Critical Enhancement and Diffusion of Non-Gaussian Fluctuations

Masakiyo Kitazawa

(Osaka U.)

Asakawa, MK, Prog. Part. Nucl. Phys. 90, 299 (2016)

MK, Luo, Phys. Rev. D95 (2017)

Nonaka, MK, Esumi, Phys. Rev. C95 (2017)

Sakaida, Asakawa, Fujii, MK, Phys. Rev. C95, 064905 (2017)

Ohnishi, MK, Asakawa, Phys. Rev. C94, 044905 (2016)

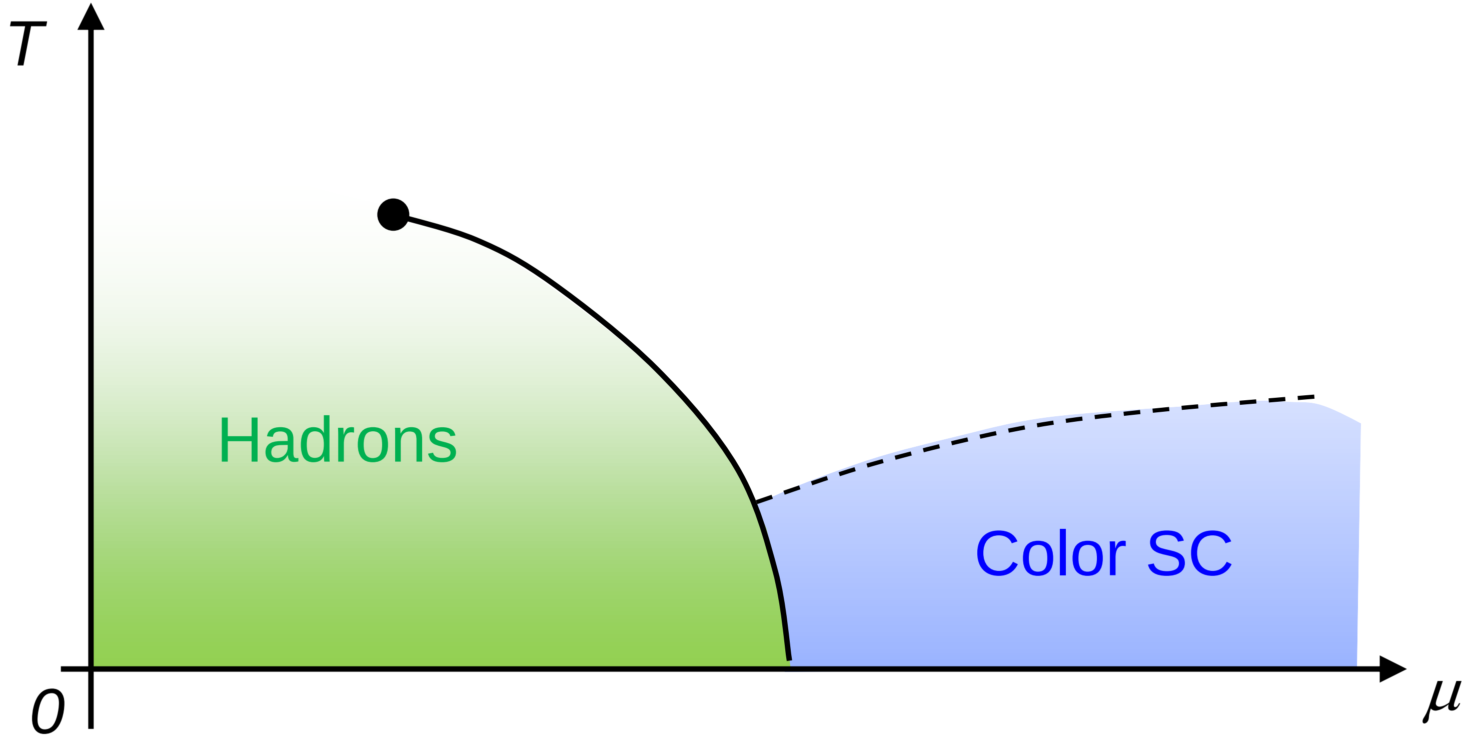
MK, Nucl. Phys. A942, 65 (2015)

Sakaida, Asakawa, MK, Phys. Rev. C90, 064901 (2014)

MK, Asakawa, Ono, Phys. Lett. B728, 386 (2014)

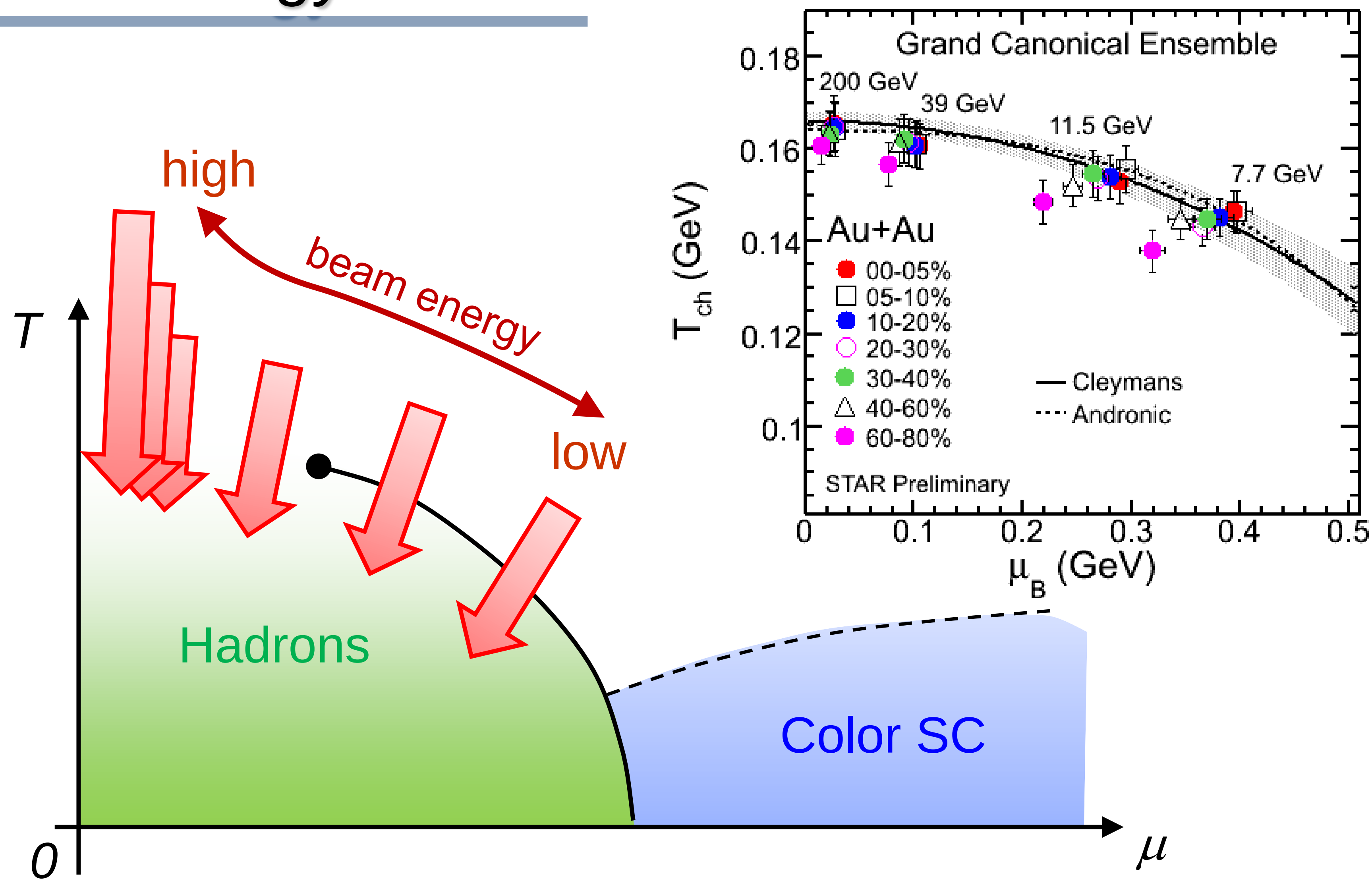
EMMI workshop on Critical Fluctuations, Oct. 10-13, 2017, CCNU, China

Beam-Energy Scan



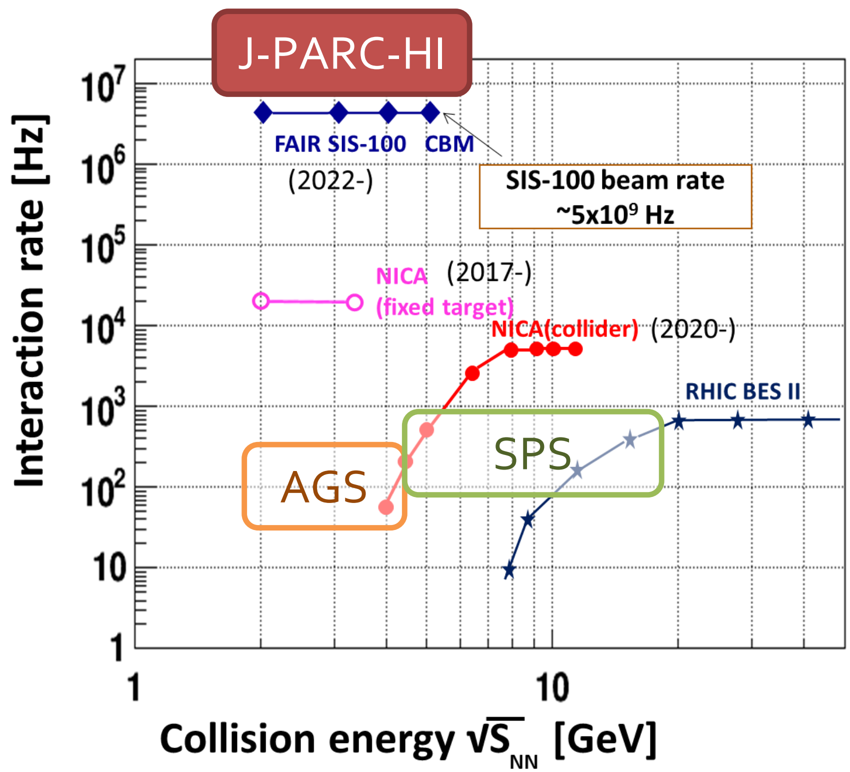
Beam-Energy Scan

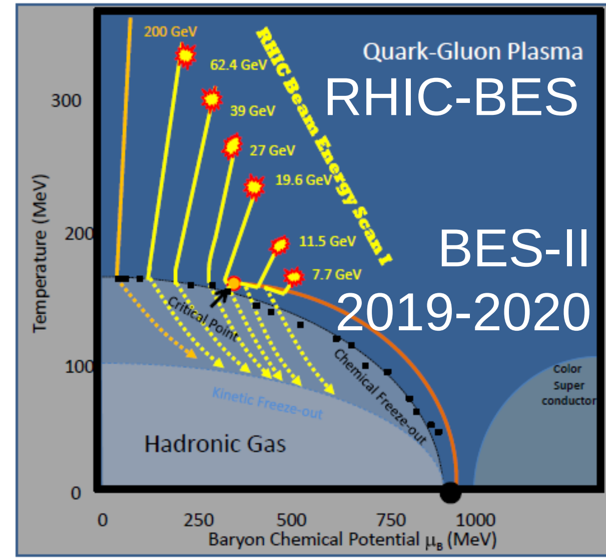
STAR 2012

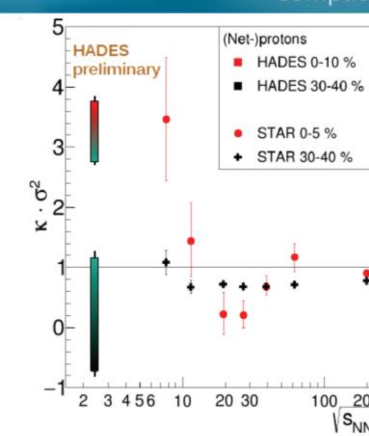


Beam-Energy Scan

Active experimental researches/plans for the beam-energy scan





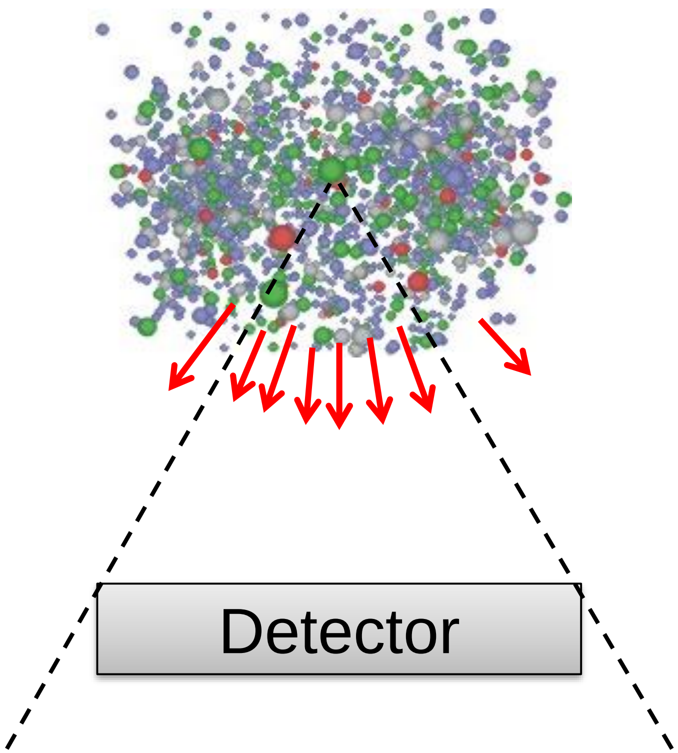


Search for QCD phase structure / critical point

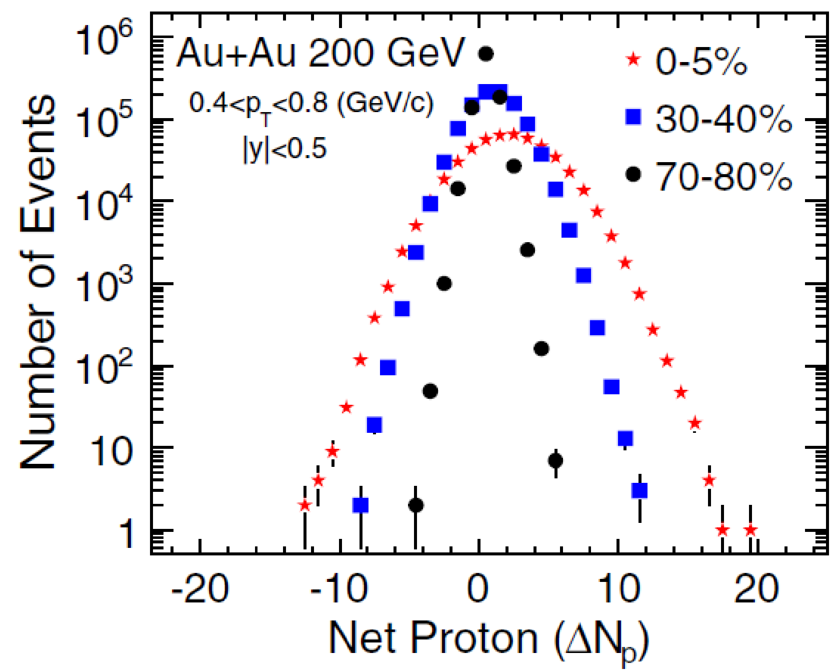
Event-by-Event Fluctuations

Review: Asakawa, MK, PPNP **90** (2016)

Fluctuations can be measured by e-by-e analysis in experiments.



STAR, PRL**105** (2010)

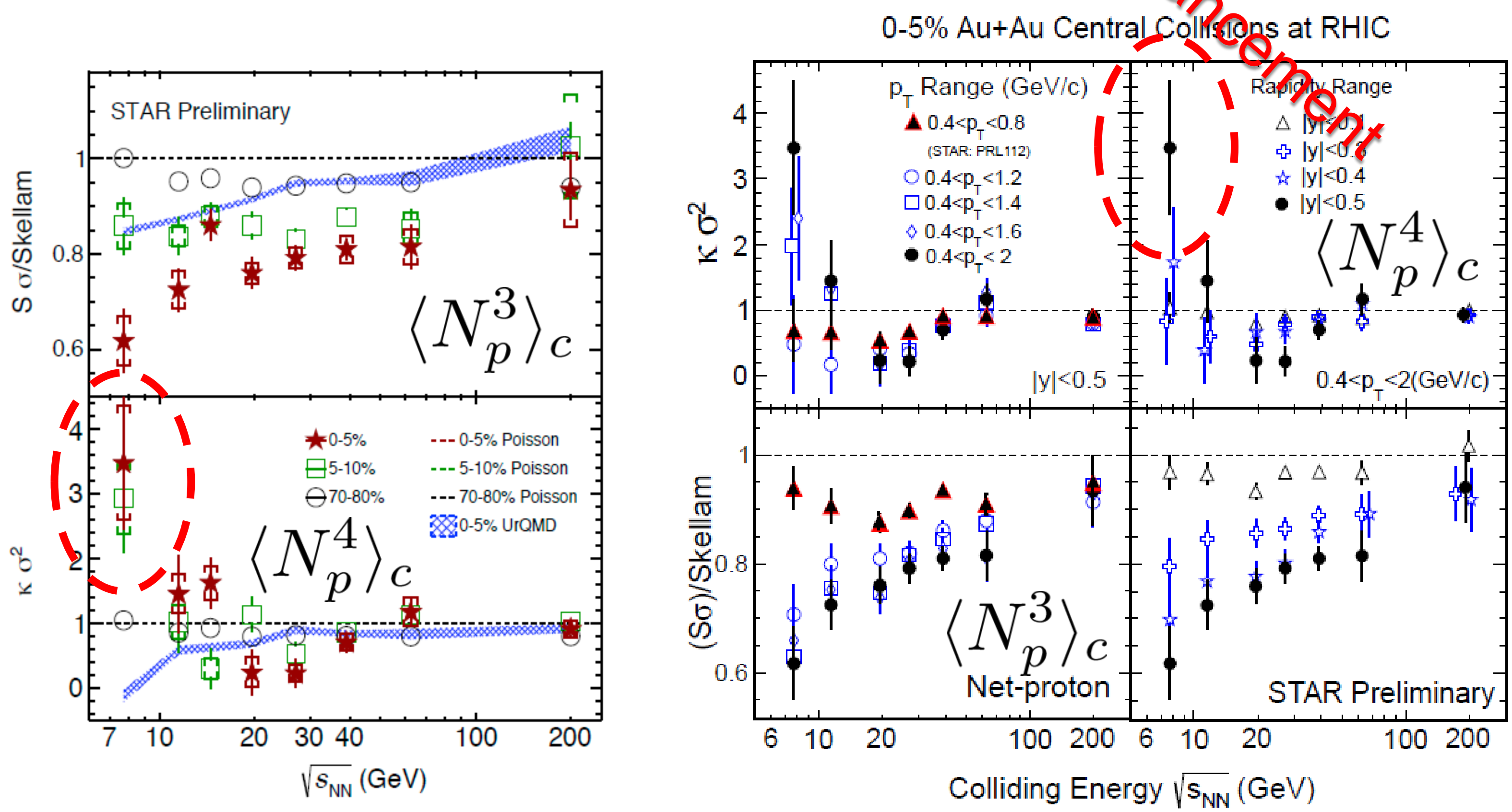


Cumulants

$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$

Higher-Order Cumulants

STAR Collab.
2010~

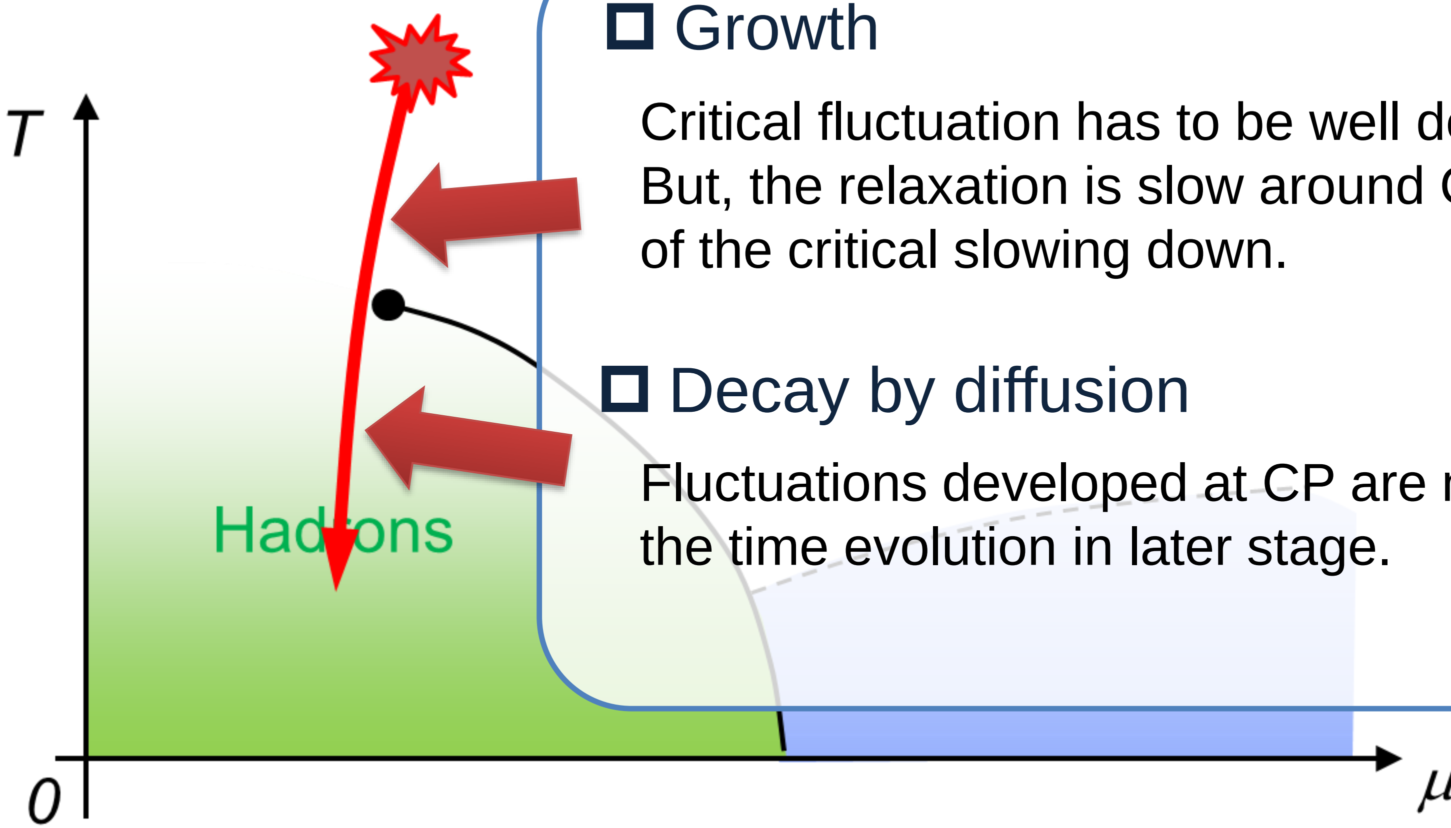


Non-zero **non**-Gaussian and **non**-Poissonian cumulants have been established!

Have we measured critical fluctuations?

Remark on Critical Fluctuation

Experiments cannot observe critical fluctuation in equilibrium directly.



☐ Growth

Critical fluctuation has to be well developed. But, the relaxation is slow around CP because of the critical slowing down.

☐ Decay by diffusion

Fluctuations developed at CP are modified by the time evolution in later stage.

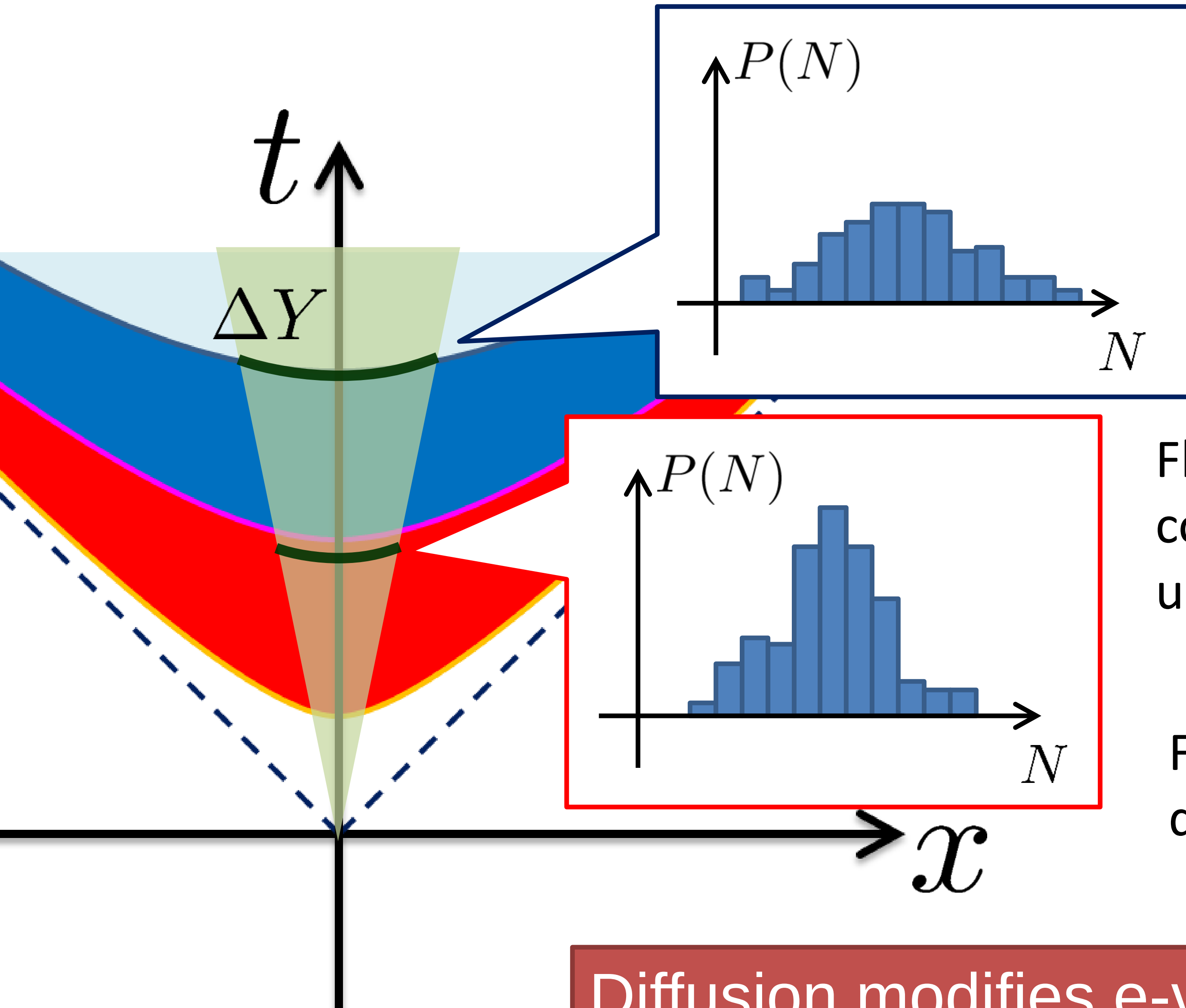
Three Topics

1. Diffusion of fluctuations
2. Analysis of fluctuations with factorial cumulants
3. Critical enhancement and diffusion

Diffusion of Fluctuations

Ohnishi, MK, Asakawa, Phys. Rev. C94, 044905 (2016)
MK, Nucl. Phys. A942, 65 (2015)
MK, Asakawa, Ono, Phys. Lett. B728, 386 (2014)

Diffusion of Fluctuations

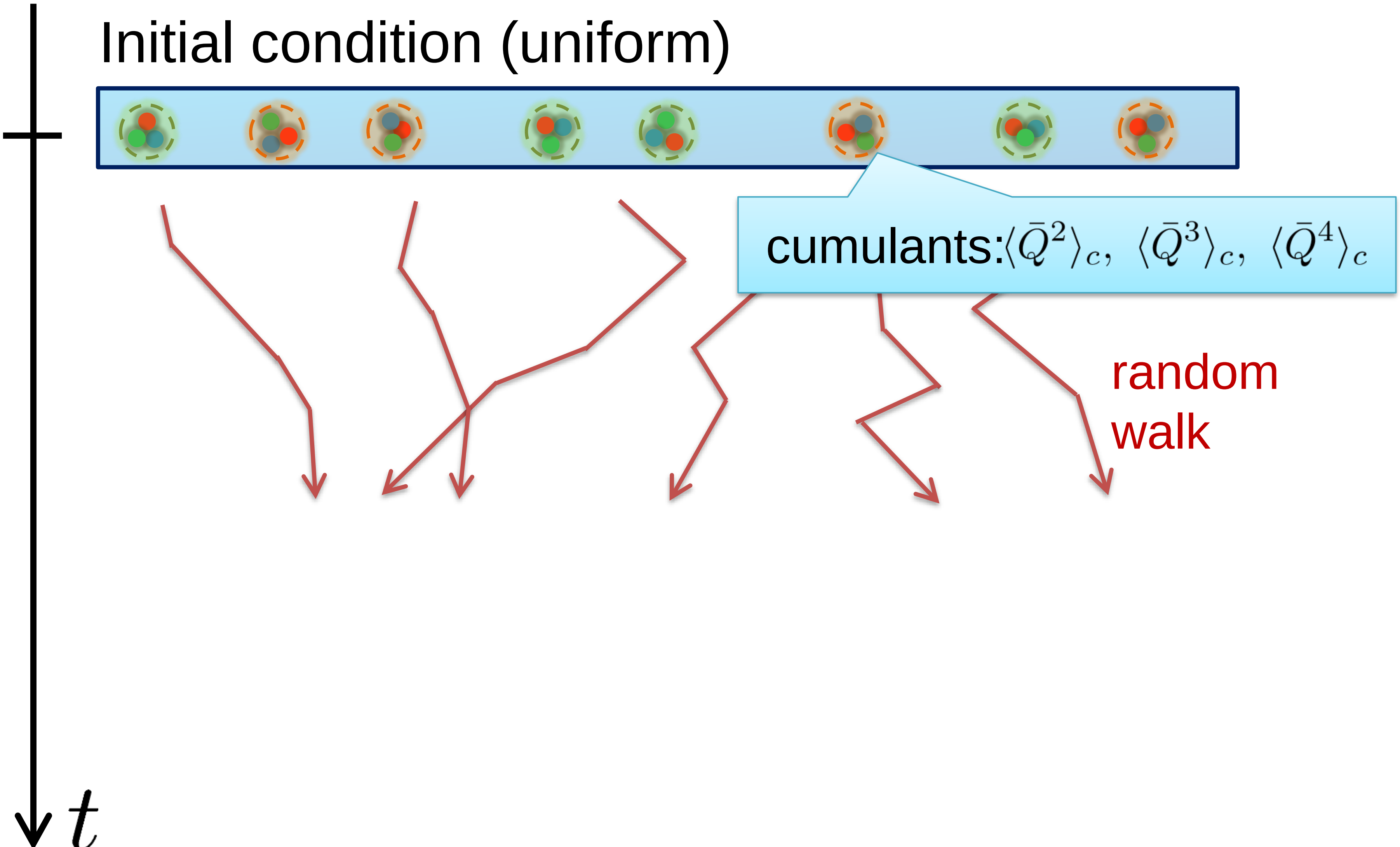


Fluctuations in ΔY continue to change until kinetic f.o.

Fluctuations are ΔY dependent

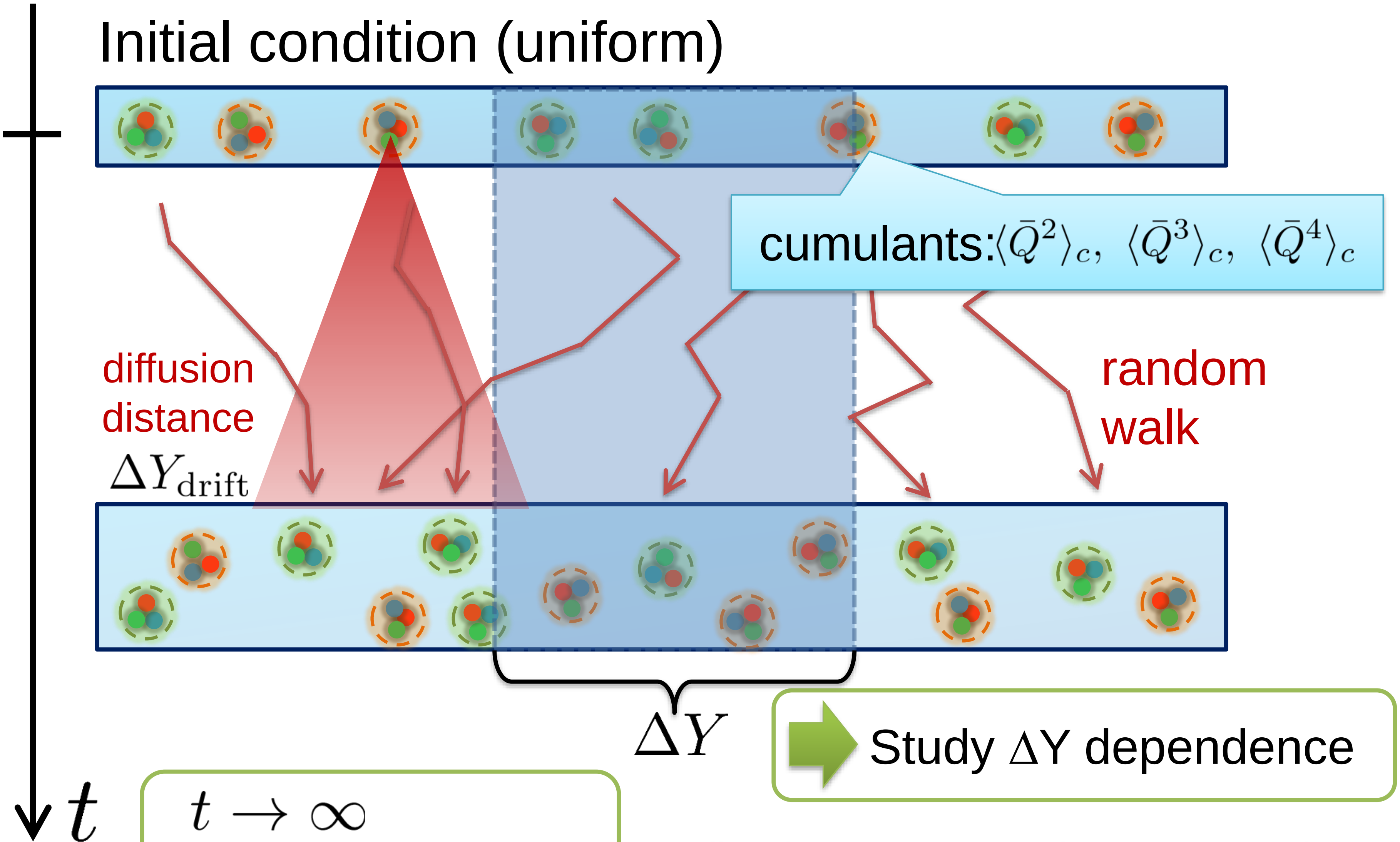
Diffusion modifies e-v-e fluctuations

(Non-Interacting) Brownian Particle Model



diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

(Non-Interacting) Brownian Particle Model

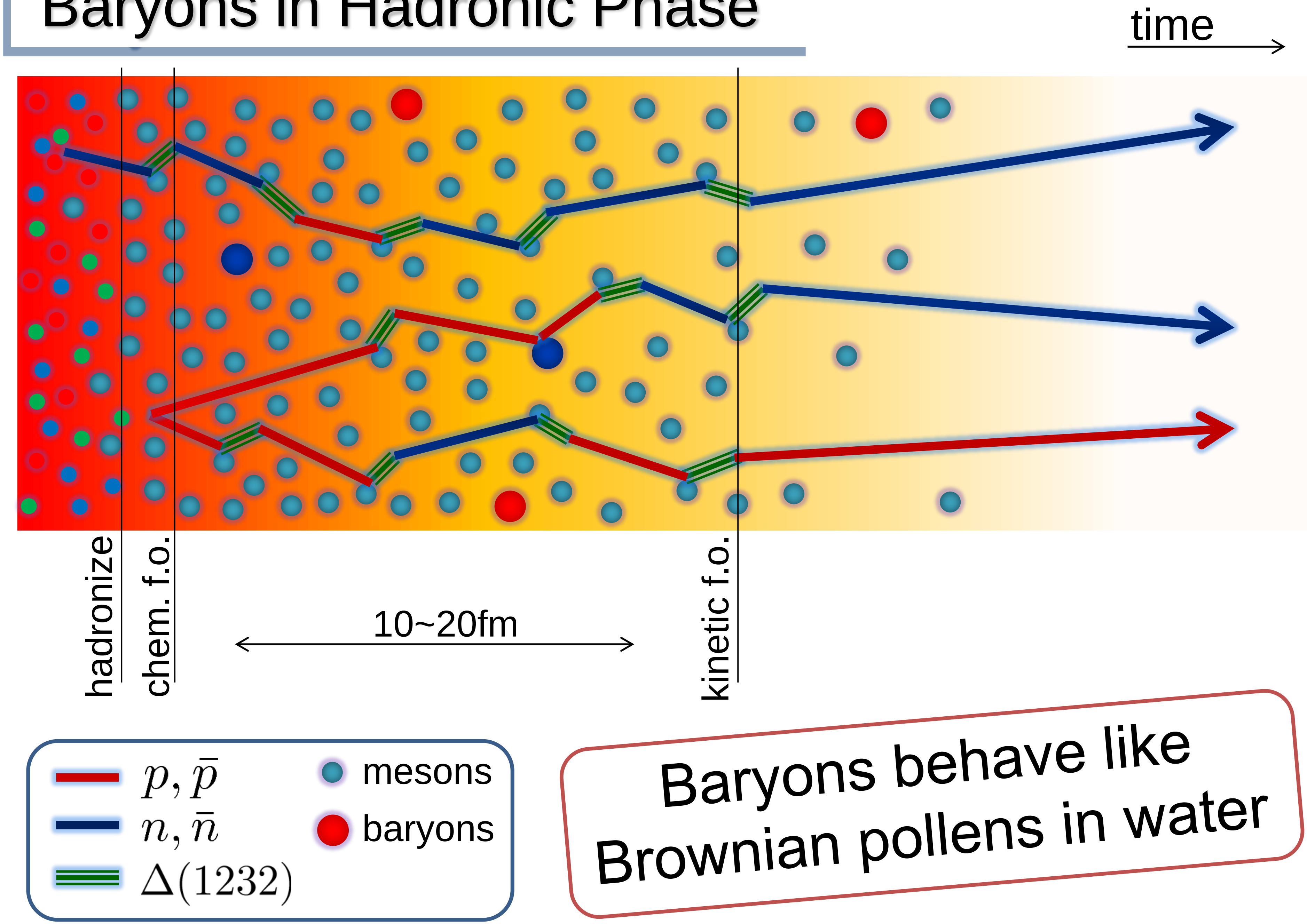


Study ΔY dependence

$t \to \infty$
Poisson distribution

diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

Baryons in Hadronic Phase



Baryons behave like Brownian pollens in water

Rapidity Window Dep.

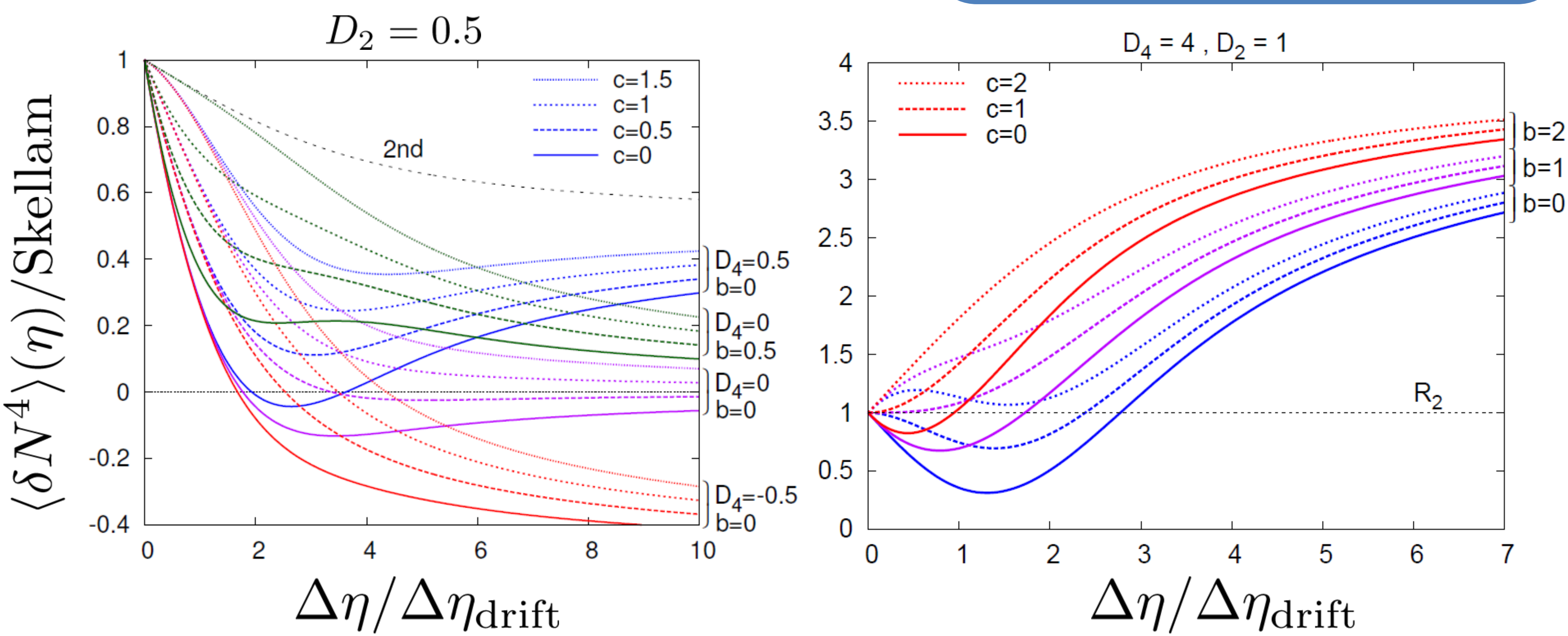
4th-order cumulant

MK+, 2014
MK, 2015

Initial Conditions

$$D_4 = \frac{\langle Q_{(\mathrm{net})}^4 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} \qquad b = \frac{\langle Q_{(\mathrm{net})}^2 Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$D_2 = \frac{\langle Q_{(\mathrm{net})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} \qquad c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$



- Different initial conditions give rise to different characteristic Δη dependence.
- Non-monotonic behaviors can appear in Δη dependence.

Finite volume effects: Sakaida+, PRC90 (2015)

Rapidity Window Dep.

4^{th}-order cumulant

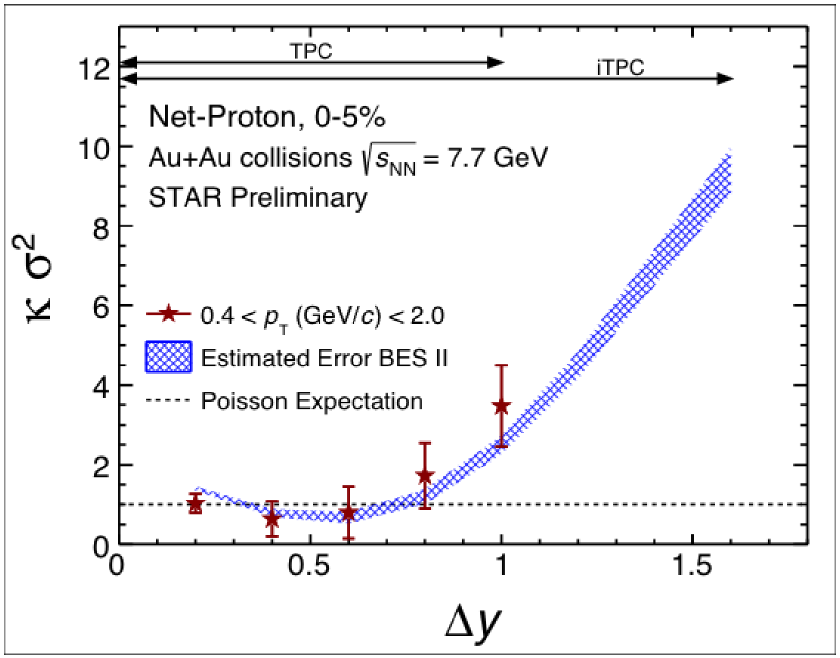
MK+, 2014
MK, 2015

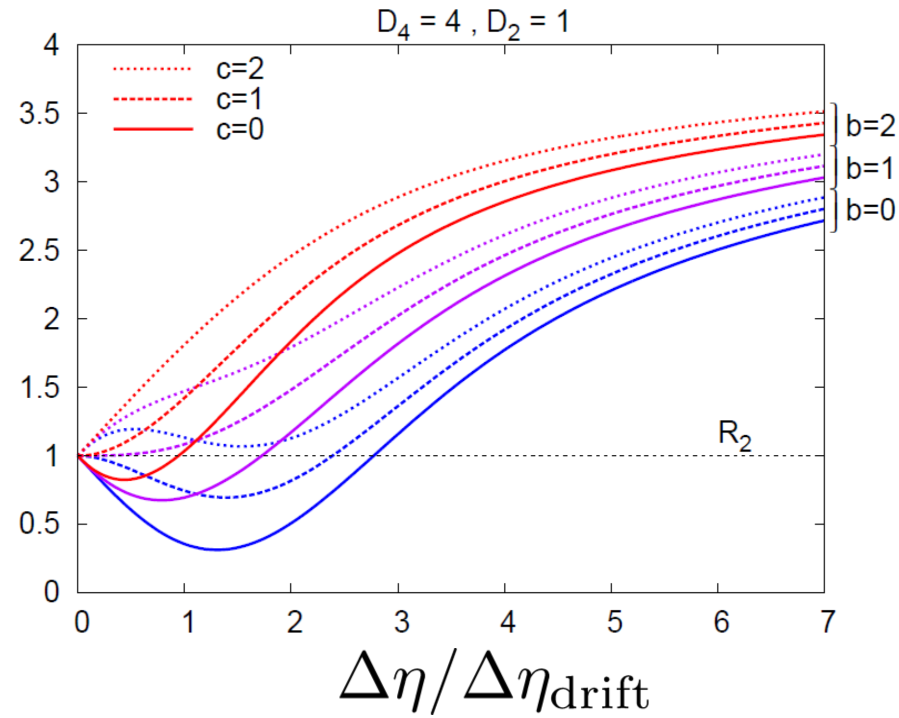
Initial Conditions

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad c = \frac{\langle Q^2_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

STAR Collab. (X. Luo, CPOD2014)





- Different initial conditions give rise to different characteristic Δη dependence.
- Non-monotonic behaviors can appear in Δη dependence.

Finite volume effects: Sakaida+, PRC90 (2015)

Analysis of Fluctuations with Factorial Cumulants

MK, Luo, Phys. Rev. C94, 044905 (2016)
Nonaka, MK, Esumi, Phys. Rev. C95, (2016)

Definition of Terminology

- Cumulants

$$\langle N \rangle_\mathrm{c} = \langle N \rangle, \quad \langle N^2 \rangle_\mathrm{c} = \langle \delta N^2 \rangle, \quad \langle N^3 \rangle_\mathrm{c} = \langle \delta N^3 \rangle,$$

$$\langle N^4 \rangle_\mathrm{c} = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2,$$

$$K(\theta) \equiv \ln \langle e^{N\theta} \rangle = \sum_m \frac{\theta^m}{m!} \langle N^m \rangle_\mathrm{c}$$

- Factorial Cumulants

$$\langle N \rangle_\mathrm{c} = \langle N \rangle, \quad \langle N^2 \rangle_\mathrm{fc} = \langle N(N-1) \rangle_\mathrm{c},$$

$$\langle N^3 \rangle_\mathrm{fc} = \langle N(N-1)(N-2) \rangle_\mathrm{c},$$

$$K_\mathrm{f}(s) \equiv \ln \langle (1+s)^N \rangle = \sum_m \frac{s^m}{m!} \langle N^m \rangle_\mathrm{fc}$$

- Correlation Functions

$$\langle n(x)n(y) \rangle_\mathrm{c}, \quad \langle n(x)n(y)n(z) \rangle_\mathrm{c},$$

Cumulants and Correlation Function

$$Q = \int_V dx n(x)$$

total charge

charge density

$$\langle \delta Q^2 \rangle = \int_V dxdy \langle \delta n(x) \delta n(y) \rangle$$

2nd order cumulant (fluctuation)

correlation function

1-to-1 correspondence

1-dim case

$$\langle \delta Q^2 \rangle_{\Delta y} = \int_{\Delta y} dy (\Delta y - |y|) \langle \delta n(y) \delta n(0) \rangle$$

Factorial Cumulants

Ling, Stephanov, 2016
Bzdak, Koch, 2017

= Cumulants after removing "self-correlation"

Correlation function

$$C(x,y) = \langle \delta n(x) \delta n(y) \rangle$$

$$C(x,y) = C^*(x,y) + \langle n(x) \rangle \delta(x-y)$$

Cumulant

$$\langle Q^2 \rangle_{\rm c} = \int_\Delta dx dy C(x,y)$$

Factorial cumulant

$$\langle Q^2 \rangle_{\rm fc} = \int_\Delta dx dy C^*(x,y)$$

Caveats:

- This interpretation is valid only for classical particle systems.
- When quasi-particles are lost, f-cumulants are not well-defined.
- Values of f-cumulants are modified by inelastic scatterings.

⇔ Cumulants of conserved charges have desirable properties.

Usage of F-Cumulants

Suggestion

F-cumulants play useful roles in revealing underlying physics described (approximately) by the binomial model.

independent probabilistic events

Binomial Model

$$P_{\rm obs}(n) = \sum_N B_p(n;N)P(N)$$

"observed" distribution

"original" distribution

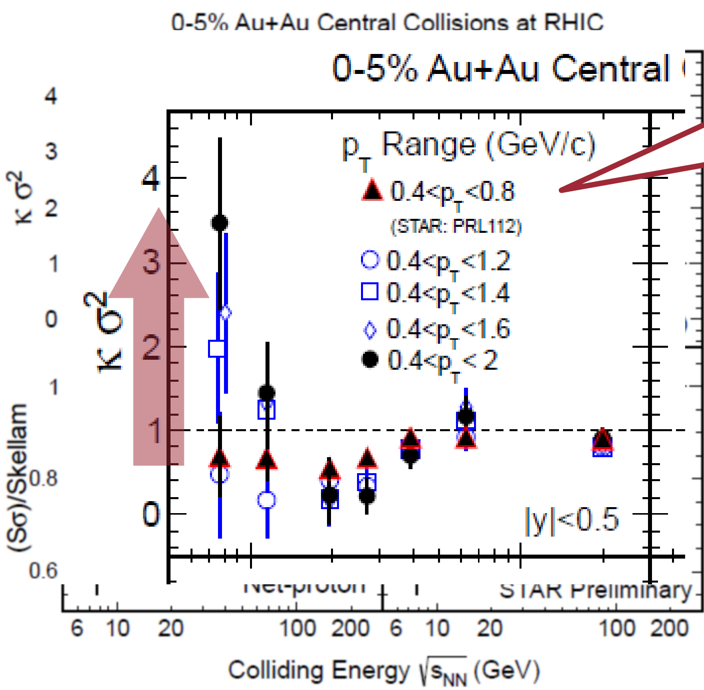
F-cumulants

$$\langle n^m \rangle_{\rm fc} = \langle p^m N^m \rangle_{\rm fc}$$

Ex: efficiency correction
Nonaka, MK, Esumi, 2017

p_T-cut Dependence

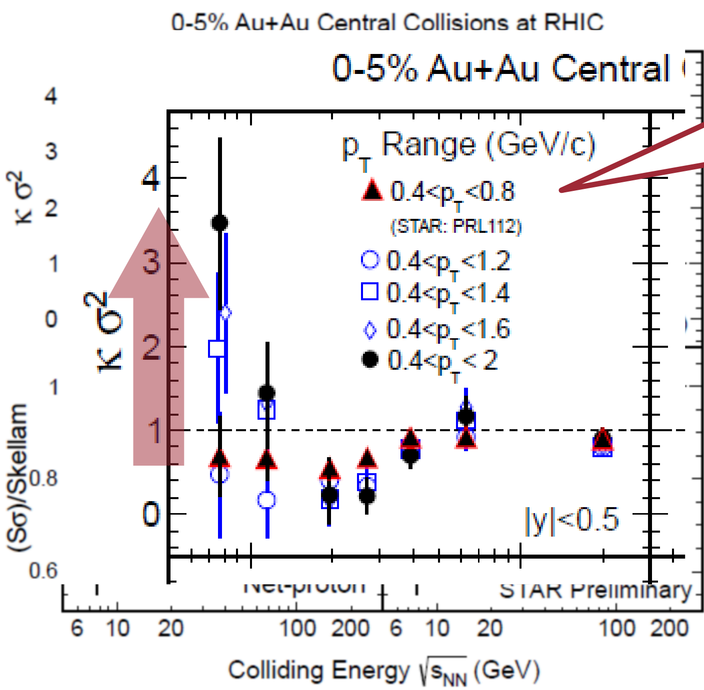
MK, Luo, PRC, 2017



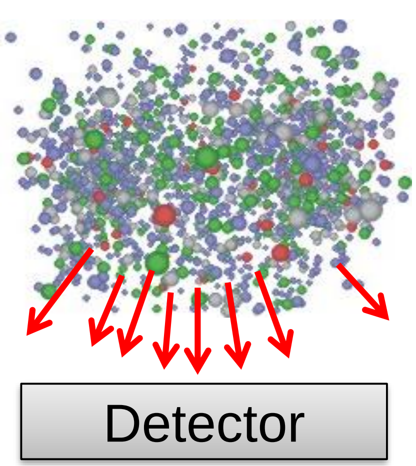
Enhancement of cumulant with increasing p_T acceptance

p_T-cut Dependence

MK, Luo, PRC, 2017



Enhancement of cumulant with increasing p_T acceptance



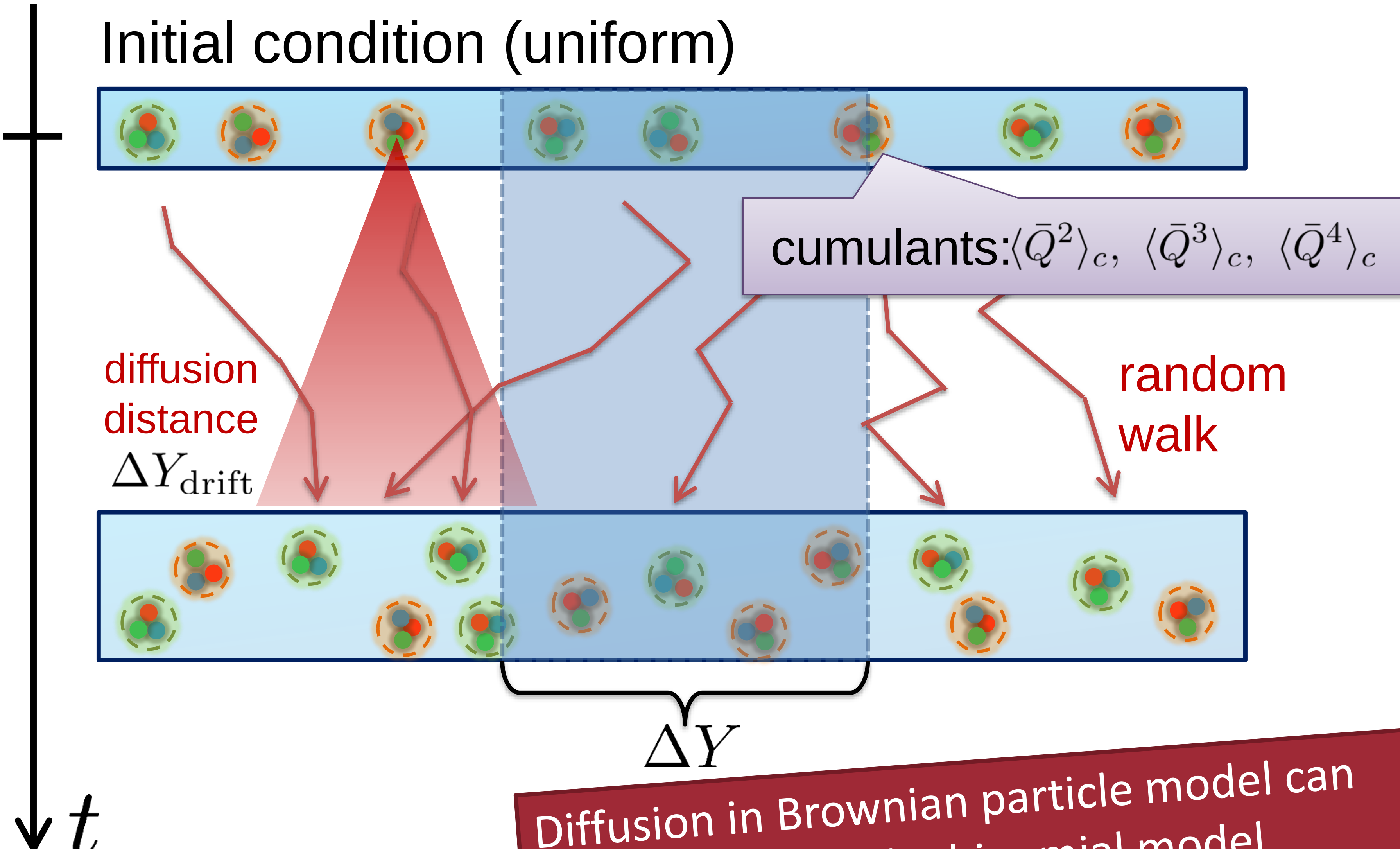
Assumption:
independent and random p_T distribution in the final state

Power-law behavior of f-cum.

$$\frac{\langle n^m \bar{n}^{\bar{m}} \rangle_{\mathrm{fc},p_T}}{\langle n \rangle_{p_T}^m \langle \bar{n} \rangle_{p_T}^{\bar{m}}} = \frac{\langle N^m \bar{N}^{\bar{m}} \rangle_{\mathrm{fc}}}{\langle N \rangle^m \langle \bar{N} \rangle^{\bar{m}}}$$

- ✓ Study particle emission mechanism
- ✓ Construction of cumulants

Non-Interacting Brownian Particle System



Diffusion in Brownian particle model can be described by the binomial model.

Δη Dependences of F-Cumulants

MK, Luo, PRC, 2017

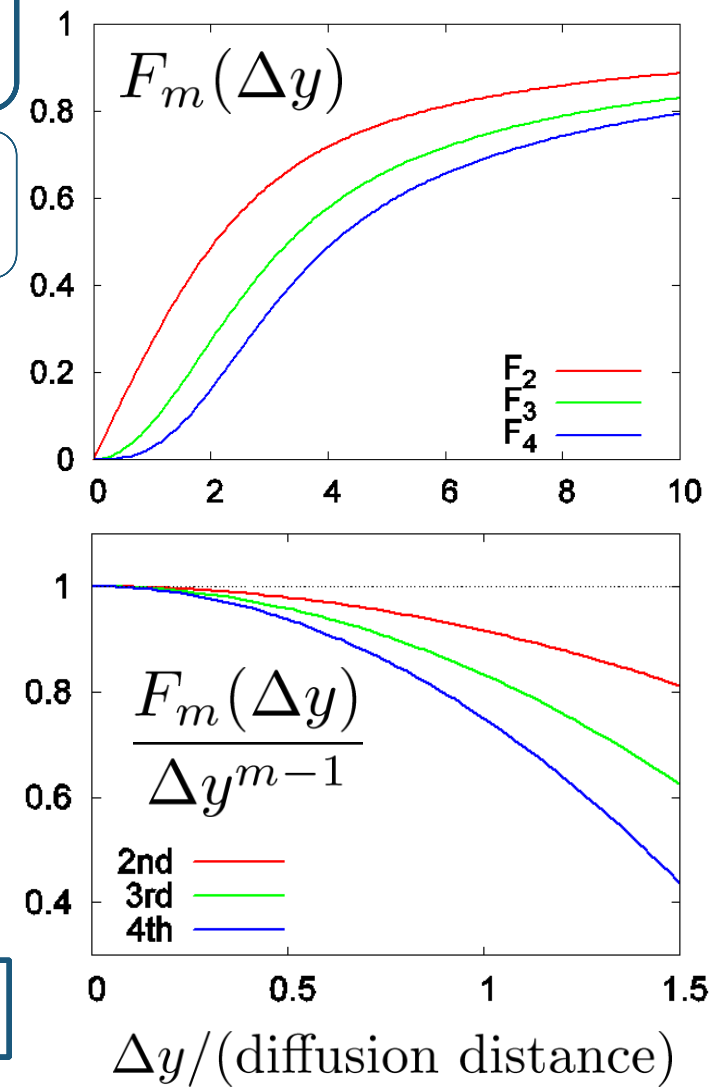
$$\langle n_p^m n_{\bar{p}}^{\bar{m}} \rangle_{\rm fc} = \kappa_{m\bar{m}} \Delta y F_{m+\bar{m}}(\Delta y/d)$$

f-cumulant at initial cond.

diffusion distance

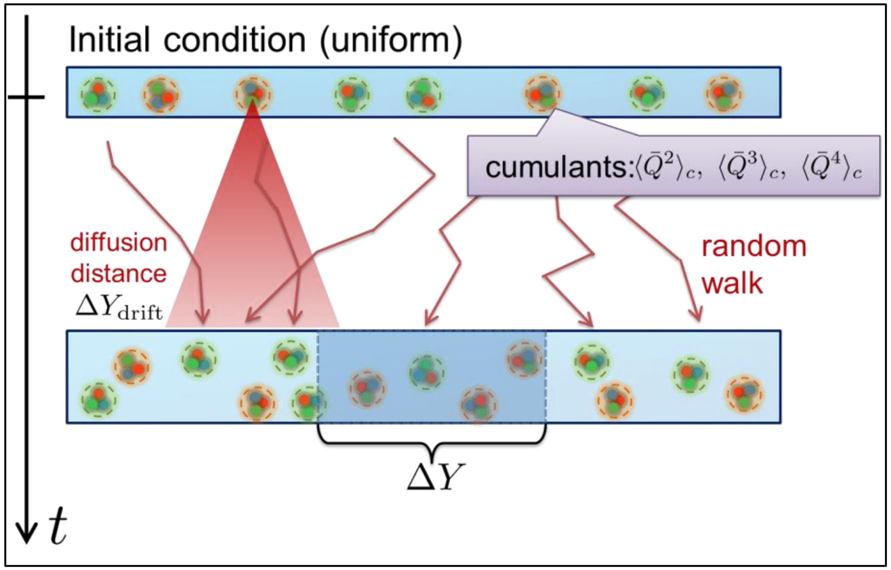
- All f-cumulants at the same order have the same Δy dependence up to normalization.
- f-cumulants and diffusion distance can be inferred.

reconstruct cumulants of conserved charges

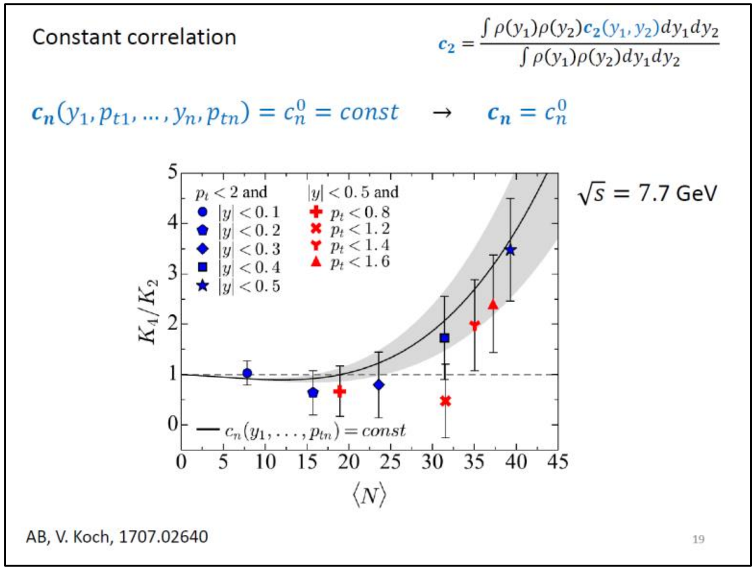


Translating Languages

Brownian particle model



From Bzdak's talk



$$\langle n^m \bar{n}^{\bar{m}} \rangle_{\mathrm{fc}} = \kappa_{m\bar{m}} \Delta y F_{m+\bar{m}}(\Delta y/d)$$

$$c^0_{m\bar{m}} = \frac{1}{2}\frac{\partial^2}{\partial \Delta y^2} \langle n^m \bar{n}^{\bar{m}} \rangle_{\mathrm{fc}} \Big|_{\Delta y \to 0} = \kappa_{m\bar{m}} \frac{\partial}{\partial \Delta y} F_{m+\bar{m}}(\Delta y/d) \Big|_{\Delta y \to 0}$$

$$= \frac{\kappa_{m\bar{m}}}{d} \frac{1}{\sqrt{(m+\bar{m})(2\pi)^{m+\bar{m}-1}}}$$

$\kappa_{m\bar{m}}$: F cumulants at initial condition

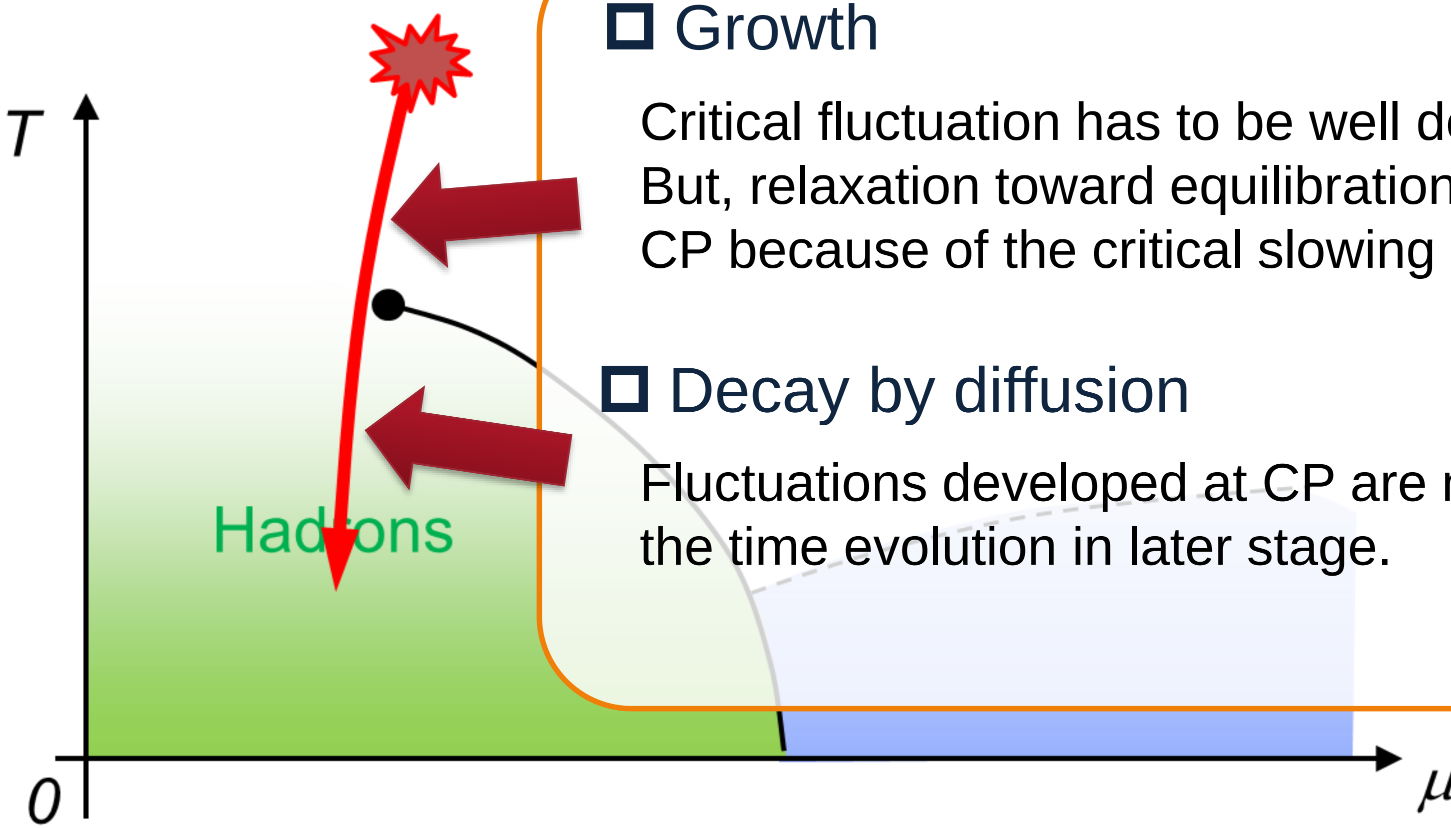
d : diffusion distance

Critical Enhancement and Diffusion

Sakaida, Asakawa, Fujii, MK, Phys. Rev. C95, 064905 (2017)

Remarks on Critical Fluctuation 1

Experiments cannot observe critical fluctuation in equilibrium directly.



☐ Growth

Critical fluctuation has to be well developed.
But, relaxation toward equilibration is slow around CP because of the critical slowing down.

☐ Decay by diffusion

Fluctuations developed at CP are modified by the time evolution in later stage.

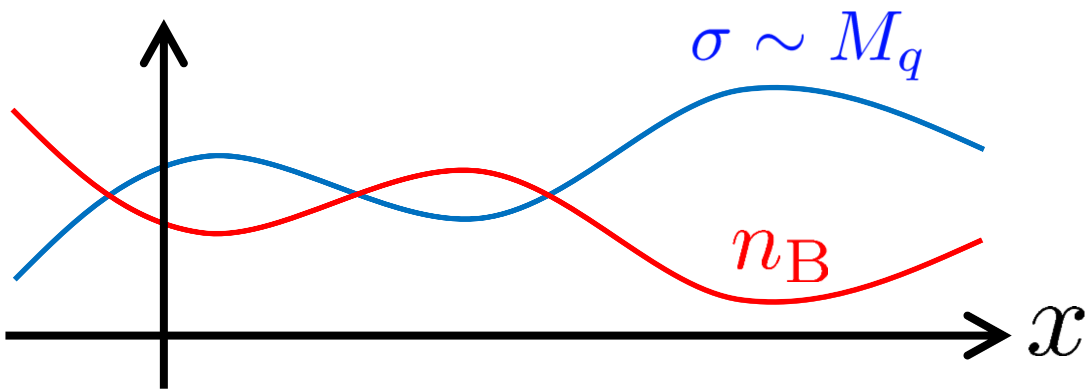
Remarks on Critical Fluctuation 2

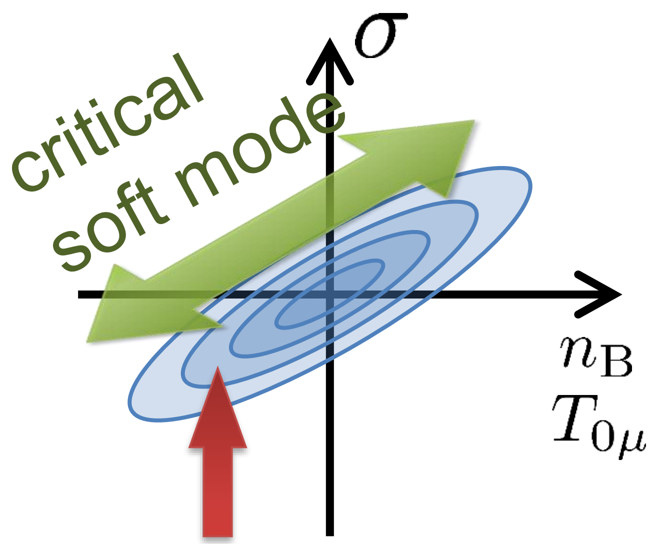
Critical fluctuation is a conserved mode!

Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

Fluctuations of σ and n_B are coupled around the CP!

$$\delta\sigma \simeq \delta n_B$$





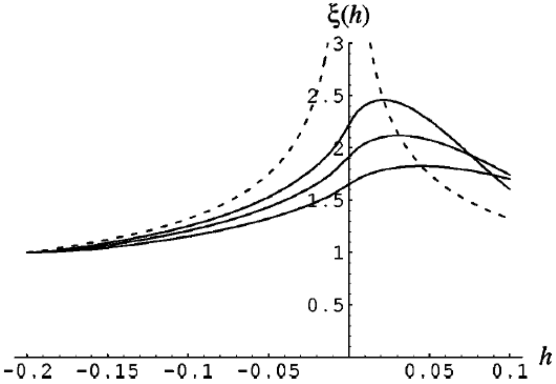
σ: fast damping

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \cdots$$

Dynamical Evolution of Critical Fluctuations

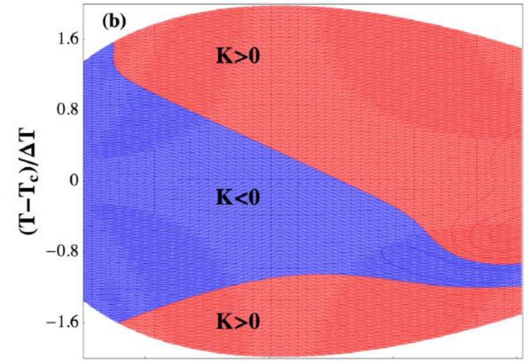
- Evolution of correlation length

  Berdnikov, Rajagopal (2000)
  Asakawa, Nonaka (2002)



- Higher orders (spatially uniform “σ” mode)

  Mukherjee, Venugopalan, Yin (2015)



- Dynamical evolution in chiral fluid model

  Nahrgang, Herold, … (2014~)

- Correlation functions

  Kapusta, Torres-Rincon (2012)

Aim of This Study

- ☐ Describe **conserved nature** of critical fluctuation.
- ☐ We want to study **experimental observables**.
  - ☐ focus on a **conserved charge (baryon number)**
  - ☐ study evolution of **conserved-charge** fluctuation
- ☐ Concentrate on **2^{nd} order** fluctuation. (not higher)
- ☐ We study
  - ☐ **rapidity window denepdence** of the cumulant
  - ☐ 2-particle **correlation function**

Our Main Conclusion

Non-monotonicity in cumulants or correlation func.	=	Signal of QCD-CP

Stochastic Diffusion Equation (SDE)

Diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n$$

- Describe a relaxation of a conserved density *n* toward uniform state without fluctuation

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

$$\langle \xi(\eta_1)\xi(\eta_2)\rangle \sim \chi\delta(\eta_1-\eta_2)$$

- Describe a relaxation toward fluctuating uniform state
- χ: susceptibility (fluctuation in equil.)

Review: Asakawa, MK, PPNP **90** (2016)

Soft Mode of QCD Critical Point

Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

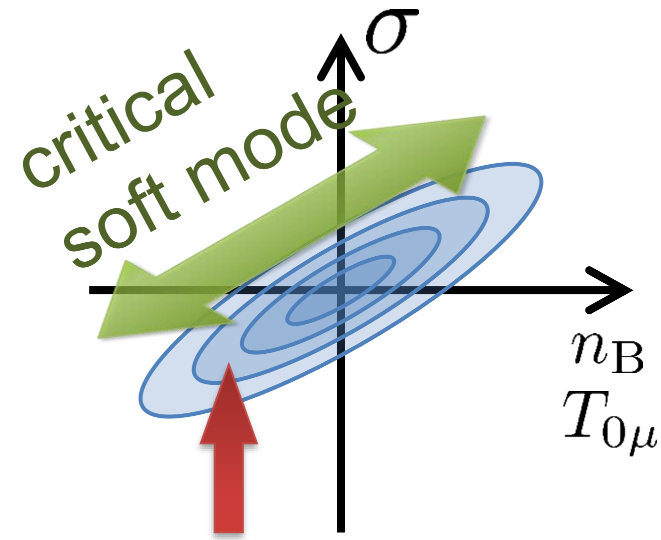
- Effective potential

$$F(\sigma,n) = A\sigma^2 + B\sigma n + Cn^2 + \cdots$$

- Time dependent Ginzburg-Landau

$$\begin{pmatrix} \dot\sigma \\ \dot n \end{pmatrix} = \begin{pmatrix} \Gamma_{\sigma\sigma} & \Gamma_{\sigma n} \\ \Gamma_{n\sigma} & \Gamma_{nn} \end{pmatrix} \begin{pmatrix} \sigma \\ n \end{pmatrix}$$

$$\sim k^2$$



σ: fast damping

For slow and long wavelength,

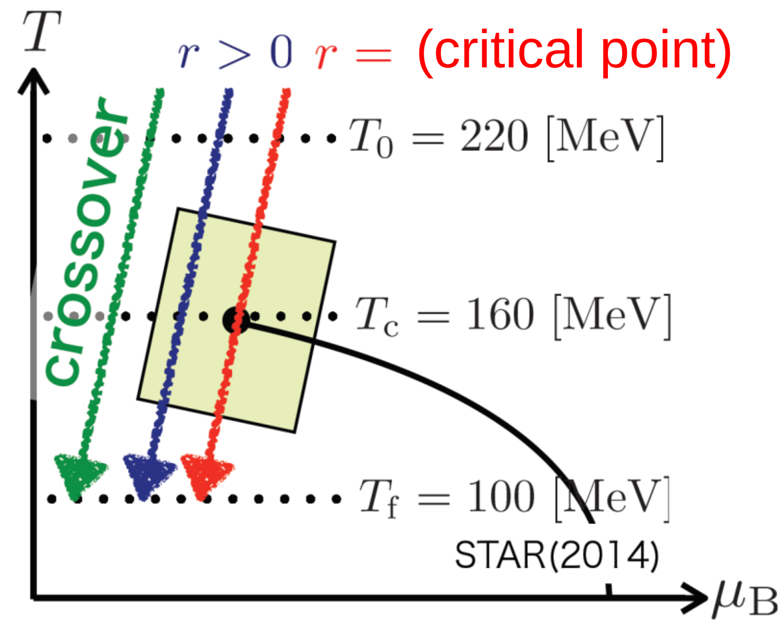
SDE $\partial_\tau n = D(\tau)\partial_\eta^2 n + \partial_\eta \xi$

singularities in $D(\tau)$ and $\chi(\tau)$

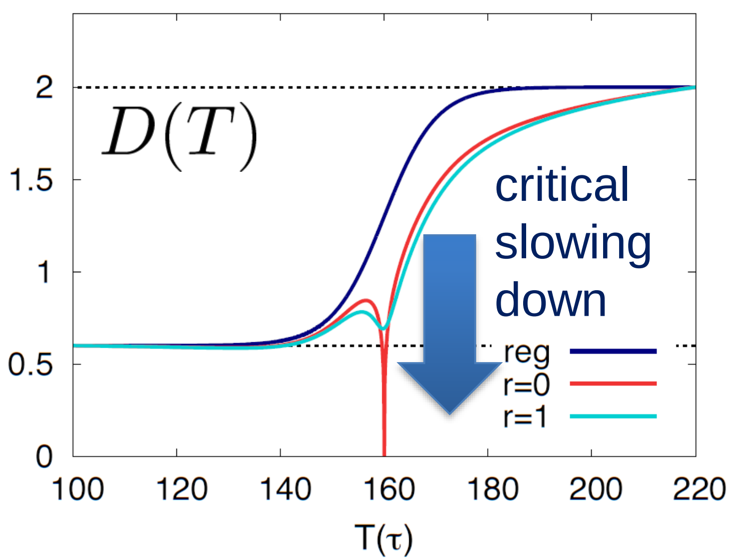
Parametrizing D(τ) and χ(τ)

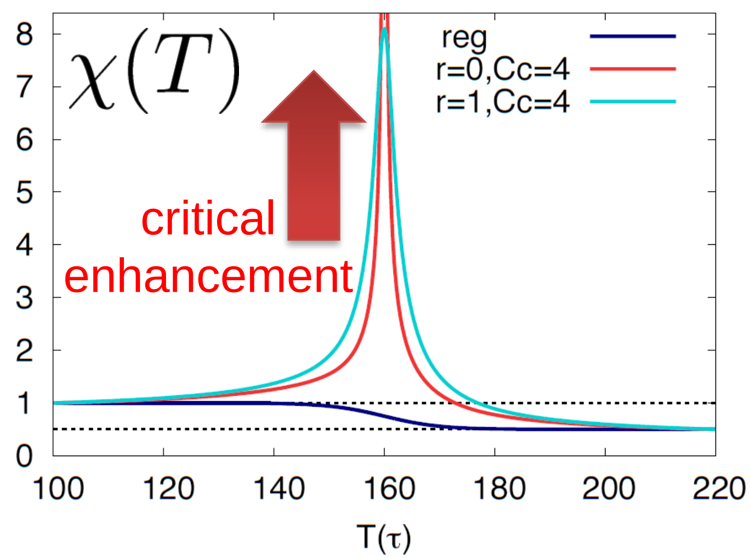
Critical behavior

- 3D Ising (r,H)
- model H



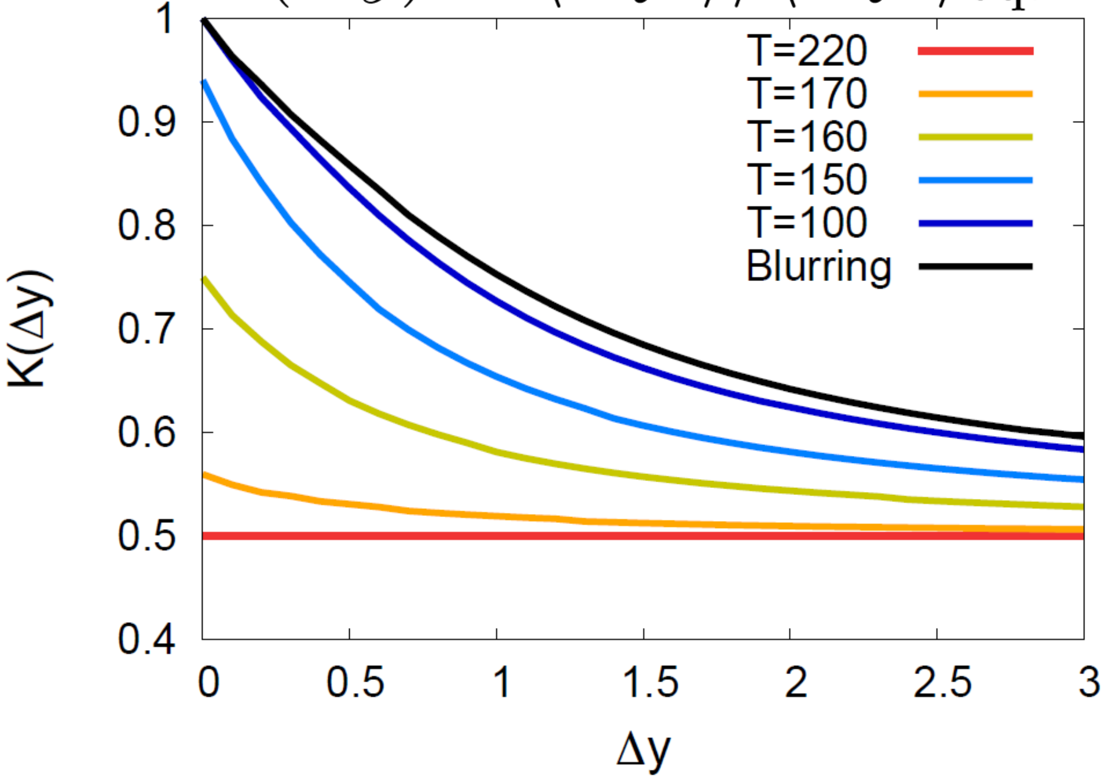
Temperature dep.

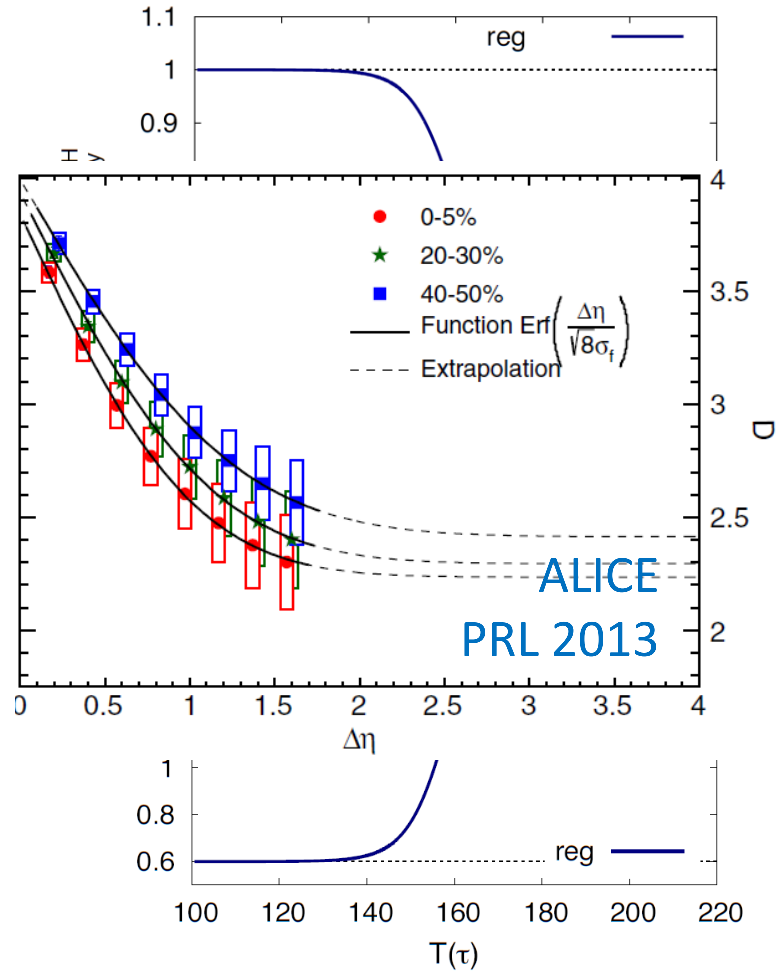




Crossover / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.}$$





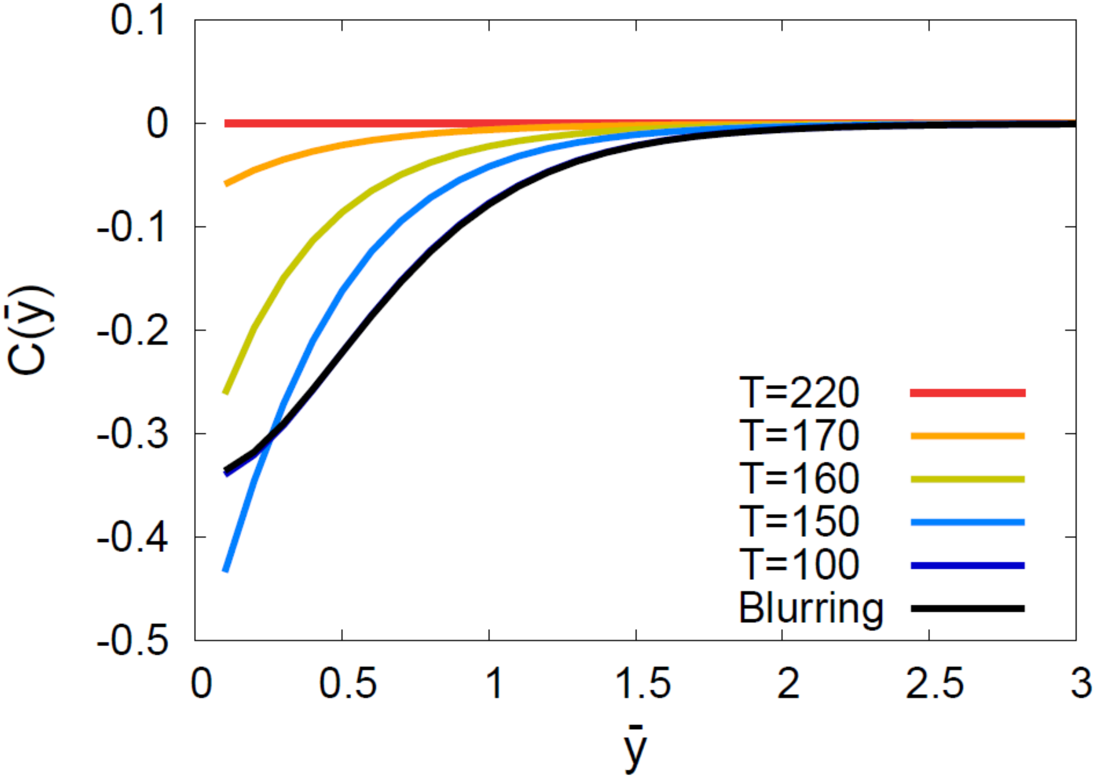
- monotonically decresing

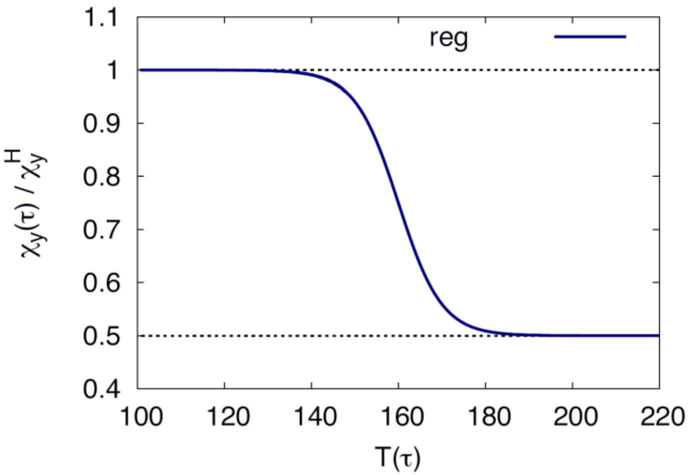
Analytic result

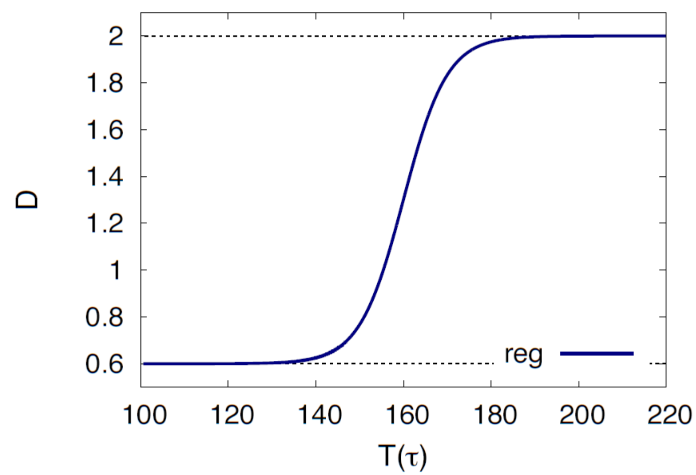
$\chi(\tau)$ monotonically increasing → $K(\Delta y)$ monotonically decreasing

Crossover / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$





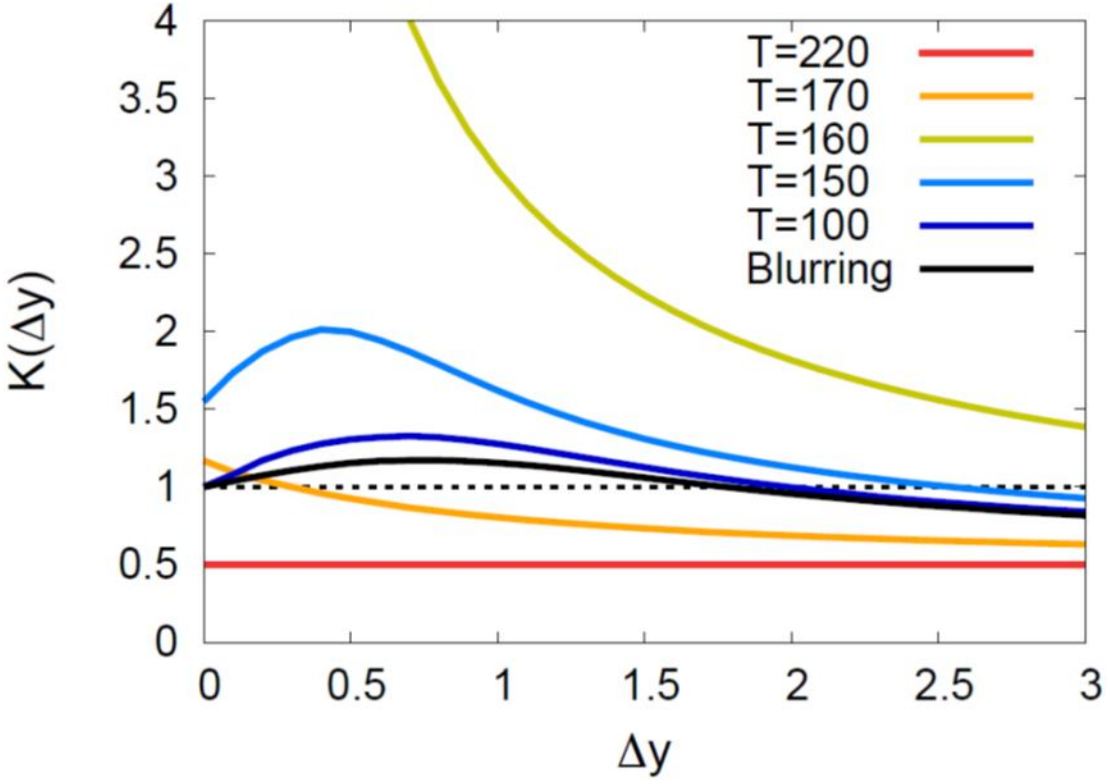


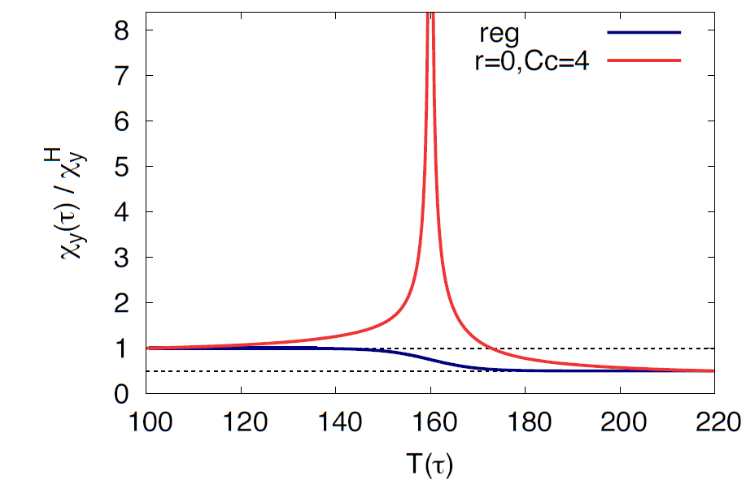
- monotonically decresing

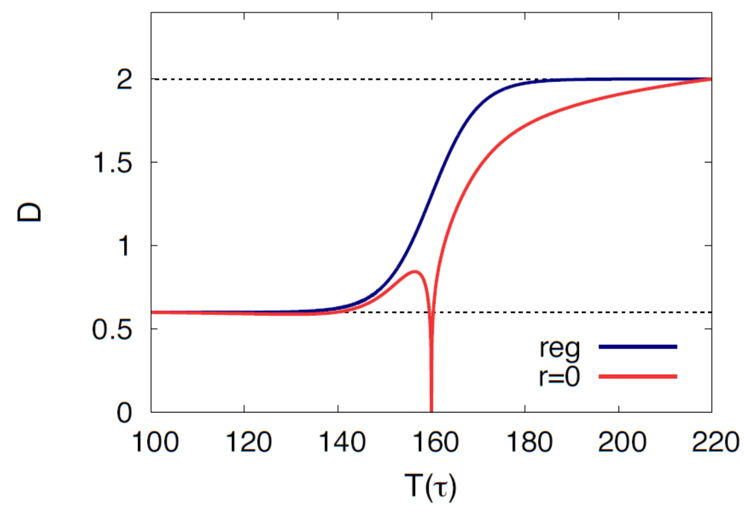
Analytic result: $\chi(\tau)$ monotonically increasing ⇒ $C(\bar{y})$ monotonically decreasing

Critical Point / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.}$$





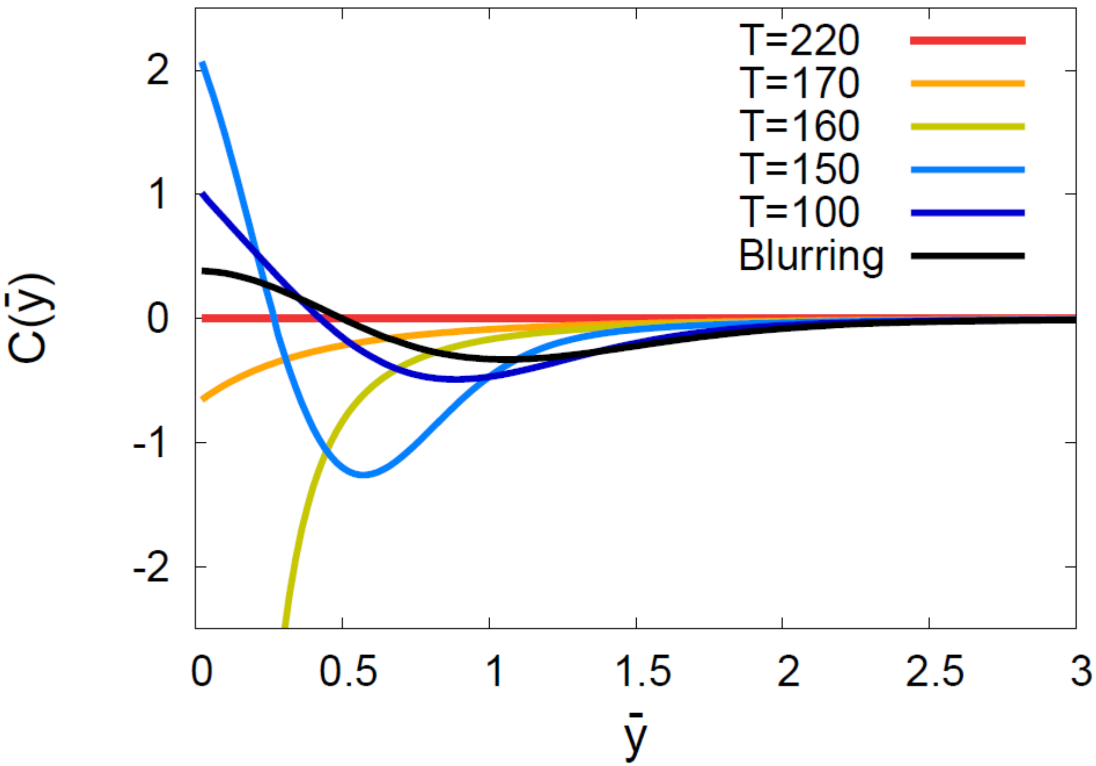


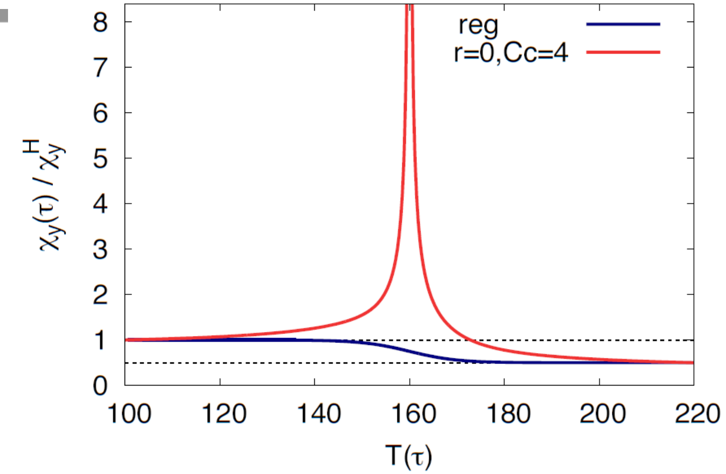
- non-monotonic Δy dep.

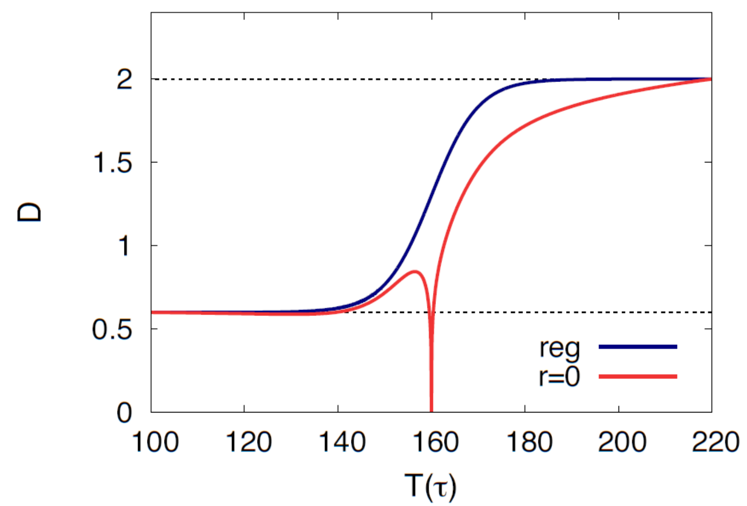
Analytic result: $K(\Delta y)$ non-monotonic → $\chi(\tau)$ non-monotonic

Criticap Point / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$





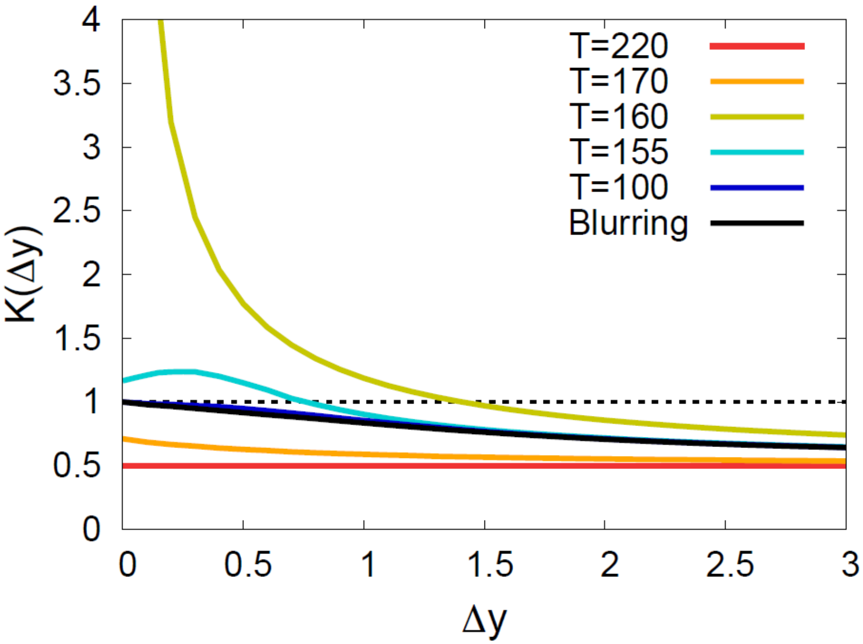


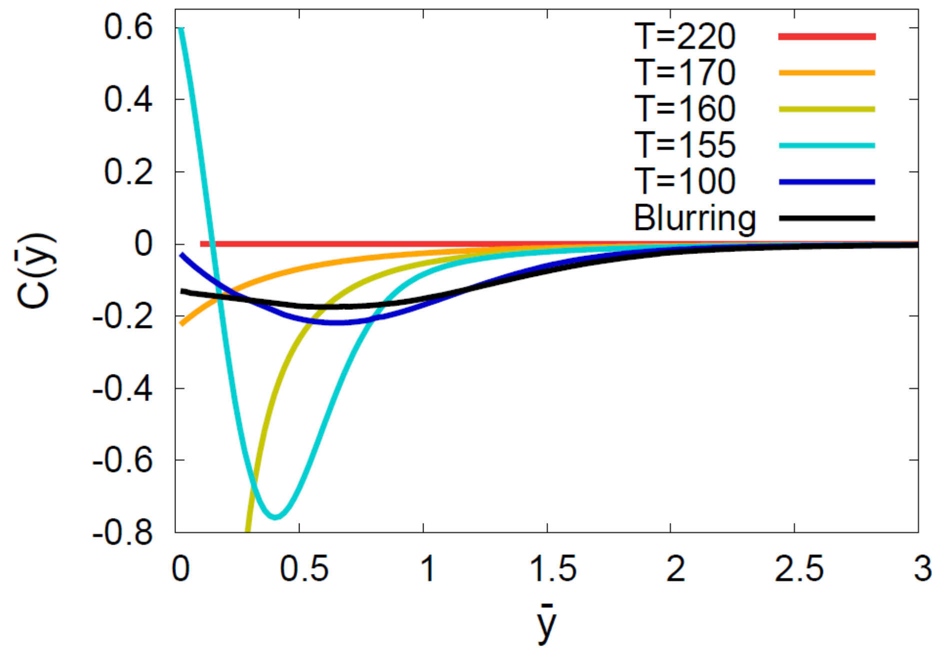
- ☐ non-monotonic Δy dep.

Analytic result	$C(\Delta y)$ non-monotonic	⇨	$\chi(\tau)$ non-monotonic

Weaker Critical Enhancement

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}} \quad C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$





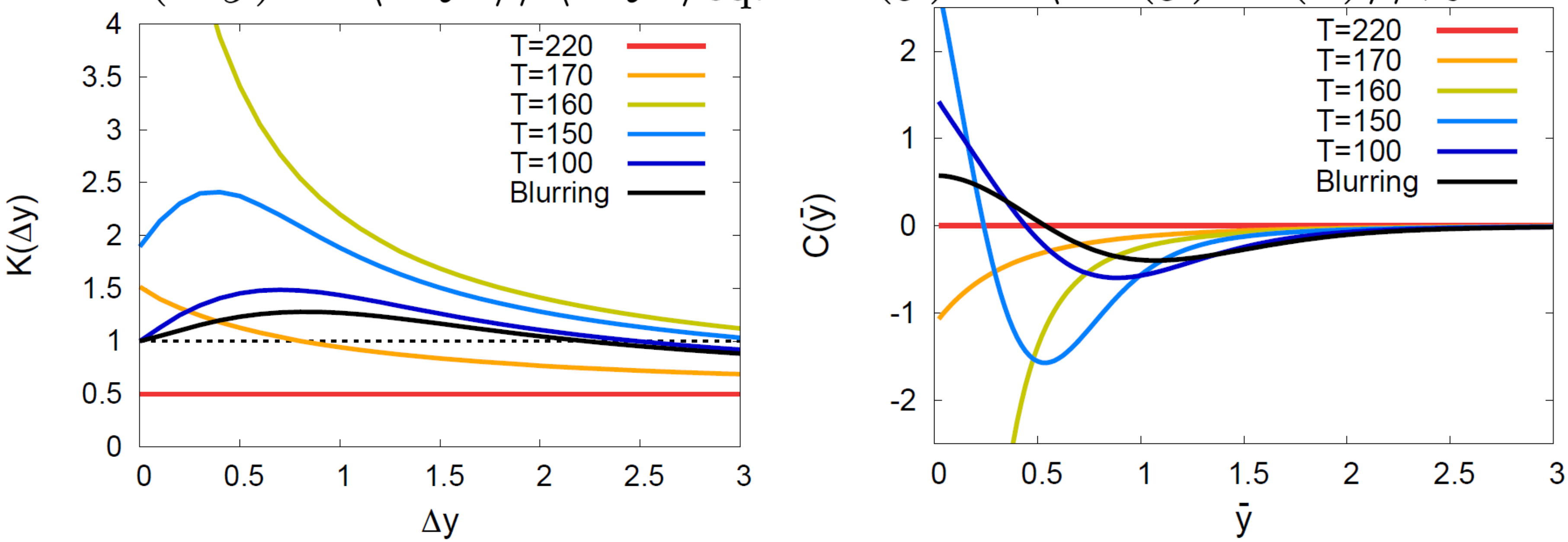
- Non-monotonicity in K(Δy) disappears.
- But C(y) is still non-monotonic.

Analytic result: $K(\Delta y),\ C(\bar{y})$ monotonic ⇒ no information on $\chi(\tau)$

- C(y) is better to see non-monotonicity.

Away from the CP

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.} \qquad C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\rm hadron}$$



- Signal of the critical enhancement can be clearer on a path away from the CP.

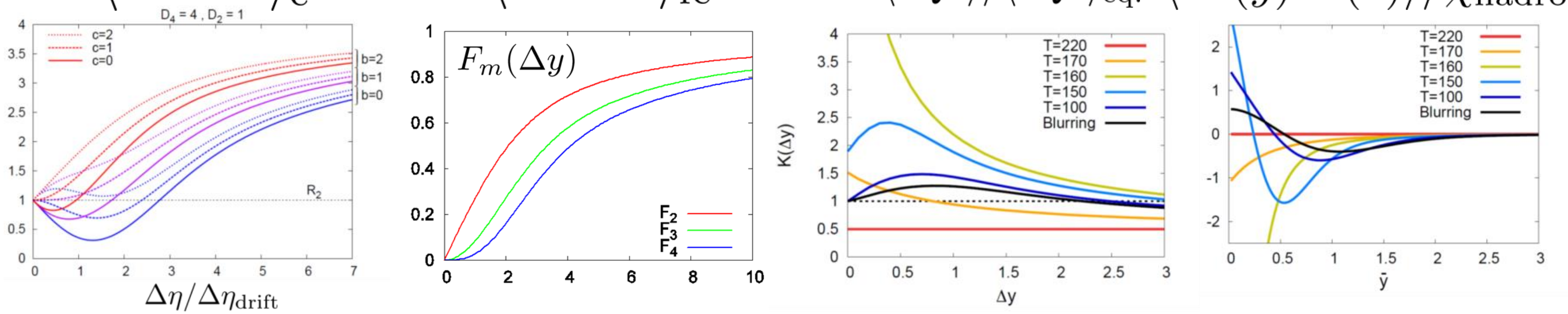
Away from the CP → Weaker critical slowing down

Summary

- Non-equilibrium dynamics is crucial for a proper understanding of experimentally-observed fluctuations.
- $\Delta\eta$ dependences of (factorial) cumulants and correlation functions are useful information to understand fluctuations.
- A non-monotonic behavior of **2nd-order** cumulant or correlation function is the signal of the critical enhancement!

Suggestion to experimentalists

$\langle n^m \bar{n}^{\bar{m}} \rangle_{\rm c}$ $\langle n^m \bar{n}^{\bar{m}} \rangle_{\rm fc}$ $\langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.}$ $\langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\rm hadron}$

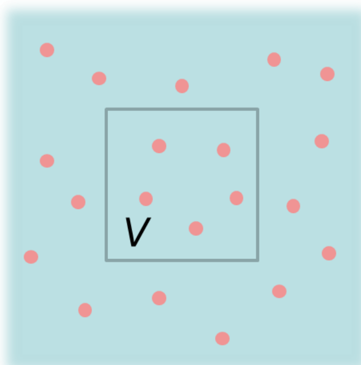


- cumulants / factorial cumulants / correlation funcs.
- $\Delta\eta$ dependence / p_T dependence / Baryon number / etc.

Fluctuations: Theory vs Experiment

Theoretical analyses

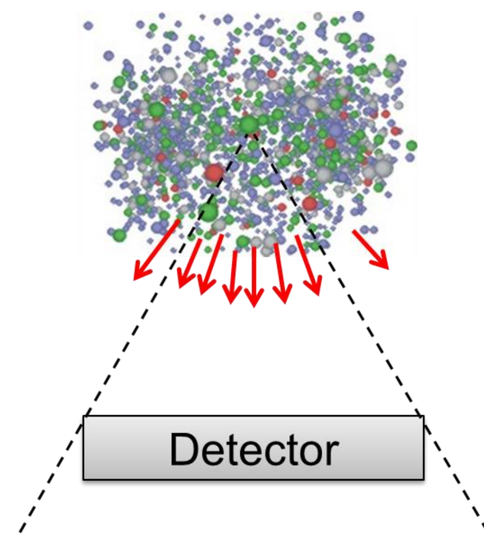
based on statistical mechanics



lattice, critical point,
effective models, …

Fluctuation in a spatial volume

Experiments



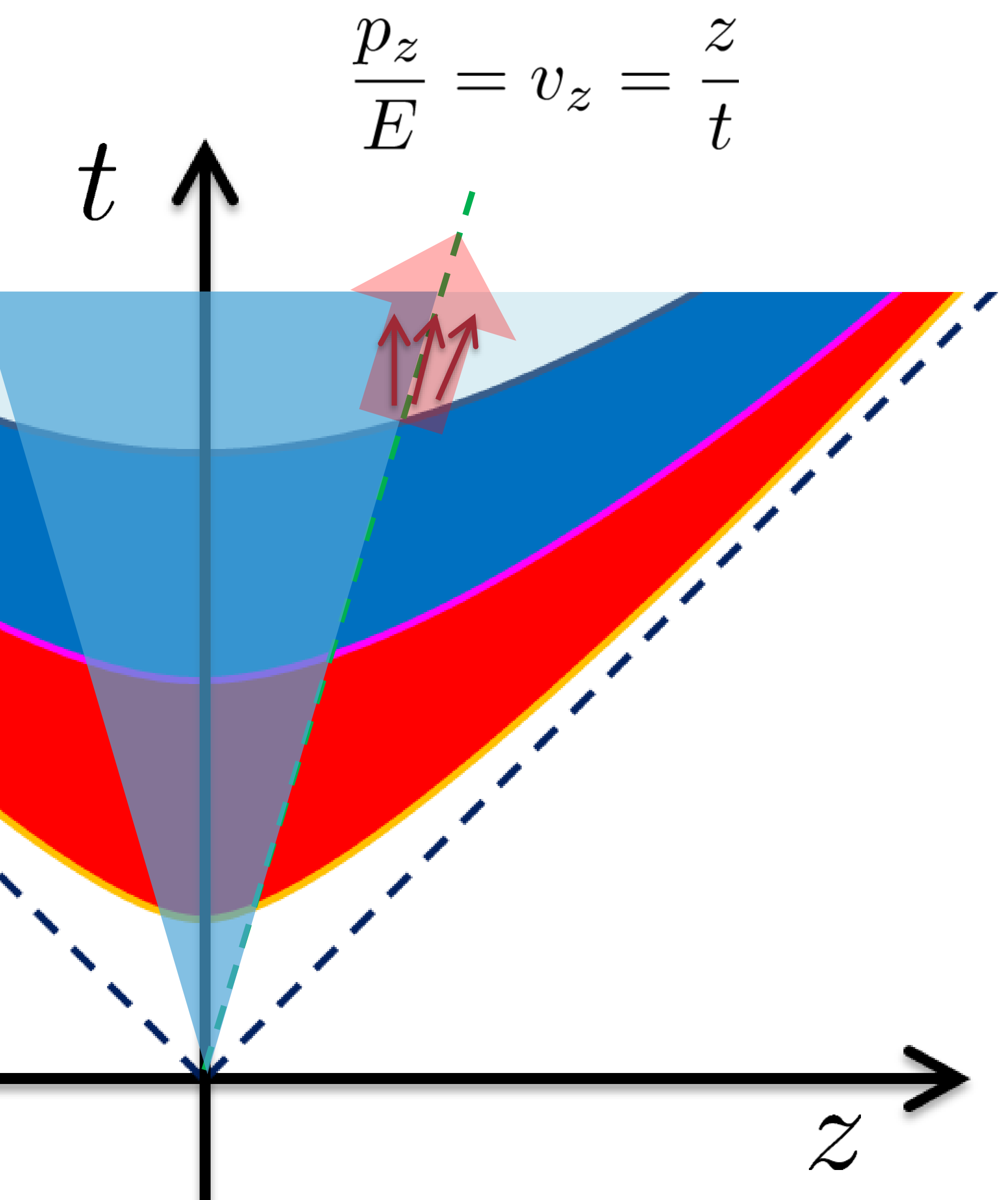
Fluctuations in a momentum space

discrepancy in phase spaces

Asakawa, Heinz, Muller, 2000; Jeon, Koch, 2000; Shuryak, Stephanov, 2001

Thermal Blurring

Ohnishi, MK, Asakawa,
PRC94, 044905 (2016)



Under Bjorken picture,

coordinate-space rapidity *Y*

||

momentum-space rapidity *y*
of medium

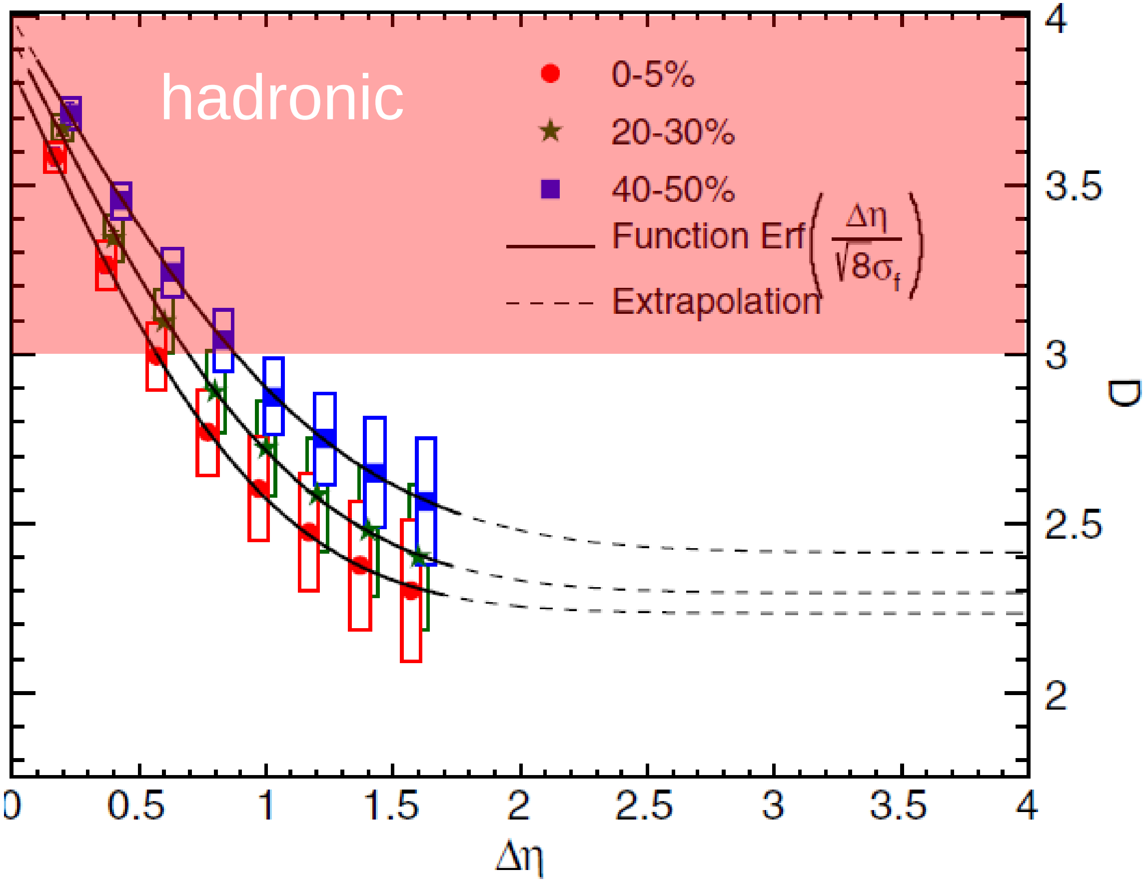
≀

momentum-space rapidity *y*
of individual particles

$$\Delta y \simeq \Delta Y$$

Δη Dependence @ ALICE

ALICE
PRL 2013

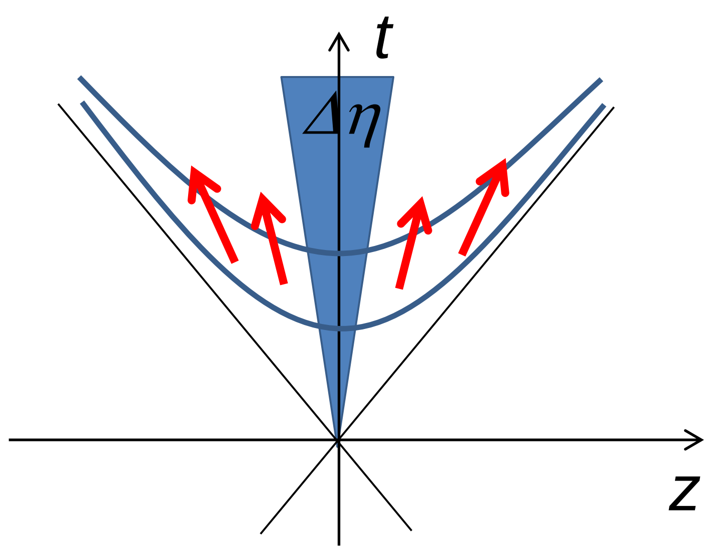


rapidity window

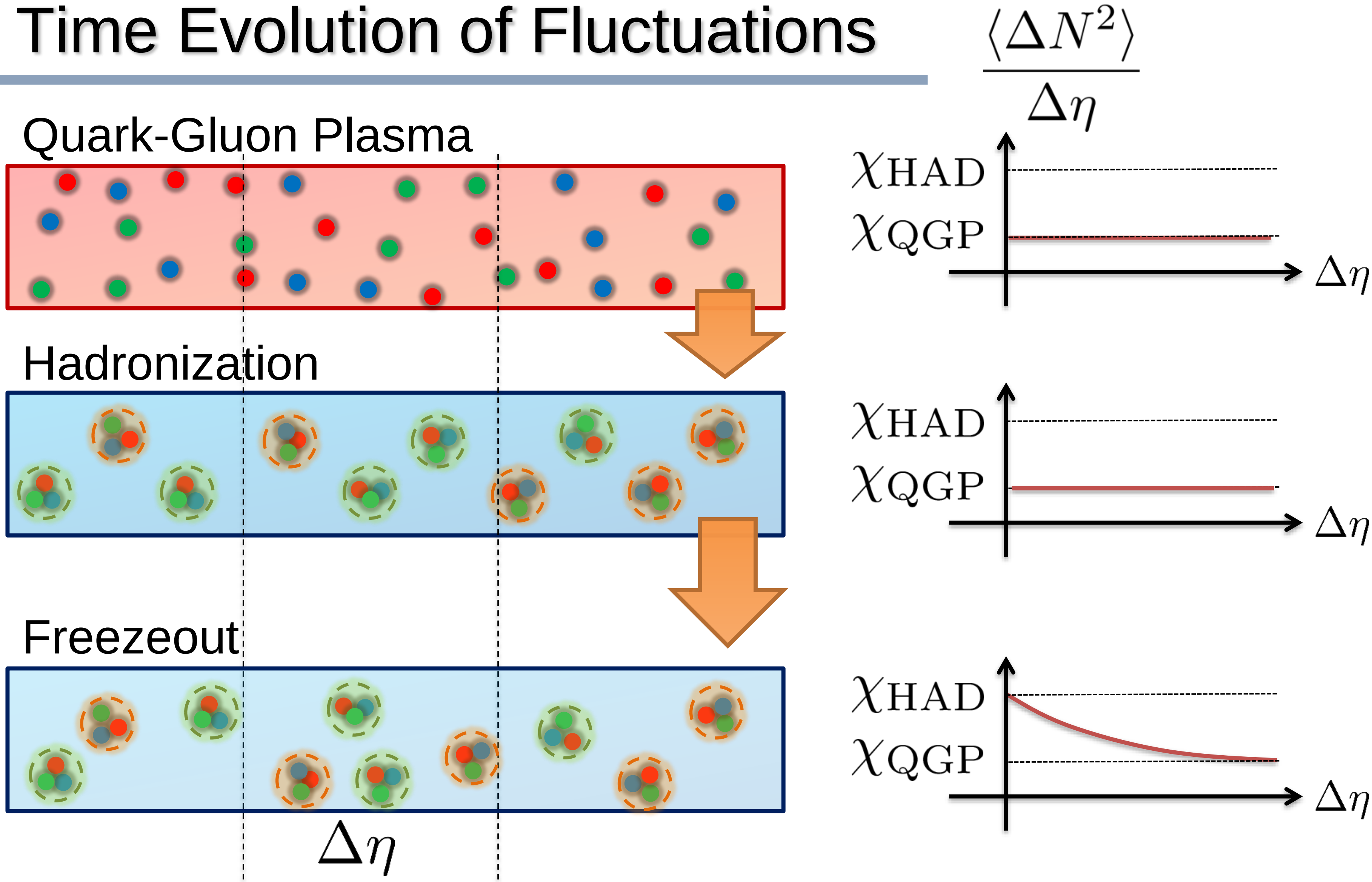
D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- *D* ~ 3-4 Hadronic
- *D* ~ 1-1.5 Quark



Time Evolution of Fluctuations



Variation of a conserved charge is achieved only through diffusion. ⇒ The larger $\Delta\eta$, the slower diffusion