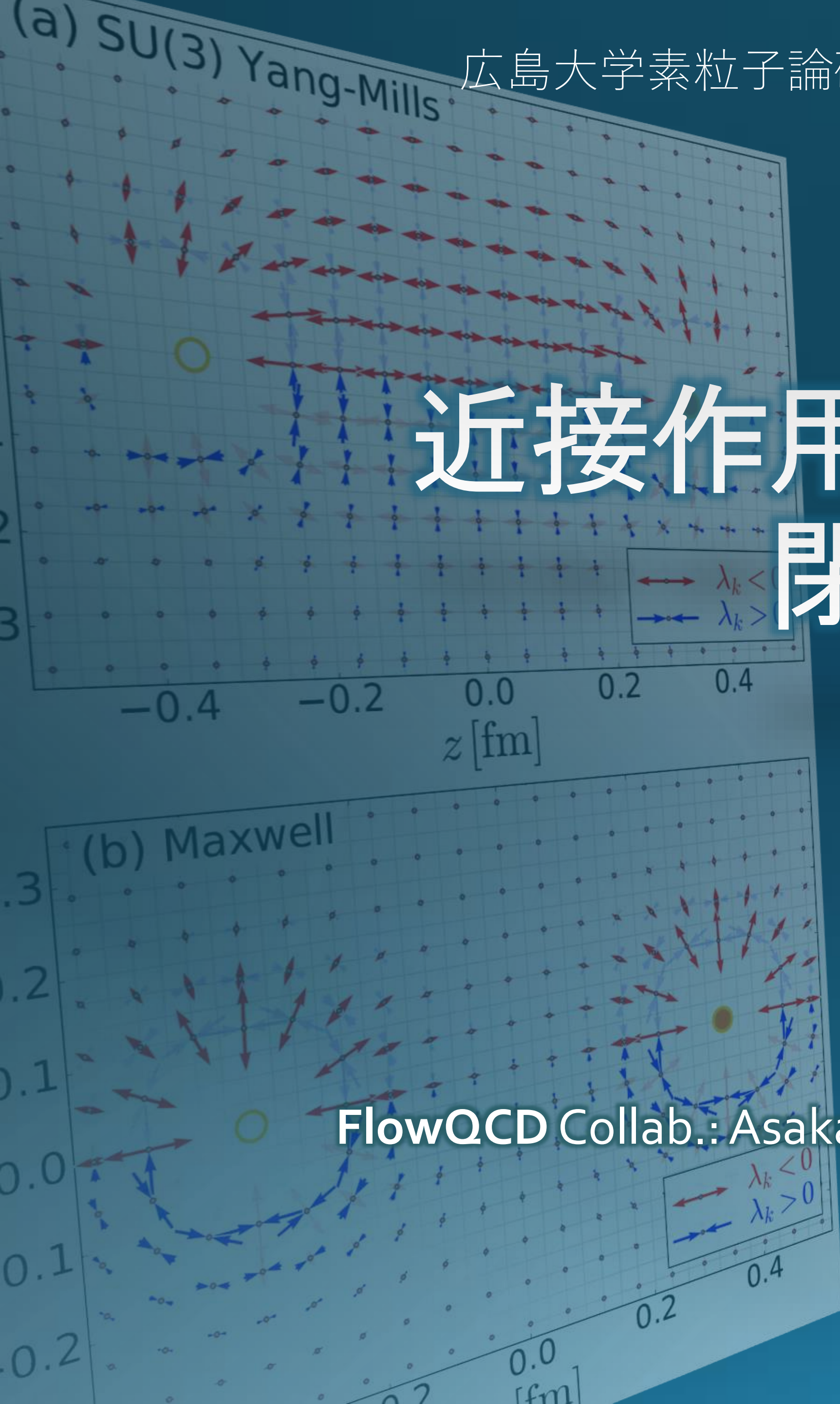


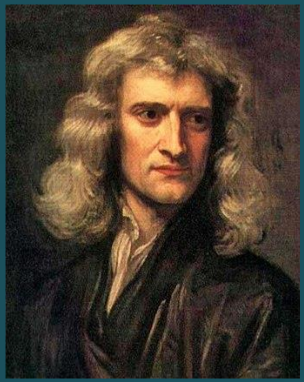
近接作用として理解する閉じ込め相互作用

北沢 正清
(大阪大学)

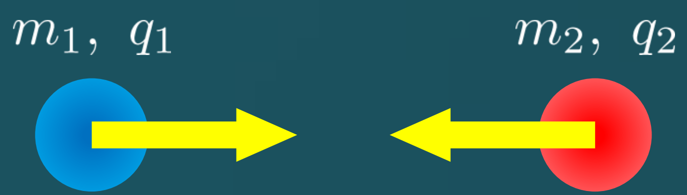
FlowQCD Collab.: Asakawa, Hatsuda, **Iritani**, Suzuki, **Yanagihara**
PLB, in press [arXiv:1803.05656]

Force

Action-at-a-distance



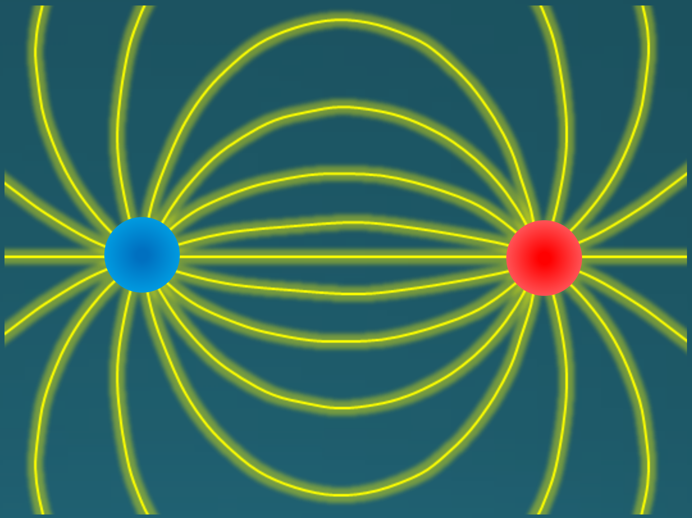
Newton
1687



$$F = -G\frac{m_1 m_2}{r^2} \qquad F = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Local interaction

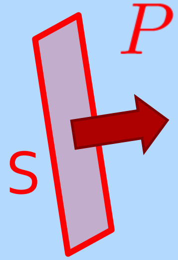
Faraday
1839



Stress = Force per Unit Area

Stress = Force per Unit Area

Pressure

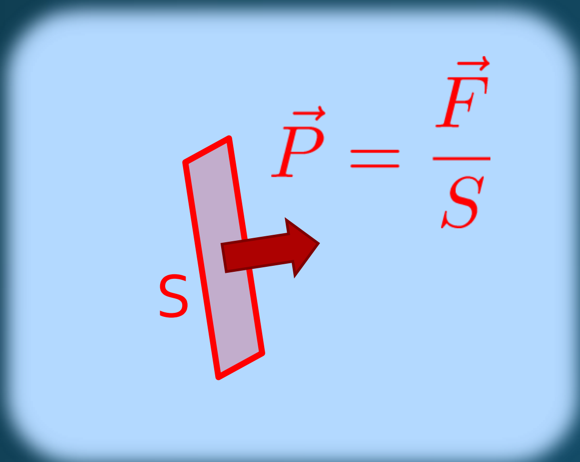


$$\vec{P} = \frac{\vec{F}}{S}$$

$$\vec{P} = P\vec{n}$$

Stress = Force per Unit Area

Pressure

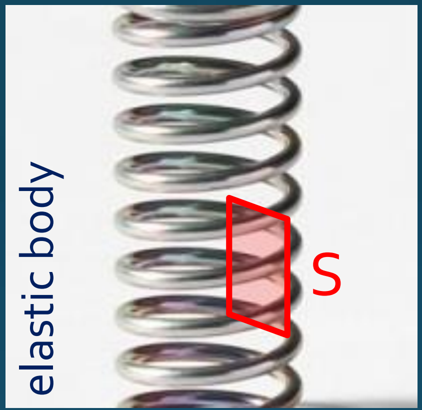


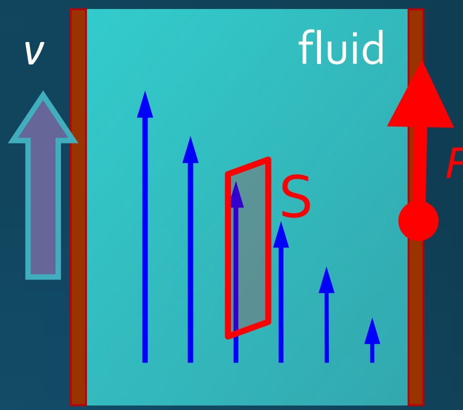
$$\vec{P} = P\vec{n}$$

In thermal medium

$$T_{ij} = P\delta_{ij}$$

Generally, F and n are not parallel





$$\frac{F_i}{S} = \boxed{\sigma_{ij}} n_j$$

Stress Tensor

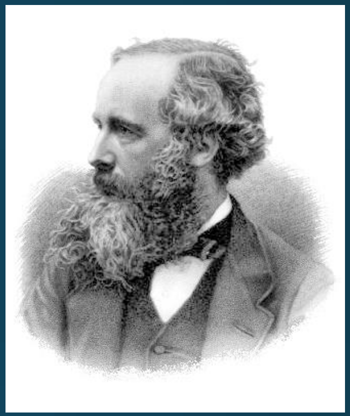
$$\sigma_{ij} = -T_{ij}$$

Landau
Lifshitz

Maxwell Stress

(in Maxwell Theory)

$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

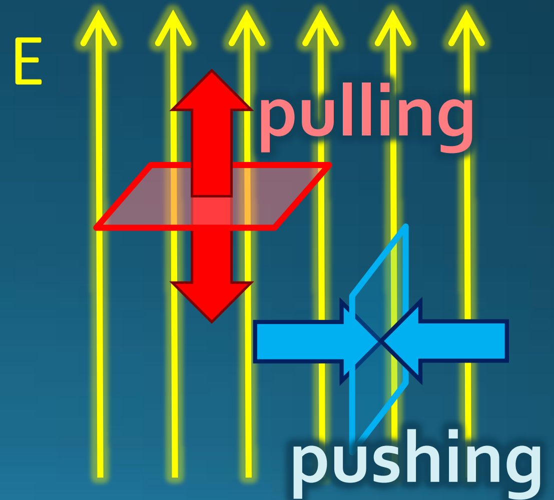


Maxwell

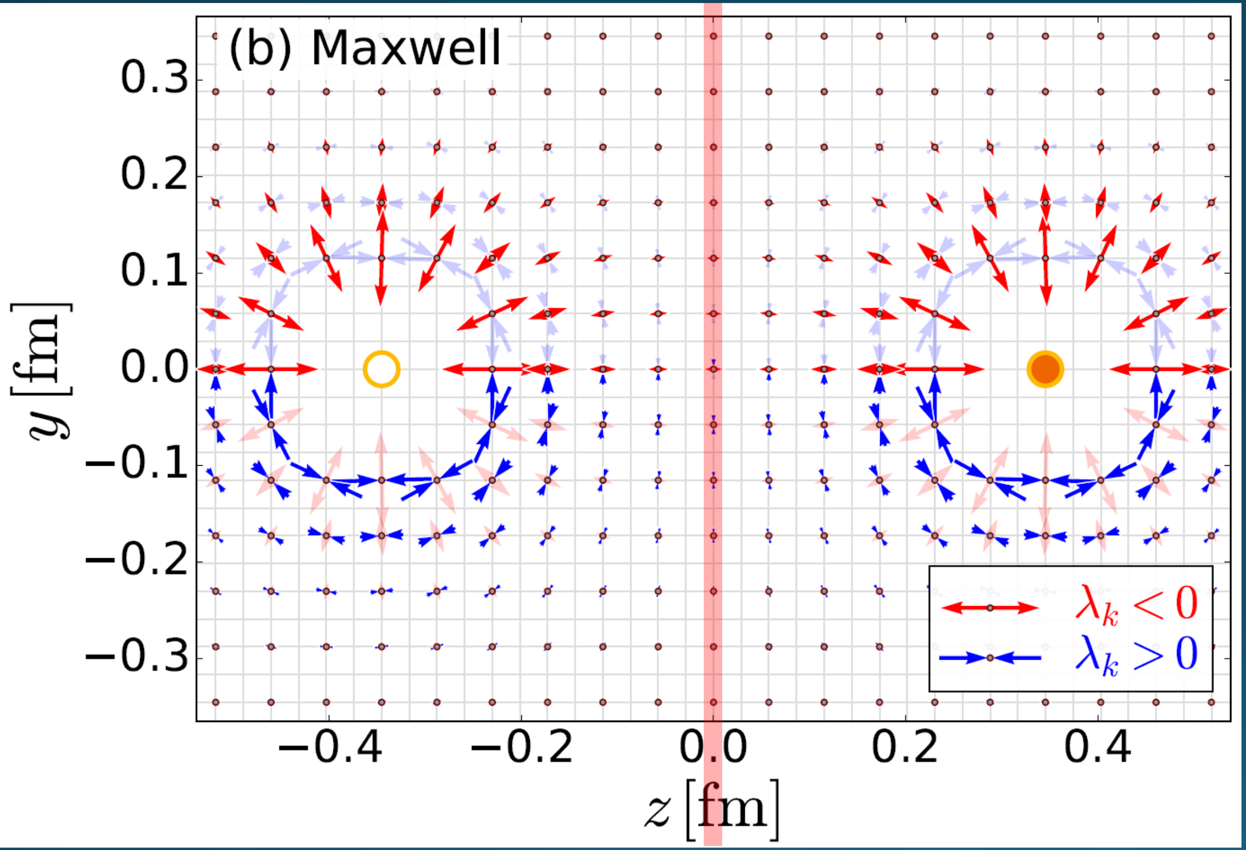
$$\vec{E} = (E, 0, 0)$$

$$T_{ij} = \begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: **Pulling**
- Vertical to field: **Pushing**



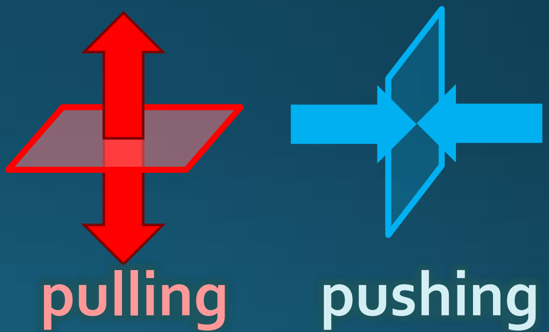
Maxwell Stress
(in Maxwell Theory)



$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$$(k = 1, 2, 3)$$

length: $\sqrt{|\lambda_k|}$

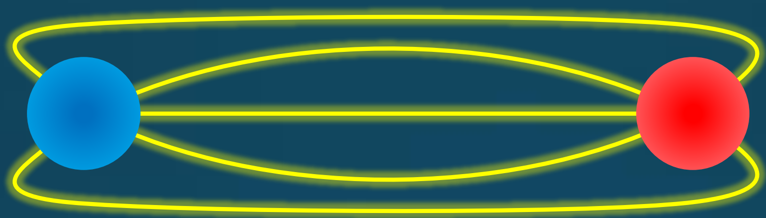


Definite physical meaning

- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction

Quark-Anti-quark system

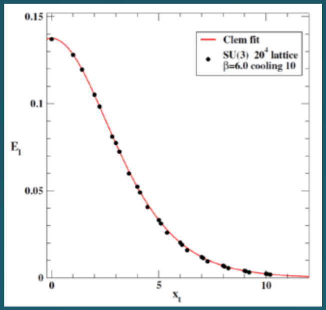
Formation of the flux tube → confinement



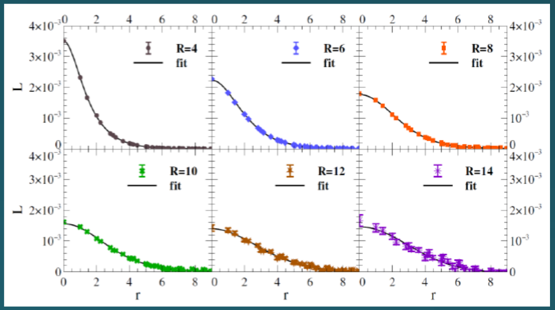
Previous Studies on Flux Tube

- ☐ Potential
- ☐ Action density
- ☐ Color-electric field

so many studies…

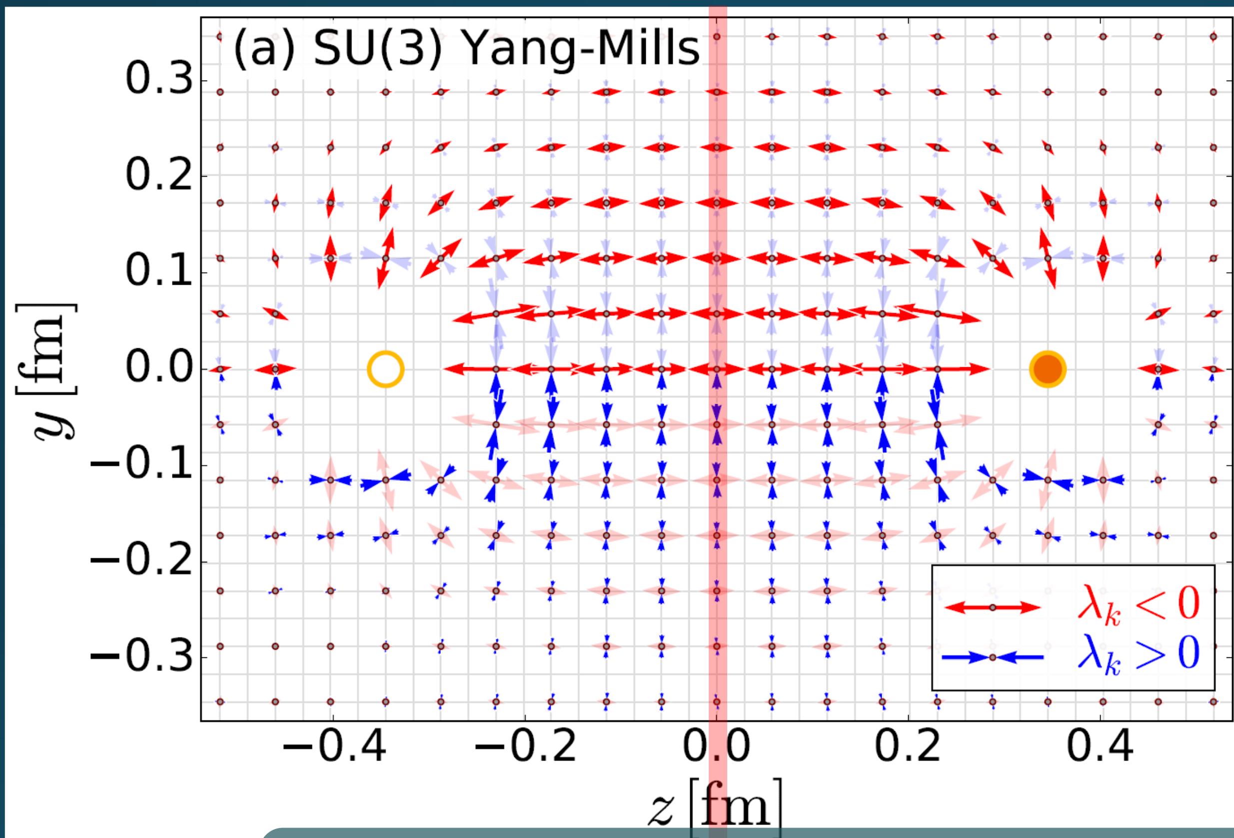


Cea+ (2012)



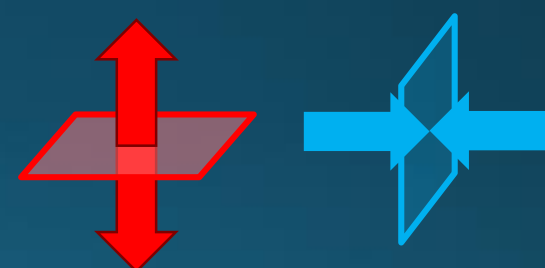
Cardoso+ (2013)

Stress Tensor in $Q\bar{Q}$ System



Yanagihara+, 1803.05656
PLB, in press

Lattice simulation
SU(3) Yang-Mills
a=0.029 fm
R=0.69 fm
t/a^2=2.0



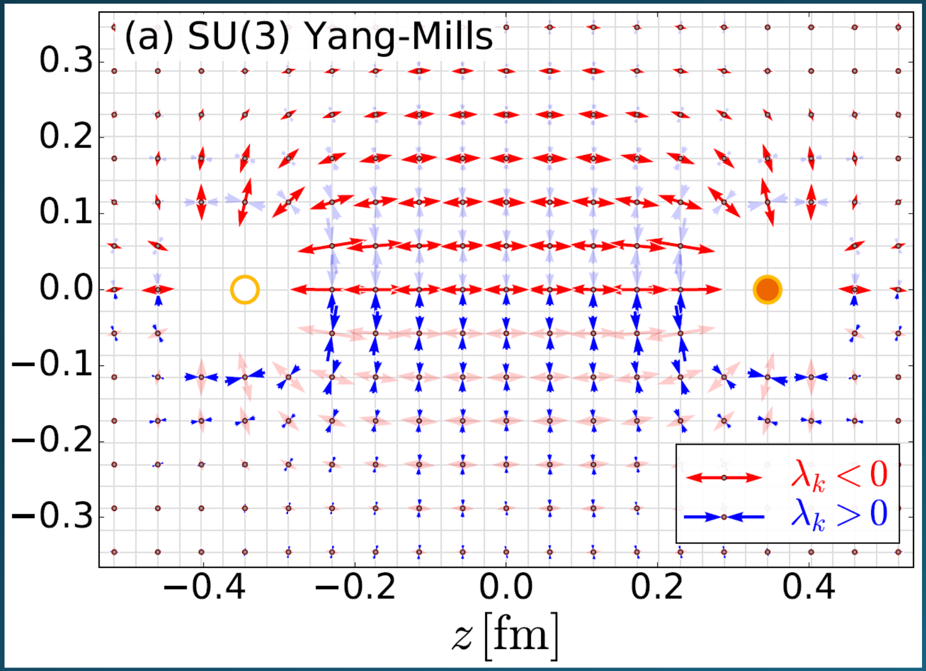
Definite physical meaning

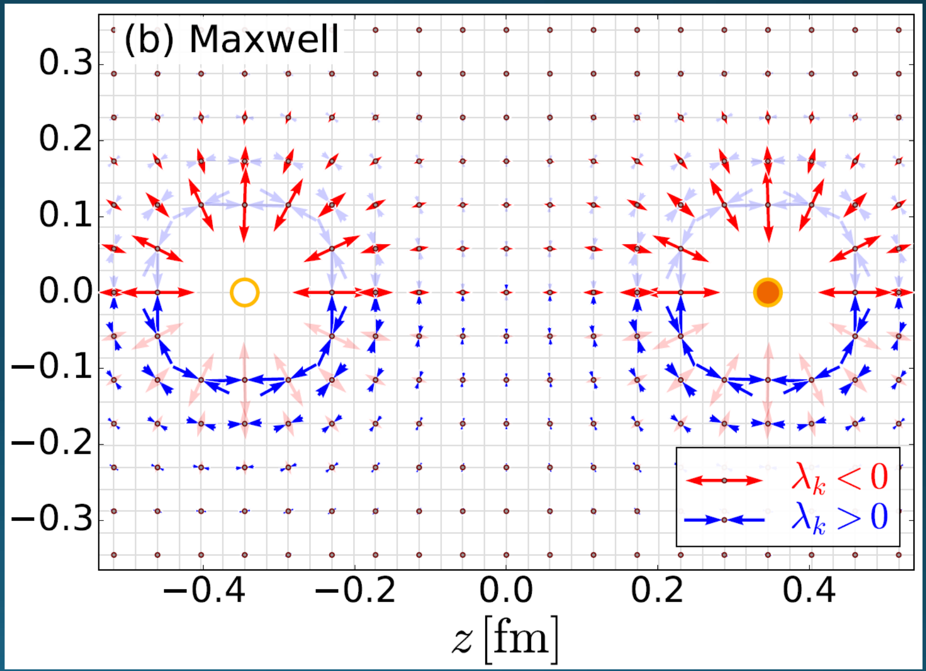
- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction
- ☐ Manifestly gauge invariant

SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)

Maxwell
(classical)





Propagation of the force is clearly different in YM and Maxwell theories!

Energy-Momentum Tensor

energy momentum

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

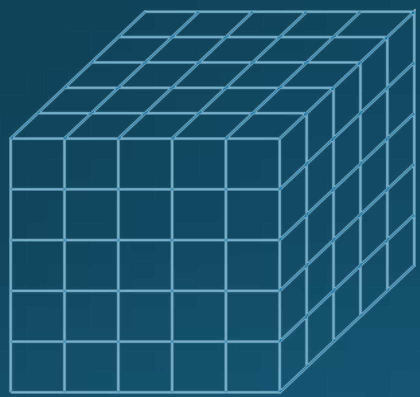
pressure

stress

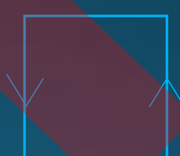
All components are important physical observables!

$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial
because of the explicit breaking of Lorentz symmetry



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

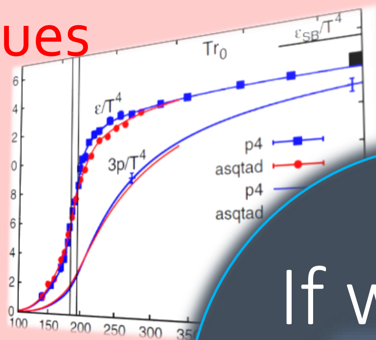
$F_{\mu\nu} =$ 

② Its measurement is extremely noisy
due to high dimensionality and etc.

Thermodynamics

direct measurement of expectation values

$$\langle T_{00}\rangle, \langle T_{ii}\rangle$$



Fluctuations and Correlations

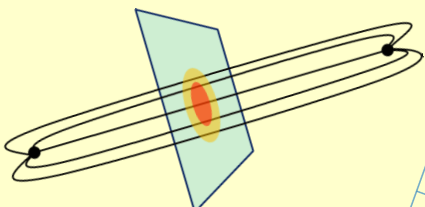
viscosity, specific heat, ...

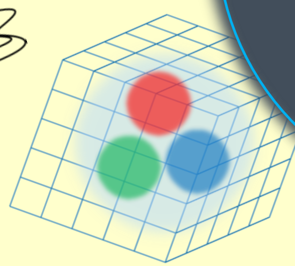
$$c_V \sim \langle \delta T_{00}^2 \rangle$$

$$\eta = \langle T_{12}; T_{12} \rangle$$

If we have

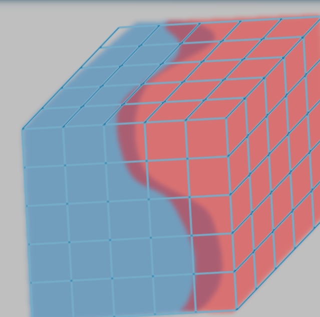
$$T_{\mu\nu}$$





- flux tube / hadrons
- EM form factors

Hadron Structure



- vacuum configuration
- mixed state on 1st transition

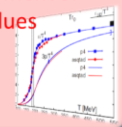
Vacuum Structure

Contents

$T_{\mu\nu}$

Constructing EMT on the lattice
with gradient flow

Thermodynamics
direct measurement of expectation values
$\langle T_{00}\rangle, \langle T_{ii}\rangle$

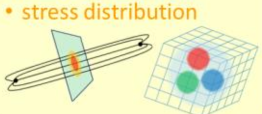


Thermodynamics

Fluctuations and Correlations
viscosity, specific heat, ...
$\eta = \int_0^\infty dt \langle T_{12}; T_{12}\rangle$
$c_V \sim \langle \delta T_{00}^2\rangle$

EMT Correlation Function

Hadron Structure
- flux tube / hadrons
- stress distribution



Stress distribution in $\bar{q}q$ system

Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

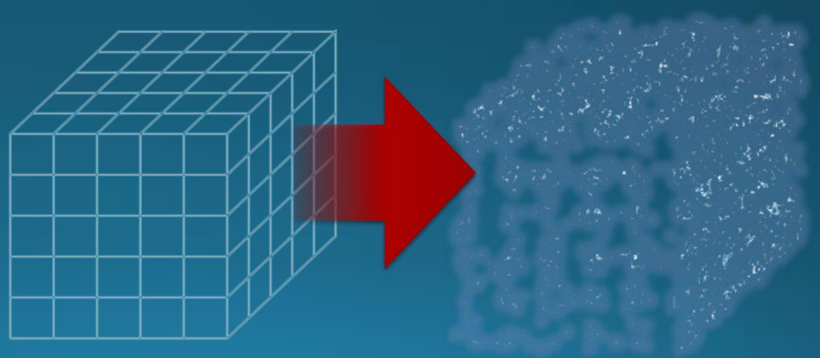
$$A_\mu(0,x) = A_\mu(x)$$

t: “flow time”
dim:[length²]

leading

$$\boxed{\partial_t A_\mu} = D_\nu G_{\mu\nu} = \boxed{\partial_\nu \partial_\nu A_\mu} + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- “continuous” cooling/smearing
- No UV divergence at t>0



Yang-Mills Gradient Flow

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t: "flow time"
dim:[length²]

leading

$$\boxed{\partial_t A_\mu} = D_\nu G_{\mu\nu} = \boxed{\partial_\nu \partial_\nu A_\mu} + \cdots$$

Applications

scale setting / topological charge / running coupling
noise reduction / defining operators / ...

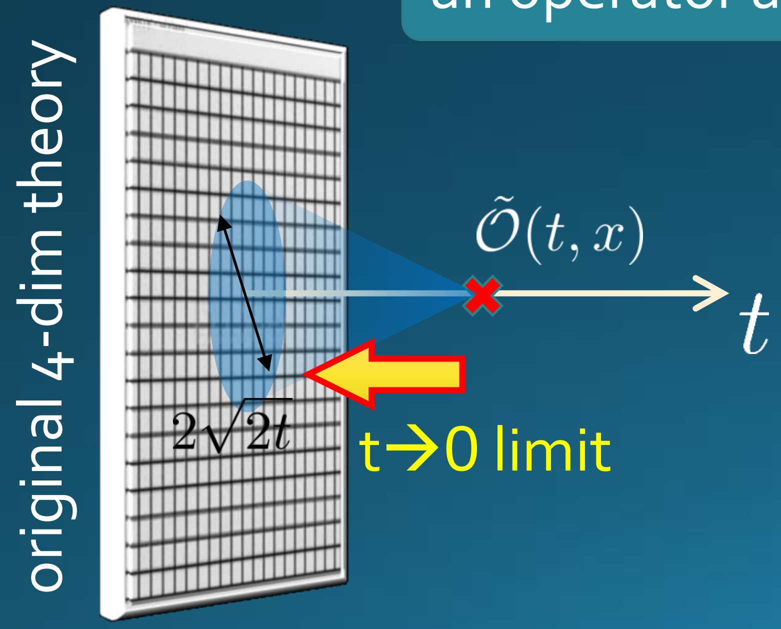
Small Flow-Time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t \to 0]{} \sum_i c_i(t) \mathcal{O}_i^R(x)$$

an operator at t>0

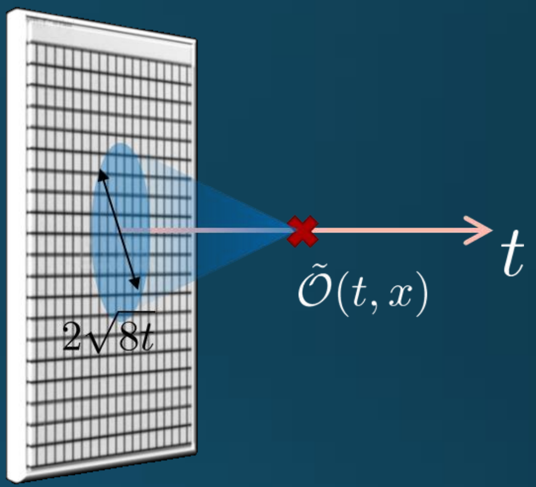
remormalized operators of original theory



Constructing EMT 1

Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t \to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$



- **Gauge-invariant dimension 4 operators**

$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

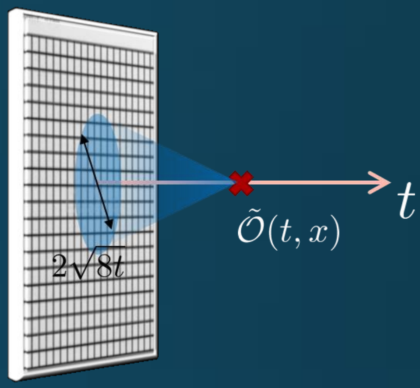
Constructing EMT 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t) T^R_{\rho\rho}(x) + \mathcal{O}(t)$$

vacuum subtr.



Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\mathrm{subt.}}\right]$$

Perturbative Coefficients

$$T_{\mu\nu}(t) = c_1(t)U_{\mu\nu}(t) + \delta_{\mu\nu}c_2(t)E(t)$$

	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○		
$c_2(t)$	× zero	○	○	

Suzuki (2013)

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, in prep.

□ Choice of the scale of g²

$$c_1(t) = c_1\left(g^2(\mu(t))\right)$$

Previous: $\mu_d(t) = 1/\sqrt{8t}$

Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E}t}$

Harlander+ (2018)

Perturbative Coefficients

$$T_{\mu\nu}(t) = c_1(t)U_{\mu\nu}(t) + \delta_{\mu\nu}c_2(t)E(t)$$

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$c_1(t)$	○	○	○	
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Suzuki (2013) Harlander+(2018)

Iritani, MK, Suzuki, Takaura, in prep.

Suzuki, PTEP 2013, 083B03
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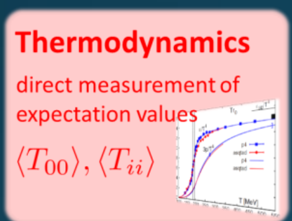
Harlander+ (2018)

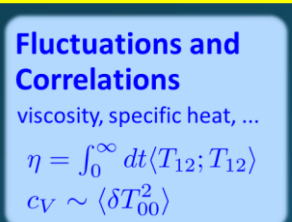
Contents

$T_{\mu\nu}$

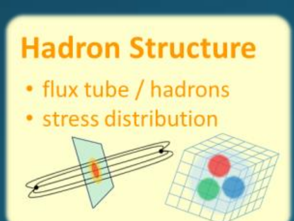
Constructing EMT on the lattice



Thermodynamics



EMT Correlation Function



Stress distribution in $\bar{q}q$ system

Thermodynamics of SU(3) YM

- **Integral method**
  - Most conventional / established
  - Use themodynamic relations

    Boyd+ 1995; Borsanyi, 2012

$$p = \frac{T}{V} \ln Z$$

$$T \frac{\partial (p/T^4)}{\partial T} = \frac{\varepsilon - 3p}{T^4}$$

- Gradient-flow method
  - Take expectation values of EMT

    FlowQCD, 2014, 2016

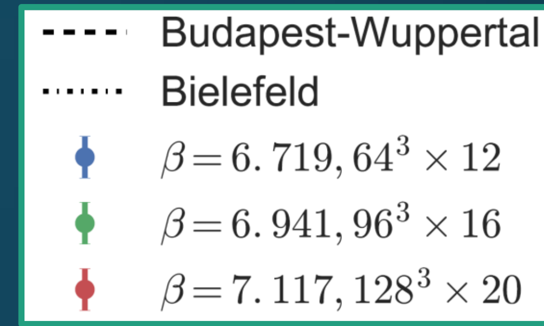
$$\begin{cases} \varepsilon = \langle T_{00} \rangle \\ p = \langle T_{11} \rangle \end{cases}$$

- Moving-frame method

  Giusti, Pepe, 2014~
- Non-equilibrium method
  - Use Jarzynski's equality Caselle+, 2016;2018
- Differential method

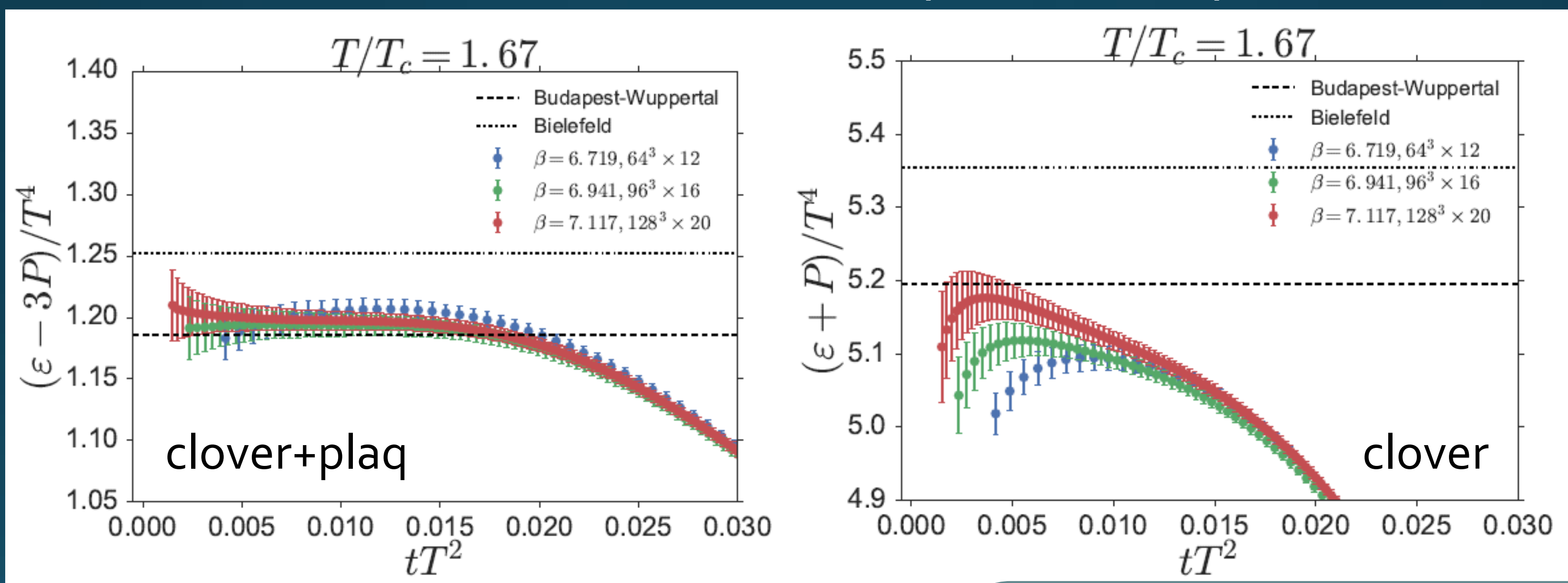
  Shirogane+(WHOT-QCD), 2016~

t, a Dependence



$$T_{\mu\nu}(t) = c_1(t)U_{\mu\nu}(t) + \delta_{\mu\nu}c_2(t)E(t)$$

FlowQCD2016 (c1: 1-loop / c2: 2-loop)



$$\begin{cases} \sqrt{8t} < a & \text{: strong discretization} \\ \sqrt{8t} > 1/(2T) & \text{: over smeared} \end{cases}$$

$a < \sqrt{8t} < 1/(2T)$

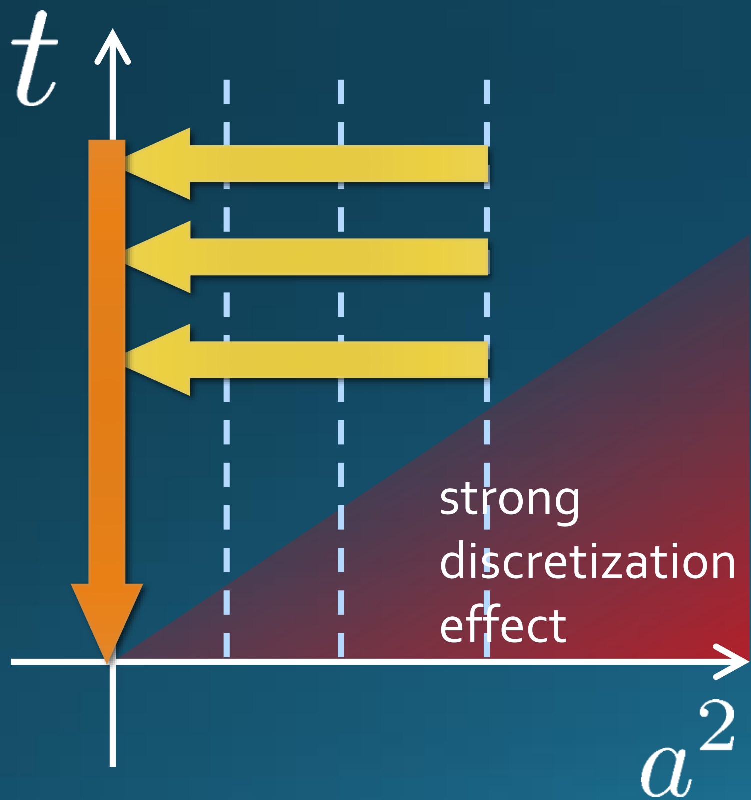
Stable t dependence

Double Extrapolation

t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + \boxed{C_{\mu\nu} t} + \boxed{D_{\mu\nu}(t)\frac{a^2}{t}}$$

O(t) terms in SFTE lattice discretization



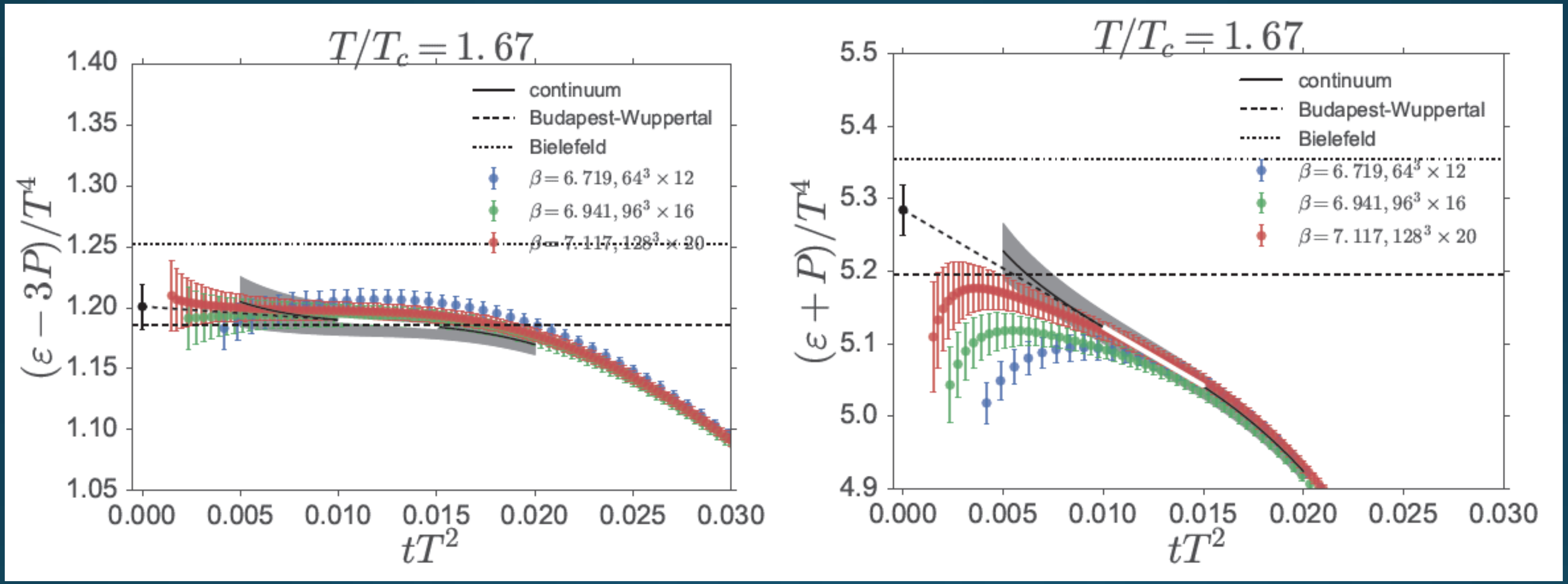
Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

Small t extrapolation

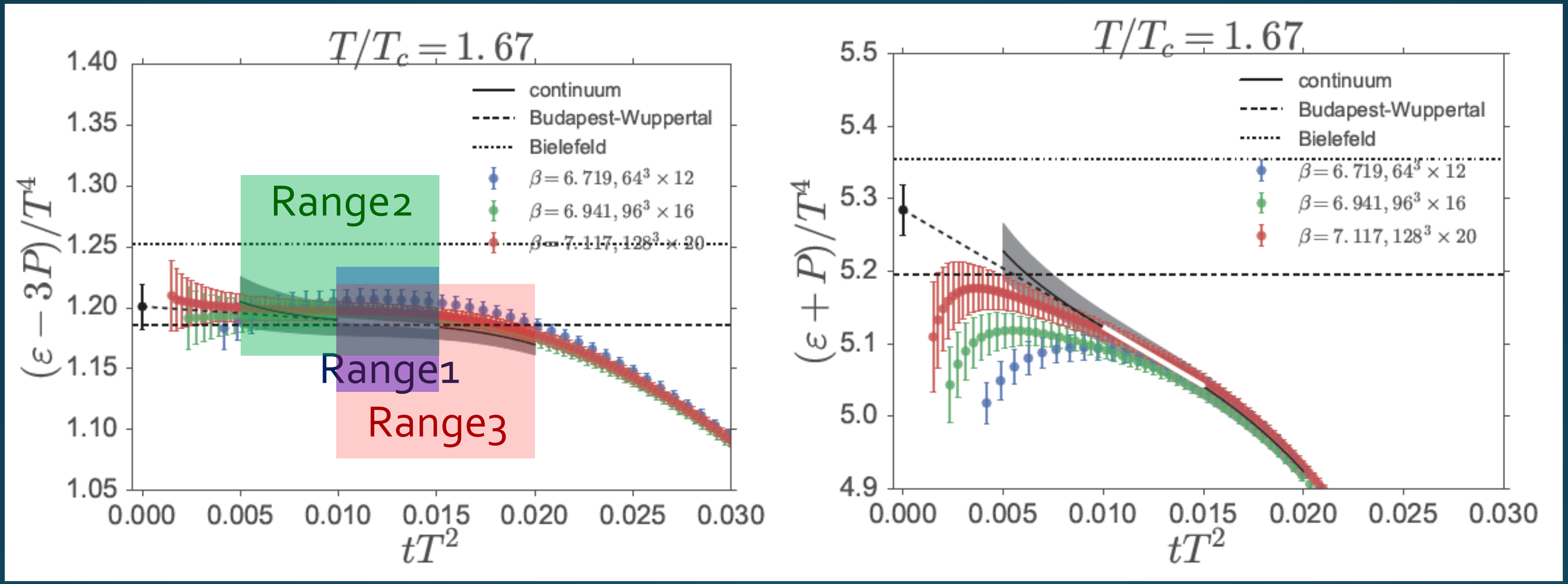
$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$

Double Extrapolation



Black line: continuum extrapolated

Double Extrapolation



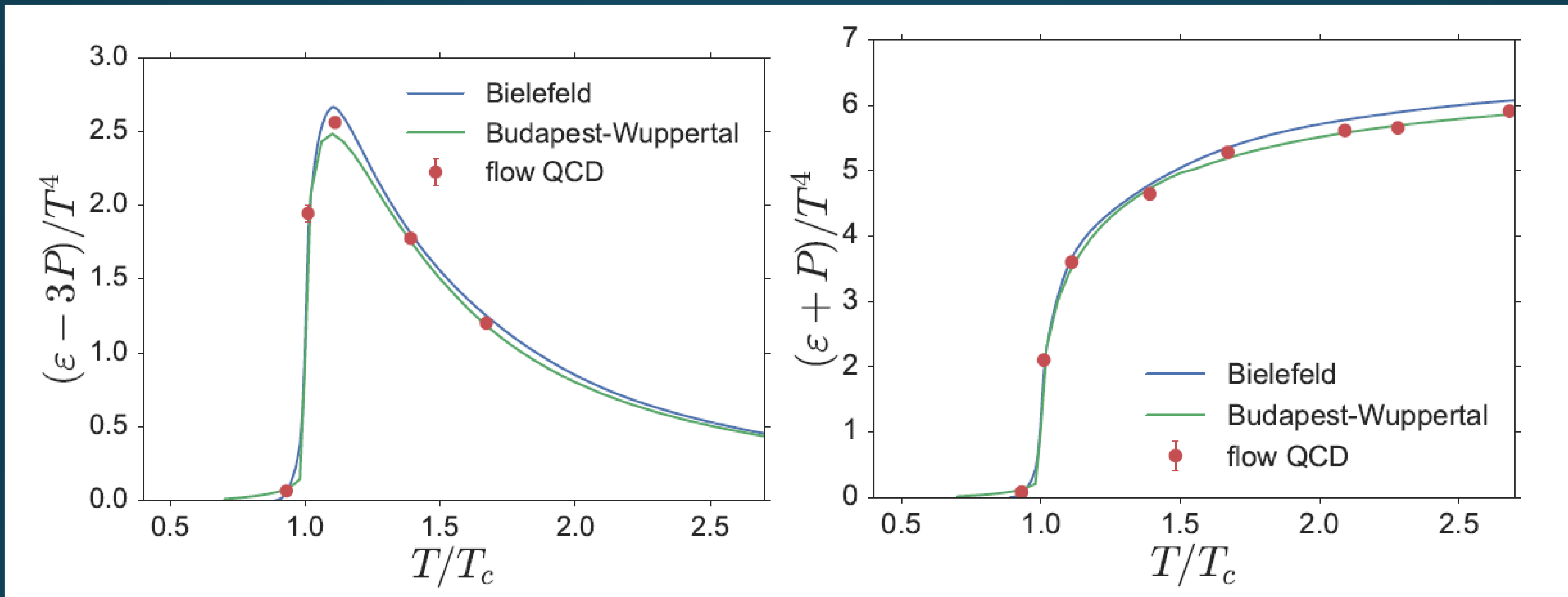
Black line: continuum extrapolated

- Fitting ranges:
  - range-1: $0.01 < tT^2 < 0.015$
  - range-2: $0.005 < tT^2 < 0.015$
  - range-3: $0.01 < tT^2 < 0.02$

Systematic error from the choice of fitting range

≈ statistical error

Temperature Dependence



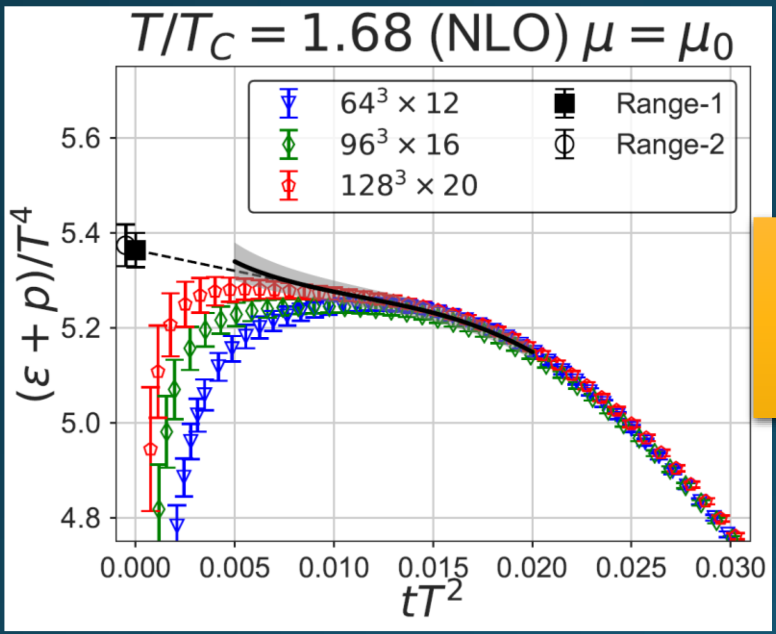
Error includes
- statistical error
- choice of t range for t→0 limit
- uncertainty in aΛ_{MS}

total error <1.5% for T>1.1T_c

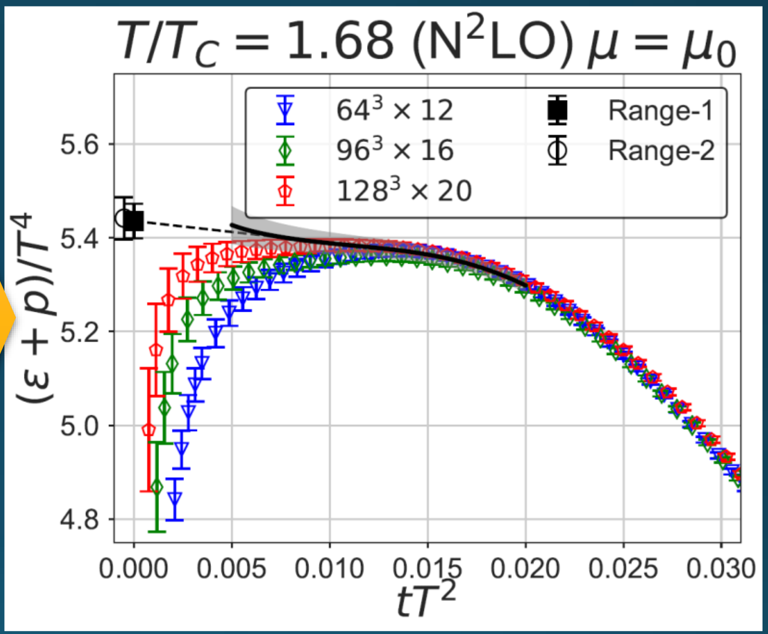
- Excellent agreement with integral method
- High accuracy only with ~2000 confs.

Higher Order Coefficient: ε+p

NLO (1-loop)



N²LO (2-loop)



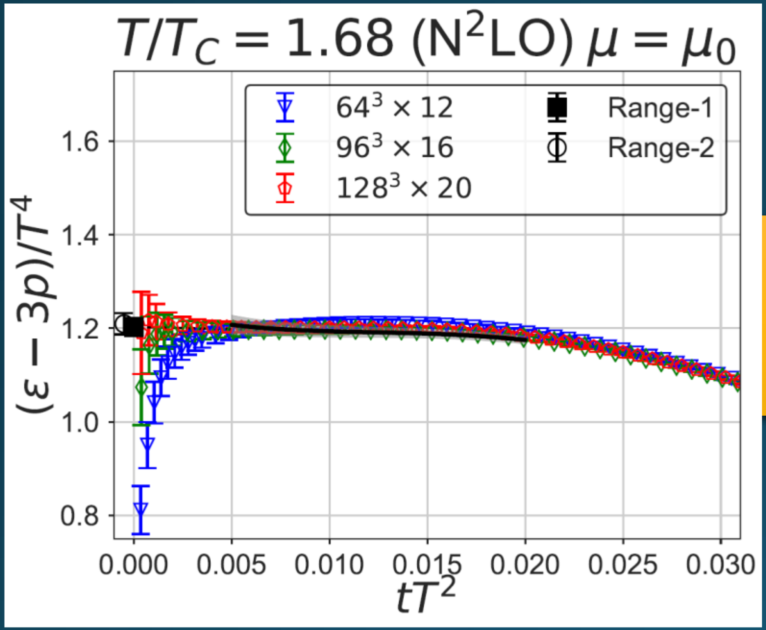
Iritani, MK, Suzuki, Takaura, in prep.

- ☐ t dependence becomes milder with higher order coeff.
- ☐ 1-loop → 2-loop : about 2% increase

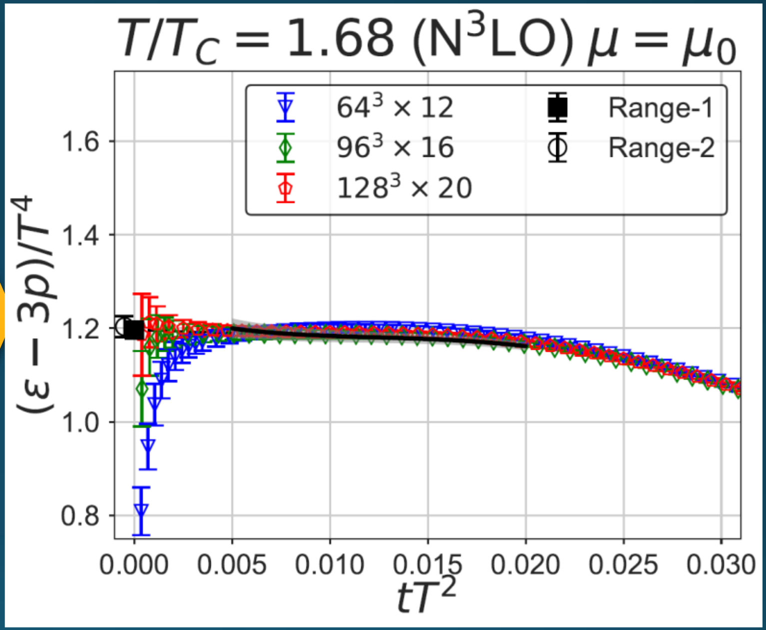
- ☐ Systematic analysis: μ_0 or μ_d, uncertainty of Λ, fit range
- ☐ Extrapolation func: linear, higher order term in c_1 ($\sim g^6$)

Higher Order Coefficient: ε-3p

N²LO (2-loop)



N³LO (3-loop)



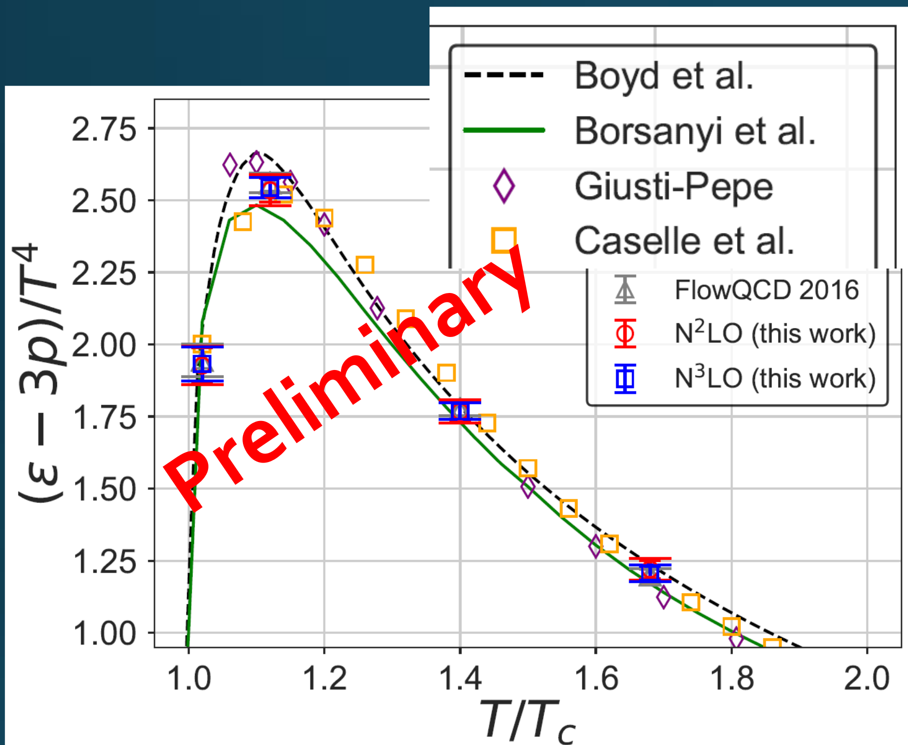
Iritani, MK, Suzuki, Takaura, in prep.

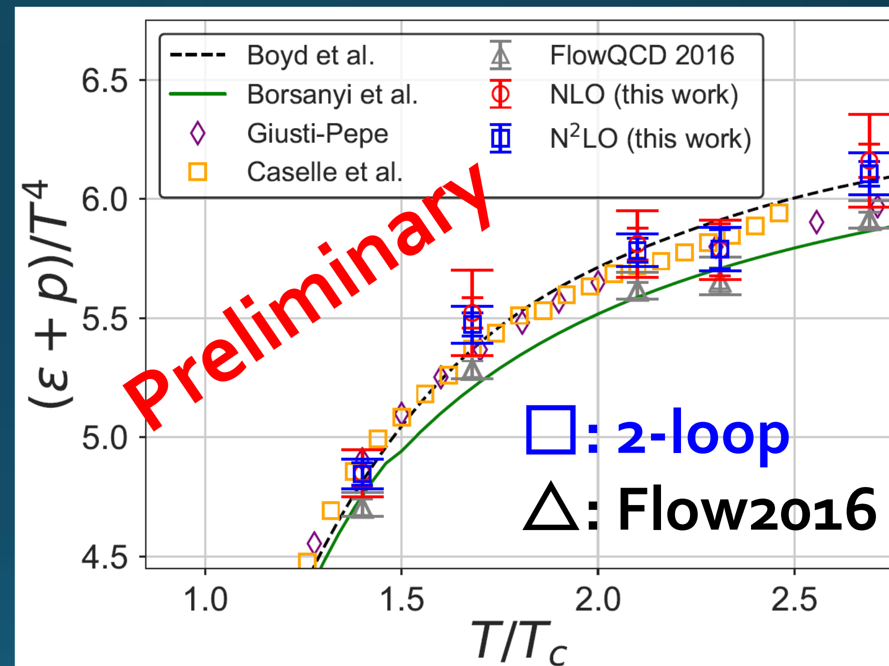
- No difference b/w 2- & 3-loops: 2-loop is already good!

- Systematic analysis: μ_0 or μ_d, uncertainty of Λ, fit range
- Extrapolation func: linear, higher order term in c_2 ($\sim g^8$)

Effect of Higher-Order Coeffs.

Iritani, MK, Suzuki, Takaura, in prep.





Systematic error: μ_0 or μ_d, Λ, t→0 function, fit range

Effect of higher order c_1 & c_2 (pure gauge)
- ☐ e-3p: negligible (<0.5%)
- ☐ e+p: ~2% increase

Gradient Flow for Fermions

$$\partial_t \psi(t,x) = D_\mu D_\mu \psi(t,x)$$

$$\partial_t \bar{\psi}(t,x) = \psi(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu$$

$$D_\mu = \partial_\mu + A_\mu(t,x)$$

Luscher, 2013
Makino, Suzuki, 2014
Taniguchi+ (WHOT) 2016; 2017

- Not "gradient" flow but a "diffusion" equation.

- Divergence in field renormalization of fermions.
- All observables are finite at t>0 once Z(t) is fixed.

$$\tilde{\psi}(t,x) = Z(t)\psi(t,x)$$

- Energy-momentum tensor from SFTE Makino, Suzuki, 2014

EMT in QCD

$$
\begin{aligned}
T_{\mu\nu}(t,x) =& c_1(t)U_{\mu\nu}(t,x) + c_2(t)\delta_{\mu\nu}\big(E(t,x) - \langle E\rangle_0\big) \\
&+ c_3(t)\big(O_{3\mu\nu}(t,x) - 2O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) \\
&+ c_4(t)\big(O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) + c_5(t)\big(O_{5\mu\nu}(t,x) - \mathrm{VEV}\big)
\end{aligned}
$$

$$
T_{\mu\nu}(x) = \lim_{t\to 0} T_{\mu\mu}(t,x)
$$

$$
\begin{aligned}
\tilde{O}^f_{3\mu\nu}(t,x) &\equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x), \\
\tilde{O}^f_{4\mu\nu}(t,x) &\equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x), \\
\tilde{O}^f_{5\mu\nu}(t,x) &\equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x),
\end{aligned}
$$

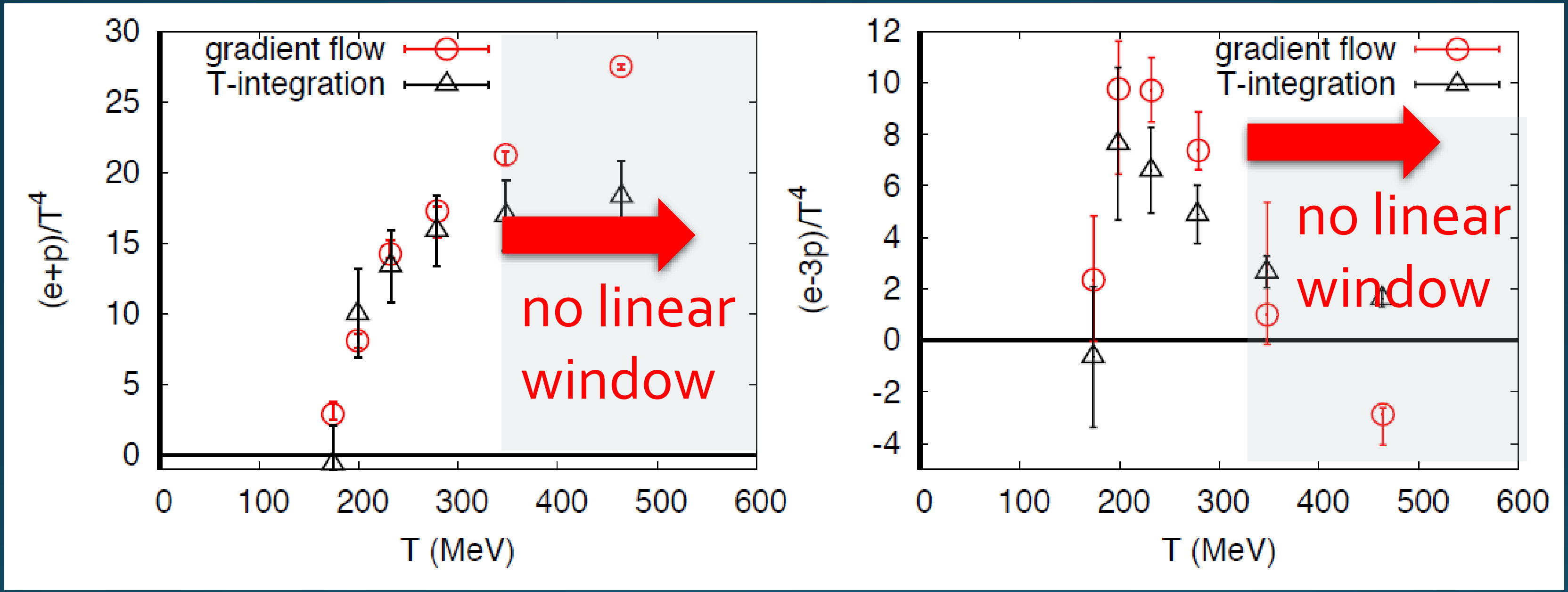
$$
\varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.
$$

$$
\begin{aligned}
c_1(t) &= \frac{1}{\bar{g}(1/\sqrt{8t})^2} - \frac{1}{(4\pi)^2}\left[9(\gamma - 2\ln 2) + \frac{19}{4}\right], \\
c_2(t) &= \frac{1}{(4\pi)^2}\frac{33}{16}, \\
c_3(t) &= \frac{1}{4}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[2 + \frac{4}{3}\ln(432)\right]\right\}, \\
c_4(t) &= \frac{1}{(4\pi)^2}\bar{g}(1/\sqrt{8t})^2, \\
c_5^f(t) &= -\bar{m}_f(1/\sqrt{8t})\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[4(\gamma - 2\ln 2) + \frac{14}{3} + \frac{4}{3}\ln(432)\right]\right\}
\end{aligned}
$$

2+1 QCD EoS from Gradient Flow

Taniguchi+ (WHOT-QCD), PR**D96**, 014509 (2017)

$m_{PS}/m_V \approx 0.63$



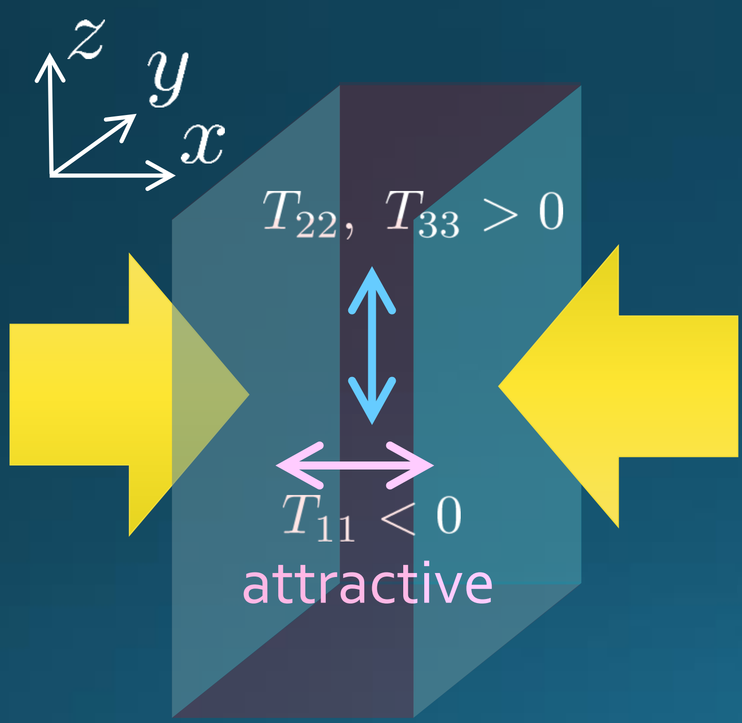
- Agreement with integral method except for N_t=4, 6
- N_t=4, 6: No stable extrapolation is possible
- Statistical error is substantially suppressed!

Physical mass: Kanaya+ (WHOT-QCD), 1710.10015

Pressure anisotropy in finite system

NEW

Casimir effect



Finite system at nonzero *T*

MK, Mogliacci, Kolbe, Horowitz, in preparation

$$V = L_x \times L_y \times L_z$$

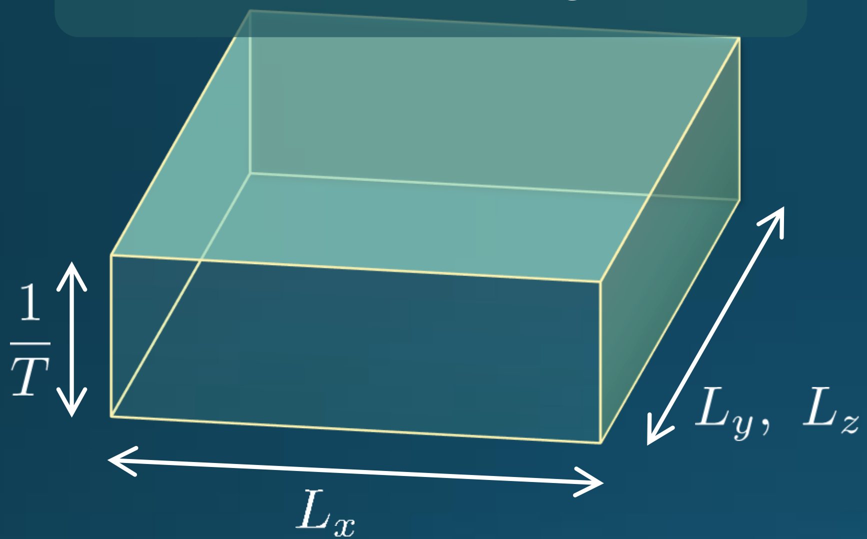
$$L_x \ll L_y = L_z$$

pressure anisotropy

$$T_{11} \neq T_{22} = T_{33}$$

Two Special Cases with PBC

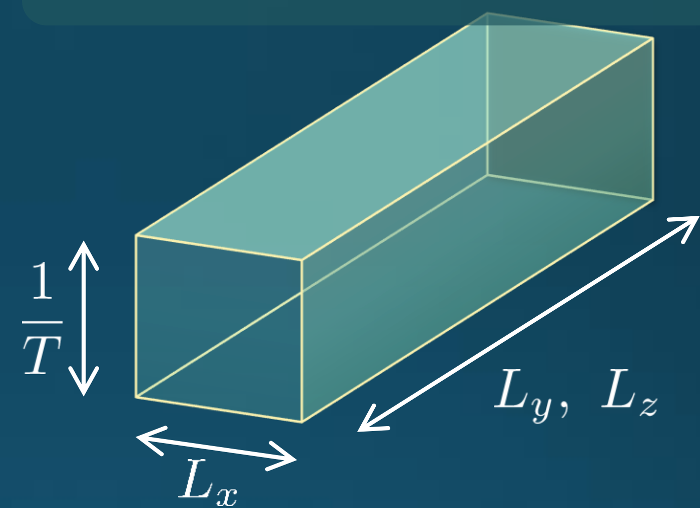
$$1/T \ll L_x = L_y = L_z$$



$$T_{11} = T_{22} = T_{33}$$

$$\frac{p_1}{p_2} = 1$$

$$1/T = L_x,\ L_y = L_z$$



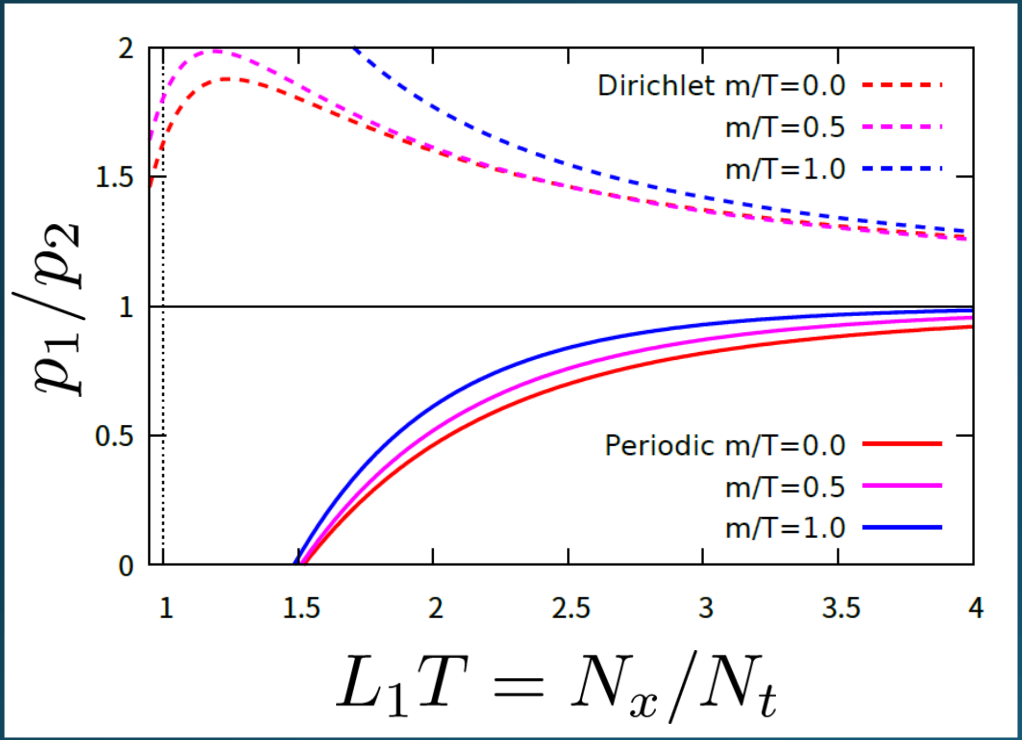
$$T_{44} = T_{11},\ T_{22} = T_{33}$$

In conformal ($\Sigma_\mu T_{\mu\mu}=0$)

$$\frac{p_1}{p_2} = -1$$

Pressure Anisotropy

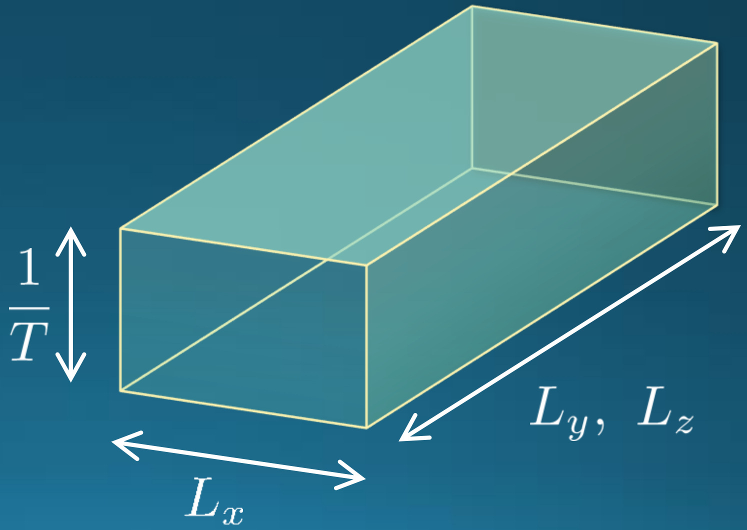
MK, Mogliacci, Kolbe, Horowitz, in prep.



Free scalar field

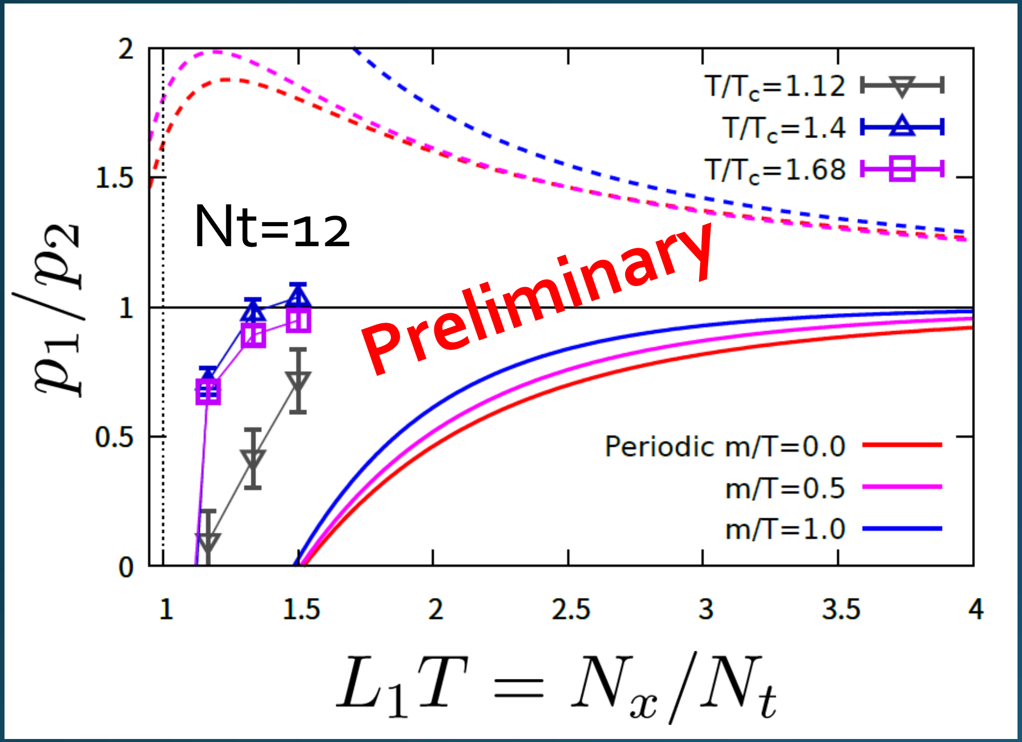
- $L_2=L_3=\infty$

Mogliacci+, 1807.07871



Pressure Anisotropy

MK, Mogliacci, Kolbe, Horowitz, in prep.



Free scalar field

- $L_2=L_3=\infty$

Mogliacci+, 1807.07871

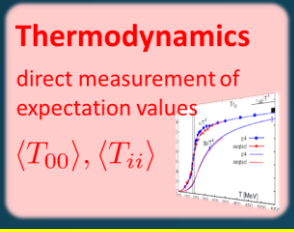
Lattice result

- Periodic BC
- $N_s^2 x N_x x N_t = 72^2 x N_x x 12$
- N_x=12, 14, 16, 18
- Only t→0 limit (fixed a)

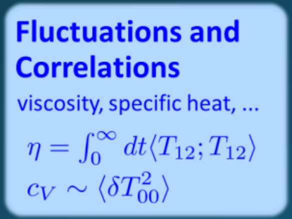
Medium near T_c is remarkably insensitive to finite size!
How do we understand??

Contents

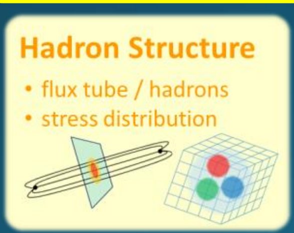
Constructing EMT on the lattice



Thermodynamics



EMT Correlation Function



Stress distribution in $\bar{q}q$ system

EMT Correlator: Motivation

- Transport Coefficient

Kubo formula → viscosity

$$\eta = \int_0^\infty dt \int_0^{1/T} d\tau \int d^3x \langle T_{12}(x, -i\tau) T_{12}(0,t) \rangle$$

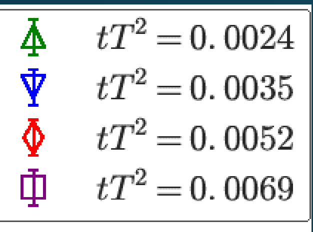
Karsch, Wyld, 1987
Nakamura, Sakai, 2005
Meyer; 2007, 2008
...
Borsanyi+, 2018
Astrakhantsev+, 2018

- Energy/Momentum Conservation

$\langle \bar{T}_{0\mu}(\tau) \bar{T}_{\rho\sigma}(0) \rangle$: τ-independent constant

- Fluctuation-Response Relations

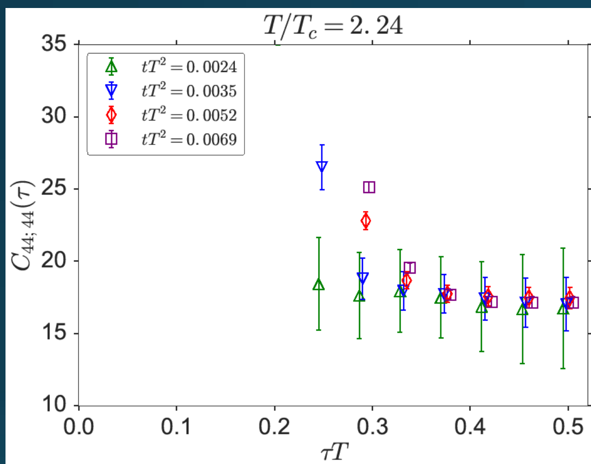
$$c_V = \frac{\langle \delta E^2 \rangle}{VT^2} \qquad E + p = \frac{\langle \bar{T}_{01}^2 \rangle}{VT} = \frac{\langle \bar{T}_{11} \bar{T}_{00} \rangle}{VT}$$



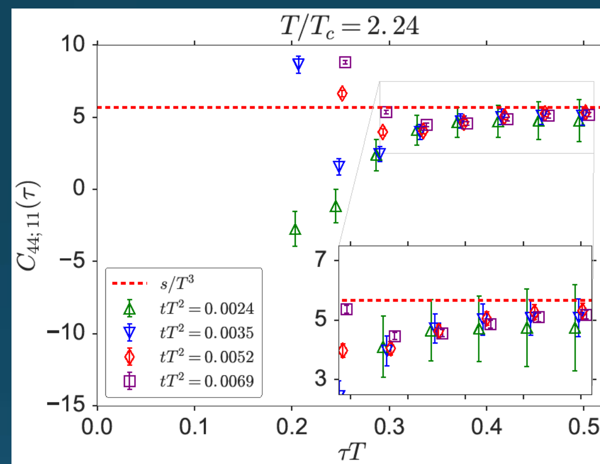
EMT Euclidean Correlator

FlowQCD, PR **D96**, 111502 (2017)

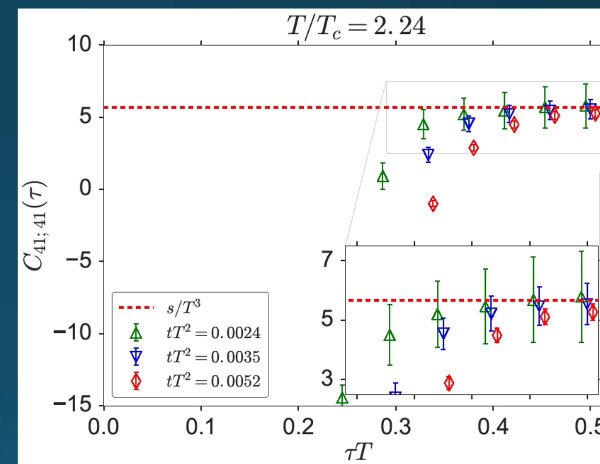
$$\langle \bar{T}_{44}(\tau)\bar{T}_{44}(0)\rangle$$



$$\langle \bar{T}_{44}(\tau)\bar{T}_{11}(0)\rangle$$



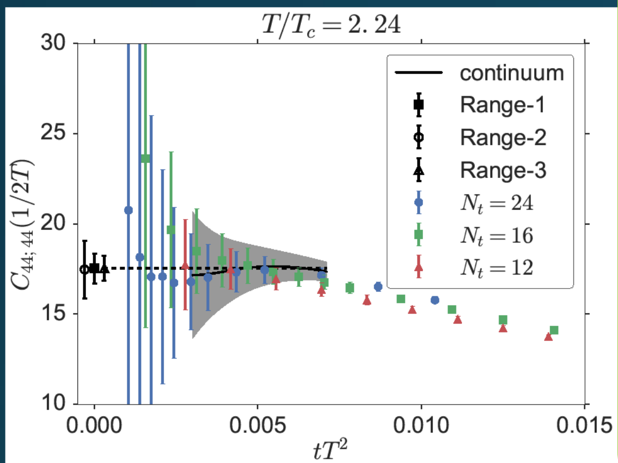
$$\langle \bar{T}_{41}(\tau)\bar{T}_{41}(0)\rangle$$



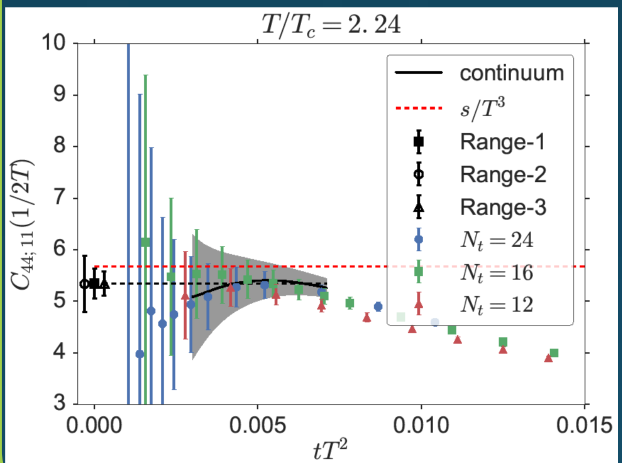
- τ-independent plateau in all channels ➔ conservation law
- Confirmation of fluctuation-response relations
- New method to measure c_V
  - Similar result for (41;41) channel: Borsanyi+, 2018
  - Perturbative analysis: Eller, Moore, 2018

Fluctuation-Response Relations

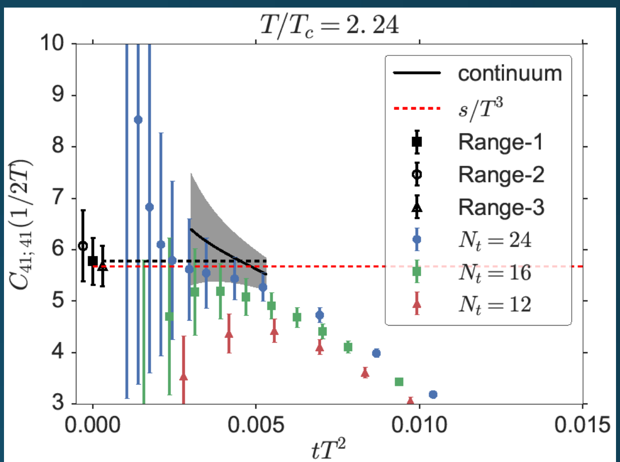
$\langle T_{44}(\tau) T_{44}(0) \rangle$



$\langle T_{44}(\tau) T_{11}(0) \rangle$



$\langle T_{41}(\tau) T_{41}(0) \rangle$



New measurement of c_V

c_V/T^3

T/T_c	$C_{44;44}(\tau_m)$	Ref.[19]	Ref.[11]	ideal gas
1.68	$17.7(8)(^{+2.1}_{-0.4})$	$22.8(7)^*$	17.7	21.06
2.24	$17.5(0.8)(^{+0}_{-0.1})$	$17.9(7)^{**}$	18.2	21.06

Confirmation of FRR

$$E + p = \frac{\langle \bar{T}_{01}^2 \rangle}{VT} = \frac{\langle \bar{T}_{11} \bar{T}_{00} \rangle}{VT}$$

2+1 QCD:
Taniguchi+ (WHOT-QCD), 1711.02262

Contents

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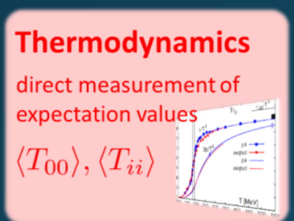

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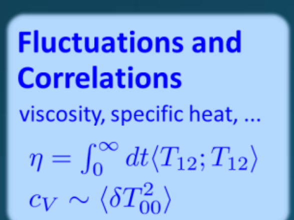

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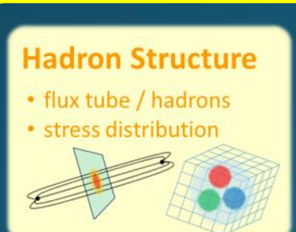
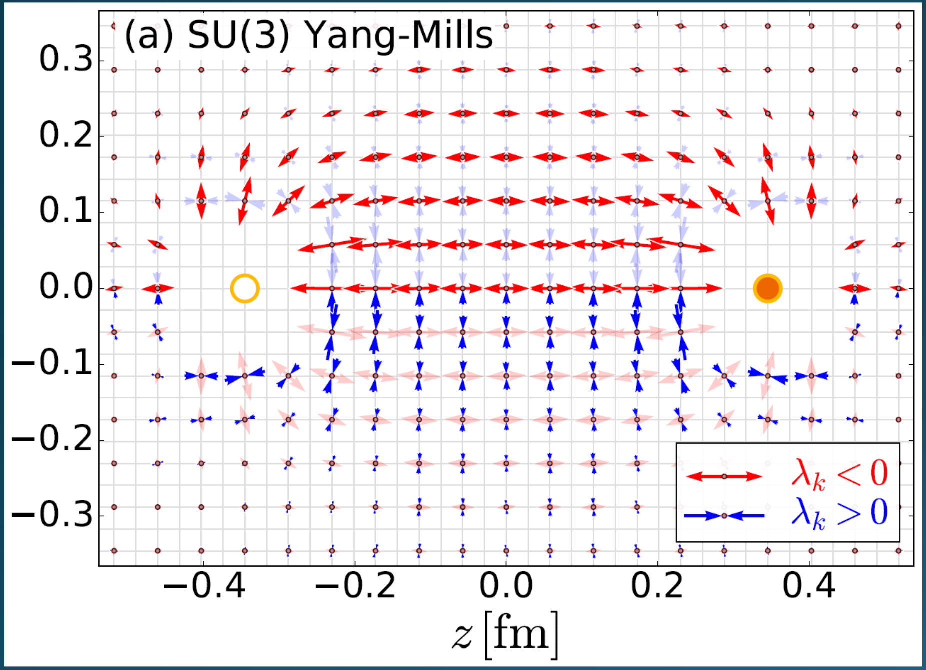


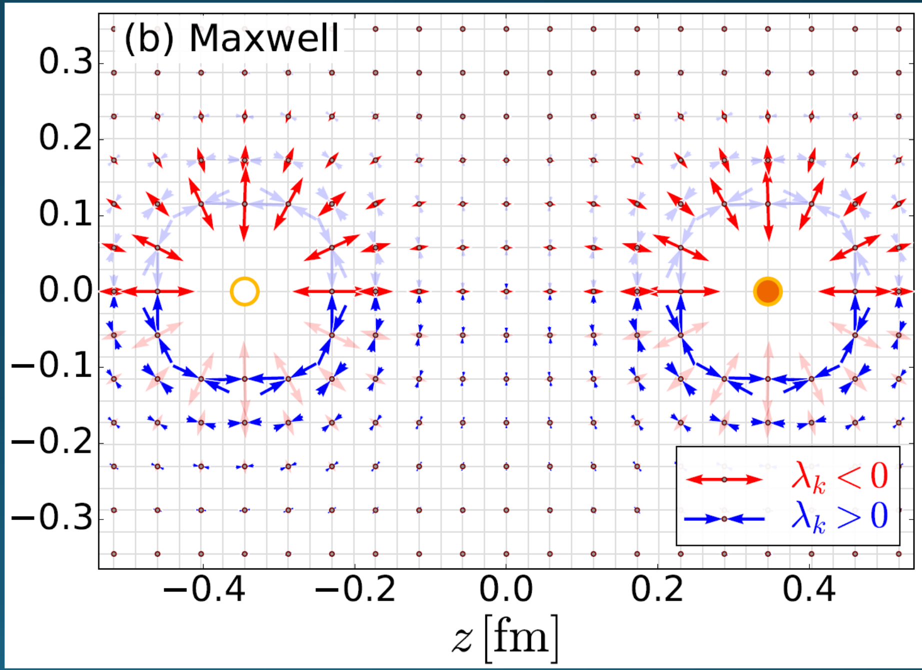
Stress distribution in $\bar{q}q$ system

SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)



Maxwell
(classical)



Propagation of the force is clearly different in YM and Maxwell theories!

Lattice Setup

- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

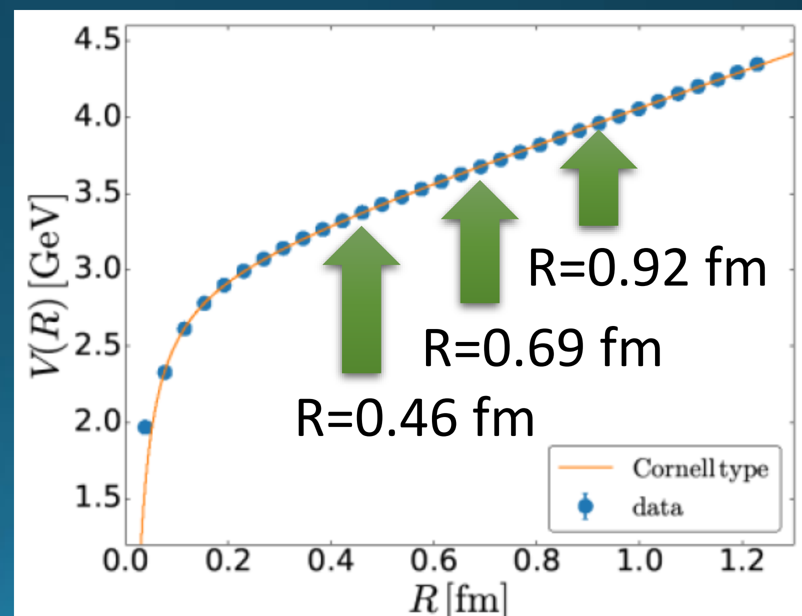
- APE smearing / multi-hit

- fine lattices (a=0.029-0.06 fm)
- continuum extrapolation

- Simulation: bluegene/Q@KEK

Yanagihara+, 1803.05656

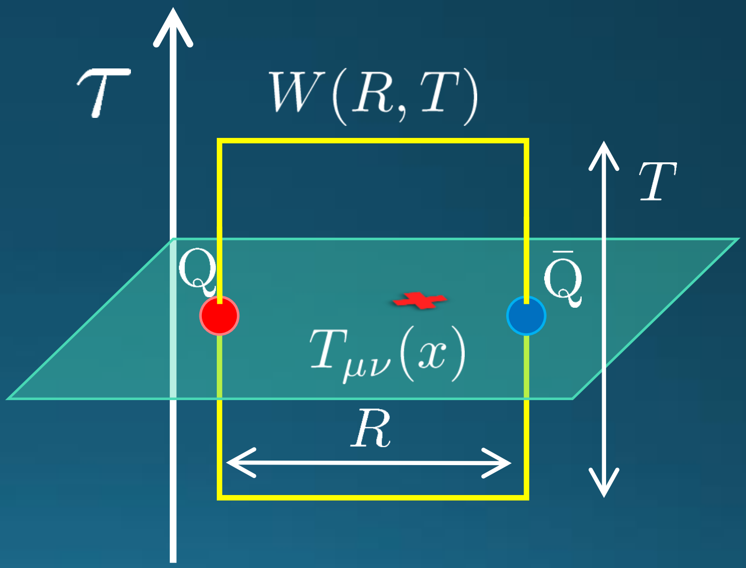
β	a [fm]	$N_{\rm size}^4$	$N_{\rm conf}$	R/a		
6.304	0.058	48^4	140	8	12	16
6.465	0.046	48^4	440	10	–	20
6.513	0.043	48^4	600	–	16	–
6.600	0.038	48^4	1,500	12	18	24
6.819	0.029	64^4	1,000	16	24	32
			R [fm]	0.46	0.69	0.92



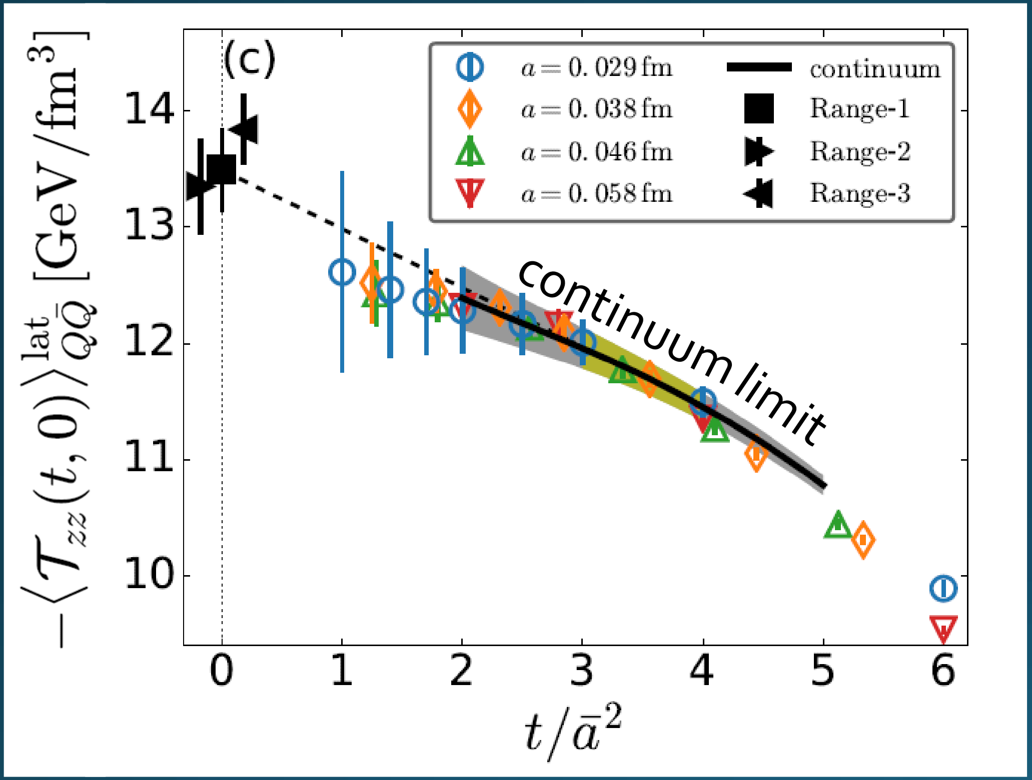
Stress Distribution in $Q\bar{Q}$

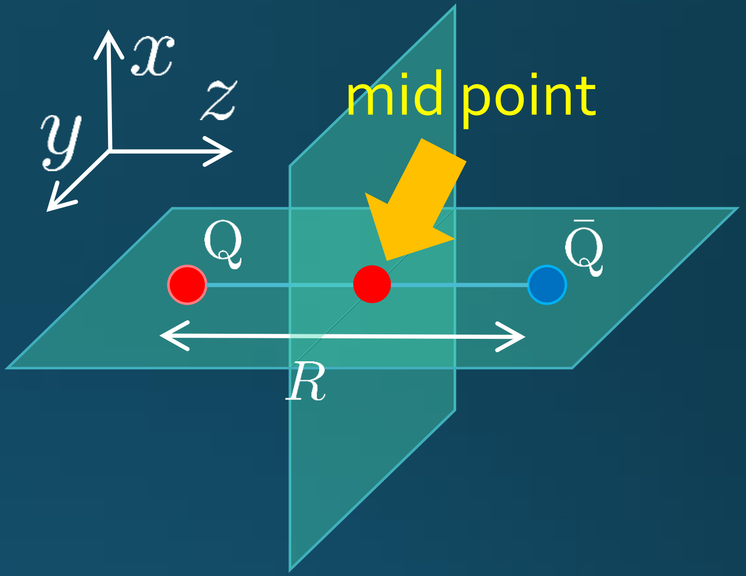
$$\langle T_{\mu\nu}(x)\rangle_{Q\bar{Q}} = \lim_{T\to\infty} \frac{\langle W(T,R)T_{\mu\nu}(x)\rangle}{\langle W(T,R)\rangle} - \langle T_{\mu\nu}(x)\rangle$$

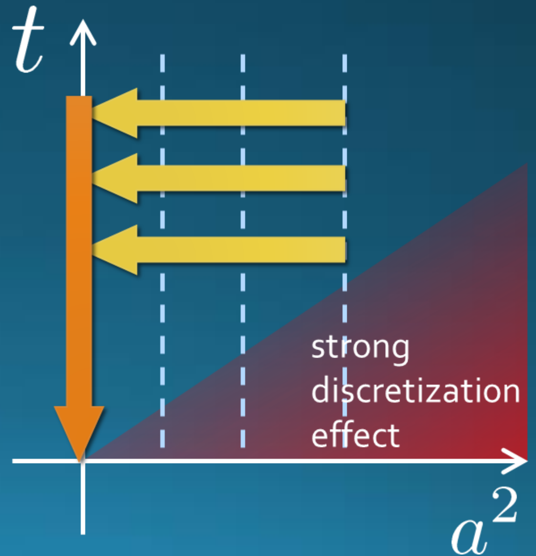
- **EMT** $T_{\mu\nu}(x)$
  - via gradient flow method
- **Wilson Loop** $W(R,T)$
  - No gradient flow for W(R,T)
  - APE smearing for spatial links
  - Multi-hit for temporal links



Continuum Extrapolation
at mid-point

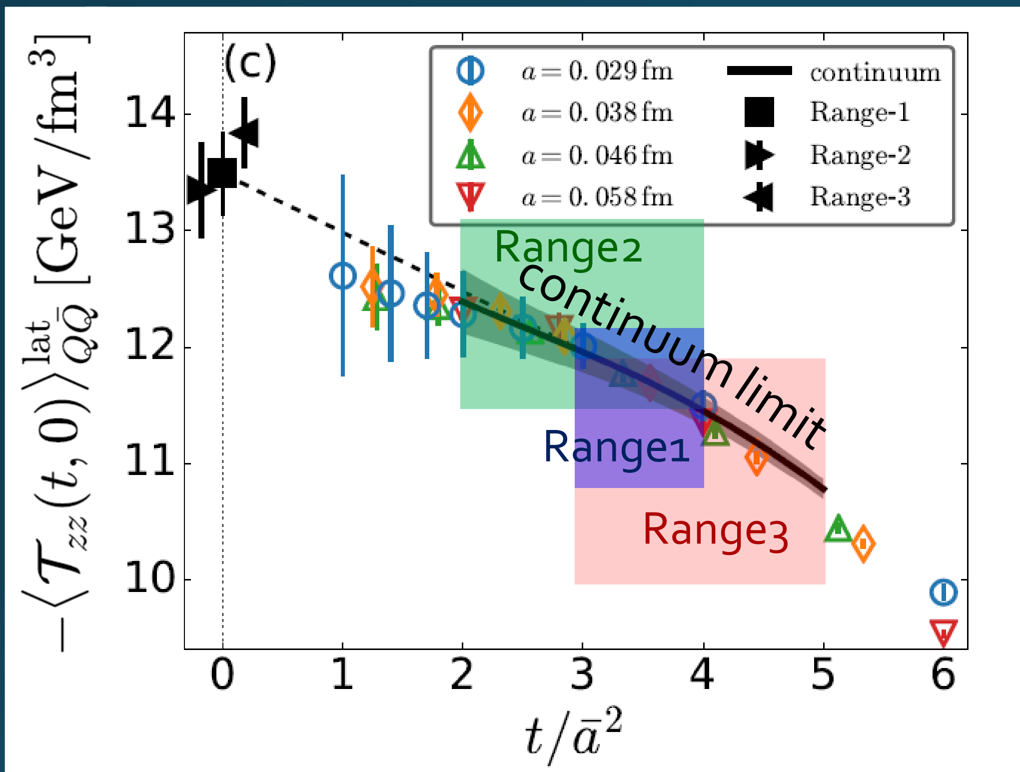


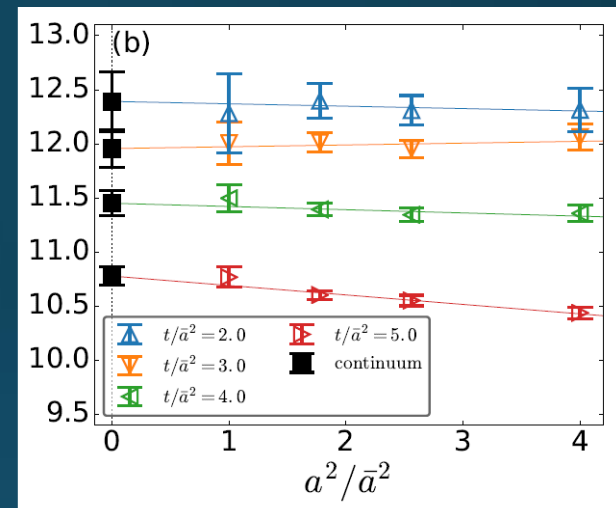


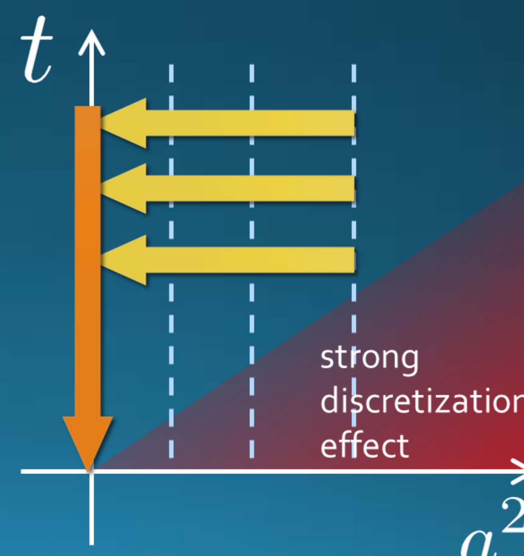


- ☐ a→0 extrapolation with fixed t

t→0 Extrapolation
at mid-point







- ☐ a→0 extrapolation with fixed t
- ☐ Then, t→0 with three ranges

Stress Distribution on Mid-Plane

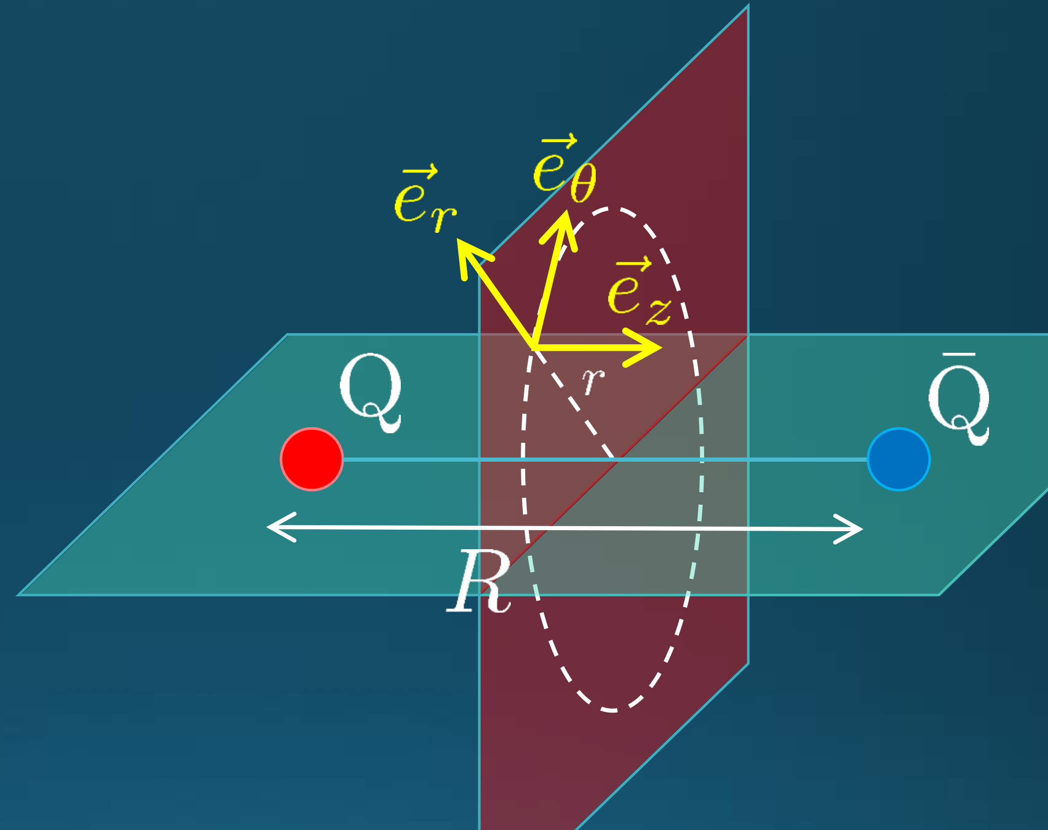
From rotational symm. & parity

EMT is diagonalized
in Cylindrical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{zz} & \\ & & & T_{44} \end{pmatrix}$$

$$T_{rr} = \vec{e}_r^{\,T} T \vec{e}_r$$

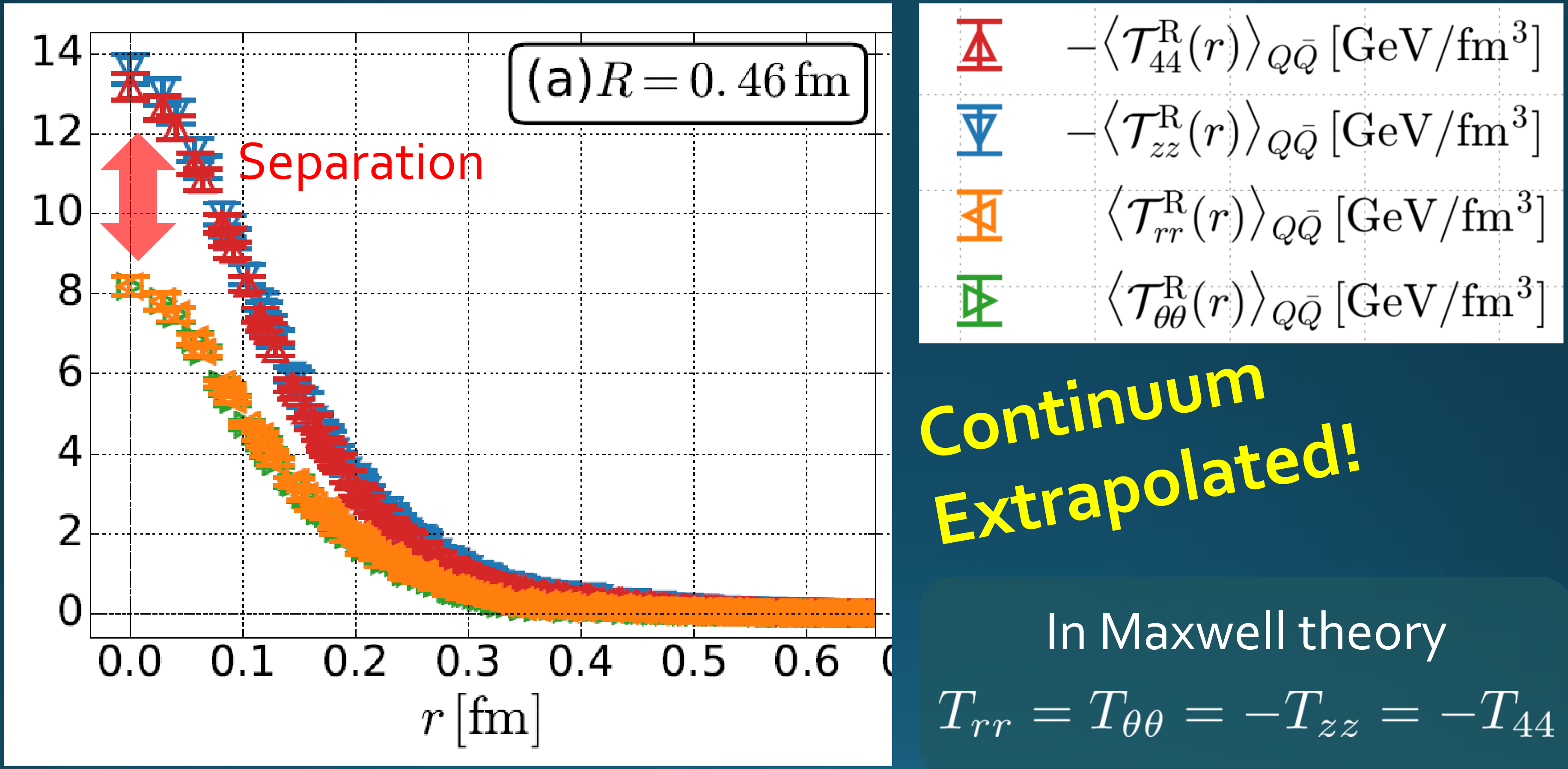
$$T_{\theta\theta} = \vec{e}_\theta^{\,T} T \vec{e}_\theta$$



Degeneracy
in Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

Mid-Plane



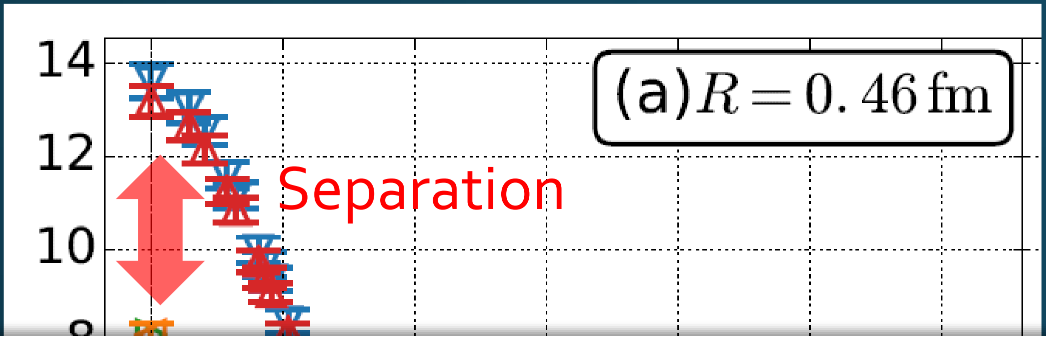
Continuum Extrapolated!

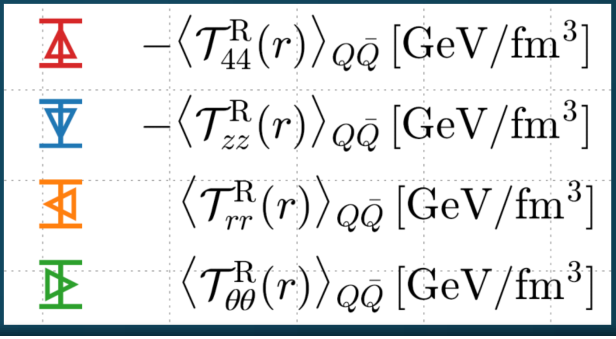
In Maxwell theory

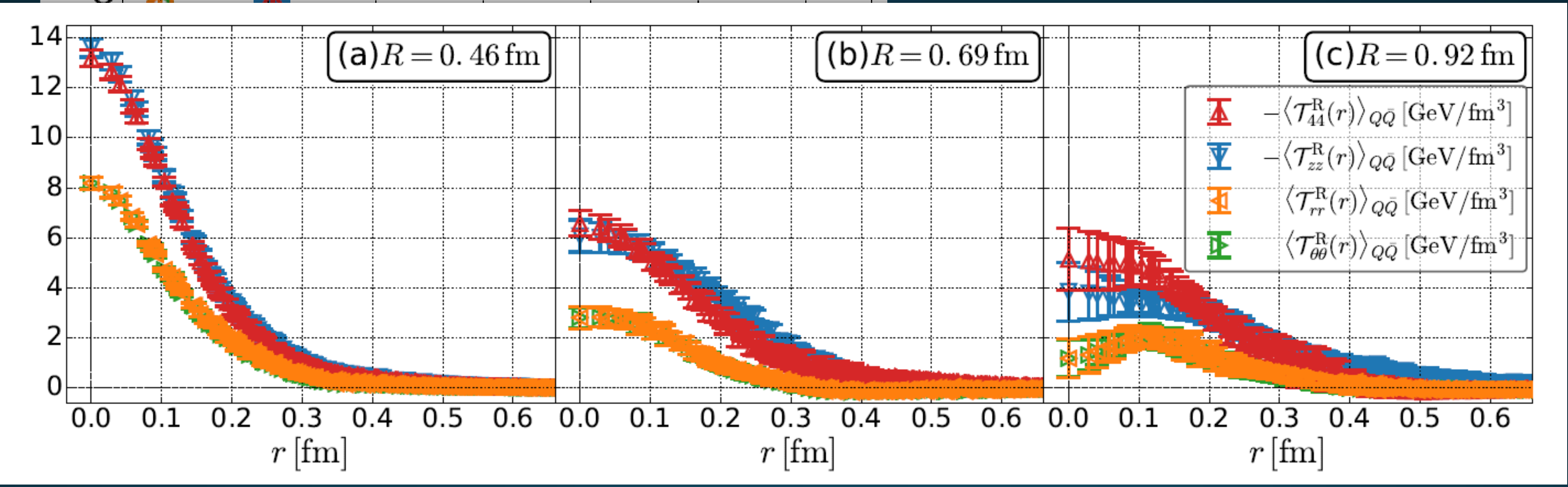
$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

- ☐ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- ☐ Separation: $T_{zz} \neq T_{rr}$
- ☐ Nonzero trace anomaly $\sum T_{cc} \neq 0$

Mid-Plane

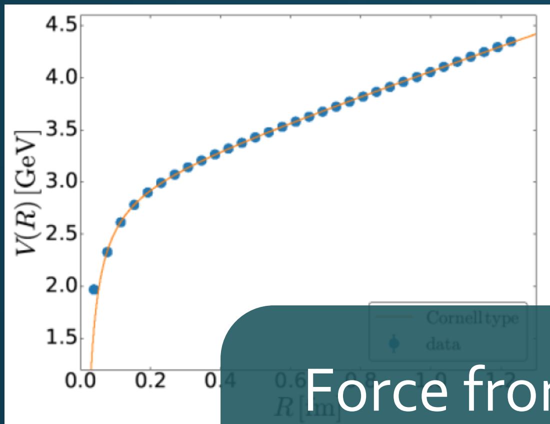


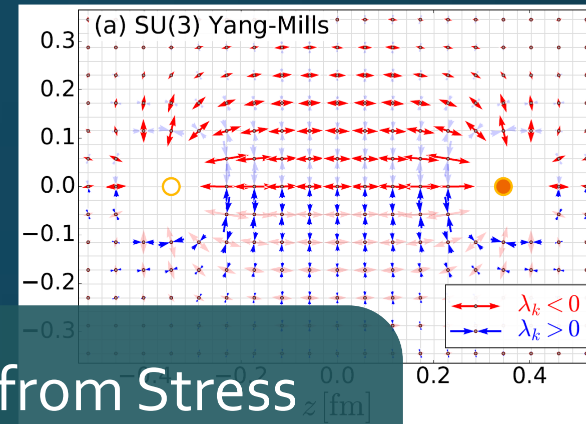




- Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- Separation: $T_{zz} \neq T_{rr}$
- Nonzero trace anomaly $\sum T_{cc} \neq 0$

Force





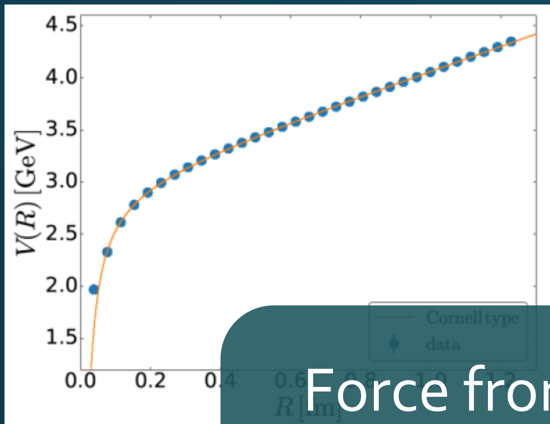
Force from Potential

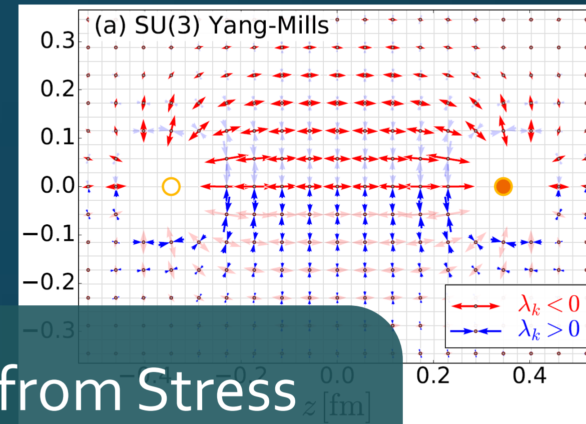
$$F_{\text{pot}} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\text{stress}} = \int_{\text{mid.}} d^2x T_{zz}(x)$$

Force





Force from Potential

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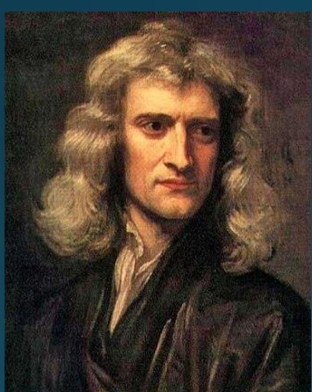
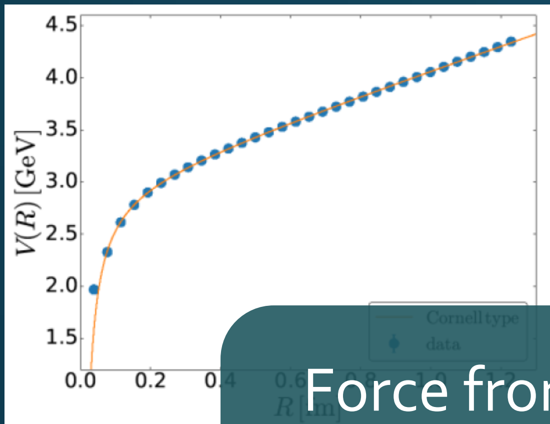
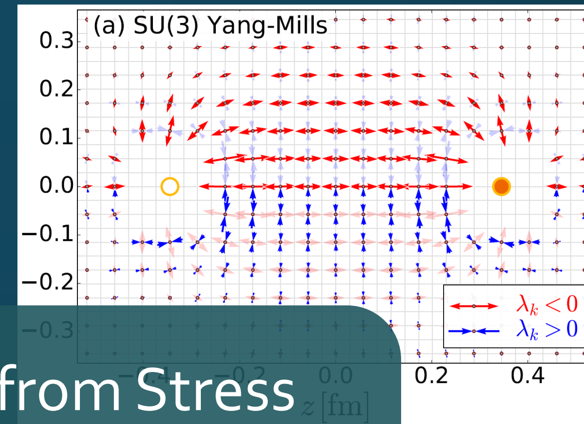


Newton
1687

Faraday
1839

Force



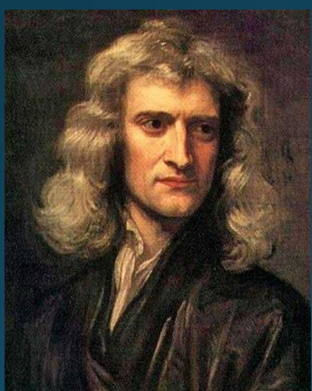


Force from Potential

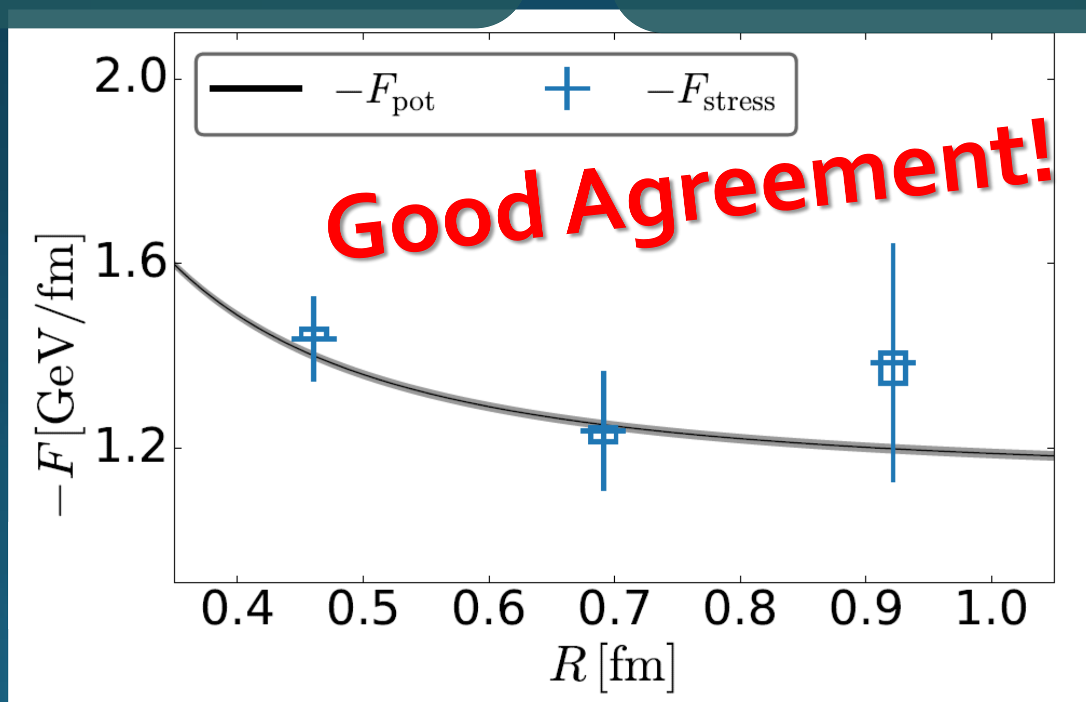
$$F_{\text{pot}} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\text{stress}} = \int_{\text{mid.}} d^2x T_{zz}(x)$$



Newton
1687



Faraday
1839

Abelian-Higgs Model

Yanagihara, Iritani, MK, in prep.

Abelian-Higgs Model

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

GL parameter: $\kappa = \sqrt{\lambda}/g$

- type-I : $\kappa < 1/\sqrt{2}$
- type-II : $\kappa > 1/\sqrt{2}$
- Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

Infinitely long tube

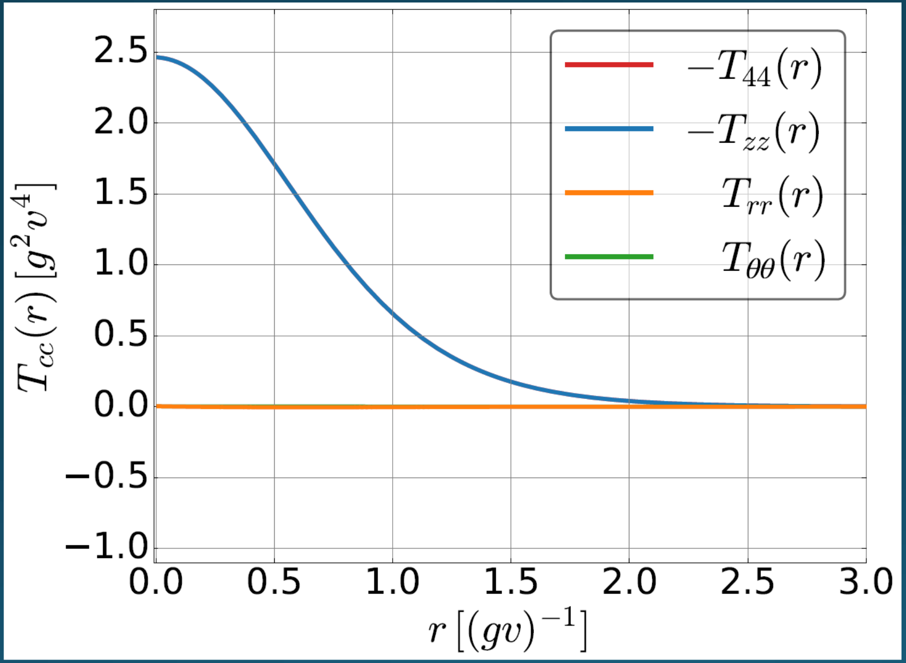
- degeneracy

  $T_{zz}(r) = T_{44}(r)$ Luscher, 1981
- momentum conservation

  $$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

Stress Tensor in AH Model
infinitely-long flux tube

Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$



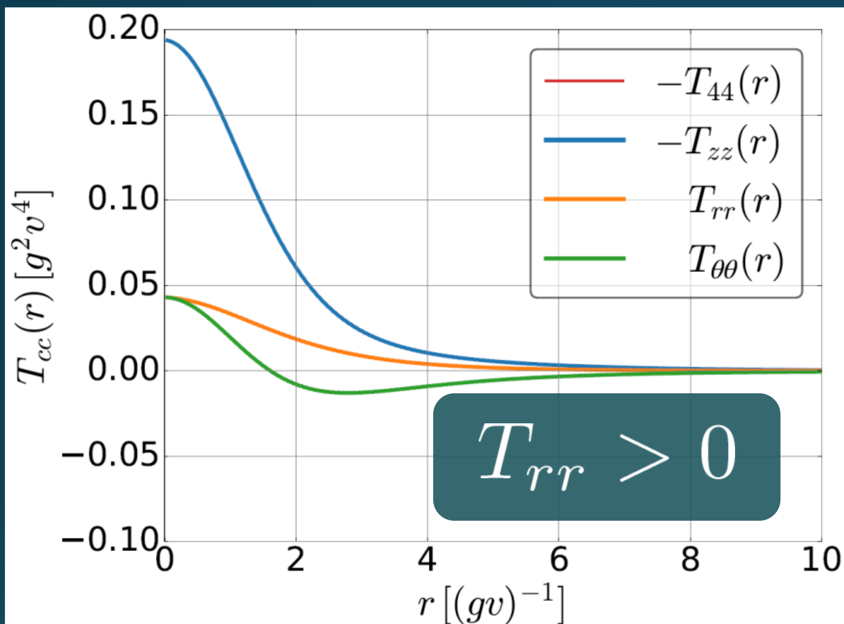
$$T_{rr} = T_{\theta\theta} = 0$$

de Vega, Schaposnik, PR**D14**, 1100 (1976).

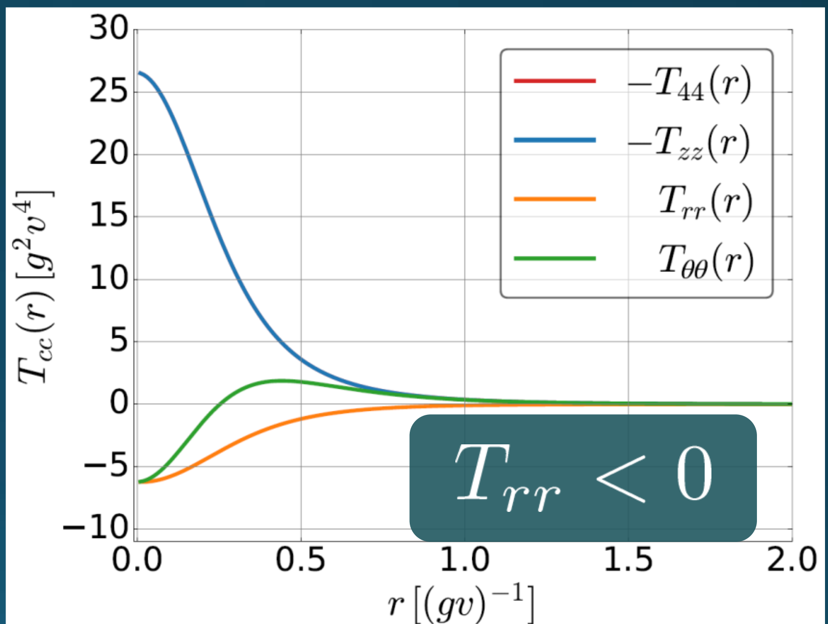
Stress Tensor in AH Model

infinitely-long flux tube

Type-I $\kappa = 0.1$



$\kappa = 3.0$ **Type-II**



- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

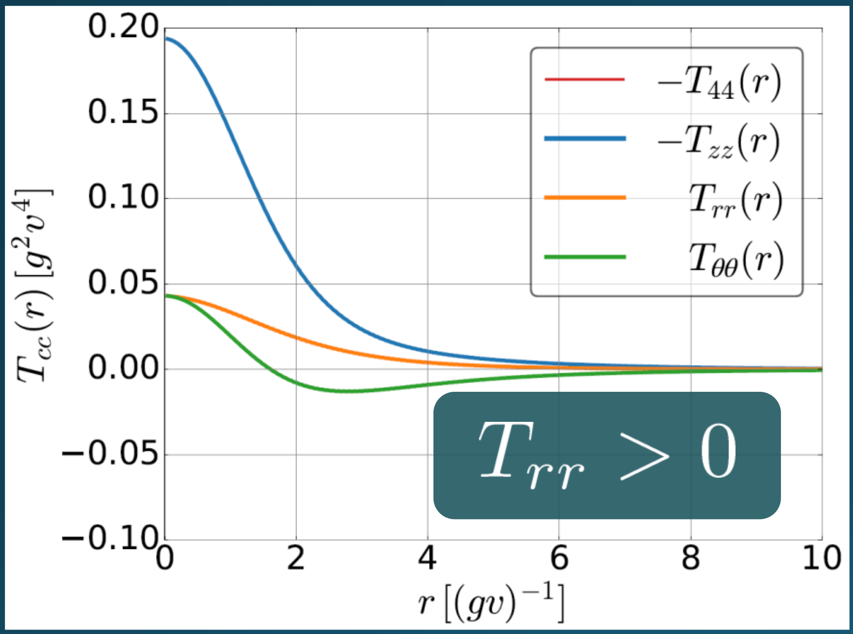
conservation law

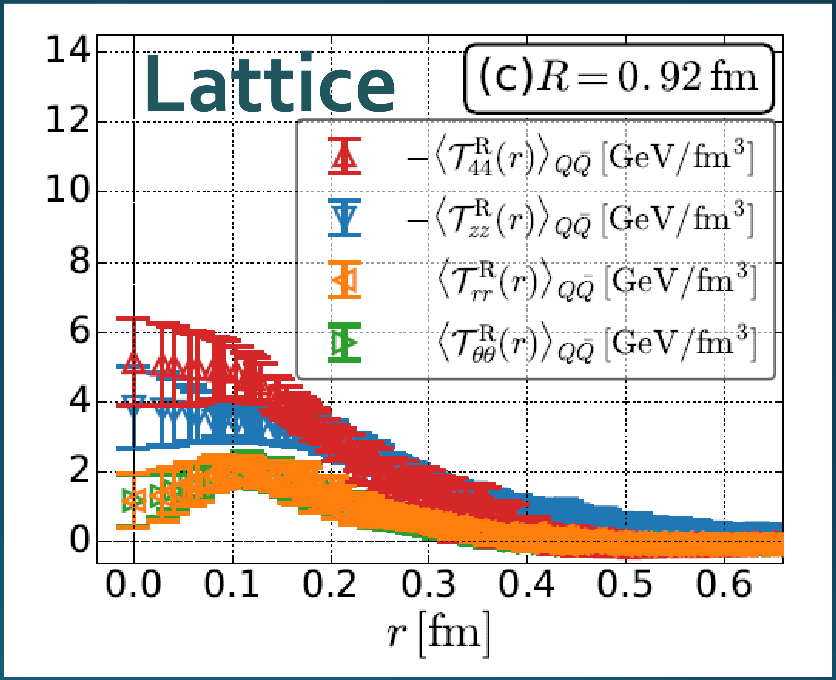
$$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

Stress Tensor in AH Model
infinitely-long flux tube

Type-I

$\kappa = 0.1$





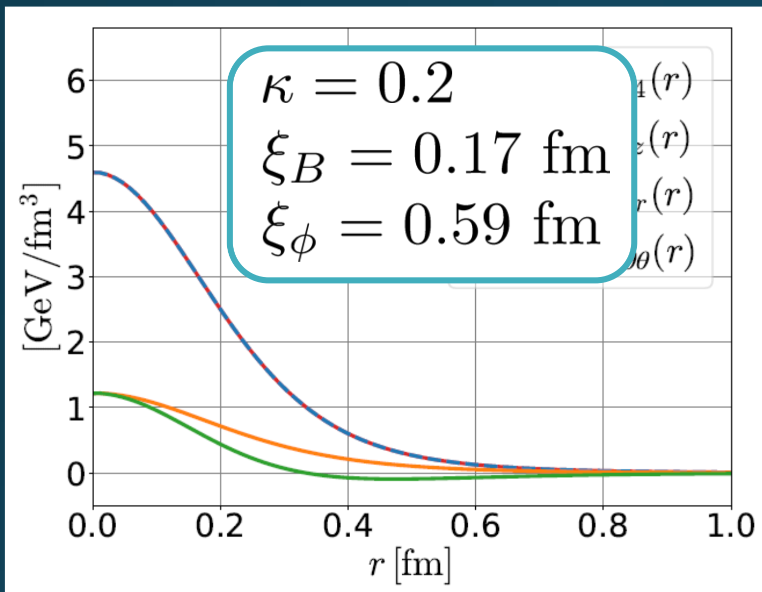
- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

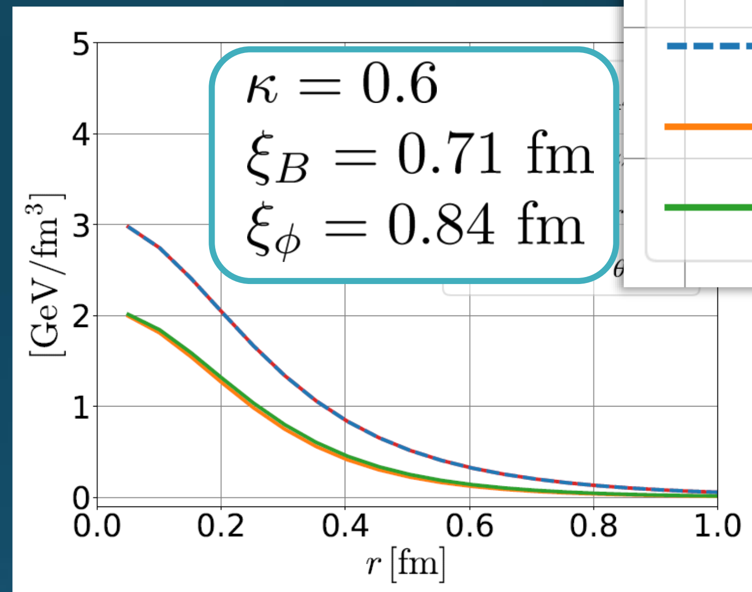
Inconsistent with lattice result

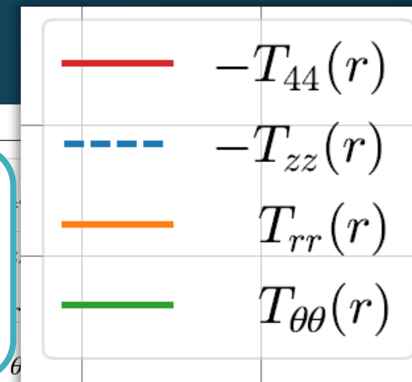
$$T_{rr} \simeq T_{\theta\theta}$$

Flux Tube with Finite Length

R=0.92 fm



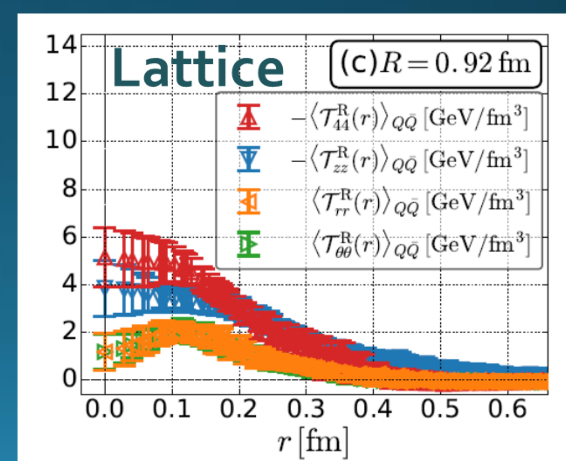




Left: $T_{zz}(0)$, $T_{rr}(0)$ reproduce lattice result

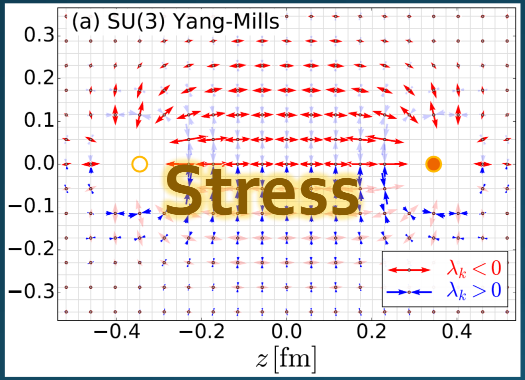
Right: A parameter satisfying $T_{rr} \approx T_{\theta\theta}$

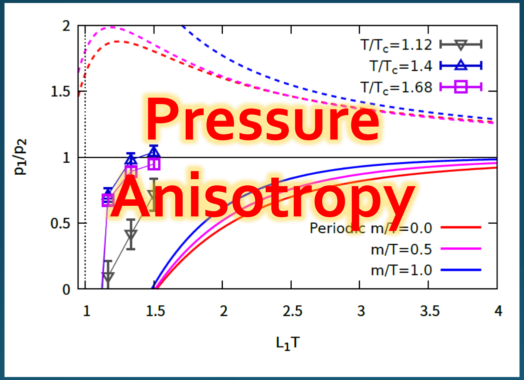
No parameter can reproduce lattice data at R=0.92fm.

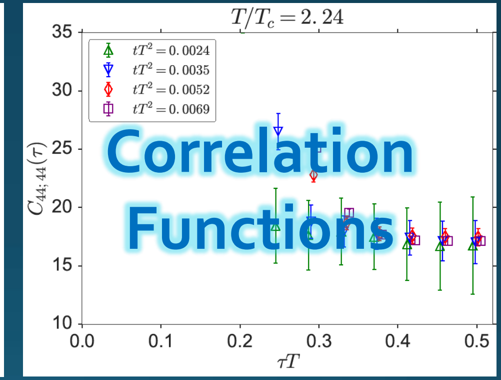


Summary

- The analysis of energy-momentum tensor on the lattice is now available, and various stuides are ongoing!
  - gradient flow method
  - higher-order perturbative coefficients







Summary

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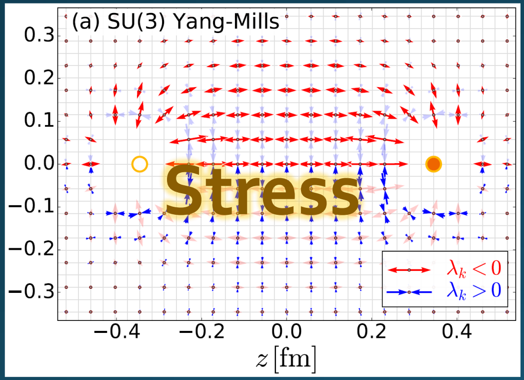
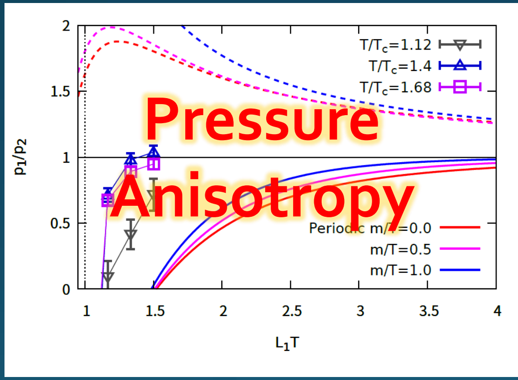
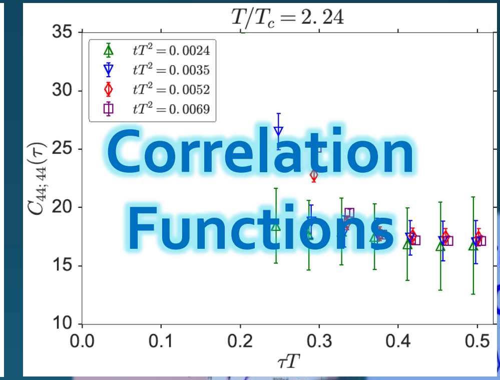
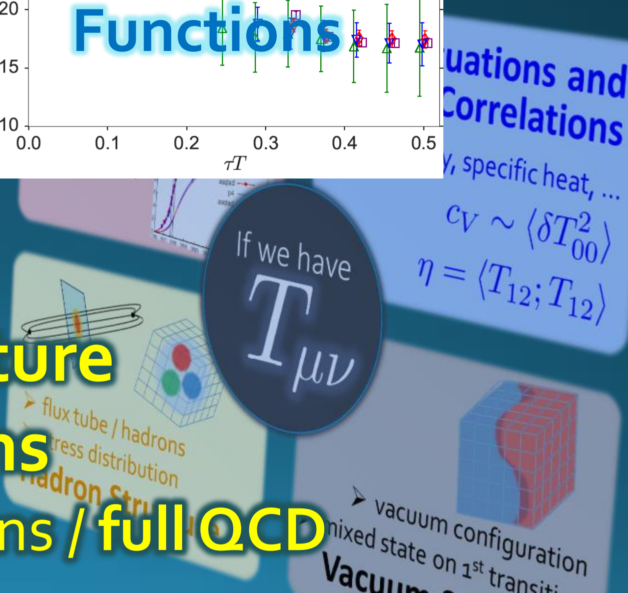






- **So many future studies**
  - Flux tube at **nonzero temperature**
  - EMT distribution **inside hadrons**
  - viscosity / other operators / instantons / **full QCD**



backup

EMT on the Lattice: Conventional

Lattice EMT Operator Caracciolo+, 1990

$$T_{\mu\nu} = Z_6 T^{[6]}_{\mu\nu} + Z_3 T^{[3]}_{\mu\nu} + Z_1 \left(T^{[1]}_{\mu\nu} - \langle T^{[1]}_{\mu\nu} \rangle \right)$$

$$T^{[6]}_{\mu\nu} = (1-\delta_{\mu\nu}) F^a_{\mu\rho} F^a_{\nu\rho}, \;\; T^{[3]}_{\mu\nu} = \delta_{\mu\nu} \left(F^a_{\mu\rho} F^a_{\nu\rho} - \frac{1}{4} F^a_{\rho\sigma} F^a_{\rho\sigma} \right), \;\; T^{[1]}_{\mu\nu} = \delta_{\mu\nu} F^a_{\rho\sigma} F^a_{\rho\sigma}$$

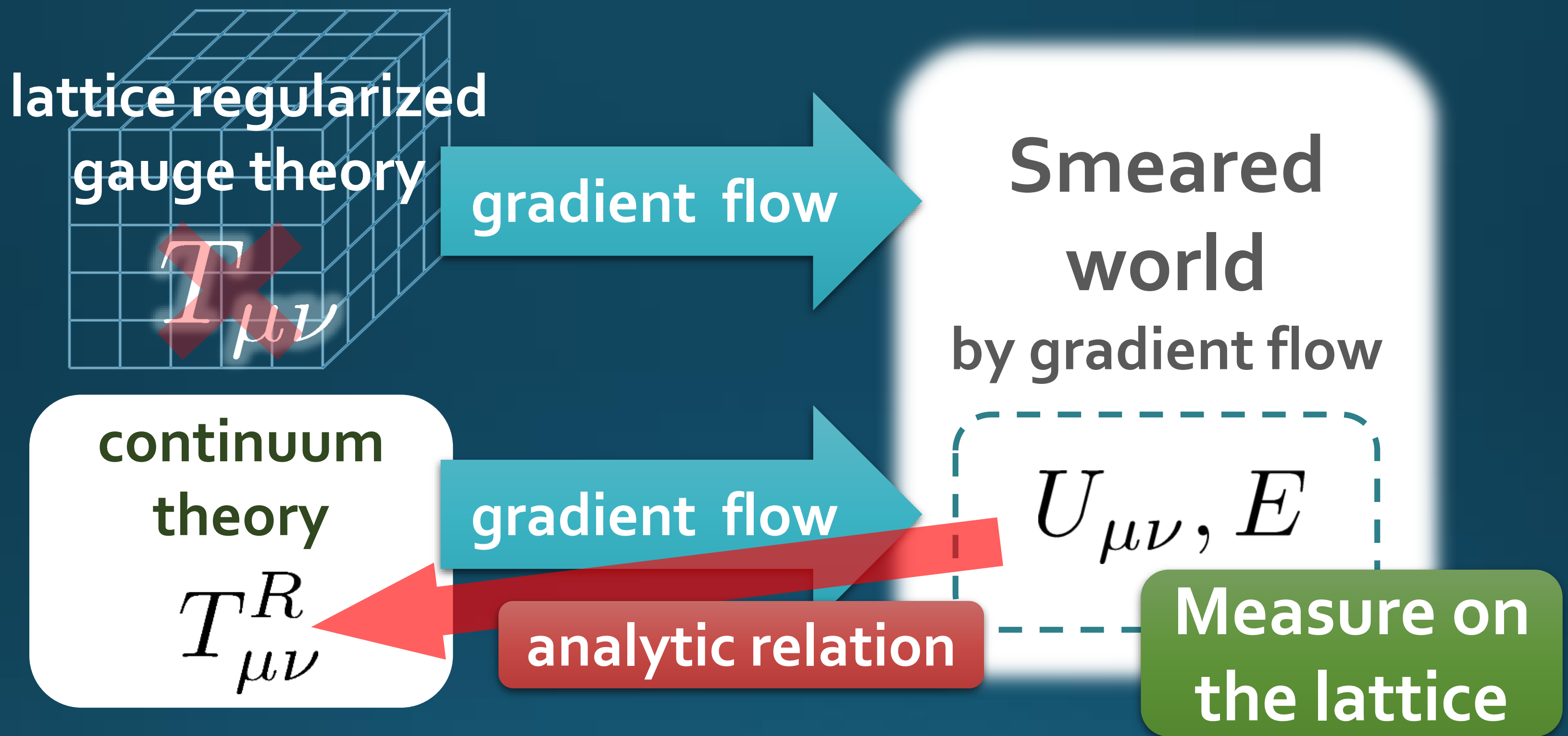
- Fit to thermodynamics: Z_3, Z_1
- Shifted-boundary method: Z_6, Z_3 Giusti, Meyer, 2011; 2013; Giusti, Pepe, 2014~; Borsanyi+, 2018

Multi-level algorithm

- effective in reducing statistical error of correlator

Meyer, 2007;
Borsanyi, 2018;
Astrakhantsev+, 2018

Gradient Flow Method



Take Extrapolation (t,a)→(0,0)

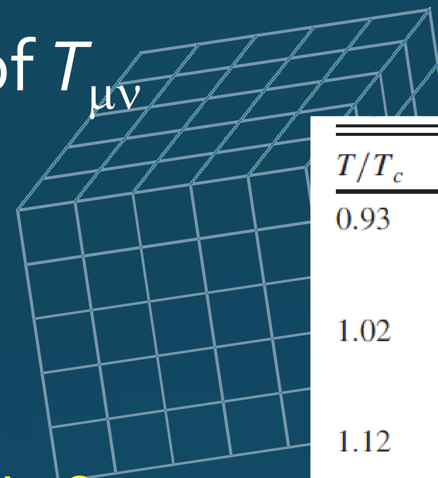
$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + \boxed{C_{\mu\nu} t} + \boxed{D_{\mu\nu}\frac{a^2}{t}} + \cdots$$

O(t) terms in SFTE lattice discretization

Numerical Simulation

FlowQCD,
PRD**94**, 114512 (2016)

- Expectation values of $T_{\mu\nu}$
- SU(3) YM theory
- Wilson gauge action
- Parameters:
  - N_t = 12, 16, 20-24
  - aspect ratio $5.3<N_s/N_t<8$
  - 1500~2000 configurations



- Scale from gradient flow

$$\rightarrow aT_c \text{ and } a\Lambda_{\rm MS}$$

FlowQCD, 1503.06516

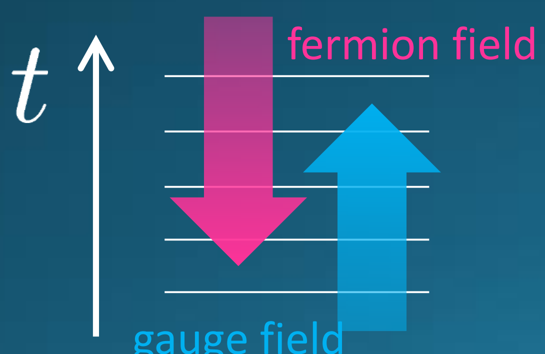
T/T_c	β	N_s	N_τ	Configurations
0.93	6.287	64	12	2125
	6.495	96	16	1645
	6.800	128	24	2040
1.02	6.349	64	12	2000
	6.559	96	16	1600
	6.800	128	22	2290
1.12	6.418	64	12	1875
	6.631	96	16	1580
	6.800	128	20	2000
1.40	6.582	64	12	2080
	6.800	128	16	900
	7.117	128	24	2000
1.68	6.719	64	12	2000
	6.941	96	16	1680
	7.117	128	20	2000
2.10	6.891	64	12	2250
	7.117	128	16	840
	7.296	128	20	2040
2.31	7.200	96	16	1490
	7.376	128	20	2020
	7.519	128	24	1970
2.69	7.086	64	12	2000
	7.317	96	16	1560
	7.500	128	20	2040

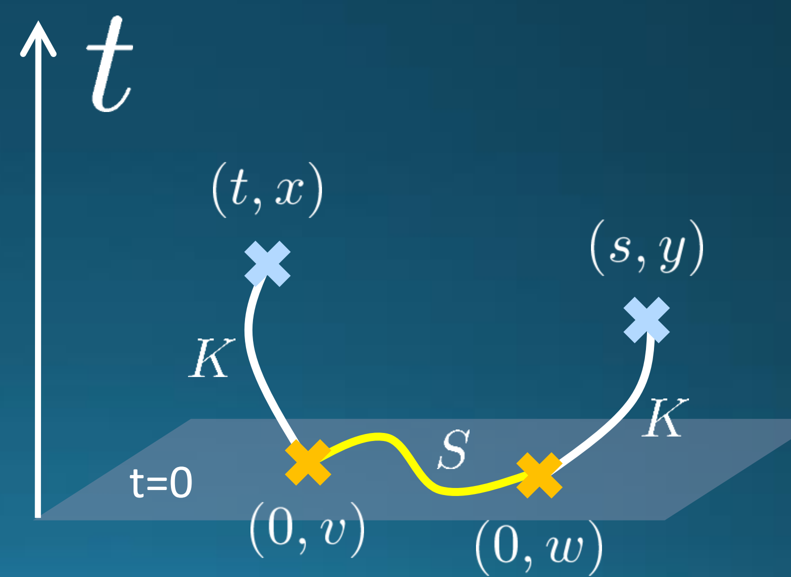
Fermion Propagator

$$S(t,x;s,y) = \langle \chi(t,x)\bar{\chi}(s,y)\rangle$$
$$= \sum_{v,w} K(t,x;0,v)S(v,w)K(s,y;0,w)^\dagger$$

$$\Big(\partial_t - D_\mu D_\mu\Big)K(t,x) = 0$$

- propagator of flow equation
- Inverse propagator is needed





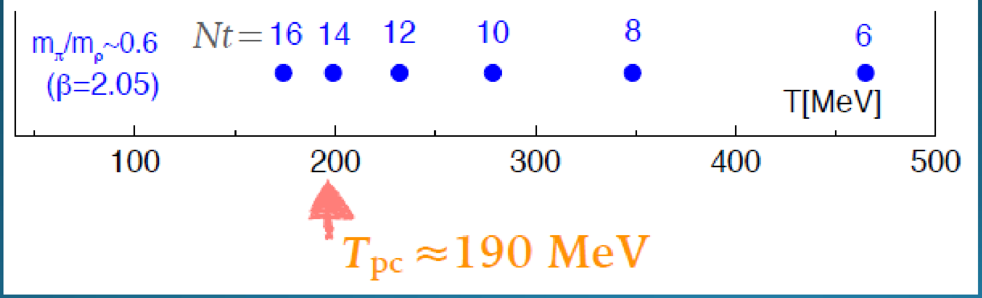
N_f=2+1 QCD Thermodynamics

Taniguchi+ (WHOT-QCD), PRD**96**, 014509 (2017)

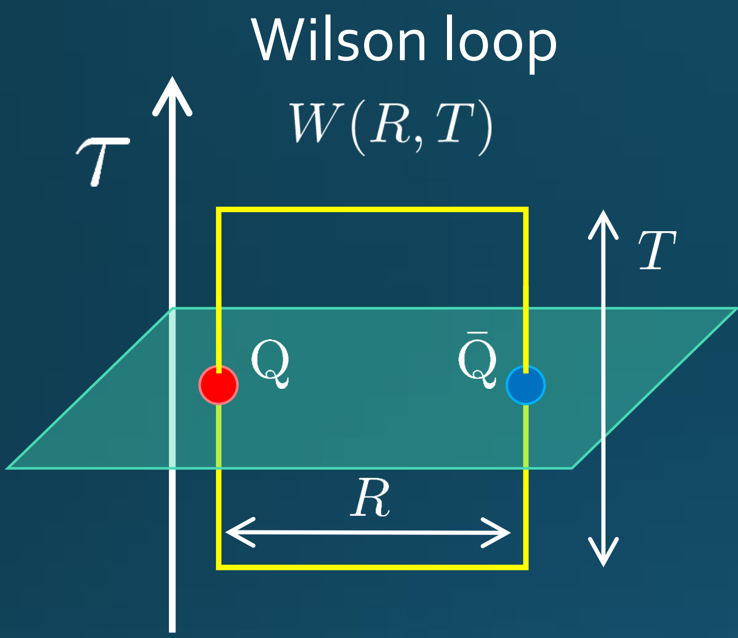
- N_f=2+1 QCD, Iwasaki gauge + NP-clover
- $m_{PS}/m_V \approx 0.63$ / almost physical s quark mass

- T=0: CP-PACS+JLQCD (ß=2.05, 28^3x56, a≈0.07fm)
- T>0: 32^3xN_t, N_t = 4, 6, … , 14, 16):
- T≈174-697MeV

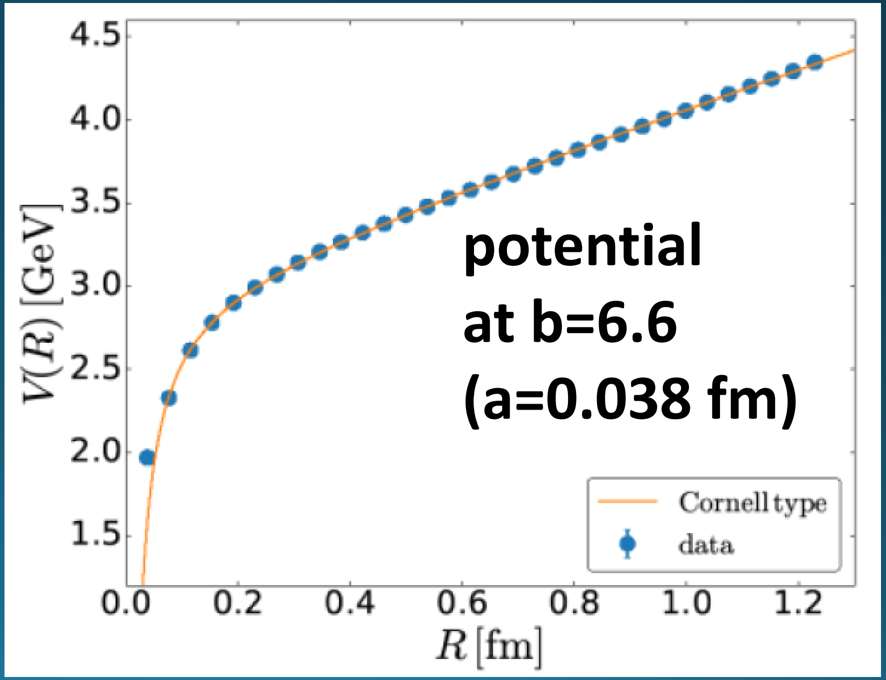
- t→0 extrapolation only (No continuum limit)



Preparing Static $Q\bar{Q}$



- APE smearing for spatial links
- Multi-hit for temporal links
- No gradient flow for W(R,T)

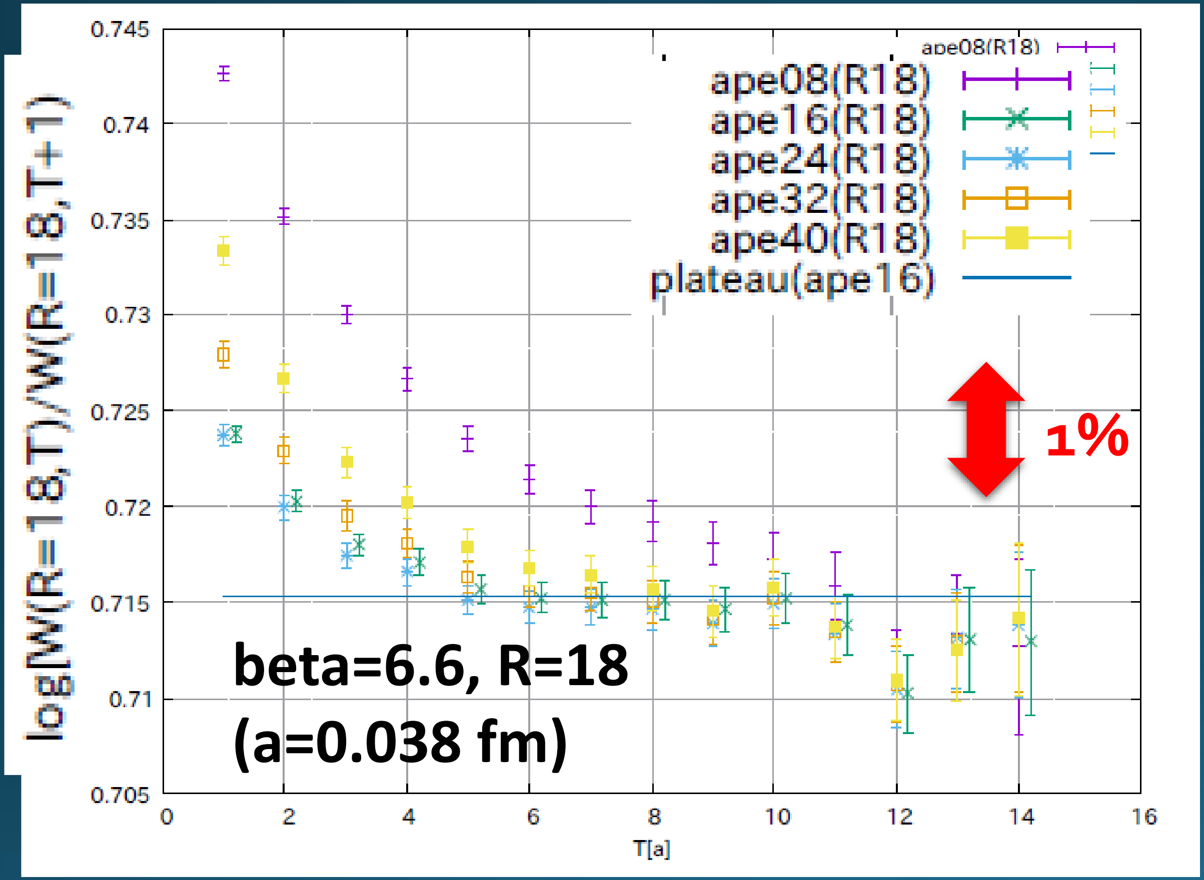


$$V(R) = -\lim_{T\to\infty} \log\langle W(R,T)\rangle$$

$$\langle O(x)\rangle_{Q\bar{Q}} = \lim_{T\to\infty} \frac{\langle \delta O(x)\delta W(R,T)\rangle}{\langle W(R,T)\rangle}$$

Ground State Saturation of W

Effective Mass Plot of W(R,T)

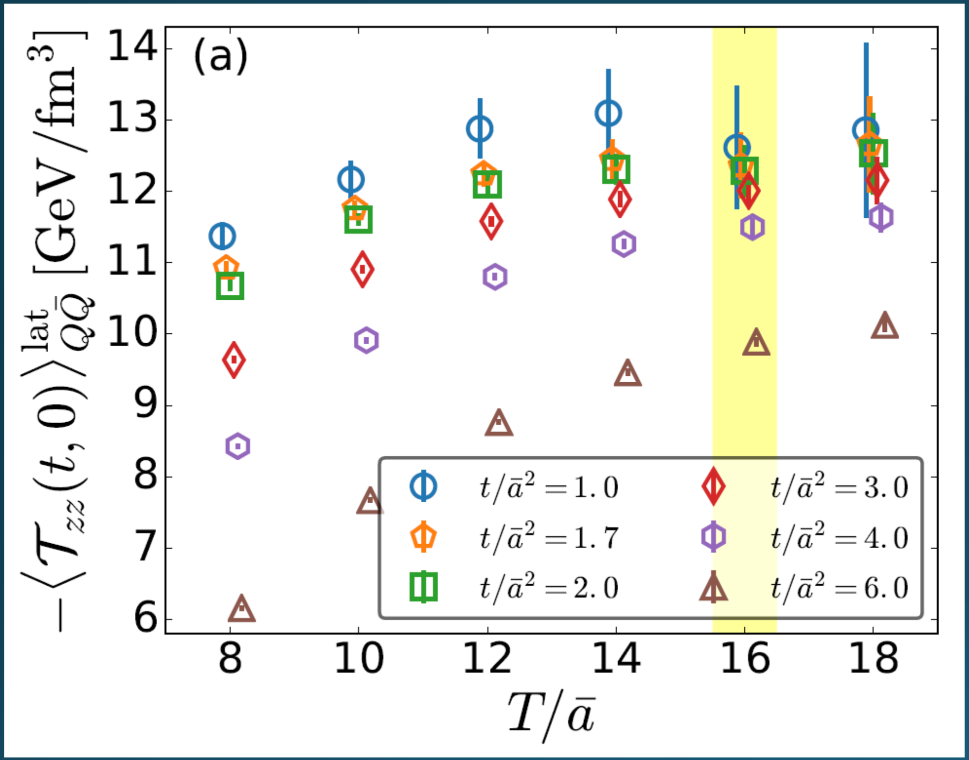


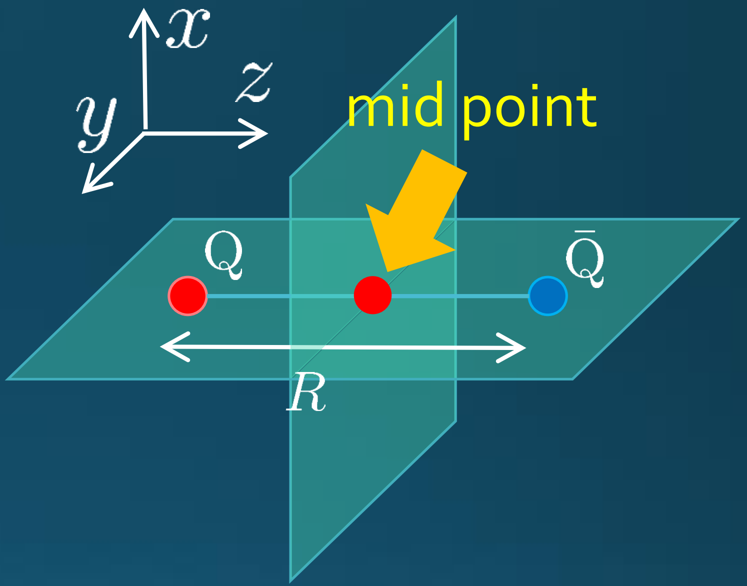
- APE smearing for spatial links
- Multi-hit for temporal links

- About 100 confs.
- Use translational symmetry

With an optimal choice of N_{APE}, ground-state saturation is already established at T/a=5 (C_0>99.5%)

Ground State Saturation

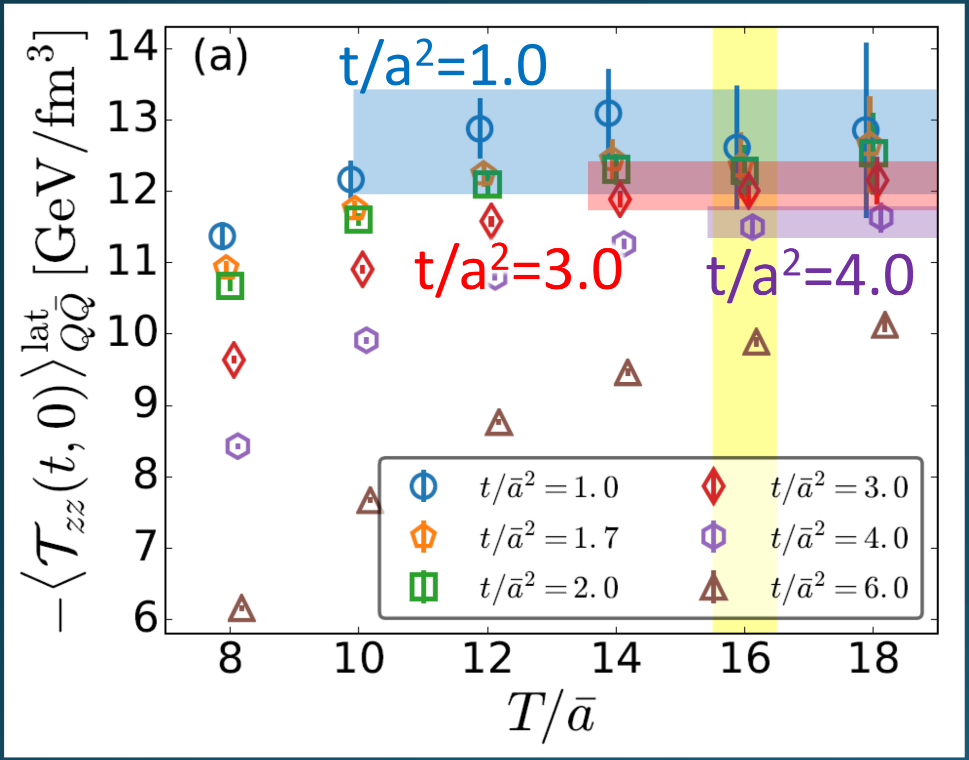


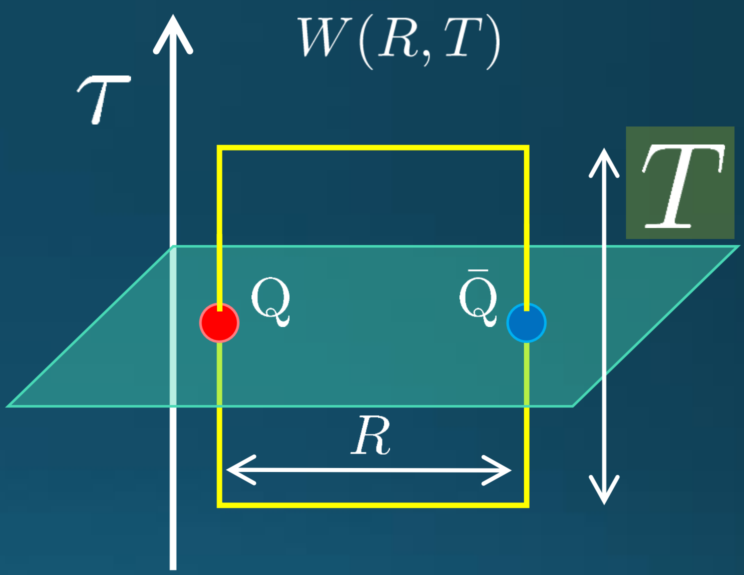


β=6.819 (a=0.029 fm), R=0.46 fm

Appearance of plateau for $t/a^2<4$, $T/a>15$ ⇒ Grand state saturation under control

Ground State Saturation





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