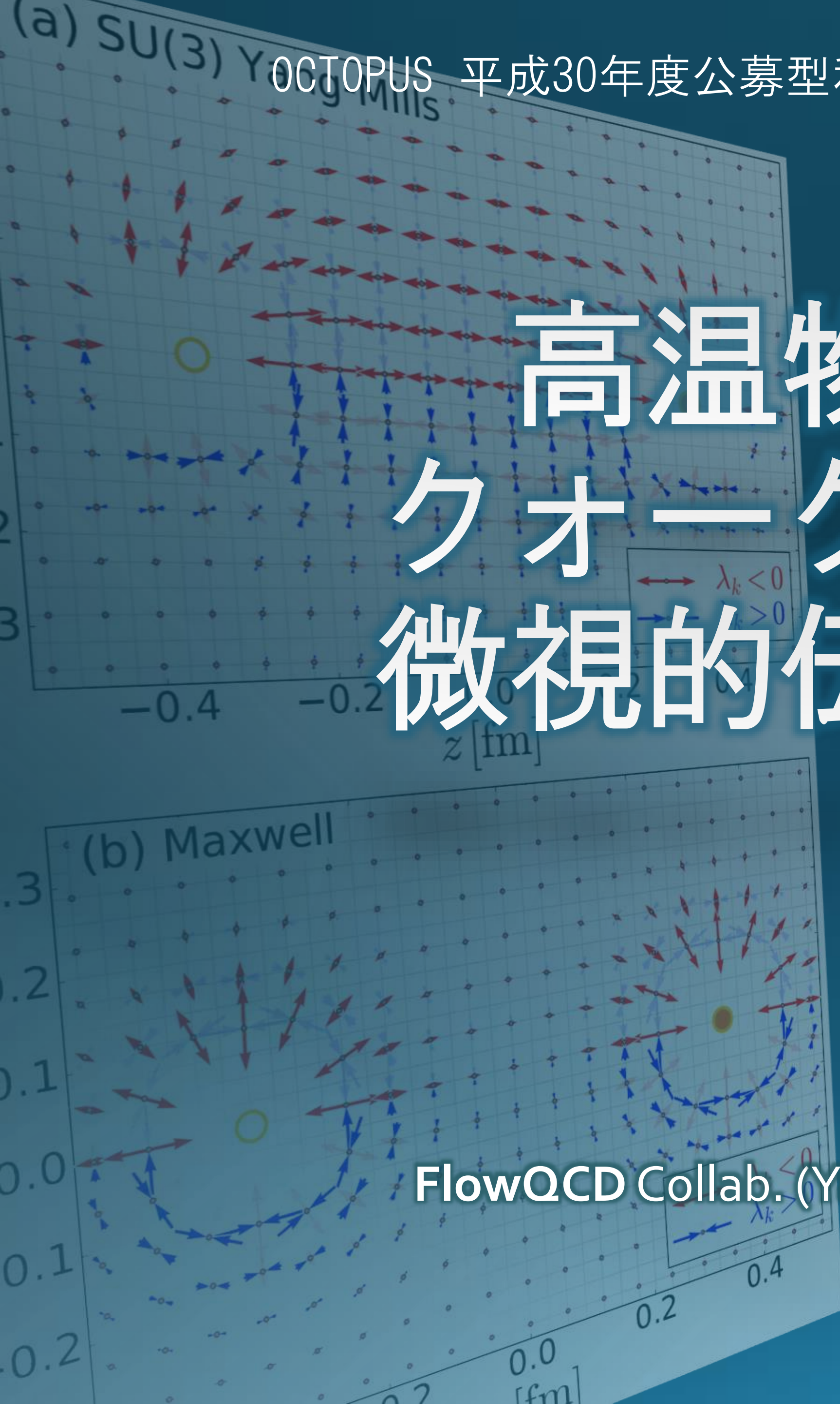
OCTOPUS 平成30年度公募型利用制度 成果報告会、2019年3月5日



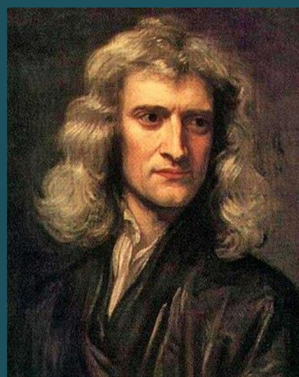
高温物質中における クォーク間相互作用の 微視的伝達機構の解明

北沢正清
(阪大理)

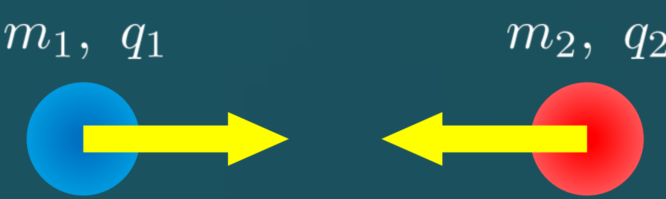
FlowQCD Collab. (Yanagihara, Iritani, MK, Asakawa, Hatsuda
Phys. Lett. **B789**, 210 (2019)

Force

Action-at-a-distance



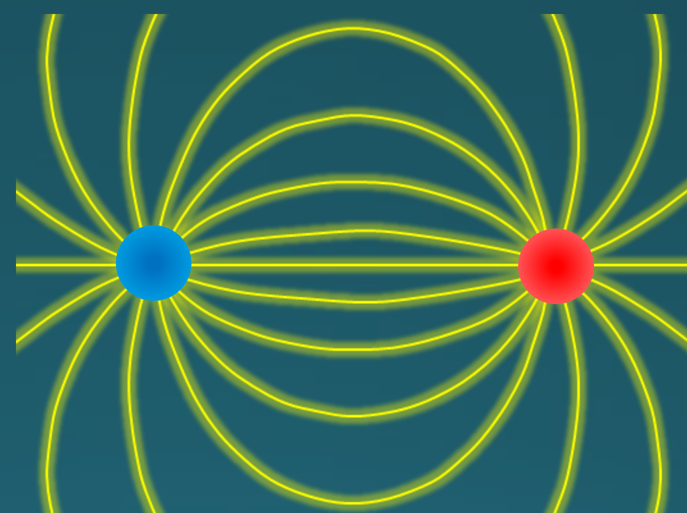
Newton
1687



$$F = -G\frac{m_1 m_2}{r^2} \qquad F = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Local interaction

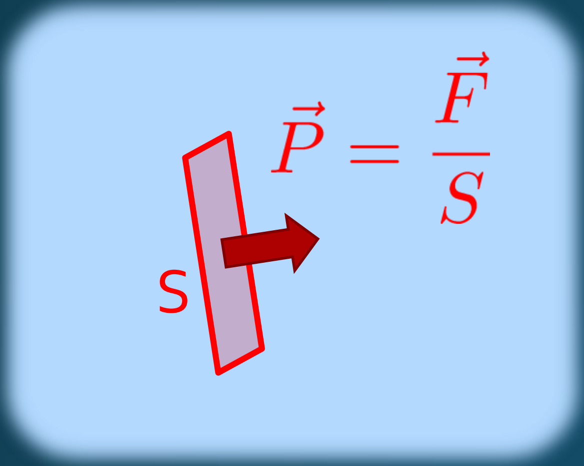
Faraday
1839



Stress = Force per Unit Area

Stress = Force per Unit Area

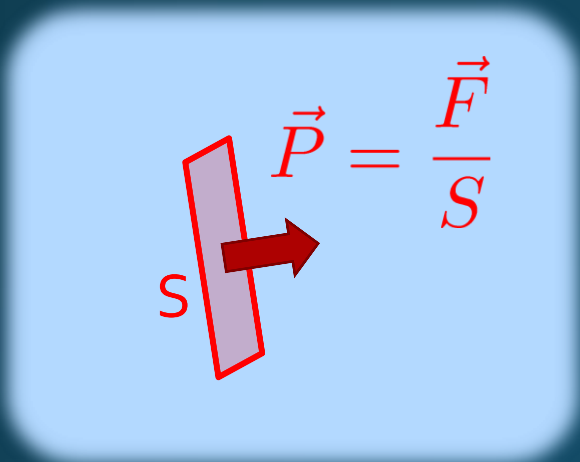
Pressure



$$\vec{P} = P\vec{n}$$

Stress = Force per Unit Area

Pressure

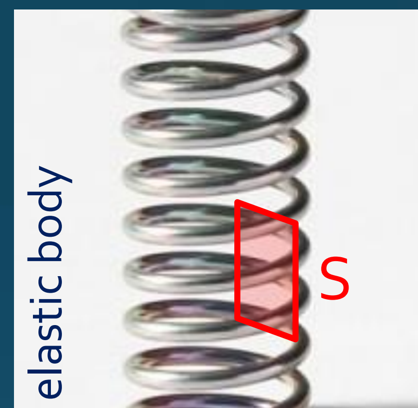


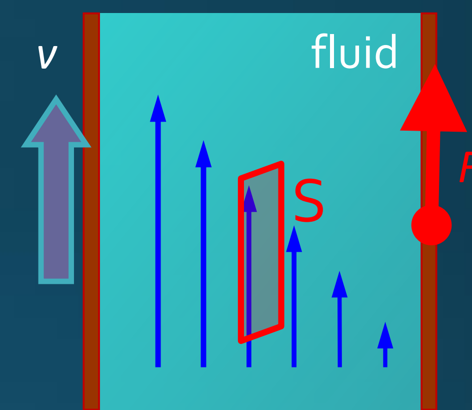
$$\vec{P} = P\vec{n}$$

In thermal medium

$$T_{ij} = P\delta_{ij}$$

Generally, F and n are not parallel





$$\frac{F_i}{S} = \sigma_{ij} n_j$$

Stress Tensor

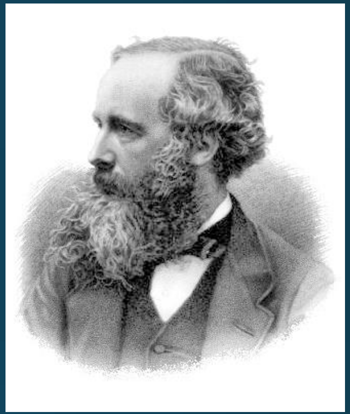
$$\sigma_{ij} = -T_{ij}$$

Landau
Lifshitz

Maxwell Stress

(in Maxwell Theory)

$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

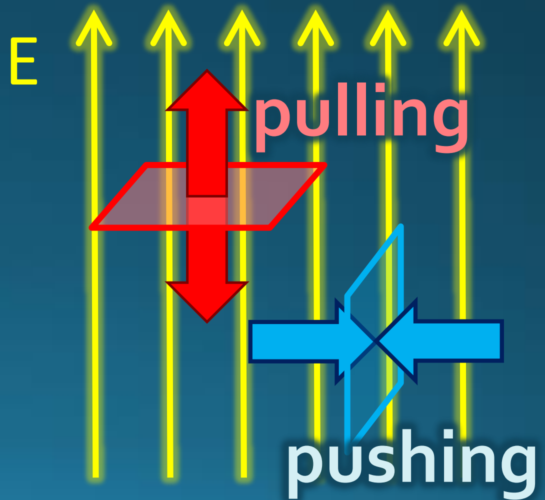


Maxwell

$$\vec{E} = (E, 0, 0)$$

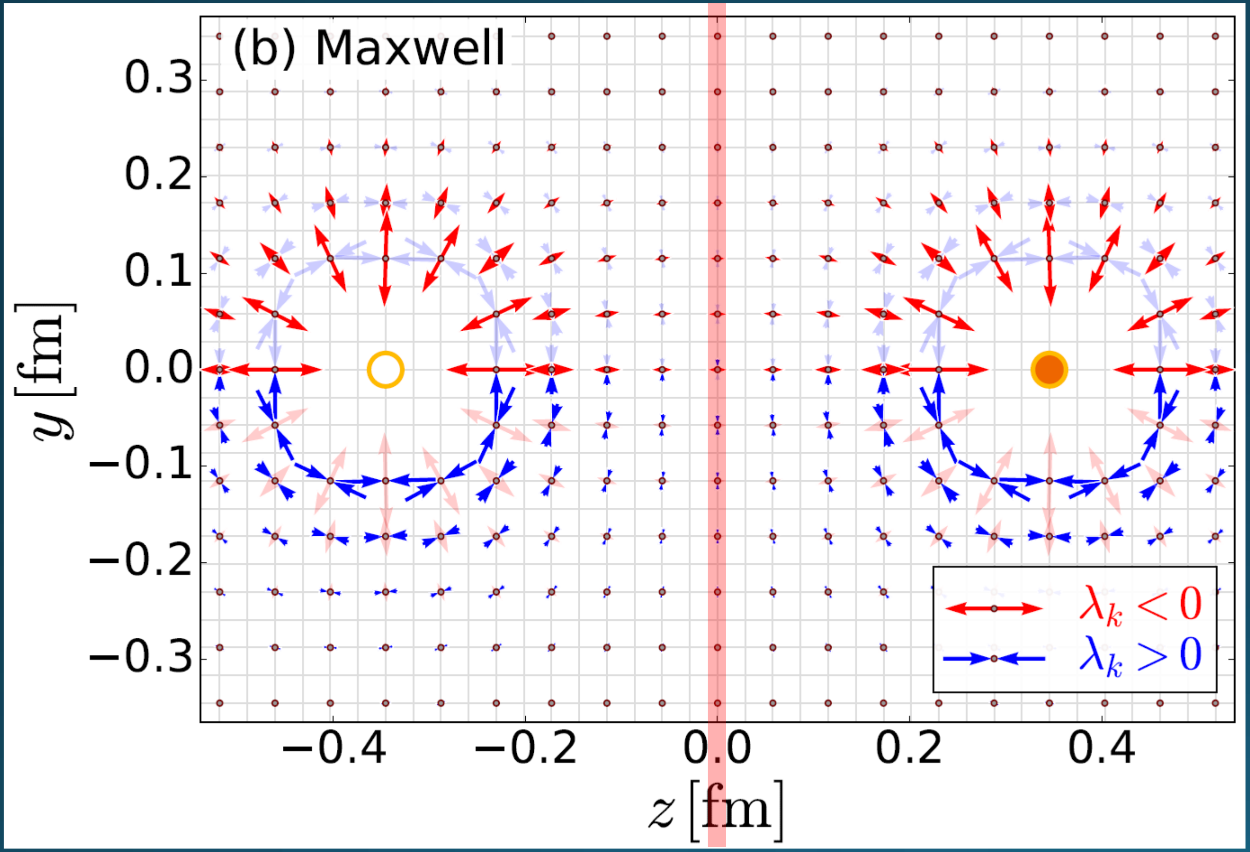
$$T_{ij} = \begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: **Pulling**
- Vertical to field: **Pushing**



Maxwell Stress

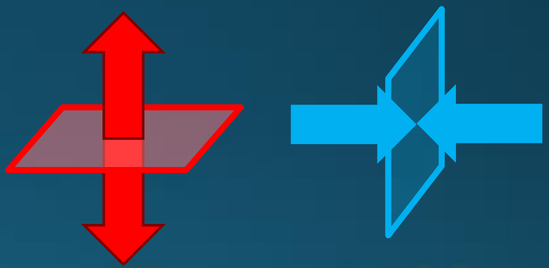
(in Maxwell Theory)



$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$$(k = 1, 2, 3)$$

length: $\sqrt{|\lambda_k|}$

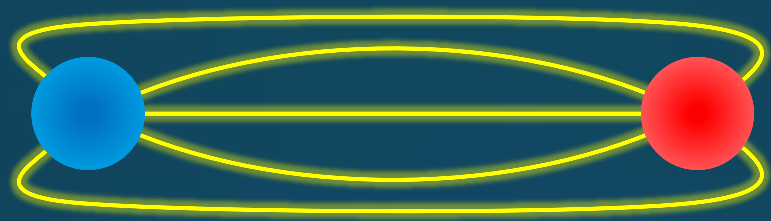


Definite physical meaning

- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction

Quark-Anti-quark system

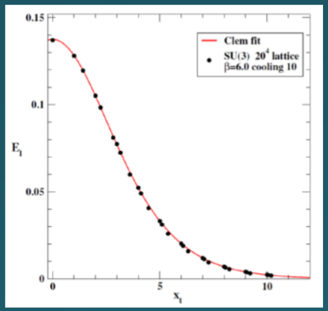
Formation of the flux tube → confinement



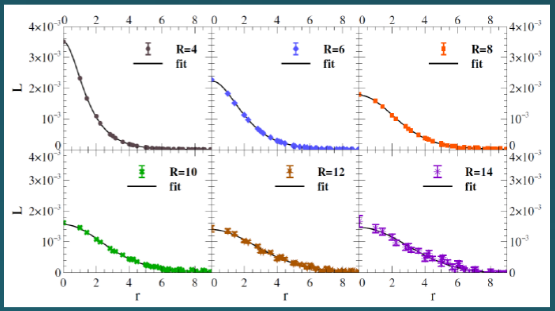
Previous Studies on Flux Tube

- ☐ Potential
- ☐ Action density
- ☐ Color-electric field

so many studies…



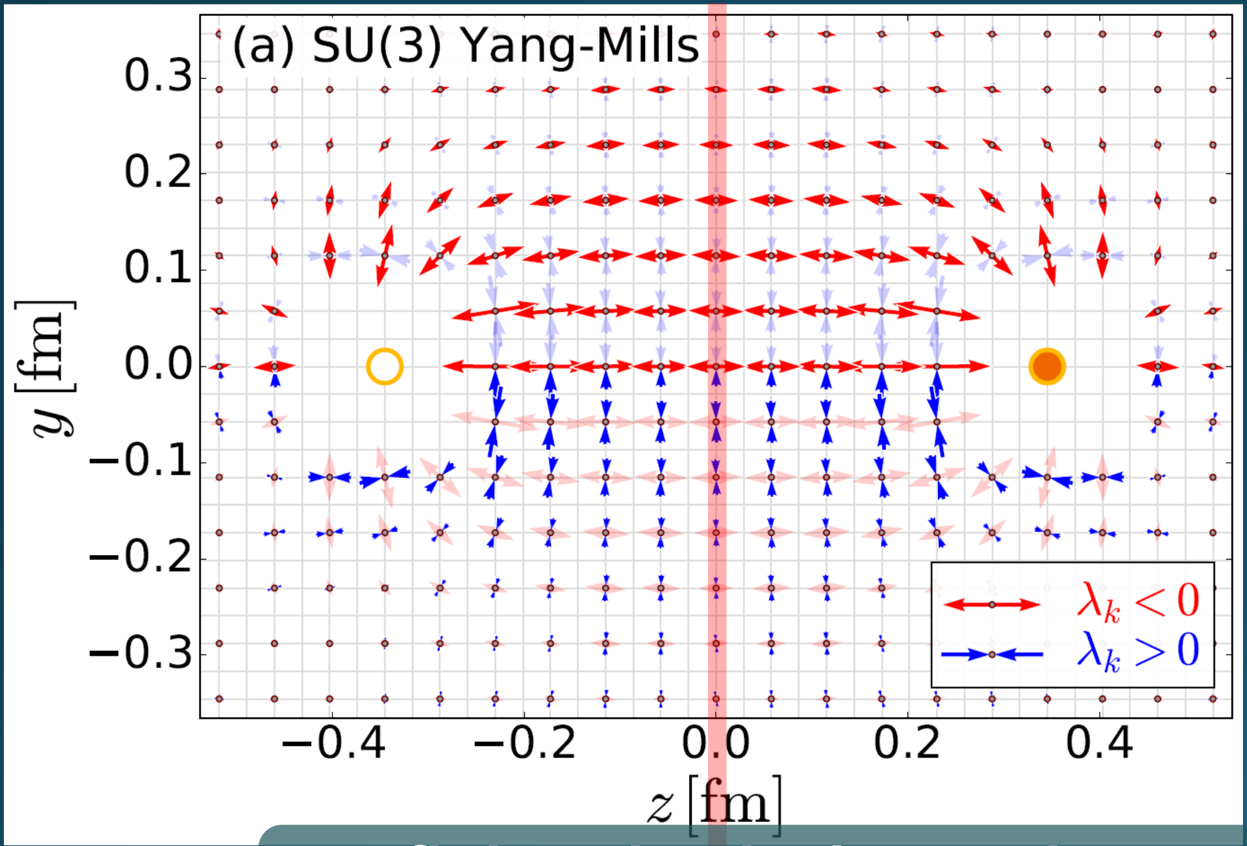
Cea+ (2012)



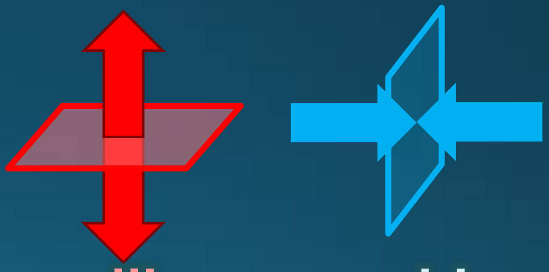
Cardoso+ (2013)

Stress Tensor in $Q\bar{Q}$ System

Yanagihara+, PLB (2019)



Lattice simulation
SU(3) Yang-Mills
a=0.029 fm
R=0.69 fm
t/a^2=2.0

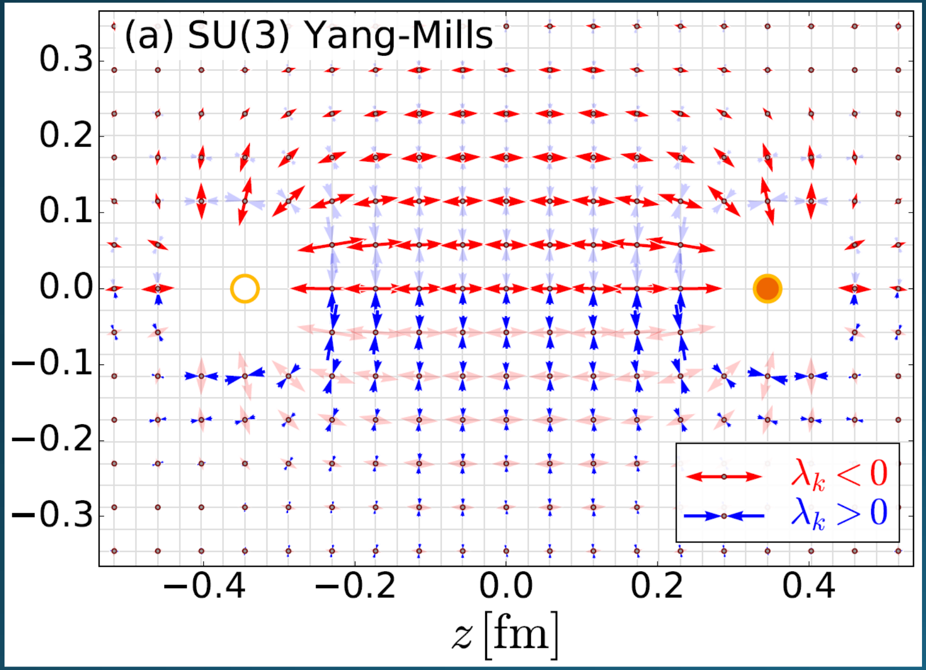


Definite physical meaning

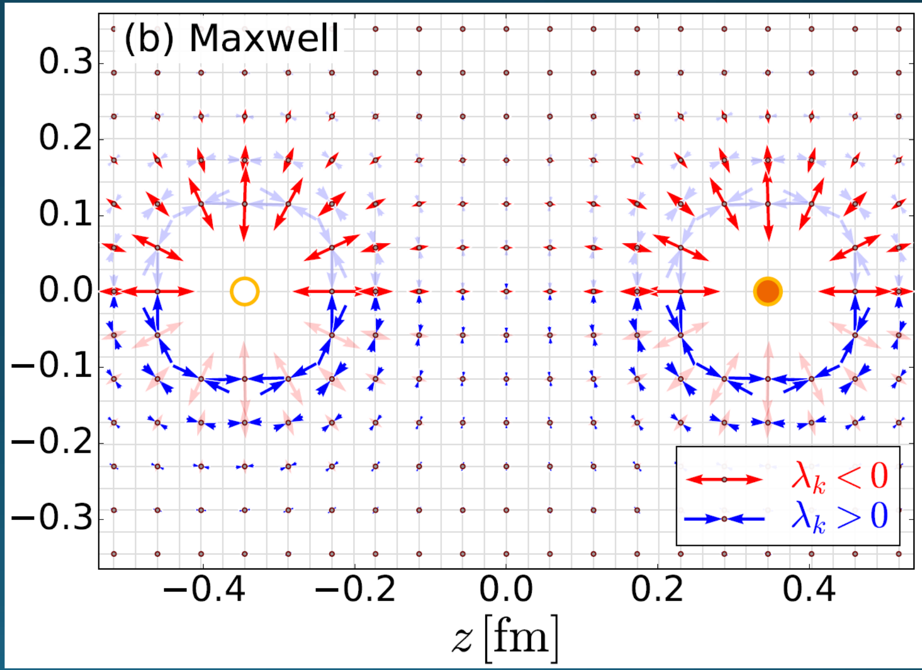
- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction
- ☐ Manifestly gauge invariant

SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)



Maxwell
(classical)



Propagation of the force is clearly different in YM and Maxwell theories!

Energy-Momentum Tensor

the most fundamental observable in physics

Einstein Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

Energy-Momentum Tensor

the most fundamental observable in physics

Einstein Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

All components are important physical observables!

energy momentum

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

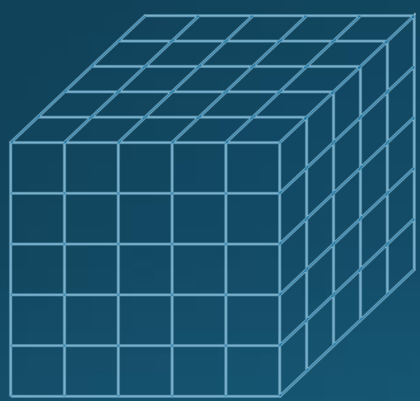
pressure

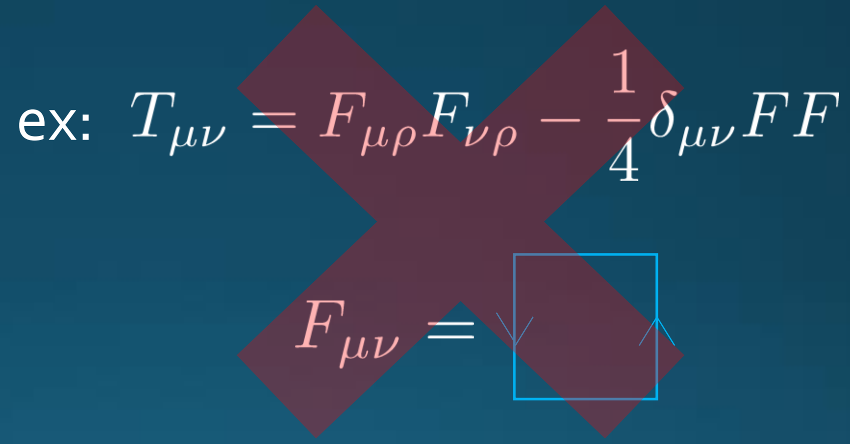
stress

$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial

because of the explicit breaking of Lorentz symmetry





② Its measurement is extremely noisy

due to high dimensionality and etc.

Energy-Momentum Tensor on the Lattice and Gradient Flow

energy momentum

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

pressure

stress

Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

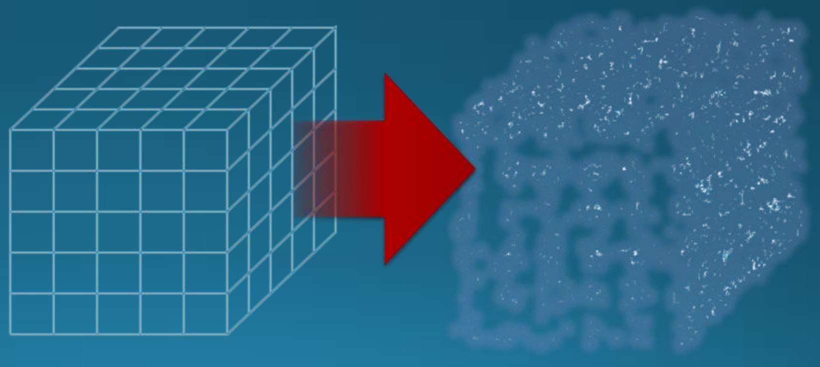
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length²]

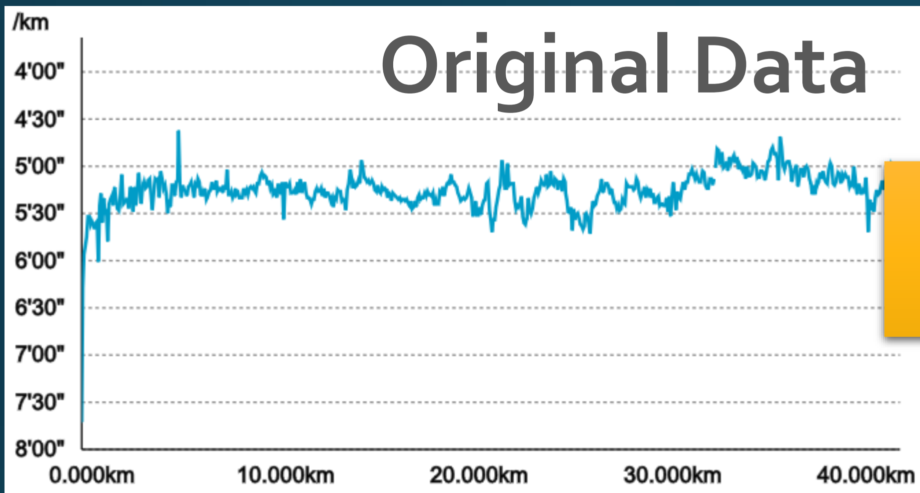
leading

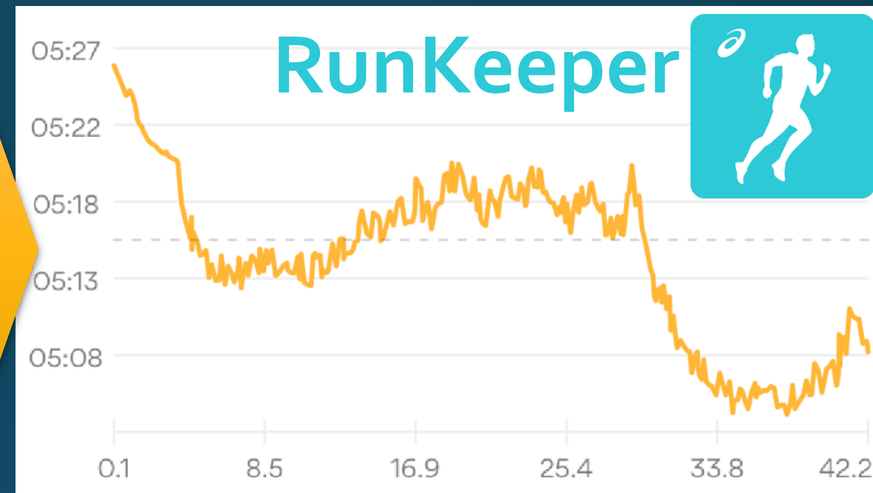
$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at t>0



Gradient Flow = Smearing





第39回篠山ABCマラソン
2019年3月3日(日)
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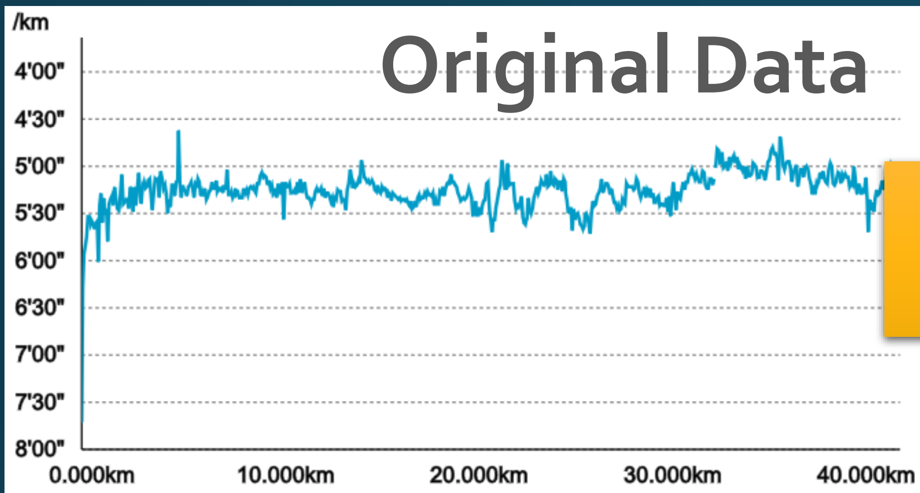
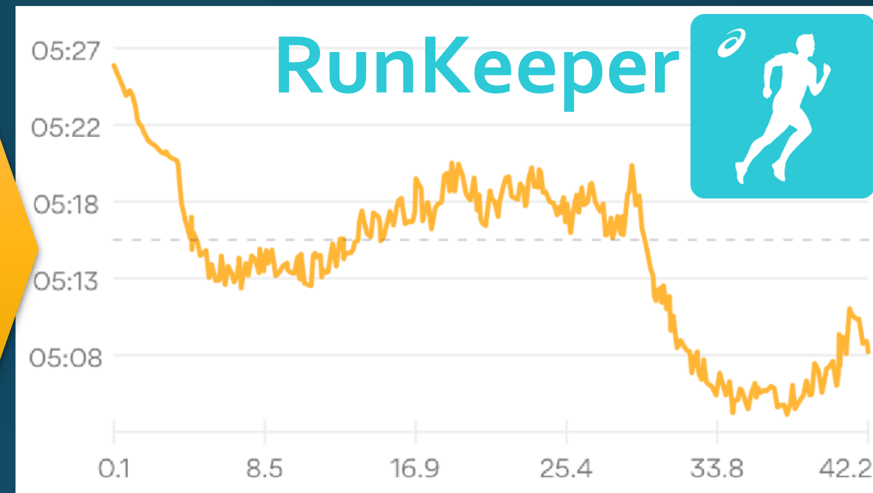
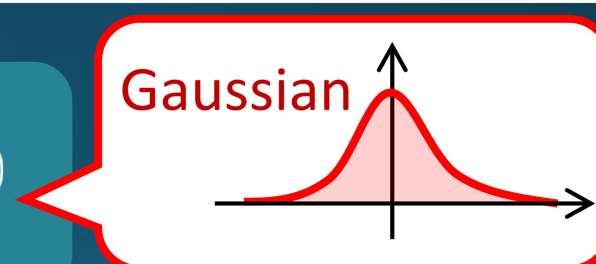
Gradient Flow = Smearing





① $x(t) \to x'(t) \sim \int dt' \exp\left[-\frac{(t-t')^2}{2\sigma^2}\right] x(t')$



$\sigma = \sqrt{2s}$

② $\frac{d}{ds} x(t;s) = \frac{d^2}{dt^2} x(t,s) \qquad x(t;0) = x(t)$

Gradient Flow

$\partial_t A_\mu = \partial_\nu \partial_\nu A_\mu + \cdots$

Two Advantages
of EMT Operator from Gradient Flow

$$\frac{\partial}{\partial t}A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

1. The operator is uniquely determined
2. Statistics is substantially improved

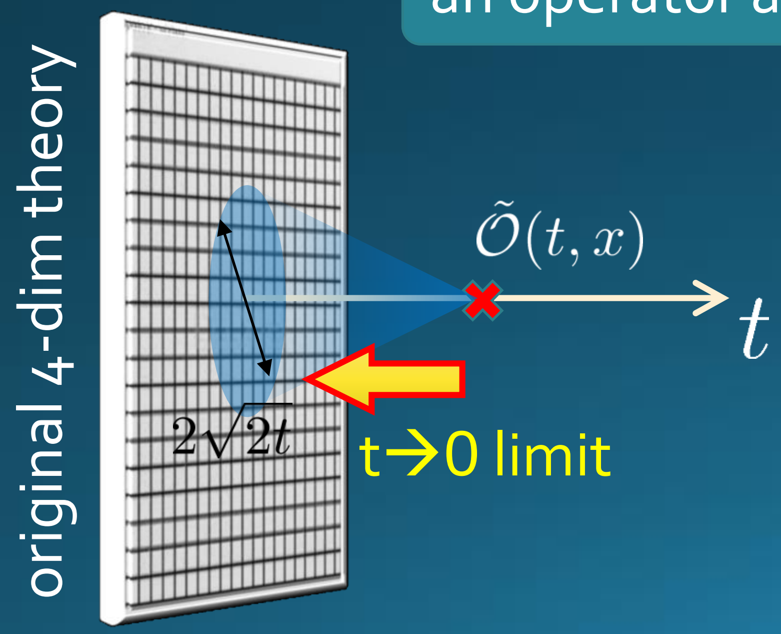
Small Flow-Time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at t>0

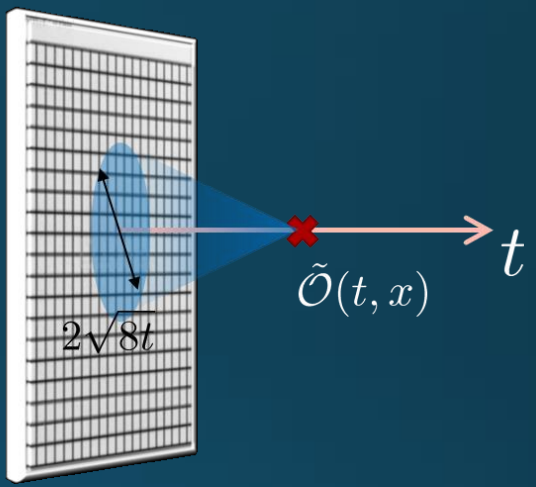
remormalized operators of original theory



Constructing EMT 1

Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t \to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$



- **Gauge-invariant dimension 4 operators**

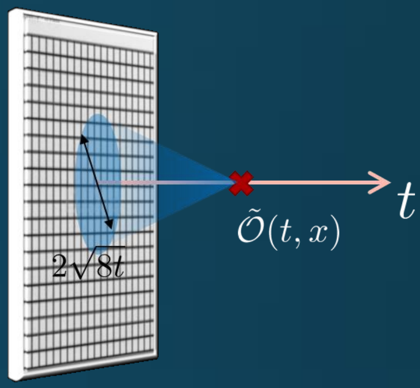
$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

Constructing EMT 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$

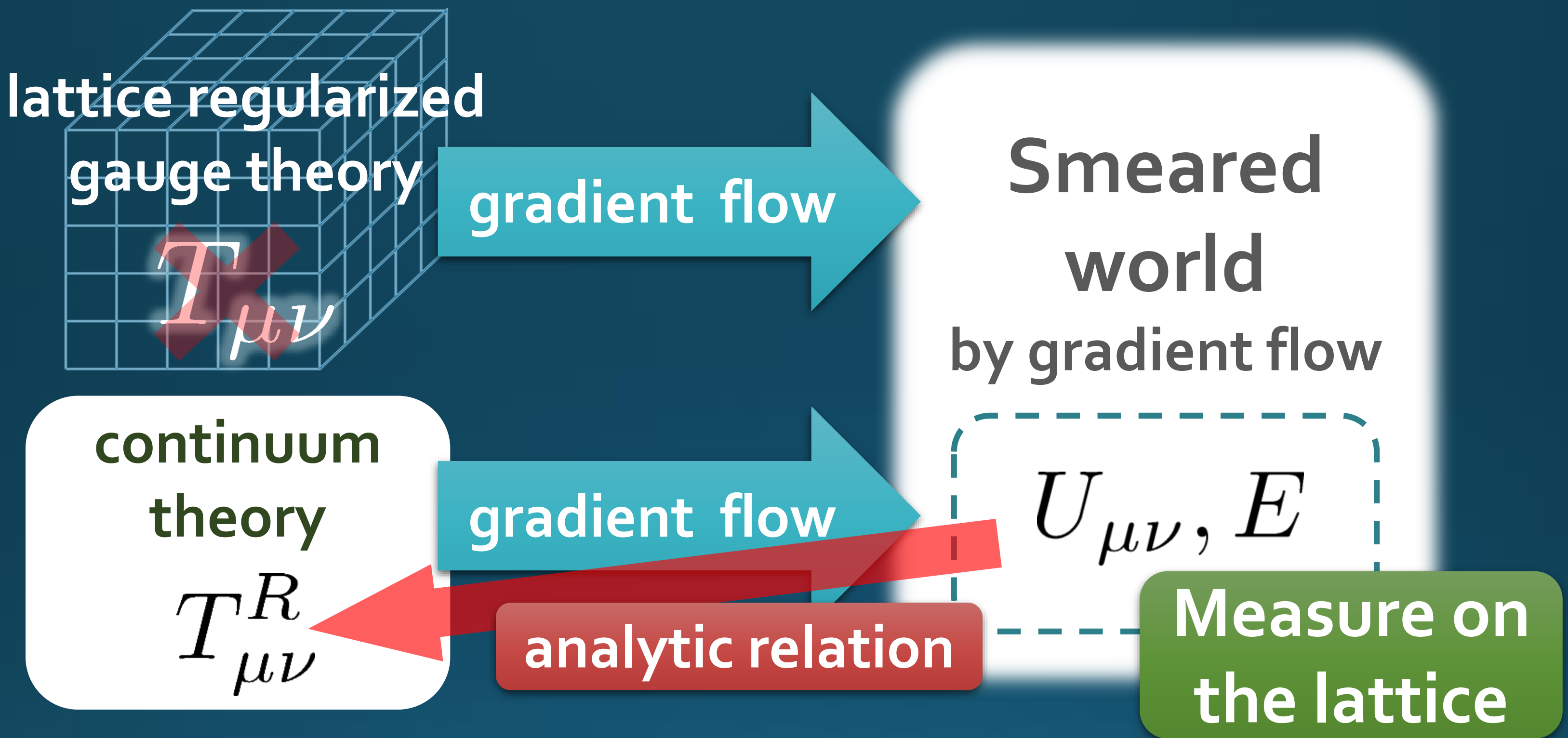


vacuum subtr.

Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\mathrm{subt.}}\right]$$

Gradient Flow Method



Take Extrapolation (t,a)→(0,0)

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}\frac{a^2}{t} + \cdots$$

O(t) terms in SFTE lattice discretization

Thermodynamics of SU(3) YM

- **Integral method**
  - Most conventional / established
  - Use themodynamic relations

    Boyd+ 1995; Borsanyi, 2012

$$p = \frac{T}{V} \ln Z$$

$$T \frac{\partial(p/T^4)}{\partial T} = \frac{\varepsilon - 3p}{T^4}$$

- Gradient-flow method
  - Take expectation values of EMT

    FlowQCD, 2014, 2016

$$\begin{cases} \varepsilon = \langle T_{00} \rangle \\ p = \langle T_{11} \rangle \end{cases}$$

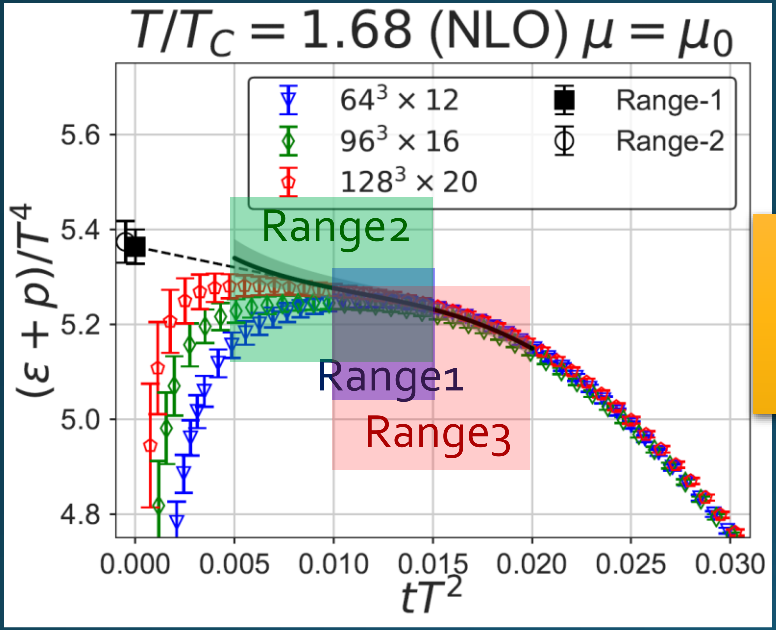
- Moving-frame method

  Giusti, Pepe, 2014~
- Non-equilibrium method
  - Use Jarzynski's equality Caselle+, 2016;2018
- Differential method

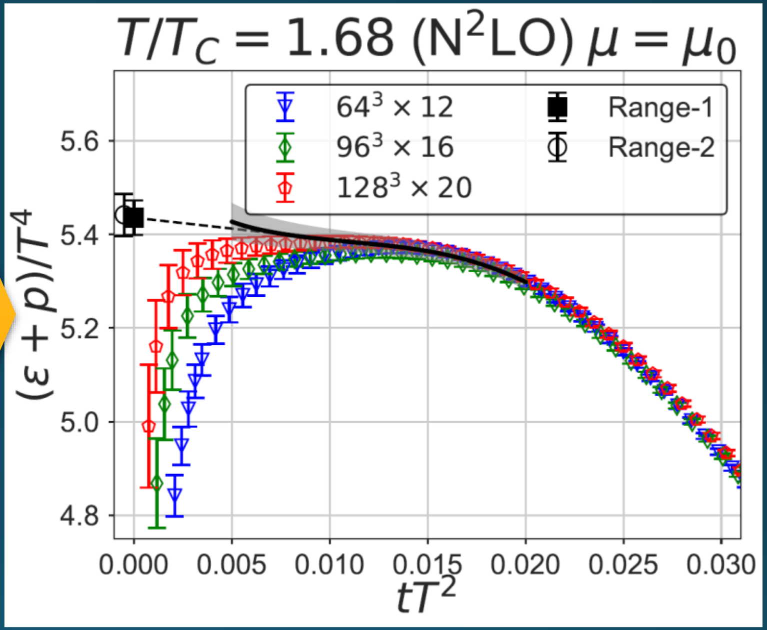
  Shirogane+(WHOT-QCD), 2016~

Higher Order Coefficient: ε+p

NLO (1-loop)



N²LO (2-loop)



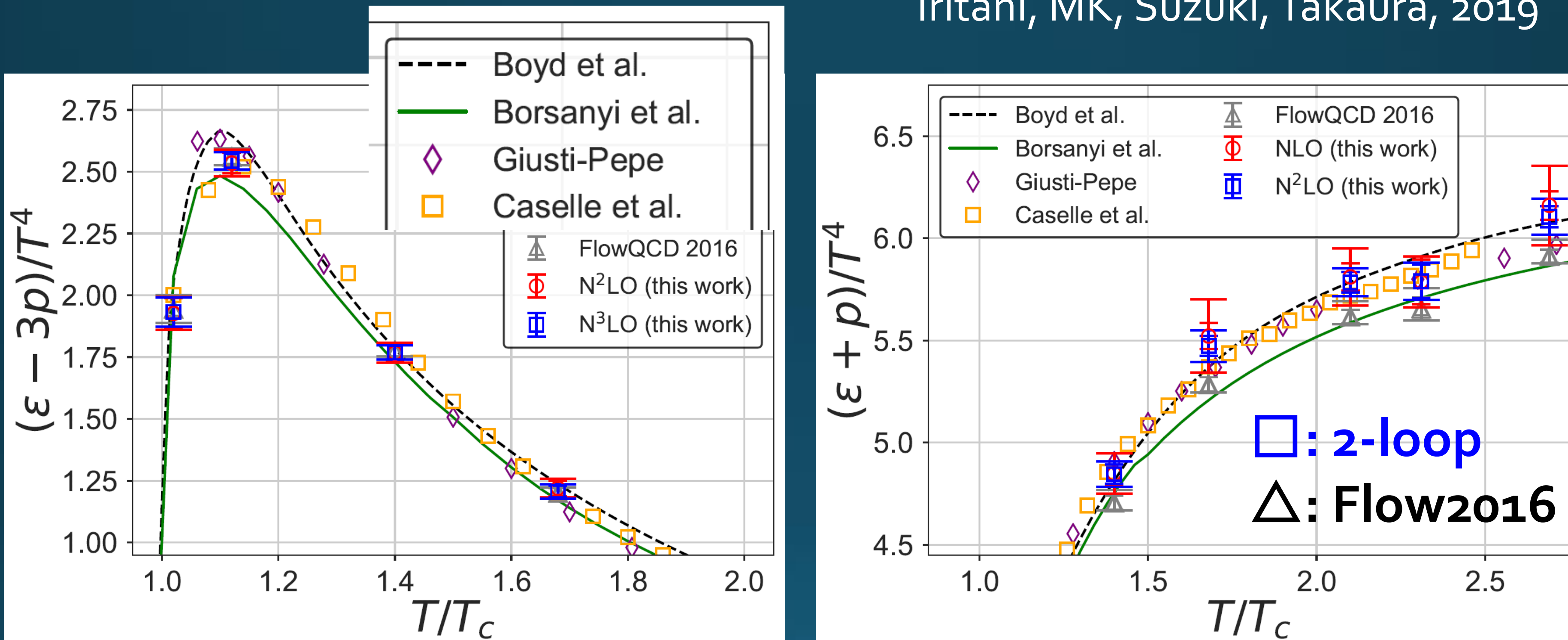
Iritani, MK, Suzuki, Takaura, 2019

- t dependence becomes milder with higher order coeff.
- 1-loop → 2-loop : about 2% increase

- Systematic analysis: μ_0 or μ_d, uncertainty of Λ, fit range
- Extrapolation func: linear, higher order term in c_1 ($\sim g^6$)

Effect of Higher-Order Coeffs.

Iritani, MK, Suzuki, Takaura, 2019



Systematic error: μ_0 or μ_d, Λ, t→0 function, fit range

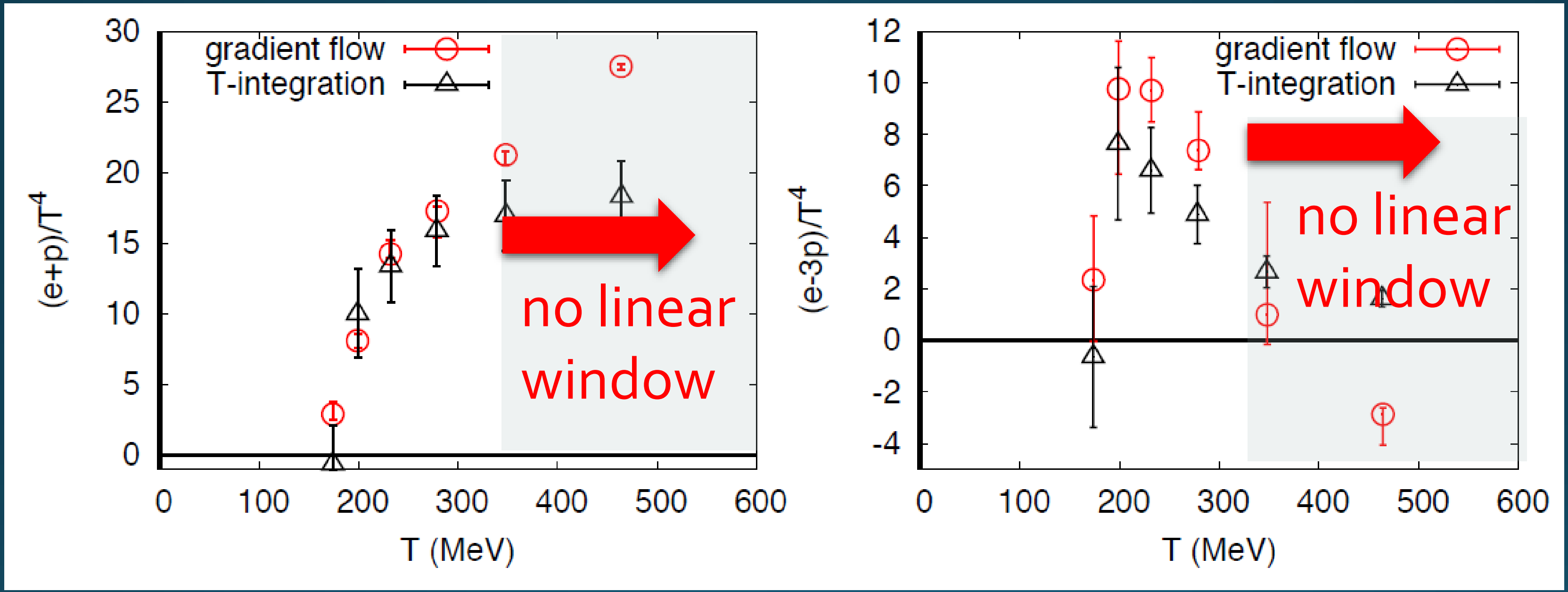
Effect of higher order c_1 & c_2 (pure gauge)

- ☐ e-3p: negligible (<0.5%)
- ☐ e+p: ~2% increase

2+1 QCD EoS from Gradient Flow

Taniguchi+ (WHOT-QCD), PR**D96**, 014509 (2017)

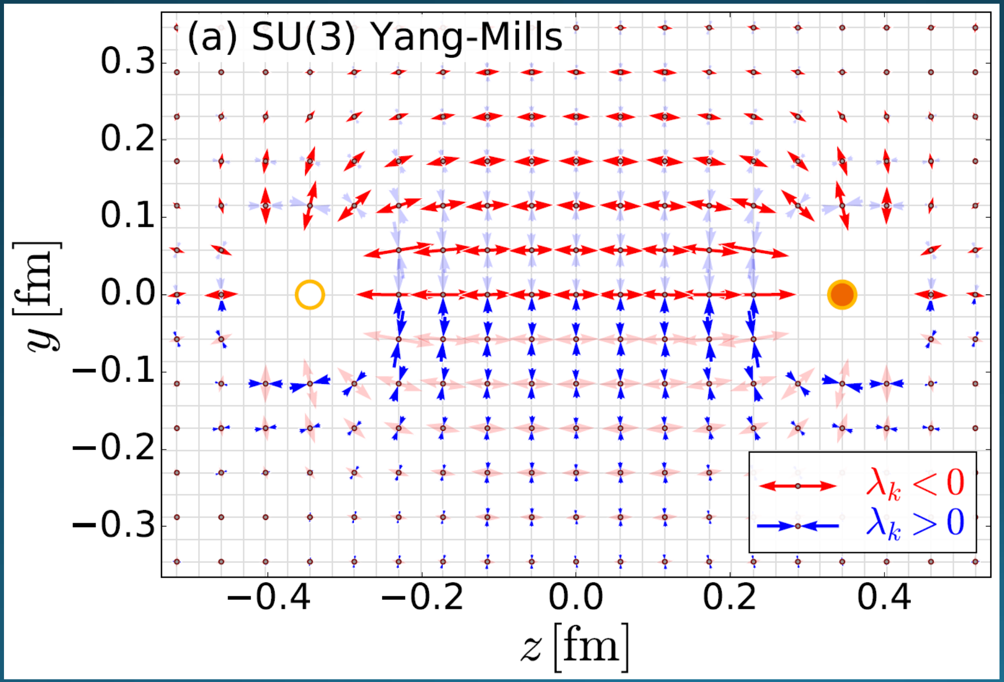
$m_{PS}/m_V \approx 0.63$



- Agreement with integral method except for N_t=4, 6
- N_t=4, 6: No stable extrapolation is possible
- Statistical error is substantially suppressed!

Physical mass: Kanaya+ (WHOT-QCD), 1710.10015

Stress Tensor Distribution
around Flux Tube



Lattice Setup

- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

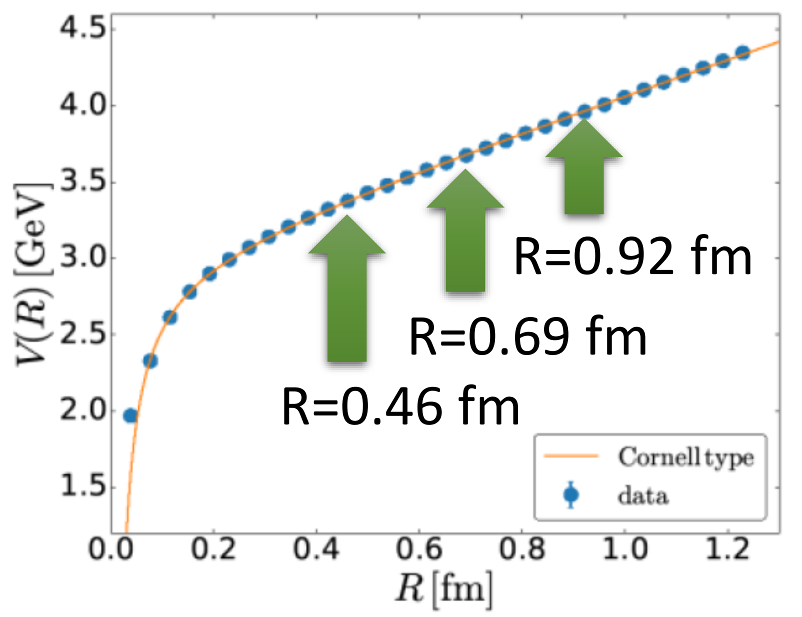
- APE smearing / multi-hit

- fine lattices (a=0.029-0.06 fm)
- continuum extrapolation

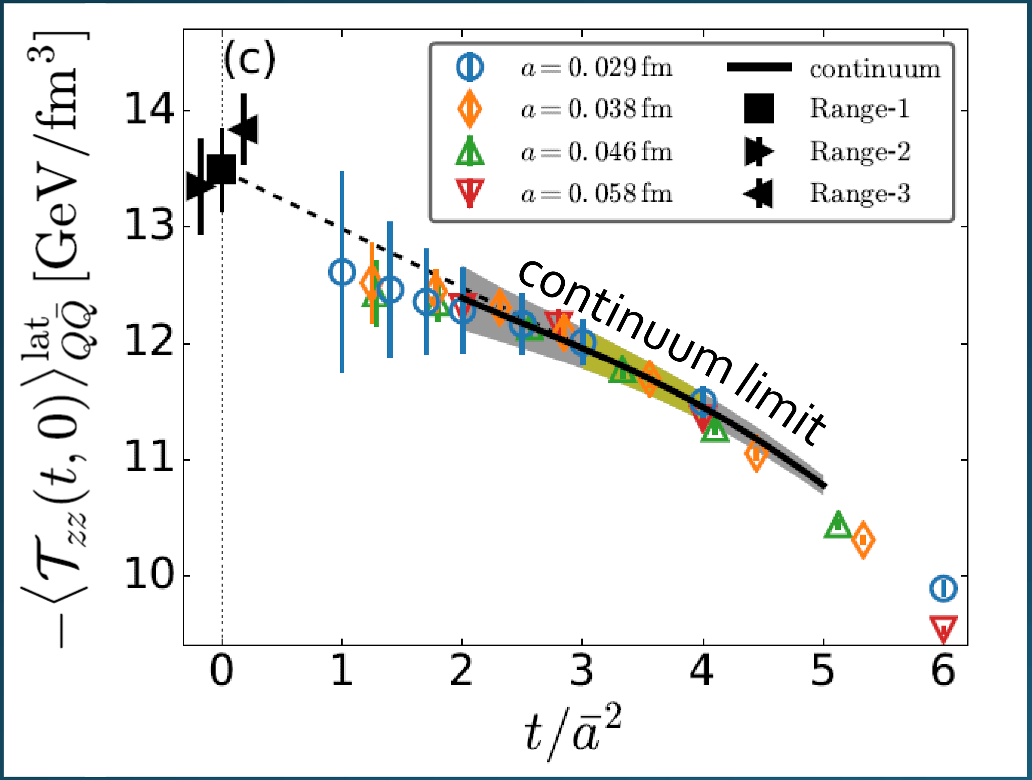
- Simulation: bluegene/Q@KEK

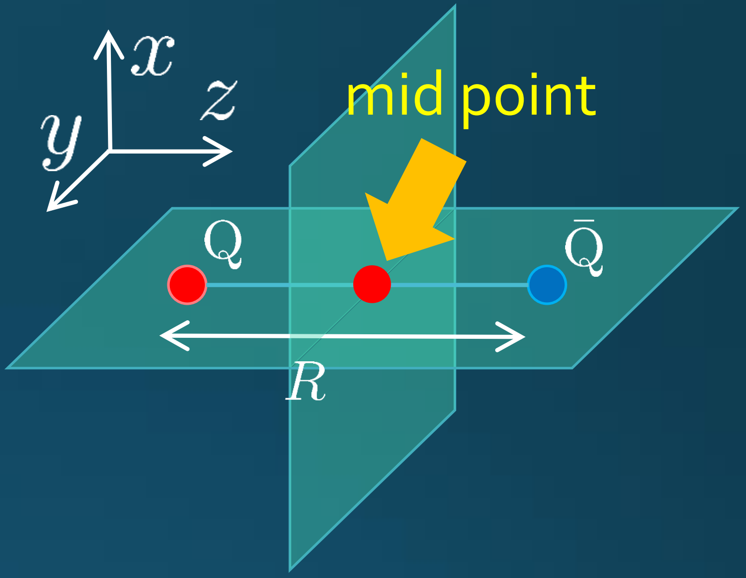
Yanagihara+, 1803.05656

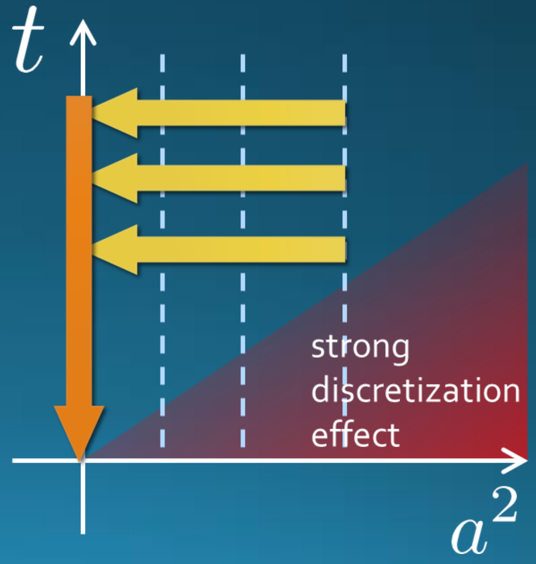
β	a [fm]	N_{size}^4	N_{conf}	R/a		
6.304	0.058	48^4	140	8	12	16
6.465	0.046	48^4	440	10	–	20
6.513	0.043	48^4	600	–	16	–
6.600	0.038	48^4	1,500	12	18	24
6.819	0.029	64^4	1,000	16	24	32
			R [fm]	0.46	0.69	0.92



Continuum Extrapolation
at mid-point

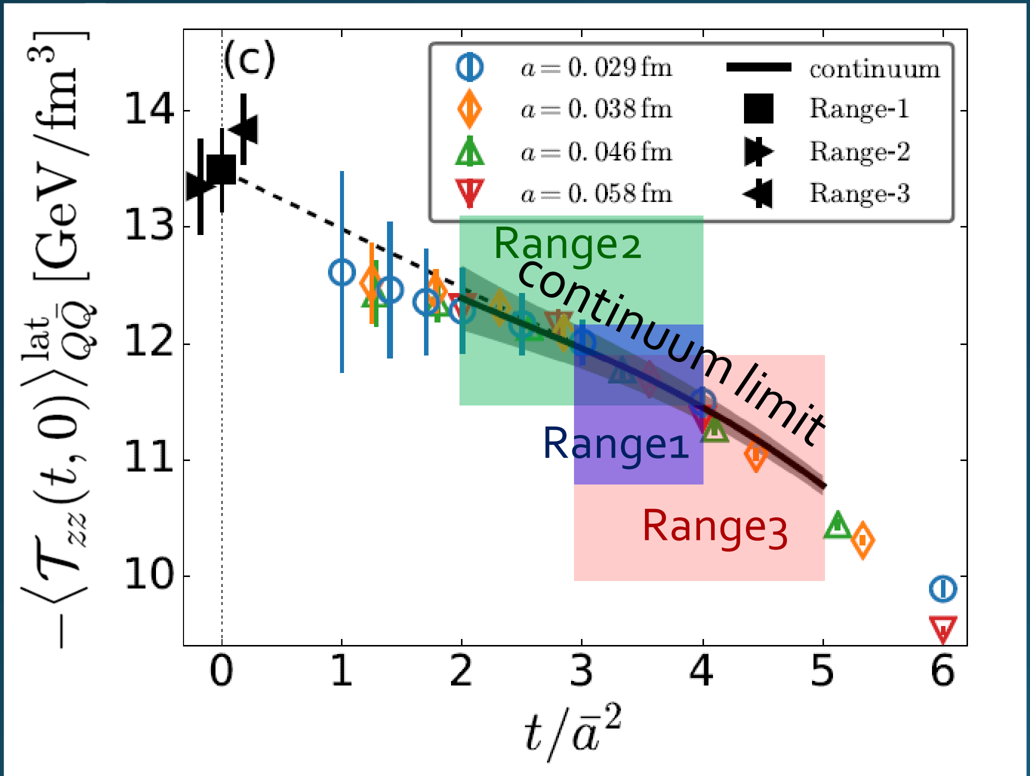


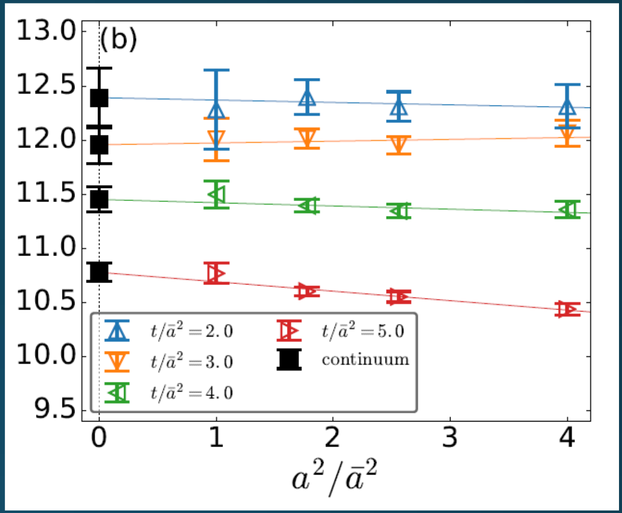


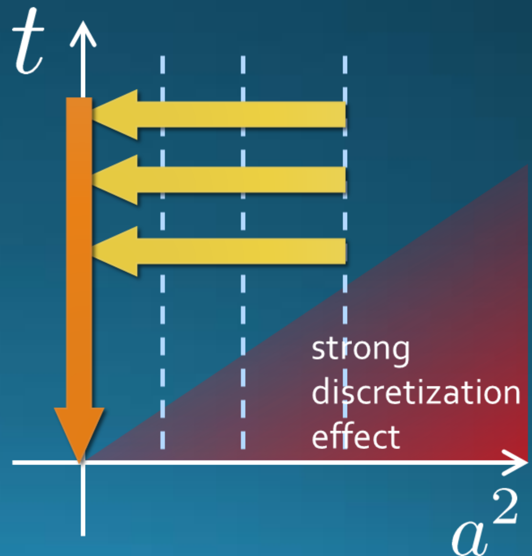


- ☐ a→0 extrapolation with fixed t

t→0 Extrapolation
at mid-point







- a→0 extrapolation with fixed t
- Then, t→0 with three ranges

Stress Distribution on Mid-Plane

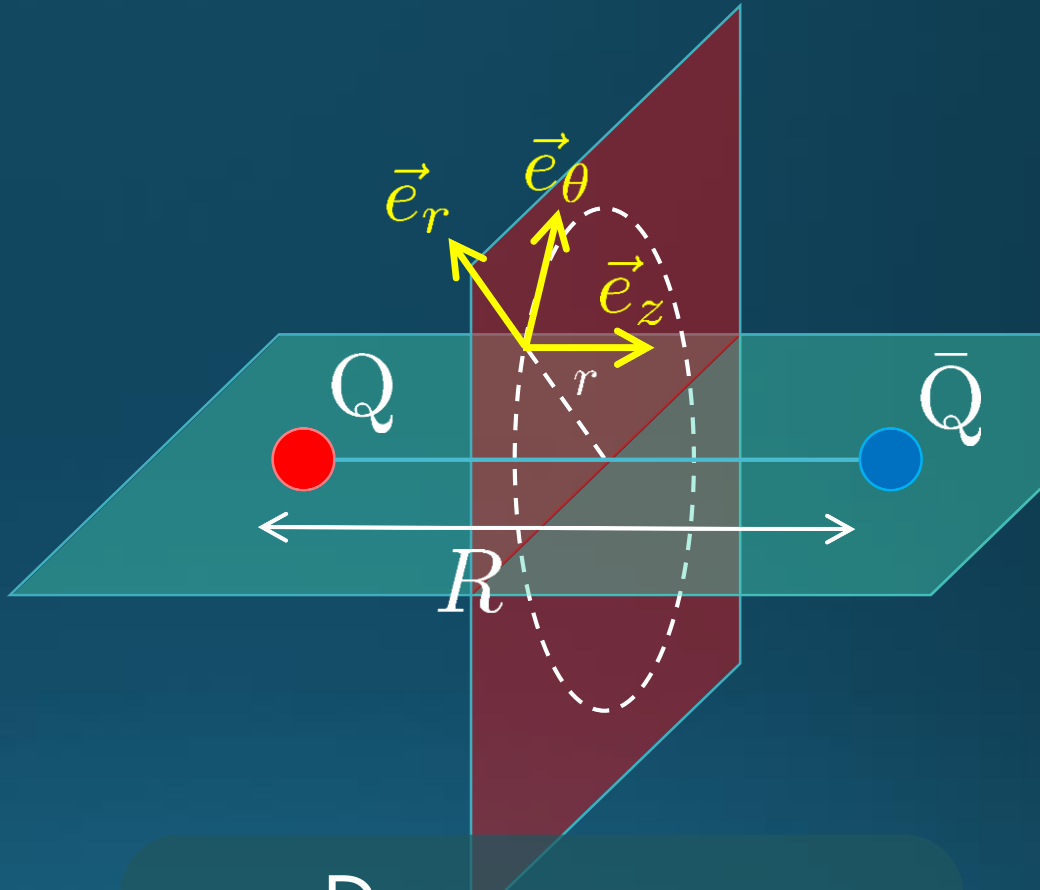
From rotational symm. & parity

EMT is diagonalized
in Cylindrical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{zz} & \\ & & & T_{44} \end{pmatrix}$$

$$T_{rr} = \vec{e}_r^{\,T} T \vec{e}_r$$

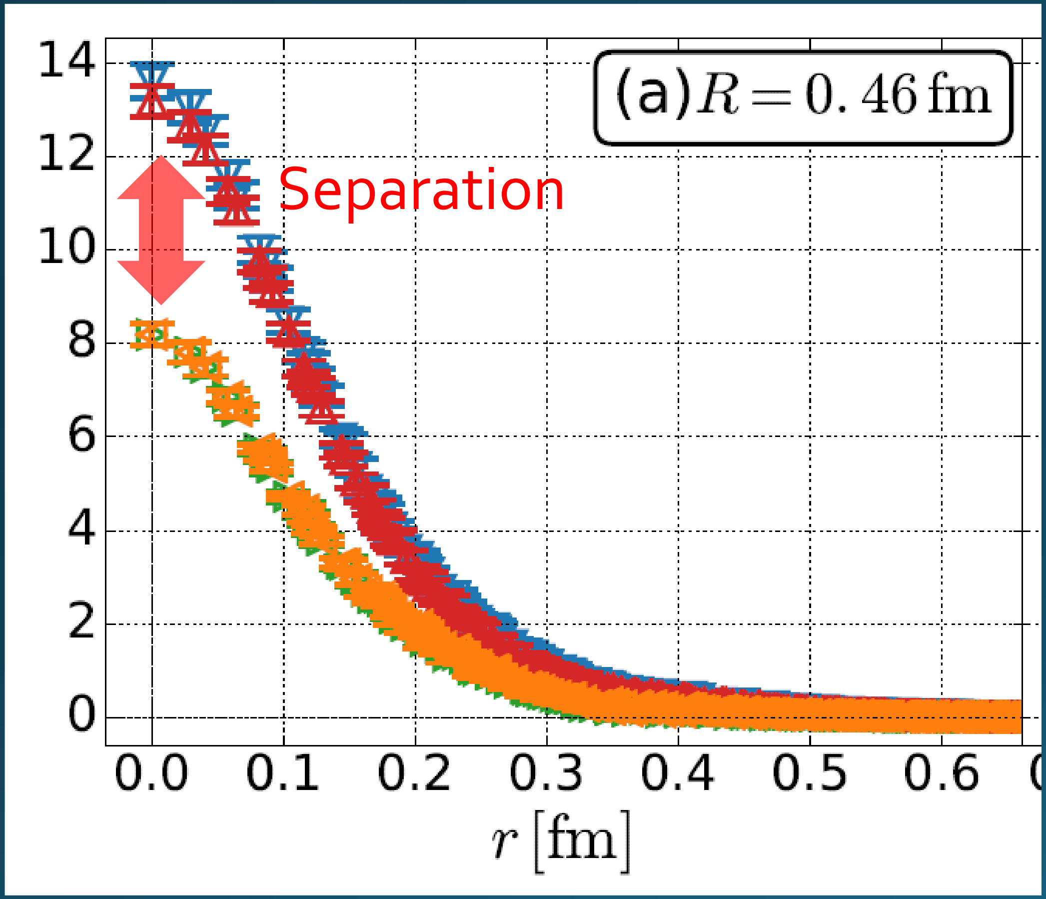
$$T_{\theta\theta} = \vec{e}_\theta^{\,T} T \vec{e}_\theta$$

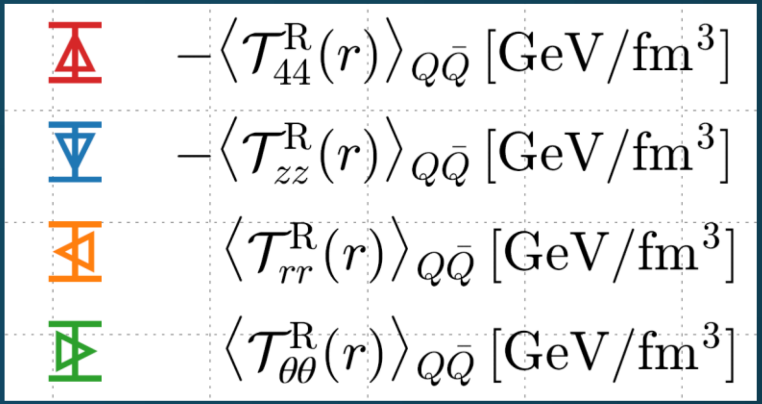


Degeneracy
in Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

Mid-Plane





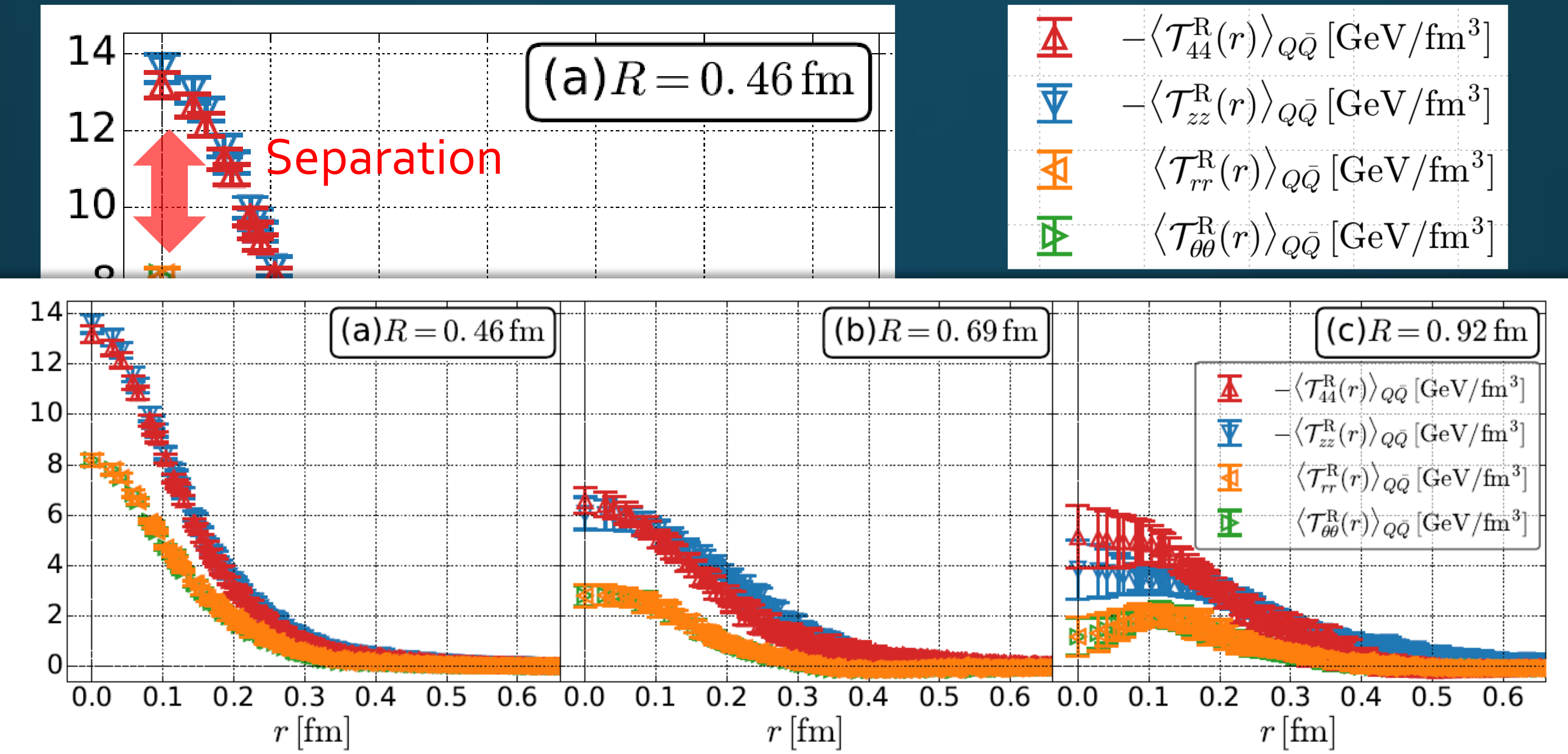
Continuum Extrapolated!

In Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

- ☐ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- ☐ Separation: $T_{zz} \neq T_{rr}$
- ☐ Nonzero trace anomaly $\sum T_{cc} \neq 0$

Mid-Plane



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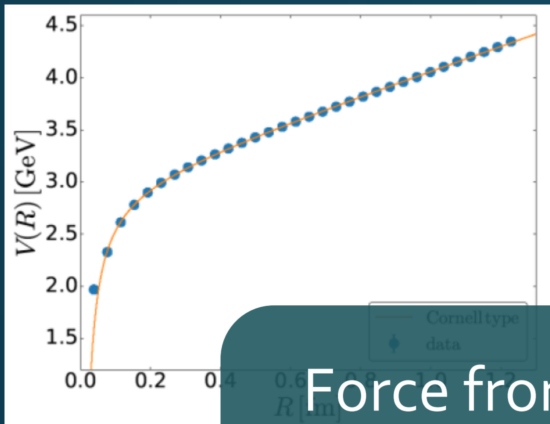
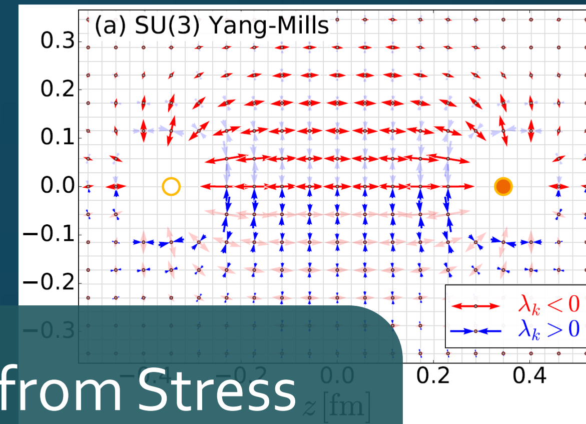
Force





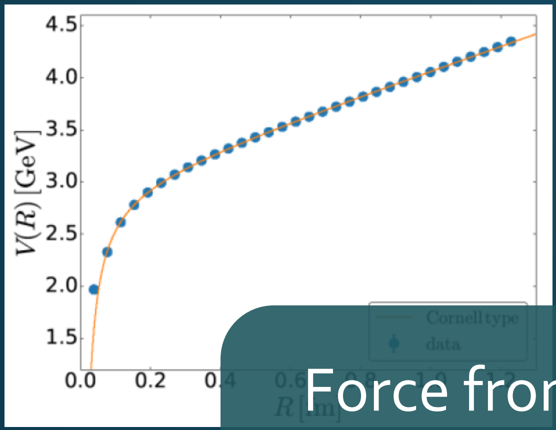
Force from Potential

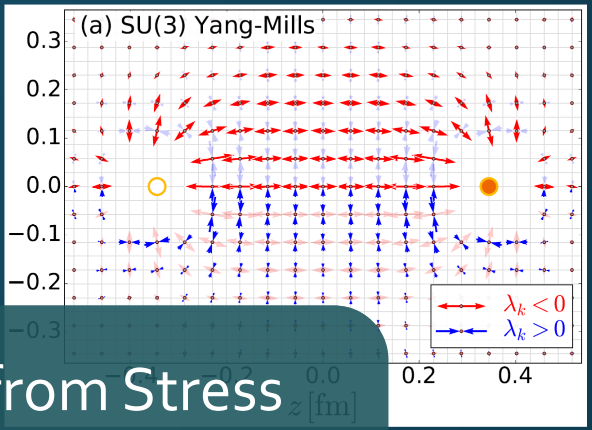
$$F_{\mathrm{pot}} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\mathrm{stress}} = \int_{\mathrm{mid.}} d^2x T_{zz}(x)$$

Force





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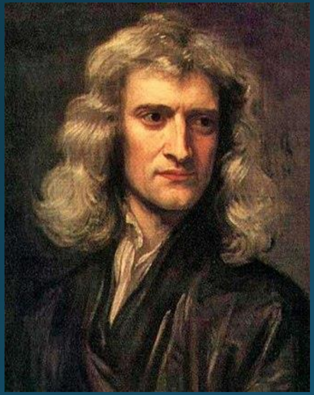
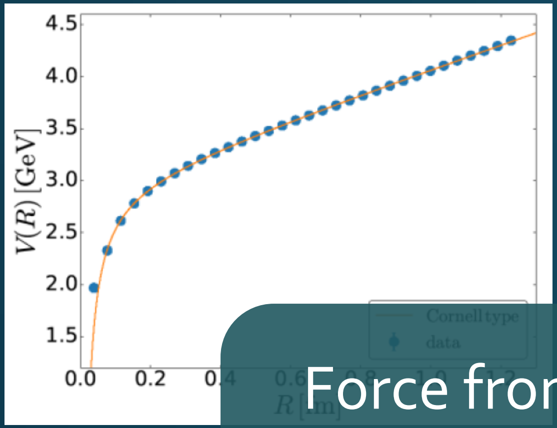
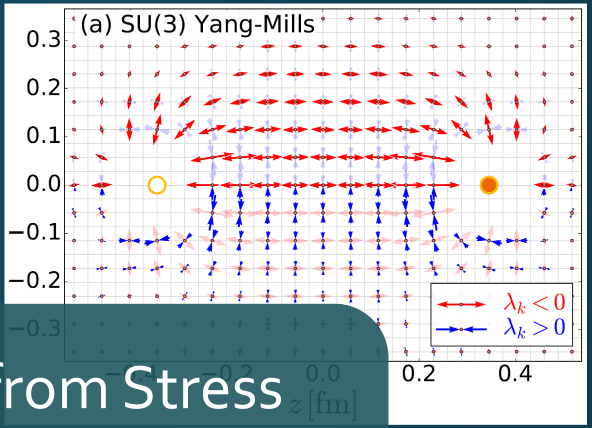


Newton
1687

Faraday
1839

Force





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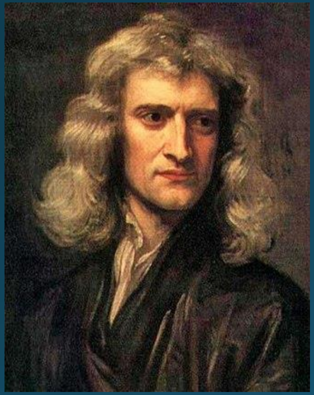
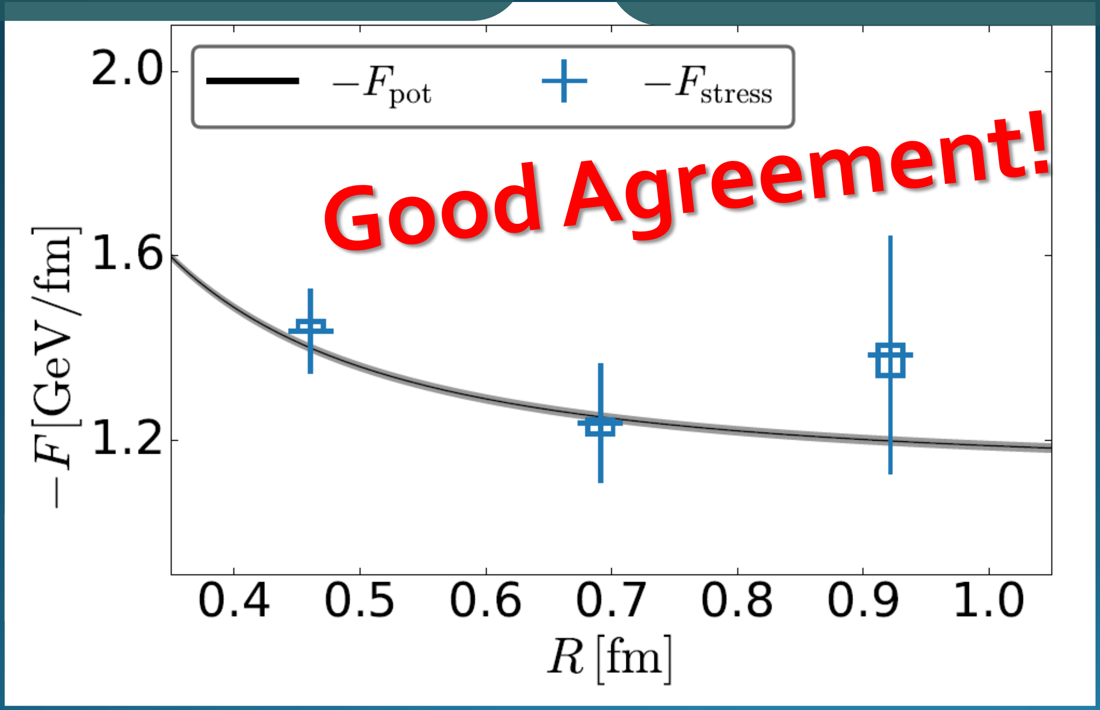


Newton
1687



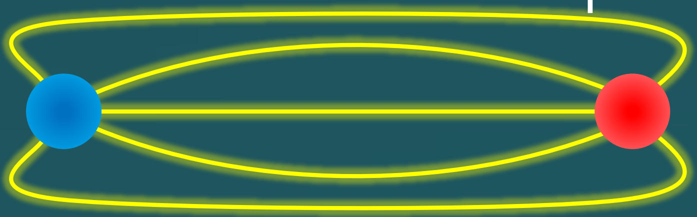
Faraday
1839

Dual Superconductor Picture

Nambu, 1970
Nielsen, Olesen, 1973
t 'Hooft, 1981
…

QCD Vacuum

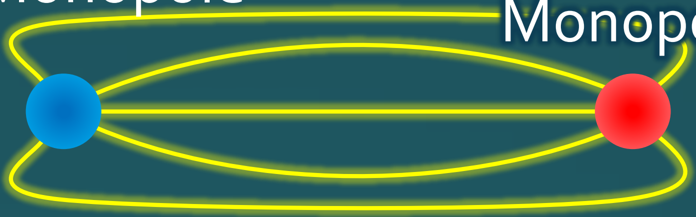
Quark
Anti-quark



Flux Tube

Superconductor

Monopole
Anti-Monopole



Magnetic Vortex

Dual (E⇔B)

Abelian-Higgs Model

Yanagihara, Iritani, MK, in prep.

Abelian-Higgs Model

$$\mathcal{L}_{\rm AH} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

GL parameter: $\kappa = \sqrt{\lambda}/g$

- type-I : $\kappa < 1/\sqrt{2}$
- type-II : $\kappa > 1/\sqrt{2}$
- Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

Infinitely long tube

- degeneracy

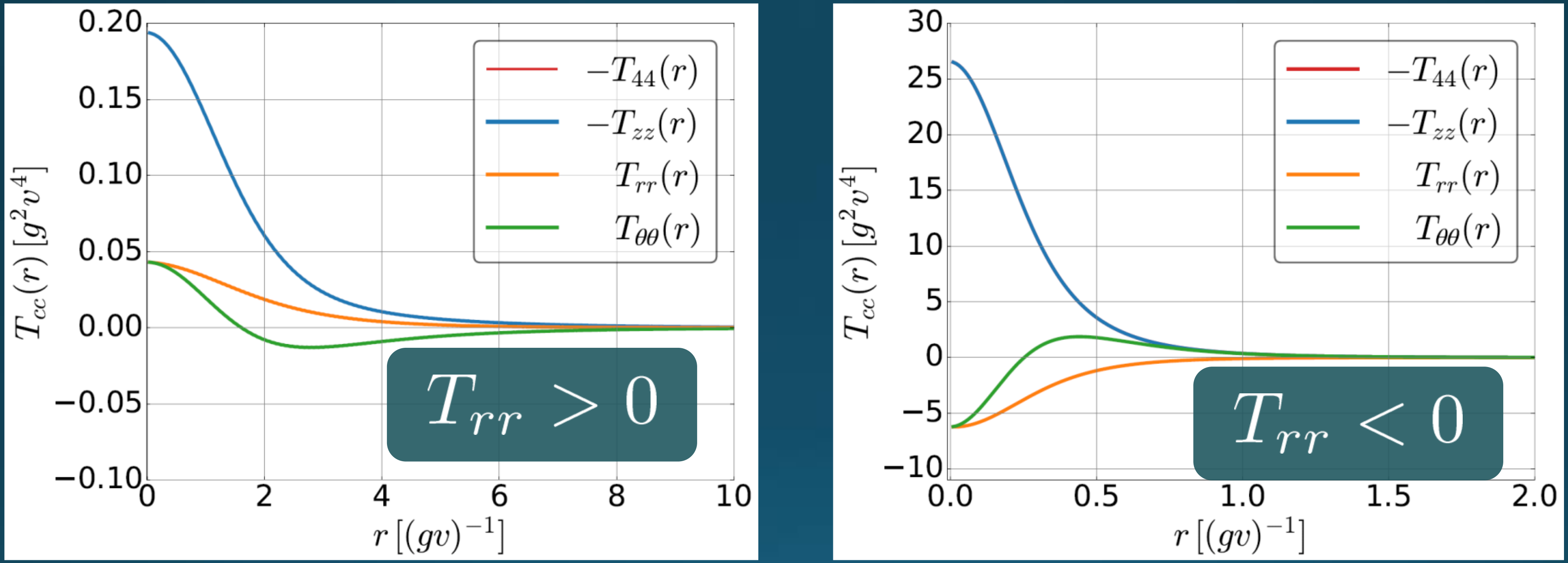
  $T_{zz}(r) = T_{44}(r)$ Luscher, 1981
- momentum conservation

  $$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

Stress Tensor in AH Model

infinitely-long flux tube

Type-I $\kappa = 0.1$ **Type-II** $\kappa = 3.0$



- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

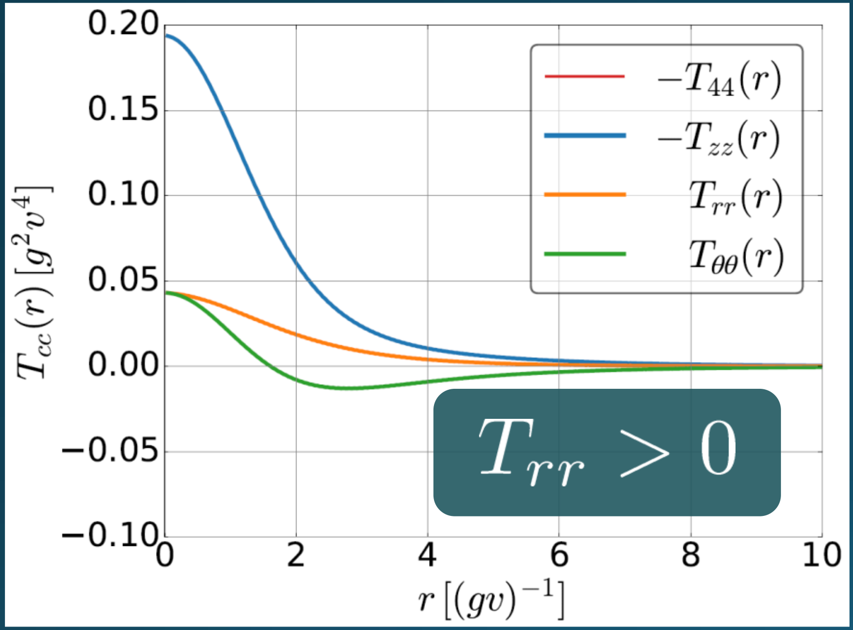
conservation law

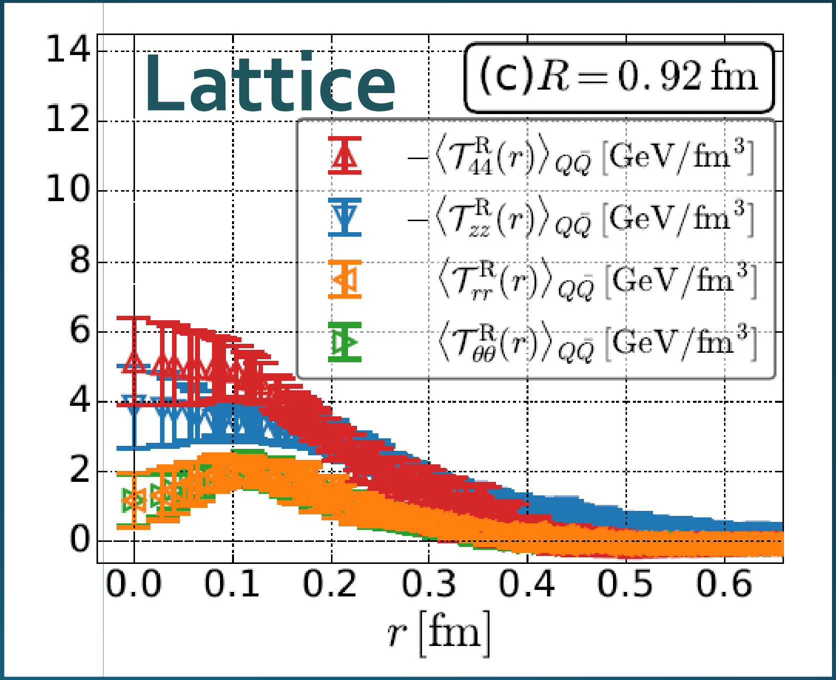
$$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

Stress Tensor in AH Model
infinitely-long flux tube

Type-I

$\kappa = 0.1$





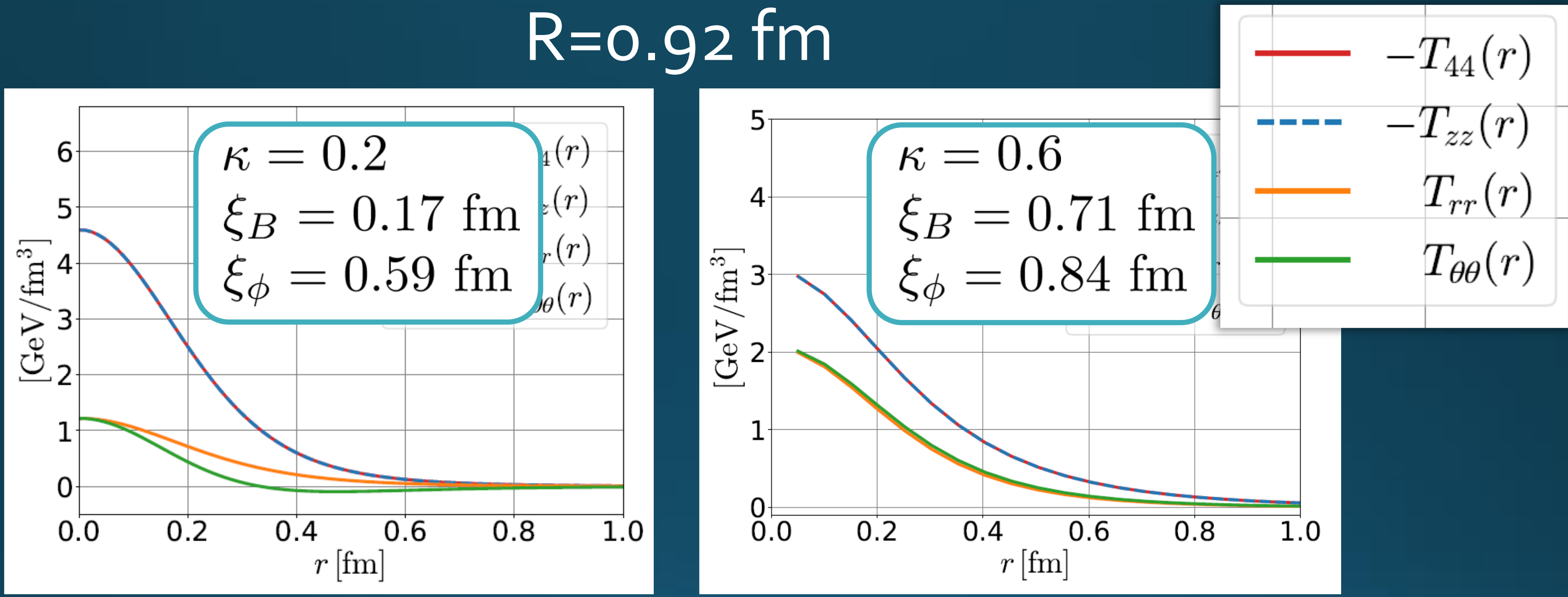
- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

Inconsistent with lattice result

$$T_{rr} \simeq T_{\theta\theta}$$

Flux Tube with Finite Length

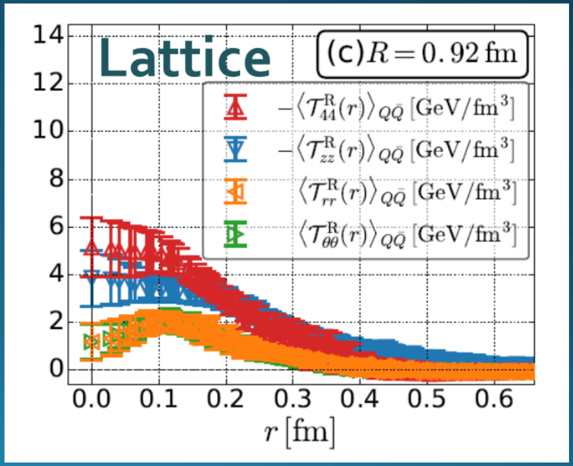
R=0.92 fm



Left: $T_{zz}(0)$, $T_{rr}(0)$ reproduce lattice result

Right: A parameter satisfying $T_{rr} \approx T_{\theta\theta}$

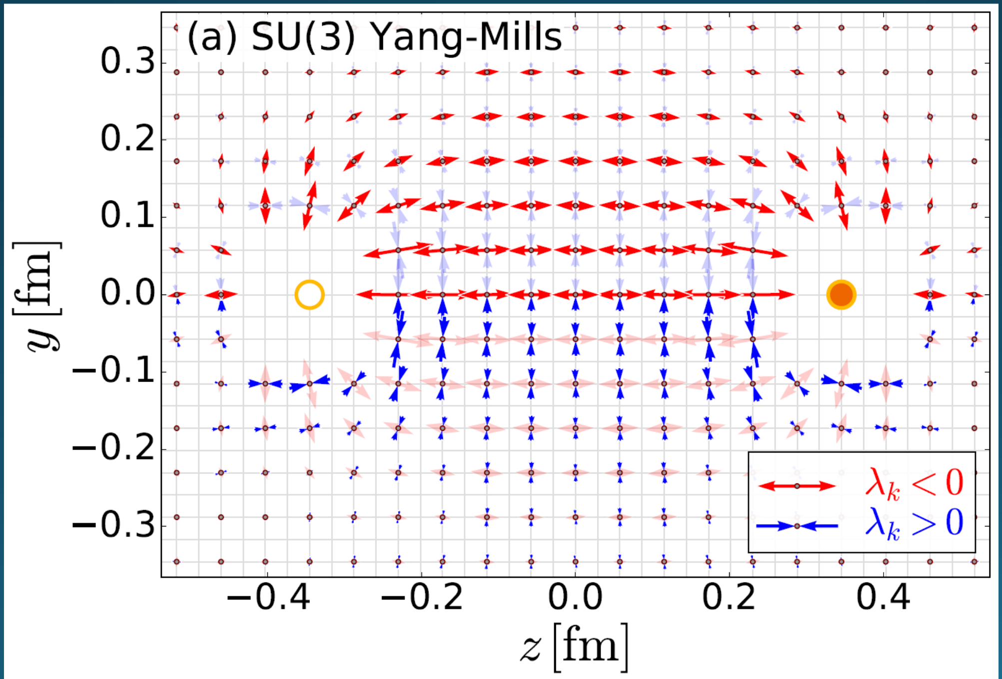
No parameter can reproduce lattice data at R=0.92fm.



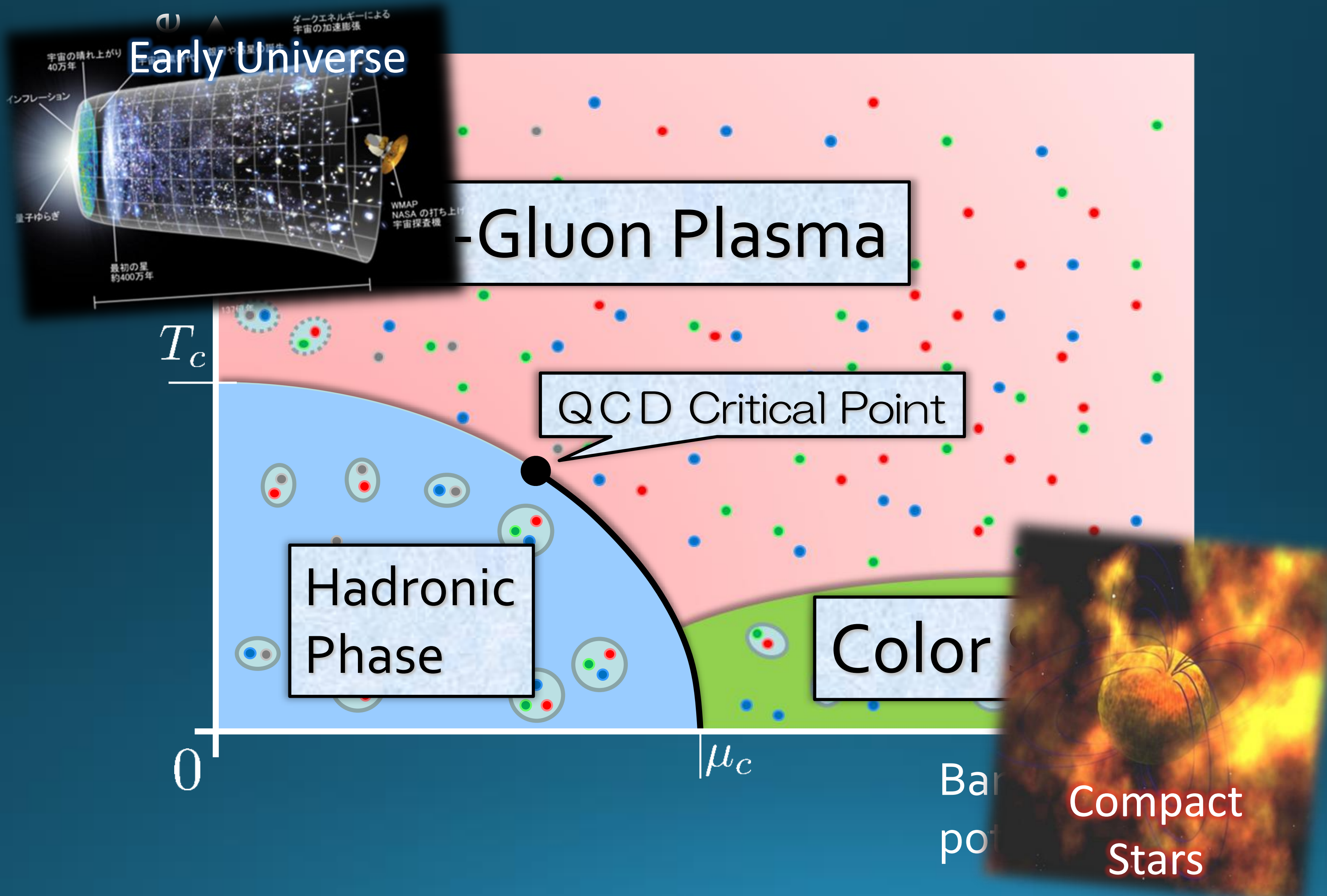
本年度の成果

Quark Anti-Quark System

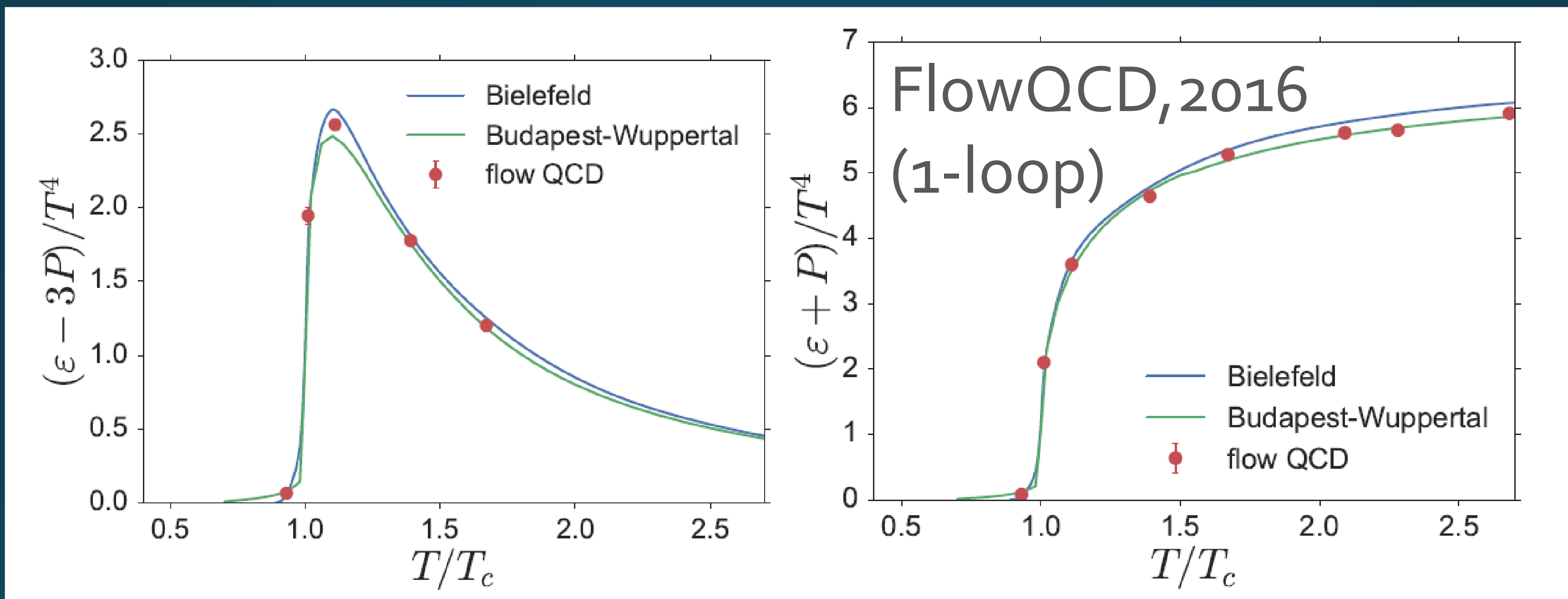
in the Early Universe



QCD Phase Diagram

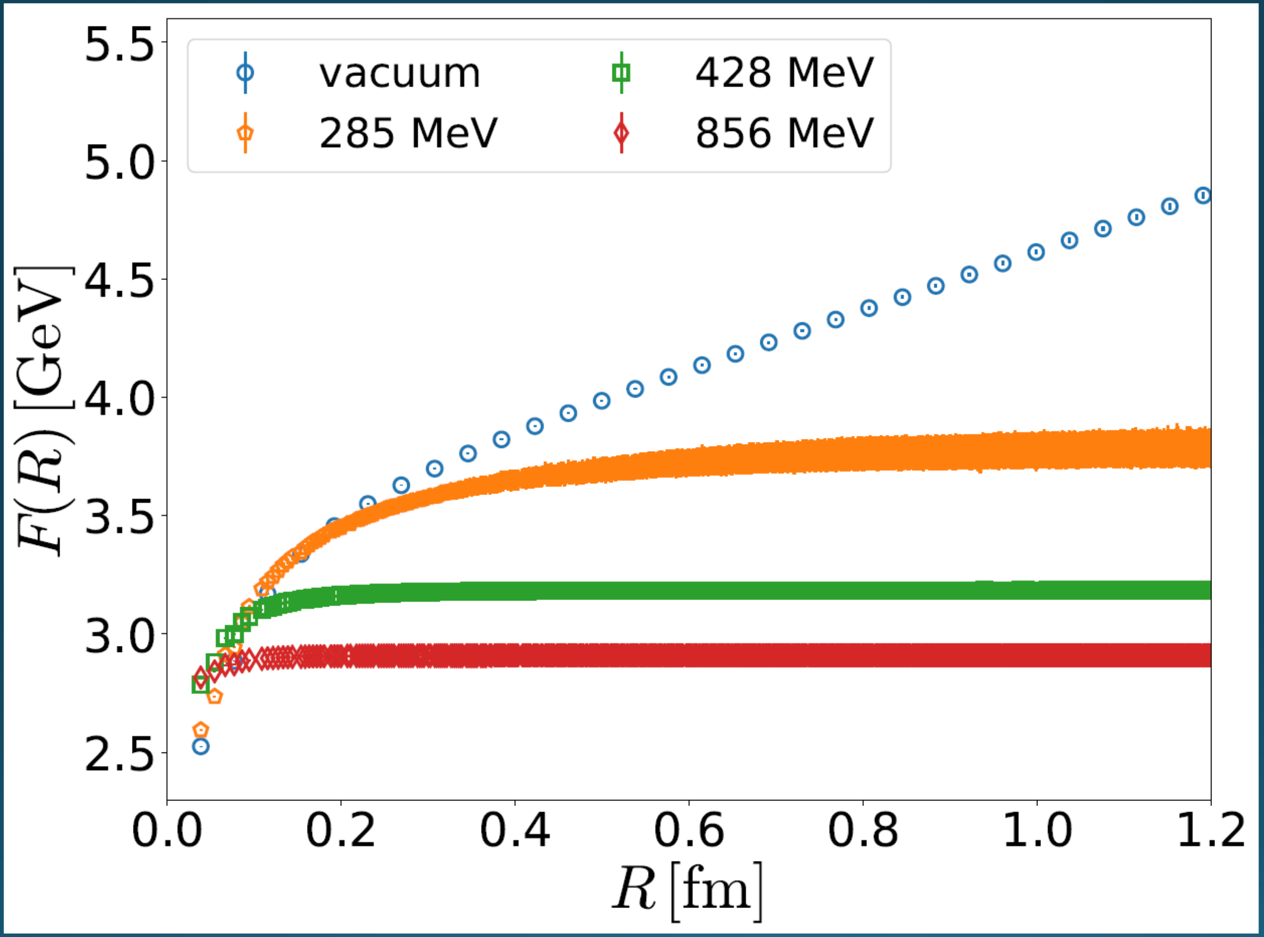


SU(3) YM Thermodynamics



- 1st order phase transition at Tc≈3x10[12] Kelvin
- Sudden increase of energy density and pressure at Tc

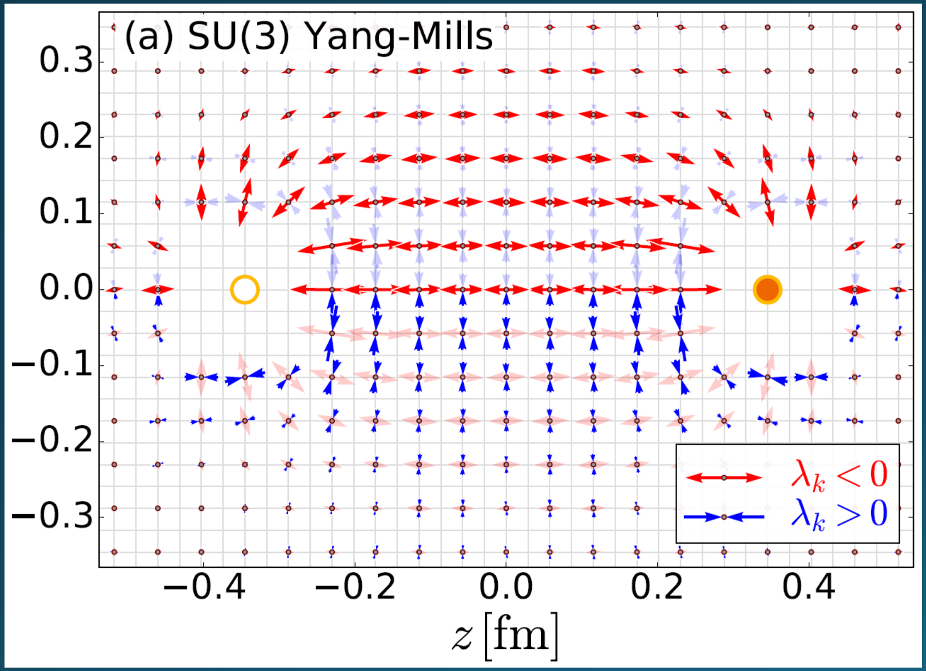
Quark Anti-Quark Potential



Q-Qbar force is screened in the deconfined phase.

Temperature Dependence

Vacuum
(Current Universe)

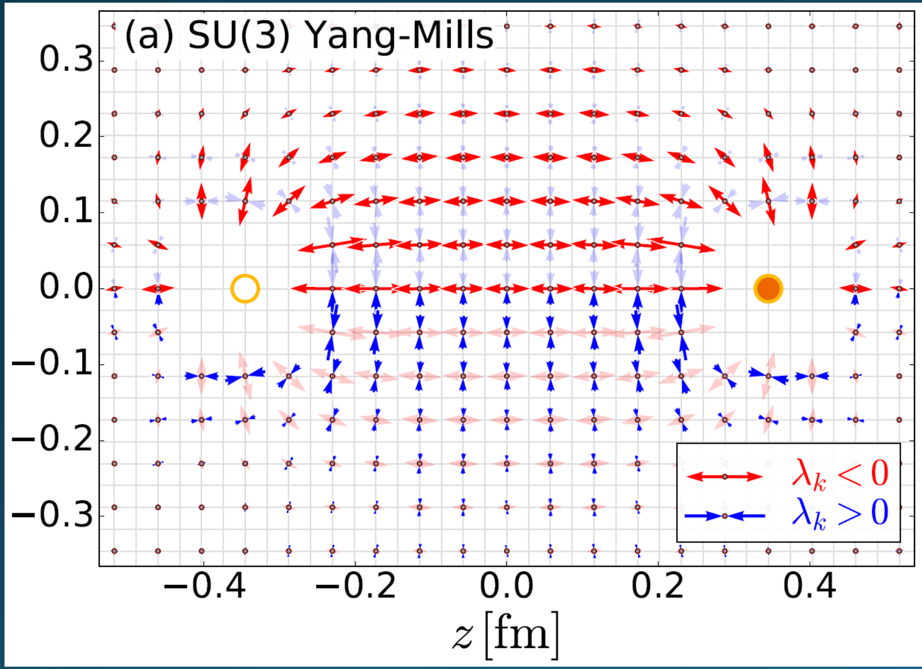


High Temperature
(Early Universe)

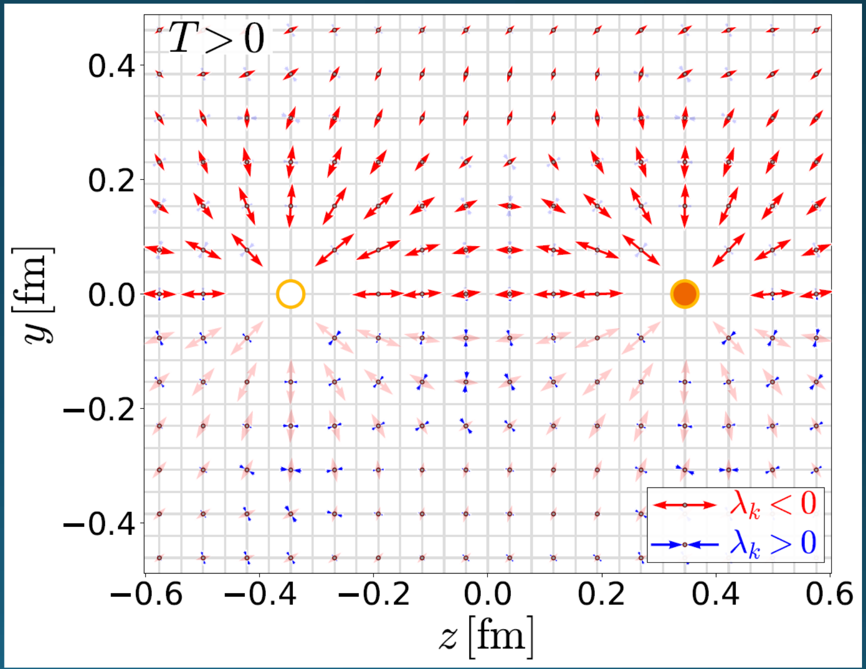
?

Temperature Dependence

Vacuum
(Current Universe)



High Temperature
(Early Universe)



T=1.42Tc

Flux-tube structure disappears above Tc.

Stress Distribution on Mid-Plane

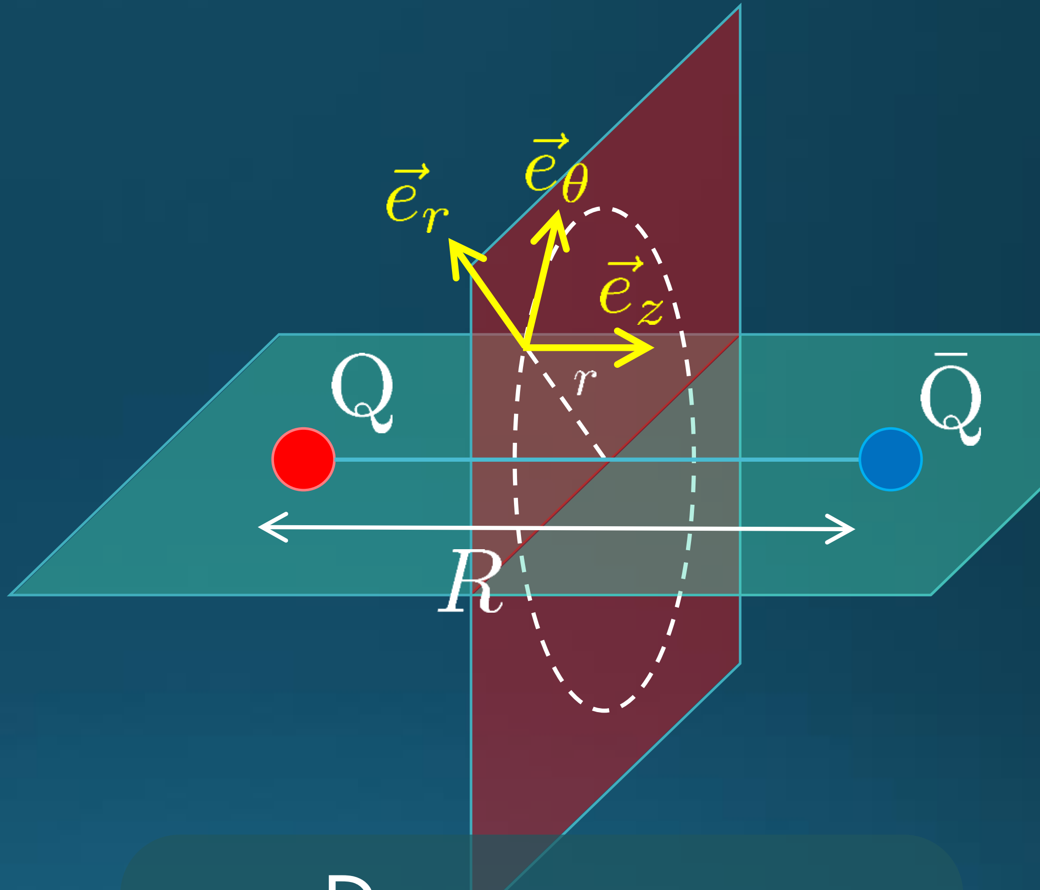
From rotational symm. & parity

EMT is diagonalized
in Cylindrical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{zz} & \\ & & & T_{44} \end{pmatrix}$$

$$T_{rr} = \vec{e}_r^{\,T} T \vec{e}_r$$

$$T_{\theta\theta} = \vec{e}_\theta^{\,T} T \vec{e}_\theta$$

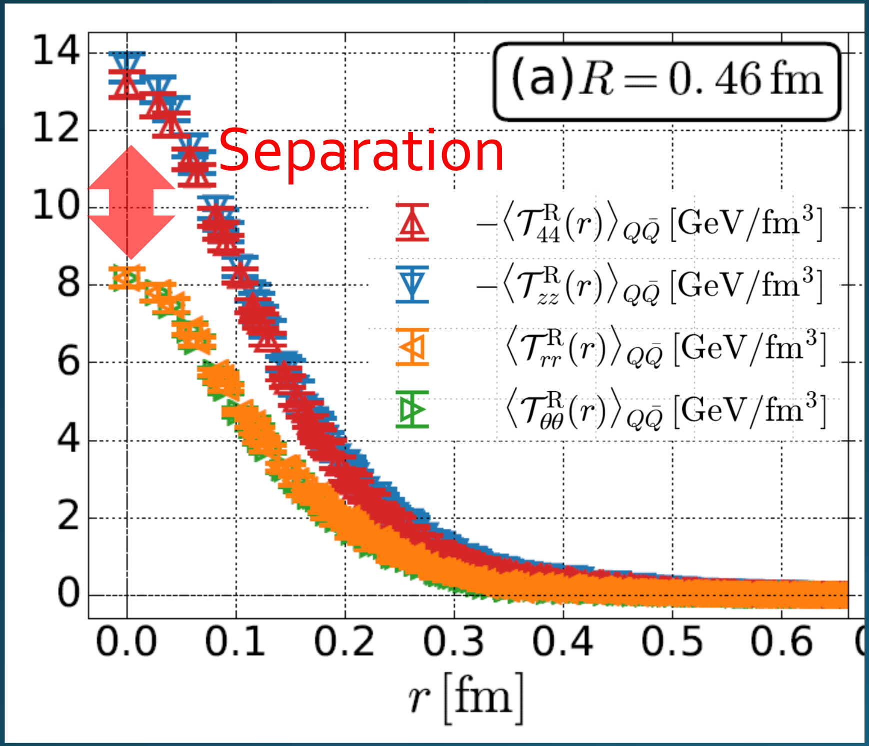


Degeneracy
in Maxwell theory

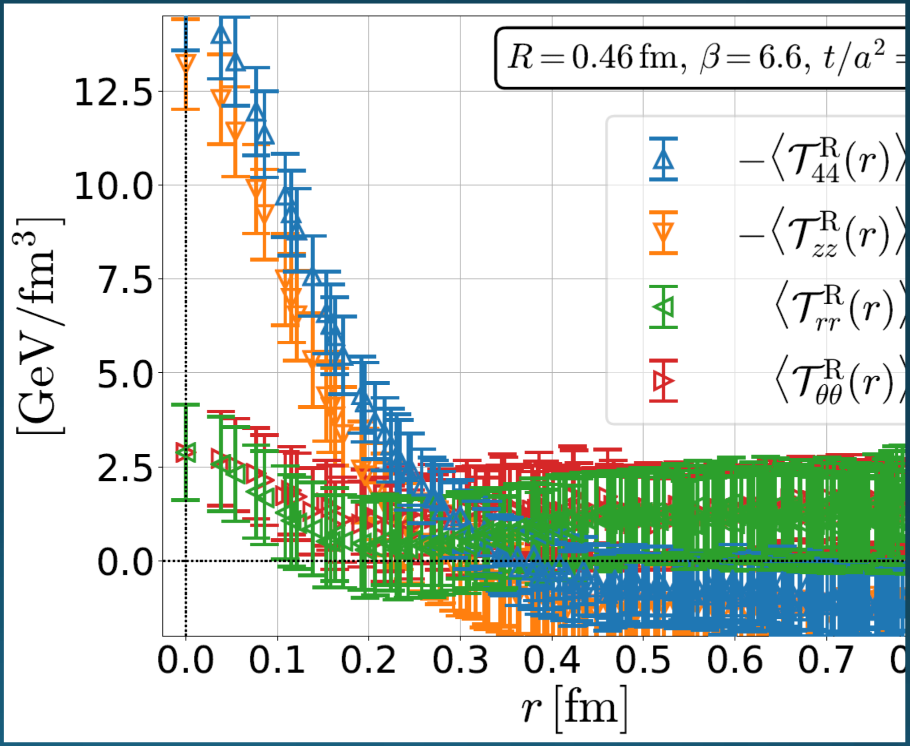
$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

Mid-Plane

Vacuum



T=1.42Tc

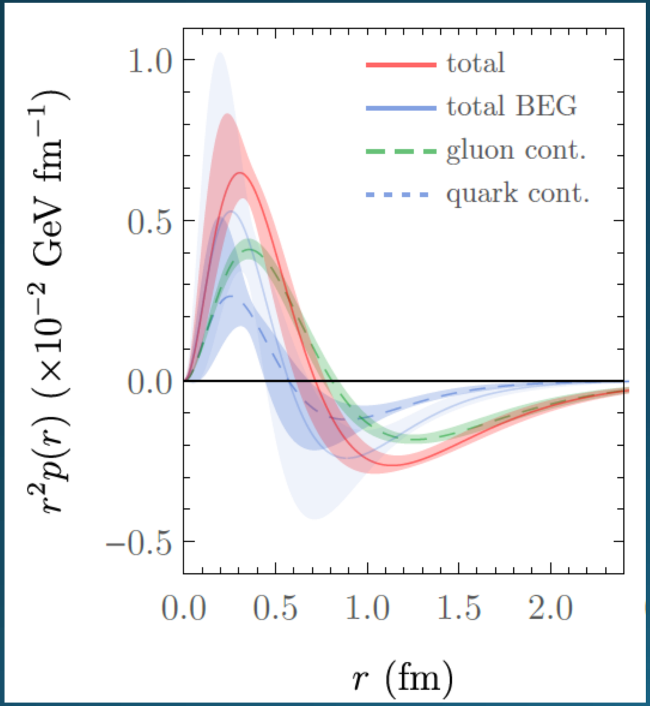


- ☐ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- ☐ Separation: $T_{zz} \neq T_{rr}$
- ☐ Too strong stress? Transverse stress is suppressed

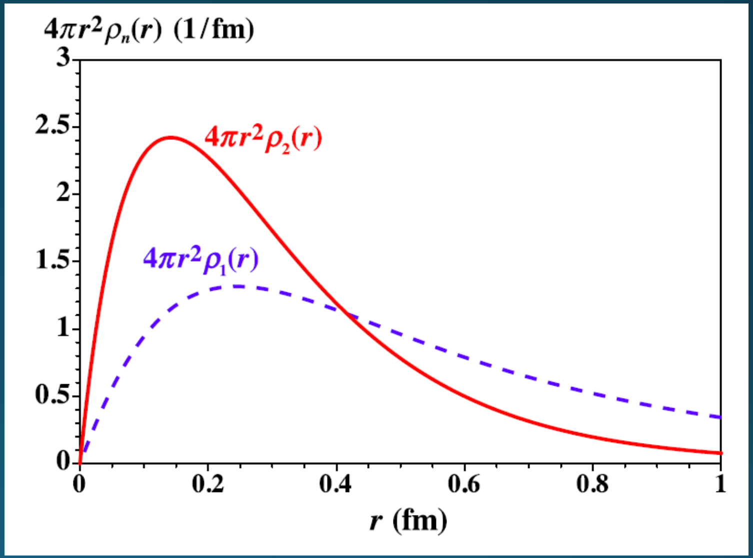
Proton EMT Distribution

EMT distribution inside hadrons now accessible??

Pressure @ proton



EMT distribution @ pion



arXiv:1810.07589
Nature, 557, 396 (2018)

Kumano, Song, Teryaev
Phys. Rev. D 97, 014020 (2018)

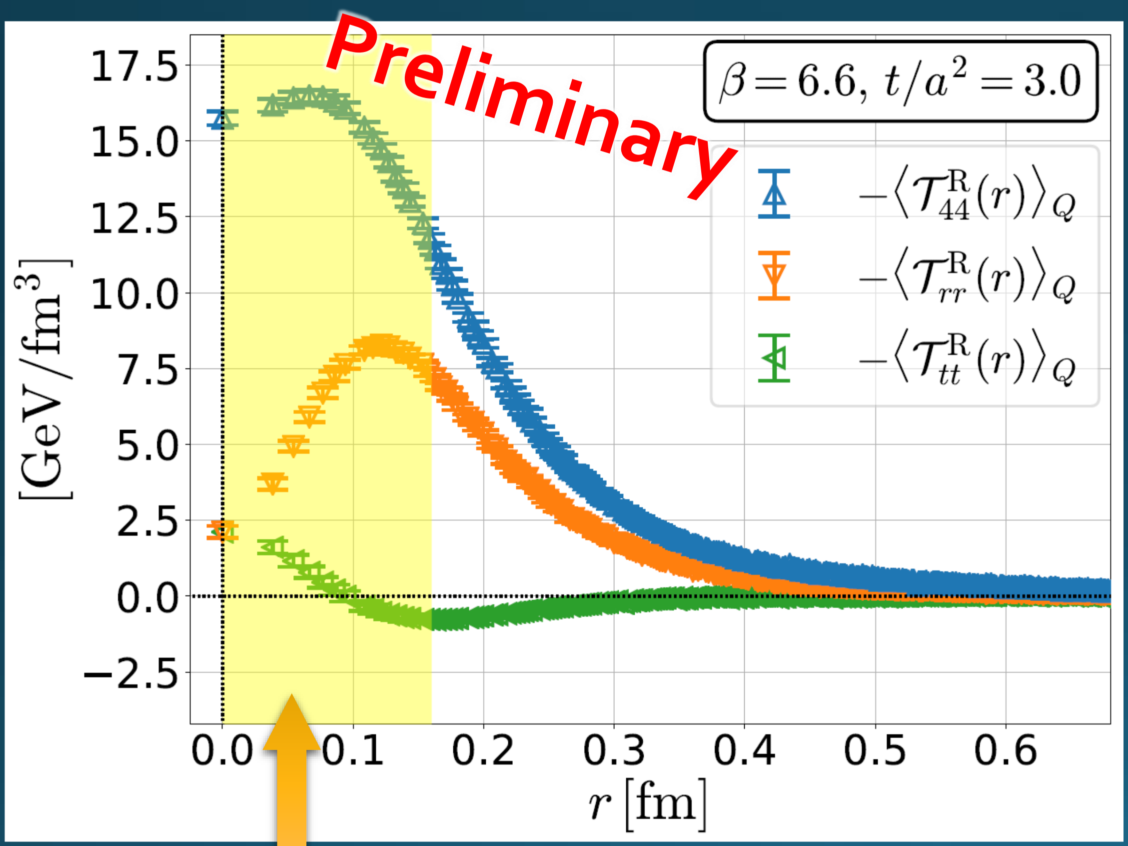
EMT around A Quark

in a deconfined phase

Q

EMT around A Quark

in a deconfined phase



not reliable

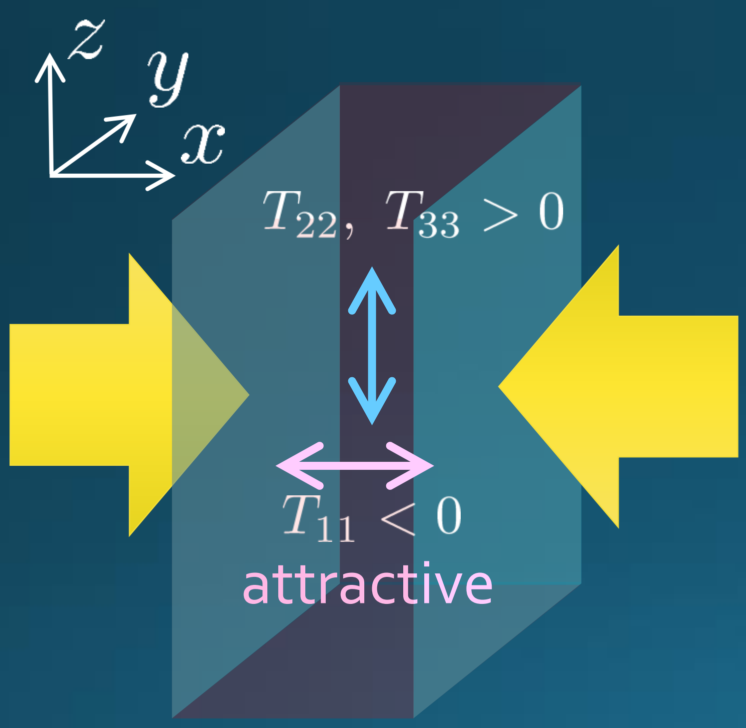
Yanagihara+, in prep.

Quenched QCD
48^3x12 ($T \approx 1.4 T_c$)
fixed t , a

- Energy density
  $-\langle T_{44}\rangle = \varepsilon$
- Longitudinal pressure
  $-\langle T_{rr}\rangle = -p(r)$
- Transverse pressure
  $-\langle T_{tt}\rangle$

Pressure anisotropy
in finite system

Casimir effect



Finite system at nonzero *T*

MK, Mogliacci, Kolbe, Horowitz, in preparation

$$V = L_x \times L_y \times L_z$$

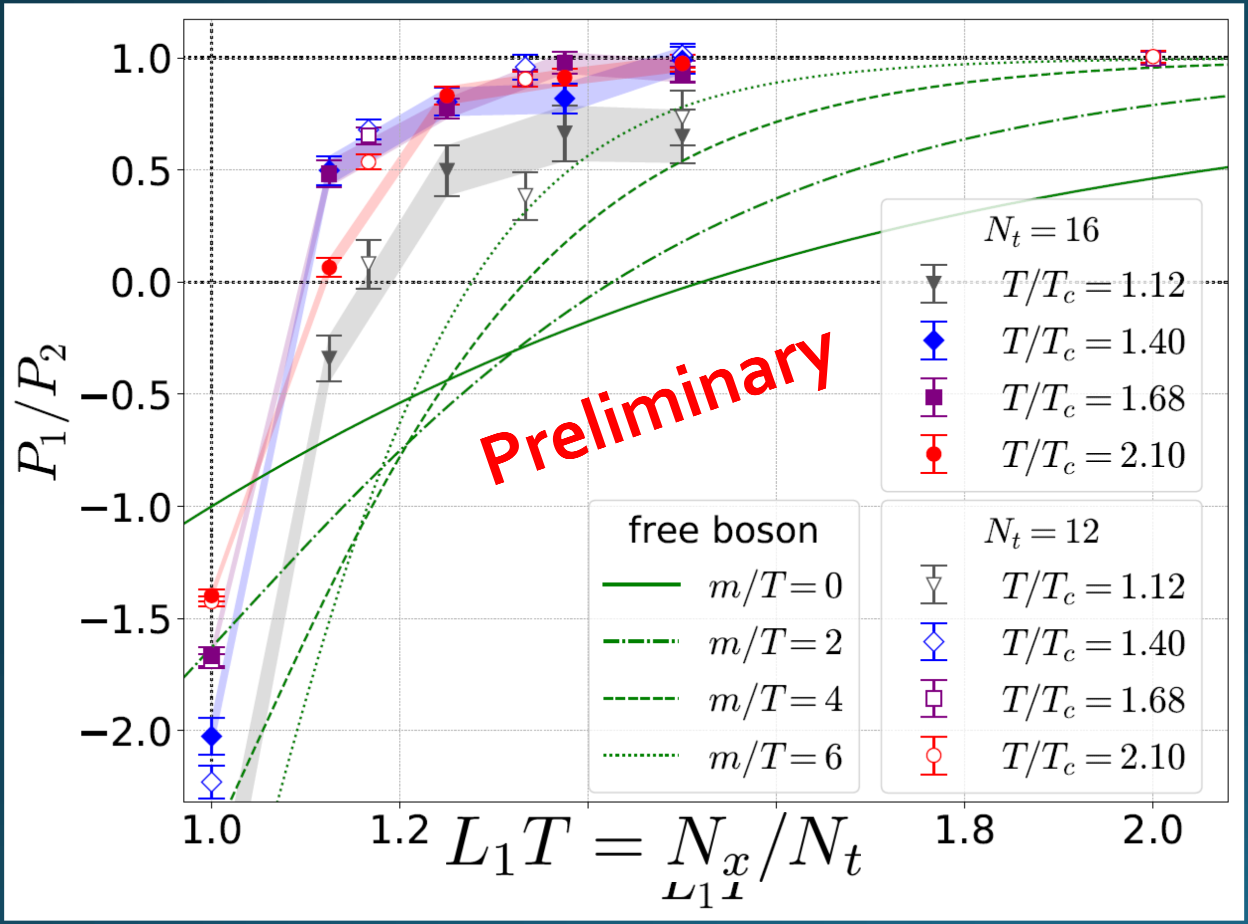
$$L_x \ll L_y = L_z$$

pressure anisotropy

$$T_{11} \neq T_{22} = T_{33}$$

Pressure Anisotropy

MK, Mogliacci, Kolbe, Horowitz, in prep.



Free scalar field

- $L_2=L_3=\infty$
  Mogliacci+, 1807.07871

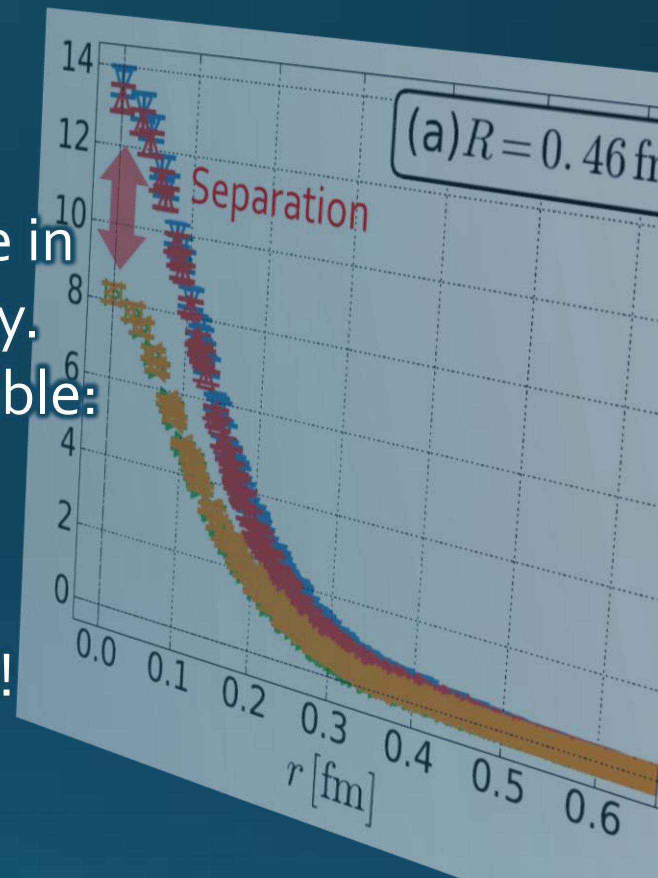
Lattice result

- Periodic BC
- Only t→0 limit

Medium near T_c is remarkably insensitive to finite size!
How do we understand??

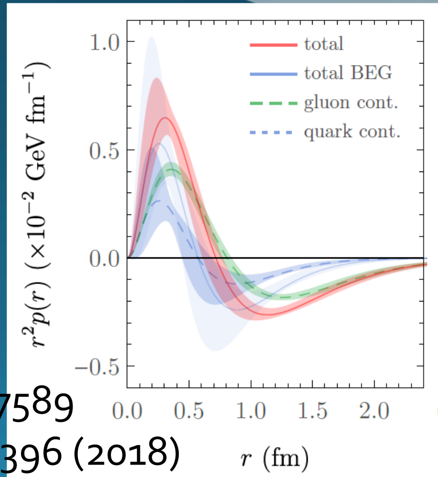
Summary

- ◻ EMT (stress tensor) is a useful observable in understanding QCD and YM gauge theory.
- ◻ EMT distribution in vacuum is now available:
  - ◻ Degeneracy: $T_{44} \approx T_{zz}$, $T_{rr} \approx T_{\theta\theta}$
  - ◻ Separation: $T_{zz} \neq T_{rr}$
- ◻ EMT distribution at nonzero T is ongoing!



◻ Future

- ◻ Continuum limit
- ◻ Temperature dependence
- ◻ Excited states / full QCD
- ◻ EMT distribution inside hadrons

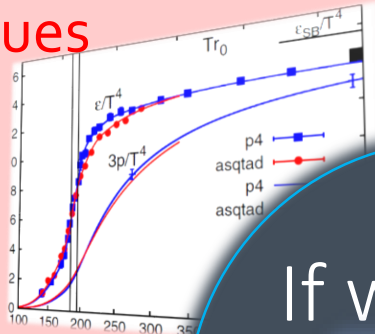


arXiv:1810.07589
Nature, 557, 396 (2018)

Thermodynamics

direct measurement of expectation values

$$\langle T_{00} \rangle, \langle T_{ii} \rangle$$



Fluctuations and Correlations

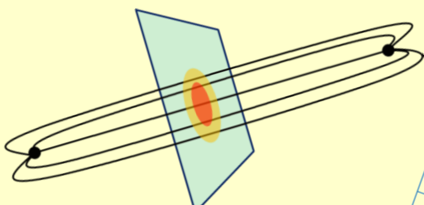
viscosity, specific heat, ...

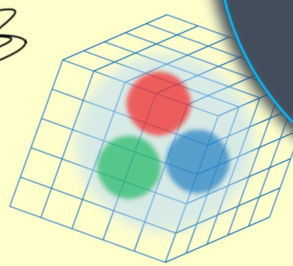
$$c_V \sim \langle \delta T_{00}^2 \rangle$$

$$\eta = \langle T_{12} ; T_{12} \rangle$$

If we have

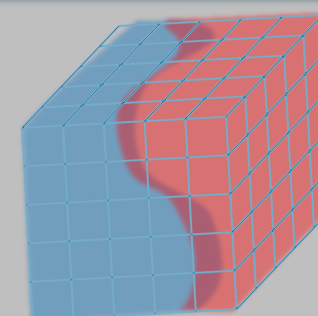
$$T_{\mu\nu}$$





- flux tube / hadrons
- EM form factors

Hadron Structure



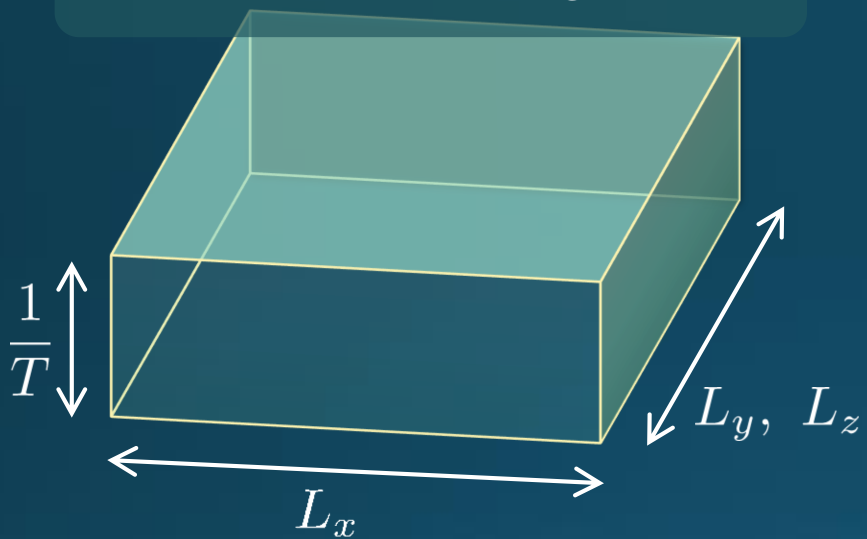
- vacuum configuration
- mixed state on 1st transition

Vacuum Structure

backup

Two Special Cases with PBC

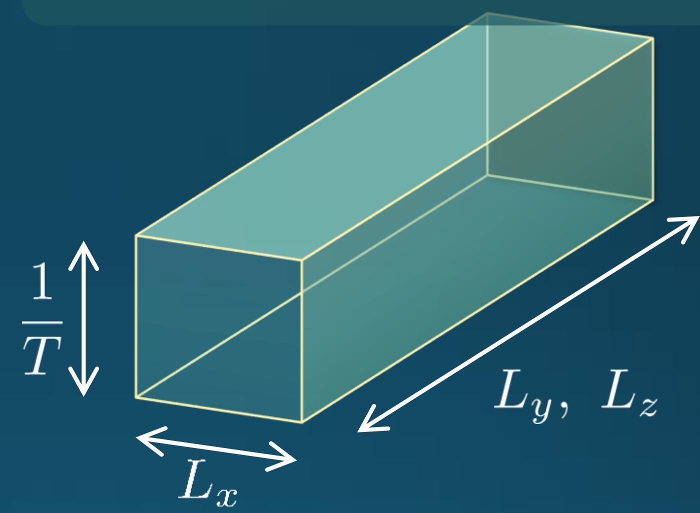
$$1/T \ll L_x = L_y = L_z$$



$$T_{11} = T_{22} = T_{33}$$

$$\frac{p_1}{p_2} = 1$$

$$1/T = L_x,\ L_y = L_z$$



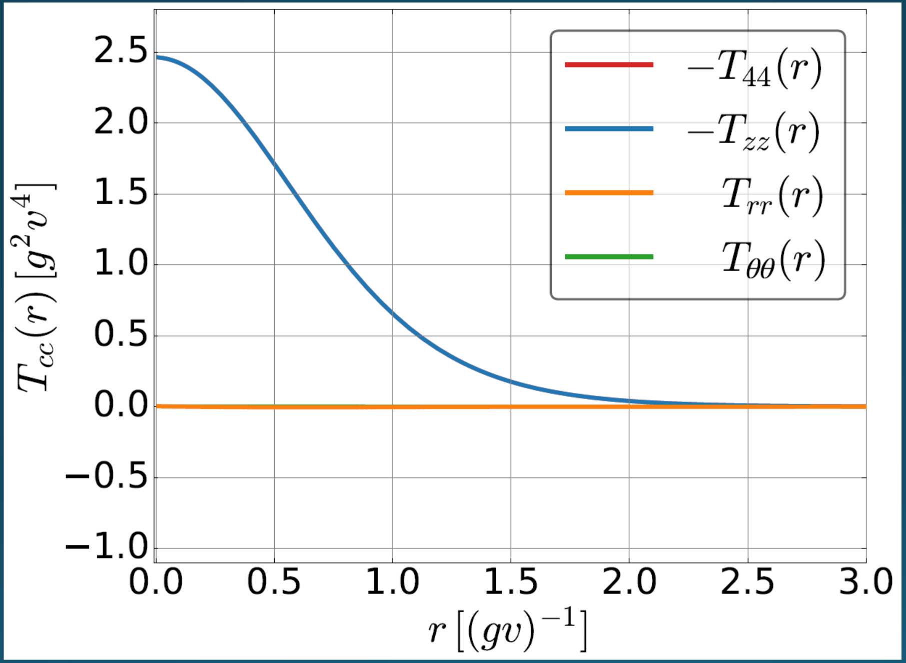
$$T_{44} = T_{11},\ T_{22} = T_{33}$$

In conformal ($\Sigma_\mu T_{\mu\mu}=0$)

$$\frac{p_1}{p_2} = -1$$

Stress Tensor in AH Model
infinitely-long flux tube

Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$



$$T_{rr} = T_{\theta\theta} = 0$$

de Vega, Schaposnik, PR**D14**, 1100 (1976).

Perturbative Coefficients

$$T_{\mu\nu}(t) = c_1(t)U_{\mu\nu}(t) + \delta_{\mu\nu}c_2(t)E(t)$$

	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○		
$c_2(t)$	× zero	○	○	

Suzuki (2013)

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

□ Choice of the scale of g²

$$c_1(t) = c_1\left(g^2(\mu(t))\right)$$

Previous: $\mu_d(t) = 1/\sqrt{8t}$

Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E}t}$

Harlander+ (2018)

Perturbative Coefficients

$$T_{\mu\nu}(t) = c_1(t)U_{\mu\nu}(t) + \delta_{\mu\nu}c_2(t)E(t)$$

	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○	○	
$c_2(t)$	× zero	○	○	○

Suzuki (2013) Harlander+(2018)

Iritani, MK, Suzuki, Takaura, 2019

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

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Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E}t}$

Harlander+ (2018)

EMT on the Lattice: Conventional

Lattice EMT Operator

Caracciolo+, 1990

$$T_{\mu\nu} = Z_6 T_{\mu\nu}^{[6]} + Z_3 T_{\mu\nu}^{[3]} + Z_1 \left(T_{\mu\nu}^{[1]} - \langle T_{\mu\nu}^{[1]} \rangle \right)$$

$$T_{\mu\nu}^{[6]} = (1-\delta_{\mu\nu}) F_{\mu\rho}^a F_{\nu\rho}^a, \quad T_{\mu\nu}^{[3]} = \delta_{\mu\nu} \left(F_{\mu\rho}^a F_{\nu\rho}^a - \frac{1}{4} F_{\rho\sigma}^a F_{\rho\sigma}^a \right), \quad T_{\mu\nu}^{[1]} = \delta_{\mu\nu} F_{\rho\sigma}^a F_{\rho\sigma}^a$$

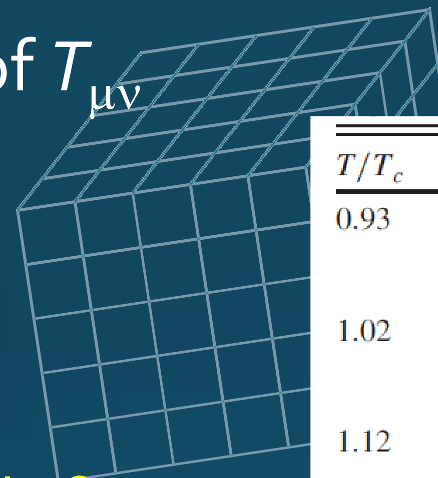
- Fit to thermodynamics: Z_3, Z_1
- Shifted-boundary method: Z_6, Z_3 — Giusti, Meyer, 2011; 2013; Giusti, Pepe, 2014~; Borsanyi+, 2018

Multi-level algorithm

- effective in reducing statistical error of correlator — Meyer, 2007; Borsanyi, 2018; Astrakhantsev+, 2018

Numerical Simulation

FlowQCD,
PRD**94**, 114512 (2016)



- ☐ Expectation values of $T_{\mu\nu}$
- ☐ SU(3) YM theory
- ☐ Wilson gauge action
- ☐ Parameters:
  - N_t = 12, 16, 20-24
  - aspect ratio 5.3<N_s/N_t<8
  - 1500~2000 configurations
- ☐ Scale from gradient flow

$$\to aT_c \text{ and } a\Lambda_{\rm MS}$$

FlowQCD, 1503.06516

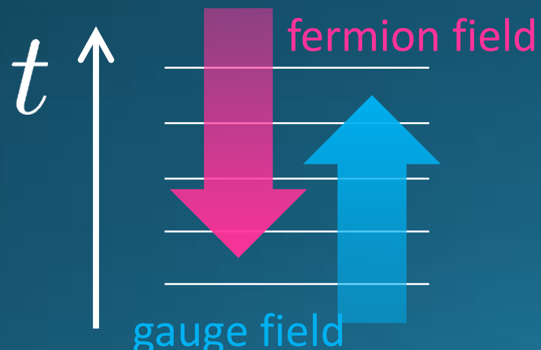
T/T_c	β	N_s	N_τ	Configurations
0.93	6.287	64	12	2125
	6.495	96	16	1645
	6.800	128	24	2040
1.02	6.349	64	12	2000
	6.559	96	16	1600
	6.800	128	22	2290
1.12	6.418	64	12	1875
	6.631	96	16	1580
	6.800	128	20	2000
1.40	6.582	64	12	2080
	6.800	128	16	900
	7.117	128	24	2000
1.68	6.719	64	12	2000
	6.941	96	16	1680
	7.117	128	20	2000
2.10	6.891	64	12	2250
	7.117	128	16	840
	7.296	128	20	2040
2.31	7.200	96	16	1490
	7.376	128	20	2020
	7.519	128	24	1970
2.69	7.086	64	12	2000
	7.317	96	16	1560
	7.500	128	20	2040

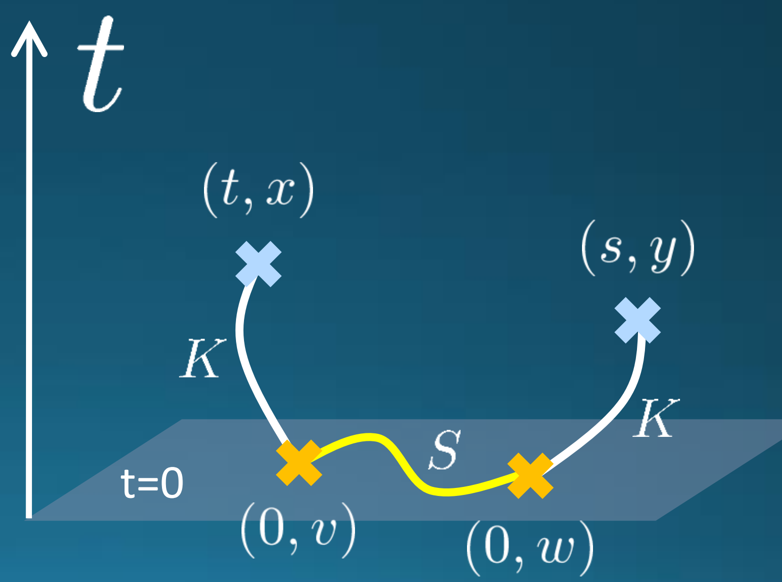
Fermion Propagator

$$S(t,x;s,y) = \langle \chi(t,x)\bar{\chi}(s,y)\rangle$$
$$= \sum_{v,w} K(t,x;0,v)S(v,w)K(s,y;0,w)^{\dagger}$$

$$\Big(\partial_t - D_\mu D_\mu\Big)K(t,x) = 0$$

- propagator of flow equation
- Inverse propagator is needed





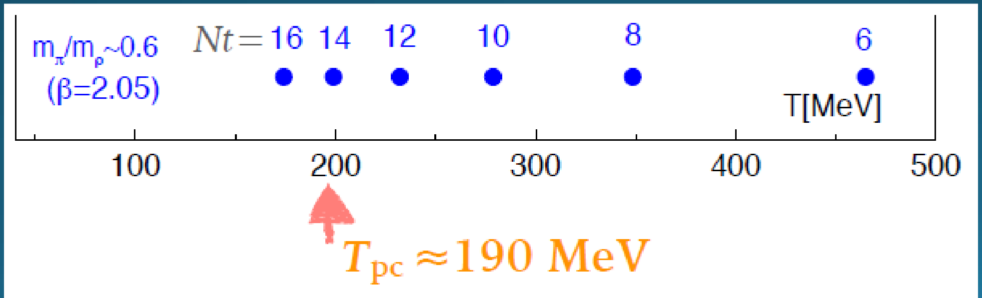
N_f=2+1 QCD Thermodynamics

Taniguchi+ (WHOT-QCD),
PRD**96**, 014509 (2017)

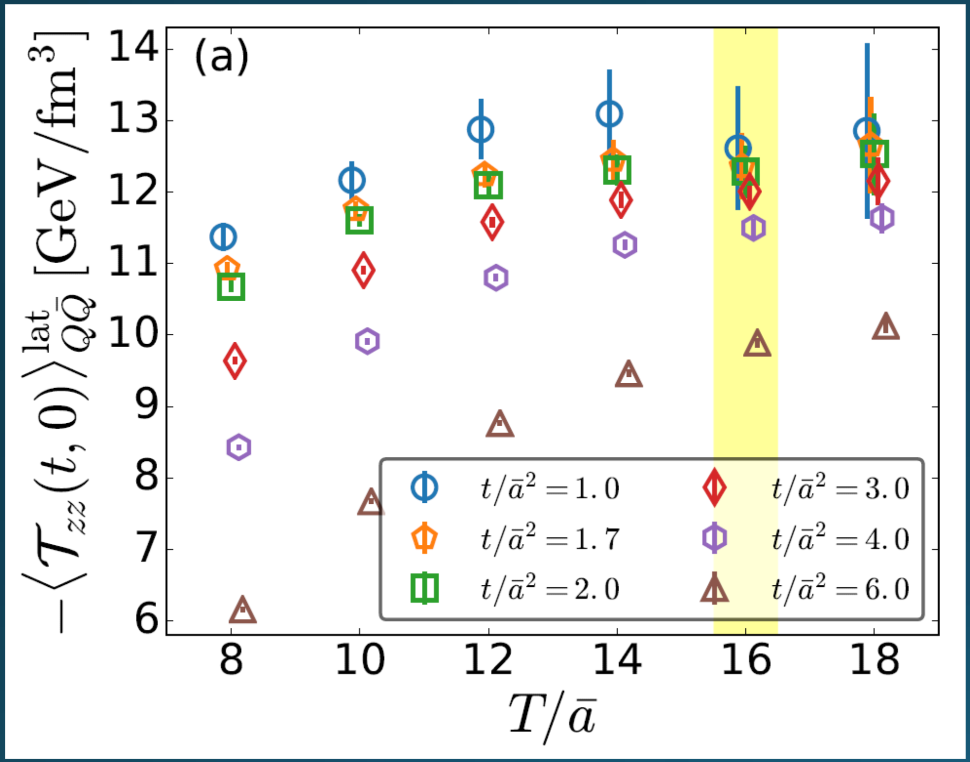
- N_f=2+1 QCD, Iwasaki gauge + NP-clover
- m_{PS}/m_V ≈0.63 / almost physical s quark mass

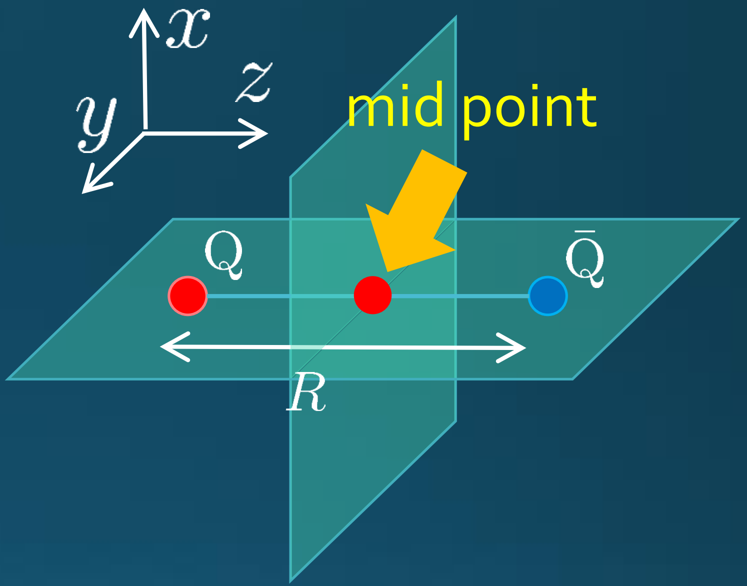
- T=0: CP-PACS+JLQCD (ß=2.05, 28^3x56, a≈0.07fm)
- T>0: 32^3xN_t, N_t = 4, 6, … , 14, 16):
- T≈174-697MeV

- t→0 extrapolation only (No continuum limit)



Ground State Saturation

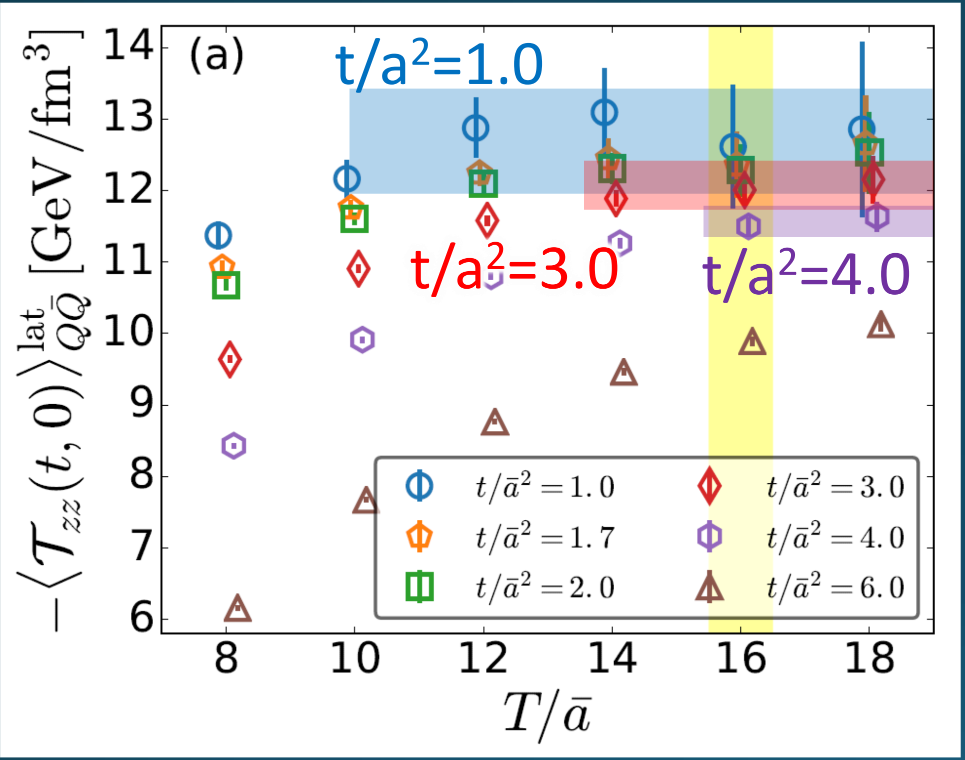


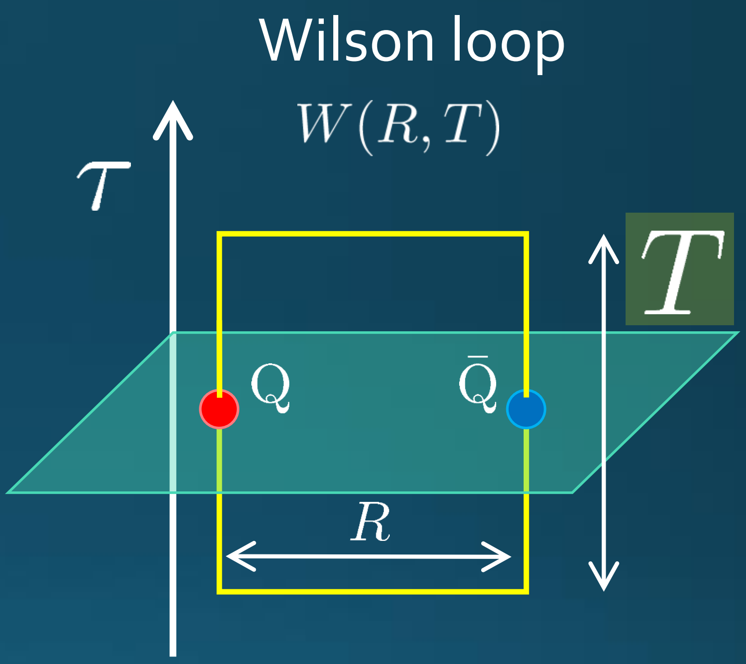


β=6.819 (a=0.029 fm), R=0.46 fm

Appearance of plateau for $t/a^2<4$, $T/a>15$ → Grand state saturation under control

Ground State Saturation

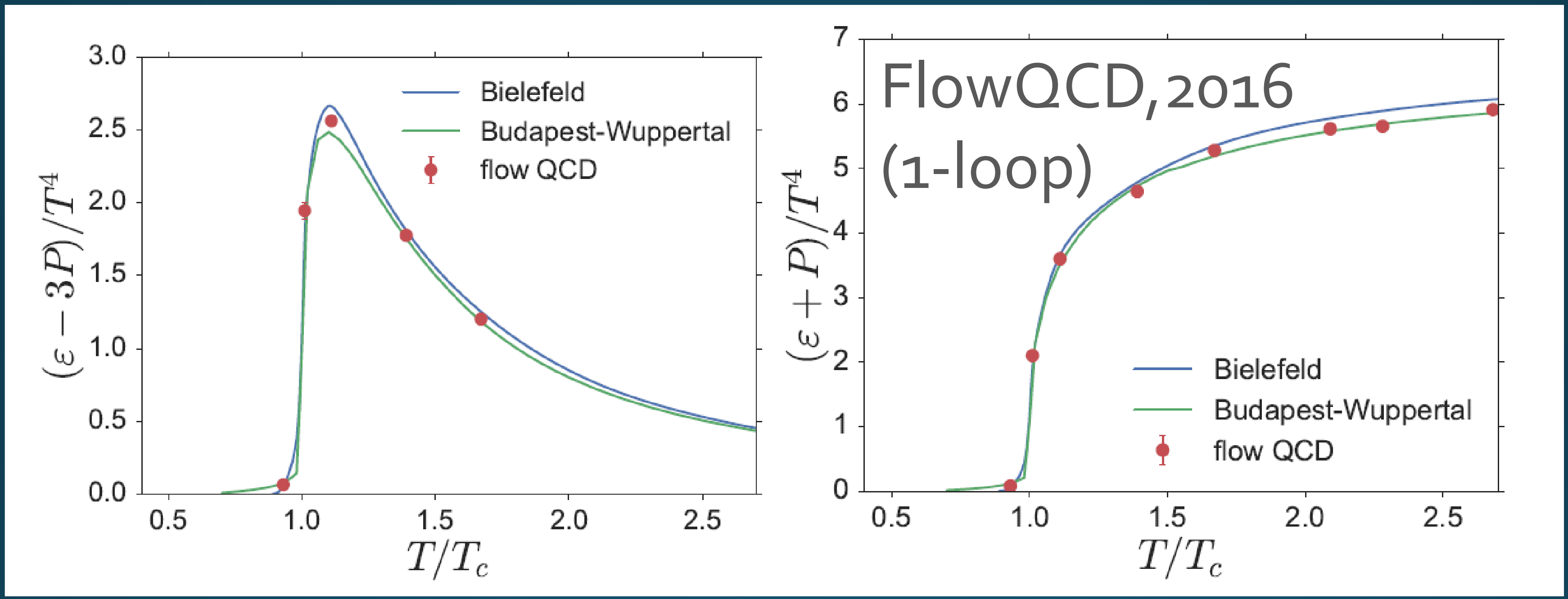




β=6.819 (a=0.029 fm), R=0.46 fm

Appearance of plateau for $t/a^2<4$, $T/a>15$ → Grand state saturation under control

Temperature Dependence



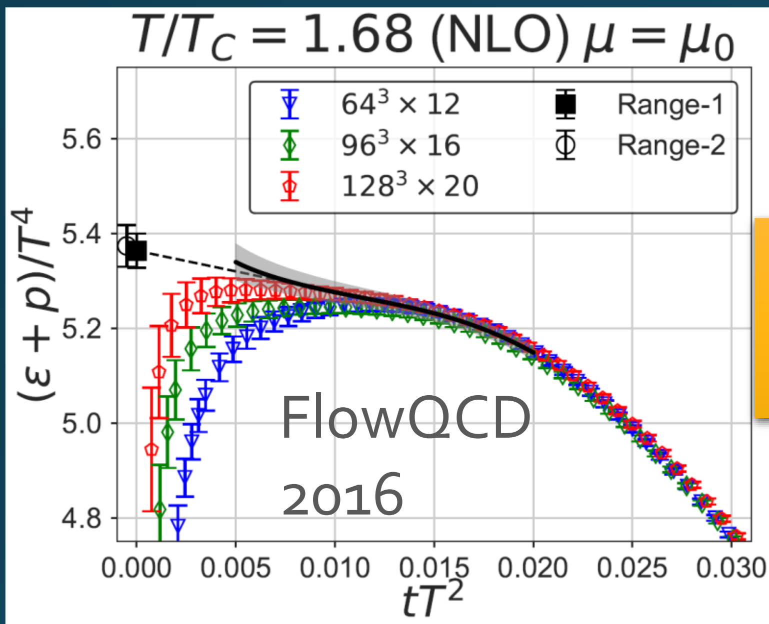
Error includes
- statistical error
- choice of t range for t→0 limit
- uncertainty in aΛ_{MS}

total error <1.5% for T>1.1T_c

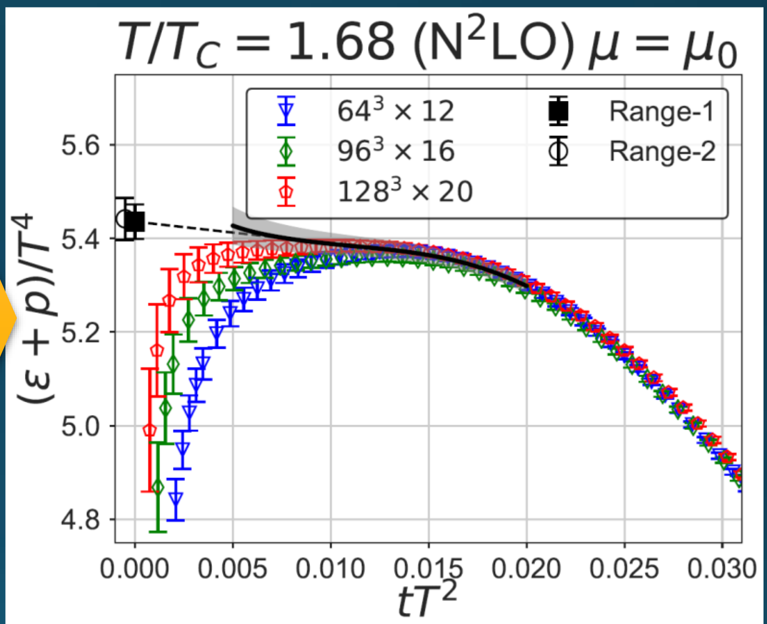
- Excellent agreement with integral method
- High accuracy only with ~2000 confs.

Higher Order Coefficient: ε+p

NLO (1-loop)



N^2LO (2-loop)



Iritani, MK, Suzuki, Takaura, 2019

- t dependence becomes milder with higher order coeff.
- 1-loop → 2-loop : about 2% increase

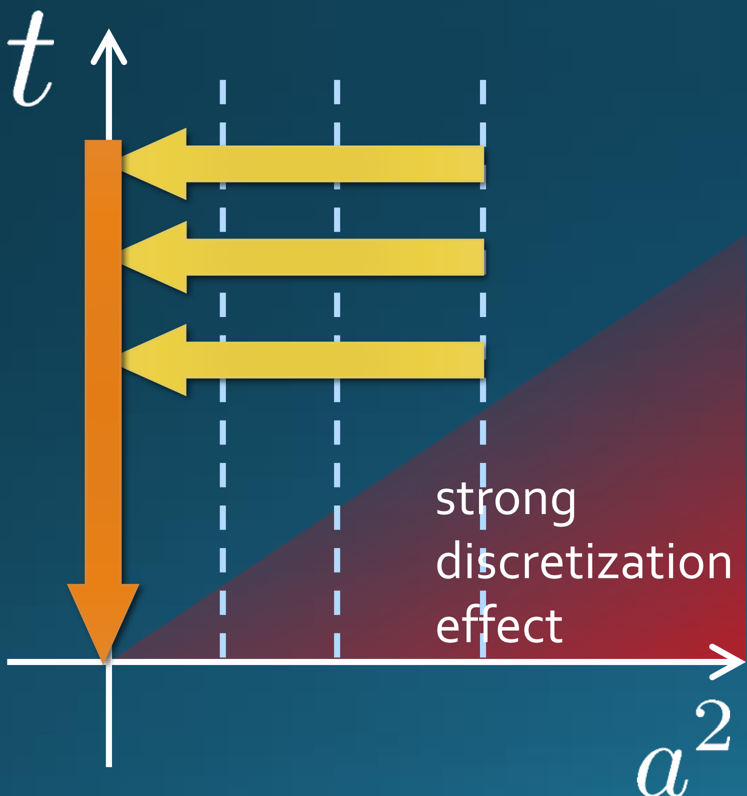
- Systematic analysis: μ_o or μ_d, uncertainty of Λ, fit range
- Extrapolation func: linear, higher order term in c_1 ($\sim g^6$)

Double Extrapolation

t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\mathrm{latt}} = \langle T_{\mu\nu}(t)\rangle_{\mathrm{phys}} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE lattice discretization



Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\mathrm{cont}} = \langle T_{\mu\nu}(t)\rangle_{\mathrm{lat}} + C(t)a^2$$

Small t extrapolation

$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$

EMT in QCD

$$\begin{aligned} T_{\mu\nu}(t,x) =& c_1(t)U_{\mu\nu}(t,x) + c_2(t)\delta_{\mu\nu}\big(E(t,x) - \langle E\rangle_0\big) \\ &+ c_3(t)\big(O_{3\mu\nu}(t,x) - 2O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) \\ &+ c_4(t)\big(O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) + c_5(t)\big(O_{5\mu\nu}(t,x) - \mathrm{VEV}\big) \end{aligned}$$

$$T_{\mu\nu}(x) = \lim_{t\to 0} T_{\mu\mu}(t,x)$$

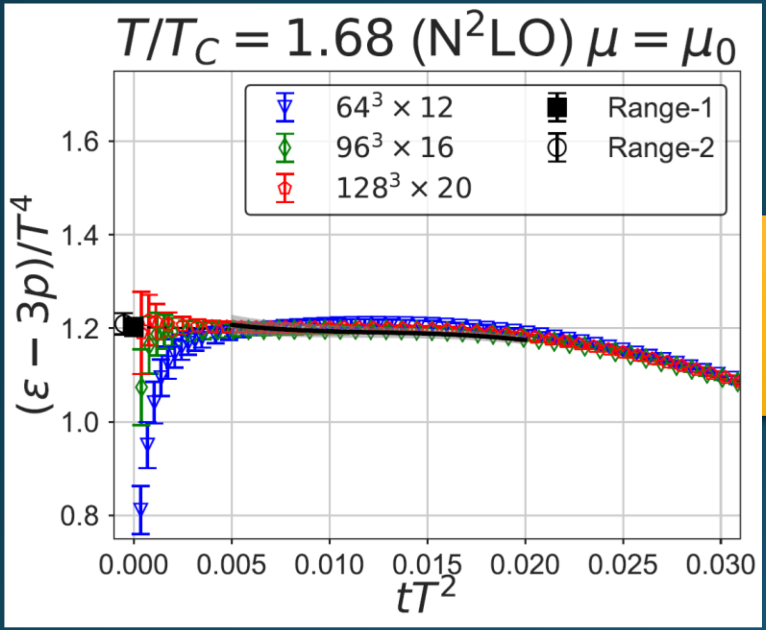
$$\begin{aligned} \tilde{O}^f_{3\mu\nu}(t,x) &\equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x), \\ \tilde{O}^f_{4\mu\nu}(t,x) &\equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x), \\ \tilde{O}^f_{5\mu\nu}(t,x) &\equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x), \end{aligned}$$

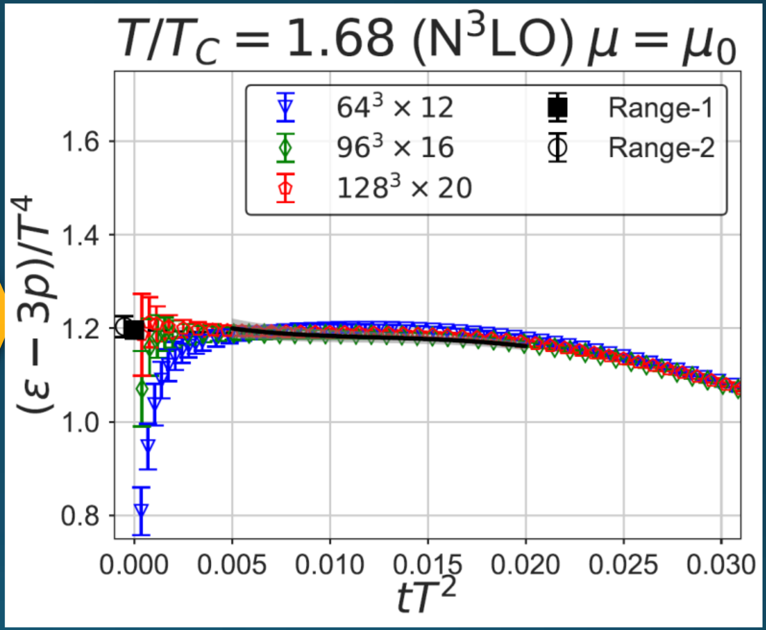
$$\varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.$$

$$\begin{aligned} c_1(t) &= \frac{1}{\bar{g}(1/\sqrt{8t})^2} - \frac{1}{(4\pi)^2}\left[9(\gamma - 2\ln 2) + \frac{19}{4}\right], \\ c_2(t) &= \frac{1}{(4\pi)^2}\frac{33}{16}, \\ c_3(t) &= \frac{1}{4}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[2 + \frac{4}{3}\ln(432)\right]\right\}, \\ c_4(t) &= \frac{1}{(4\pi)^2}\bar{g}(1/\sqrt{8t})^2, \\ c^f_5(t) &= -\bar{m}_f(1/\sqrt{8t})\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[4(\gamma - 2\ln 2) + \frac{14}{3} + \frac{4}{3}\ln(432)\right]\right\} \end{aligned}$$

Higher Order Coefficient: ε-3p

N²LO (2-loop) → **N³LO (3-loop)**





Iritani, MK, Suzuki, Takaura, in prep.

- No difference b/w 2- & 3-loops: 2-loop is already good!

- Systematic analysis: μ_0 or μ_d, uncertainty of Λ, fit range
- Extrapolation func: linear, higher order term in c_2 ($\sim g^8$)

Gradient Flow for Fermions

$$\partial_t \psi(t,x) = D_\mu D_\mu \psi(t,x)$$

$$\partial_t \bar\psi(t,x) = \psi(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu$$

$$D_\mu = \partial_\mu + A_\mu(t,x)$$

Luscher, 2013
Makino, Suzuki, 2014
Taniguchi+ (WHOT) 2016; 2017

- Not "gradient" flow but a "diffusion" equation.

- Divergence in field renormalization of fermions.
- All observables are finite at t>0 once Z(t) is fixed.

$$\tilde\psi(t,x) = Z(t)\psi(t,x)$$

- Energy-momentum tensor from SFTE Makino, Suzuki, 2014

Abelian-Higgs Model

Abelian-Higgs Model

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

GL parameter: $\kappa = \sqrt{\lambda}/g$

- type-I : $\kappa < 1/\sqrt{2}$
- type-II : $\kappa > 1/\sqrt{2}$
- Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

Infinitely long tube

- degeneracy

  $T_{zz}(r) = T_{44}(r)$ Luscher, 1981

- conservation law

$$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$