Non-Gaussian Fluctuations in Relativistic Heavy-Ion Collisions

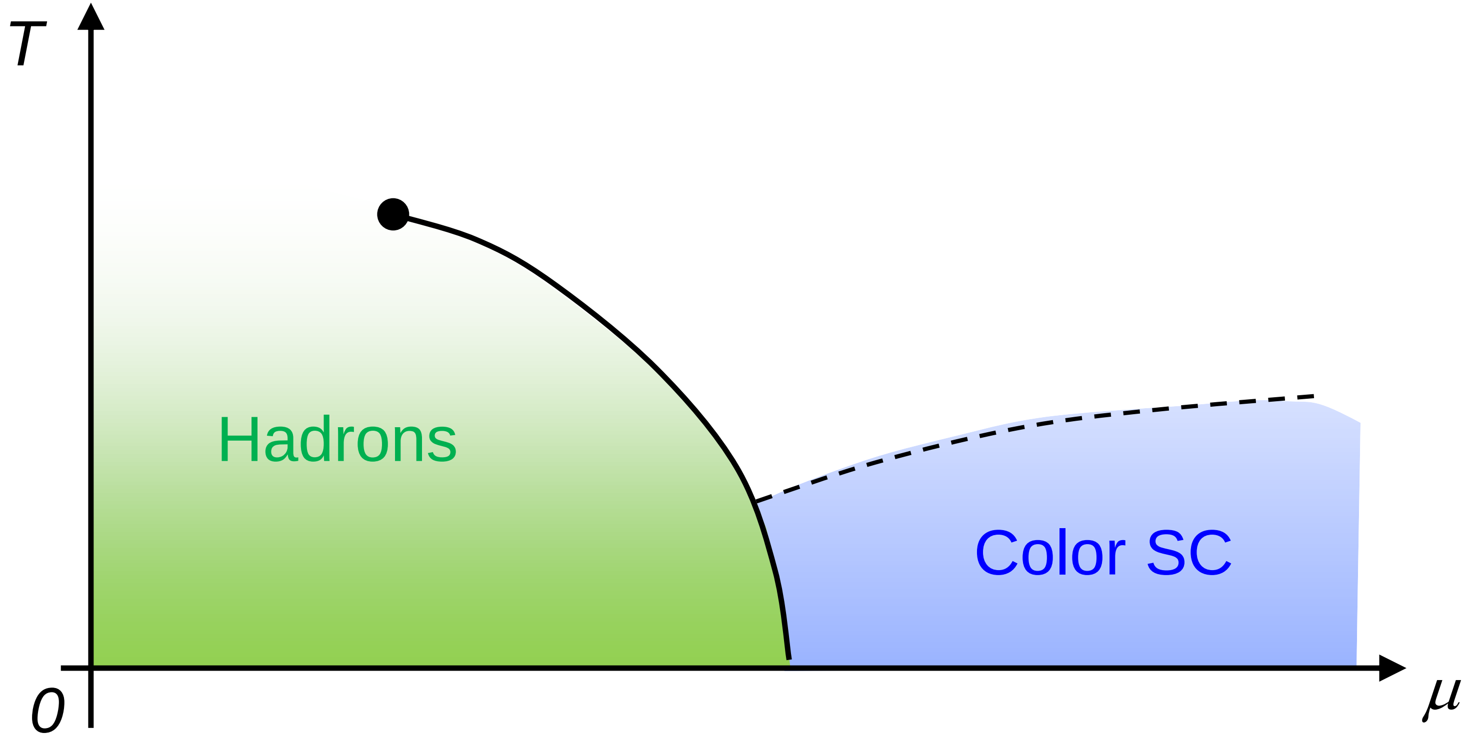
Masakiyo Kitazawa

(Osaka U.)

Asakawa, MK, Prog. Part. Nucl. Phys. 90, 299 (2016)
Sakaida, Asakawa, Fujii, MK, to appear in PRC
MK, Nucl. Phys. A942, 65 (2015)
Sakaida, Asakawa, MK, PRC90, 064911 (2014)
MK, Asakawa, Ono, Phys. Lett. B728, 386-392 (2014)

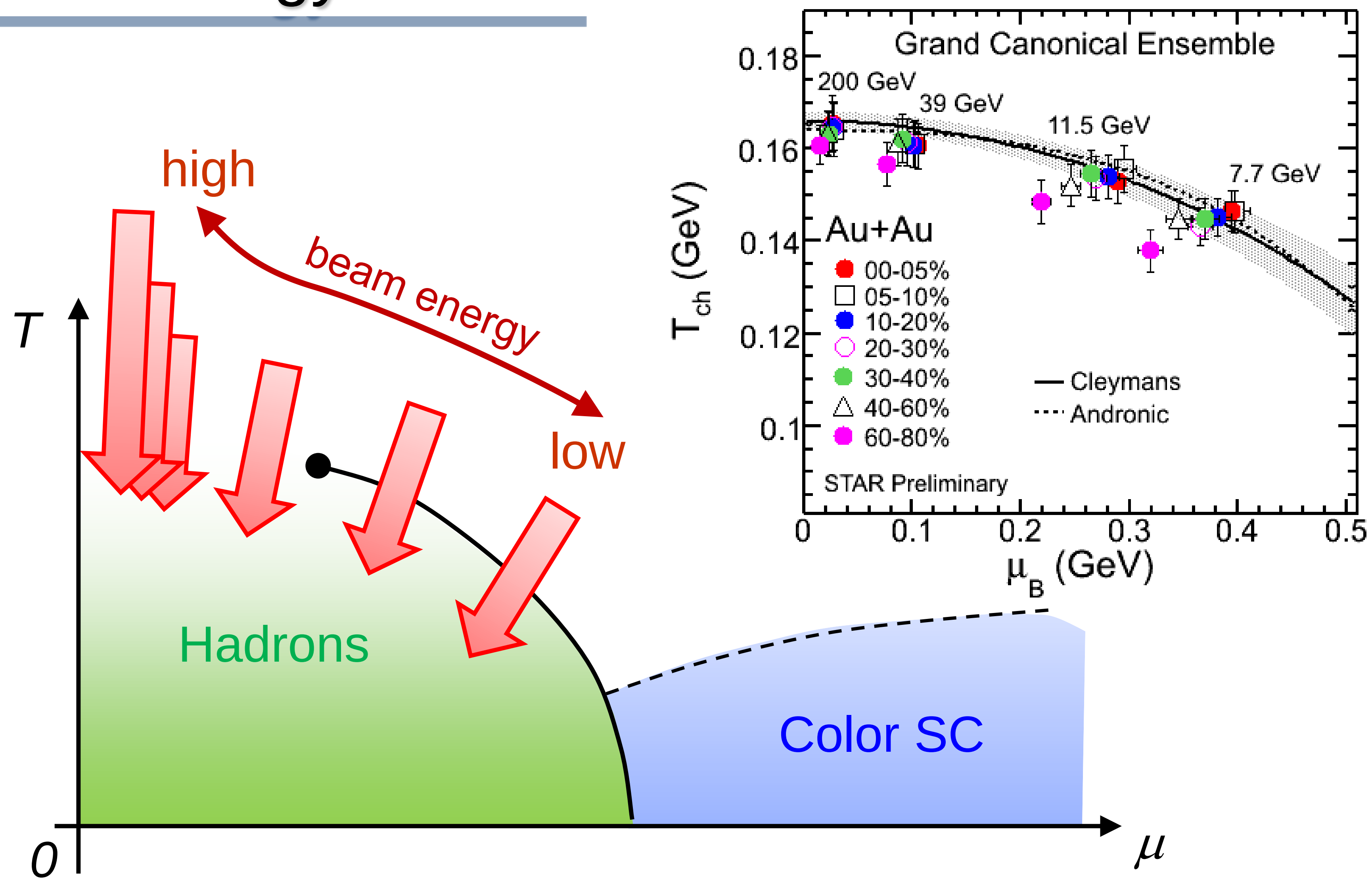
Nagoya Seminar, Nagoya U., 31/May/2017

Beam-Energy Scan



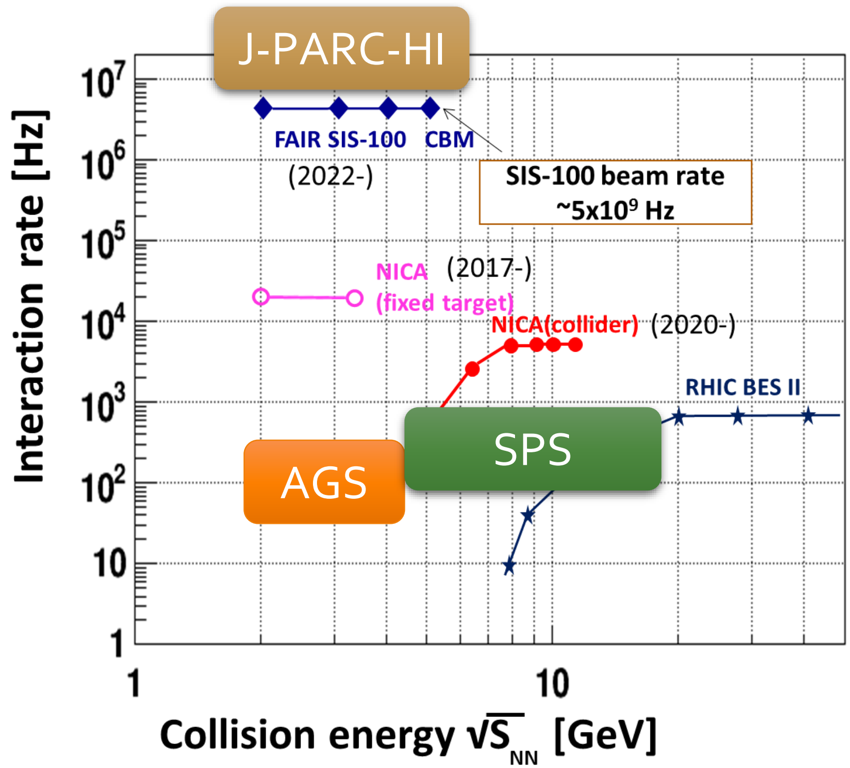
Beam-Energy Scan

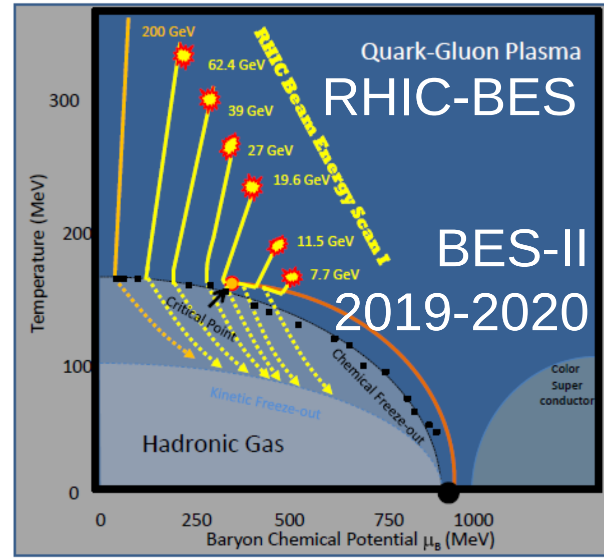
STAR 2012

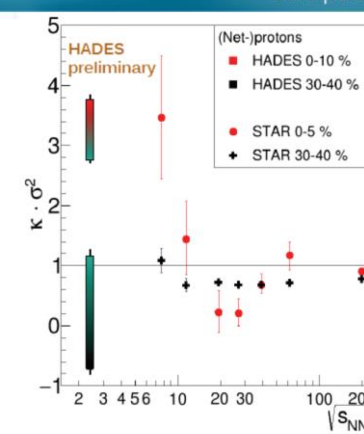


Beam-Energy Scan

Active experimental researches/plans for the beam-energy scan





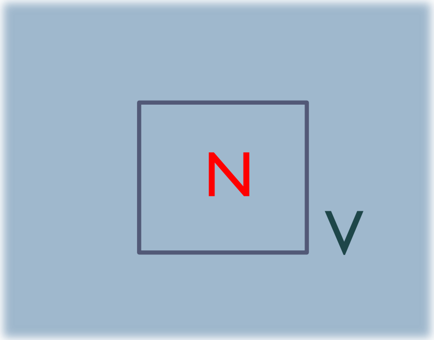


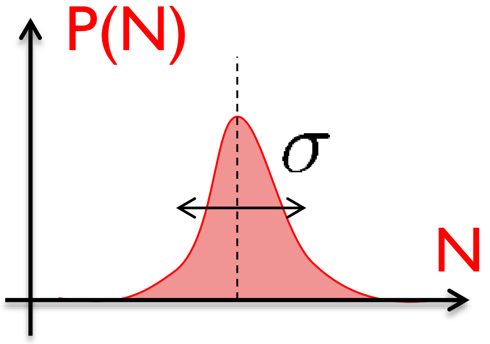
Search for QCD phase structure / critical point

Fluctuations

Thermal Fluctuations

Observables are fluctuating even in an equilibrated medium.





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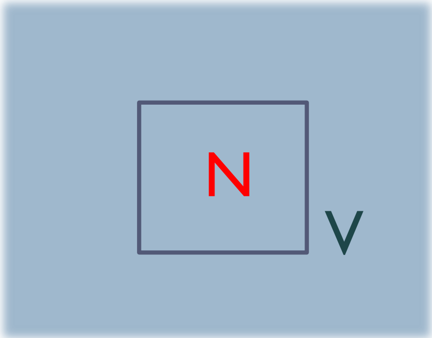
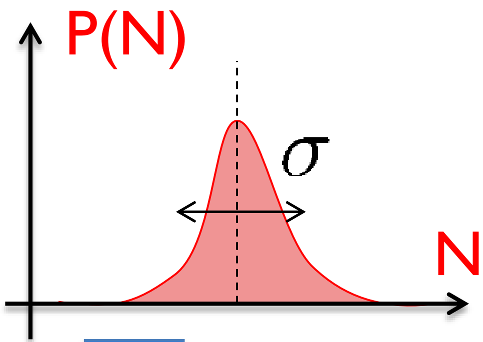




$$\delta N = N - \langle N \rangle$$

- Variance: $\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \dfrac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

Non-Gaussianity

The noise is the signal.

R. Landauer
1998

A Coin Game

① Bet 250 JPY
② You get head coins of

A. 100 x 5 JPY

B. 50 x 10 JPY

Same expectation value.

A Coin Game

① Bet 250 JPY
② You get head coins of

A. 100 x 5 JPY

B. 50 x 10 JPY

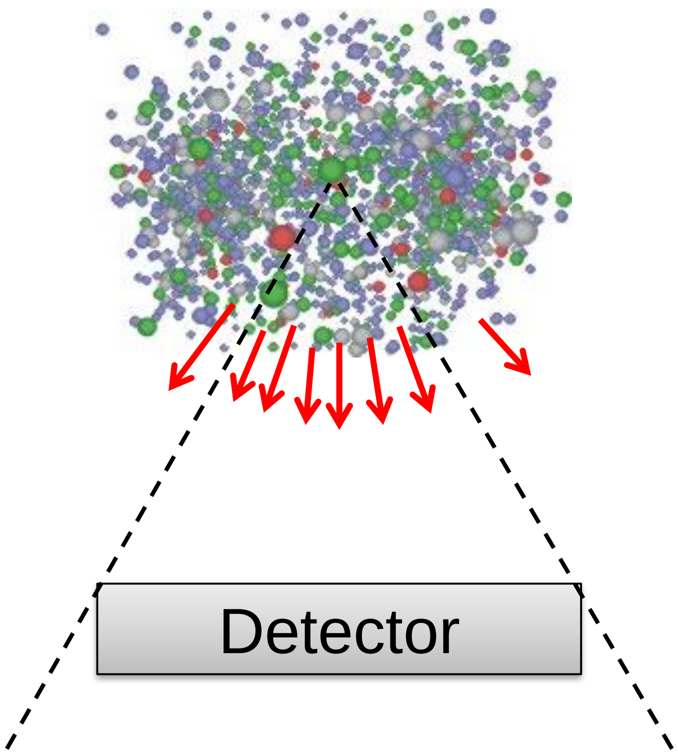
C. 1 x 500 JPY

Same expectation valu
But, different fluctuation

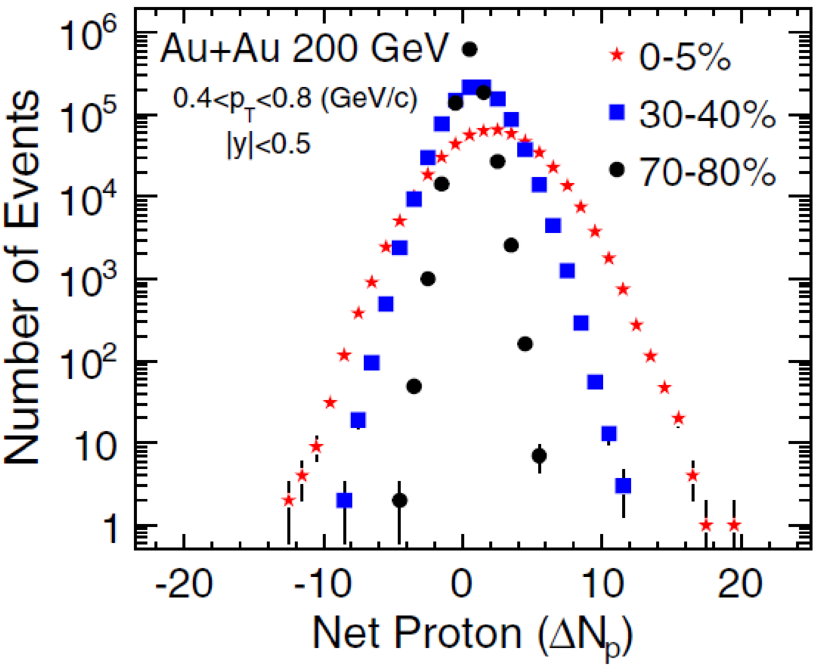
Event-by-Event Fluctuations

Review: Asakawa, MK, PPNP **90** (2016)

Fluctuations can be measured by e-by-e analysis in experiments.



STAR, PRL**105** (2010)

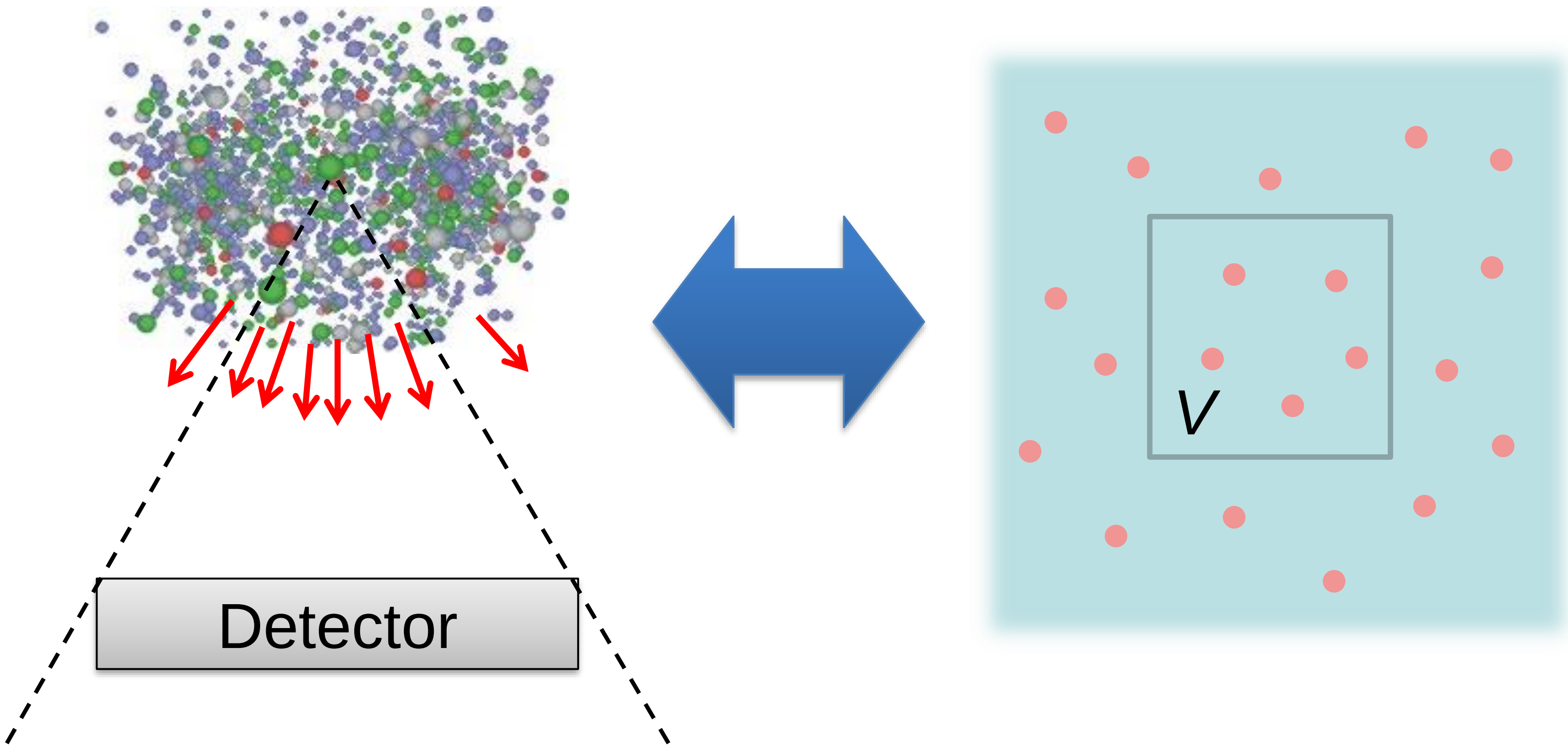


Cumulants

$$\langle\delta N_p^2\rangle, \langle\delta N_p^3\rangle, \langle\delta N_p^4\rangle_c$$

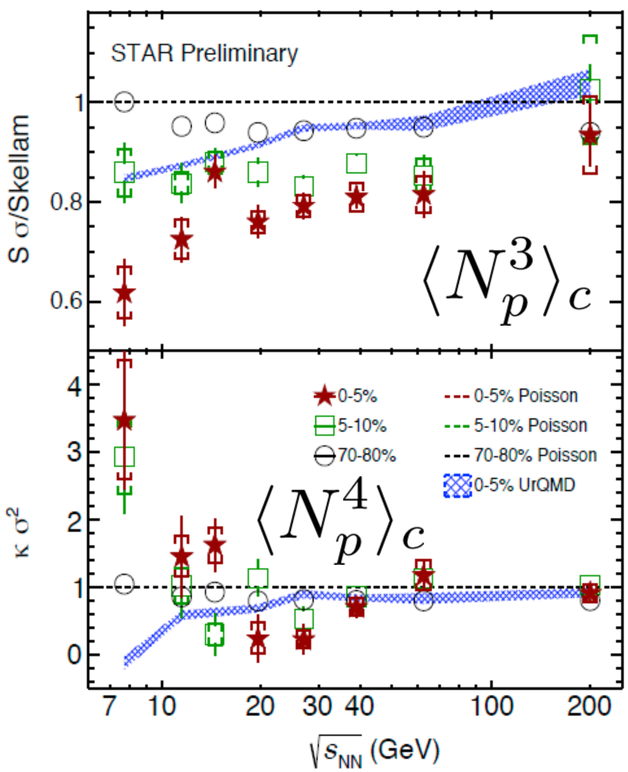
Event-by-Event Analysis @ HIC

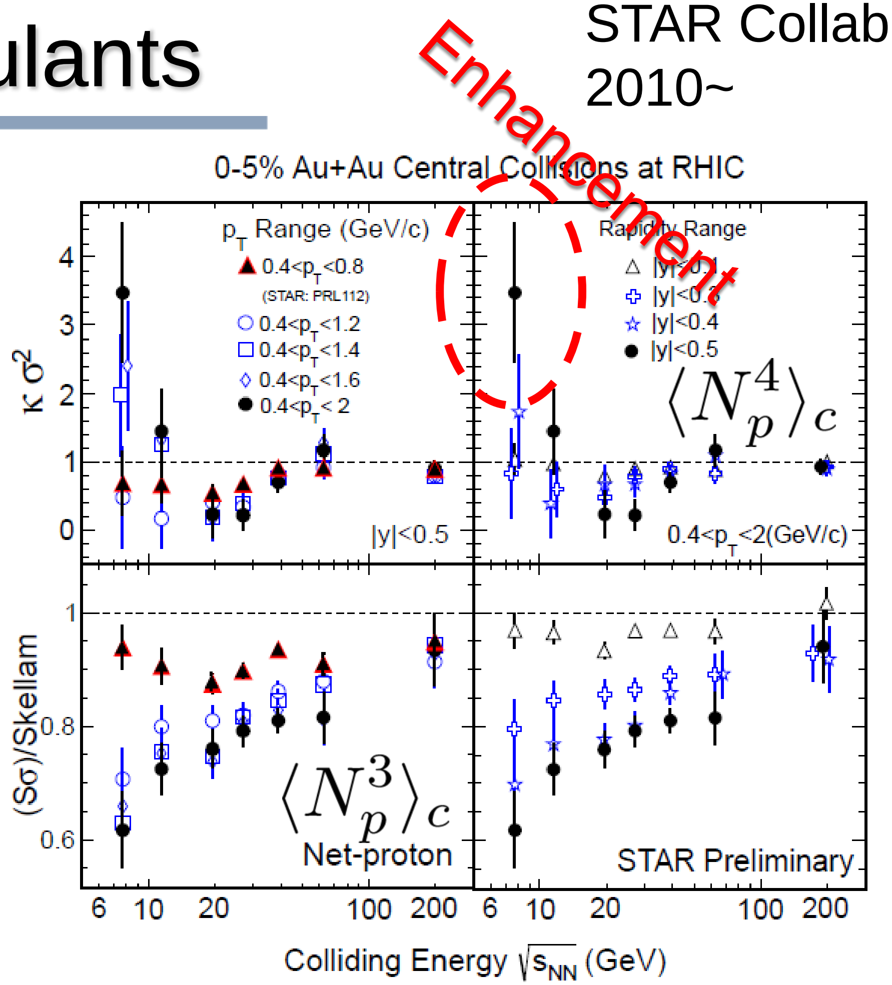
Fluctuations can be measured by e-by-e analysis in experiments.



Higher-Order Cumulants

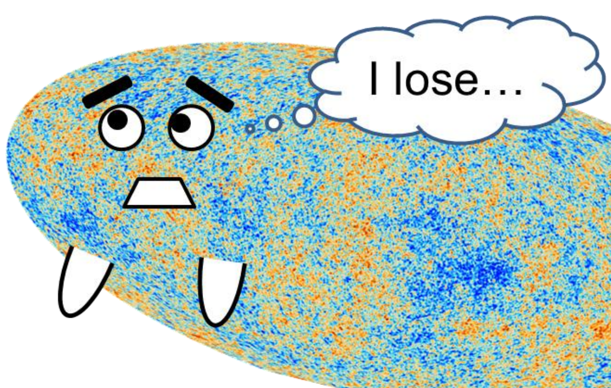
STAR Collab.
2010~





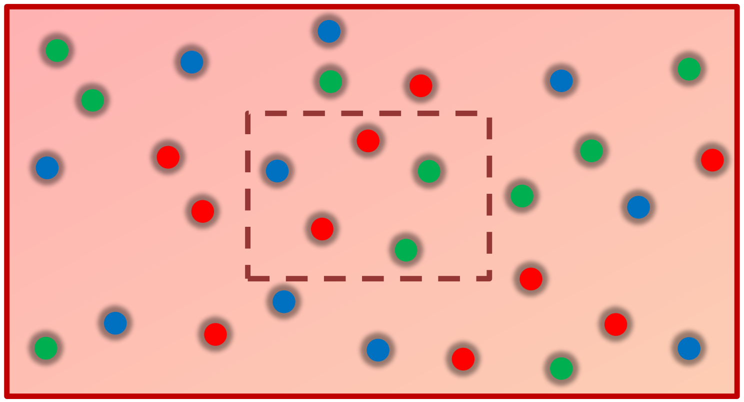
Non-zero **non**-Gaussian cumulants have been established!

Have we measured critical fluctuations?



Fluctuations and Elemental Charge

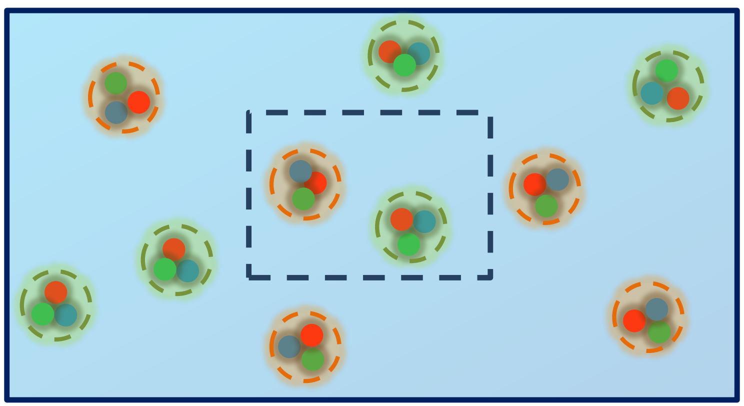
Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000
Ejiri, Karsch, Redlich, 2005



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$



$$\langle \delta N_B^n \rangle_c = \langle N_B \rangle$$

Free Boltzmann → Poisson

$$\langle \delta N^n \rangle_c = \langle N \rangle$$

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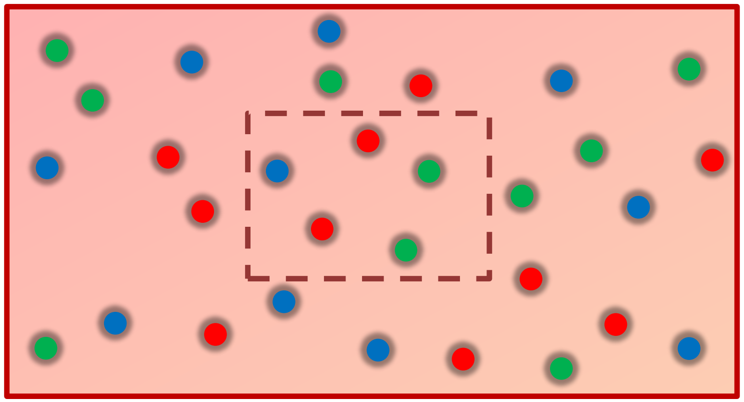

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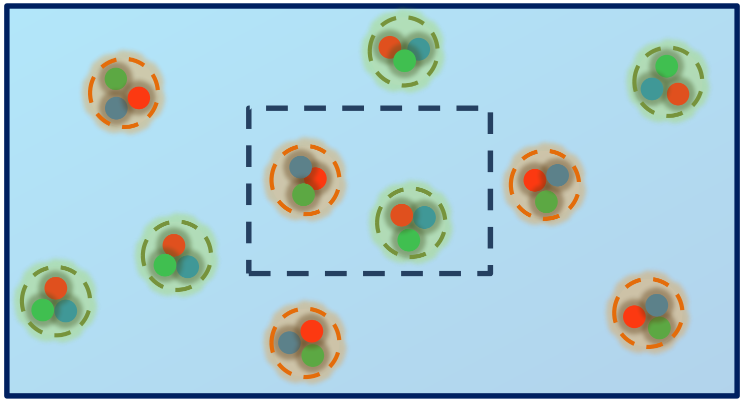

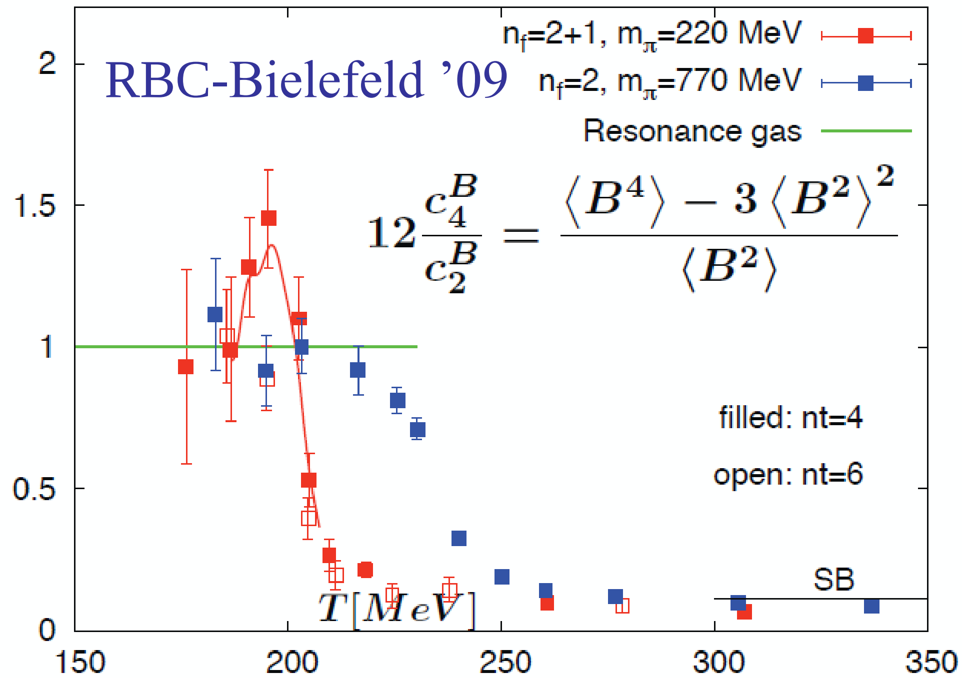

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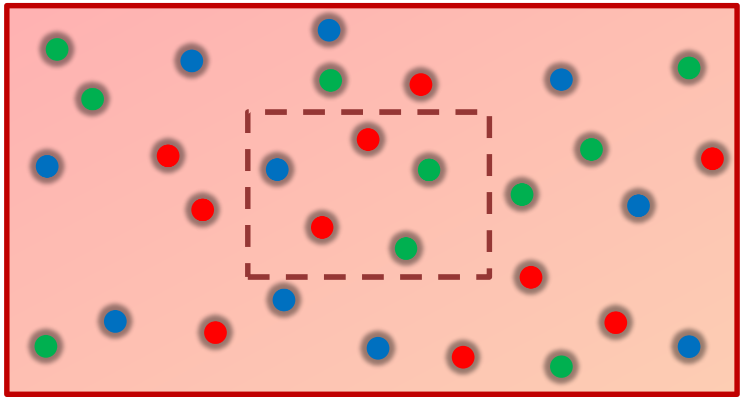

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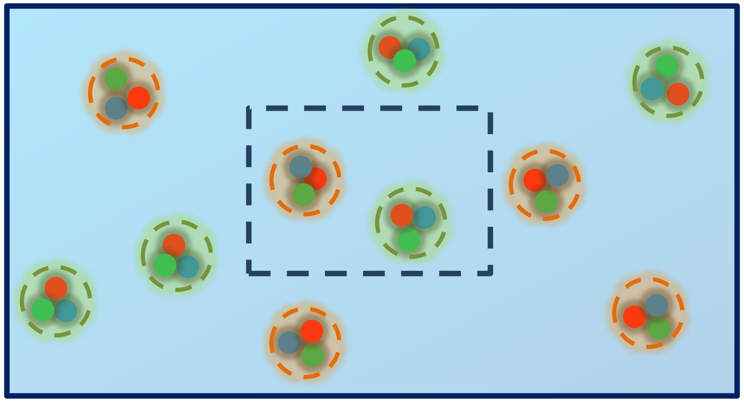
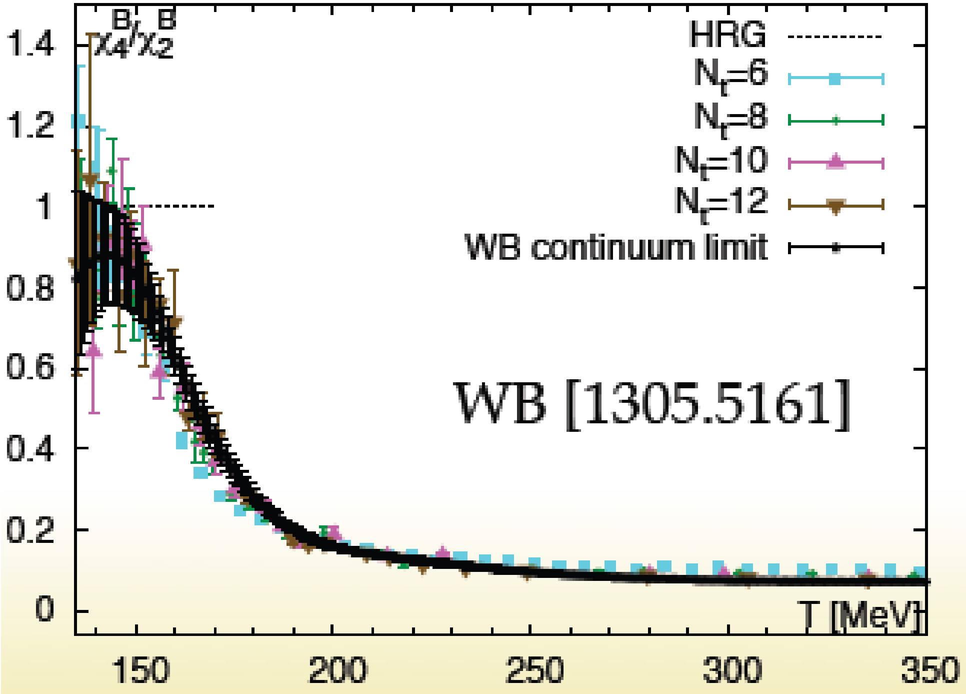
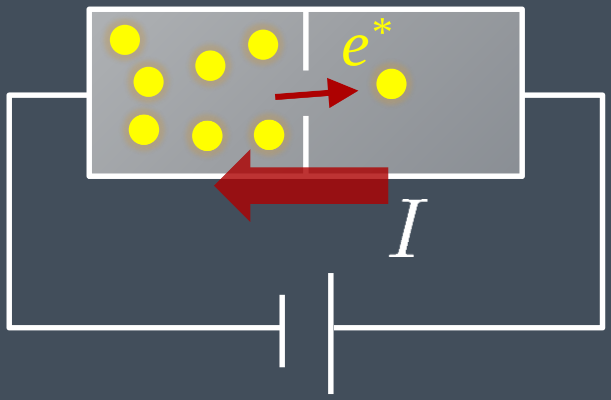




Shot Noise



$$S_{\mathrm{shot}} \sim \langle \delta I^2 \rangle$$

$$S_{\mathrm{shot}} = 2e^* \langle I \rangle$$

charge of quasi-particles

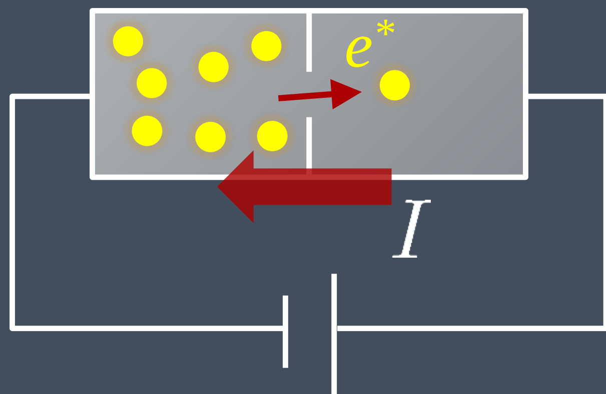
Total charge Q:

$$Q = e\langle N \rangle$$

$$\langle \delta Q^2 \rangle = e^2 \langle \delta N^2 \rangle = e^2 \langle N \rangle = eQ$$

$$\frac{\langle \delta Q^2 \rangle}{Q} = e$$

Shot Noise



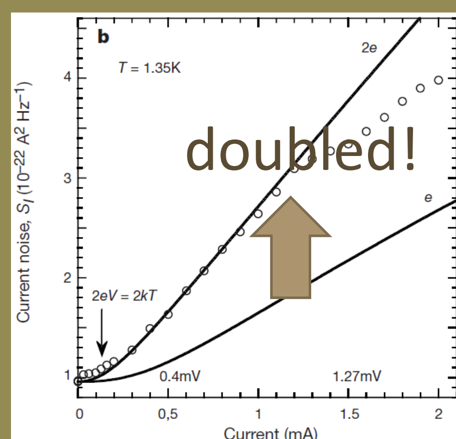
$$S_{\text{shot}} \sim \langle \delta I^2 \rangle$$
$$S_{\text{shot}} = 2e^* \langle I \rangle$$

charge of quasi-particles

Superconductors with Cooper Pairs

$$e^* = 2e$$

Jehl+, Nature **405**,50 (2000)



Fractional Quantum Hall Systems

$$e^* = \frac{q}{p} e$$

Saminadayar+, PRL**79**,2526 (1997)

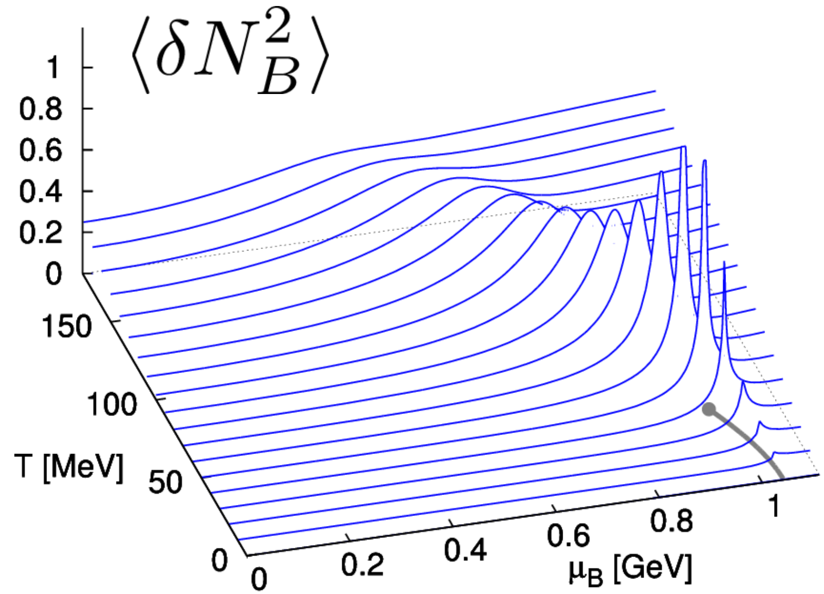
Higher order cumulants:

3rd order: ex. Beenakker+, PRL90,176802(2003)

up to 5th order: Gustavsson+, Surf.Sci.Rep.**64**,191(2009)

Fluctuation and QCD Critical Point

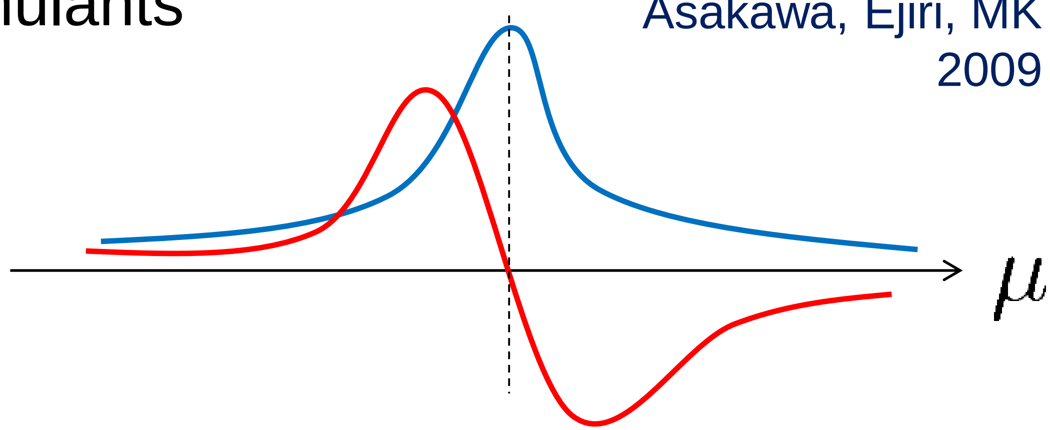
Fluctuations diverge at the QCD critical point



- Geometric interpretation to signs of higher order cumulants

$$\langle \delta N^3 \rangle = T \frac{\partial \langle \delta N^2 \rangle}{\partial \mu}$$

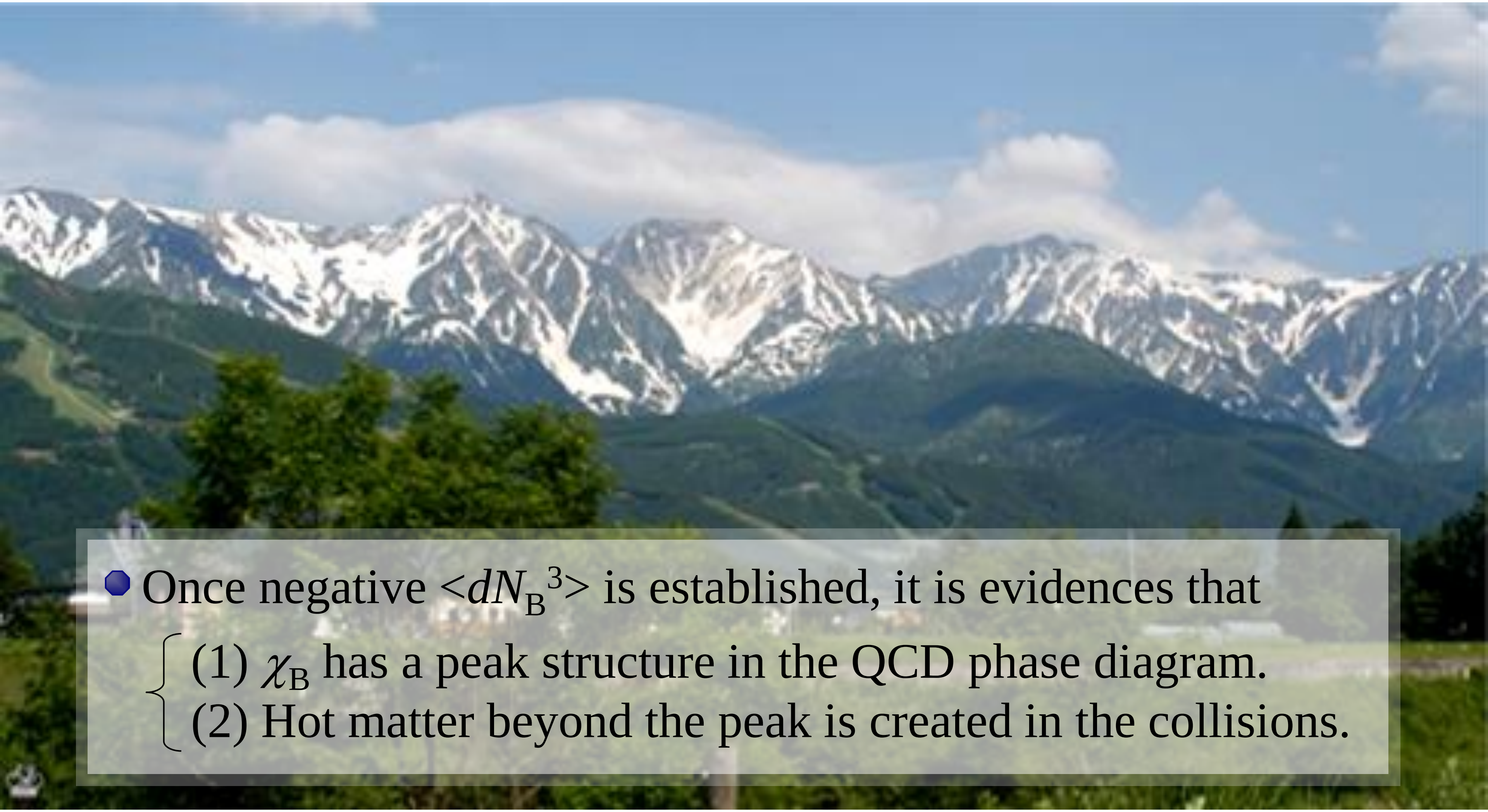
Asakawa, Ejiri, MK 2009



- More severe divergence for higher-order cumulants

Stephanov, 2009

Impact of Negative Third Cumulants



- Once negative $<dN_B^3>$ is established, it is evidences that
  - (1) χ_B has a peak structure in the QCD phase diagram.
  - (2) Hot matter beyond the peak is created in the collisions.

-
  - **No** dependence on any specific models.
  - Just the sign! **No** normalization (such as by N_{ch}).

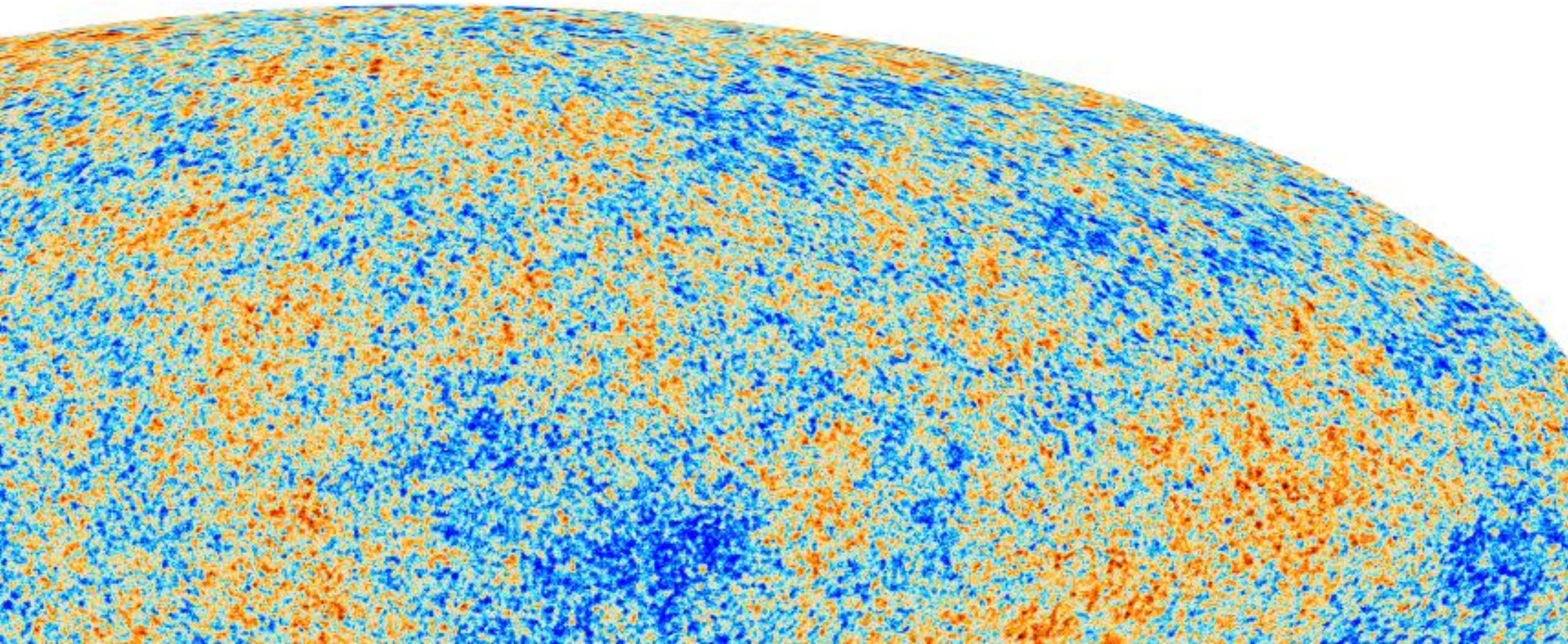
In “haiku”, Japanese short-style poem, a poet wrote…

Even on one blade of grass the cool wind lives

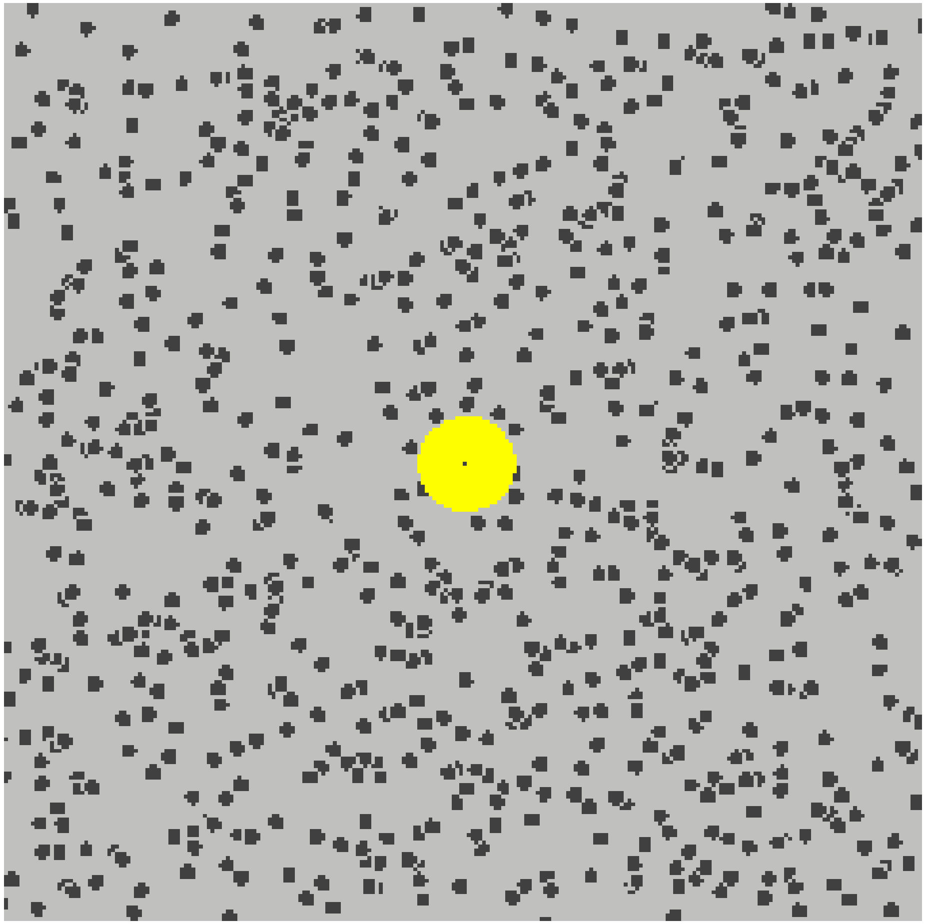
Issa Kobayashi
1814

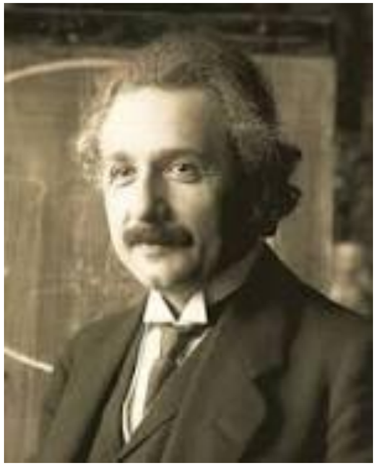
一本の草も涼風宿りけり
小林一茶

Physicists can feel hot early Universe
13 800 000 000 years ago
in tiny fluctuations of
cosmic microwave



Physicists can feel the existence of microscopic atoms behind random fluctuations of Brownian pollens





A. Einstein
1905

Zwicky 1933

Virial theorem

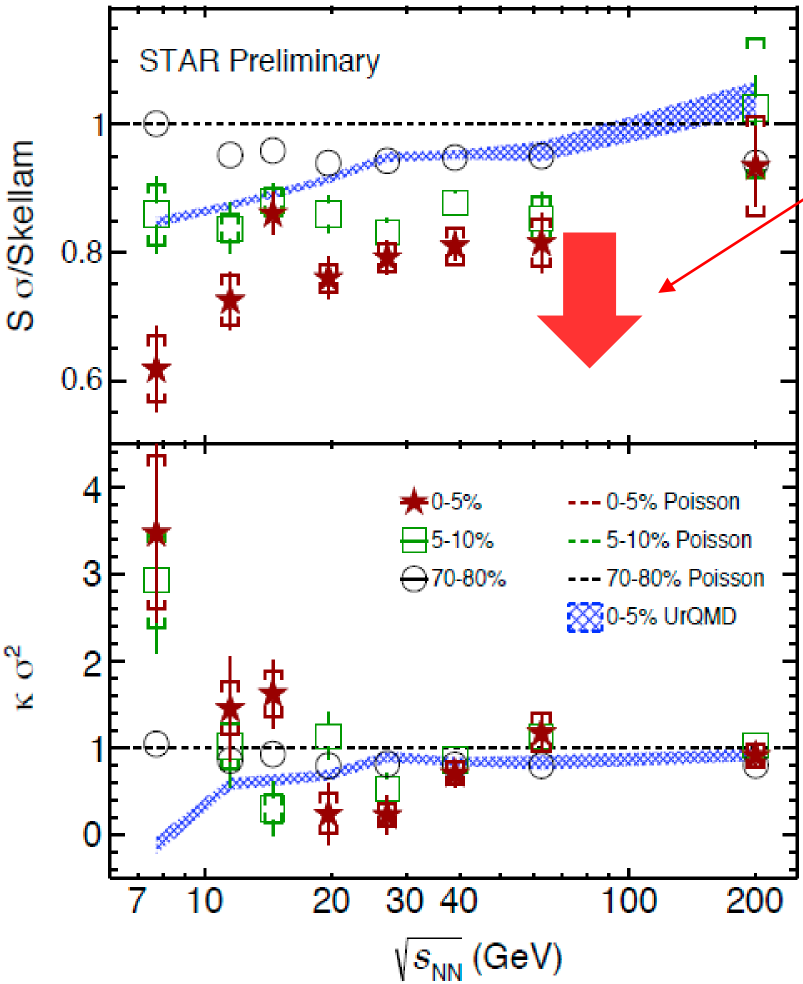
$$2K = -U$$

Physicists can feel dark matter behind fluctuations of galaxies billion light years away

Feel hot quark wind behind fluctuations
in relativistic heavy ion collisions

2010-

X. Luo, STAR, QM2015



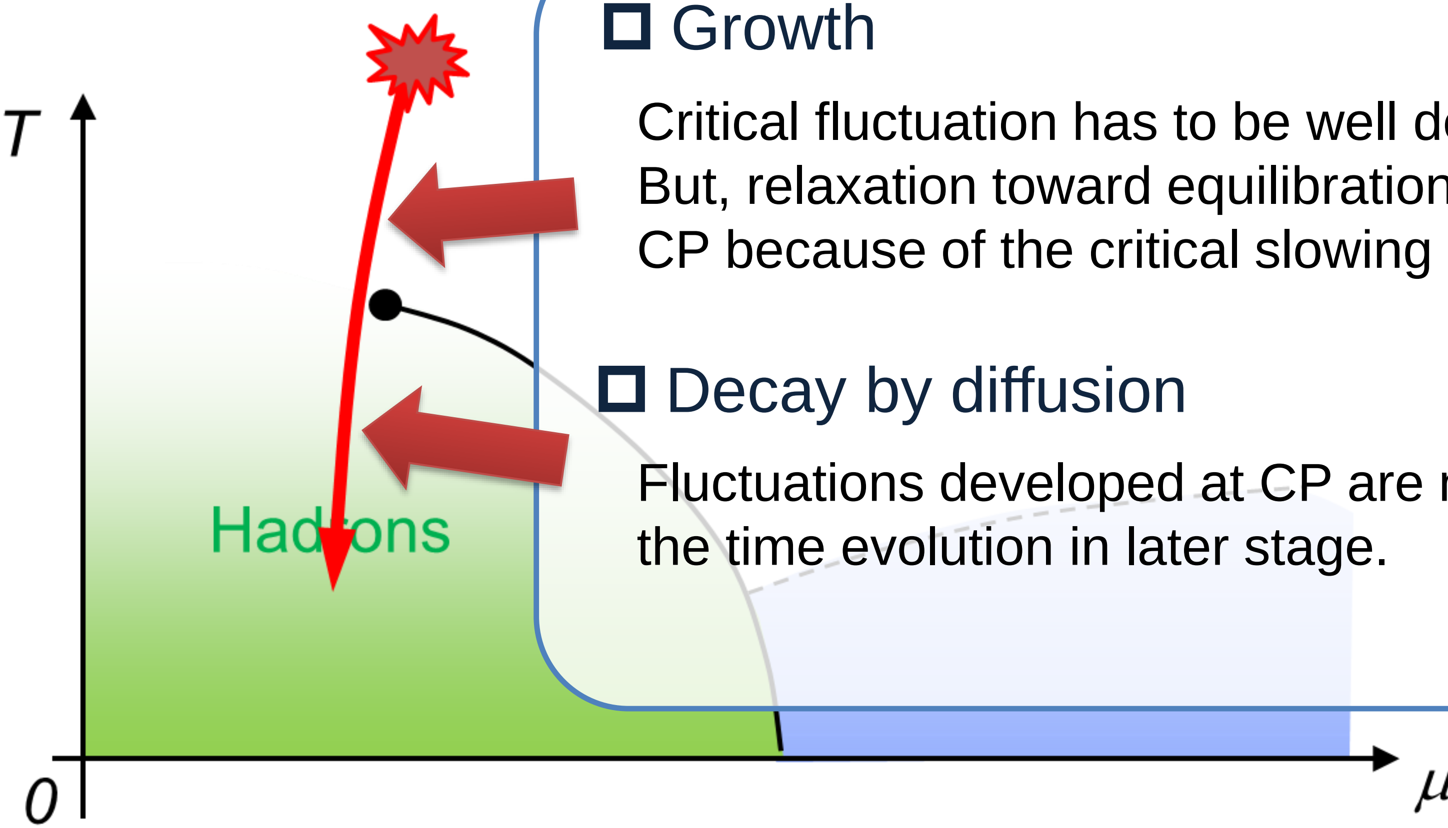
Clear suppression!

ex. Asakawa, Ejiri, MK, 2009

Rapidity Window Dependences of Fluctuations

Remarks on Critical Fluctuation

Experiments cannot observe critical fluctuation in equilibrium directly.



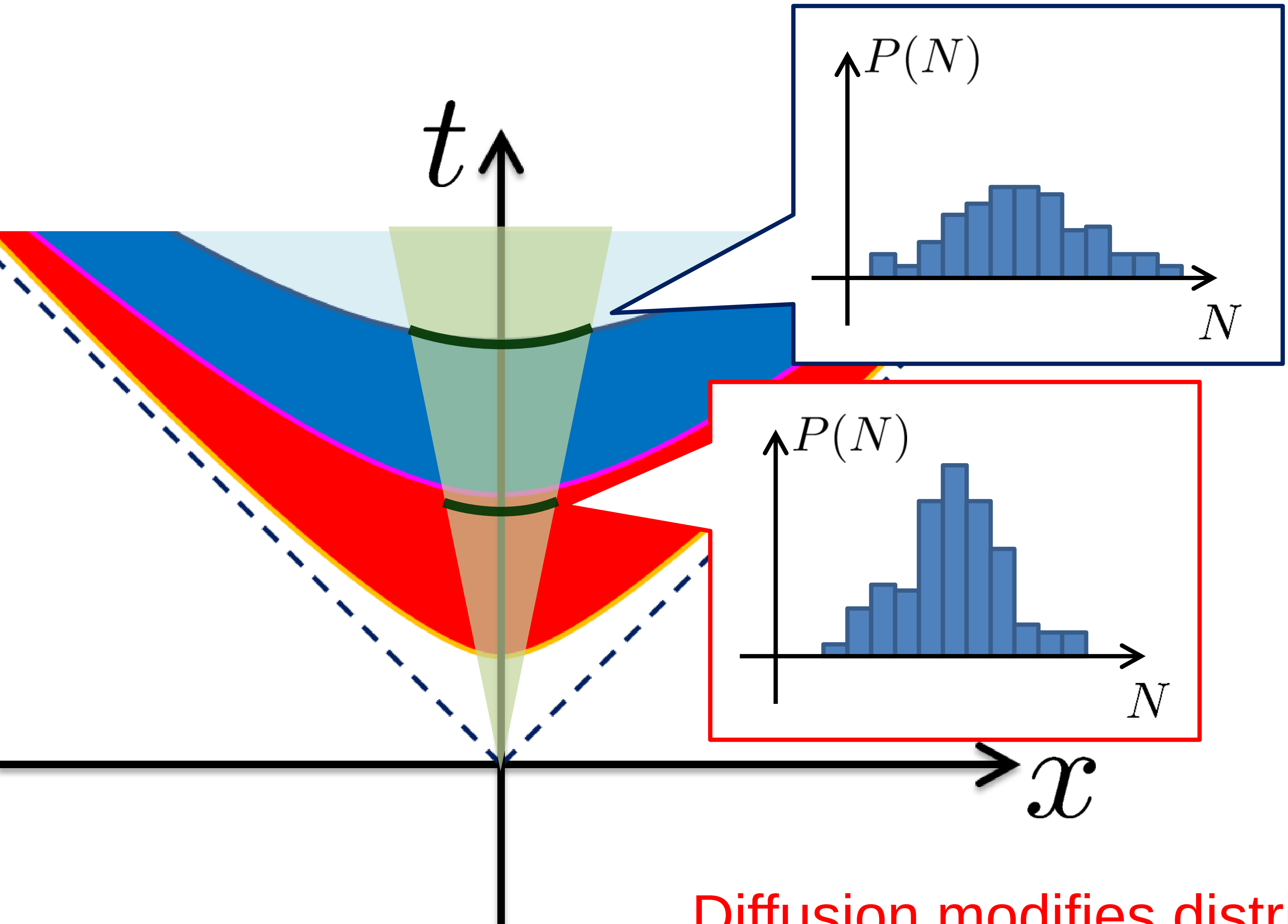
☐ Growth

Critical fluctuation has to be well developed. But, relaxation toward equilibration is slow around CP because of the critical slowing down.

☐ Decay by diffusion

Fluctuations developed at CP are modified by the time evolution in later stage.

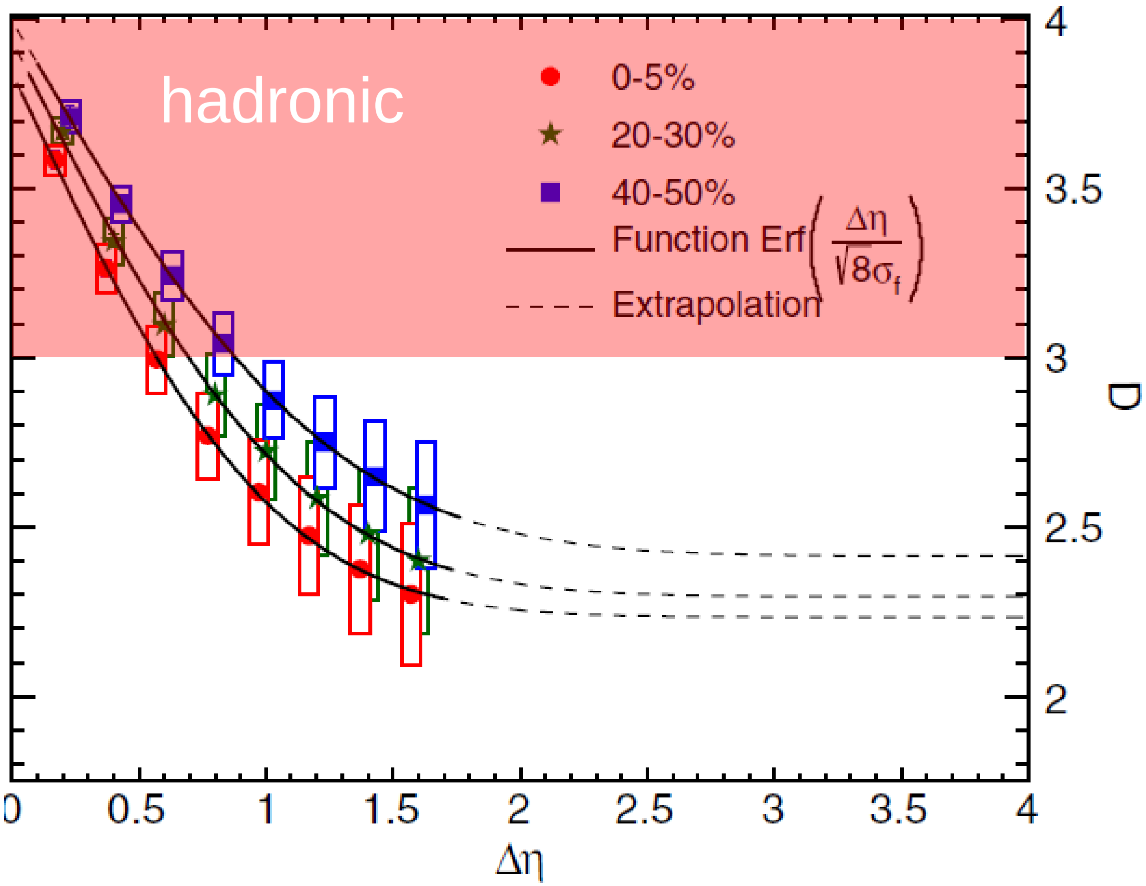
Time Evolution of Fluctuations



Diffusion modifies distribution

Δη Dependence @ ALICE

ALICE
PRL 2013

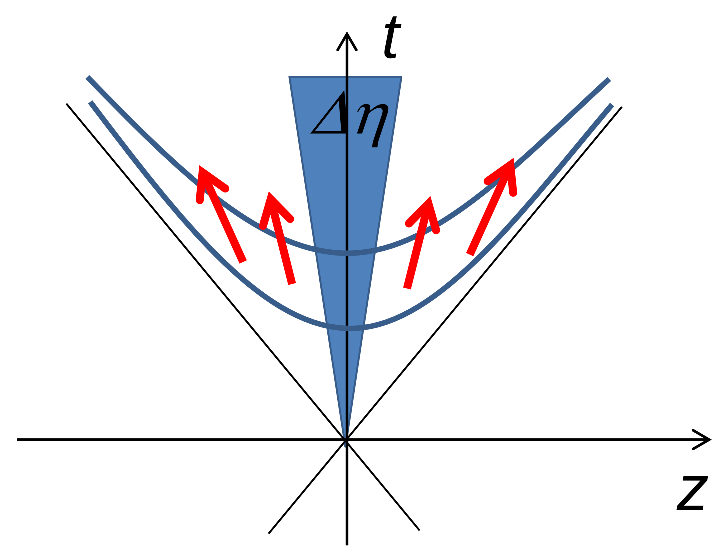


rapidity window

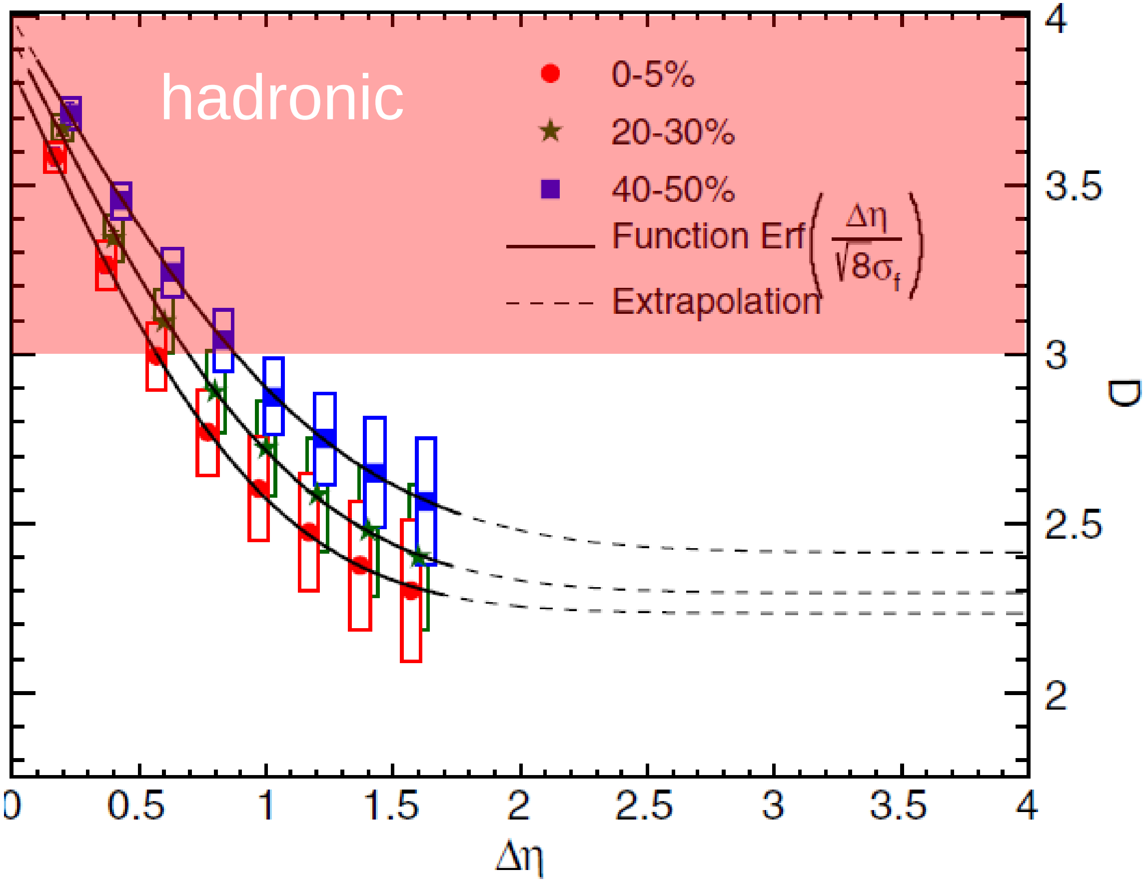
D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- *D* ~ 3-4 Hadronic
- *D* ~ 1-1.5 Quark



Δη Dependence @ ALICE



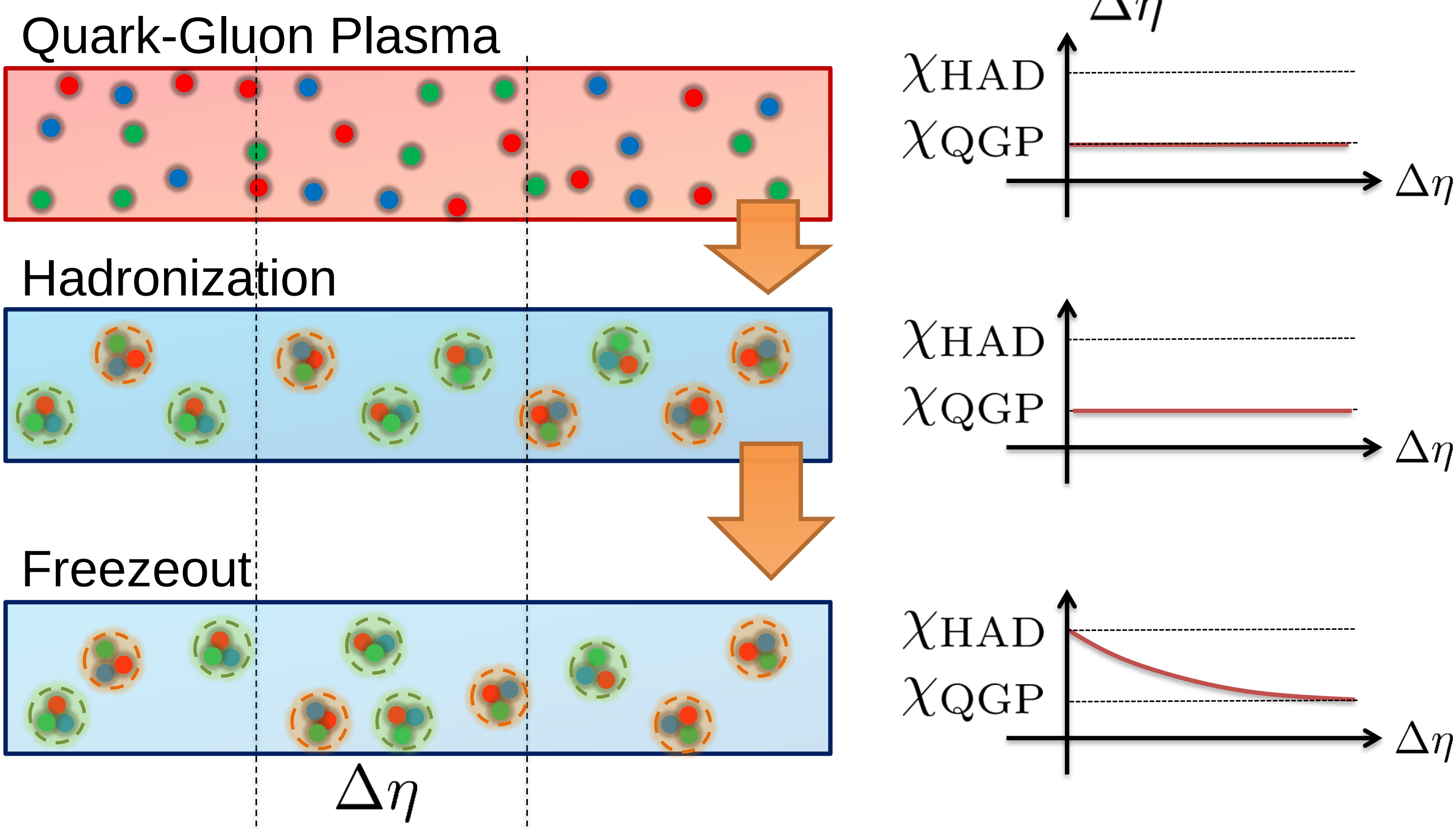
rapidity window

$$D \sim \frac{\langle \delta N_Q \rangle^2}{\Delta\eta}$$

has to be a constant in equil. medium

Fluctuation of N_Q at ALICE is not the equilibrated one.

Time Evolution of Fluctuations

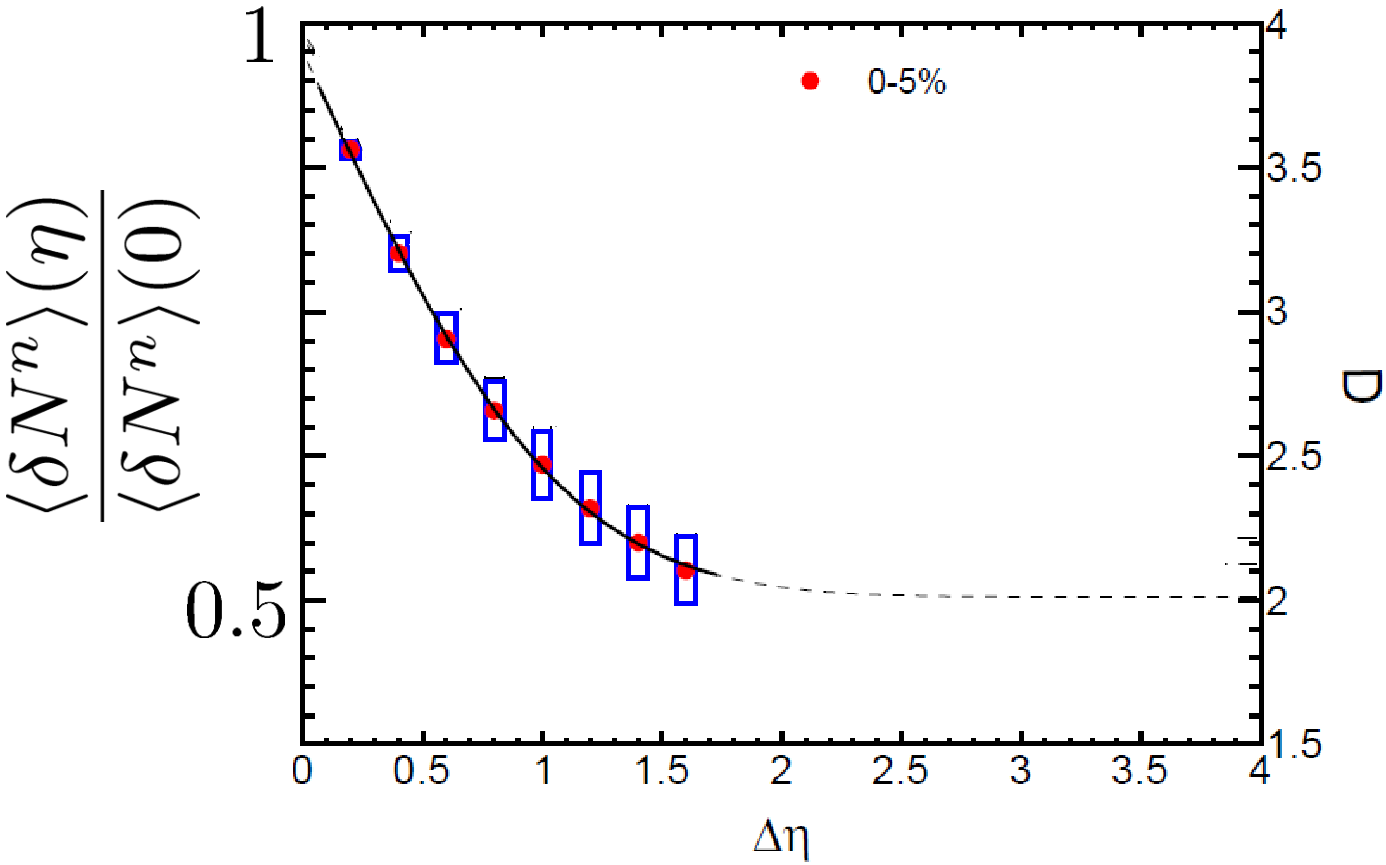


Variation of a conserved charge is achieved only through diffusion. ⇨ The larger $\Delta\eta$, the slower diffusion

$\langle \delta N_Q^4 \rangle$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

suppression or enhancement



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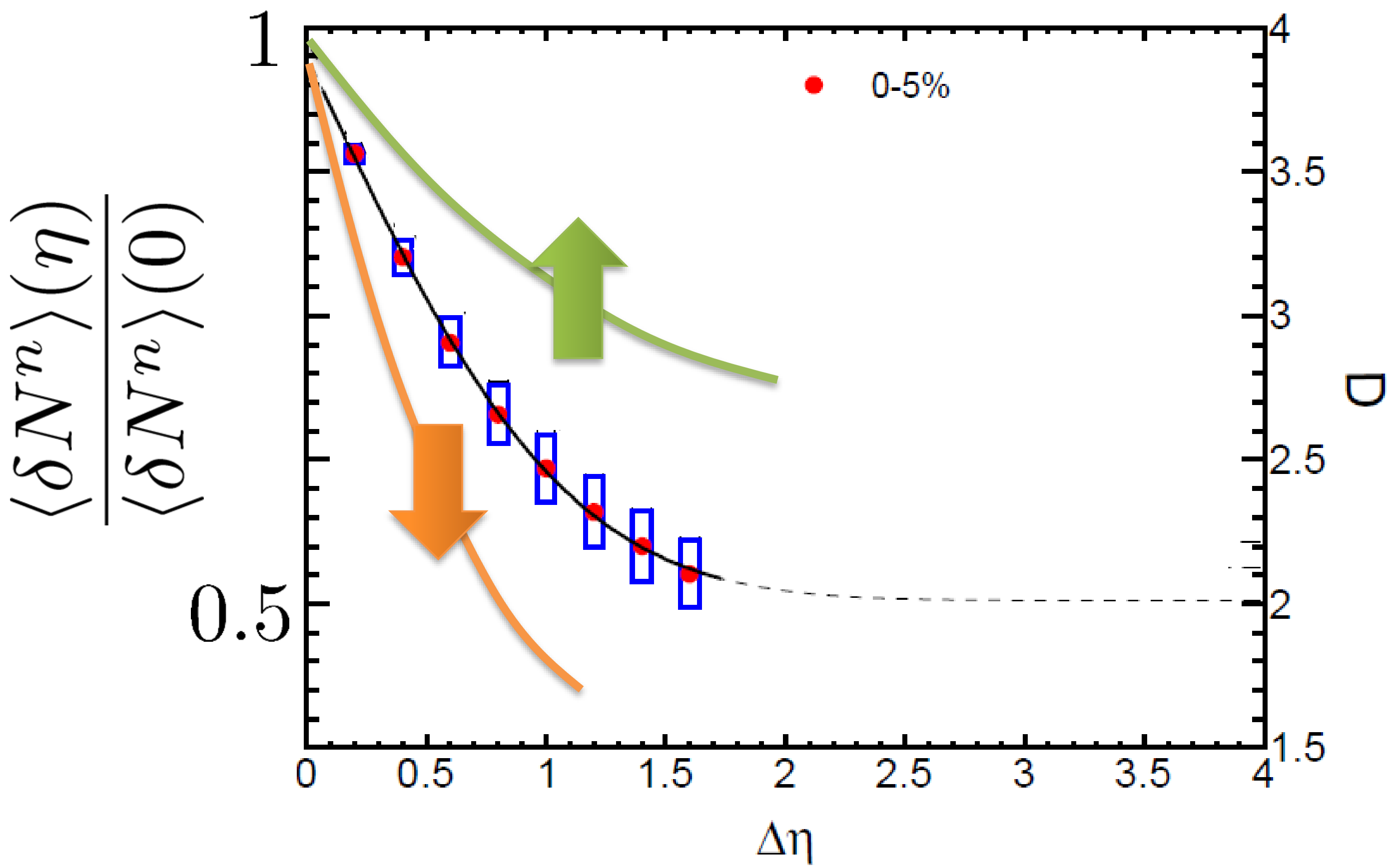


Stochastic Diffusion Equation (SDE)

Diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n$$

- Describe a relaxation of a conserved density *n* toward uniform state without fluctuation

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

$$\langle \xi(\eta_1)\xi(\eta_2)\rangle \sim \chi\delta(\eta_1-\eta_2)$$

- Describe a relaxation toward fluctuating uniform state
- χ: susceptibility (fluctuation in equil.)

Review: Asakawa, MK, PPNP **90** (2016)

Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Shuryak, Stephanov, 2001

Markov (white noise)
+
continuity

↓

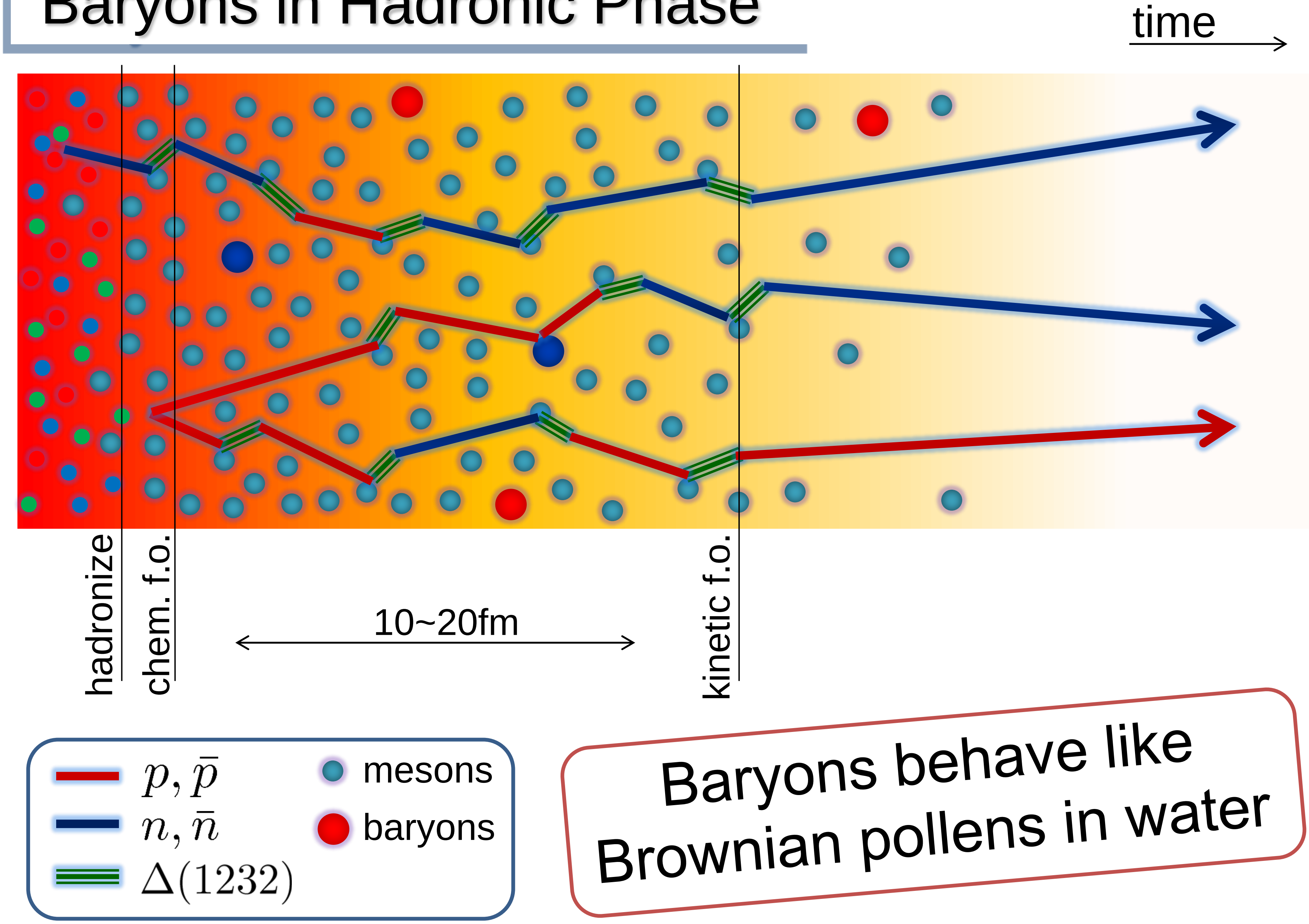
Gaussian noise

cf) Gardiner, “Stochastic Methods”

↓

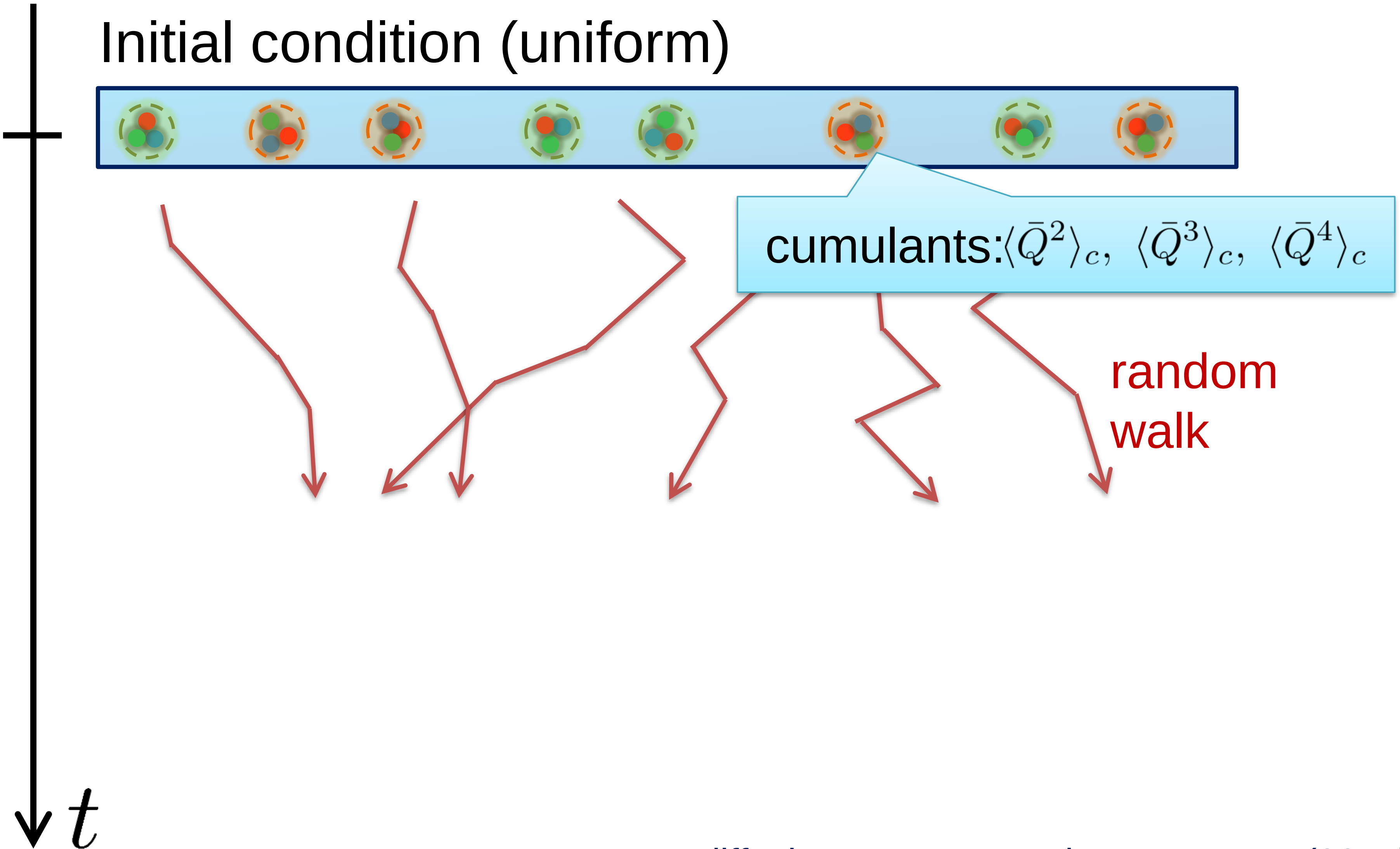
Fluctuation of *n* is
Gaussian in equilibrium

Baryons in Hadronic Phase



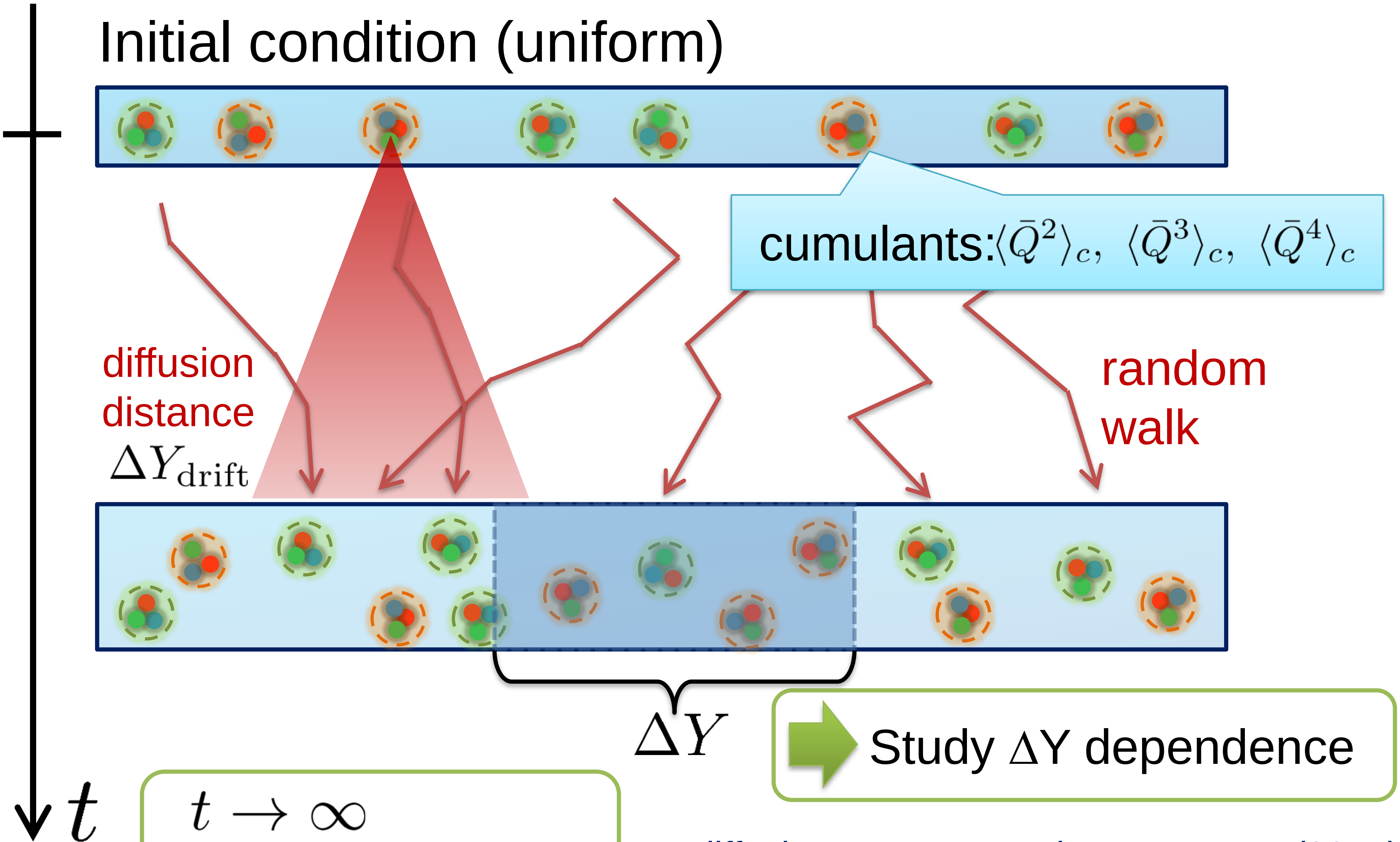
Baryons behave like Brownian pollens in water

Non-Interacting Brownian Particle System



diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

Non-Interacting Brownian Particle System



Study ΔY dependence

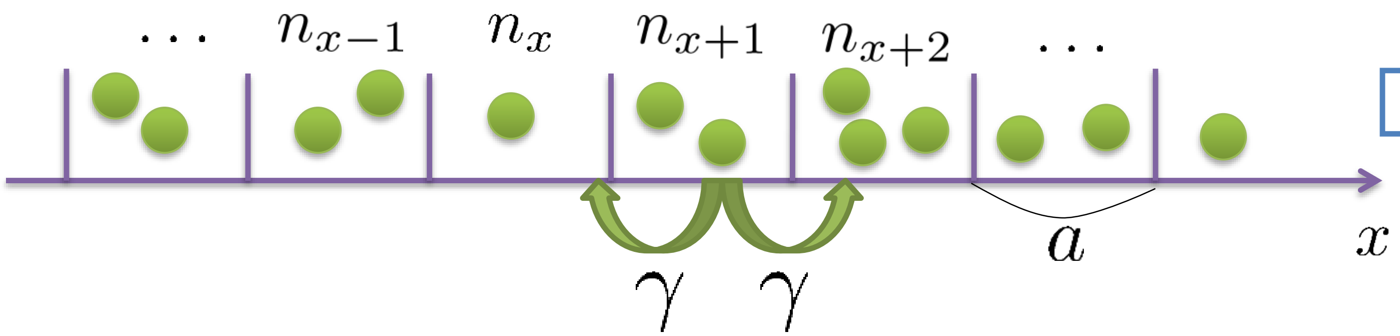
$t \to \infty$
Poisson distribution

diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

Diffusion Master Equation

MK, Asakawa, Ono, 2014
MK, 2015

Divide spatial coordinate into discrete cells



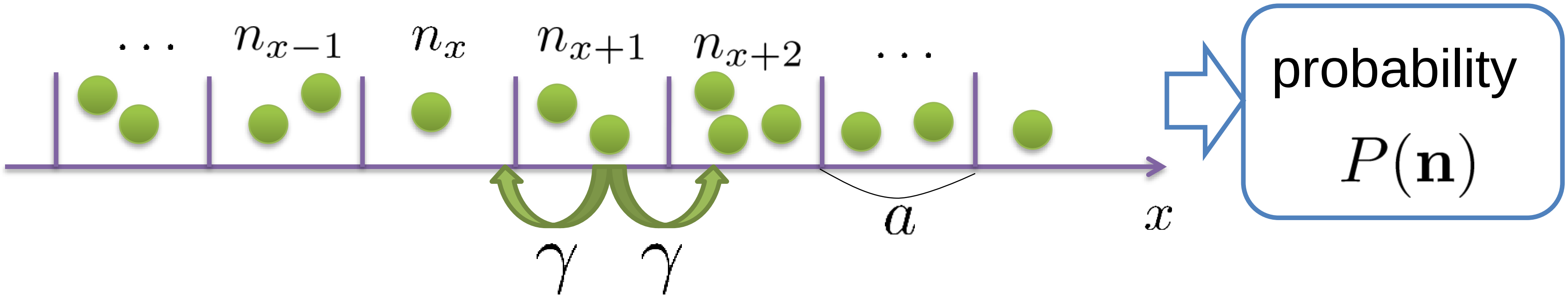
probability

$$P(\mathbf{n})$$

Diffusion Master Equation

MK, Asakawa, Ono, 2014
MK, 2015

Divide spatial coordinate into discrete cells



Master Equation for $P(n)$

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x+1)\{P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x+1})+P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x-1})\} - 2n_x P(\mathbf{n})]$$

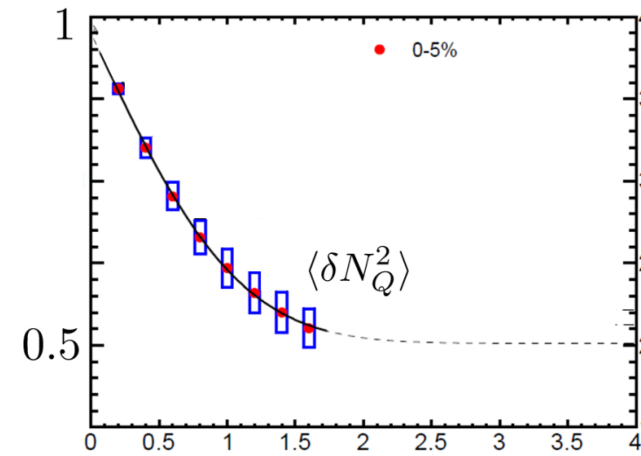
Solve the DME **exactly**, and take $a \to 0$ limit

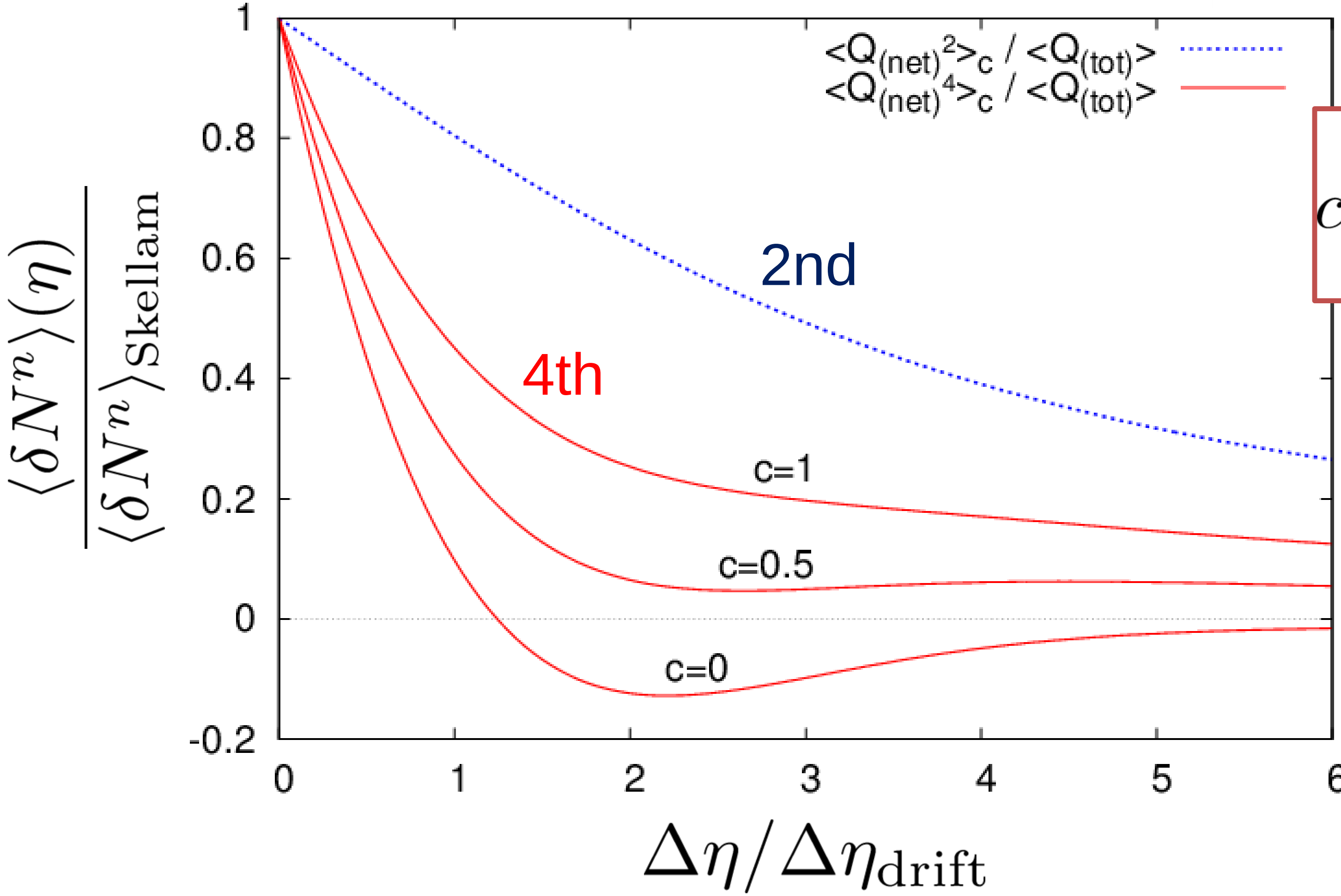
No approx., ex. van Kampen's system size expansion

Rapidity Window Dependence

No initial net fluctuation:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$



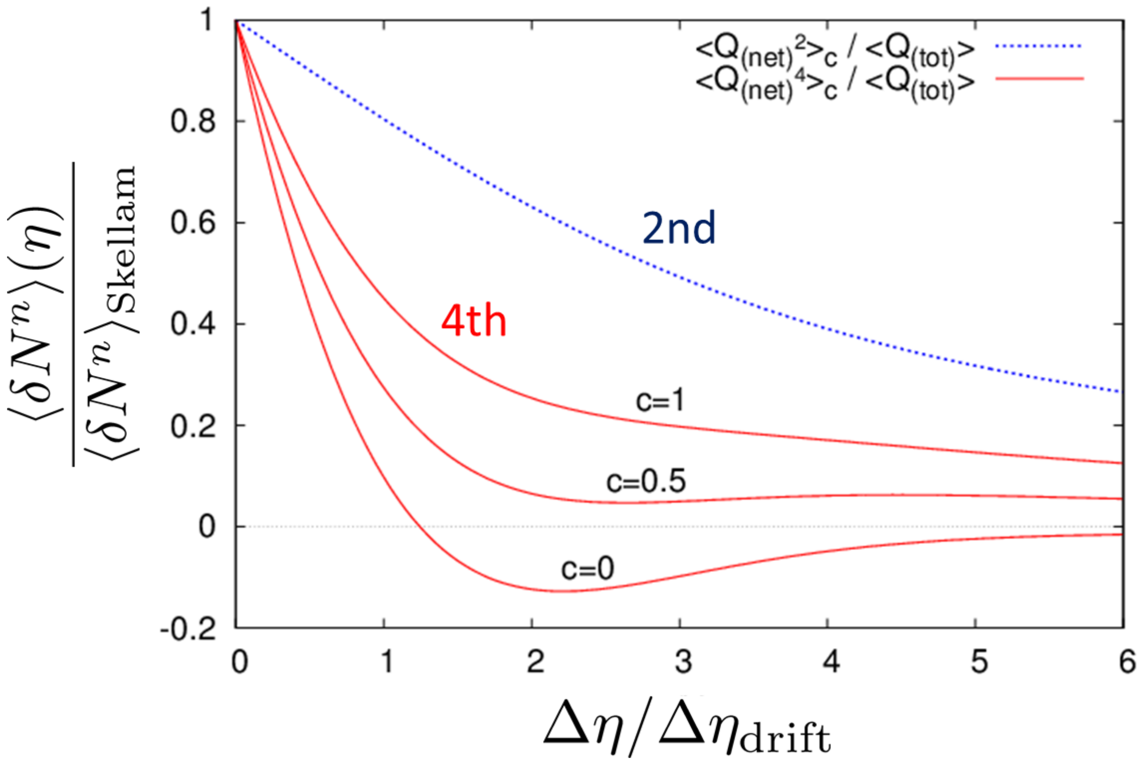


$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

Rapidity Window Dependence

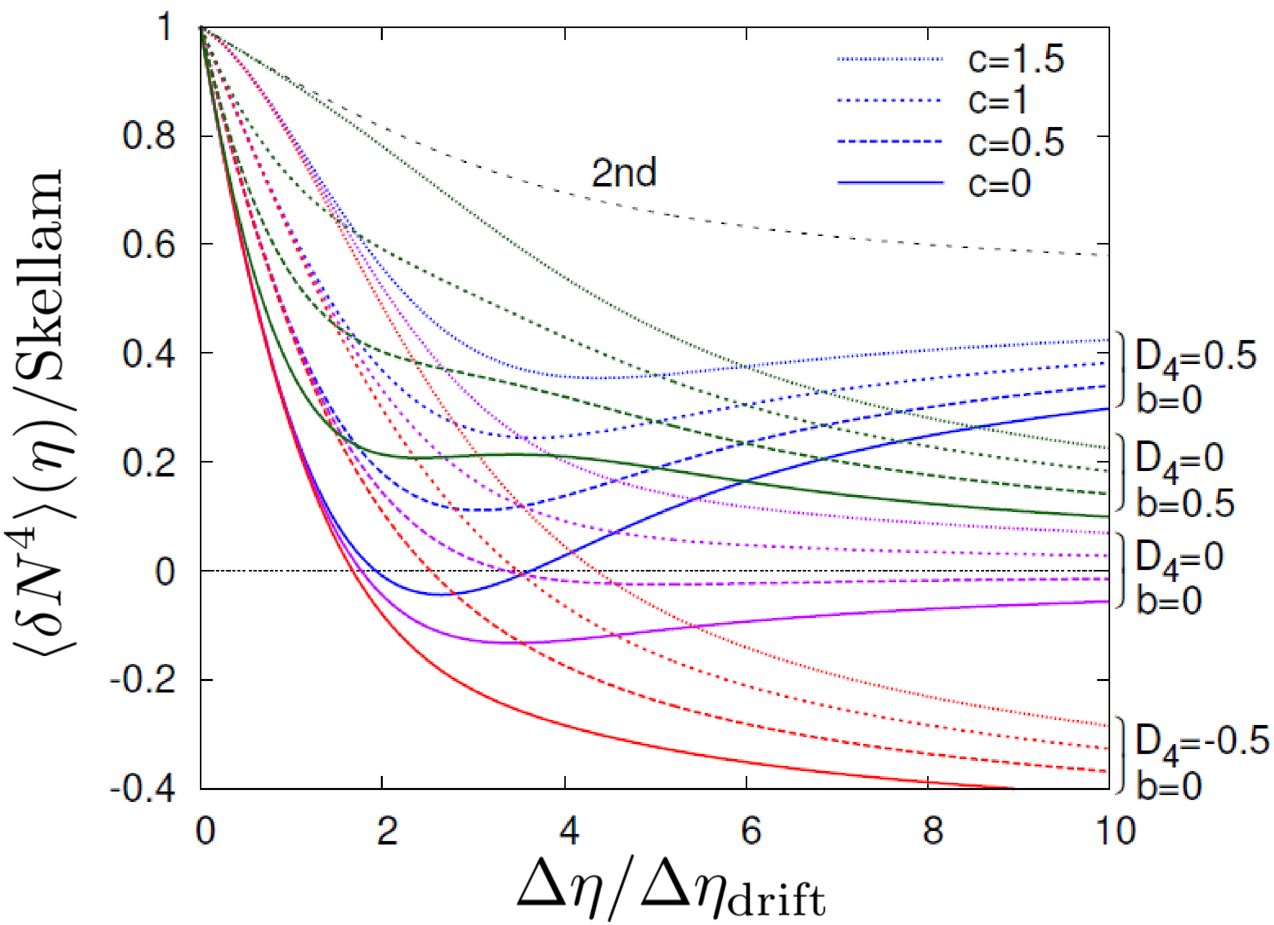
MK+, PLB(2014)



- Cumulants at finite Δy is different from initial value.
- 4th cumulant can have a sign change.
- 4th cumulant can have non-monotonic behavior.

Δη Dependence: 4th order

MK, NPA(2015)



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

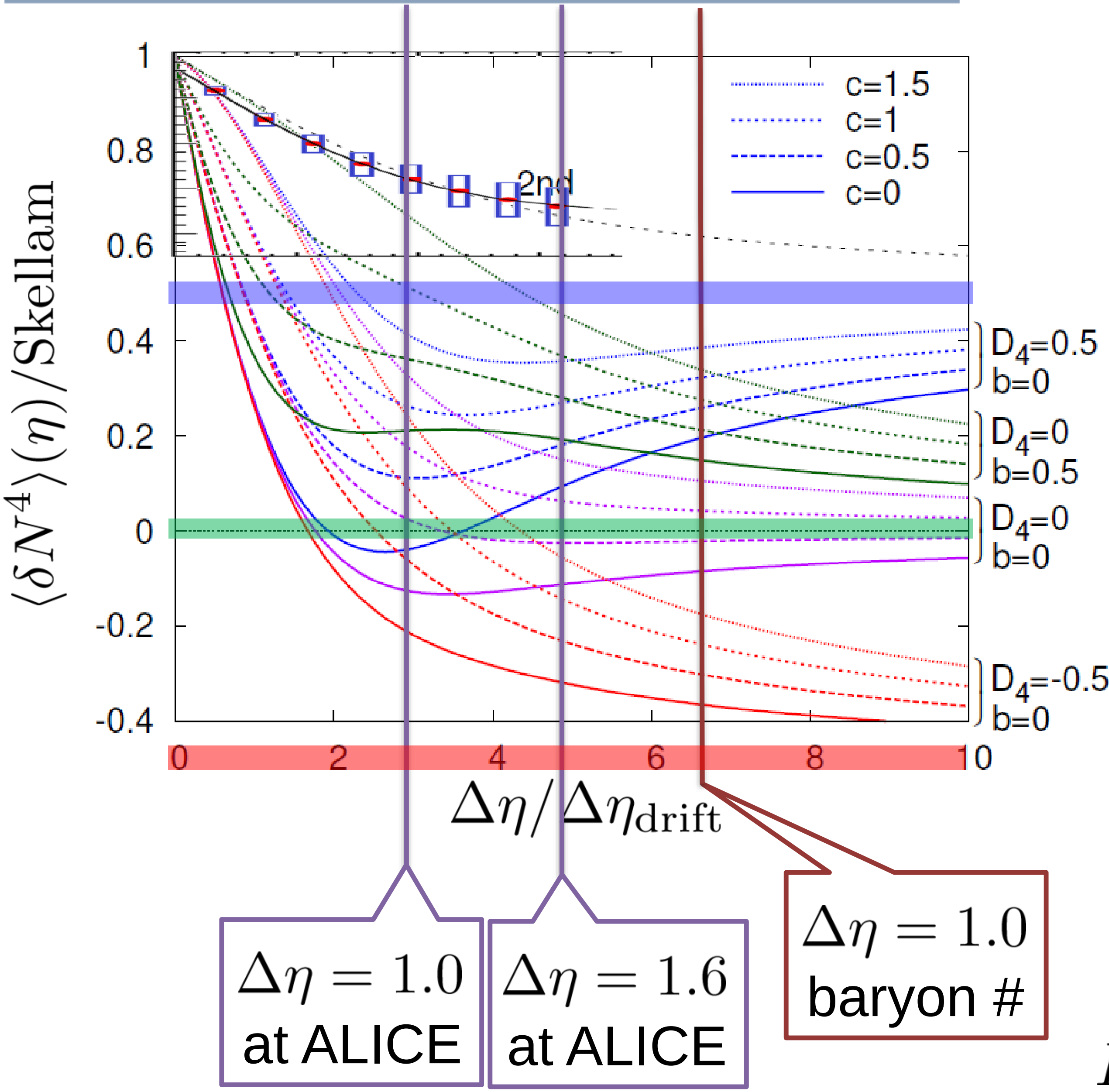
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

Characteristic Δη dependences!

Δη Dependence: 4th order

MK, NPA(2015)



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

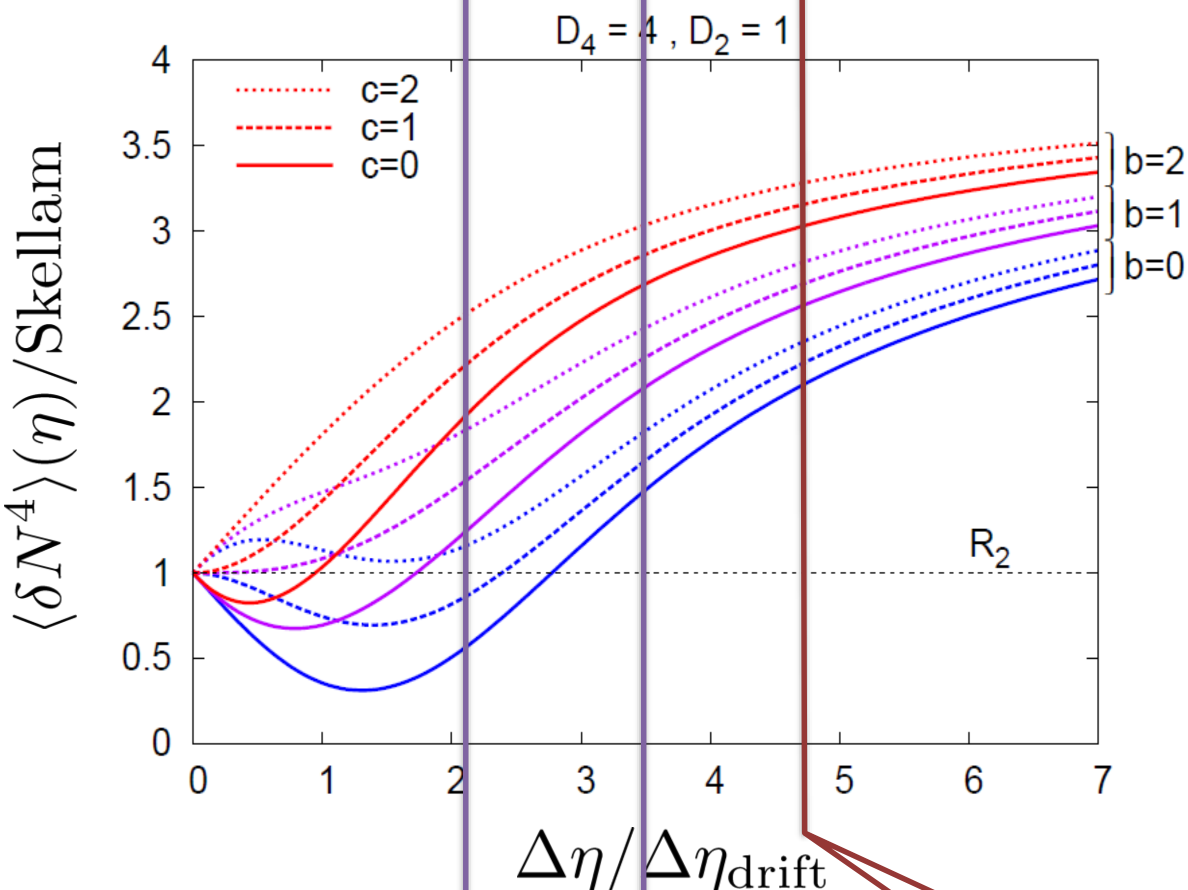
$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

$$c = \frac{\langle Q^2_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} = 0.5$$

$$D \sim M^{-1}$$

4^{th} order : w/ Critical Fluctuation



$\Delta\eta = 1.0$ at ALICE

$\Delta\eta = 1.6$ at ALICE

$\Delta\eta = 1.0$ baryon #

Initial Condition

$$D_4 = \frac{\langle Q^4_{(\rm net)} \rangle_c}{\langle Q_{(\rm tot)} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\rm net)} Q_{(\rm tot)} \rangle_c}{\langle Q_{(\rm net)} \rangle}$$

$$c = \frac{\langle Q^2_{(\rm tot)} \rangle_c}{\langle Q_{(\rm tot)} \rangle}$$

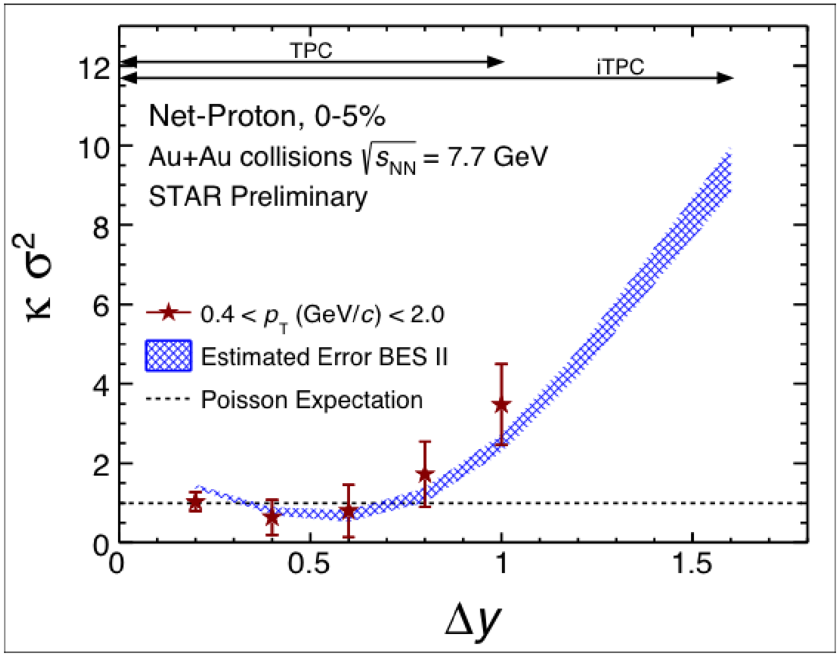
$$D_2 = \frac{\langle Q^2_{(\rm net)} \rangle_c}{\langle Q_{(\rm tot)} \rangle} = 1$$

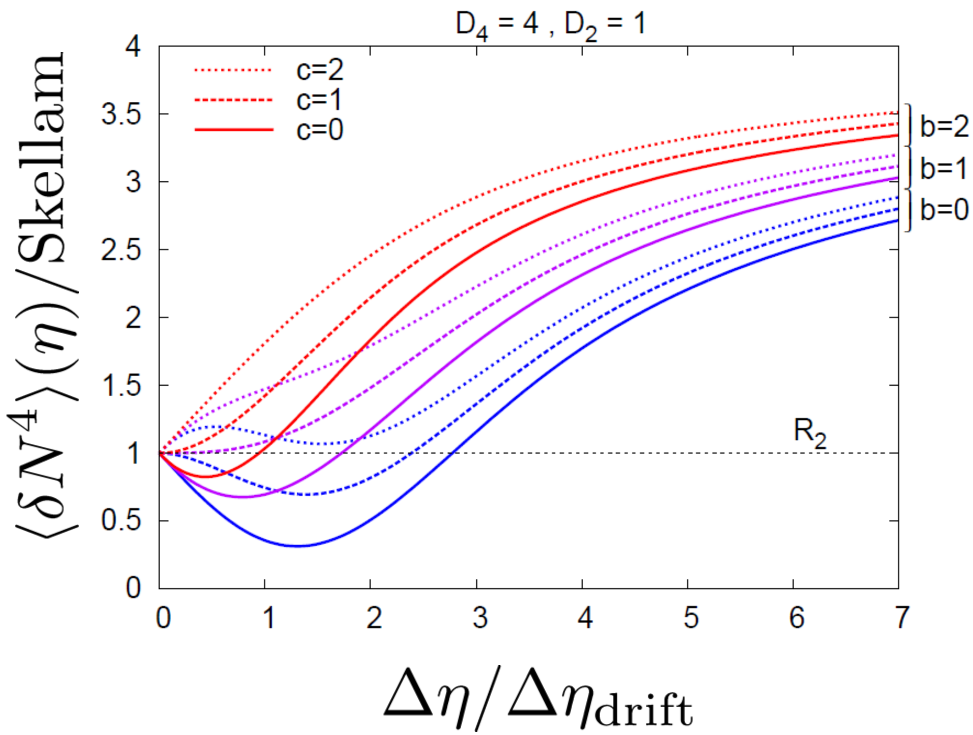
$$D \sim M^{-1}$$

Δη Dependence @ STAR

MK+ (2014)
MK (2015)

X. Luo, CPOD2014

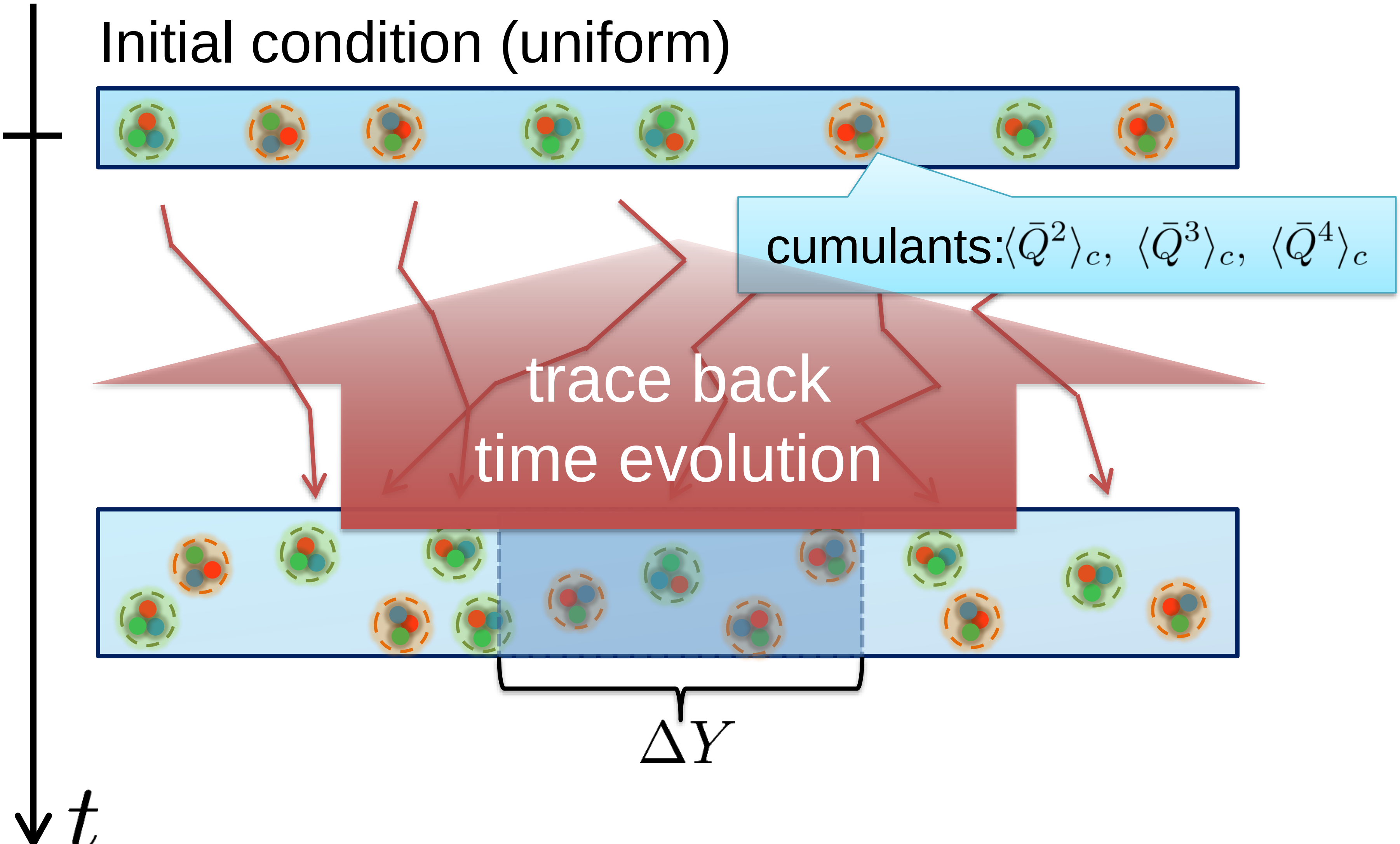




- Non monotonic behavior of cumulants.
- Approach initial value as Δy→large

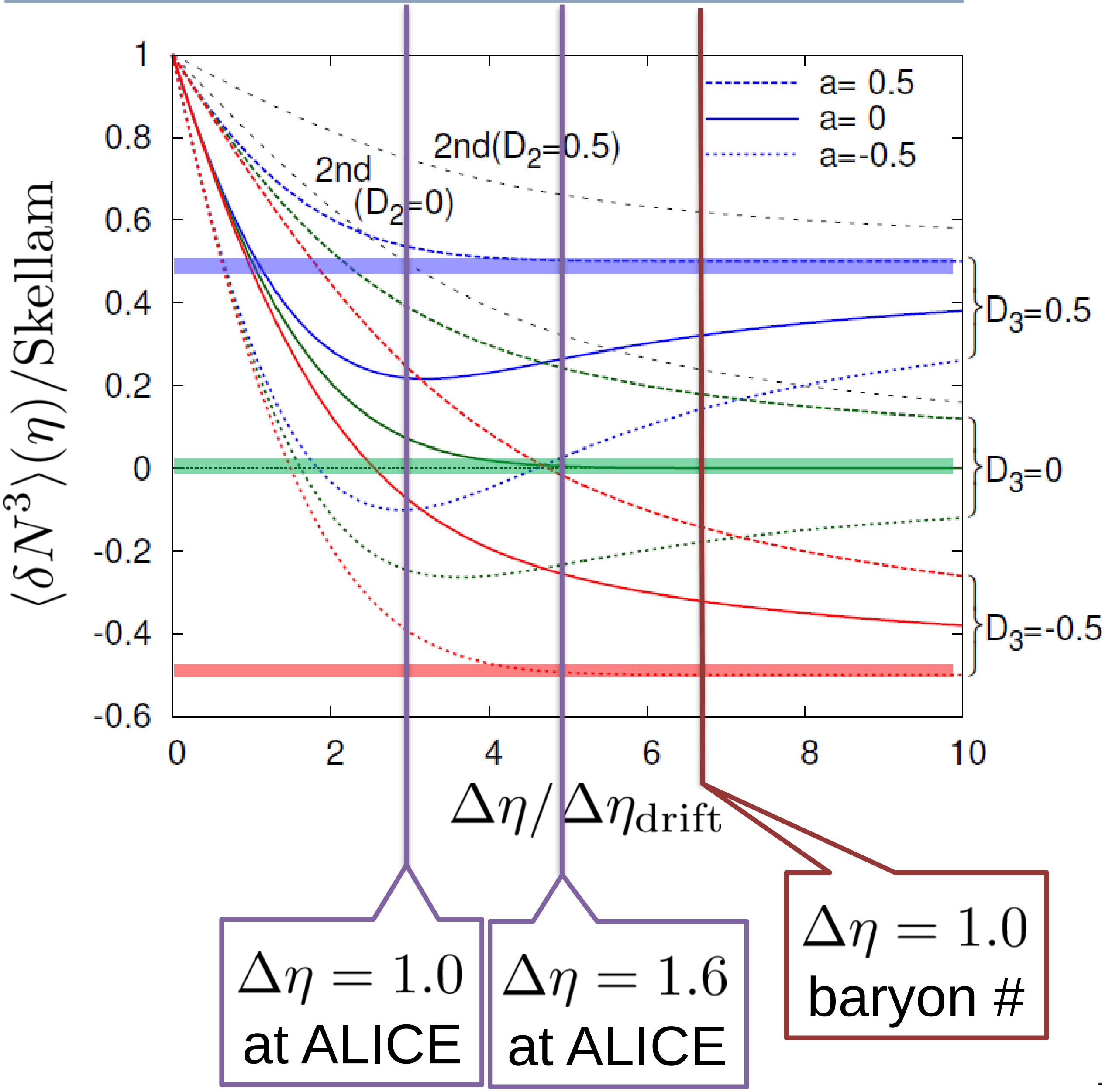
finite volume effect: Sakaida+, PRC064911(2014)
More sophisticated analysis with **factorial cumulants**, MK, Luo (2017)

Non-Interacting Brownian Particle System



diffusion distance ←→ diffusion coefficient

$\Delta\eta$ Dependence: 3[rd] order



Initial Condition

$$D_3 = \frac{\langle Q^3_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$a = \frac{\langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

$$D \sim M^{-1}$$

Summary

- Non-Gaussian fluctuations are one of the most interesting topics in relativistic heavy ion collisions.

- Using fluctuation observables, we can explore early thermodynamics and QCD phase structure.

- Rapidity window dependences of higher-order cumulants encode various information on fluctuation.

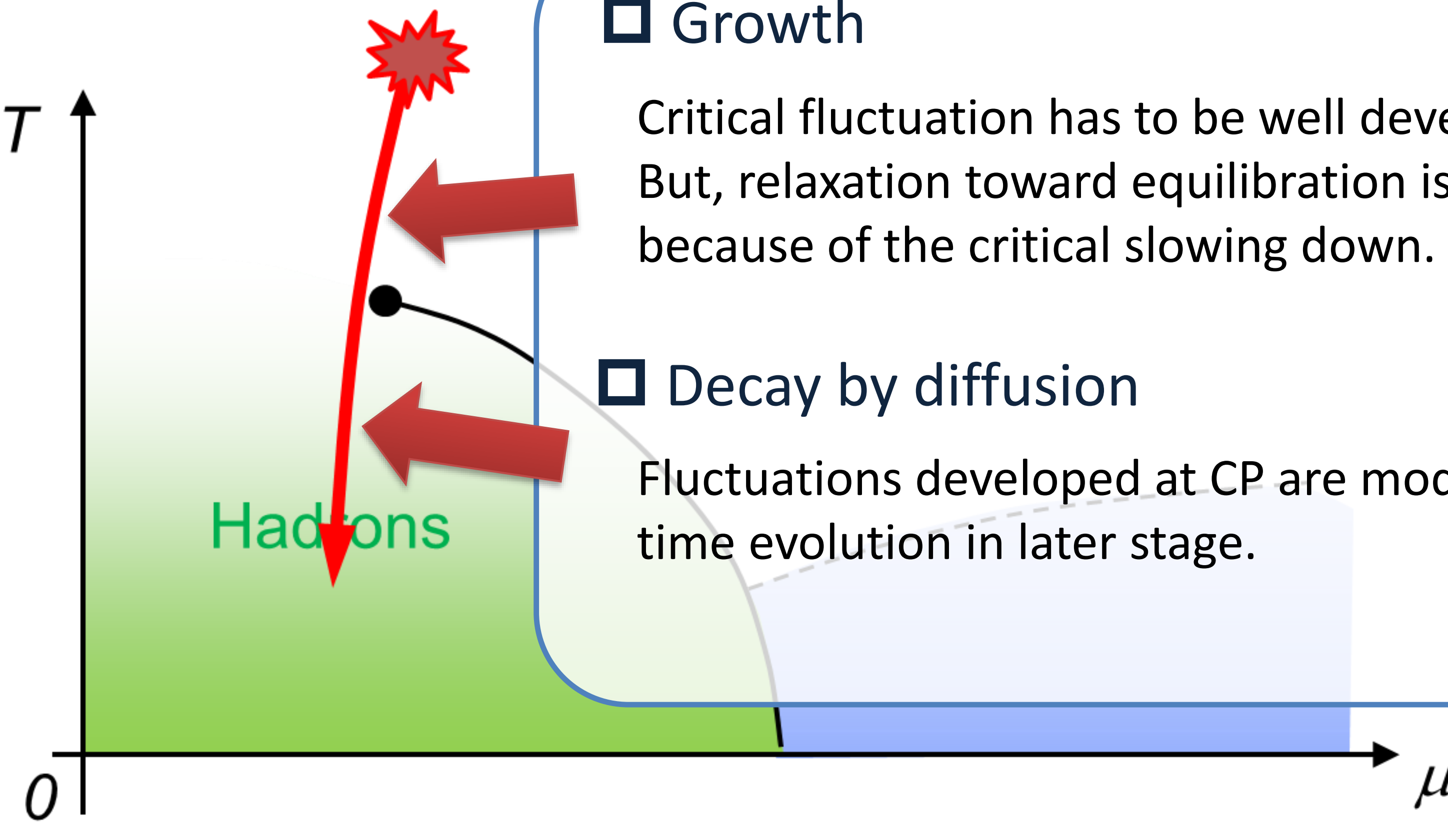
- More information in future experiments. More theoretical studies are required!

Search for QCD Critical Point

Sakaida, Asakawa, Fujii, MK, to appear in PRC
arXiv:1703.08008

Remarks on Critical Fluctuation 1

Experiments cannot observe
critical fluctuation in equilibrium directly.



- **Growth**

  Critical fluctuation has to be well developed.
  But, relaxation toward equilibration is slow around CP because of the critical slowing down.

- **Decay by diffusion**

  Fluctuations developed at CP are modified by the time evolution in later stage.

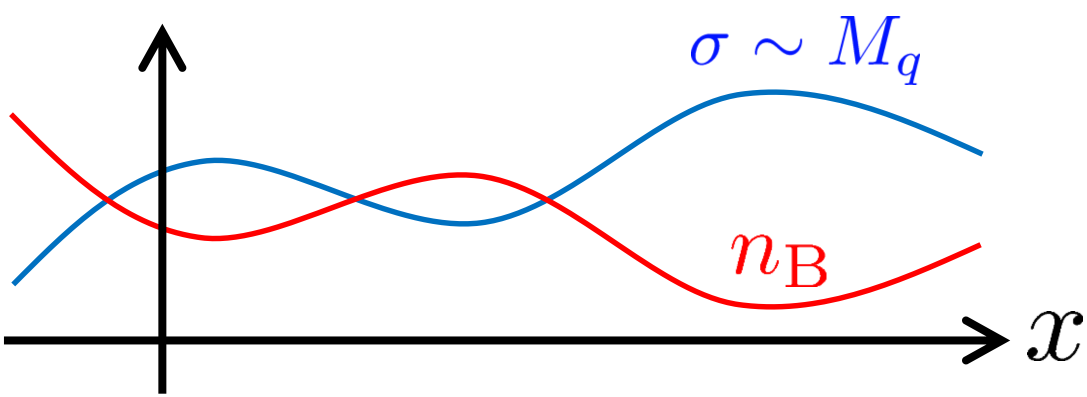
Remarks on Critical Fluctuation 2

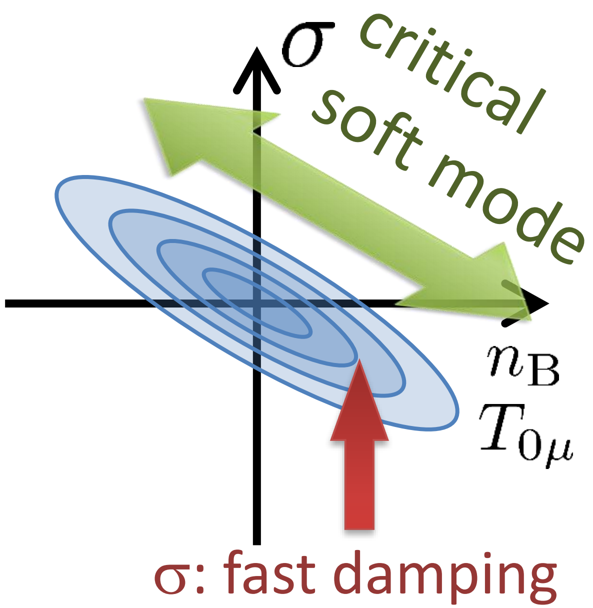
Critical fluctuation is a conserved mode!

Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

Fluctuations of σ and n_B are coupled around the CP!

$$\delta\sigma \simeq \delta n_B$$



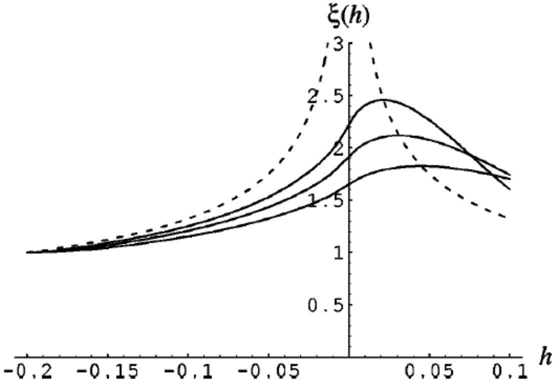


$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \cdots$$

Dynamical Evolution of Critical Fluctuations

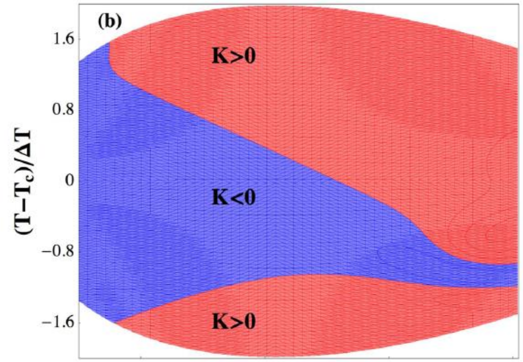
- Evolution of correlation length

  Berdnikov, Rajagopal (2000)
  Asakawa, Nonaka (2002)



- Higher orders (spatially uniform “σ” mode)

  Mukherjee, Venugopalan, Yin (2015)



- Dynamical evolution in chiral fluid model

  Nahrgang, Herold, … (2014~)

- Correlation functions

  Kapusta, Torres-Rincon (2012)

Aim of This Study

- Describe **conserved nature** of critical fluctuation.
- We want to study **experimental observables**.
  - focus on a **conserved charge** **(baryon number)**
  - study evolution of **conserved-charge** fluctuation
- Concentrate on **2nd order** fluctuation. (not higher)
- We study
  - **rapidity window denepdence** of the cumulant
  - 2-particle **correlation function**

Our Main Conclusion

Non-monotonicity in cumulants or correlation func.	=	Signal of QCD-CP

Cumulants and Correlation Function

$$Q = \int_V dx\, n(x)$$

total charge

charge density

$$\langle \delta Q^2 \rangle = \int_V dx dy \langle \delta n(x) \delta n(y) \rangle$$

2nd order cumulant (fluctuation)

correlation function

1-to-1 correspondence

1-dim case

$$\langle \delta Q^2 \rangle_{\Delta y} = \int_{\Delta y} dy (\Delta y - |y|) \langle \delta n(y) \delta n(0) \rangle$$

Stochastic Diffusion Equation (SDE)

Diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n$$

- Describe a relaxation of a conserved density *n* toward uniform state without fluctuation

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

$$\langle \xi(\eta_1)\xi(\eta_2)\rangle \sim \chi\delta(\eta_1 - \eta_2)$$

- Describe a relaxation toward fluctuating uniform state
- χ: susceptibility (fluctuation in equil.)

Review: Asakawa, MK, PPNP **90** (2016)

Soft Mode of QCD Critical Point

Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

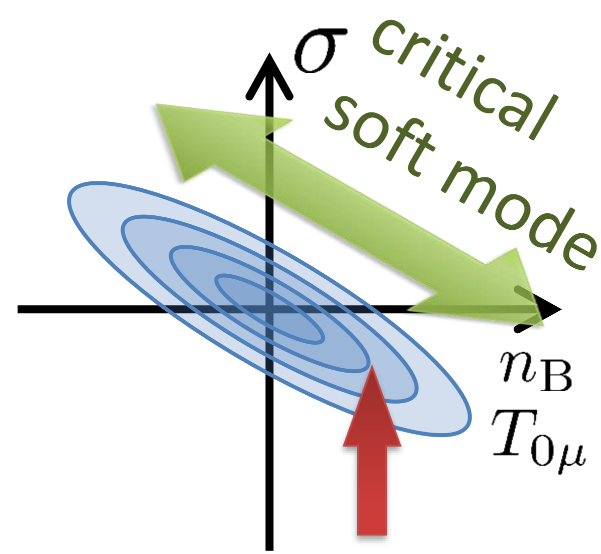
- Effective potential

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \cdots$$

- Time dependent Ginzburg-Landau

$$\begin{pmatrix} \dot\sigma \\ \dot n \end{pmatrix} = \begin{pmatrix} \Gamma_{\sigma\sigma} & \Gamma_{\sigma n} \\ \Gamma_{n\sigma} & \Gamma_{nn} \end{pmatrix} \begin{pmatrix} \sigma \\ n \end{pmatrix}$$

$\sim k^2$



For slow and long wavelength,

SDE

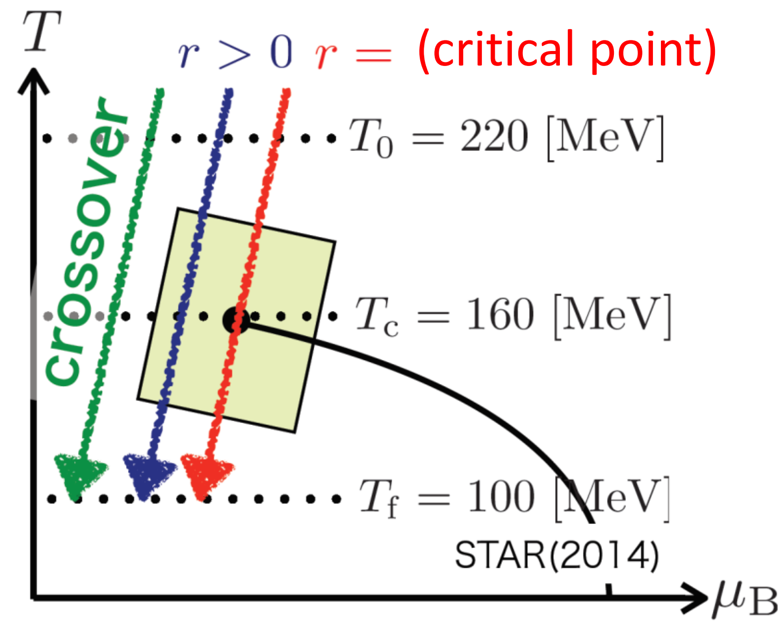
$$\partial_\tau n = D(\tau)\partial_\eta^2 n + \partial_\eta \xi$$

$$\langle \xi(\eta_1)\xi(\eta_2)\rangle \sim \chi(\tau)\delta(\eta_1 - \eta_2)$$

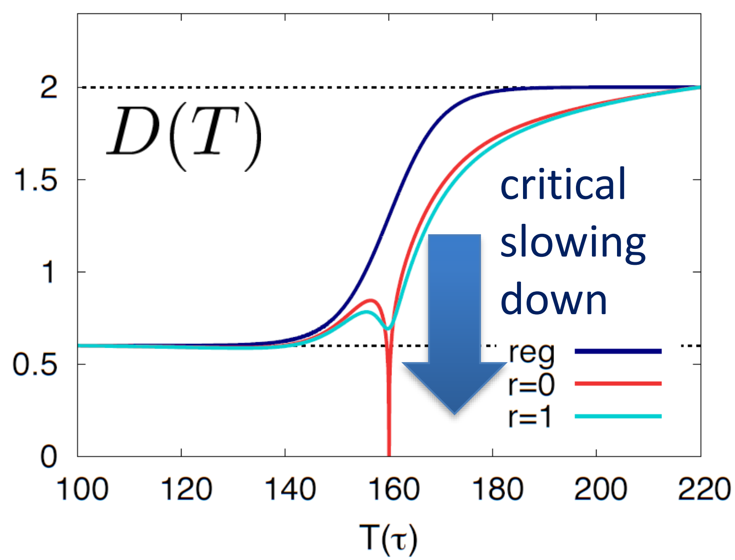
singularities in $D(\tau)$ and $\chi(\tau)$

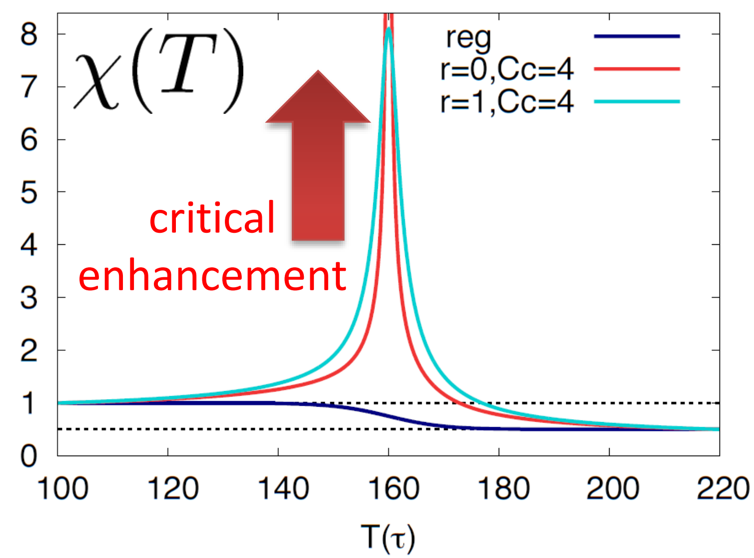
Parametrizing D(τ) and χ(τ)

- **Critical behavior**
  - 3D Ising (r,H)
  - model H



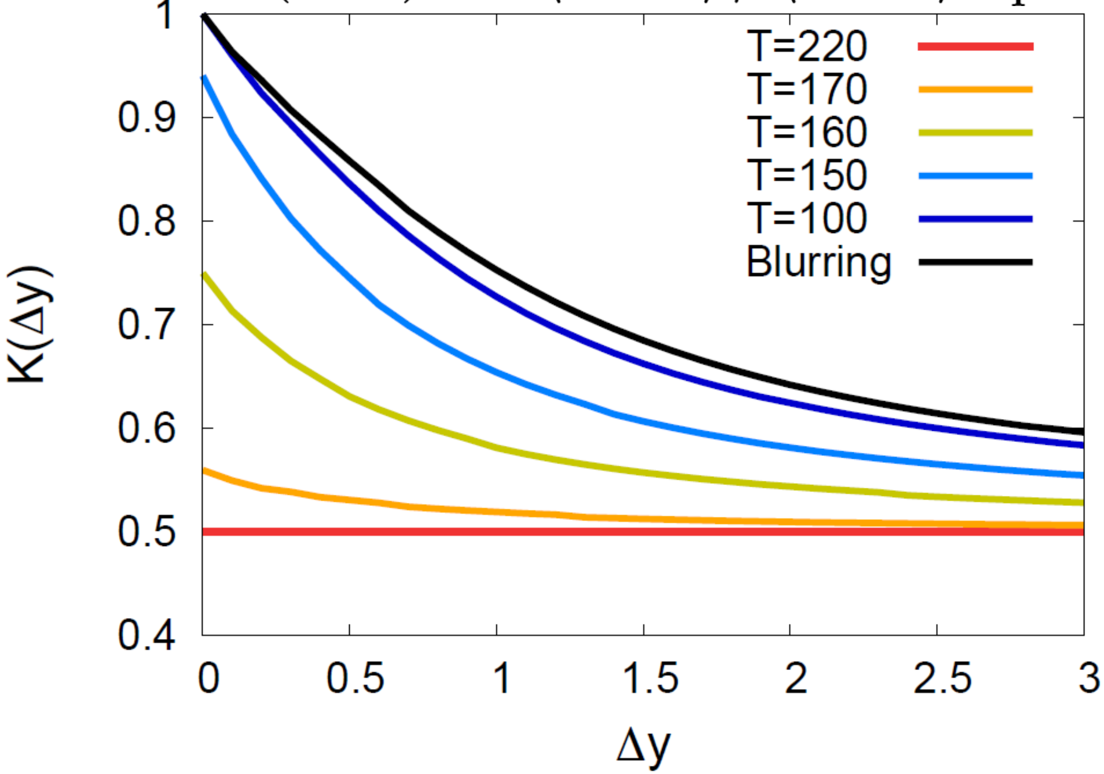
- **Temperature dep.**

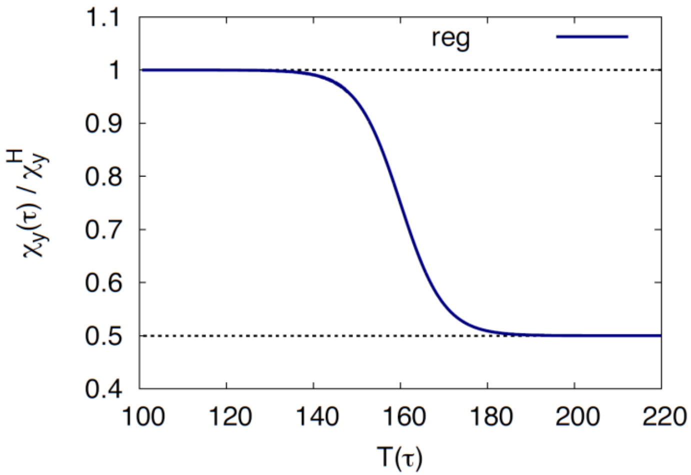


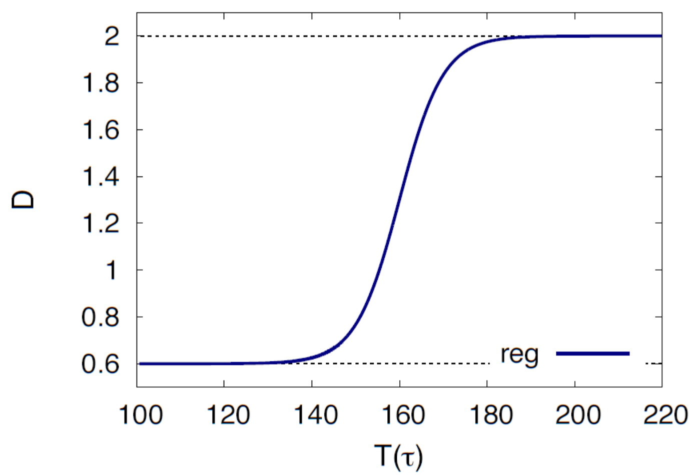


Crossover / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.}$$





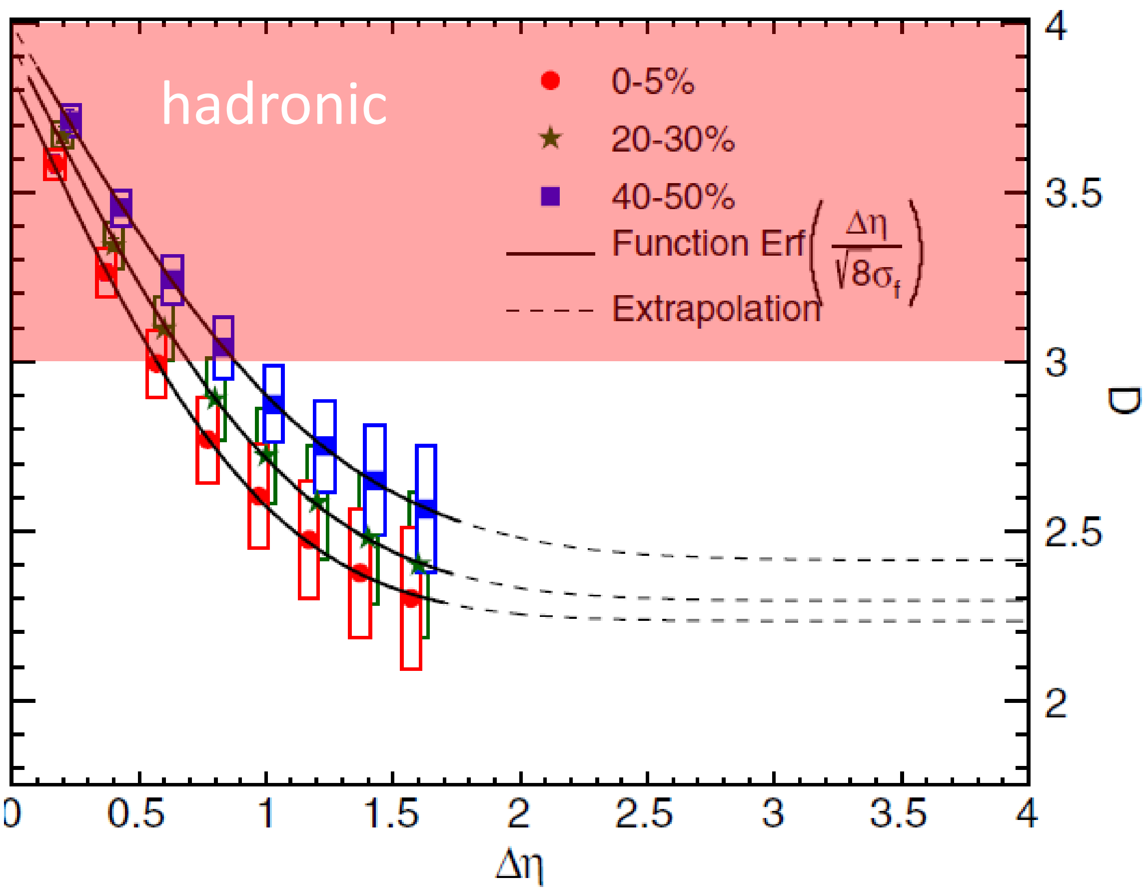


- monotonically decresing

Analytic property: $\chi(\tau)$ monotonically increasing ⇒ $K(\Delta y)$ monotonically decreasing

Δη Dependence @ ALICE

ALICE
PRL 2013

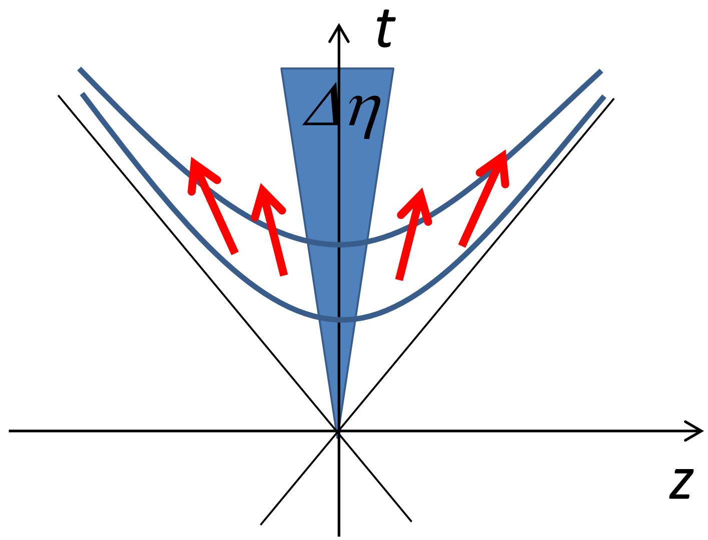


rapidity window

D-measure

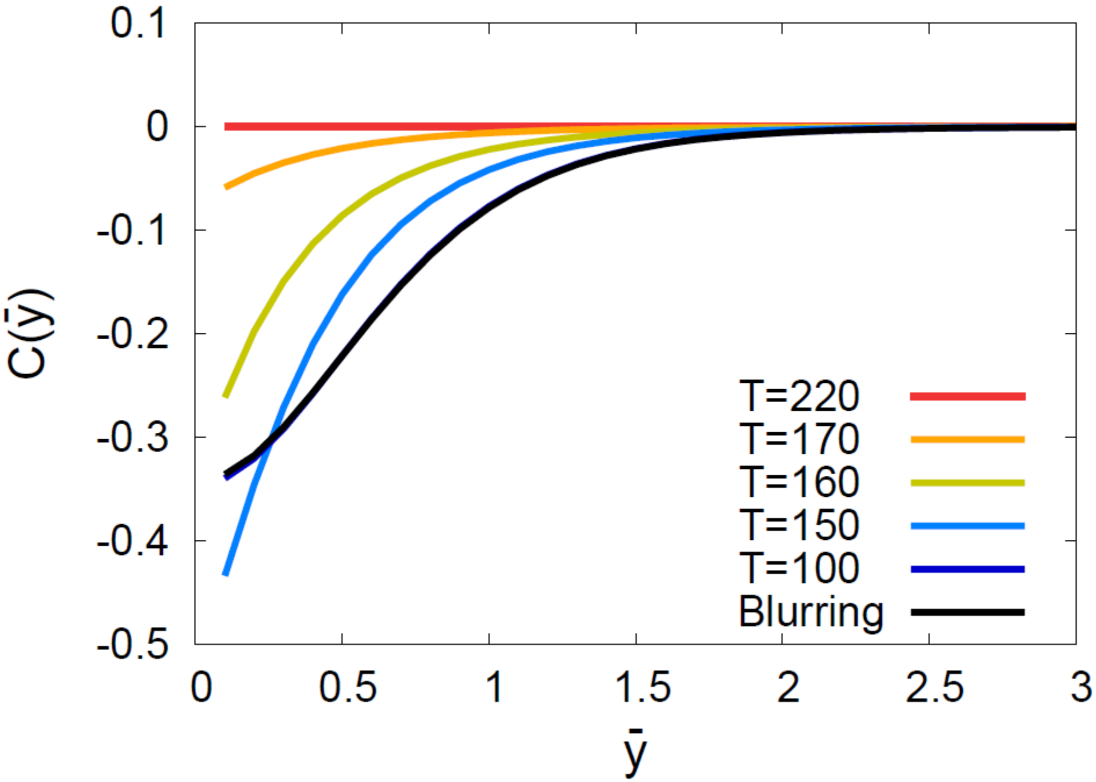
$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

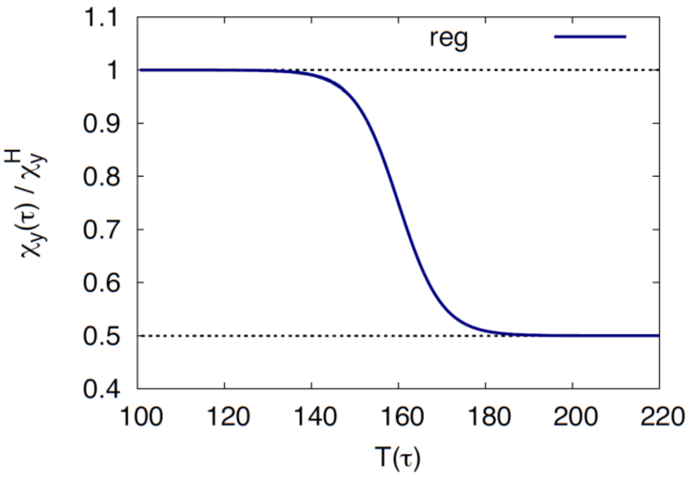
- *D* ~ 3-4 Hadronic
- *D* ~ 1-1.5 Quark

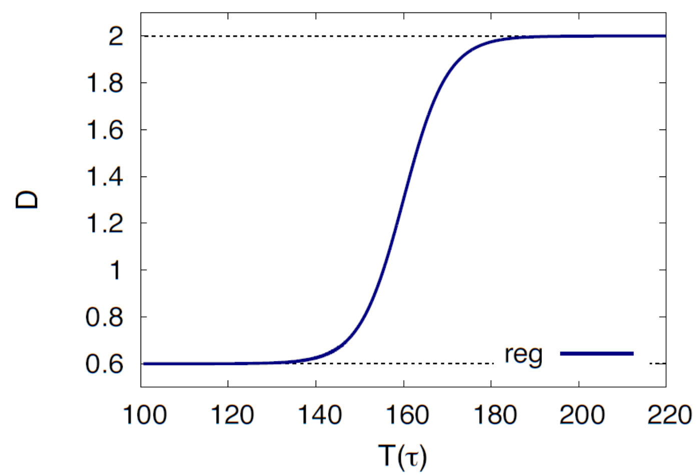


Crossover / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$





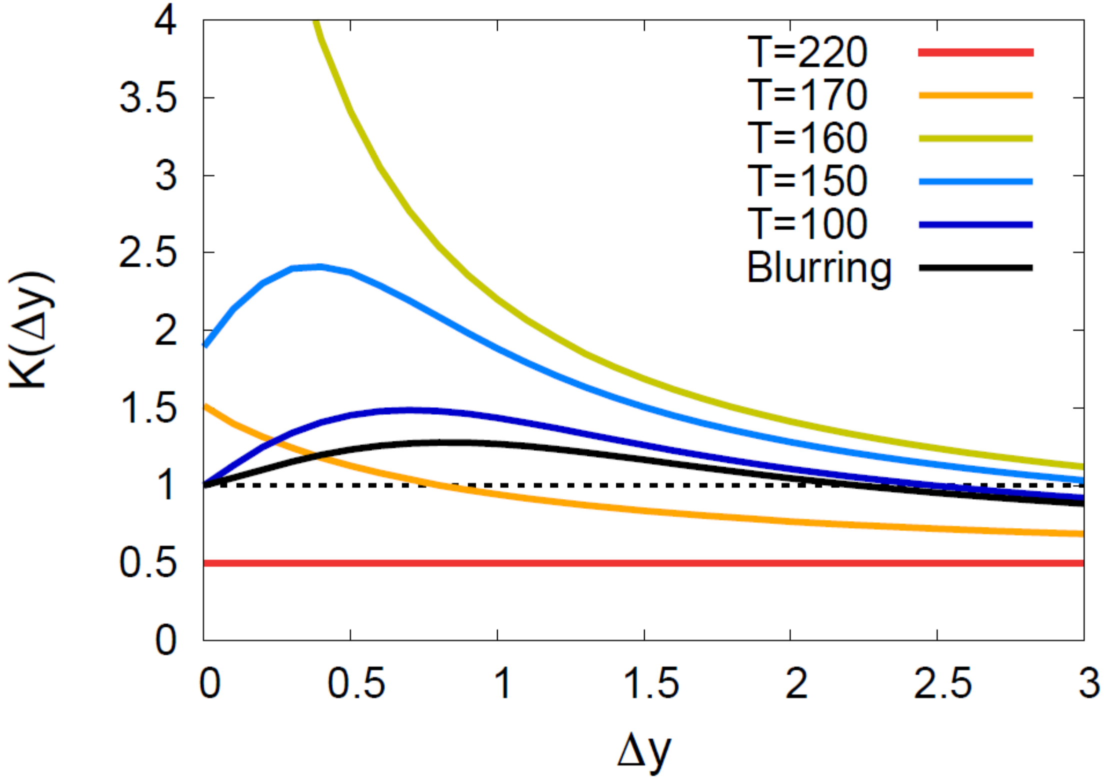


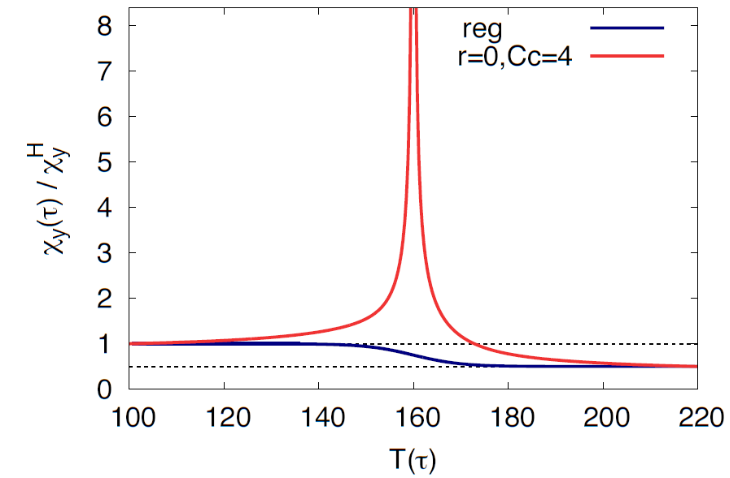
- monotonically decresing

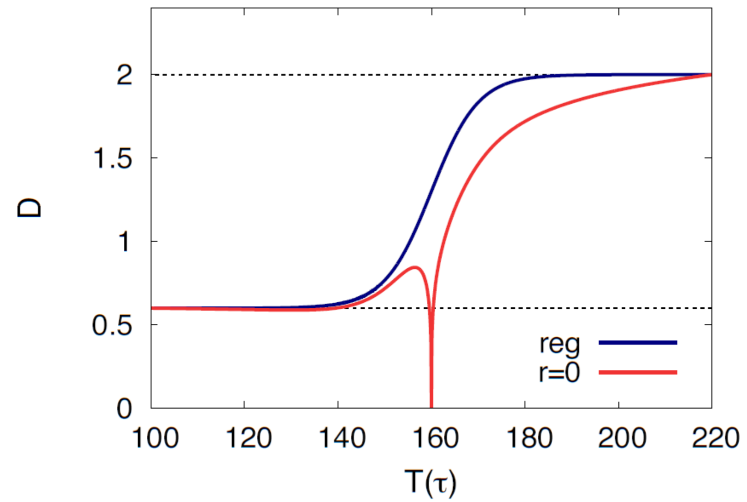
Analytic property: $\chi(\tau)$ monotonically increasing ⇒ $C(\bar{y})$ monotonically decreasing

Critical Point / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$





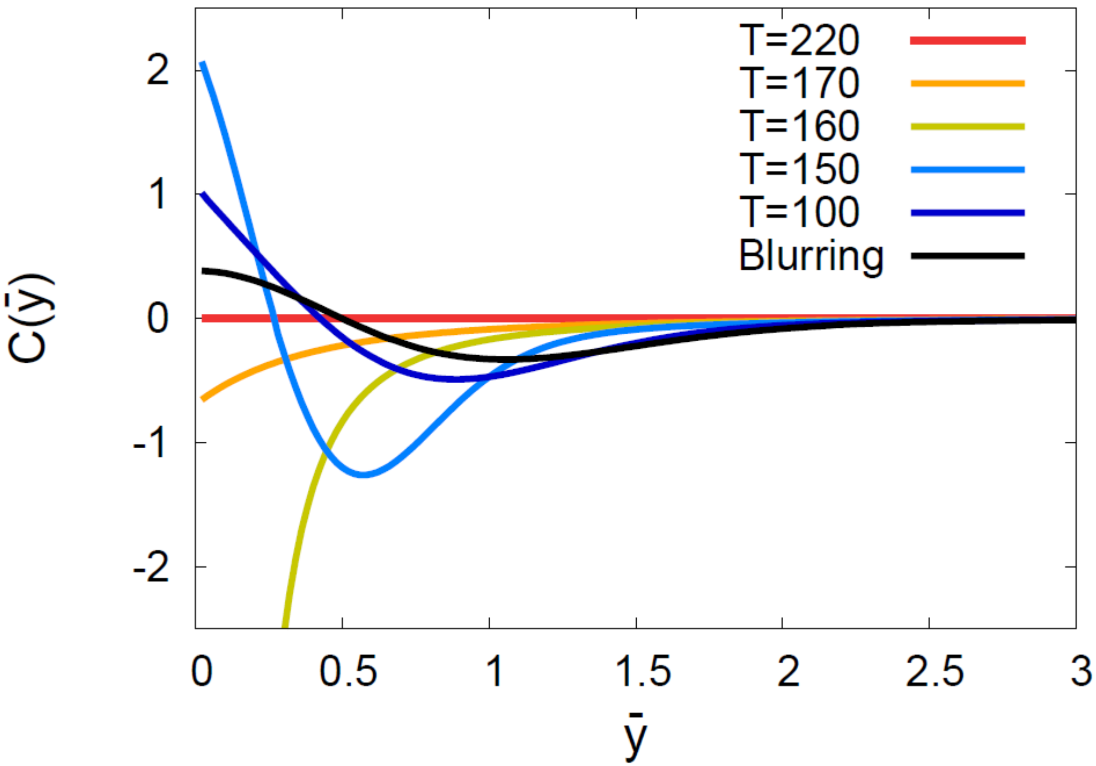


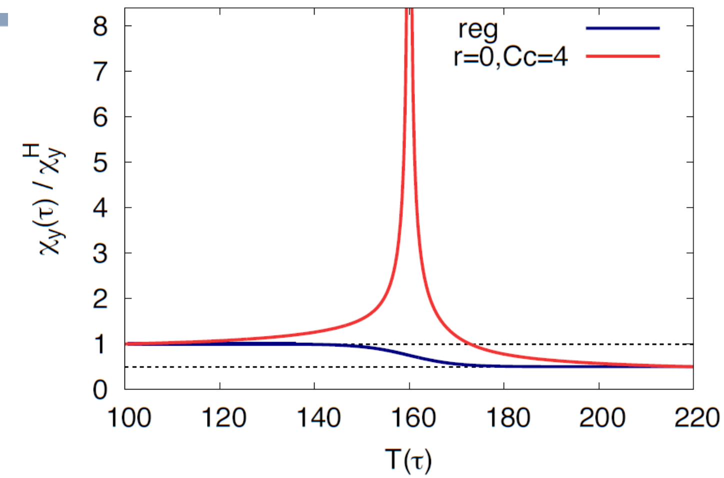
- non-monotonic Δy dep.

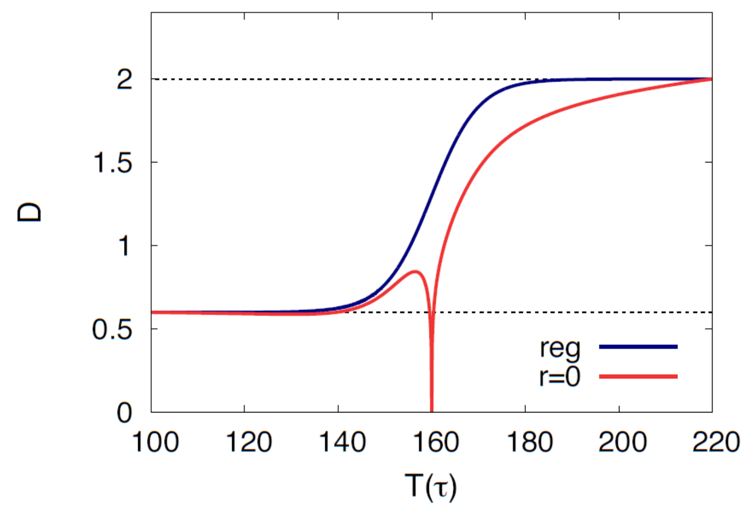
Analytic property	$K(\Delta y)$ non-monotonic	⇨	$\chi(\tau)$ non-monotonic

Criticap Point / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$





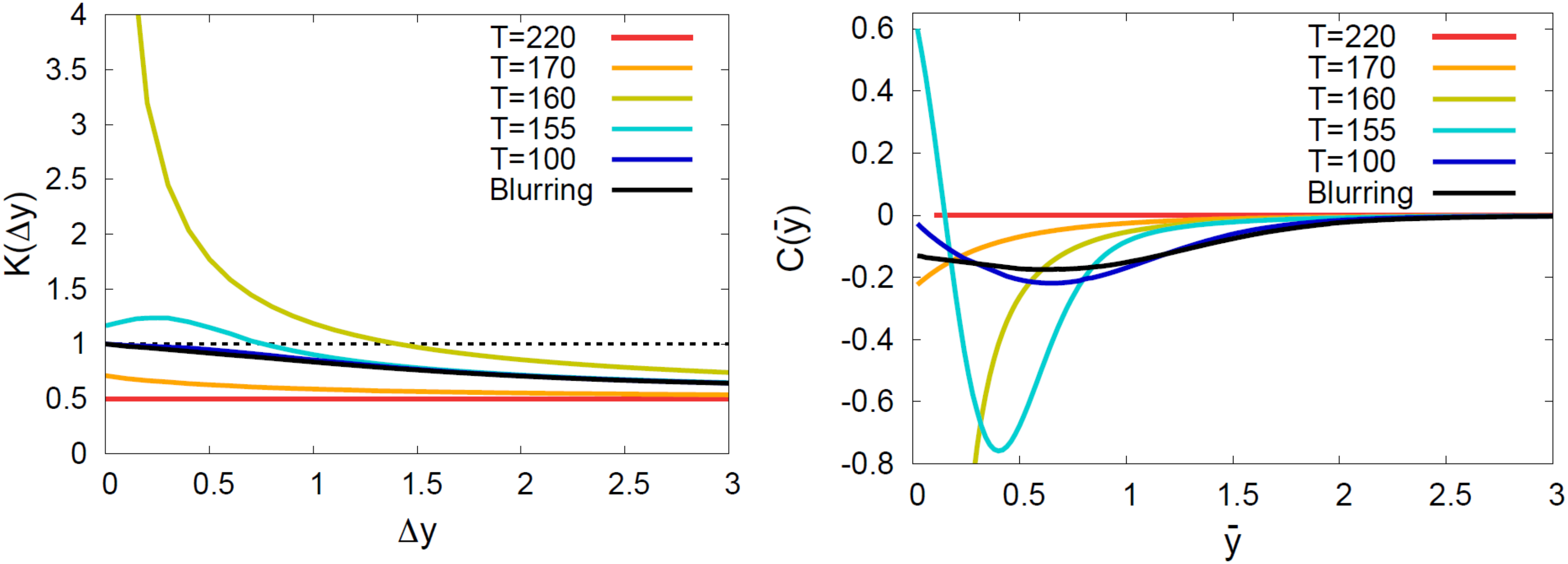


- non-monotonic Δy dep.

Analytic property: $C(\Delta y)$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

Weaker Critical Enhancement

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\mathrm{eq.}} \qquad C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\mathrm{hadron}}$$



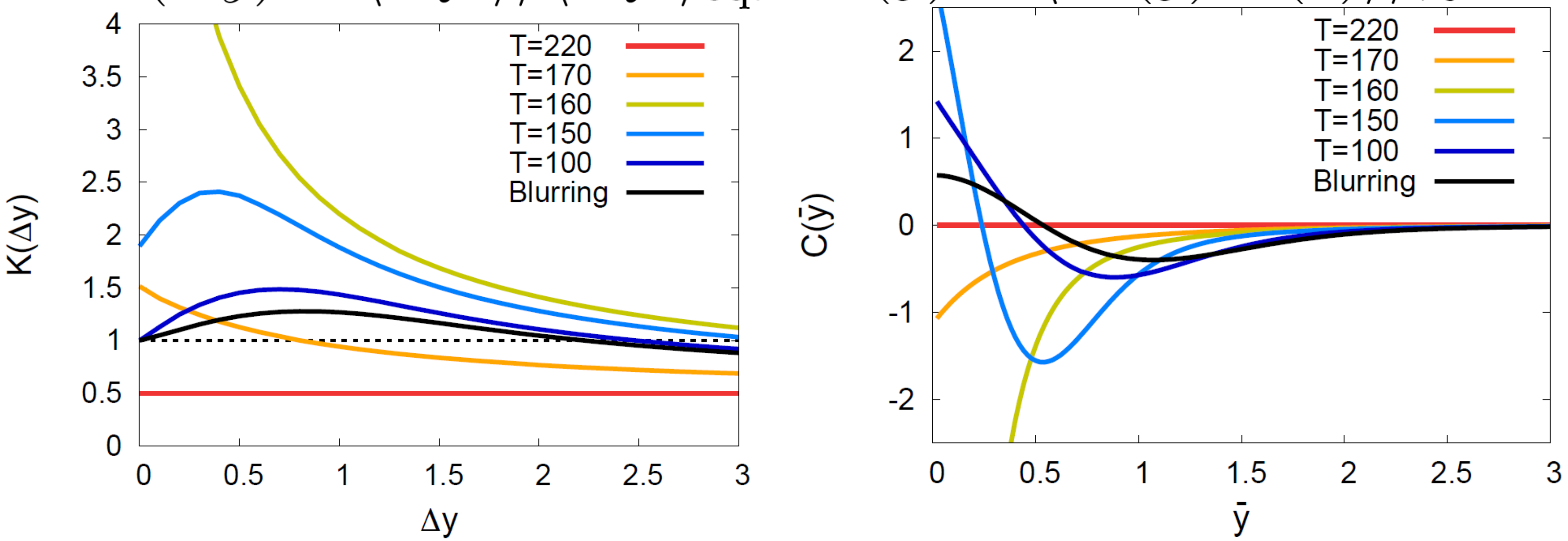
- Non-monotonicity in K(Δy) disappears.
- But C(y) is still non-monotonic.

Analytic property: $K(\Delta y),\ C(\bar{y})$ monotonic ⇒ no information on $\chi(\tau)$

- C(y) is better to see non-monotonicity.

Away from the CP

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.} \qquad C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\rm hadron}$$



- Signal of the critical enhancement can be clearer on a path away from the CP.

Away from the CP → Weaker critical slowing down

Summary

- Soft mode of the QCD critical point is a conserved mode. Its time evolution depends on the size defining the charge.
- Time evolution of conserved charges (especially baryon number) is well described by the stochastic diffusion equation.
- A non-monotonic behavior of cumulant or correlation function is the signal of the critical enhancement!

Suggestion to experimentalists

- To find the CP, measure
  - Δy dep. of 2nd order cumulant
  - y dep. of correlation function
- Study lower-order fluctuation in more detail

Future Studies

Experimental side:

- rapidity window dependences
- baryon number cumulants
- BES for SPS- to LHC-energies

Theoretical side:

- rapidity window dependences in dynamical models
- description of non-equilibrium non-Gaussianity
- accurate measurements on the lattice

Both sides:

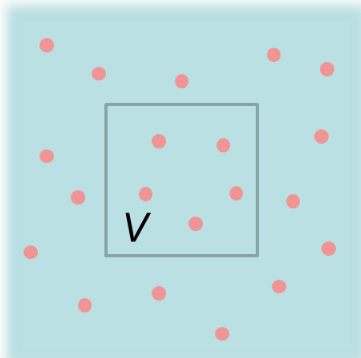
- Compare theory and experiment carefully
- Let’s accelerate our understanding on fluctuations!

Themal Blurring

Ohnishi, MK, Asakawa, PRC, in press

Fluctuations: Theory vs Experiment

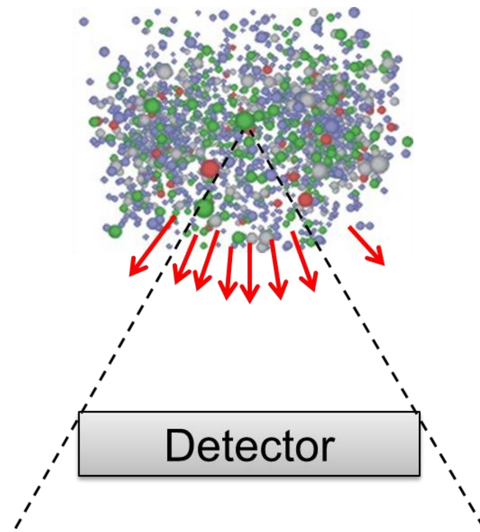
Theoretical analyses
based on statistical mechanics



lattice, critical point,
effective models, …

Fluctuation in a spatial volume

Experiments



Fluctuations in a momentum space

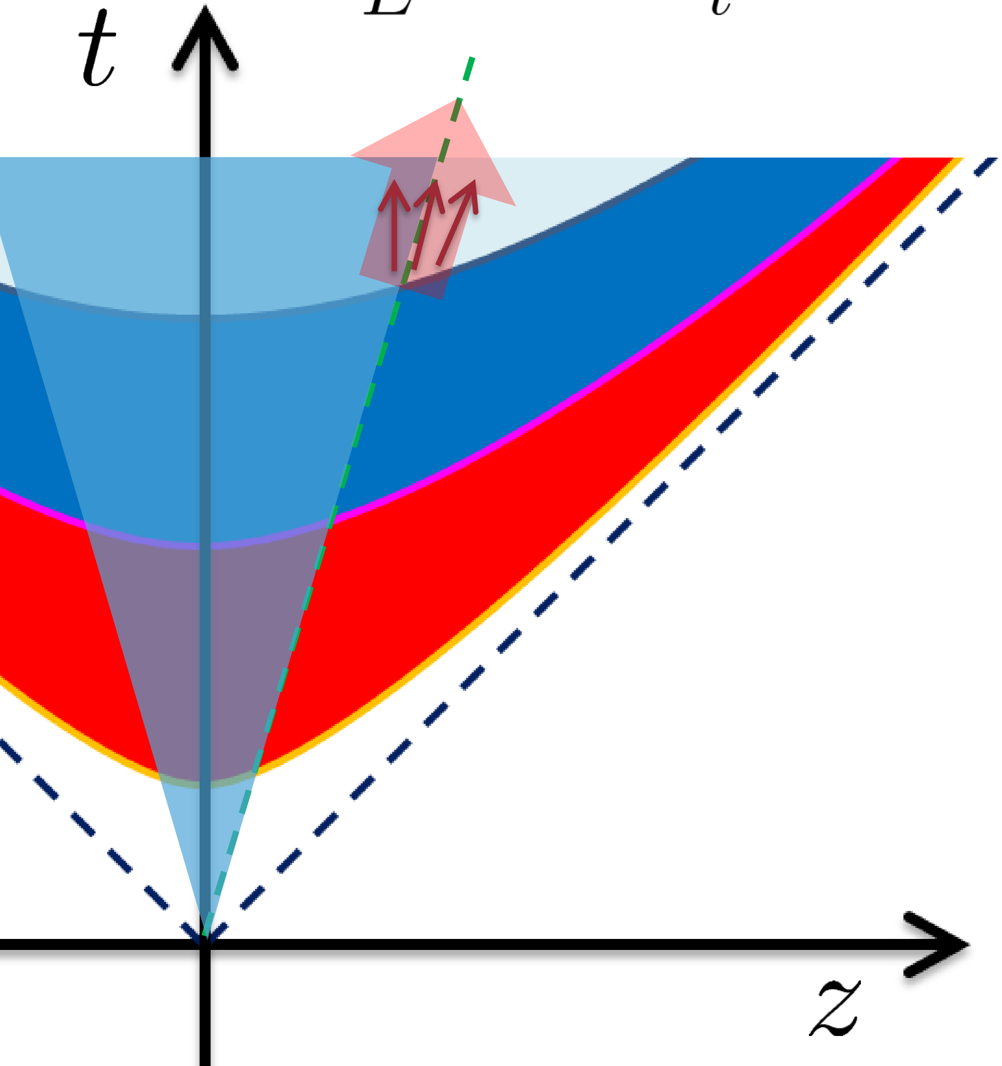
discrepancy in phase spaces

Asakawa, Heinz, Muller, 2000; Jeon, Koch, 2000; Shuryak, Stephanov, 2001

Connecting Phase Spaces

Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000

$$\frac{p_z}{E} = v_z = \frac{z}{t}$$



Under Bjorken picture,

coordinate-space rapidity *Y*

||

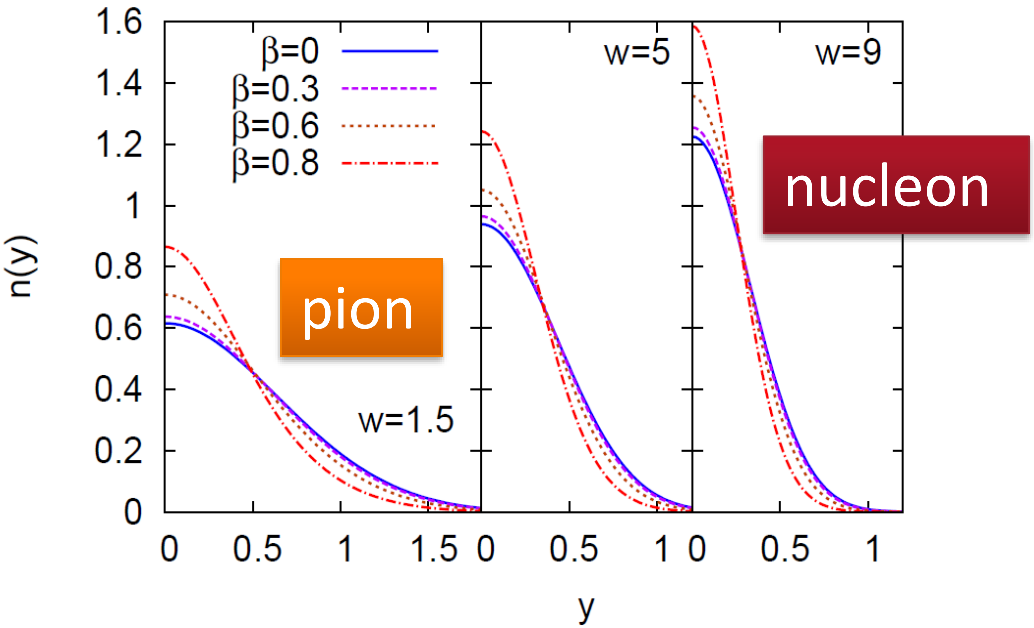
momentum-space rapidity *y*
of medium

≀

momentum-space rapidity *y*
of individual particles

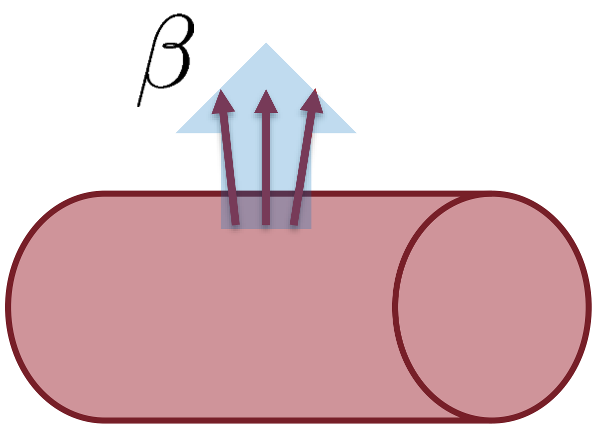
$$\Delta y \simeq \Delta Y$$

Thermal distribution in y space

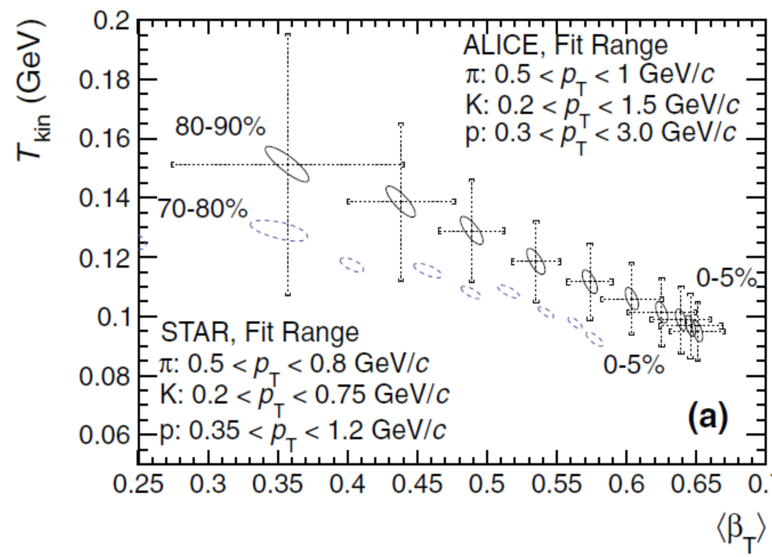


$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$

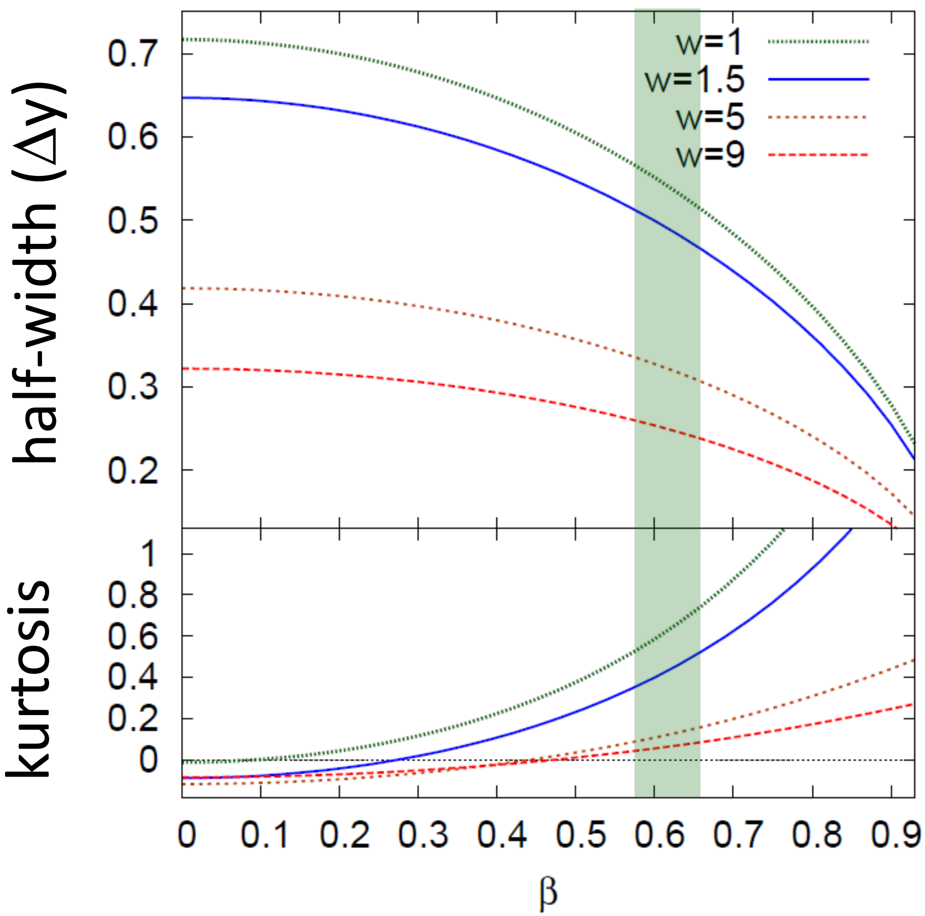


Blast wave squeezes the distribution in rapidity space



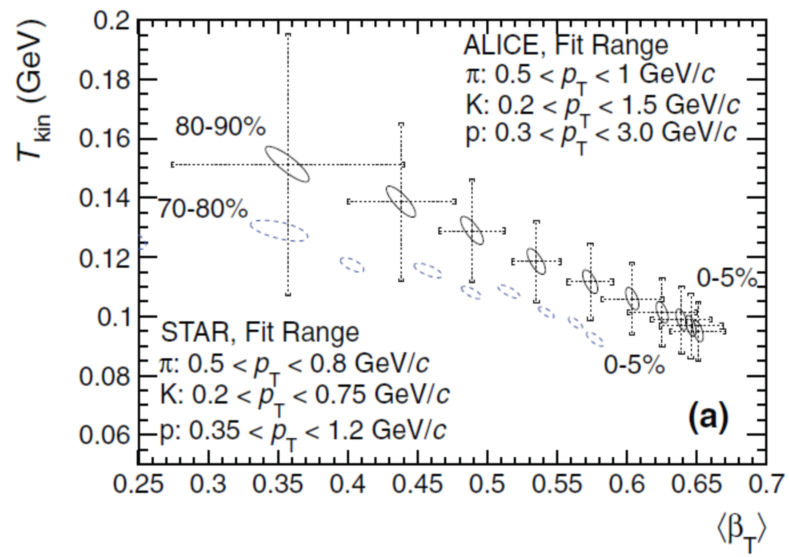
- blast wave
- flat freezeout surface

Thermal distribution in y space



$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$



Rapidity distribution can be well approximated by Gaussian.

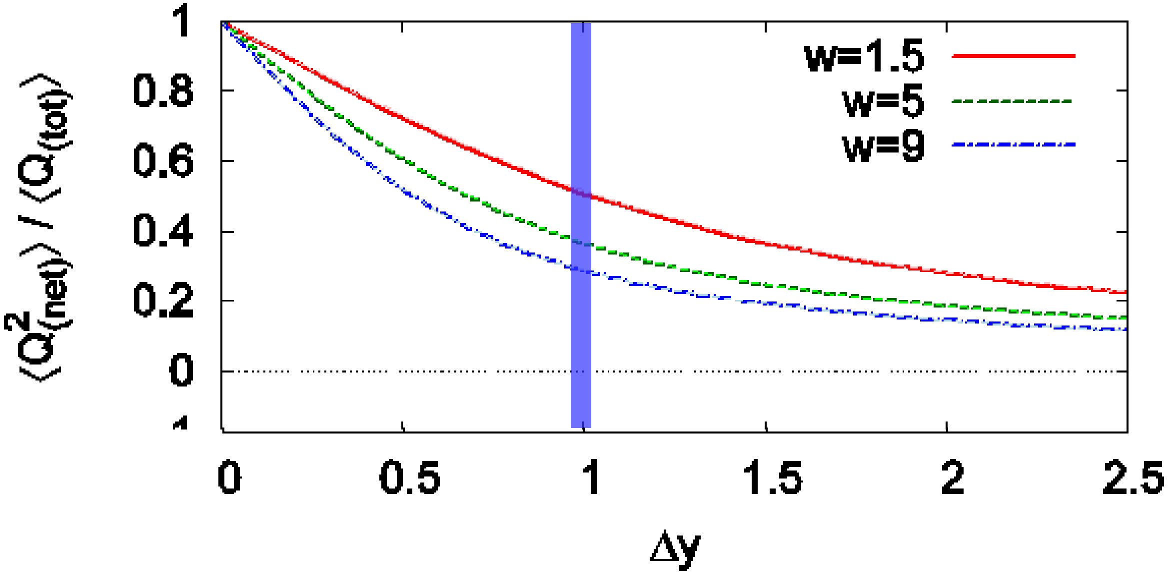
- blast wave
- flat freezeout surface

Δη Dependence

Initial condition
(before blurring)
no e-v-e fluctuations

Cumulants after blurring
can take nonzero values

Cumulants after blurring



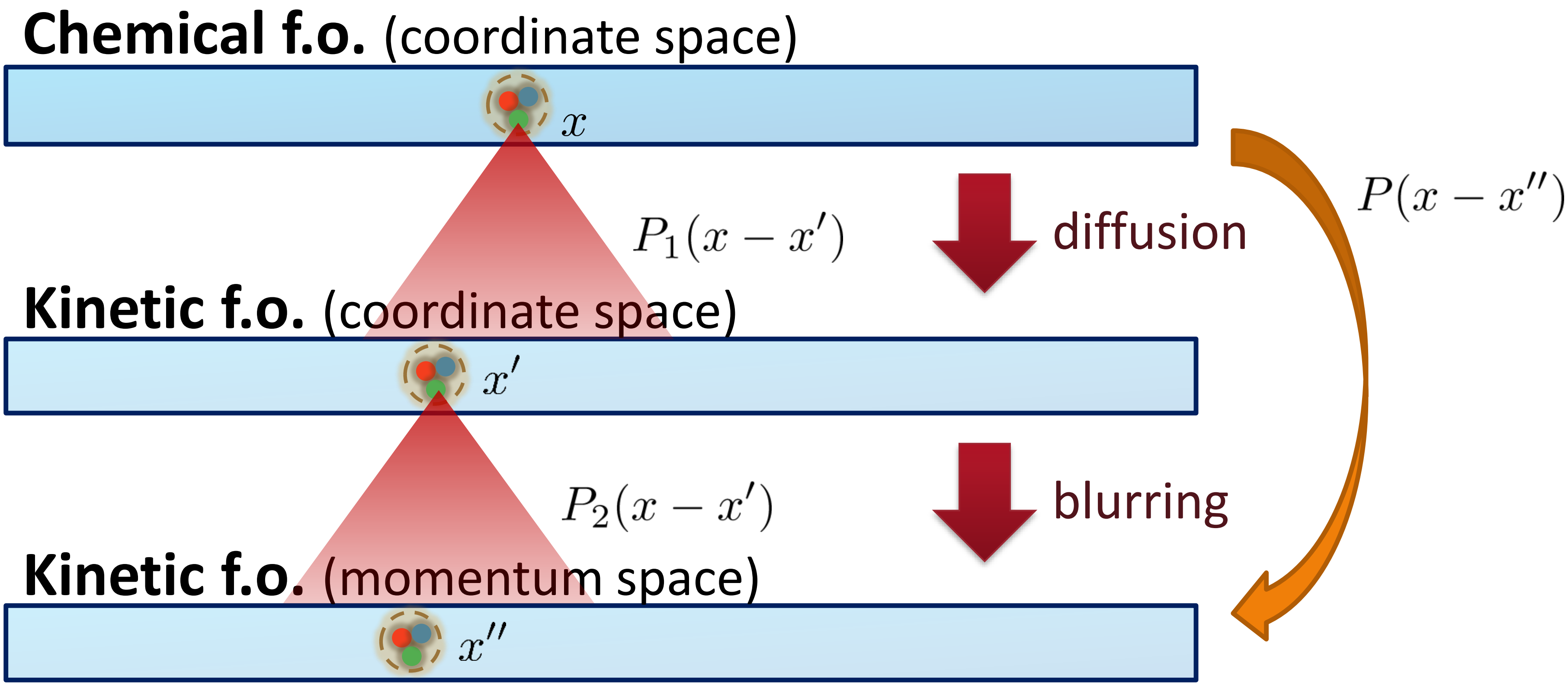
With Δy=1, the effect is not well suppressed

$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$

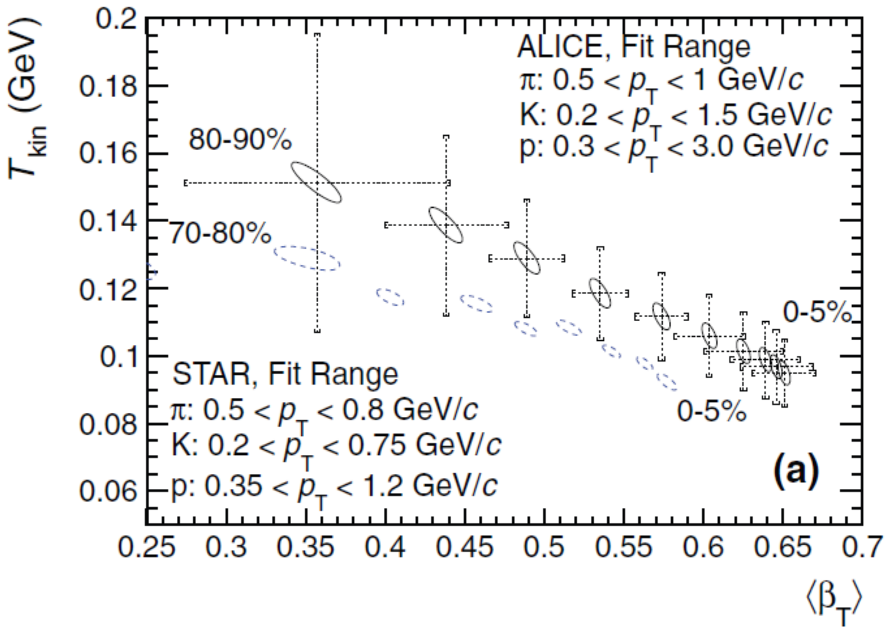
Diffusion + Thermal Blurring

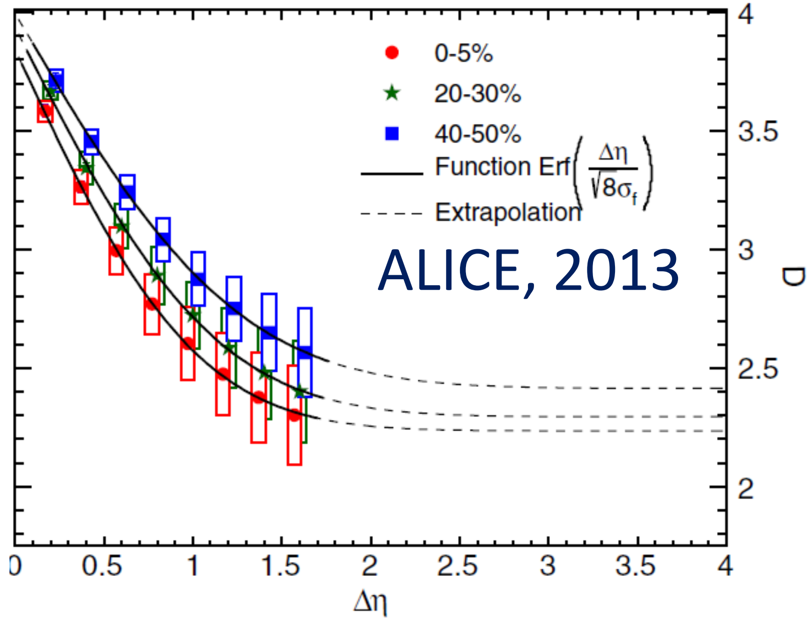
Thermal blurring can be regarded as a part of diffusion



Total diffusion: $P(x - x'') = \int dx' P_1(x - x') P_2(x' - x'')$

Centrality Dependence





More central ⇒ { lower T, larger β } ⇒ Weaker blurring

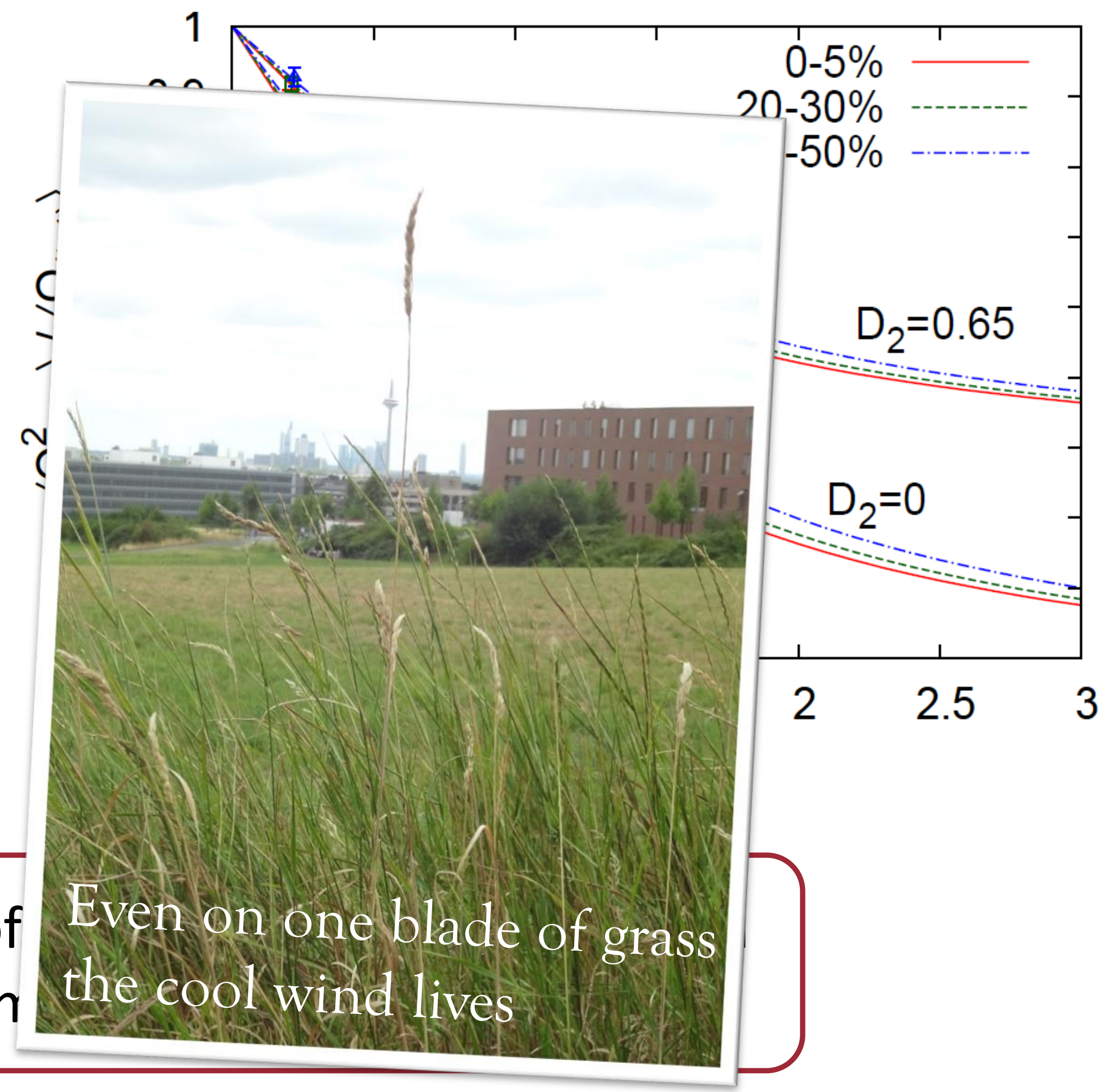
Is the centrality dependence understood solely by the thermal blurring at kinetic f.o.?

Centrality Dependence @ ALICE

$$D_2 = \frac{\langle \delta N_Q^2 \rangle}{\langle \delta N_Q^2 \rangle_{eq.}}$$

Assumptions:

- Centrality independent cumulant at kinetic f.o.
- Thermal blurring at kinetic f.o.



☐ Centrality dep. of
by a simple therm