Critical Fluctuations in HIC
~ Theory ~

FLUCTUAT NEC MERGITUR

Masakiyo Kitazawa

(Osaka U.)

RRTF "Dynamics of Critical Fluctuations: Theory – Phenomenology – HIC"
GSI, Darmstadt, 8/Apr./2019

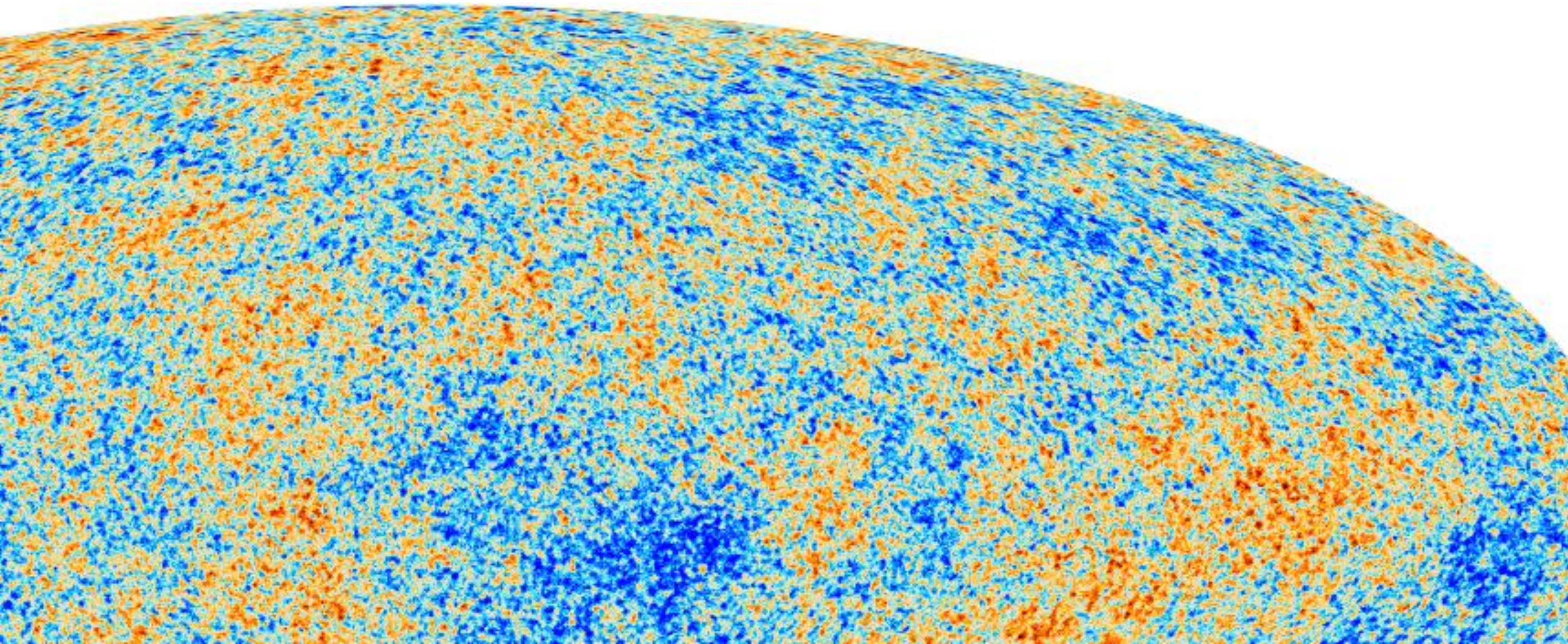
Fluctuat nec mergitur

"[She] is tossed by the waves, but does not sink"
from Wikipedia

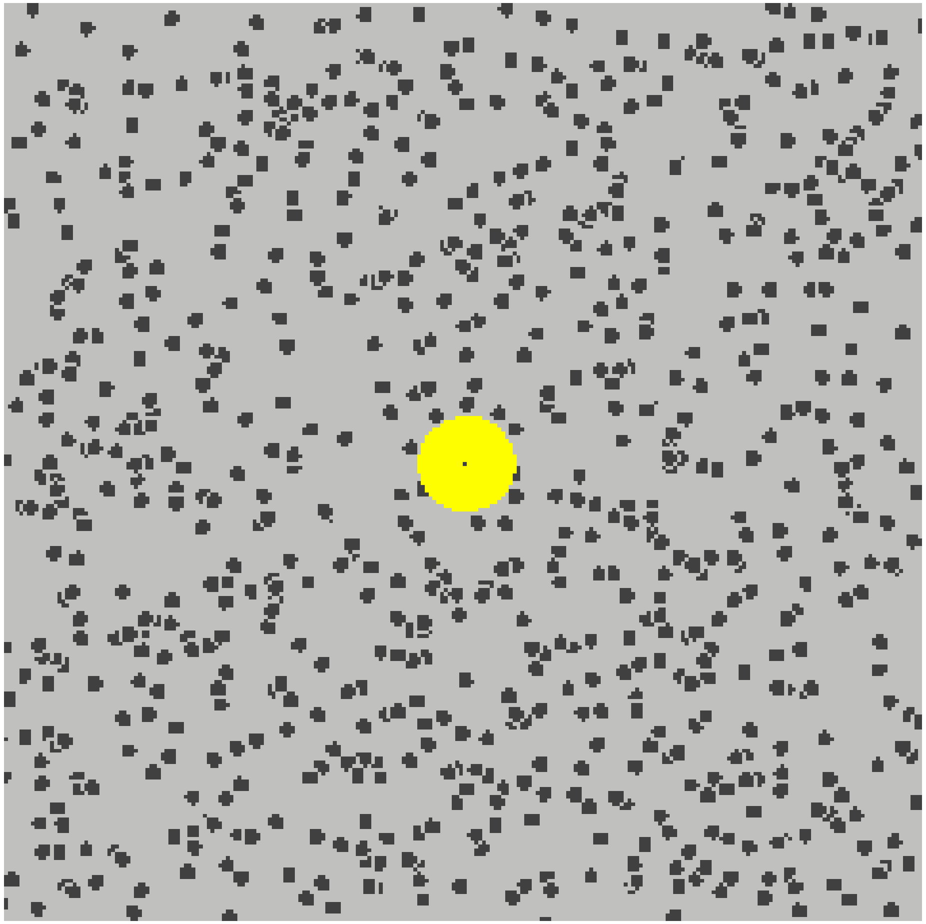
Coat of arms of Paris

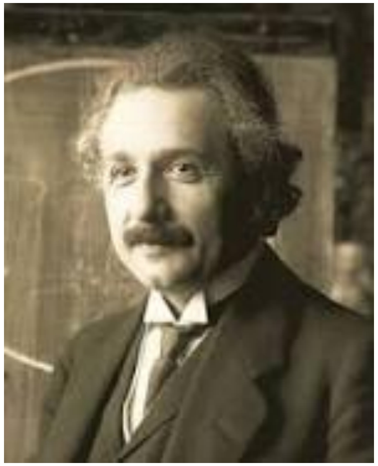
Why Fluctuations?

Physicists can feel hot early Universe
13 800 000 000 years ago
in tiny fluctuations of
cosmic microwave



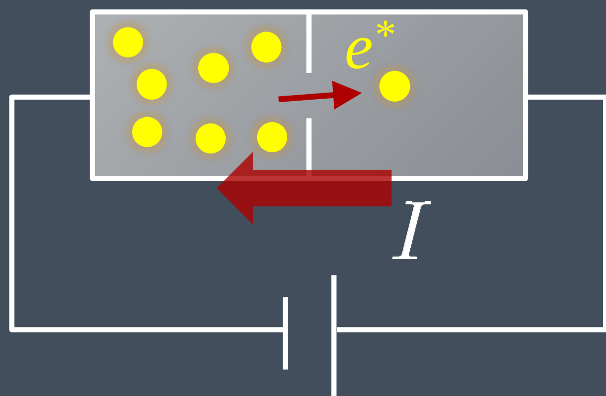
Physicists can feel the existence of microscopic atoms behind random fluctuations of Brownian pollens





A. Einstein
1905

Shot Noise



$$S_{\text{shot}} \sim \langle \delta I^2 \rangle$$

$$S_{\text{shot}} = 2e^* \langle I \rangle$$

charge of quasi-particles

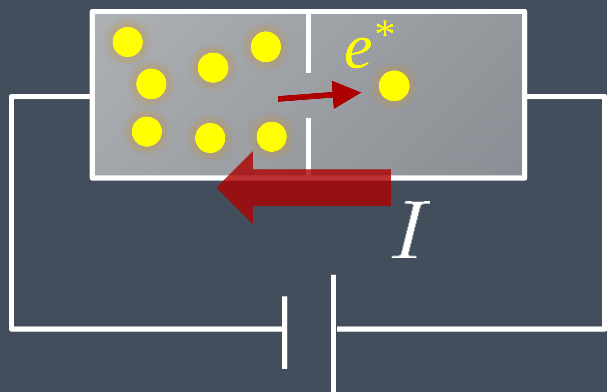
Total charge Q:

$$Q = e\langle N \rangle$$

$$\langle \delta Q^2 \rangle = e^2 \langle \delta N^2 \rangle = e^2 \langle N \rangle = eQ$$

$$\frac{\langle \delta Q^2 \rangle}{Q} = e$$

Shot Noise



$$S_{\rm shot} \sim \langle \delta I^2 \rangle$$

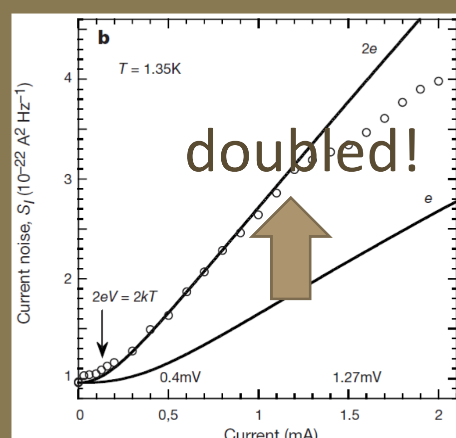
$$S_{\rm shot} = 2e^* \langle I \rangle$$

charge of quasi-particles

Superconductors with Cooper Pairs

$$e^* = 2e$$

Jehl+, Nature **405**,50 (2000)



Fractional Quantum Hall Systems

$$e^* = \frac{q}{p} e$$

Saminadayar+, PRL**79**,2526 (1997)

Higher order cumulants:

3rd order: ex. Beenakker+, PRL90,176802(2003)

up to 5th order: Gustavsson+, Surf.Sci.Rep.**64**,191(2009)

The noise is the signal.

R. Landauer
1998

A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro

B. 25 x 2 Euro

Same expectation value.

A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro

B. 25 x 2 Euro

C. 1 x 50 Euro

Same expectation value.
But, different fluctuation.

Fluctuations in HIC: 2^{nd} Order

Search for QCD CP

Fluctuation
increases

Stephanov, Rajagopal, Shuryak, 1998; 1999

Onset of QGP

Fluctuation
decreases

Asakawa, Heinz, Muller, 2000;
Jeon, Koch, 2000

Higher-order Cumulants

A. 50 x 1 Euro

B. 25 x 2 Euro

$$2\langle \delta€^2 \rangle$$  $$= \langle \delta€^2 \rangle$$ 

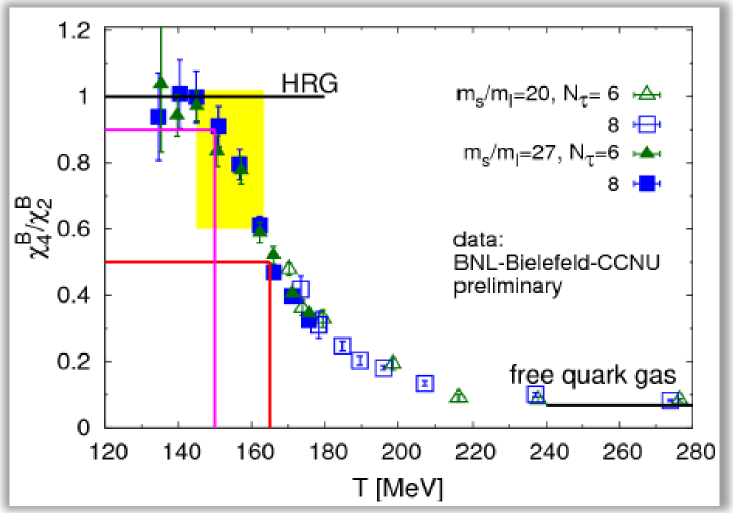
$$4\langle \delta€^3 \rangle$$  $$= \langle \delta€^3 \rangle$$ 

$$8\langle €^4 \rangle_c$$  $$= \langle €^4 \rangle_c$$ 

Asakawa, MK,
PPNP 90, 299
(2016)

Non-Gaussian Fluctuations

Onset of QGP



Fluctuation
decreases

Ejiri, Karsch, Redlich, 2006

Search for QCD CP

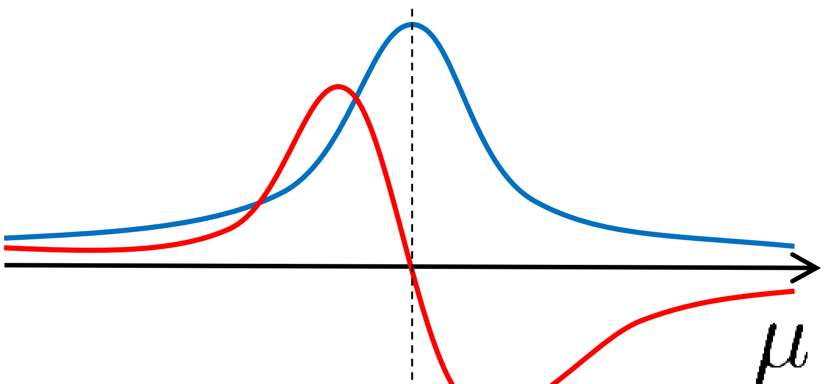
Fluctuation
increases

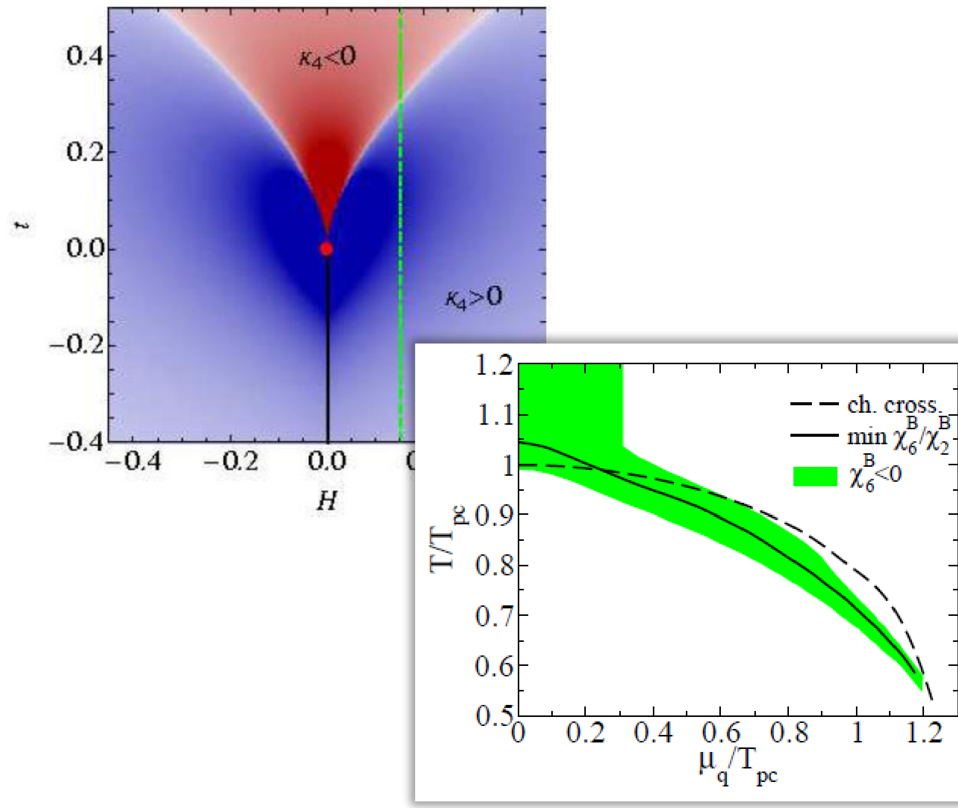
Stephanov, 2009

Sign of Higher-order Cumulants

Higher order cumulants can change sign near CP.

$$\langle \delta N^3 \rangle = T \frac{\partial \langle \delta N^2 \rangle}{\partial \mu}$$

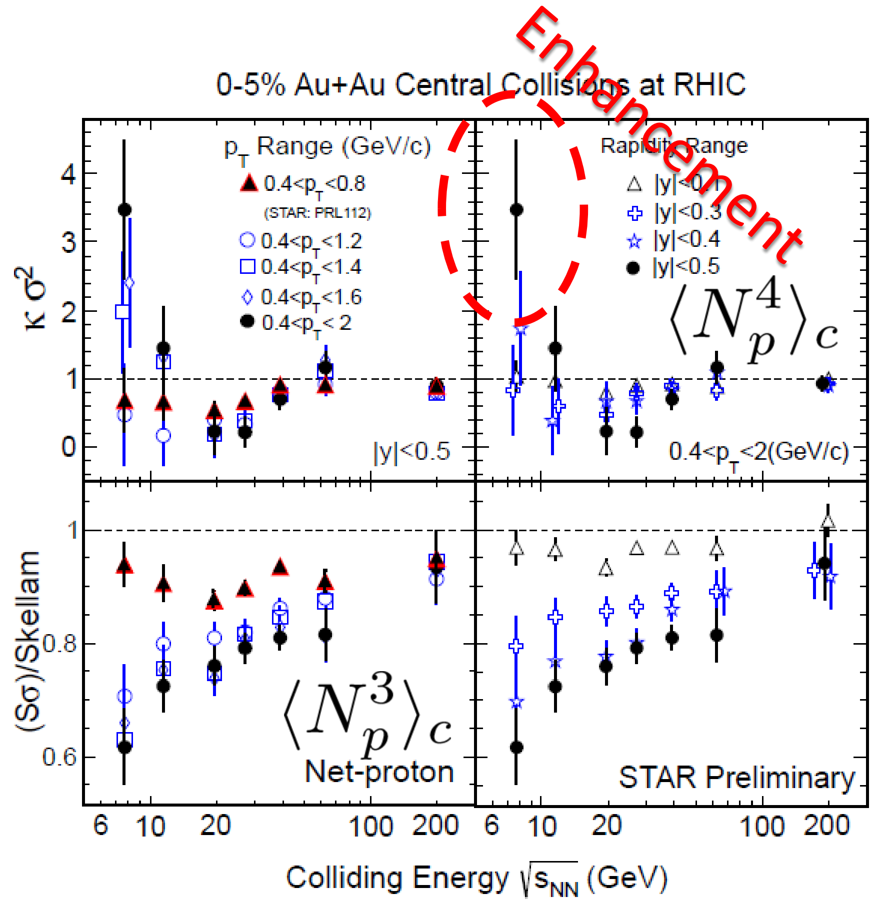




Asakawa, Ejiri, MK, 2009

Stephanov, 2011;
Friman, Karsch, Redlich, Skokov, 2011; …

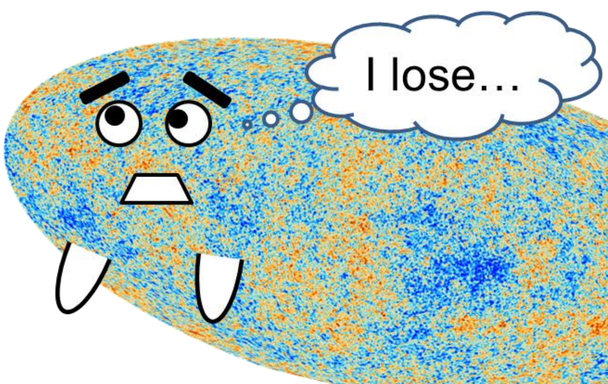
Higher-Order Cumulants



Non-zero **non**-Gaussian cumulants have been established!

STAR Collab.
2010~

This is a great achievement in physics!!



Theory/Lattice vs HIC

discussion so far

Thermal System

↔

real experiments

Dynamical System

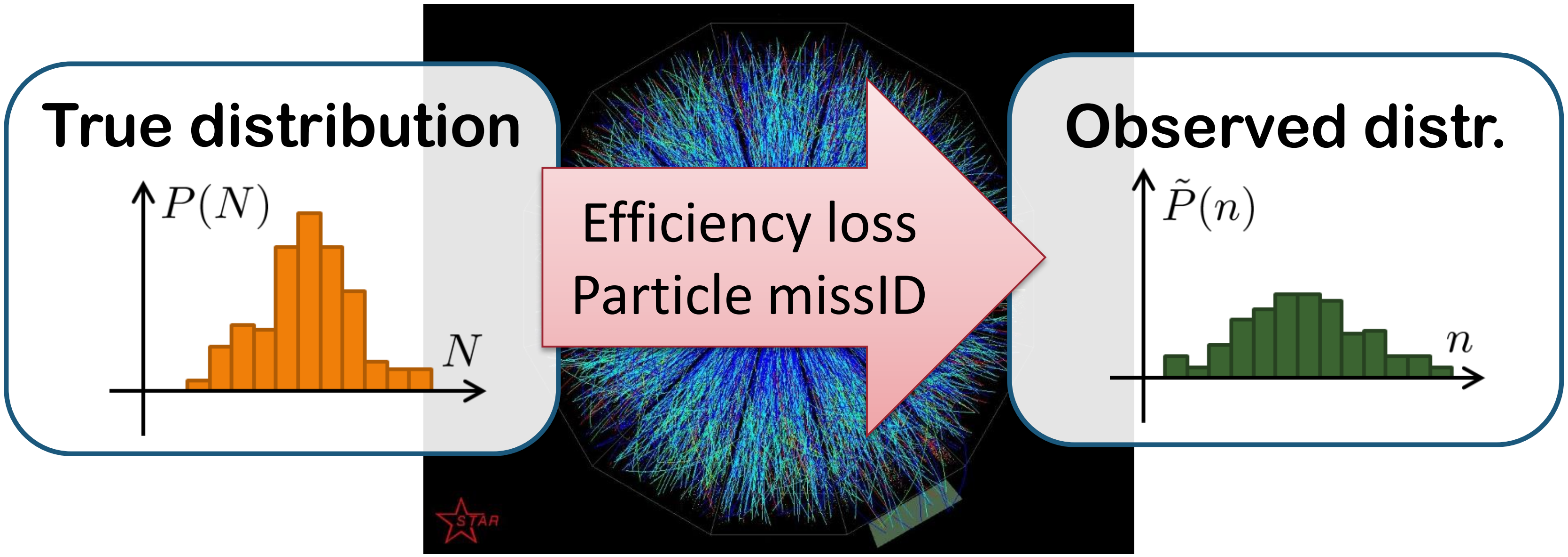
- ☐ Initial fluctuation
- ☐ Time evolution
- ☐ Correct experimental measurement

Many Problems to be Considered

- initial fluctuation: volume, collision dynamics
- time evolution in fireballs
- rapidity vs space-time rapidity
- clustering / resonance decays
- global charge conservation
- effect of jets
- classical vs quantum
- evolution on dynamical models
- discrete vs continuous: particlization
- detector-response correction
- acceptance cut such as p_T
- …

Detector-Response Correction

Detector-Response Correction



☐ Correction assuming a binomial response

Bialas, Peschanski (1986);
MK, Asakawa (2012); Bzdak, Koch (2012); ….

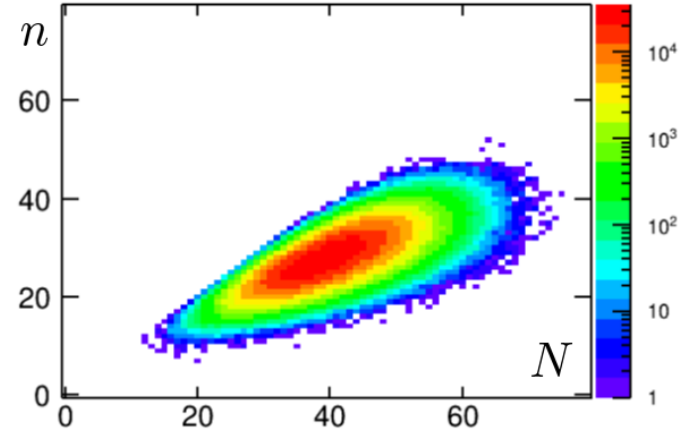
But, the response of the detector is not binomial…

Non-Binomial Correction

Nonaka, MK, Esumi (2018)

- Response matrix

$$\tilde{P}(n) = \sum_N \mathcal{R}(n;N)P(N)$$



Reconstruction for any R(n;N)

with moments of R(n;N)

$$\langle n^m \rangle_R = \sum_n n^m \mathcal{R}(n;N)$$

- Caveats:
  - Correction can be made. But,
  - Need accurate properties of the detector.
  - The higher order, the larger error.
  - Huge experimental statistics / numerical cost

Result for Toy-Model

Binomial w/ multiplicity-dependent efficiency

$$\epsilon(N) = \epsilon_0 + (N - N_{\rm ave})\epsilon'$$

Holtzman, Bzdak, Koch (16)

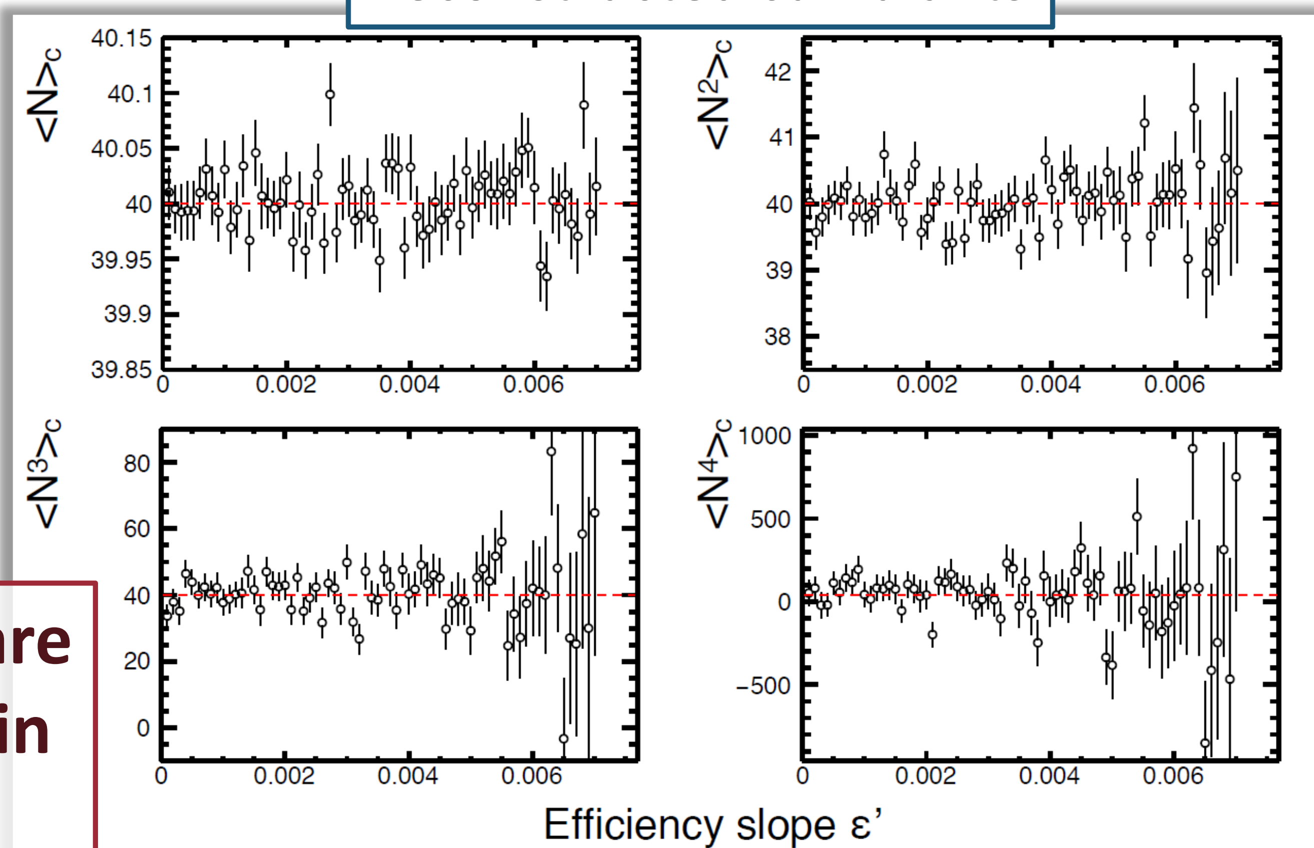
Nonaka, MK, Esumi (2018)

Input P(N): Poisson(λ=40)

$\epsilon_0 = 0.7$

Red: true cumulant

Reconstructed cumulants



True cumulants are reproduced within statistics!

Proton vs Baryon Cumulants

MK, Asakawa, 2012; 2012

Experiments

proton number cumulants

$$\langle N_p^n \rangle_c$$

Many theories

baryon number cumulants

$$\langle N_B^n \rangle_c$$

measurement with 50% efficiency loss

- Clear difference b/w these cumulants.
- Isospin randomization justifies the reconstruction of $\langle N_B^n \rangle_c$ via the binomial model.
- Similar problem on the **momentum cut**…

Fragile Higher Orders

Ex.: Relation b/w baryon & proton # cumulants

(with approximations)

MK, Asakawa, 2012

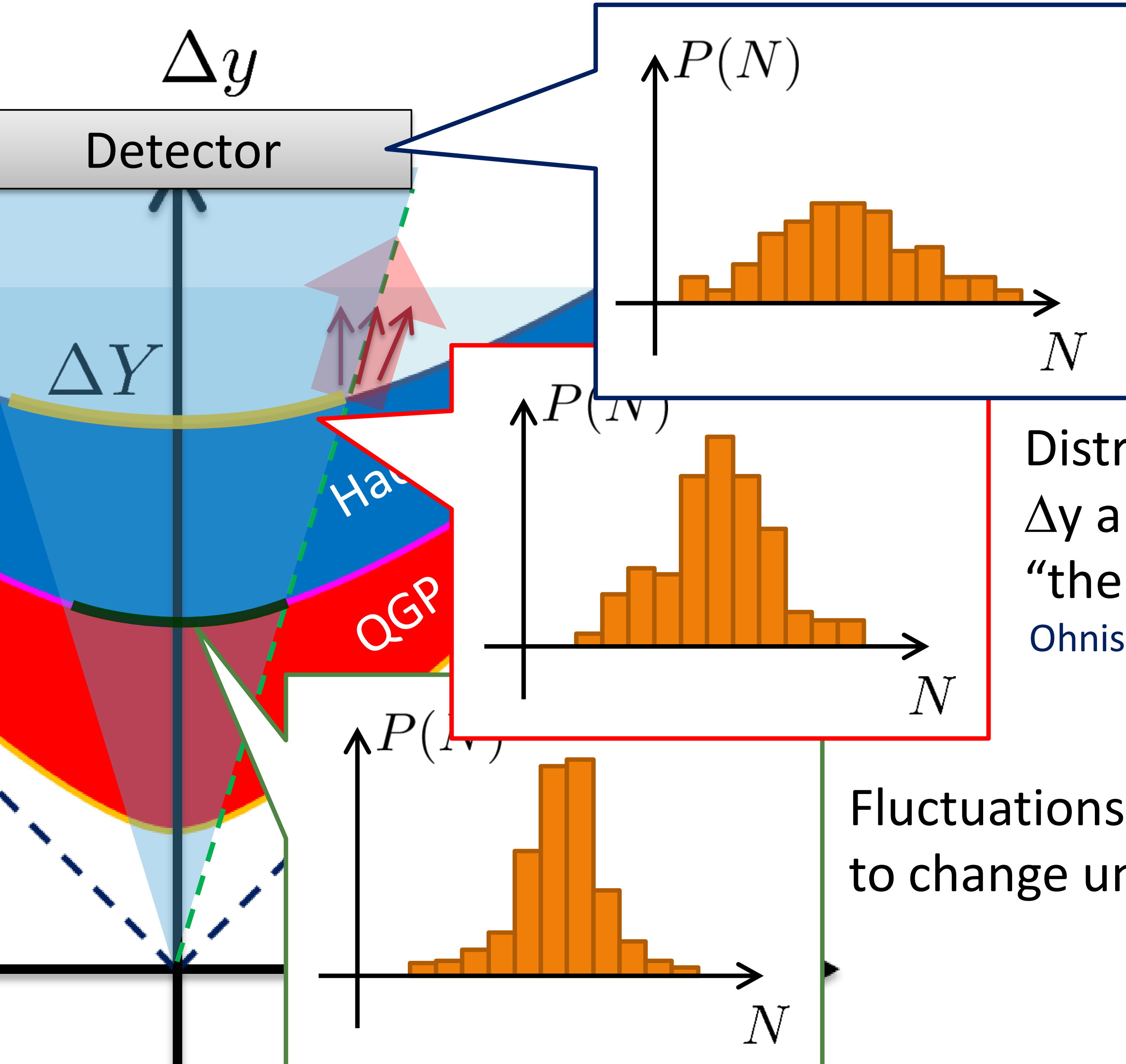
$$\left\{\begin{array}{l} 2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\ 2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\ 2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots \end{array}\right.$$

genuine info. Poisson noise

Higher orders are more seriously affected by efficiency loss.

Evolution of Fluctuations

Time Evolution of Fluctuations



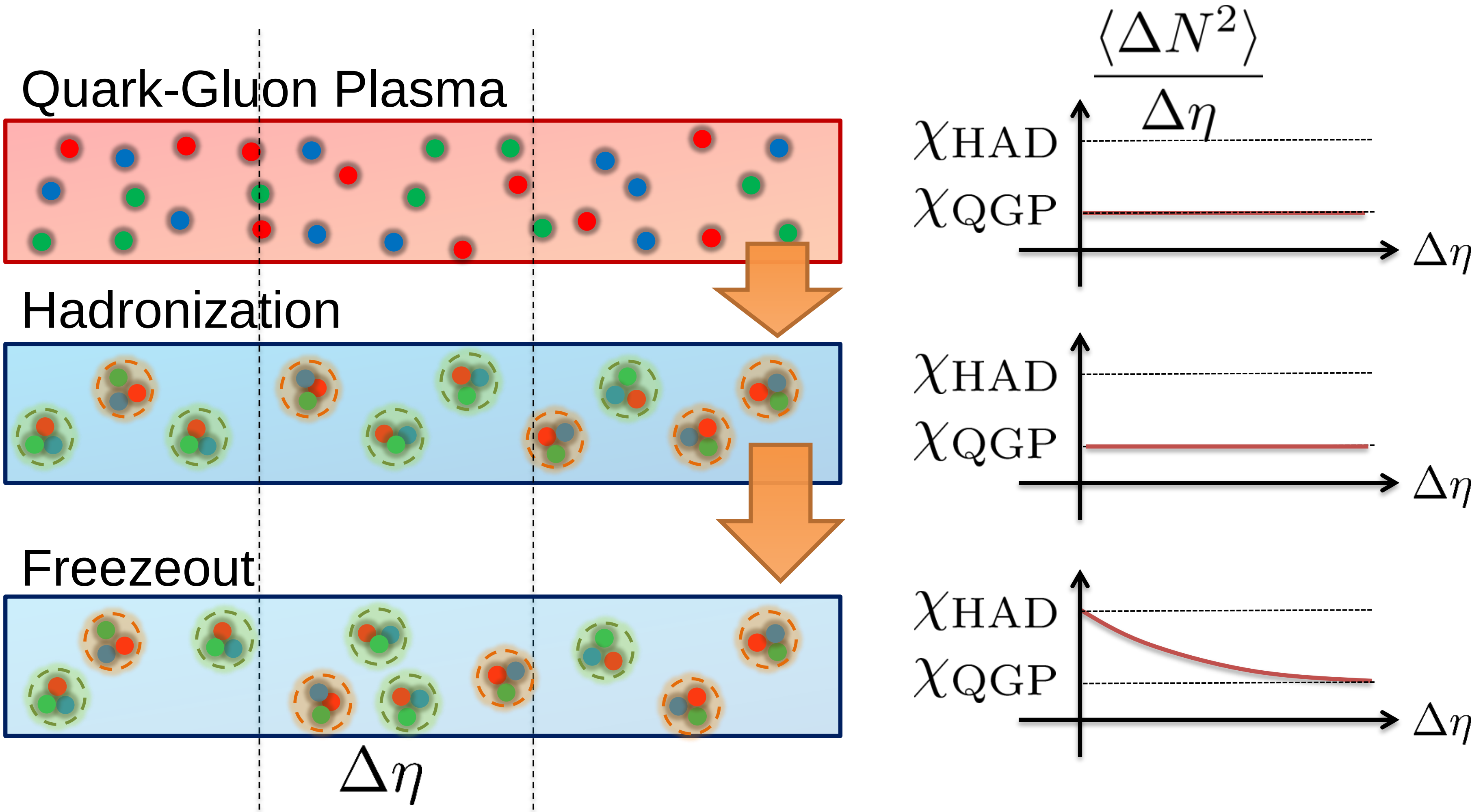
Distributions in ΔY and Δy are different due to "thermal blurring".

Ohnishi, MK, Asakawa, PRC(2016)

Fluctuations in ΔY continue to change until kinetic f.o.

Rapidity-window Dependence

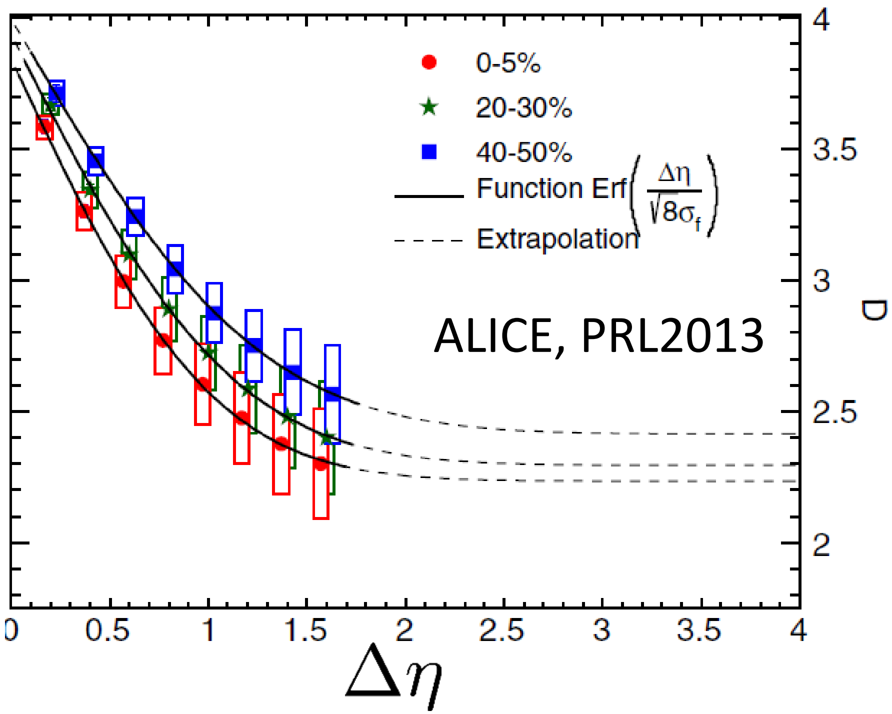
Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000; Shuryak, Stephanov, 2001



The larger $\Delta\eta$, the slower diffusion.

2^{nd} Order @ ALICE

Net charge fluctuation

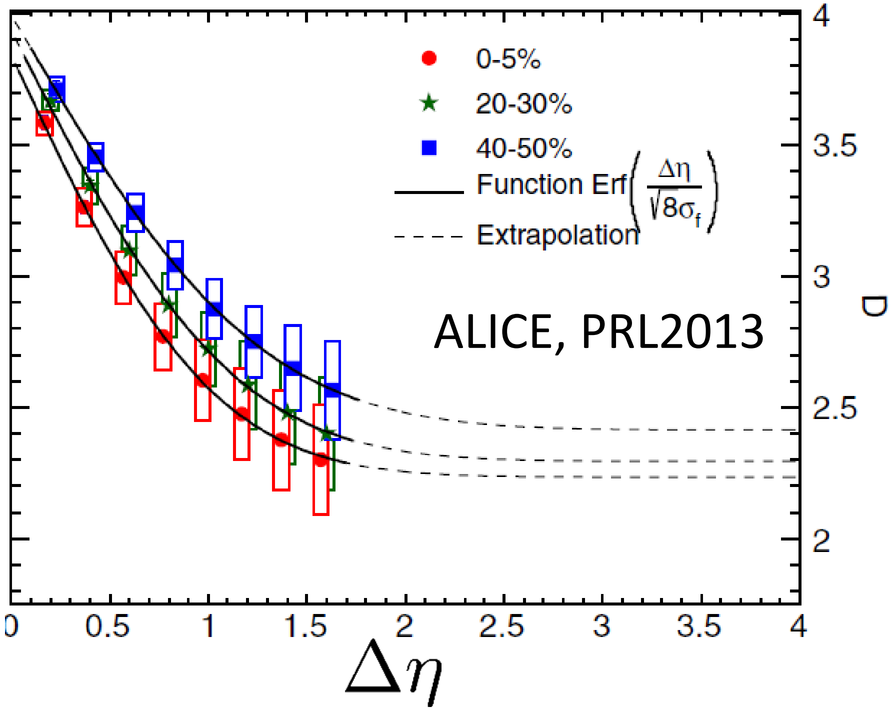


D-measure

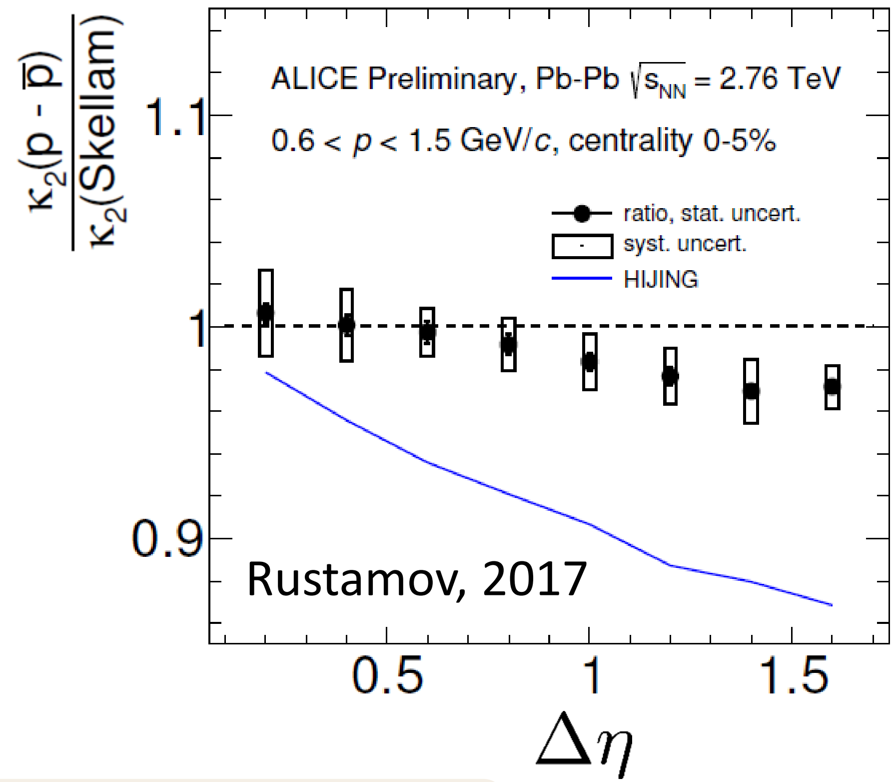
$$D \simeq 4\frac{\langle \delta N_Q^2 \rangle}{\langle \delta N_Q^2 \rangle_{\rm HRG}}$$

2nd Order @ ALICE

Net charge fluctuation



Net proton fluctuation

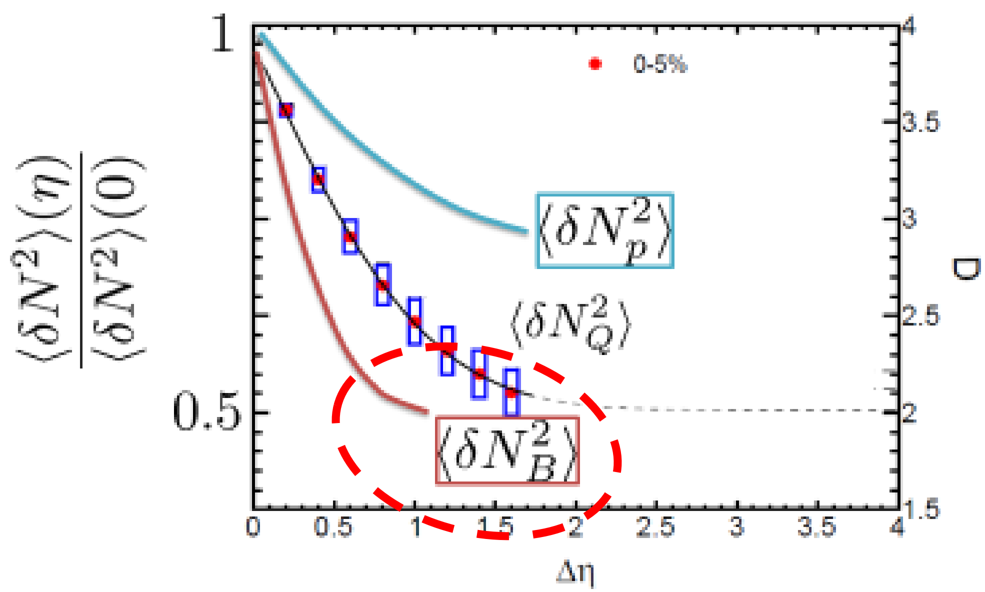


- Net-charge fluctuation has a suppression,
- but net-proton fluctuation does not. Why??

From My Old Prediction

$\langle \delta N_B^2 \rangle$ and $\langle \delta N_p^2 \rangle$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$ should have different $\Delta\eta$ dependence.



Baryon # cumulants are experimentally observable! MK, Asakawa, 2011;2012

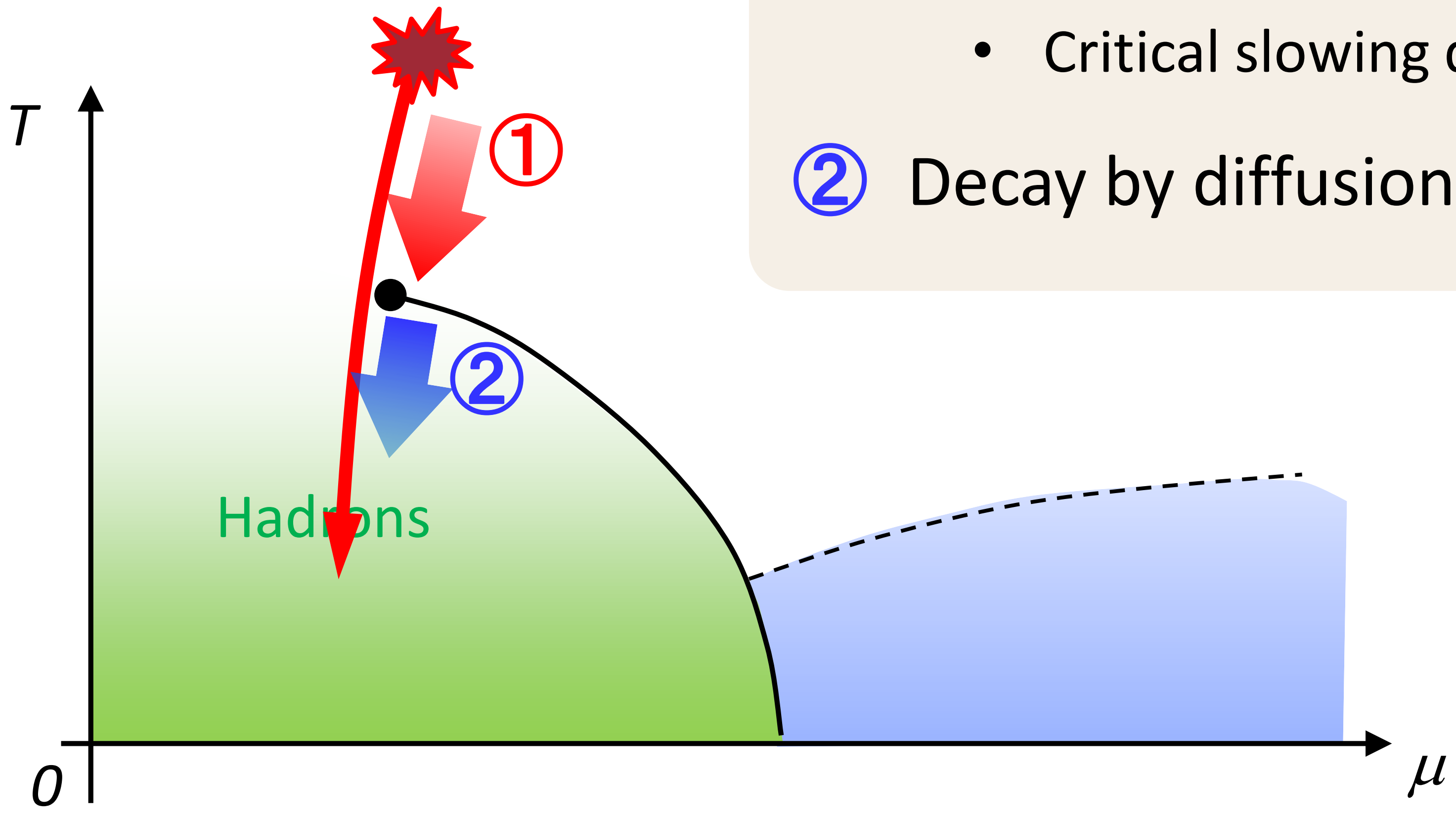
GSI, Jan. 2013
Berkeley, Sep. 2014
FIAS, Jul. 2015
GSI, Jan. 2016
…

Baryon #

- ☐ slower diffusion
- ☐ more sensitive to phase transition

How to understand proton/charge fluctuations @ ALICE ??

Critical Fluctuation



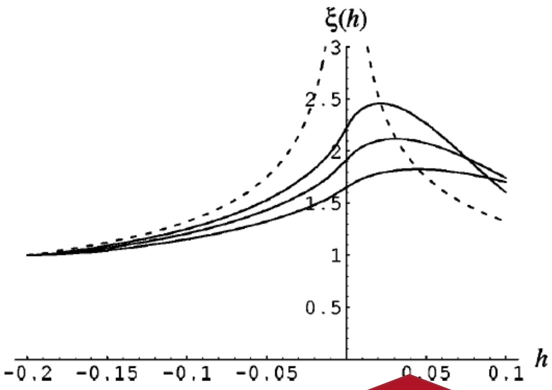
① Growth of critical fluctuation

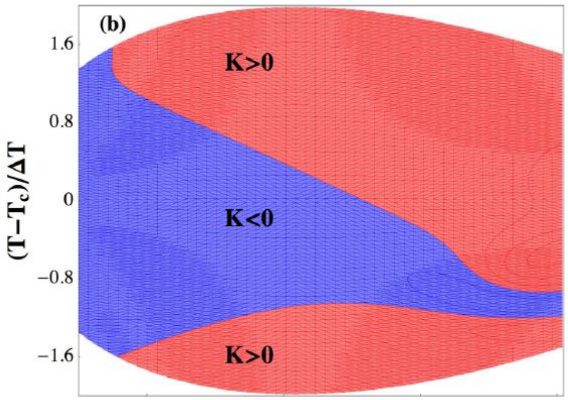
- Critical slowing down

② Decay by diffusion

Dynamical Evolution of Critical Fluctuations

- Evolution of **spatially uniform "σ" mode**





Berdnikov, Rajagopal (2000)
Asakawa, Nonaka (2002)
Mukherjee+ (2015)
…

Evolution of **conserved charge fluctuations**

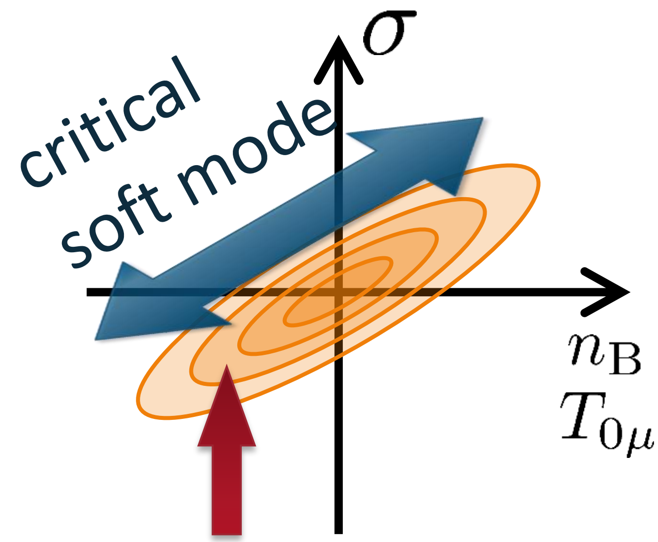
1. Conserved charges are directly observable.
2. Soft mode at QCD-CP is a conserved mode.

Soft Mode of QCD-CP = Conserved Mode

Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

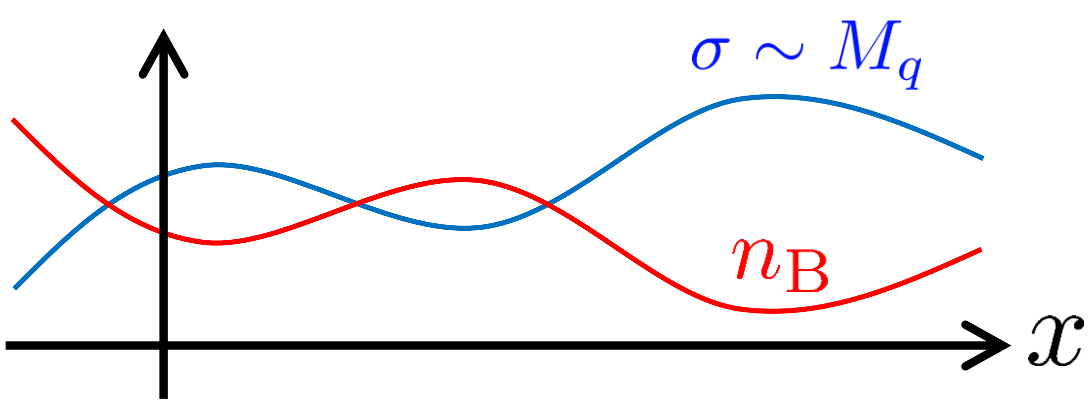
Fluctuations of σ and n_B are coupled around the CP!

$$\delta\sigma \simeq \delta n_B$$



σ: fast damping

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \cdots$$



Analysis of 2nd-order Cumulant

Sakaida+ (2017)

Evolution of baryon number density

Stochastic Diffusion Equation

$$\partial_t n = D(t)\partial_x^2 n + \partial_x \xi$$

$$\langle \xi(x_1,t_1)\xi(x_2,t_2)\rangle = 2D\chi_2\delta^{(2)}(1-2)$$

$D(t),\ \chi_2(t)$:parameters characterizing criticality

- Analytic solution is obtained.
- Study 2nd order cumulant & correlation function.

Our Main Conclusion

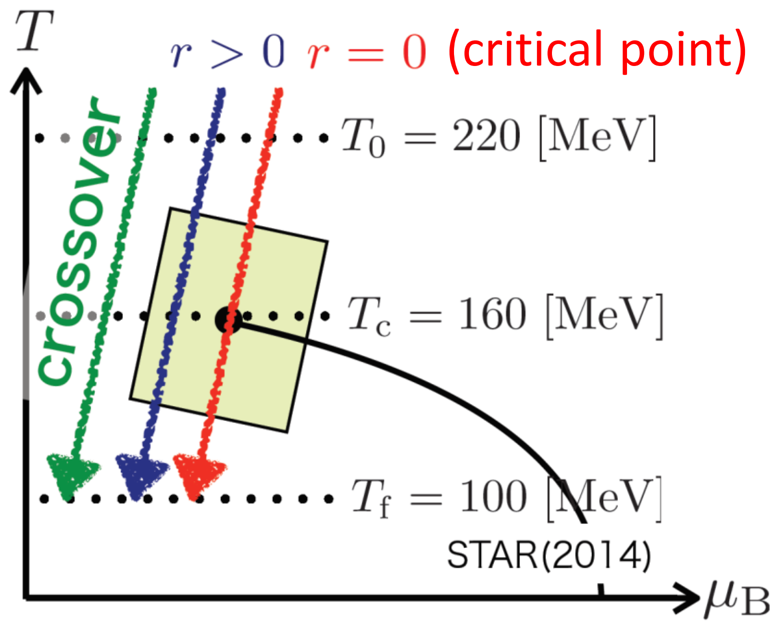
Non-monotonicity in cumulants or correlation func. = Signal of QCD-CP

Parametrizing D(τ) and χ(τ)

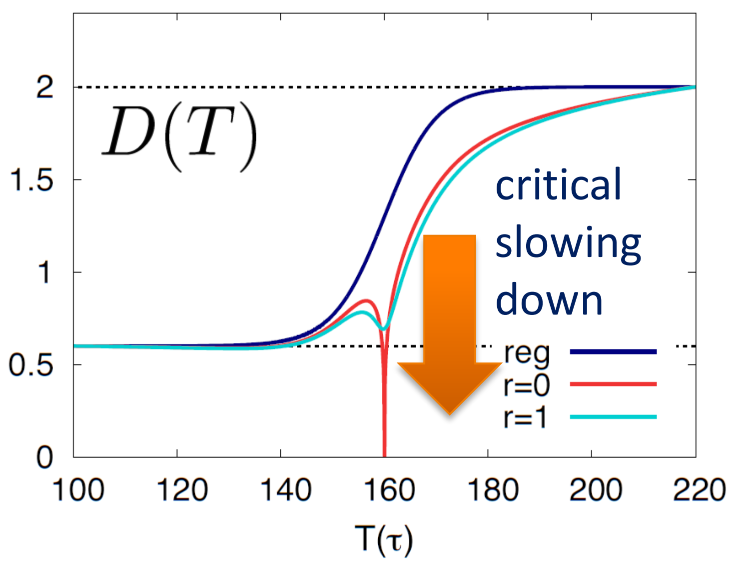
Critical behavior

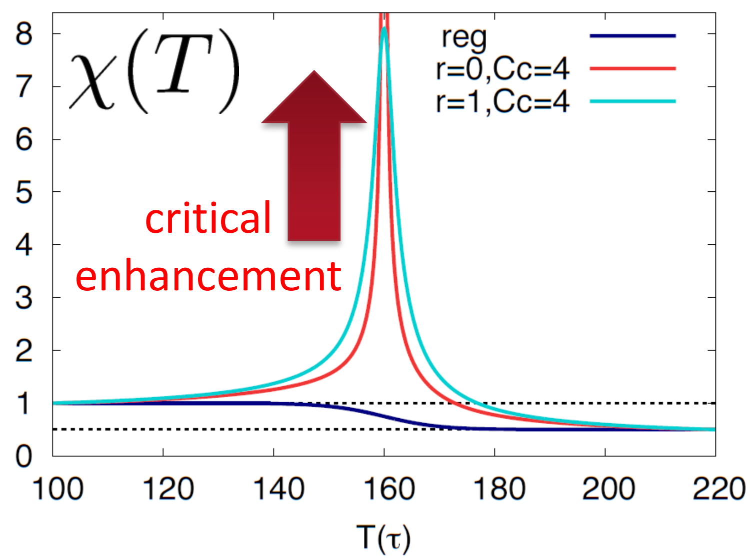
- 3D Ising (r,H)
- model H

Berdnikov, Rajagopal (2000)
Stephanov (2011); Mukherjee+(2015)



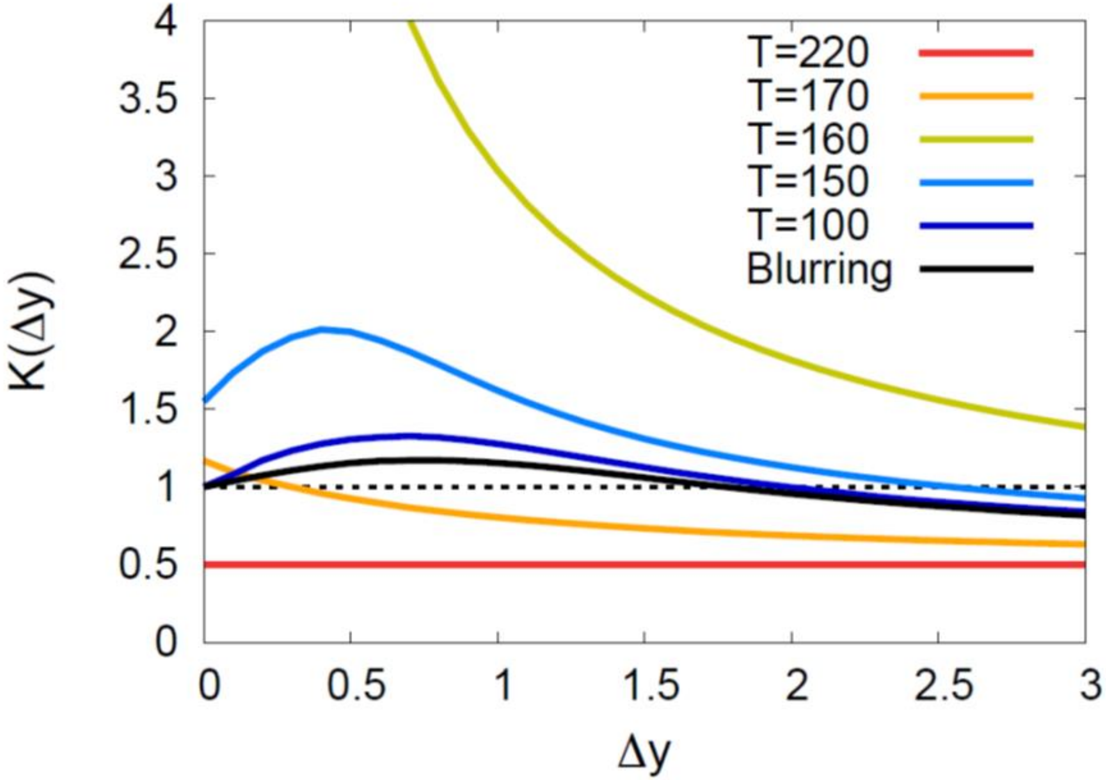
Temperature dep.

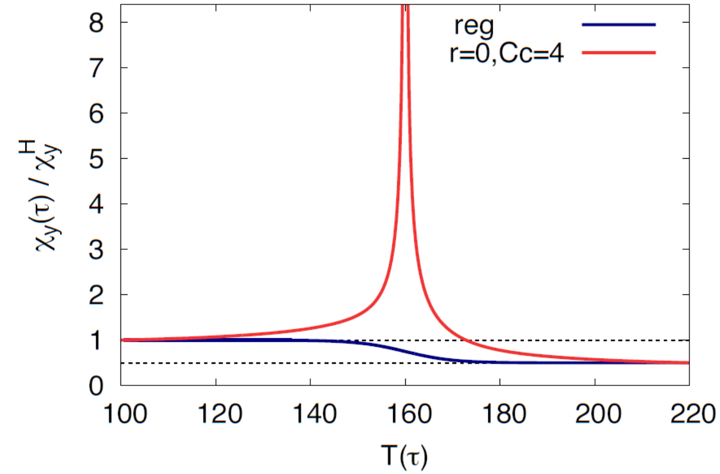


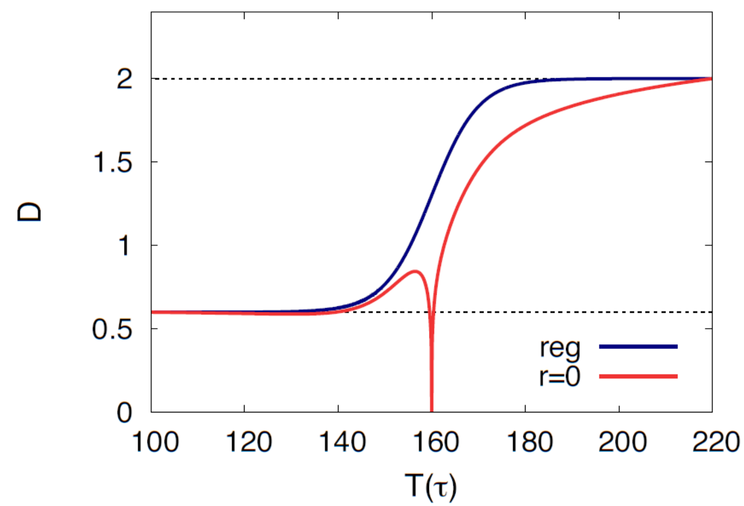


Critical Point / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.}$$







- non-monotonic Δy dep.

Analytic result: $K(\Delta y)$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

See also,
Wu, Song
arXiv: 1903.06075

Cumulants and Correlation Function

$$Q = \int_V dx\, n(x)$$

total charge

charge density

$$\langle \delta Q^2 \rangle = \int_V dx dy \langle \delta n(x) \delta n(y) \rangle$$

2nd order cumulant (fluctuation)

correlation function

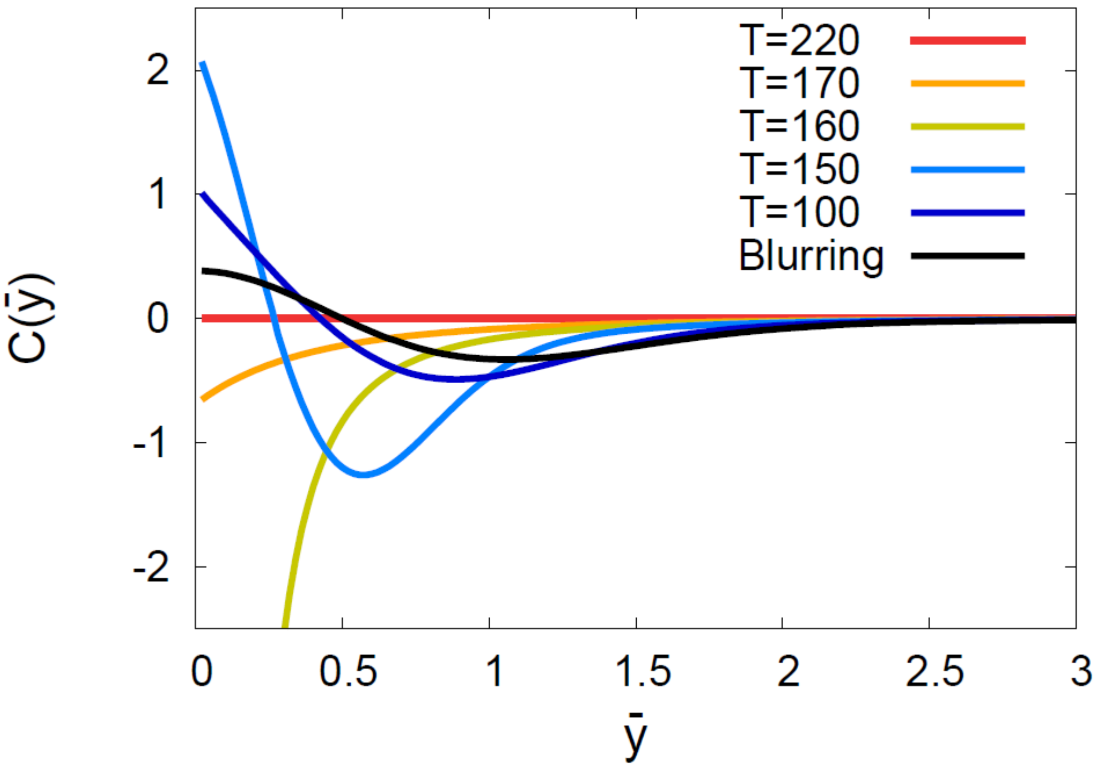
1-to-1 correspondence

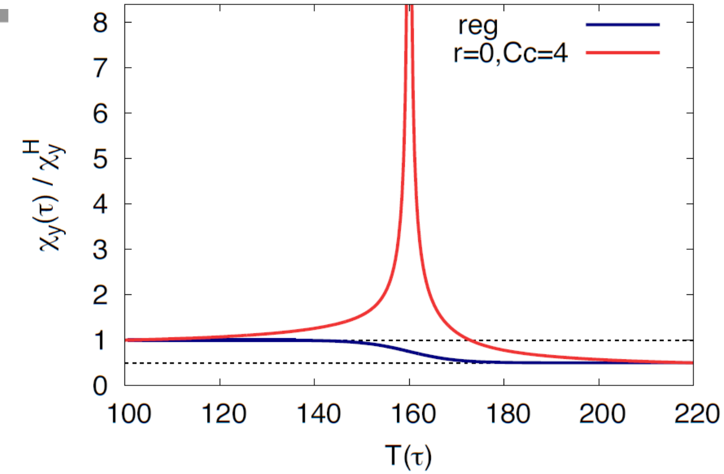
1-dim case

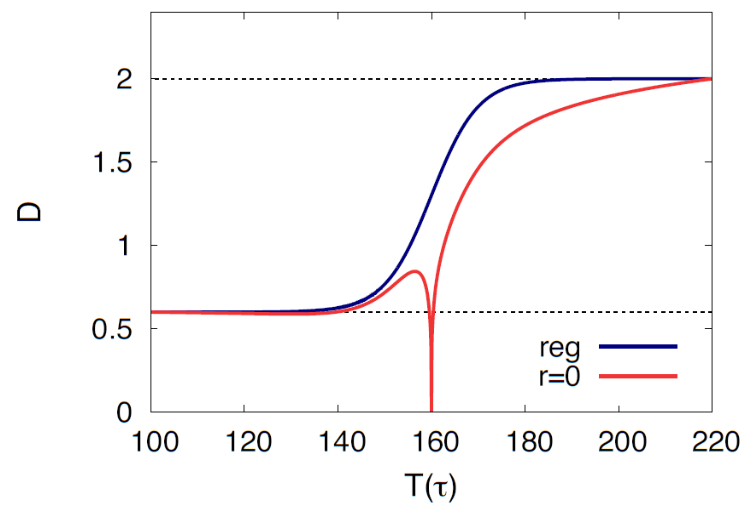
$$\langle \delta Q^2 \rangle_{\Delta y} = \int_{\Delta y} dy (\Delta y - |y|) \langle \delta n(y) \delta n(0) \rangle$$

Criticap Point / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\mathrm{hadron}}$$







- non-monotonic Δy dep.

Analytic result: $C(\Delta y)$ non-monotonic → $\chi(\tau)$ non-monotonic

See also,
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Describing Non-Gaussianity

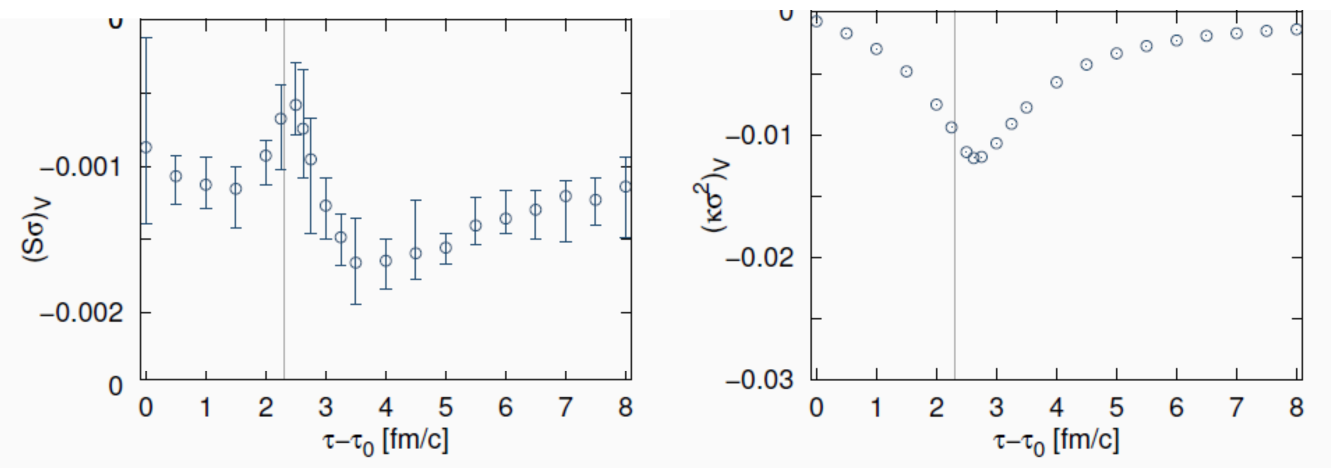
Diffusion Eq. with Non-linear Terms

$$\partial_\tau n = \Gamma(n)\partial_Y^2 \frac{\delta F[n]}{\delta n(Y)} + \partial_Y \xi$$

$$\langle \xi(Y_1,\tau_1)\xi(Y_2,\tau_2)\rangle = 2A\delta^{(2)}(1-2)$$

$$f(n) = k(\nabla n)^2 + a\Delta n^2 + b\Delta n^3 + c\Delta n^4 + \cdots$$

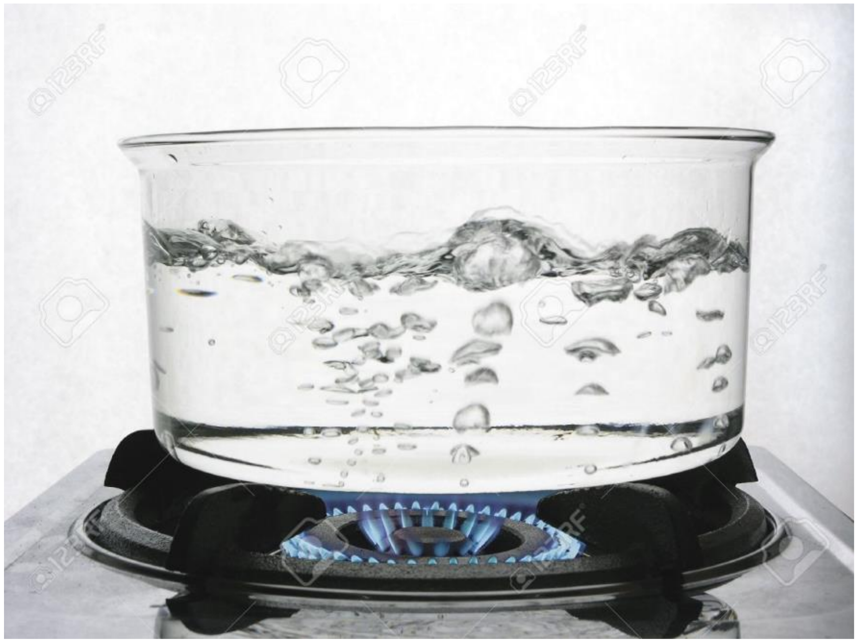
Nahrgang, Bluhm, Schaefer, Bass, arxiv:1804.05728

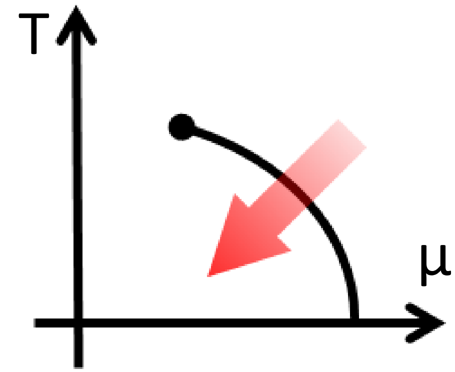


Application to 1^{st} order transition:
Nonaka, Akamatsu, Bluhm, MK, Nahrgang, Wednesday

☐ Proper description of higher order cumulants

1^{st}-Order Transition





- ☐ Domain formation
- ☐ Non-uniform system

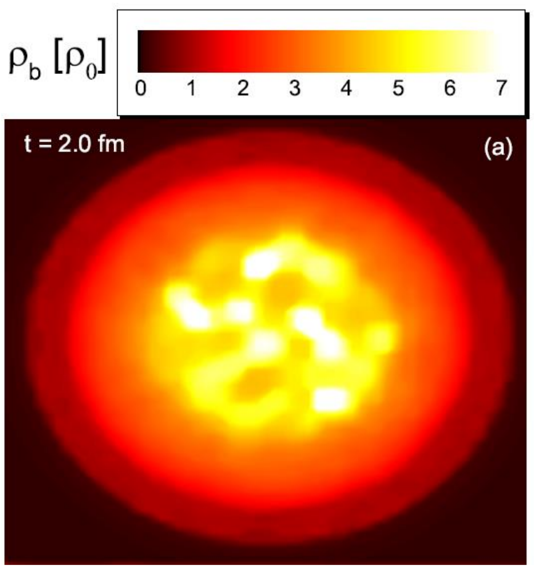
Finding a line: Easier than the search for a point.

Herold, Nahrgang, et al. (2011~); Steinheimer, Randrup (2012; 2013)

Dynamical Simulations

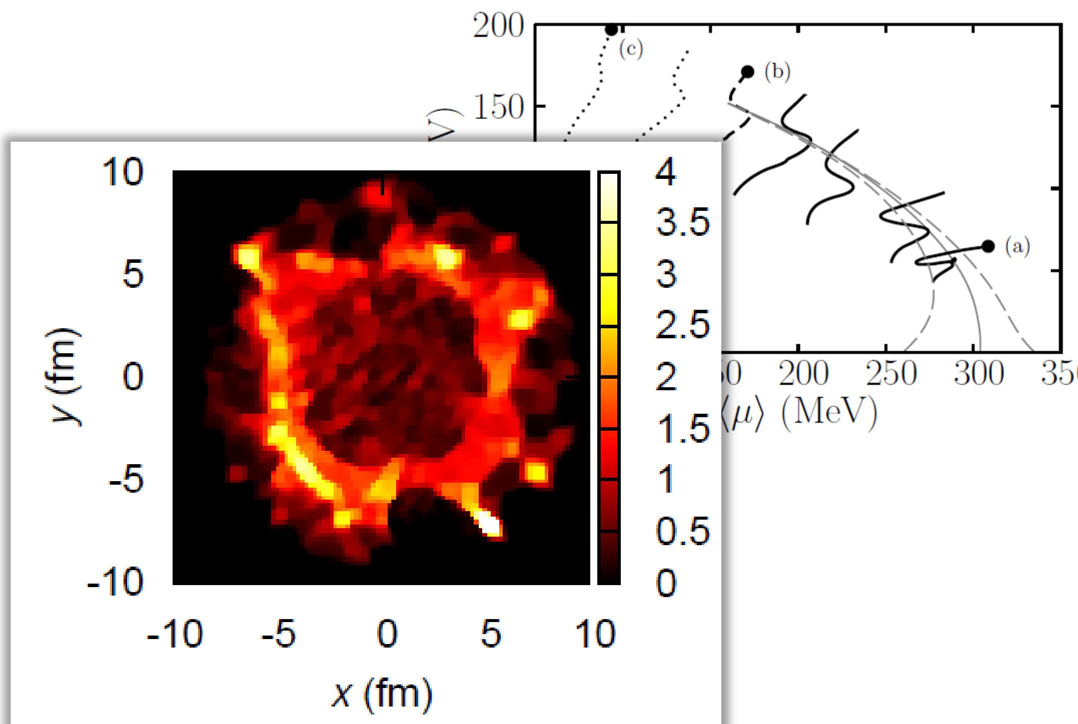
Hydro with unstable EoS

Steinheimer, Randrup, Koch, 2014



Chiral Fluid Dynamics

Herold, Nahrgang, Mishustin, Bleicher, 2014



- ☐ Good observables to see domain formation: $<\rho^n>/<\rho>$, and ?

Many Problems to be Considered

- initial fluctuation: volume, collision dynamics
- time evolution in fireballs
- rapidity vs space-time rapidity
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- effect of jets
- classical vs quantum
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- discrete vs continuous: particlization
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Coat of arms of Paris

"[She] is tossed by the waves, but does not sink"
from Wikipedia

Let's continue our voyage to QCD-CP!

In place of Summary

- ☐ Let's consider lower order cumulants more seriously before exploring higher orders.

- ☐ Experimental analysis becomes more difficult and less reliable for higher orders.
- ☐ Puzzle of Proton/charge fluctuations at ALICE.