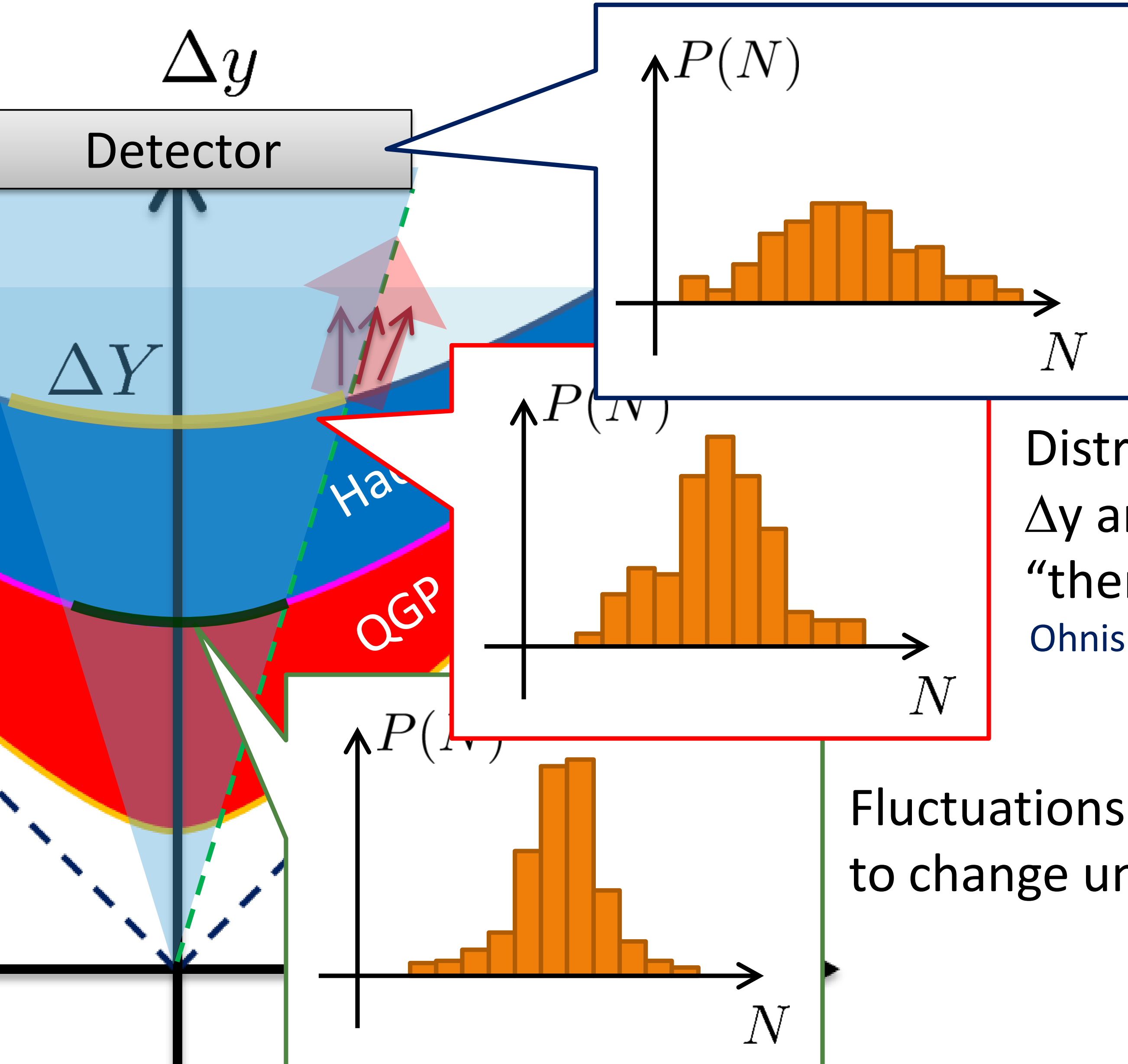
Critical Diffusion Dynamics

Masakiyo Kitazawa

(Osaka U.)

Rapid Reaction Task Force on
Dynamics of Critical Fluctuations: Theory – Phenomenology – HIC
GSI, Darmstadt, Germany, 11/Apr./2019

Time Evolution of Fluctuations



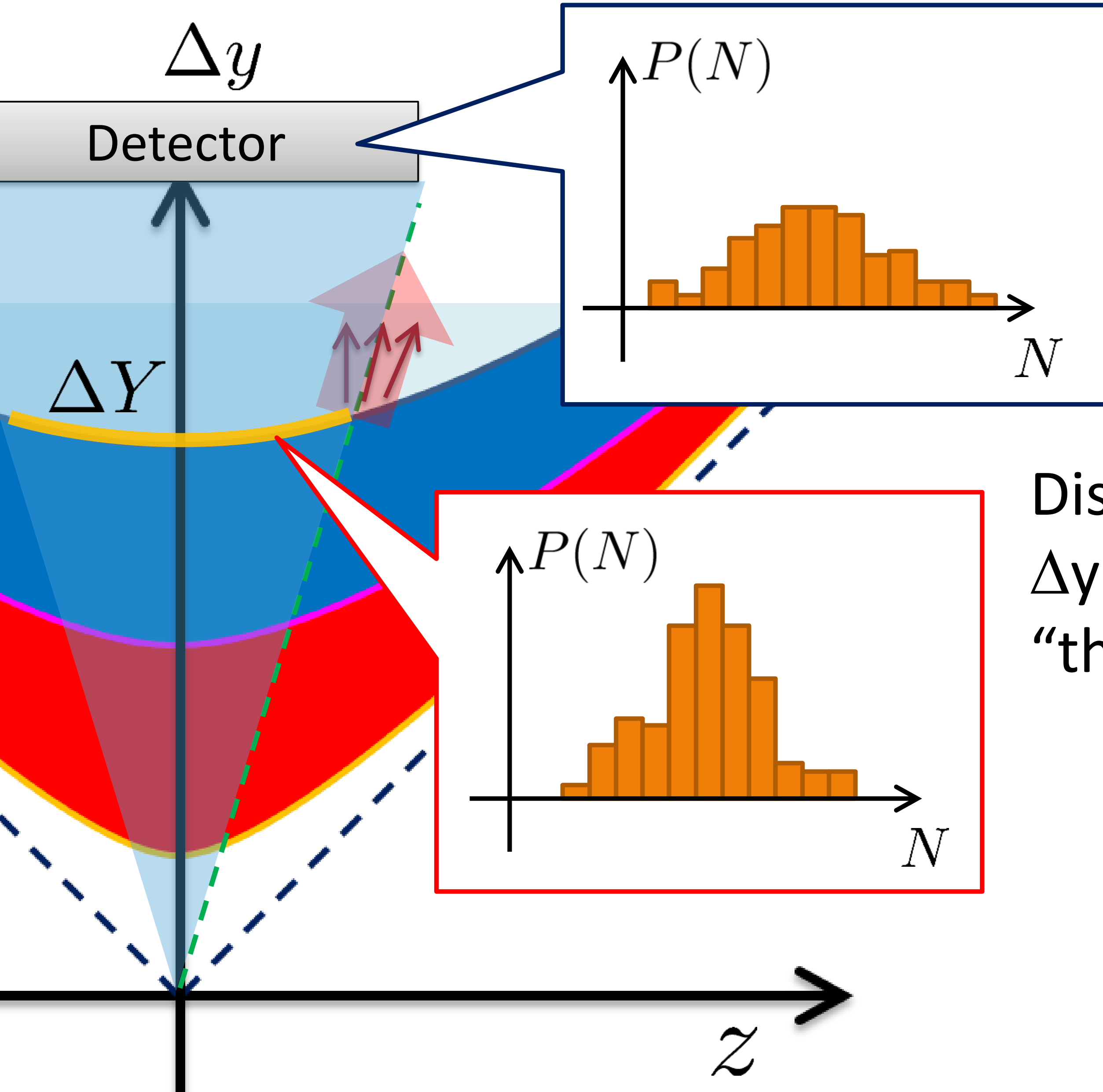
Distributions in ΔY and Δy are different due to "thermal blurring".

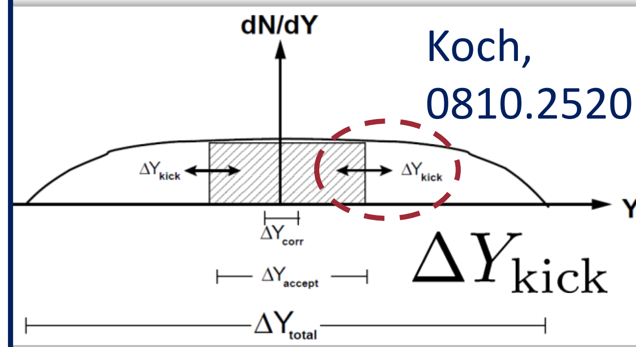
Ohnishi, MK, Asakawa, PRC(2016)

Fluctuations in ΔY continue to change until kinetic f.o.

Thermal Blurring

Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000

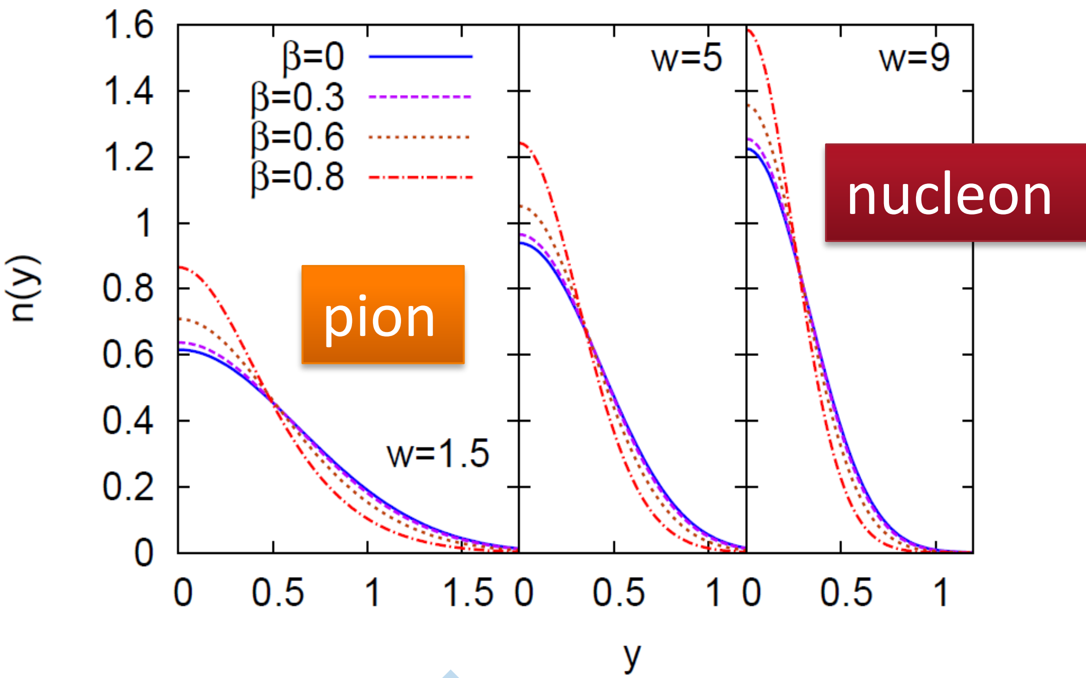




Distributions in ΔY and Δy are different due to “thermal blurring”.

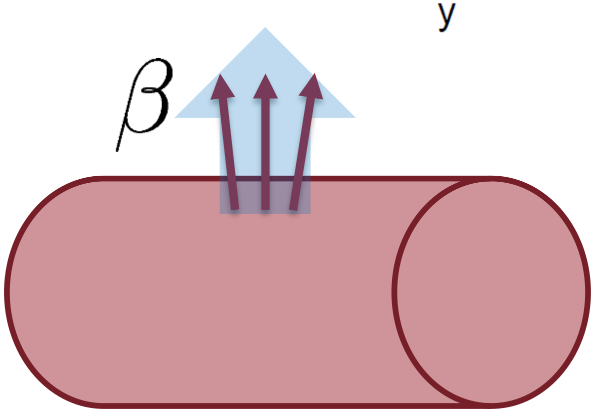
Thermal distribution in y space

Ohnishi, MK, Asakawa
PRC, 2016

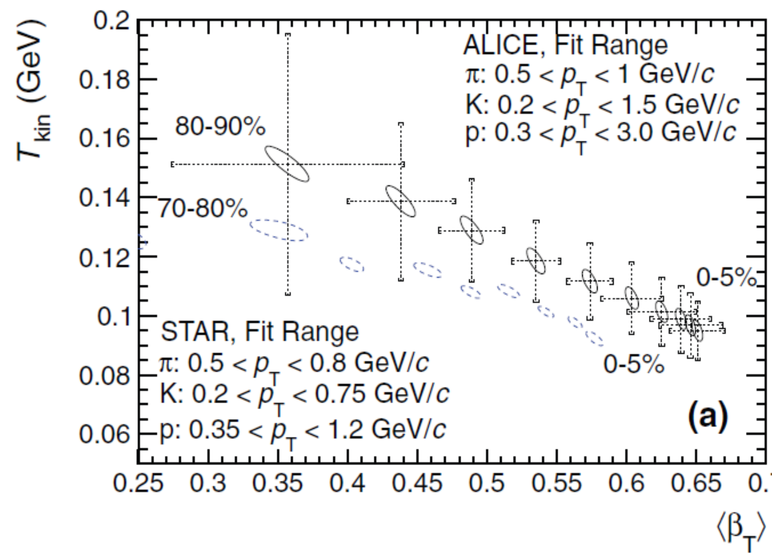


$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$



Blast wave squeezes the distribution in rapidity space



- blast wave
- flat freezeout surface

Thermal distribution in y space

Ohnishi, MK, Asakawa
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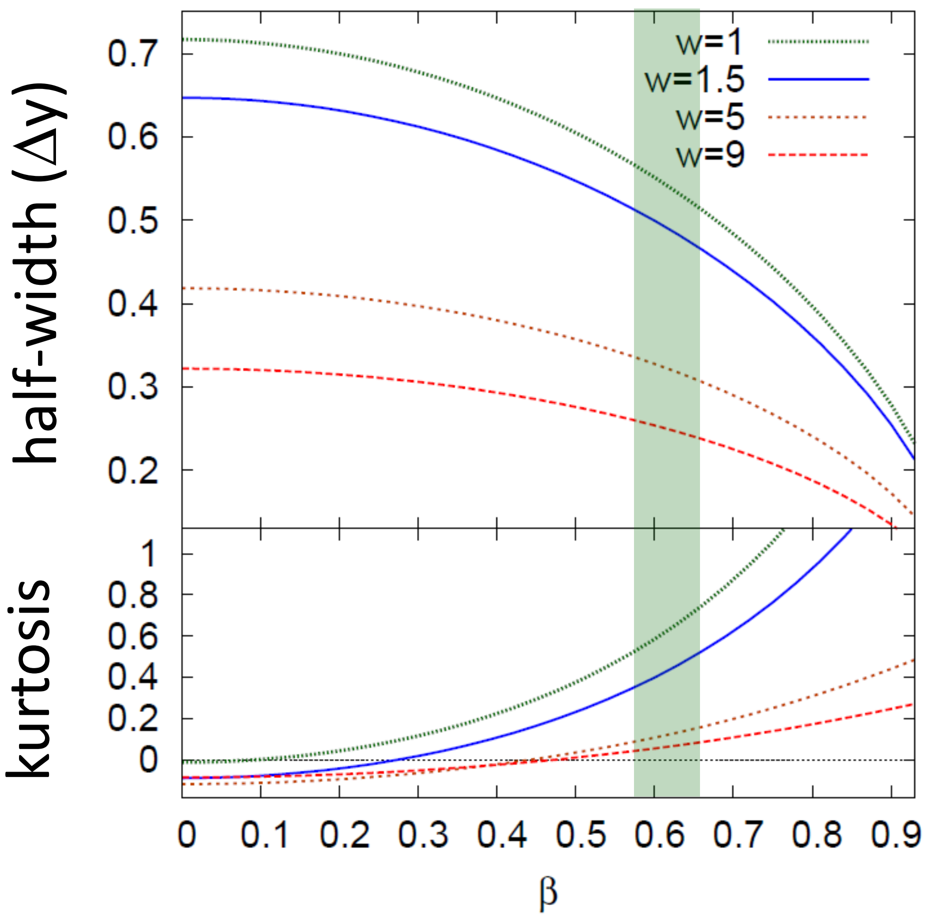

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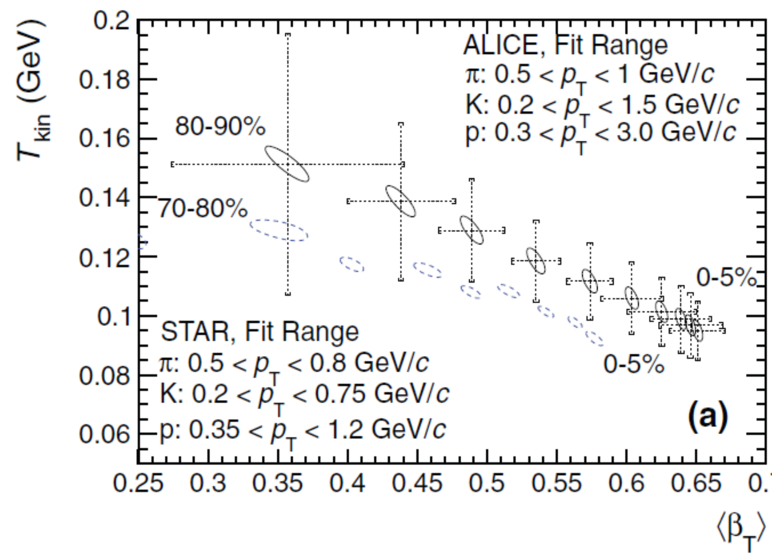
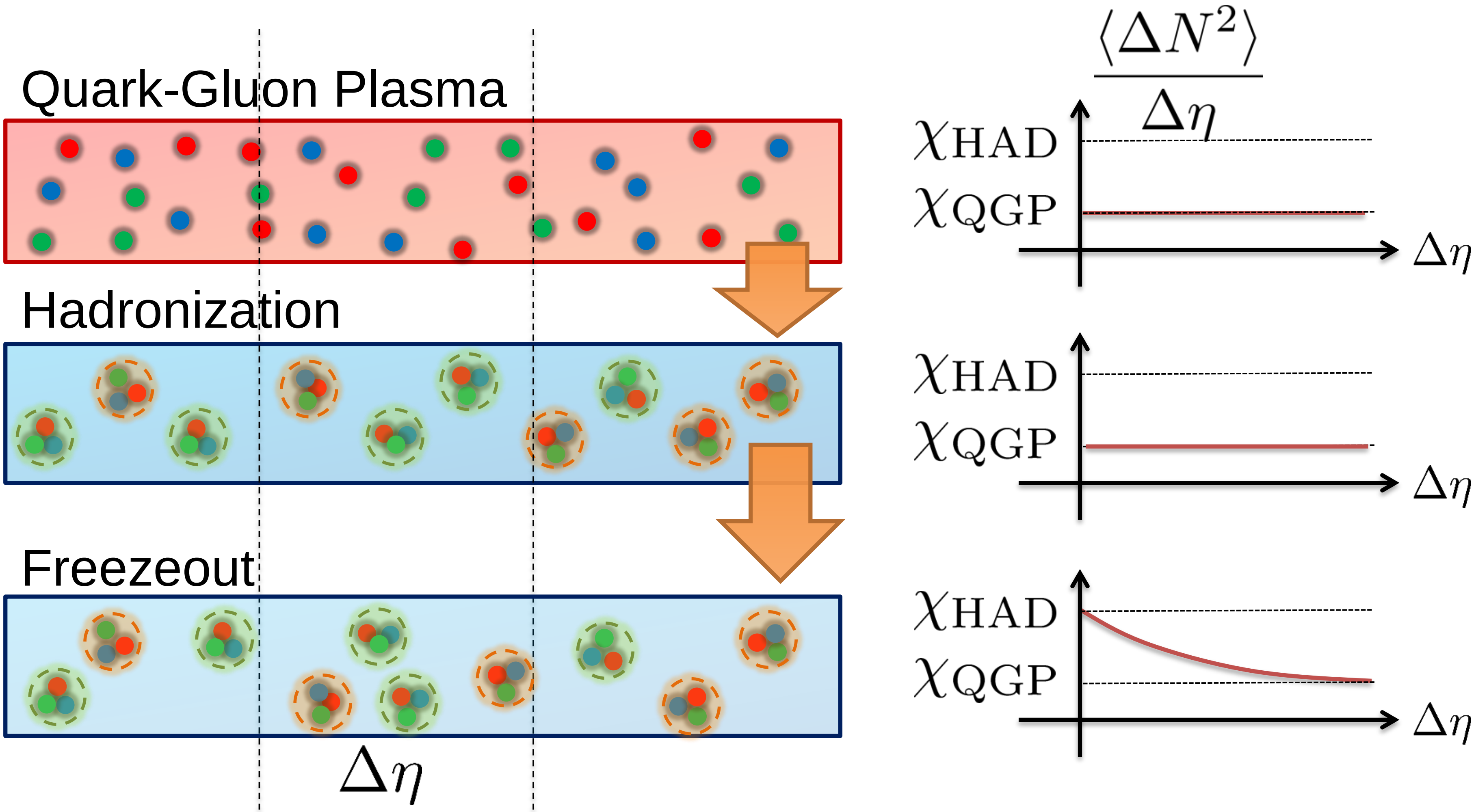


Rapidity distribution can be well approximated by Gaussian.

- blast wave
- flat freezeout surface

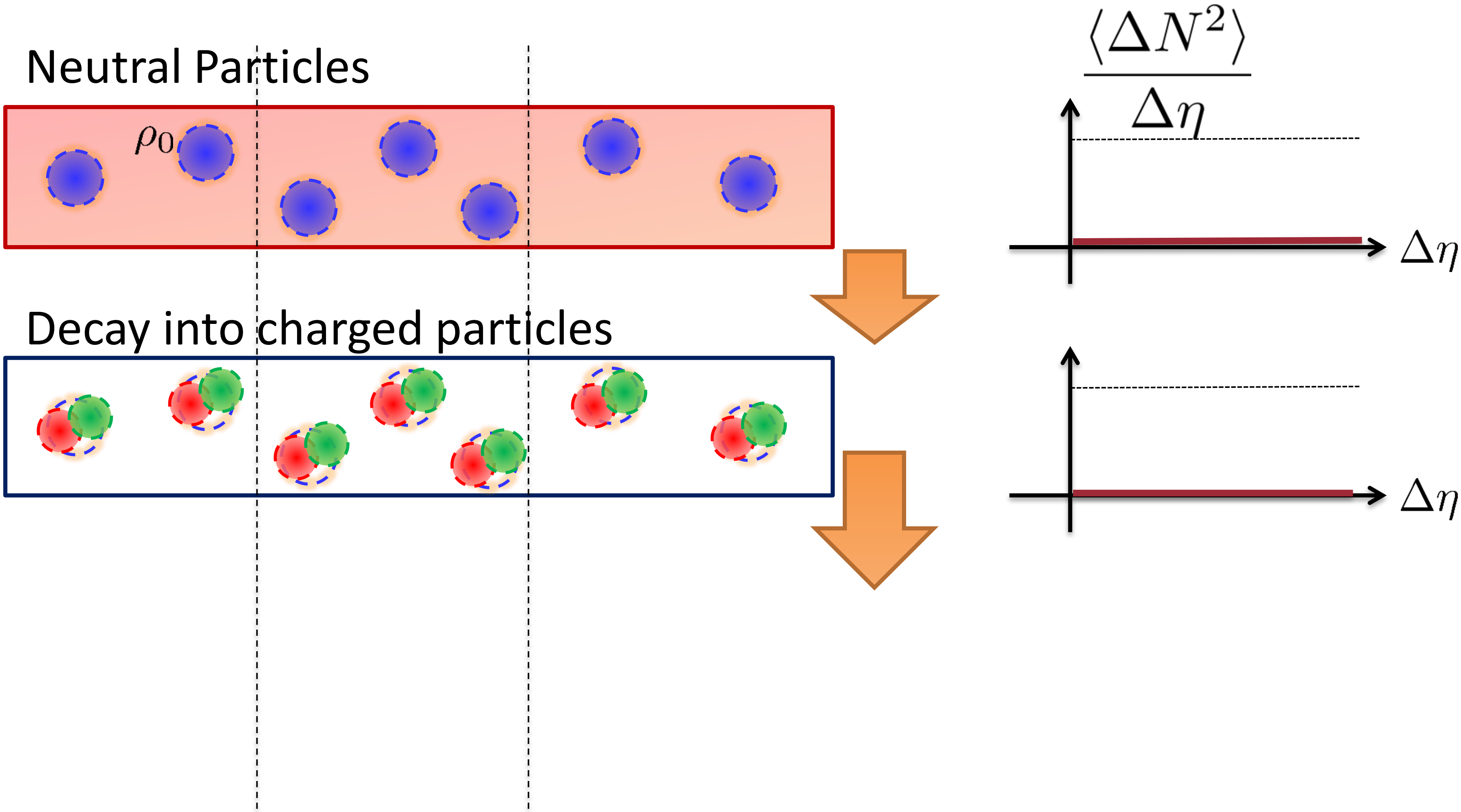
Rapidity-window Dependence

Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000; Shuryak, Stephanov, 2001

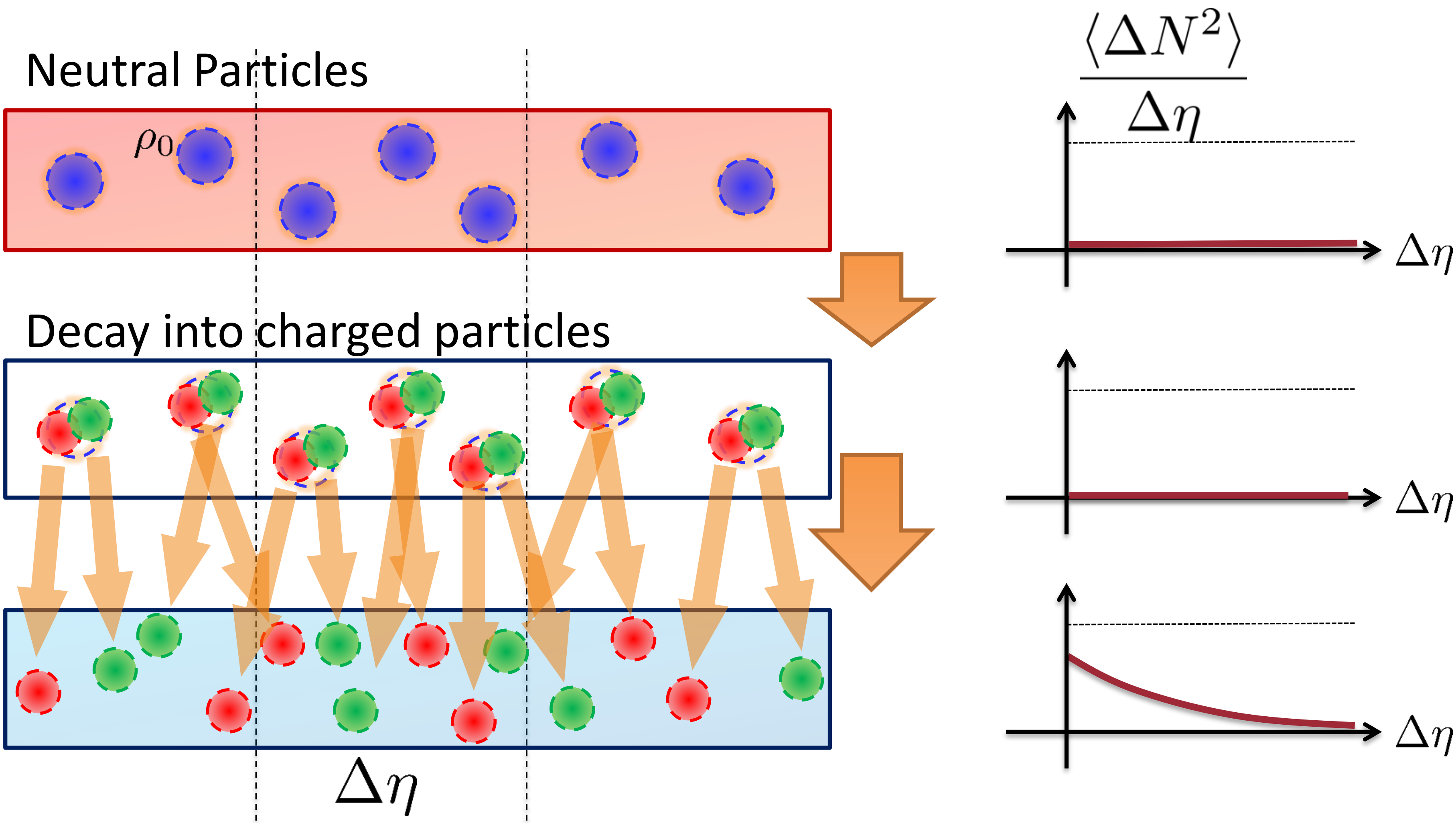


The larger Δη, the slower diffusion.

Resonance Decay



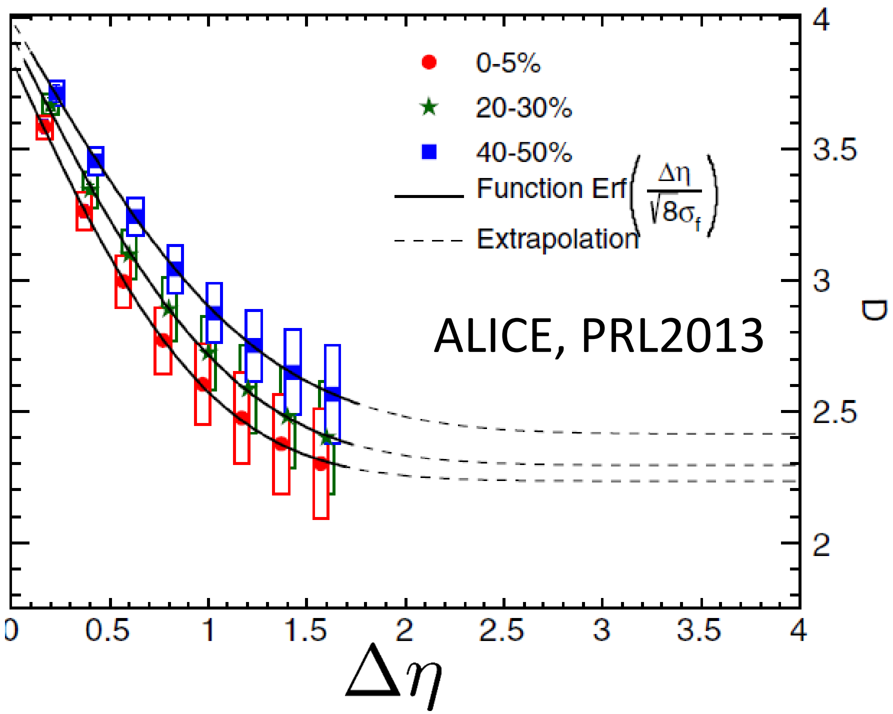
Resonance Decay



The larger Δη, the slower diffusion.

2nd Order @ ALICE

Net charge fluctuation

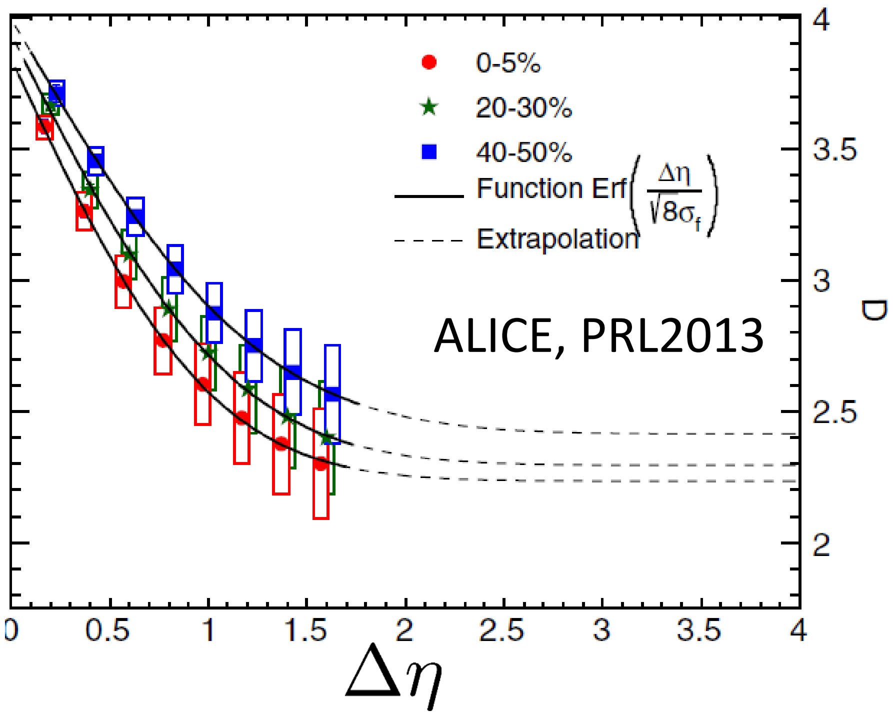


D-measure

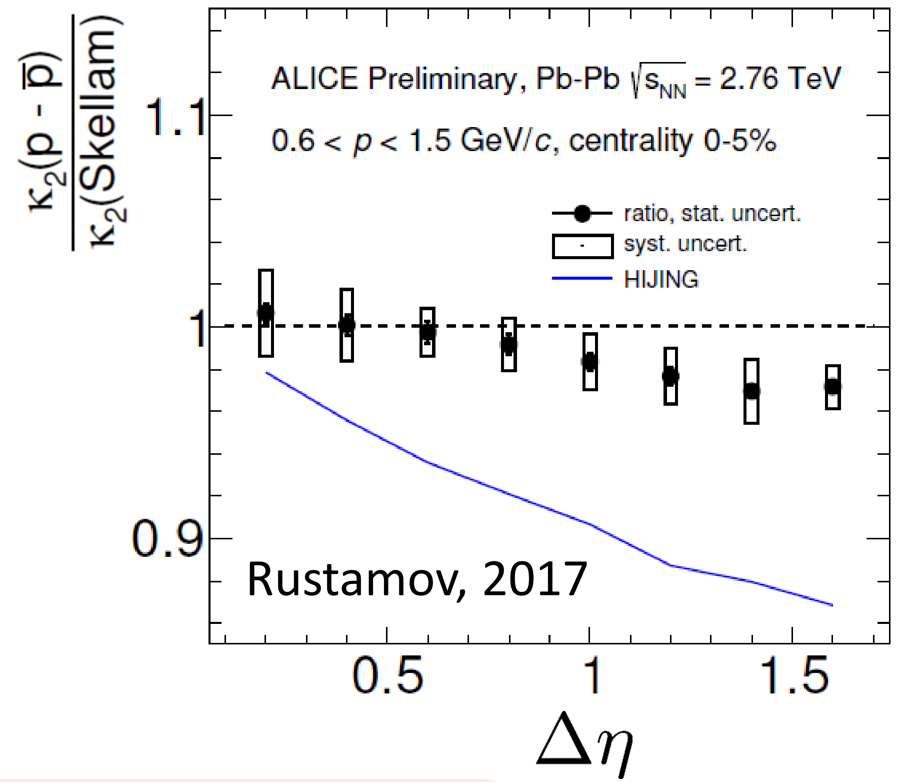
$$D \simeq 4\frac{\langle \delta N_Q^2 \rangle}{\langle \delta N_Q^2 \rangle_{\rm HRG}}$$

2^{nd} Order @ ALICE

Net charge fluctuation



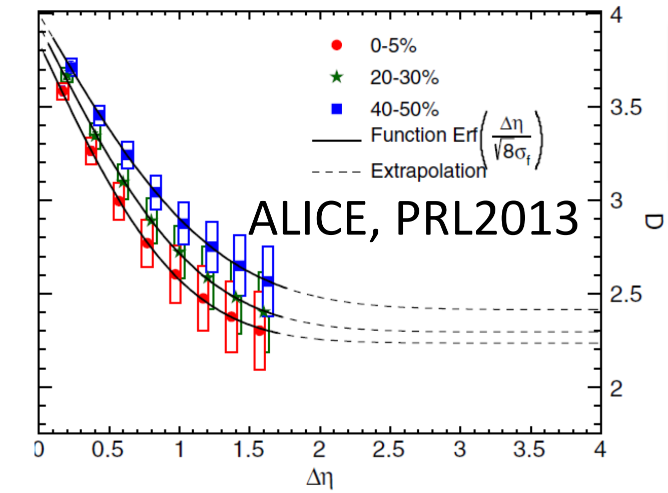
Net proton fluctuation



- Net-charge fluctuation has a suppression,
- but net-proton fluctuation does not. Why??

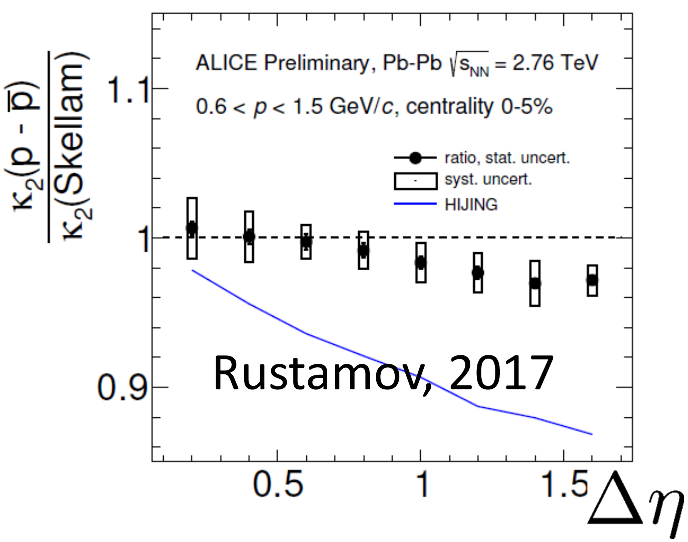
Suggestion

Net charge fluctuation



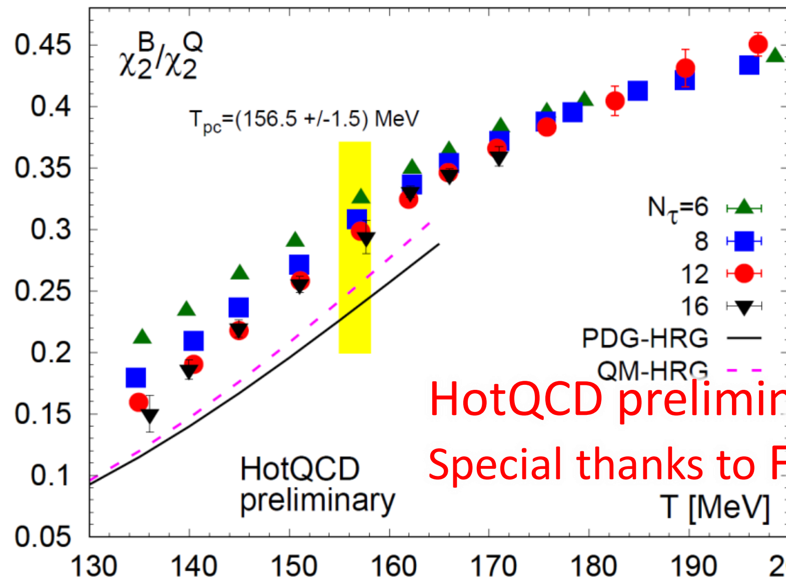
ALICE, PRL2013

Net proton fluctuation



Rustamov, 2017

- ☐ Construct $\langle \delta N_B^2 \rangle$ $(\langle \delta N_N^2 \rangle)$, $\langle \delta N_Q^2 \rangle$
- ☐ Then, take ratio $\langle \delta N_B^2 \rangle / \langle \delta N_Q^2 \rangle$
- ☐ Compare it with lattice



HotQCD preliminary

Special thanks to F. Karsch

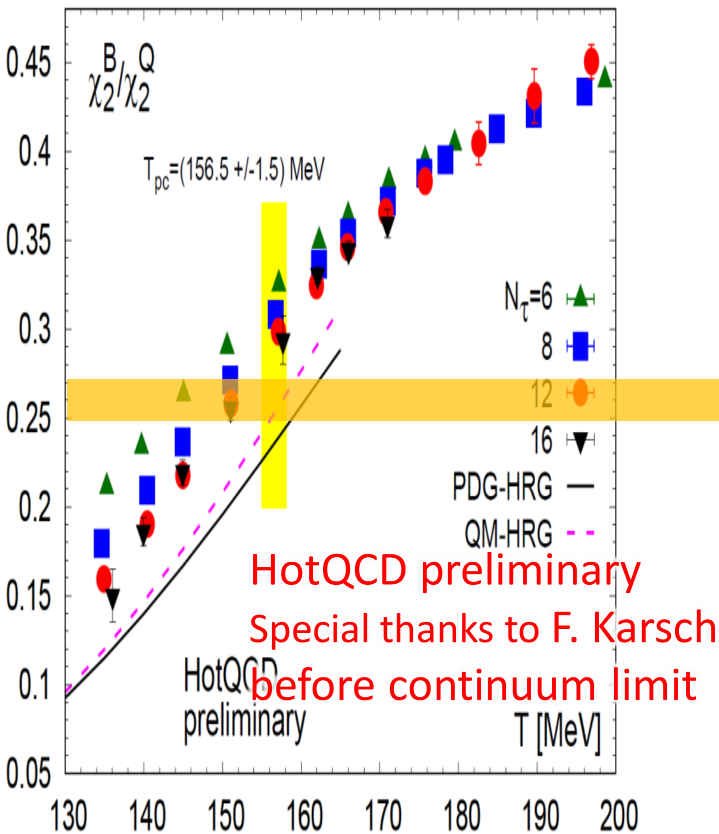
✓ **linear T dependence near Tc !!**

✓ **only 2nd order: reliable !!**

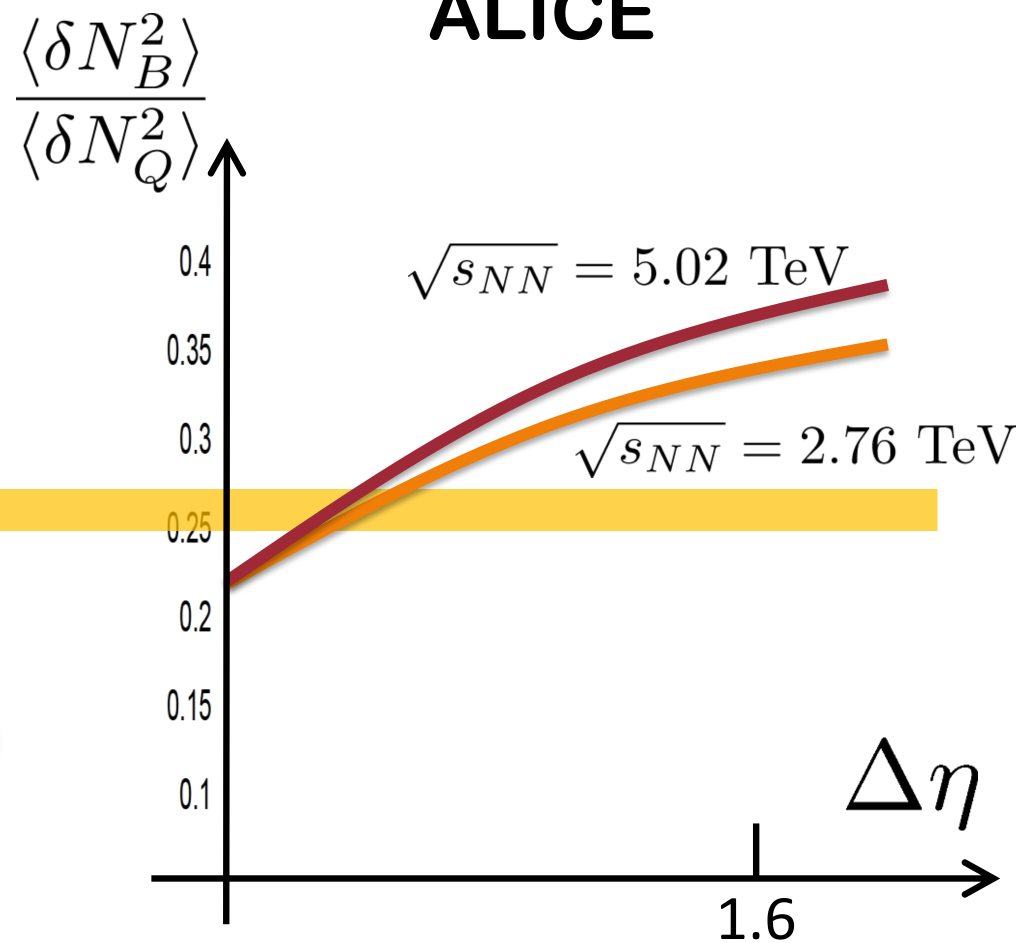
First reliable comparison of LAT/HIC

Prediction

LATTICE



ALICE



Δη dependence for tracing back the history!

Contents of
Critical Diffusion Dynamics

1. in **Hadronic Stage**

   MK, Ono, Asakawa, PLB (2014); MK(2015)

2. around the **Critical Point**

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Time Evolution of Fluctuations

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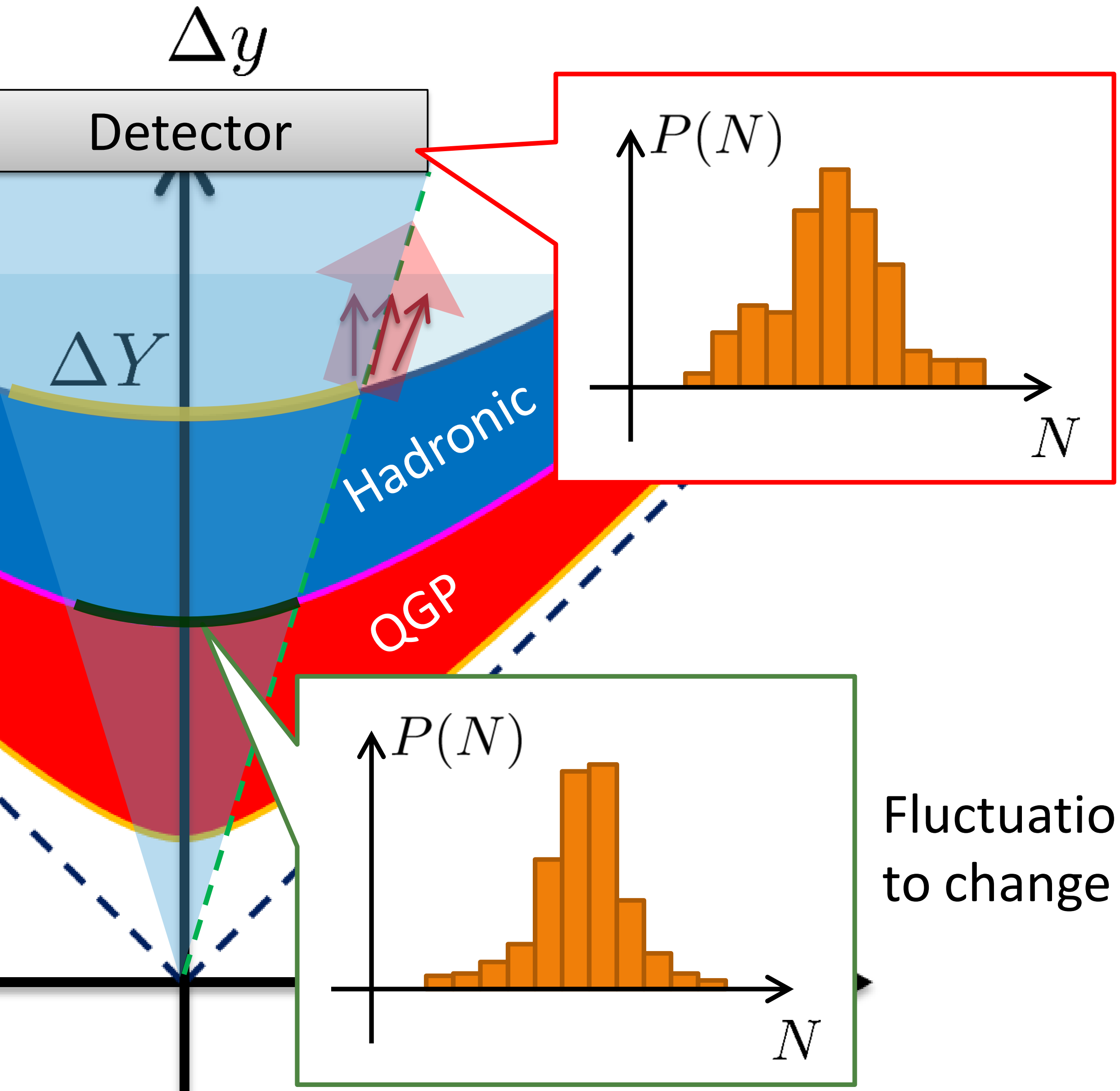
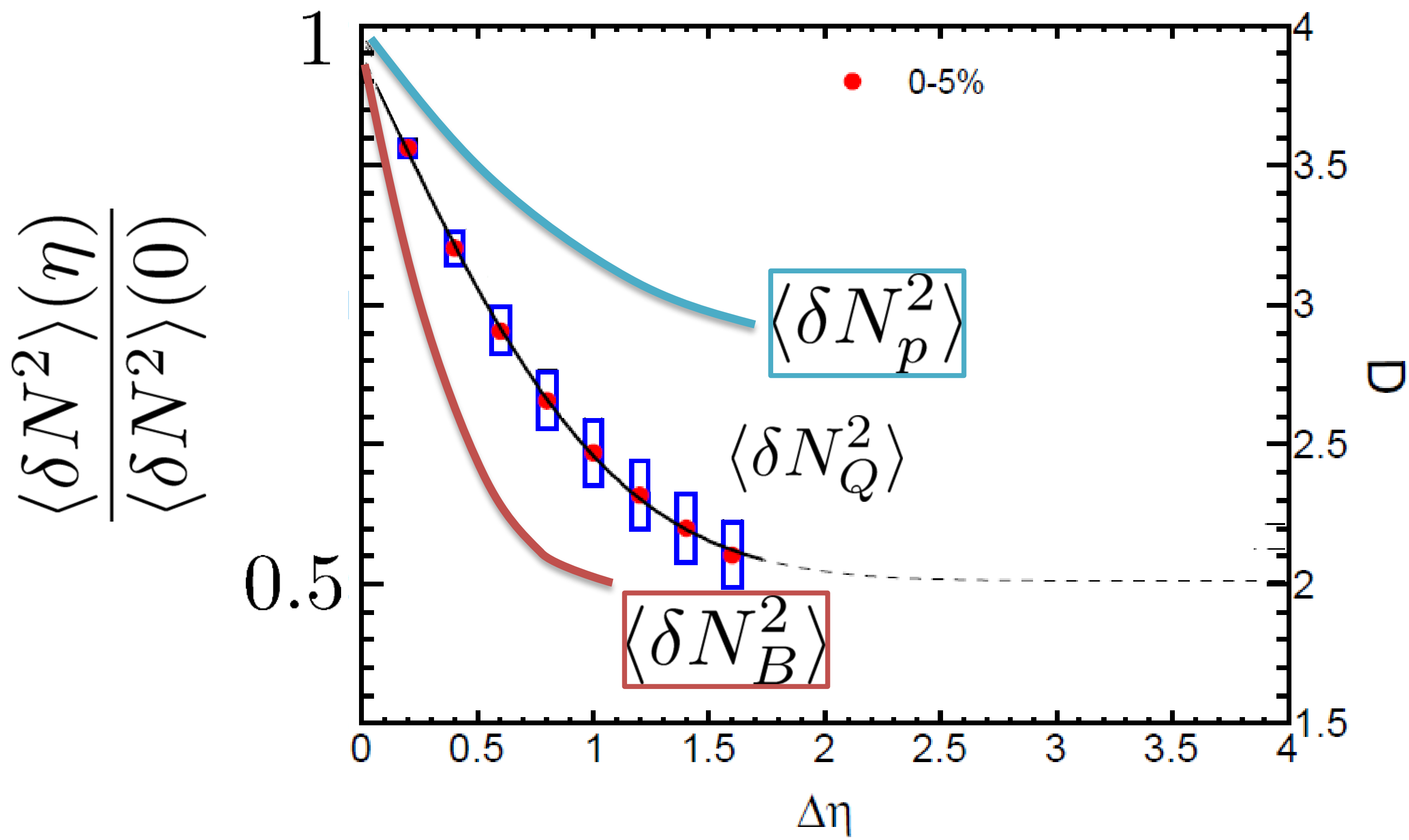


Fluctuations in ΔY continue to change until kinetic f.o.

$<\delta N_B^2>$ and $< \delta N_p^2 >$ @ LHC ?

MK, presentations
GSI, Jan. 2013
Berkeley, Sep. 2014
FIAS, Jul. 2015
GSI, Jan. 2016
…

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
should have different Δη dependence.



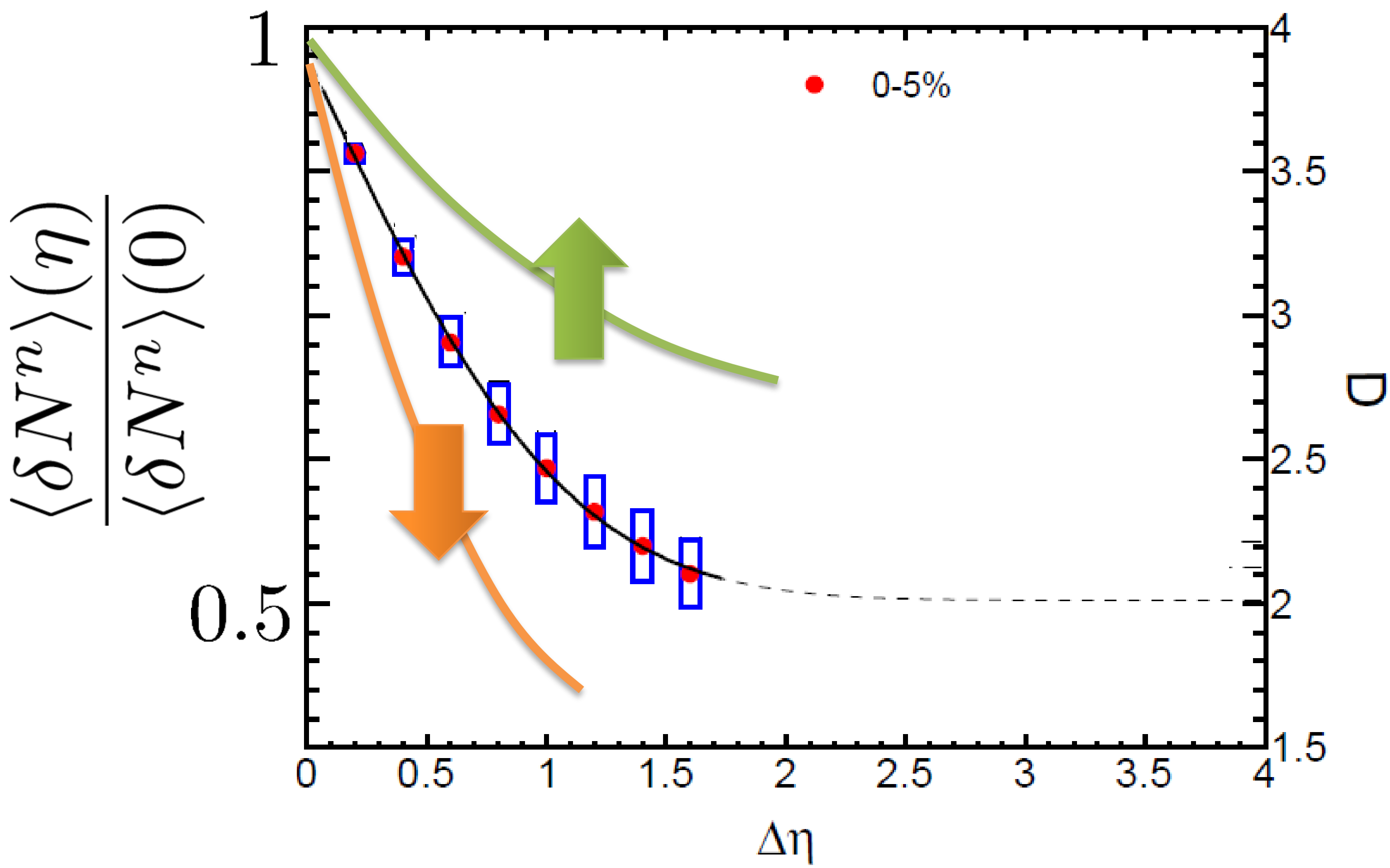
Baryon # cumulants are experimentally observable! MK, Asakawa, 2012

$<\delta N_Q{}^4>$ @ LHC ?

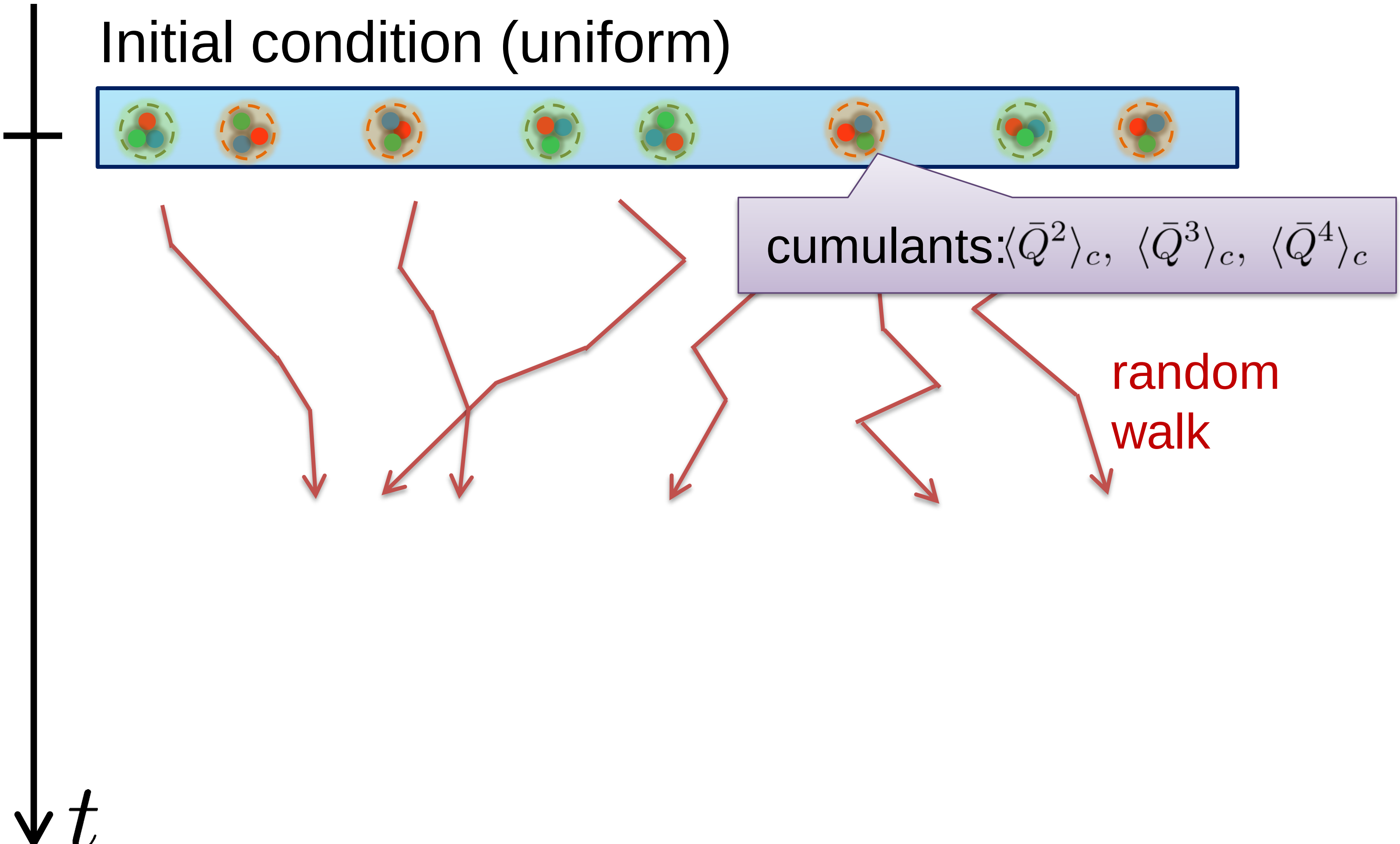
MK, presentations
GSI, Jan. 2013
Berkeley, Sep. 2014
FIAS, Jul. 2015
GSI, Jan. 2016
…

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function o

suppression or enhancement

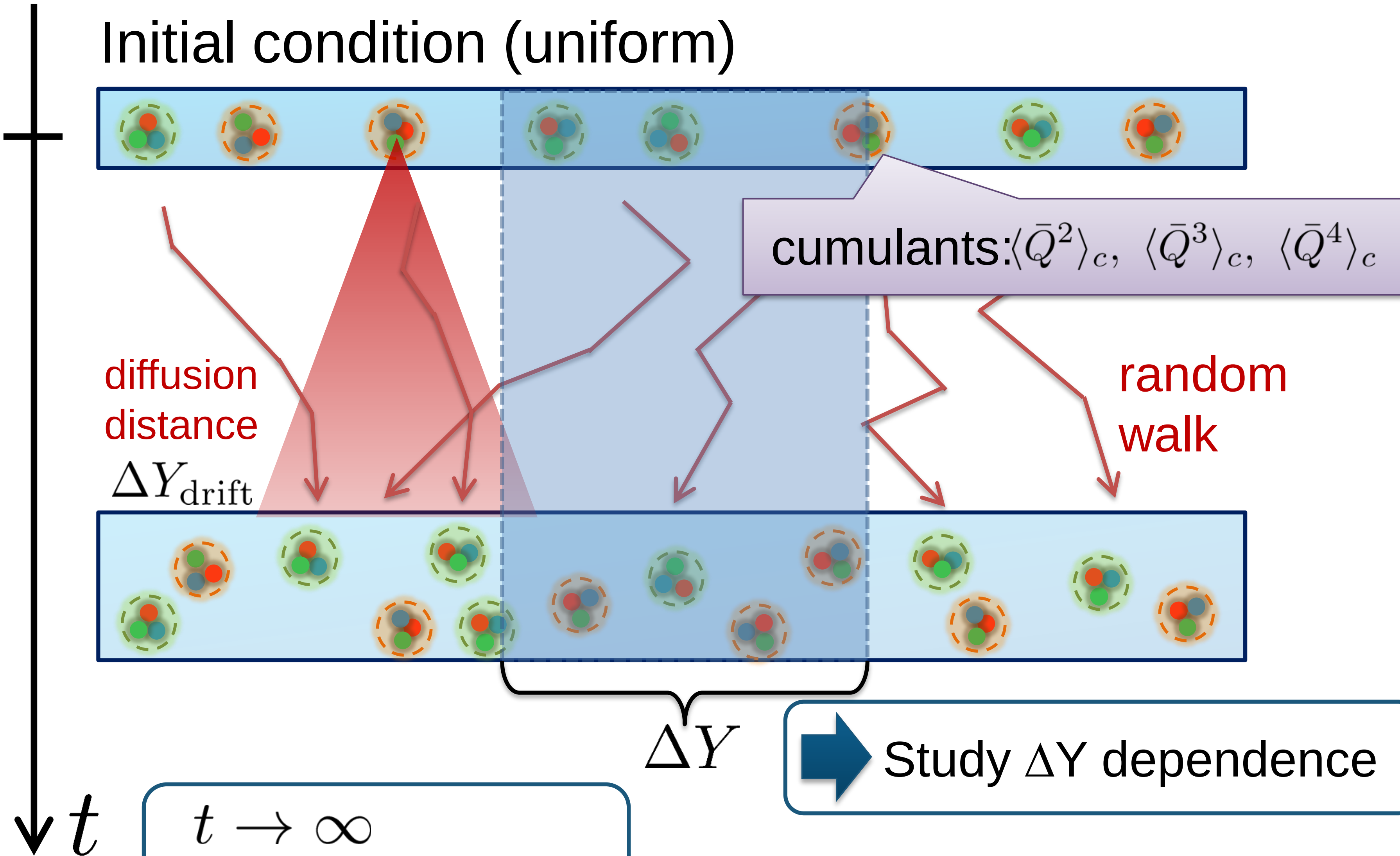


(Non-Interacting) Brownian Particle Model



diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

(Non-Interacting) Brownian Particle Model

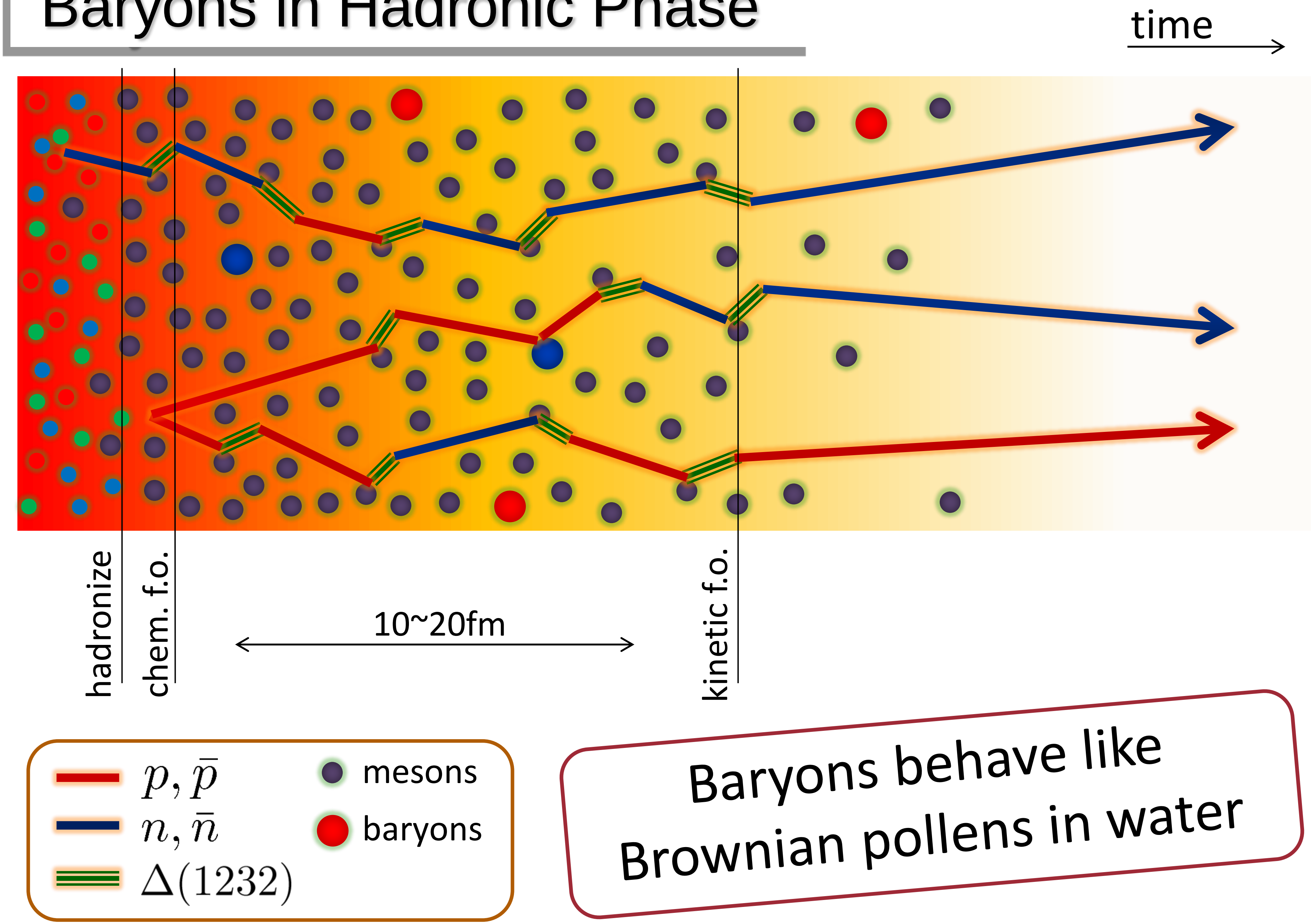


$t \to \infty$
Poisson distribution

Study ΔY dependence

diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

Baryons in Hadronic Phase

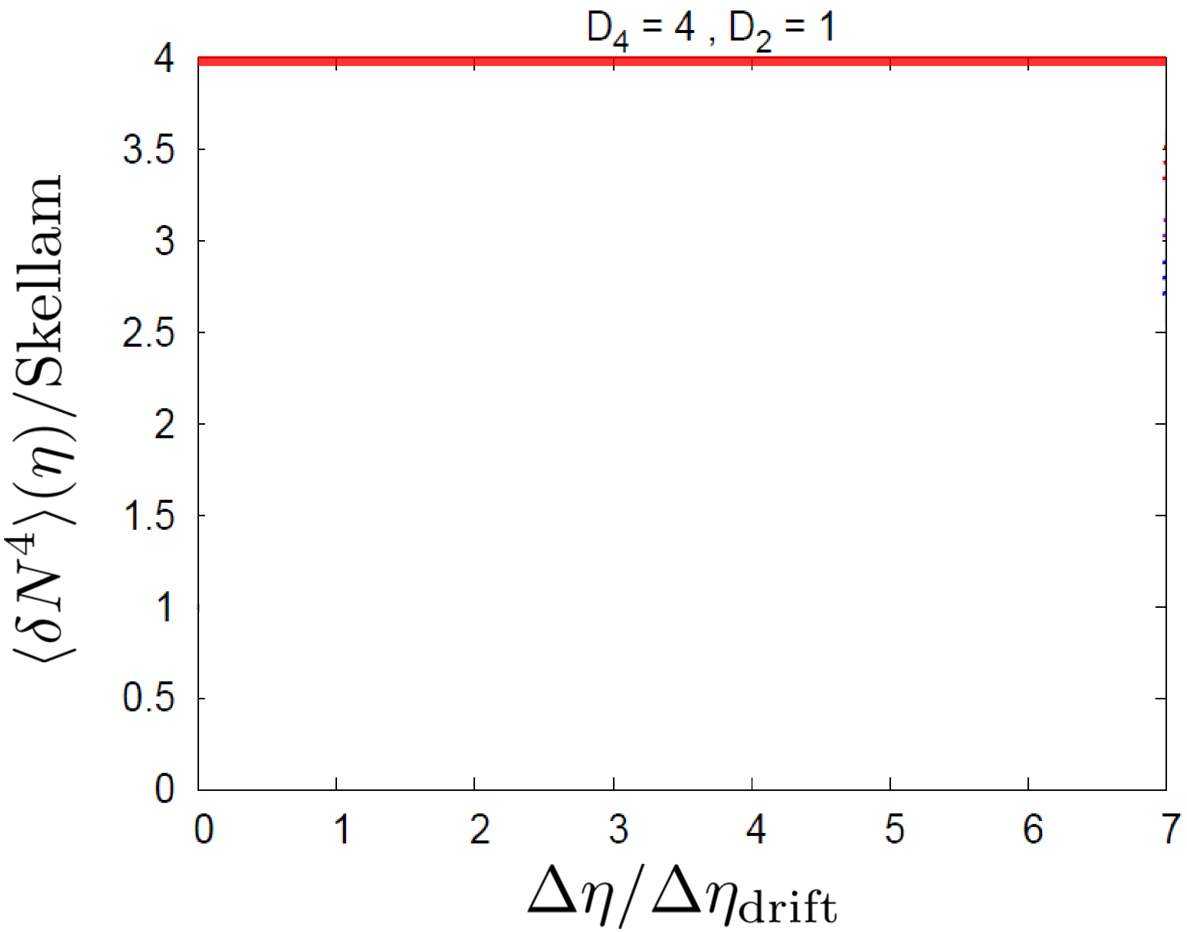


Baryons behave like
Brownian pollens in water

4^{th} Order Cumulant

MK+ (2014)
MK (2015)

Before the diffusion



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

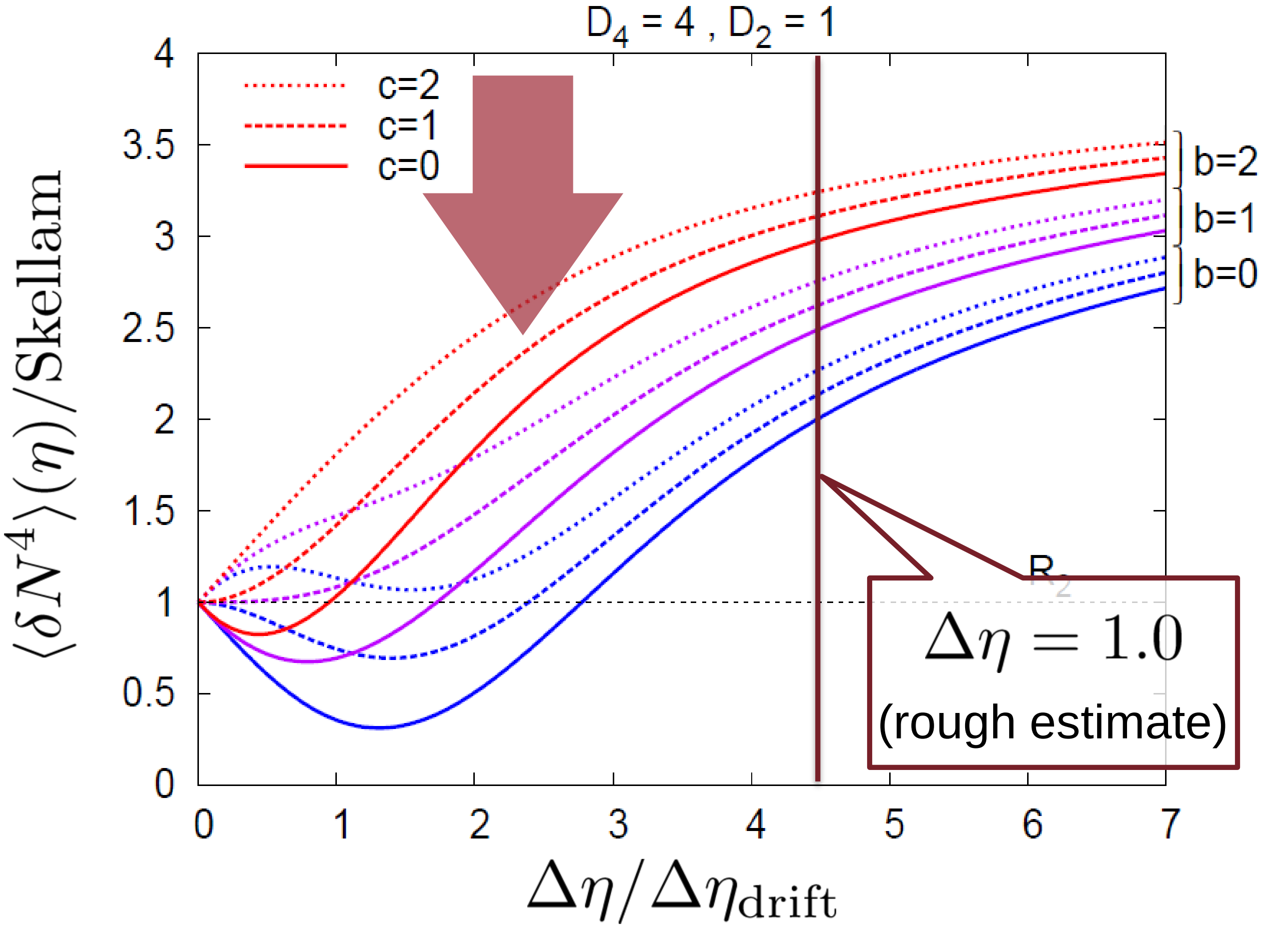
$$c = \frac{\langle Q^2_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} = 1$$

4^{th} Order Cumulant

MK+ (2014)
MK (2015)

After the diffusion



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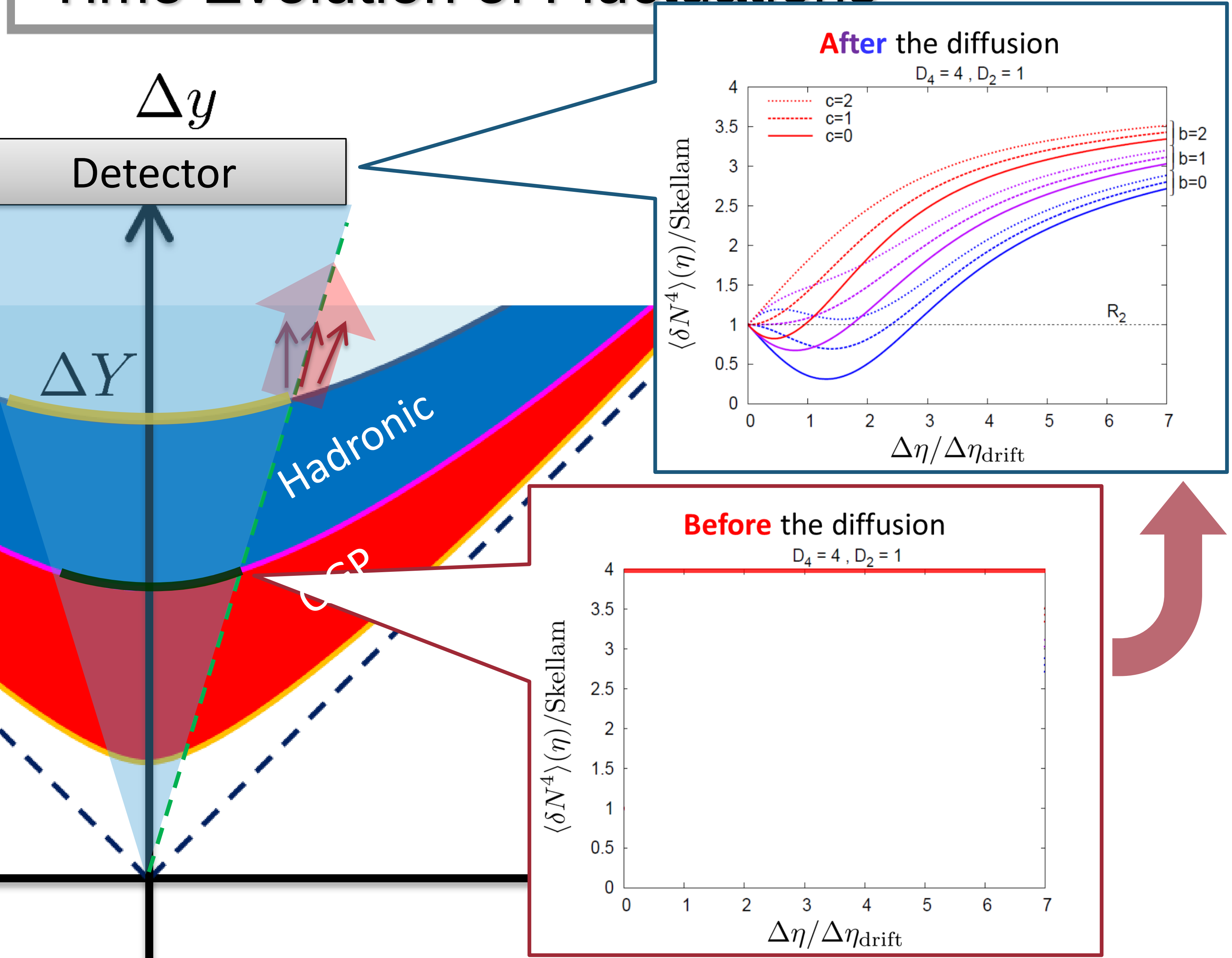
- ☐ Cumulant at small $\Delta\eta$ is modified toward a Poisson value.
- ☐ Non-monotonic behavior can appear.

Time Evolution of Fluctuations



As a result of a simple random walk...

Rapidity Window Dep.

4th-order cumulant

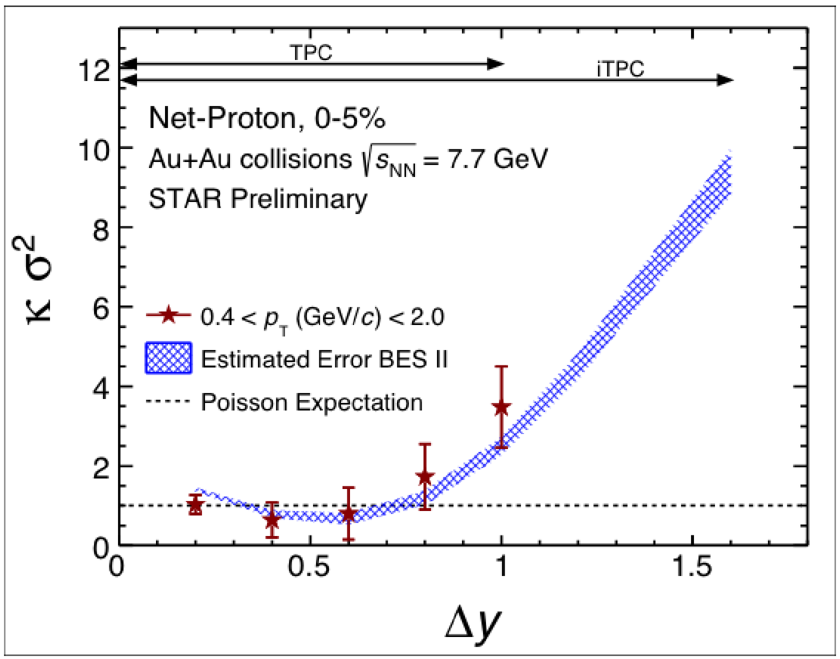
MK+, 2014
MK, 2015

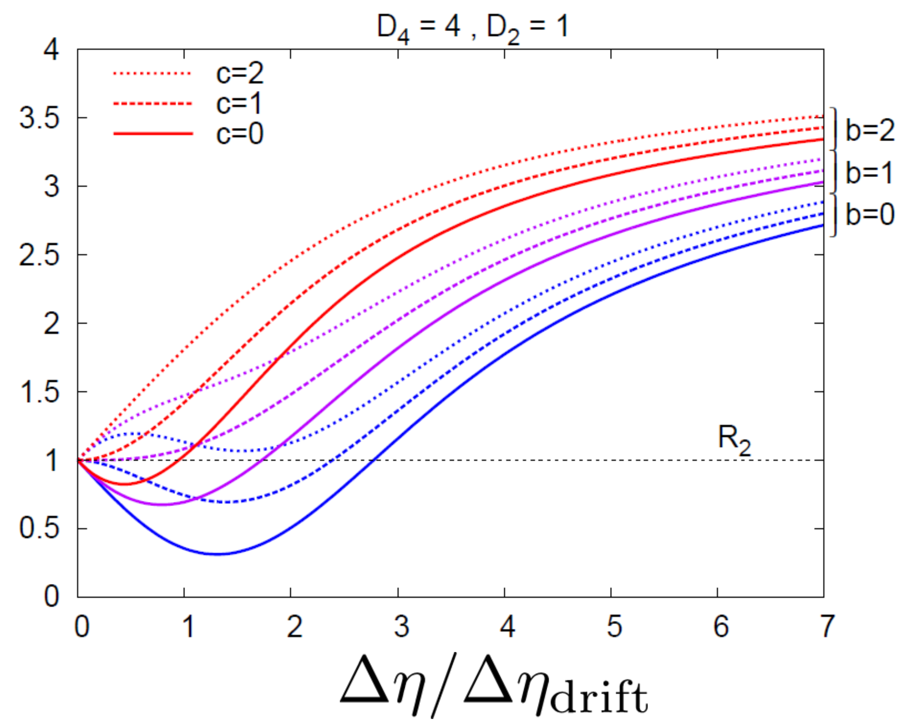
Initial Conditions

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad c = \frac{\langle Q^2_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

STAR Collab. (X. Luo, CPOD2014)



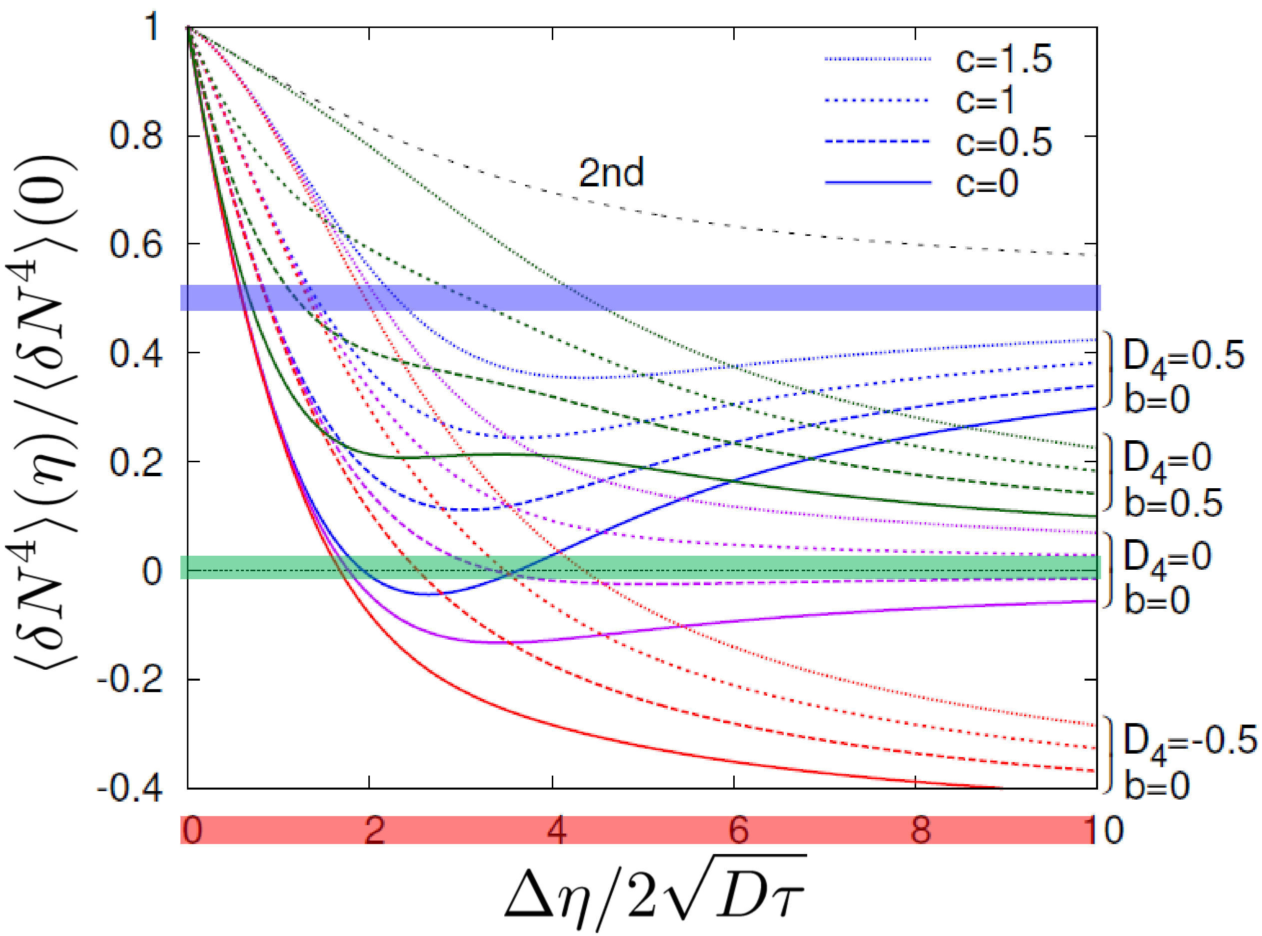


- Is non-monotonic $\Delta\eta$ dependence already observed?
- Different initial conditions give rise to different characteristic $\Delta\eta$ dependence. → Study initial condition

Finite volume effects: Sakaida+, PRC90 (2015)

Δη Dependence: 4^{th} order

MK, arXiv:1505.04349



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$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

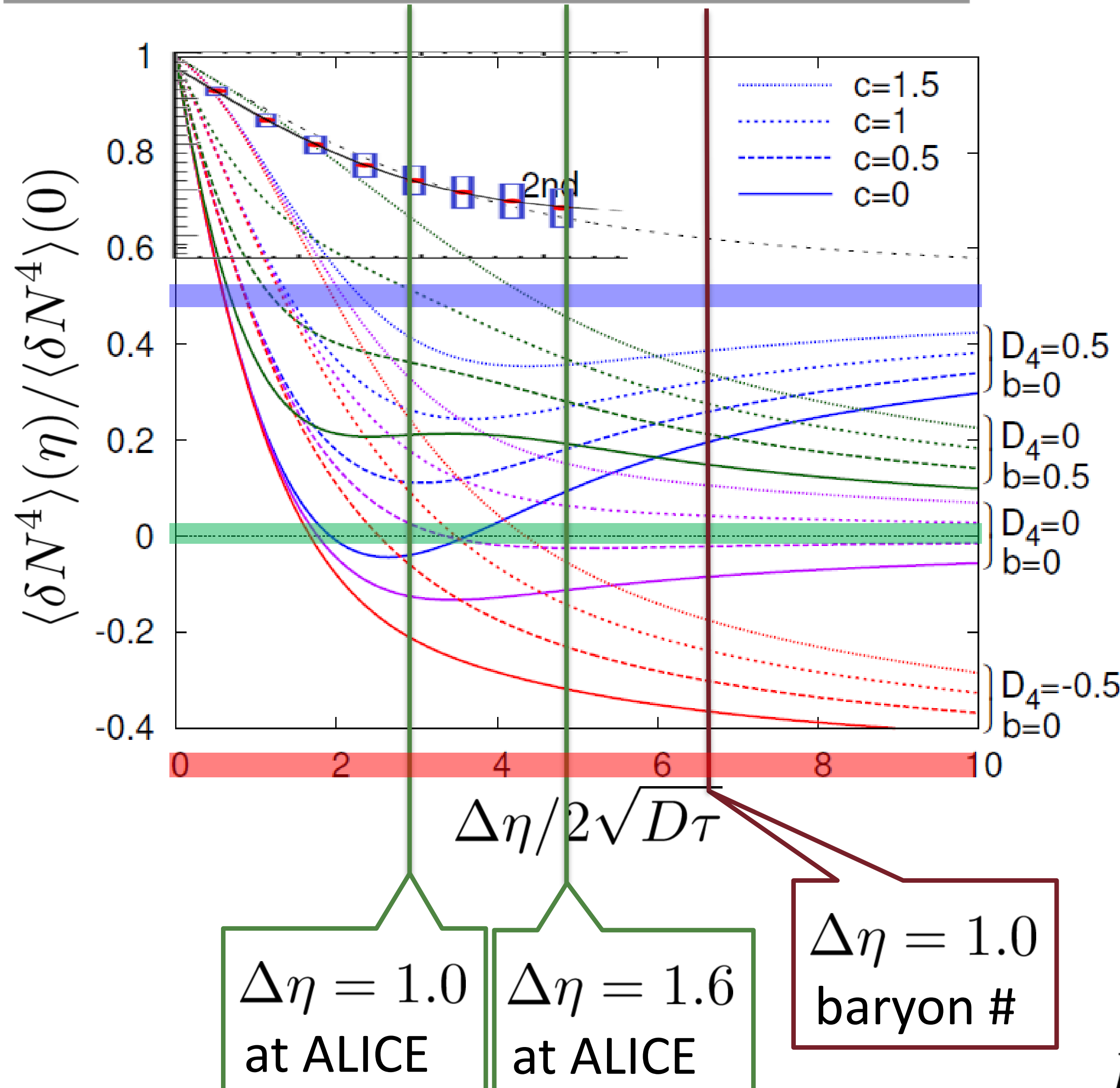
$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

Charcteristic Δη dependences!

Cumulants with a Δη is not the initial value.

Δη Dependence: 4th order

MK, arXiv:1505.04349



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$$D \sim M^{-1}$$

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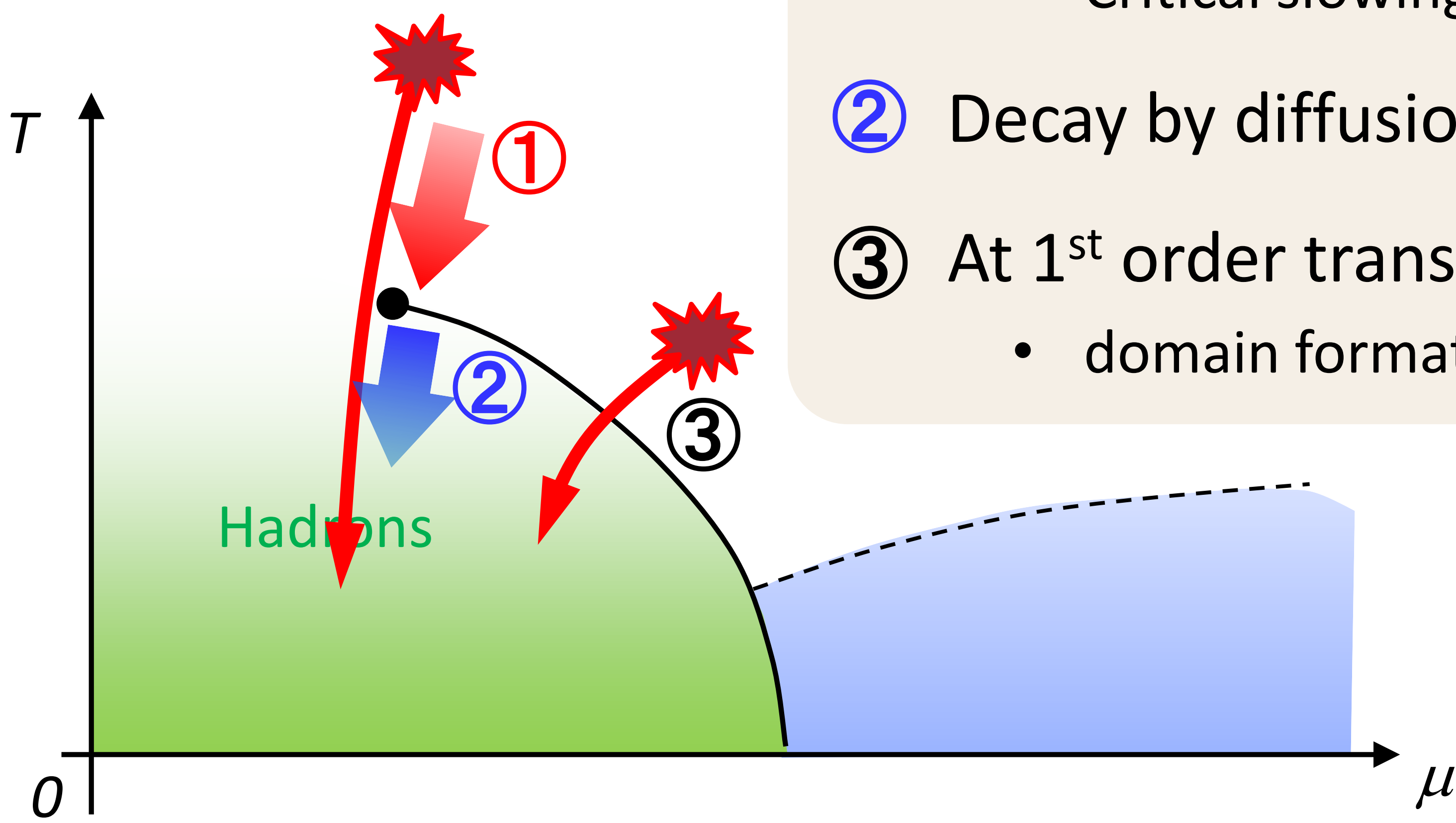
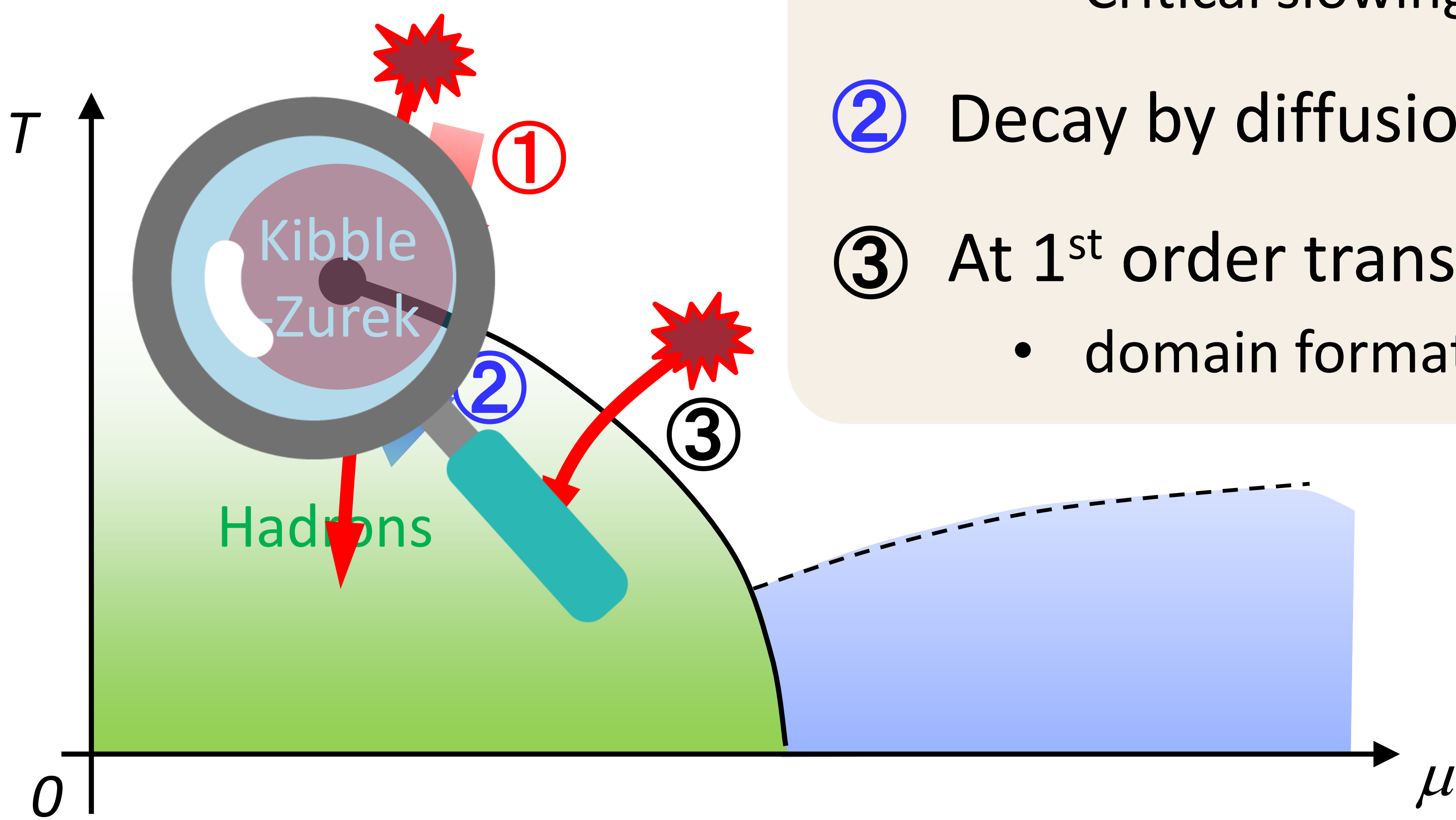


① Growth of critical fluctuation
- Critical slowing down

② Decay by diffusion

③ At 1st order transition
- domain formation

Critical Fluctuation



① Growth of critical fluctuation
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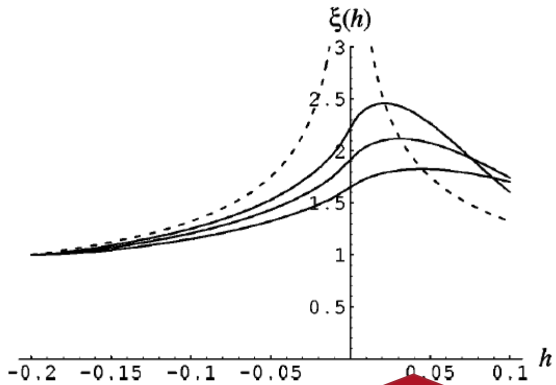
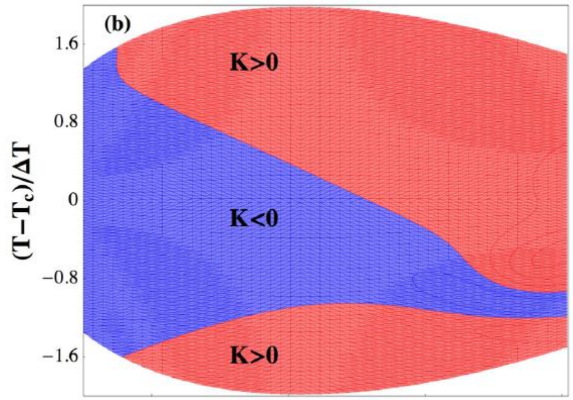
Dynamical Evolution of Critical Fluctuations

Evolution of **spatially uniform "σ" mode**





Berdnikov, Rajagopal (2000)
Asakawa, Nonaka (2002)
Mukherjee+ (2015)
…

Model A
Model B

THIS STUDY

Evolution of **conserved charge fluctuations**

Sakaida+, PRC2017; Murata, MK, in prep.

1. Conserved charges are directly observable.
2. Soft mode at QCD-CP is a conserved mode.

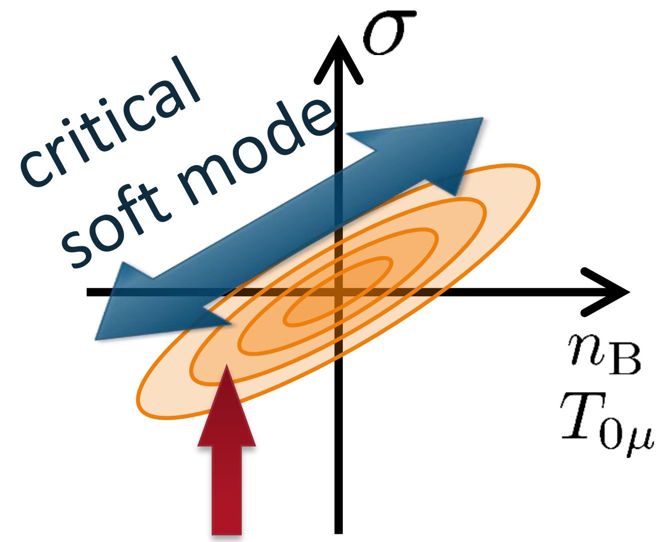
See also, Kapusta, Torres-Rincon (2012); Herold, Nahrgang, … (2015); …

Soft Mode of QCD-CP = Conserved Mode

Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

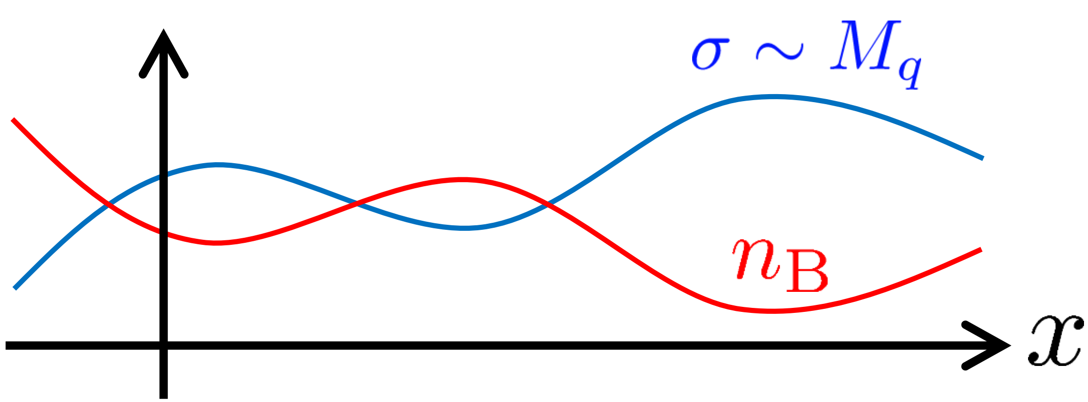
Fluctuations of σ and n_B are coupled around the CP!

$$\delta\sigma \simeq \delta n_B$$



σ: fast damping

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \cdots$$



Analysis of 2nd-order Cumulant

Sakaida+ (2017)

Evolution of baryon number density

Stochastic Diffusion Equation

$$\partial_t n = D(t)\partial_x^2 n + \partial_x \xi$$

$$\langle \xi(x_1,t_1)\xi(x_2,t_2)\rangle = 2D\chi_2\delta^{(2)}(1-2)$$

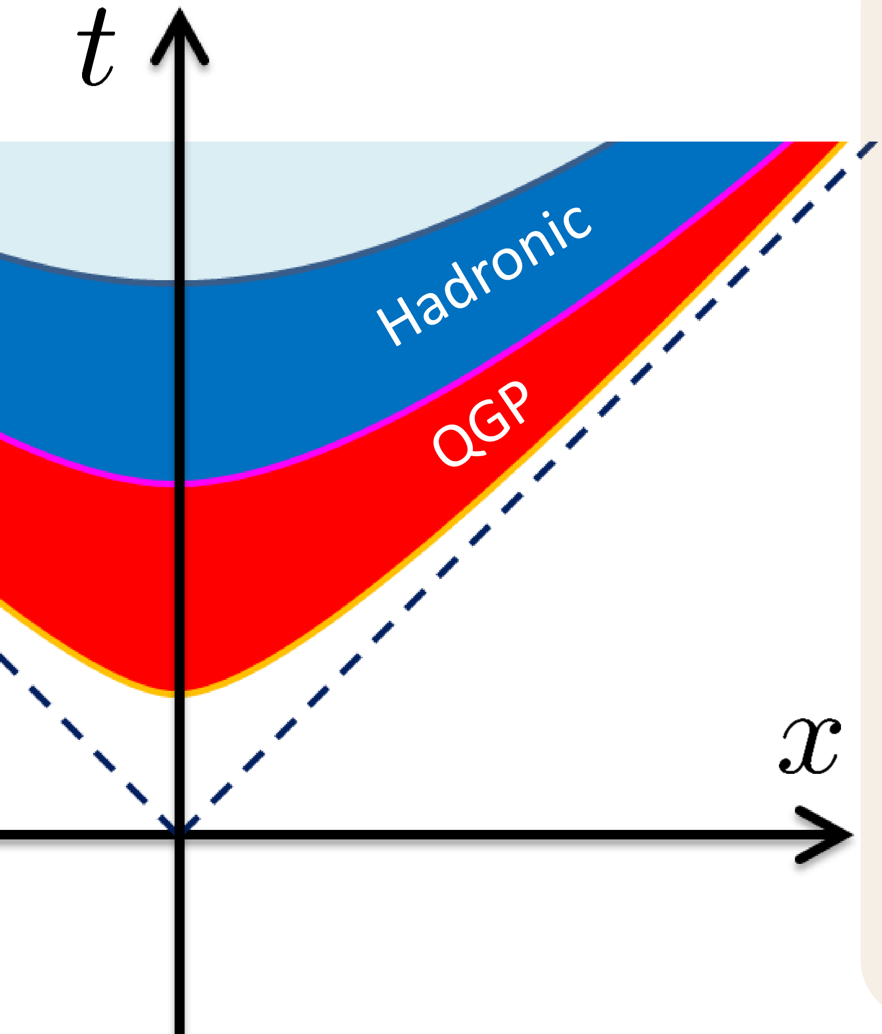
$D(t),\ \chi_2(t)$:parameters characterizing criticality

- Analytic solution is obtained.
- Study 2nd order cumulant & correlation function.

Our Main Conclusion

Non-monotonicity in cumulants or correlation func.	=	Signal of QCD-CP

Bjorken Expansion



Cartesian coordinates

$$\partial_t n = D(t)\partial_x^2 n + \partial_x \xi$$

Milne coordinates

$$\partial_\tau n = \frac{D(t)}{\tau^2}\partial_Y^2 n + \frac{1}{\tau}\partial_Y \xi - \frac{n}{\tau}$$

suppression of diffusion

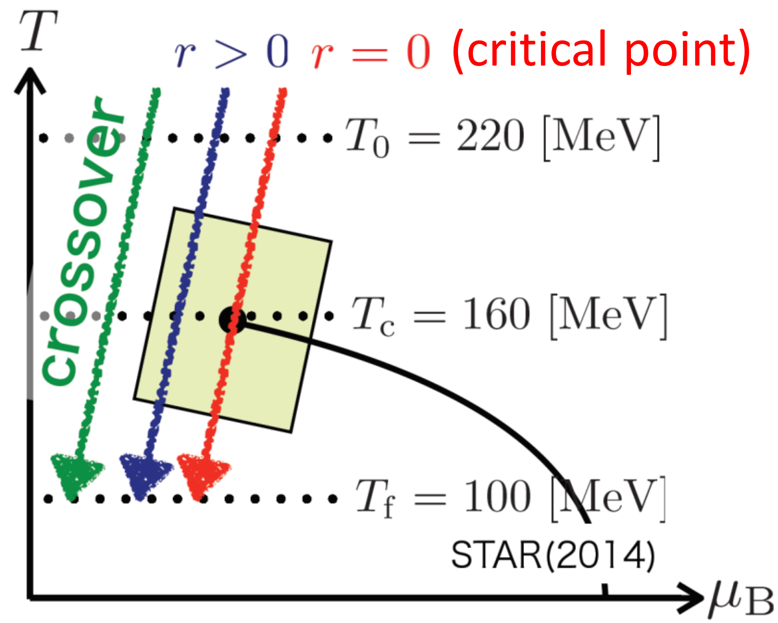
density reduction

Parametrizing D(τ) and χ(τ)

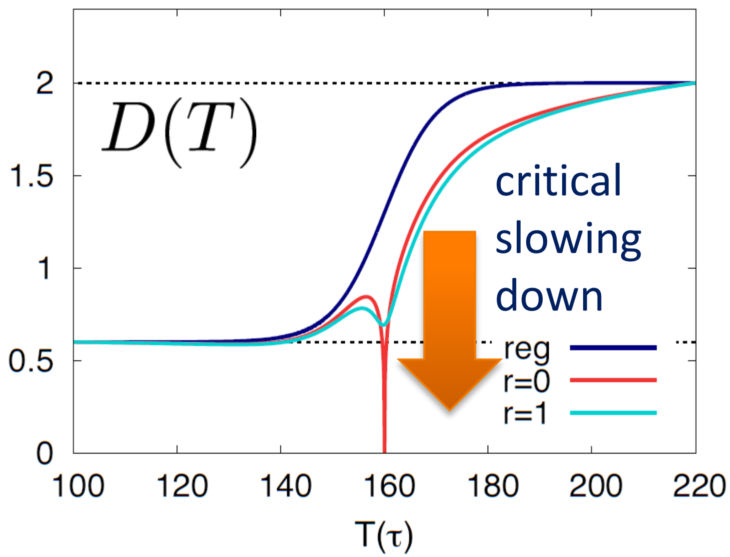
☐Critical behavior

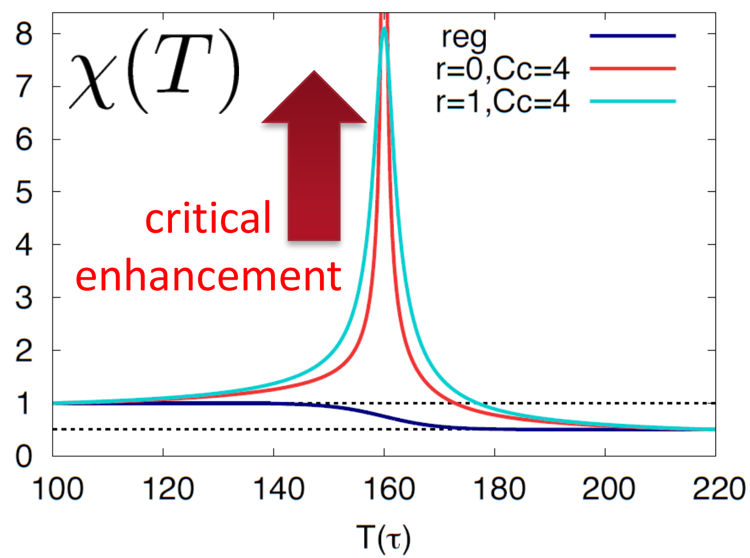
- 3D Ising (r,h)
- model H

Berdnikov, Rajagopal (2000)
Stephanov (2011); Mukherjee+(2015)



☐Temperature dep.





Assumptions

Evolution of baryon number density

Stochastic Diffusion Equation

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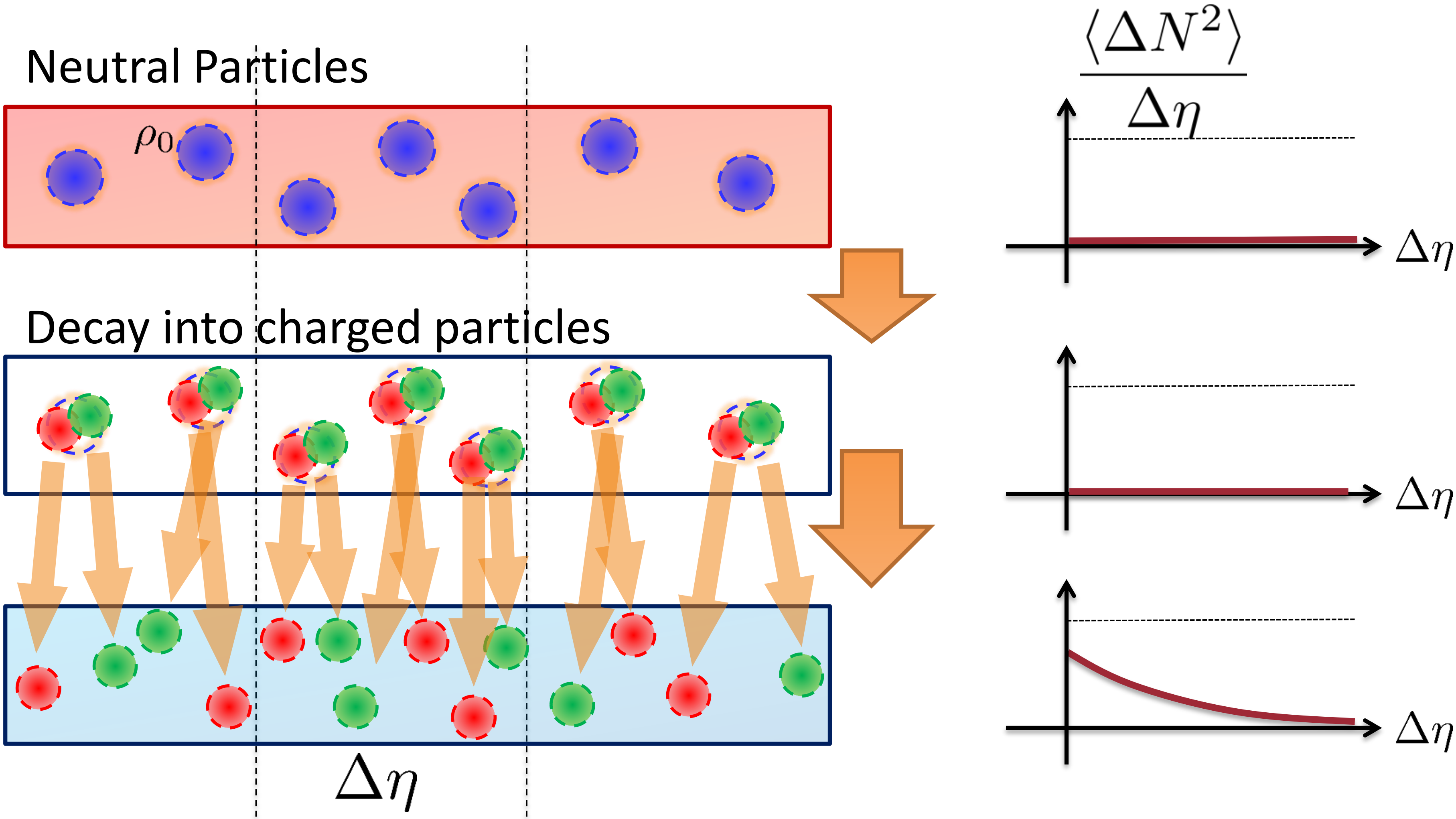
$$\langle \xi(x_1,t_1)\xi(x_2,t_2)\rangle = 2D\chi_2\delta^{(2)}(1-2)$$

$D(t),\ \chi_2(t)$:parameters characterizing criticality

- ☐ Uniform / infinitely long system
- ☐ Near equilibrium: $\delta N_\mu << N_0$
- ☐ Short correlation length
- ☐ Slow diffusion

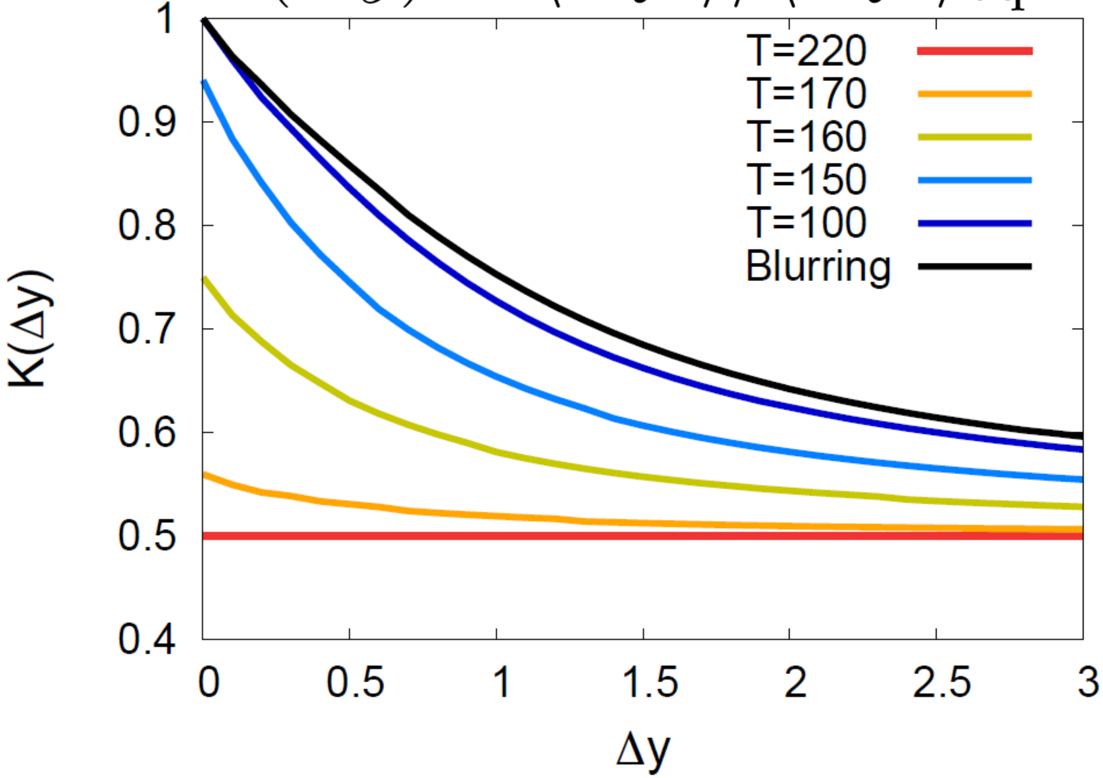
Resonance Decay

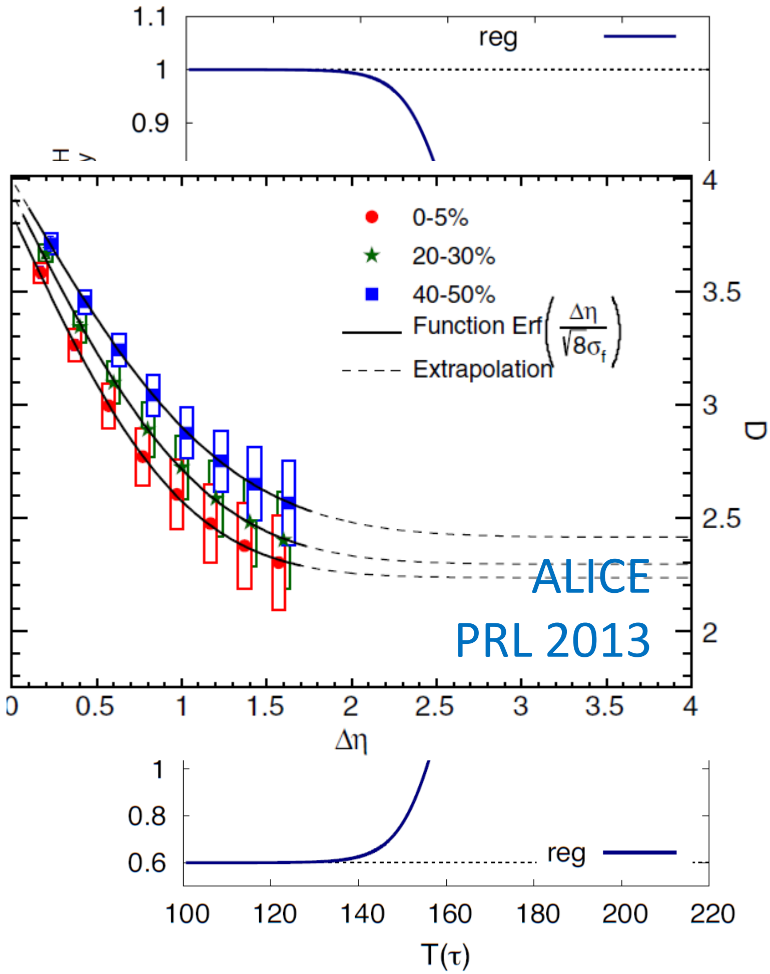


The larger Δη, the slower diffusion.

Crossover / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\mathrm{eq.}}$$



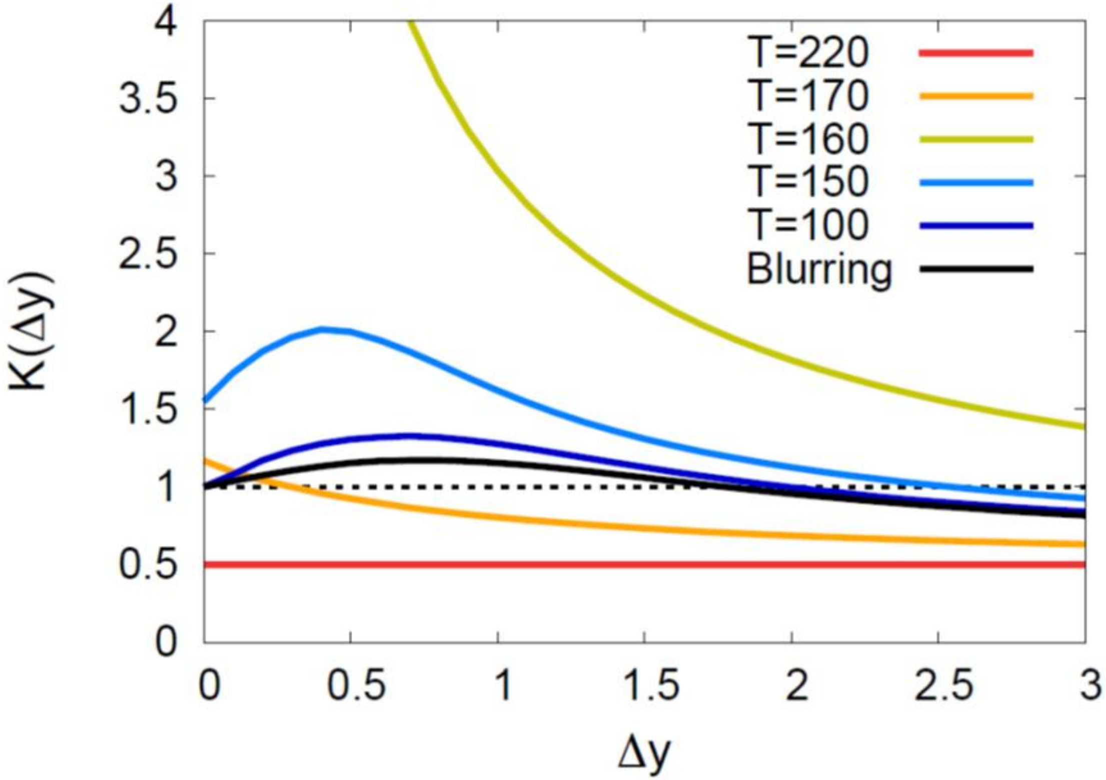


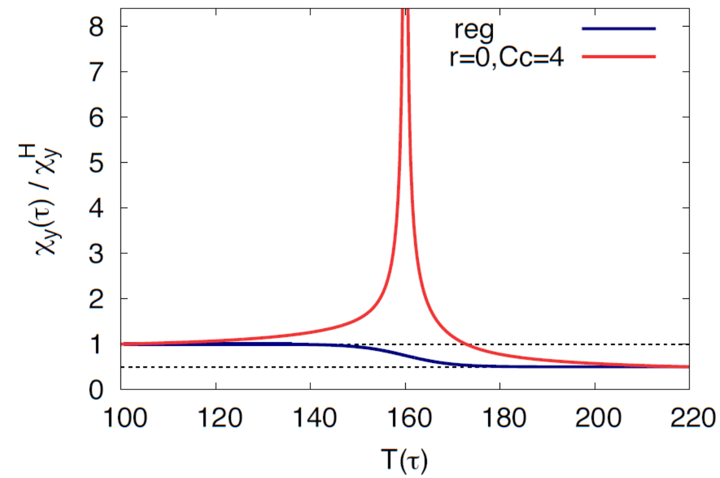
- ☐ monotonically decresing

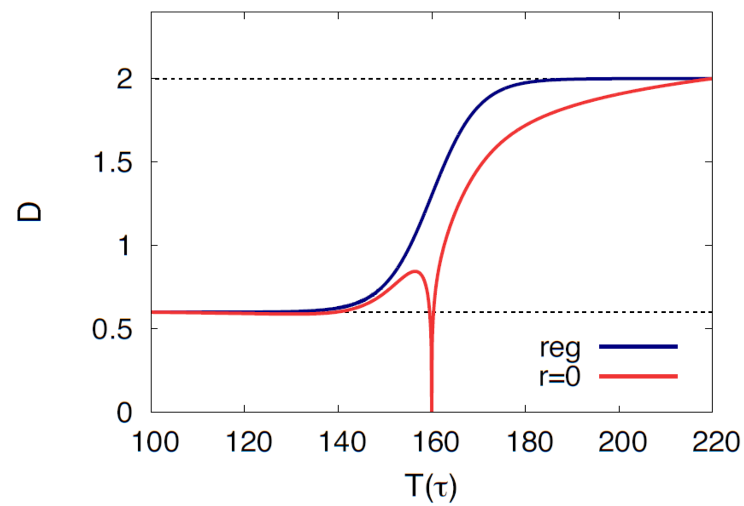
Analytic result	$\chi(\tau)$ monotonically increasing	⇨	$K(\Delta y)$ monotonically decreasing

Critical Point / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.}$$







- non-monotonic Δy dep.

Analytic result: $K(\Delta y)$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

See also,
Wu, Song
arXiv: 1903.06075

Cumulants and Correlation Function

$$Q = \int_V dx\, n(x)$$

total charge

charge density

$$\langle \delta Q^2 \rangle = \int_V dx dy \langle \delta n(x) \delta n(y) \rangle$$

2nd order cumulant (fluctuation)

correlation function

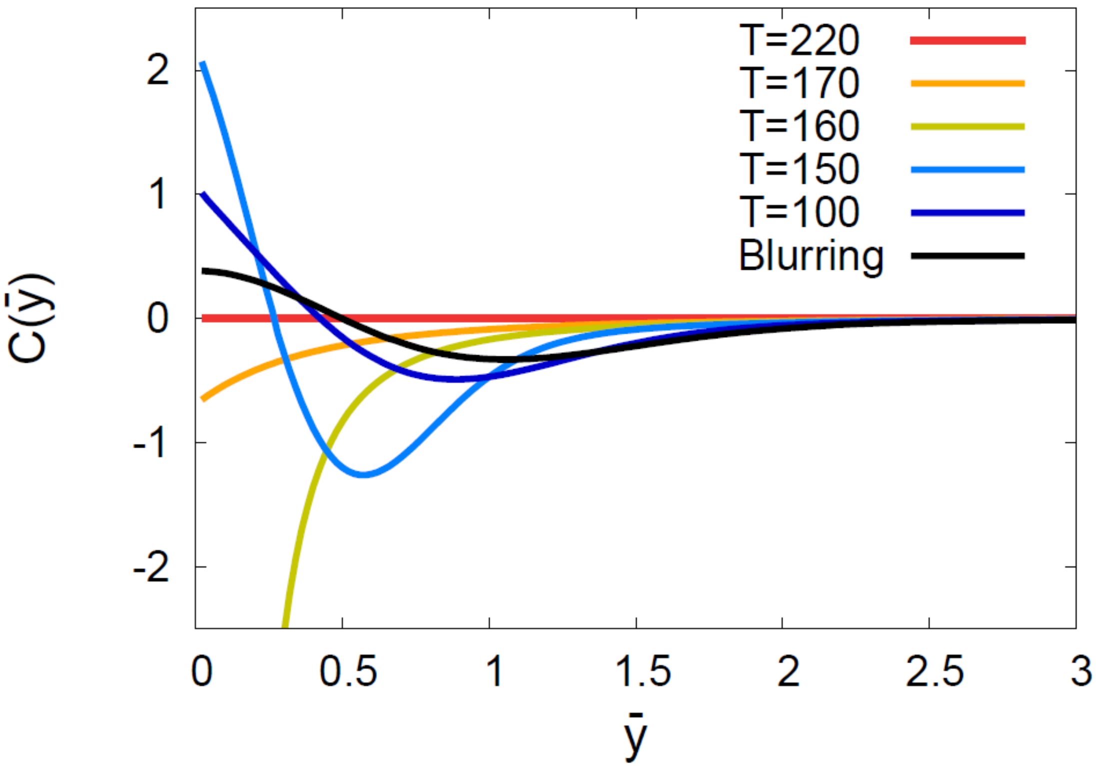
1-to-1 correspondence

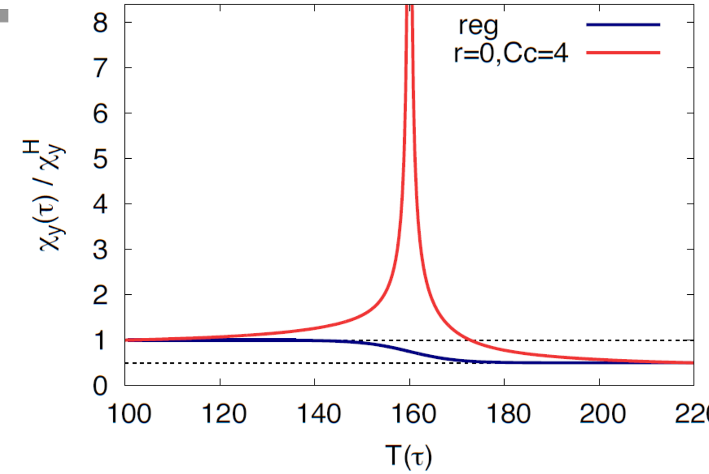
1-dim case

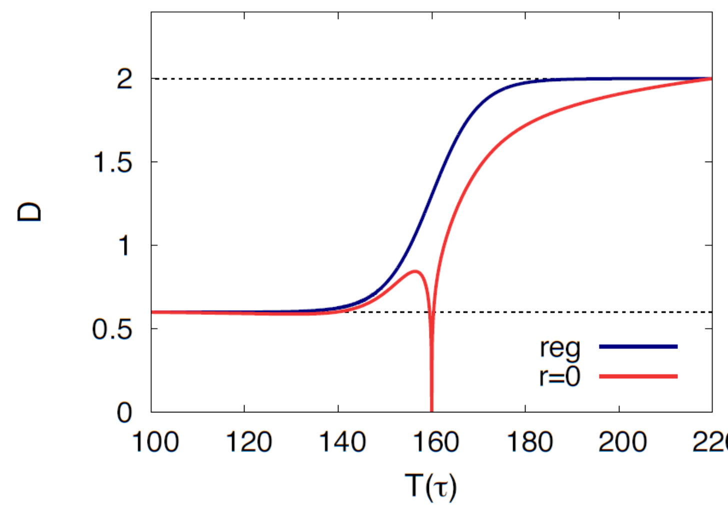
$$\langle \delta Q^2 \rangle_{\Delta y} = \int_{\Delta y} dy (\Delta y - |y|) \langle \delta n(y) \delta n(0) \rangle$$

Criticap Point / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\mathrm{hadron}}$$





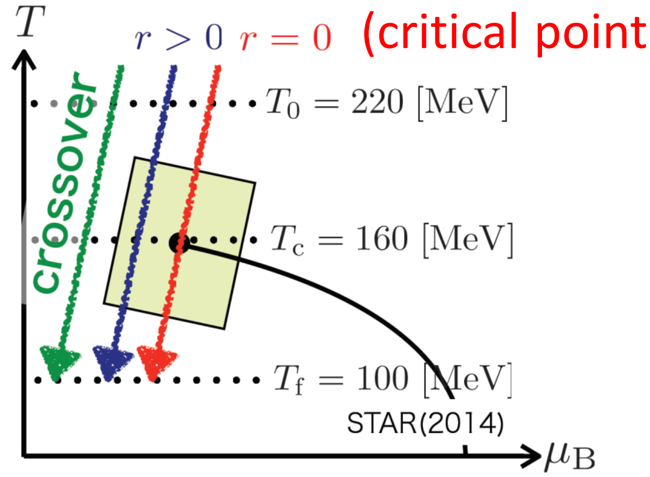


- non-monotonic Δy dep.

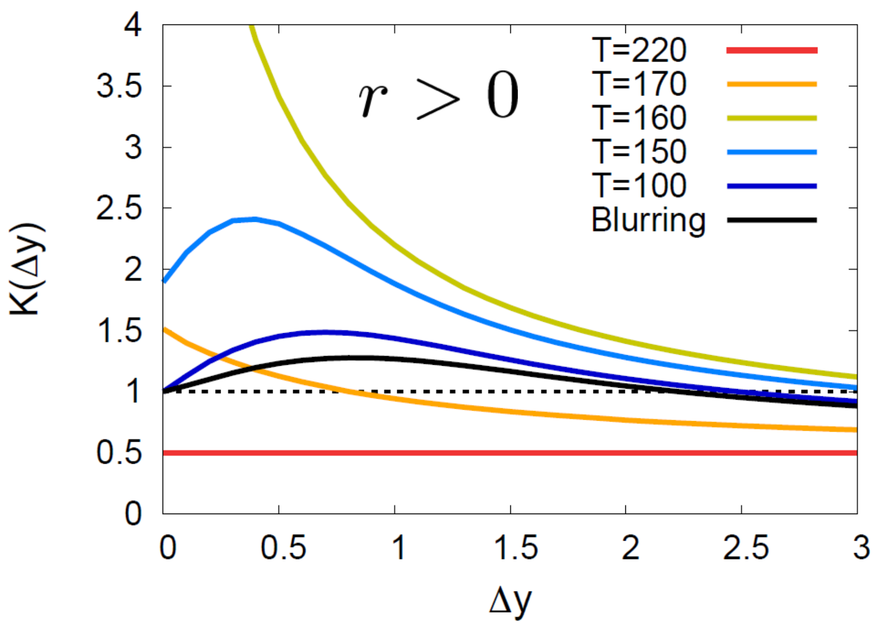
Analytic result: $C(\Delta y)$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

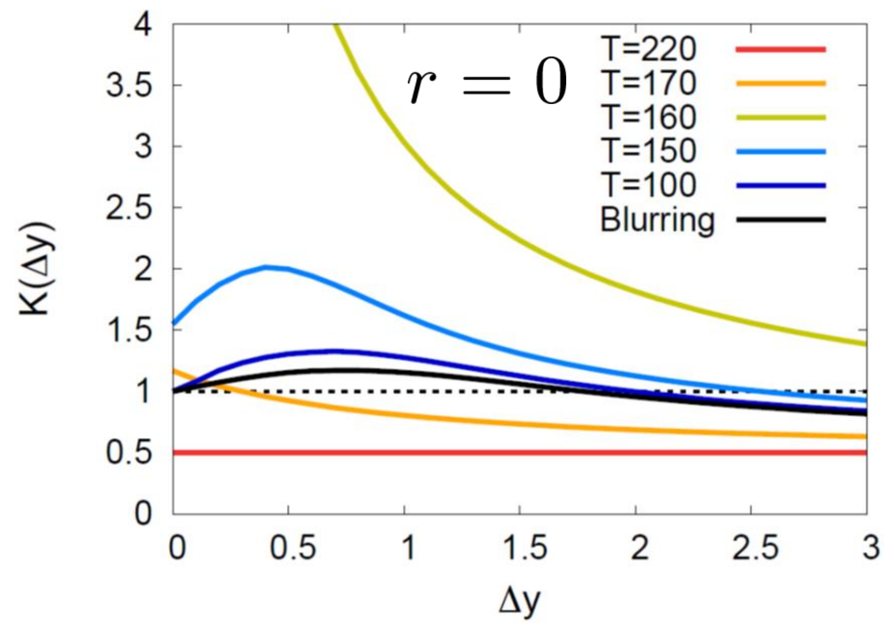
See also,
Wu, Song
arXiv: 1903.06075

Away from the CP



$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\mathrm{eq.}}$$





- Signal of the critical enhancement can be clearer on a path away from the CP.

Away from the CP → Weaker critical slowing down

Describing Non-Gaussianity

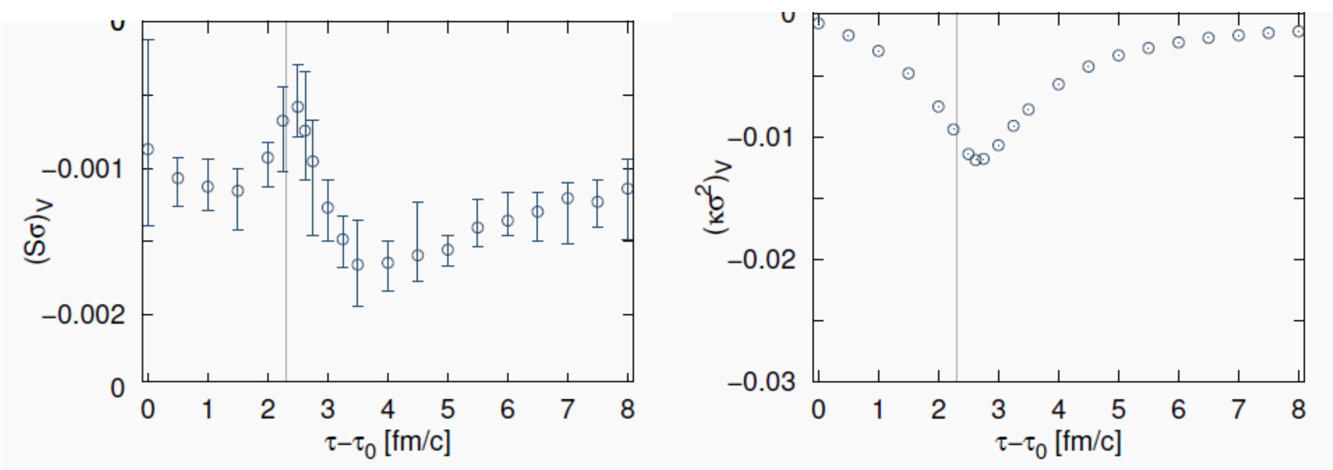
Diffusion Eq. with Non-linear Terms

$$\partial_\tau n = \Gamma(n)\partial_Y^2 \frac{\delta F[n]}{\delta n(Y)} + \partial_Y \xi$$

$$\langle \xi(Y_1,\tau_1)\xi(Y_2,\tau_2)\rangle = 2A\delta^{(2)}(1-2)$$

$$f(n) = k(\nabla n)^2 + a\Delta n^2 + b\Delta n^3 + c\Delta n^4 + \cdots$$

Nahrgang, Bluhm, Schaefer, Bass
arxiv:1804.05728



Application to
1st order transition:
Nonaka, Akamatsu, Bluhm, MK, Nahrgang, Wednesday

- ☐ Proper description of higher order cumulants

Contents of
Critical Diffusion Dynamics

1. in **Hadronic Stage**

   MK, Ono, Asakawa, PLB (2014); MK(2015)

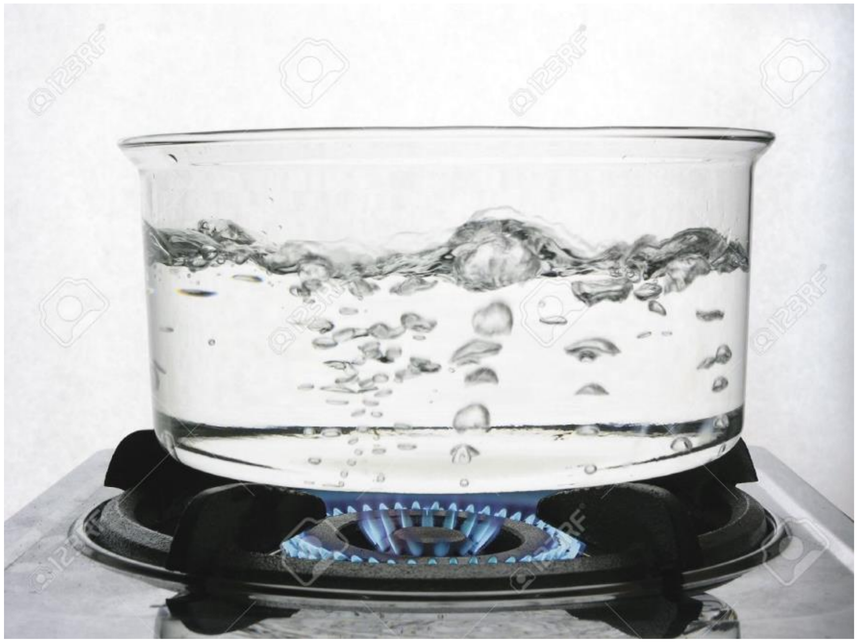
2. around the **Critical Point**

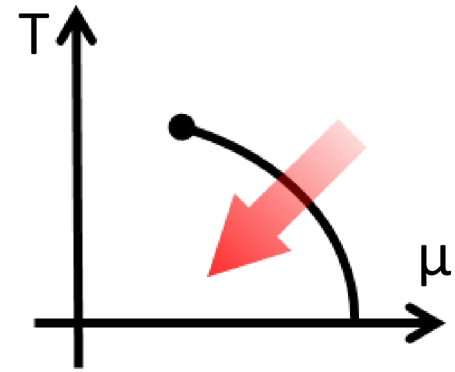
   Sakaida, Asakawa, Fujii, MK (2017)

3. at **First Order Transition**

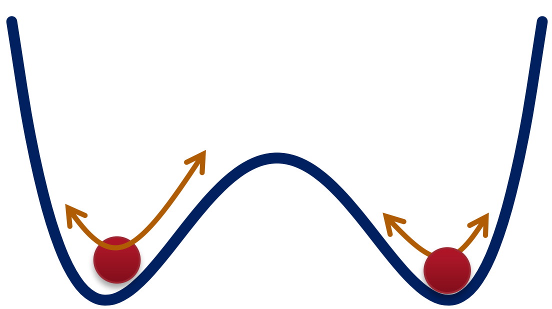
   Nonaka, MK, Akamatsu, Bluhm, Nahrgang, in prep.

1^{st}-Order Transition





- ☐ Domain formation
- ☐ Non-uniform system



Herold, Nahrgang, et al. (2011~); Steinheimer, Randrup (2012; 2013)

Including Non-Linearity

Nahrgang, Bluhm, Schafer, Bass (2018)

$$\partial_\tau n = \frac{D}{\tau^2}\partial_Y^2 n + \frac{1}{\tau}\partial_Y \xi(\eta,\tau) - \frac{n}{\tau}$$

Include non-linear effects

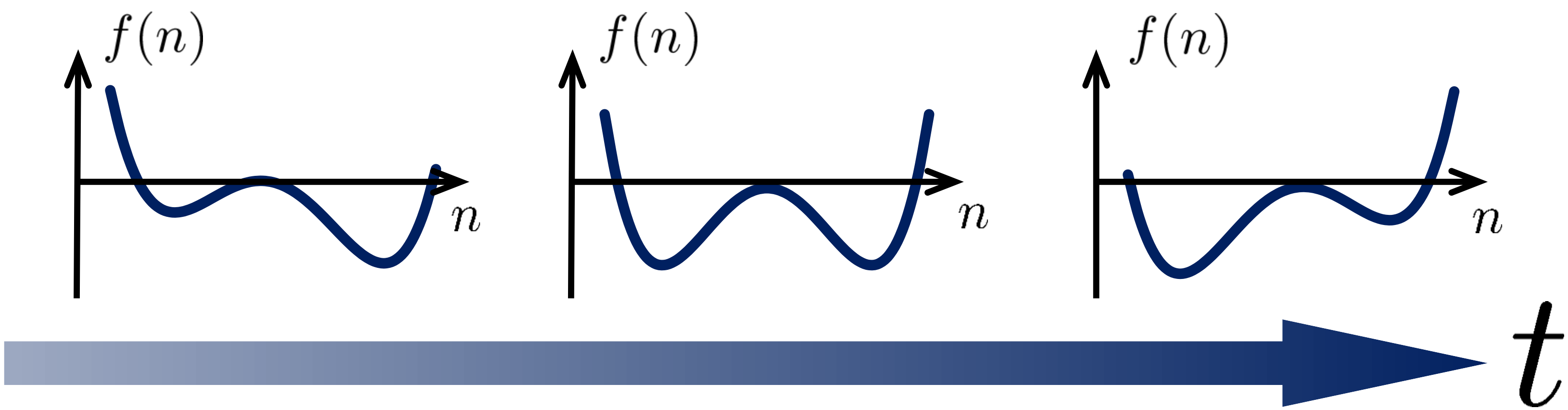
$$\partial_\tau n = \frac{\Gamma(n)}{\tau^2}\partial_Y^2 \frac{\delta F[n]}{\delta n(x)} + \frac{1}{\tau}\partial_Y \xi(\eta,\tau) - \frac{n}{\tau}$$

$$F[n(x)] = \int dx f(x)$$

- Diffusion equation: $f(n) = \frac{a}{2}n^2, \quad D = \Gamma a$
- solve numerically

Free Energy

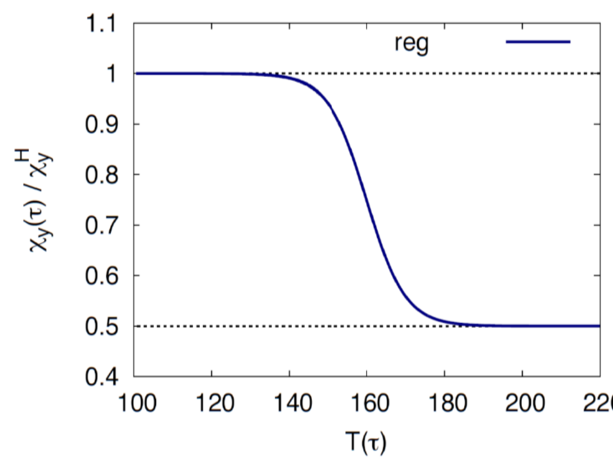
- **At 1st transition point**



- **Large and small n**

$$\chi(n) = \frac{\partial^2 f}{\partial n^2} \quad \to \chi_{\mathrm{QGP}} \;\; (n \to \infty)$$

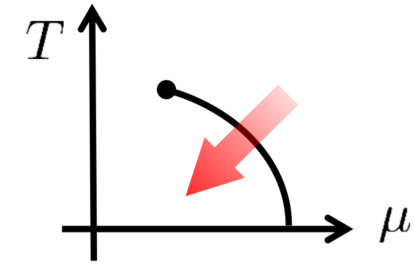
$$\to \chi_{\mathrm{hadron}} \;\; (n \to 0)$$ Poisson



Modeling 1st Transition

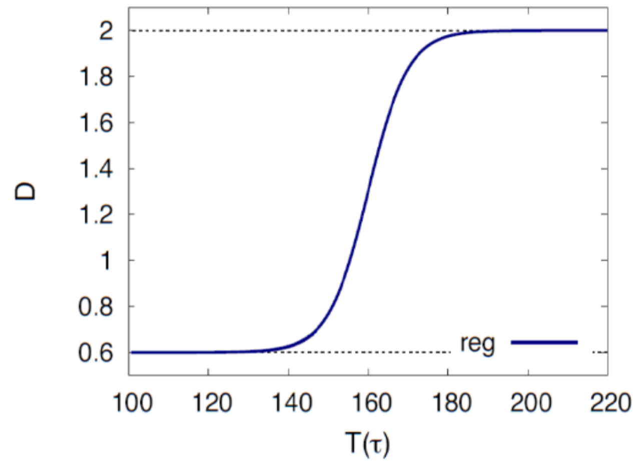
$$\partial_\tau n = \Gamma(n)\partial_Y^2 \frac{\delta F[n]}{\delta n(Y)} + \partial_Y \xi$$

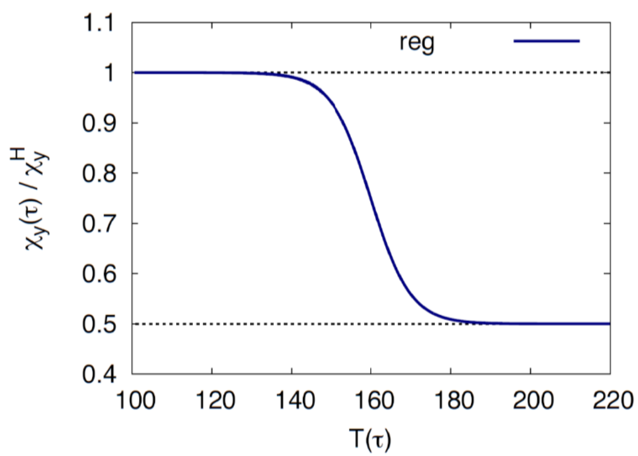
$$\langle \xi(Y_1,\tau_1)\xi(Y_2,\tau_2)\rangle = 2A\delta^{(2)}(1-2)$$



- $$f(n) = \frac{1}{2}a(n-n_s)^2 + \frac{1}{4}b(n-n_s)^4 + c(\tau)n + k(\partial_Y n)^2$$
- Γ: positive
- adjust Γ and A to reproduce the behavior of D at small and large n

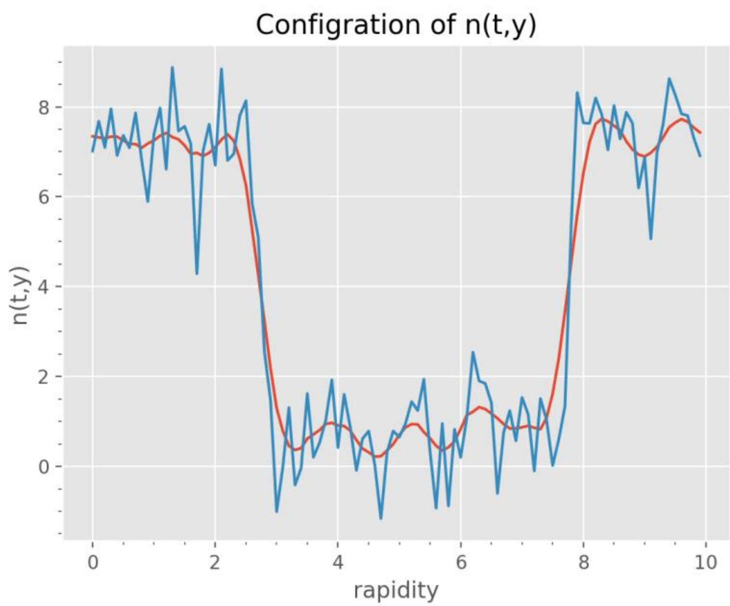
$$\tilde{D} = \Gamma\left(\frac{\partial^2 f}{\partial n^2} + X\right) \qquad A = 2D\chi_2$$

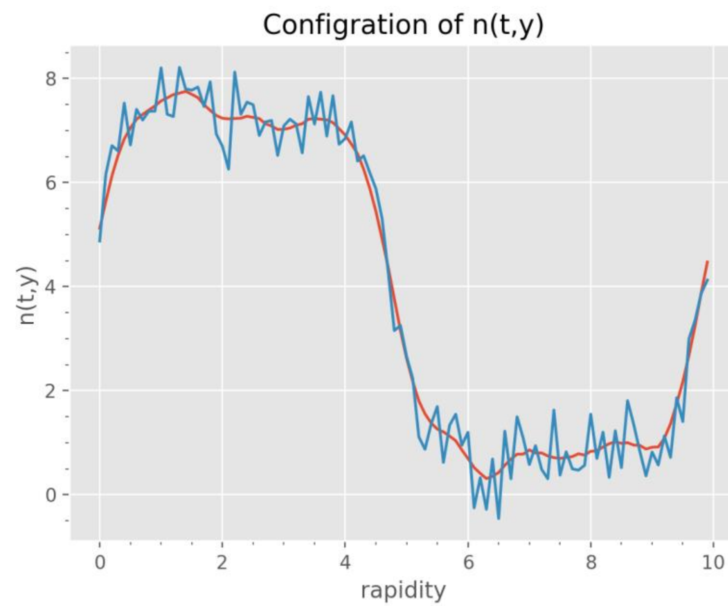




Configurations in Equilibrium

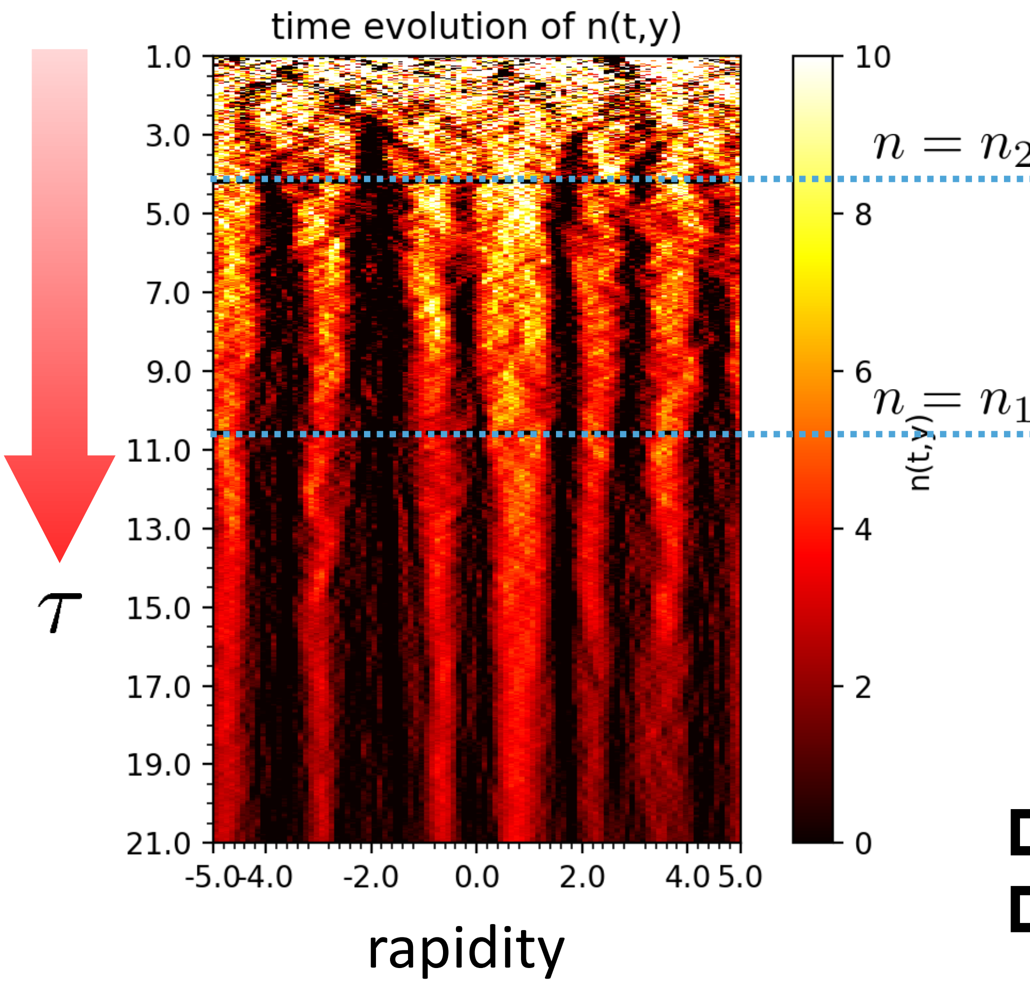
$$\partial_\tau n = \frac{\Gamma(n)}{\tau^2}\partial_Y^2 \frac{\delta F[n]}{\delta n(x)} + \frac{1}{\tau}\partial_Y \xi(\eta,\tau) - \frac{n}{\tau}$$

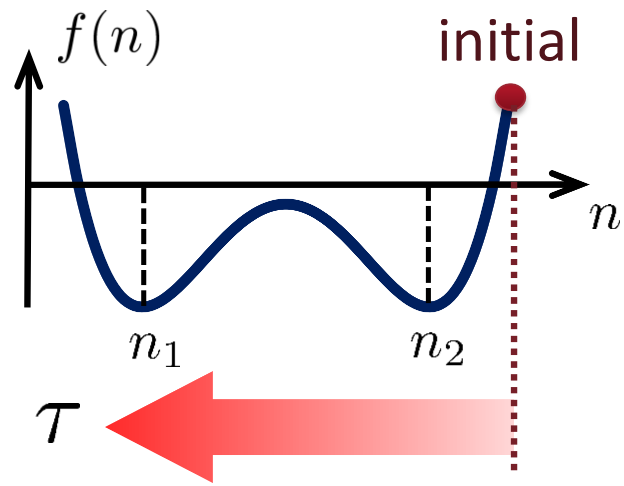




- ☐ Domain formation
- ☐ Surface: thickness √(2k/a), surface tension

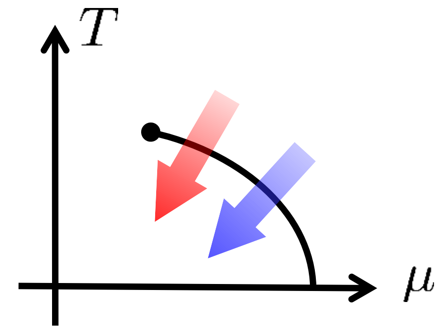
Time Evolution

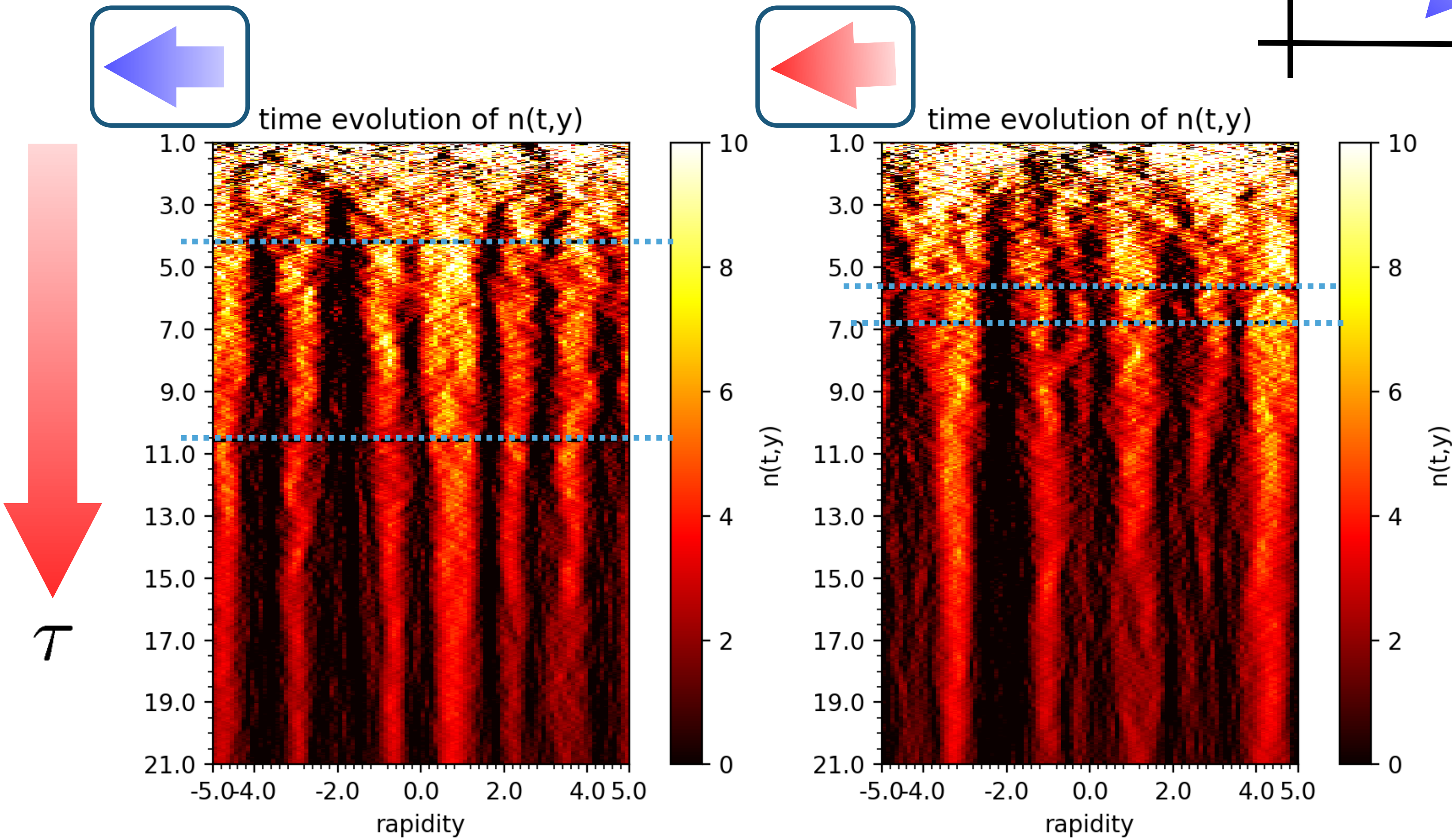




- ☐ Dynamical domain formation
- ☐ Domains survive even after 1st transition

Time Evolution



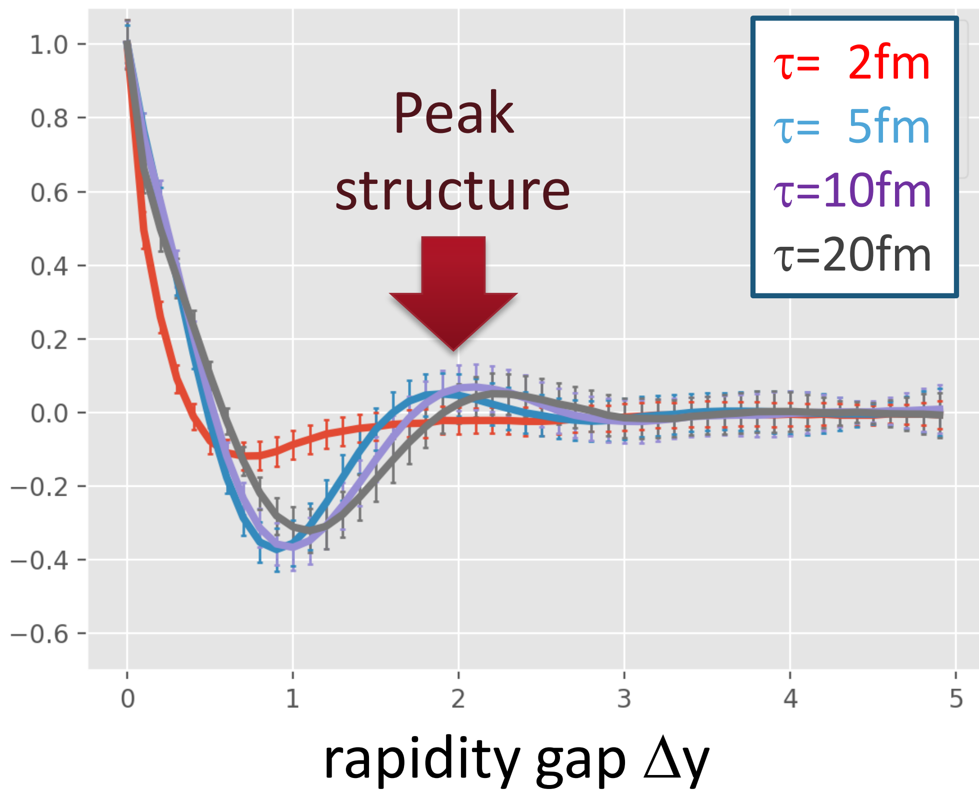


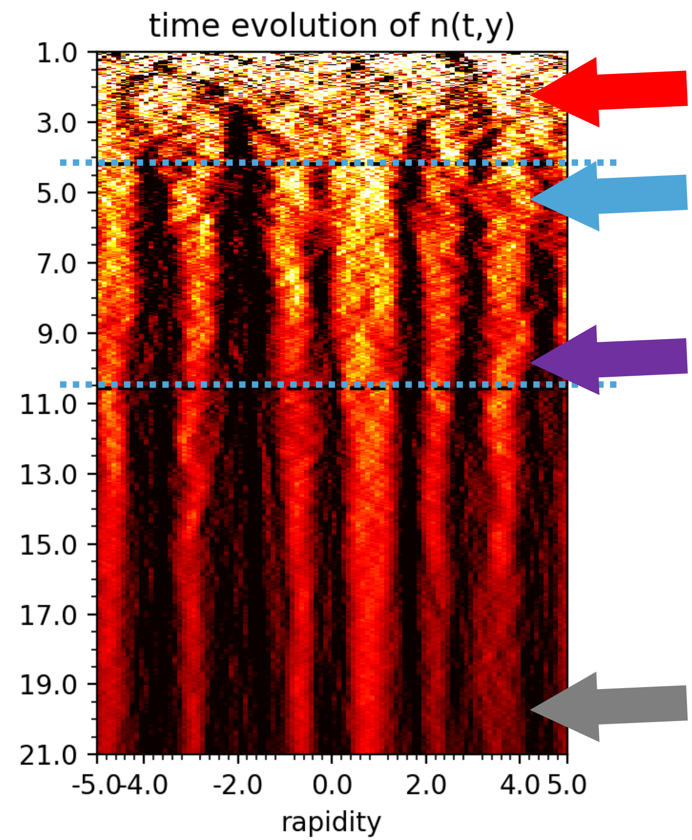
- Weaker 1st transition can also lead to formation of domains.

Correlation Function

Correlation Function

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\rm hadron}$$



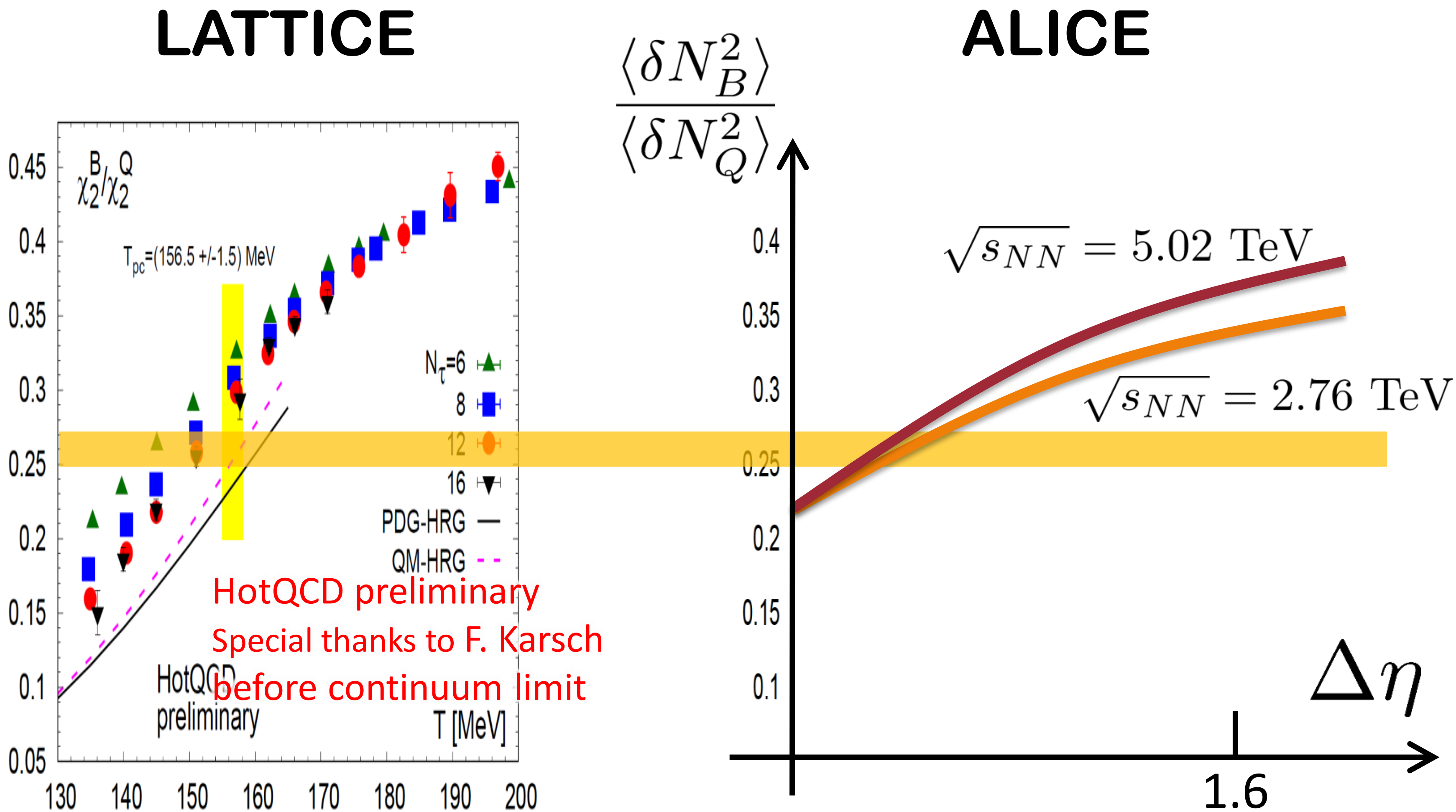


- Domain leads to a peak structure in C(y).
- The peak can survive even in the final state.

Summary

- Fluctuations observed in HIC are not in equilibrium.
- Plenty of information in rapidity window dependences of higher-order cumulants.
- 2nd-order cumulant (correlation function) already contains interesting information.
- Future
  - Evolution of higher-order cumulants around the critical point / 1st transition
  - combination to momentum (model-H)
  - more realistic model (dimension, Y dependence, …)

Prediction



Δη dependence for tracing back the history!