Evolution of Critical Fluctuations / Non-binomial Efficiency correction

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(Osaka U.)

GSI Workshop
Constraining the Phase Boundary with Data from HIC
GSI, Darmstadt, 12/Feb./2018

Contents

1. Diffusion

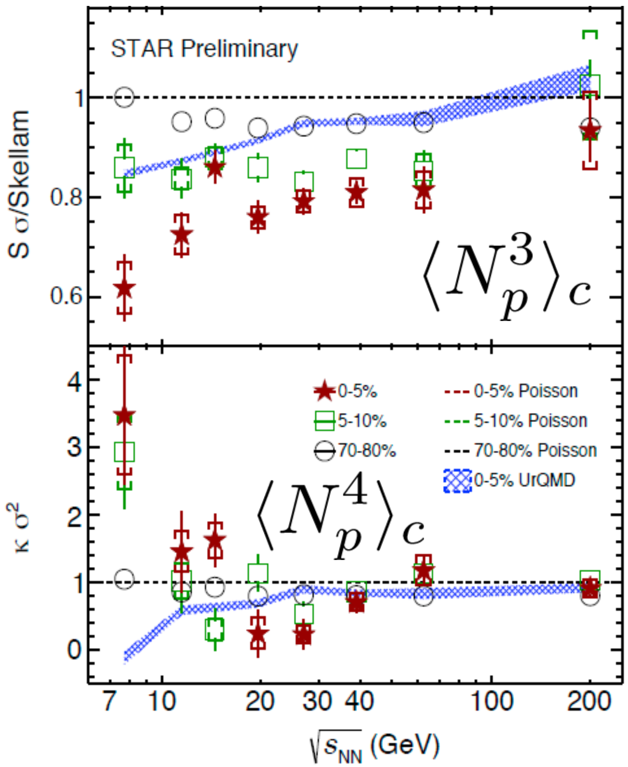
2. Evolution of Critical Fluctuations: 2nd order
3. Evolution of Critical Fluctuations: 3rd order

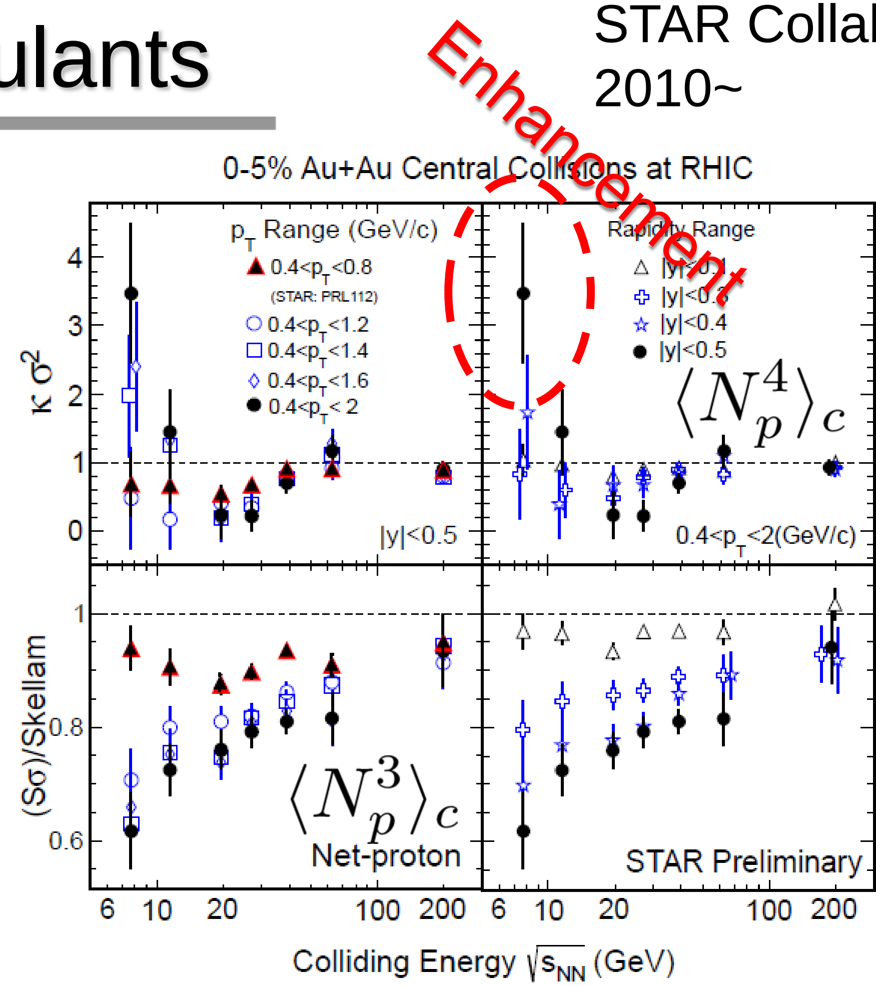
4. Non-binomial Efficiency Correction
   - Previous methods
   - New general method

General Review: Asakawa, MK, PPNP (2016)

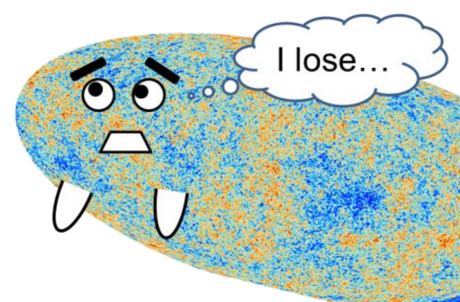
Higher-Order Cumulants

STAR Collab.
2010~





Non-zero **non**-Gaussian cumulants
have been established!



Have we measured critical fluctuations?

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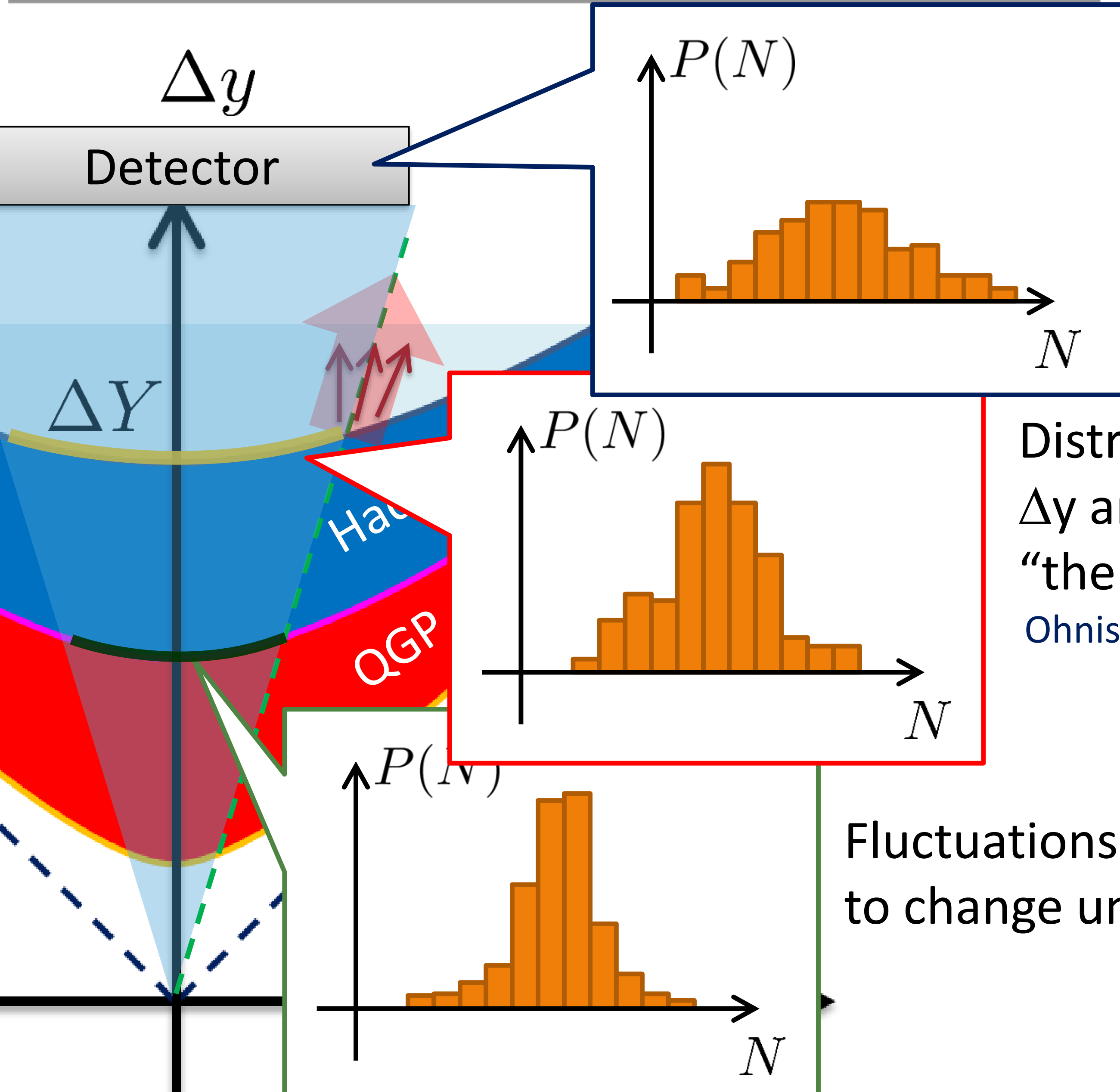
MK, Asakawa, Ono, PLB 728, 386 (2014)
Sakaida, Asakawa, MK, PRL90, 064911 (2014)
MK, NPA942, 65 (2015)

Time Evolution of Fluctuations

Asakawa, Heinz, Muller (2000)
Jeon, Koch (2000)
Shuryak, Stephanov (2001)

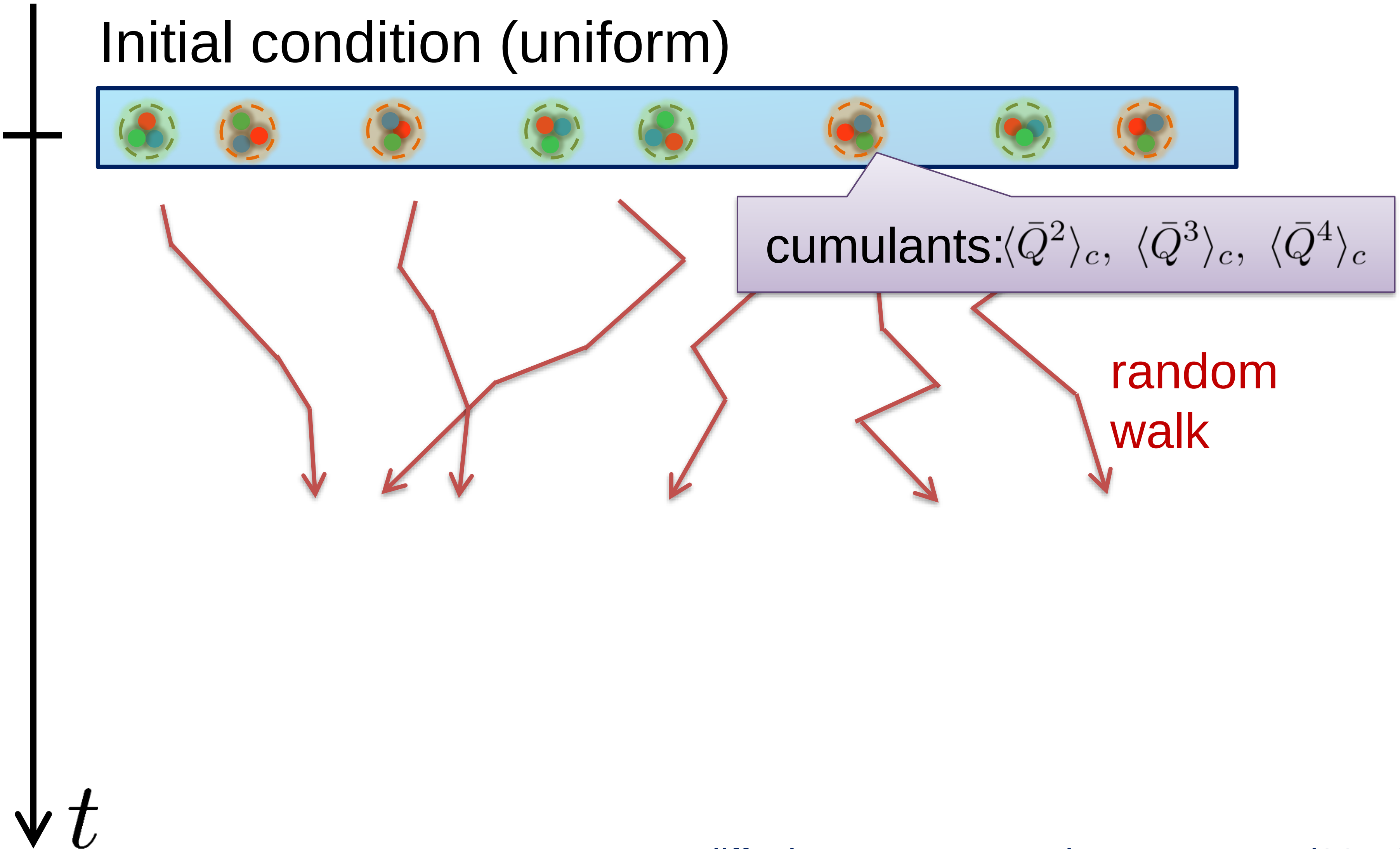


Distributions in ΔY and Δy are different due to "thermal blurring".

Ohnishi, MK, Asakawa, PRC(2016)

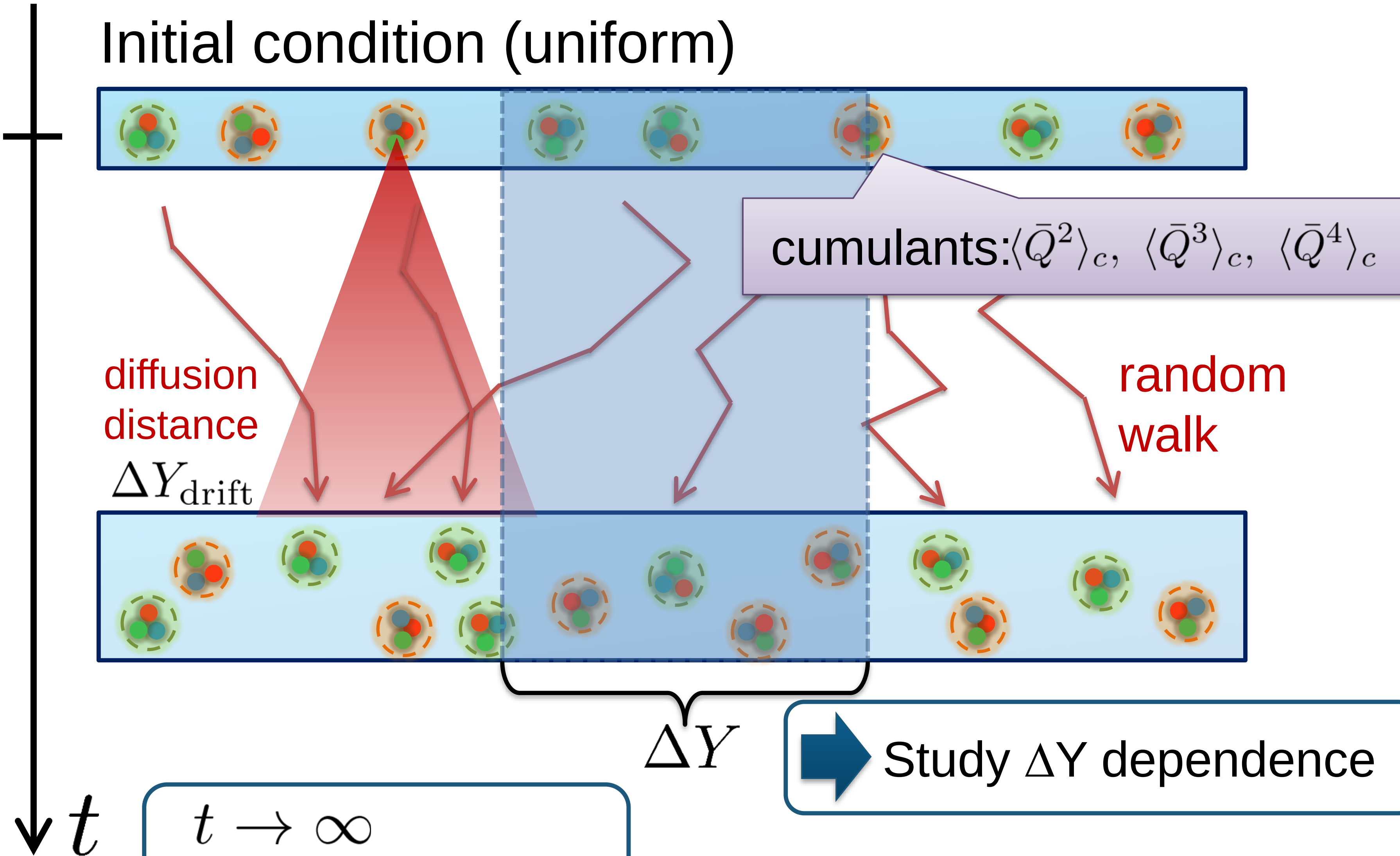
Fluctuations in ΔY continue to change until kinetic f.o.

(Non-Interacting) Brownian Particle Model



diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

(Non-Interacting) Brownian Particle Model



$t \to \infty$
Poisson distribution

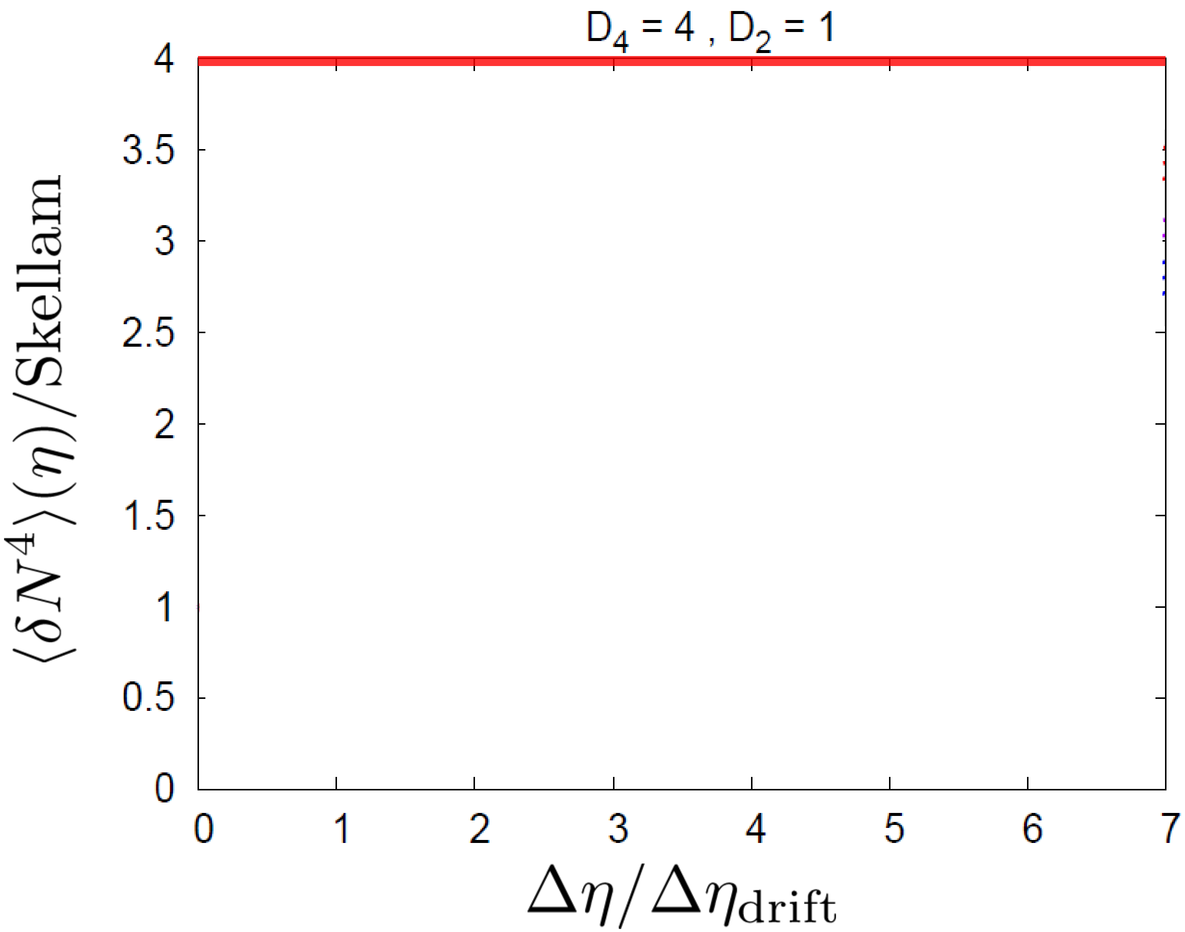
Study ΔY dependence

diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

4th Order Cumulant

MK+ (2014)
MK (2015)

Before the diffusion



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

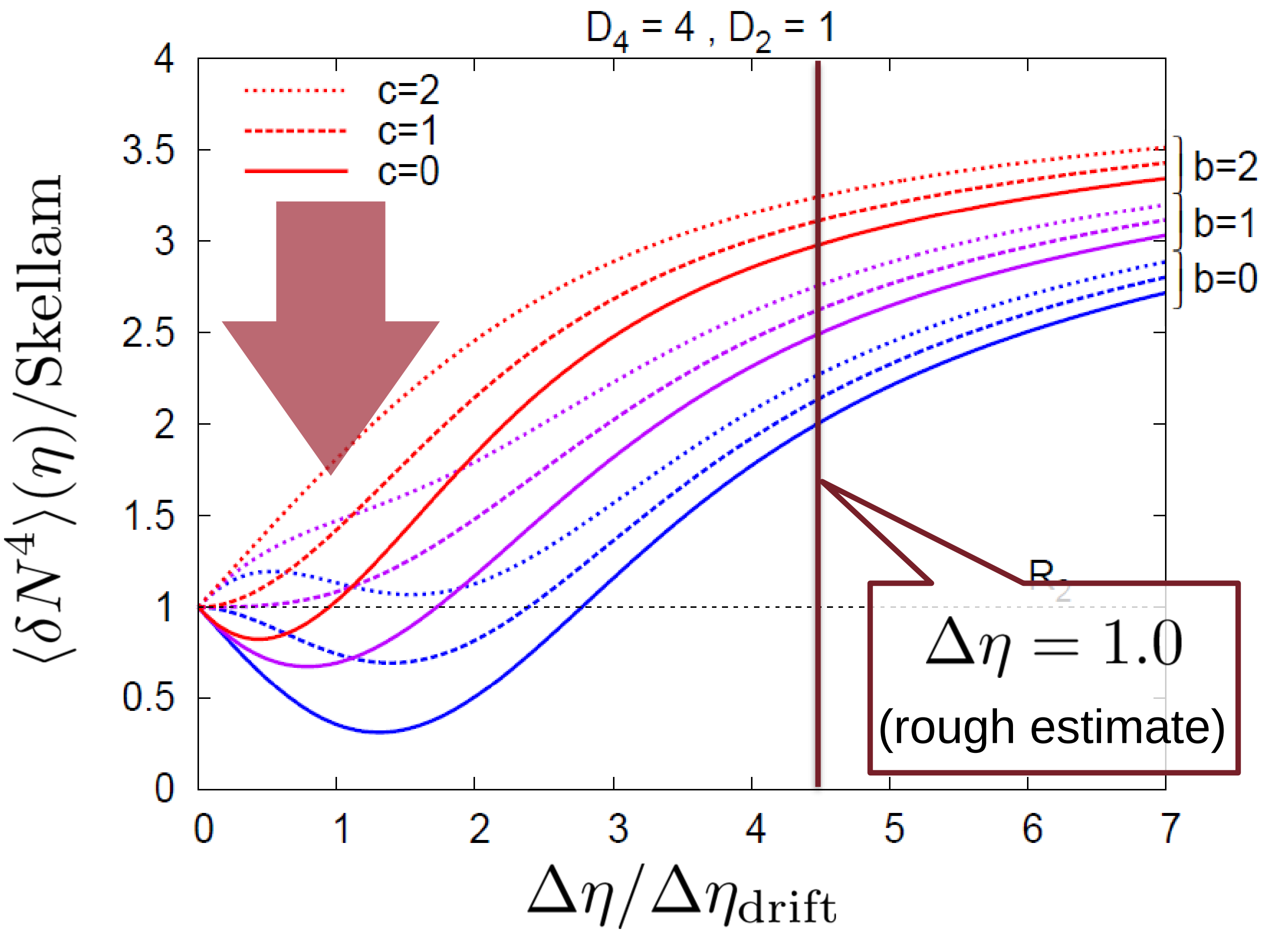
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

4^{th} Order Cumulant

MK+ (2014)
MK (2015)

After the diffusion



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

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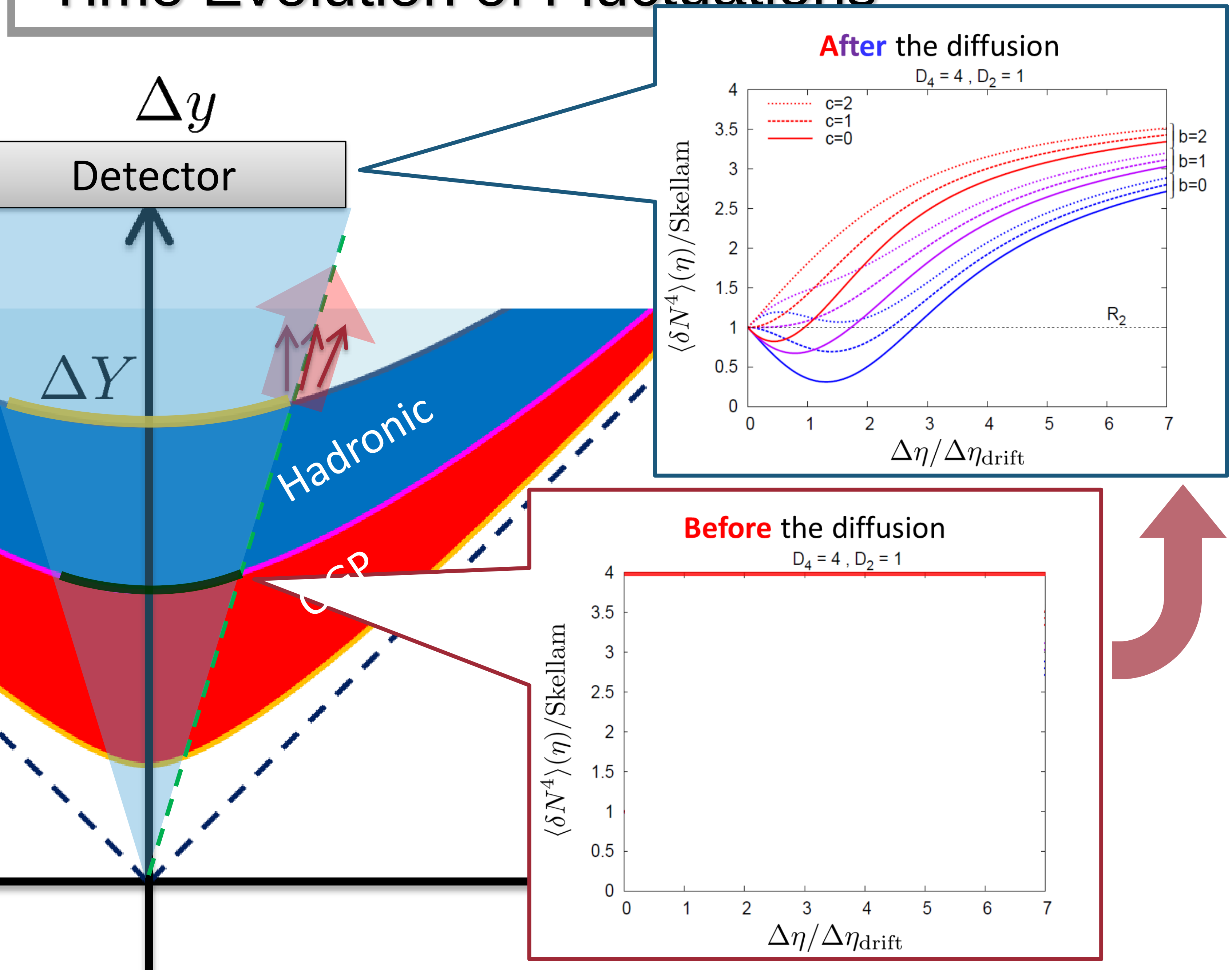
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

- Cumulant at small $\Delta\eta$ is modified toward a Poisson value.
- Non-monotonic behavior can appear.

Time Evolution of Fluctuations



As a result of a simple random walk...

Rapidity Window Dep.

4th-order cumulant

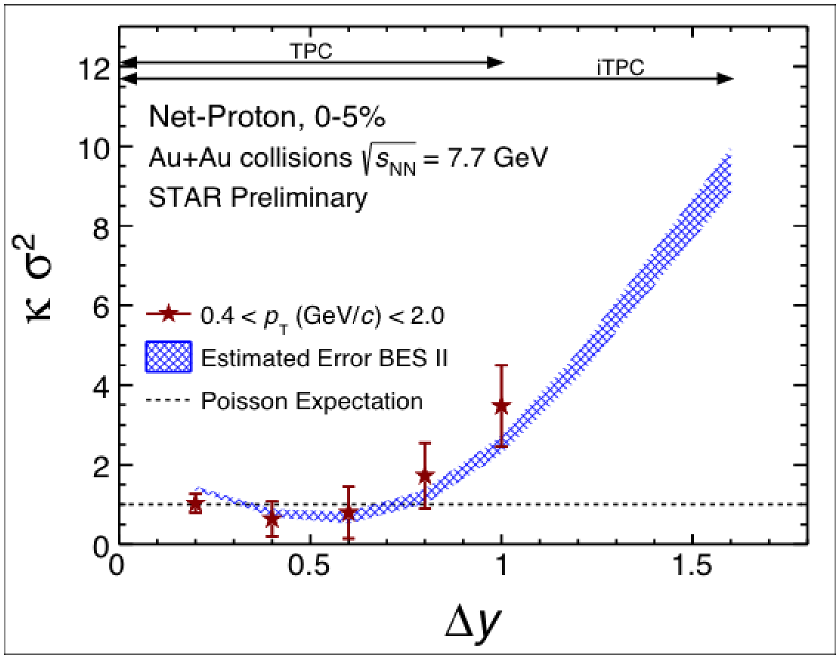
MK+, 2014
MK, 2015

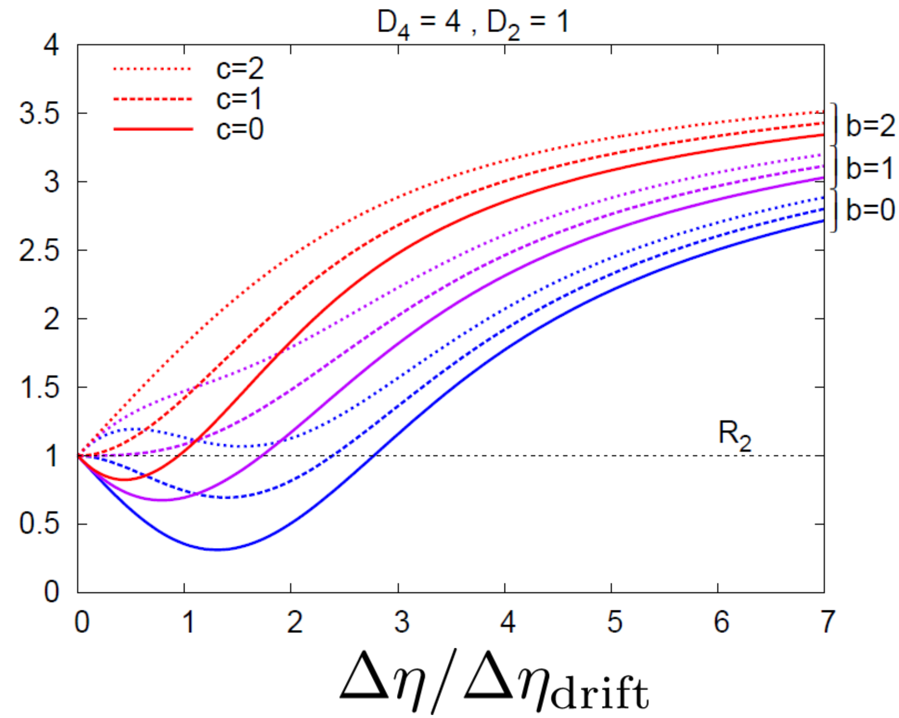
Initial Conditions

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad c = \frac{\langle Q^2_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

STAR Collab. (X. Luo, CPOD2014)





- Is non-monotonic $\Delta\eta$ dependence already observed?
- Different initial conditions give rise to different characteristic $\Delta\eta$ dependence. → Study initial condition

Finite volume effects: Sakaida+, PRC90 (2015)

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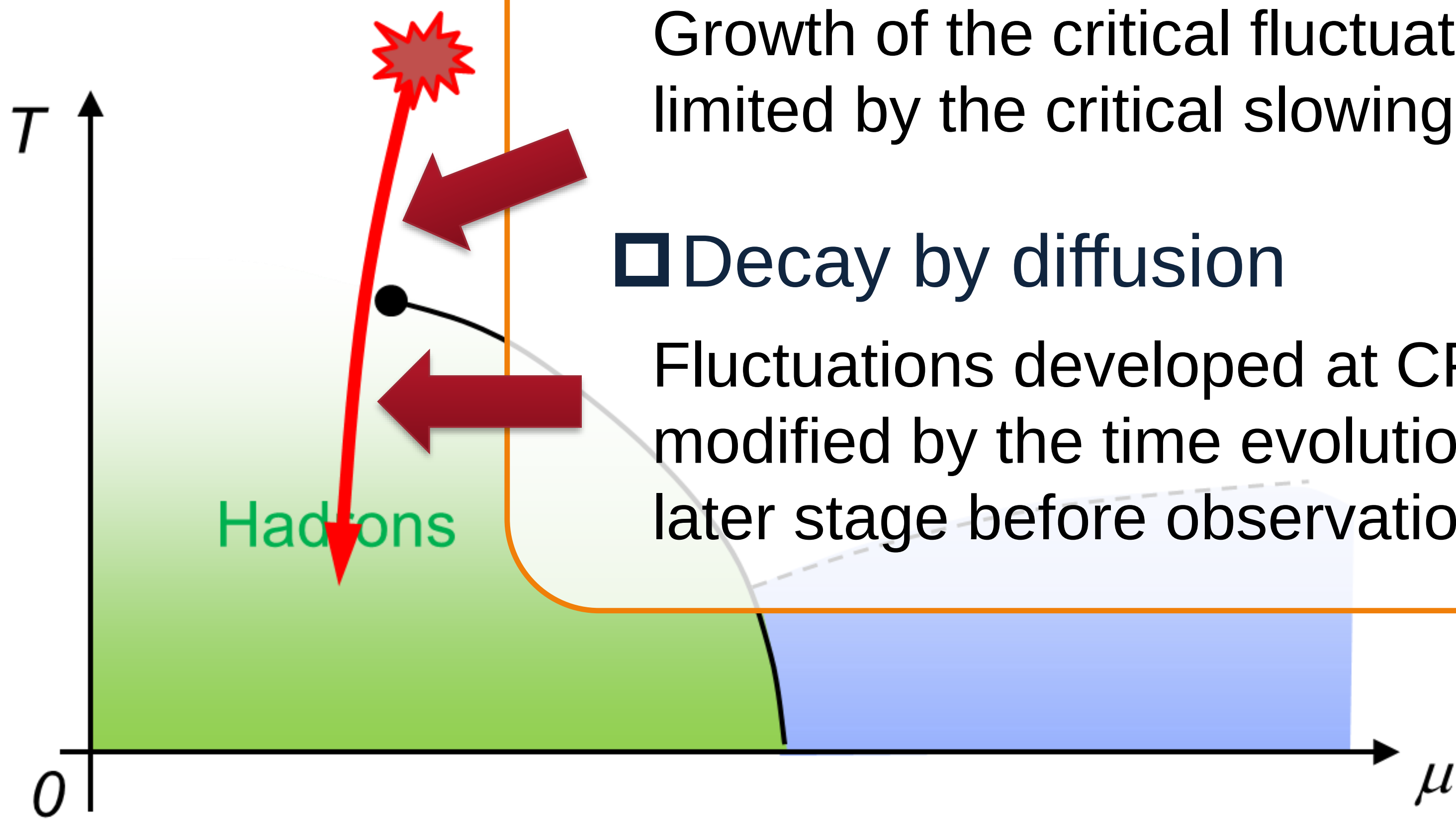
Sakaida, Asakawa, Fujii, MK, PRC95,064905(2017)

Effect of Dynamical Evolution



☐ Growth

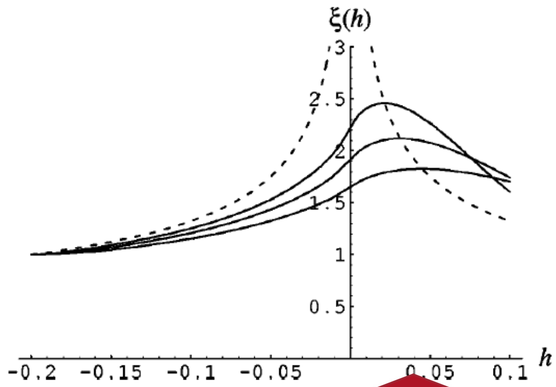
Growth of the critical fluctuation is limited by the critical slowing down.

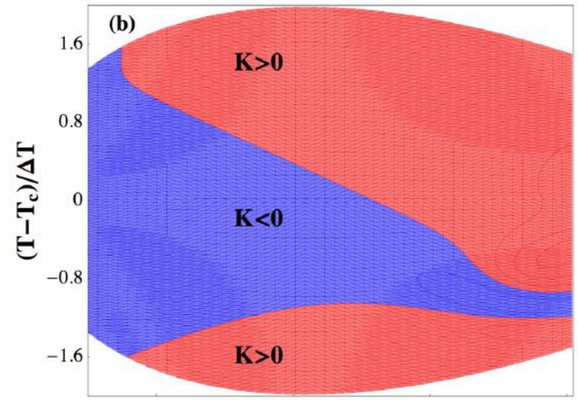
☐ Decay by diffusion

Fluctuations developed at CP are modified by the time evolution in later stage before observation.

Dynamical Evolution of Critical Fluctuations

- **Evolution of spatially uniform "σ" mode**





Berdnikov, Rajagopal (2000)
Asakawa, Nonaka (2002)
Mukherjee+ (2015)

THIS STUDY

Evolution of **conserved charge fluctuations**

Sakaida+, PRC2017; Murata, MK, in prep.

1. Conserved charges are directly observable.
2. Soft mode at QCD-CP is a conserved mode.

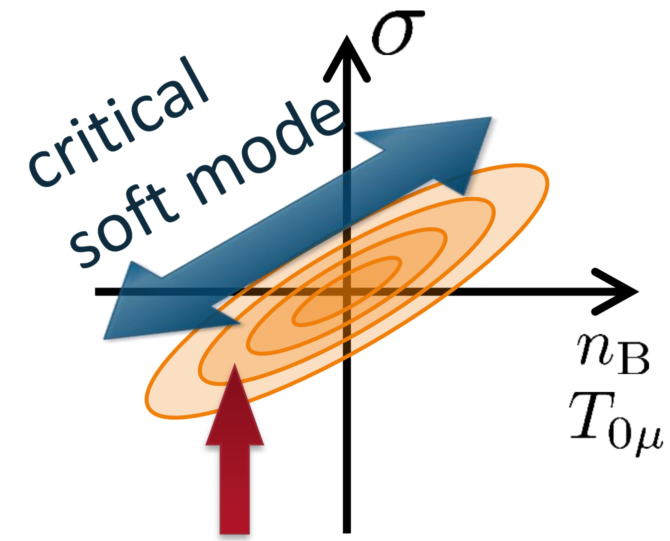
See also, Kapusta, Torres-Rincon (2012); Herold, Nahrgang, … (2015)

Soft Mode of QCD-CP = Conserved Mode

Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

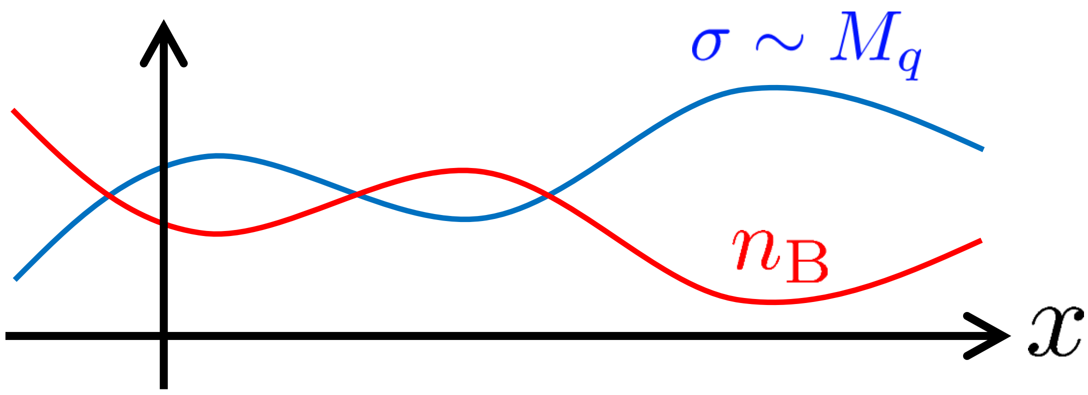
Fluctuations of σ and n_B are coupled around the CP!

$$\delta\sigma \simeq \delta n_B$$



σ: fast damping

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \cdots$$



Analysis of 2nd-order Cumulant

Sakaida+ (2017)

Evolution of baryon number density

Stochastic Diffusion Equation

$$\partial_t n = D(t)\partial_x^2 n + \partial_x \xi$$

$$\langle \xi(x_1,t_1)\xi(x_2,t_2)\rangle = \chi_2(t)\delta^{(2)}(1-2)$$

$D(t),\ \chi_2(t)$:parameters characterizing criticality

We study the 2nd order cumulant as well as correlation function.

Our Main Conclusion

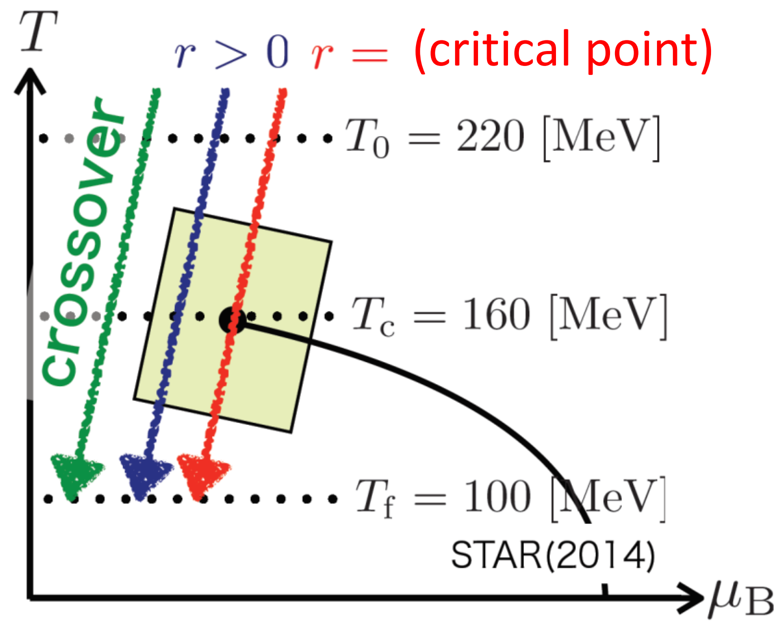
Non-monotonicity in cumulants or correlation func. = Signal of QCD-CP

Parametrizing D(τ) and χ(τ)

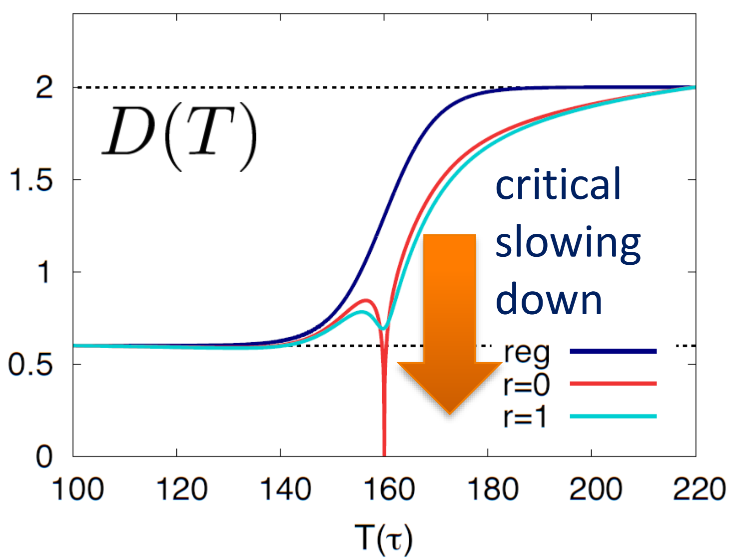
Critical behavior

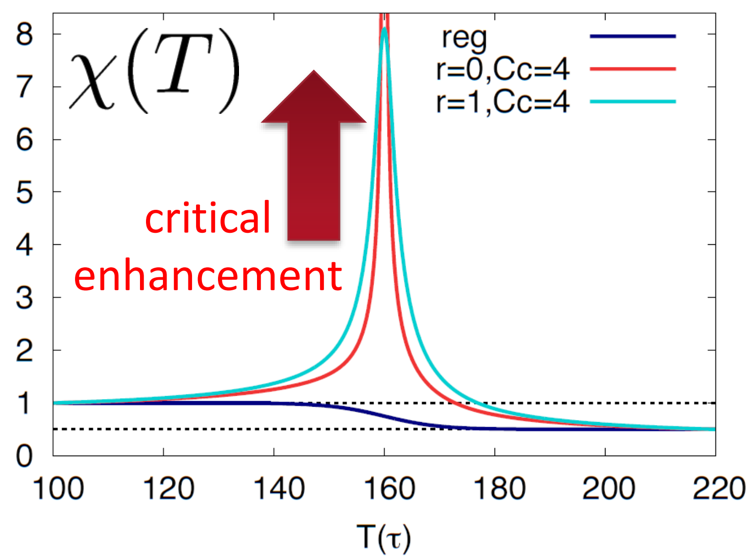
- 3D Ising (r,H)
- model H

Berdnikov, Rajagopal (2000)
Stephanov (2011); Mukherjee+(2015)



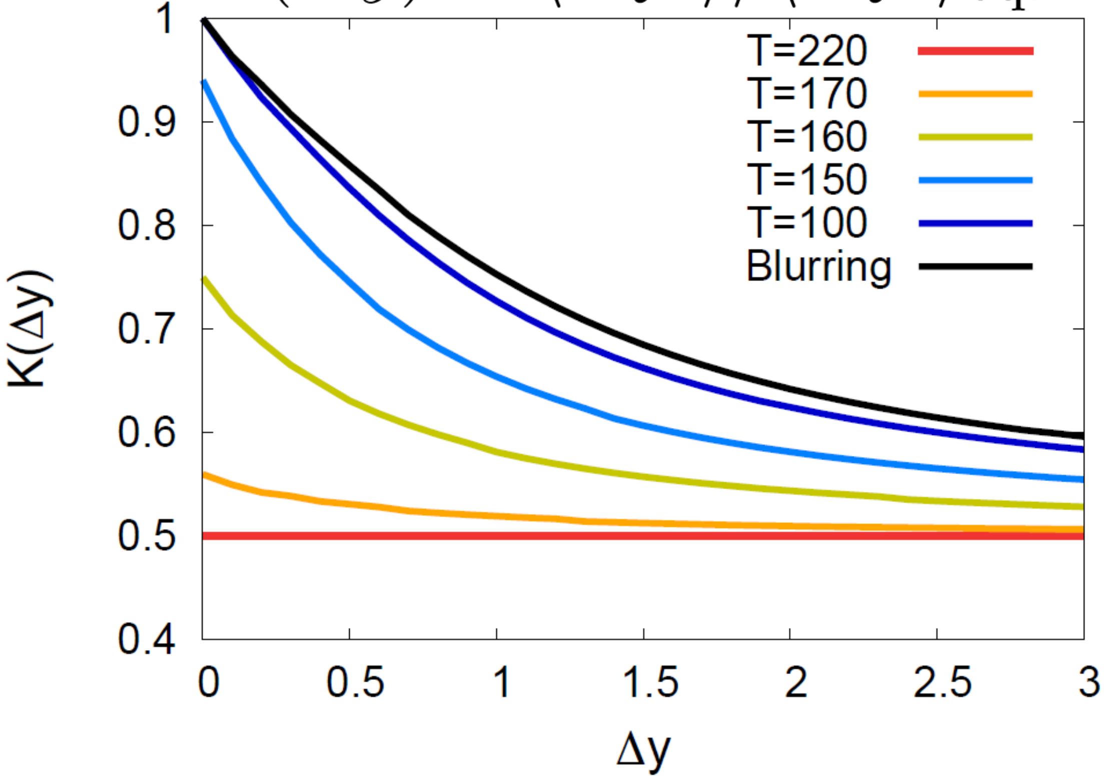
Temperature dep.

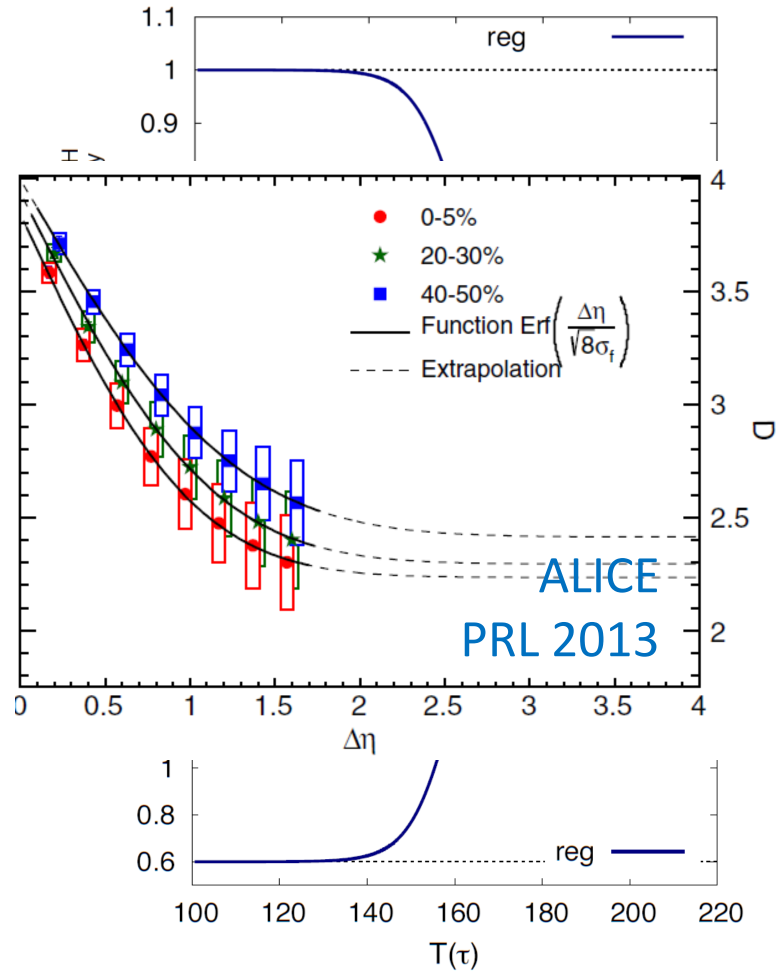




Crossover / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\mathrm{eq.}}$$





- monotonically decresing

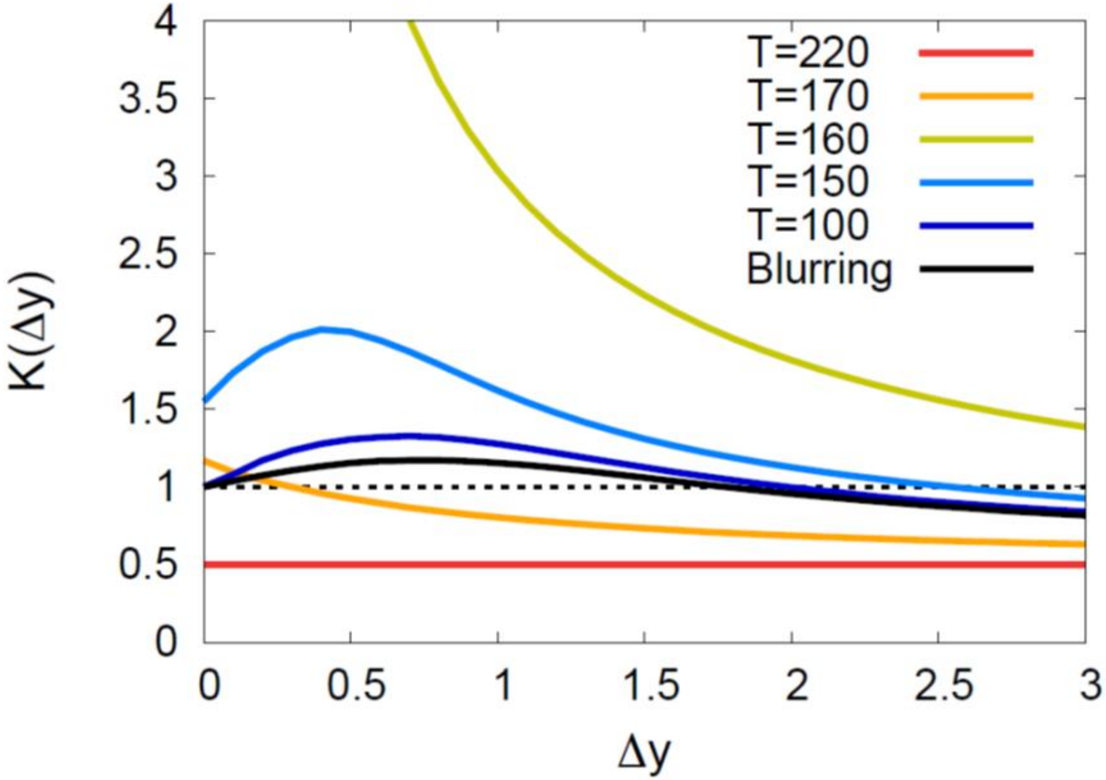
Analytic result

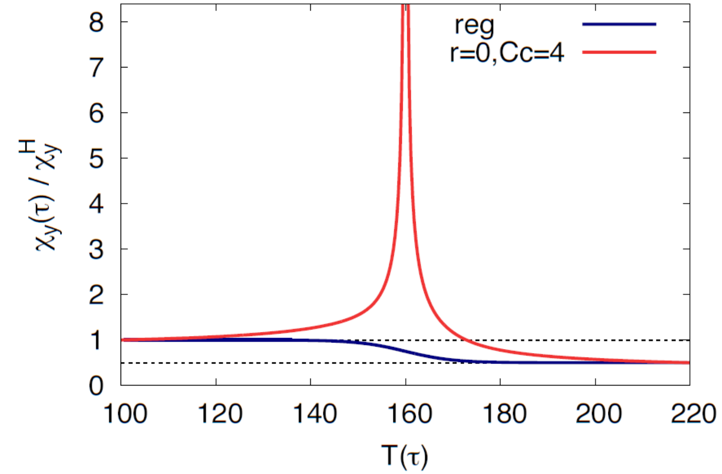
$\chi(\tau)$ monotonically increasing

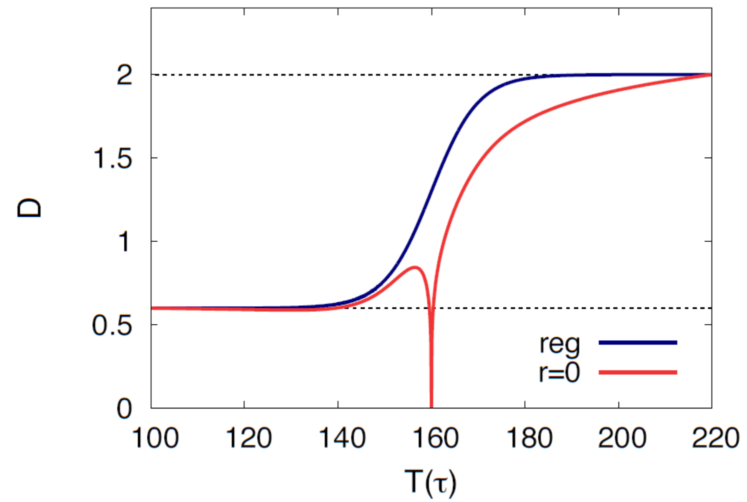
$K(\Delta y)$ monotonically decreasing

Critical Point / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\mathrm{eq.}}$$







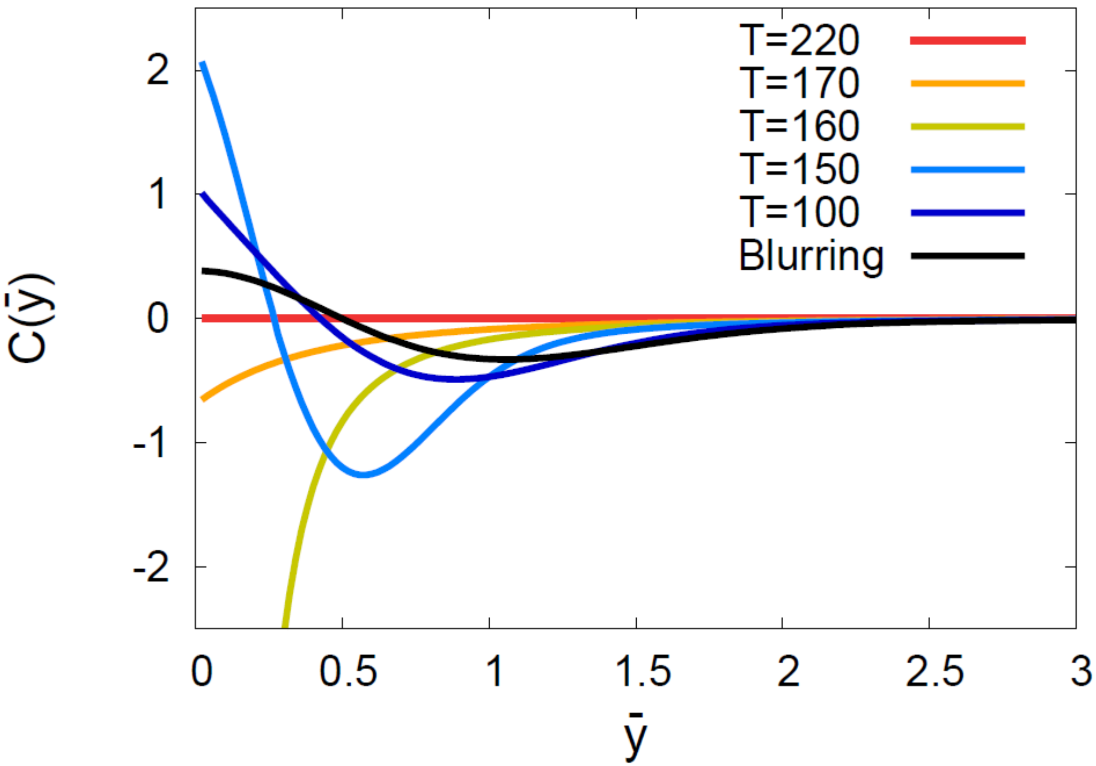
- non-monotonic Δy dep.

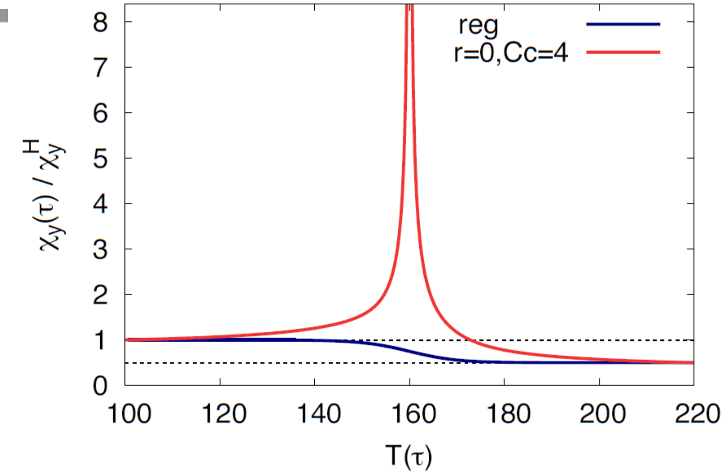
Analytic result

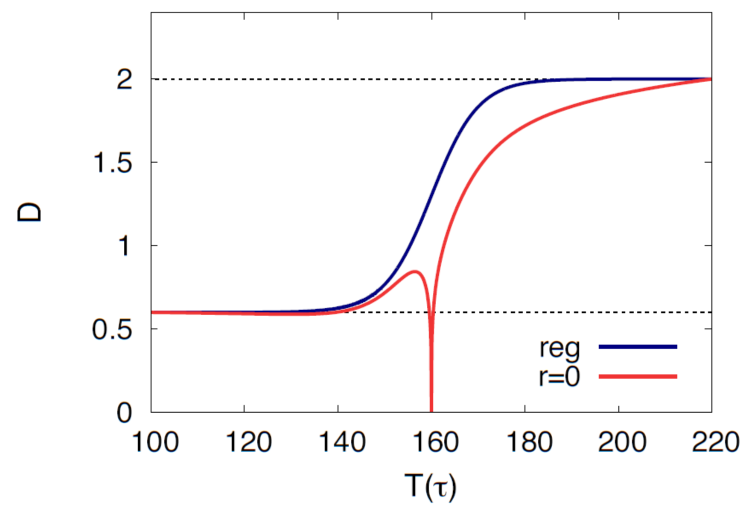
$K(\Delta y)$ non-monotonic → $\chi(\tau)$ non-monotonic

Criticap Point / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$







- non-monotonic Δy dep.

Analytic result

$C(\Delta y)$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

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Murata, MK, in preparation

Analysis of 3rd-order Cumulant

Murata, MK
in preparation

SDE: Higher order cumulants vanish in equi.

Include a non-linear effect into SDE

$$\partial_t n = D(t)\partial_x^2 \frac{\delta\Omega[n]}{\delta n(x)} + \partial_x \xi$$

$$\Omega[n] = \int dx(\lambda_2 n(x)^2 + \lambda_3 n(x)^3)$$

See, Nahrgang, QM2017

$$\lambda_3 = \frac{\chi_3}{\chi_2^3}$$

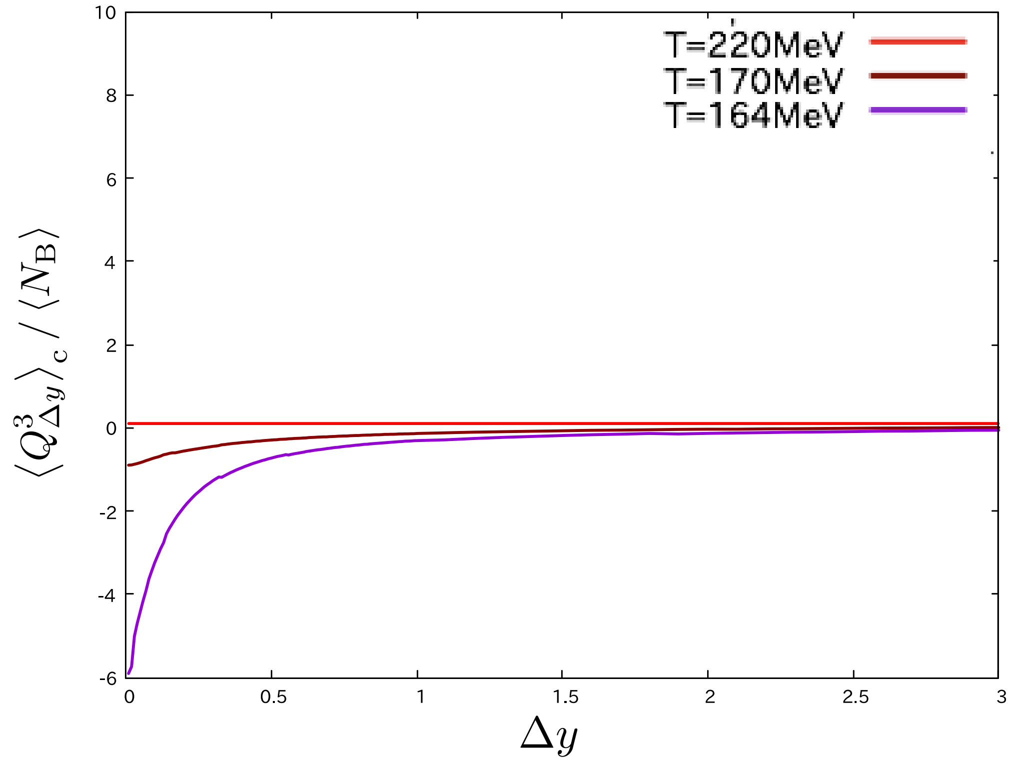
$\lambda_3 = 0$
SDE

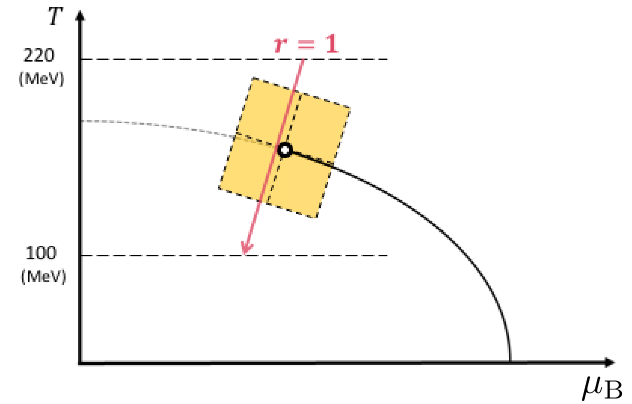
Analytic solution at the leading order in λ_3 for

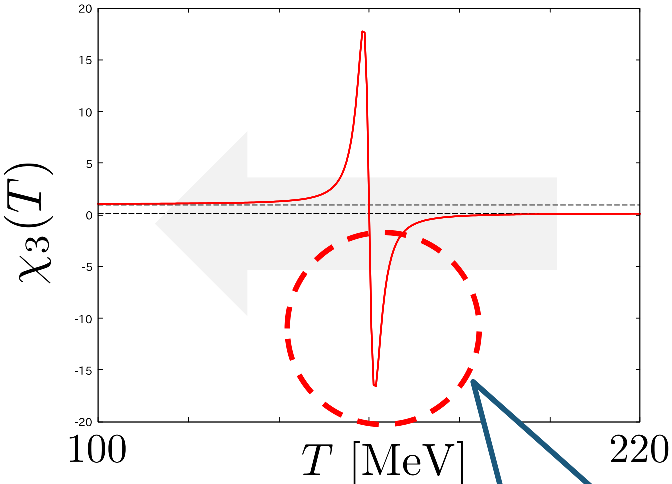
$$\langle N^3 \rangle_c, \ \langle \delta n(x_1) n(x_2) n(x_3) \rangle$$

Time Evolution: Near CP

Murata, MK
in preparation





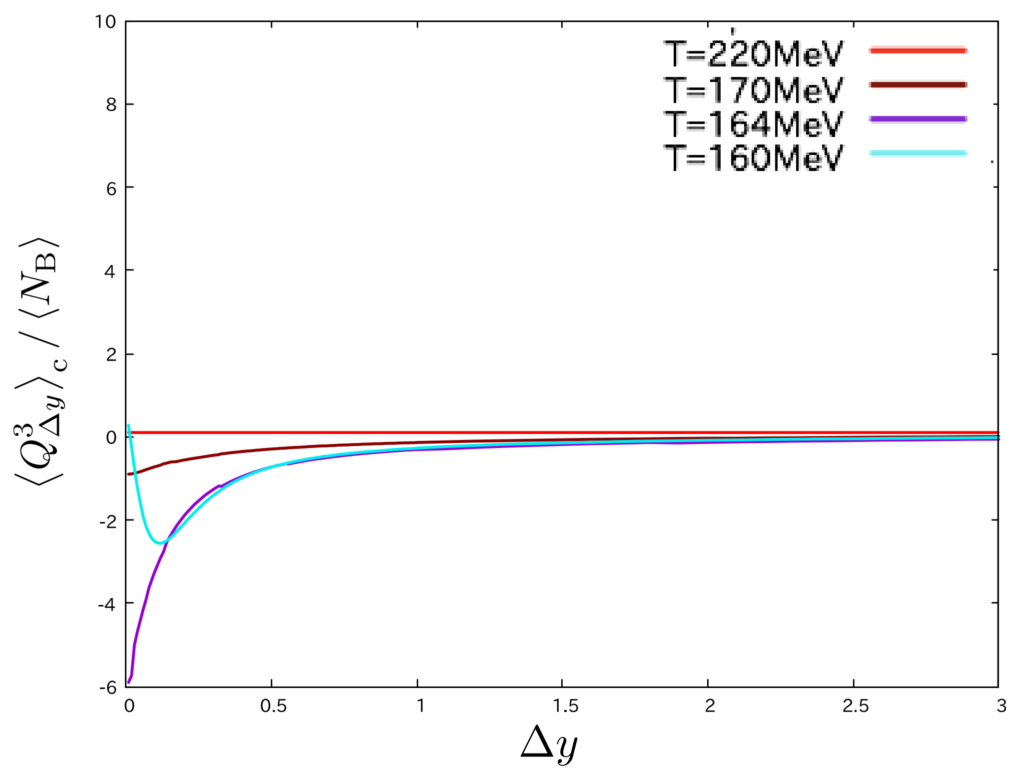


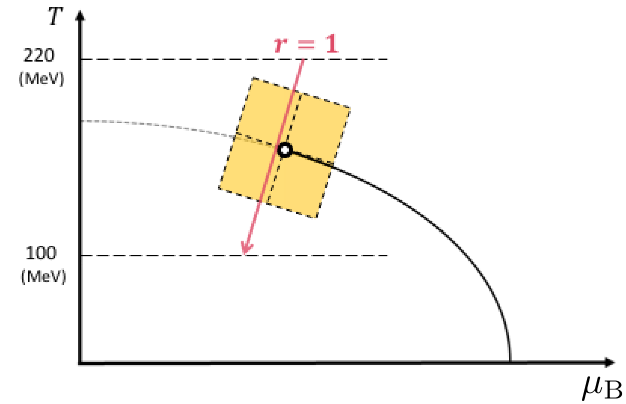
negative

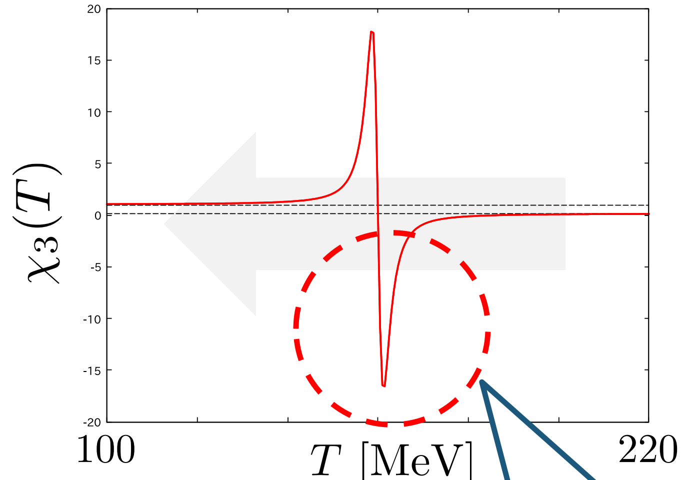
Asakawa, Ejiri, MK (2009)

Time Evolution: Near CP

Murata, MK
in preparation



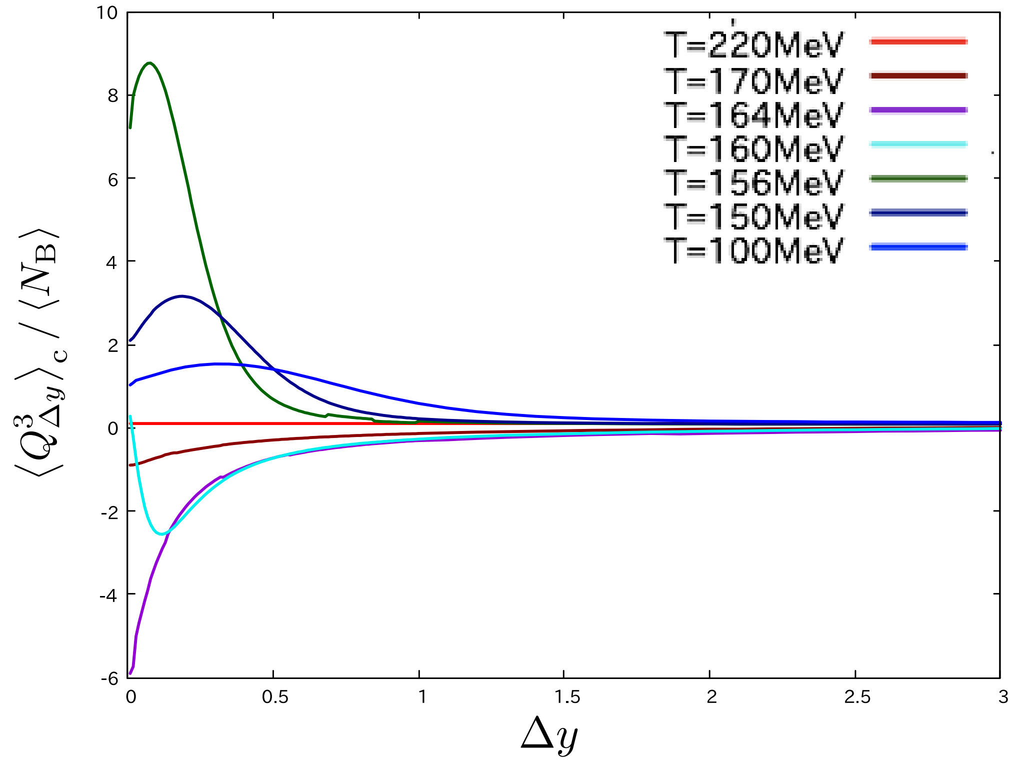


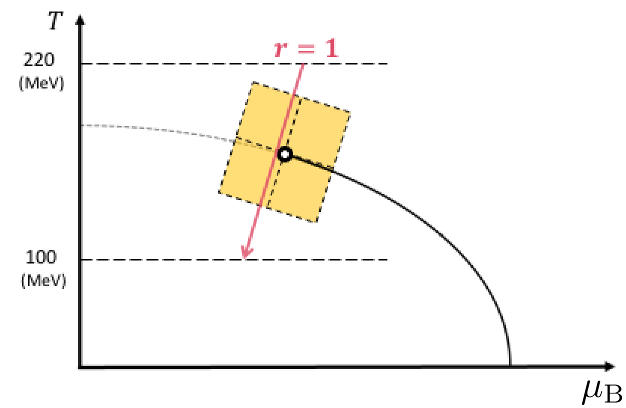


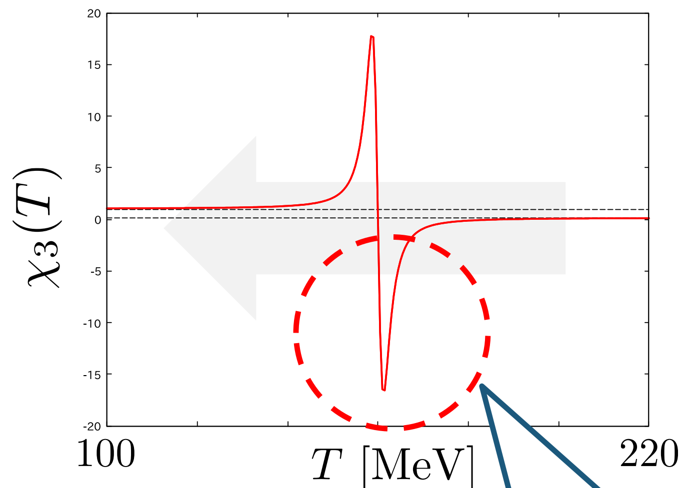
negative
Asakawa, Ejiri, MK
(2009)

Time Evolution: Near CP

Murata, MK
in preparation







negative
Asakawa, Ejiri, MK (2009)

- Sharp peak survives as a remnant of criticarity
- Negative 3rd cumulant is buried by the diffusion.

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Nonaka, Esumi, MK, to appear soon

Efficiency / Efficiency Correction

Experimental detectors have miscounting & misidentification...

True particle # distribution

$P(N)$

Detector

Observed particle # distribution

$\tilde{P}(n)$

We need cumulants of this distribution.

We can measure this distribution only.

True cumulants have to be reconstructed from experimental data.

Response Matrix

Fixed injected particle #
N

Detector

Probability observing n particles
$\mathcal{R}(n; N)$

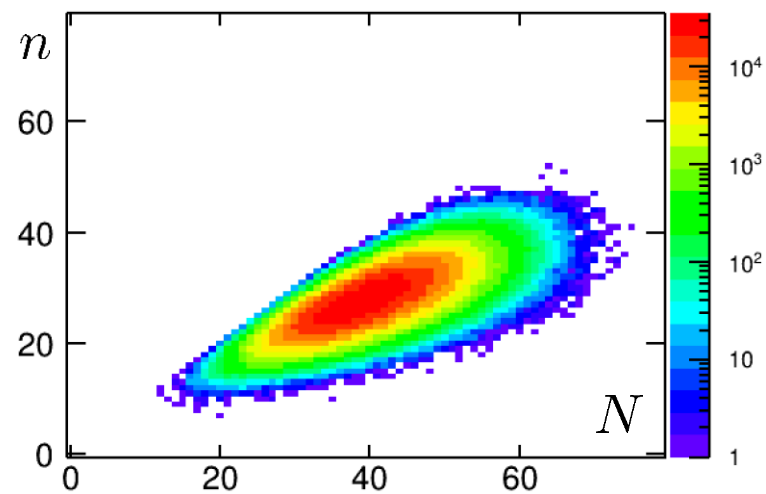
If each measurement is uncorrelated…

$$\tilde{P}(n) = \sum_N \mathcal{R}(n; N) P(N)$$

$\mathcal{R}(n; N)$

Response matrix

model of detector



Efficiency Correction

Binomial model

Independence of efficiency loss for individual particles

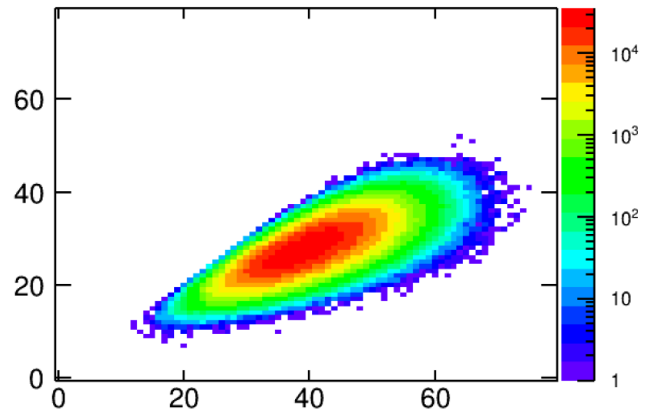
➡ $\mathcal{R}(n;N)$:Binomial distribution func.

Cumulants of n can be represented by those of N

Bialas, Peschanski (1986); MK, Asakawa (2012); Bzdak, Koch (2012)

Unfolding

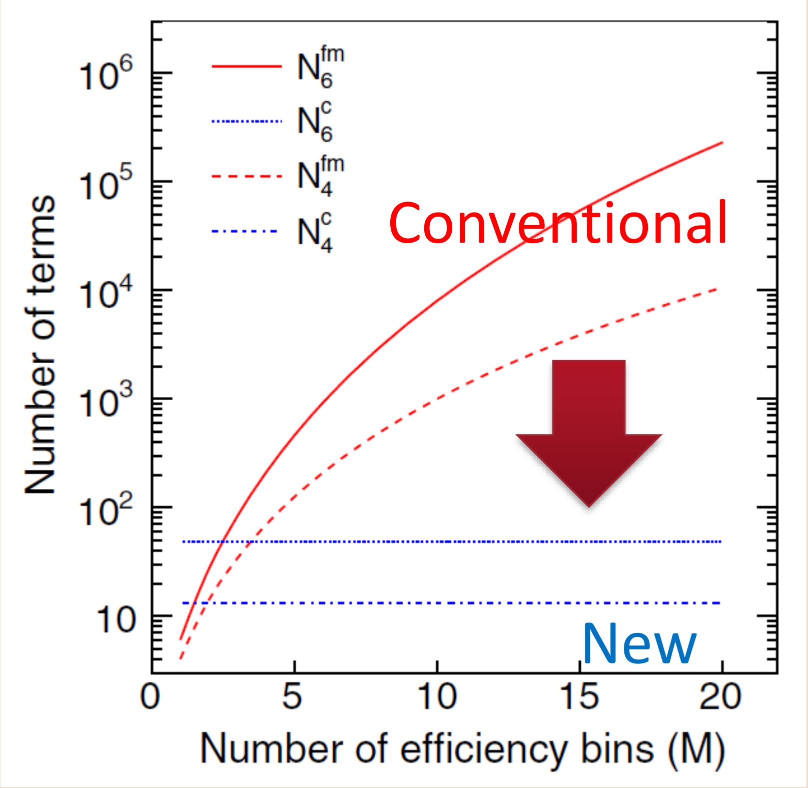
- Construct true distribution func.
- Numerically demanding



Binomial Model

An efficient algorithm for multi-variable system
Nonaka, MK, Esumi, PRC2017

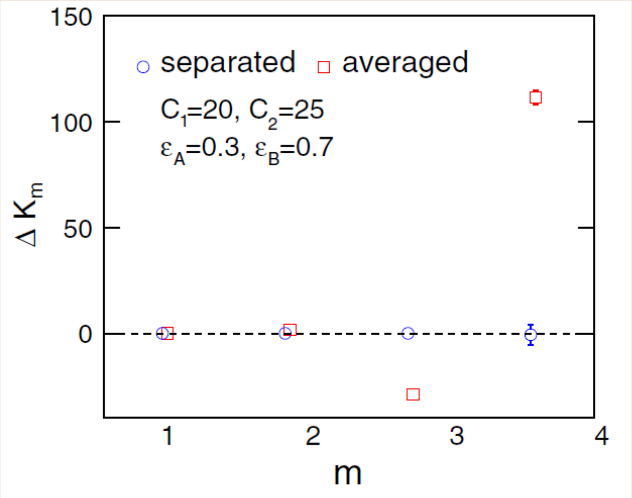
Numerical Cost



A Toy Model Test

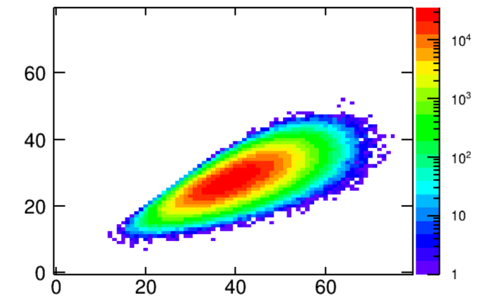
source → ε_A=0.3
source → ε_B=0.7

deviation from true cumulant



General Efficiency Correction

Nonaka, Esumi, MK, to appear soon



Use moments of R(n;N)

$$\langle n^m \rangle_R = \sum_n n^m \mathcal{R}(n;N)$$

Taylor expand

$$\langle n^m \rangle_R = r'_{m0} + r'_{m1}(N - N_0) + r'_{m2}(N - N_0)^2 + \cdots$$

Relation b/w true and observed moments

$$\begin{bmatrix} \langle n \rangle \\ \langle n^2 \rangle \\ \vdots \end{bmatrix} = R \begin{bmatrix} \langle N \rangle \\ \langle N^2 \rangle \\ \vdots \end{bmatrix} \Rightarrow \begin{bmatrix} \langle N \rangle \\ \langle N^2 \rangle \\ \vdots \end{bmatrix} = R^{-1} \begin{bmatrix} \langle n \rangle \\ \langle n^2 \rangle \\ \vdots \end{bmatrix}$$

By truncating the Taylor exp. at *m*th order,
"true" moments up to *m*th order are obtained.

A Toy-Model Analysis

Nonaka, Esumi, MK
to appear soon

Binomial model w/ multiplicity-dependent efficiency

$$\epsilon(N) = \epsilon_0 + (N - N_{\rm ave})\epsilon'$$

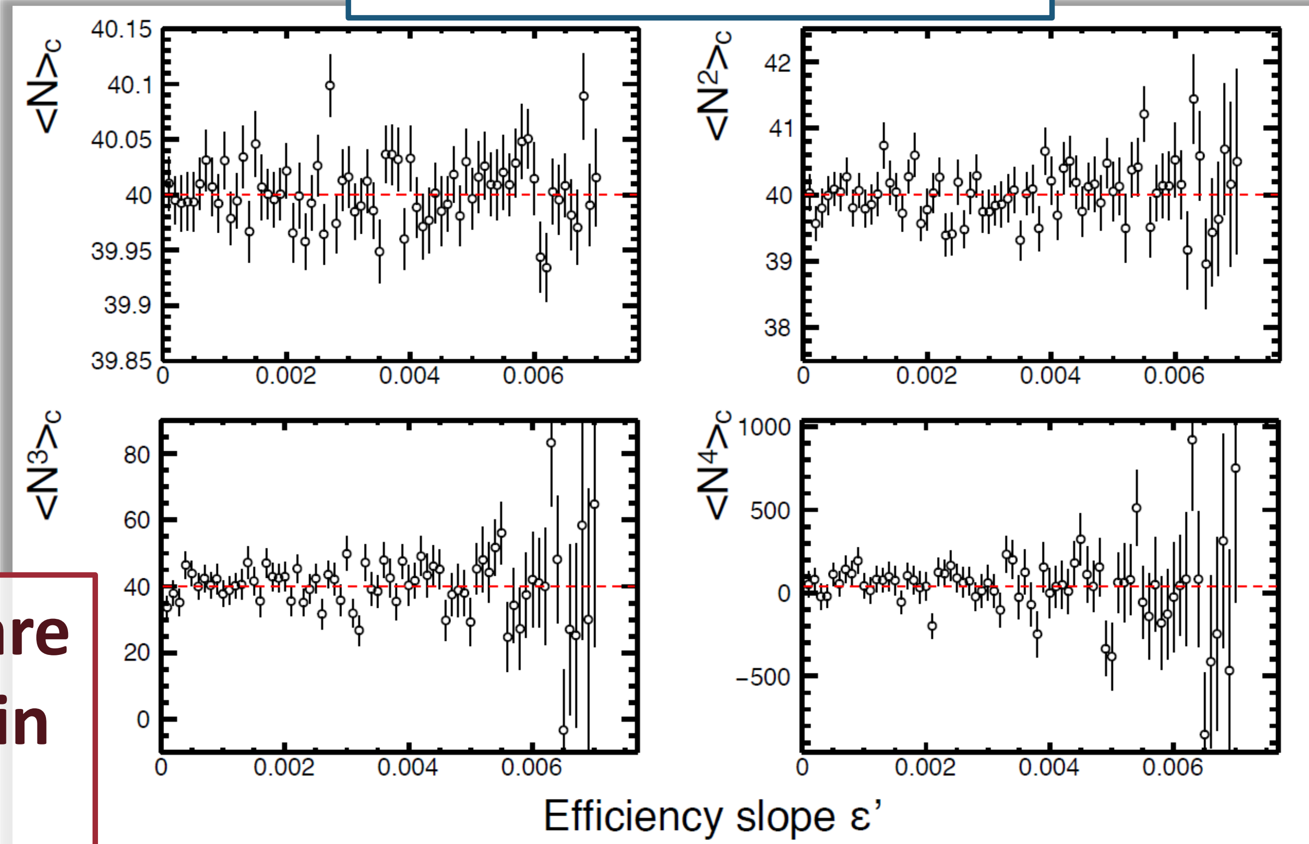
Holtzman, Bzdak, Koch (16)

Input P(N):
Poisson(λ=40)

$\epsilon_0 = 0.7$

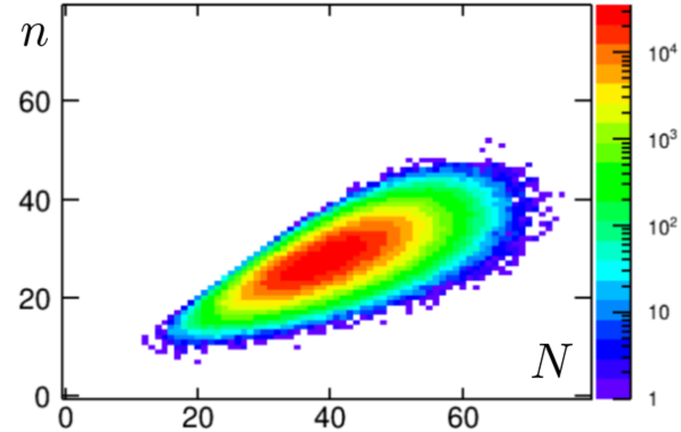
Red:
true cumulant

Reconstructed cumulants



True cumulants are reproduced within statistics!

Comments



- $\langle n^m \rangle_R = \sum_n n^m \mathcal{R}(n;N)$
  can be obtained from R(n;N)

- The truncation has to be well justified.

- Some distributions are automatically truncated.
  - Correct efficiency correction is possible.
  - binomial, hyper-geometric, beta-binomial, … , binomial with fluctuating probability He, Luo, last Friday

- Compared to unfolding method,
  - numerically cheaper and would be more stable
  - origin of error is more apparent

- Extension to multi-variable case is straightforward.

Proton v.s. Baryon Number Cumulants

MK, Asakawa, 2012; 2012

Experiments

proton number cumulants

$$\langle N_p^n \rangle_c$$

Many theories

baryon number cumulants

$$\langle N_{\mathrm{B}}^n \rangle_c$$

measurement with
50% efficiency loss

- The difference would be large.
- Reconstruction of $\langle N_B^n \rangle_c$ is possible using the binomial model.
- The use of binomial model is justified by "isospin randomization."
- And the loss due to momentum cut…

Summary

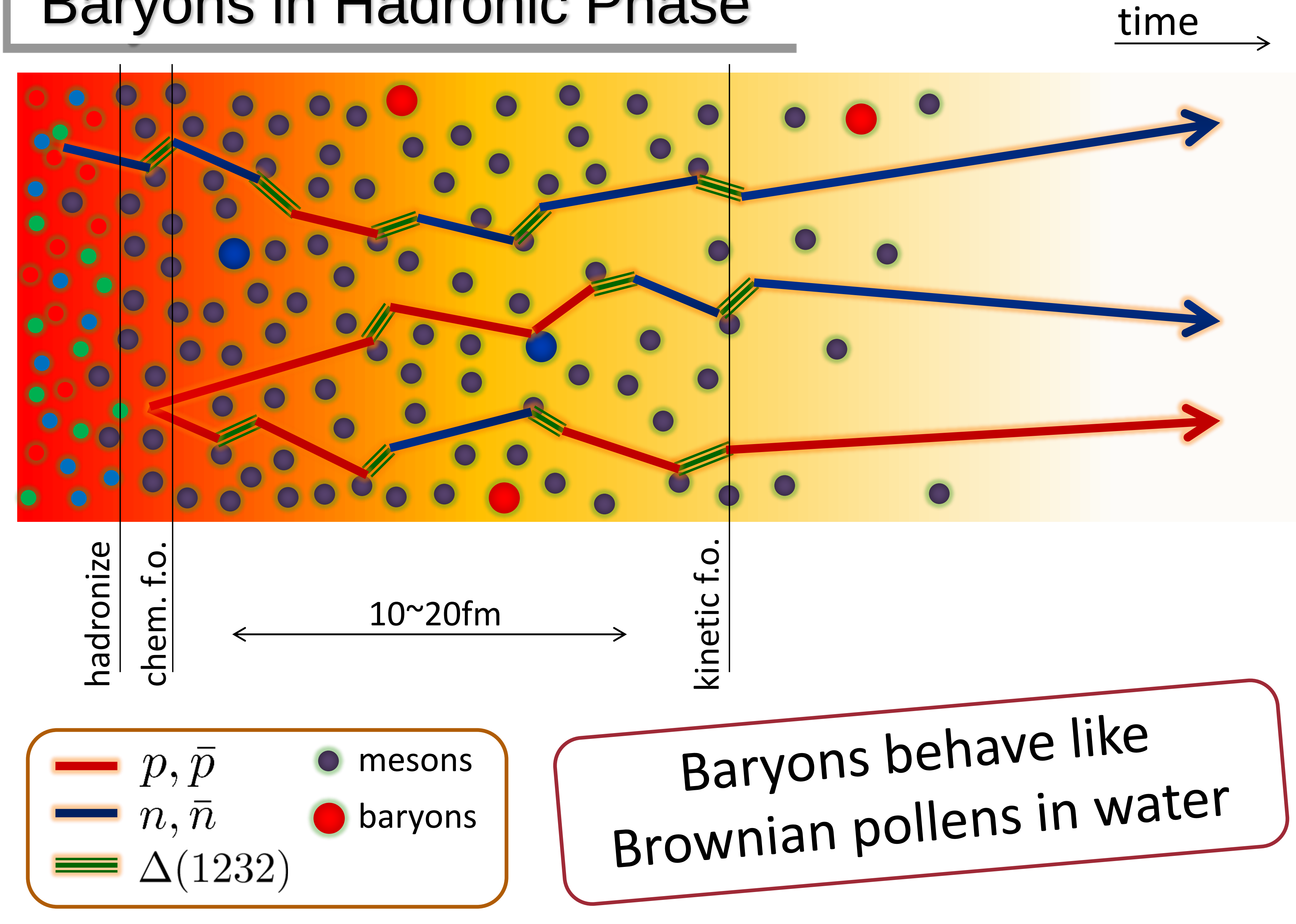
- Understanding dynamical aspects of fluctuations is important!
- Plenty information in Δy dependence of cumulants:
  - Higher order cumulants can behave non-monotonically.
  - → can be used for constraining parameters.
  - Non-monotonicity in 2^{nd} order cumulant is an experimental signal for the existence of the QCD-CP.

- A general algorithm for the efficiency correction:
  - Correct reconstruction for non-binomial response.
  - Smaller numerical cost than unfolding methods.

- Reonstructing baryon # cumulants is important!

- Let’s continue the search for the QCD-CP!

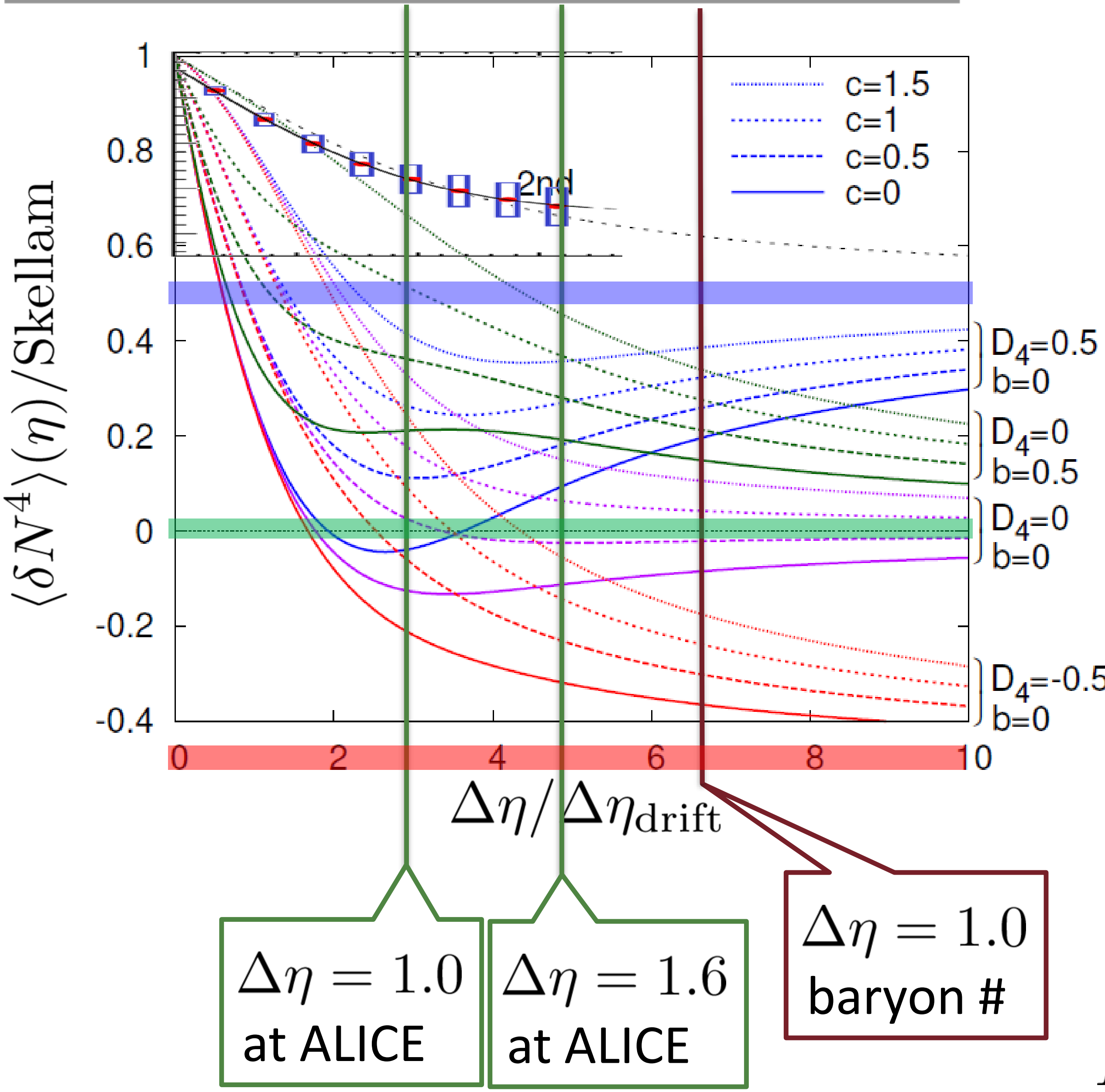
Baryons in Hadronic Phase



Baryons behave like
Brownian pollens in water

Δη Dependence: 4th order

MK, NPA(2015)



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

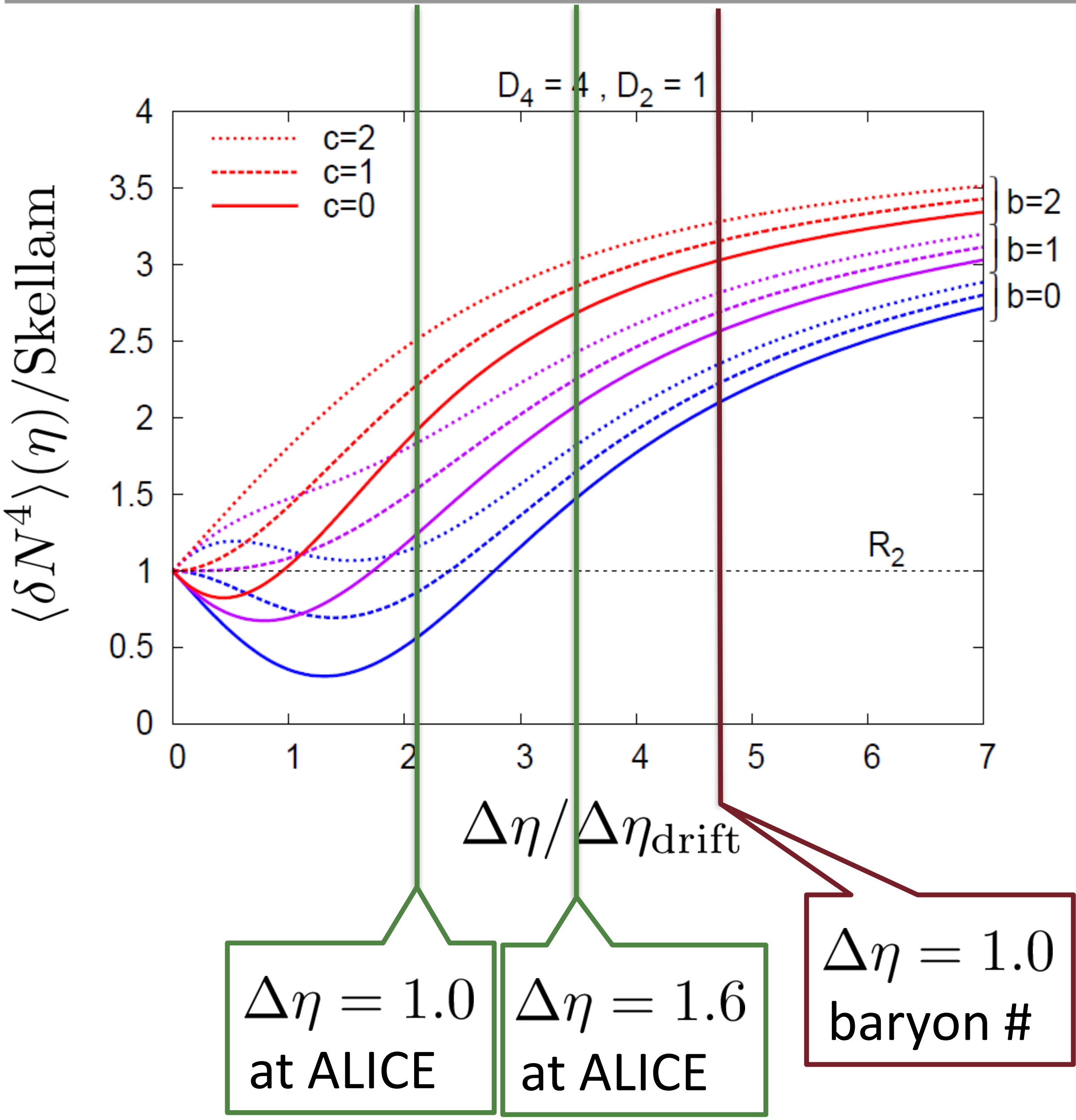
$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

$$D \sim M^{-1}$$

4^{th} order : w/ Critical Fluctuation



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

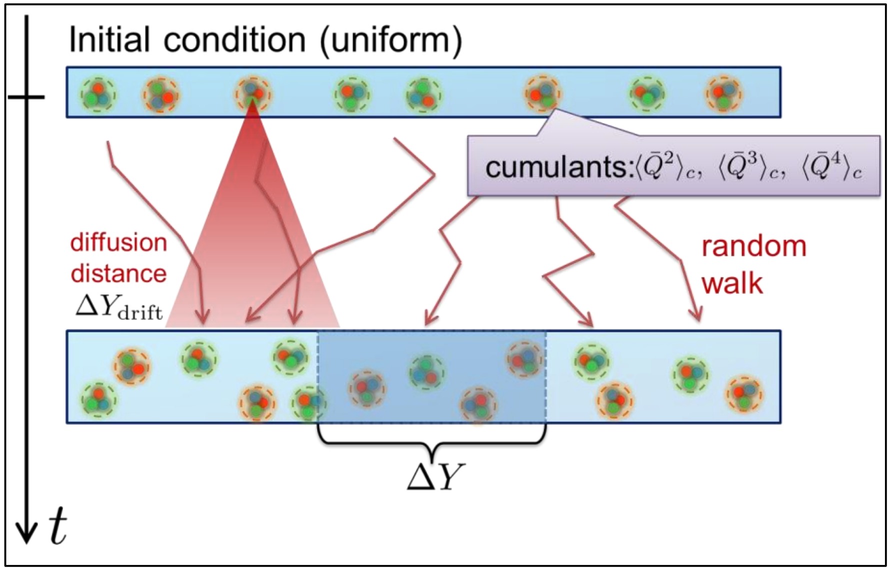
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

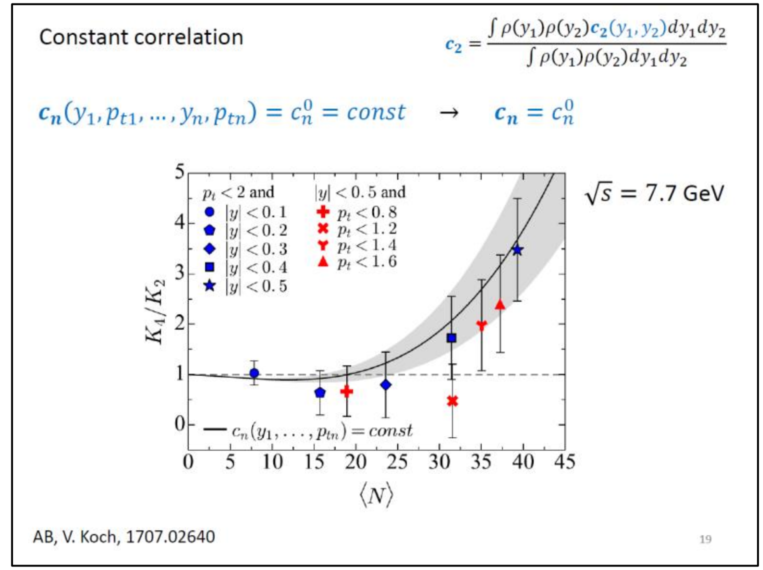
$$D \sim M^{-1}$$

Translating Languages

Brownian particle model



From Bzdak's talk



$$\langle n^m \bar{n}^{\bar{m}}\rangle_{\rm fc} = \kappa_{m\bar{m}} \Delta y F_{m+\bar{m}}(\Delta y/d)$$

$$c^0_{m\bar{m}} = \frac{1}{2}\frac{\partial^2}{\partial \Delta y^2}\langle n^m \bar{n}^{\bar{m}}\rangle_{\rm fc}\Big|_{\Delta y\to 0} = \kappa_{m\bar{m}}\frac{\partial}{\partial \Delta y}F_{m+\bar{m}}(\Delta y/d)\Big|_{\Delta y\to 0}$$

$$= \frac{\kappa_{m\bar{m}}}{d}\frac{1}{\sqrt{(m+\bar{m})(2\pi)^{m+\bar{m}-1}}}$$

$\kappa_{m\bar{m}}$: F cumulants at initial condition

d : diffusion distance