Equilibration of Higher-order Cumulants

Masakiyo Kitazawa

(Osaka U.)

GSI Workshop on
Probing the Phase Structure of Strongly Interacting Matter: Theory and Experiment
GSI, Darmstadt, 26/Mar./2019

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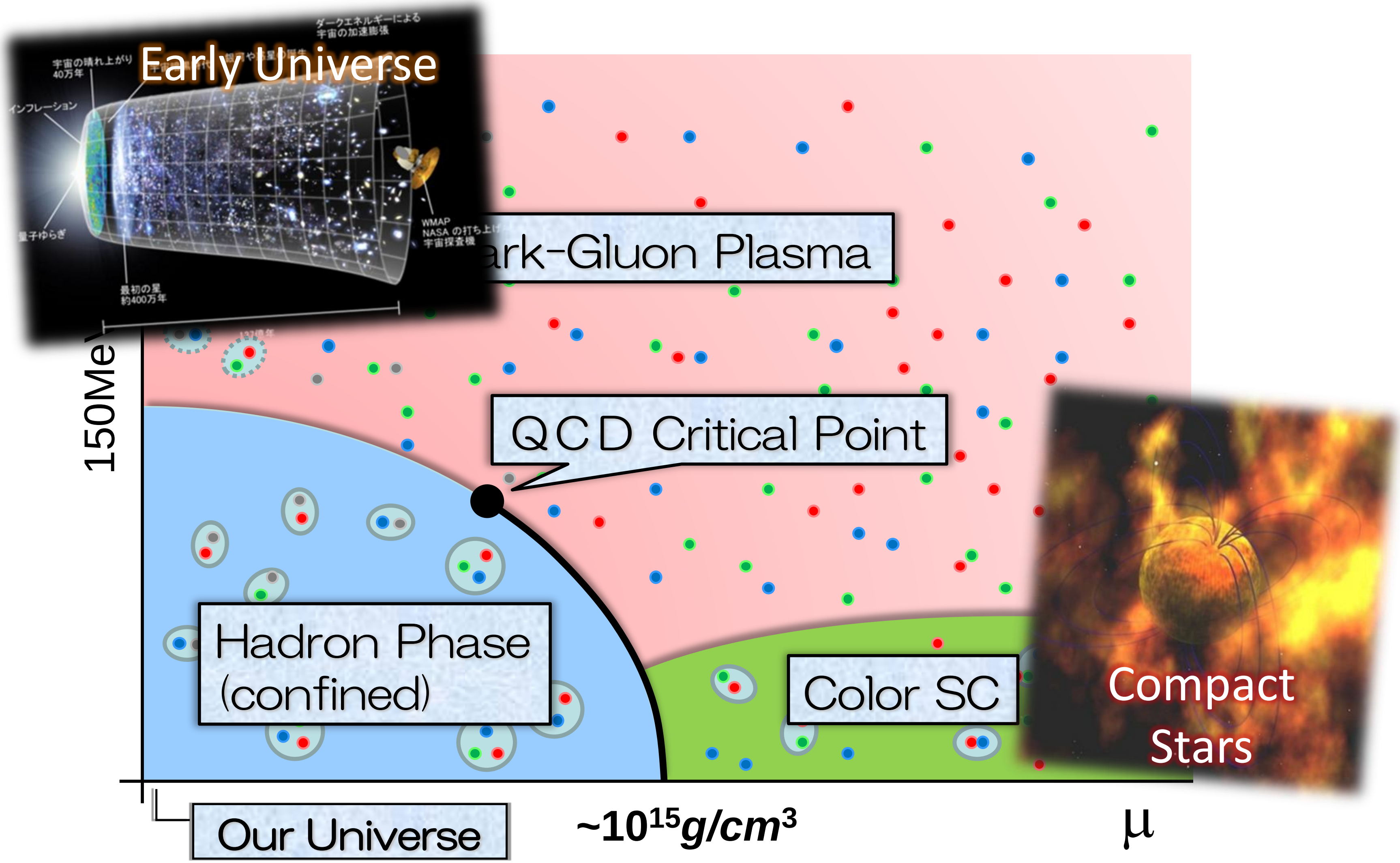
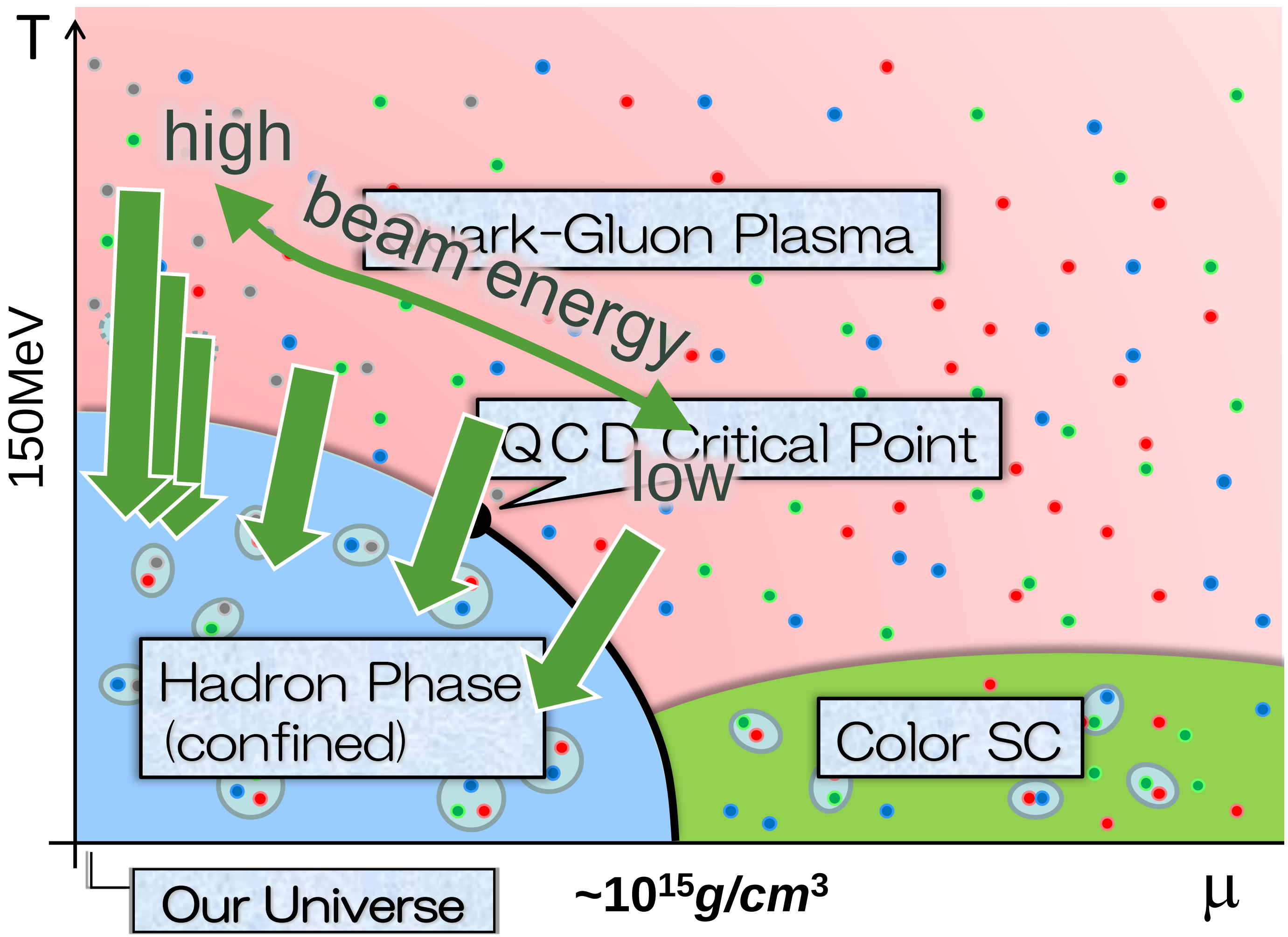
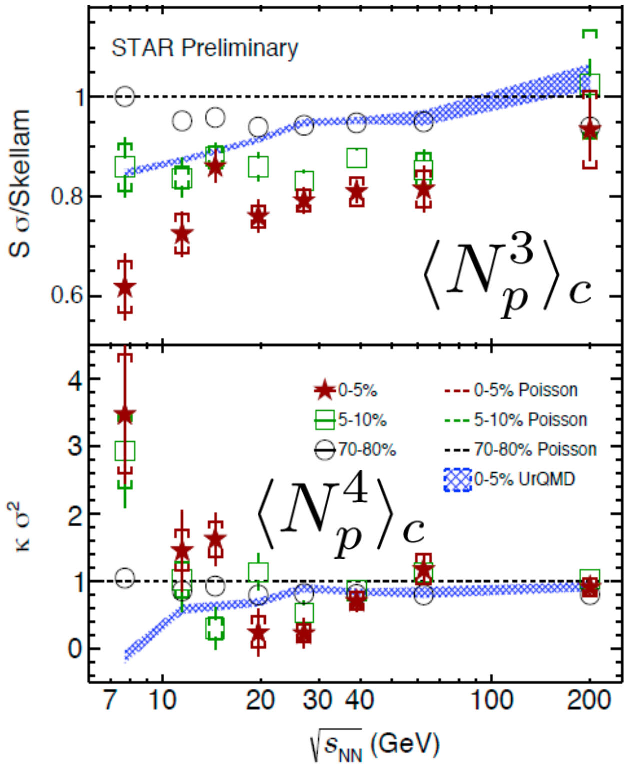
QCD Phase Diagram

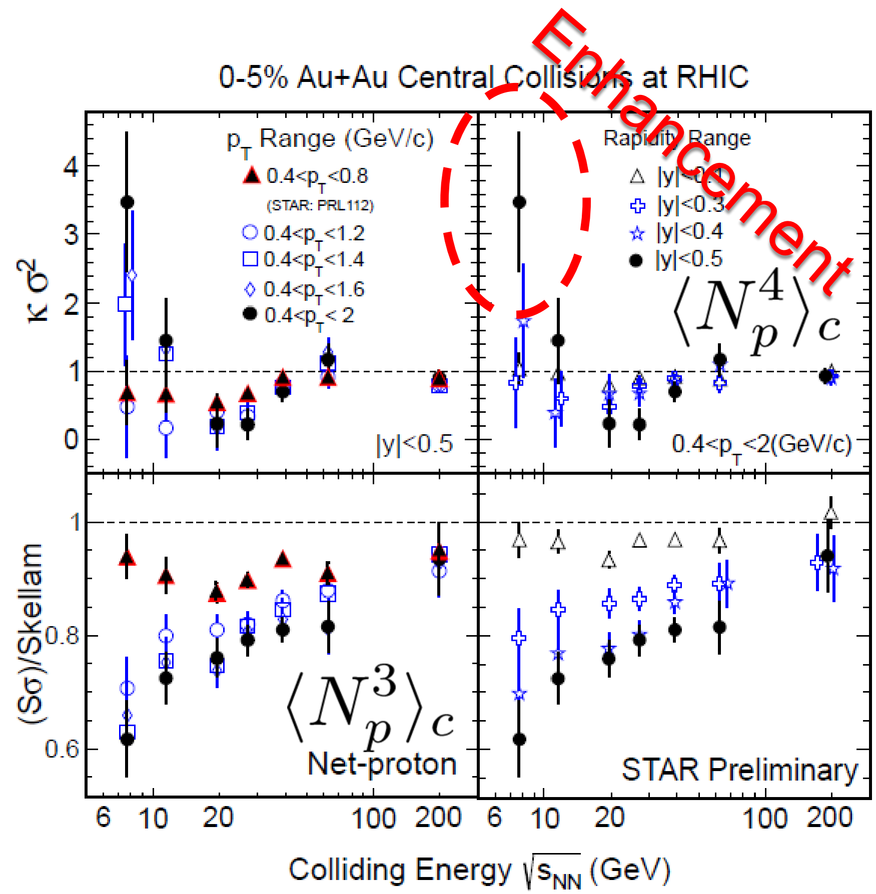


Beam-Energy Scan Program in Heavy-Ion Collisions



Higher-Order Cumulants



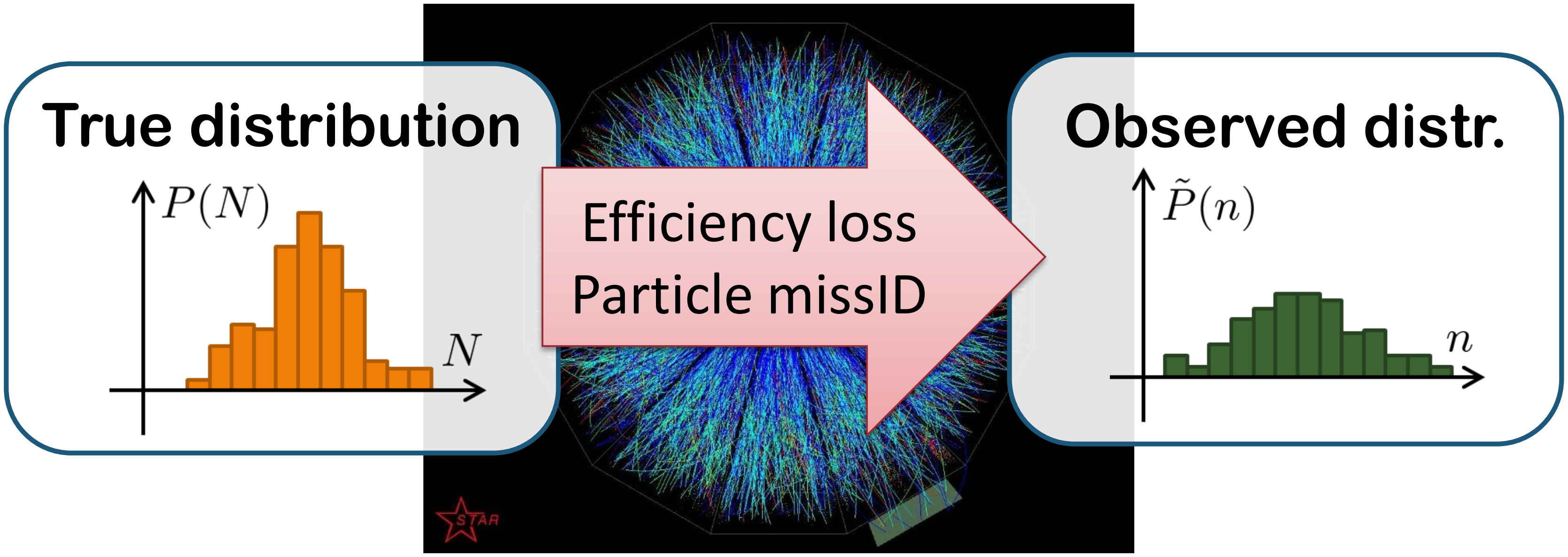


STAR
2010~

Non-zero **non**-Gaussian cumulants have been established!

General Review:
Asakawa, MK, PPNP (2016)

Detector-Response Correction



- ☐ **Correction assuming a binomial response**

  Bialas, Peschanski (1986);
  MK, Asakawa (2012); Bzdak, Koch (2012); ….

  But, the response of the detector is not binomial…

Non-Binomial Correction

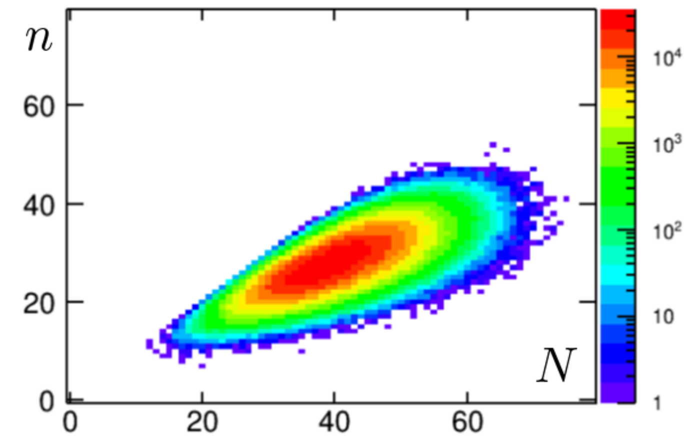
Nonaka, MK, Esumi (2018)

- Response matrix

$$\tilde{P}(n) = \sum_N \mathcal{R}(n;N) P(N)$$

Reconstruction for any R(n;N) with moments of R(n;N)

$$\langle n^m \rangle_R = \sum_n n^m \mathcal{R}(n;N)$$



- Caveats:
  - R(n;N) describes the property of the detector.
  - Detailed properties of the detector have to be known.
  - Multi-distribution function can be handled.
  - Huge numerical cost would be required.
  - Truncation is required in general: another systematics?

Result for Toy-Model

Binomial w/ multiplicity-dependent efficiency

$$\epsilon(N) = \epsilon_0 + (N - N_{\rm ave})\epsilon'$$

Holtzman, Bzdak, Koch (16)

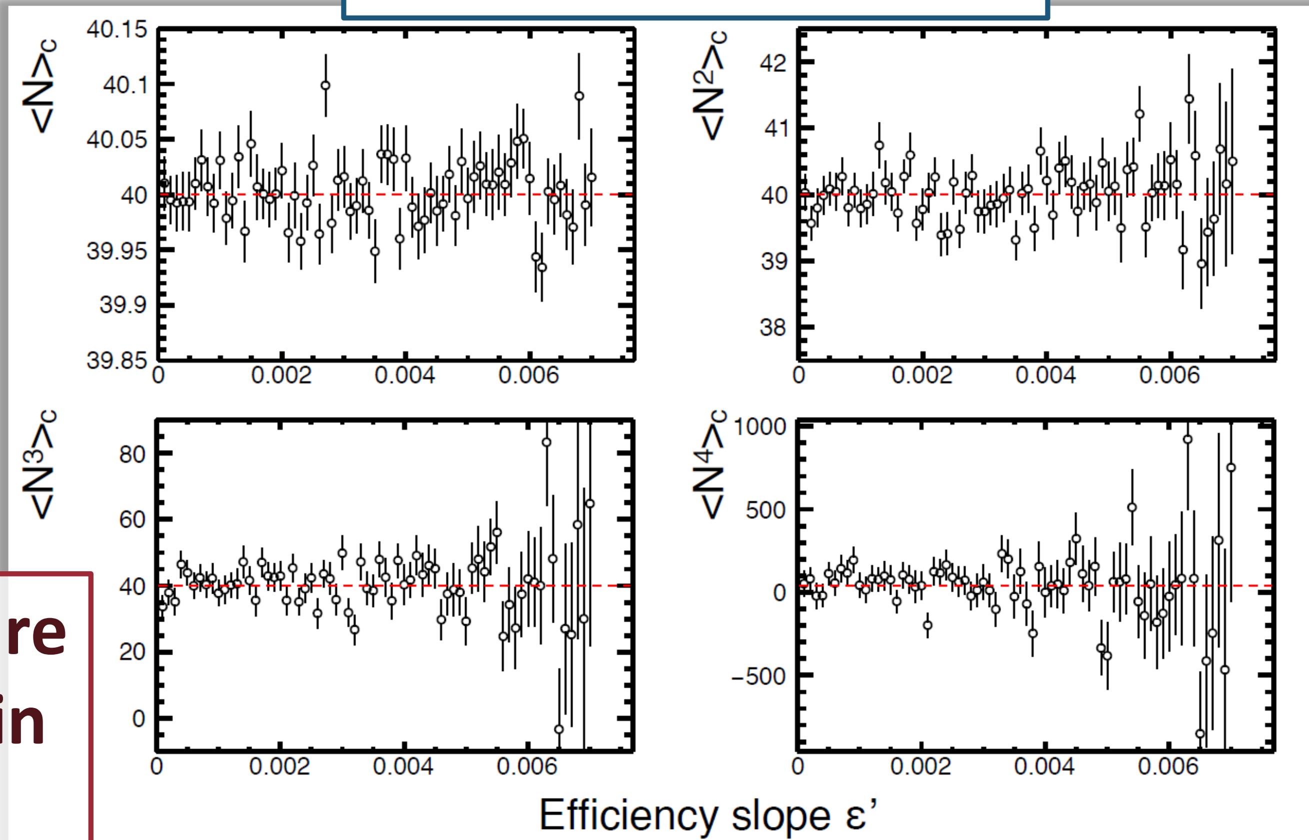
Nonaka, MK, Esumi (2018)

Input P(N): Poisson(λ=40)

$\epsilon_0 = 0.7$

Red: true cumulant

Reconstructed cumulants



True cumulants are reproduced within statistics!

Proton vs Baryon Cumulants

MK, Asakawa, 2012; 2012

Experiments
proton number cumulants

$$\langle N_p^n \rangle_c$$

Many theories
baryon number cumulants

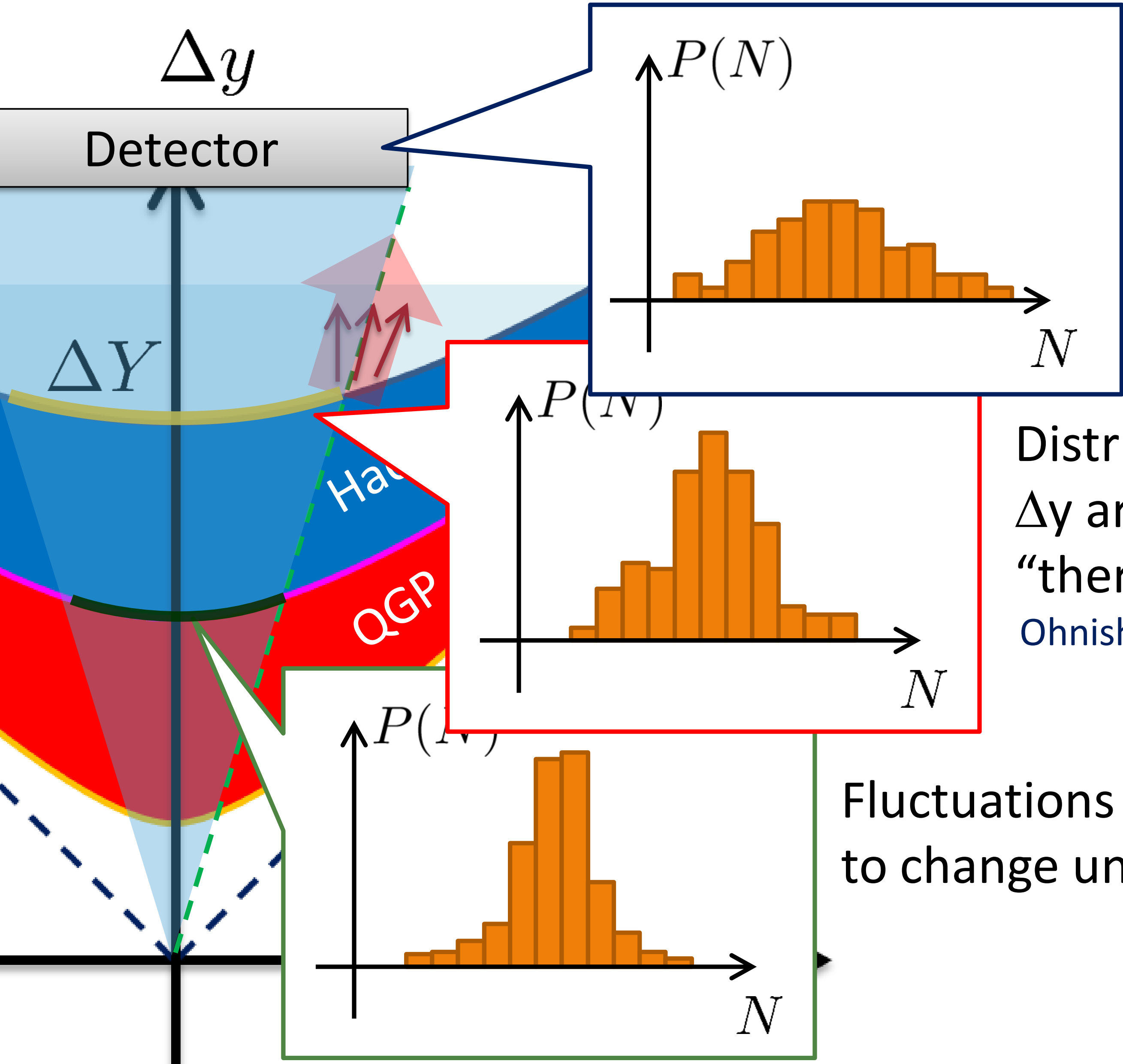
$$\langle N_{\rm B}^n \rangle_c$$

measurement with
50% efficiency loss

- ☐ Clear difference b/w these cumulants.
- ☐ Isospin randomization justifies the reconstruction of $\langle N_B^n \rangle_c$ via the binomial model.
- ☐ Similar problem on the **momentum cut**…

Non- Equilibration of Higher-order Cumulants

Time Evolution of Fluctuations

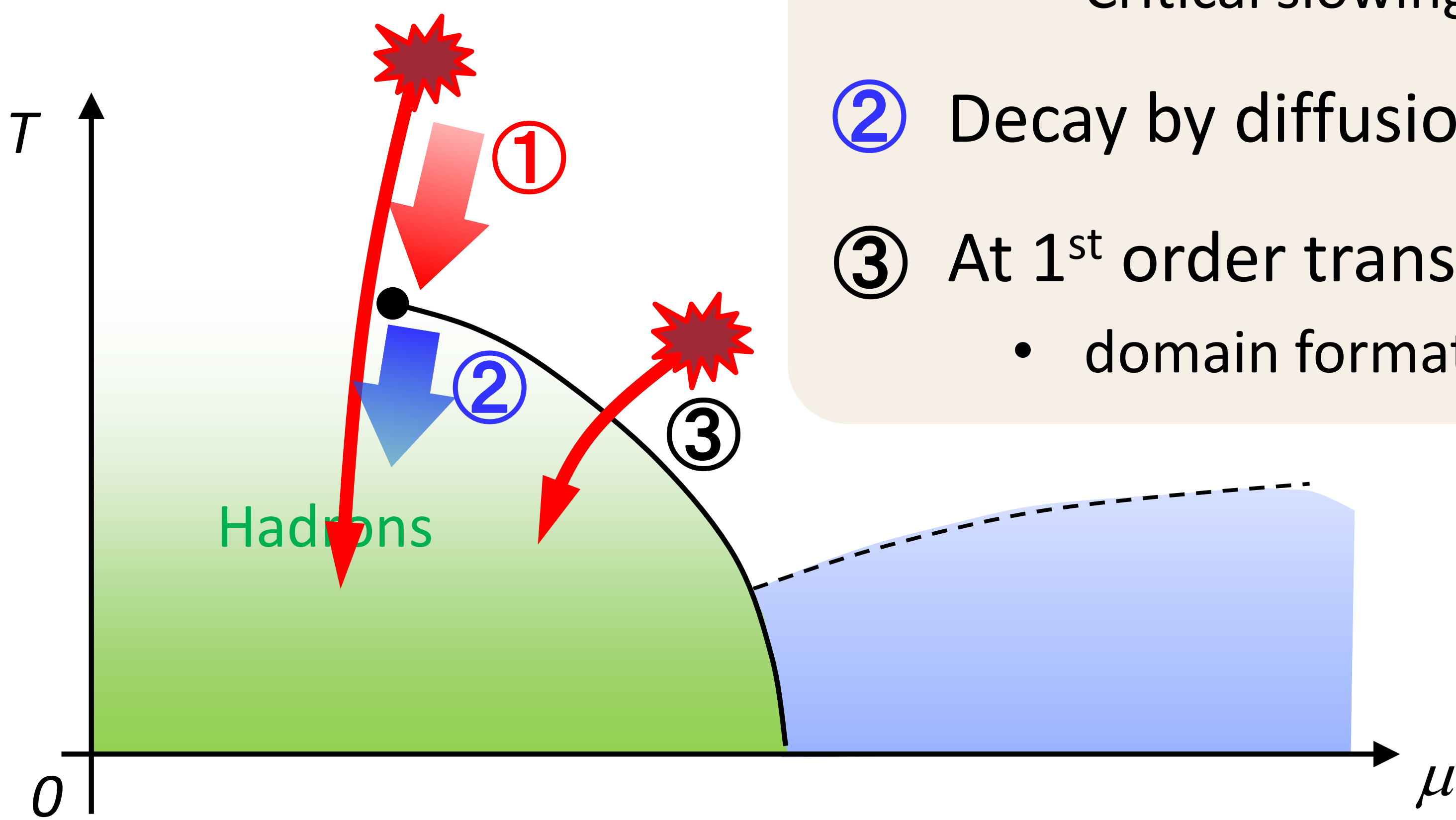


Distributions in ΔY and Δy are different due to “thermal blurring”.

Ohnishi, MK, Asakawa, PRC(2016)

Fluctuations in ΔY continue to change until kinetic f.o.

Critical Fluctuation



① Growth of critical fluctuation

- Critical slowing down

② Decay by diffusion

③ At 1st order transition

- domain formation

Contents of

Evolution of Fluctuations

1. in **Hadronic Stage**

   MK, Ono, Asakawa, PLB (2014); MK(2015)

2. around the **Critical Point**

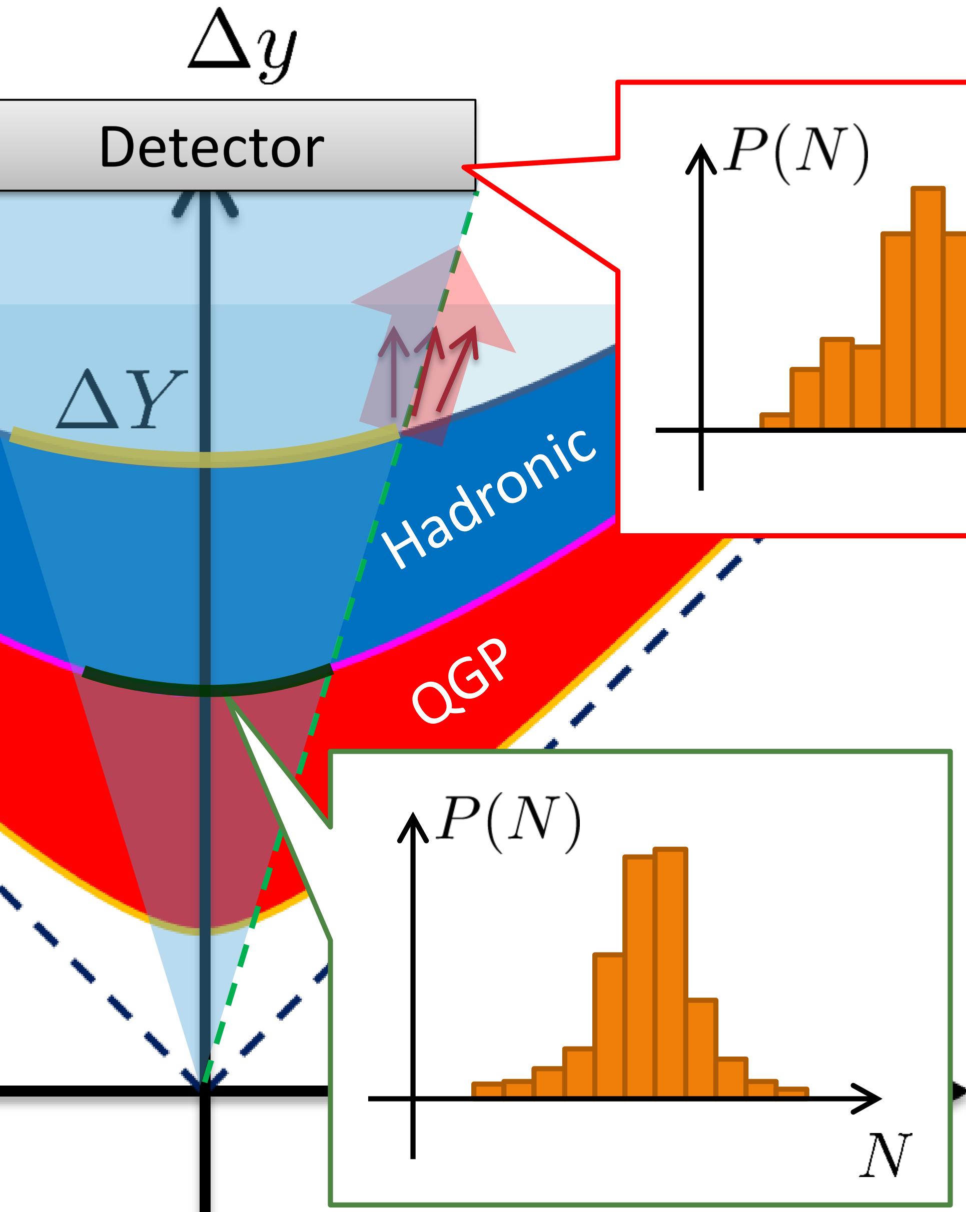
   Sakaida, Asakawa, Fujii, MK (2017)

3. at **First Order Transition**

   Nonaka, MK, Akamatsu, Bluhm, Nahrgang, in prep.

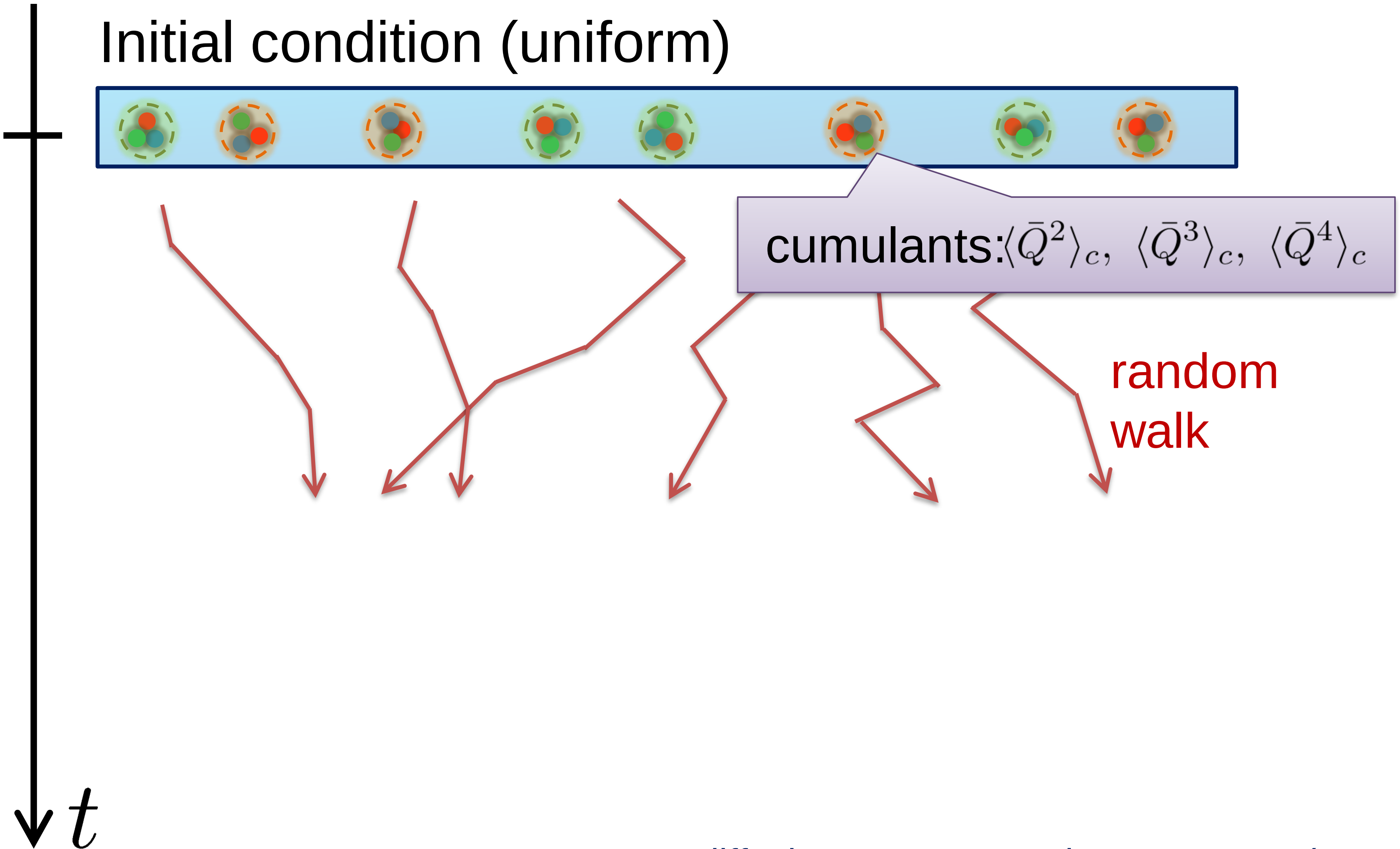
Time Evolution of Fluctuations

Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000
Shuryak, Stephanov, 2001



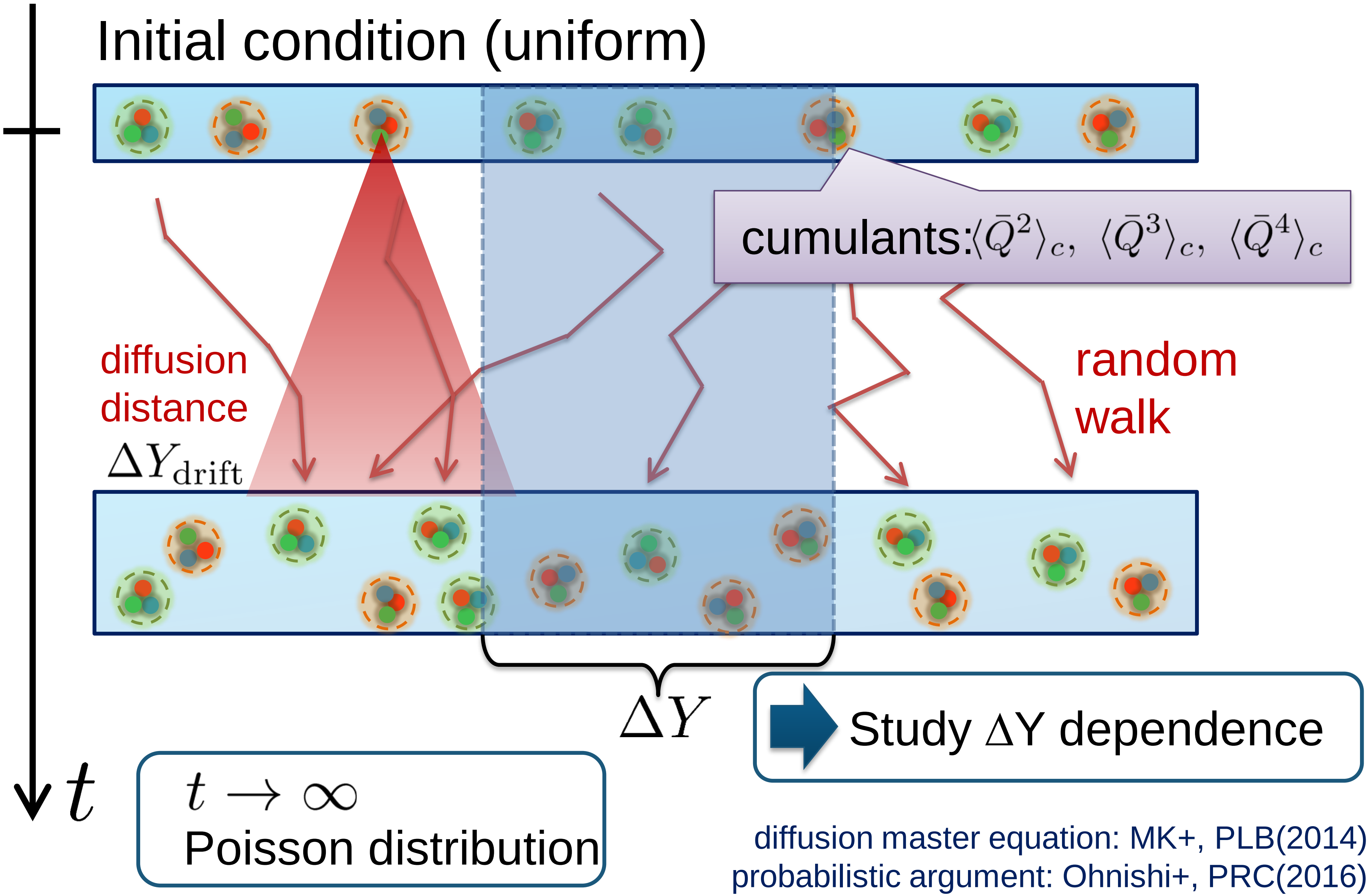
Fluctuations in ΔY continue to change until kinetic f.o.

(Non-Interacting) Brownian Particle Model



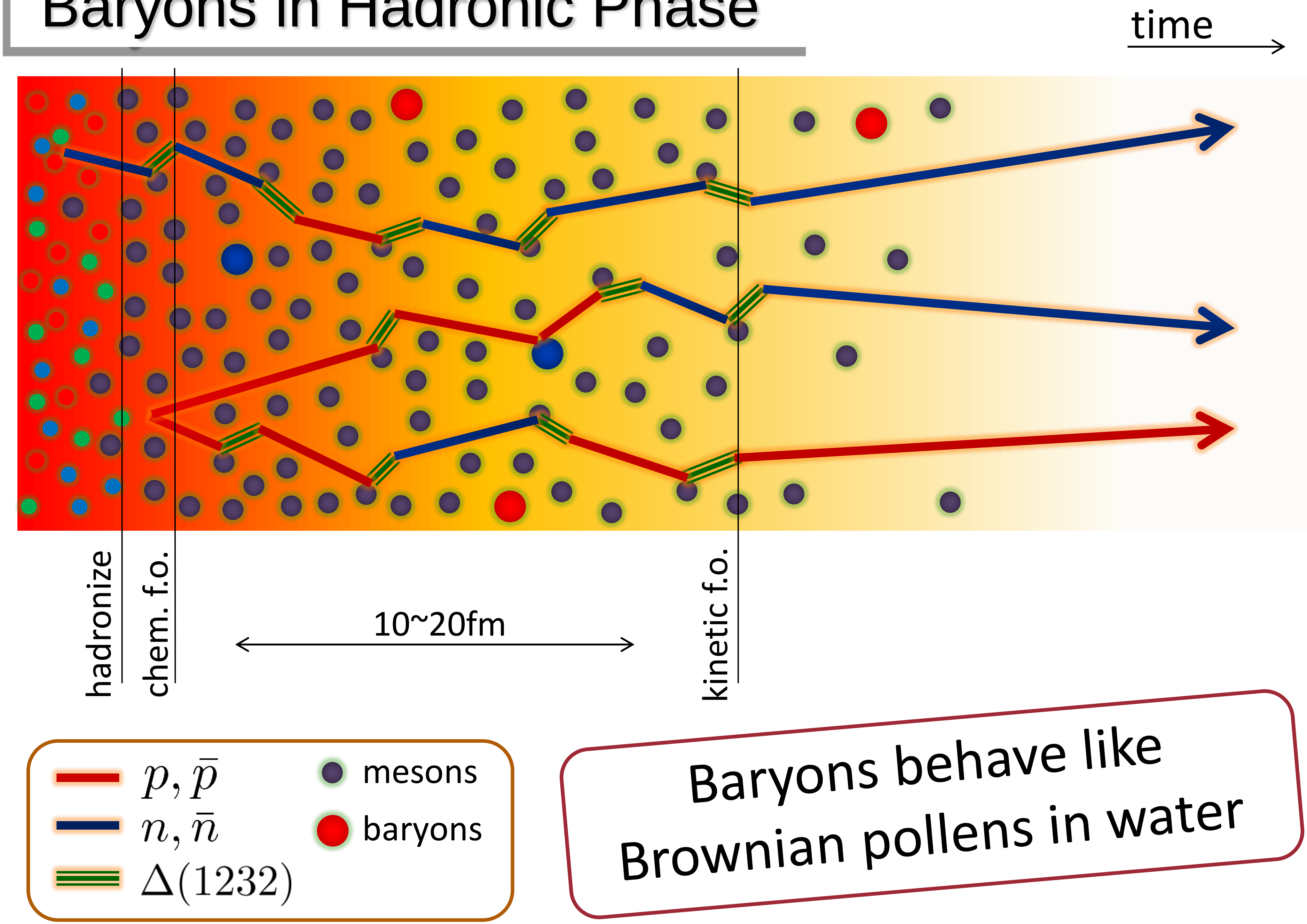
diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

(Non-Interacting) Brownian Particle Model



diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

Baryons in Hadronic Phase

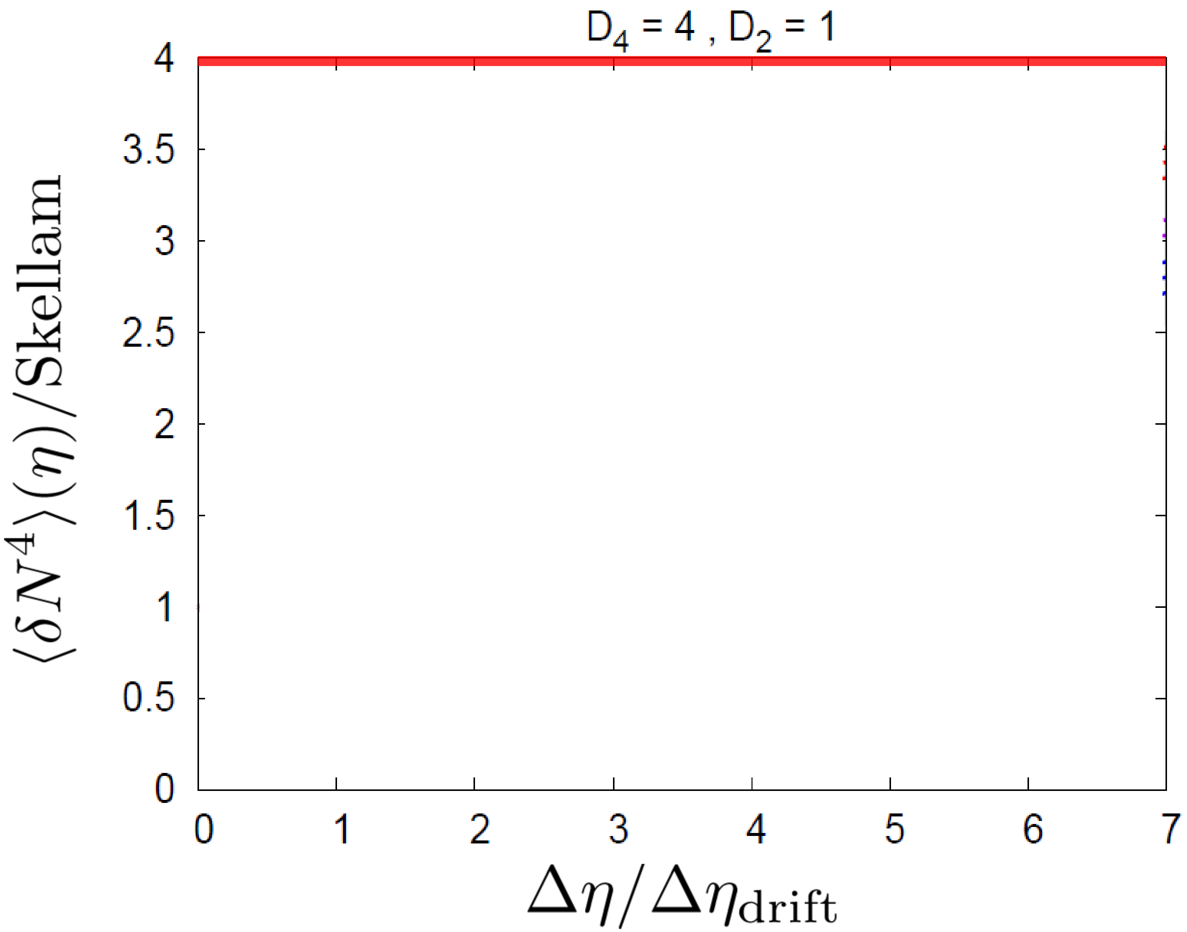


Baryons behave like Brownian pollens in water

4^{th} Order Cumulant

MK+ (2014)
MK (2015)

Before the diffusion



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

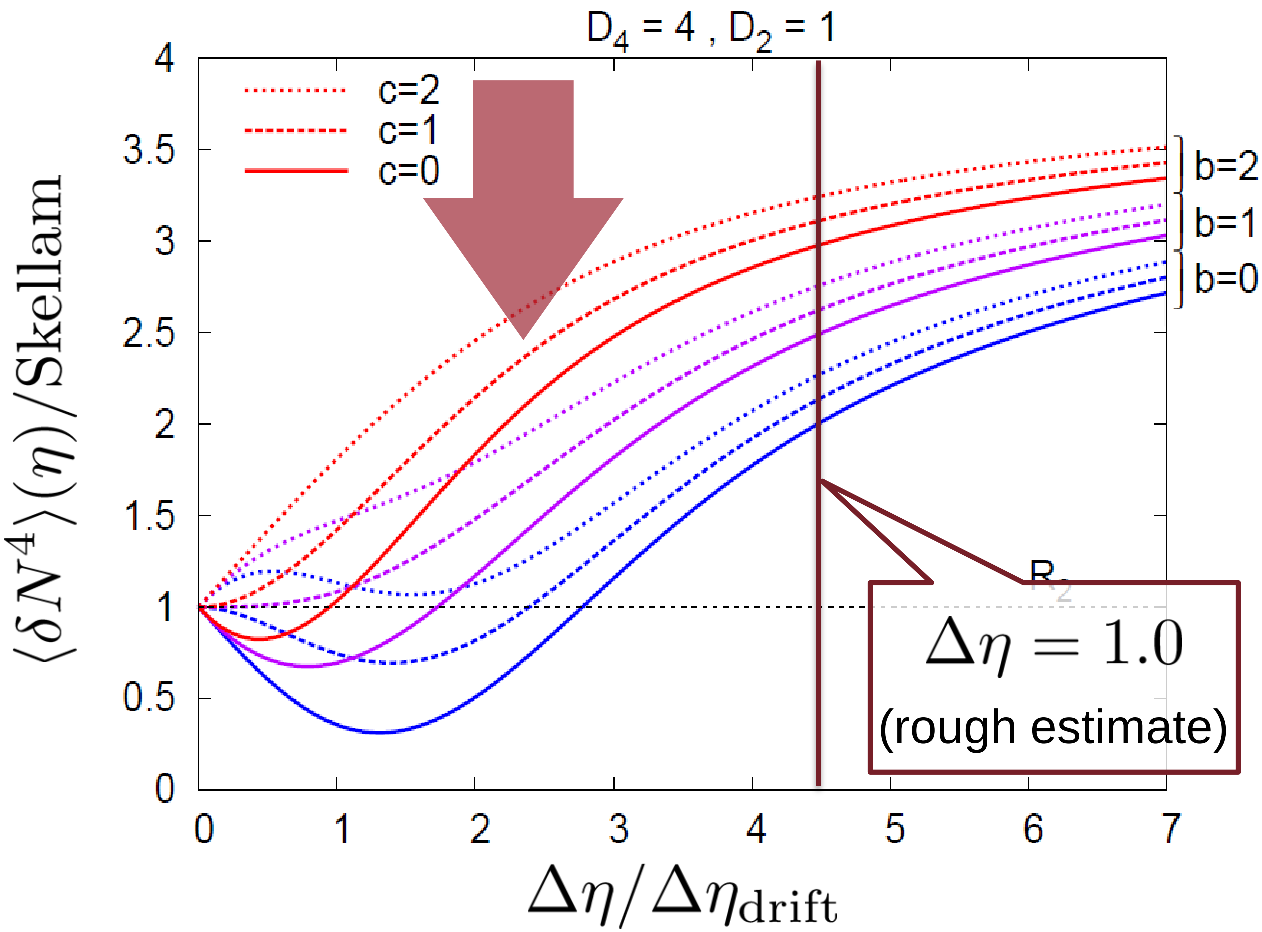
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

4^{th} Order Cumulant

MK+ (2014)
MK (2015)

After the diffusion



Initial Condition

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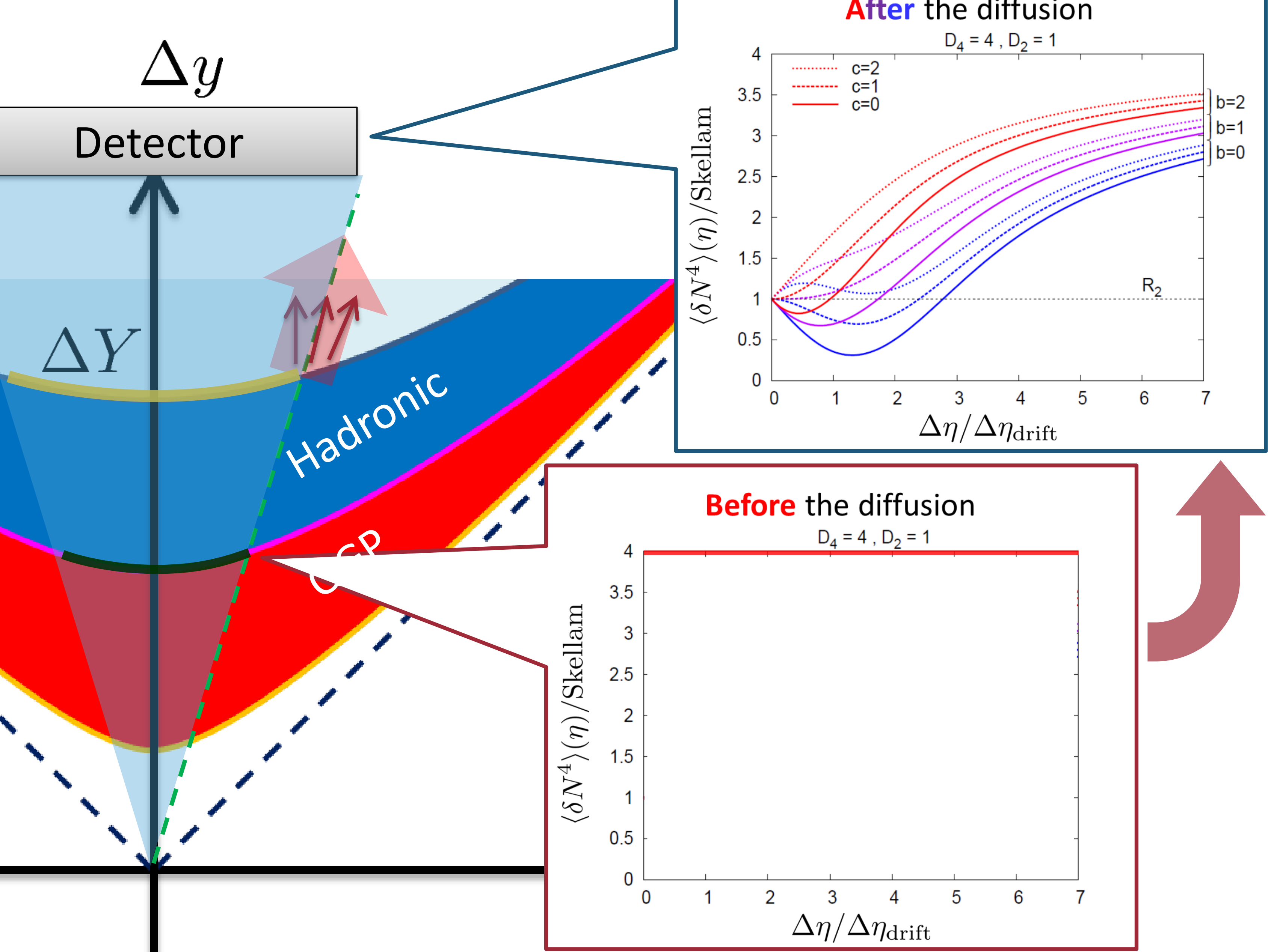
$$c = \frac{\langle Q^2_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} = 1$$

- Cumulant at small $\Delta\eta$ is modified toward a Poisson value.
- Non-monotonic behavior can appear.

Time Evolution of Fluctuations



As a result of a simple random walk…

Rapidity Window Dep.

4th-order cumulant

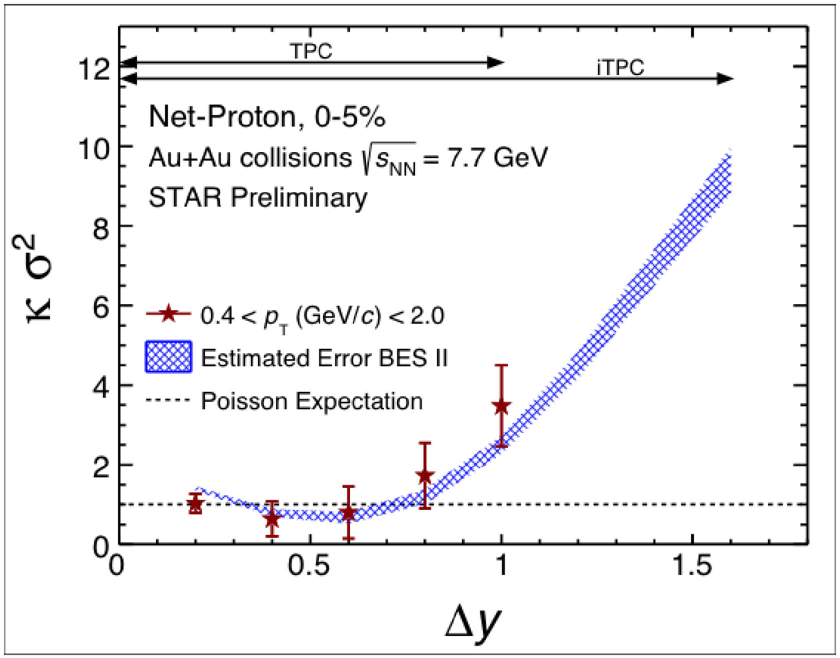
MK+, 2014
MK, 2015

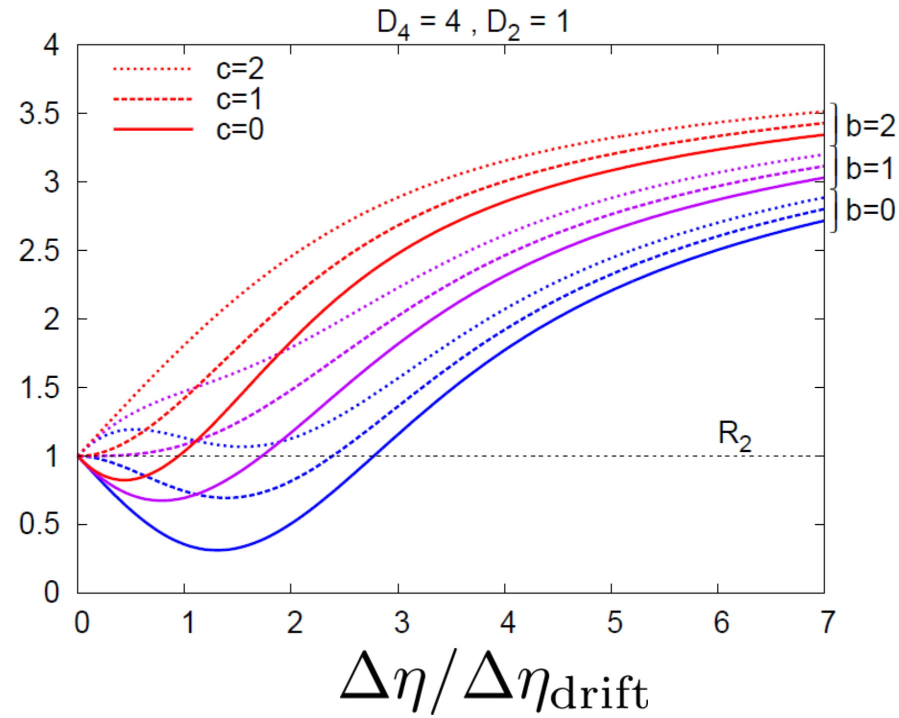
Initial Conditions

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad c = \frac{\langle Q^2_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

STAR Collab. (X. Luo, CPOD2014)





- Is non-monotonic Δη dependence already observed?
- Different initial conditions give rise to different characteristic Δη dependence. → Study initial condition

Finite volume effects: Sakaida+, PRC90 (2015)

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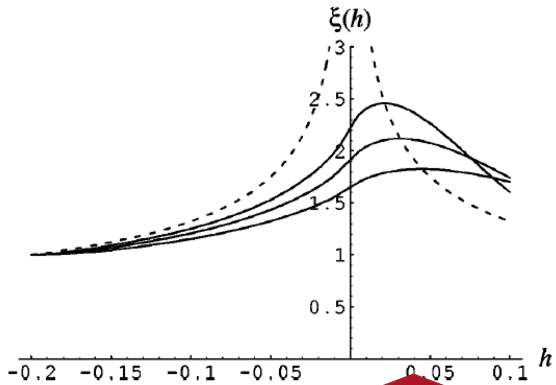
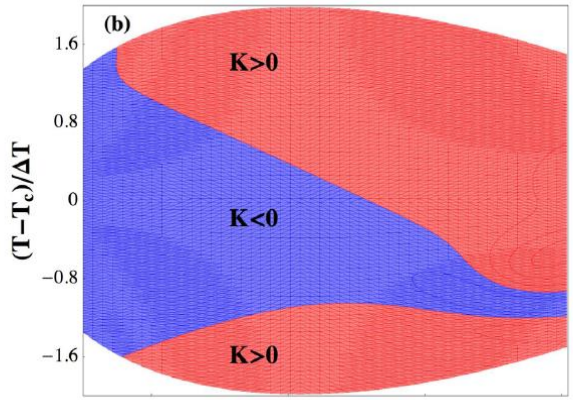
Dynamical Evolution of Critical Fluctuations

☐ Evolution of **spatially uniform "σ" mode**





Berdnikov, Rajagopal (2000)
Asakawa, Nonaka (2002)
Mukherjee+ (2015)
…

Model A
Model B

THIS STUDY

Evolution of **conserved charge fluctuations**

Sakaida+, PRC2017; Murata, MK, in prep.

1. Conserved charges are directly observable.
2. Soft mode at QCD-CP is a conserved mode.

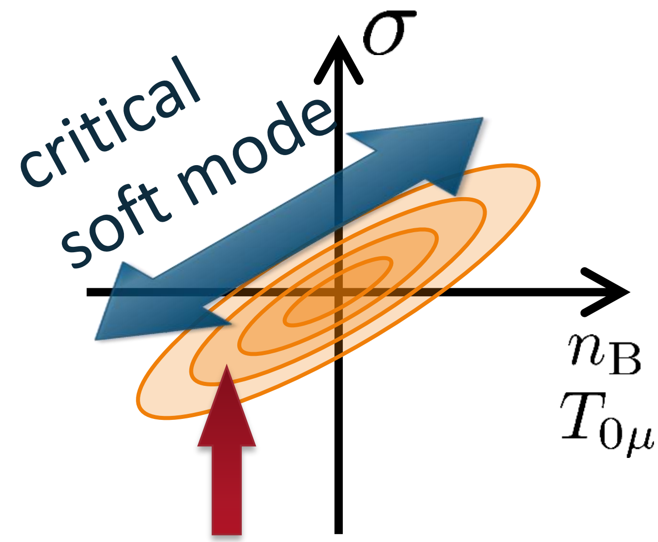
See also, Kapusta, Torres-Rincon (2012); Herold, Nahrgang, … (2015); …

Soft Mode of QCD-CP = Conserved Mode

Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

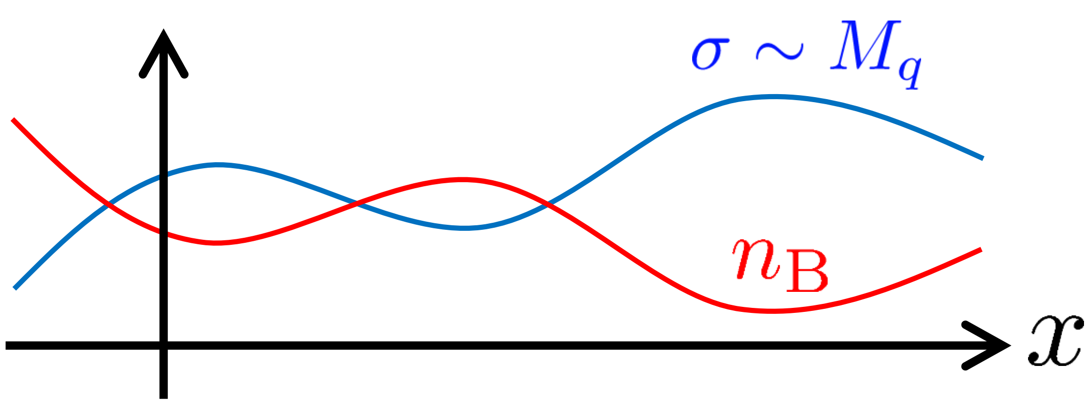
Fluctuations of σ and n_B are coupled around the CP!

$$\delta\sigma \simeq \delta n_B$$



σ: fast damping

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \cdots$$



Analysis of 2nd-order Cumulant

Sakaida+ (2017)

Evolution of baryon number density

Stochastic Diffusion Equation

$$\partial_t n = D(t)\partial_x^2 n + \partial_x \xi$$

$$\langle \xi(x_1,t_1)\xi(x_2,t_2)\rangle = 2D\chi_2\delta^{(2)}(1-2)$$

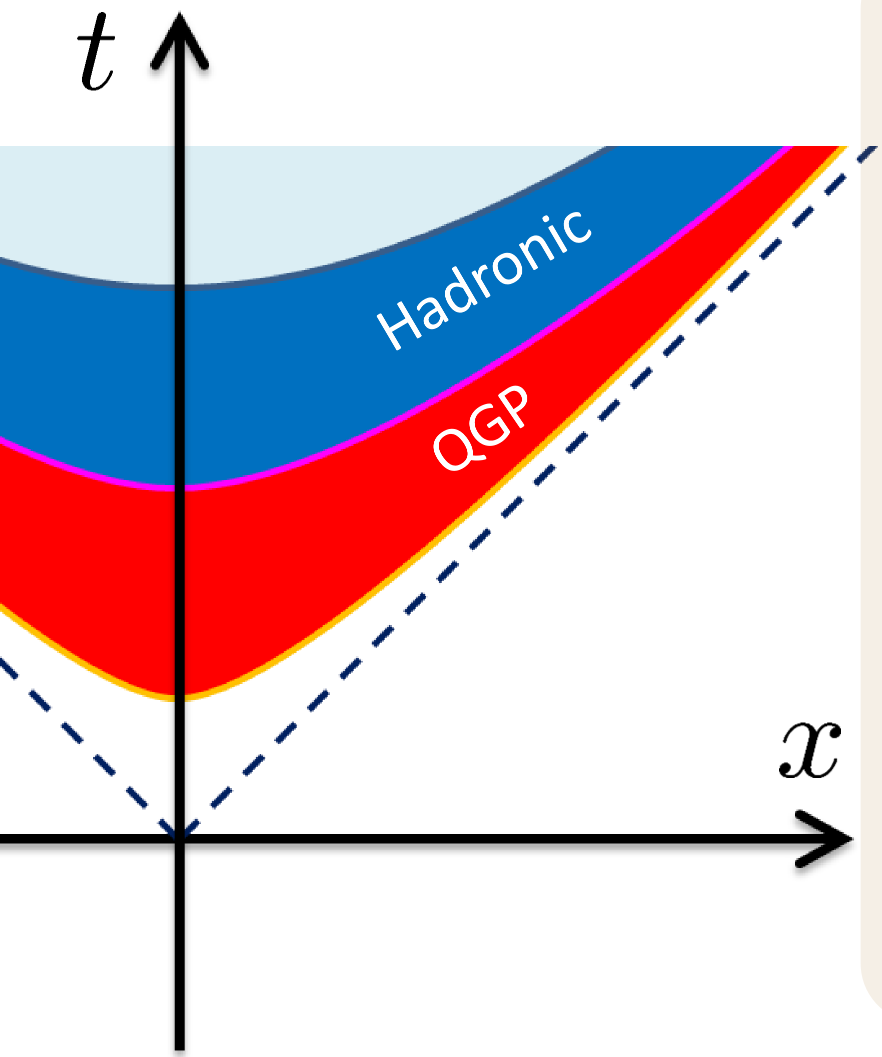
$D(t),\ \chi_2(t)$:parameters characterizing criticality

- Analytic solution is obtained.
- Study 2nd order cumulant & correlation function.

Our Main Conclusion

Non-monotonicity in cumulants or correlation func. = Signal of QCD-CP

Bjorken Expansion



Cartesian coordinates

$$\partial_t n = D(t)\partial_x^2 n + \partial_x \xi$$

Milne coordinates

$$\partial_\tau n = \frac{D(t)}{\tau^2}\partial_Y^2 n + \frac{1}{\tau}\partial_Y \xi - \frac{n}{\tau}$$

suppression of diffusion

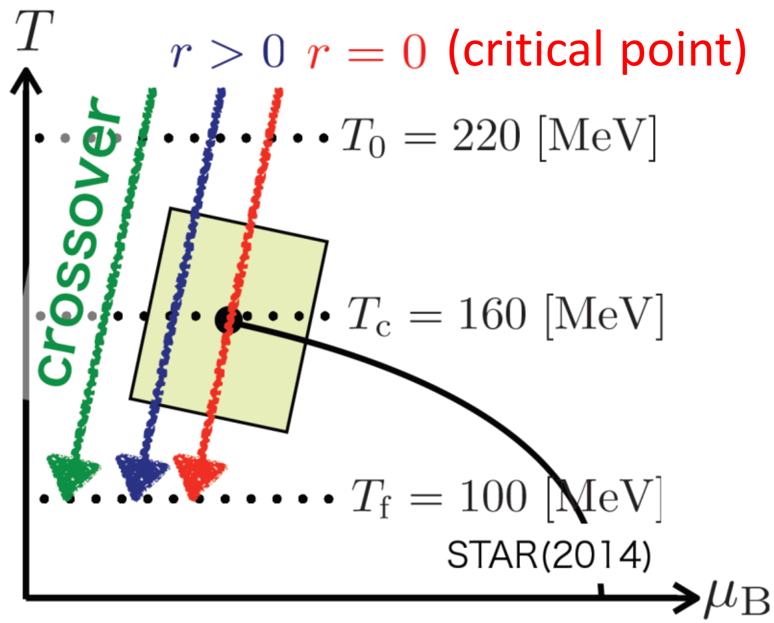
density reduction

Parametrizing D(τ) and χ(τ)

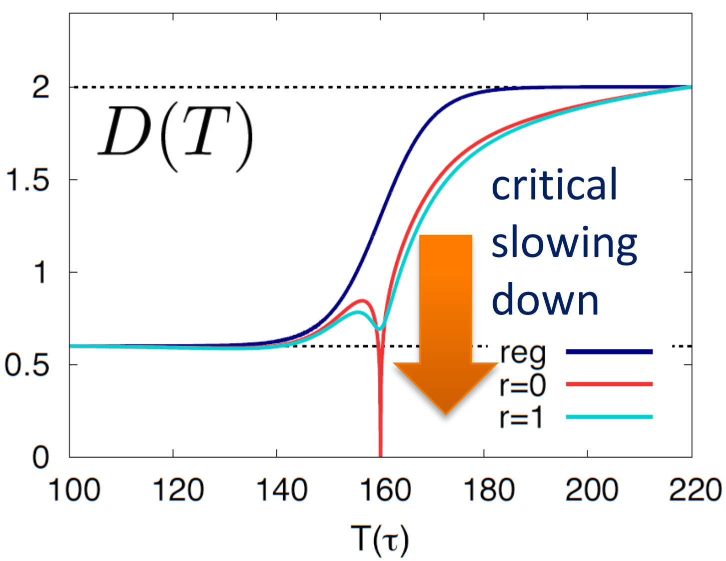
Critical behavior

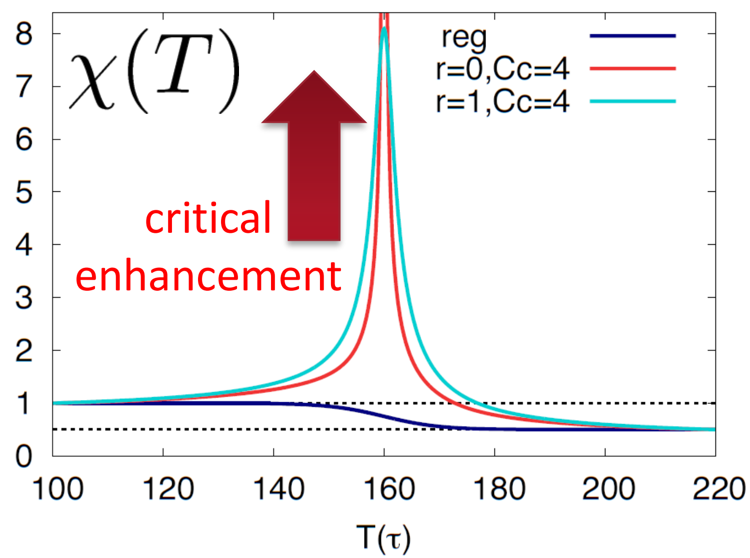
- 3D Ising (r,H)
- model H

Berdnikov, Rajagopal (2000)
Stephanov (2011); Mukherjee+(2015)



Temperature dep.





Cumulants and Correlation Function

$$Q = \int_V dx\, n(x)$$

total charge

charge density

$$\langle \delta Q^2 \rangle = \int_V dx dy \langle \delta n(x) \delta n(y) \rangle$$

2nd order cumulant (fluctuation)

correlation function

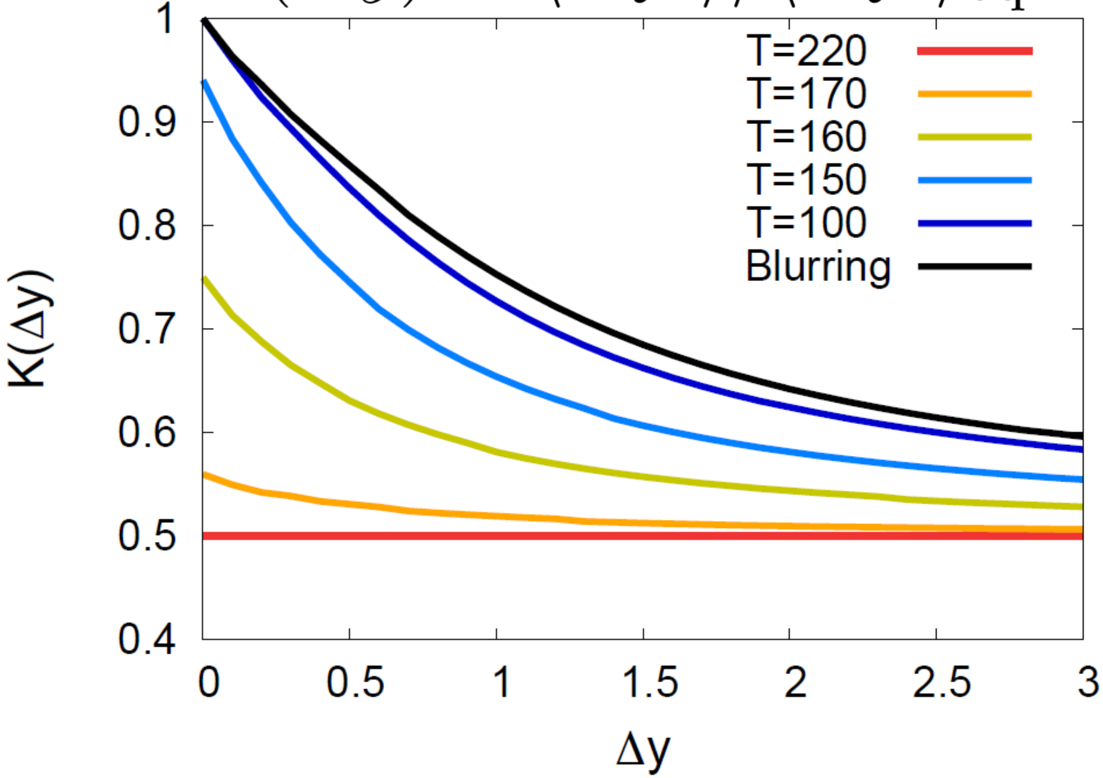
1-to-1 correspondence

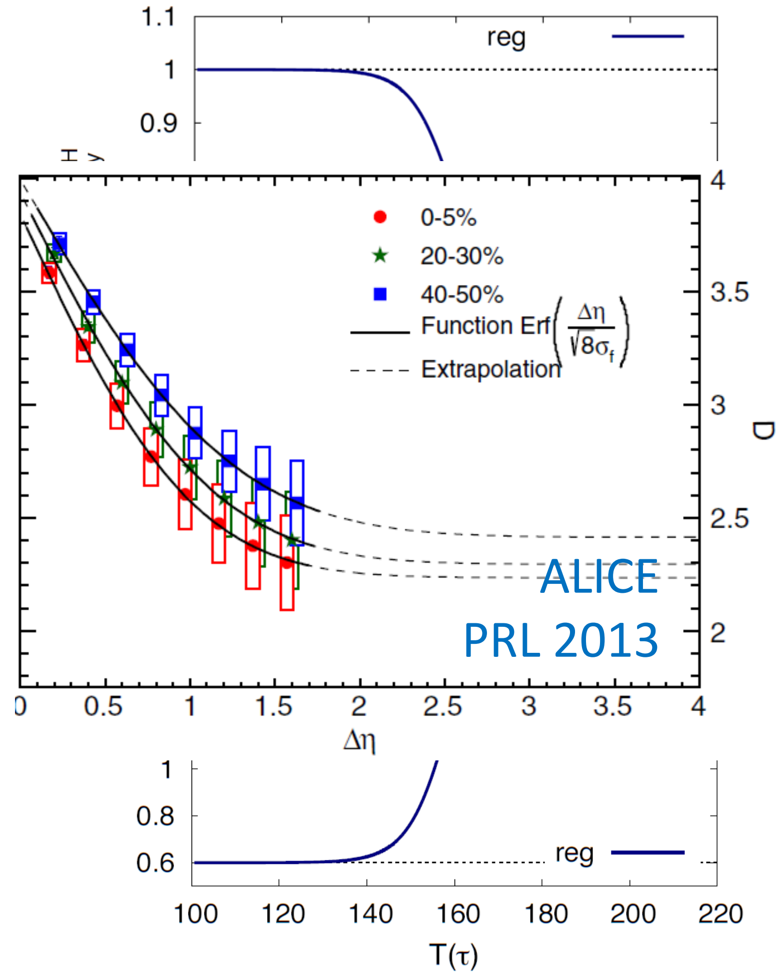
1-dim case

$$\langle \delta Q^2 \rangle_{\Delta y} = \int_{\Delta y} dy (\Delta y - |y|) \langle \delta n(y) \delta n(0) \rangle$$

Crossover / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$



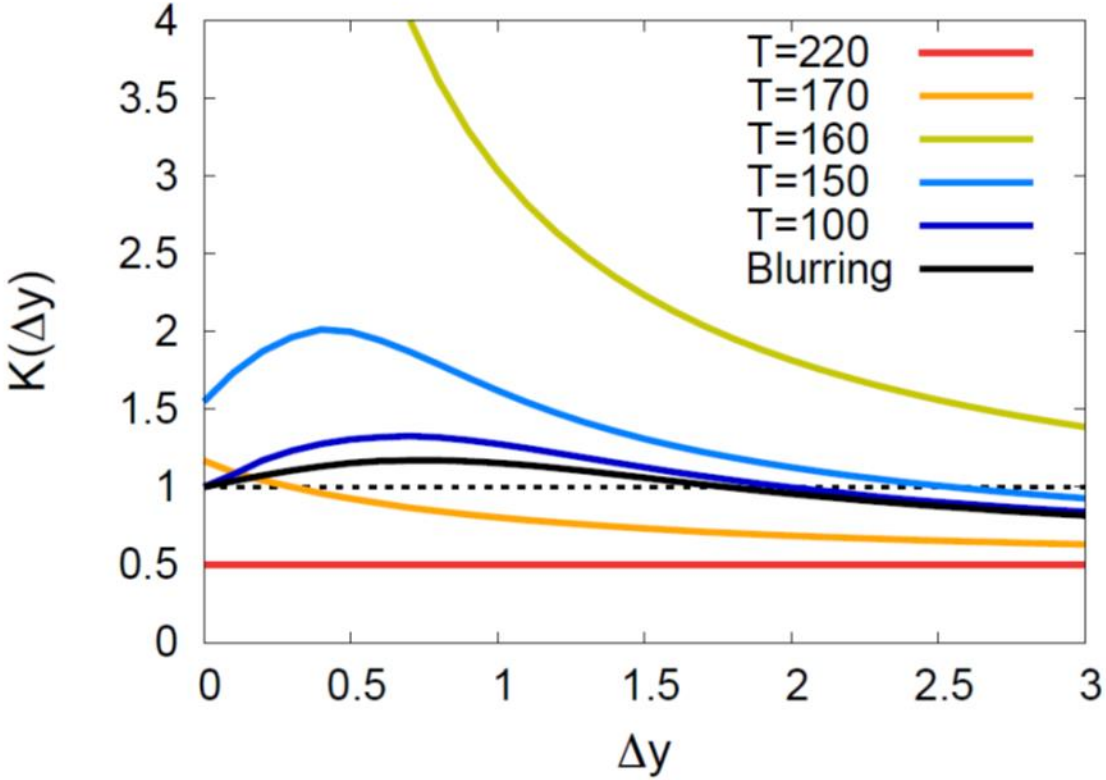


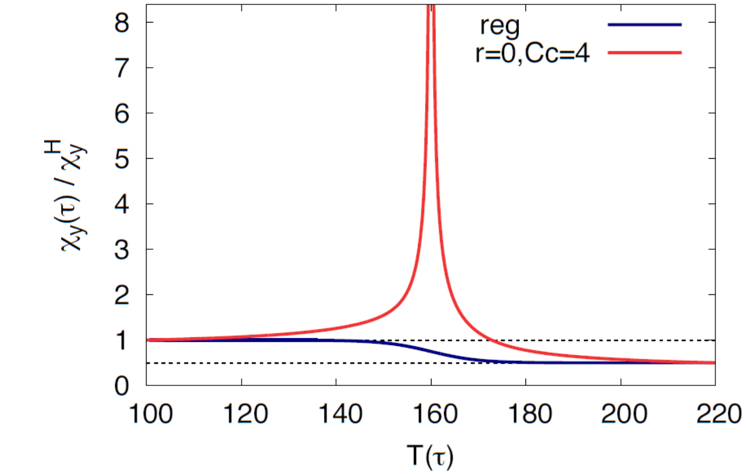
- ☐ monotonically decresing

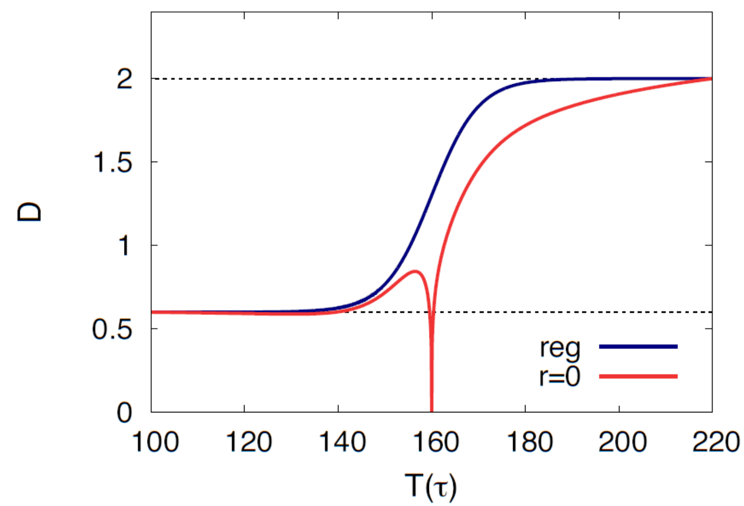
Analytic result	$\chi(\tau)$ monotonically increasing	⇨	$K(\Delta y)$ monotonically decreasing

Critical Point / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\mathrm{eq.}}$$







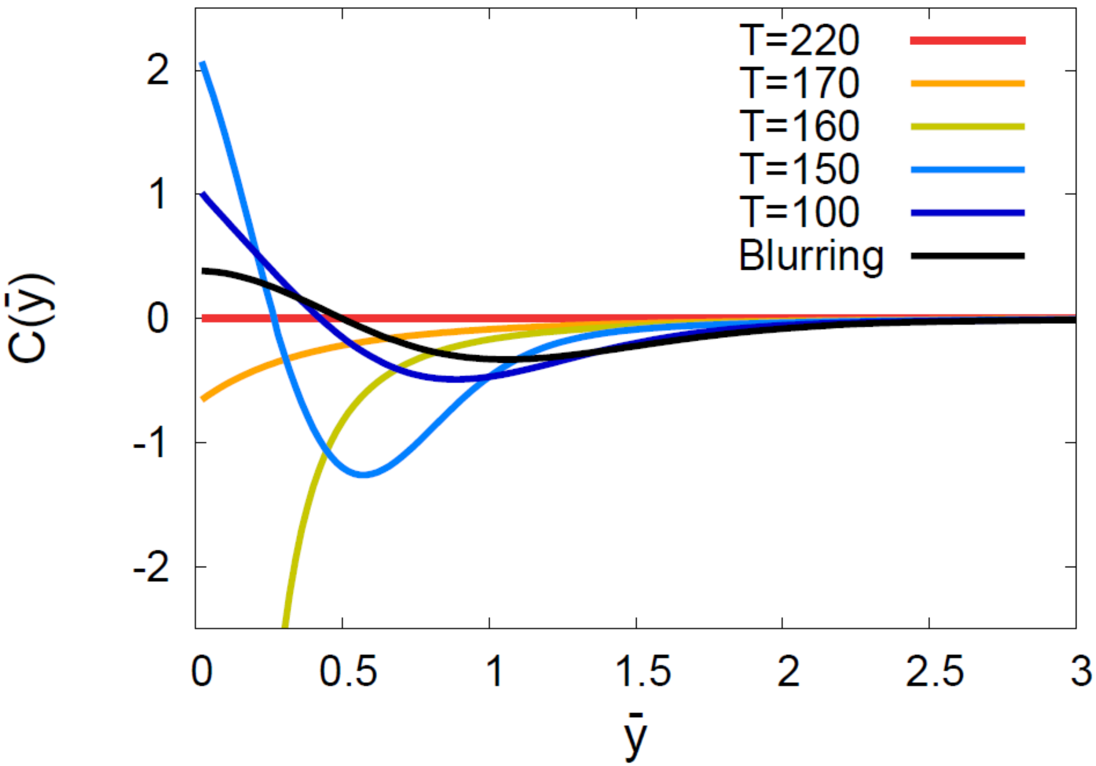
- non-monotonic Δy dep.

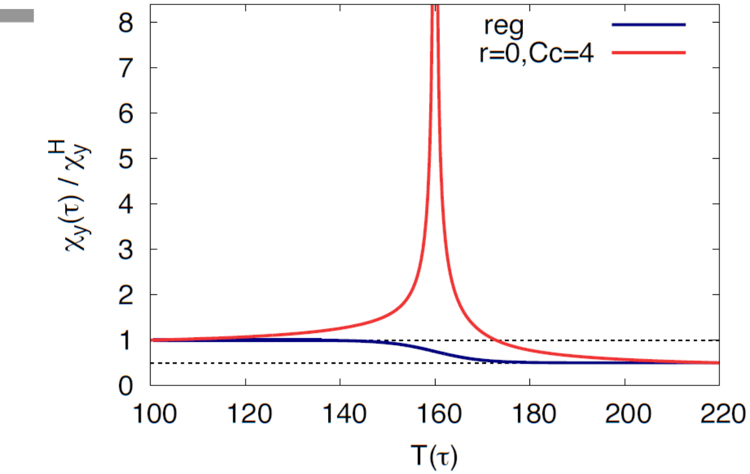
Analytic result: $K(\Delta y)$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

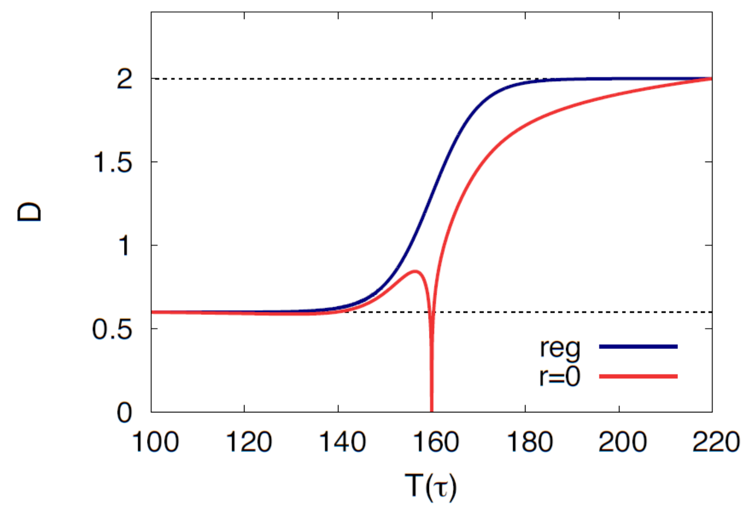
See also,
Wu, Song
arXiv: 1903.06075

Criticap Point / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\mathrm{hadron}}$$







- non-monotonic Δy dep.

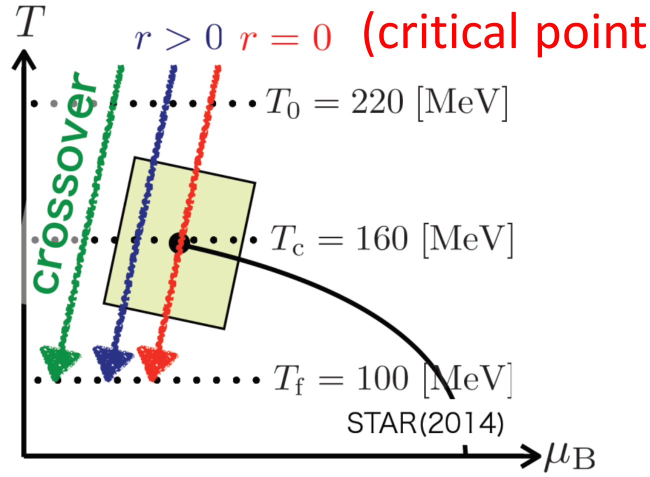
Analytic result

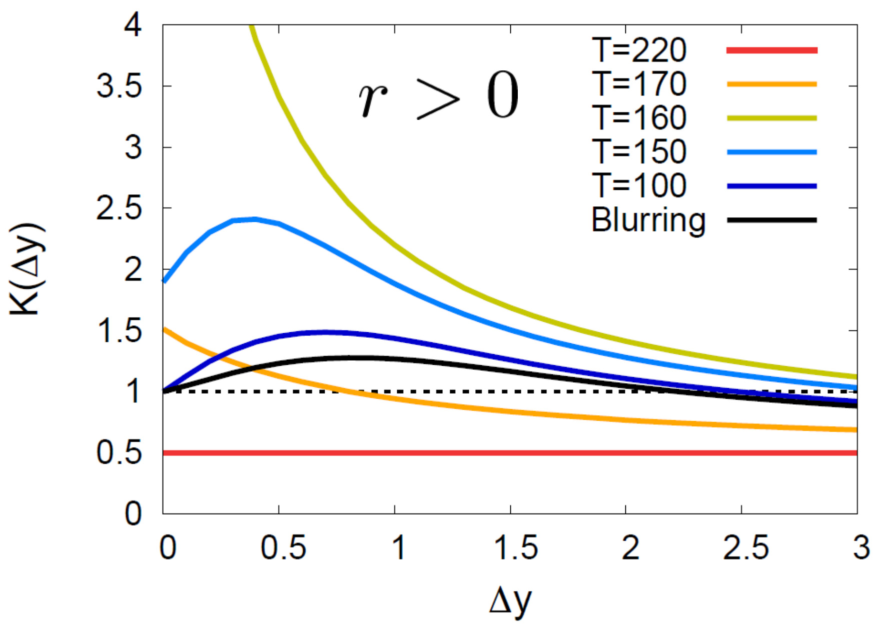
$C(\Delta y)$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

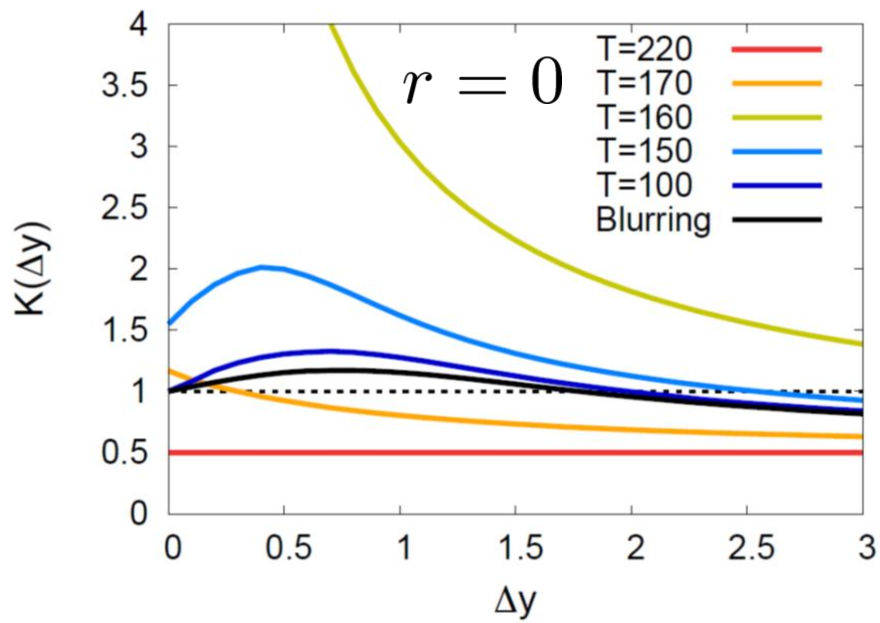
See also,
Wu, Song
arXiv: 1903.06075

Away from the CP

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$







- Signal of the critical enhancement can be clearer on a path away from the CP.

Away from the CP → Weaker critical slowing down

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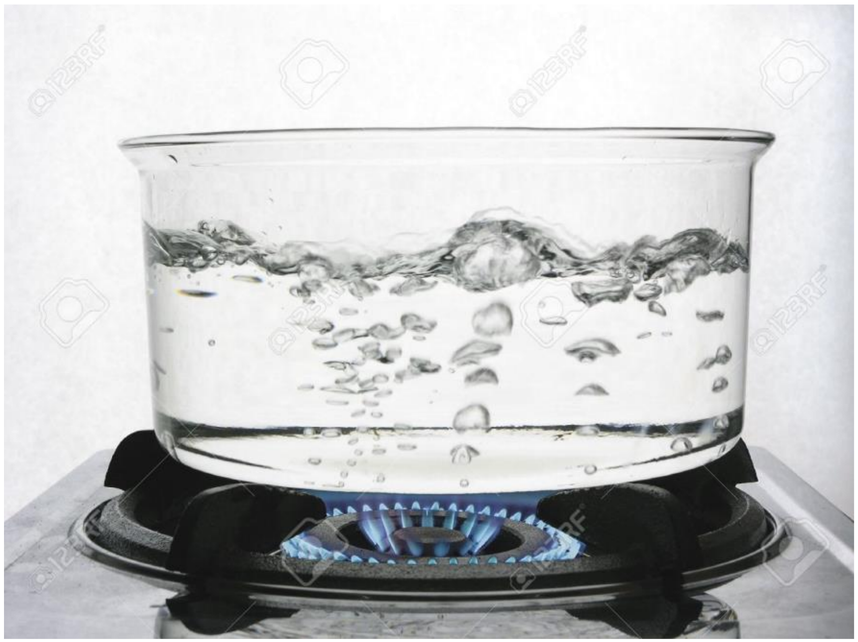
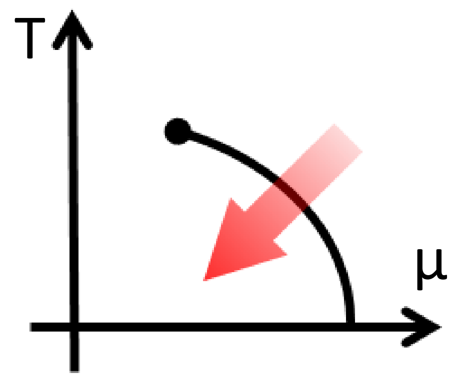
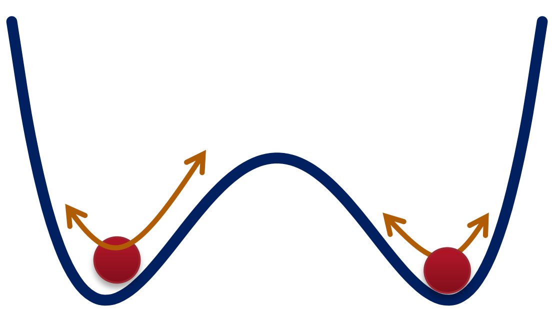
1^{st}-Order Transition





- ☐ Domain formation
- ☐ Non-uniform system



Herold, Nahrgang, et al. (2011~); Steinheimer, Randrup (2012; 2013)

Including Non-Linearity

Nahrgang, Bluhm, Schafer, Bass (2018)

$$\partial_\tau n = \frac{D}{\tau^2}\partial_Y^2 n + \frac{1}{\tau}\partial_Y \xi(\eta,\tau) - \frac{n}{\tau}$$

Include non-linear effects

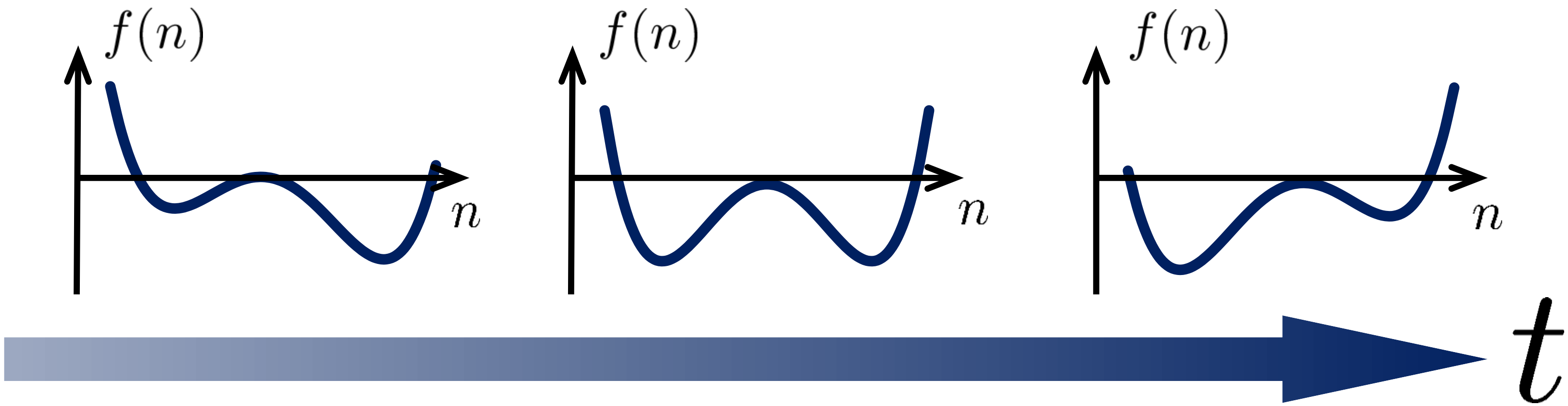
$$\partial_\tau n = \frac{\Gamma(n)}{\tau^2}\partial_Y^2 \frac{\delta F[n]}{\delta n(x)} + \frac{1}{\tau}\partial_Y \xi(\eta,\tau) - \frac{n}{\tau}$$

$$F[n(x)] = \int dx f(x)$$

- Diffusion equation: $f(n) = \frac{a}{2}n^2, \quad D = \Gamma a$
- solve numerically

Free Energy

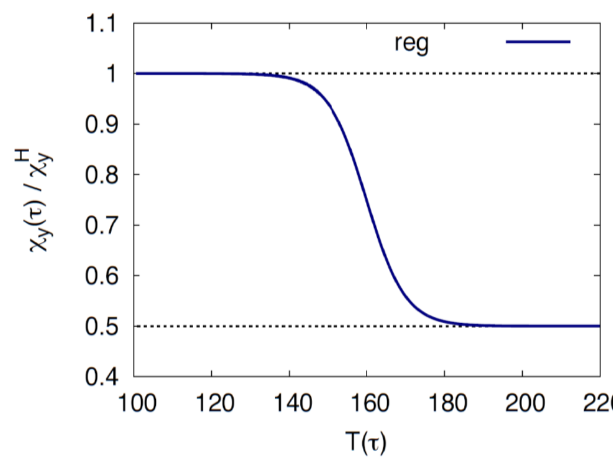
- **At 1st transition point**



- **Large and small n**

$$\chi(n) = \frac{\partial^2 f}{\partial n^2} \quad \to \chi_{\mathrm{QGP}} \ (n \to \infty)$$

$$\to \chi_{\mathrm{hadron}} \ (n \to 0)$$ Poisson



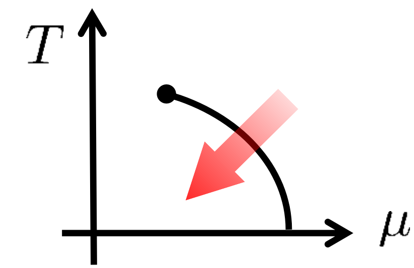
Modeling 1st Transition

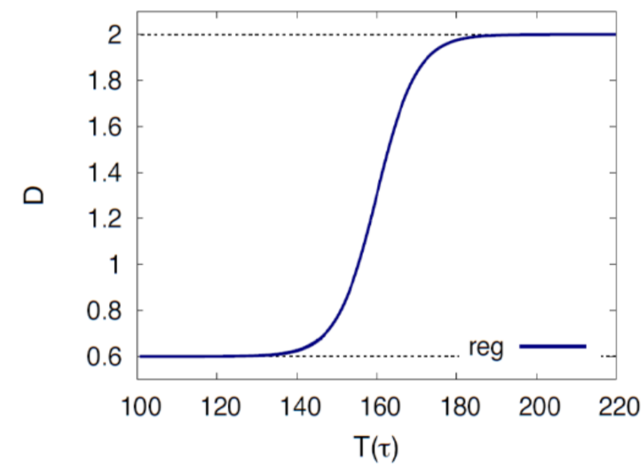
$$\partial_\tau n = \Gamma(n)\partial_Y^2 \frac{\delta F[n]}{\delta n(Y)} + \partial_Y \xi$$

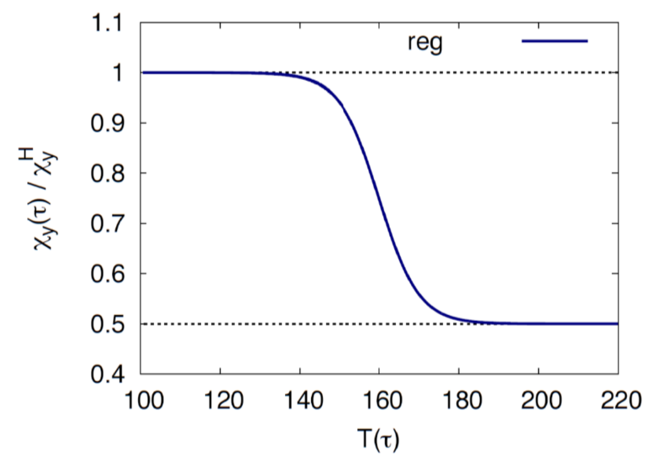
$$\langle \xi(Y_1,\tau_1)\xi(Y_2,\tau_2)\rangle = 2A\delta^{(2)}(1-2)$$

- $$f(n) = \frac{1}{2}a(n-n_s)^2 + \frac{1}{4}b(n-n_s)^4 + c(\tau)n + k(\partial_Y n)^2$$
- Γ: positive
- adjust Γ and A to reproduce the behavior of D at small and large n

$$\tilde{D} = \Gamma\left(\frac{\partial^2 f}{\partial n^2} + X\right) \qquad A = 2D\chi_2$$

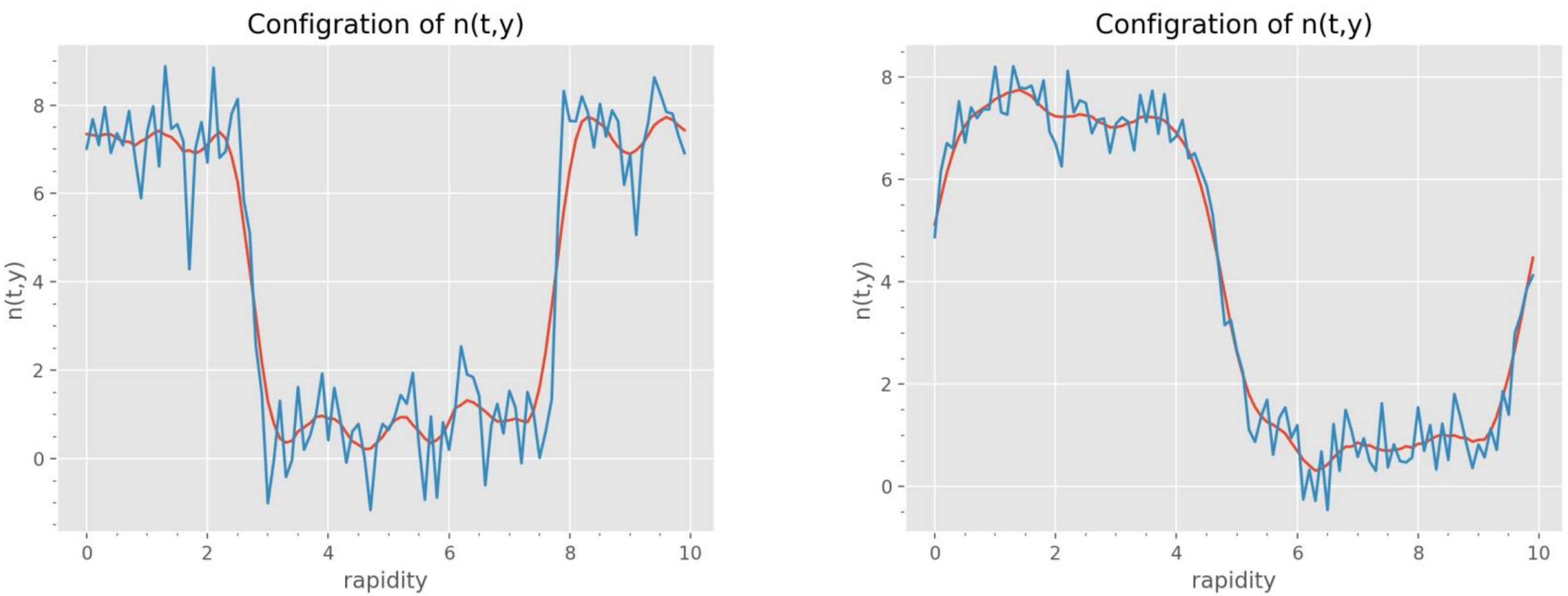






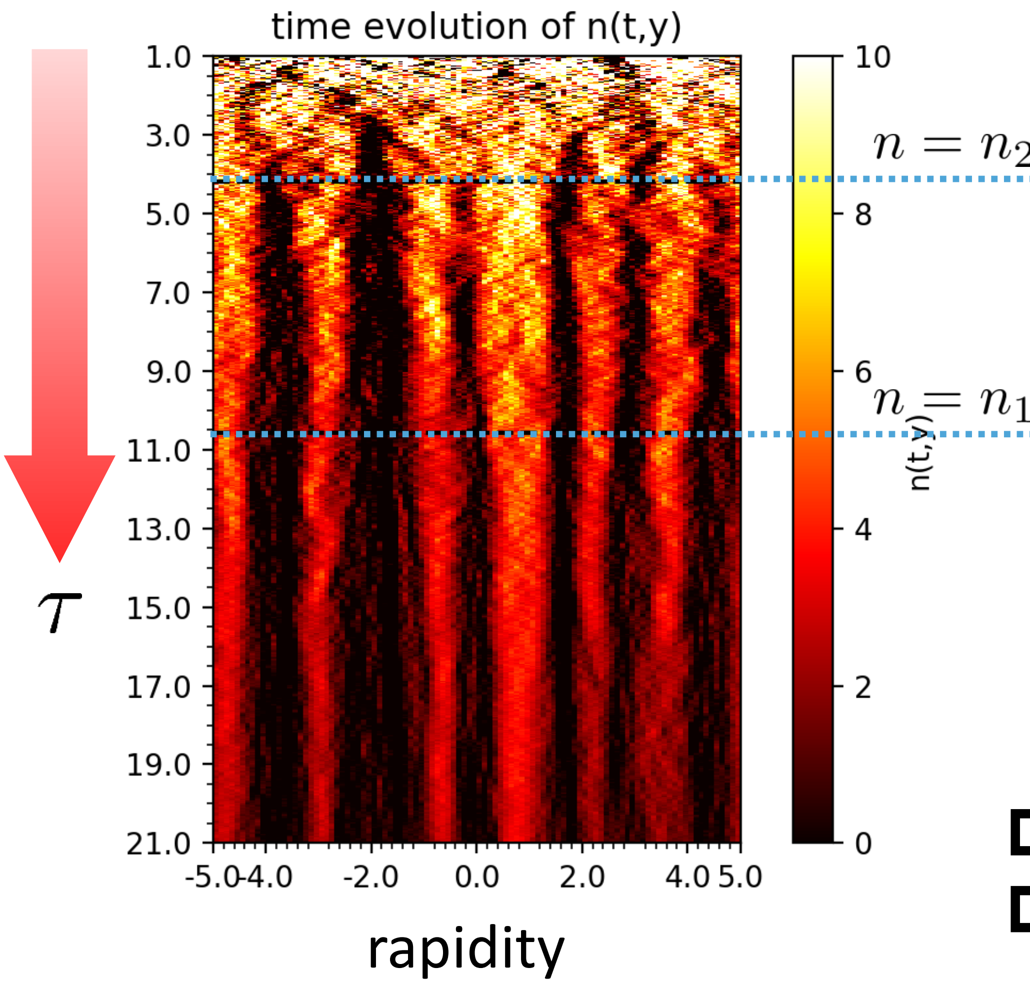
Configurations in Equilibrium

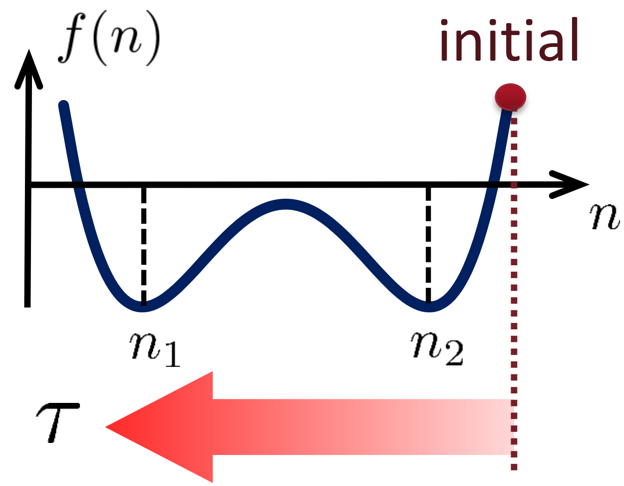
$$\partial_\tau n = \frac{\Gamma(n)}{\tau^2}\partial_Y^2 \frac{\delta F[n]}{\delta n(x)} + \frac{1}{\tau}\partial_Y \xi(\eta,\tau) - \frac{n}{\tau}$$



- ❑ Domain formation
- ❑ Surface: thickness √(2k/a), surface tension

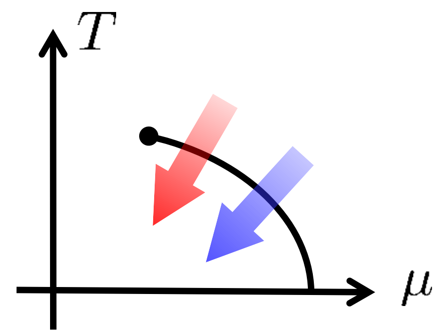
Time Evolution

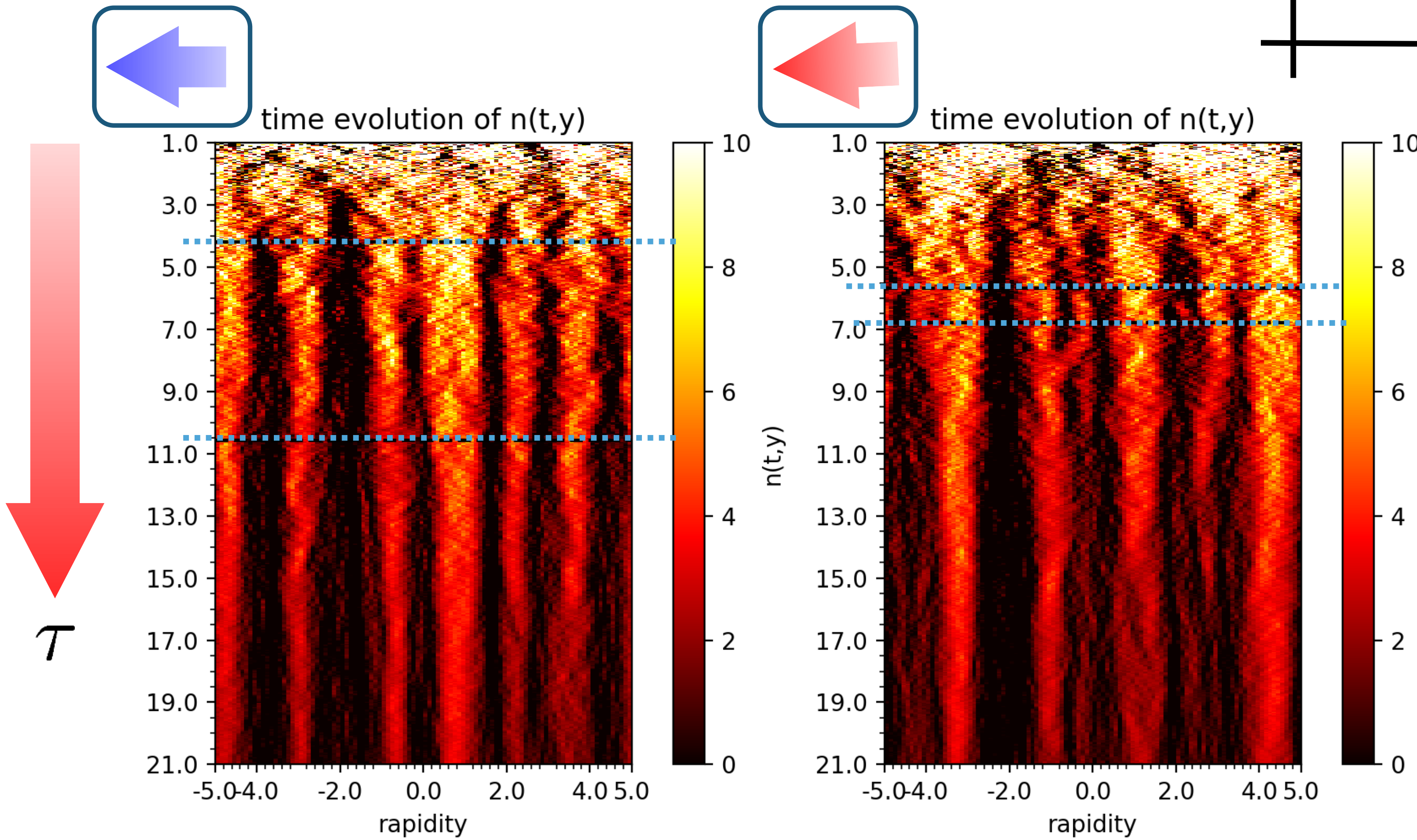




- ☐ Dynamical domain formation
- ☐ Domains survive even after 1st transition

Time Evolution



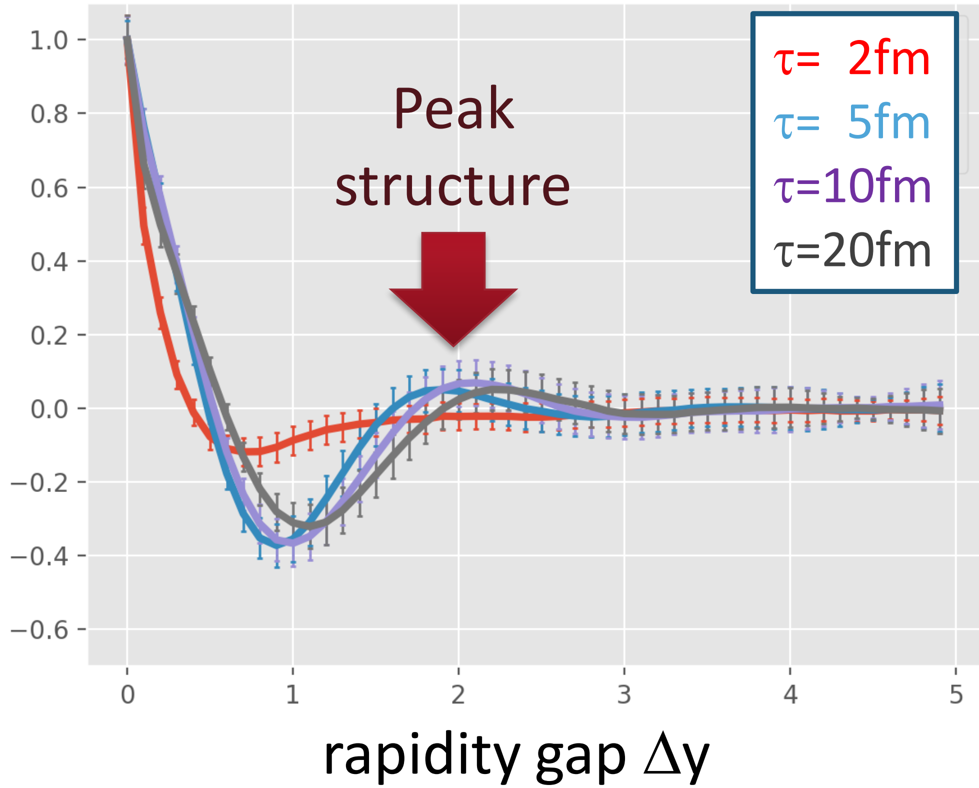


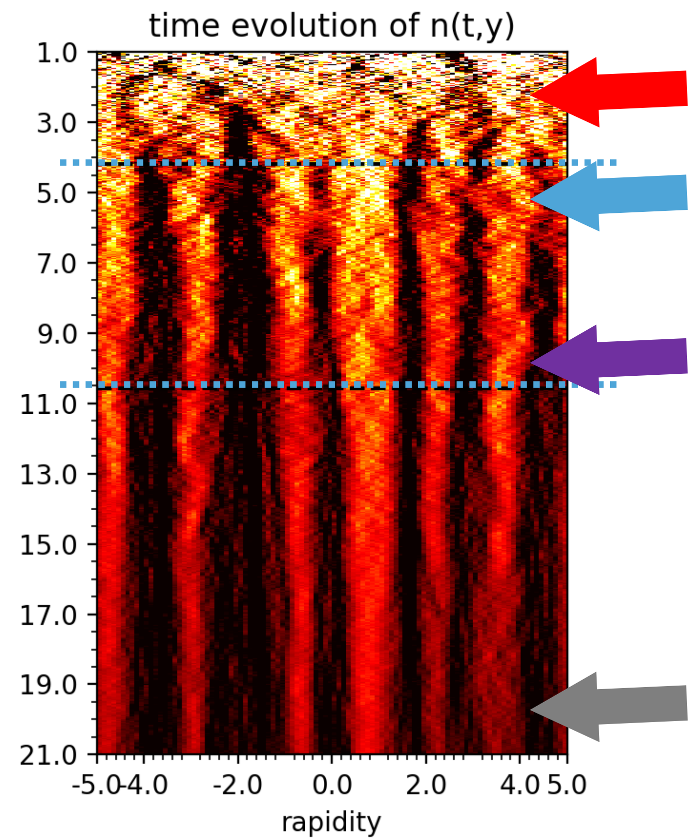
- Weaker 1st transition can also lead to formation of domains.

Correlation Function

Correlation Function

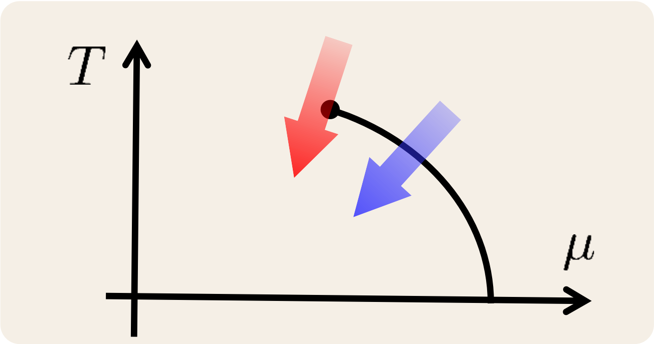
$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$



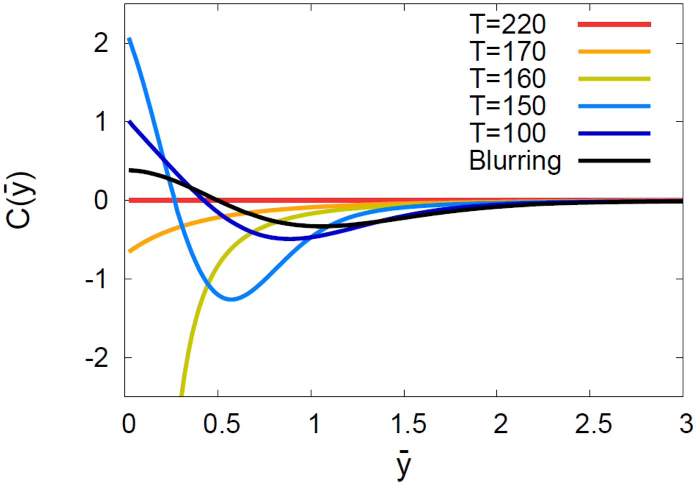


- Domain leads to a peak structure in C(y).
- The peak can survive even in the final state.

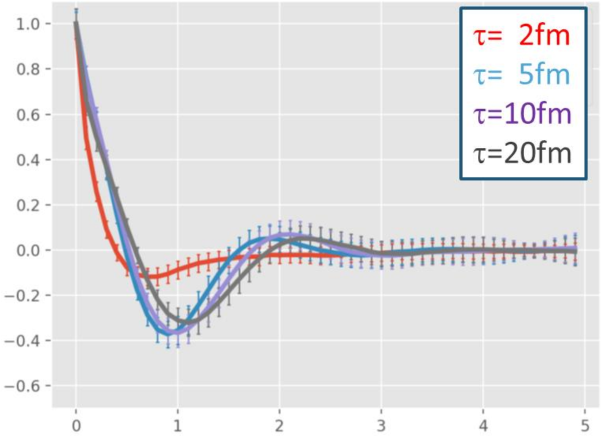
C(y) and Phase Diagram



Critical point/ crossover



1st order transition



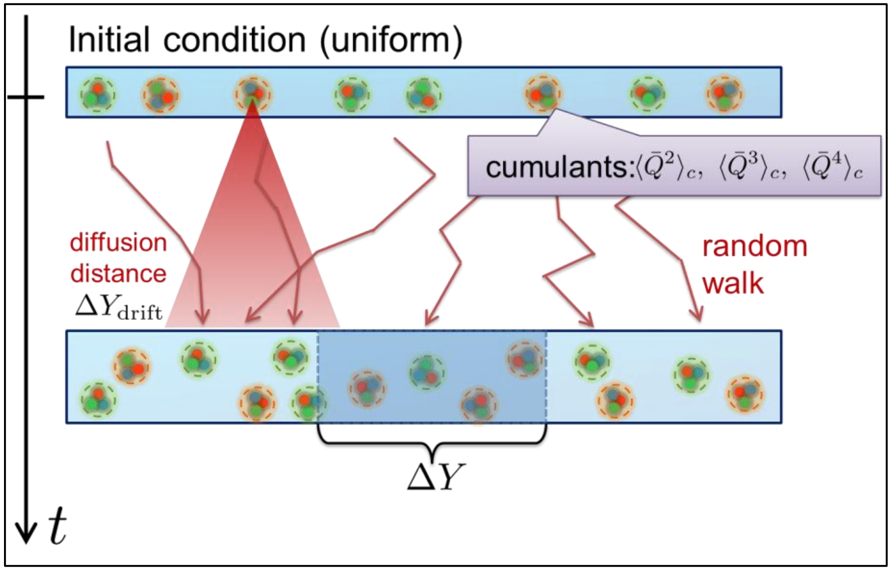
- ☐ Both critical phenomena & 1st transition lead to a minimum of the C(y) with different mechanisms.

Summary

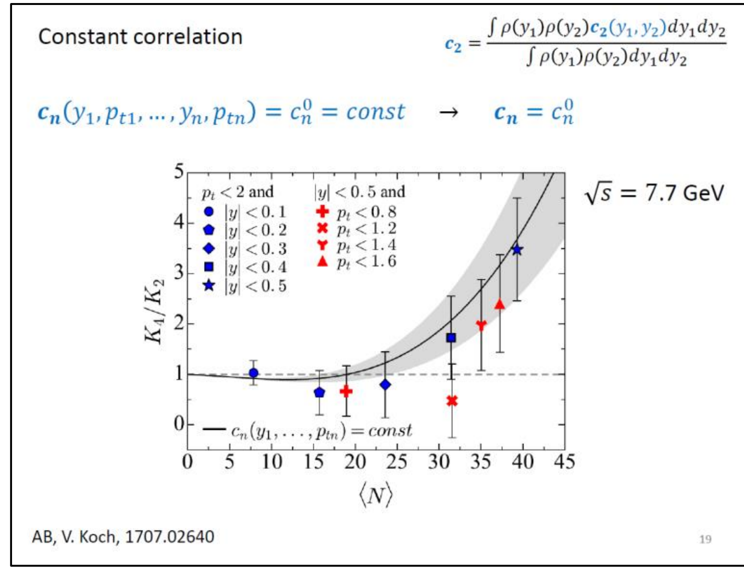
- Fluctuations observed in HIC are not in equilibrium.
- Plenty of information in rapidity window dependences of higher-order cumulants.
- 2nd-order cumulant (correlation function) already contains interesting information.
- Future
  - Evolution of higher-order cumulants around the critical point / 1st transition
  - combination to momentum (model-H)
  - more realistic model (dimension, Y dependence, …)

Translating Languages

Brownian particle model



From Bzdak's talk



$$\langle n^m \bar{n}^{\bar{m}} \rangle_{\rm fc} = \kappa_{m\bar{m}} \Delta y F_{m+\bar{m}}(\Delta y/d)$$

$$c^0_{m\bar{m}} = \frac{1}{2} \frac{\partial^2}{\partial \Delta y^2} \langle n^m \bar{n}^{\bar{m}} \rangle_{\rm fc} \Big|_{\Delta y \to 0} = \kappa_{m\bar{m}} \frac{\partial}{\partial \Delta y} F_{m+\bar{m}}(\Delta y/d) \Big|_{\Delta y \to 0}$$

$$= \frac{\kappa_{m\bar{m}}}{d} \frac{1}{\sqrt{(m+\bar{m})(2\pi)^{m+\bar{m}-1}}}$$

$\kappa_{m\bar{m}}$: F cumulants at initial condition

d : diffusion distance