

# Dynamical evolution of critical fluctuations and its observation in heavy ion collisions

Masakiyo Kitazawa  
(Osaka U.)

**Sakaida, Asakawa, Fujii, MK, arXiv:1703.08008**

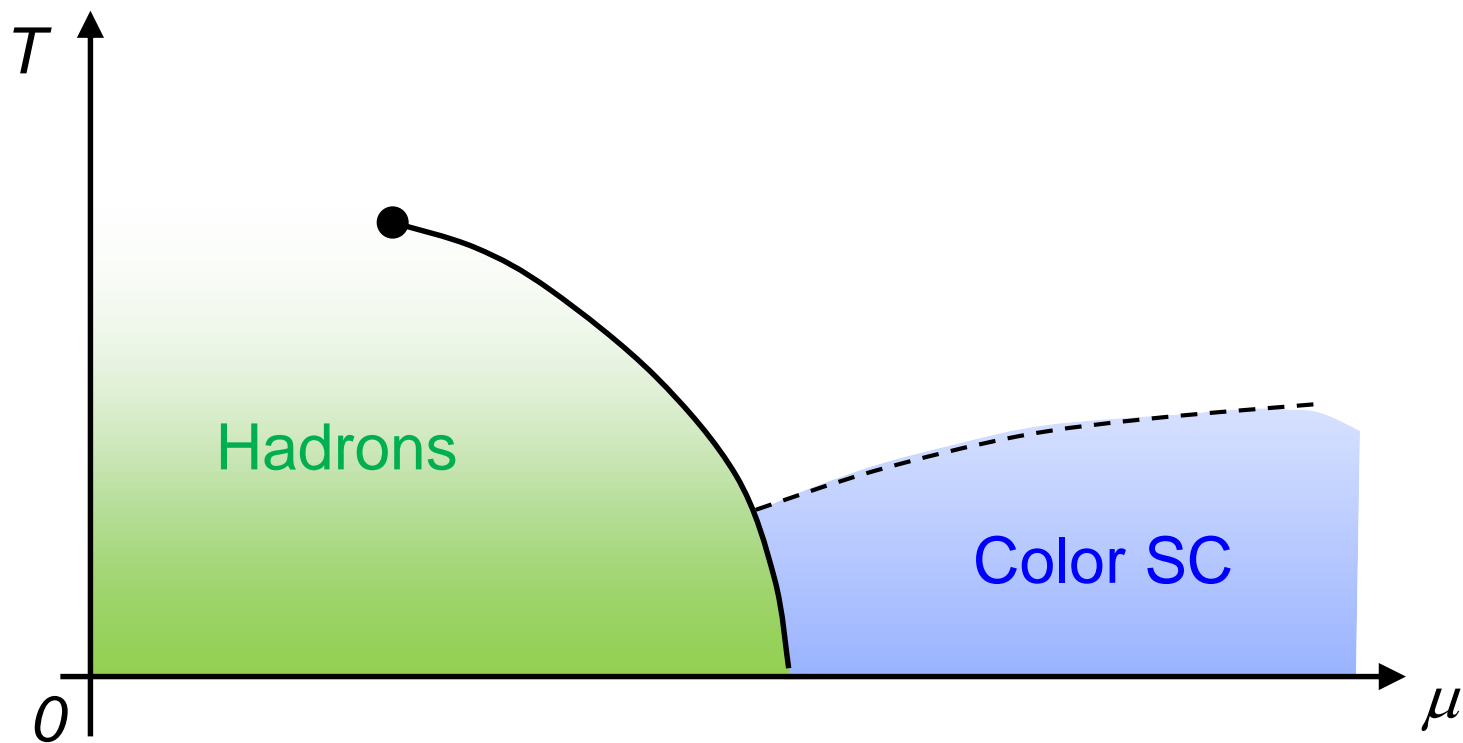
See also,

**Asakawa, MK, Prog. Part. Nucl. Phys. 90, 299 (2016)**

Ohnishi, MK, Asakawa, Phys. Rev. C94, 044905 (2016)

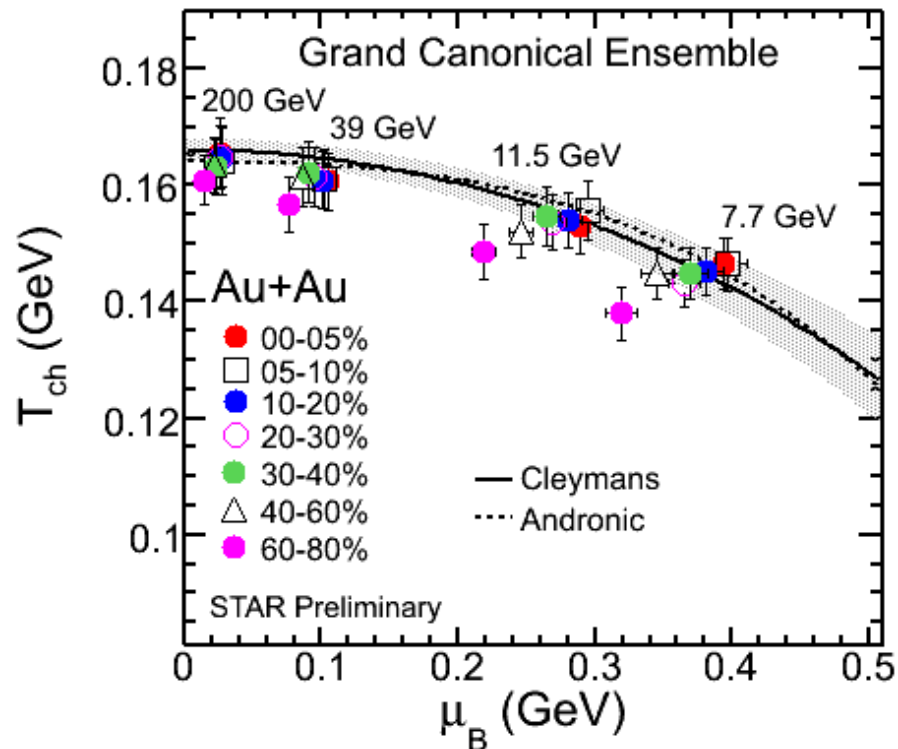
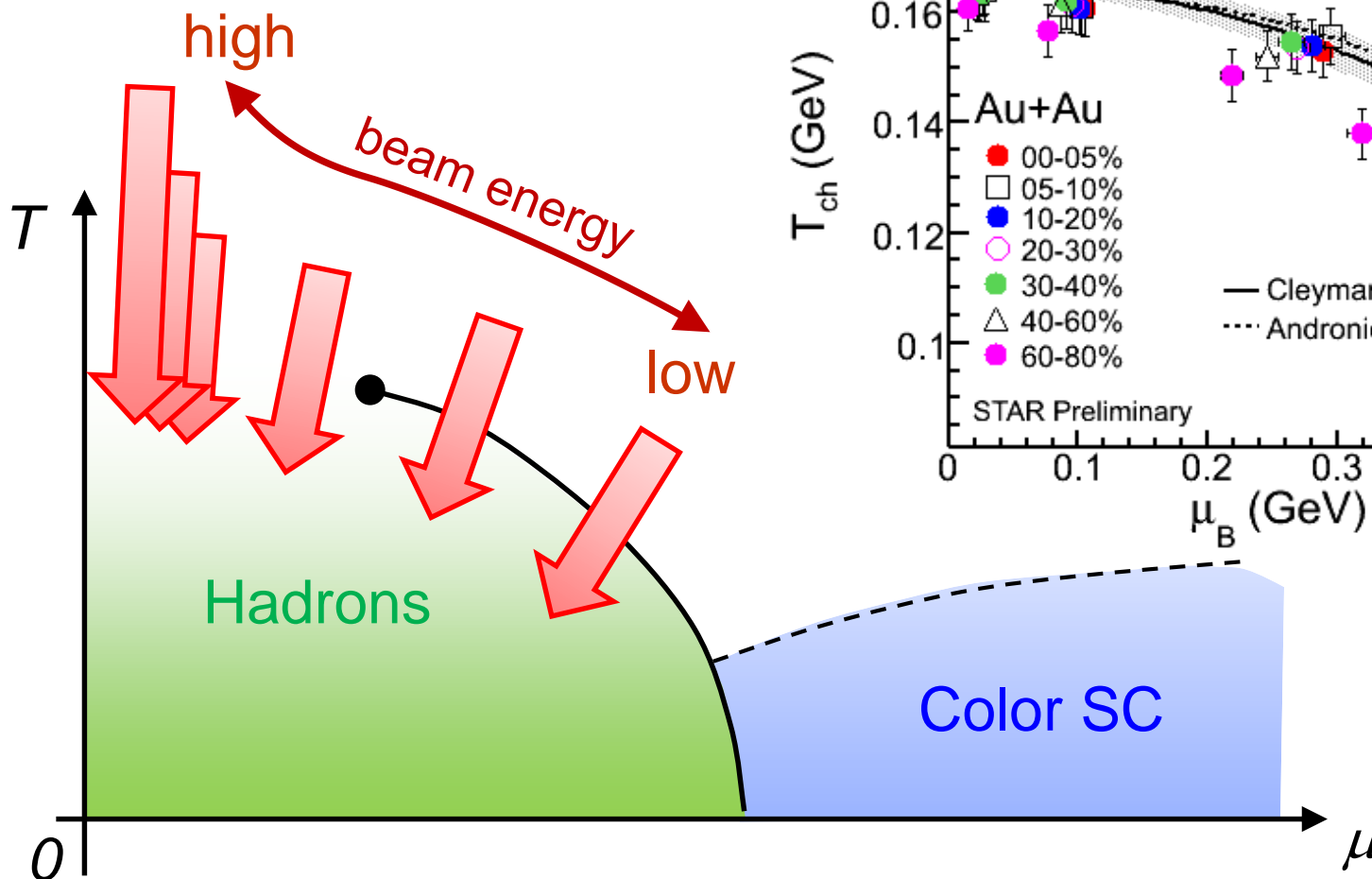
MK, Nucl. Phys. A942, 65 (2015)

# Beam-Energy Scan



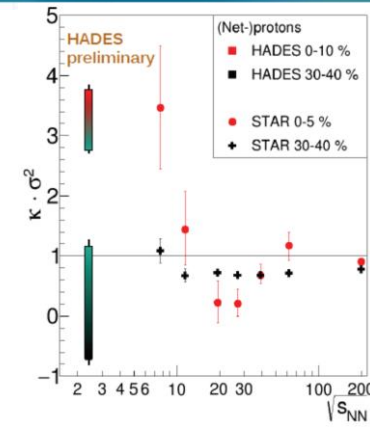
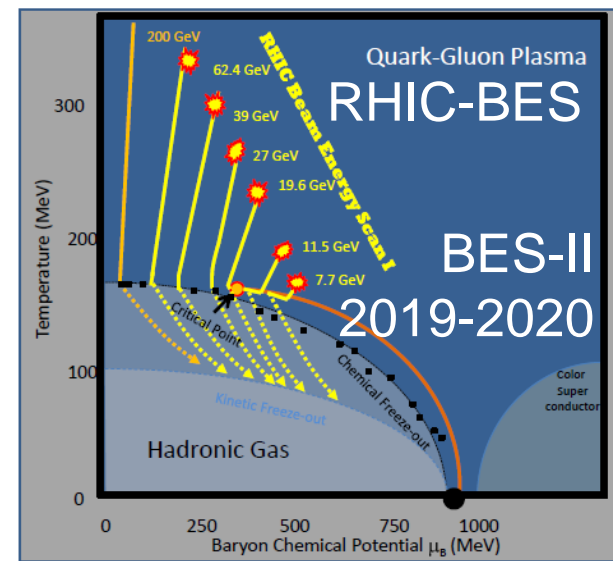
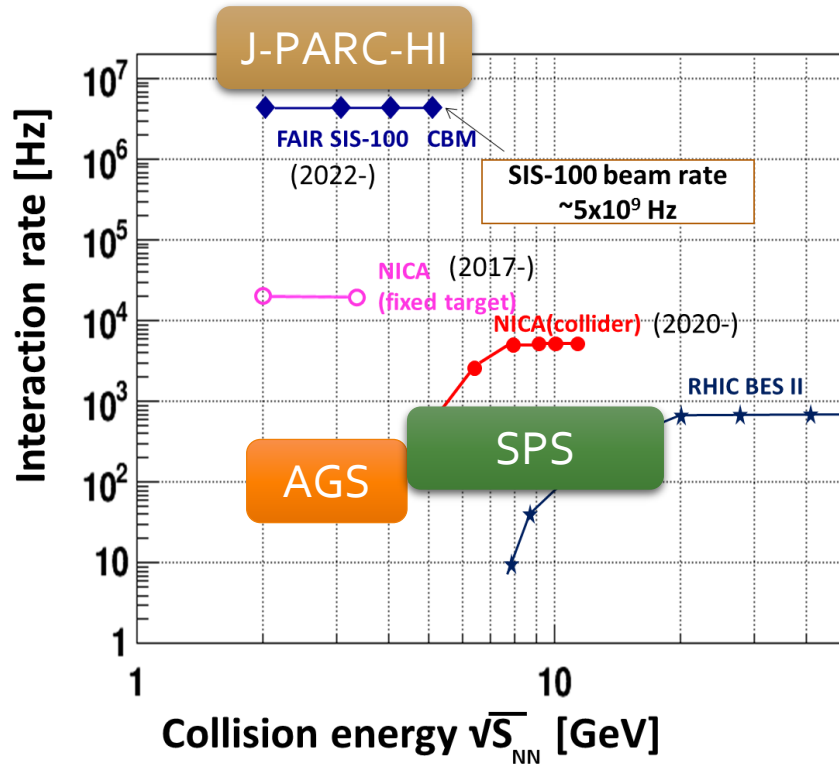
# Beam-Energy Scan

STAR 2012



# Beam-Energy Scan

Active experimental researches/plans for the beam-energy scan

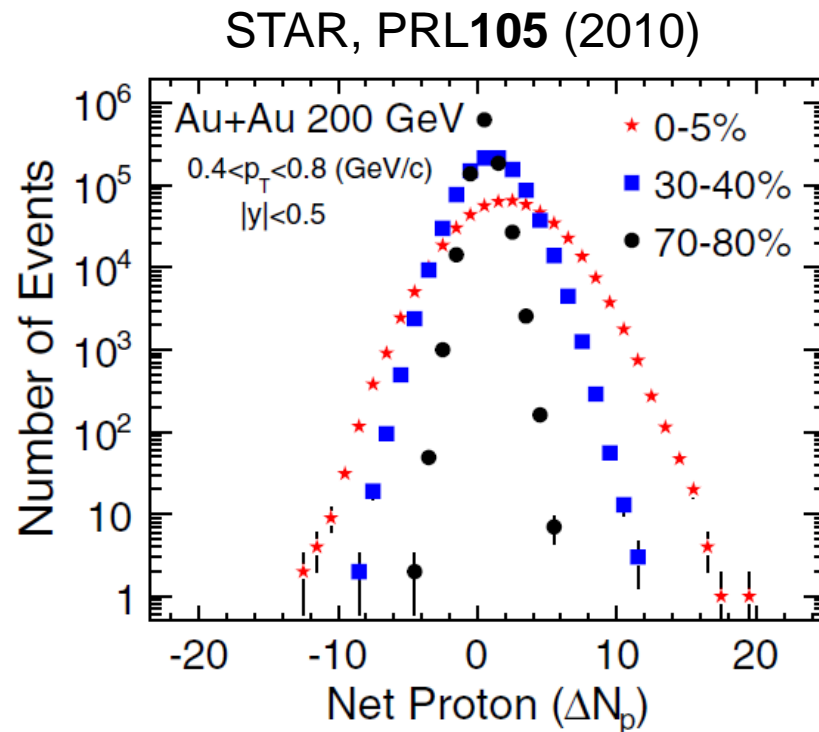
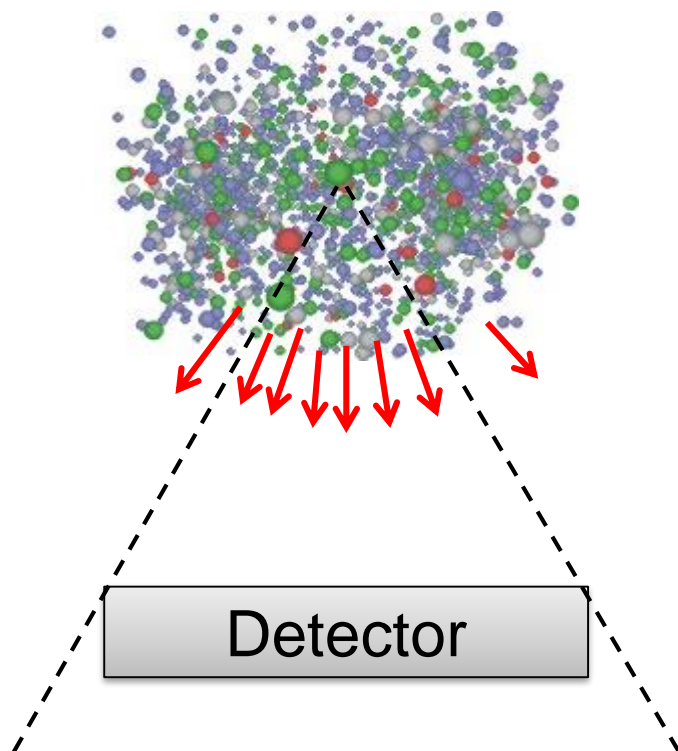


➔ Search for QCD phase structure / critical point

# Event-by-Event Fluctuations

Review: Asakawa, MK, PPNP **90** (2016)

Fluctuations can be measured by e-by-e analysis in experiments.



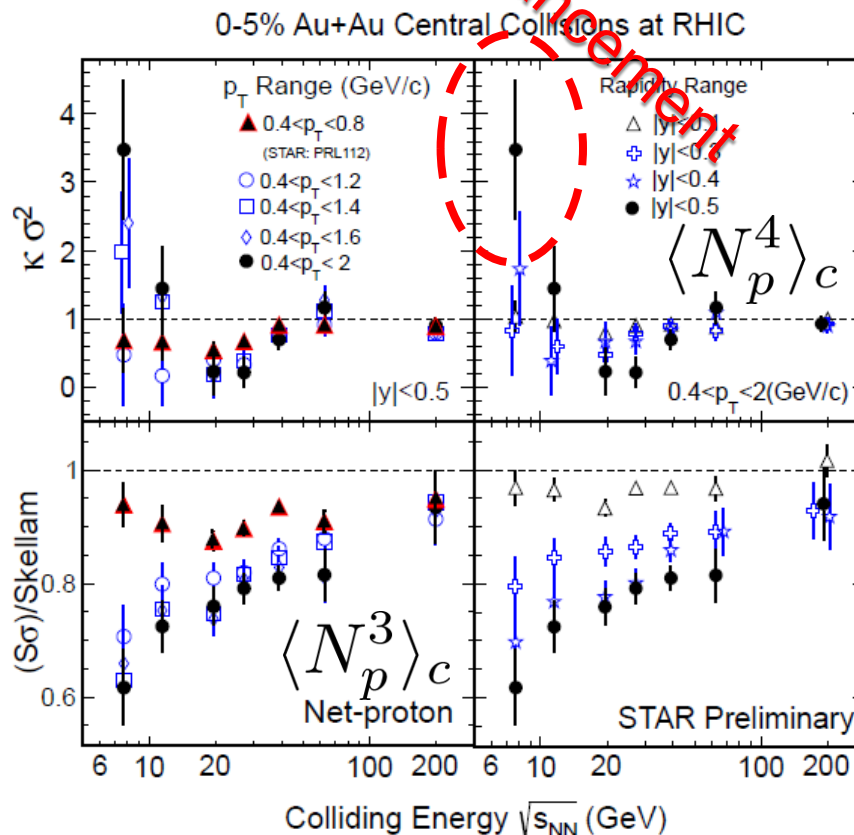
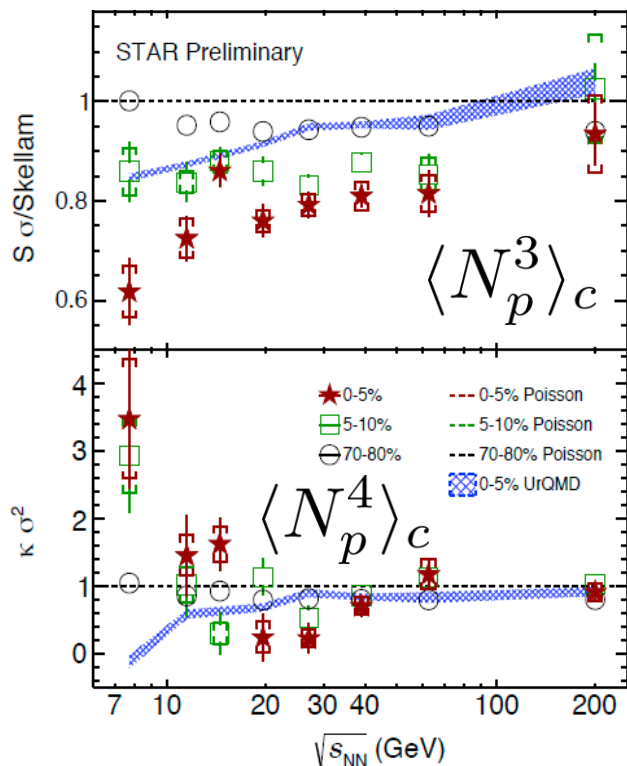
Cumulants

$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$



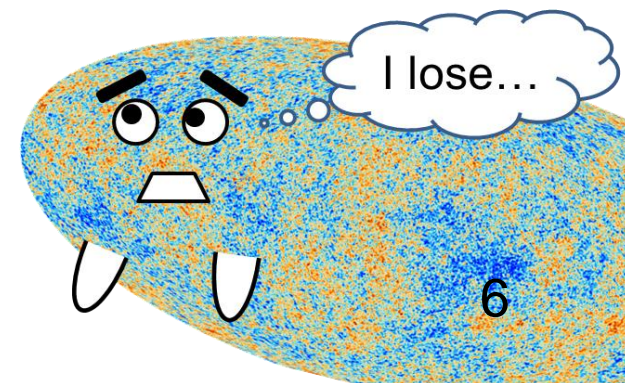
# Higher-Order Cumulants

STAR Collab.  
2010~



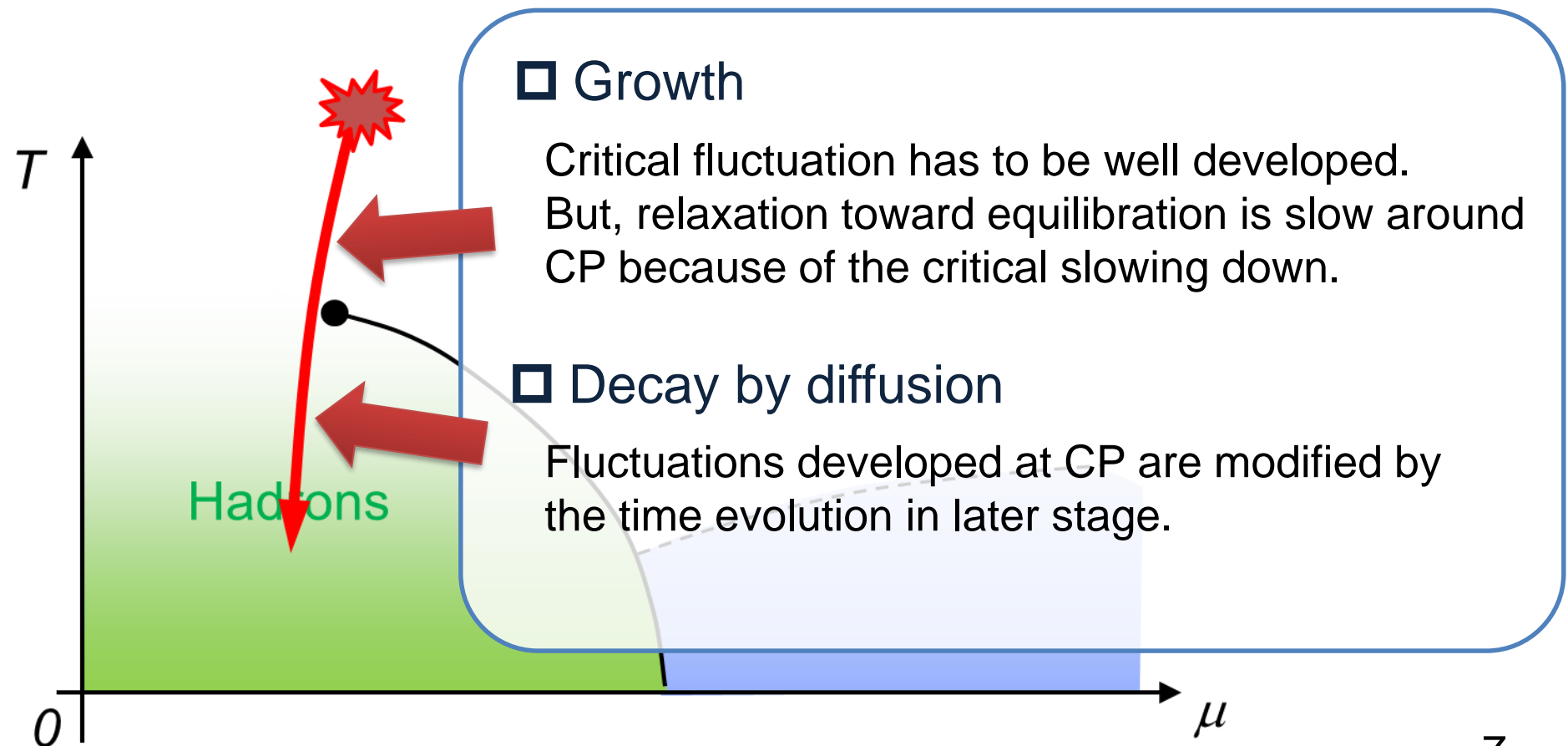
**Non-zero non-Gaussian** cumulants  
have been established!

Have we measured critical fluctuations?



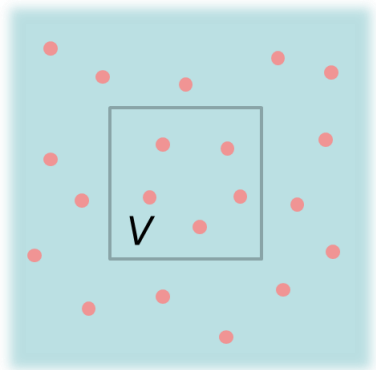
# Remarks on Critical Fluctuation 1

Experiments cannot observe critical fluctuation in equilibrium directly.



# Fluctuations: Theory vs Experiment

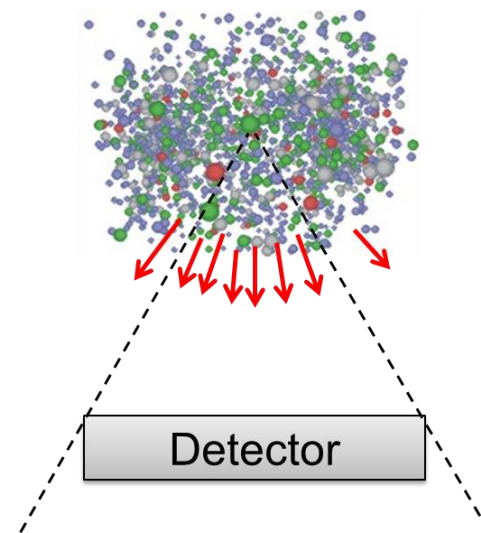
Theoretical analyses  
based on statistical mechanics



lattice, critical point,  
effective models, ...

Fluctuation in  
a spatial volume

Experiments



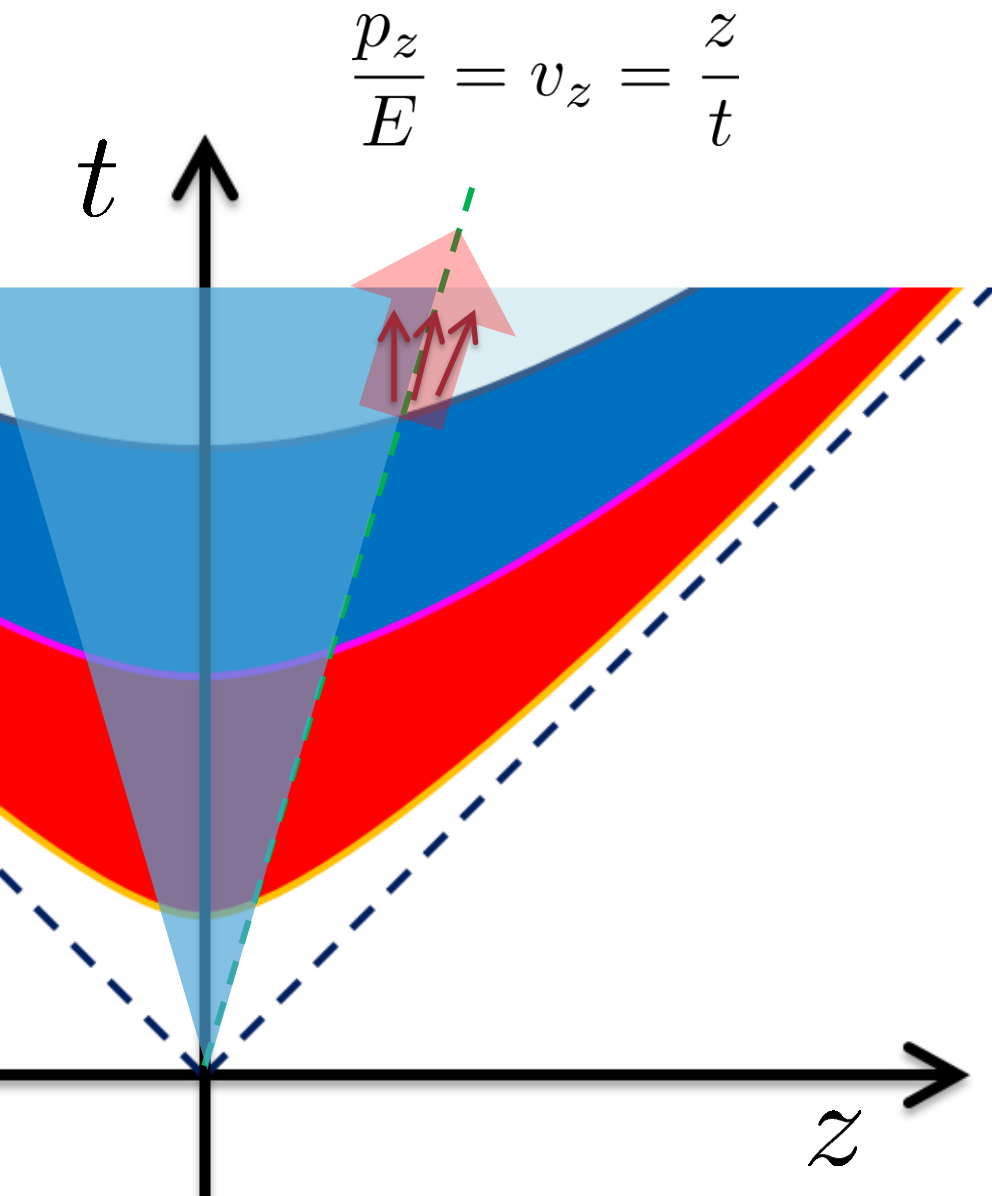
Fluctuations in  
a momentum space

discrepancy in phase spaces



# Thermal Blurring

Ohnishi, MK, Asakawa,  
PRC94, 044905 (2016)



Under Bjorken picture,

coordinate-space rapidity  $Y$

||

momentum-space rapidity  $y$   
of **medium**

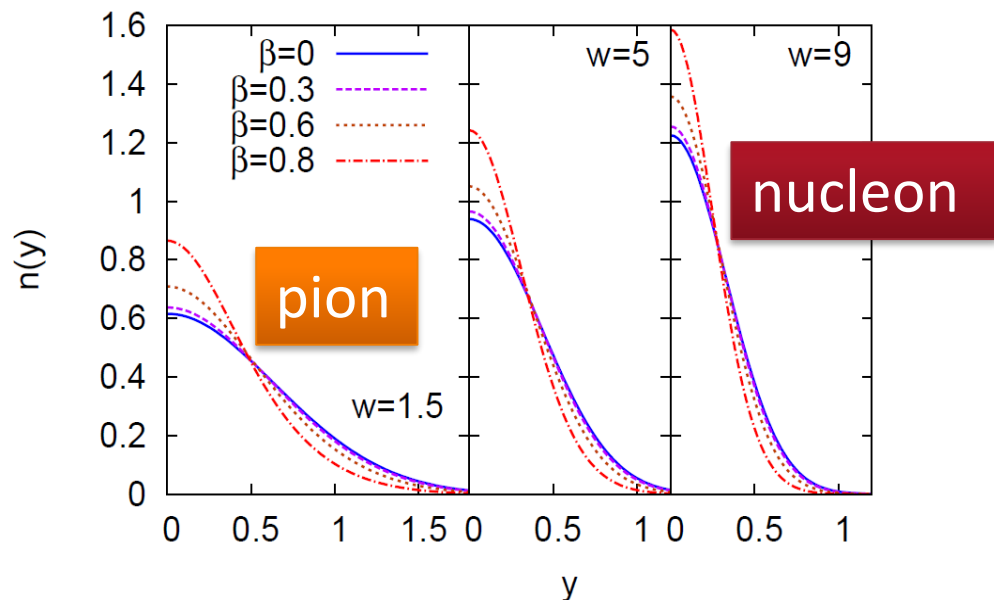
|}

momentum-space rapidity  $y$   
of **individual particles**

$$\Delta y \simeq \Delta Y$$

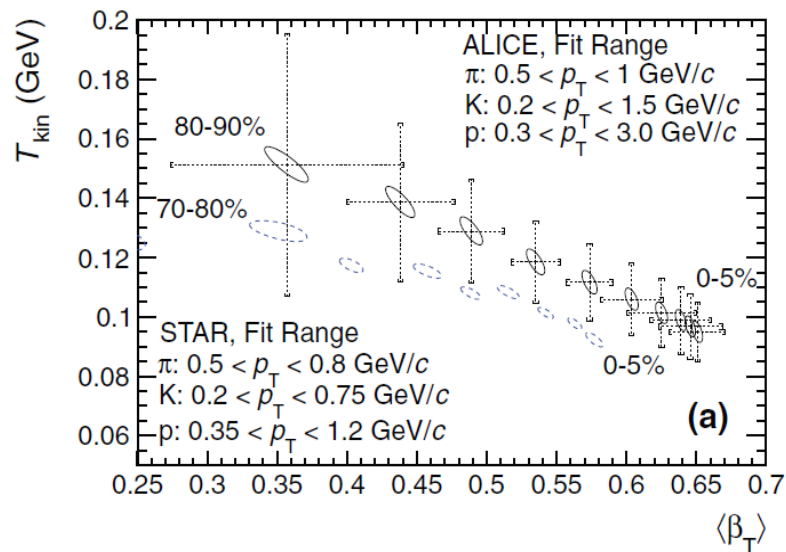
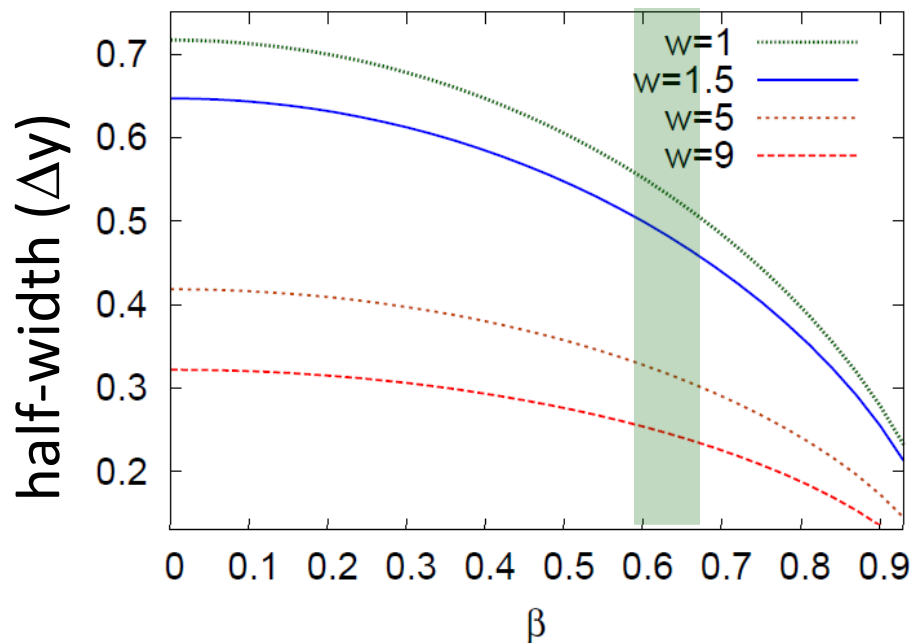
# Thermal distribution in y space

Ohnishi, MK, Asakawa,  
PRC94, 044905 (2016)



$$w = \frac{m}{T}$$

- pions  $w \simeq 1.5$
- nucleons  $w \simeq 9$



- blast wave
- flat freezeout surface<sup>10</sup>

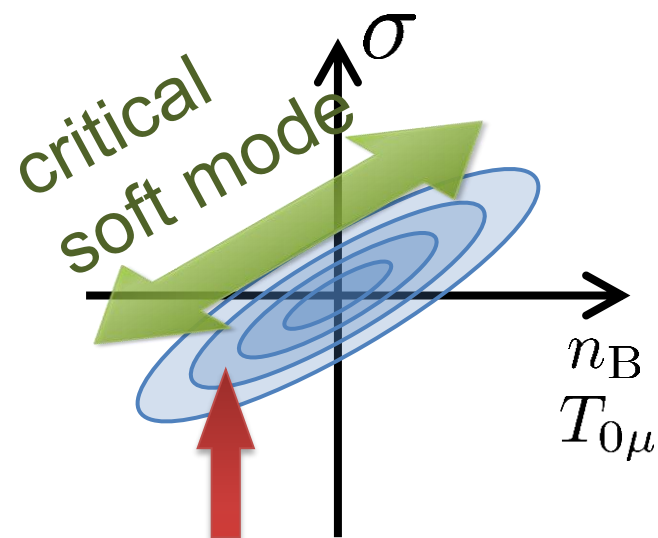
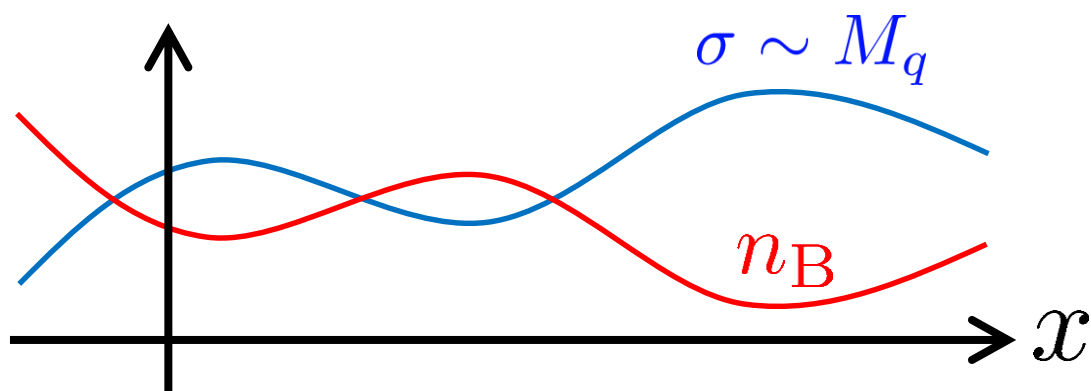
# Remarks on Critical Fluctuation 2

Critical fluctuation is a conserved mode!

Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

Fluctuations of  $\sigma$  and  $n_B$  are coupled around the CP!

$$\delta\sigma \simeq \delta n_B$$



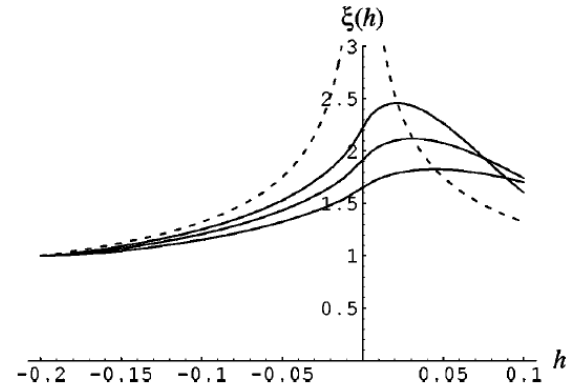
$\sigma$ : fast damping

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \dots$$

# Dynamical Evolution of Critical Fluctuations

## □ Evolution of correlation length

Berdnikov, Rajagopal (2000)  
Asakawa, Nonaka (2002)

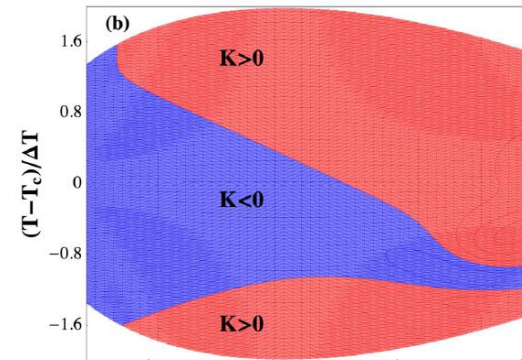


## □ Higher orders (spatially uniform “ $\sigma$ ” mode)

Mukherjee, Venugopalan, Yin (2015)

## □ Dynamical evolution in chiral fluid model

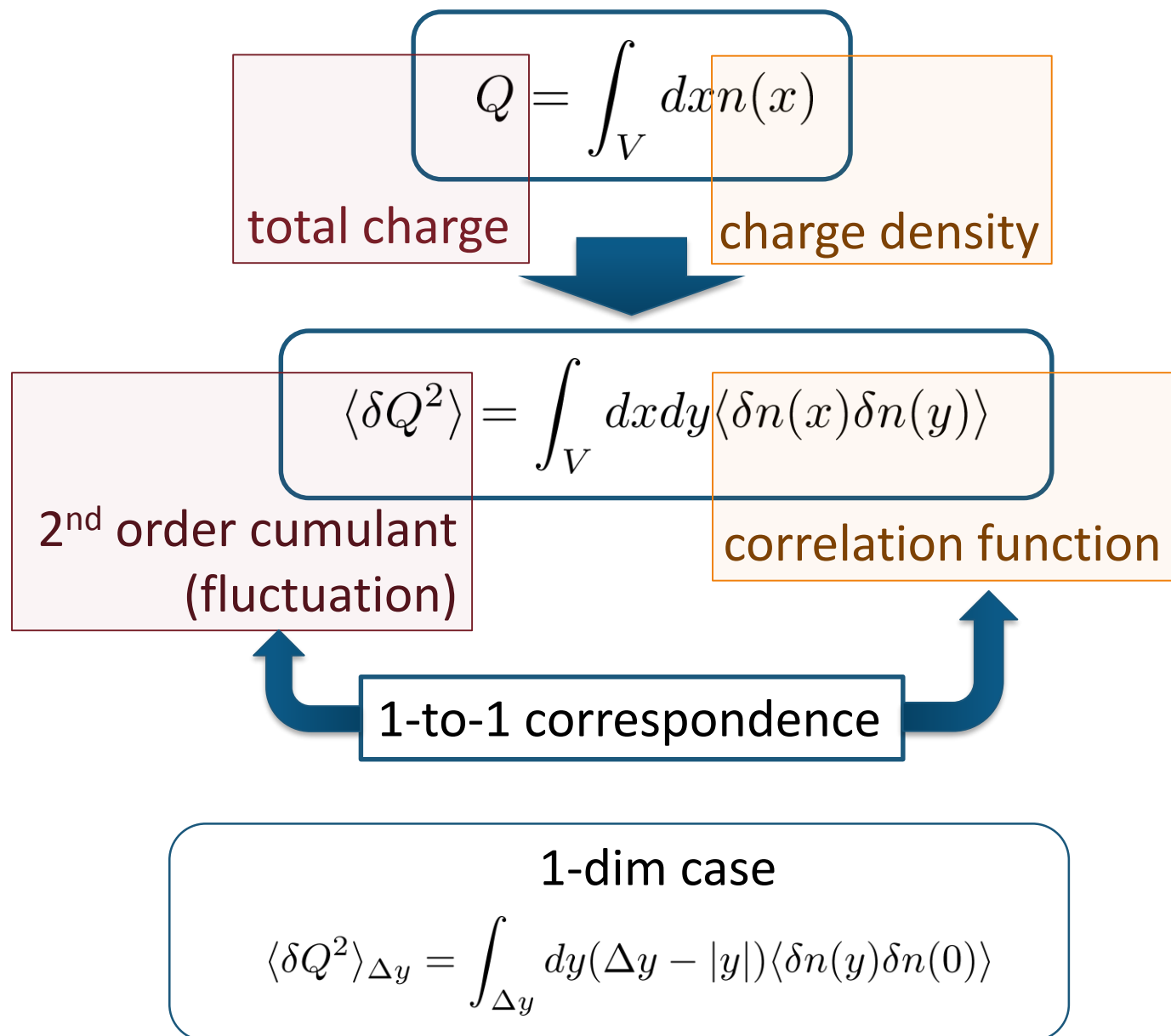
Nahrgang, Herold, ... (2014~)



## □ Correlation functions

Kapusta, Torres-Rincon (2012)

# Cumulants and Correlation Function



# Aim of This Study

- ❑ Describe **conserved nature** of critical fluctuation.
- ❑ We want to study **experimental observables**.
  - ❑ focus on a **conserved charge (baryon number)**
  - ❑ study evolution of **conserved-charge** fluctuation
- ❑ Concentrate on **2<sup>nd</sup> order** fluctuation. (not higher)
- ❑ We study
  - ❑ **rapidity window dependence** of the cumulant
  - ❑ 2-particle **correlation function**

## Our Main Conclusion

Non-monotonicity in  
cumulants or correlation func.

=

Signal of  
QCD-CP

# Stochastic Diffusion Equation (SDE)

## □ Diffusion equation

$$\partial_{\tau} n = D \partial_{\eta}^2 n$$

- Describe a relaxation of a conserved density  $n$  toward uniform state **without fluctuation**

## □ Stochastic diffusion equation

$$\partial_{\tau} n = D \partial_{\eta}^2 n + \partial_{\eta} \xi(\eta, \tau)$$

$$\langle \xi(\eta_1) \xi(\eta_2) \rangle \sim \chi \delta(\eta_1 - \eta_2)$$

- Describe a relaxation toward **fluctuating** uniform state
- $\chi$ : susceptibility (fluctuation in equil.)

# Soft Mode of QCD Critical Point

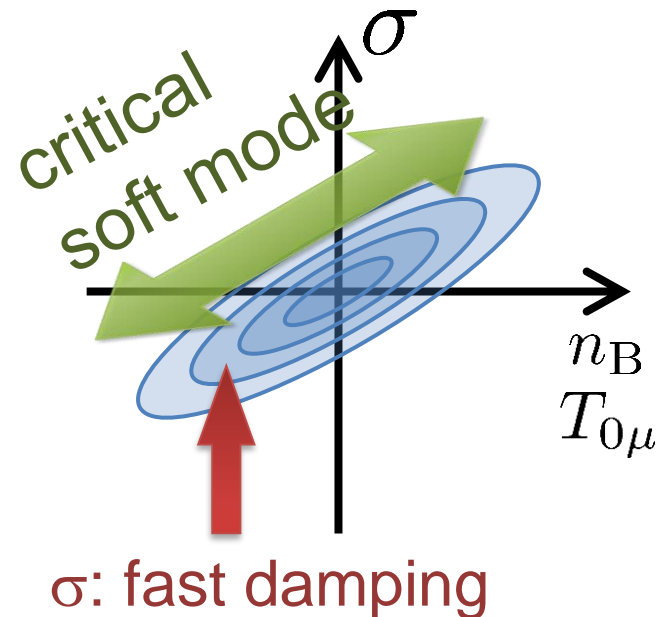
Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

## □ Effective potential

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \dots$$

## □ Time dependent Ginzburg-Landau

$$\begin{pmatrix} \dot{\sigma} \\ \dot{n} \end{pmatrix} = \begin{pmatrix} \Gamma_{\sigma\sigma} & \Gamma_{\sigma n} \\ \Gamma_{n\sigma} & \Gamma_{nn} \end{pmatrix} \begin{pmatrix} \sigma \\ n \end{pmatrix} \sim k^2$$



For slow and long wavelength,

$$\text{SDE} \quad \partial_\tau n = D(\tau) \partial_\eta^2 n + \partial_\eta \xi$$

singularities in  $D(\tau)$  and  $\chi(\tau)$

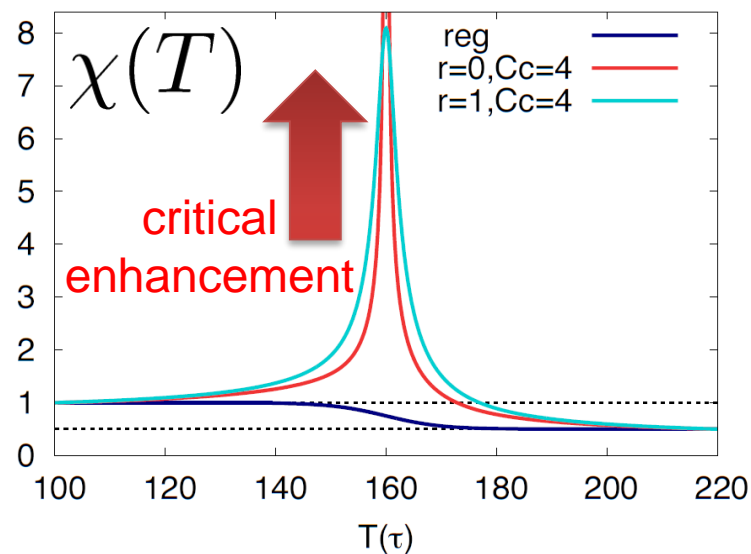
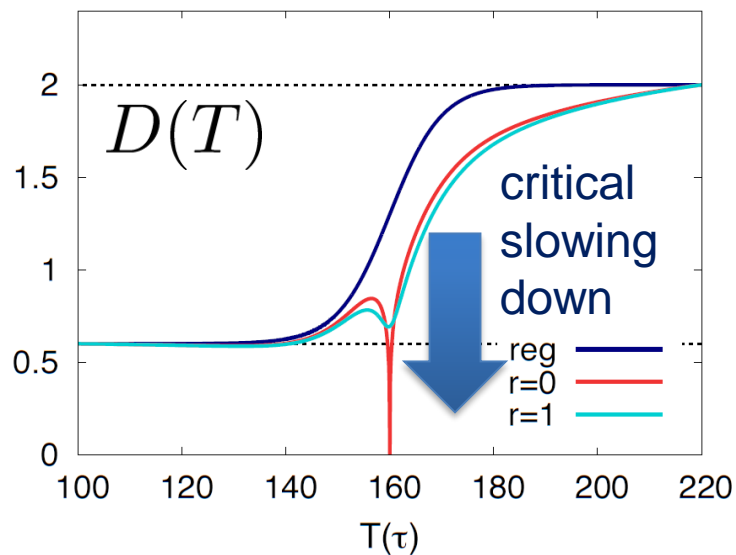
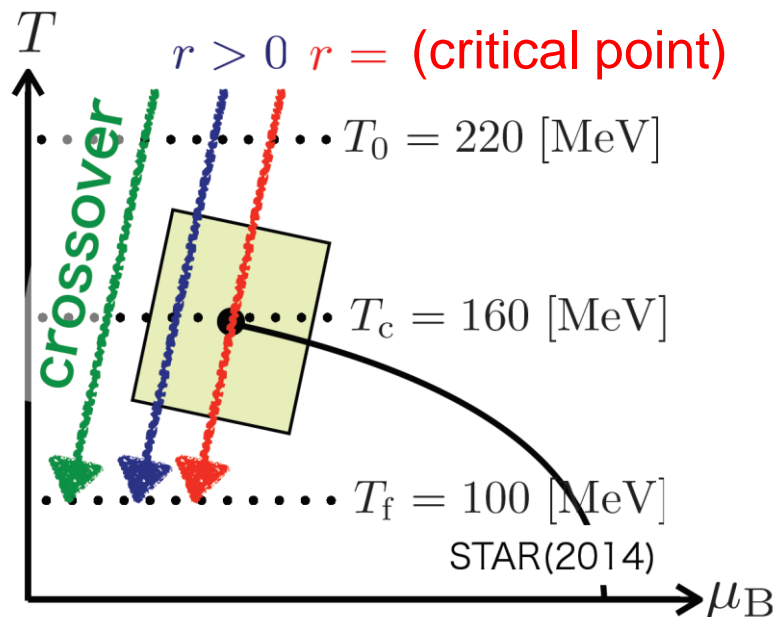


# Parametrizing $D(\tau)$ and $\chi(\tau)$

## □ Critical behavior

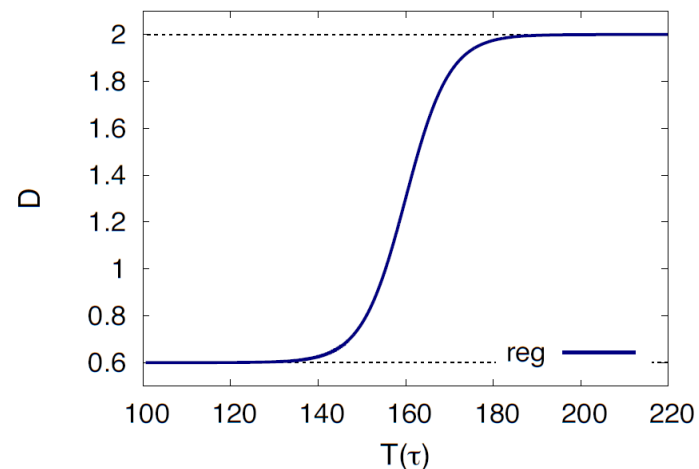
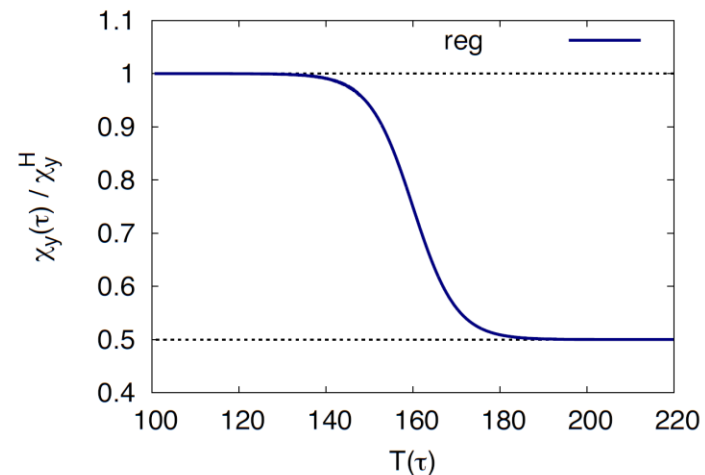
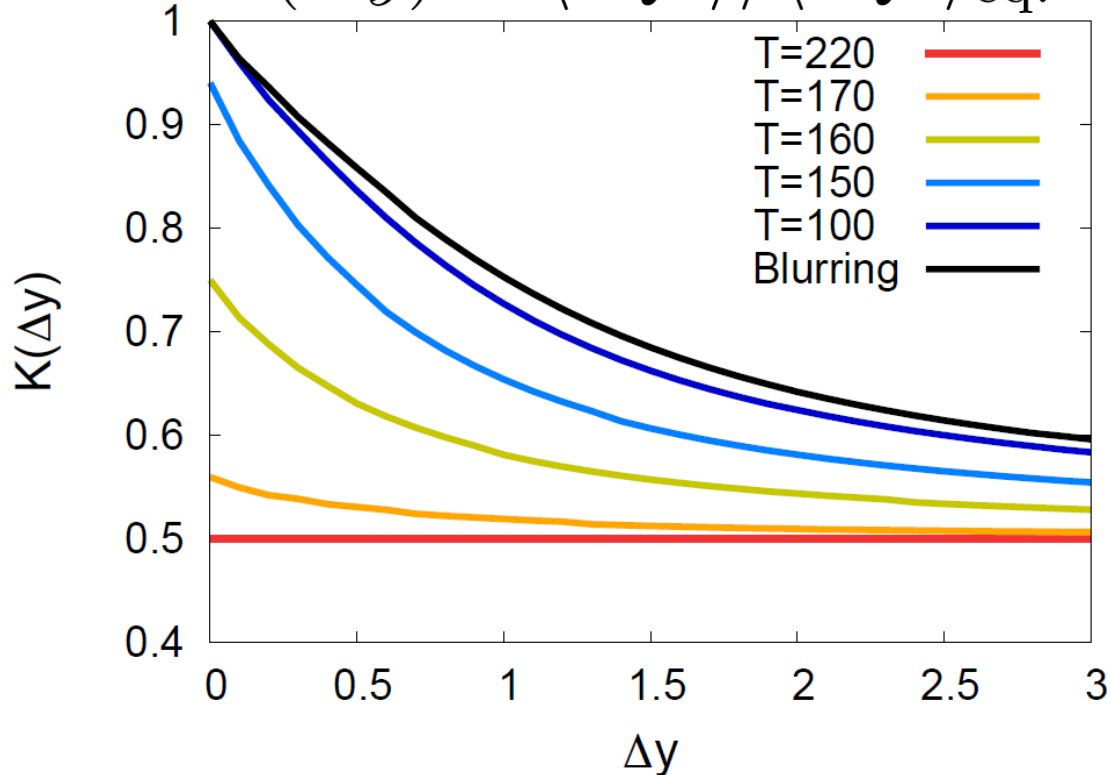
- 3D Ising ( $r, H$ )
- model H

## □ Temperature dep.

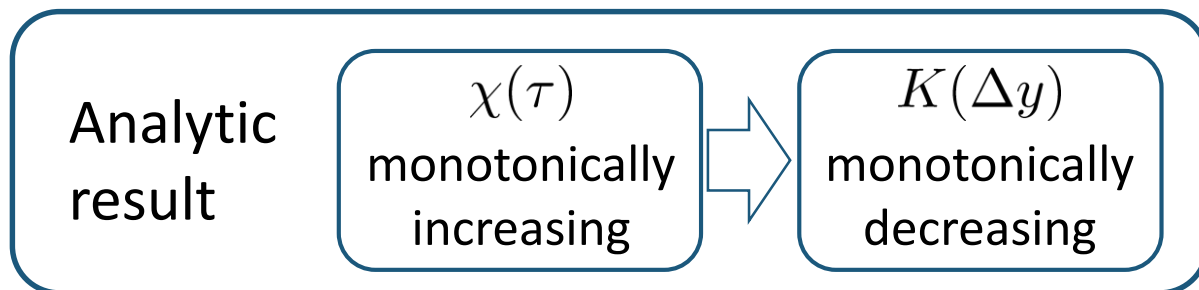


# Crossover / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$

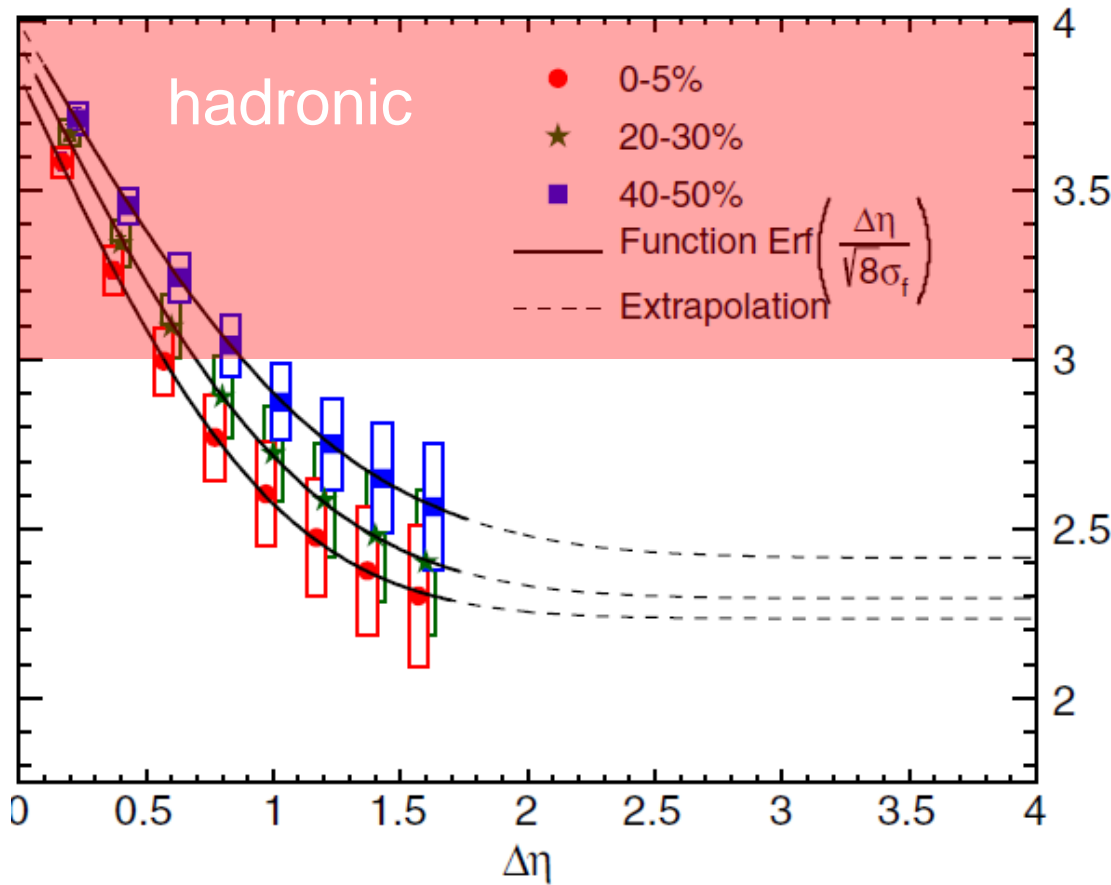


□ monotonically decreasing



# $\Delta\eta$ Dependence @ ALICE

ALICE  
PRL 2013



$\Delta\eta$

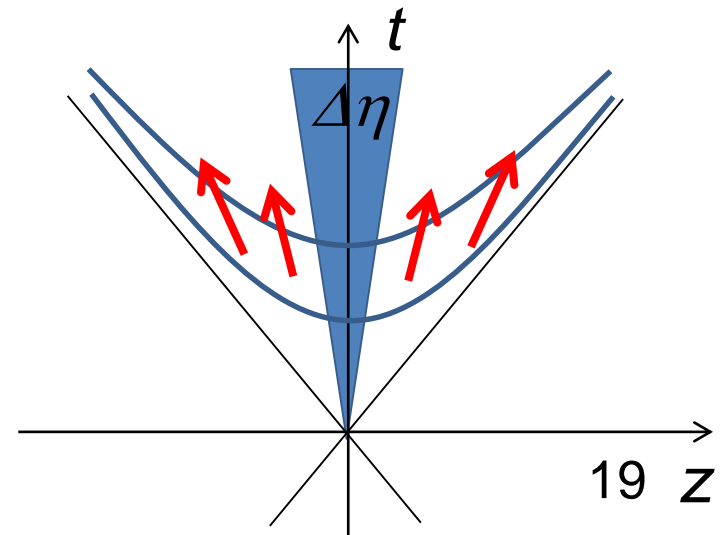
↑

rapidity window

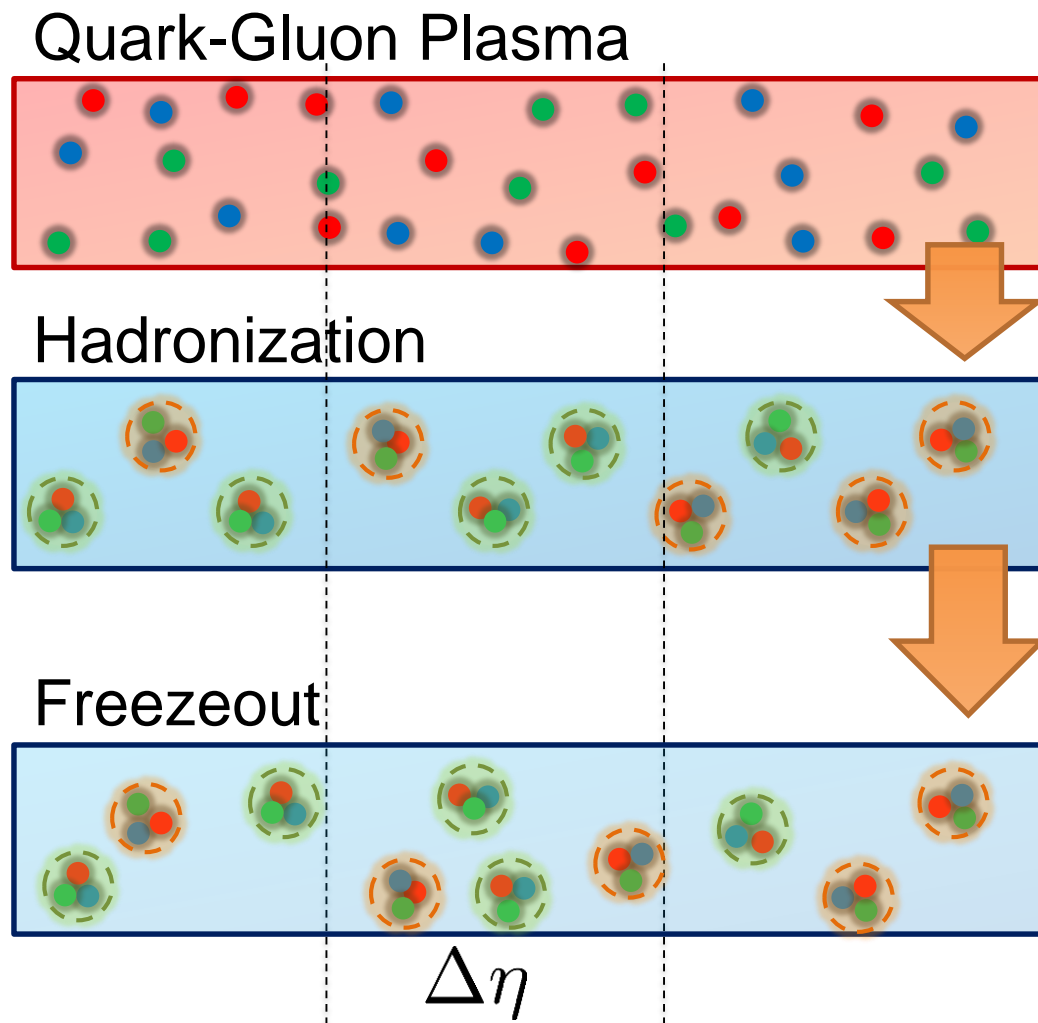
## D-measure

$$D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

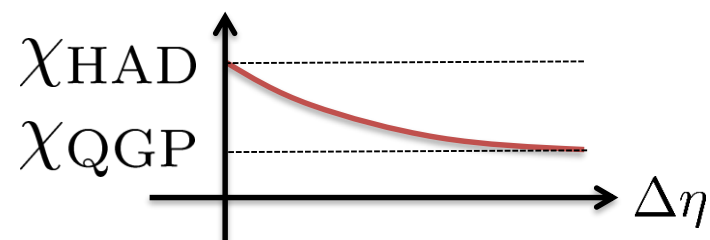
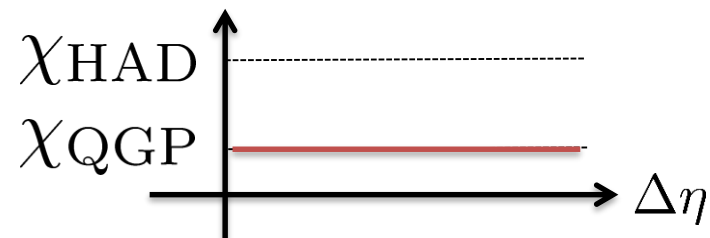
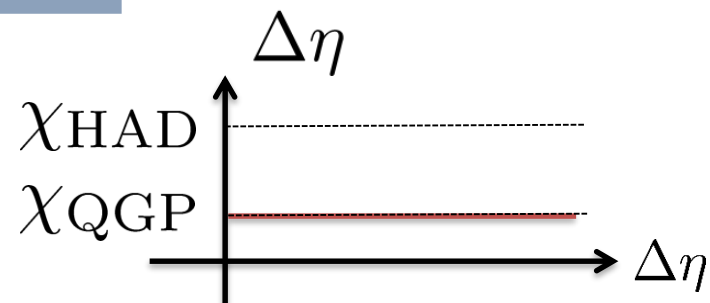
- $D \sim 3-4$  Hadronic
- $D \sim 1-1.5$  Quark



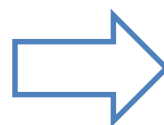
# Time Evolution of Fluctuations



$$\frac{\langle \Delta N^2 \rangle}{\Delta\eta}$$



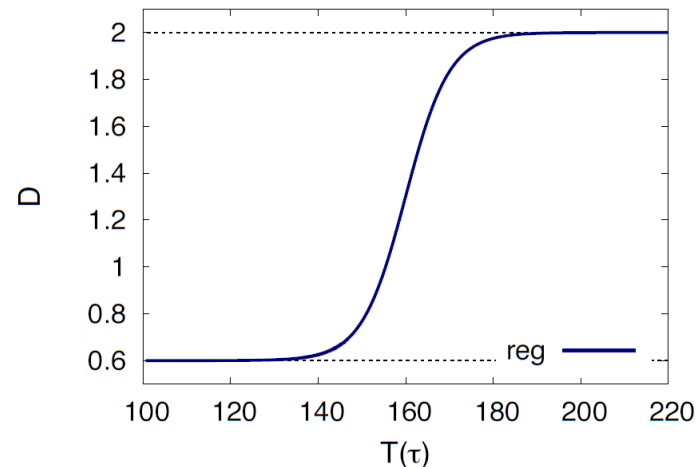
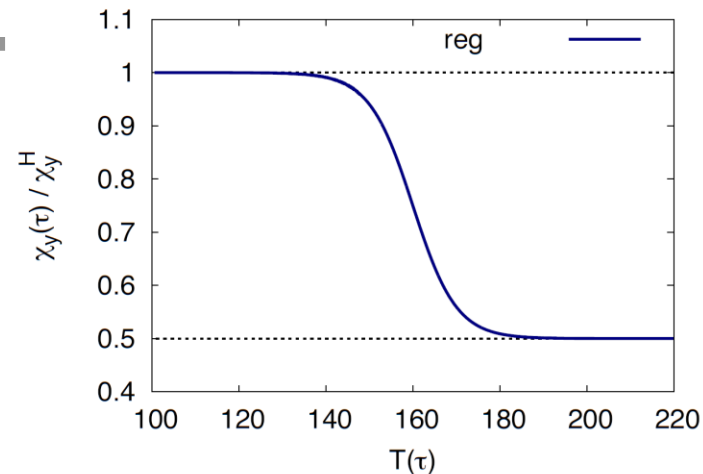
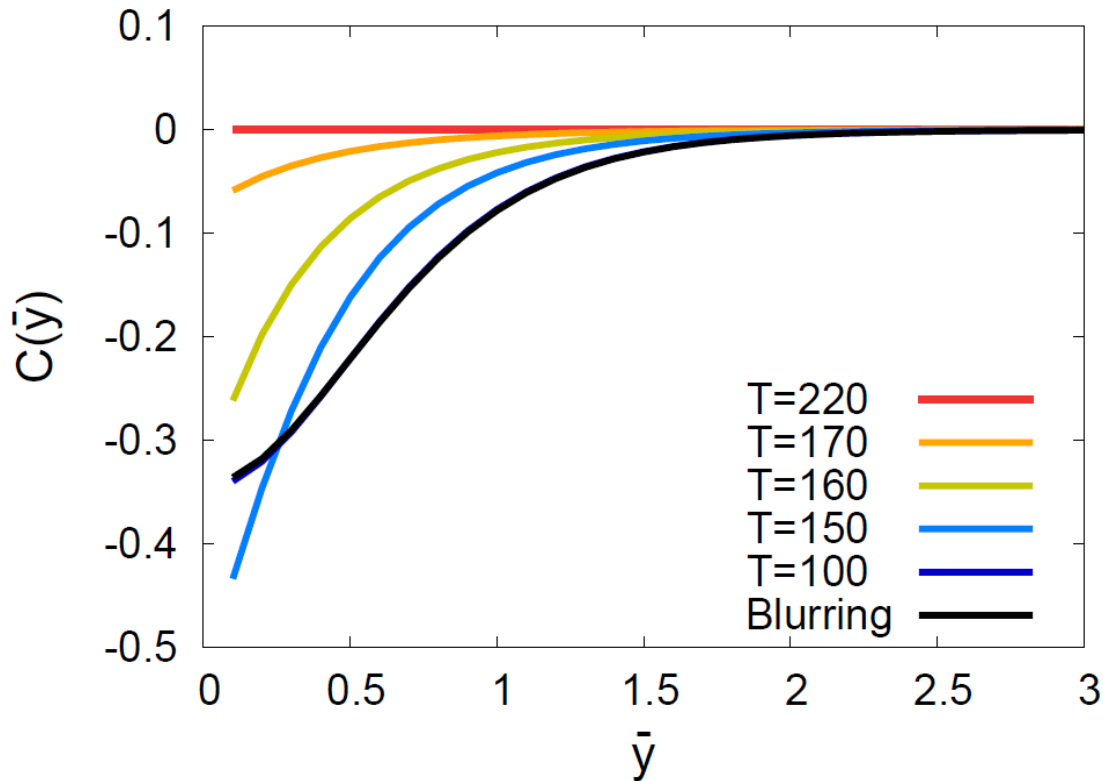
Variation of a conserved charge is achieved only through diffusion.



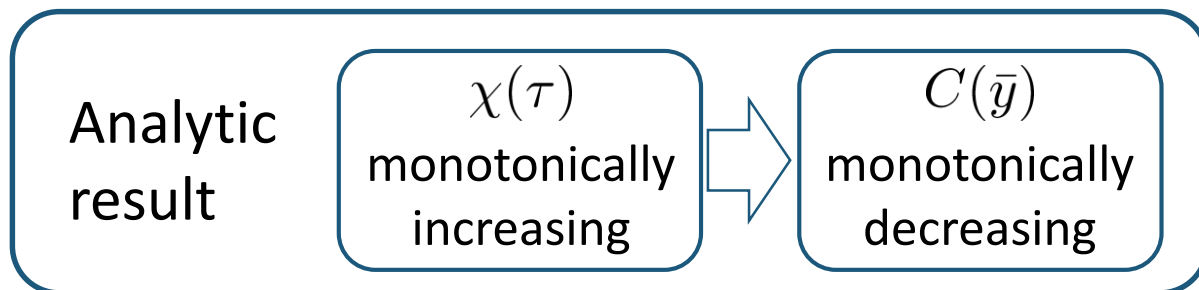
The larger  $\Delta\eta$ ,  
the slower diffusion

# Crossover / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$

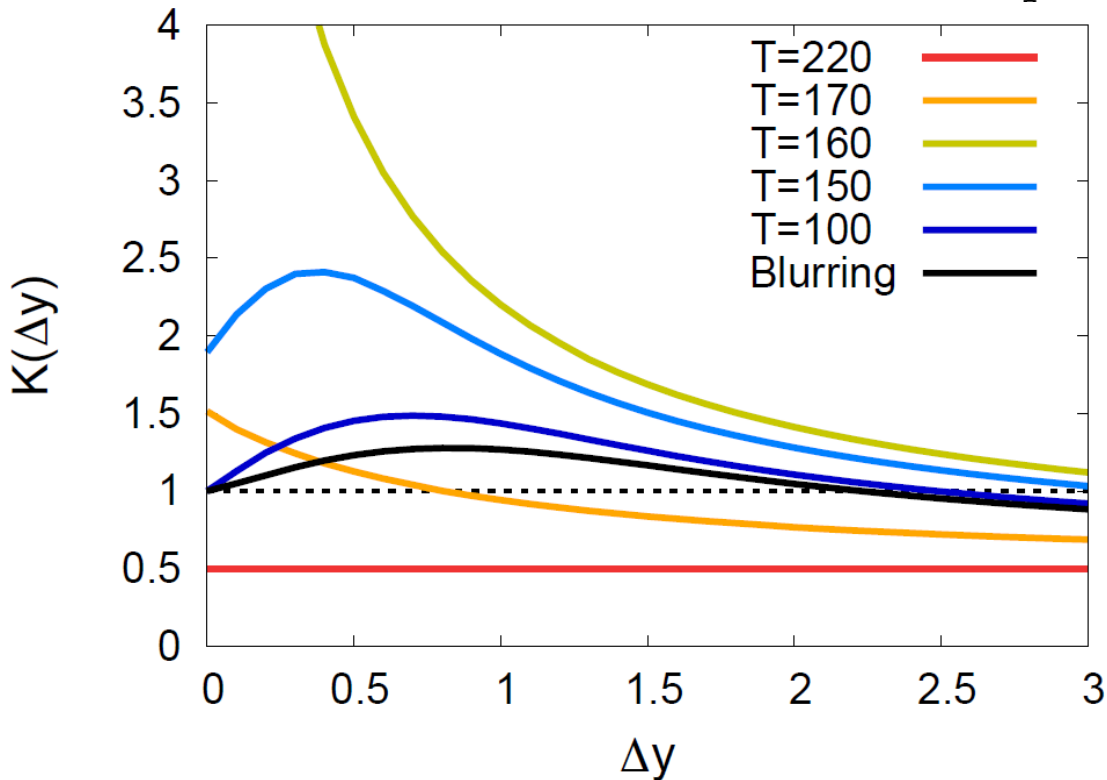


□ monotonically decreasing

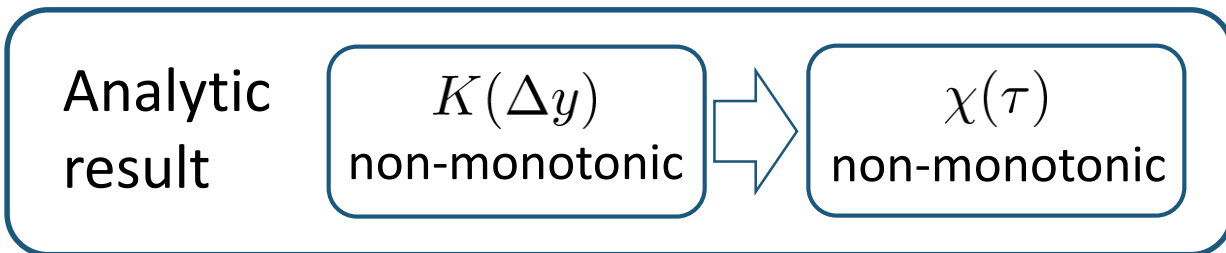
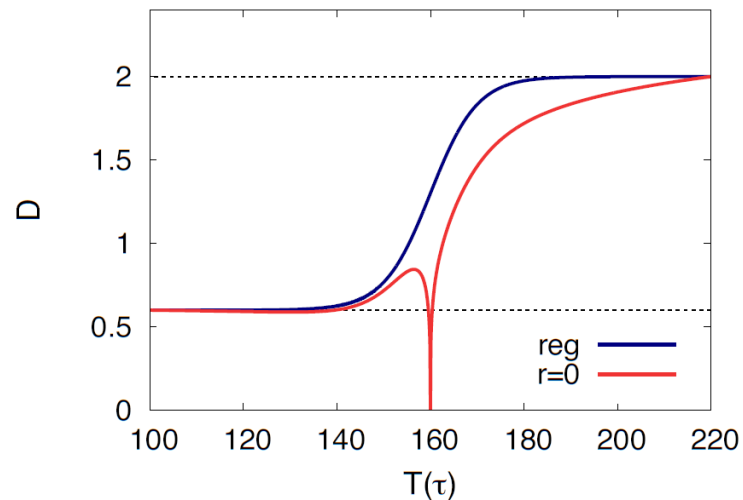
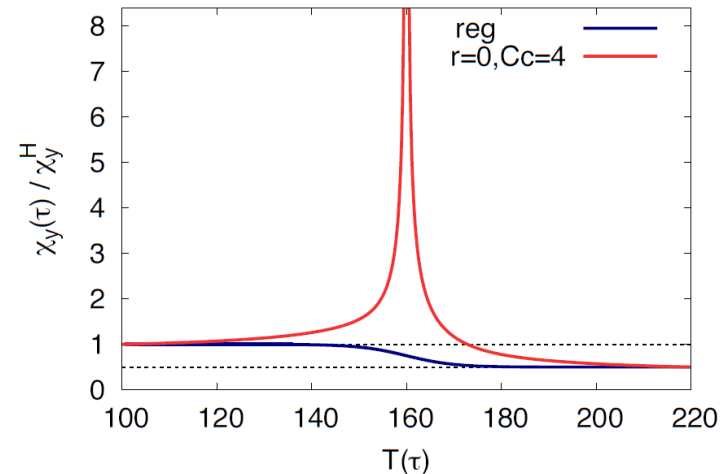


# Critical Point / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$

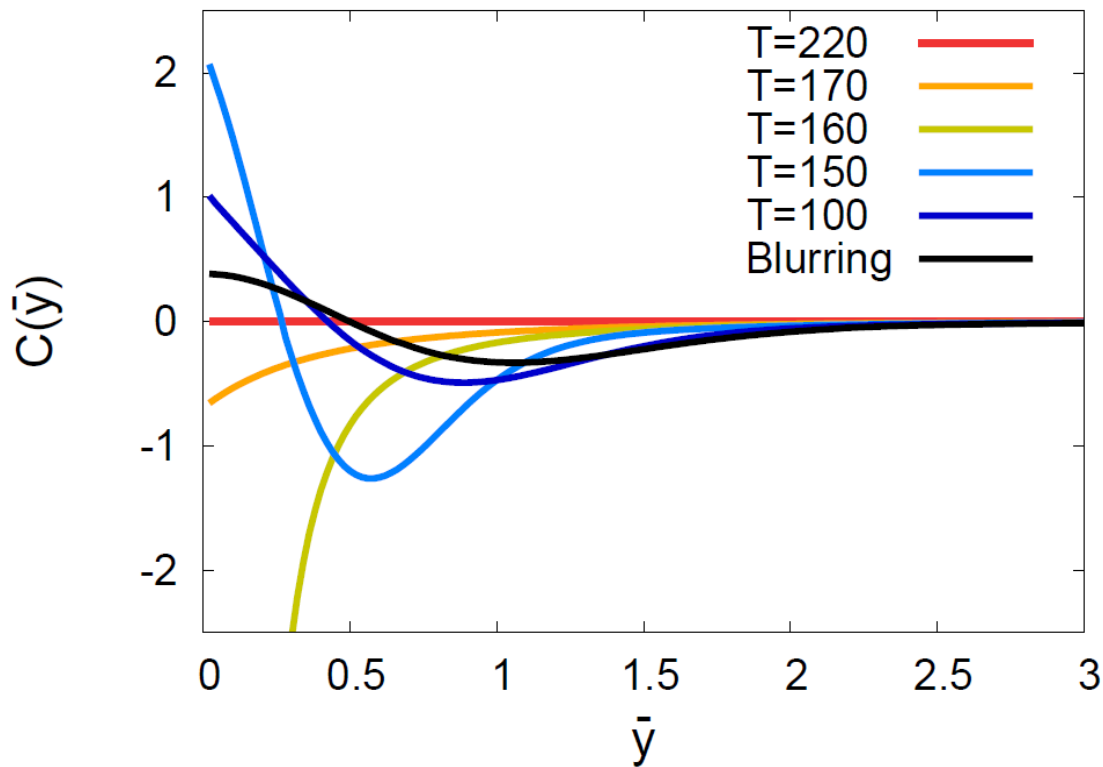


□ non-monotonic  $\Delta y$  dep.

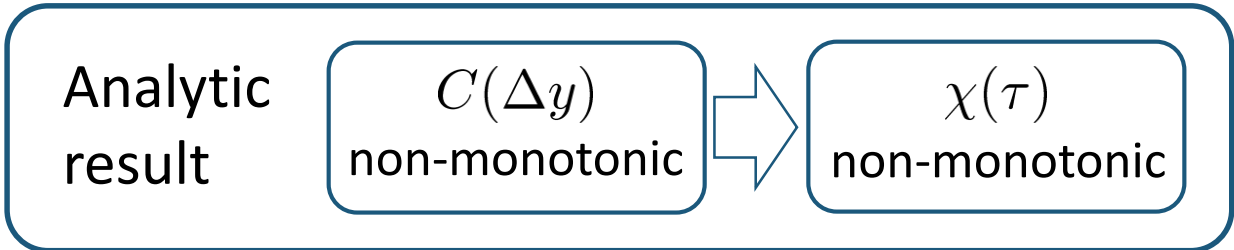
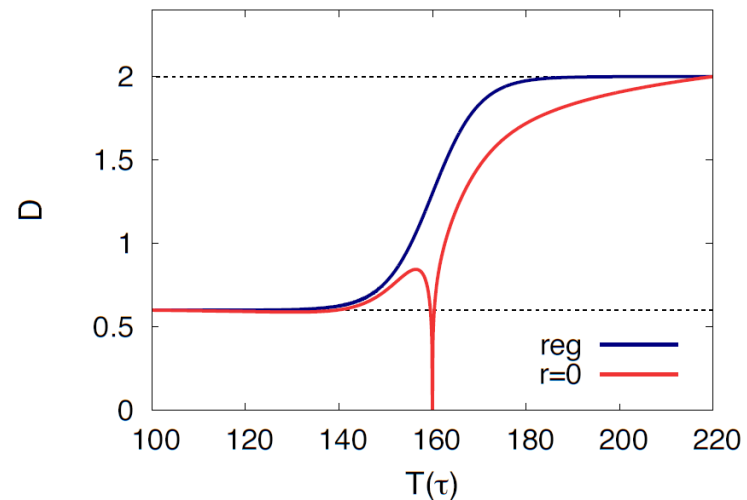
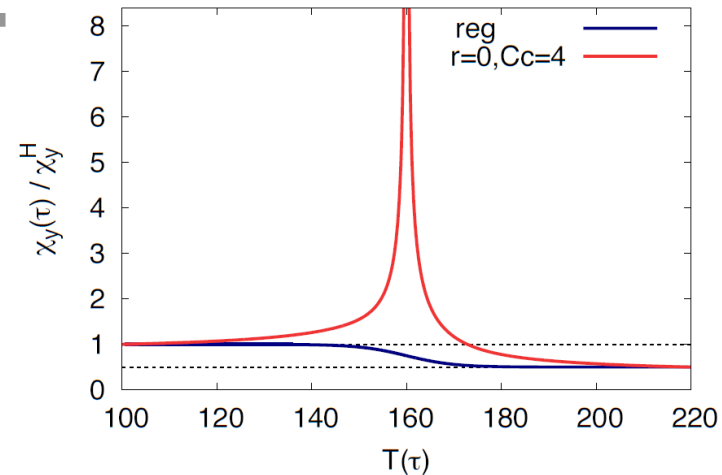


# Criticap Point / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$

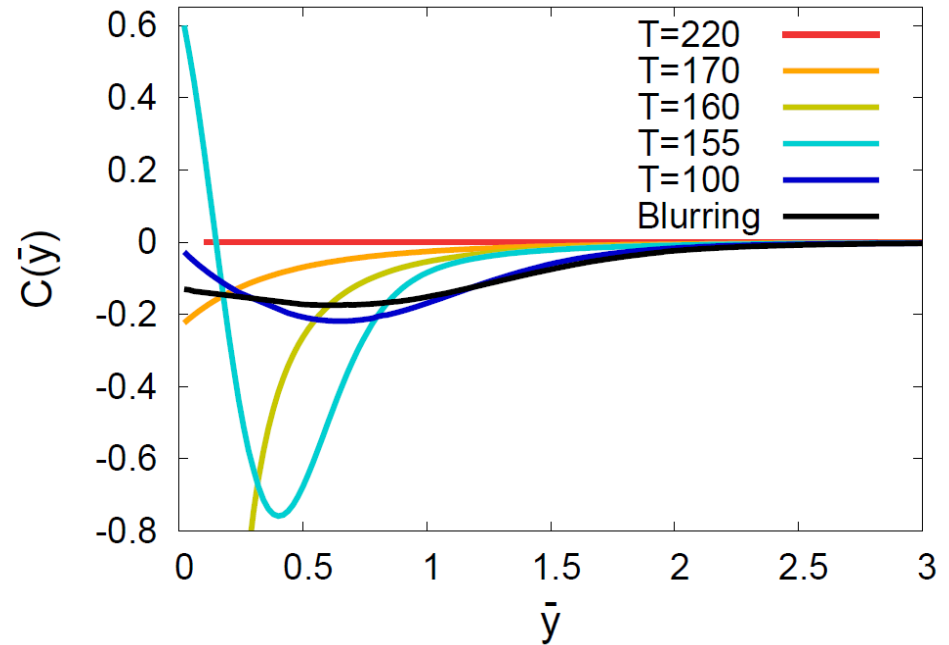
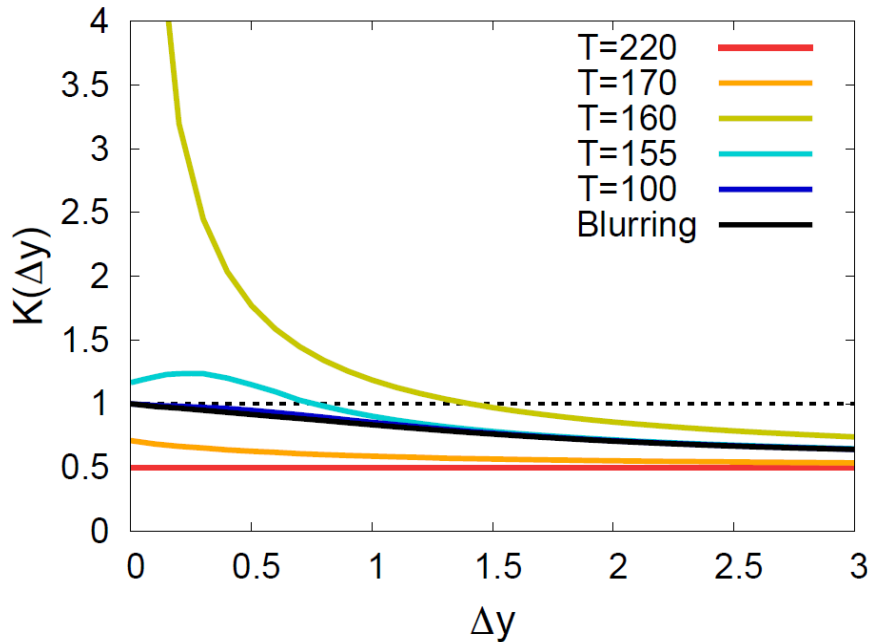


□ non-monotonic  $\Delta y$  dep.



# Weaker Critical Enhancement

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}} \quad C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$



□ Non-monotonicity in  $K(\Delta y)$  disappears.

□ But  $C(y)$  is still non-monotonic.

Analytic  
result

$K(\Delta y), C(\bar{y})$   
monotonic



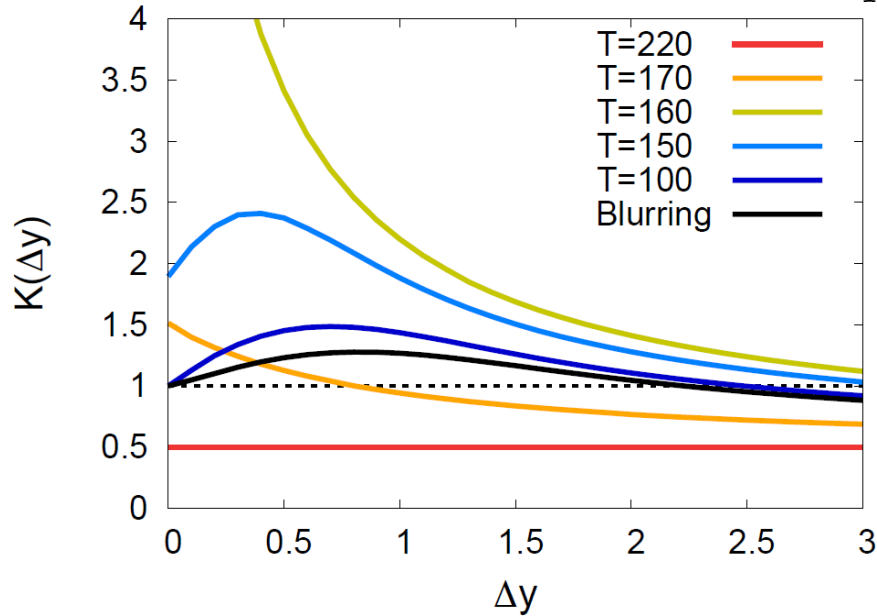
no information on  
 $\chi(\tau)$

□  $C(y)$  is better to see non-monotonicity.

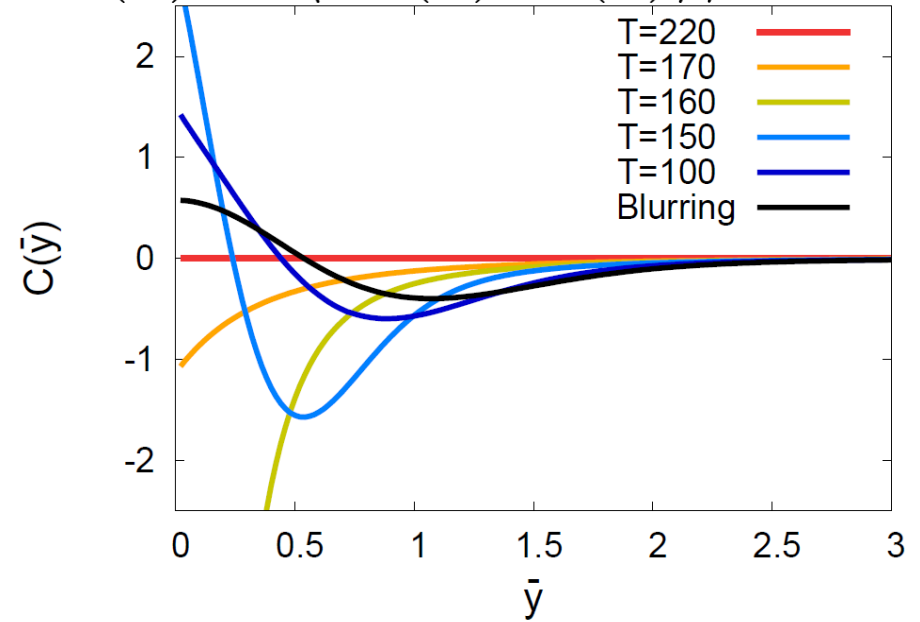


# Away from the CP

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$



$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$



□ Signal of the critical enhancement can be clearer on a path away from the CP.

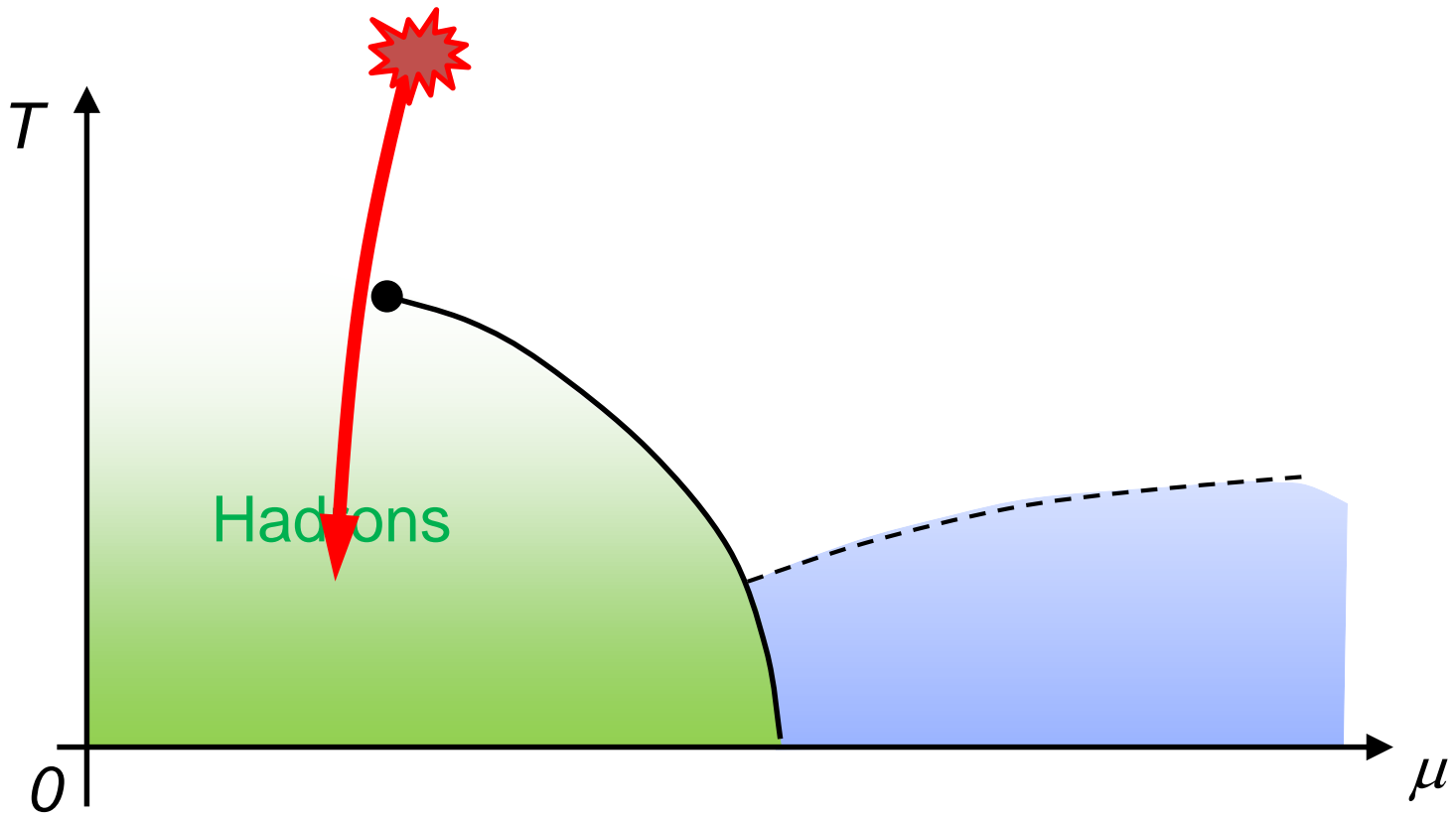
Away from the CP  $\rightarrow$  Weaker critical slowing down

# Summary

- ❑ Soft mode of the QCD critical point is a conserved mode. Its time evolution depends on the size defining the charge.
- ❑ Time evolution of conserved charges (especially baryon number) is well described by the stochastic diffusion equation.
- ❑ A non-monotonic behavior of cumulant or correlation function is the signal of the critical enhancement!

## Suggestion to experimentalists

- ❑ To find the CP, measure  $\left\{ \begin{array}{l} \bullet \Delta y \text{ dep. of } 2^{\text{nd}} \text{ order cumulant} \\ \bullet y \text{ dep. of correlation function} \end{array} \right.$
- ❑ Study lower-order fluctuation in more detail

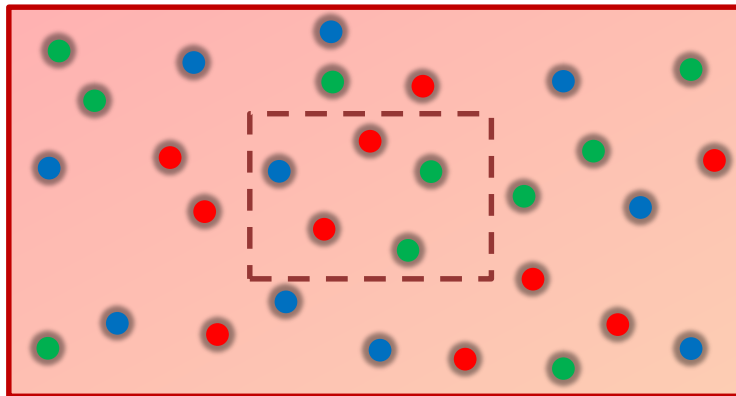


# Fluctuations and Elemental Charge

Asakawa, Heinz, Muller, 2000

Jeon, Koch, 2000

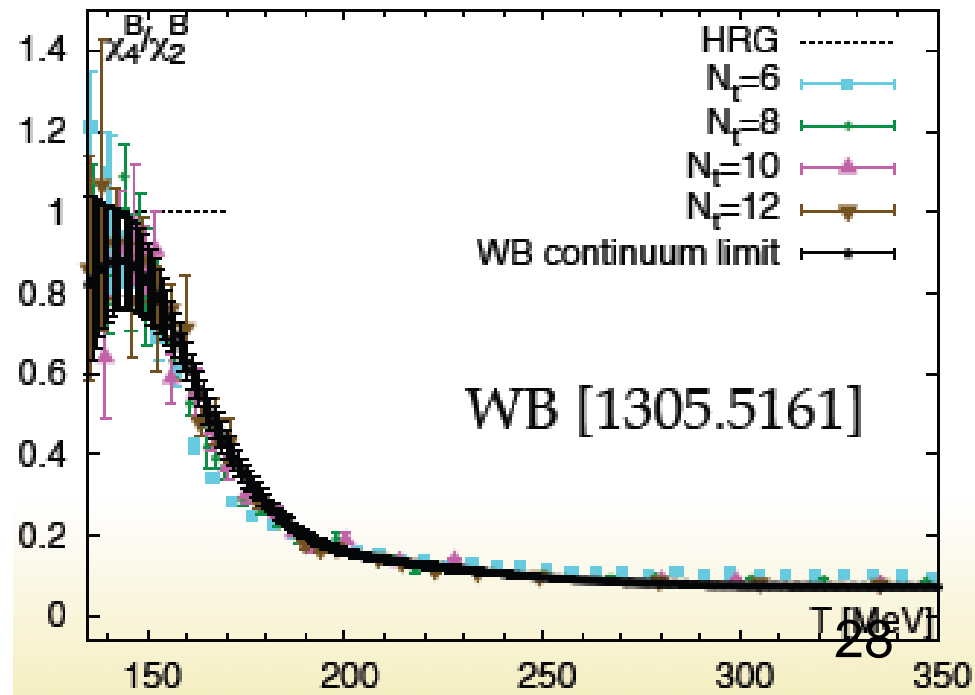
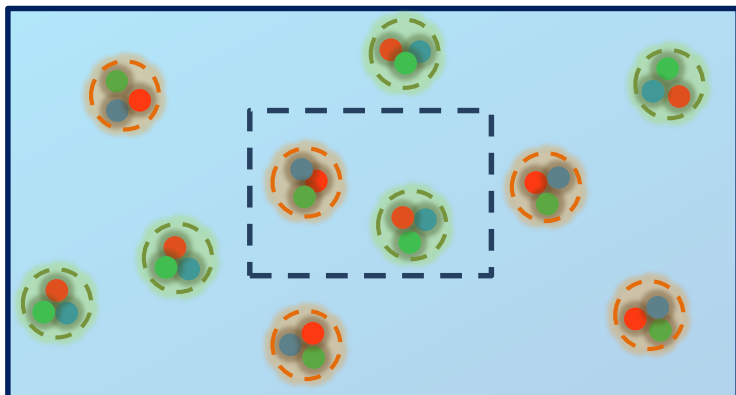
Ejiri, Karsch, Redlich, 2005



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

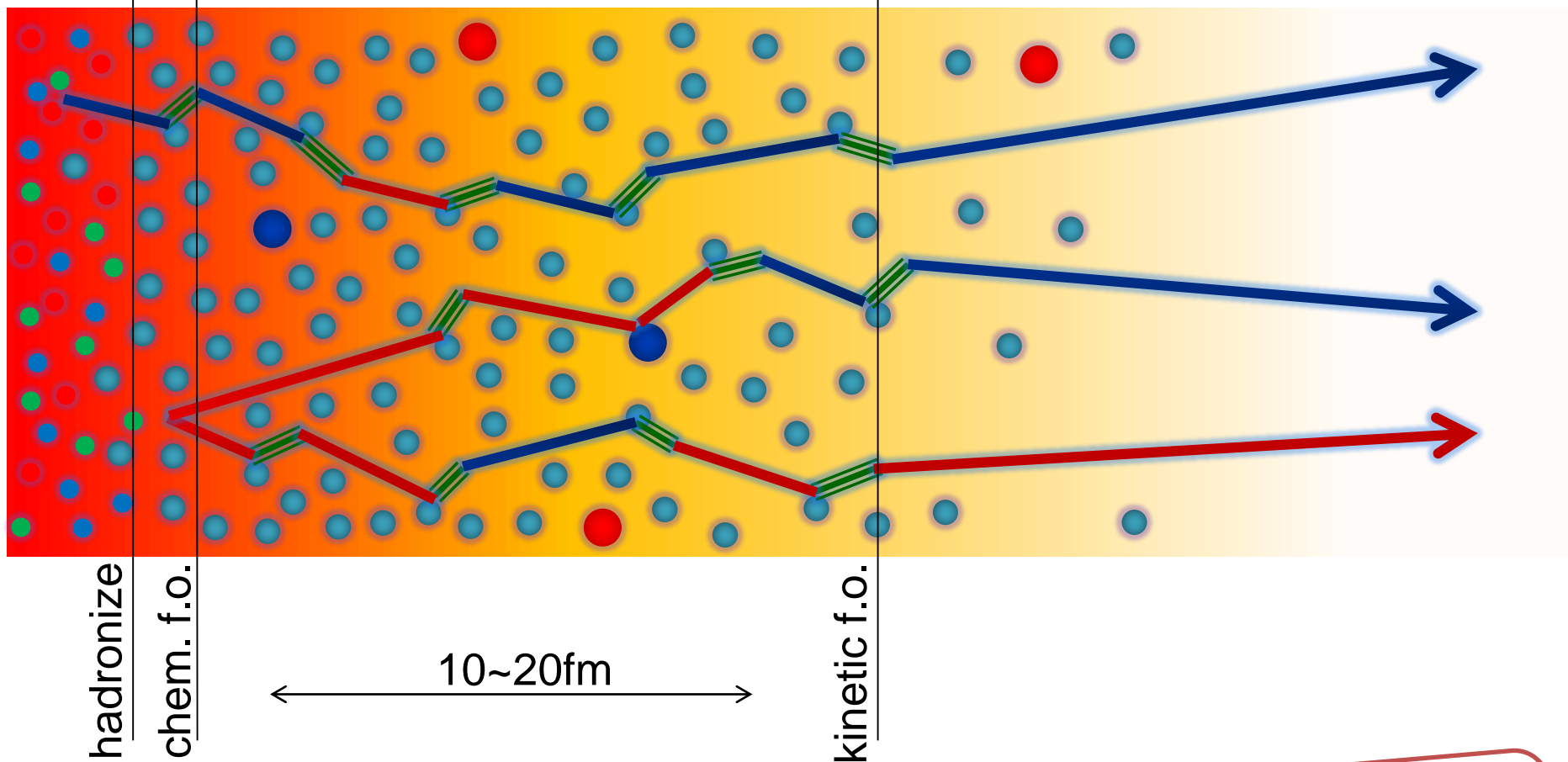
$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$



# Baryons in Hadronic Phase






time →



hadronize  
chem. f.o.

← 10~20fm →

kinetic f.o.

-   $p, \bar{p}$
-   $n, \bar{n}$
-   $\Delta(1232)$
-  mesons
-  baryons

Baryons behave like  
Brownian pollens in water