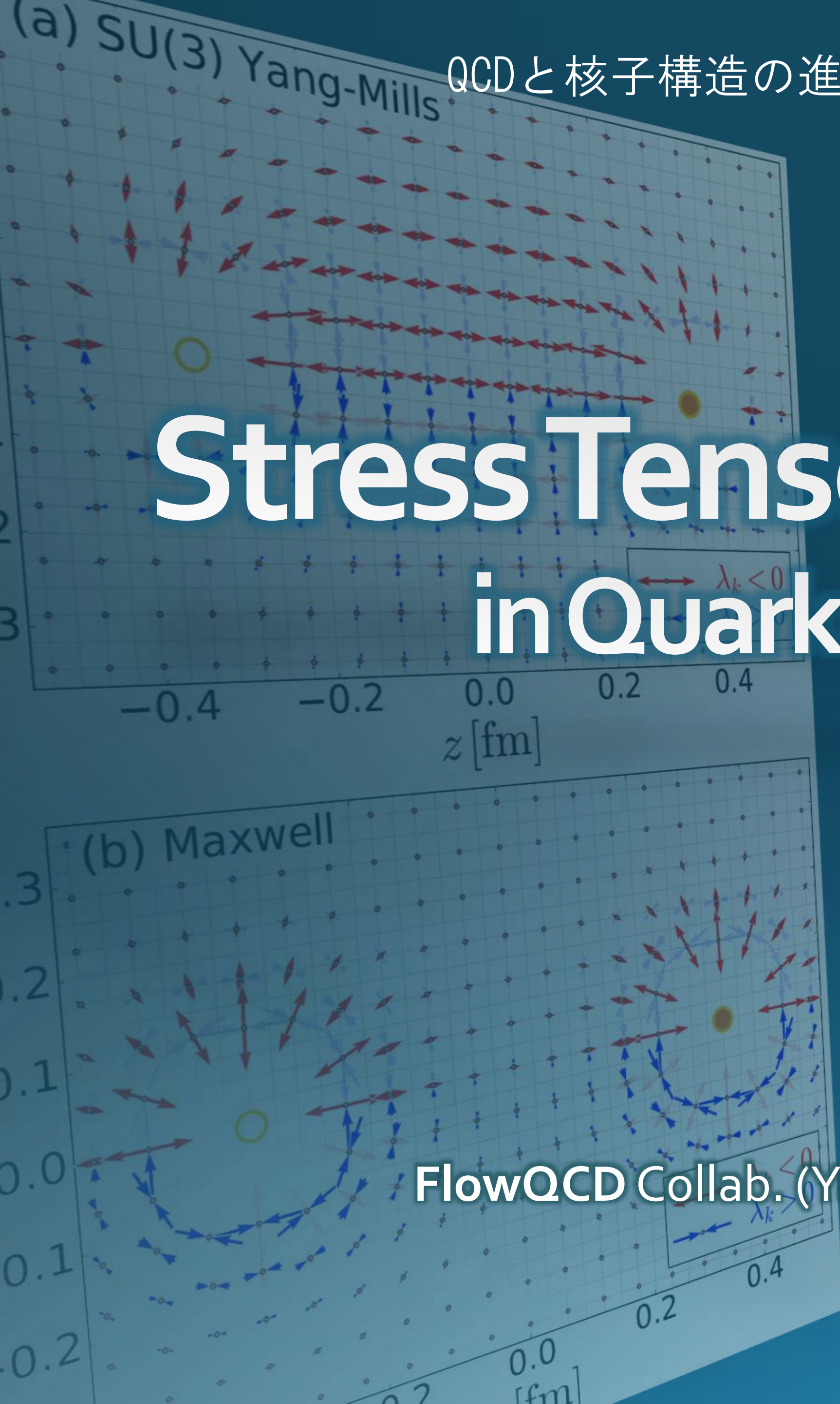
QCDと核子構造の進展2019、KEK、2019年3月1日



Stress Tensor Distribution in Quark-Anti-Quark System

Masakiyo Kitazawa

(Osaka University)

FlowQCD Collab. (Yanagihara, Iritani, MK, Asakawa, Hatsuda

Phys. Lett. **B789**, 210 (2019)

Energy-Momentum Tensor

energy momentum

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

pressure

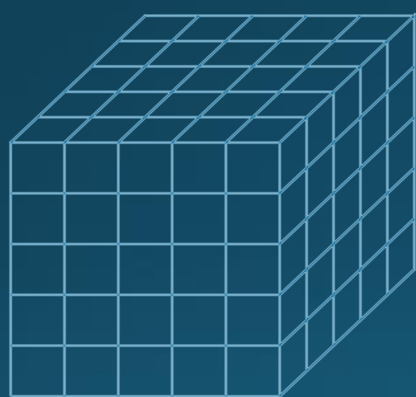
stress

All components are important physical observables!

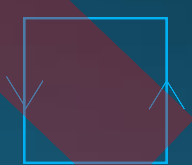
$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial

because of the explicit breaking of Lorentz symmetry



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

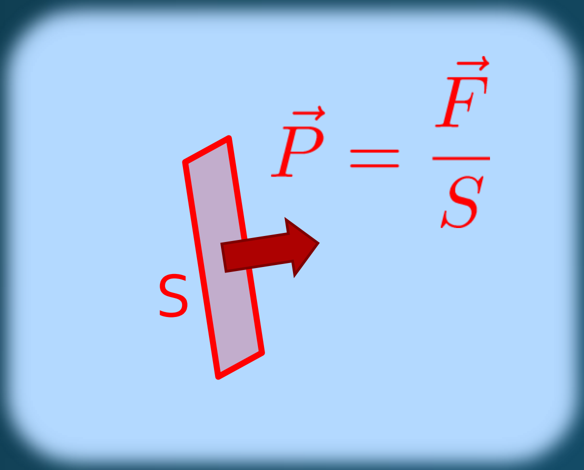
② Its measurement is extremely noisy

due to high dimensionality and etc.

Stress = Force per Unit Area

Stress = Force per Unit Area

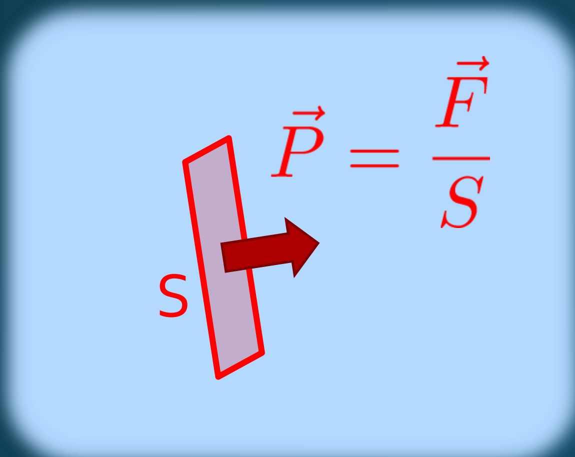
Pressure



$$\vec{P} = P\vec{n}$$

Stress = Force per Unit Area

Pressure

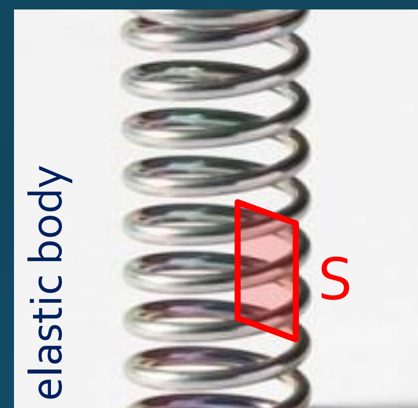


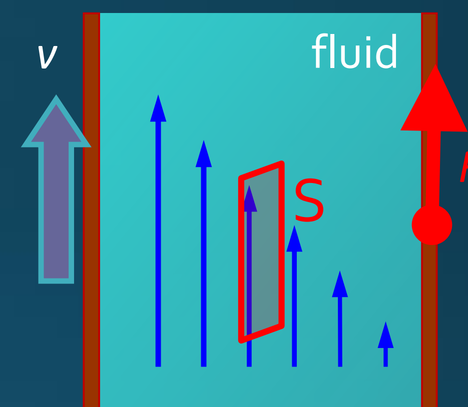
$$\vec{P} = P\vec{n}$$

In thermal medium

$$T_{ij} = P\delta_{ij}$$

Generally, F and n are not parallel





$$\frac{F_i}{S} = \sigma_{ij} n_j$$

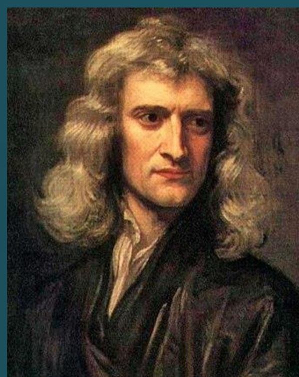
Stress Tensor

$$\sigma_{ij} = -T_{ij}$$

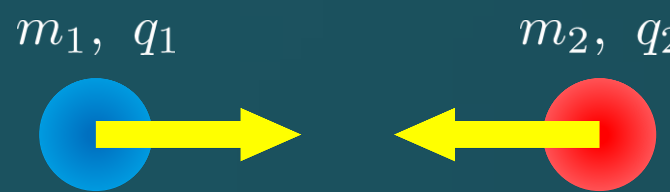
Landau
Lifshitz

Force

Action-at-a-distance



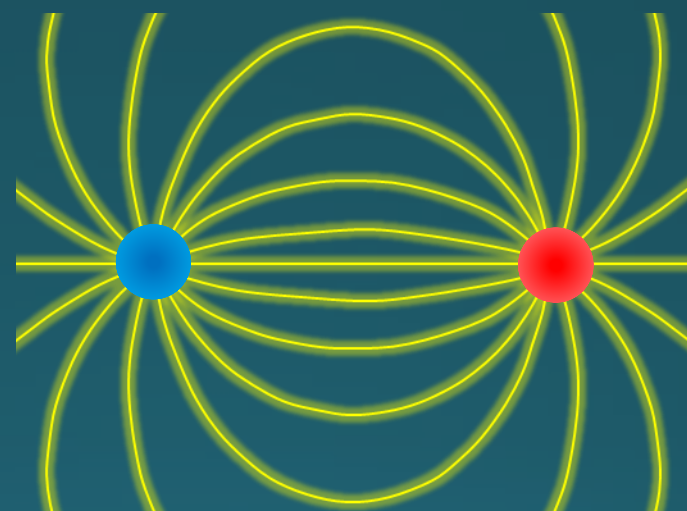
Newton
1687



$$F = -G\frac{m_1 m_2}{r^2} \qquad F = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Local interaction

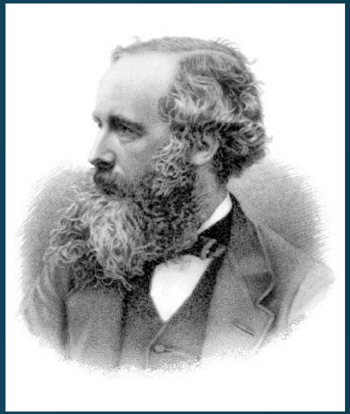
Faraday
1839



Maxwell Stress

(in Maxwell Theory)

$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

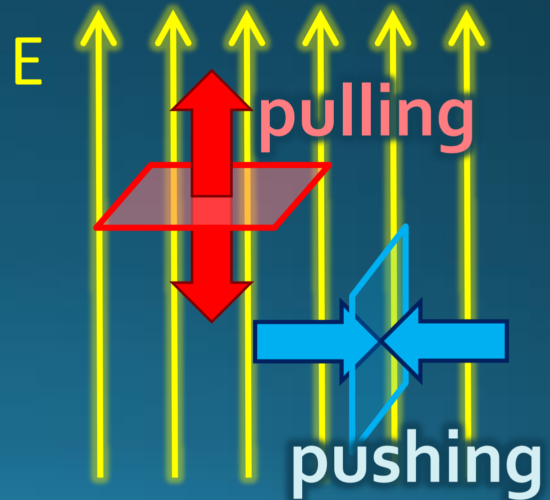


Maxwell

$$\vec{E} = (E, 0, 0)$$

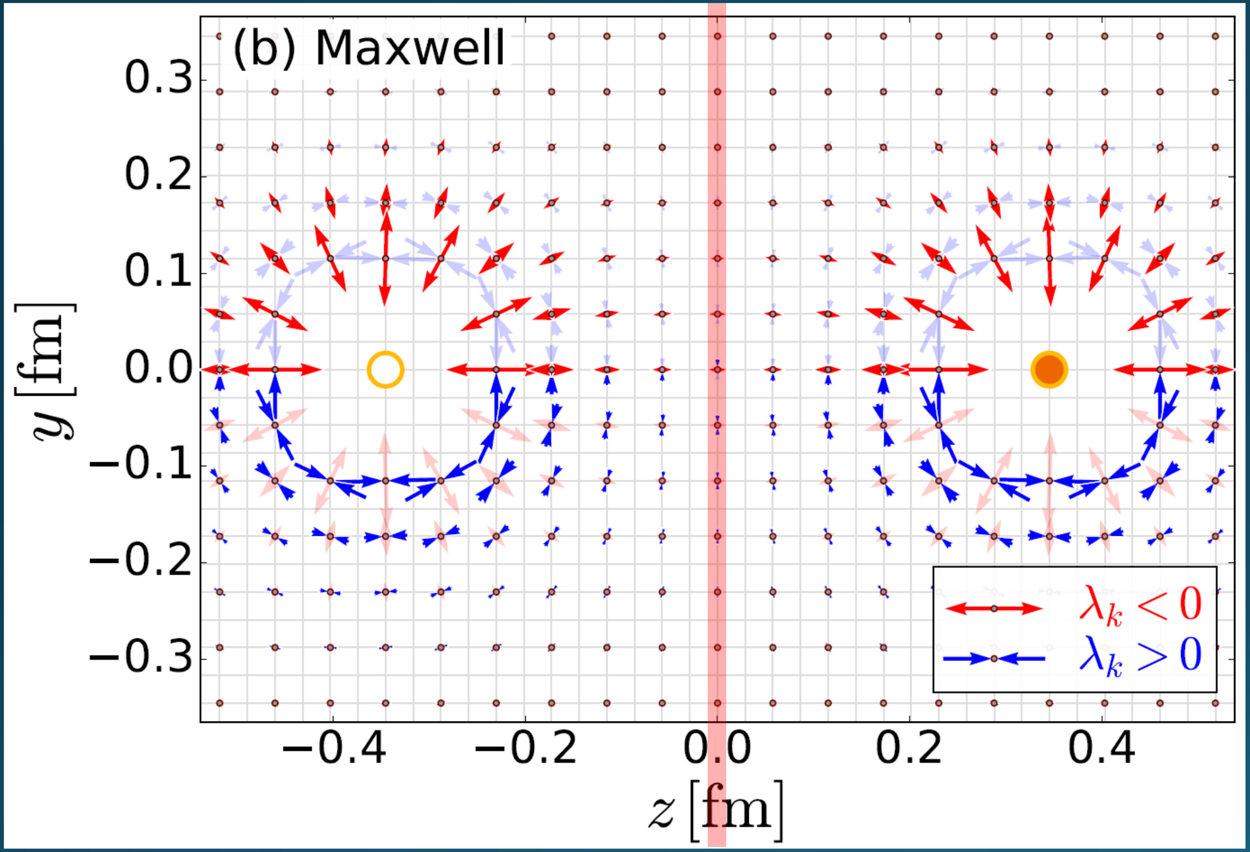
$$T_{ij} = \begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: **Pulling**
- Vertical to field: **Pushing**



Maxwell Stress

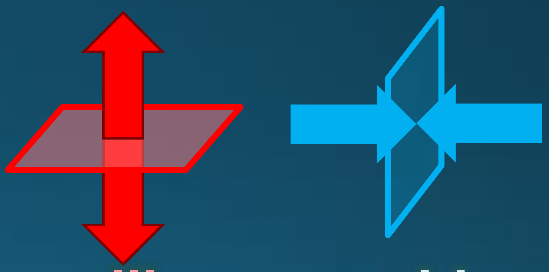
(in Maxwell Theory)



$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$$(k = 1, 2, 3)$$

length: $\sqrt{|\lambda_k|}$

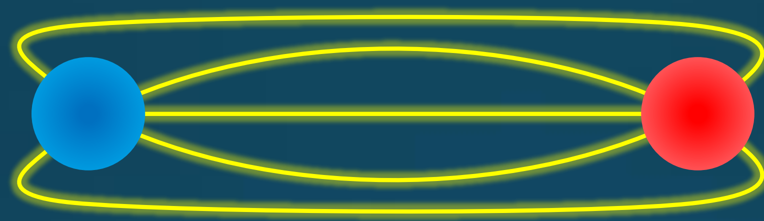


Definite physical meaning

- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction

Quark-Anti-quark system

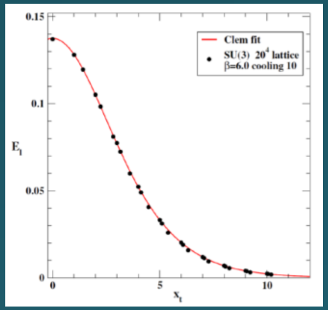
Formation of the flux tube → confinement



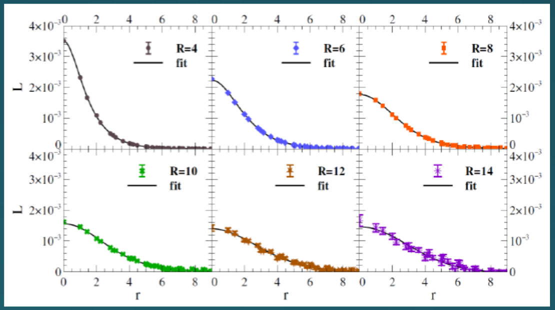
Previous Studies on Flux Tube

- ☐ Potential
- ☐ Action density
- ☐ Color-electric field

so many studies…

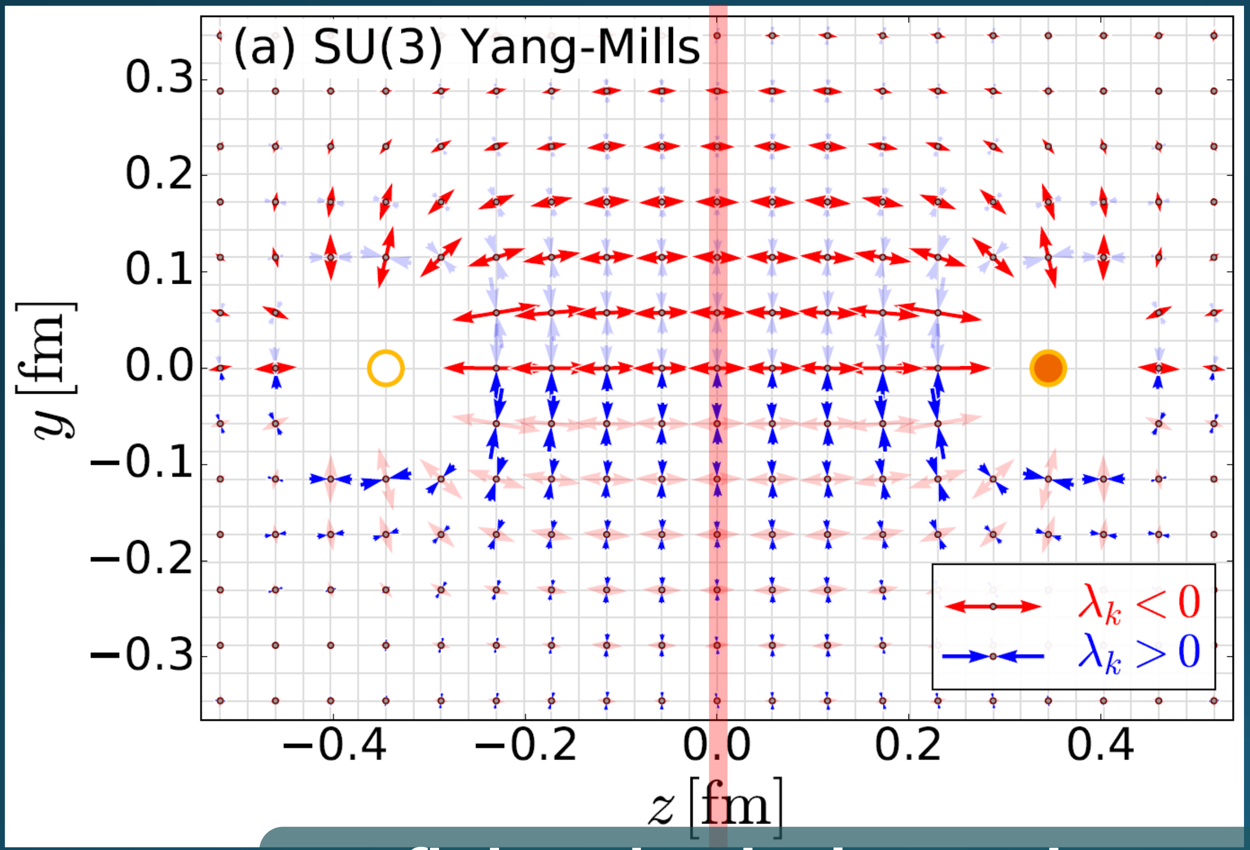


Cea+ (2012)



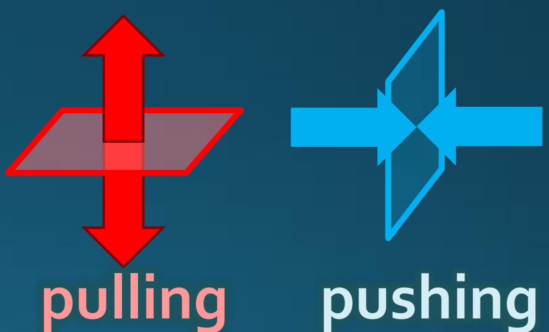
Cardoso+ (2013)

Stress Tensor in $Q\bar{Q}$ System



Yanagihara+, 1803.05656
PLB, in press

Lattice simulation
SU(3) Yang-Mills
a=0.029 fm
R=0.69 fm
t/a^2=2.0

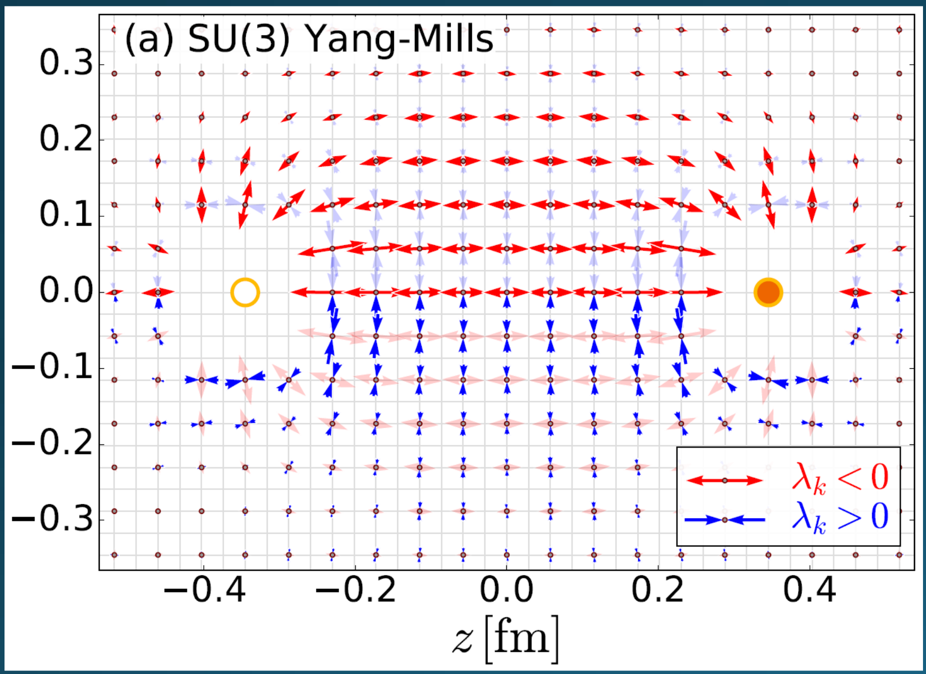


Definite physical meaning

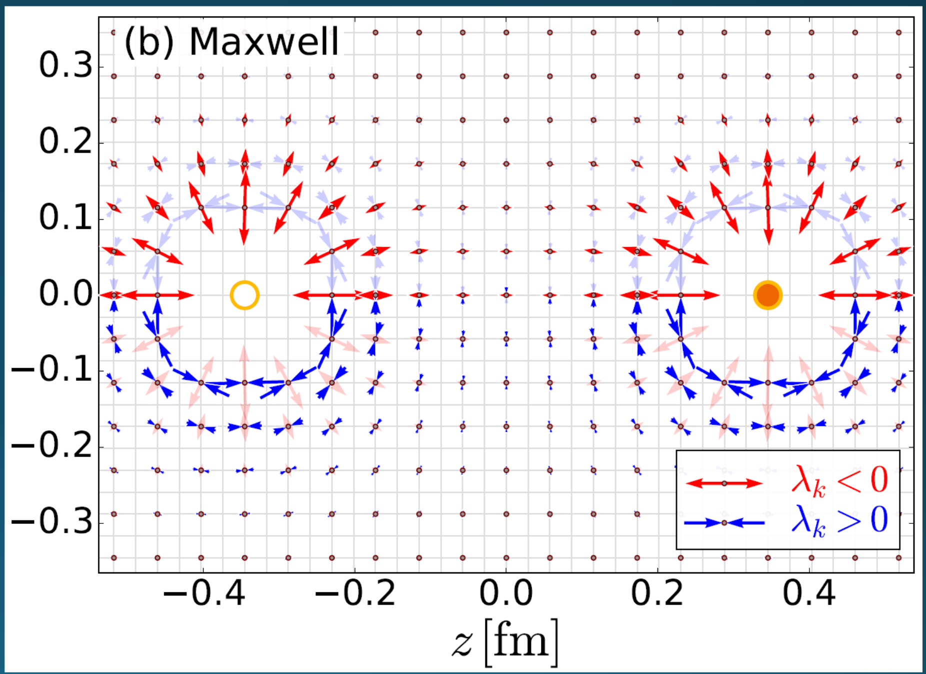
- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction
- ☐ Manifestly gauge invariant

SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)

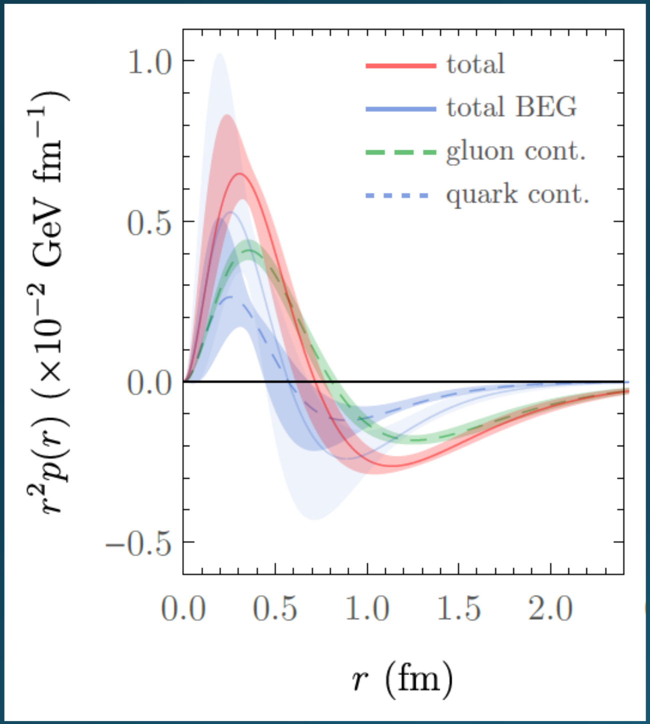


Maxwell
(classical)



Propagation of the force is clearly different
in YM and Maxwell theories!

Proton EMT Distribution



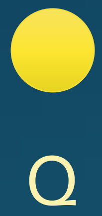
Pressure distribution in proton is now accessible??

arXiv:1810.07589
Nature, 557, 396 (2018)

Kumano+,2018

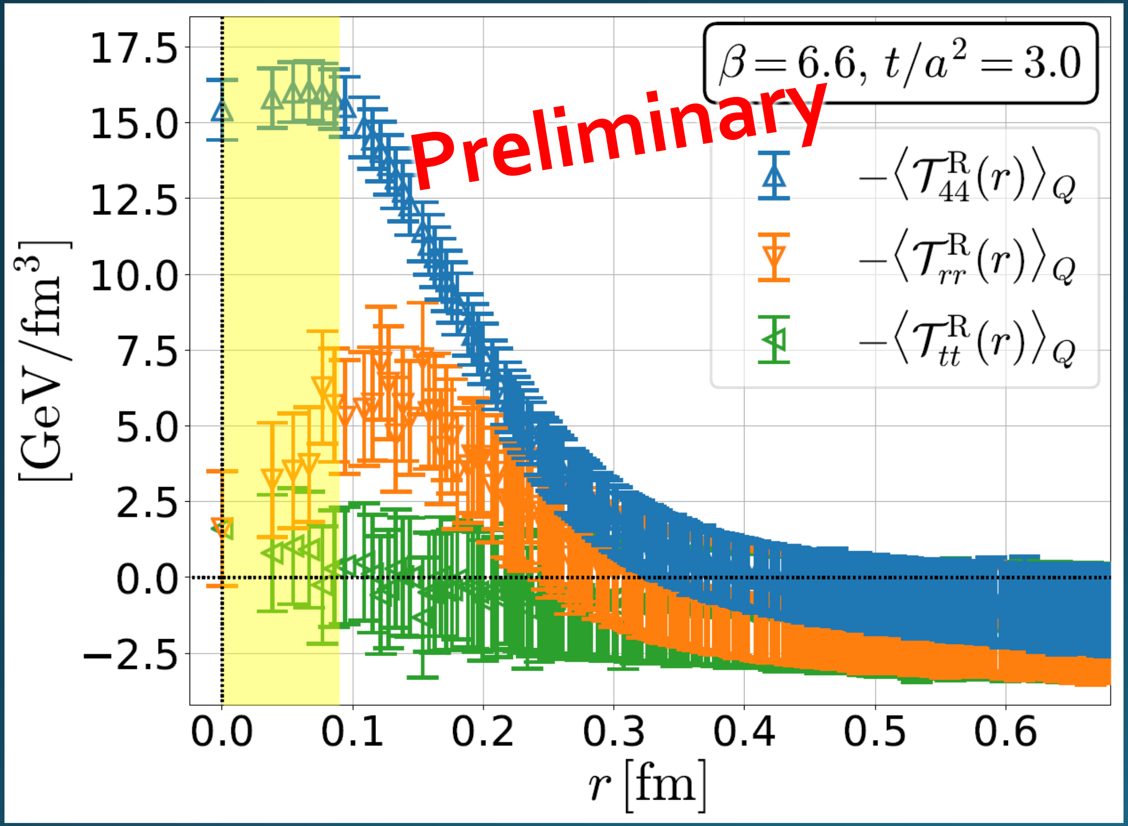
EMT around A Quark

in a deconfined phase



EMT around A Quark

in a deconfined phase



Yanagihara+, in prep.

Quenched QCD

48^3x12 (T≈1.4T_c)

fixed t , a

- Energy density

  $-\langle T_{44}\rangle = \varepsilon$

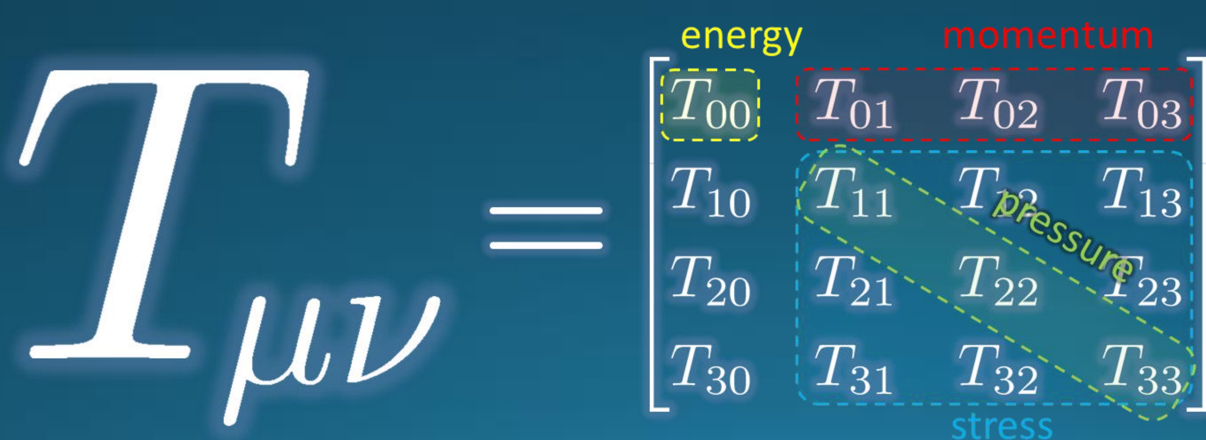
- Longitudinal pressure

  $-\langle T_{rr}\rangle = -p(r)$

- Transverse pressure

  $-\langle T_{tt}\rangle$

Energy-Momentum Tensor on the Lattice and Gradient Flow



Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

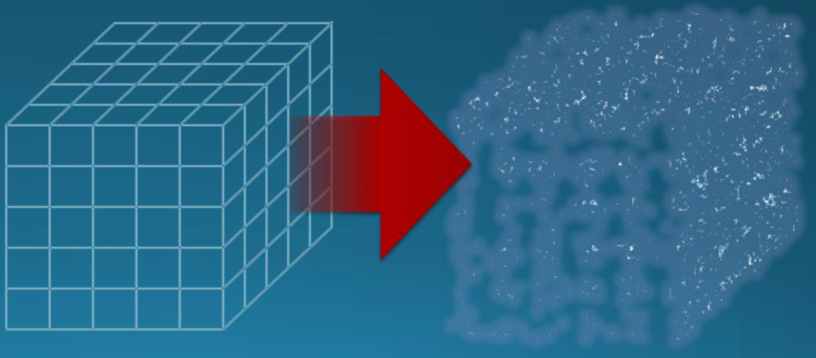
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length²]

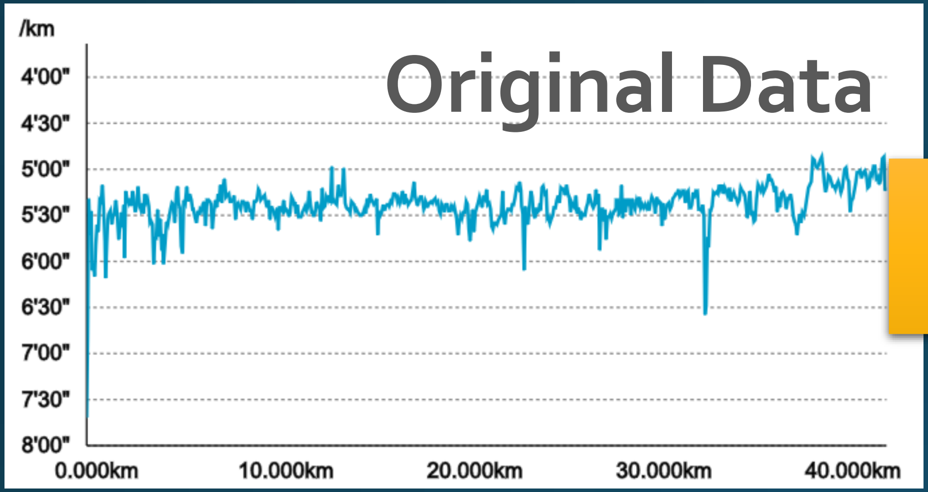
leading

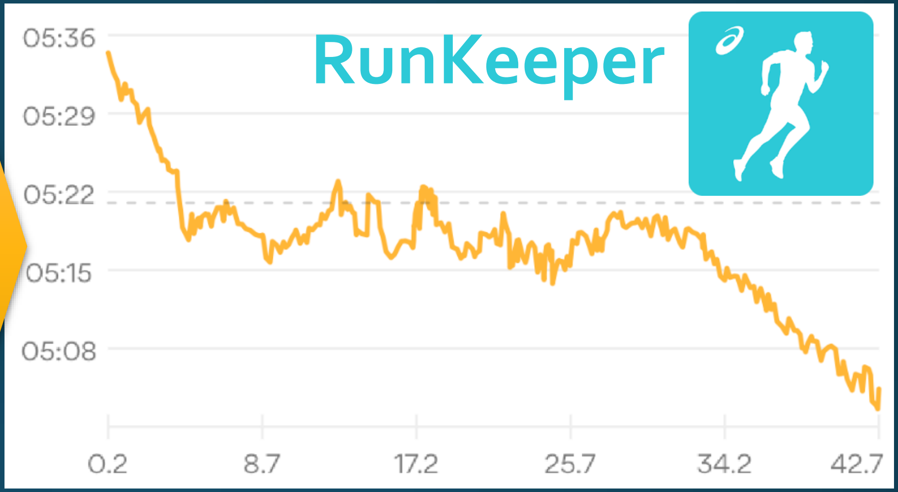
$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at t>0

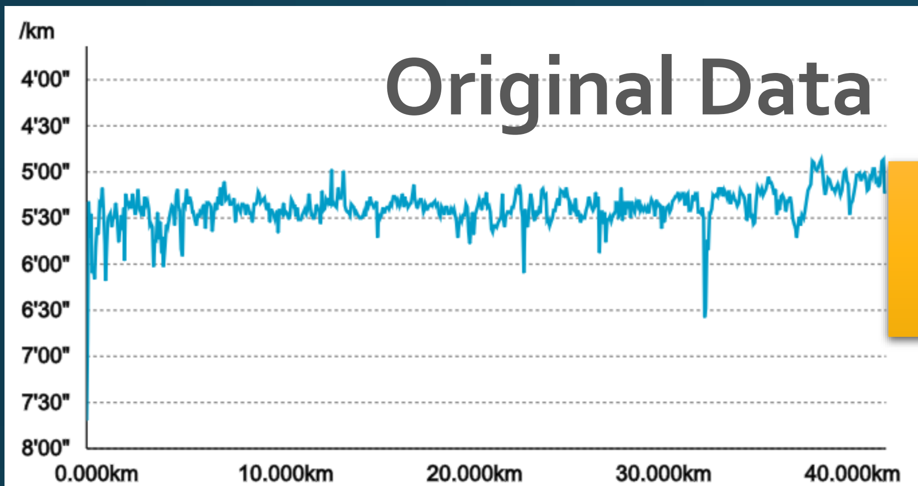


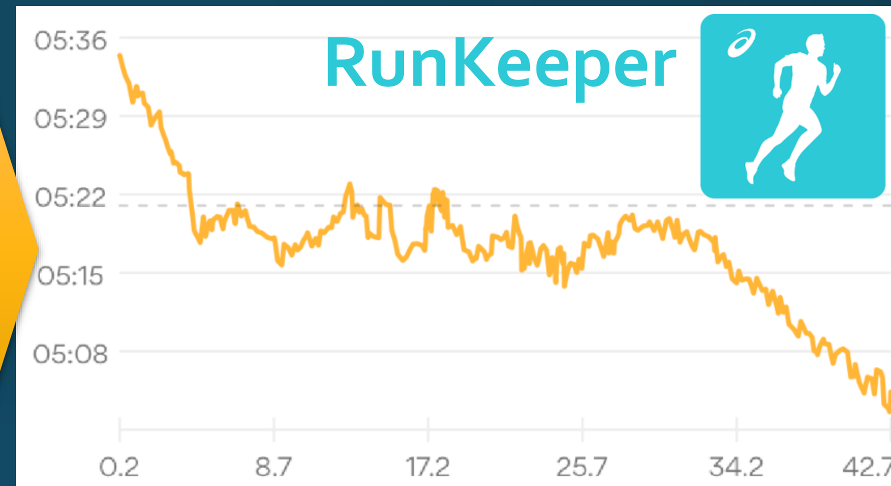
Gradient Flow = Smearing



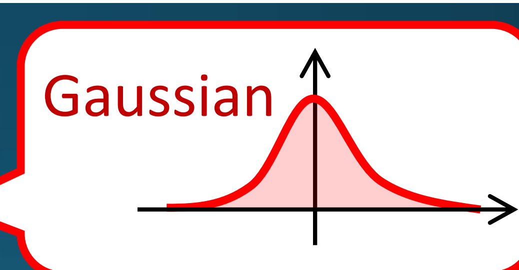


Gradient Flow = Smearing





① $x(t) \to x'(t) \sim \int dt' \exp\left[-\frac{(t-t')^2}{2\sigma^2}\right] x(t')$



$\sigma = \sqrt{2s}$

② $\frac{d}{ds} x(t;s) = \frac{d^2}{dt^2} x(t,s) \qquad x(t;0) = x(t)$

Gradient Flow

$\partial_t A_\mu = \partial_\nu \partial_\nu A_\mu + \cdots$

Two Advantages
of EMT Operator from Gradient Flow

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

1. The operator is uniquely determined
2. Statistics is substantially improved

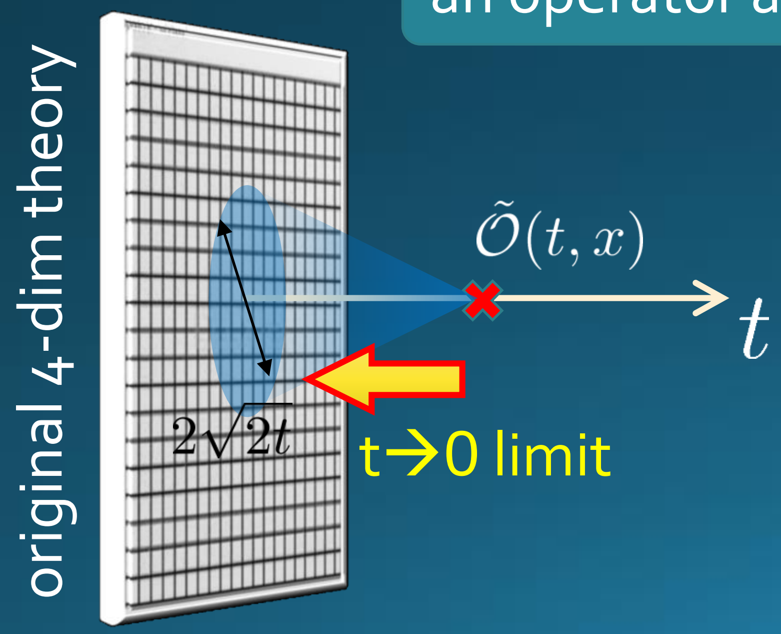
Small Flow-Time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at t>0

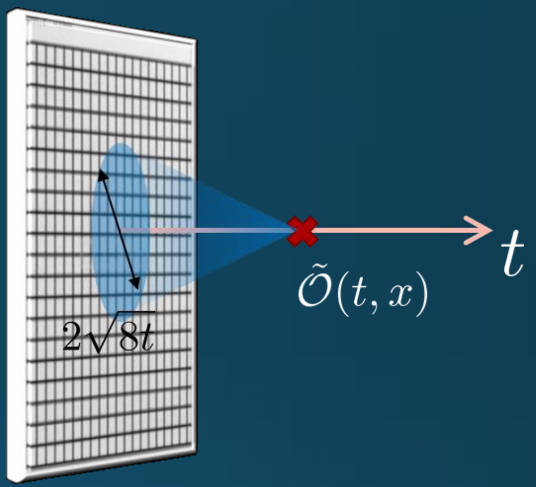
remormalized operators of original theory



Constructing EMT 1

Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$



- **Gauge-invariant dimension 4 operators**

$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

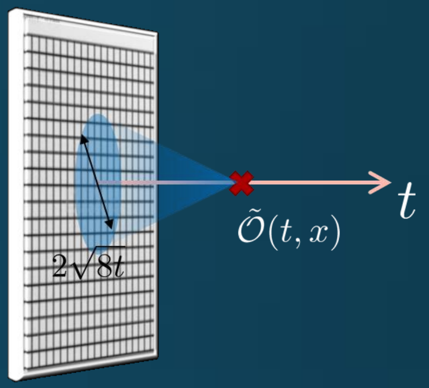
Constructing EMT 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$

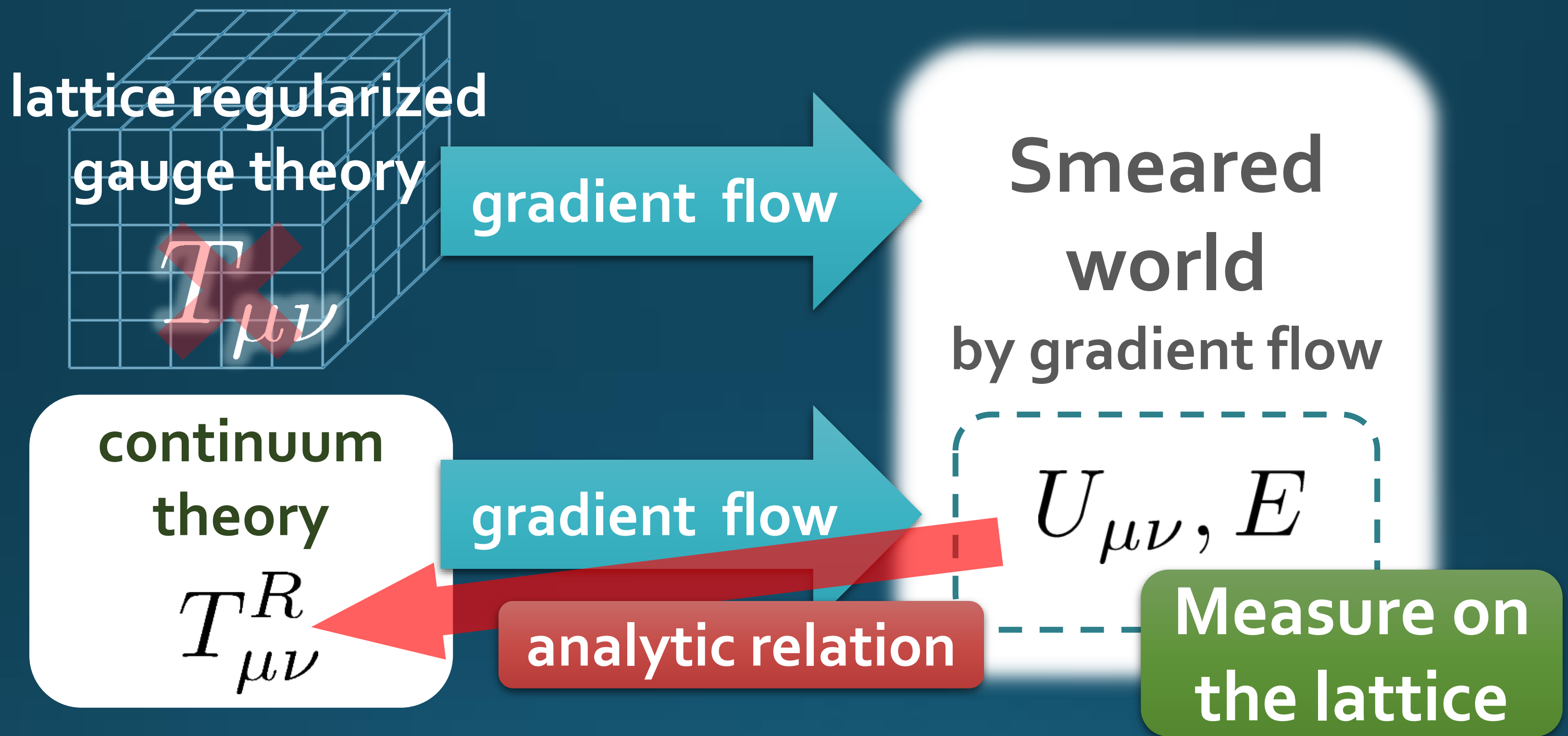
vacuum subtr.



Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\text{subt.}}\right]$$

Gradient Flow Method



Take Extrapolation (t,a)→(0,0)

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu} t + D_{\mu\nu}\frac{a^2}{t} + \cdots$$

O(t) terms in SFTE lattice discretization

Perturbative Coefficients

$$T_{\mu\nu}(t) = c_1(t) U_{\mu\nu}(t) + \delta_{\mu\nu} c_2(t) E(t)$$

	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○		
$c_2(t)$	× zero	○	○	

Suzuki (2013)

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

☐ Choice of the scale of g²

$$c_1(t) = c_1\Big(g^2\big(\mu(t)\big)\Big)$$

Previous: $\mu_d(t) = 1/\sqrt{8t}$
Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E} t}$

Harlander+ (2018)

Perturbative Coefficients

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

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	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○	○	
$c_2(t)$	× zero	○	○	○

Suzuki (2013)   Harlander+(2018)   Iritani, MK, Suzuki, Takaura, 2019

☐ Choice of the scale of g²

$$c_1(t) = c_1\Big(g^2\big(\mu(t)\big)\Big)$$

Previous: $\mu_d(t) = 1/\sqrt{8t}$

Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E} t}$

Harlander+ (2018)

Thermodynamics of SU(3) YM

- **Integral method**
  - Most conventional / established
  - Use themodynamic relations
    Boyd+ 1995; Borsanyi, 2012

$$p = \frac{T}{V} \ln Z$$

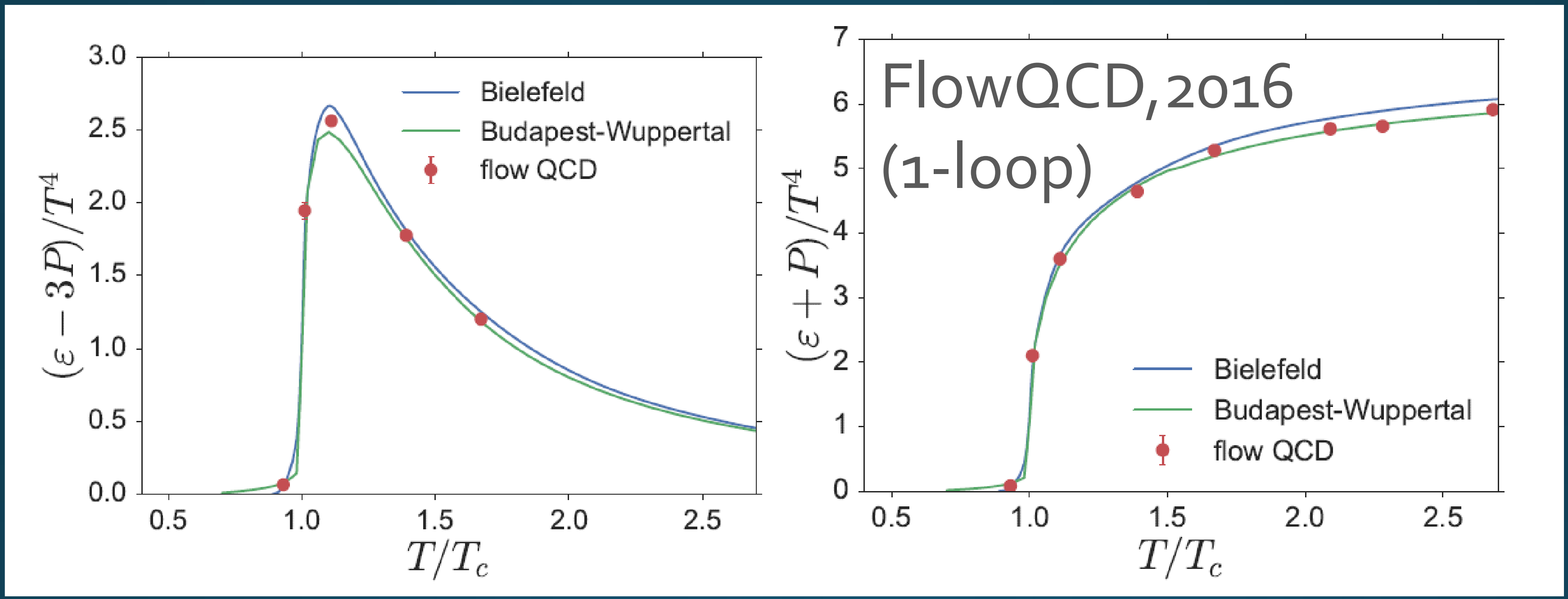
$$T \frac{\partial (p/T^4)}{\partial T} = \frac{\varepsilon - 3p}{T^4}$$

- Gradient-flow method
  - Take expectation values of EMT
    FlowQCD, 2014, 2016

$$\begin{cases} \varepsilon = \langle T_{00} \rangle \\ p = \langle T_{11} \rangle \end{cases}$$

- Moving-frame method
  Giusti, Pepe, 2014~
- Non-equilibrium method
  - Use Jarzynski's equality Caselle+, 2016;2018
- Differential method
  Shirogane+(WHOT-QCD), 2016~

Temperature Dependence



Error includes

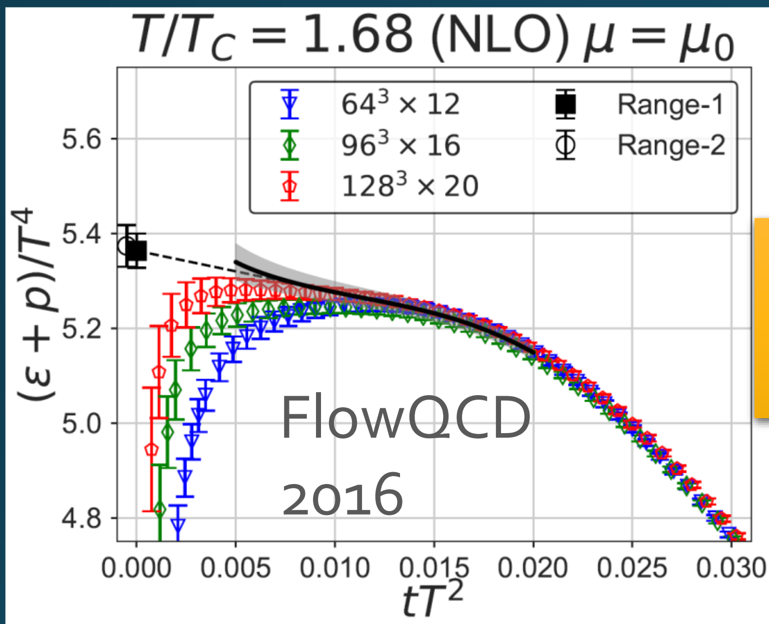
- statistical error
- choice of t range for t→0 limit
- uncertainty in $a\Lambda_{MS}$

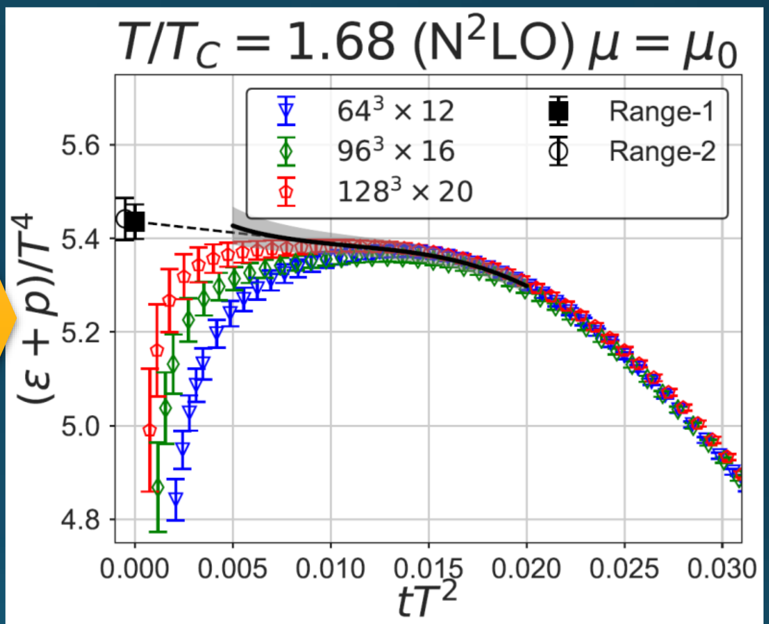
total error <1.5% for $T>1.1T_c$

- Excellent agreement with integral method
- High accuracy only with ~2000 confs.

Higher Order Coefficient: ε+p

NLO (1-loop) **N²LO (2-loop)**





Iritani, MK, Suzuki, Takaura, 2019

- ☐ t dependence becomes milder with higher order coeff.
- ☐ 1-loop → 2-loop : about 2% increase

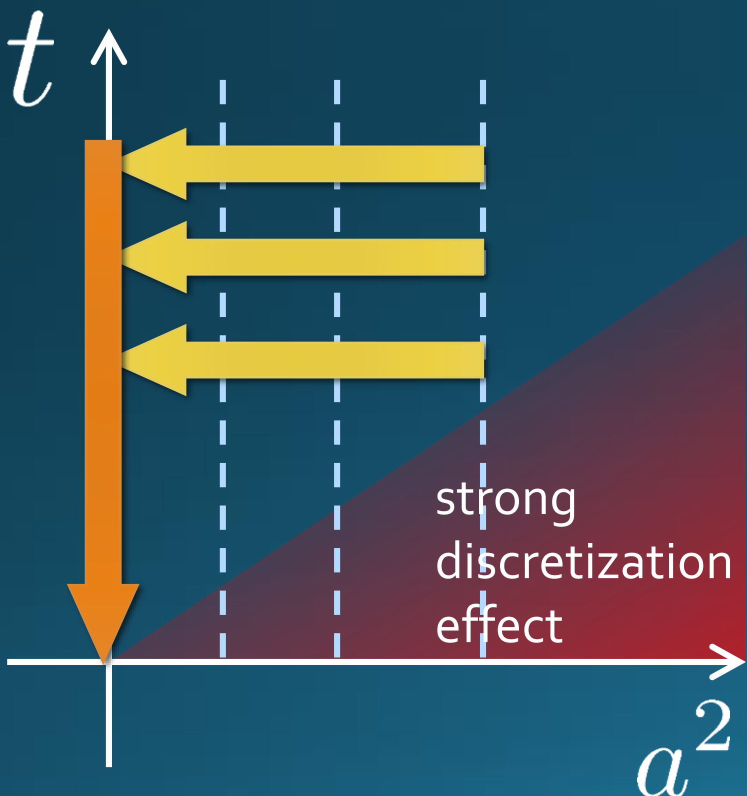
- ☐ Systematic analysis: μ_0 or μ_d, uncertainty of Λ, fit range
- ☐ Extrapolation func: linear, higher order term in c_1 ($\sim g^6$)

Double Extrapolation

t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE   lattice discretization



Continuum extrapolation

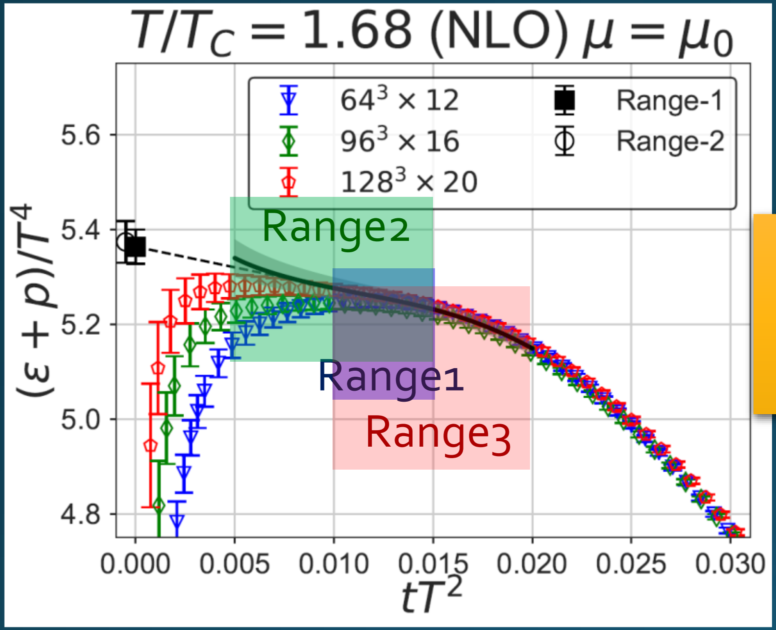
$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

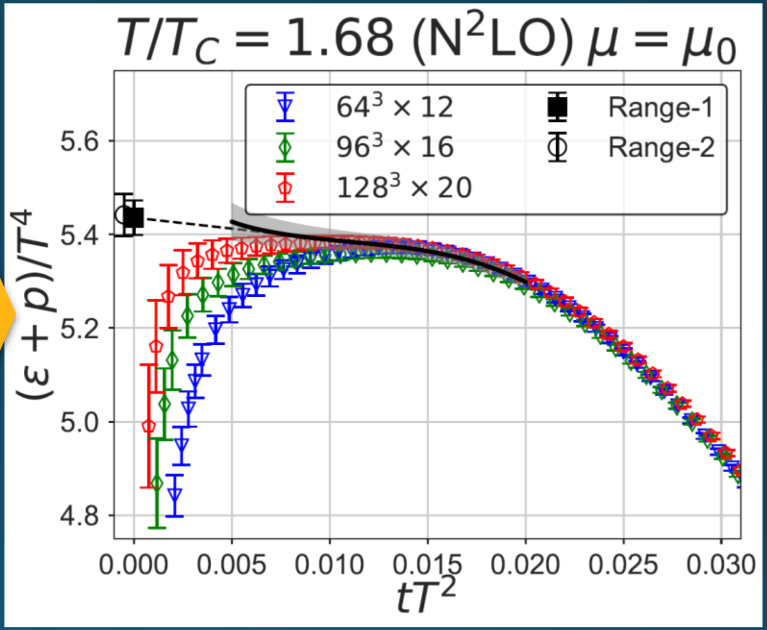
Small t extrapolation

$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$

Higher Order Coefficient: ε+p

NLO (1-loop) **N²LO (2-loop)**





Iritani, MK, Suzuki, Takaura, 2019

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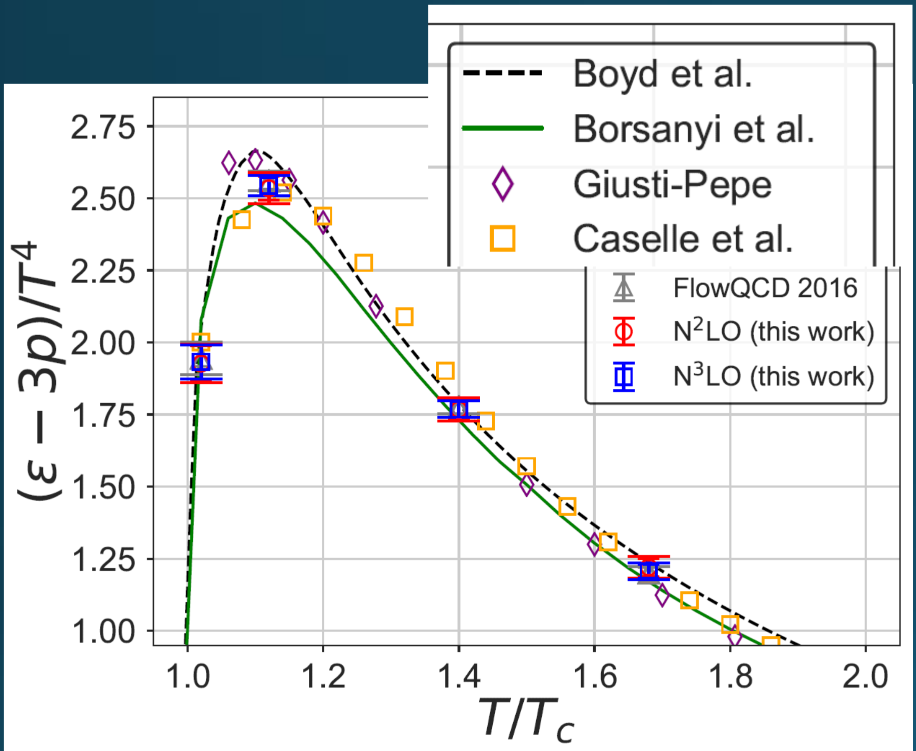
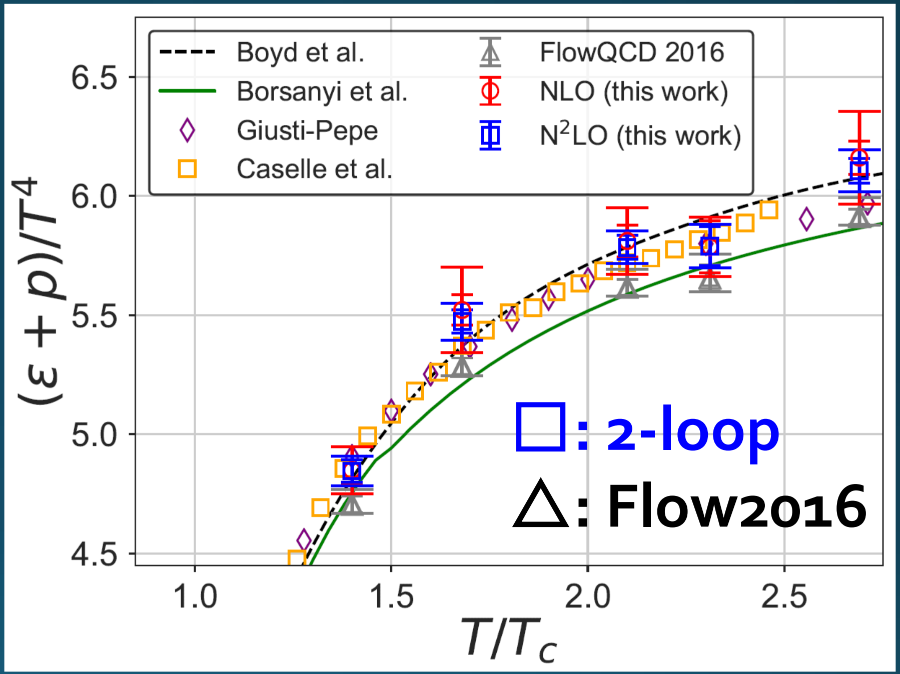
Effect of Higher-Order Coeffs.

Iritani, MK, Suzuki, Takaura, 2019





Systematic error: μ_0 or μ_d, Λ, t→0 function, fit range

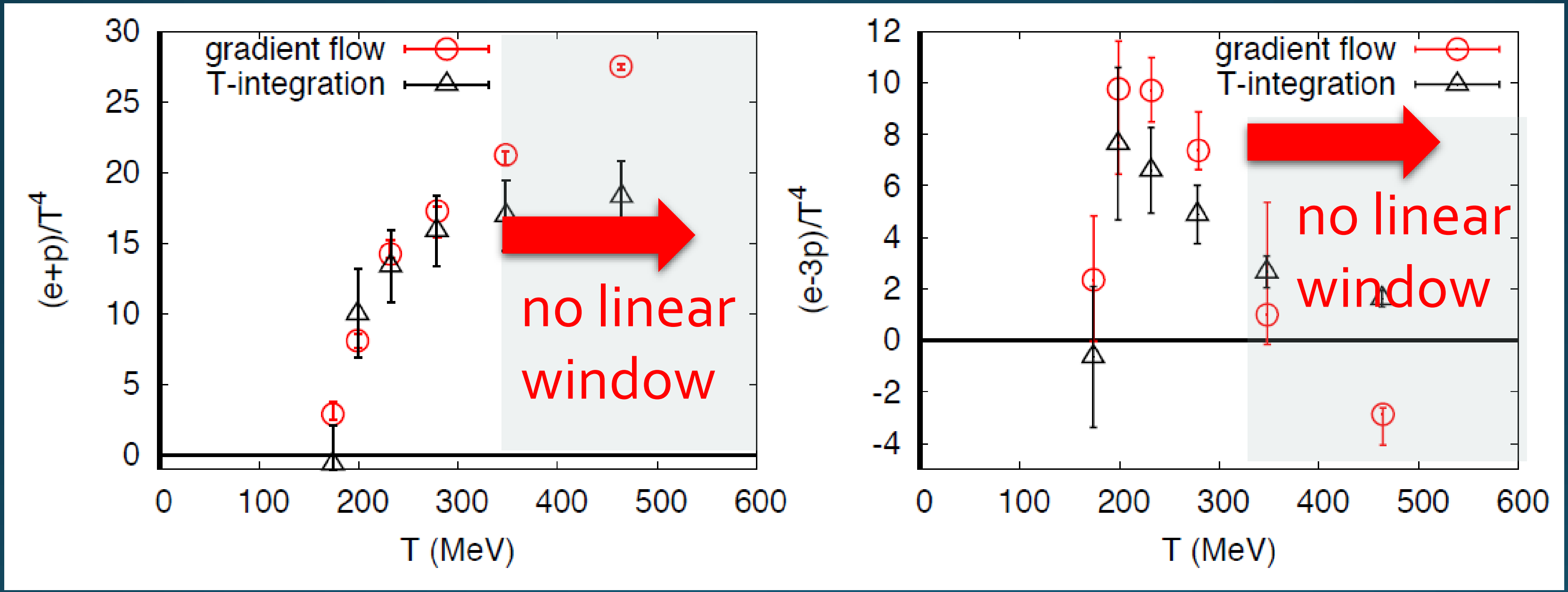
Effect of higher order c_1 & c_2 (pure gauge)

- e-3p: negligible (<0.5%)
- e+p: ~2% increase

2+1 QCD EoS from Gradient Flow

Taniguchi+ (WHOT-QCD), PR**D96**, 014509 (2017)

$m_{PS}/m_V \approx 0.63$



- Agreement with integral method except for N_t=4, 6
- N_t=4, 6: No stable extrapolation is possible
- Statistical error is substantially suppressed!

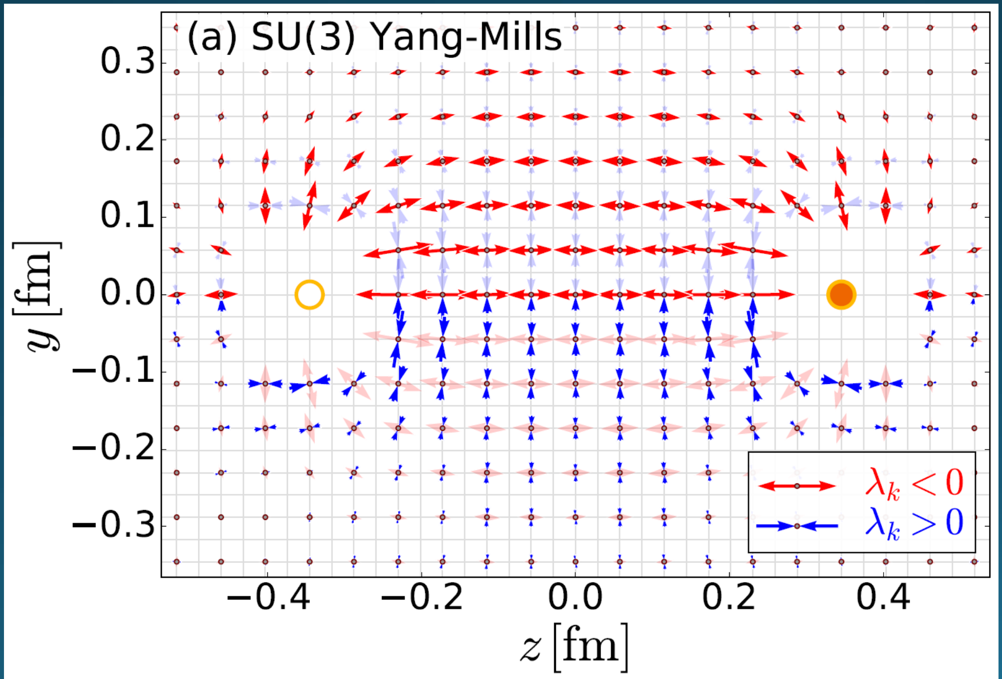
Physical mass: Kanaya+ (WHOT-QCD), 1710.10015

見小利則大事不成

小利を見ればすなわち大事成らず
Miss the wood for the trees

孔子
（論語、子路13）

Stress Tensor Distribution
around Flux Tube



Lattice Setup

- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

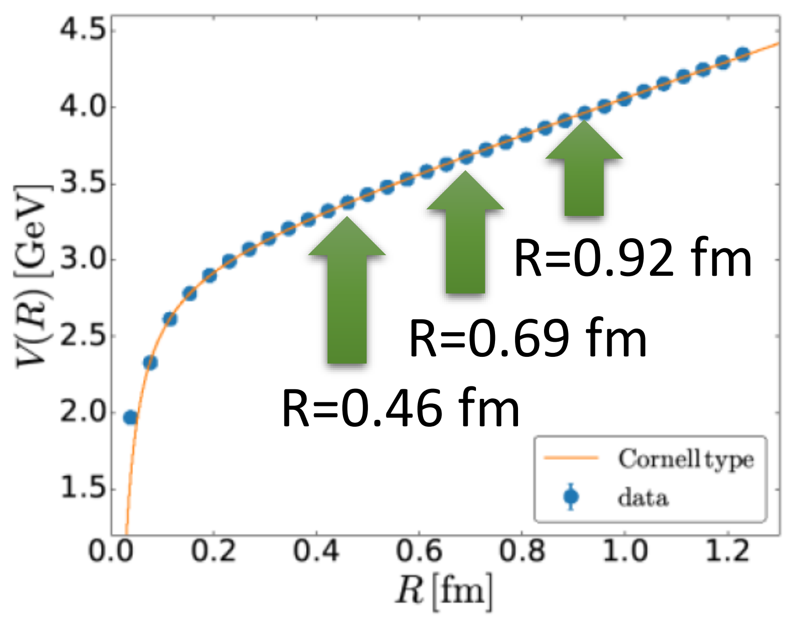
- APE smearing / multi-hit

- fine lattices (a=0.029-0.06 fm)
- continuum extrapolation

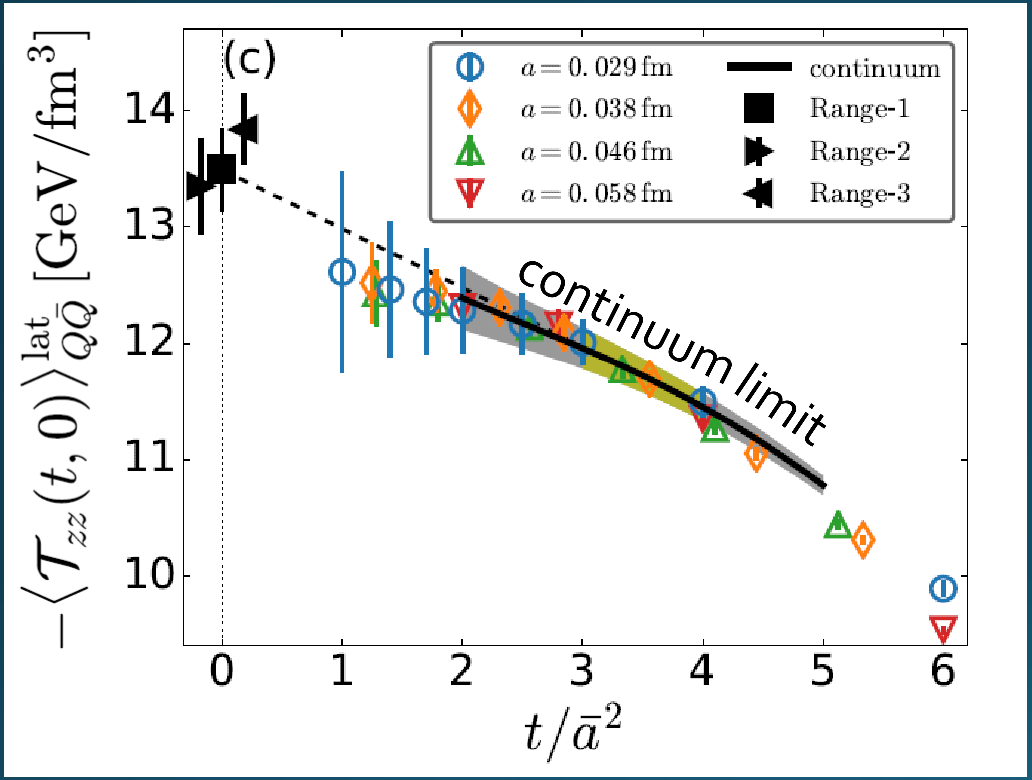
- Simulation: bluegene/Q@KEK

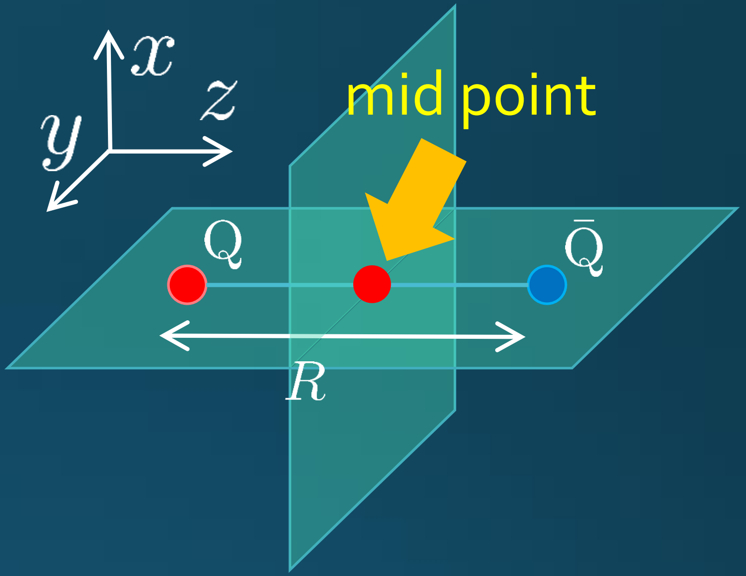
Yanagihara+, 1803.05656

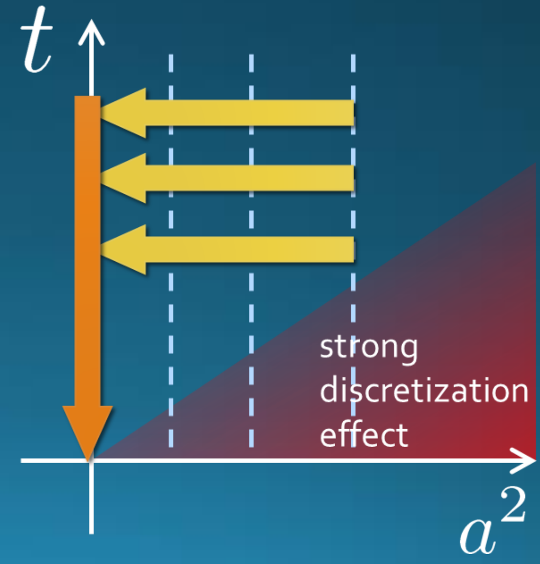
β	a [fm]	N_{size}^4	N_{conf}	R/a		
6.304	0.058	48^4	140	8	12	16
6.465	0.046	48^4	440	10	–	20
6.513	0.043	48^4	600	–	16	–
6.600	0.038	48^4	1,500	12	18	24
6.819	0.029	64^4	1,000	16	24	32
			R [fm]	0.46	0.69	0.92



Continuum Extrapolation
at mid-point

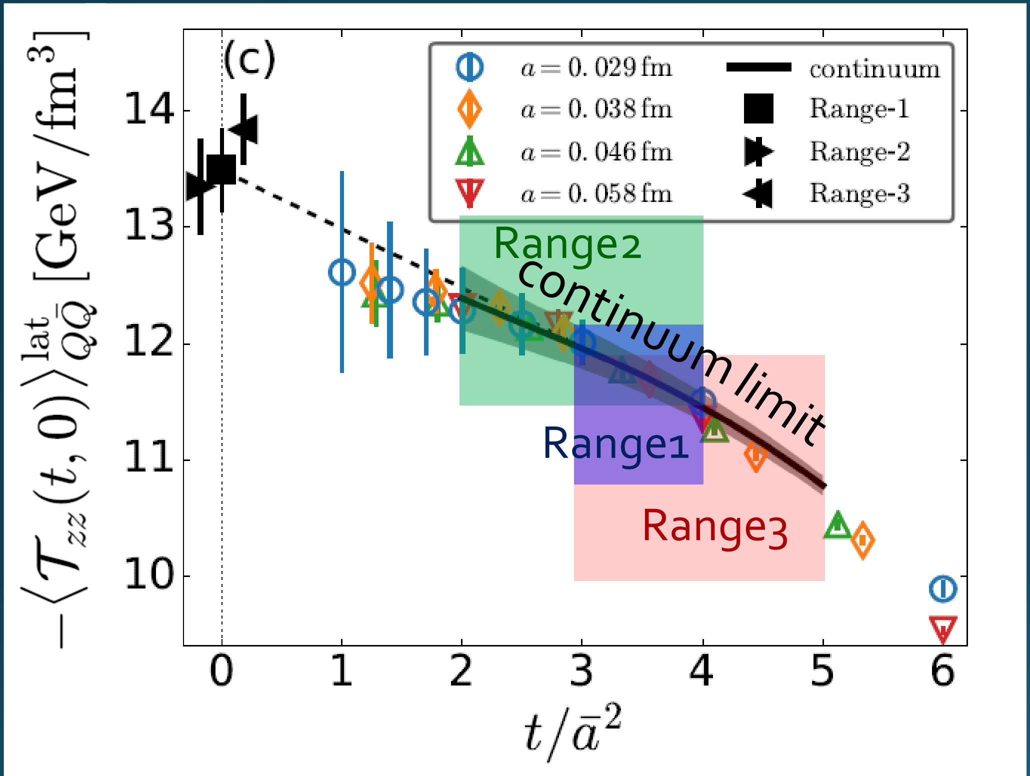


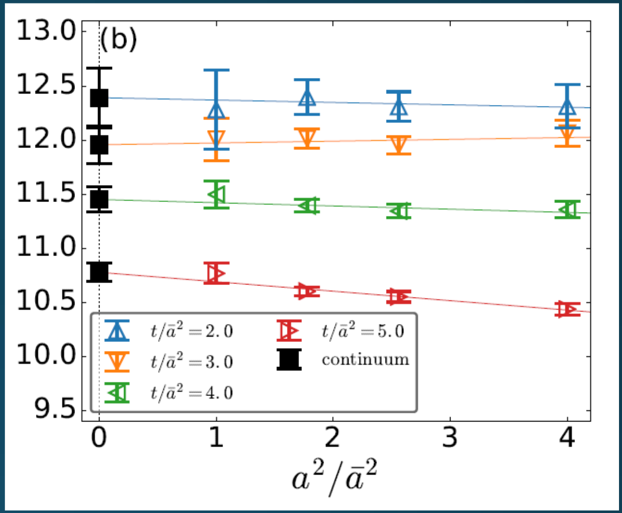


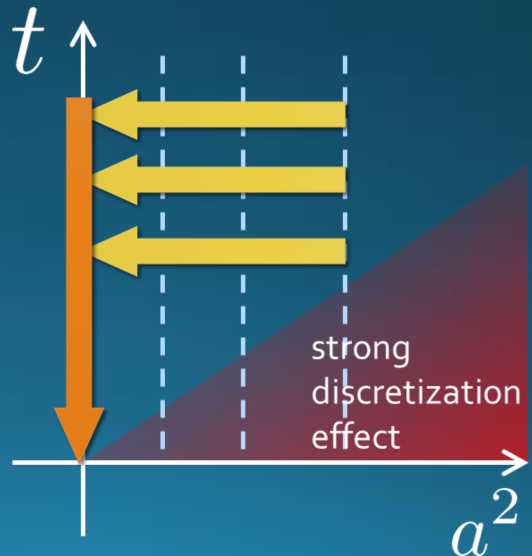


- ☐ a→0 extrapolation with fixed t

t→0 Extrapolation
at mid-point







- ☐ a→0 extrapolation with fixed t
- ☐ Then, t→0 with three ranges

Stress Distribution on Mid-Plane

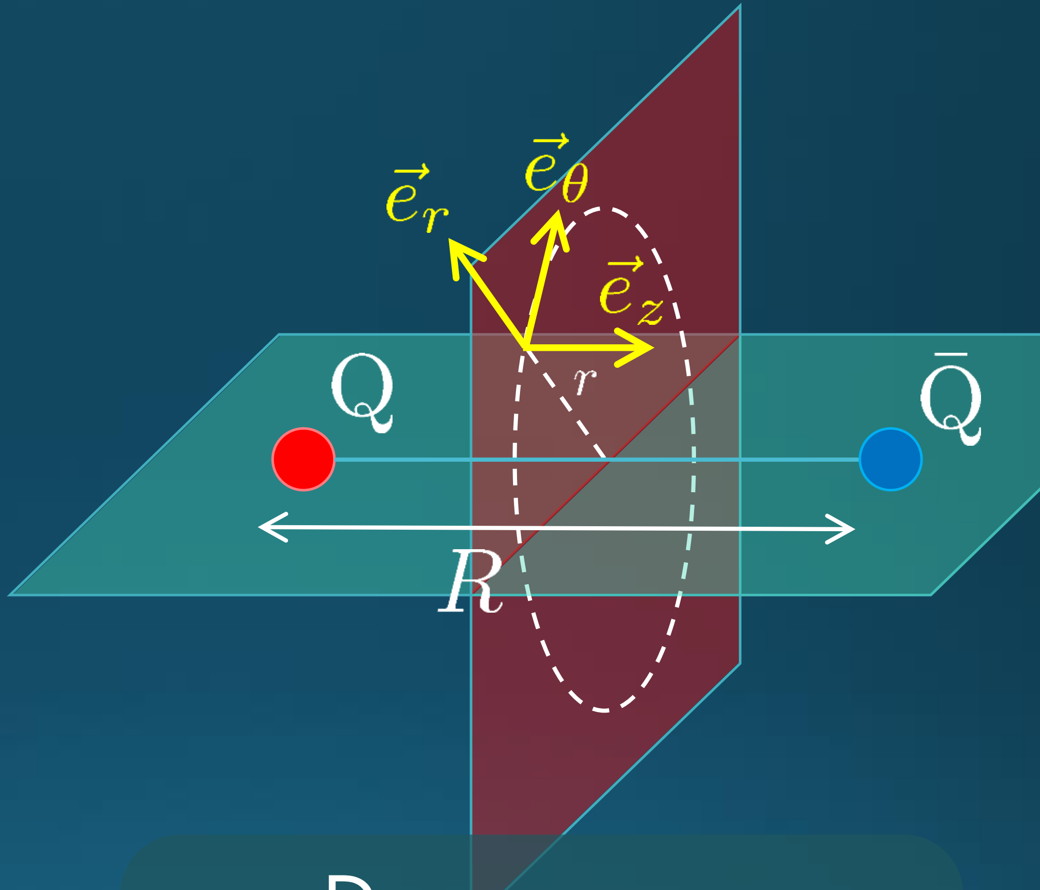
From rotational symm. & parity

EMT is diagonalized
in Cylindrical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{zz} & \\ & & & T_{44} \end{pmatrix}$$

$$T_{rr} = \vec{e}_r^{\,T} T \vec{e}_r$$

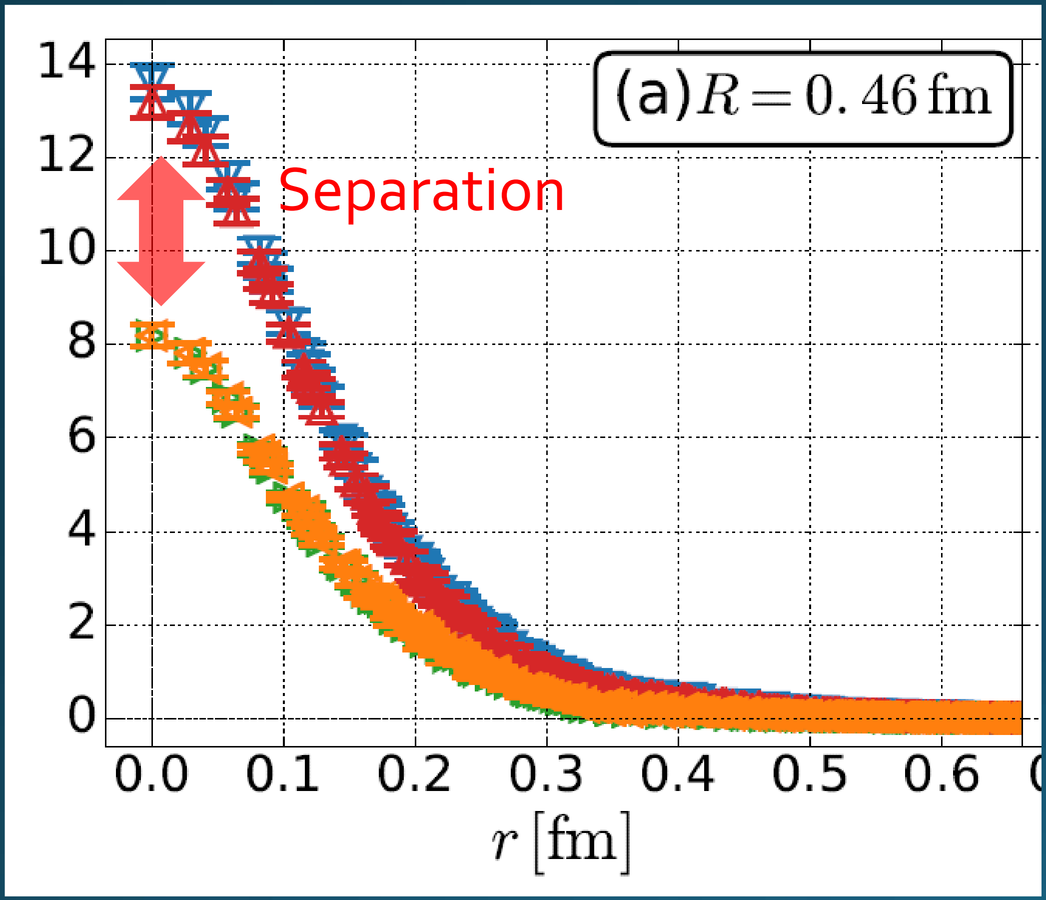
$$T_{\theta\theta} = \vec{e}_\theta^{\,T} T \vec{e}_\theta$$

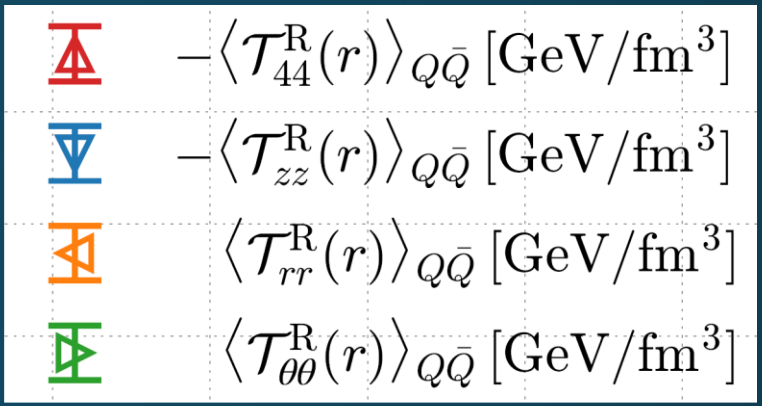


Degeneracy
in Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

Mid-Plane





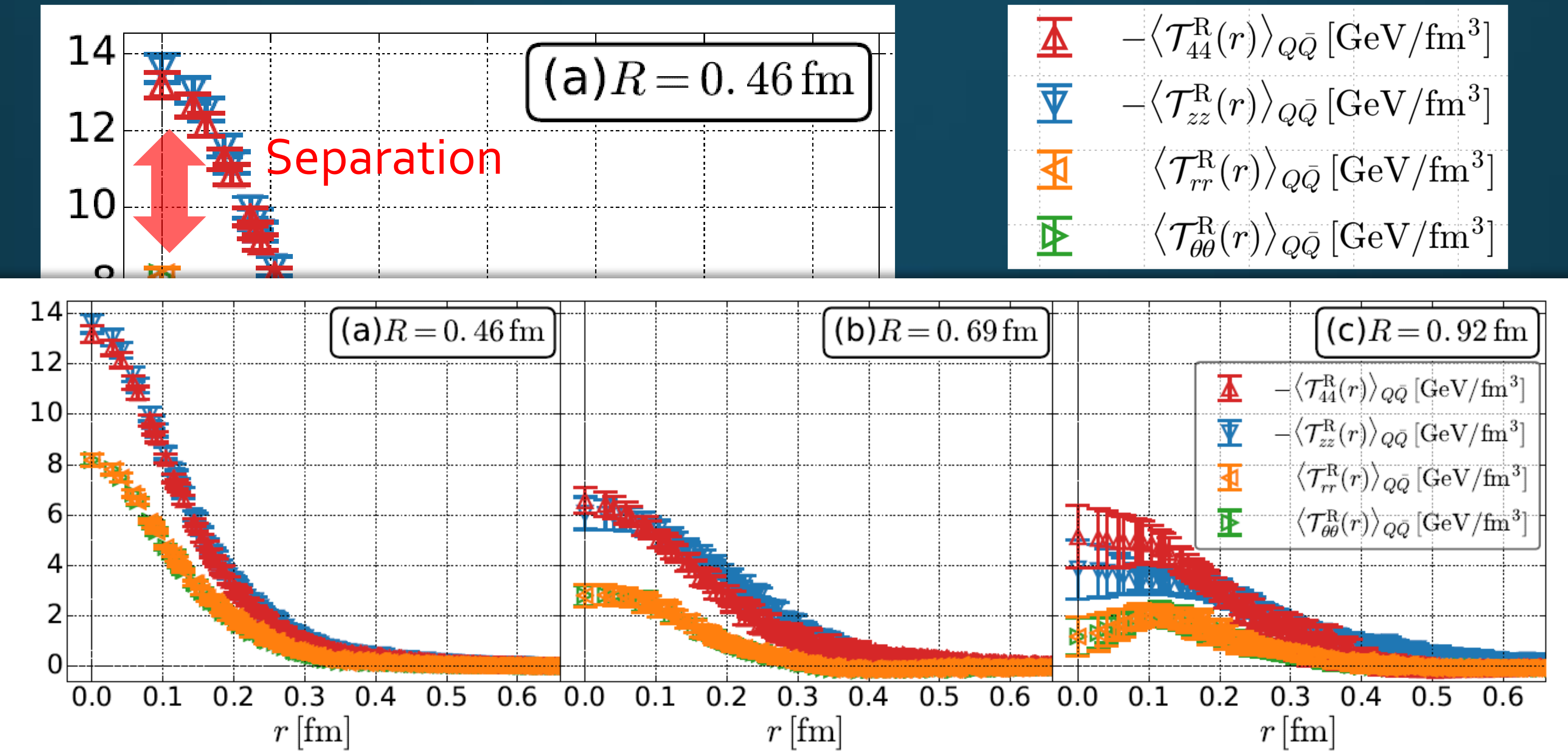
Continuum Extrapolated!

In Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

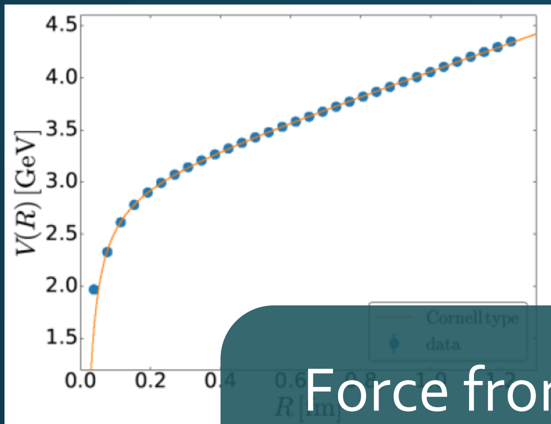
- Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- Separation: $T_{zz} \neq T_{rr}$
- Nonzero trace anomaly $\sum T_{cc} \neq 0$

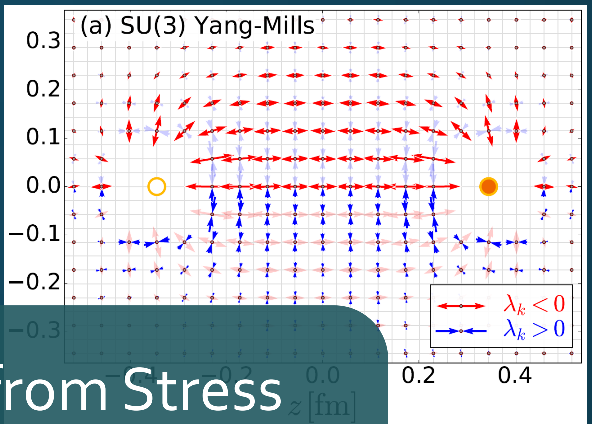
Mid-Plane



- ☐ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- ☐ Separation: $T_{zz} \neq T_{rr}$
- ☐ Nonzero trace anomaly $\sum T_{cc} \neq 0$

Force





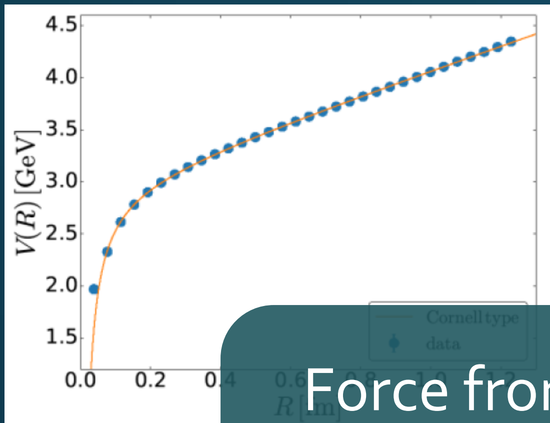
Force from Potential

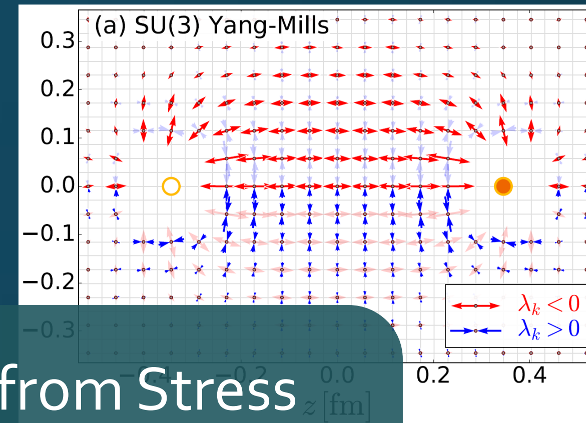
$$F_{\text{pot}} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\text{stress}} = \int_{\text{mid.}} d^2x T_{zz}(x)$$

Force





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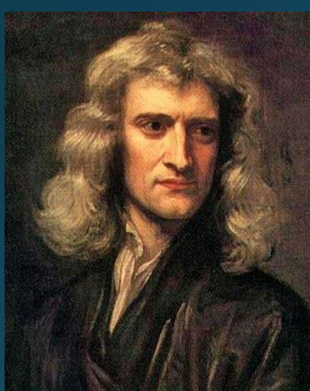
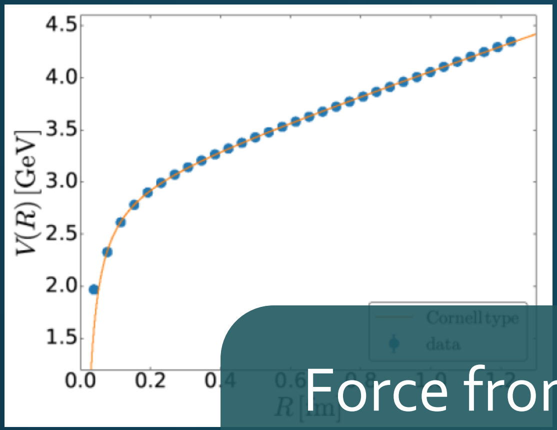
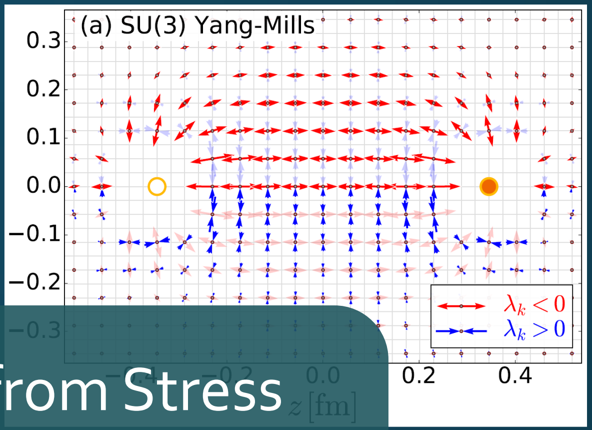


Newton
1687

Faraday
1839

Force





Force from Potential

$$F_{\rm pot} = -\frac{dV}{dR}$$

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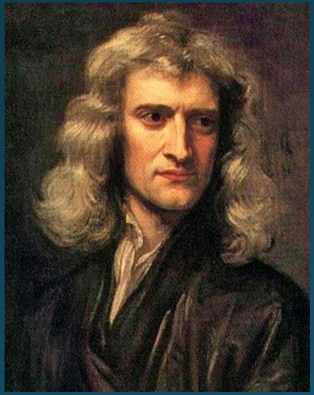
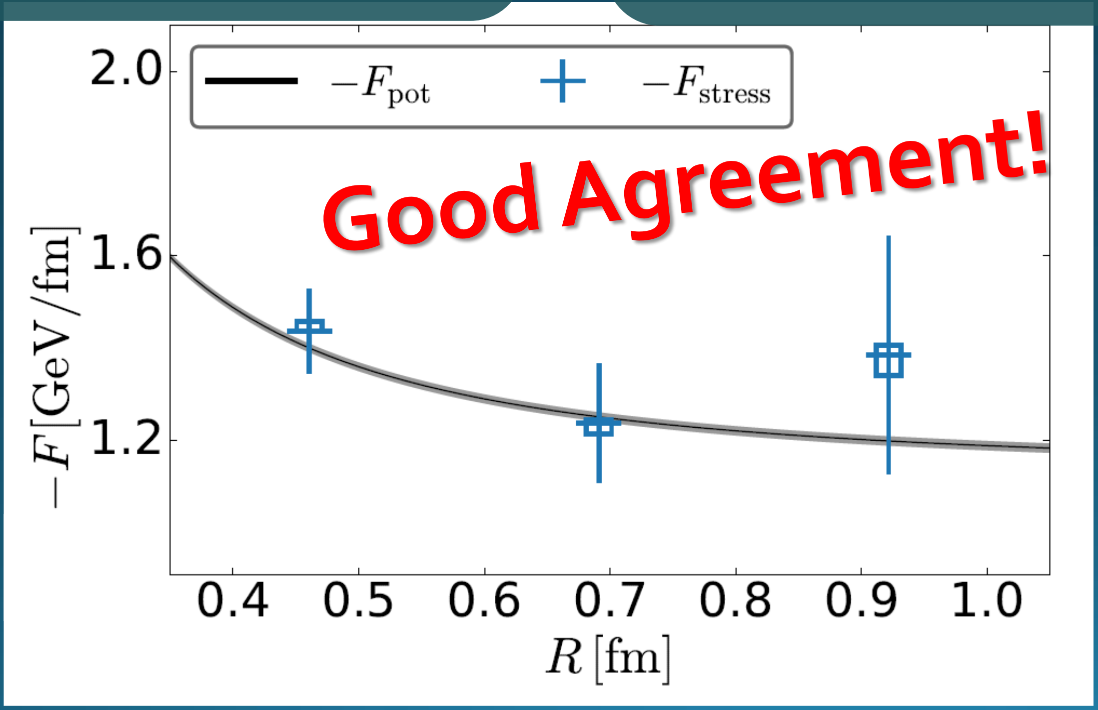


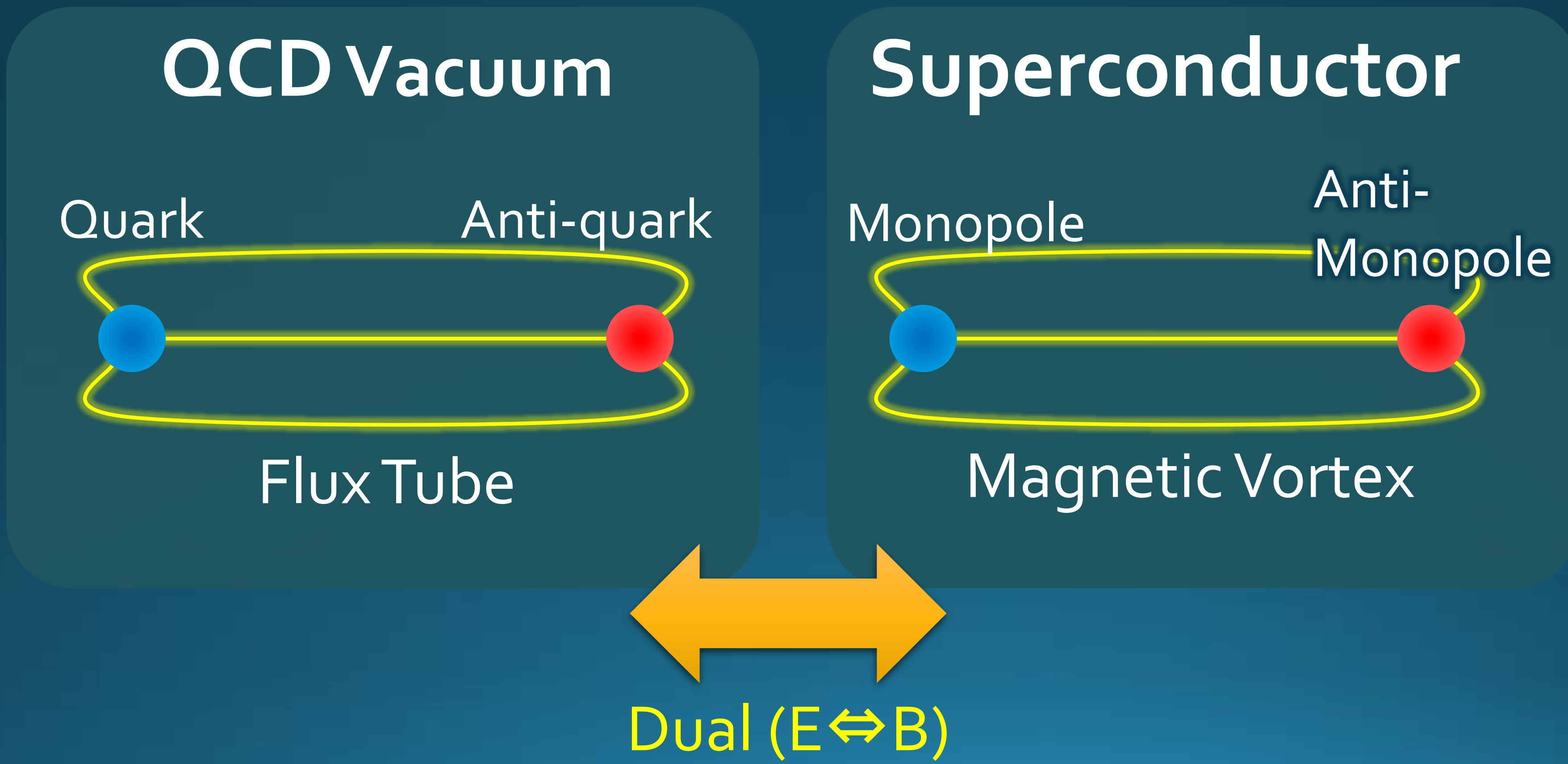
Newton
1687



Faraday
1839

Dual Superconductor Picture

Nambu, 1970
Nielsen, Olesen, 1973
t 'Hooft, 1981
...



Abelian-Higgs Model

Yanagihara, Iritani, MK, in prep.

Abelian-Higgs Model

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

GL parameter: $\kappa = \sqrt{\lambda}/g$

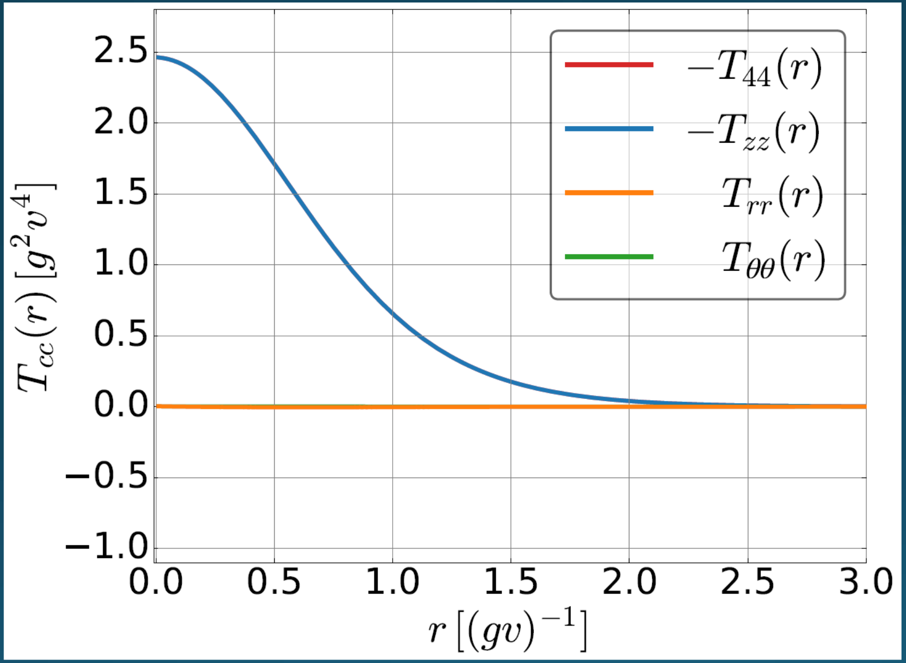
- type-I : $\kappa < 1/\sqrt{2}$
- type-II : $\kappa > 1/\sqrt{2}$
- Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

Infinitely long tube

- degeneracy
  $T_{zz}(r) = T_{44}(r)$ Luscher, 1981
- momentum conservation
  $$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

Stress Tensor in AH Model
infinitely-long flux tube

Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$



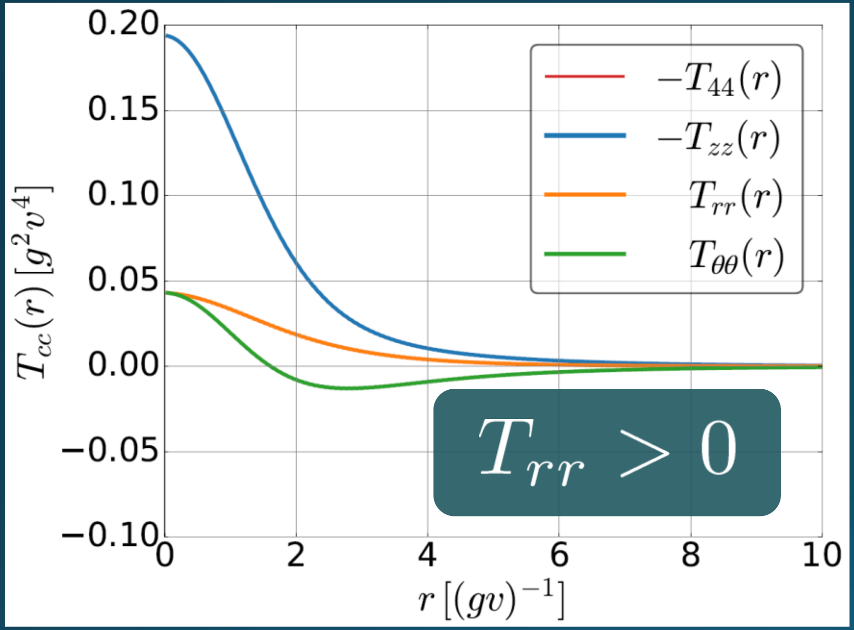
$$T_{rr} = T_{\theta\theta} = 0$$

de Vega, Schaposnik, PRD**14**, 1100 (1976).

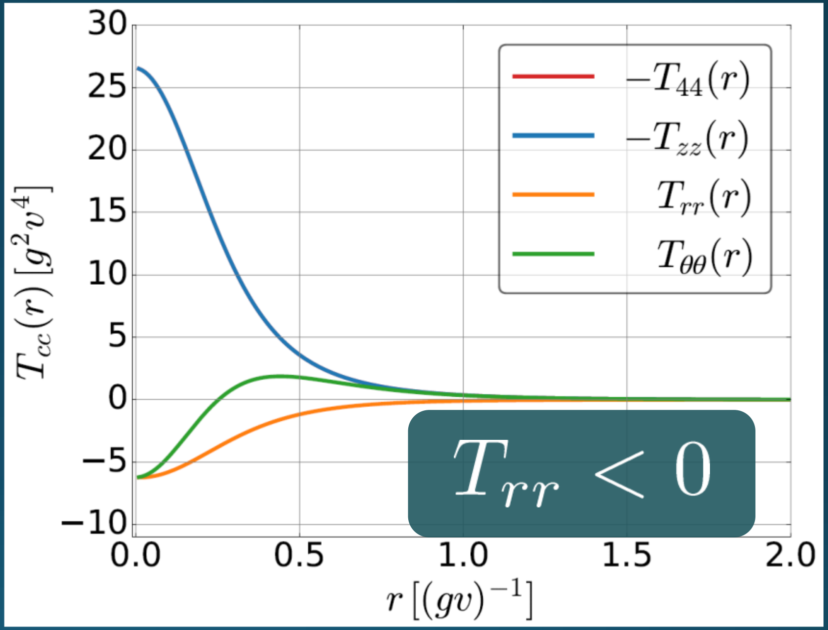
Stress Tensor in AH Model

infinitely-long flux tube

Type-I $\kappa = 0.1$



$\kappa = 3.0$ **Type-II**



- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

conservation law

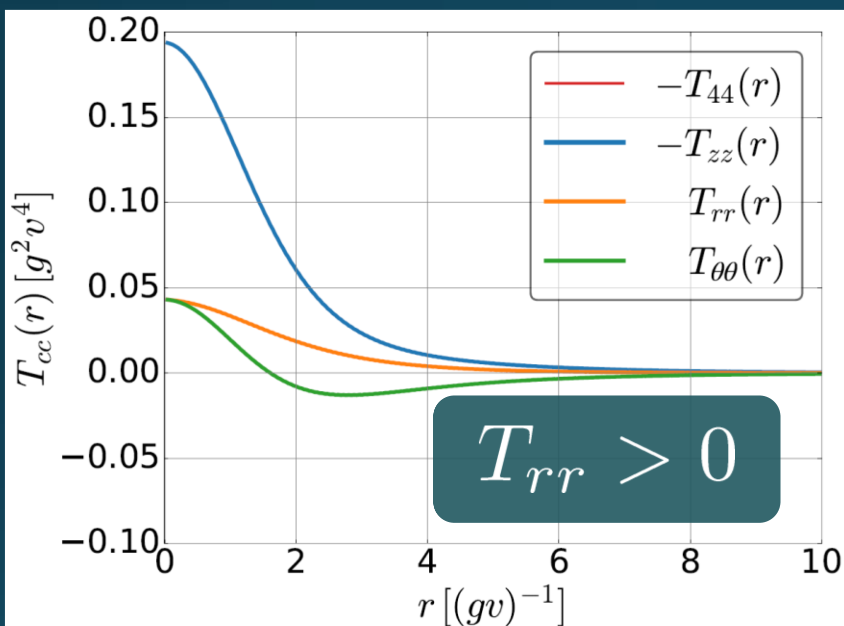
$$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

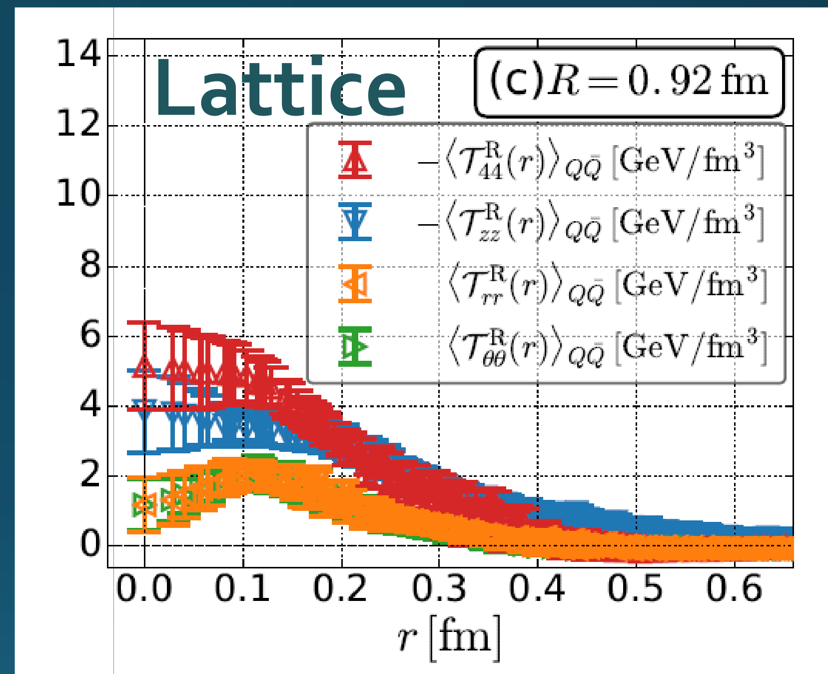
Stress Tensor in AH Model

infinitely-long flux tube

Type-I

$\kappa = 0.1$





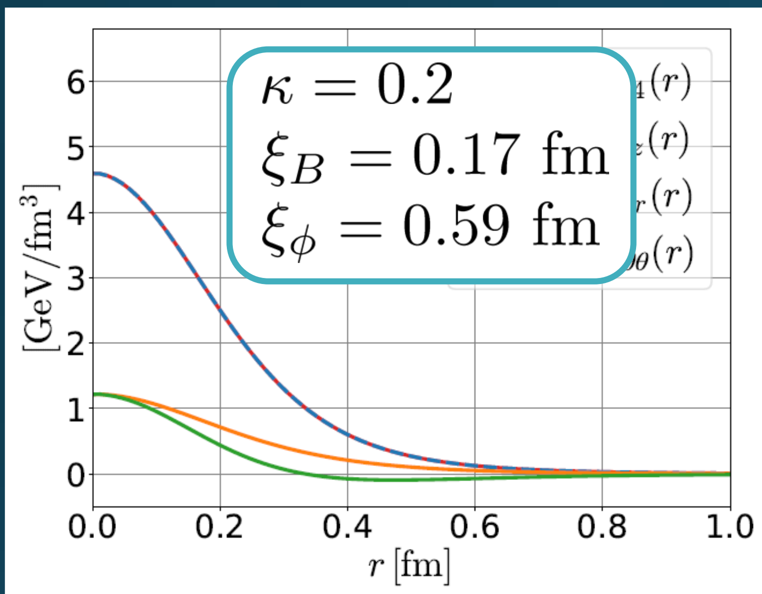
- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

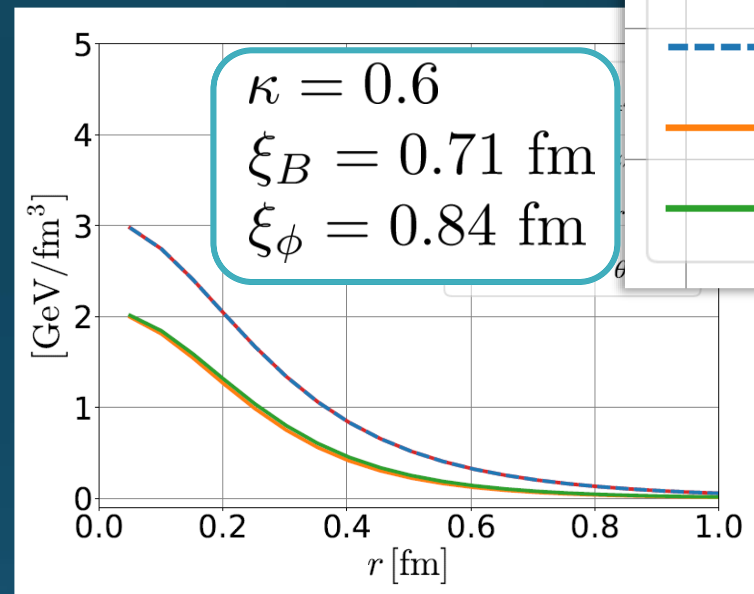
Inconsistent with lattice result

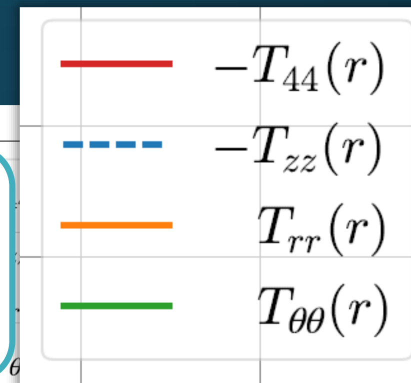
$$T_{rr} \simeq T_{\theta\theta}$$

Flux Tube with Finite Length

R=0.92 fm



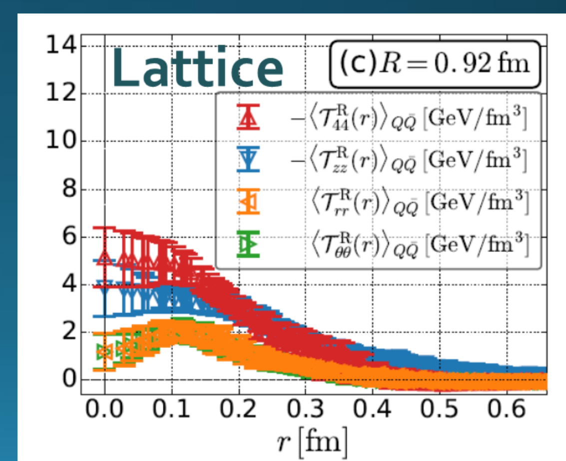




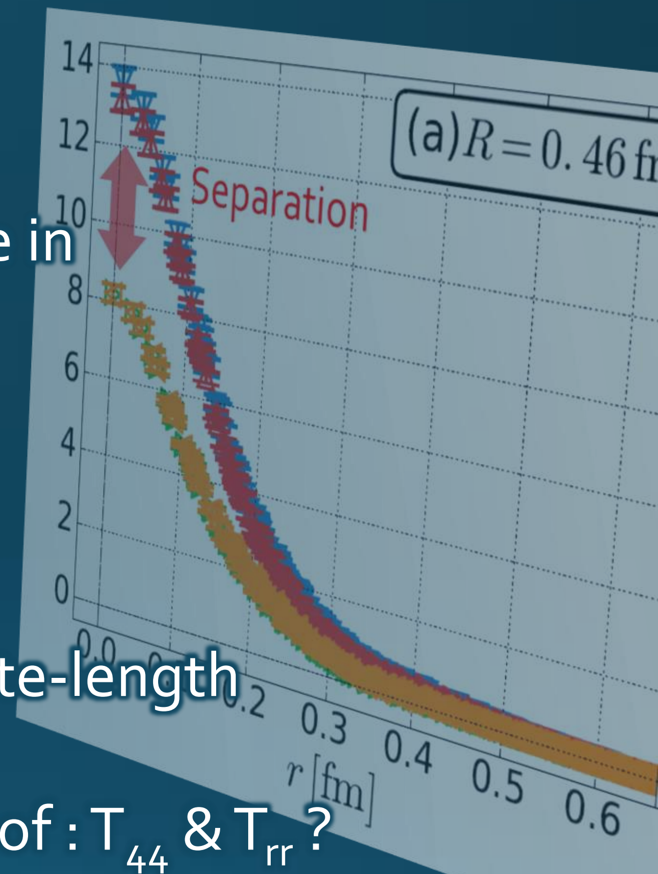
Left: $T_{zz}(0)$, $T_{rr}(0)$ reproduce lattice result

Right: A parameter satisfying $T_{rr} \approx T_{\theta\theta}$

No parameter can reproduce lattice data at R=0.92fm.



Summary

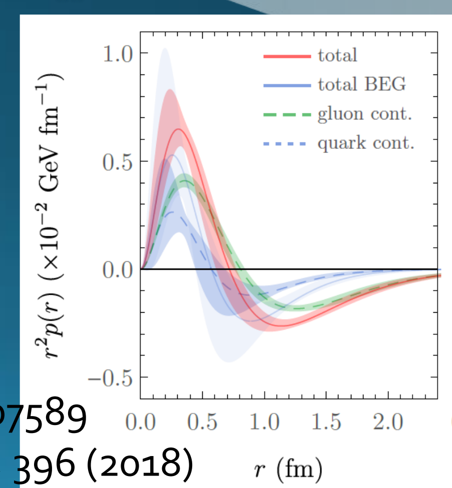


- EMT (stress tensor) is a useful observable in understanding the flux tube.
- Lattice data suggest that
  - Degeneracy: $T_{44} \approx T_{zz}$, $T_{rr} \approx T_{\theta\theta}$
  - Separation: $T_{zz} \neq T_{rr}$

- Degeneracy of T_{rr} & $T_{\theta\theta}$ suggests that finite-length effect of the flux tube is not negligible.
- Are there a priori reason for degeneragy of : T_{44} & T_{rr} ?

Future

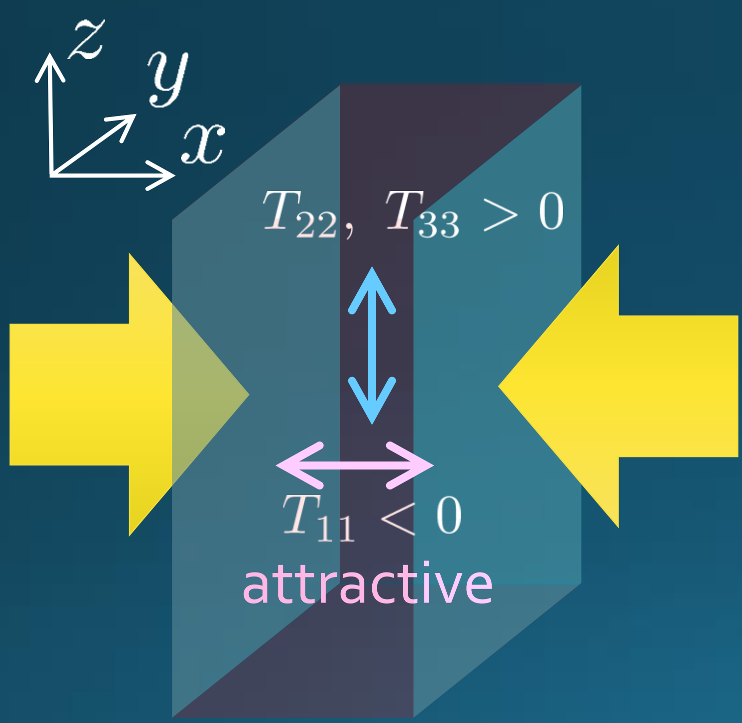
- Excited states / nonzero T / full QCD
- stress distribution inside hadrons



arXiv:1810.07589
Nature, 557, 396 (2018)

Pressure anisotropy
in finite system

Casimir effect



Finite system at nonzero *T*

MK, Mogliacci, Kolbe, Horowitz, in preparation

$$V = L_x \times L_y \times L_z$$

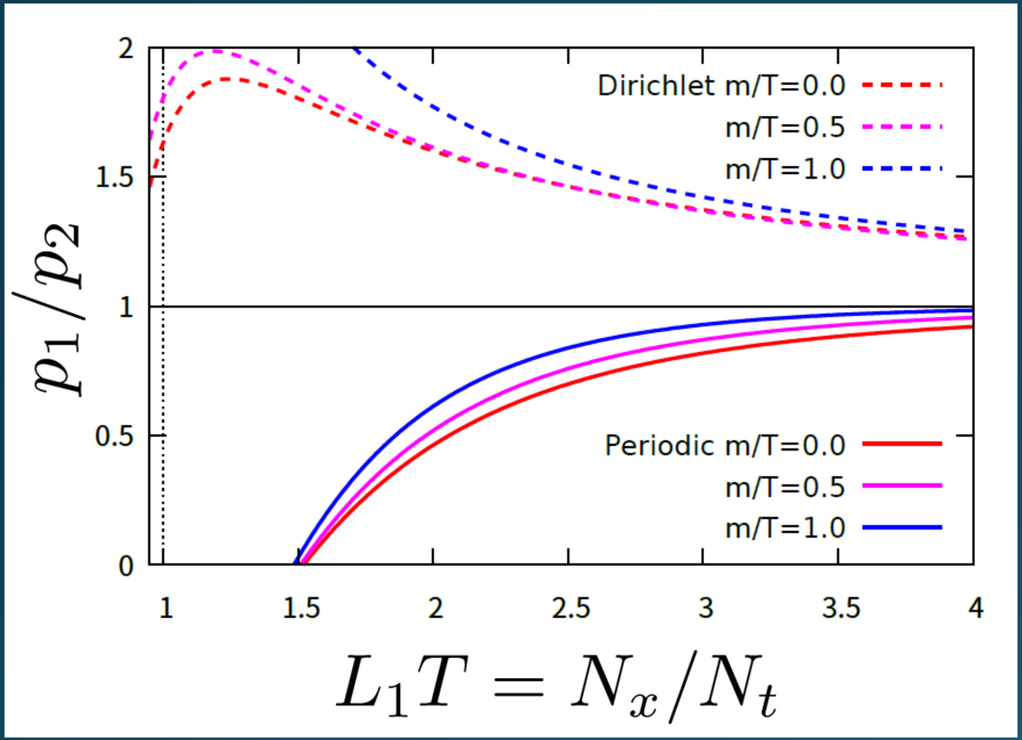
$$L_x \ll L_y = L_z$$

pressure anisotropy

$$T_{11} \neq T_{22} = T_{33}$$

Pressure Anisotropy

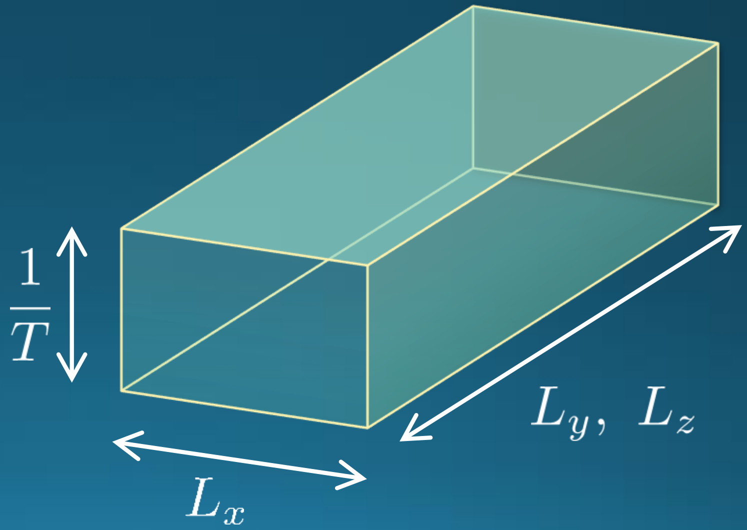
MK, Mogliacci, Kolbe, Horowitz, in prep.



Free scalar field

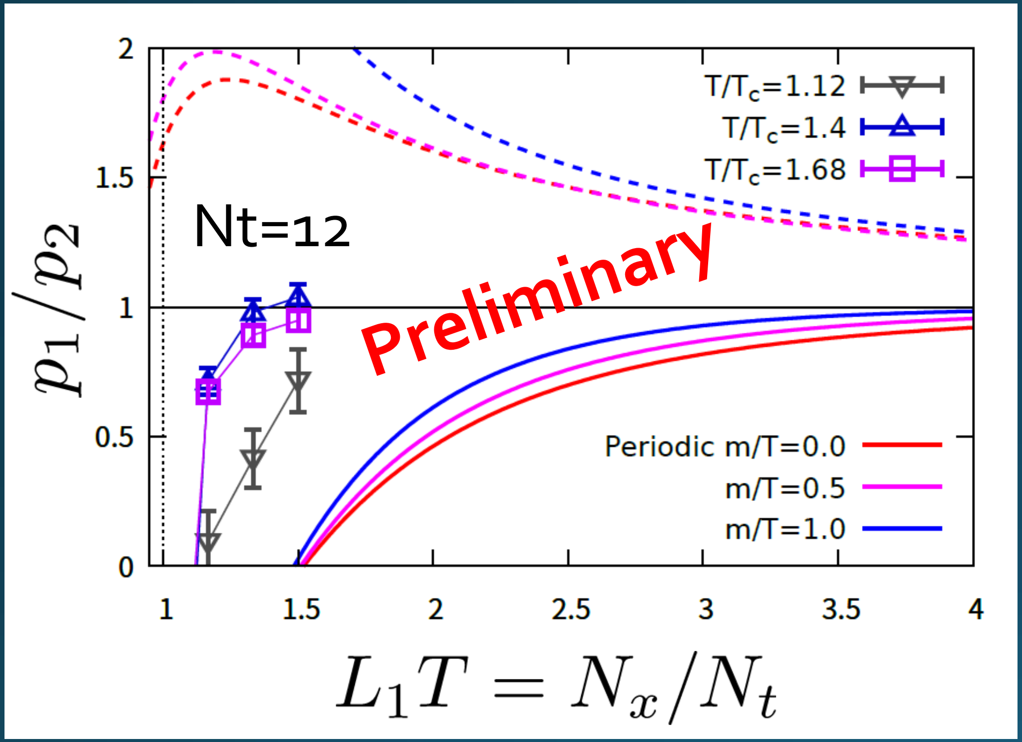
- $L_2=L_3=\infty$

Mogliacci+, 1807.07871



Pressure Anisotropy

MK, Mogliacci, Kolbe, Horowitz, in prep.



Free scalar field

- ☐ $L_2=L_3=\infty$

Mogliacci+, 1807.07871

Lattice result

- ☐ Periodic BC
- ☐ $N_s^2 x N_x x N_t = 72^2 x N_x x 12$
- ☐ N_x=12, 14, 16, 18
- ☐ Only t→0 limit (fixed a)

Medium near T_c is remarkably insensitive to finite size! How do we understand??

backup

EMT on the Lattice: Conventional

Lattice EMT Operator

Caracciolo+, 1990

$$T_{\mu\nu} = Z_6 T^{[6]}_{\mu\nu} + Z_3 T^{[3]}_{\mu\nu} + Z_1 \left(T^{[1]}_{\mu\nu} - \langle T^{[1]}_{\mu\nu} \rangle \right)$$

$$T^{[6]}_{\mu\nu} = (1-\delta_{\mu\nu}) F^a_{\mu\rho} F^a_{\nu\rho}, \;\; T^{[3]}_{\mu\nu} = \delta_{\mu\nu} \left(F^a_{\mu\rho} F^a_{\nu\rho} - \frac{1}{4} F^a_{\rho\sigma} F^a_{\rho\sigma} \right), \;\; T^{[1]}_{\mu\nu} = \delta_{\mu\nu} F^a_{\rho\sigma} F^a_{\rho\sigma}$$

- Fit to thermodynamics: Z_3, Z_1
- Shifted-boundary method: Z_6, Z_3 — Giusti, Meyer, 2011; 2013; Giusti, Pepe, 2014~; Borsanyi+, 2018

Multi-level algorithm

- effective in reducing statistical error of correlator — Meyer, 2007; Borsanyi, 2018; Astrakhantsev+, 2018

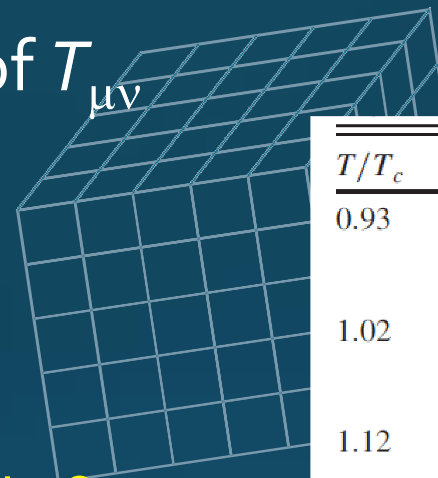
Numerical Simulation

- Expectation values of $T_{\mu\nu}$
- SU(3) YM theory
- Wilson gauge action
- Parameters:
  - N_t = 12, 16, 20-24
  - aspect ratio $5.3<N_s/N_t<8$
  - 1500~2000 configurations

- Scale from gradient flow

$$\rightarrow aT_c \text{ and } a\Lambda_{\rm MS}$$

FlowQCD, 1503.06516



FlowQCD, PRD**94**, 114512 (2016)

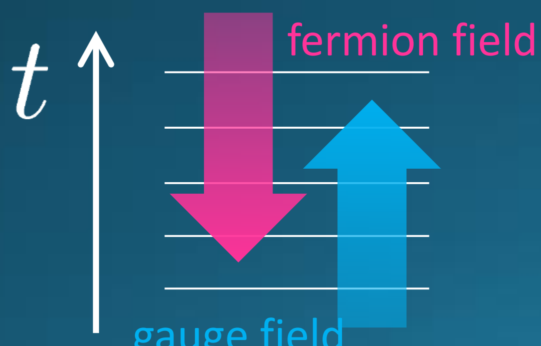
T/T_c	β	N_s	N_τ	Configurations
0.93	6.287	64	12	2125
	6.495	96	16	1645
	6.800	128	24	2040
1.02	6.349	64	12	2000
	6.559	96	16	1600
	6.800	128	22	2290
1.12	6.418	64	12	1875
	6.631	96	16	1580
	6.800	128	20	2000
1.40	6.582	64	12	2080
	6.800	128	16	900
	7.117	128	24	2000
1.68	6.719	64	12	2000
	6.941	96	16	1680
	7.117	128	20	2000
2.10	6.891	64	12	2250
	7.117	128	16	840
	7.296	128	20	2040
2.31	7.200	96	16	1490
	7.376	128	20	2020
	7.519	128	24	1970
2.69	7.086	64	12	2000
	7.317	96	16	1560
	7.500	128	20	2040

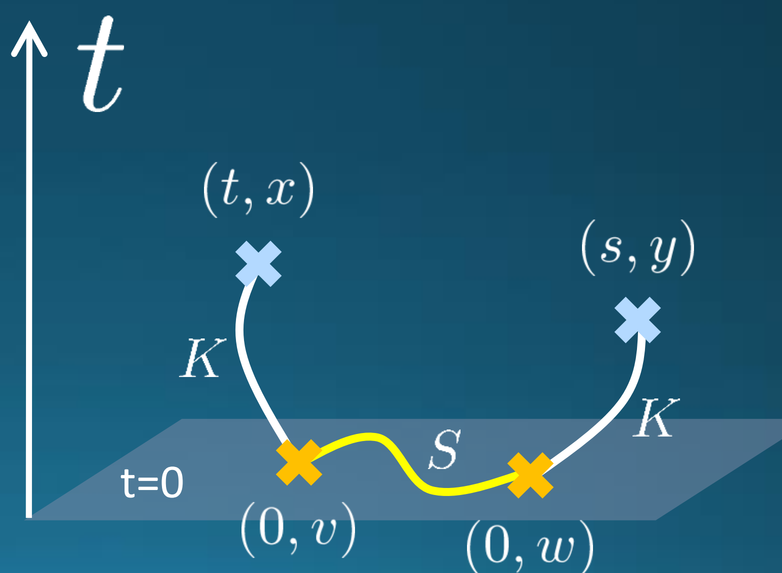
Fermion Propagator

$$S(t,x;s,y) = \langle \chi(t,x)\bar{\chi}(s,y)\rangle$$
$$= \sum_{v,w} K(t,x;0,v)S(v,w)K(s,y;0,w)^{\dagger}$$

$$\Big(\partial_t - D_\mu D_\mu\Big)K(t,x) = 0$$

- propagator of flow equation
- Inverse propagator is needed





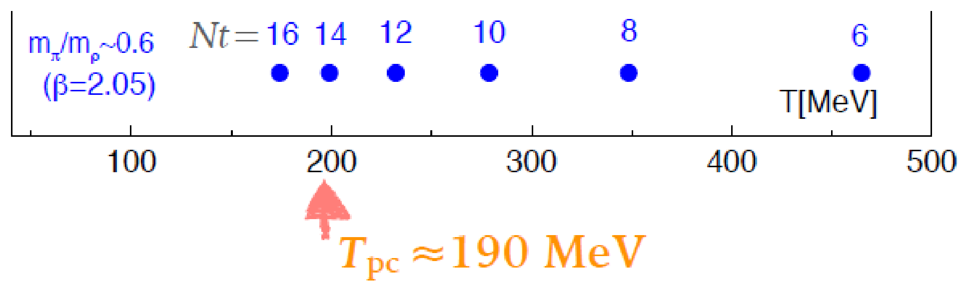
N_f=2+1 QCD Thermodynamics

Taniguchi+ (WHOT-QCD), PRD**96**, 014509 (2017)

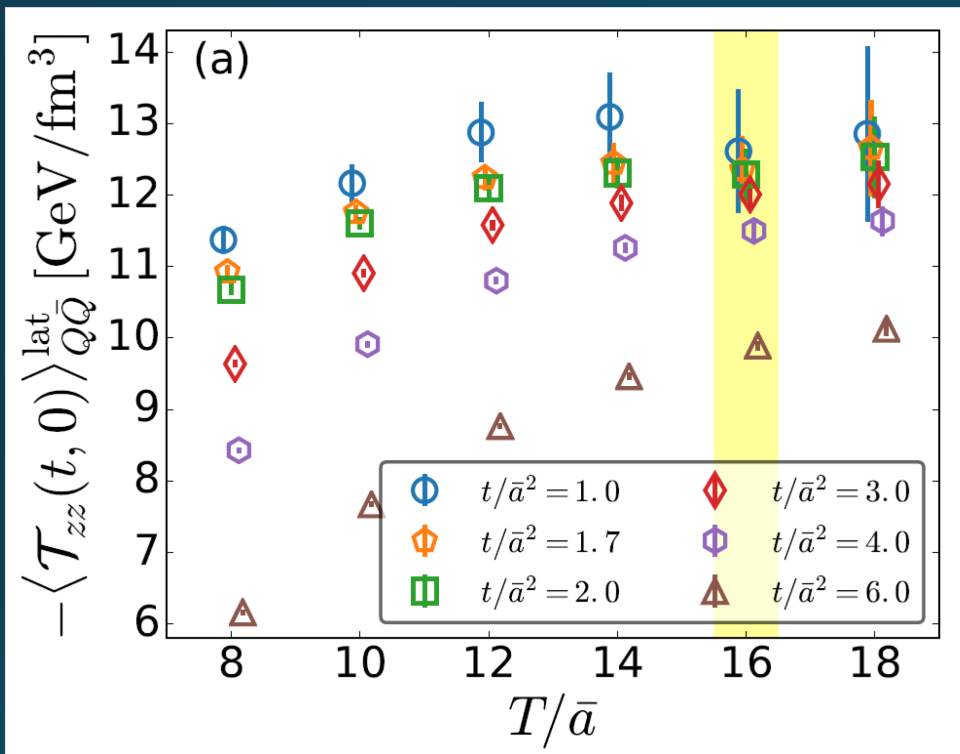
- N_f=2+1 QCD, Iwasaki gauge + NP-clover
- $m_{PS}/m_V \approx 0.63$ / almost physical s quark mass

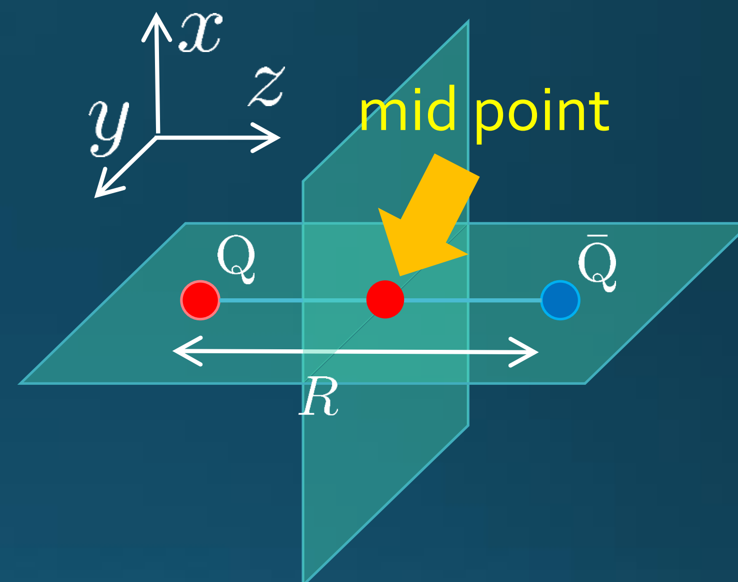
- T=0: CP-PACS+JLQCD (ß=2.05, 28^3x56, a≈0.07fm)
- T>0: 32^3xN_t, N_t = 4, 6, … , 14, 16):
- T≈174-697MeV

- t→0 extrapolation only (No continuum limit)



Ground State Saturation

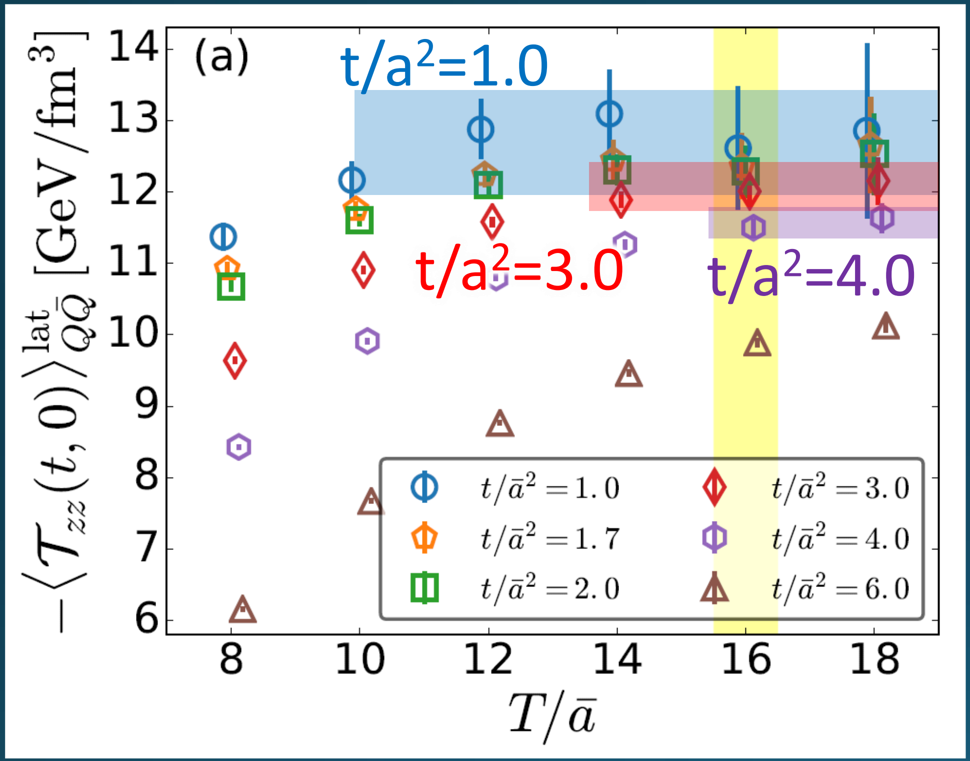


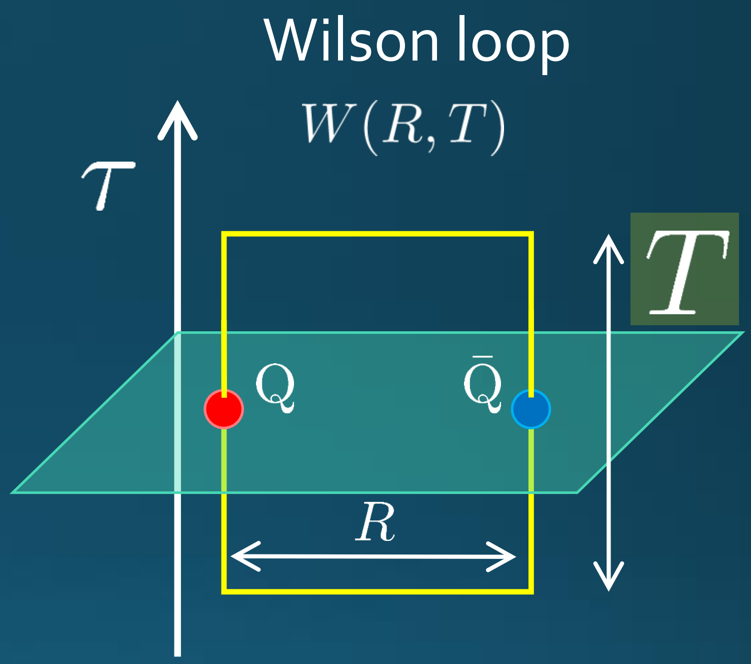


β=6.819 (a=0.029 fm), R=0.46 fm

Appearance of plateau for $t/a^2<4$, $T/a>15$ ⇒ Grand state saturation under control

Ground State Saturation





β=6.819 (a=0.029 fm), R=0.46 fm

Appearance of plateau for $t/a^2<4$, $T/a>15$ ⇒ Grand state saturation under control

EMT in QCD

$$\begin{aligned} T_{\mu\nu}(t,x) = & c_1(t)U_{\mu\nu}(t,x) + c_2(t)\delta_{\mu\nu}\big(E(t,x) - \langle E\rangle_0\big) \\ & + c_3(t)\big(O_{3\mu\nu}(t,x) - 2O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) \\ & + c_4(t)\big(O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) + c_5(t)\big(O_{5\mu\nu}(t,x) - \mathrm{VEV}\big) \end{aligned}$$

$$T_{\mu\nu}(x) = \lim_{t\to 0} T_{\mu\mu}(t,x)$$

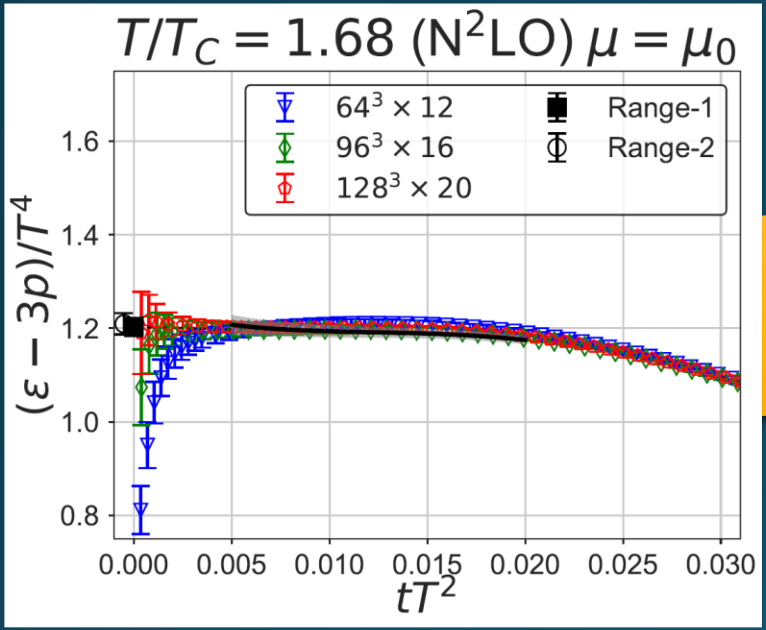
$$\begin{aligned} \tilde{O}^f_{3\mu\nu}(t,x) &\equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x), \\ \tilde{O}^f_{4\mu\nu}(t,x) &\equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x), \\ \tilde{O}^f_{5\mu\nu}(t,x) &\equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x), \end{aligned}$$

$$\varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.$$

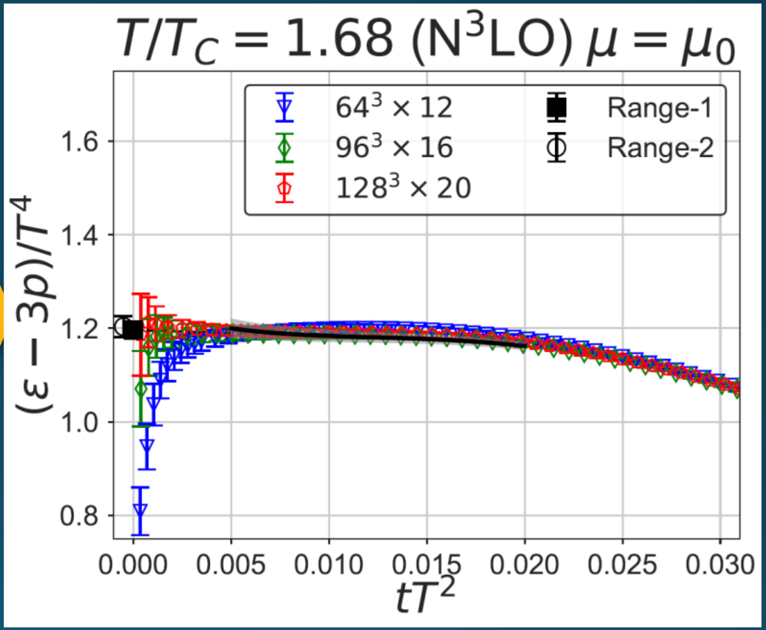
$$\begin{aligned} c_1(t) &= \frac{1}{\bar{g}(1/\sqrt{8t})^2} - \frac{1}{(4\pi)^2}\left[9(\gamma - 2\ln 2) + \frac{19}{4}\right], \\ c_2(t) &= \frac{1}{(4\pi)^2}\frac{33}{16}, \\ c_3(t) &= \frac{1}{4}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[2 + \frac{4}{3}\ln(432)\right]\right\}, \\ c_4(t) &= \frac{1}{(4\pi)^2}\bar{g}(1/\sqrt{8t})^2, \\ c_5^f(t) &= -\bar{m}_f(1/\sqrt{8t})\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[4(\gamma - 2\ln 2) + \frac{14}{3} + \frac{4}{3}\ln(432)\right]\right\} \end{aligned}$$

Higher Order Coefficient: ε-3p

N^2LO (2-loop)



N^3LO (3-loop)



Iritani, MK, Suzuki, Takaura, in prep.

- No difference b/w 2- & 3-loops: 2-loop is already good!
- Systematic analysis: μ_0 or μ_d, uncertainty of Λ, fit range
- Extrapolation func: linear, higher order term in c_2 ($\sim g^8$)

Gradient Flow for Fermions

$$\partial_t \psi(t,x) = D_\mu D_\mu \psi(t,x)$$

$$\partial_t \bar{\psi}(t,x) = \psi(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu$$

$$D_\mu = \partial_\mu + A_\mu(t,x)$$

Luscher, 2013
Makino, Suzuki, 2014
Taniguchi+ (WHOT) 2016; 2017

- Not “gradient” flow but a “diffusion” equation.

- Divergence in field renormalization of fermions.
- All observables are finite at t>0 once Z(t) is fixed.

$$\tilde{\psi}(t,x) = Z(t)\psi(t,x)$$

- Energy-momentum tensor from SFTE Makino, Suzuki, 2014

Abelian-Higgs Model

Abelian-Higgs Model

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

GL parameter: $\kappa = \sqrt{\lambda}/g$

- type-I : $\kappa < 1/\sqrt{2}$
- type-II : $\kappa > 1/\sqrt{2}$
- Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

Infinitely long tube

- degeneracy

  $T_{zz}(r) = T_{44}(r)$ Luscher, 1981
- conservation law

  $$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$