Dispersion Relation of Charmonia above $T_c$

Masakiyo Kitazawa (Osaka U.)

Ikeda, Asakawa, MK, PRD95 (2017) 014504
Charm Quarks in HIC

Impurity of QGP
= unique experimental probe

- production only in first stage
- small abundance
- $J/\psi$ suppression
- transport property
- heavy-quark potential

Figs. from arXiv:1506.03981
Charmonia above Tc

- Property of charmonia at rest
  - Melting temperature
  - Mass shift?
Charmonia above Tc

- Property of charmonia at rest
  - Melting temperature
  - Mass shift?

- Property of moving charmonia
  - Dispersion relation
  - Residue
  - Decay rate

In heavy-ion collisions, charmonia are typically moving!
Charmonia above Tc

- Property of charmonia at rest
  - Melting temperature
  - Mass shift?
- Property of moving charmonia
  - Dispersion relation
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Let’s study on the lattice

In heavy-ion collisions, charmonia are typically moving!
Nonzero-\(p\) Spectral Func.

In vacuum: Lorentz symmetry

- Tensor structure (V) \(\rho_{\mu\nu}(\omega, \vec{p}) = \left(\frac{p_\mu p_\nu}{p^2} - g_{\mu\nu}\right)\rho_V(p)\)
- Bound-state pole \(\sim Z\delta(\omega^2 - E(\vec{p})^2) = \frac{Z}{2E(\vec{p})}\delta(\omega - 2E(\vec{p}))\)
- Dispersion relation \(E(\vec{p}) = \sqrt{m^2 + \vec{p}^2}\)
Nonzero-\(p\) Spectral Func.

### In vacuum: Lorentz symmetry

- Tensor structure \((V)\)
  \[
  \rho_{\mu\nu}(\omega, \vec{p}) = \left( \frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} \right) \rho_V(p)
  \]

- Bound-state pole
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  \sim Z \delta(\omega^2 - E(\vec{p})^2) = \frac{Z}{2E(\vec{p})} \delta(\omega - 2E(\vec{p}))
  \]

- Dispersion relation
  \[
  E(\vec{p}) = \sqrt{m^2 + \vec{p}^2}
  \]

### In medium

- Transverse and longitudinal splitting
  \[
  \rho_{\mu\nu}(\omega, \vec{p}) = \rho_T(\omega, \vec{p}) \Lambda_T + \rho_L(\omega, \vec{p}) \Lambda_L
  \]

- Dispersion relation can be modified
- \(Z\) no longer be a constant

---

\[\begin{array}{c}
\omega \\
\sqrt{m^2 + \vec{p}^2}
\end{array}\]

\[m \quad \begin{array}{c}
p
\end{array}\]
Maximum Entropy Method

Lattice data

\[ G(\tau) = \int_0^\infty d\omega \frac{\cosh(\frac{1}{2T} - \tau)\omega}{\sinh(\omega/2T)} \rho(\omega) \]

“ill-posed problem”
Maximum Entropy Method

Lattice data

Bayes theorem

Prior probability
- Shannon-Jaynes entropy
- default model $m(\omega)$

Probability of $\rho(\omega)$

Spectral Function

$\rho(\omega)/\omega^2$

$T = 1.49T_c$
Maximum Entropy Method

Lattice data

Bayes theorem

Prior probability
- Shannon-Jaynes entropy
- default model $m(\omega)$

Probability of $\rho(\omega)$

$P[\rho(\omega), \alpha]$

Spectral Function

expectation value

$\langle \rho(\omega) \rangle_P$

$\langle \mathcal{O} \rangle_P = \int d\alpha \int [d\rho] P[\rho, \alpha] \mathcal{O}$
Error in MEM

MEM error = variance in \( P[\rho(\omega), \alpha] \) space

\[
W = \int d\omega f(\omega) \rho(\omega)
\]

- exp. val.: \( \langle W \rangle_P = \int d\omega f(\omega) \langle \rho(\omega) \rangle_P \)
- error: \( \Delta W = \sqrt{(W - \langle W \rangle_P)^2} \)
Error in MEM

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\[ W = \int d\omega f(\omega) \rho(\omega) \]

- exp. val.: $\langle W \rangle_P = \int d\omega f(\omega) \langle \rho(\omega) \rangle_P$
- error: $\Delta W = \sqrt{(W - \langle W \rangle_P)^2}$

\[ f(\omega') = \delta(\omega' - \omega) \]
\[ \rho_{out}(\omega) = \langle \rho(\omega) \rangle_P \]
\[ \Delta \rho_{out}(\omega) = \sqrt{\langle (\delta \rho(\omega))^2 \rangle_P} \]

typically huge
Error in MEM

MEM error = variance in $P[\rho(\omega), \alpha]$ space

$$W = \int d\omega f(\omega) \rho(\omega)$$

- exp. val.: $\langle W \rangle_P = \int d\omega f(\omega) \langle \rho(\omega) \rangle_P$
- error: $\Delta W = \sqrt{(W - \langle W \rangle_P)^2}$

$$f(\omega') = \delta(\omega' - \omega)$$

$$\rho_{out}(\omega) = \langle \rho(\omega) \rangle_P$$

$$\Delta \rho_{out}(\omega) = \sqrt{\langle (\delta \rho(\omega))^2 \rangle_P}$$

NOTE

- SPC obtained by MEM is just an image. No robust meaning.
- MEM error is more conservative than statistical one.
Defining Peak Position

Center of weight in a range $[\omega_{\text{min}} : \omega_{\text{max}}]$

$$
\bar{E} = \frac{\int_{\omega_{\text{min}}}^{\omega_{\text{max}}} d\omega \omega \left( \rho(\omega) / \omega^2 \right)}{\int_{\omega_{\text{min}}}^{\omega_{\text{max}}} d\omega \rho(\omega) / \omega^2}
$$

- Represent peak position for a sufficiently sharp peak
- Error analysis in MEM is possible!
- $[\omega_{\text{min}}, \omega_{\text{max}}]$ dependence has to be checked
Defining Peak Position

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\]

- Represent peak position for a sufficiently sharp peak
- Error analysis in MEM is possible!
- \([\omega_{\text{min}}, \omega_{\text{max}}]\) dependence has to be checked

Residue: \(\bar{Z} = \int_{\omega_1}^{\omega_2} d\omega 2\omega \rho(\omega)\)
Lattice Setup

- quenched simulation
- Wilson fermion / gauge
- anisotropic lattice ($a_\sigma/a_\tau=4$)

\[ \beta = 7.0, \quad \gamma_F = 3.476, \quad \kappa_\sigma = 0.8282 \]
\[ a_\sigma = 0.00975[\text{fm}], \quad a_\sigma/a_\tau = 4 \]

Asakawa, Hatsuda, 2004

<table>
<thead>
<tr>
<th>$N_\tau$</th>
<th>$T/T_c$</th>
<th>$N_\sigma$</th>
<th>$L_\sigma[\text{fm}]$</th>
<th>$N_{\text{conf}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>1.86</td>
<td>64</td>
<td>2.5</td>
<td>500x8</td>
</tr>
<tr>
<td>46</td>
<td>1.62</td>
<td>64</td>
<td>2.5</td>
<td>500x8</td>
</tr>
<tr>
<td>50</td>
<td>1.49</td>
<td>64</td>
<td>2.5</td>
<td>500x8</td>
</tr>
<tr>
<td>96</td>
<td>0.78</td>
<td>64</td>
<td>2.5</td>
<td>500x8</td>
</tr>
</tbody>
</table>

8 measurements on each conf.

BlueGene/Q@KEK
fermion part: Iroiro++

- Large spatial volume $\rightarrow$ high momentum resolution
- Large $N_\tau$ / high statistics $\rightarrow$ high MEM precision
Spectral Func. @ T=0.78T

\[ \rho(\omega)/\omega^2 \]

- \( \rho_T \) and \( \rho_L \) channels degenerate
- although correlators are different

\[ \vec{p} = (p, 0, 0) \]

\[ G_L = \frac{\omega^2 - p^2}{\omega^2} G_{11} \]

\[ G_T = G_{22} = G_{33} \]
Spectral Func. @ T=0.78T

- $\rho_T$ and $\rho_L$ channels degenerate
- although correlators are different

$$G_{ii}(\tau, \vec{p})/G_V(\tau, \vec{p} = 0)$$

*p = (p, 0, 0)*

$$G_L = \frac{\omega^2 - p^2}{\omega^2} G_{11}$$

$$G_T = G_{22} = G_{33}$$
Bound state peaks seem to exist at $T=1.62T_c$. 
Spectral Func. @ T=1.62T

- $\rho_T$ and $\rho_L$ channels seem to degenerate.
- Peak exists for all momentum.
Dispersion Relation

Energy interval $[\omega_{\text{min}}, \omega_{\text{max}}]$ for disp. rel.

- $\omega_{\text{min}} = 3\,\text{GeV}$
- $\omega_{\text{max}}$: first minimum

Vector, $T = 1.62T_c$
Clear mass enhancement in medium.

Dispersion relation is consistent with the Lorentz covariant form even at $T=1.62T_c$.

<table>
<thead>
<tr>
<th>$T/T_c$</th>
<th>0.78</th>
<th>1.49</th>
<th>1.62</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J/\psi$</td>
<td>3.24(6)</td>
<td>4.30(16)</td>
<td>4.47(16)</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>3.19(5)</td>
<td>4.24(31)</td>
<td>4.49(48)</td>
</tr>
</tbody>
</table>

mass $E(p=0)$ [GeV]
Dependence on $[\omega_{\text{min}} : \omega_{\text{max}}]$}

- $\omega_{\text{max}}$ dependence is well suppressed.
- No $\omega_{\text{min}}$ dependence for $\omega_{\text{min}} < 3 \text{GeV}$. 

$Lattice 2017$
Test: $N_t$ Dependence

**Correlator**

$T = 0.78T_c, \quad N_\tau = 96$

**Reconstructed Cor.**

$T = 0.78T_c, \quad N_\tau = 48$

- Peak position does not shift with the change of $N_t$

Special thanks to A. Rothkopf

\[
\begin{array}{c}
\text{correlator} \\
G(\tau, T) \\
G^{\text{reco}}(\tau, 2T; T) \\
\text{Vector, } |p| = 0, \quad T = 0.78T_c
\end{array}
\]

<table>
<thead>
<tr>
<th>$m$</th>
<th>$G(\tau, 0; T)$</th>
<th>$G^{\text{reco}}(\tau, 0; 2T; T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>3.24(6)</td>
<td>3.40(90)</td>
</tr>
</tbody>
</table>
Residue

**Vector**

- No $p$ dependence of $Z$ even for $T=1.62T_c$
- No $T/L$ splitting in vector channel

**PS**

- No $p$ dependence of $Z$ even for $T=1.62T_c$
- No $T/L$ splitting in vector channel
Summary

- We analyzed the peak positions in SPC with MEM error by defining them in terms of the center of weight.
- Charmonia have significant mass enhancement.
- Dispersion relations are consistent with Lorentz covariant form even at $T=1.62T_c$.

Future Work

- much finer p resolution
- $m_q$ dependence
- comparison with potential models and etc.