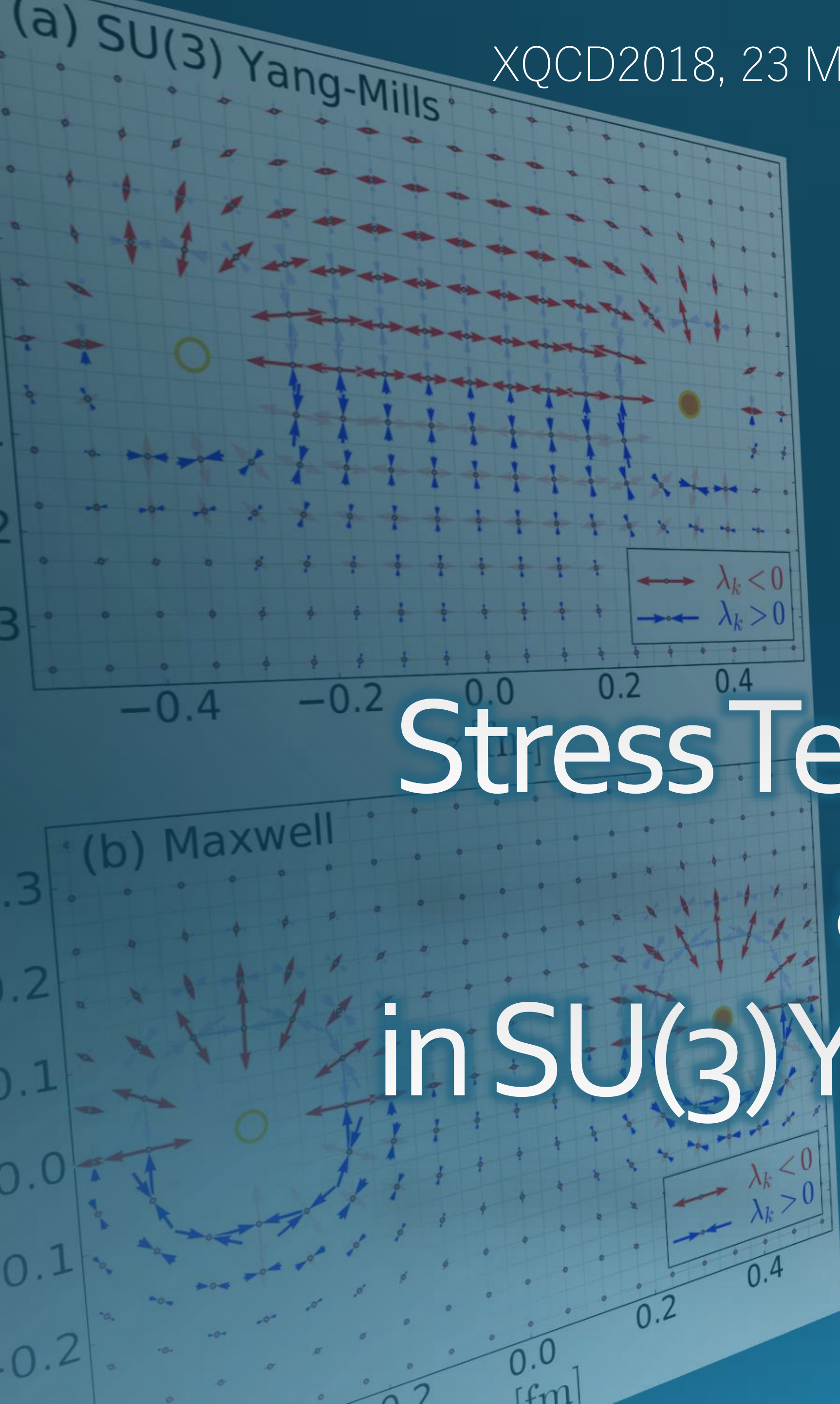
XQCD2018, 23 May, 2018, Frankfurt, Germany



Masakiyo Kitazawa
for FlowQCD Collaboration

Stress Tensor Distribution around Flux Tube in SU(3) Yang-Mills Theory

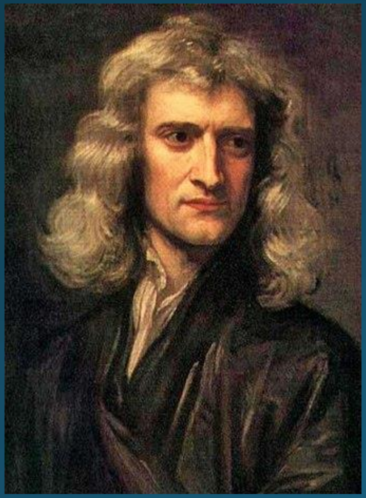
Yanagihara+ (FlowQCD Collab.)
arXiv:1803.01234

Force

Force

$$F = -G\frac{m_1 m_2}{r^2}$$

$$F = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$



Newton

Action-at-a-distance

Force

Faraday

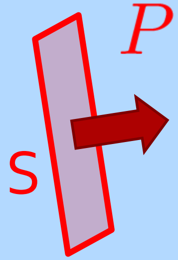
Local interaction

"Action through medium"

Stress = Force per Unit Area

Stress = Force per Unit Area

Pressure

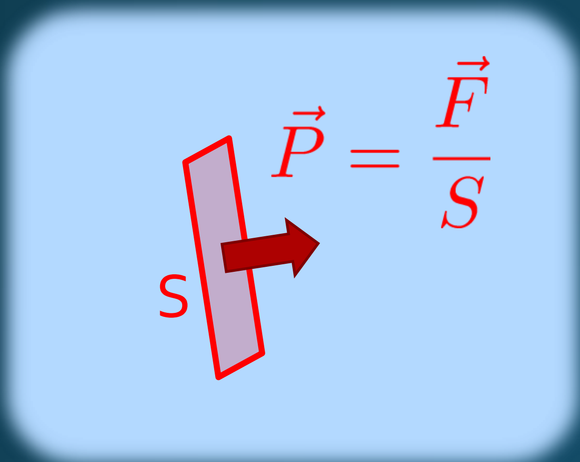


$$\vec{P} = \frac{\vec{F}}{S}$$

$$\vec{P} = P\vec{n}$$

Stress = Force per Unit Area

Pressure

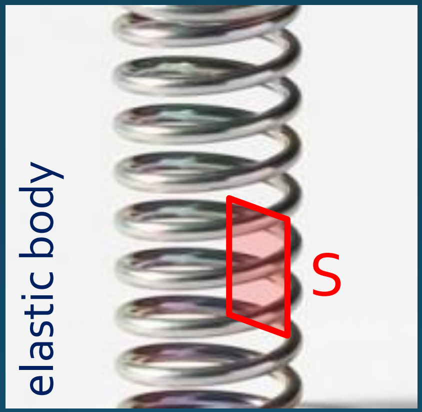


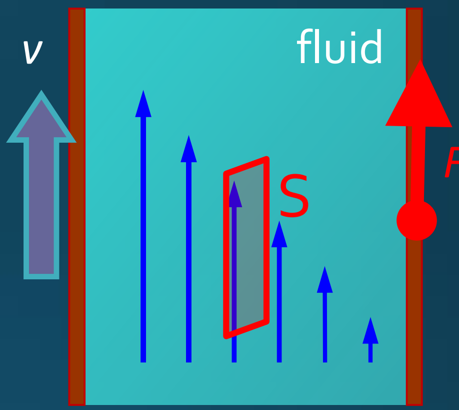
$$\vec{P} = P\vec{n}$$

In thermal medium

$$T_{ij} = P\delta_{ij}$$

Generally, F and n are not parallel





$$\frac{F_i}{S} = \sigma_{ij} n_j$$

Stress Tensor

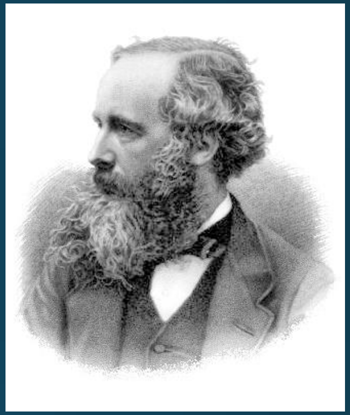
$$\sigma_{ij} = -T_{ij}$$

Landau
Lifshitz

Maxwell Stress

(in Maxwell Theory)

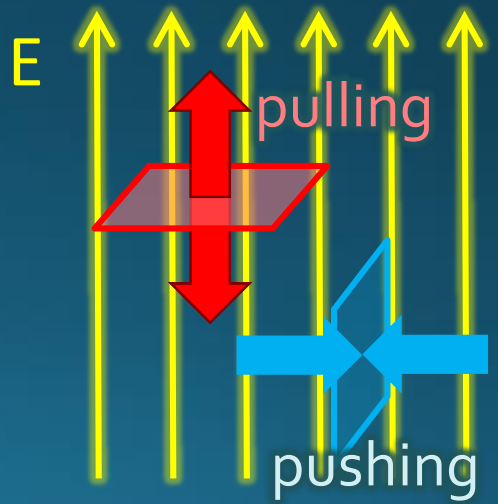
$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$



$$\vec{E} = (E, 0, 0)$$

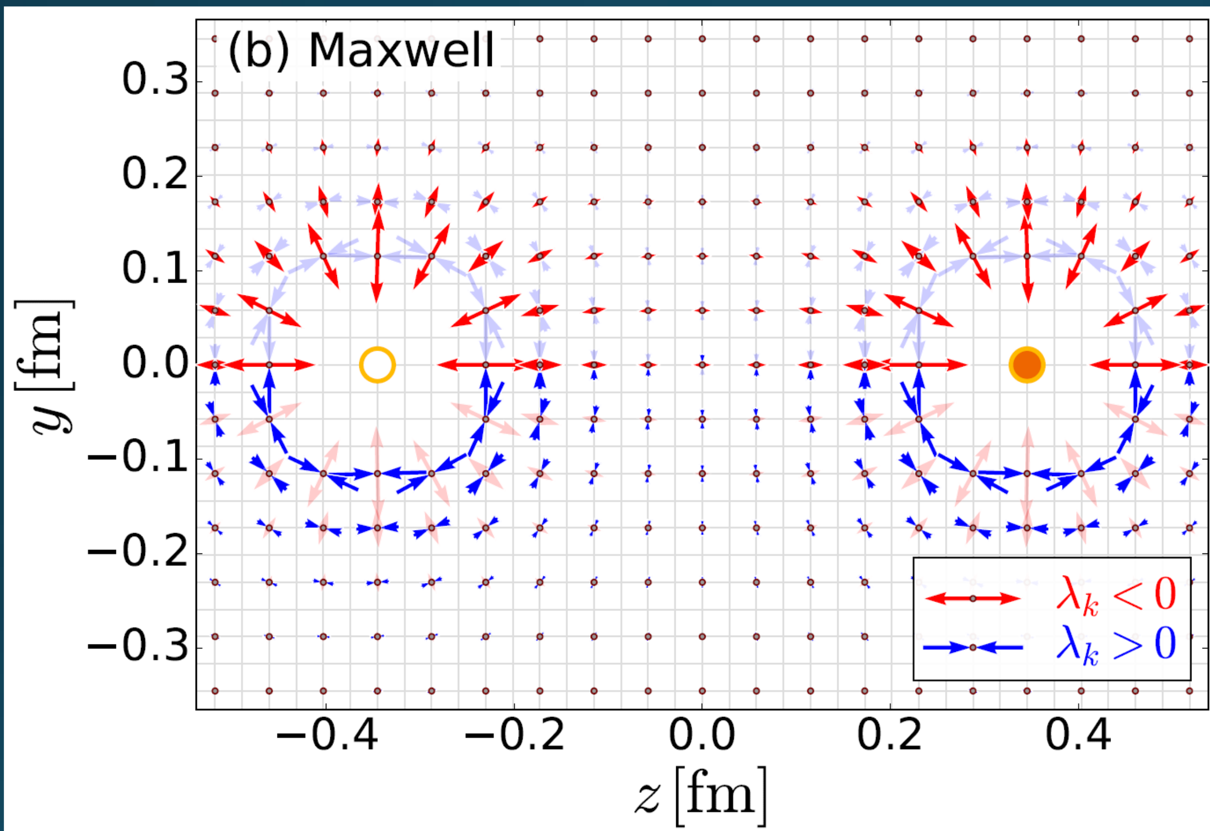
$$T_{ij} = \begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: Pulling
- Vertical to field: Pushing



Maxwell Stress

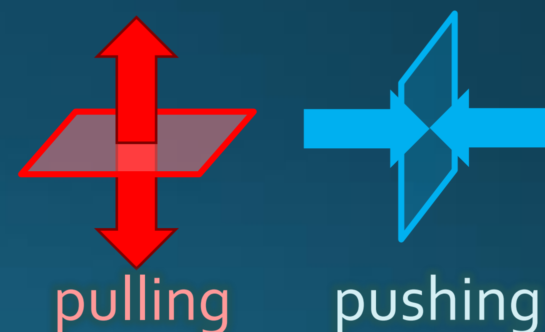
(in Maxwell Theory)



$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$$(k = 1, 2, 3)$$

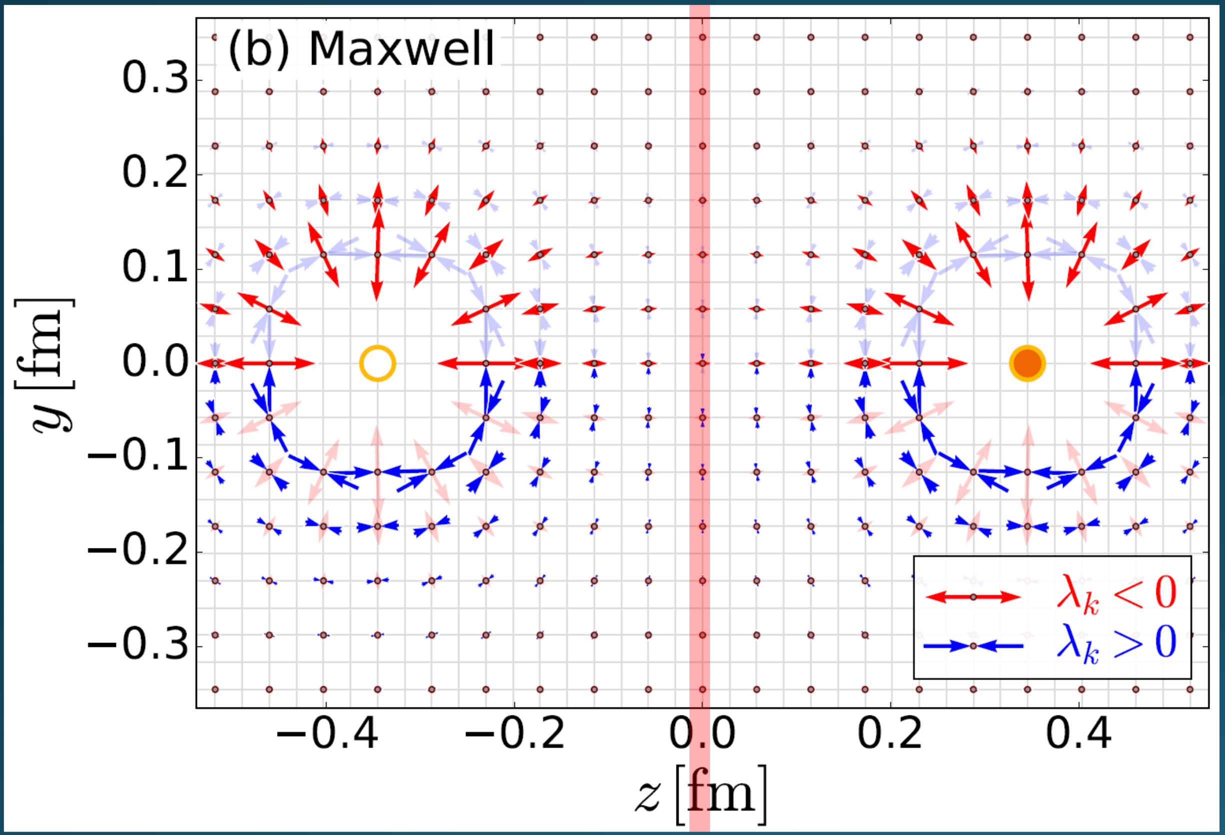
length: $\sqrt{|\lambda_k|}$



- ☐ Distortion of field, line of the force
- ☐ Propagation of the force as local interaction
- ☐ Absolute value of the force between sources

Maxwell Stress

(in Maxwell Theory)



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- ☐ Distortion of field, line of the force
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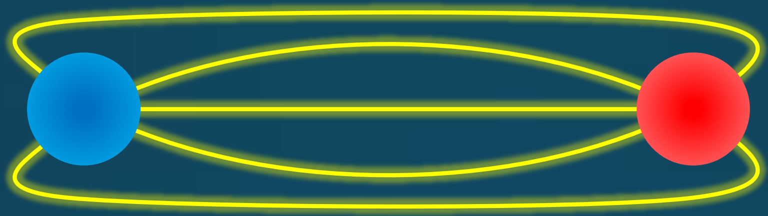
- ☐ Absolute value of the force between sources

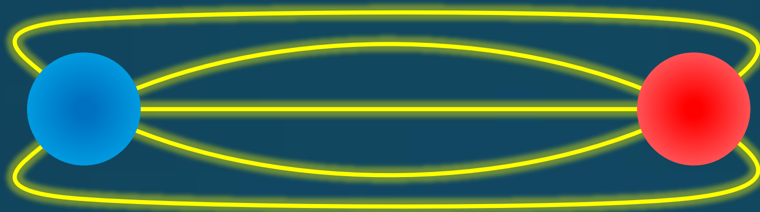
Quark—Anti-quark system

Formation of the flux tube → confinement



Quark—Anti-quark system

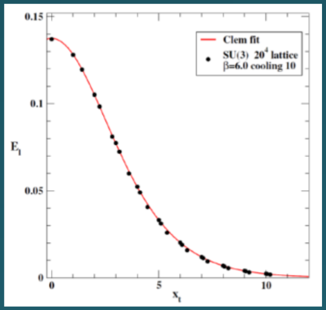
Formation of the flux tube → confinement



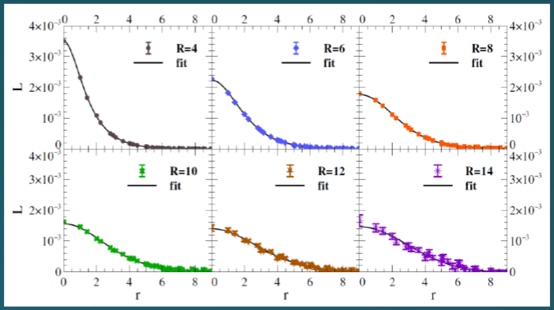
Previous Studies on Flux Tube

- ☐ Potential
- ☐ Action density
- ☐ Color-electric field

so many studies…

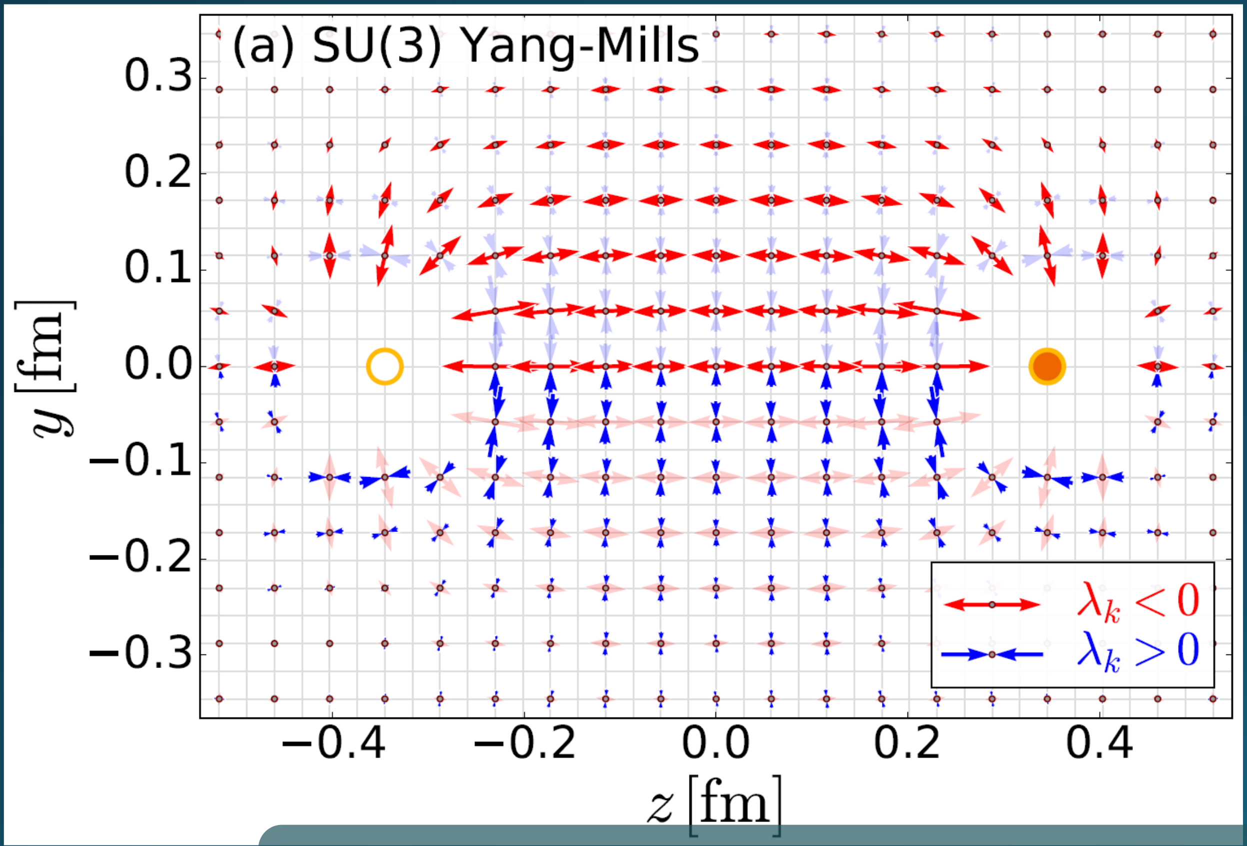


Cea+ (2012)



Cardoso+ (2013)

Spatial Distribution of Stress Tensor
in the QQ System



Lattice simulation
SU(3) Yang-Mills
a=0.029 fm
R=0.69 fm
t/a^2=2.0

$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$
$$(k = 1, 2, 3)$$

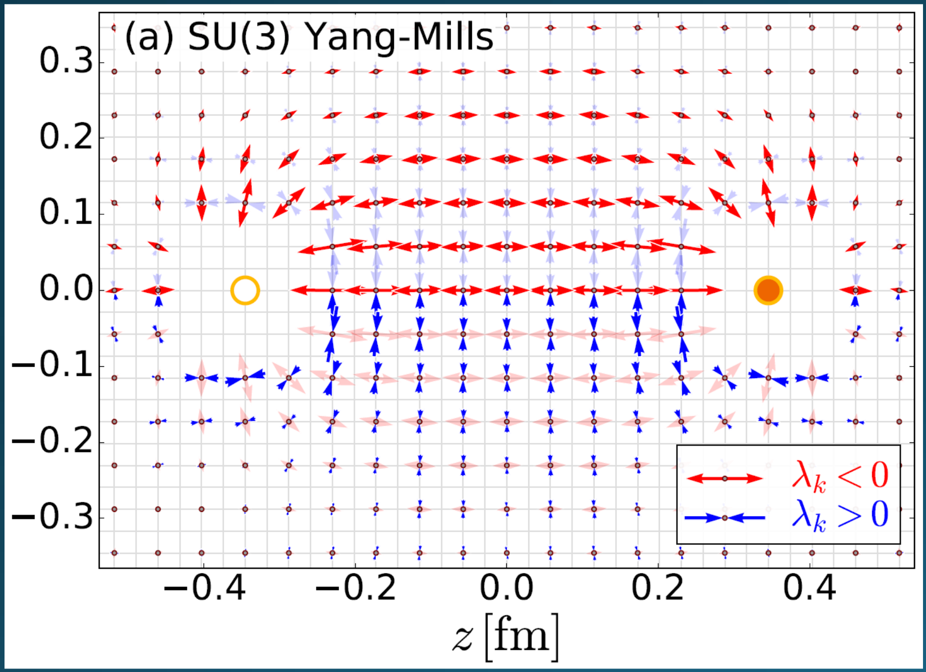
length: $\sqrt{|\lambda_k|}$

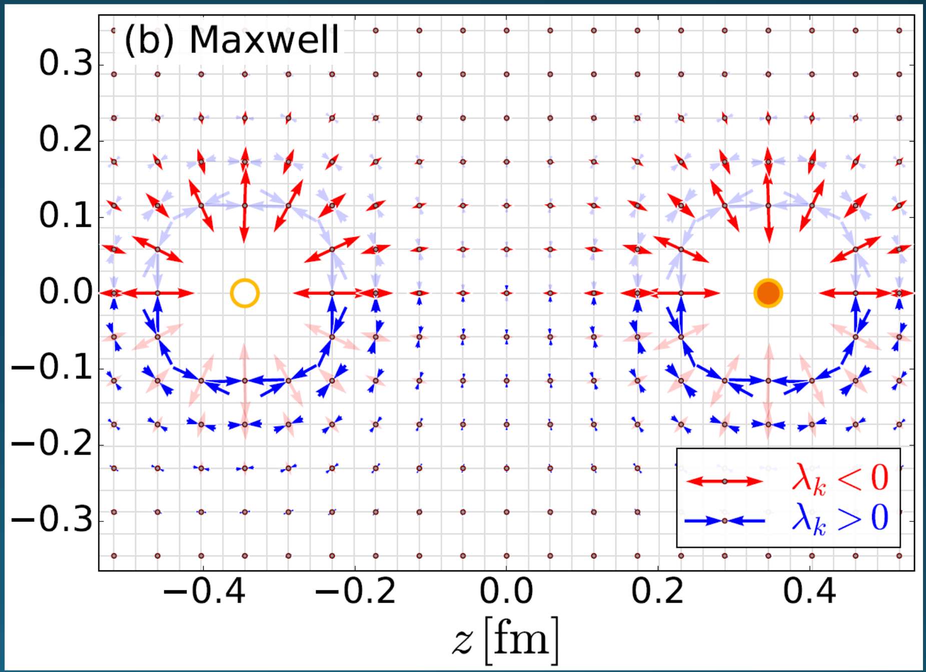
- ☐ Clearly gauge invariant
- ☐ Distortion of field, line of the force
- ☐ Propagation of the force as local interaction
- ☐ Absolute value of the force between sources

Comparison: SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)

Maxwell
(classical)





Propagation of the force is clearly different
in YM and Maxwell theories!

Energy-Momentum Tensor on the Lattice and Gradient Flow

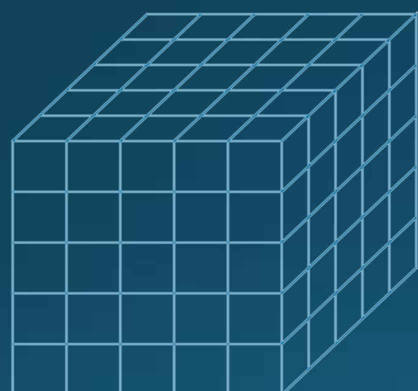
$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

energy
momentum
pressure
stress

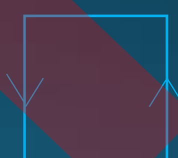
$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial

because of the explicit breaking of Lorentz symmetry



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

② Its measurement is extremely noisy

due to high dimensionality and etc.

(Yang-Mills) Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

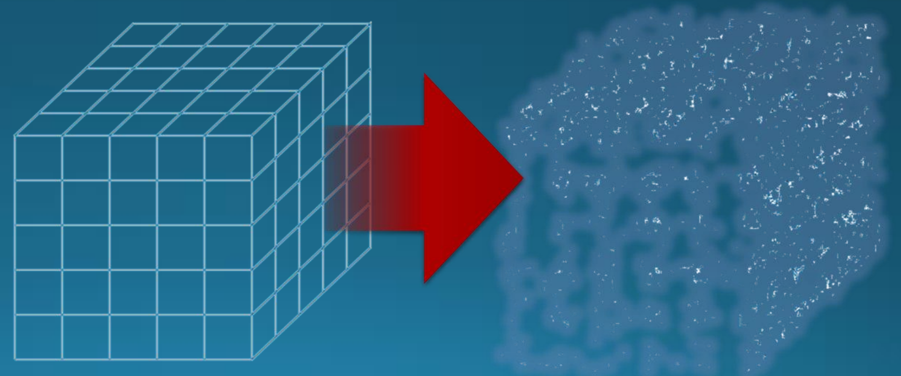
$$A_\mu(0,x) = A_\mu(x)$$

t: “flow time”
dim:[length²]

leading

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- “continuous” cooling/smearing



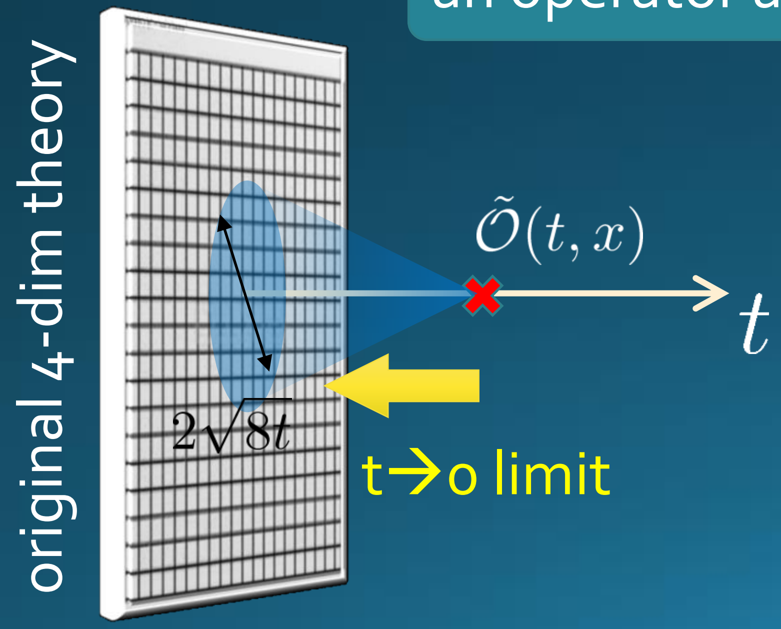
Small Flow-Time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

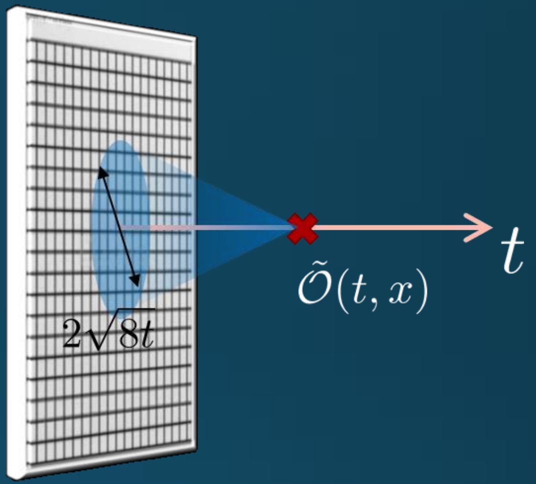
an operator at t>0

remormalized operators of original theory



Constructing EMT 1

$$\tilde{\mathcal{O}}(t,x) \underset{t\to 0}{\longrightarrow} \sum_i c_i(t)\mathcal{O}_i^R(x)$$



- **gauge-invariant dimension 4 operators**

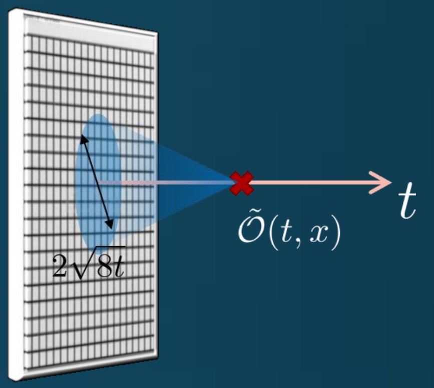
$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

Constructing EMT 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$



Suzuki coeffs.

$$\begin{cases} \alpha_U(t) = g^2\left[1 + 2b_0 s_1 g^2 + O(g^4)\right] \\ \alpha_E(t) = \dfrac{1}{2b_0}\left[1 + 2b_0 s_2 g^2 + O(g^4)\right] \end{cases}$$

$g = g(1/\sqrt{8t})$

$s_1 = 0.03296\ldots$

$s_2 = 0.19783\ldots$

Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\text{subt.}}\right]$$

Application to Thermodynamics

FlowQCD, PRD**94**, 114512 (2016)

Conventional
Integral Method

Thermodynamic relations

$$\frac{\partial \ln Z}{\partial a} = \frac{\partial \beta}{\partial a}\frac{\partial \ln Z}{\partial \beta} \sim \frac{\partial \beta}{\partial a}\langle S \rangle$$

$$T\frac{\partial (p/T^4)}{\partial T} = \frac{\varepsilon - 3p}{T^4}$$

Our Approach
Gradient Flow Method

Take expectation values

$$\begin{cases} \varepsilon = \langle T_{00} \rangle \\ p = \langle T_{11} \rangle \end{cases}$$

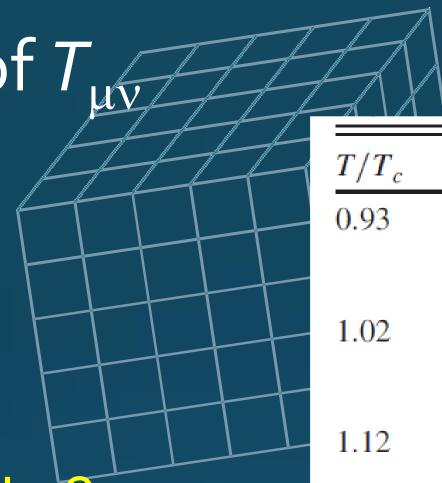
Other progress: shifted boundary Giusti and Pepe (2014~)
Jarzynski's equality Caselle+ (2018); Talk by Nada, Monday

Numerical Simulation

- Expectation values of $T_{\mu\nu}$
- SU(3) YM theory
- Wilson gauge action
- Parameters:
  - N_t = 12, 16, 20-24
  - aspect ratio $5.3<N_s/N_t<8$
  - 1500~2000 configurations
- Scale from gradient flow

$$\rightarrow aT_c \text{ and } a\Lambda_{\rm MS}$$

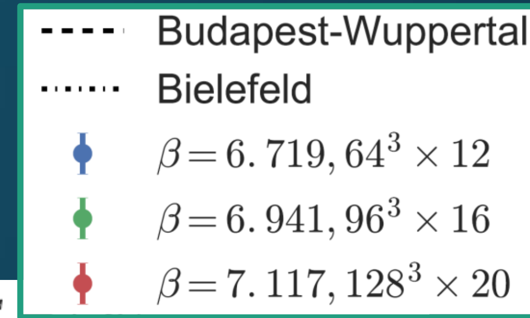
FlowQCD 1503.06516

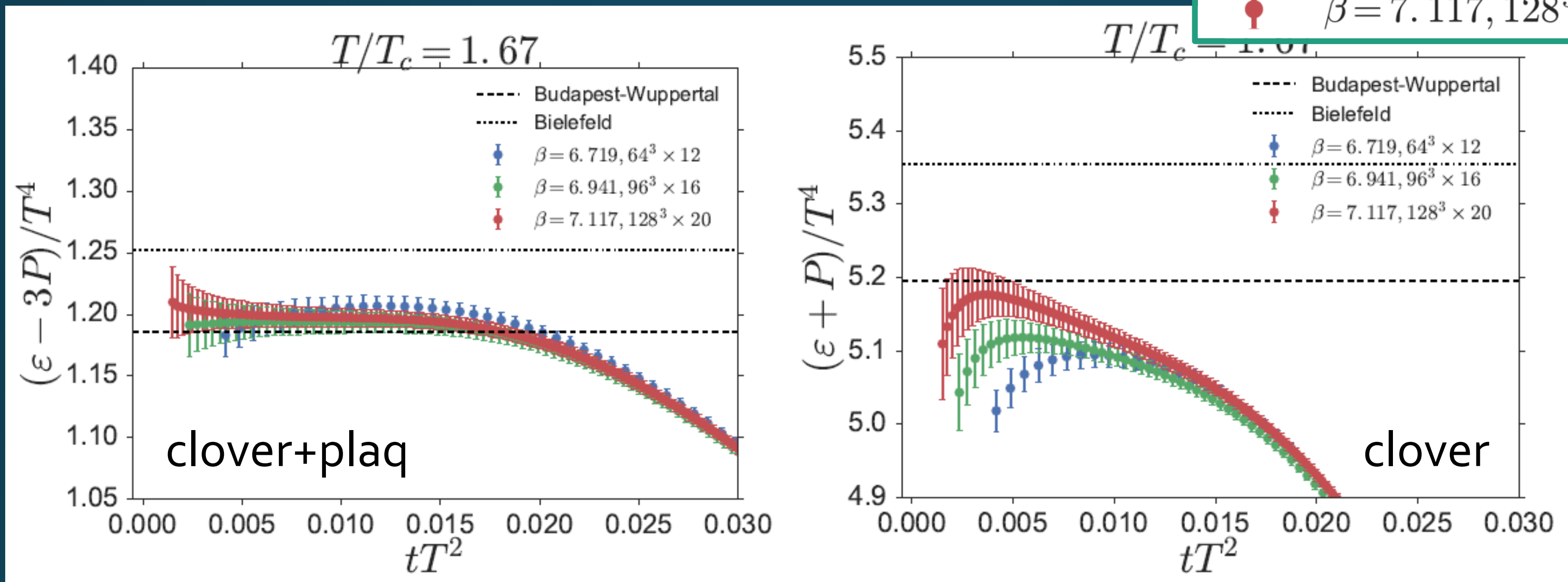


FlowQCD,
PR**D94**, 114512 (2016)

T/T_c	β	N_s	N_τ	Configurations
0.93	6.287	64	12	2125
	6.495	96	16	1645
	6.800	128	24	2040
1.02	6.349	64	12	2000
	6.559	96	16	1600
	6.800	128	22	2290
1.12	6.418	64	12	1875
	6.631	96	16	1580
	6.800	128	20	2000
1.40	6.582	64	12	2080
	6.800	128	16	900
	7.117	128	24	2000
1.68	6.719	64	12	2000
	6.941	96	16	1680
	7.117	128	20	2000
2.10	6.891	64	12	2250
	7.117	128	16	840
	7.296	128	20	2040
2.31	7.200	96	16	1490
	7.376	128	20	2020
	7.519	128	24	1970
2.69	7.086	64	12	2000
	7.317	96	16	1560
	7.500	128	20	2040

t, a Dependence





$\sqrt{8t} < a$: strong discretization effect

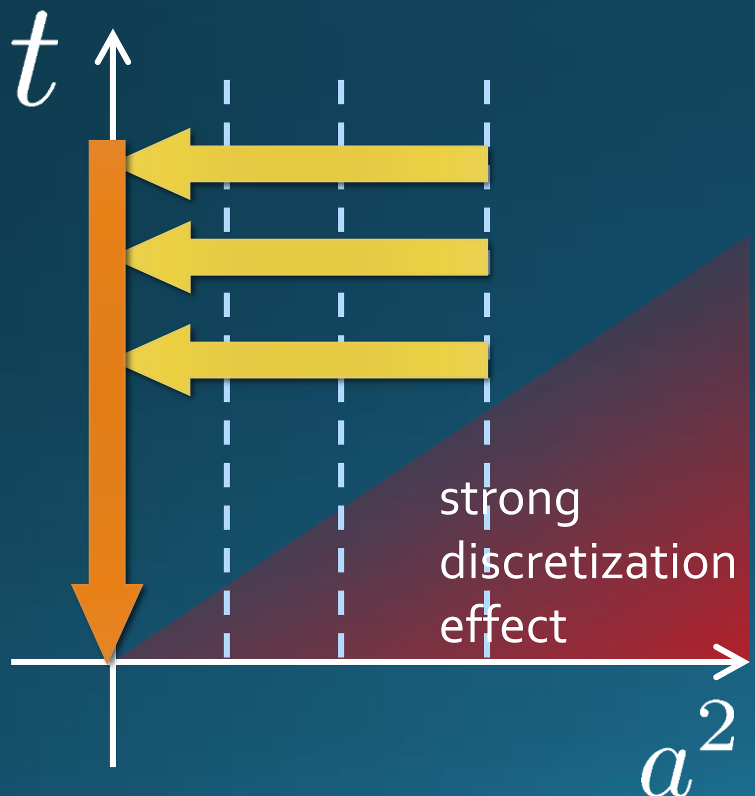
$\sqrt{8t} > 1/(2T)$: over smeared

$a < \sqrt{8t} < 1/(2T)$: Linear t dependence

Double Extrapolation

t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm cont} + C_{\mu\nu} t + D_{\mu\nu}(t)\frac{a^2}{t}$$



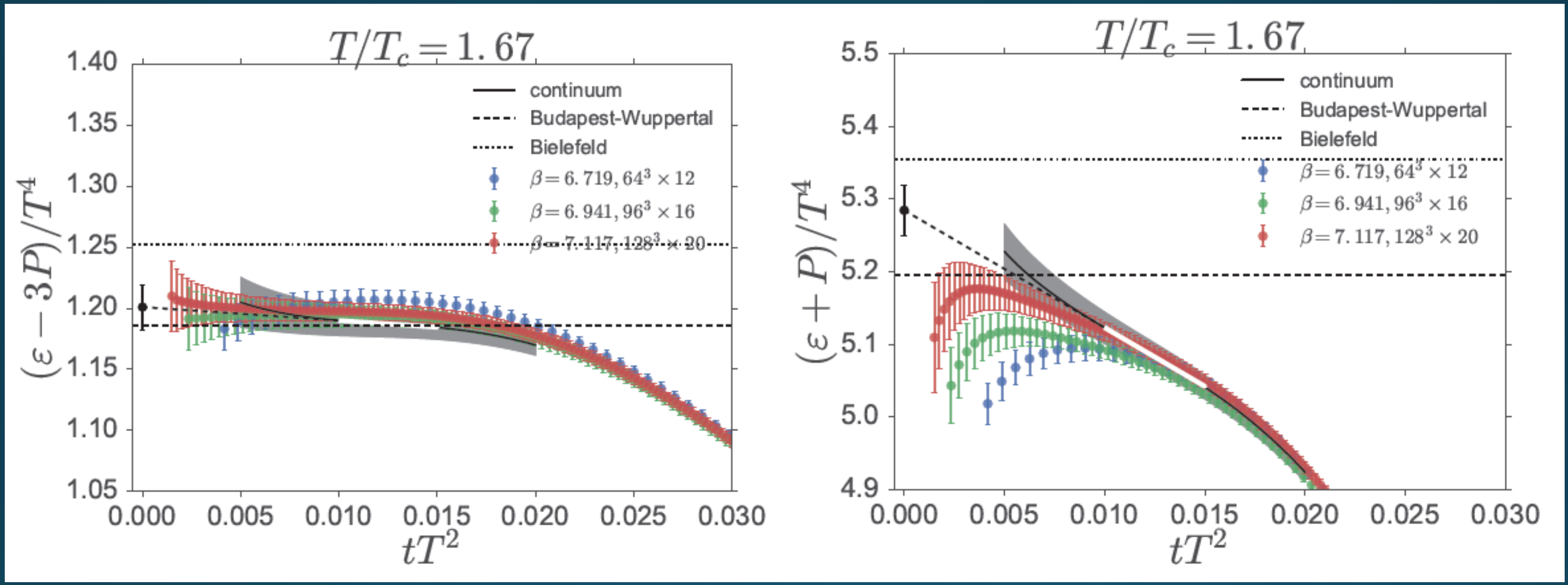
Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

Small t extrapolation

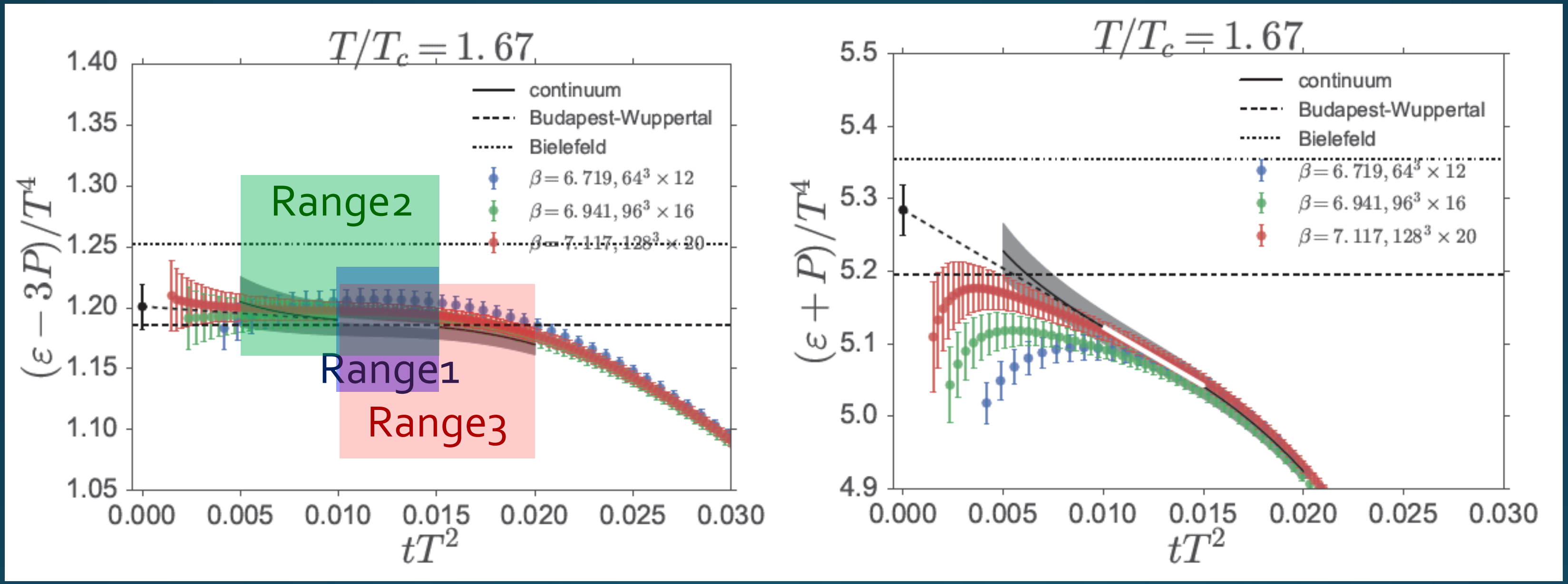
$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C' t$$

Double Extrapolation



Black line: continuum extrapolated

Double Extrapolation

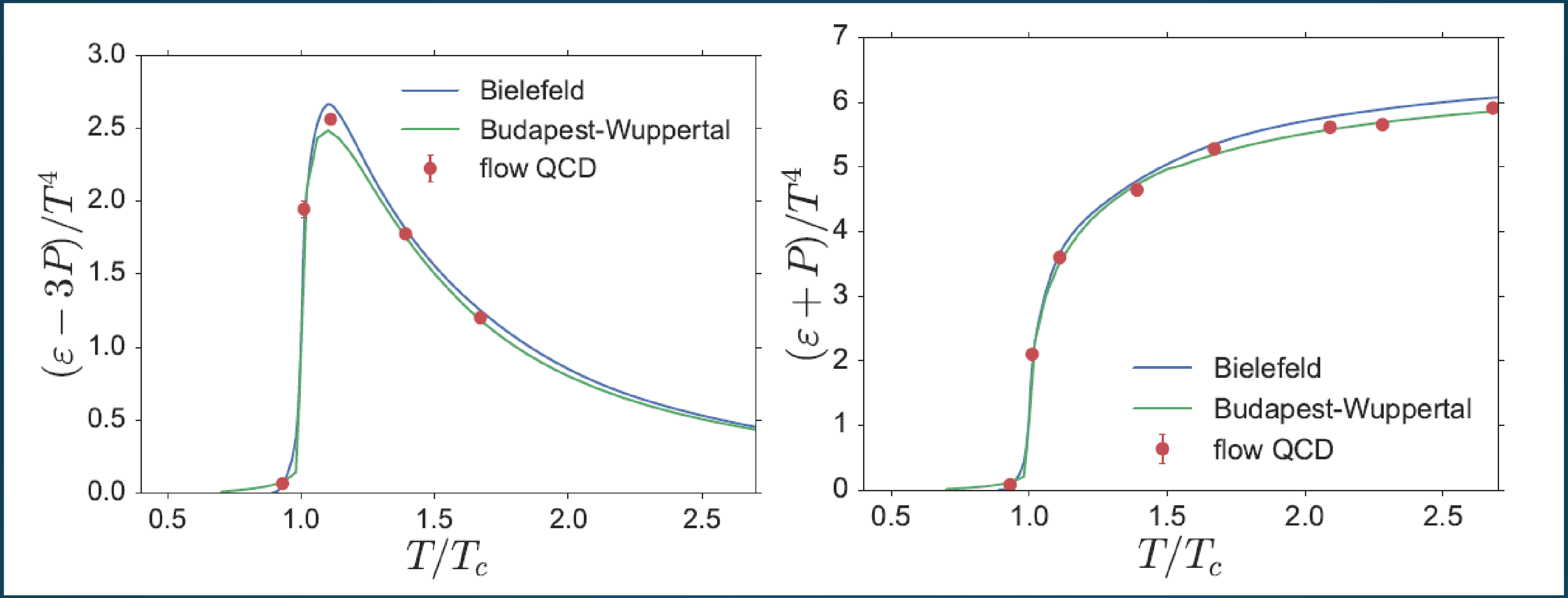


Black line: continuum extrapolated

- Fitting ranges:
  - range-1: $0.01 < tT^2 < 0.015$
  - range-2: $0.005 < tT^2 < 0.015$
  - range-3: $0.01 < tT^2 < 0.02$

Systematic error from the choice of fitting range ≈ statistical error

Temperature Dependence



Error includes

- statistical error
- choice of t range for t→0 limit
- uncertainty in $a\Lambda_{MS}$

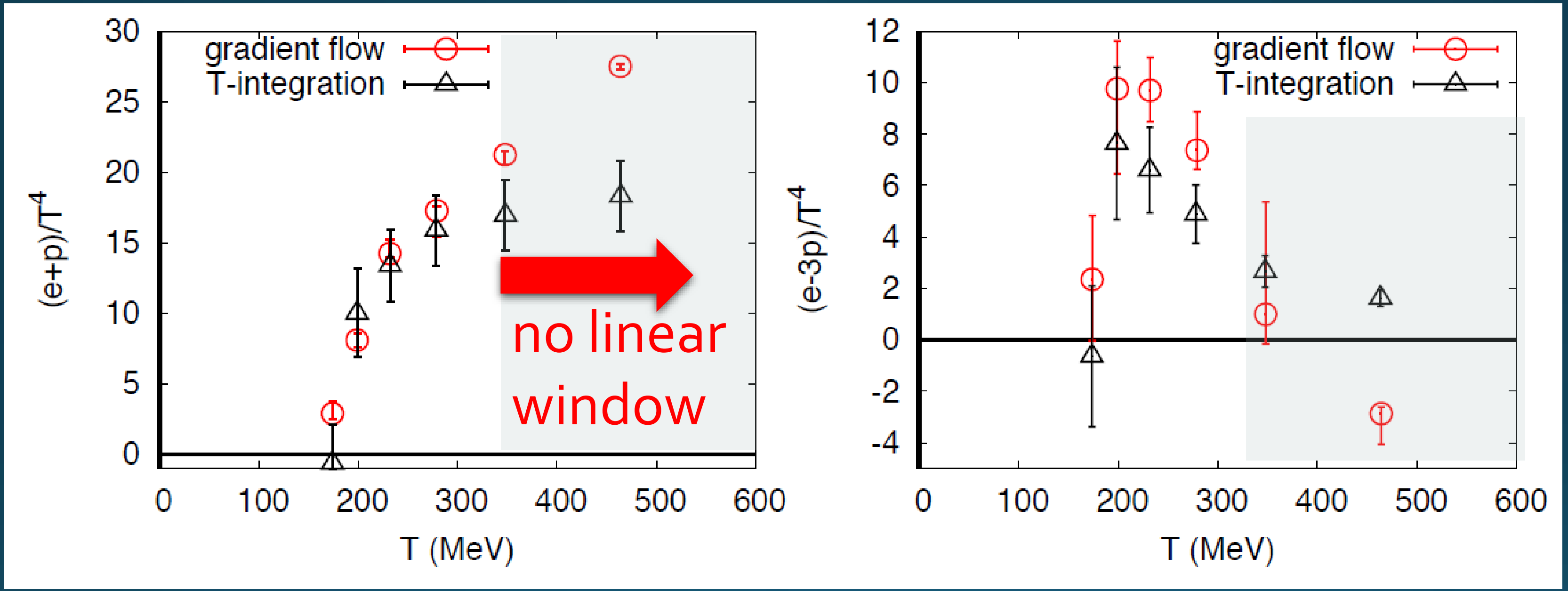
total error <1.5% for T>1.1T_c

- Excellent agreement with integral method
- High accuracy only with ~2000 confs.

See also, talk by Nada, Monday

Full QCD Result

Taniguchi+ (WHOT-QCD), PRD**96**, 014509 (2017)



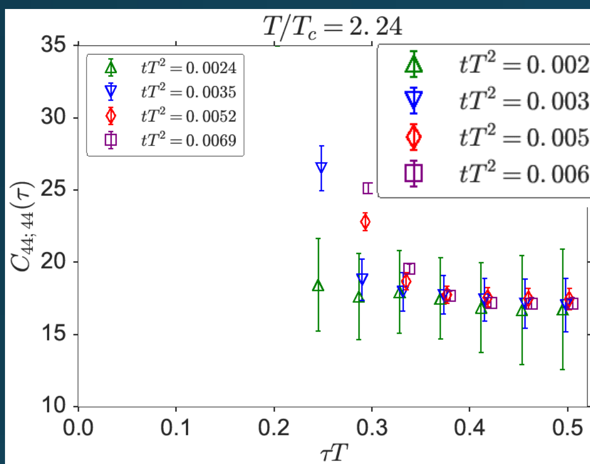
- Agreement with integral method except for N_t=4, 6
- No stable extrapolation for N_t=4, 6
- Suppression of statistical error

Physical mass: Kanaya+ (WHOT-QCD), 1710.10015

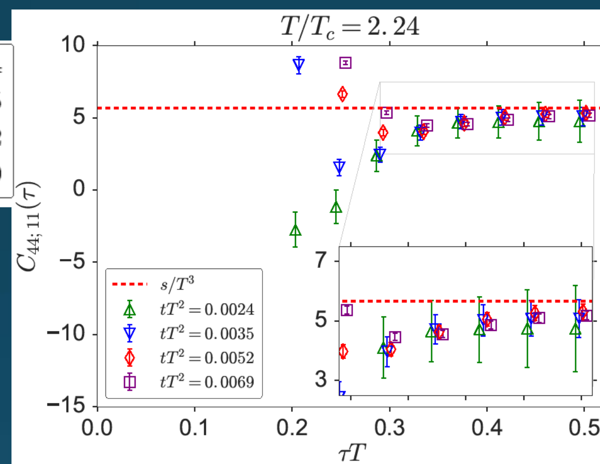
EMT Euclidean Correlator

FlowQCD, PR **D96**, 111502 (2017)

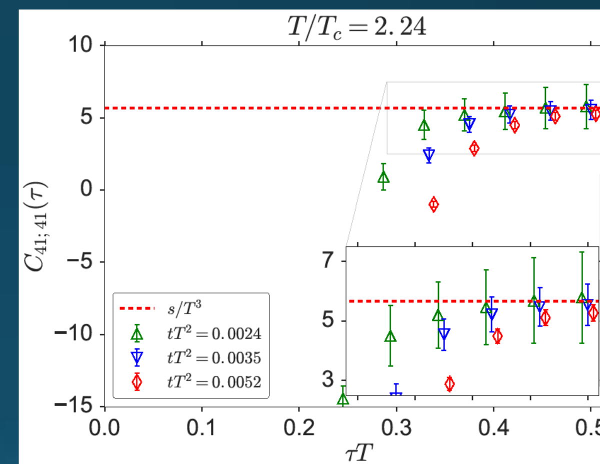
$\langle \bar{T}_{44}(\tau)\bar{T}_{44}(0)\rangle$



$\langle \bar{T}_{44}(\tau)\bar{T}_{11}(0)\rangle$



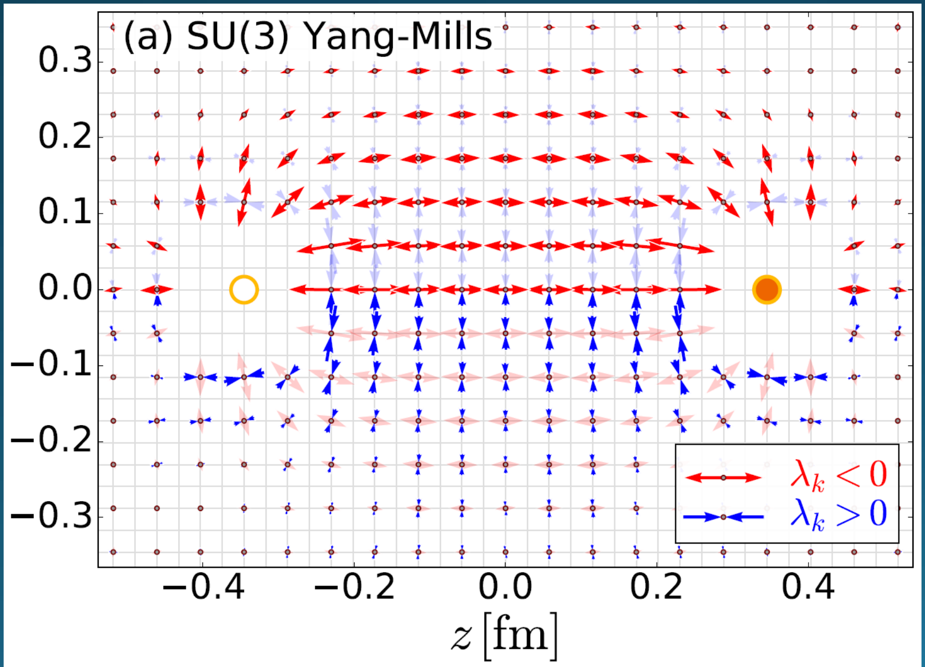
$\langle \bar{T}_{41}(\tau)\bar{T}_{41}(0)\rangle$



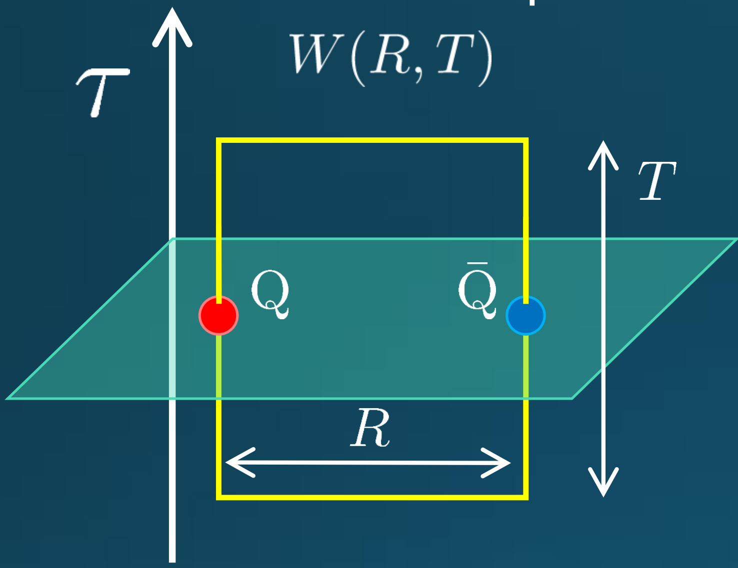
- τ-independent plateau in all channels ➔ conservation law
- Confirmation of linear-response relations
- New analysis of specific heat

$$\frac{s}{T^3} = \frac{\langle \bar{T}_{44}(\tau)\bar{T}_{11}(0)\rangle}{VT^5} = \frac{\langle \bar{T}_{41}(\tau)\bar{T}_{41}(0)\rangle}{VT^5} \qquad c_V = \frac{\langle \bar{T}_{00}^2\rangle}{VT^2}$$

Analysis of Stress Tensor in $Q\bar{Q}$ System



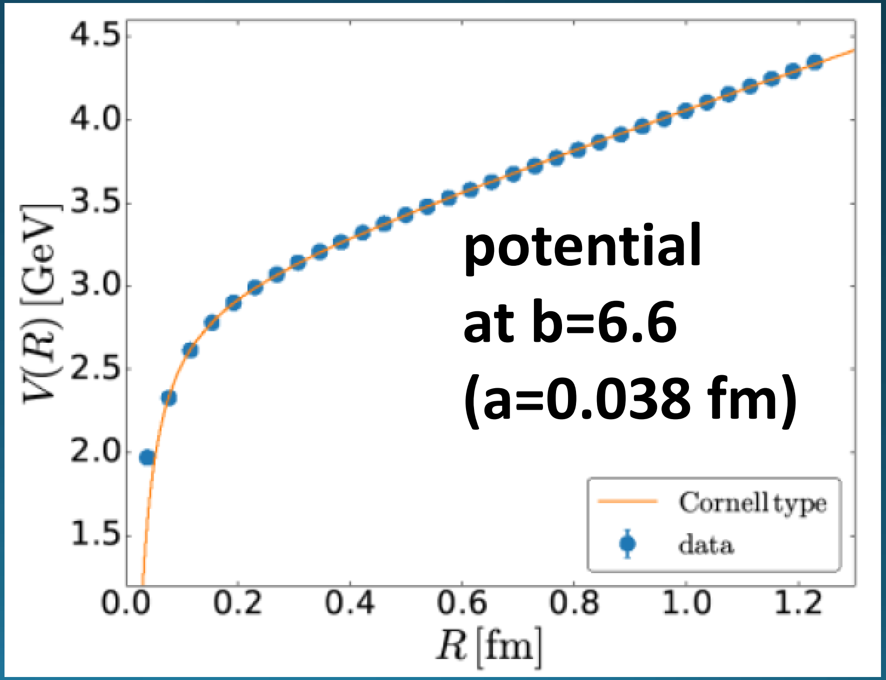
Preparing Static $Q\bar{Q}$



- APE smearing for spatial links
- Multi-hit for temporal links
- No gradient flow for W(R,T)

$$V(R) = -\lim_{T\to\infty} \log\langle W(R,T)\rangle$$

$$\langle O(x)\rangle_{Q\bar{Q}} = \lim_{T\to\infty} \frac{\langle \delta O(x)\delta W(R,T)\rangle}{\langle W(R,T)\rangle}$$



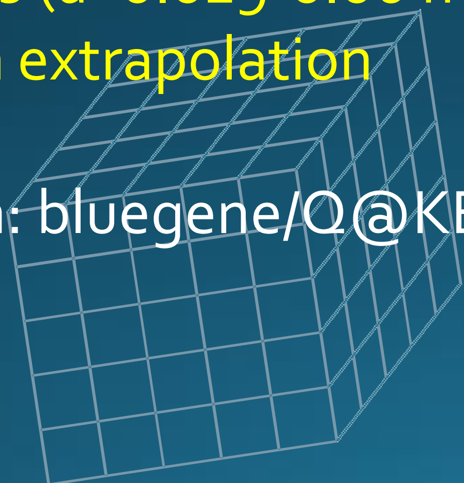
Lattice Setup

- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

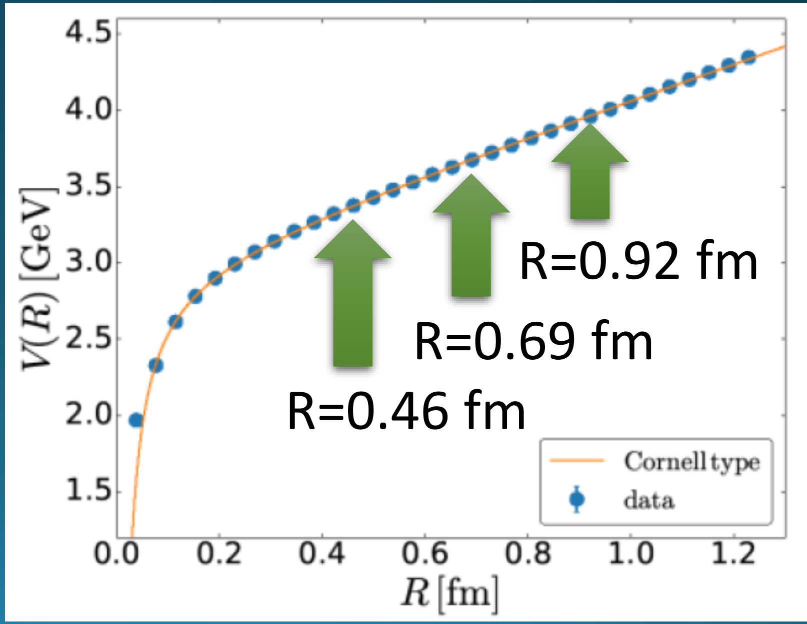
- APE smearing / multi-hit

- fine lattices (a=0.029-0.06 fm)
- continuum extrapolation

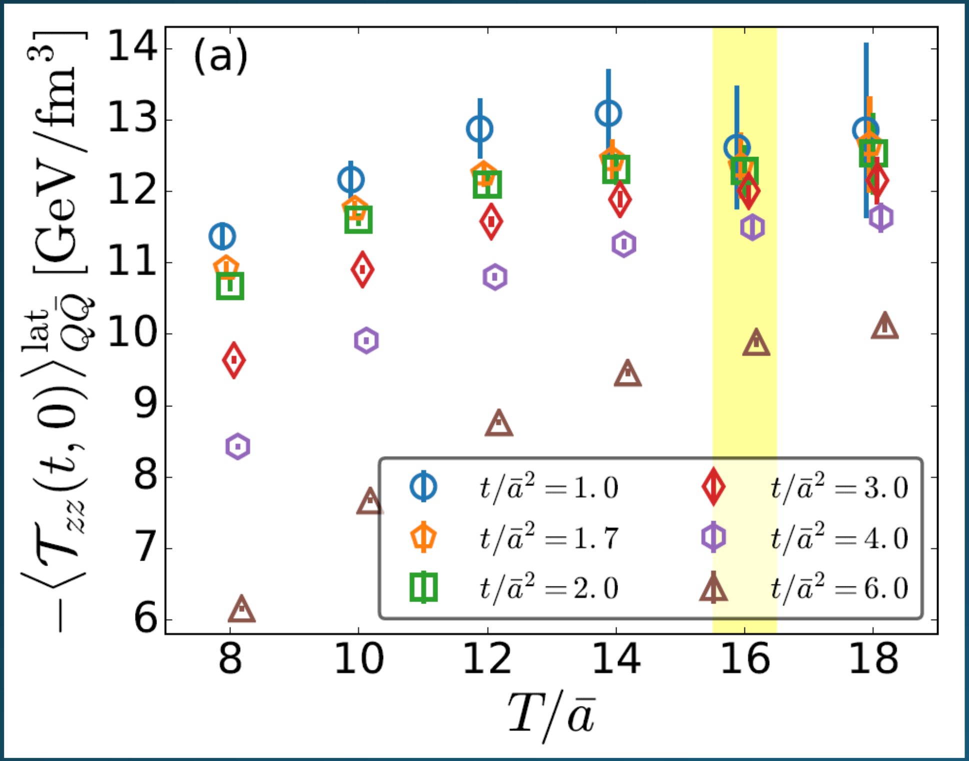
- Simulation: bluegene/Q@KEK

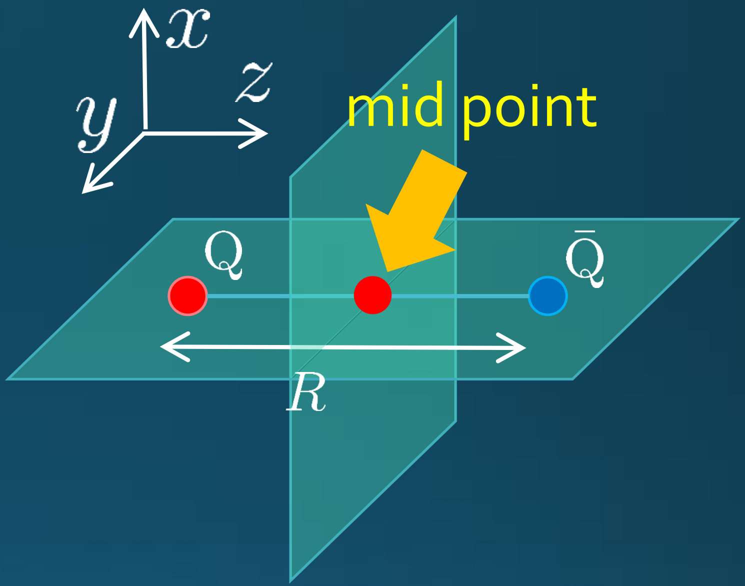


β	a [fm]	$N_{\rm size}^4$	$N_{\rm conf}$	R/a		
6.304	0.058	48^4	140	8	12	16
6.465	0.046	48^4	440	10	–	20
6.513	0.043	48^4	600	–	16	–
6.600	0.038	48^4	1,500	12	18	24
6.819	0.029	64^4	1,000	16	24	32
			R [fm]	0.46	0.69	0.92



Ground State Saturation

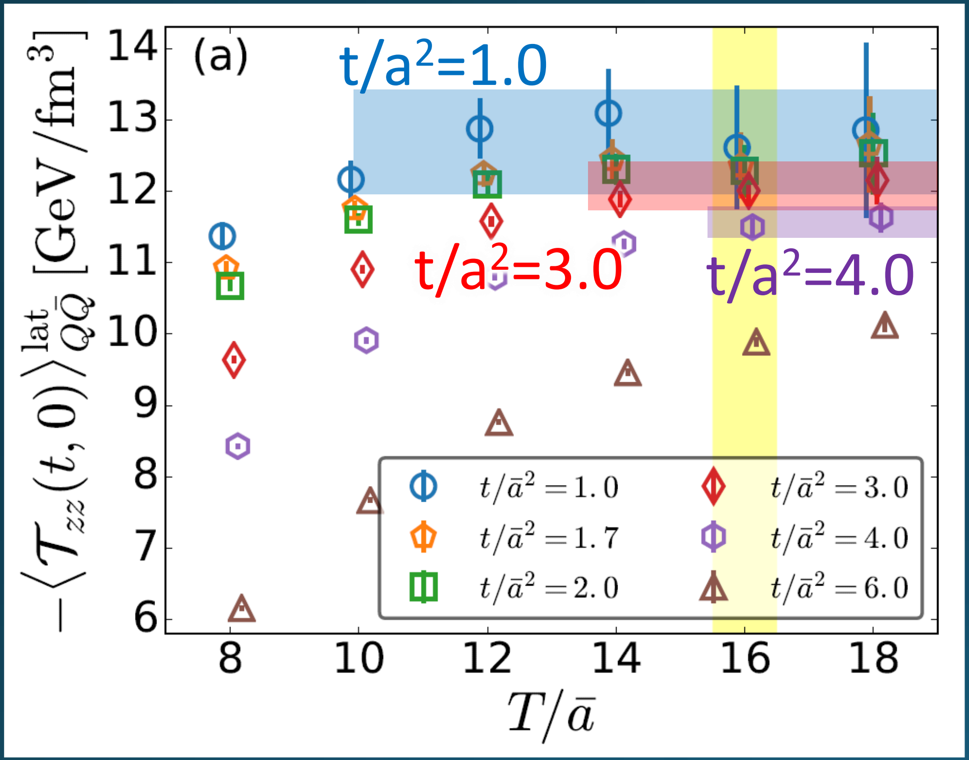


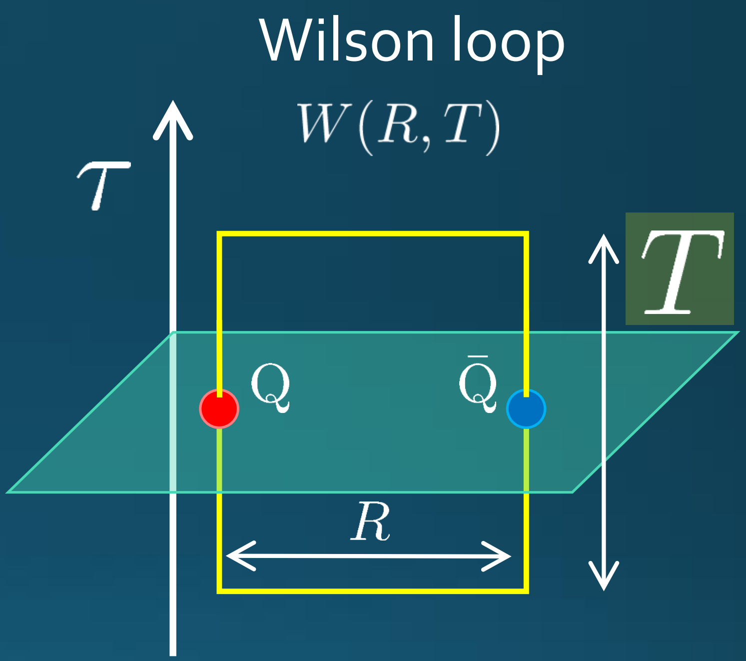


β=6.819 (a=0.029 fm), R=0.46 fm

Appearance of plateau for $t/a^2<4$, $T/a>15$ → Grand state saturation under control

Ground State Saturation





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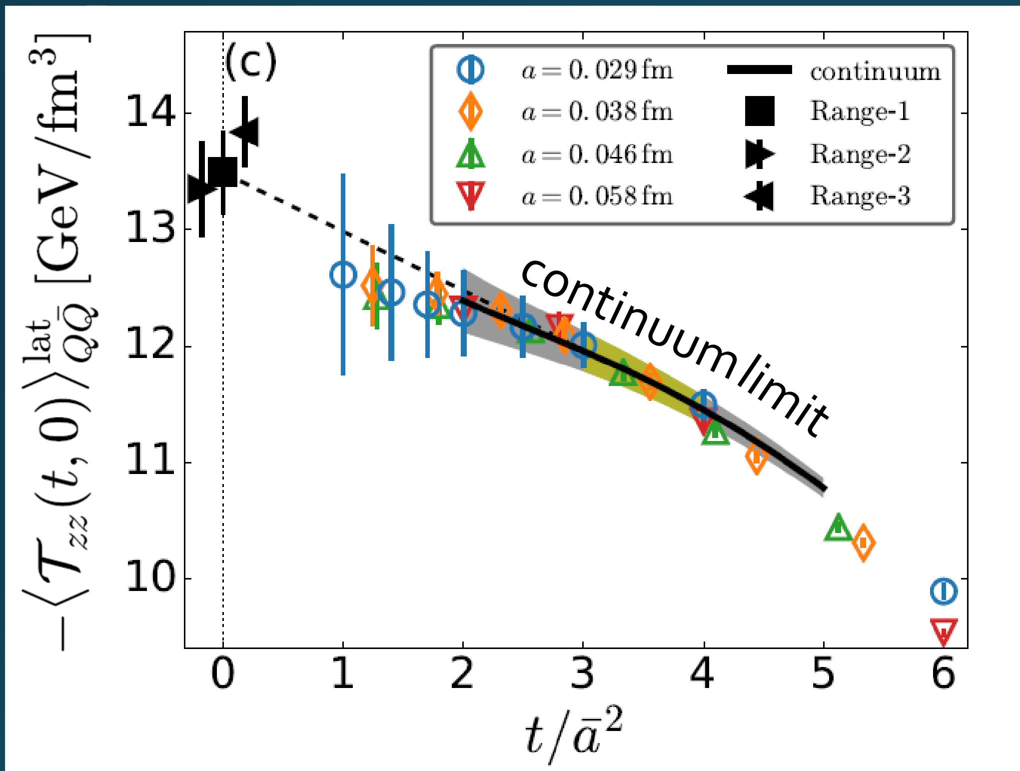
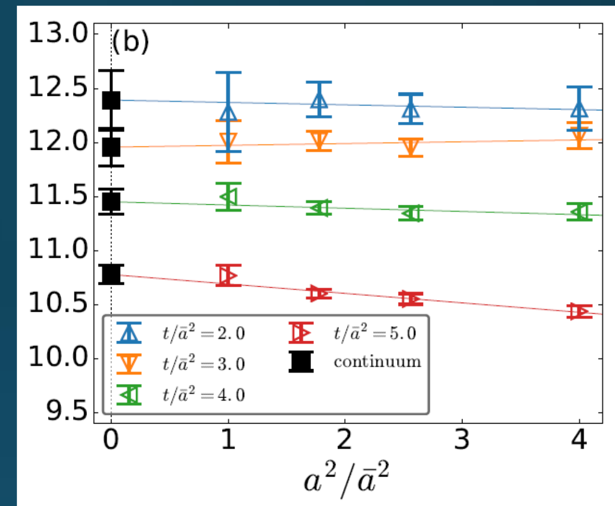
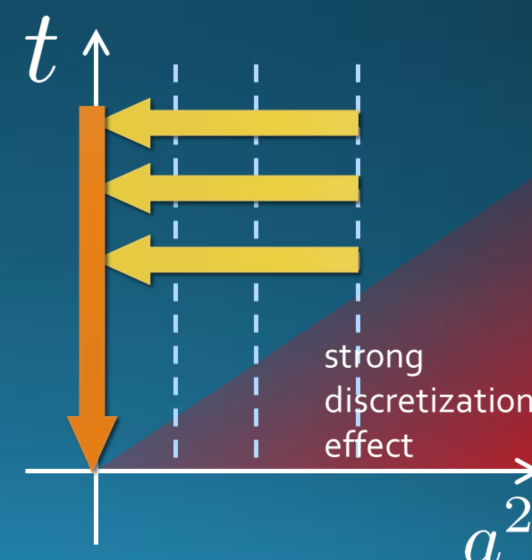
Continuum Extrapolation

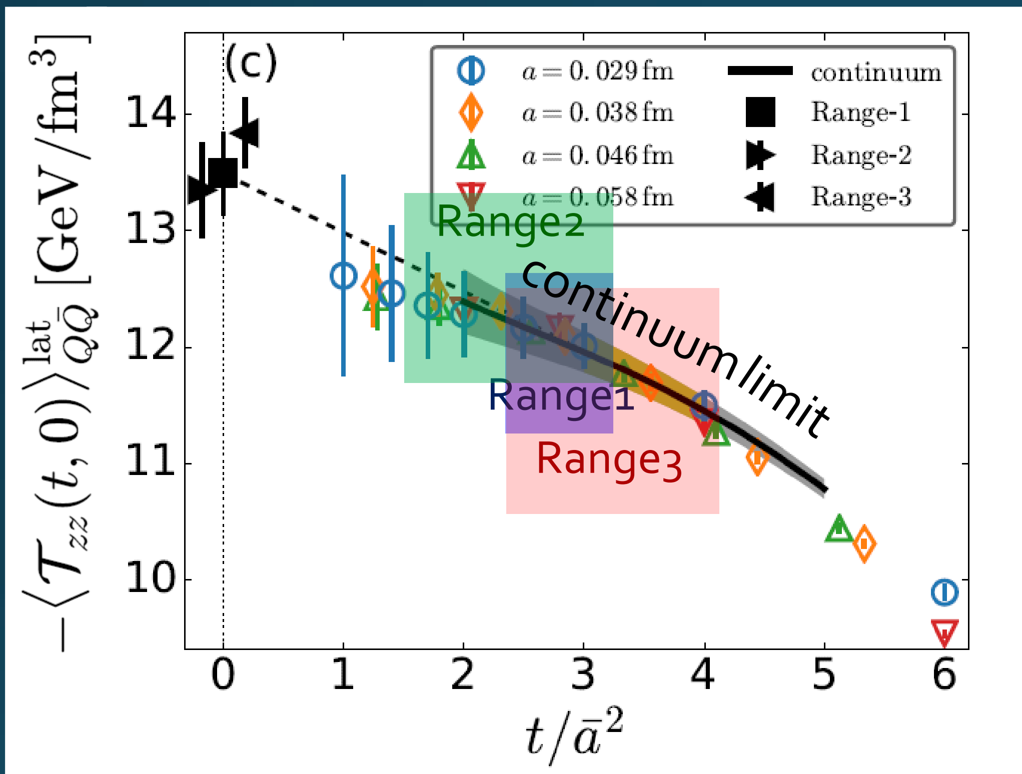


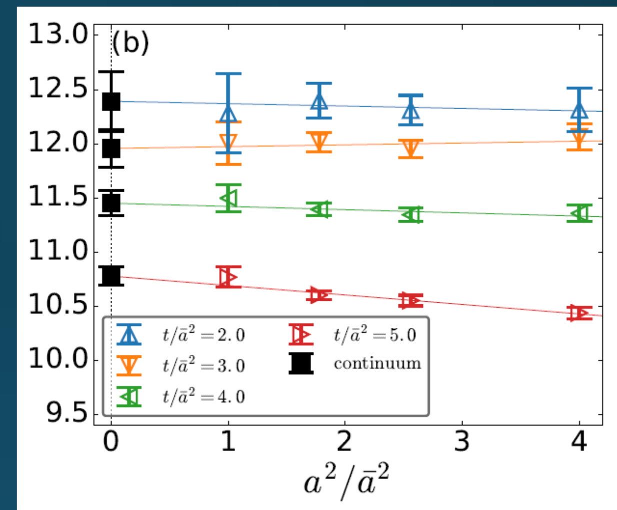


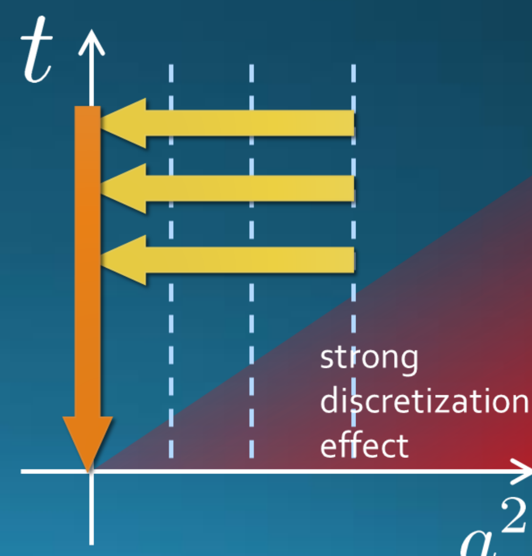


- ☐ a→0 extrapolation with fixed t

t→0 Extrapolation







- ☐ a→0 extrapolation with fixed t
- ☐ Then, t→0 with three ranges

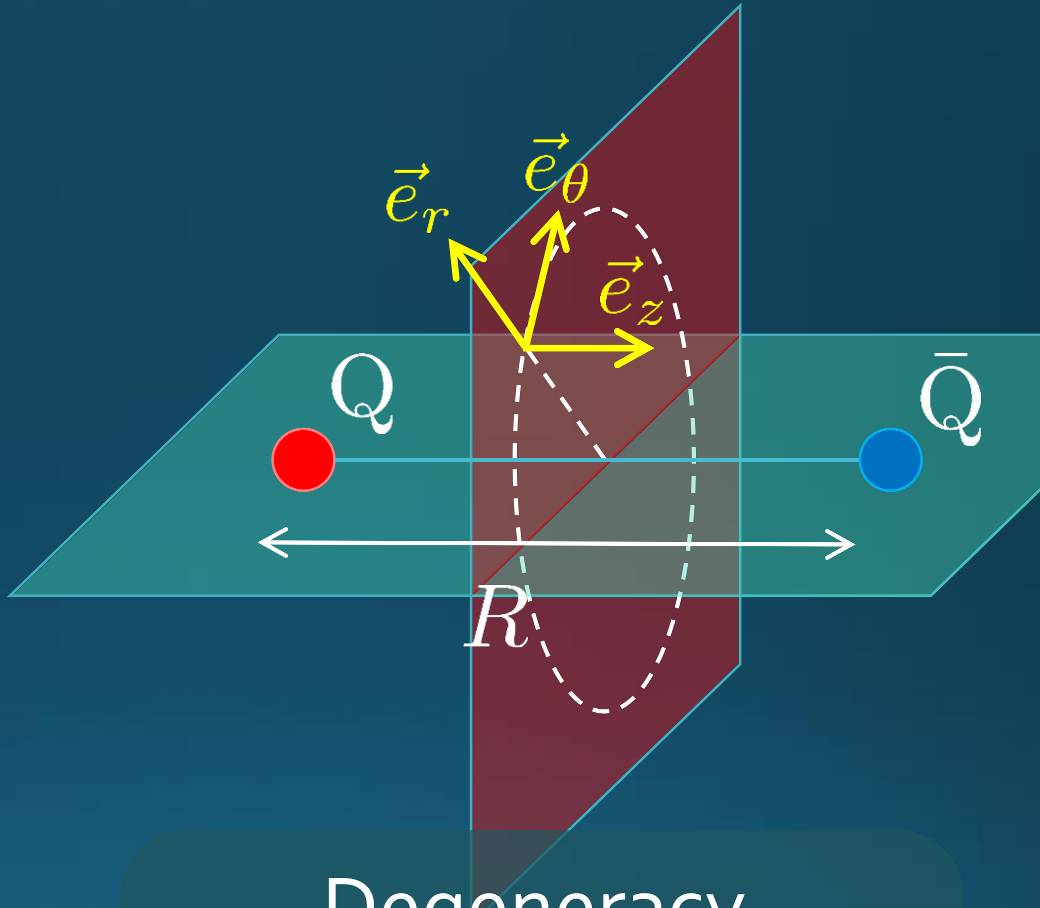
Symmetry on Mid-Plane

From rotational symm. & parity

EMT is diagonalized
in Cylindrical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{zz} & \\ & & & T_{44} \end{pmatrix}$$

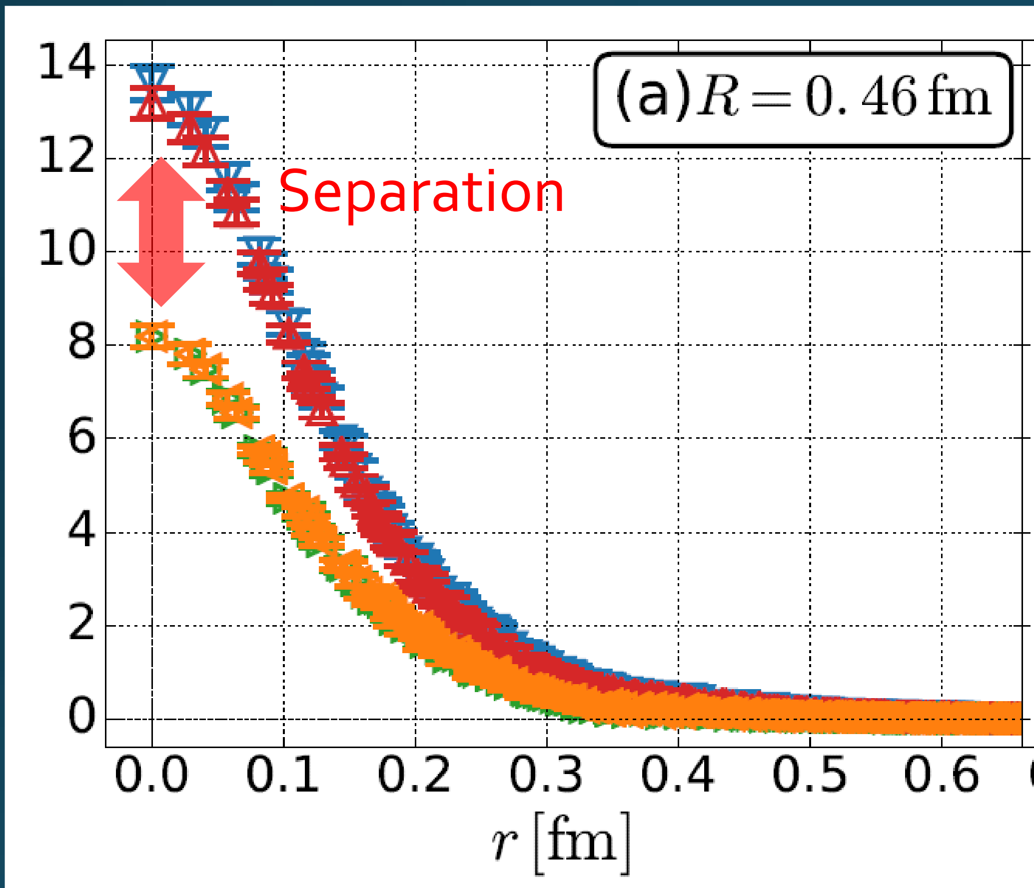
$$T_{\theta\theta} = \vec{e}_\theta^T T \vec{e}_\theta$$

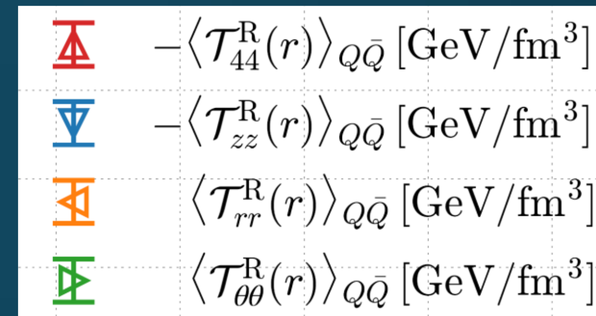


Degeneracy
in Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

Mid-Plane

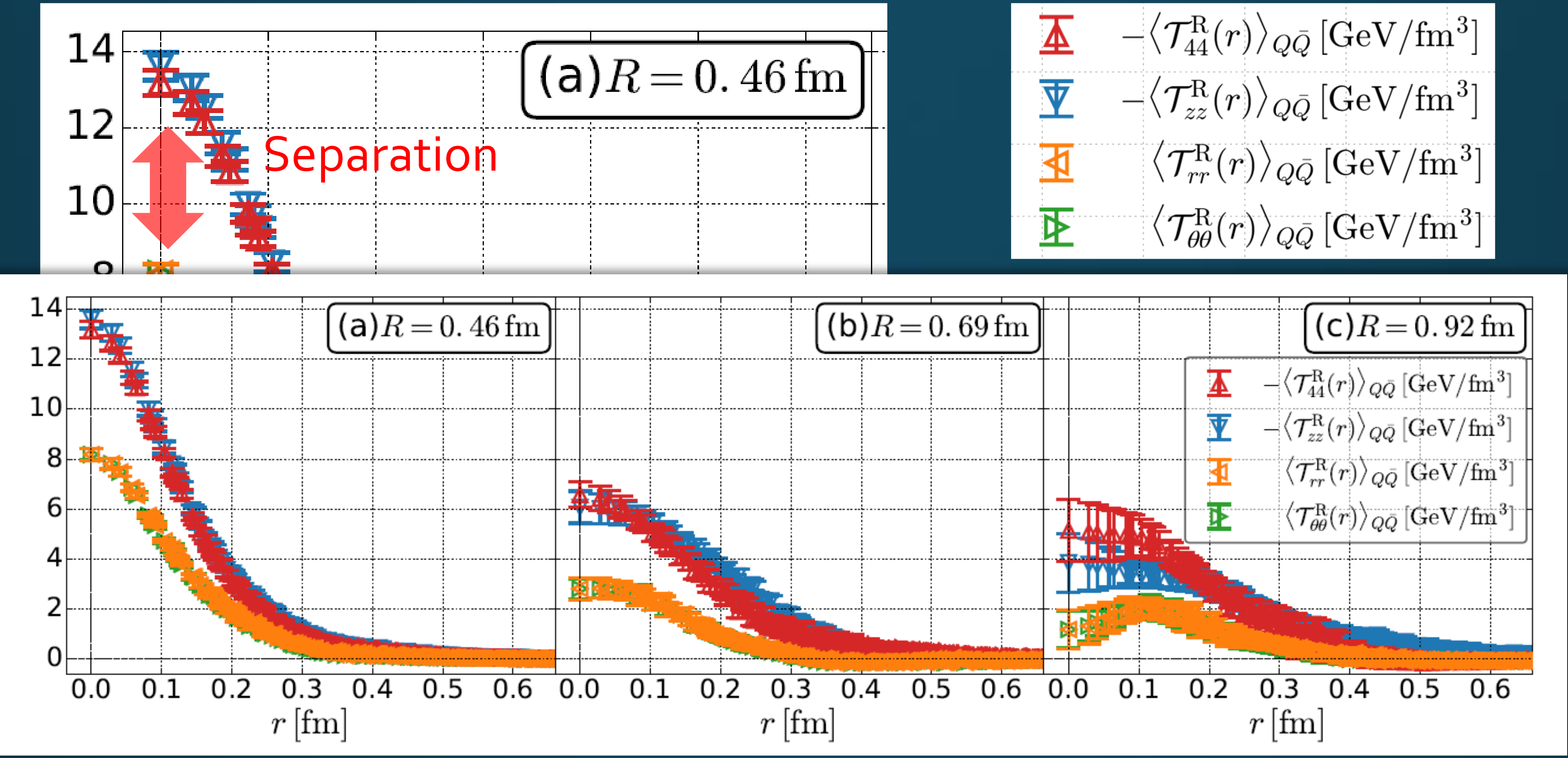




Continuum Extrapolated!

- Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- Separation: $T_{zz} \neq T_{rr}$
- Nonzero trace anomaly $\sum T_{cc} \neq 0$

Mid-Plane



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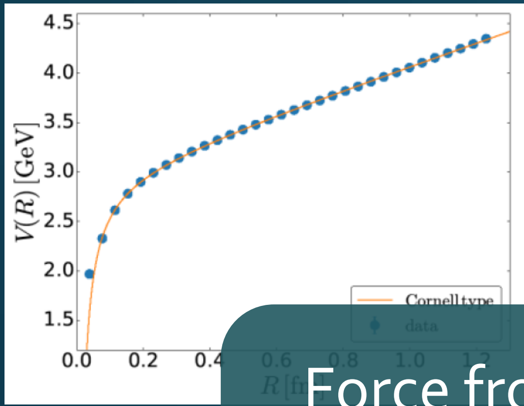
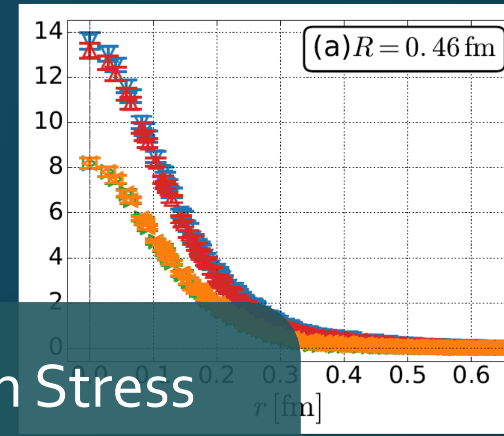
Force





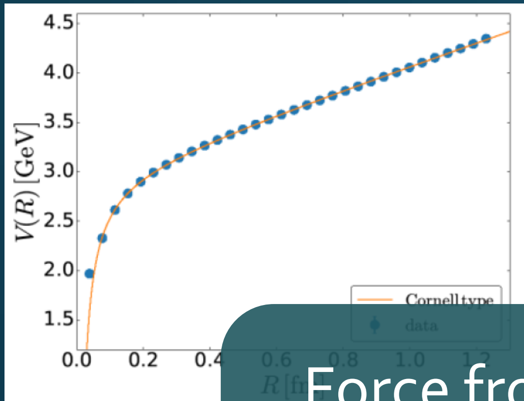
Force from Potential

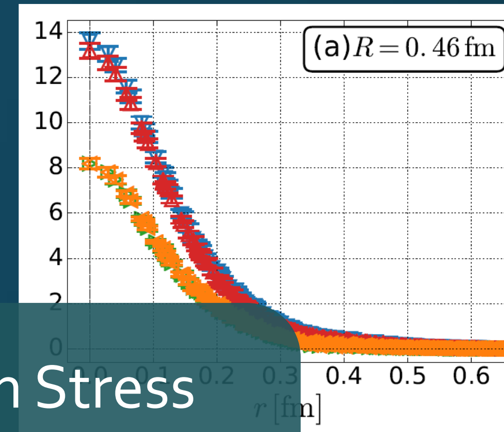
$$F_{\text{pot}} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\text{stress}} = \int_{\text{mid.}} d^2x T_{zz}(x)$$

Force





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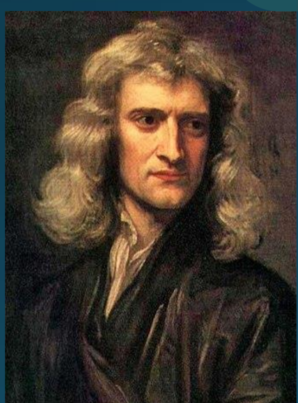
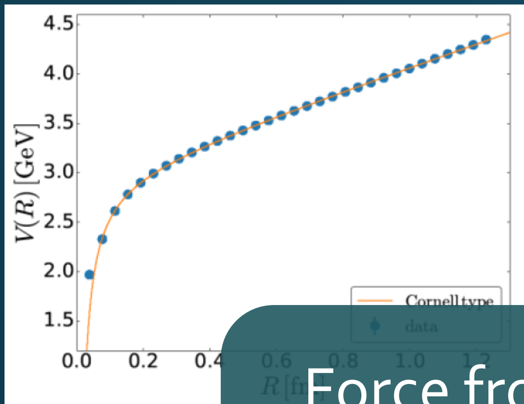
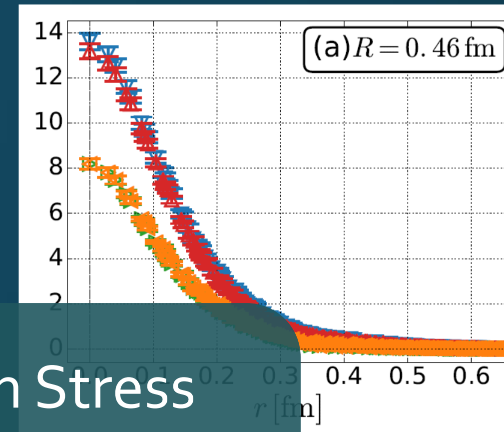


Force





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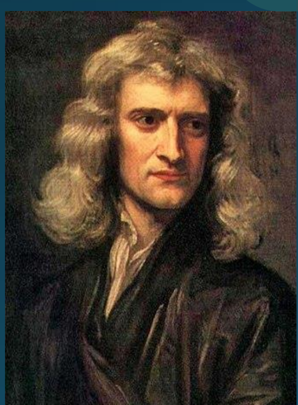
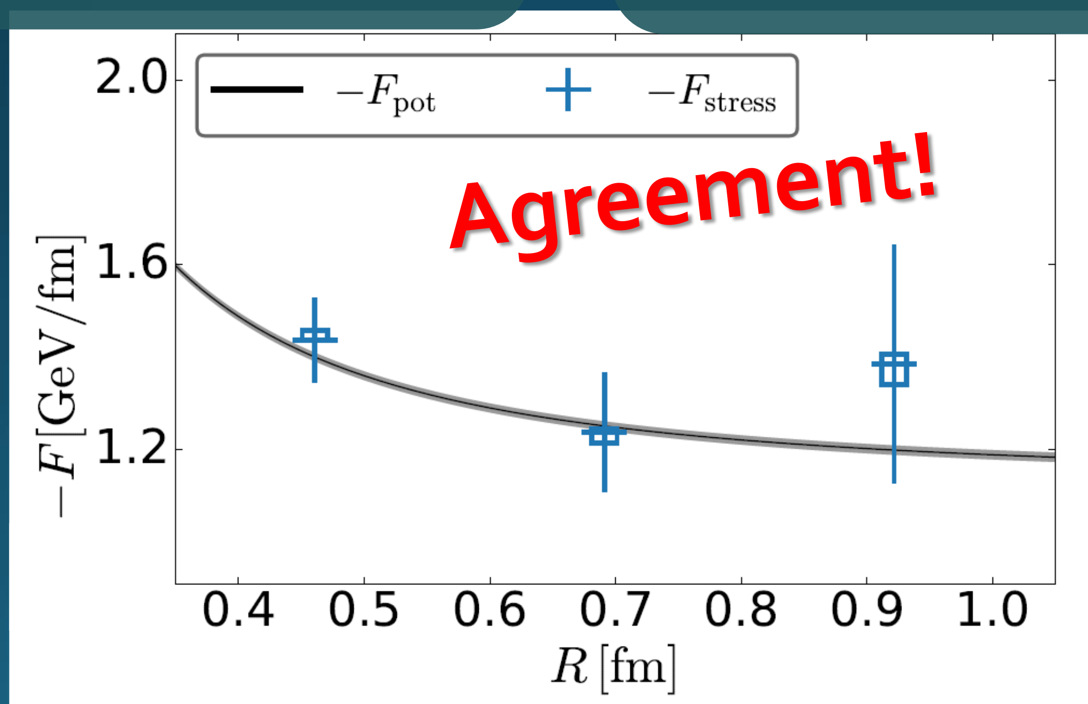




Stress Tensor Distribution in Dual Abelian-Higgs Model

Yanagihara+, in prep.

Abelian-Higgs Model

Abelian-Higgs Model

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

- flux-tube solution w/ monopoles Nielsen, Olesen (1973)
- model for QCD vacuum (dual-Ginzburg-Landau)
- describe symmetry breaking/restoration
- nonzero trace anomaly

Abelian-Higgs Model

Abelian-Higgs Model

$$\mathcal{L}_{\rm AH} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

GL parameter: $\kappa = \sqrt{\lambda}/g$

- type-I : $\kappa < 1/\sqrt{2}$
- type-II : $\kappa > 1/\sqrt{2}$
- Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

Infinitely long tube

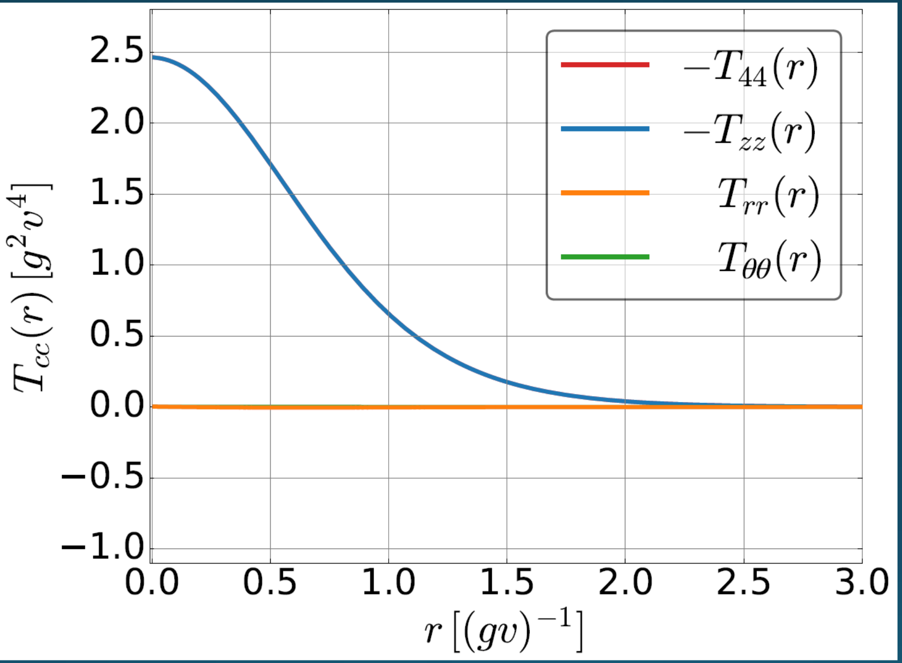
- degeneracy

  $T_{zz}(r) = T_{44}(r)$ Luscher, 1981
- conservation law

$$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

Stress Tensor in AH Model

Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

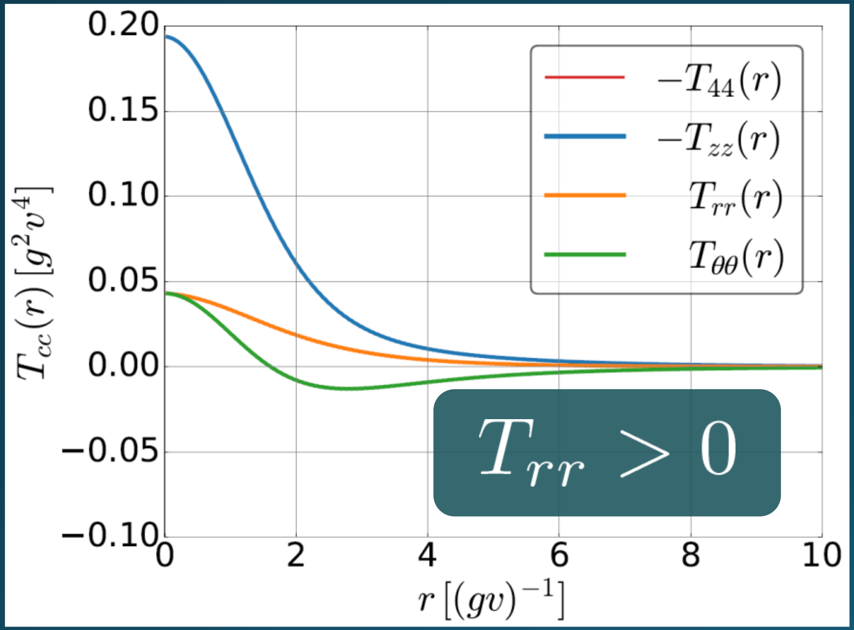


$$T_{rr} = T_{\theta\theta} = 0$$

de Vega, Schaposnik, PR**D14**, 1100 (1976).

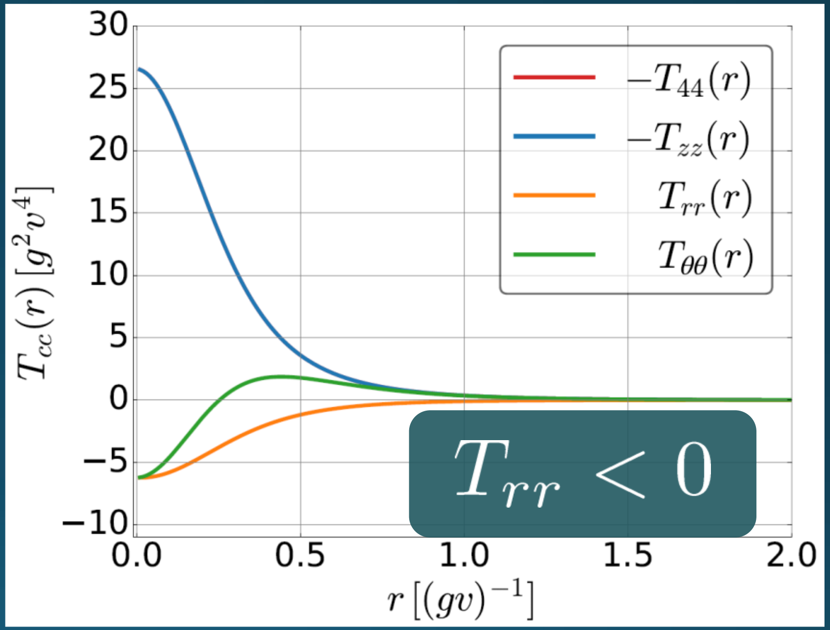
Stress Tensor in AH Model

Type-I $\kappa = 0.1$



Gauge dominant

$\kappa = 3.0$ **Type-II**

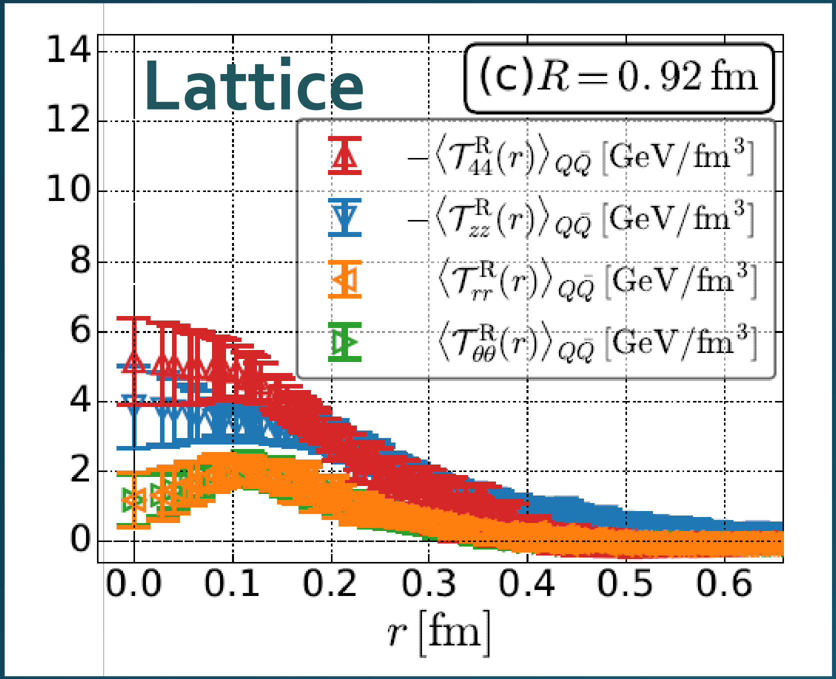


Higgs dominant

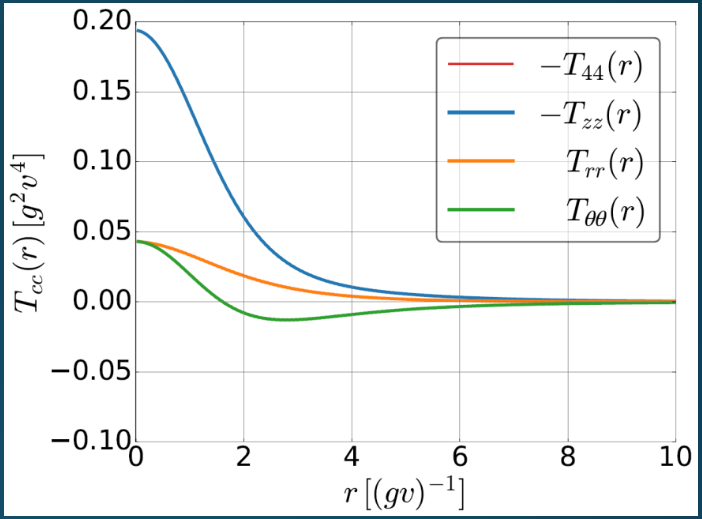
- No degeneracy bw T_{rr} & $T_{\theta\theta}$ ← conservation law

$$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

Comparison



Abelian-Higgs



Type-I; infinitely long

- $T_{rr} > 0$
  → Suggest type-I (if dual-SC picture is correct)?
- $T_{rr} \simeq T_{\theta\theta}$
  → Translationally-invariant flux tube is not formed.

Summary

- First non-perturbative analysis of the stress tensor distribution in quark-anti-quark systems
- New insights into the nature of the flux tube
- "Action-at-a-distance" force is given by the sum of local interaction.
- Non-trivial degeneracy in mid-plane
- $T_{rr}>0$ → type-I vacuum? / $T_{rr}\sim T_{\theta\theta}$ → Is R still small?

Summary

- First non-perturbative analysis of the stress tensor distribution in quark-anti-quark systems
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- Non-trivial degeneracy in mid-plane
- $T_{rr}>0$ → type-I vacuum? / $T_{rr}\sim T_{\theta\theta}$ → Is R still small?
- So many future studies
  - Nonzero temperature / excited states
  - EMT distribution inside hadrons
  - Model study of the flux tube with finite length