勾配流を用いた格子QCD数値解析

北沢正清（大阪大学）

Special thanks to:

FlowQCD Collab.(柳原良亮, Asakawa, Hatsuda, Iritani, Itou, Suzuki)

WHOT Collab.(Kanaya, Ejiri, Taniguchi, Umeda, Shirogane, Suzuki, …)

Hadron Spectroscopy Café, 東工大, 2017年7月10日

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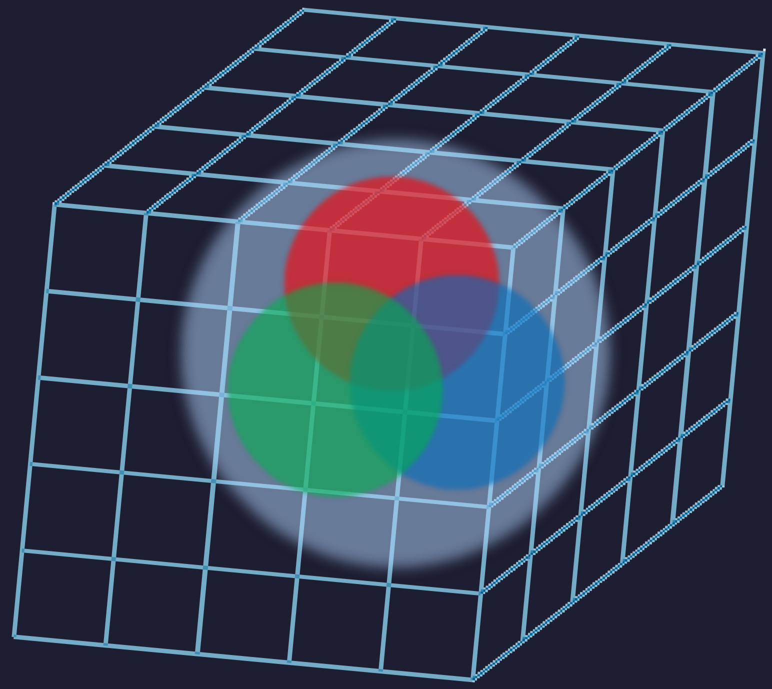
格子QCD数値解析



量子色力学（QCD）を
第一原理的に取り扱う現状唯一の手段

hadron spectra, chiral symmetry, phase transition, etc.

勾配流

Luscher, 2010-
Narayanan, Neuberger, 2006

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

様々な解析への応用

- スケール設定
- トポロジー解析
- Running coupling
- 演算子の定義
  エネルギー運動量テンソル
- 他もろもろ

LATTICE2014, D. Nogradiより

Why care?

- Tuesday 14:55 – Nathan Brown – Gradient Flow Analysis on MILC HISQ Ensembles
- Tuesday 14:35 – Andrea Shindler – Beyond the Standard Model Matrix Elements with the gradient flow
- Tuesday 14:35 – Liam Keegan – TEK twisted gradient flow running coupling
- Wednesday 09:00 – Anna Hasenfratz – Improved gradient flow for step scaling function and scale setting
- Wednesday 09:20 – Jarno Rantaharju – The gradient flow running coupling in SU2 with 8 flavors
- Wednesday 11:10 – Marco Ce – Testing the WittenVeneziano mechanism with the YangMills gradient flow on the lattice
- Thursday 14:55 – Agostino Patella – Energy-momentum tensor on the lattice and Wilson flow
- Thursday 15:15 – Masanori Okawa – String tension from smearing and Wilson flow methods
- Thursday 15:55 – Stefan Sint – How to reduce $O(a^2)$ effects in gradient flow observables
- Friday 10:15 – Alberto Ramos – Wilson flow and renormalization
- Saturday 09:30 – Kitazawa Masakiyo – Measurement of thermodynamics using Gradient Flow

$$T_{\mu\nu}$$

Poincare symmetry

$$T_{\mu\nu}$$

energy

momentum

$$\begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

stress

pressure

Einstein Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

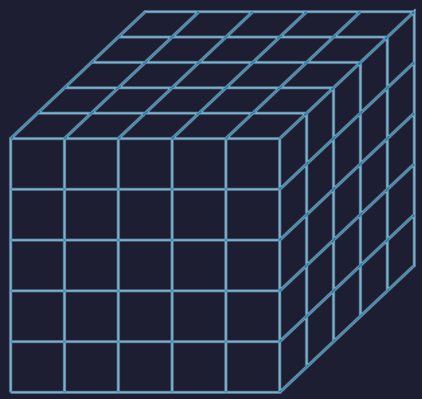
Hydrodynamic Eq.

$$\partial_\mu T_{\mu\nu} = 0$$

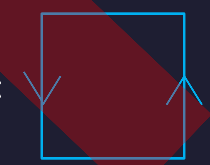
$T_{\mu\nu}$: nontrivial observable on the lattice

① **Definition of the operator is nontrivial**

because of the explicit breaking of Lorentz symmetry



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

② **Its measurement is extremely noisy**

due to high dimensionality and etc.

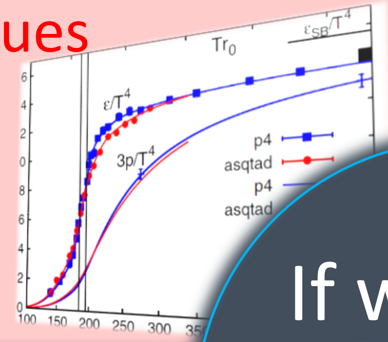
If we have

$$T_{\mu\nu}$$

Thermodynamics

direct measurement of expectation values

$$\langle T_{00} \rangle, \langle T_{ii} \rangle$$



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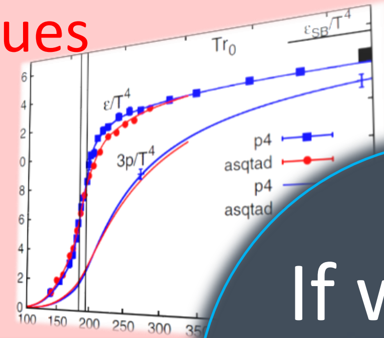


Fluctuations and Correlations

viscosity, specific heat, ...

$$\eta = \int_0^\infty dt \langle T_{12} ; T_{12} \rangle$$

$$c_V \sim \langle \delta T_{00}^2 \rangle$$

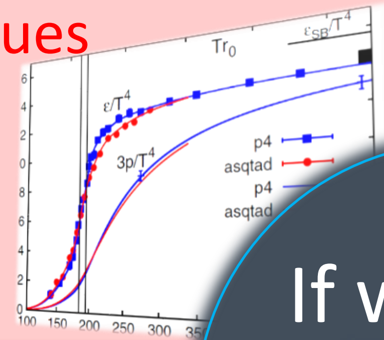
If we have

$T_{\mu\nu}$

Thermodynamics

direct measurement of expectation values

$$\langle T_{00} \rangle, \langle T_{ii} \rangle$$



Fluctuations and Correlations

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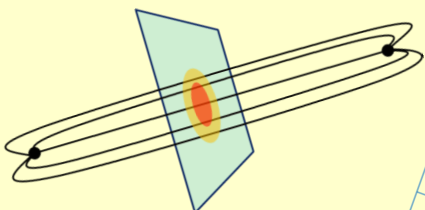
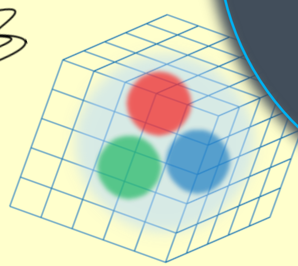




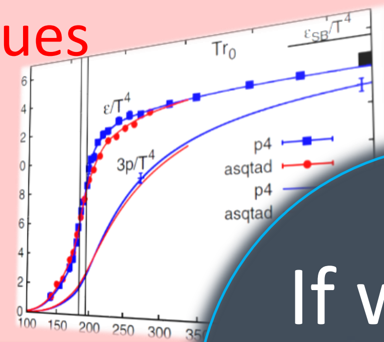
- confinement string
- EM distribution in hadrons

Hadron Structure

Thermodynamics

direct measurement of expectation values

$\langle T_{00} \rangle, \langle T_{ii} \rangle$



Fluctuations and Correlations

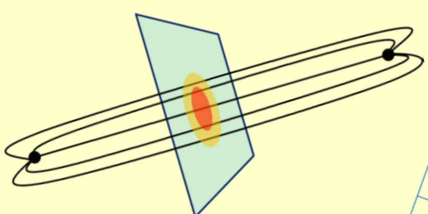
viscosity, specific heat, ...

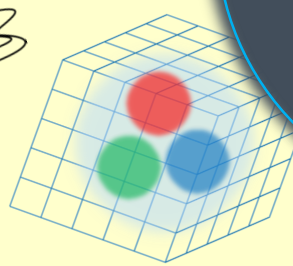
$\eta = \int_0^\infty dt \langle T_{12}; T_{12} \rangle$

$c_V \sim \langle \delta T_{00}^2 \rangle$

If we have

$T_{\mu\nu}$





- confinement string
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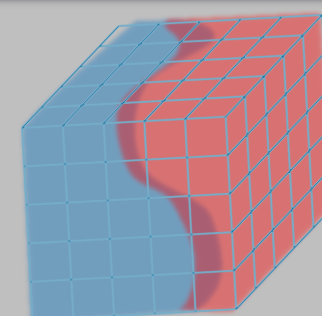
Hadron Structure

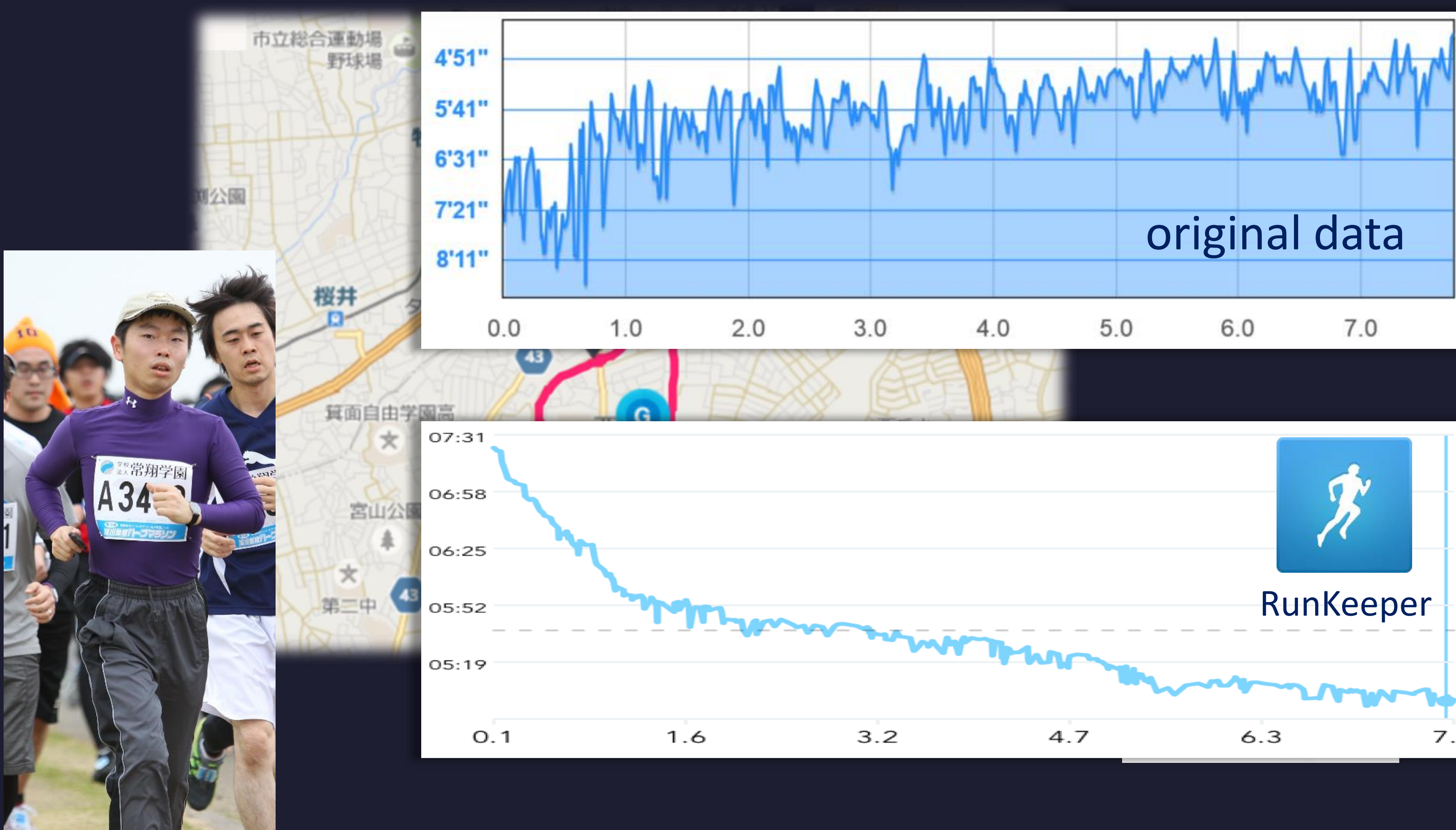


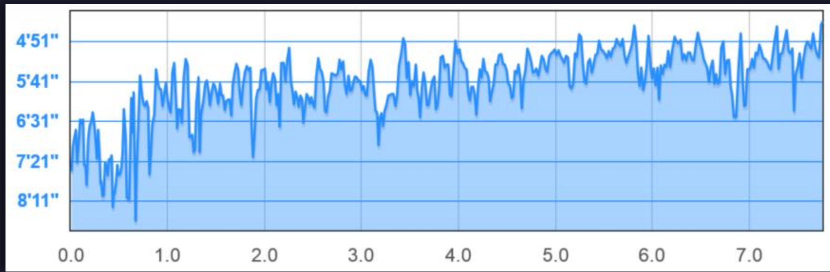
- vacuum configuration
- mixed state on 1st transition

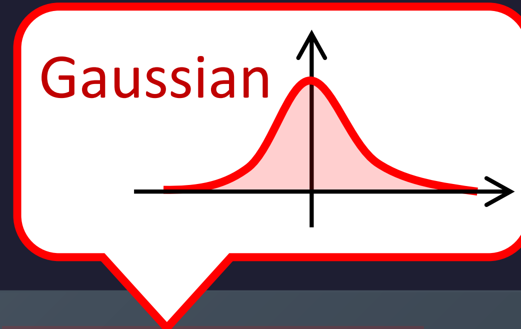
Vacuum Structure

勾配流とは？
～マラソンと勾配流の奇妙な関係～

勾配流とは？
～マラソンと勾配流の奇妙な関係～



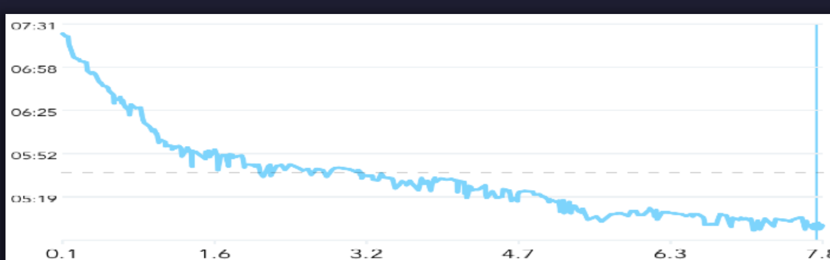


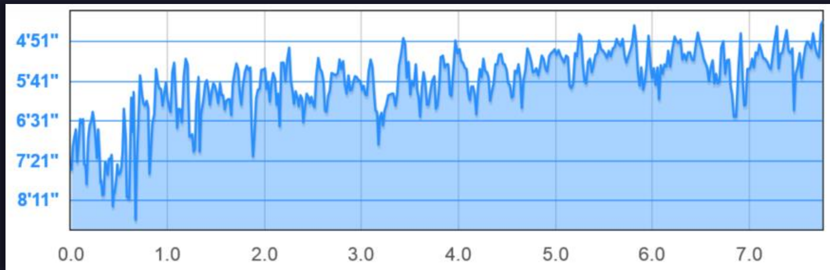


① $$x(t) \to x'(t) \sim \int dt' \exp\left[-\frac{(t-t')^2}{2\sigma^2}\right] x(t')$$

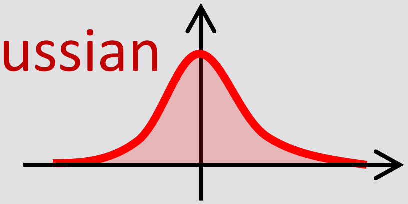
$$\sigma = \sqrt{2s}$$

② $$\frac{d}{ds} x(t;s) = \frac{d^2}{dt^2} x(t,s) \qquad x(t;0) = x(t)$$





Gaussian



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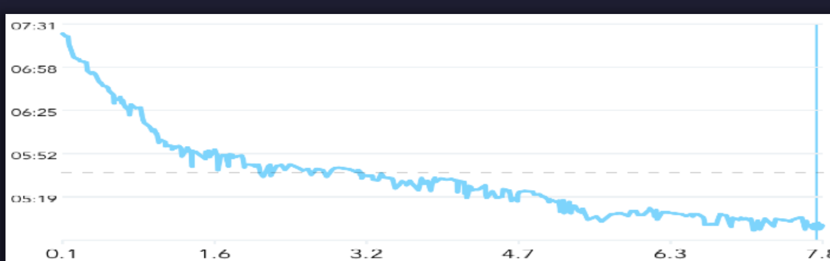


YM Gradient Flow

$$\partial_t A_\mu = D_\nu G_{\mu\nu}$$
$$= \partial_\nu \partial_\nu A_\mu + \cdots$$

Gauge invariant version of
4-dim. diffusion equation

勾配流=場の連続的cooling

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

$$A_\mu(0,x) = A_\mu(x)$$

t: “flow time”
dim:[length²]

Tree level

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- 4次元空間の拡散方程式
- 平均拡散長 $d \sim \sqrt{8t}$

見小利則大事不成

小利を見ればすなわち大事成らず
Miss the wood for the trees

孔子
（論語、子路13）

見小利則大事不成

孔子（論語、子路13）

- 格子QCD数値解析では、連続極限への外挿が必要
- 格子間隔が狭くなるほど、測定に伴う誤差が増大

- 例：エネルギー運動量テンソルの期待値 $\langle T_{\mu\mu} \rangle$

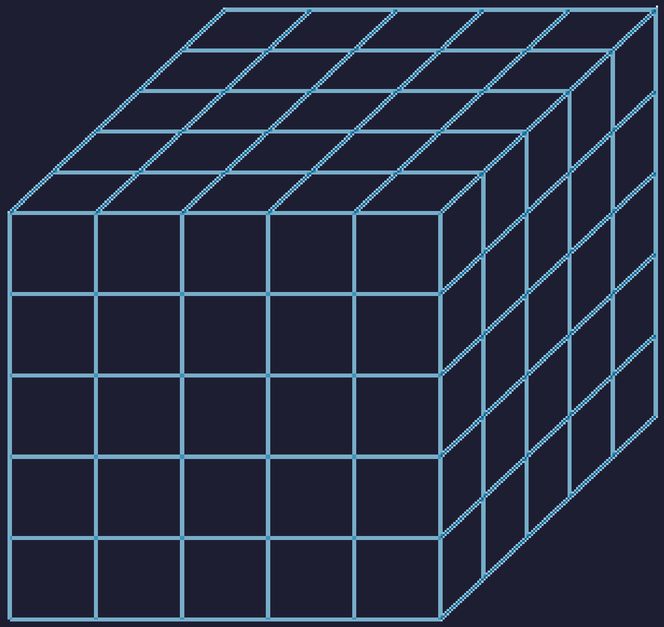
誤差 :$\Delta \langle T_{\mu\mu} \rangle \sim a^{-2}$

⇒ 小利（紫外領域のゆらぎ）の中に
大事（マクロな観測量）が埋もれてしまう

データ容量と物理

ゲージ配位

128^4



$$128^4 \times 4 \times 9 \times 2 \times 8 \text{ Bytes}$$

$$= 144 \text{ GB}$$

データ容量と物理

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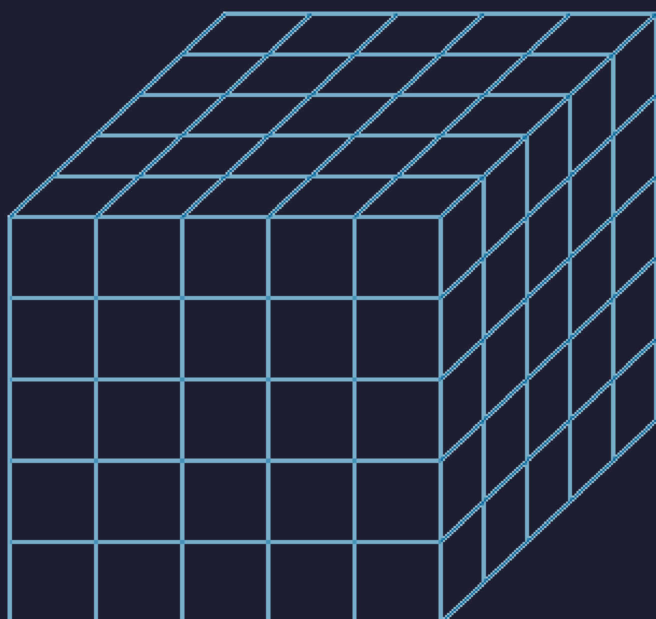

$$128^4 \times 4 \times 9 \times 2 \times 8 \text{ Bytes}$$

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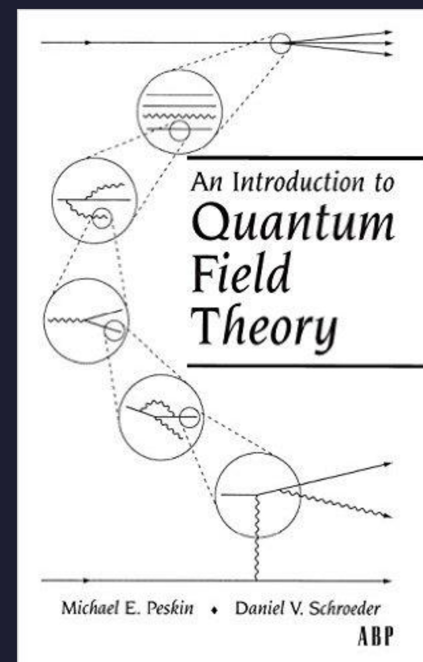
場の理論の教科書

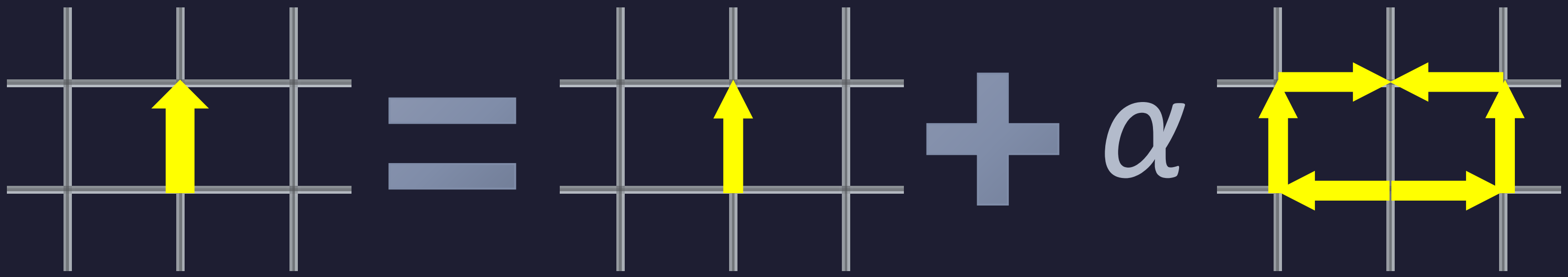
Peskin-Schroeder



約10MB

格子上の場のCooling

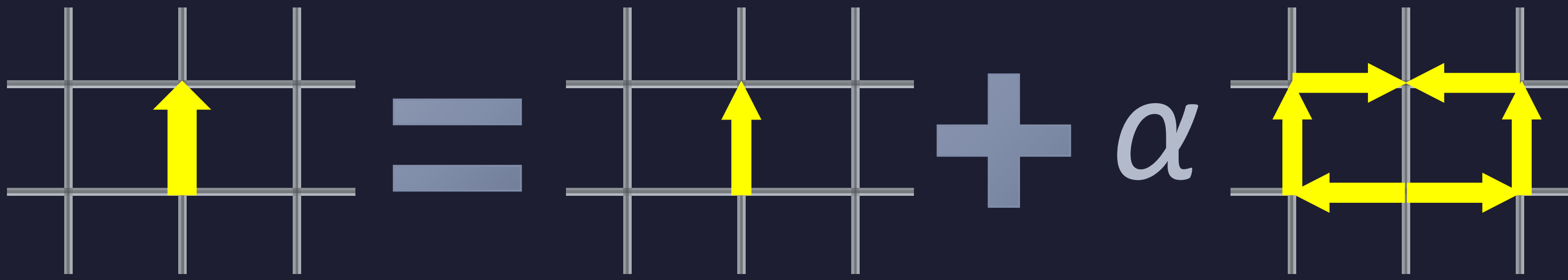
例：APEスメアリング



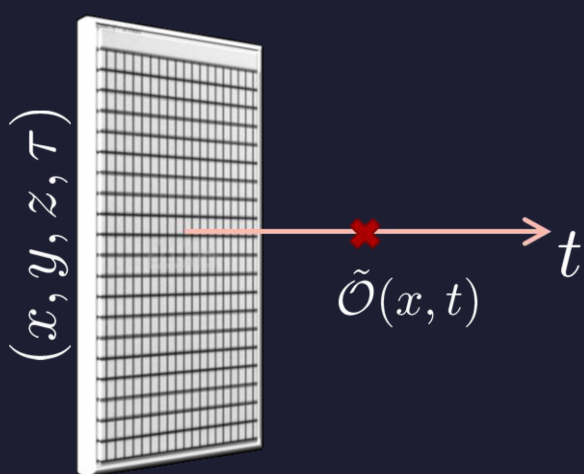
- 従来のcoolingは離散的に行われていた。

格子上の場のCooling

例：APEスメアリング



- 従来のcoolingは離散的に行われていた。
- 「連続変換」に置き換えたのが革命。



$$O(x) \Longleftrightarrow O(x, t)$$

４次元理論と、有限tの関係が確立

Luscher, Weiss, 2011

勾配流による場の変換

- 場の変換の数学的構造が明確
  - 各種期待値等のt依存性が摂動論的に評価可能

- t>0における全ての観測量が紫外有限 Luscher,Weisz,2011

  ⇒ t>0での場は、繰り込まれている
  a→0の極限が安全に取れる

- 勾配流で変換された理論は、元のゲージ理論とは別物。
  - 勾配流は、近似解析用の粗視化手段**ではない。**
  - ただし、統計誤差の抑制には著しく効果的。

勾配流 for Fermion場

Luscher, 2013
Makino, Suzuki, 2014
Taniguchi+ (WHOT) 2016

$$\partial_t \psi(t,x) = D_\mu D_\nu \psi(t,x)$$

$$\partial_t \bar{\psi}(t,x) = \psi(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\nu$$

$$D_\mu = \partial_\mu + A_\mu(t,x)$$

- もはや「勾配」流ではない。
- ゲージ場の勾配流はフェルミオンと独立。

- フェルミオン場の波動関数繰り込みに発散。
- Z(t)を決めれば、全ての観測量が有限。

$$\tilde{\psi}(t,x) = Z(t)\psi(t,x)$$

呼称について

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

- gradient flow
- Yang-Mills gradient flow
- Wilson flow
- Symanzik, Iwasaki flow, …
- 勾配流（変換）？

Contents

a 格子間隔：

格子上の最も基本的な量

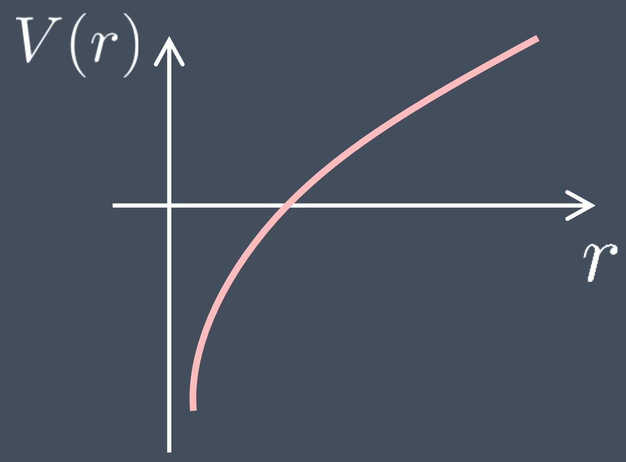
- 物理次元の導入
- 連続外挿

に必須

物理次元を持った観測量（スケール）が必要

String Tension / Sommer Scale

重クォークポテンシャル



- 弦張力

  V(r)の遠方での傾きσ

- Sommer scale

$$r_0^2 \frac{dV(r_0)}{dr} = 1.65$$

となるr_0を求める

デメリット
- V(r)を決めるため、フィット解析が必要
- 統計誤差が大きい
- 非局所的な数値解析が必要

勾配流を用いたスケール設定

Luscher, 2010

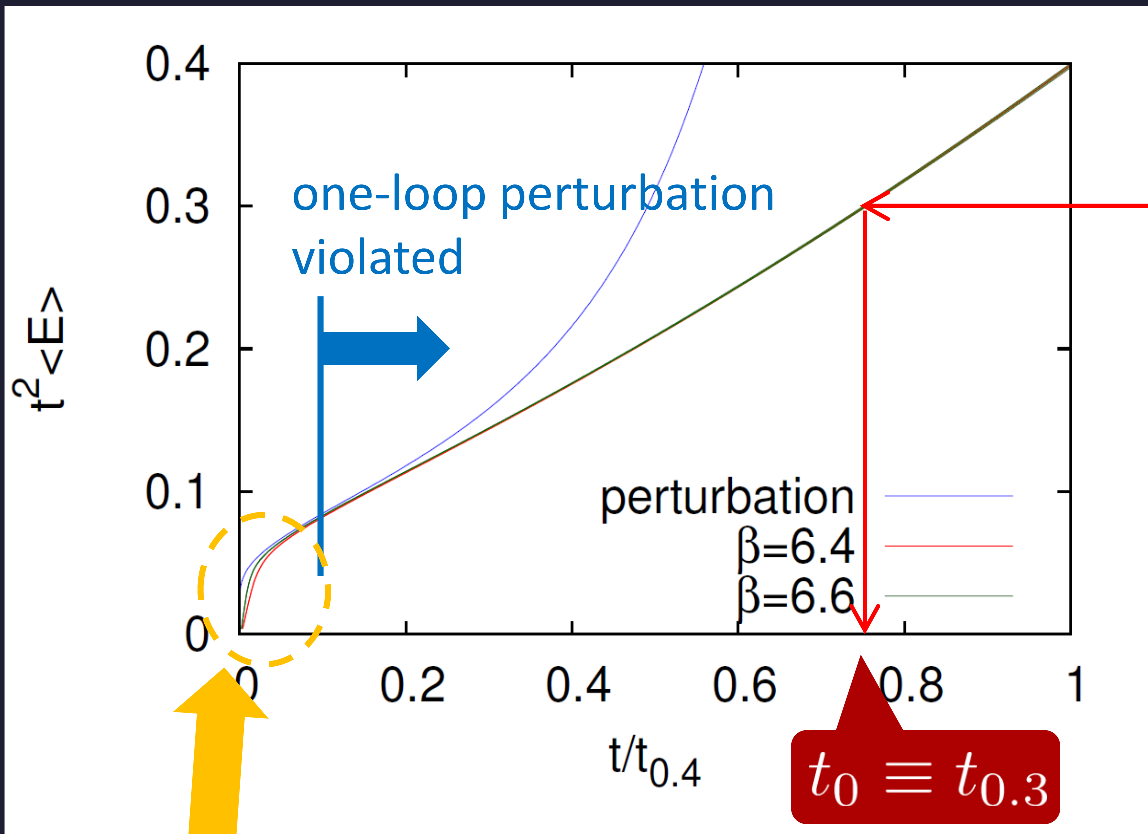
$\langle \mathcal{O}(t) \rangle$: 適当な無次元演算子の期待値
格子間隔には依存しない

$\mathcal{O}$: an observable

$$\langle \mathcal{O}(t_0) \rangle = \mathrm{const} \Rightarrow t_0 = \hat{t} a^2$$

- standard choice of *O*: $\mathcal{O}(t) = \frac{1}{4} t^2 F_{\mu\nu} F^{\mu\nu} \equiv t^2 E$
- perturbative formula: $t^2 \langle E \rangle = \frac{3}{(4\pi)^2} g^2 (1 + k_1 g^2 + \cdots)$ $g = g(1/\sqrt{8t})$

A Dimensionless Choice: $t^2\langle E\rangle$



$$t^2\langle E\rangle|_{t=t_0} = 0.3$$

lattice discretization effect

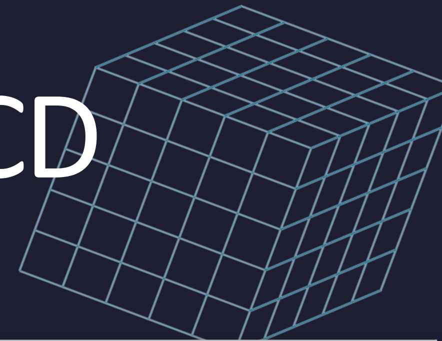
Another choice: w_0

$$t\frac{d}{dt}t^2\langle E\rangle|_{t=w_x^2} = x$$

Budapest-Wuppertal 2012

weaker *a* dep.

スケール設定 by FlowQCD



- SU(3) YM theory
- Wilson gauge action
- w_0 scaling
- $6.3<\beta<7.4$

- 少ない配位で高統計

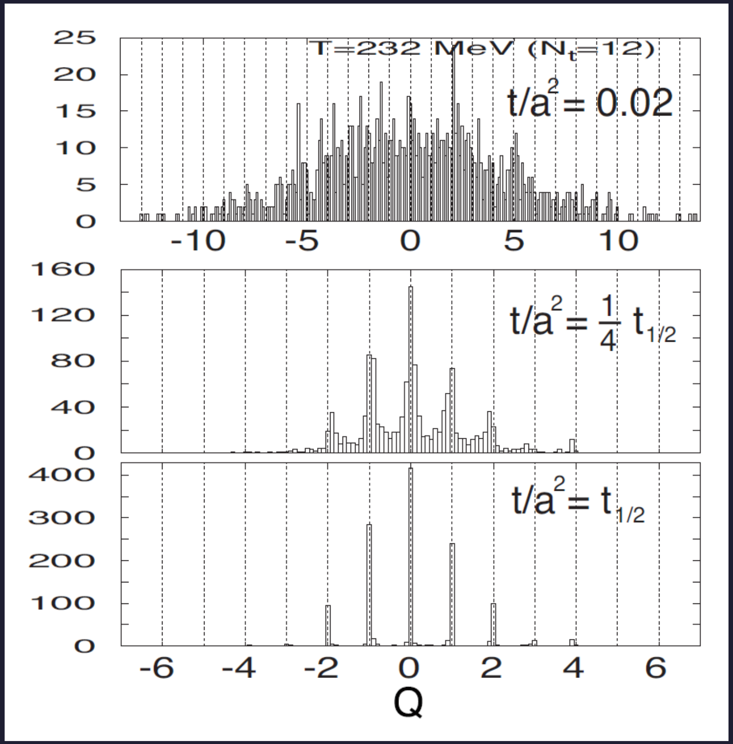
β	size	N_{conf}	β	size	N_{conf}
6.3	64^4	30	6.9	64^4	30
6.4	64^4	100	7.0	96^4	60
6.5	64^4	49	7.2	96^4	53
6.6	64^4	100	7.4	128^4	40
6.7	64^4	30			
6.8	64^4	100			

$$\frac{w_0}{a} = \exp\left(\frac{4\pi^2}{33}\beta - 9.1268 + \frac{41.806}{\beta} - \frac{158.26}{\beta^2}\right) \times [1 \pm 0.004(\mathrm{stat})].$$

FlowQCD, PRD94 (2016); see also 1503.06516

勾配流の応用例

- スケール設定
- トポロジカル電荷
- running coupling
- etc.



Taniguchi+, WHOT-QCD, 2016

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$$T_{\mu\nu}$$

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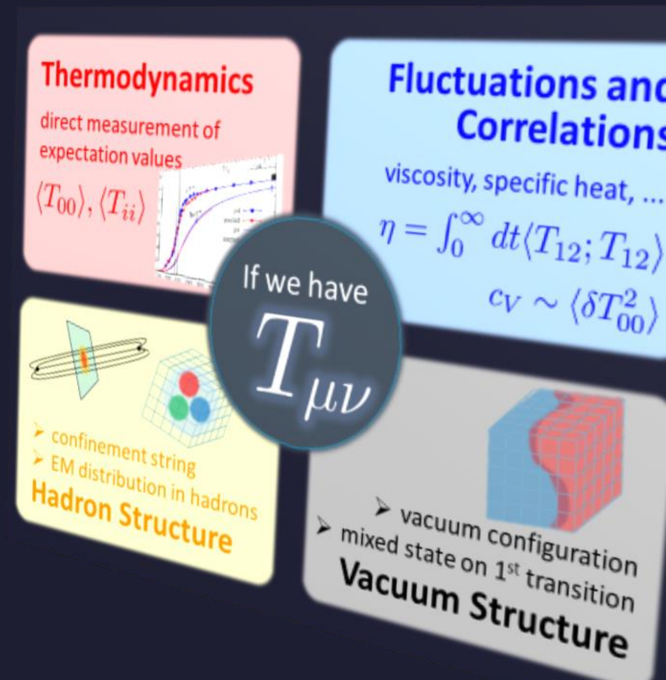
2. 勾配流の応用：スケール設定
3. 勾配流を用いたEMTの構成
4. 熱力学量の測定
5. EMT相関関数の測定
6. flux tube周辺の応力構造の解析



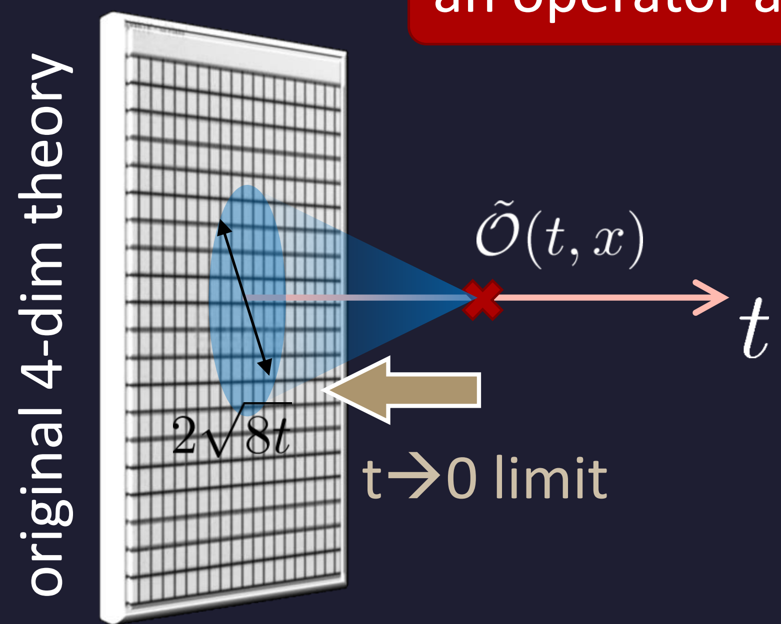
微少フロー時間展開

Luescher, Weisz, 2011

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t \to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at t>0

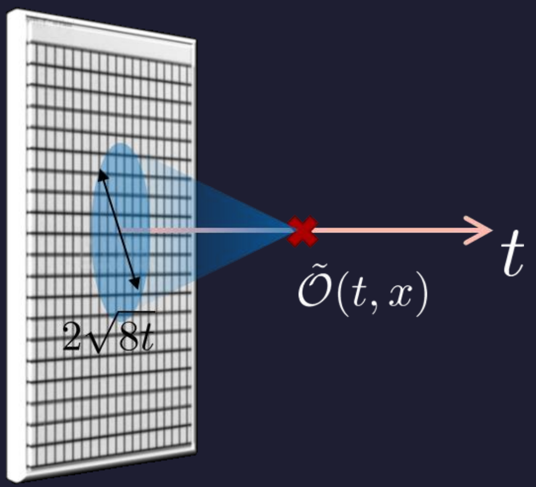
remormalized operators of original theory



EMTの構築

Suzuki, 2013
DelDebbio,Patella,Rago,2013

$$\tilde{\mathcal{O}}(t,x) \underset{t\to 0}{\longrightarrow} \sum_i c_i(t)\mathcal{O}_i^R(x)$$



□ **gauge-invariant dimension 4 operators**

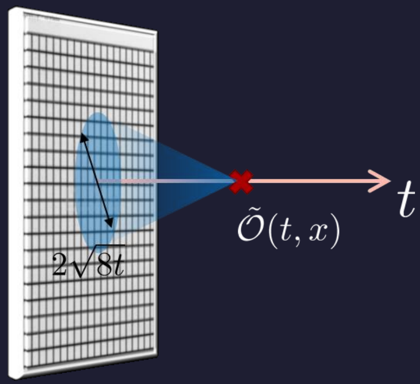
$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

EMTの構築 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$



Suzuki coeffs.

$$\begin{cases} \alpha_U(t) = g^2\left[1 + 2b_0 s_1 g^2 + O(g^4)\right] \\ \alpha_E(t) = \dfrac{1}{2b_0}\left[1 + 2b_0 s_2 g^2 + O(g^4)\right] \end{cases}$$

$g = g(1/\sqrt{8t})$

$s_1 = 0.03296\ldots$

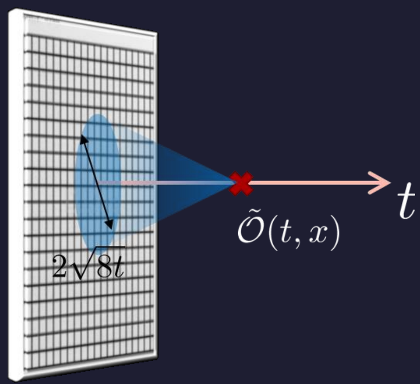
$s_2 = 0.19783\ldots$

EMTの構築 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$



Suzuki coeffs.

$$\begin{cases} \alpha_U(t) = g^2\left[1 + 2b_0 s_1 g^2 + O(g^4)\right] \\ \alpha_E(t) = \dfrac{1}{2b_0}\left[1 + 2b_0 s_2 g^2 + O(g^4)\right] \end{cases}$$

$g = g(1/\sqrt{8t})$

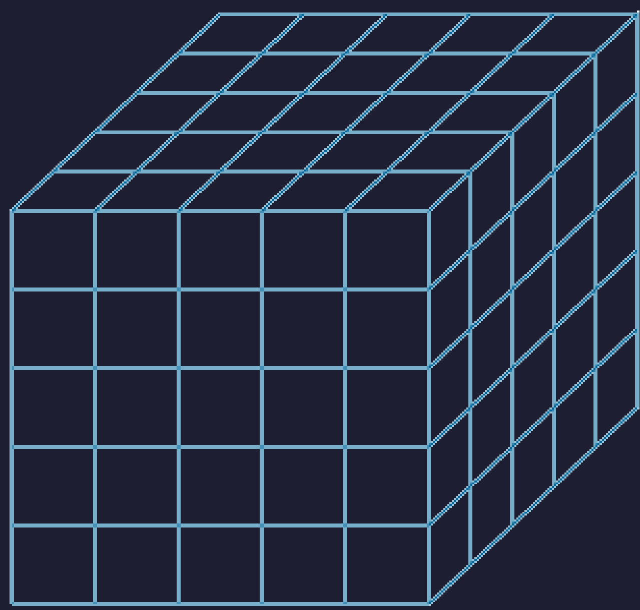
$s_1 = 0.03296\ldots$

$s_2 = 0.19783\ldots$

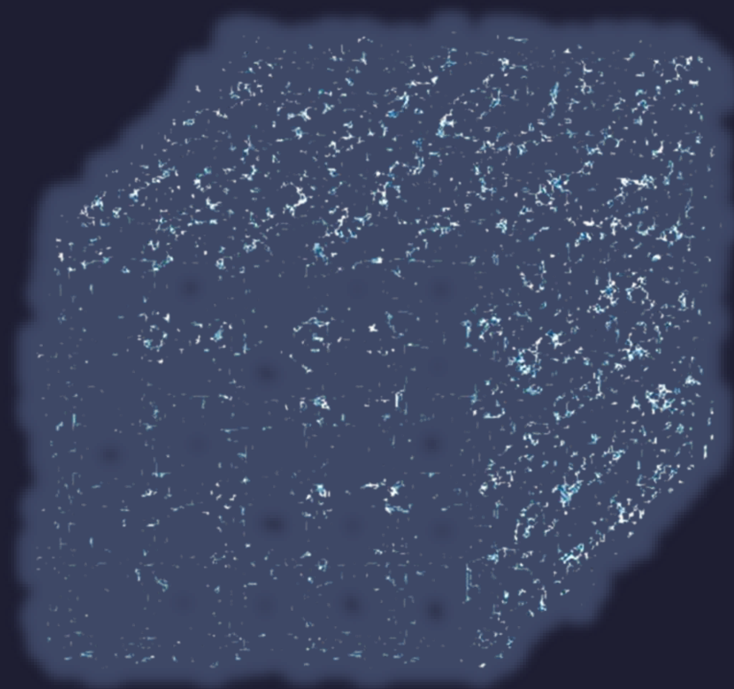
Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\text{subt.}}\right]$$

Rough Idea



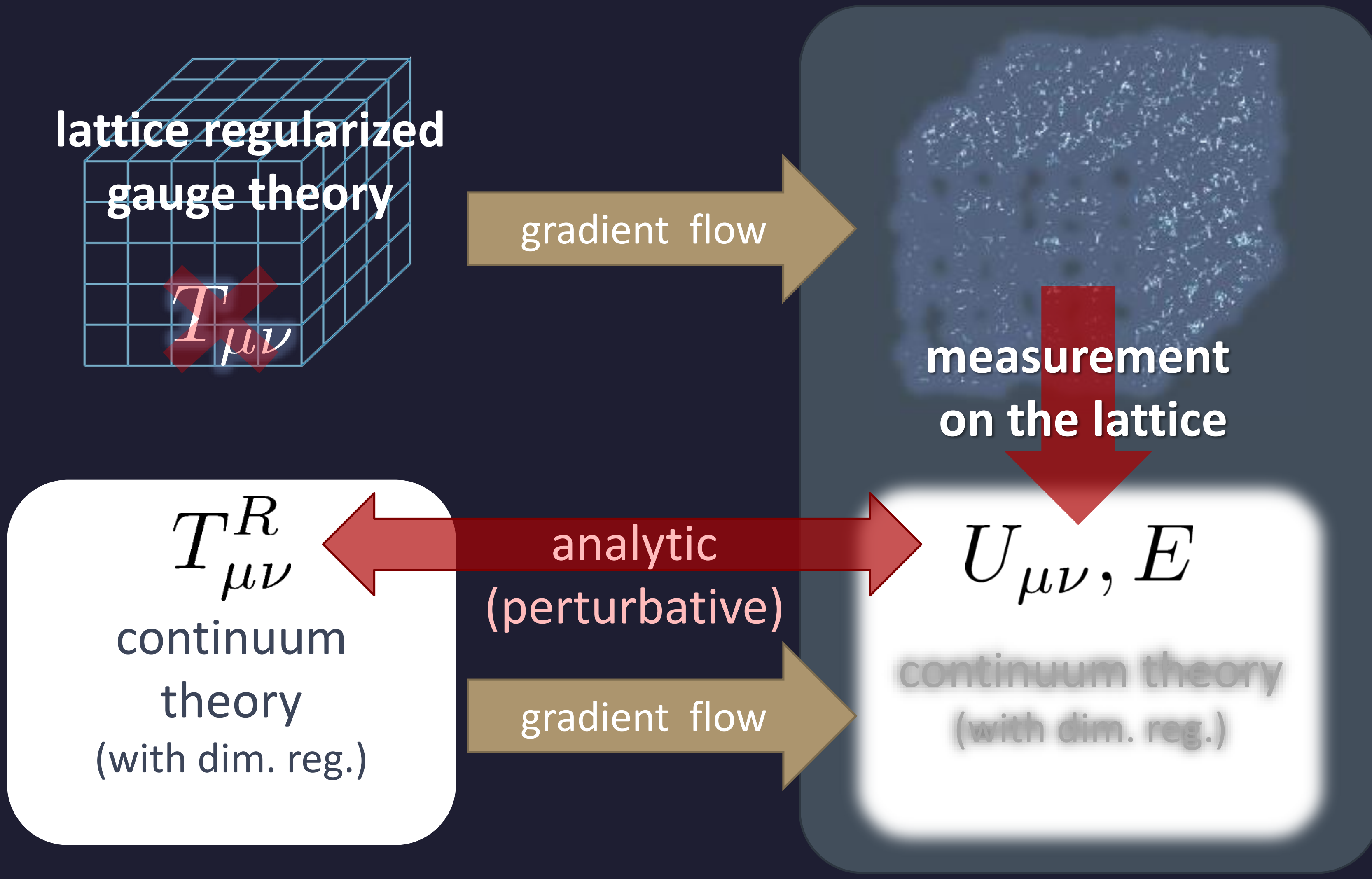
coarse
graining



no translational
invariance

translational symmetry
is recovered!

勾配流によるEMT測定



格子上のEMT(素朴な構築)

$$T_{\mu\nu}(x) = Z_1\left(F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma}\right) + Z_2\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma} + Z_3\delta_{\mu\nu}F_{\mu\rho}F_{\nu\rho}$$

- Z_1, Z_2, Z_3を非摂動的に決定すればEMTは構築可能
- Z_1, Z_3の測定は近年高精度化 Giusti, Pepe, 2014-, BW, 2016

- 純ゲージ理論では、multi-level法で効率的な測定

フェルミオン場の導入

$$T_{\mu\nu}(t,x) = c_1(t)U_{\mu\nu}(t,x) + c_2(t)\delta_{\mu\nu}\big(E(t,x) - \langle E\rangle_0\big)$$
$$+ c_3(t)\big(O_{3\mu\nu}(t,x) - 2O_{4\mu\nu}(t,x) - \mathrm{VEV}\big)$$
$$+ c_4(t)\big(O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) + c_5(t)\big(O_{5\mu\nu}(t,x) - \mathrm{VEV}\big)$$

$$T_{\mu\nu}(x) = \lim_{t\to 0} T_{\mu\mu}(t,x)$$

$$\tilde{O}^f_{3\mu\nu}(t,x) \equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x),$$
$$\tilde{O}^f_{4\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x),$$
$$\tilde{O}^f_{5\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x),$$

$$\varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.$$

$$c_1(t) = \frac{1}{\bar{g}(1/\sqrt{8t})^2} - \frac{1}{(4\pi)^2}\left[9(\gamma - 2\ln 2) + \frac{19}{4}\right],$$
$$c_2(t) = \frac{1}{(4\pi)^2}\frac{33}{16},$$
$$c_3(t) = \frac{1}{4}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[2 + \frac{4}{3}\ln(432)\right]\right\},$$
$$c_4(t) = \frac{1}{(4\pi)^2}\bar{g}(1/\sqrt{8t})^2,$$
$$c^f_5(t) = -\bar{m}_f(1/\sqrt{8t})\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[4(\gamma - 2\ln 2) + \frac{14}{3} + \frac{4}{3}\ln(432)\right]\right\}$$

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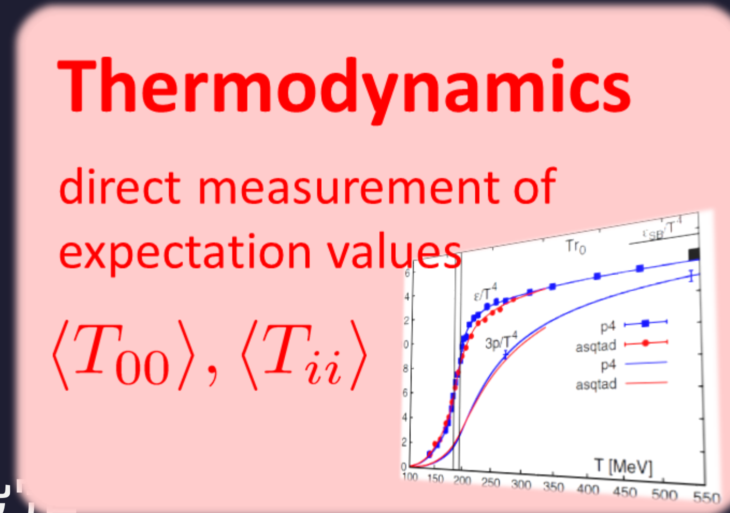
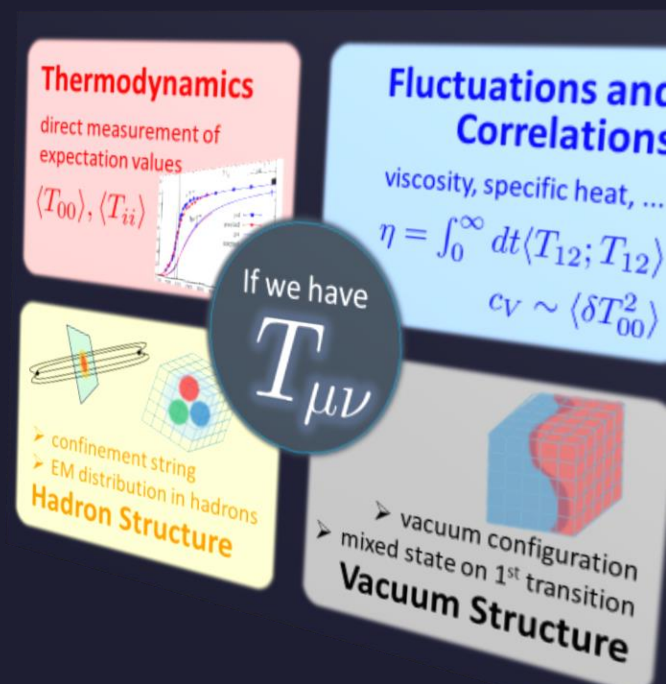
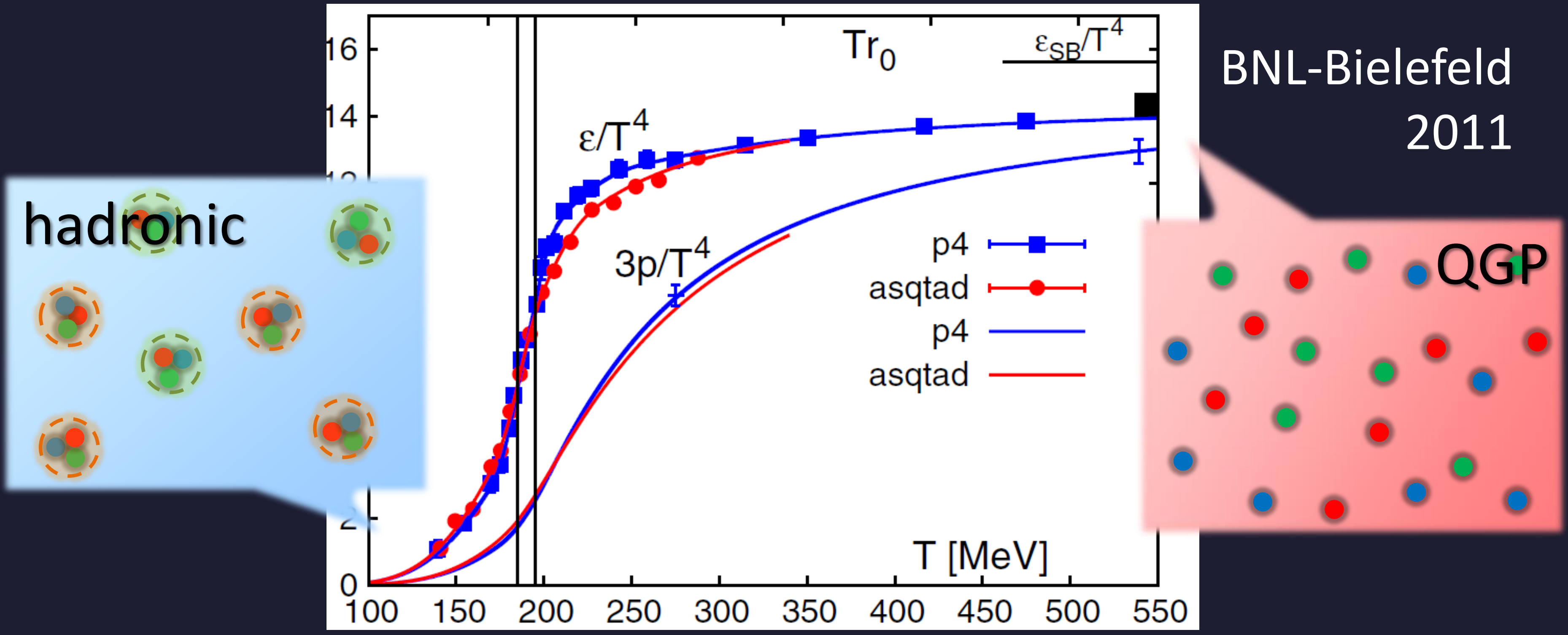




QCD EoS
(Energy Density, Pressure)



BNL-Bielefeld
2011

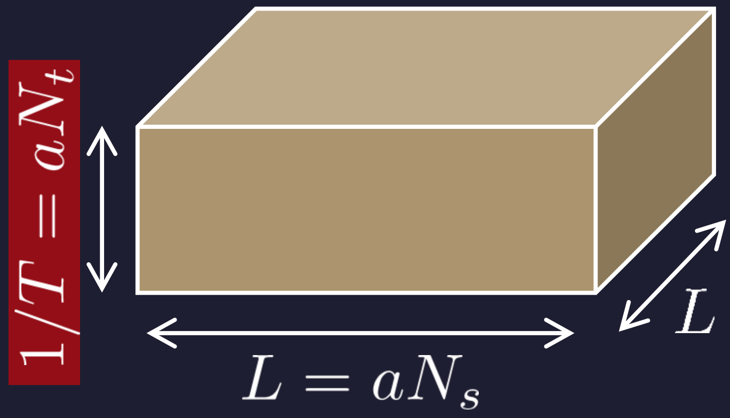
- Rapid increase of ε/T^4 around T=150-200 MeV
- Crossover transition
- Low T: hadron resonance gas model / High T: perturbative QCD

QCD Thermodynamics

熱力学関係式

$$\varepsilon = \frac{T^2}{V}\frac{\partial \ln Z}{\partial T} \qquad p = T\frac{\partial \ln Z}{\partial V}$$

$$Z(T) = \mathrm{Tr}\left[e^{-H/T}\right] = \int \mathcal{D}A \exp\left[-\int_0^{1/T} d\tau \int_V d^3x \mathcal{L}_E\right]$$



ln ZのT、V微分を求められれば、熱力学量が決まる

格子間隔を変化させてみる

Changing lattice spacing a ⇨ 1/T and V change

$$\left\{\begin{array}{l} \dfrac{\partial \ln Z}{\partial a} \sim \varepsilon - 3p \\ \dfrac{\partial \ln Z}{\partial a} = \dfrac{\partial \beta}{\partial a}\dfrac{\partial \ln Z}{\partial \beta} \sim \dfrac{\partial \beta}{\partial a}\langle S \rangle \\ \beta = 2N_c/g^2 \end{array}\right.$$

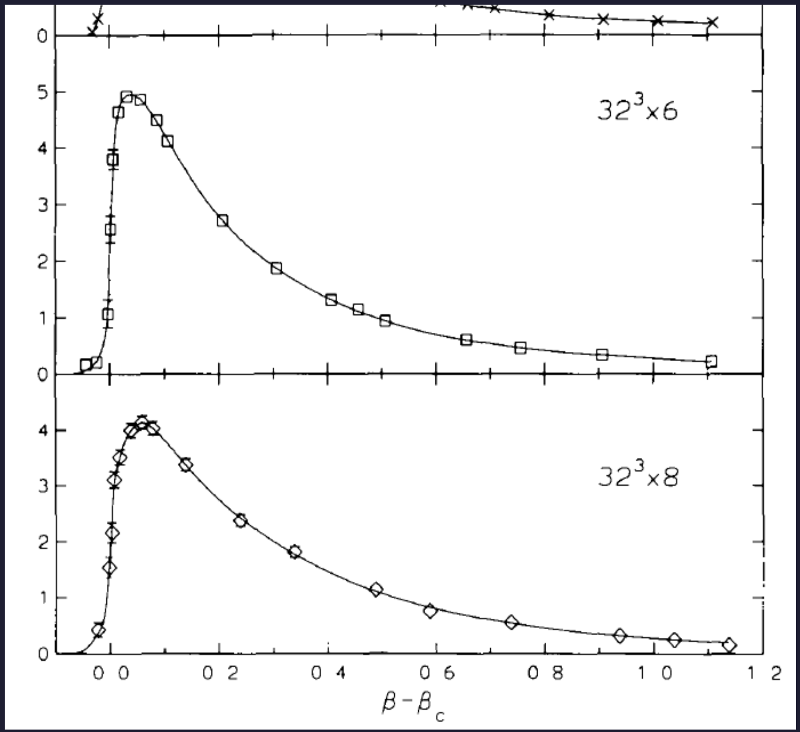
$\dfrac{\partial \beta}{\partial a}$, $\langle S \rangle$ ⇨ e-3pが求まる

積分法

$$T\frac{\partial(p/T^4)}{\partial T} = \frac{\varepsilon - 3p}{T^4}$$

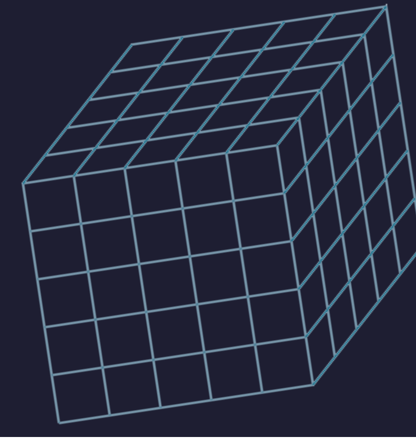
$$\frac{p}{T^4} = \int_{T_0}^{T} dT \frac{\varepsilon - 3p}{T^5}$$

Boyd+ 1996



- measurements of e-3p for many T
- vacuum subtraction for each T
- information on beta function

Numerical Simulation



- SU(3) YM theory
- Wilson gauge action
- Parameters:
  - N_t = 12, 16, 20-24
  - aspect ratio $5.3<N_s/N_t<8$
  - 1500~2000 configurations

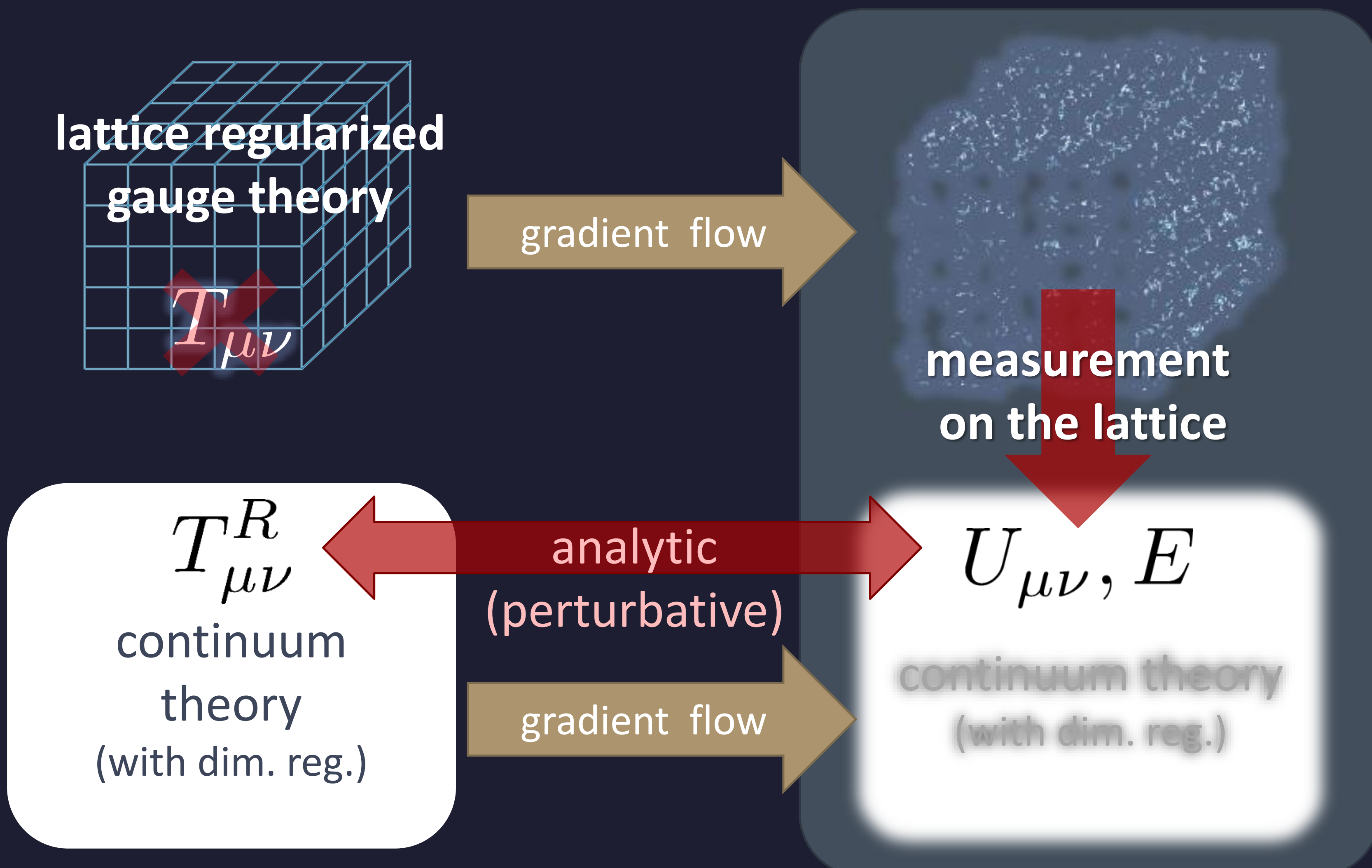
- Scale from gradient flow

$$\rightarrow aT_c \text{ and } a\Lambda_{\mathrm{MS}}$$

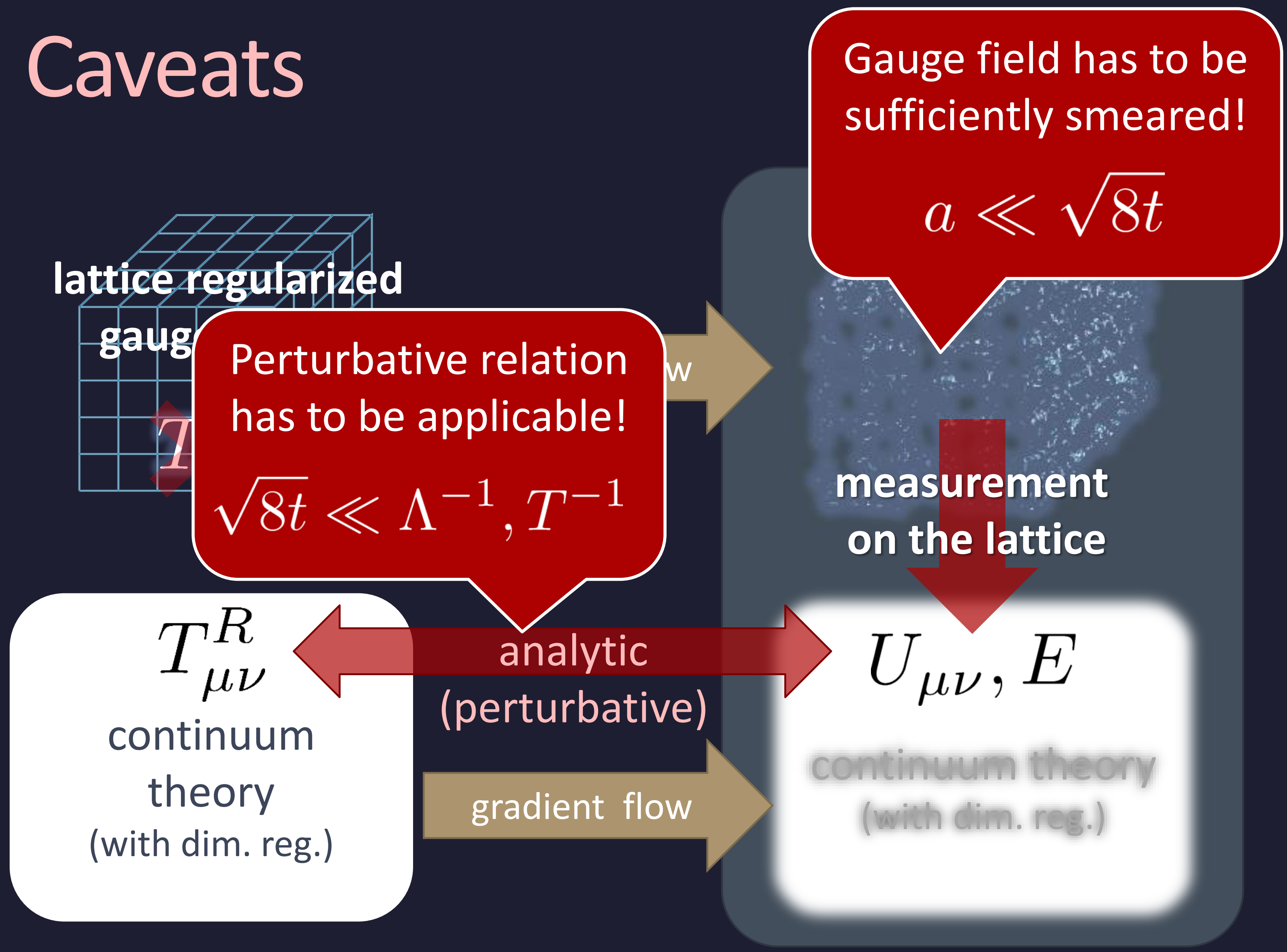
FlowQCD 1503.06516

T/T_c	β	N_s	N_τ	Configurations
0.93	6.287	64	12	2125
	6.495	96	16	1645
	6.800	128	24	2040
1.02	6.349	64	12	2000
	6.559	96	16	1600
	6.800	128	22	2290
1.12	6.418	64	12	1875
	6.631	96	16	1580
	6.800	128	20	2000
1.40	6.582	64	12	2080
	6.800	128	16	900
	7.117	128	24	2000
1.68	6.719	64	12	2000
	6.941	96	16	1680
	7.117	128	20	2000
2.10	6.891	64	12	2250
	7.117	128	16	840
	7.296	128	20	2040
2.31	7.200	96	16	1490
	7.376	128	20	2020
	7.519	128	24	1970
2.69	7.086	64	12	2000
	7.317	96	16	1560
	7.500	128	20	2040

Gradient Flow Method

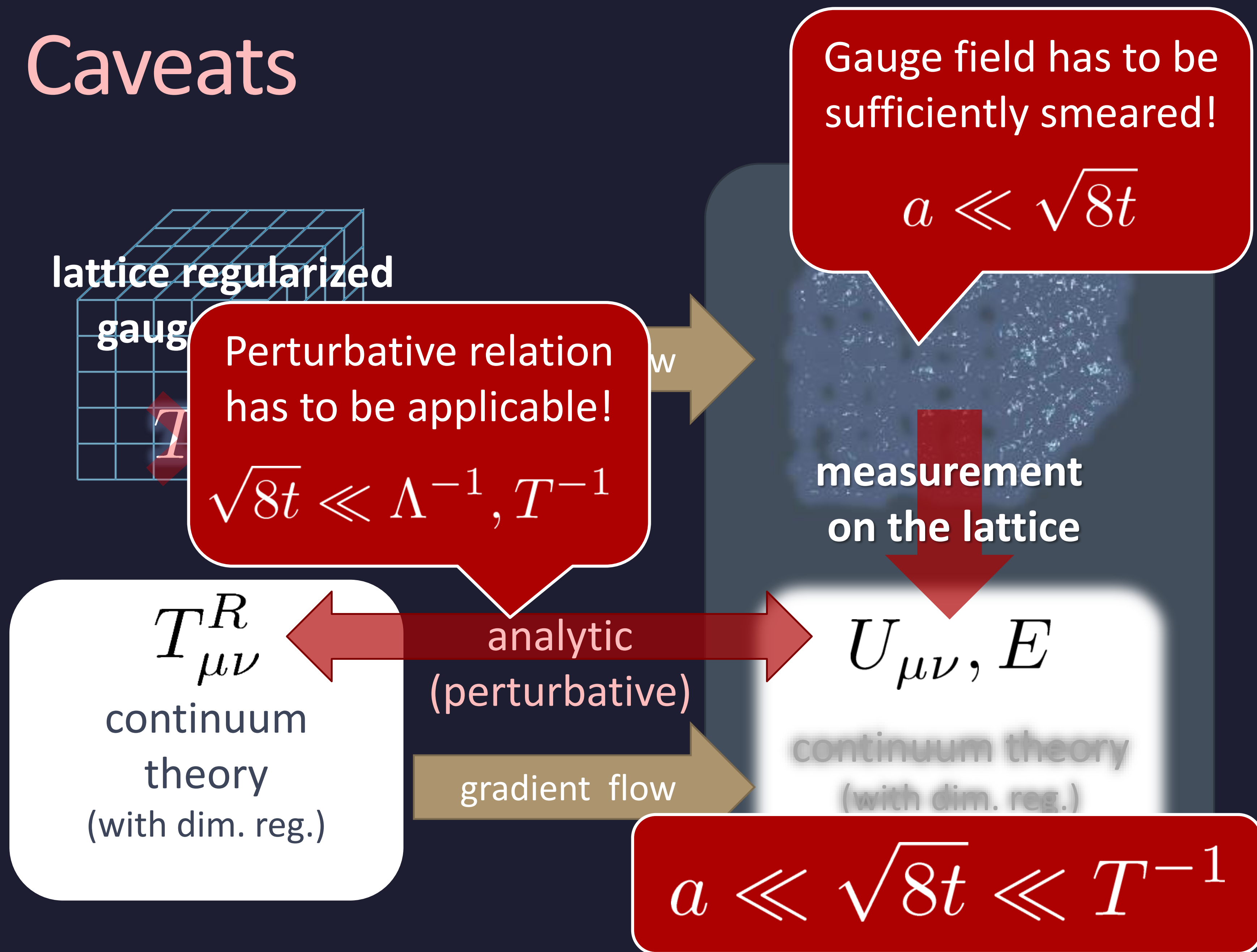


Caveats



Gauge field has to be sufficiently smeared!
$a \ll \sqrt{8t}$
lattice regularized
Perturbative relation has to be applicable!
$\sqrt{8t} \ll \Lambda^{-1}, T^{-1}$
measurement on the lattice
$T^R_{\mu\nu}$
continuum theory
(with dim. reg.)
analytic
(perturbative)
$U_{\mu\nu}, E$
continuum theory
(with dim. reg.)
gradient flow

Caveats



Gauge field has to be sufficiently smeared!
$a \ll \sqrt{8t}$
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gaug
Perturbative relation has to be applicable!
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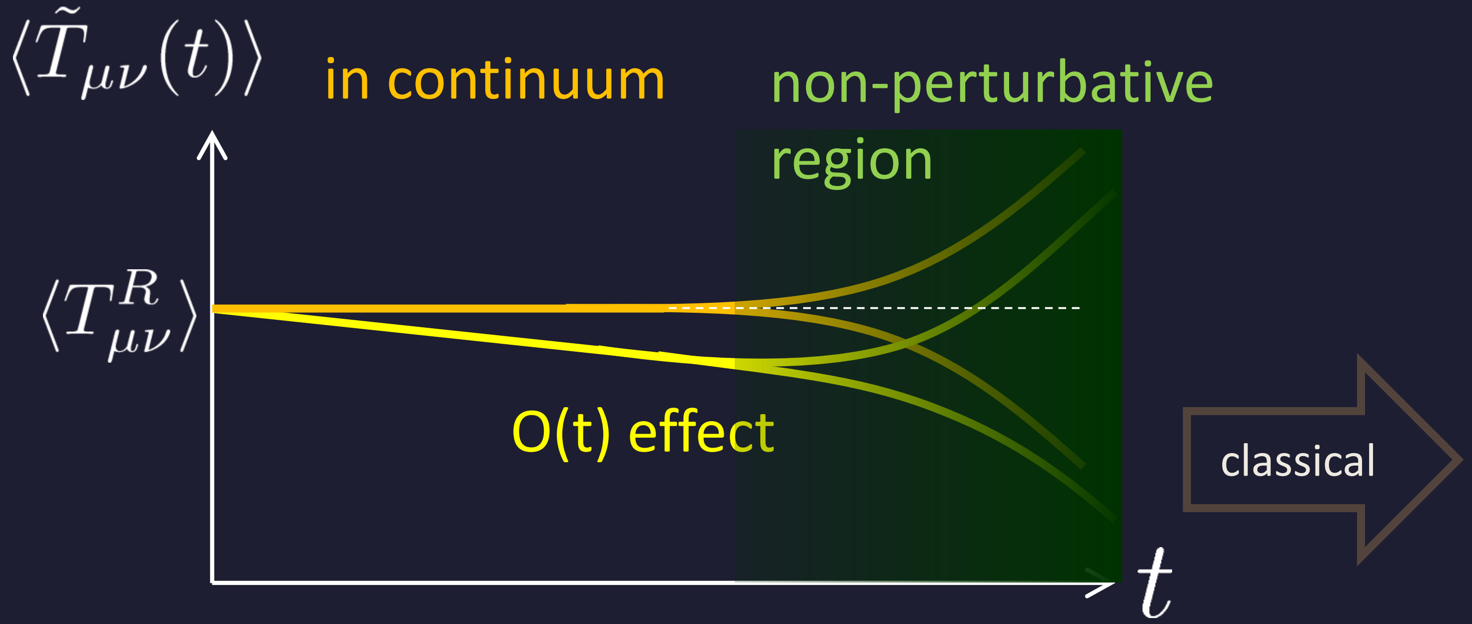
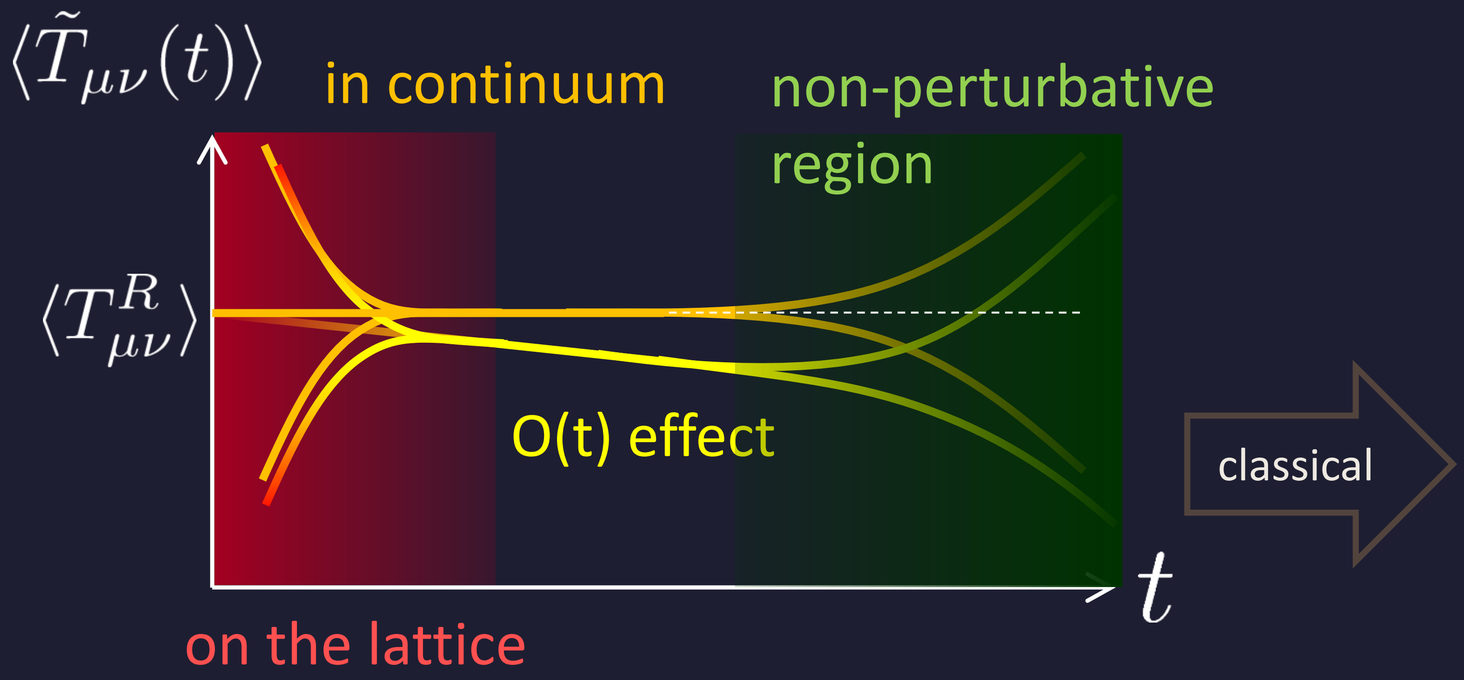
measurement on the lattice
$T^R_{\mu\nu}$
continuum theory
(with dim. reg.)
analytic
(perturbative)
$U_{\mu\nu}, E$
continuum theory
(with dim. reg.)
gradient flow
$a \ll \sqrt{8t} \ll T^{-1}$

$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\rm subt.} \qquad T^R_{\mu\nu} = \lim_{t\to 0} \tilde{T}_{\mu\nu}(t)$$

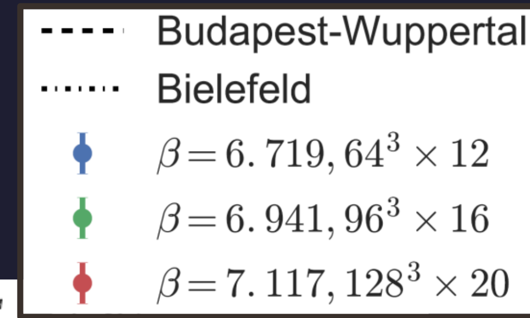


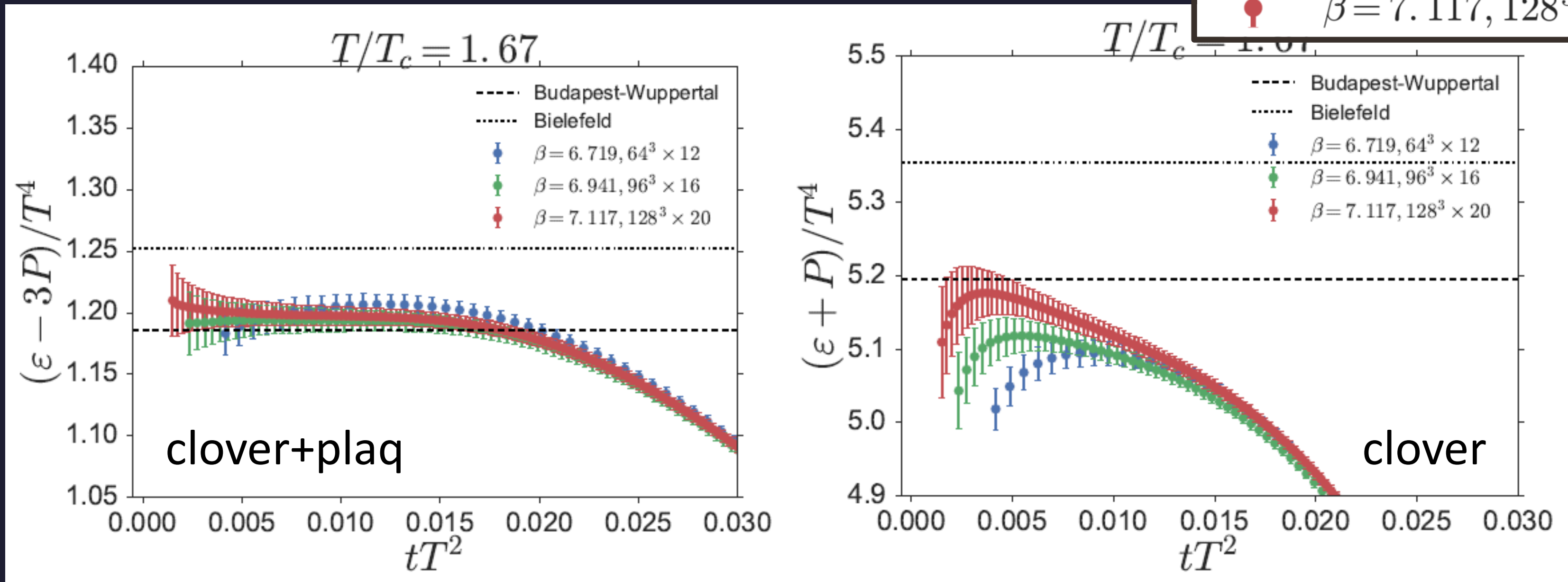
$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\rm subt.} \qquad T^R_{\mu\nu} = \lim_{t\to 0} \tilde{T}_{\mu\nu}(t)$$



- Double extrapolation t→0, a→0 required
- Extrapolation has to be taken keeping $t \gg a^2$

t Dependence



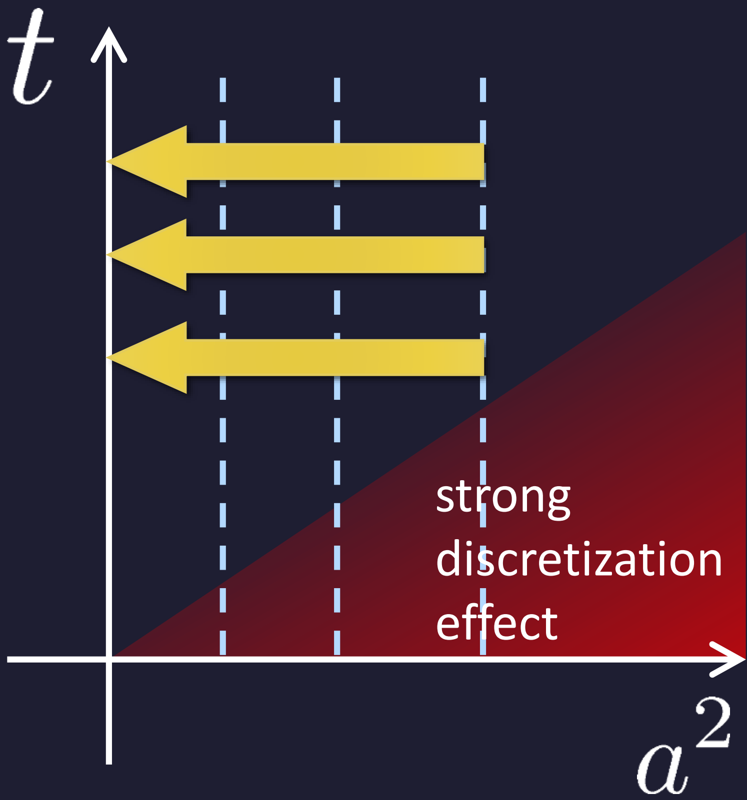


$\sqrt{8t} < a$: strong discretization effect

$\sqrt{8t} > 1/(2T)$: oversmeared

$a < \sqrt{8t} < 1/(2T)$: Linear t dependence

Double Extrapolation

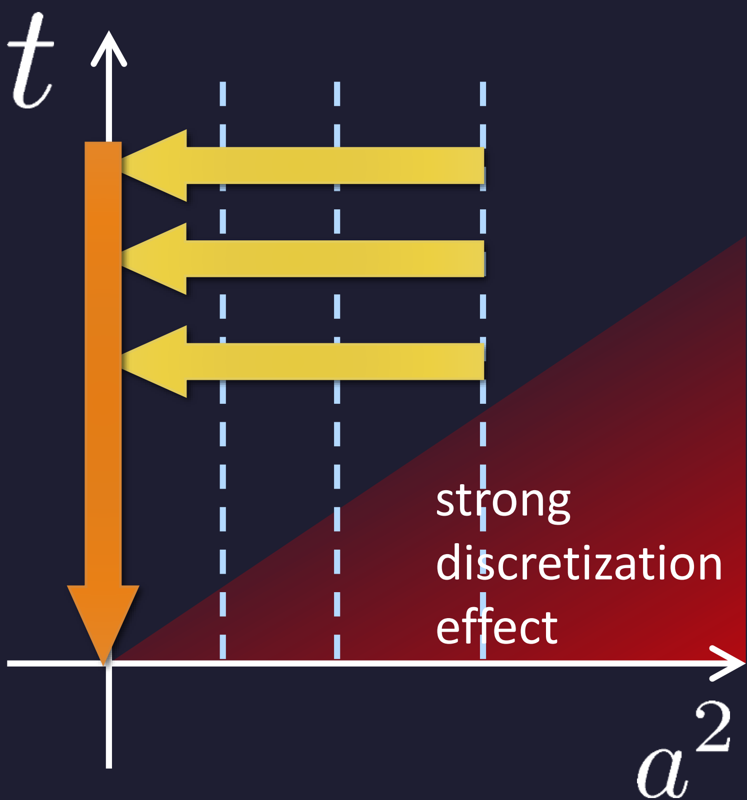


Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\mathrm{cont}} = \langle T_{\mu\nu}(t)\rangle_{\mathrm{lat}} + C(t)a^2$$

Note: FlowQCD, 2014: continuum extrapolation only
WHOT-QCD, 2016: small t limit only

Double Extrapolation



Continuum extrapolation

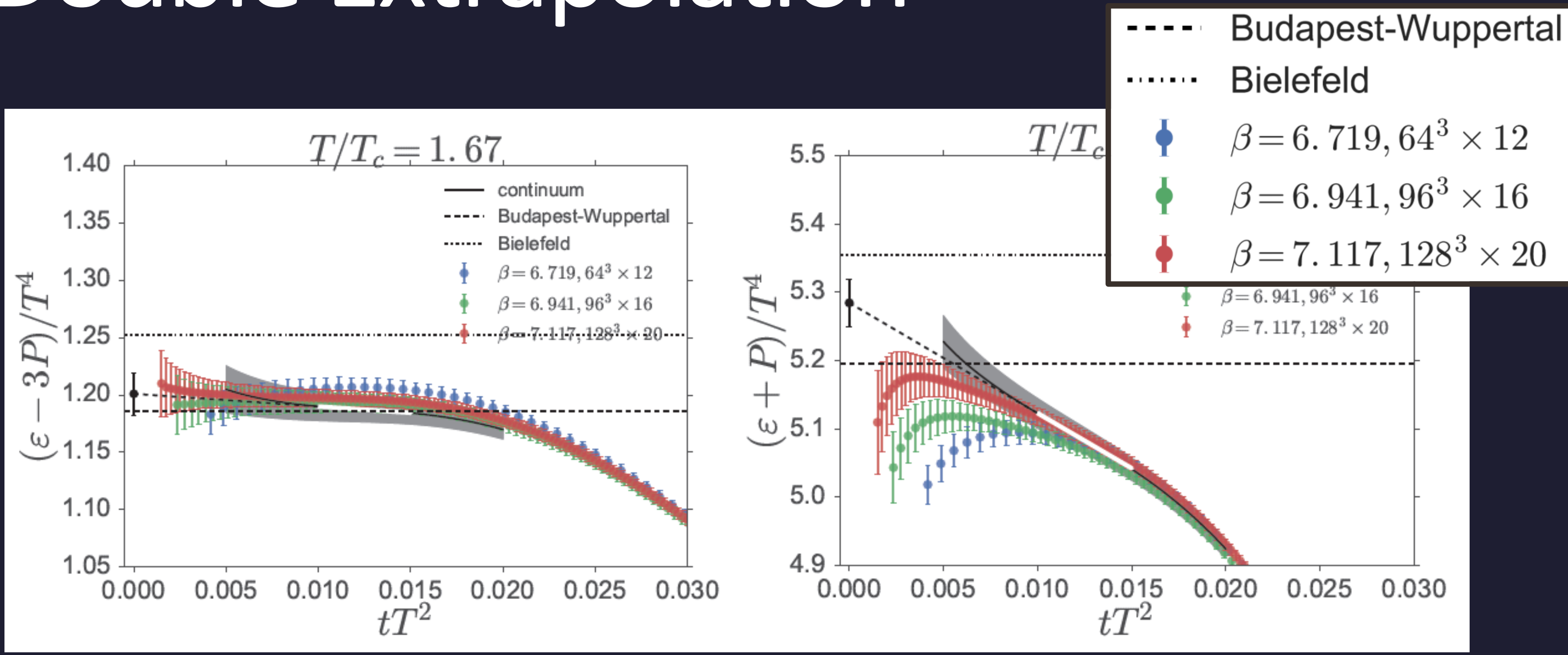
$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

Small t extrapolation

$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C'(t)t$$

Note: FlowQCD, 2014: continuum extrapolation only
WHOT-QCD, 2016: small t limit only

Double Extrapolation



Black band: continuum extrapolated

- ☐ range of t for fitting:

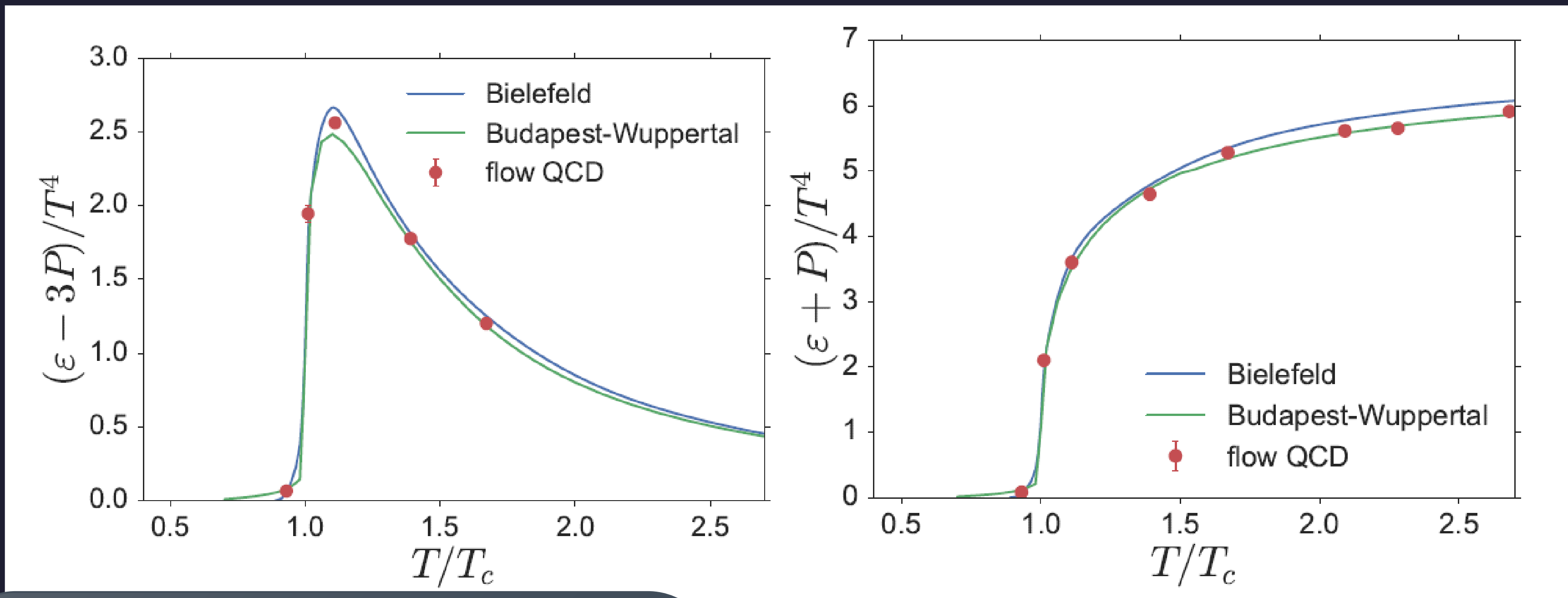
$$0.005 < tT^2 < 0.015$$

$$0.01 < tT^2 < 0.015$$

$$0.01 < tT^2 < 0.02$$

T Dependence

FlowQCD, PRD, 2016



Error includes

- statistical error
- choice of t range for t→0 limit
- uncertainty in $a\Lambda_{MS}$

total error <1.5% for $T>1.1T_c$

- ☐ Excellent agreement with integral method
- ☐ High accuracy only with ~2000 confs.

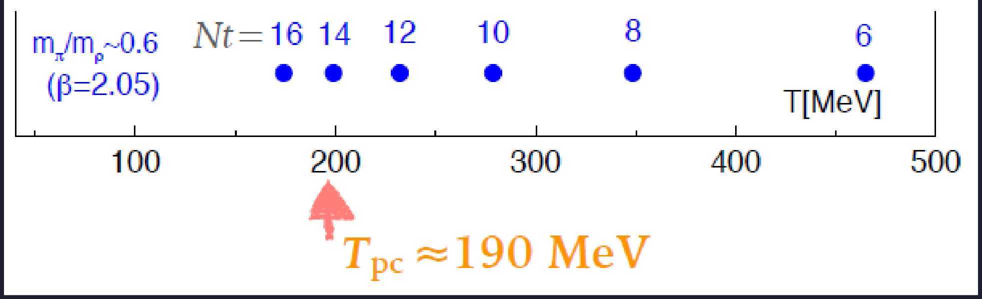
N_f=2+1 QCD 熱力学の解析

Taniguchi+, WHOT-QCD, PRD, 2017

- N_f=2+1 QCD, Iwasaki gauge + NP-clover
- m_{PS}/m_V ≈0.63 with ≈physical s quark

- T=0: CP-PACS+JLQCD (ß=2.05, 28^3x56, a≈0.07fm)
- T>0: 32^3xN_t, N_t = 4, 6, ... , 14, 16):
- T≈174-697MeV

- t→0 extrapolation only (No continuum limit)

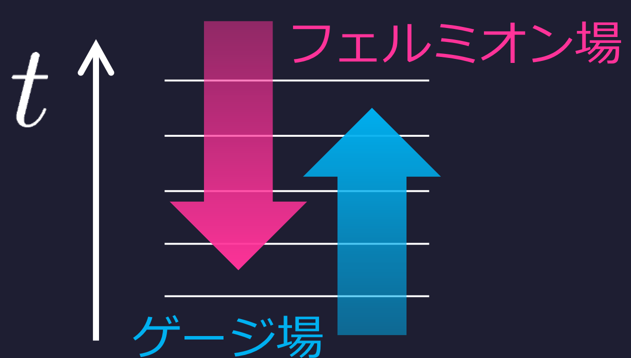


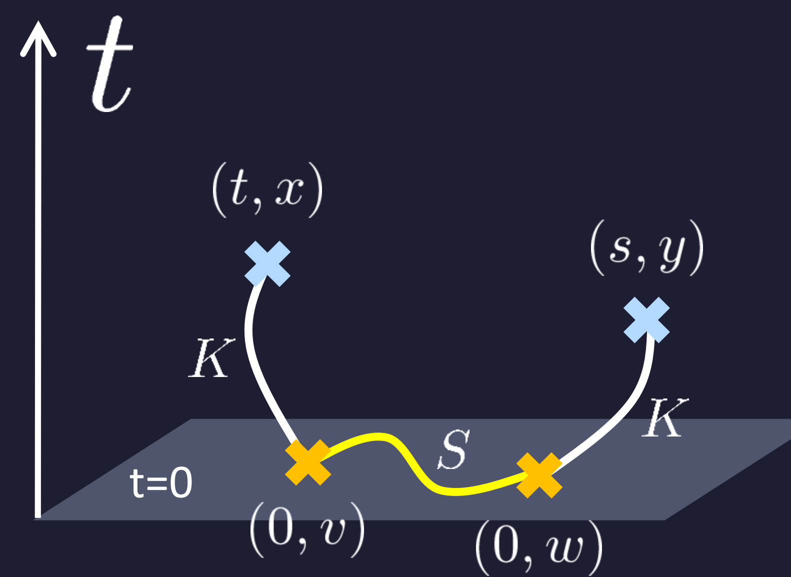
Fermion Propagator

$$S(t,x;s,y) = \langle \chi(t,x)\bar{\chi}(s,y)\rangle$$
$$= \sum_{v,w} K(t,x;0,v)S(v,w)K(s,y;0,w)^{\dagger}$$

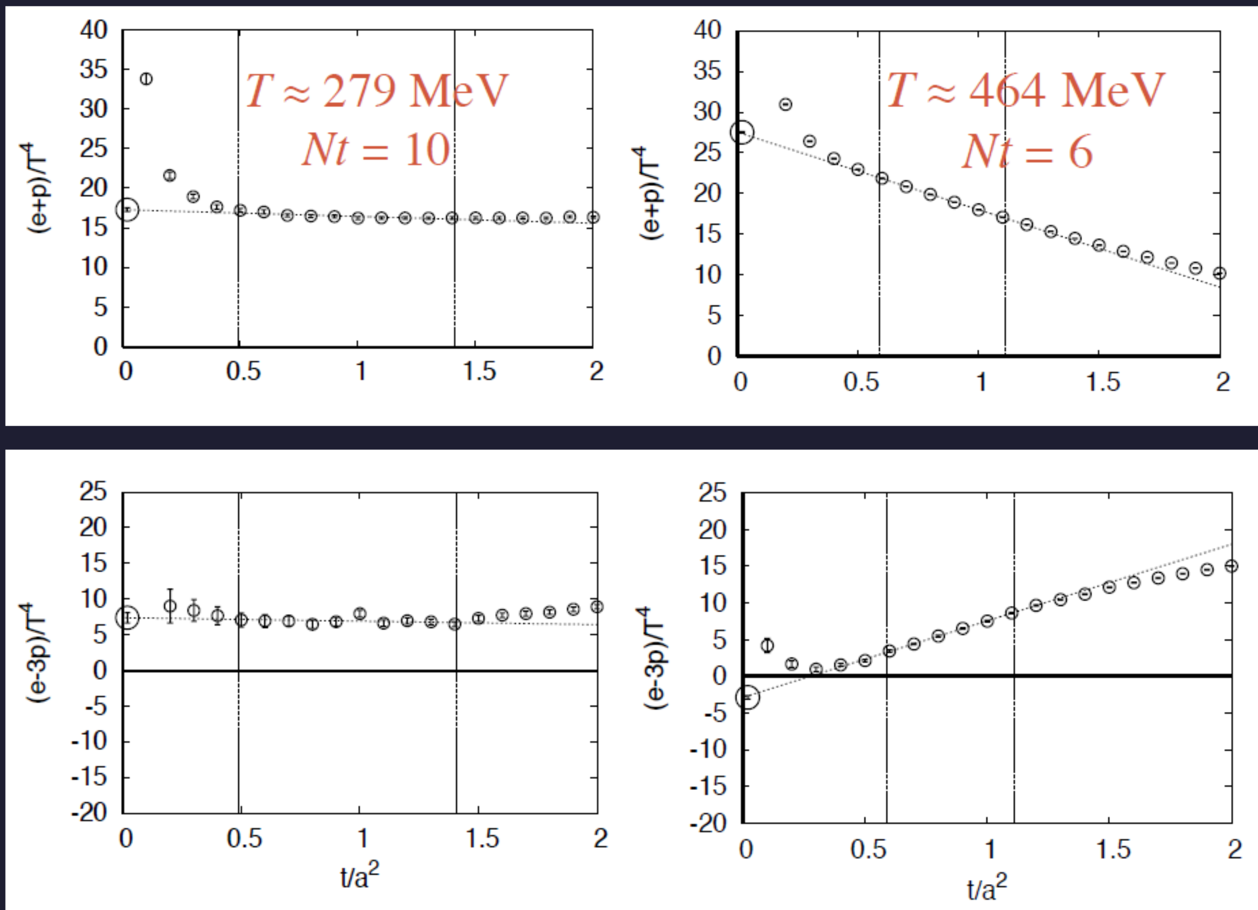
$$\Big(\partial_t - D_\mu D_\mu\Big)K(t,x) = 0$$

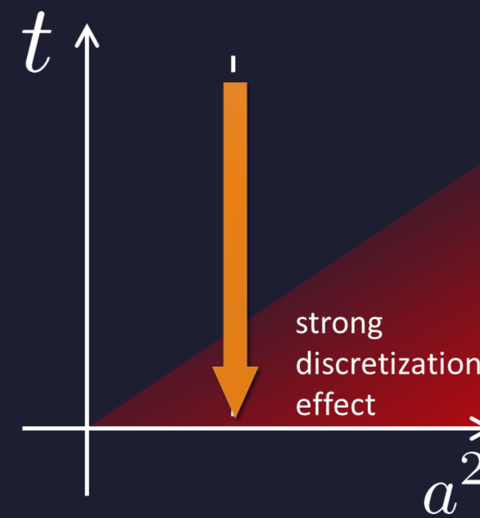
- 勾配流方程式に対する伝搬関数
- 逆向き伝搬関数が一つは必要





t→0外挿



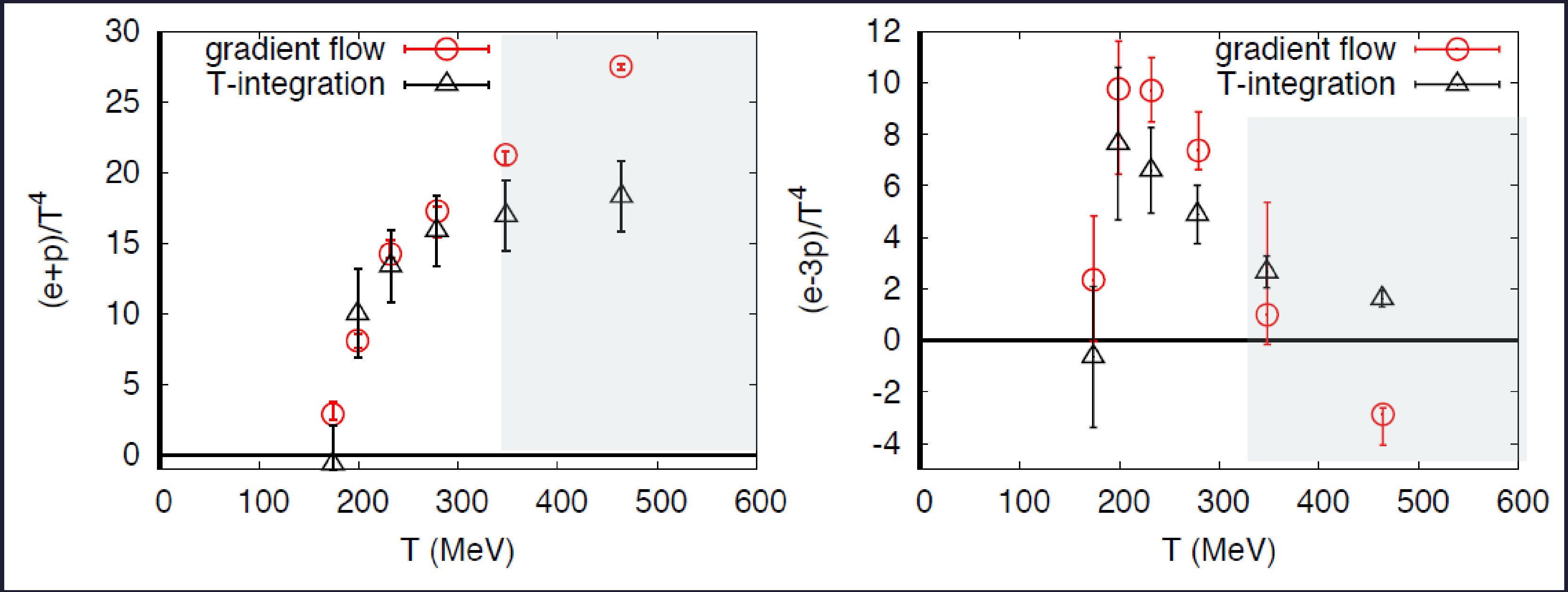


$$\begin{cases} \dfrac{t}{a^2} \geq 1 \\ \sqrt{8t} \leq \dfrac{1}{2T} \end{cases}$$

- □ Nt>6では、”linear window”が存在
- □ フィット範囲、a^2/t項の考慮

N_f=2+1 状態方程式

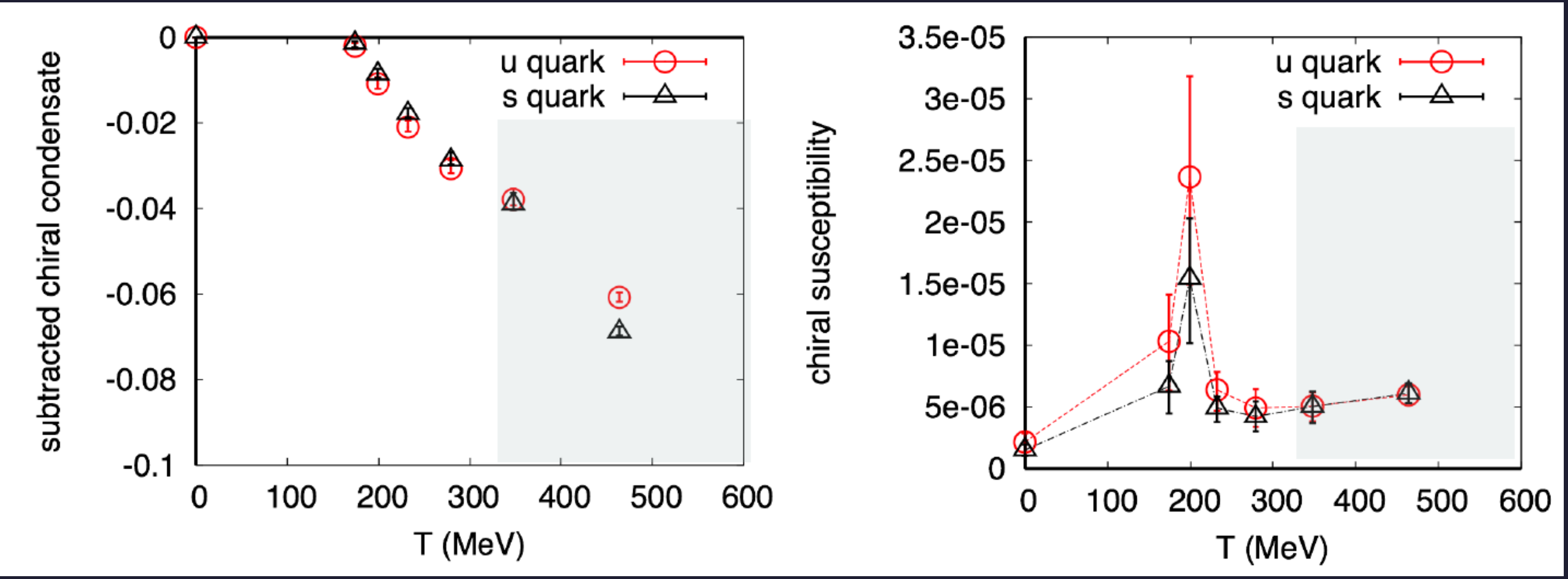
Taniguchi+, WHOT-QCD, PRD, 2017



- N_t=4, 6を除き、積分法の結果と一致
- N_t=4, 6では安定した外挿t領域が得られない
- 積分法と比べ、誤差の抑制

カイラル凝縮・感受率

Taniguchi+, WHOT-QCD, PRD, 2017

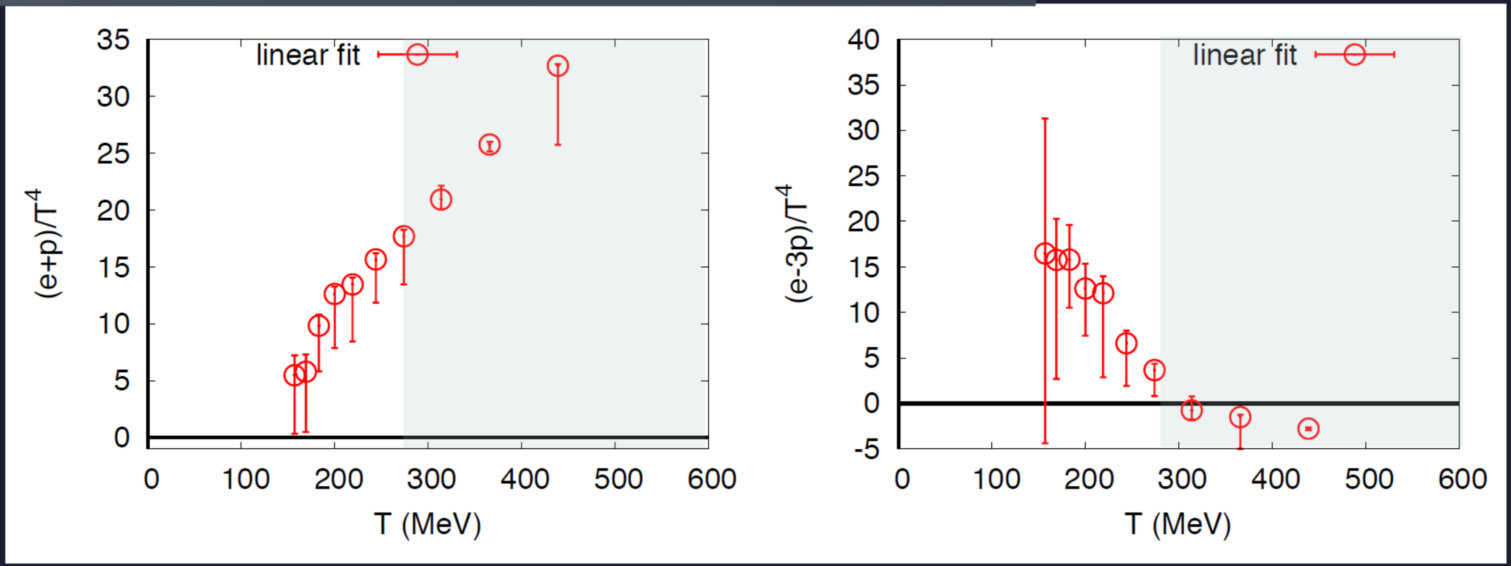


- カイラル凝縮は、T>Tcで減少の傾向
- カイラル感受率は、T=Tc付近で鋭いピーク

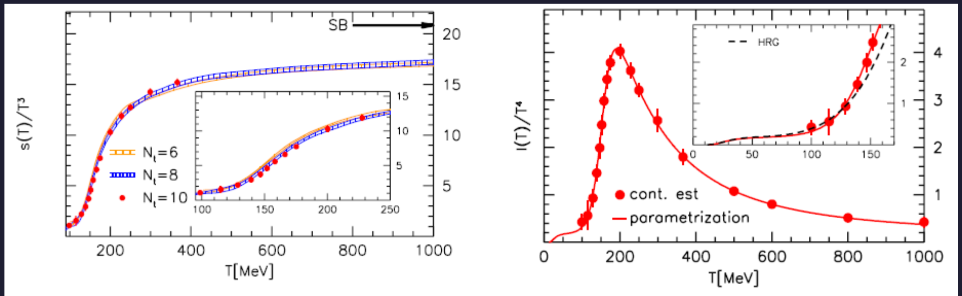
物理点シミュレーション

- Nf=2+1 QCD, Iwasaki gauge + NP-clover
- T=0: PACS-CS (ß=1.9, 32^3x64, a≈0.09fm)
- Fine-tuned to the phys.pt. by reweighting.
- T>0: 32^3xNt, Nt = 4, 5, ... , 14, T≈157--549MeV

WHOT-QCD, Preliminary



Budapest-Wuppertal
2010



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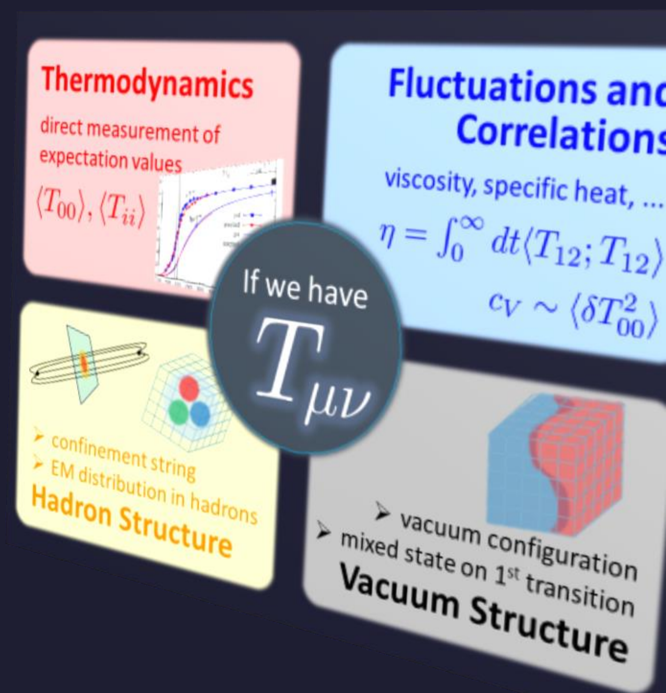
Fluctuations and Correlations

viscosity, specific heat, ...

$$\eta = \int_0^\infty dt \langle T_{12}; T_{12} \rangle$$

$$c_V \sim \langle \delta T_{00}^2 \rangle$$



なぜEMT相関関数？

- □ Kubo Formula: T_{12} correlator ←→shear viscosity

$$\eta = \int_0^\infty dt \int_0^{1/T} d\tau \int d^3x \langle T_{12}(x, -i\tau) T_{12}(0, t) \rangle$$

  - ➢ Hydrodynamics describes long range behavior of $T_{\mu\nu}$

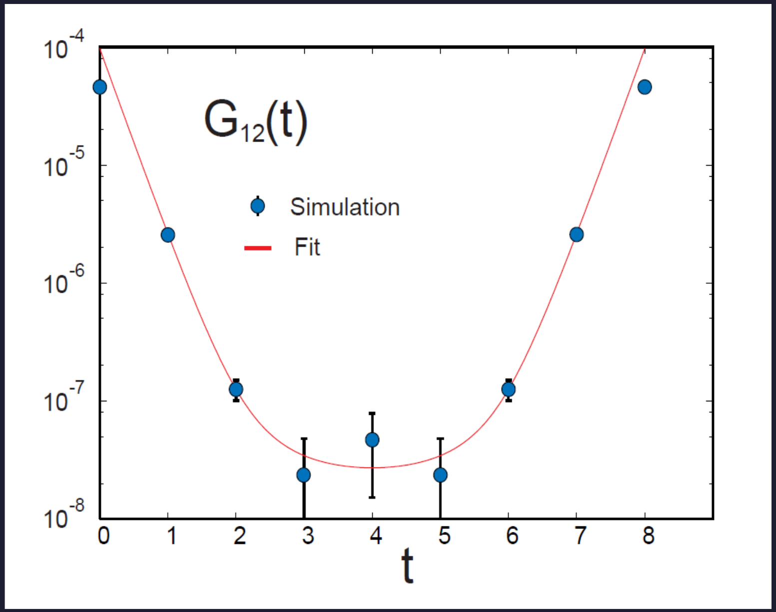
- □ Energy fluctuation ←→specific heat

$$c_V = \frac{\langle \delta E^2 \rangle}{VT^2}$$

EMT Correlator : Noisy...

With naïve EMT operators

$$\langle T_{12}(\tau) T_{12}(0) \rangle$$



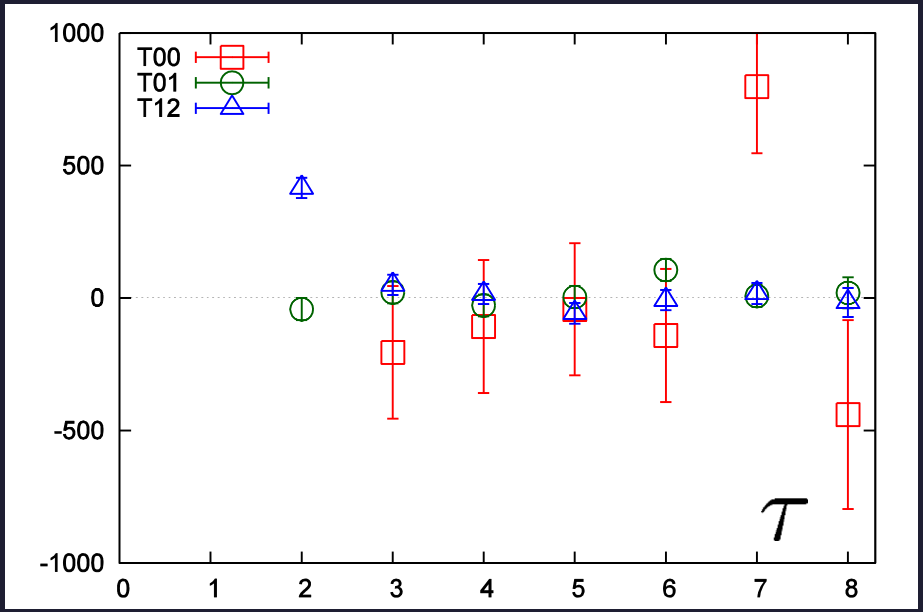
Nakamura, Sakai, PRL,2005

N_t=8

improved action

~10^6 configurations

$$\langle T_{\mu\nu}(\tau) T_{\mu\nu}(0) \rangle$$



Nt=16

standard action

5x10^4 configurations

... no signal

Conservation Law

$$\frac{\partial}{\partial \tau}\bar{T}_{00} = 0$$

$$\frac{\partial}{\partial \tau}\bar{T}_{01} = 0$$

$$\frac{\partial}{\partial \tau}\langle \bar{T}_{00}(\tau)\bar{T}_{00}(0)\rangle = 0$$

$$\frac{\partial}{\partial \tau}\langle \bar{T}_{00}(\tau)\bar{T}_{11}(0)\rangle = 0$$

$$\frac{\partial}{\partial \tau}\langle \bar{T}_{01}(\tau)\bar{T}_{01}(0)\rangle = 0$$

$$(\tau \neq 0)$$

- 保存電荷を含む相関関数は、τによらない定数となる

Linear Response Relations

$$c_V = \frac{d}{dT}\langle E \rangle = \frac{\langle \bar{T}_{00}^2 \rangle}{VT^2}$$

Specific heat

$$s = \frac{d}{dT}P = \frac{\langle \bar{T}_{11}\bar{T}_{00} \rangle}{VT^2}$$

entropy density

Giusti, Meyer, 2011

$$\varepsilon + p = \frac{\langle \bar{T}_{01}^2 \rangle}{VT}$$

enthalpy density

Minami, Hidaka, 2012

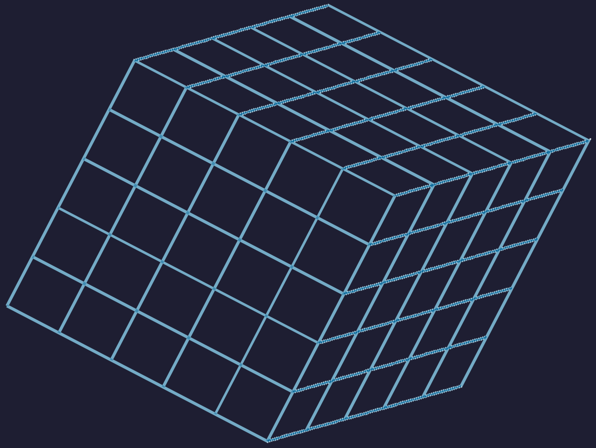
導出

$$\langle \hat{O} \rangle = \frac{1}{Z}\mathrm{Tr}\left[\hat{O}e^{-\beta\hat{H}}\right] \Rightarrow \frac{d}{d\beta}\langle \hat{O} \rangle = -\langle \delta\hat{O}\delta\hat{H} \rangle$$

格子解析

FlowQCD, to appear soon

- SU(3) ゲージ理論
- Wilson作用／クローバー演算子
- アスペクト比：Ns/Nt=4
- 統計数：180,000

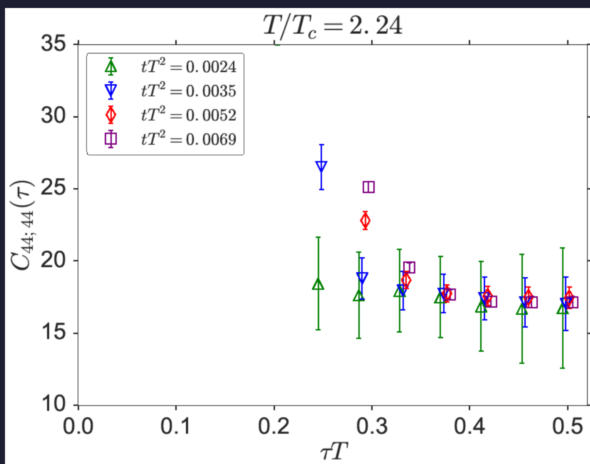


β	$T=1.66T_c$	$T=2.22T_c$
48^3x12	6.719	6.943
64^3x16	6.941	7.170
96^3x24	7.265	7.500

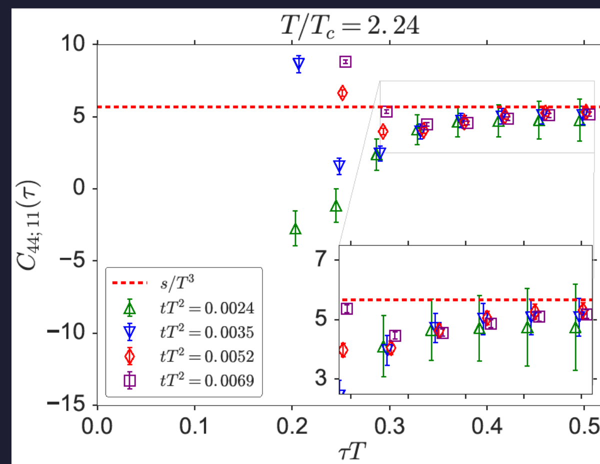
数値解析：Bluegene/Q @KEK

虚時間相関関数@T=2.24Tc

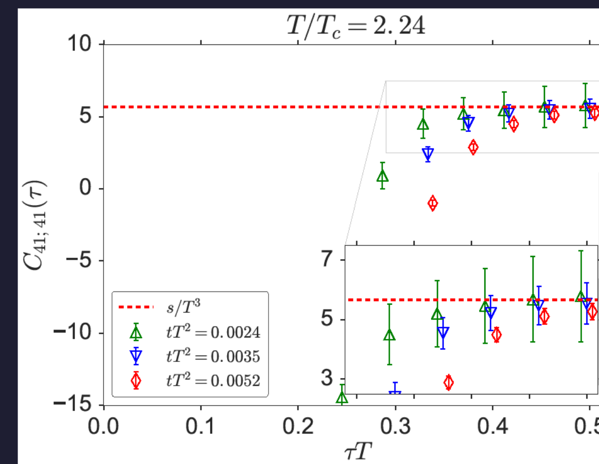
$\langle T_{44}(\tau)T_{44}(0)\rangle$



$\langle T_{44}(\tau)T_{11}(0)\rangle$



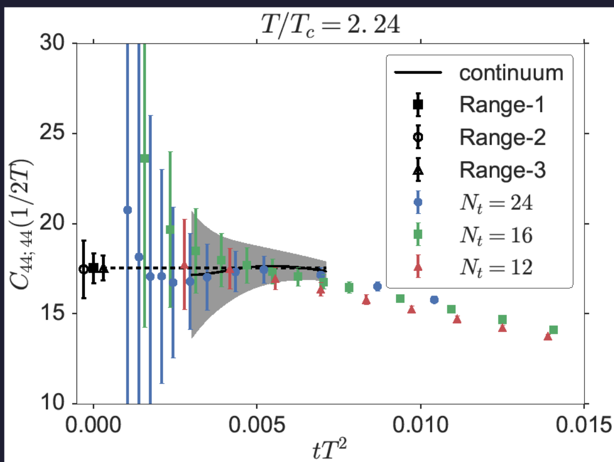
$\langle T_{41}(\tau)T_{41}(0)\rangle$



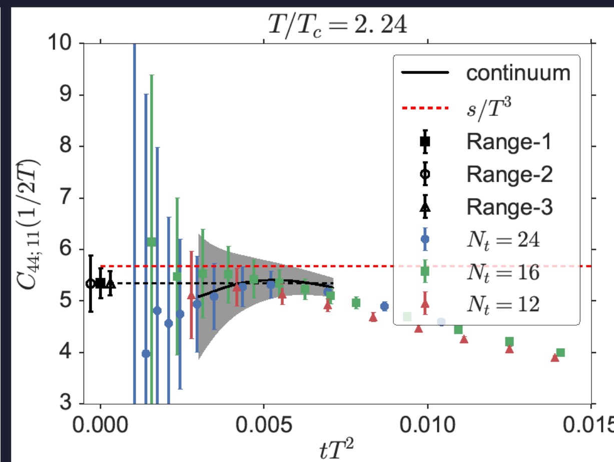
- 全てのチャンネルに、保存則に対応する平坦領域が存在
- tが小さい領域では、演算子のオーバーラップが発生

中点相関関数@T=2.24Tc

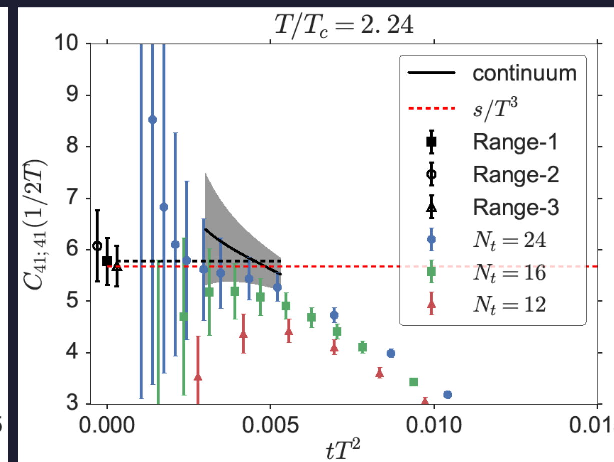
$\langle T_{44}(\tau) T_{44}(0) \rangle$



$\langle T_{44}(\tau) T_{11}(0) \rangle$



$\langle T_{41}(\tau) T_{41}(0) \rangle$



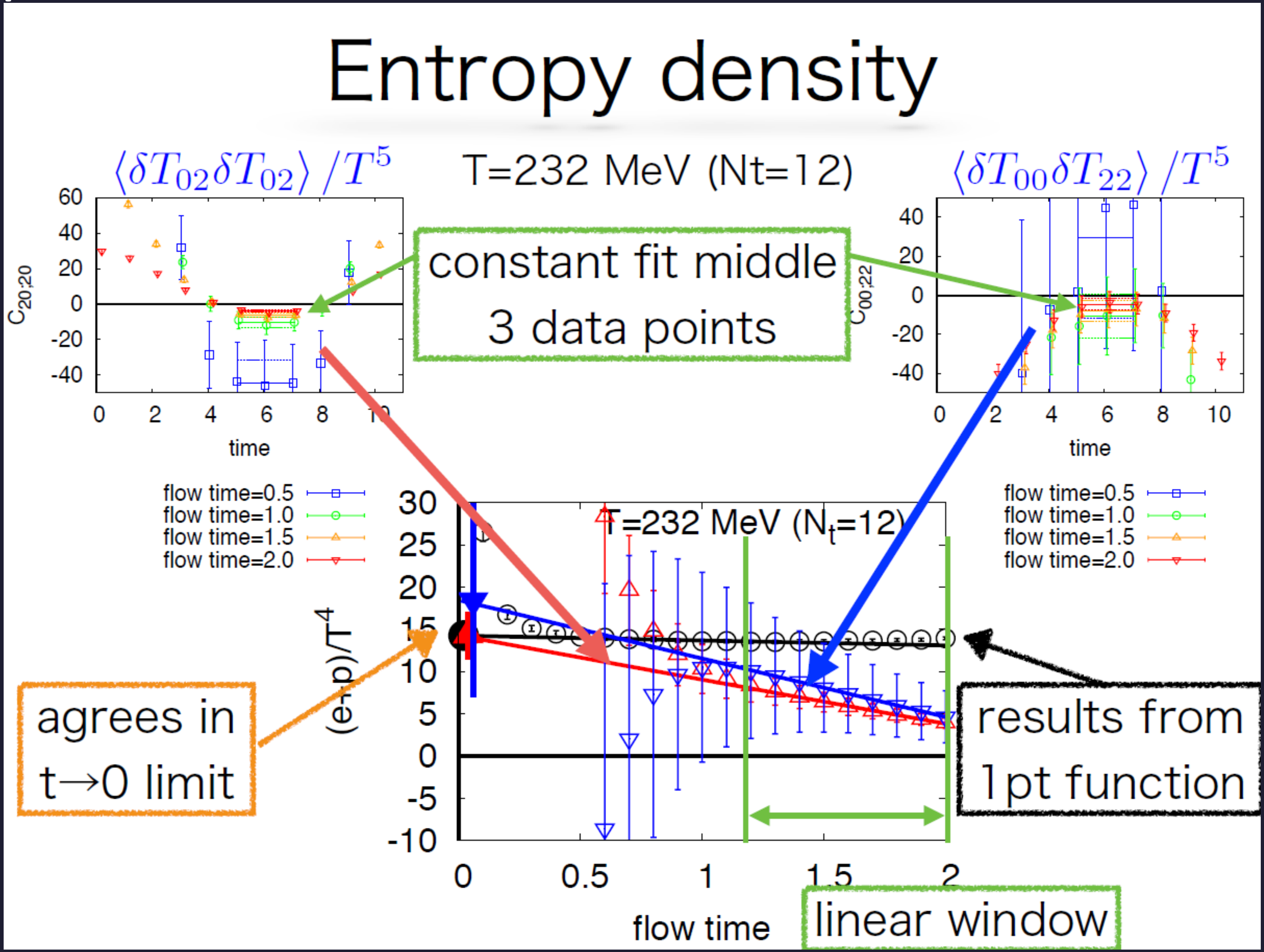
- (44;11), (41;41)チャンネル：線形応答関係の確認
- (44;44)チャンネルからは、比熱の測定が可能！

c_V/T^3

T/T_c	$C_{44;44}(\tau_m)$	Ref.[19]	Ref.[11]	ideal gas
1.68	$17.7(8)(^{+2.1}_{-0.4})$	22.8(7)*	17.7	21.06
2.24	$17.5(0.8)(^{+0}_{-0.1})$	17.9(7)**	18.2	21.06

N_f=2+1 QCD

WHOT-QCD, Preliminary



Slide from Y. Taniguchi, LATTICE2017/XQCD2017

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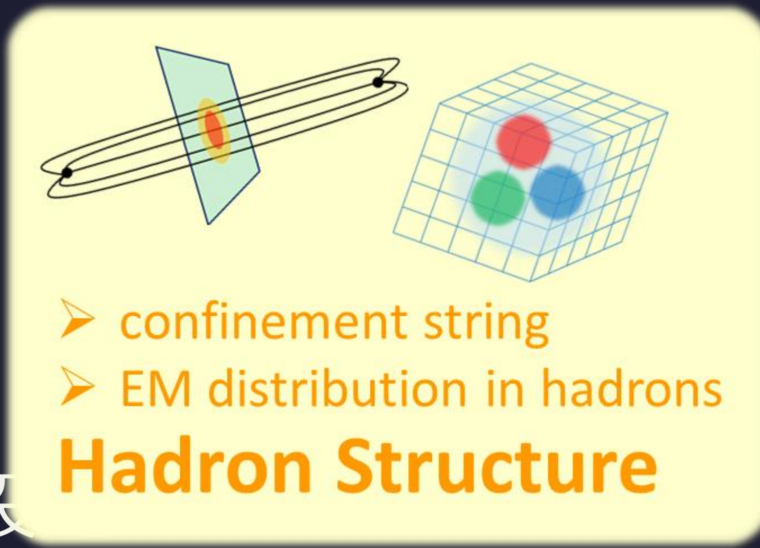
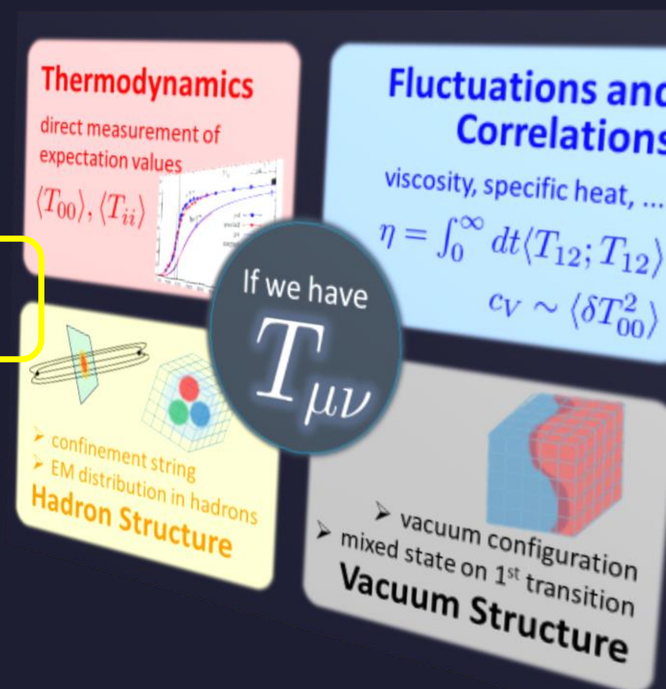


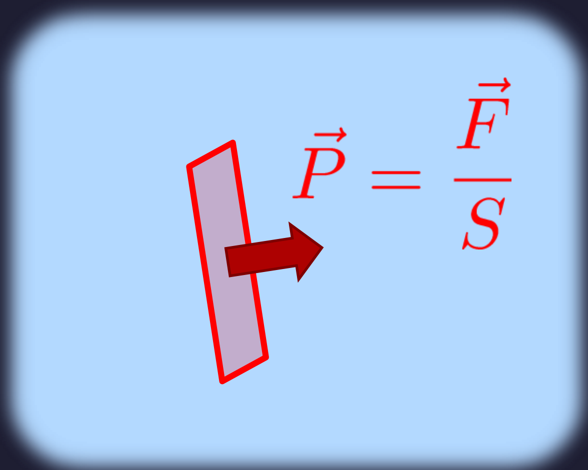
1. 勾配流とは
2. 勾配流の応用：スケール設
3. 勾配流を用いたEMTの構成
4. 熱力学量の測定
5. EMT相関関数の測定
6. flux tube周辺の応力構造の解析

柳原良亮 for FlowQCD, in progress



応力とは

圧力

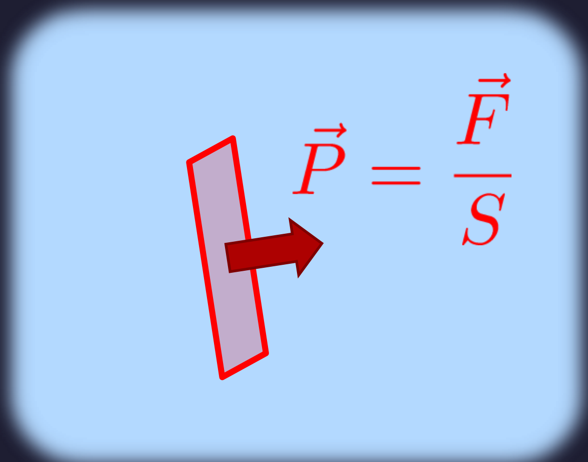


ある面に働く
単位体積当たりの力

$$\vec{P} = P\vec{n}$$

応力とは

圧力

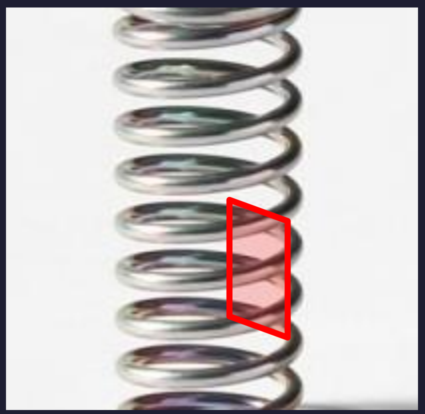


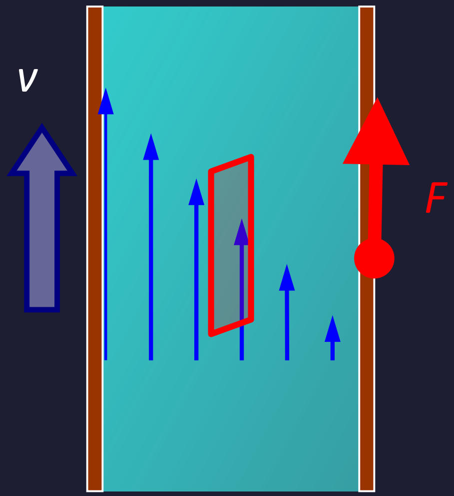
ある面に働く
単位体積当たりの力

$$\vec{P} = P\vec{n}$$

応力

一般にPとnは平行でない→応力



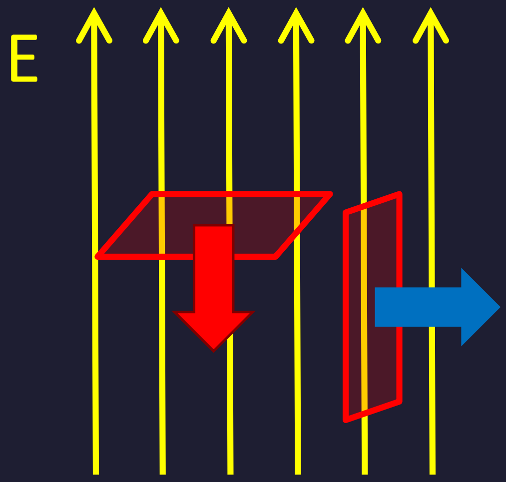


応力テンソル

$$P_i = T_{ij} n_j$$

Maxwell応力

$$T_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$



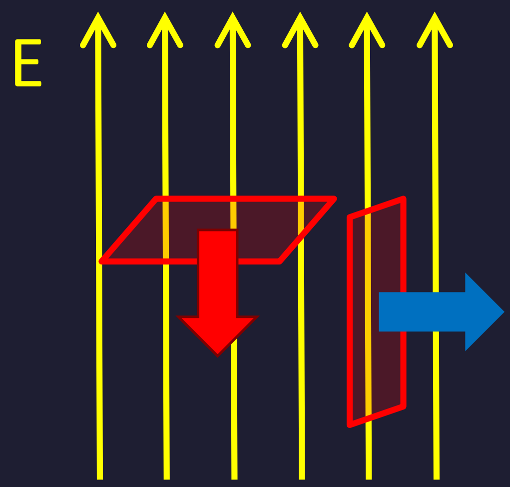
$$\vec{E} = (E, 0, 0)$$

$$T = \frac{1}{2}\begin{bmatrix} E^2 & 0 & 0 \\ 0 & -E^2 & 0 \\ 0 & 0 & -E^2 \end{bmatrix}$$

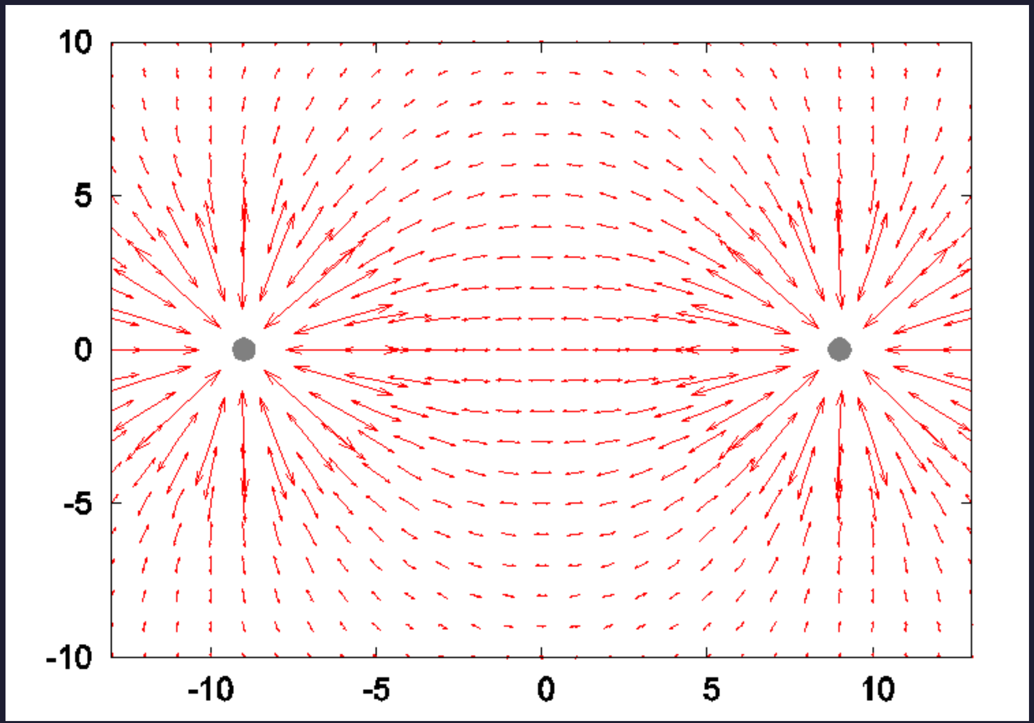
電場と平行方向：引力
電場と垂直方向：斥力

Maxwell応力

$$T_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$



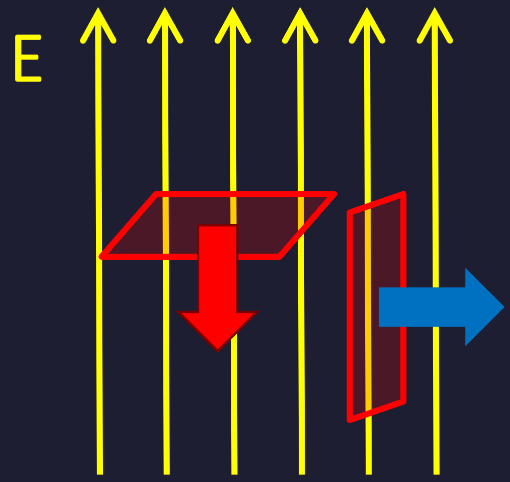
電場と平行方向：引力
電場と垂直方向：斥力



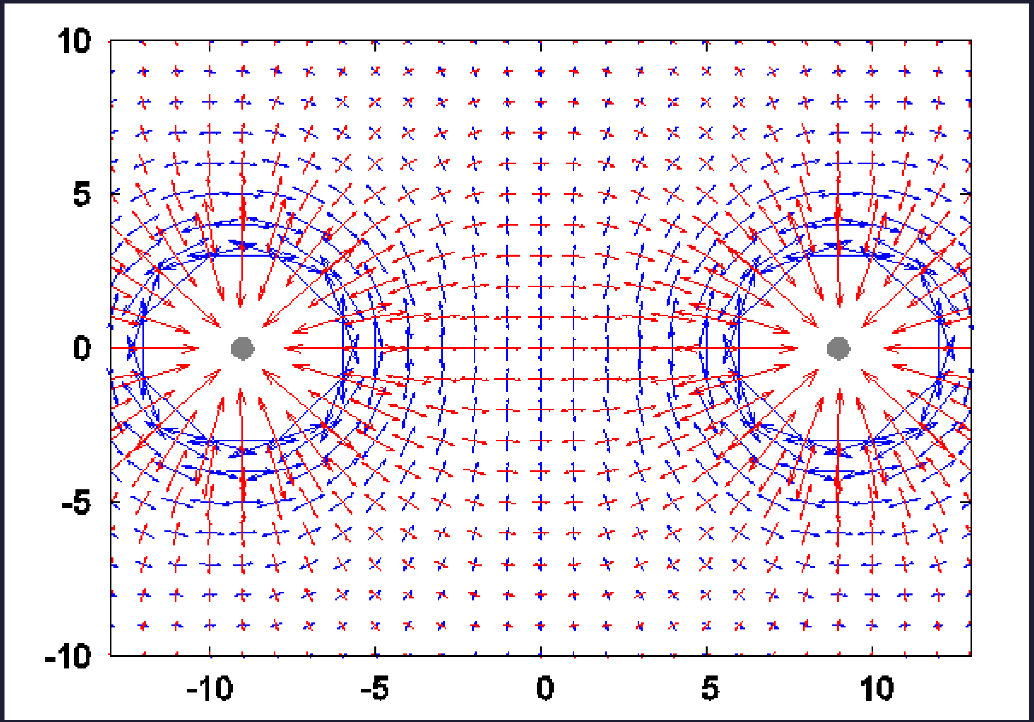
異なる電荷間の応力分布

Maxwell応力

$$T_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$



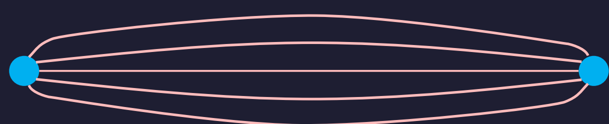
電場と平行方向：引力
電場と垂直方向：斥力



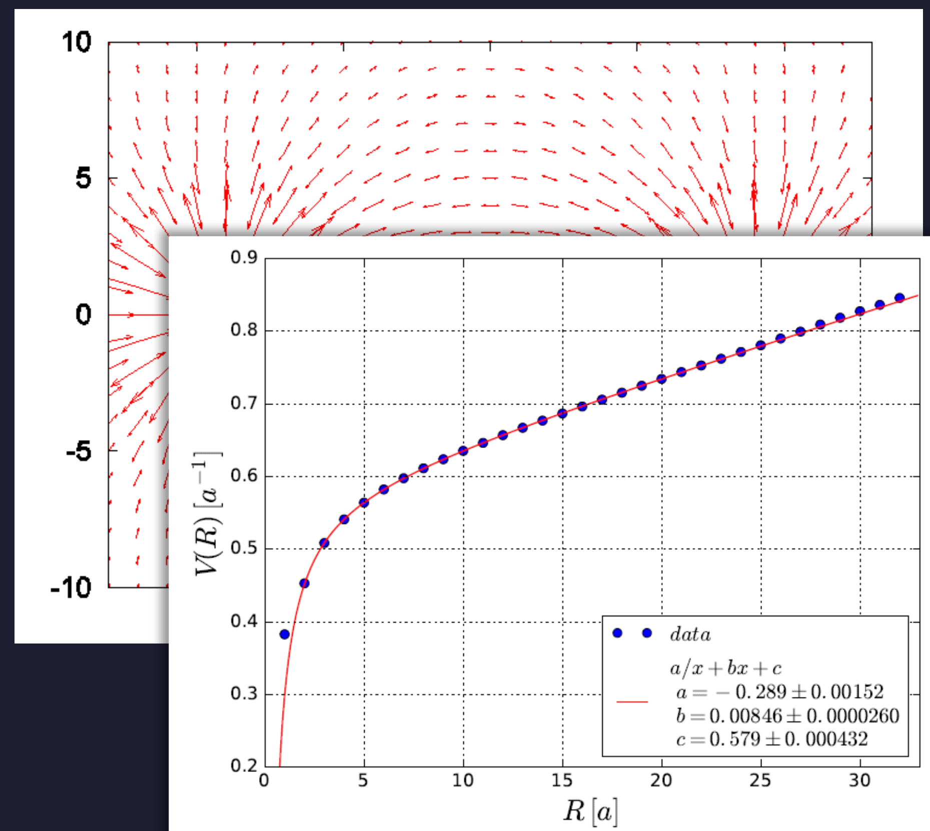
異なる電荷間の応力分布

クォーク・反クォーク系
＝Flux tubeの形成

QCD



QED



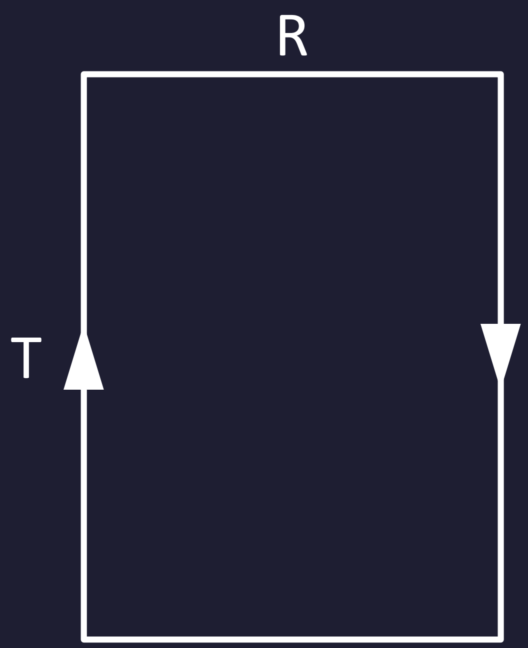
力線は、flux tube内に閉じ込められる

↓

クォーク・反クォークポテンシャルは
遠方で線形的に振る舞う。

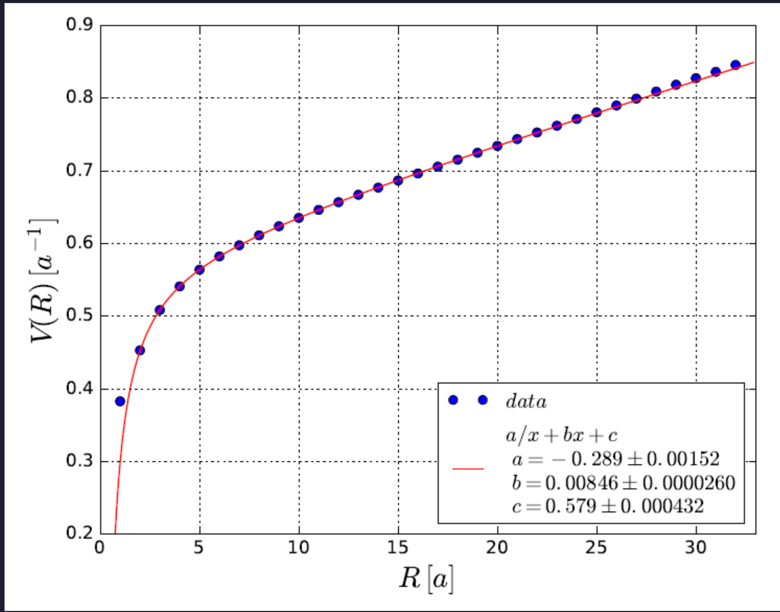
クォーク間ポテンシャル ←Wilson Loop

$$W(R,T) = \frac{1}{3}\mathrm{Tr}[U_0(x)U_0(x+\hat{0})\cdots]$$

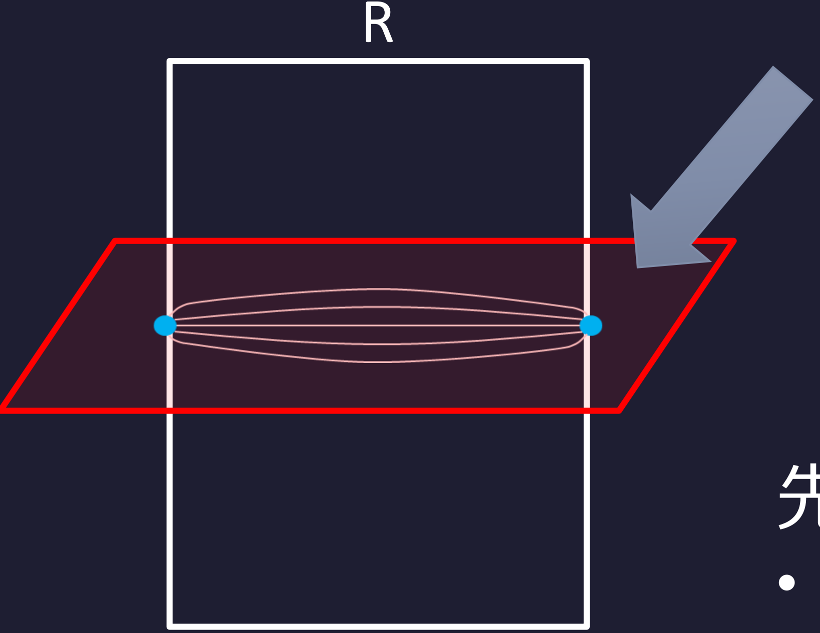


$$W(R,T) = C_0 e^{-V_0(R)T} + C_1 e^{-V_1(R)T} + \cdots$$

$$V(R) = \lim_{T\to\infty} \frac{1}{T} \log W(R,T)$$



Flux Tubeの構造



Wilson loopの中間時刻平面上で物理量を測定

$$\langle O(x)\rangle_W = \frac{\langle O(x)W(R,T)\rangle}{\langle W(R,T)\rangle} - \langle O(x)\rangle$$

先行研究

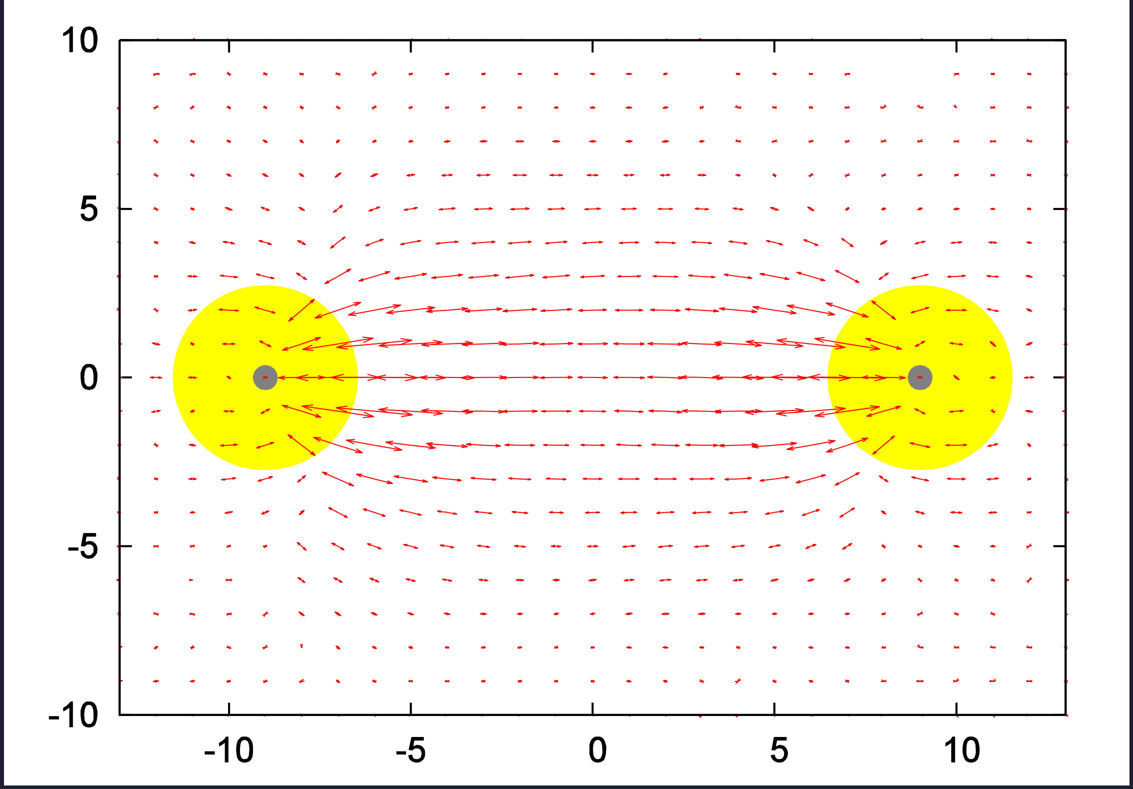
- 作用密度分布
- エネルギー密度分布（形状のみ）
- カラー電磁場（形状のみ）
- (カラー電磁場)^2（形状のみ）

$$\langle \vec{E}\rangle,\ \langle \mathrm{Tr}[\vec{E}^2]\rangle$$

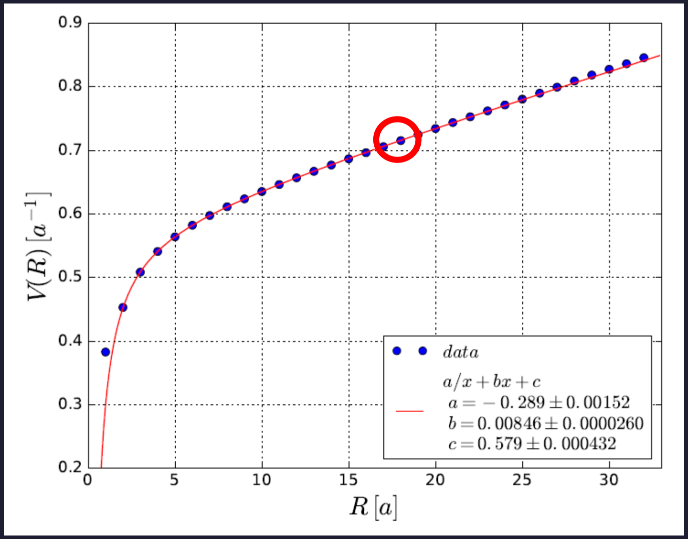
本研究：エネルギー密度・応力分布を絶対値を含めて測定

q q̄ 系の応力分布

beta=6.6
R/a=18
APE: 180 (alpha=2.3)
MF temporal link
t/a2=1.5 (no t→0 limit)

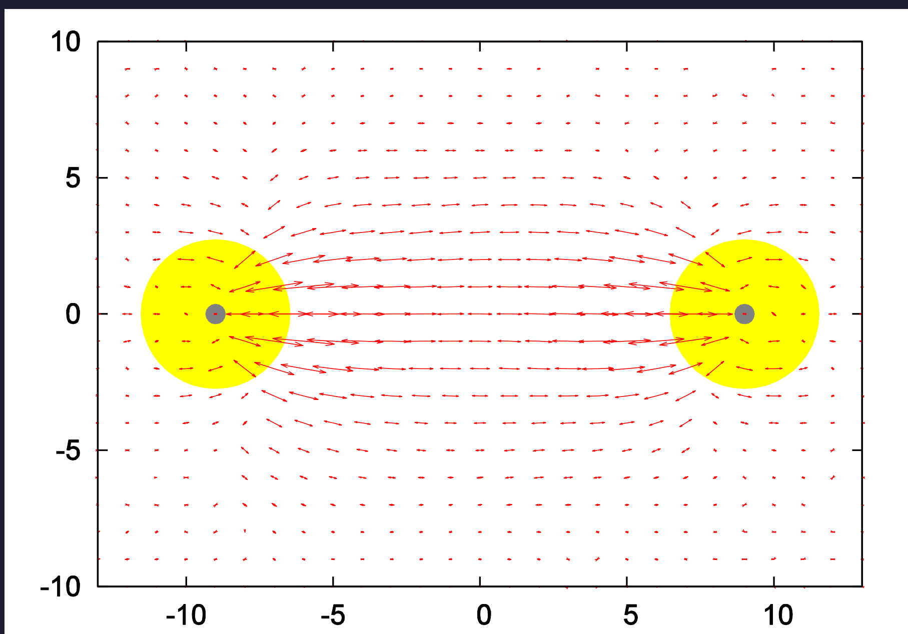


R=0.684fm

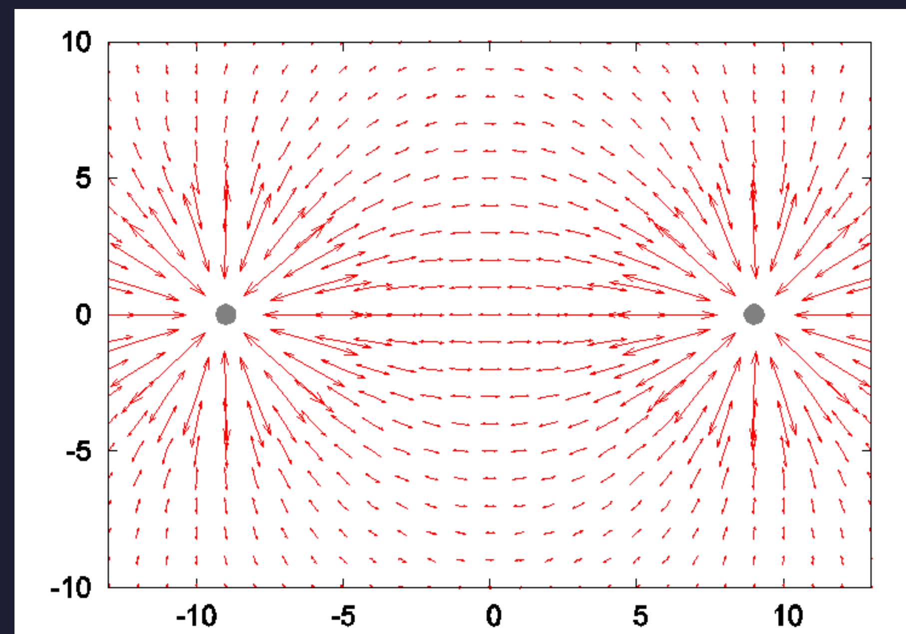


$q\bar{q}$系の応力分布

QCD



QED

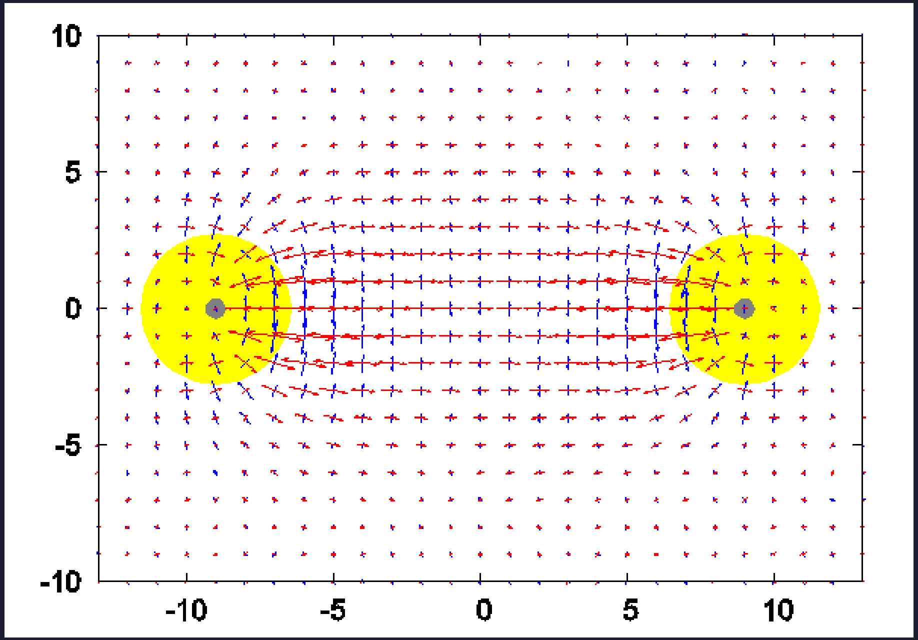


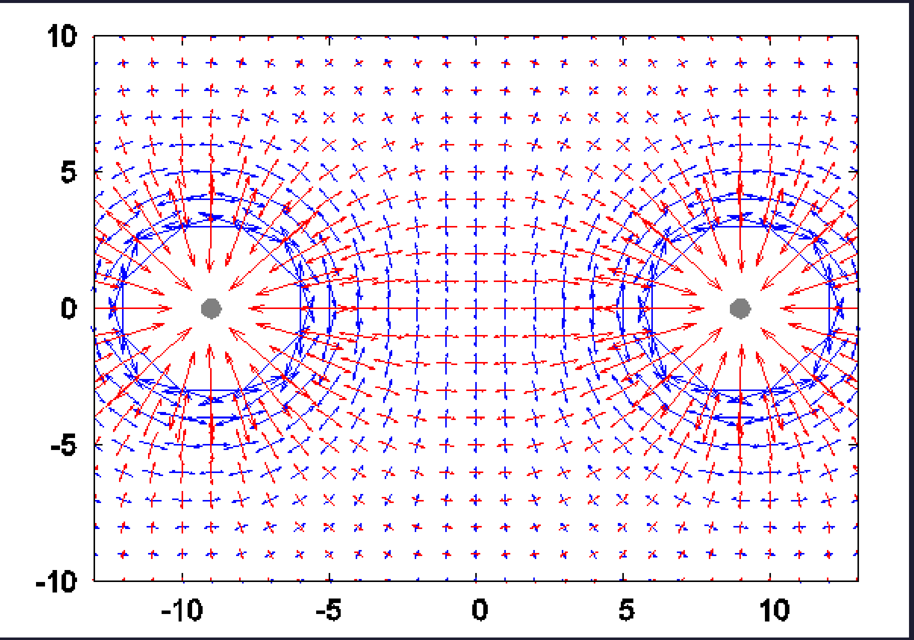
R=0.684fm

$q\bar{q}$ 系の応力分布

QCD

QED





R=0.684fm

まとめ

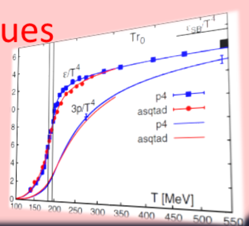
- 勾配流とは、場を連続的ににじませる変換。
- 格子QCD数値解析上の多様な問題に応用されている。

- 勾配流を用いると、格子上でEMTを定義できる。
- この演算子を用いた、様々な解析が始まっている。

Thermodynamics

direct measurement of expectation values

$\langle T_{00}\rangle, \langle T_{ii}\rangle$

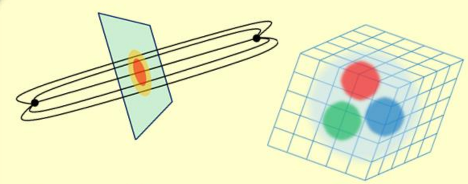


Fluctuations and Correlations

viscosity, specific heat, ...

$\eta = \int_0^\infty dt \langle T_{12}; T_{12}\rangle$

$c_V \sim \langle \delta T_{00}^2 \rangle$

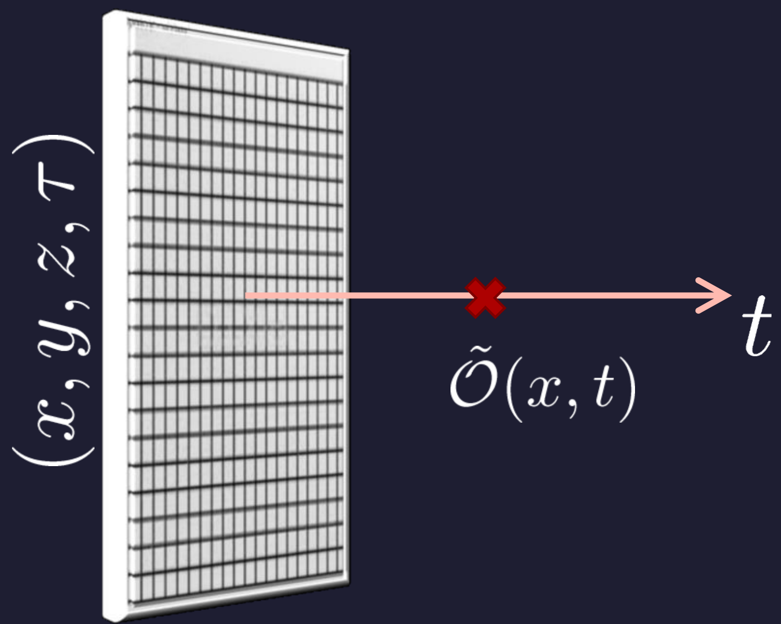


- ➢ confinement string
- ➢ EM distribution in hadrons

Hadron Structure

今後の課題：輸送係数、ハドロン内部のEM/応力分布、
非閉じ込め相転移の検証、情報削減への利用、

(x,y,z,t,s) ５次元理論



$O(x)$ ⟺ $O(x,t)$

４次元理論と、有限tの関係が確立

Luscher, Weiss, 2011

トポロジカル電荷