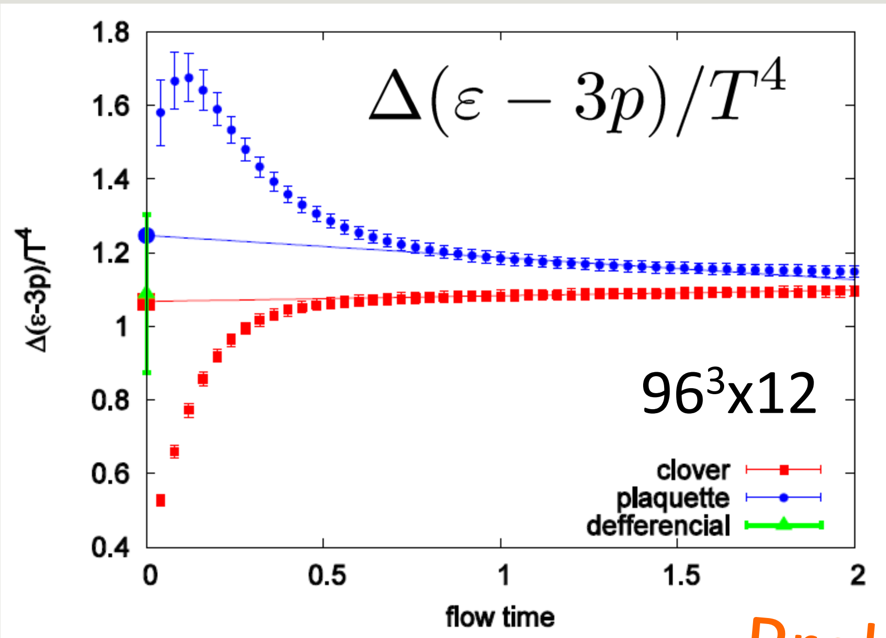


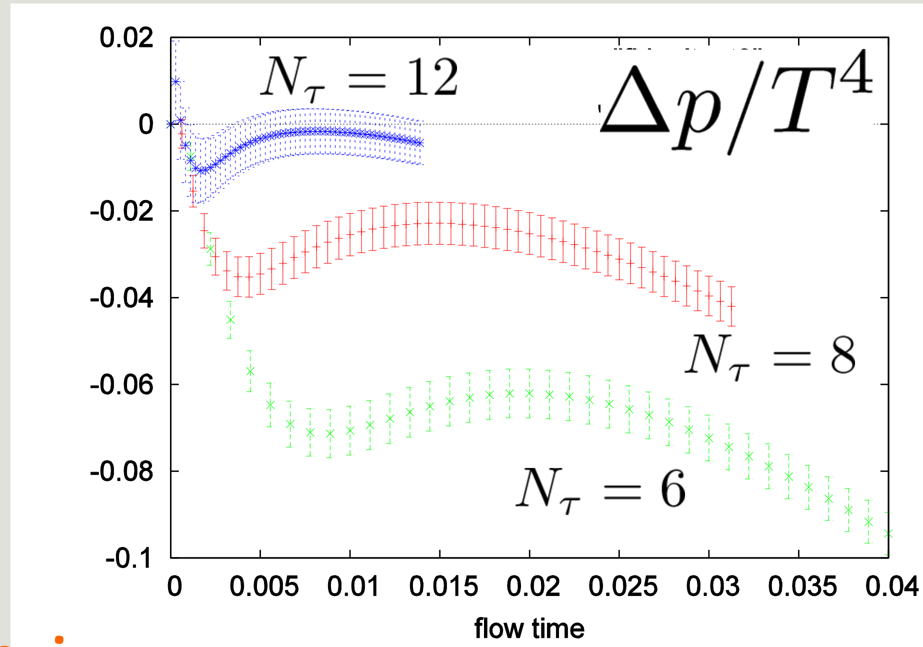
- Nt=4, 6, 8, 12
- nonperturbatively determined Karsch coefficients
- continuum extrapolation

- no pressure gap within statistics

Gradient Flow Method

$p/T^4 \simeq 0.22\ (T = T_c)$





t/a^2 Preliminary tT^2

- Latent heats in two methods agree with each other.
- Statistical error is drastically reduced by GF method.
- Nonzero pressure gap exists, but it becomes smaller for larger N_t.

➡ Visit Shirogane's poster

Correlation Functions

FlowQCD, in preparation

Zero-momentum correlator

$$\bar{T}_{\mu\nu}(\tau) = \int d^3x \big(T_{\mu\nu}(x,\tau) - \langle T_{\mu\nu} \rangle\big)$$

Conservation Law

$$\frac{\partial}{\partial \tau}\bar{T}_{00} = 0$$

$$\frac{\partial}{\partial \tau}\bar{T}_{01} = 0$$

$$\frac{\partial}{\partial \tau}\langle \bar{T}_{00}(\tau)\bar{T}_{00}(0)\rangle = 0$$

$$\frac{\partial}{\partial \tau}\langle \bar{T}_{00}(\tau)\bar{T}_{11}(0)\rangle = 0$$

$$\frac{\partial}{\partial \tau}\langle \bar{T}_{01}(\tau)\bar{T}_{01}(0)\rangle = 0$$

$$(\tau \neq 0)$$

Linear Response Relations

$$c_V = \frac{d}{dT}\langle E \rangle = \frac{\langle \bar{T}_{00}^2 \rangle}{VT^2}$$

Specific heat

$$s = \frac{d}{dT}P = \frac{\langle \bar{T}_{11}\bar{T}_{00} \rangle}{VT^2}$$

entropy density

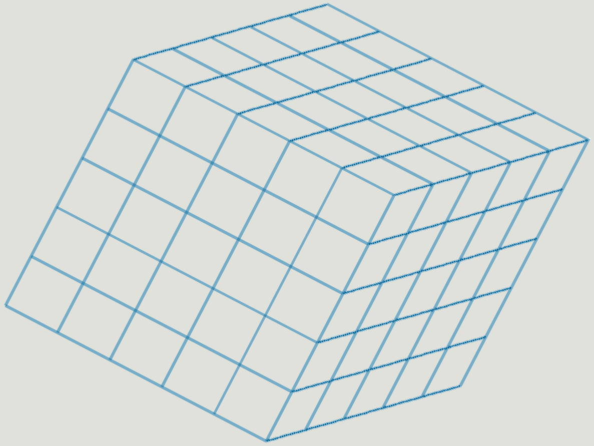
Giusti, Meyer, 2011

$$\varepsilon + p = \frac{\langle \bar{T}_{01}^2 \rangle}{VT}$$

enthalpy density

Numerical Simulation

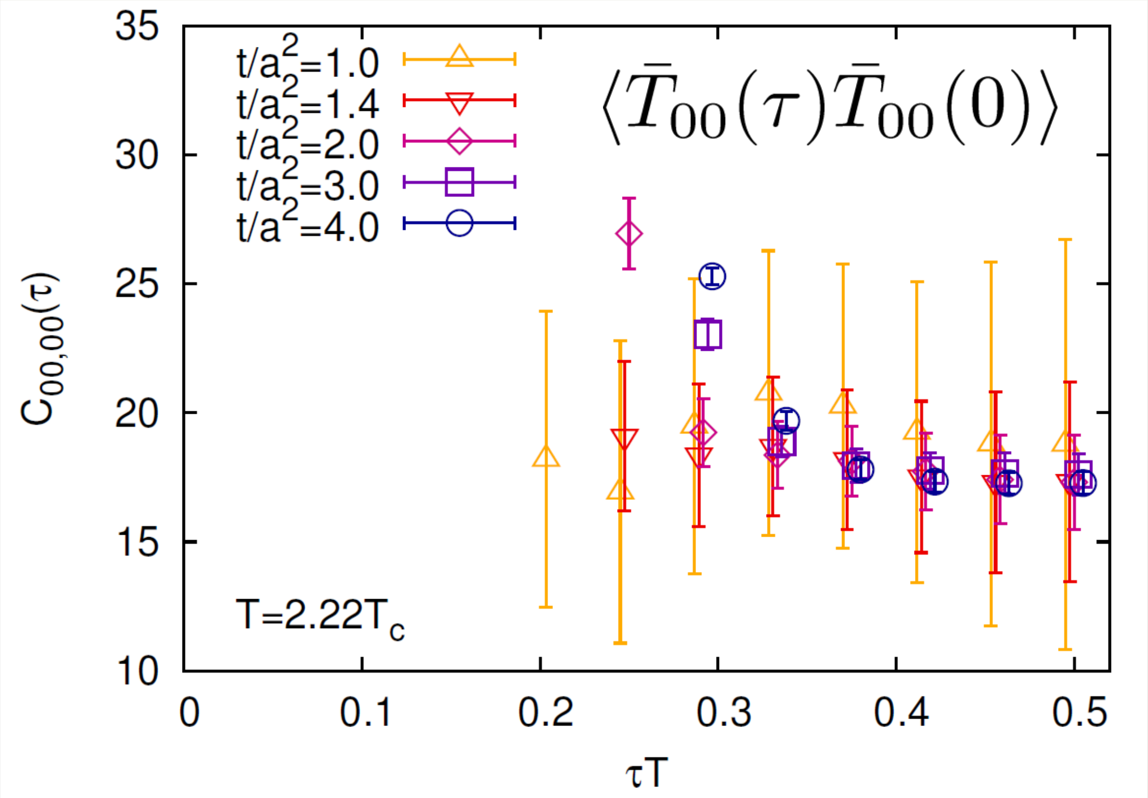
- Wilson gauge action
- clover operator
- aspect ratio $N_s/N_t=4$
- 200,000 configurations



β	$T=1.67T_c$	$T=2.22T_c$
48^3x12	6.719	6.943
64^3x16	6.941	7.170
96^3x24	7.265	7.500

Numerical analysis: Bluegene/Q @KEK

Energy-energy correlator

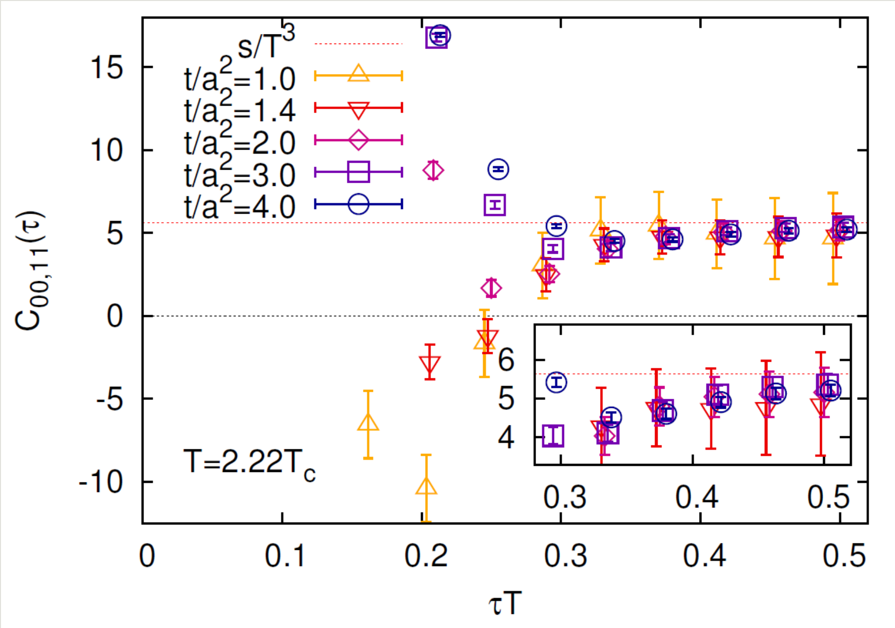


Nt=24
T=2.22Tc

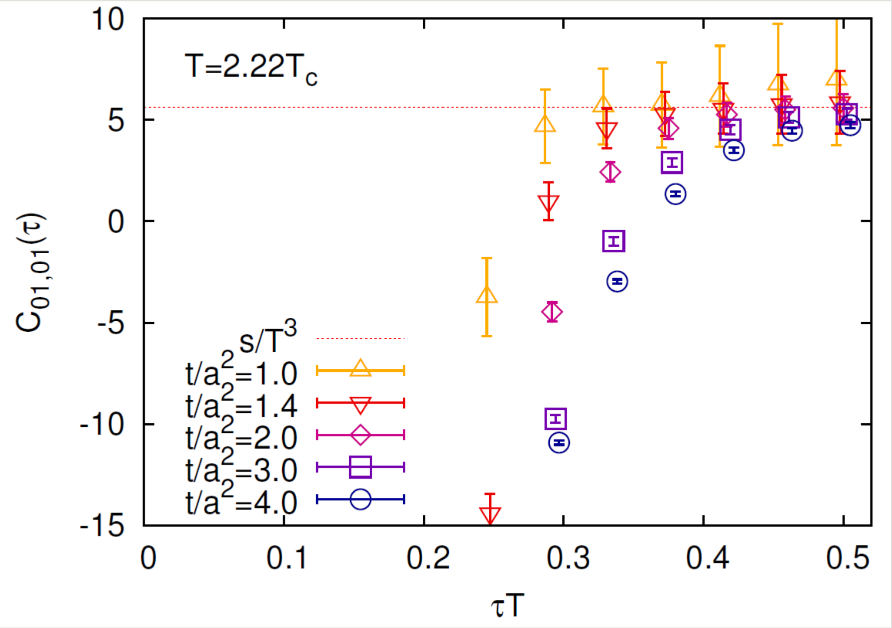
- Existence of a plateau → energy conservation
- Larger flow time ←→ smaller error
- Oversmearing for $\tau < 2\sqrt{8t}$

Correlation Functions

$$\langle \bar{T}_{00}(\tau)\bar{T}_{11}(0)\rangle$$



$$\langle \bar{T}_{01}(\tau)\bar{T}_{01}(0)\rangle$$



- The value of constant is consistent with $(e+p)/T^4$.
- Oversmearing effect is much stroner.

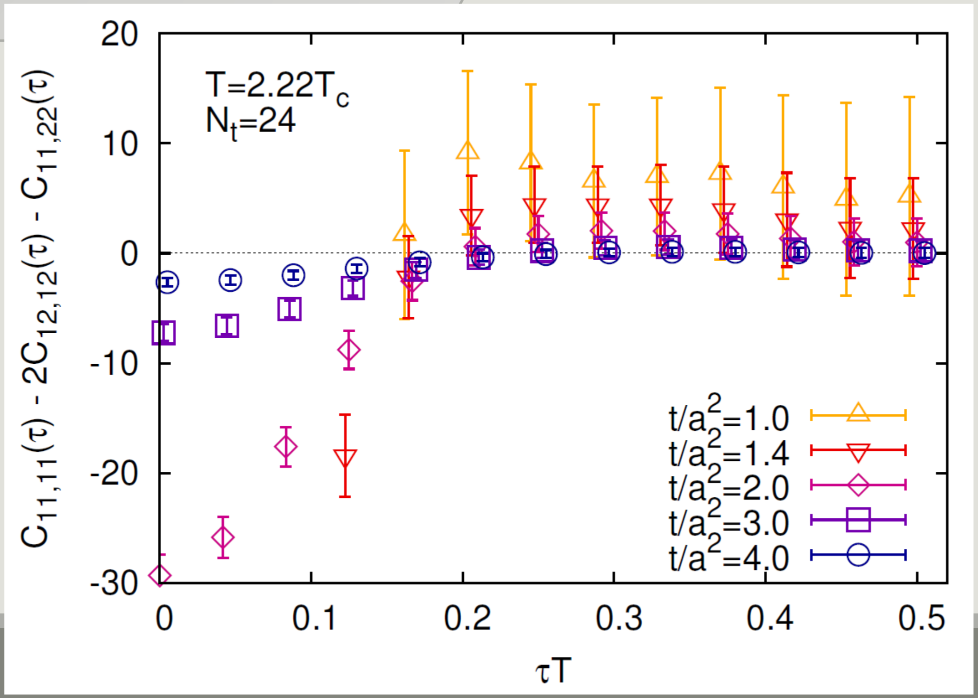
Constraint from Rot. Symm.

$$\langle \bar{T}_{ij}\bar{T}_{kl}\rangle = A\delta_{ij}\delta_{kl} + B(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$i,j,k,l = 1,2,3$$

$$\langle \bar{T}_{11}\bar{T}_{11}\rangle - \langle \bar{T}_{11}\bar{T}_{22}\rangle - 2\langle \bar{T}_{12}\bar{T}_{12}\rangle = 0$$

Karsch, Wyld, 1987



zero within statistics
for τ/a>4

Summary

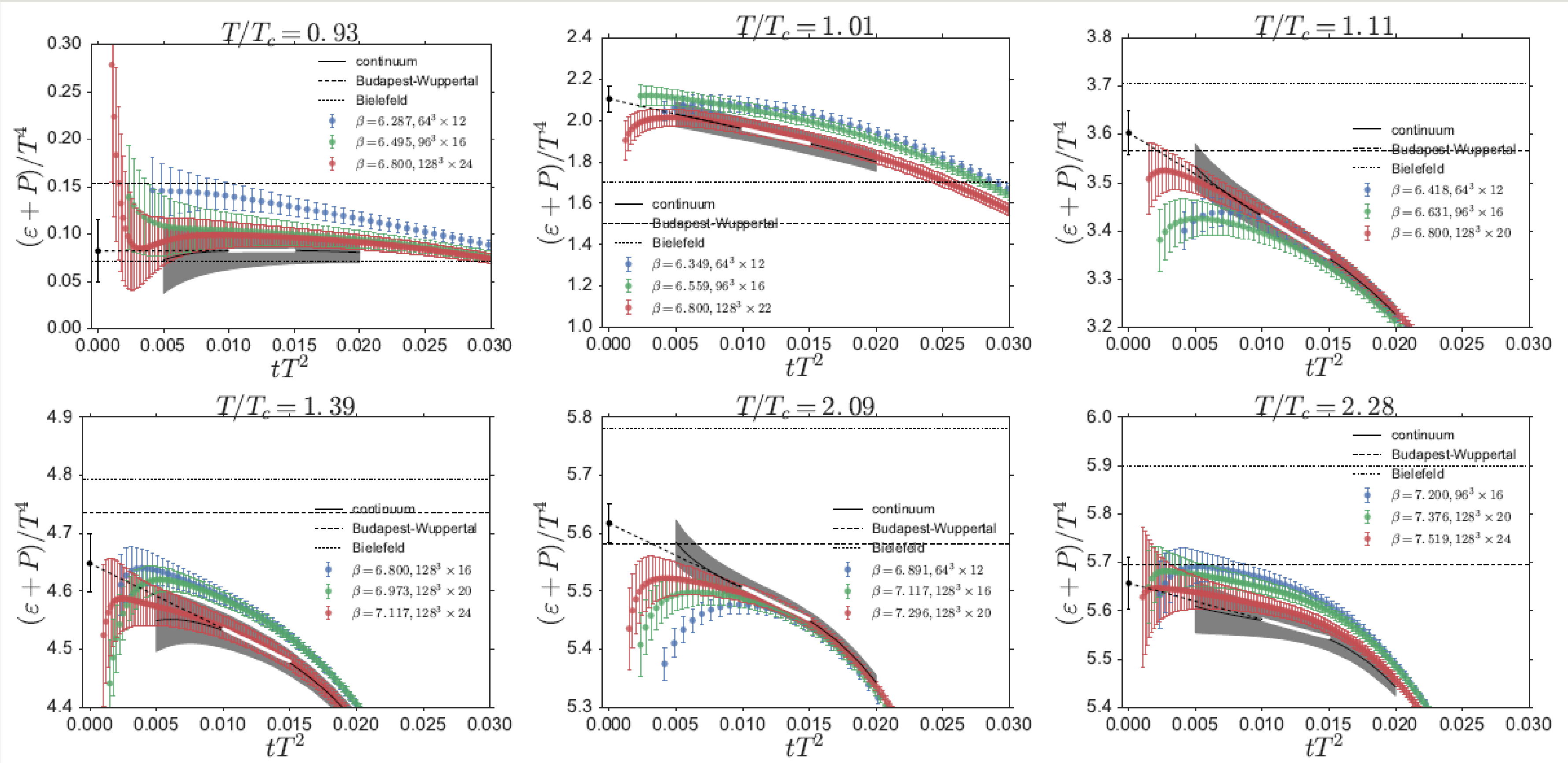
- We measured EMT expectation value and correlation function of SU(3) gauge theory by using the operator constructed from gradient flow.

- Expectation values of EMT agrees well with the previous results on thermodynamics.

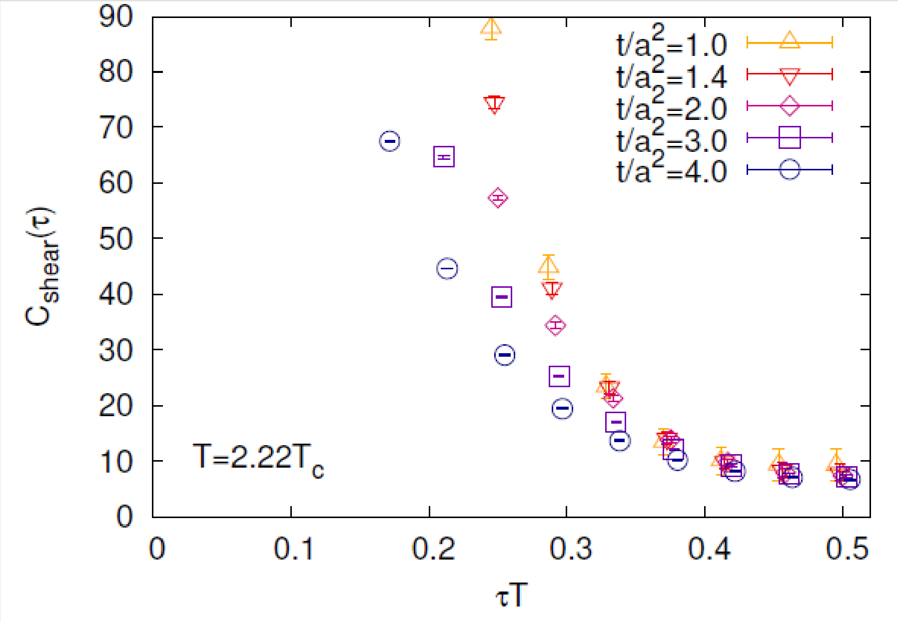
- Behaviors of the correlation functions are consistent with the conservation law and linear response relations.

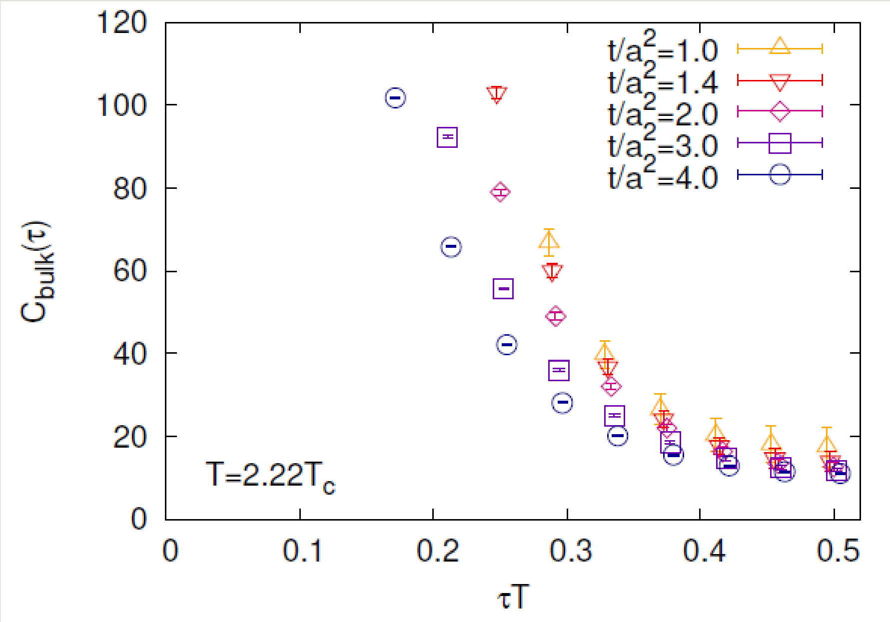
- Our method can be applied to full QCD → Next talk

Double Extrapolation



Shear and Bulk Channels





- No plateau behavior in non-conserved channels