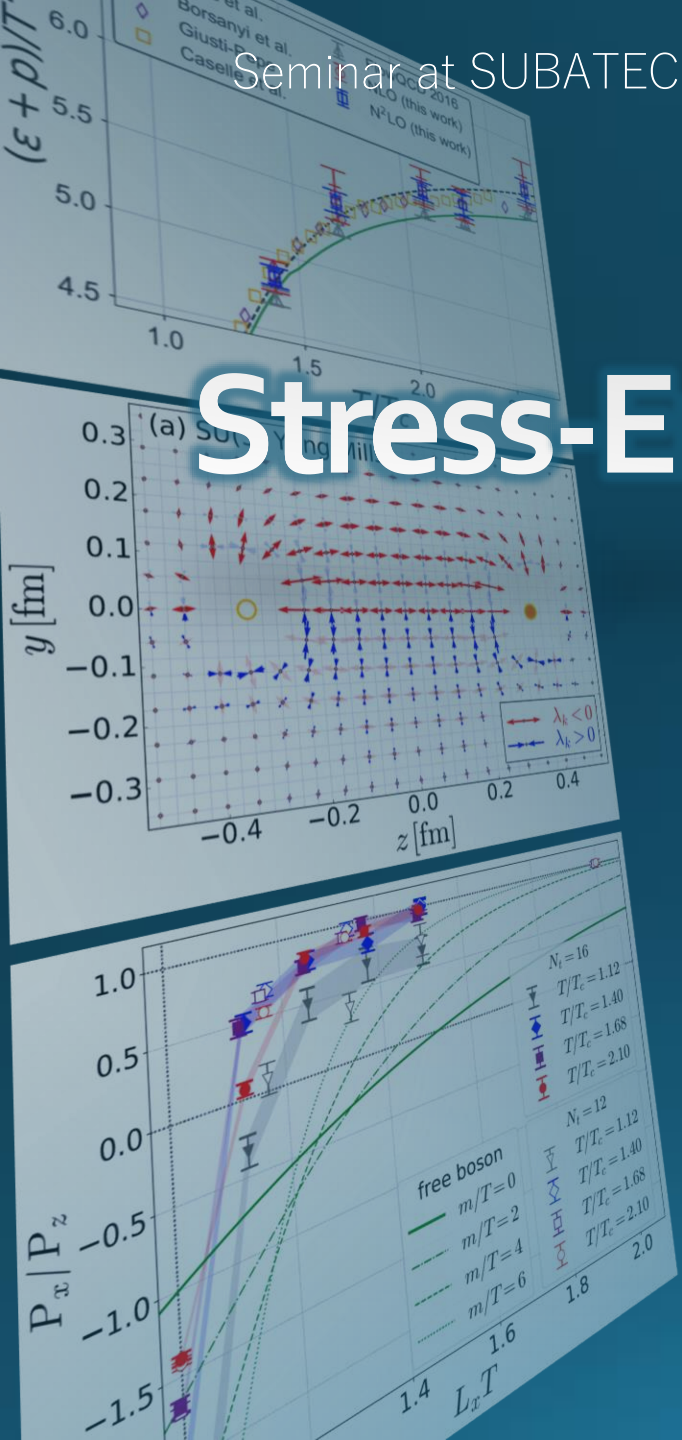
Seminar at SUBATECH, IMT Atlantique, Nantes, France, Sep./25/2019




# Stress-Energy-Momentum Tensor on the Lattice


Masakiyo Kitazawa
(Osaka U.)

# Energy-Momentum Tensor

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

energy

momentum

pressure

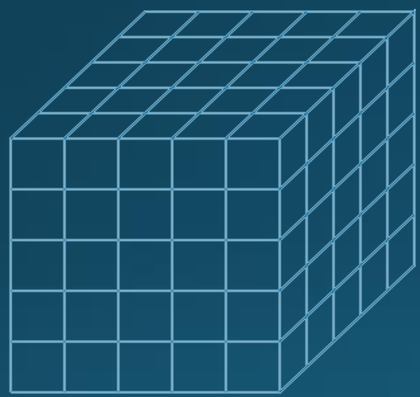
stress

All components are important physical observables!

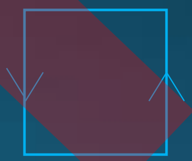
# $T_{\mu\nu}$ : nontrivial observable on the lattice

① Definition of the operator is nontrivial

because of the explicit breaking of Lorentz symmetry



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

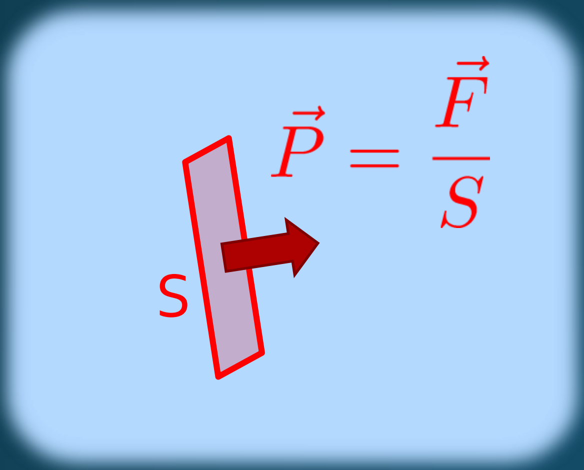
② Its measurement is extremely noisy

due to high dimensionality and etc.

# Stress = Force per Unit Area

# Stress = Force per Unit Area

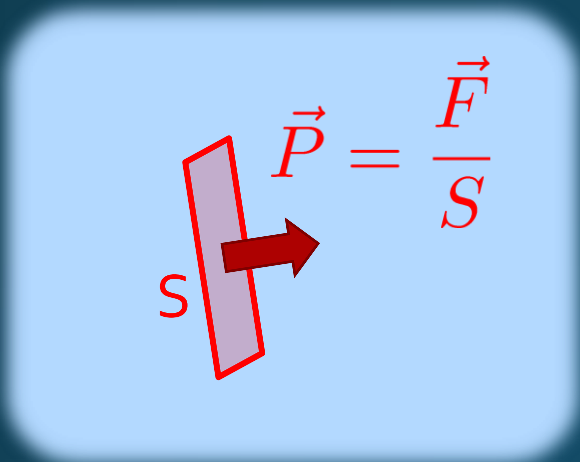
## Pressure




$$\vec{P} = P\vec{n}$$

# Stress = Force per Unit Area

## Pressure

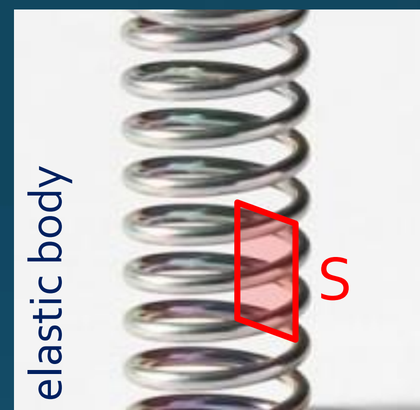


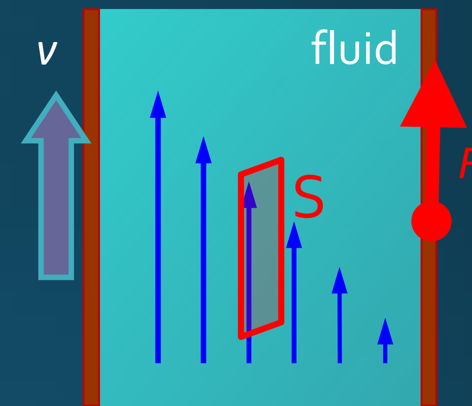

$$\vec{P} = P\vec{n}$$

**In thermal medium**

$$T_{ij} = P\delta_{ij}$$

## Generally, F and n are not parallel




$$\frac{F_i}{S} = \sigma_{ij} n_j$$

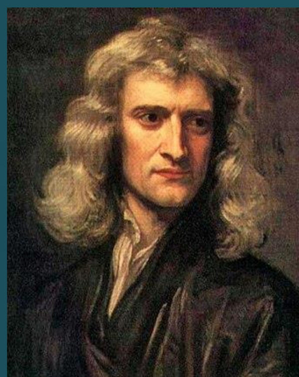
**Stress Tensor**

$$\sigma_{ij} = -T_{ij}$$

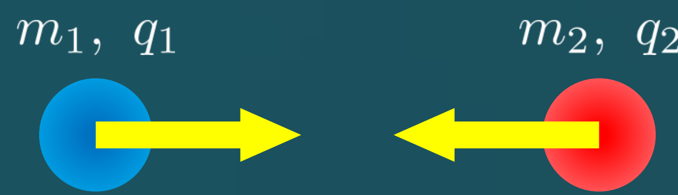
Landau
Lifshitz

# Force

## Action-at-a-distance



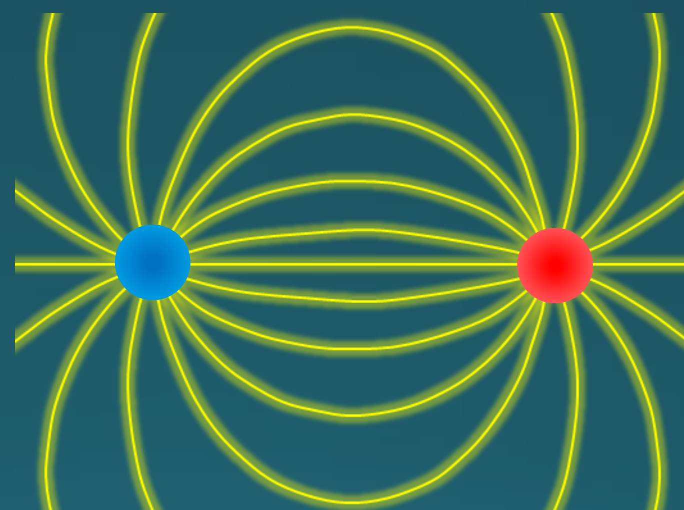
Newton
1687




$$F = -G\frac{m_1 m_2}{r^2} \qquad F = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

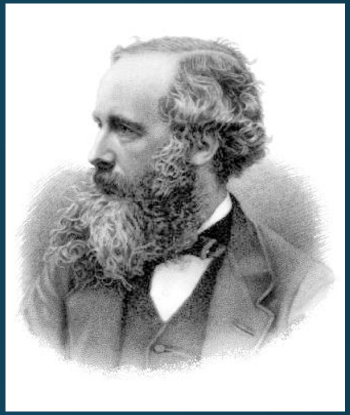
## Local interaction

Faraday
1839



# Maxwell Stress
(in Maxwell Theory)

$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

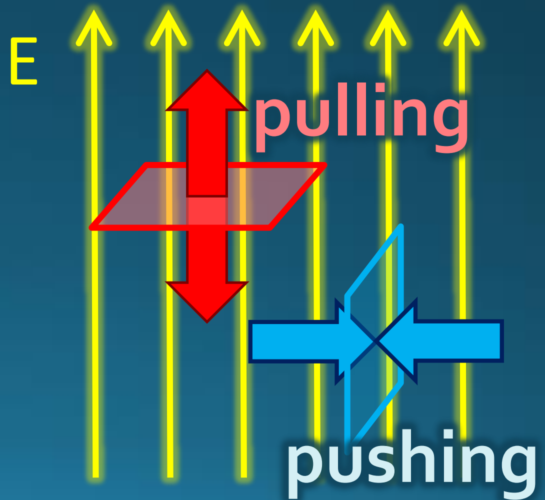


Maxwell

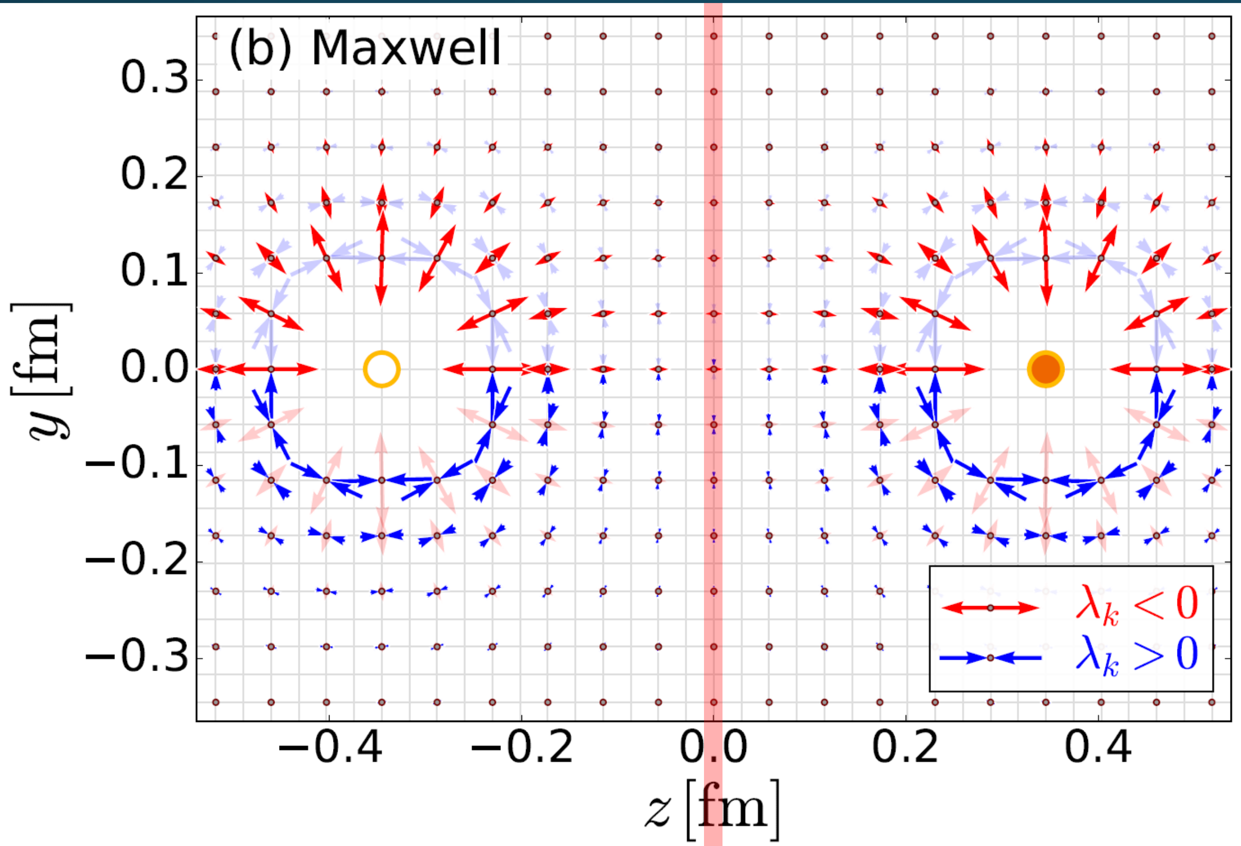
$$\vec{E} = (E, 0, 0)$$

$$T_{ij} = \begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: **Pulling**
- Vertical to field: **Pushing**



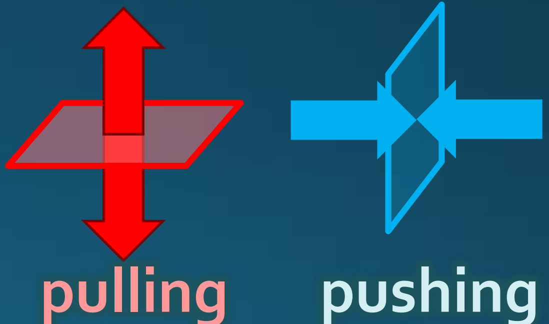
# Maxwell Stress
## (in Maxwell Theory)




$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$$(k = 1, 2, 3)$$

length: $\sqrt{|\lambda_k|}$

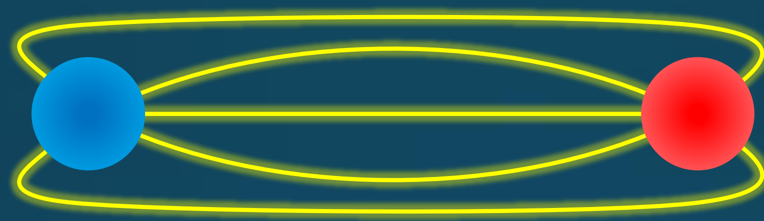



**Definite physical meaning**

- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction

# Quark-Anti-quark system

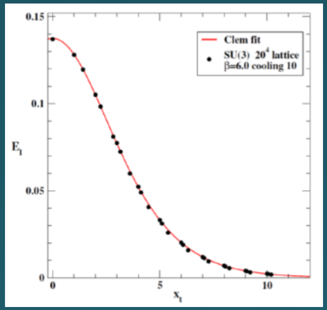
Formation of the flux tube → confinement



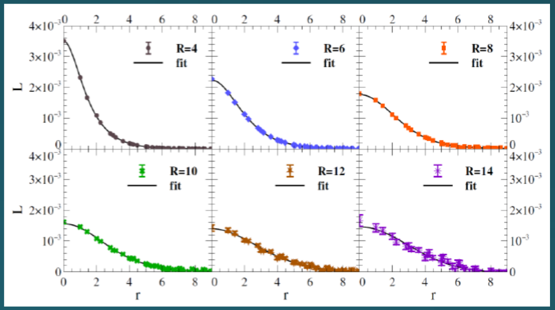
## Previous Studies on Flux Tube

- ☐ Potential
- ☐ Action density
- ☐ Color-electric field

so many studies…



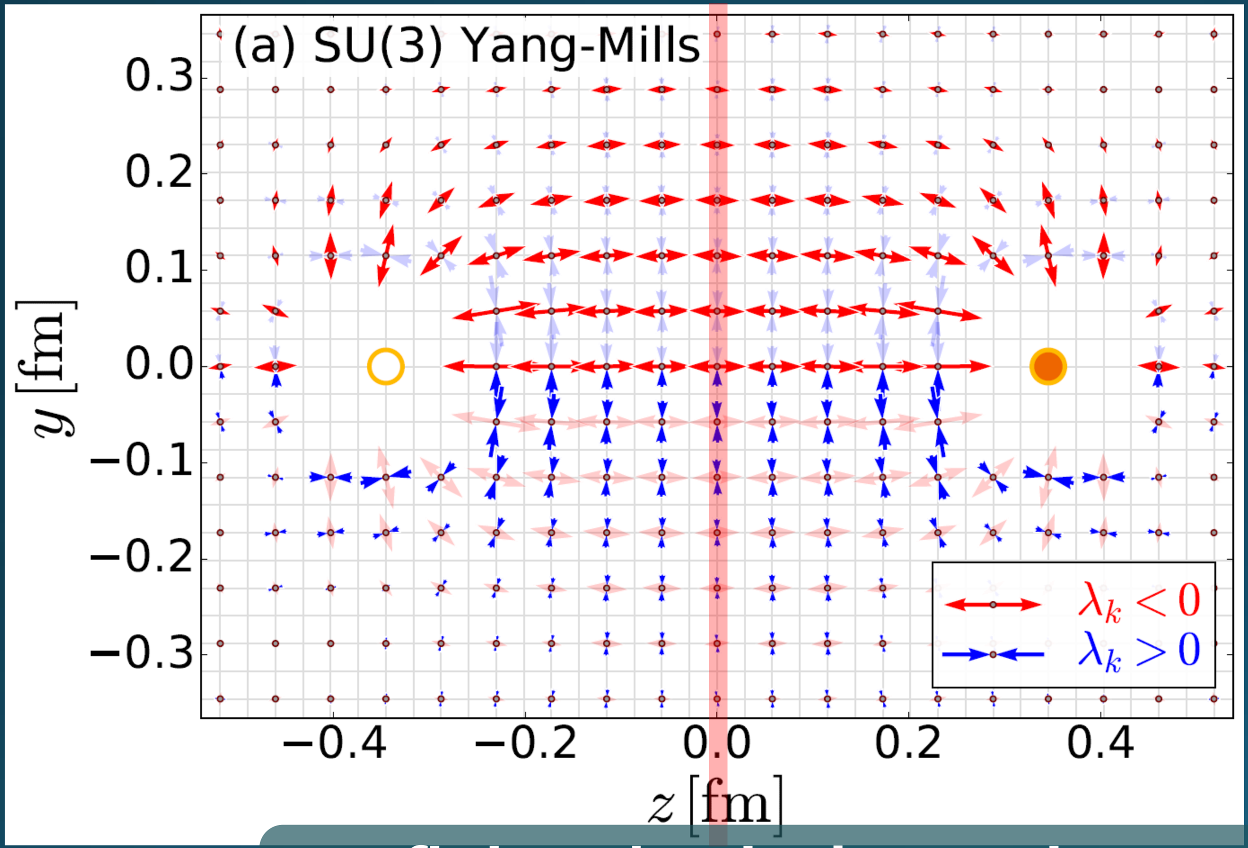

Cea+ (2012)



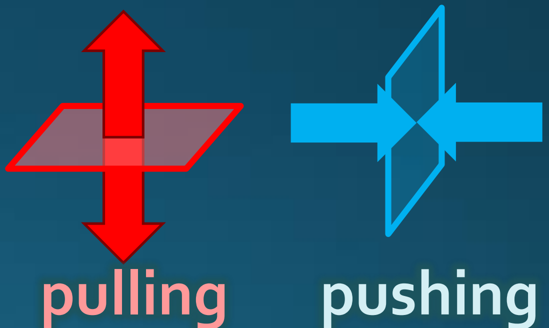

Cardoso+ (2013)

# Stress Tensor in $Q\bar{Q}$ System

Yanagihara+, PLB, 2019




Lattice simulation
SU(3) Yang-Mills
a=0.029 fm
R=0.69 fm
$t/a^2$=2.0

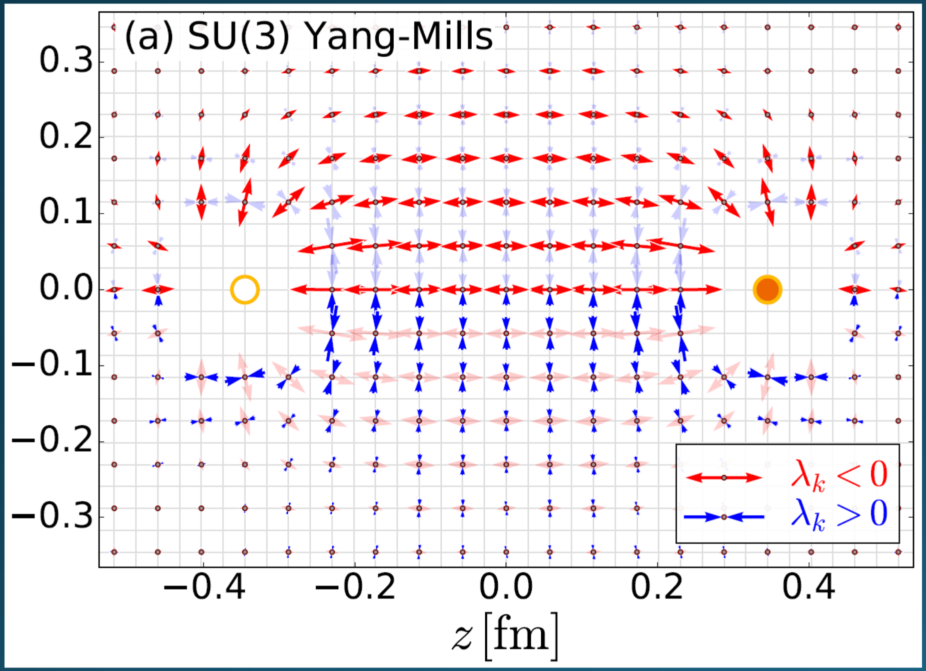



**Definite physical meaning**

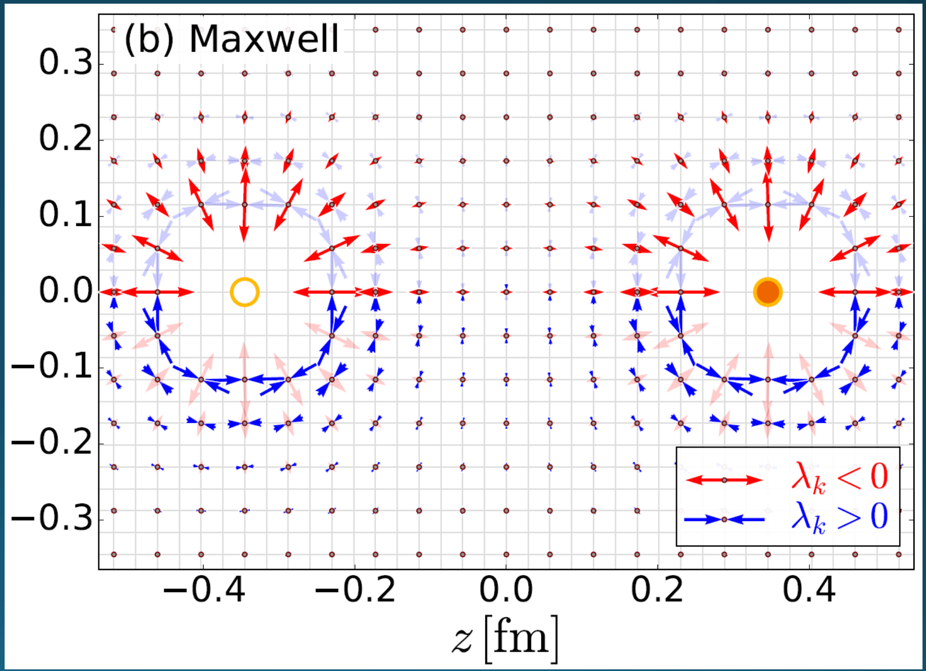
- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction
- ☐ Manifestly gauge invariant

# SU(3) YM vs Maxwell

**SU(3) Yang-Mills**
(quantum)



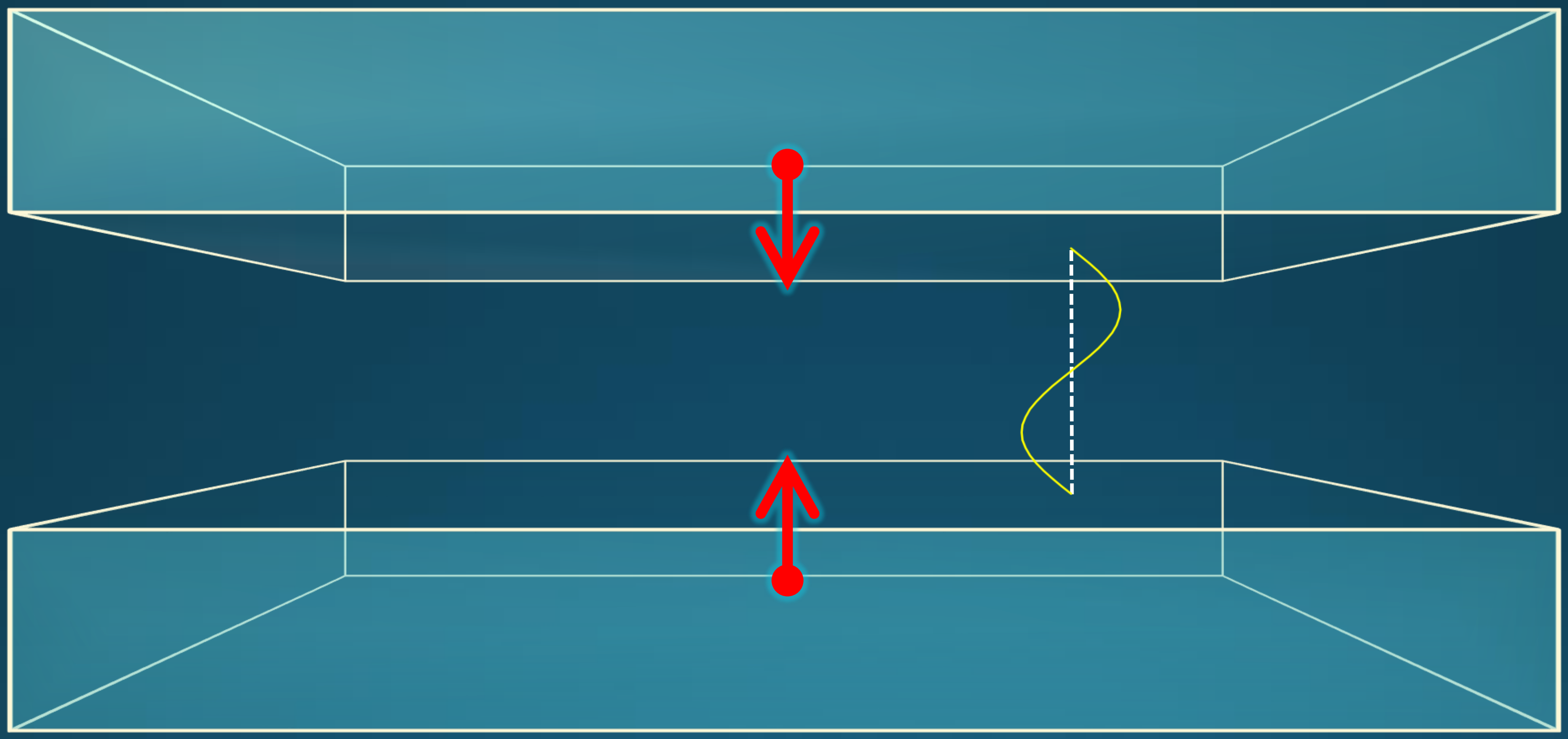

**Maxwell**
(classical)




Propagation of the force is clearly different in YM and Maxwell theories!

# Casimir Effect

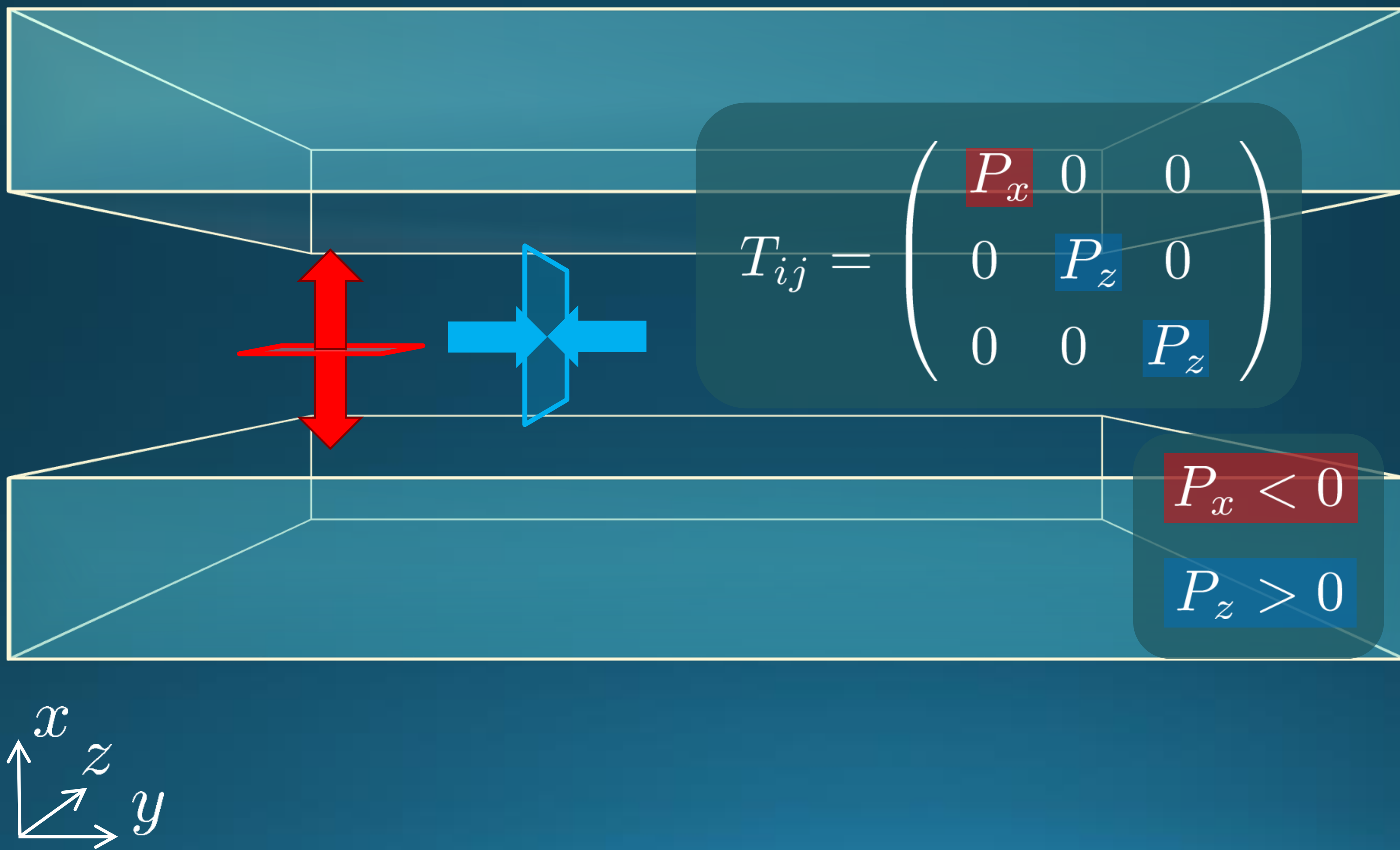
# Casimir Effect



attractive force between two conductive plates

# Casimir Effect

Brown, Maclay
1969



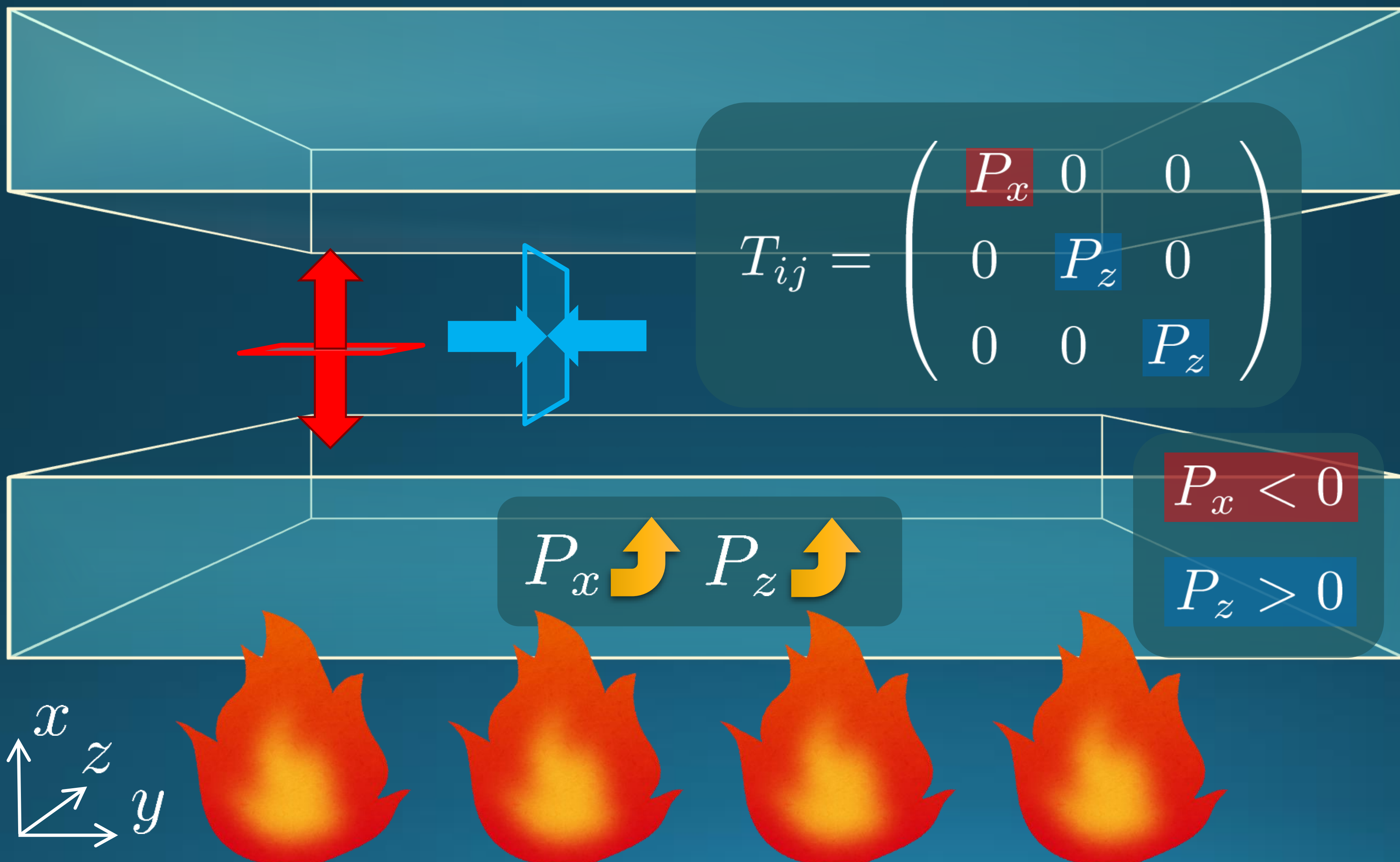

$$T_{ij} = \begin{pmatrix} P_x & 0 & 0 \\ 0 & P_z & 0 \\ 0 & 0 & P_z \end{pmatrix}$$

$$P_x < 0$$

$$P_z > 0$$

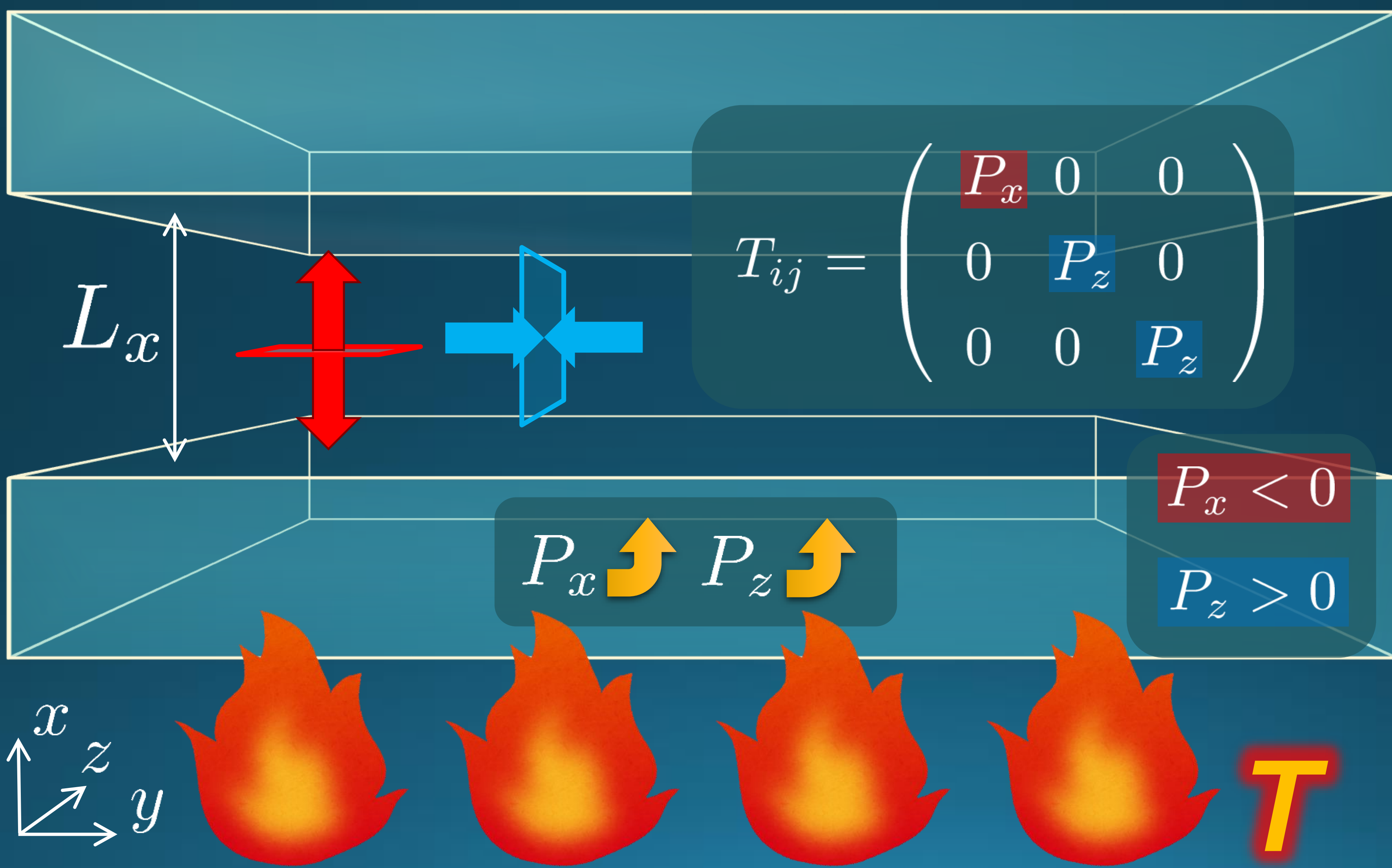
# Casimir Effect

Brown, Maclay
1969



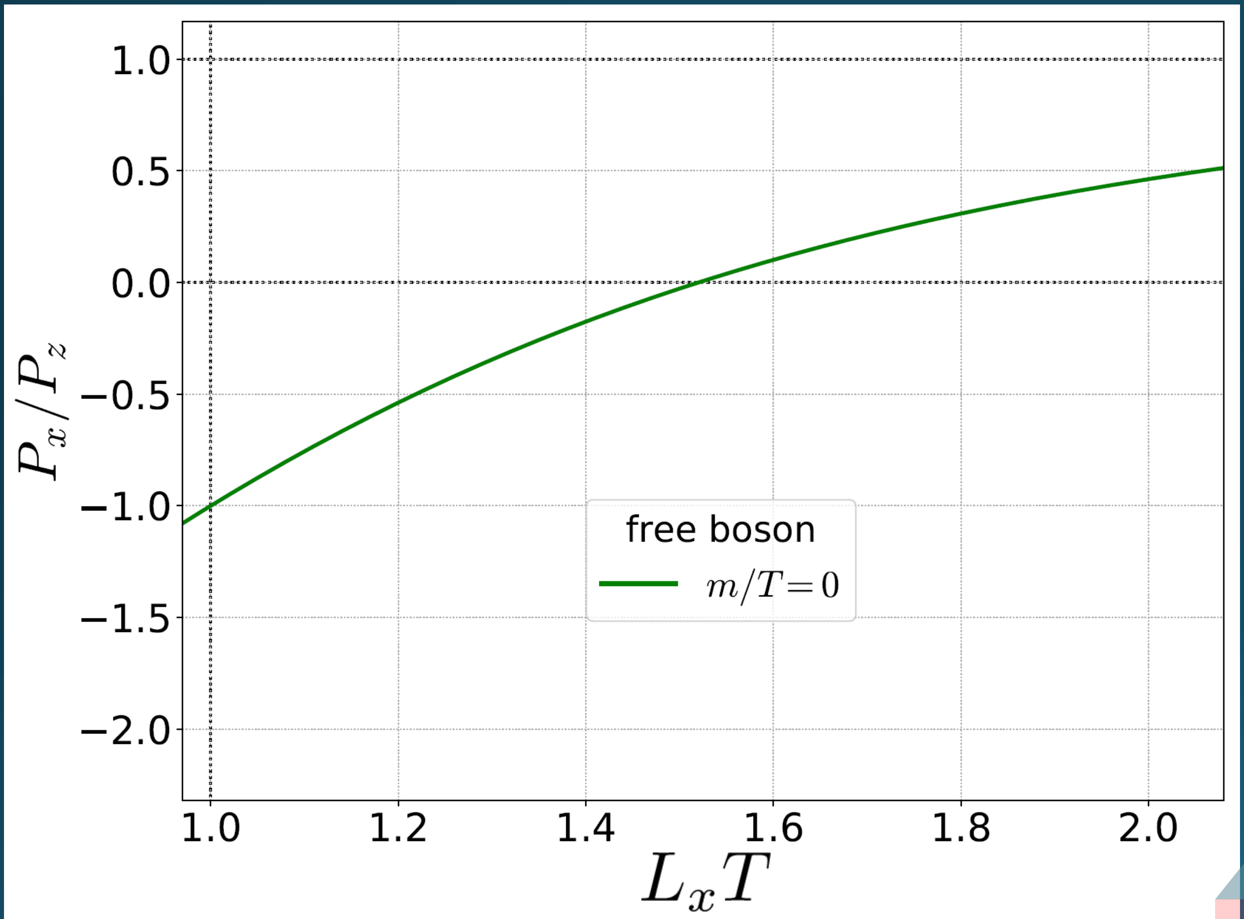
# Casimir Effect

Brown, Maclay
1969



# Pressure Anisotropy @ T≠0

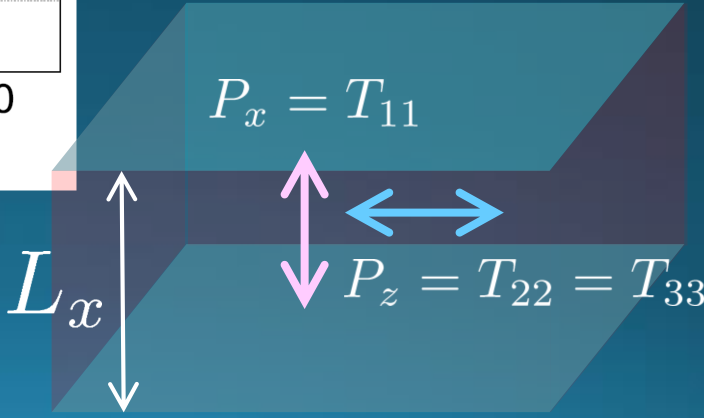
MK, Mogliacci, Kolbe, Horowitz, PRD, 2019




## Free scalar field

- $L_2=L_3=\infty$
- Periodic BC

Mogliacci+, 1807.07871



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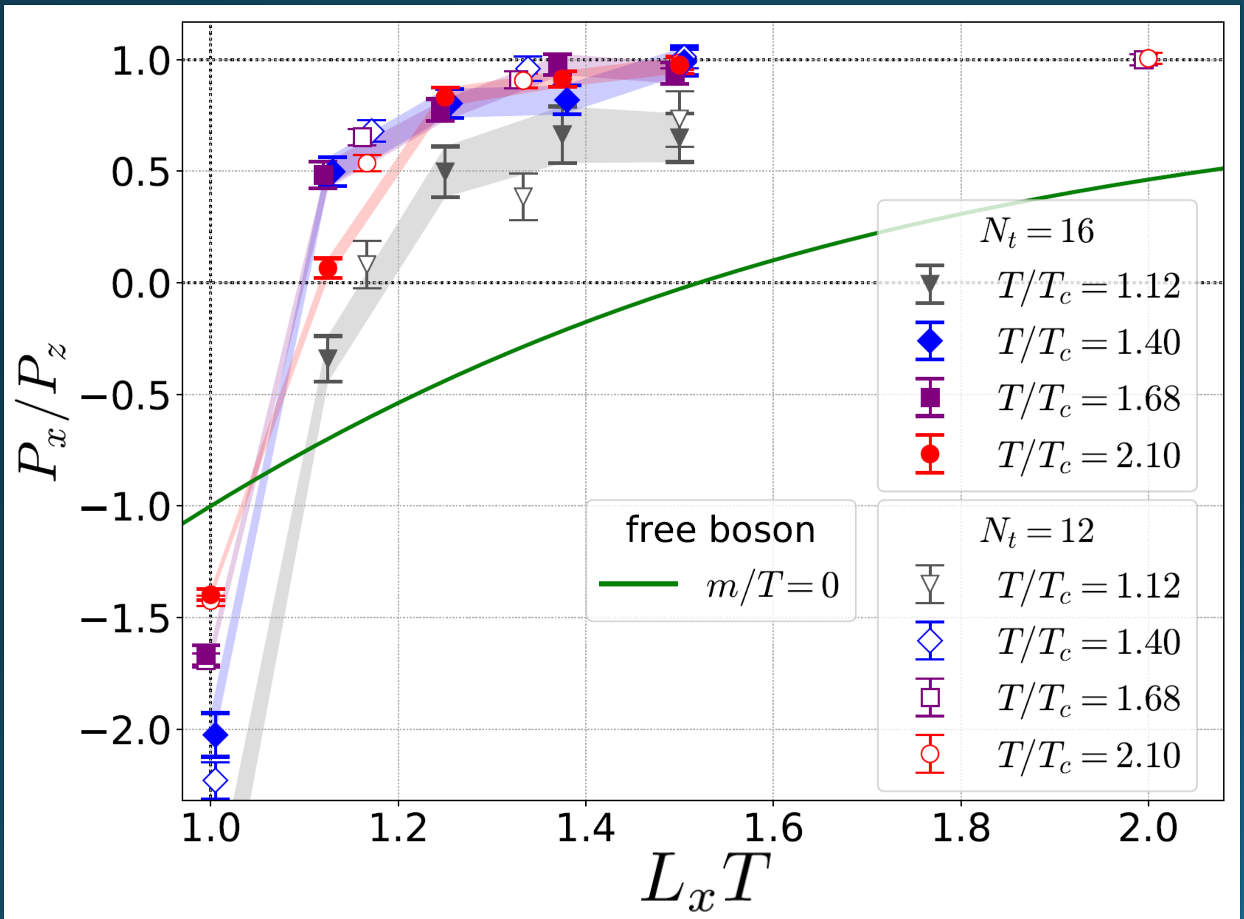



## Free scalar field

- ☐ $L_2=L_3=\infty$
- ☐ Periodic BC

Mogliacci+, 1807.07871

## Lattice result

- ☐ Periodic BC
- ☐ Only $t\to 0$ limit
- ☐ Error: stat.+sys.

**Medium near $T_c$ is remarkably insensitive to finite size!**

# Contents

1. Constructing EMT
2. Thermodynamics

FlowQCD, PR**D90**,011501 (2014); PR**D94**, 114512 (2016);
WHOT-QCD, PR**D96**, 014509 (2017); Iritani+, PTEP **2019**, 023B02 (2019)

3. Flux Tube

FlowQCD, PLB789, 210 (2019); Yanagihara+, in prep.

4. Static Quark Systems at Nonzero T

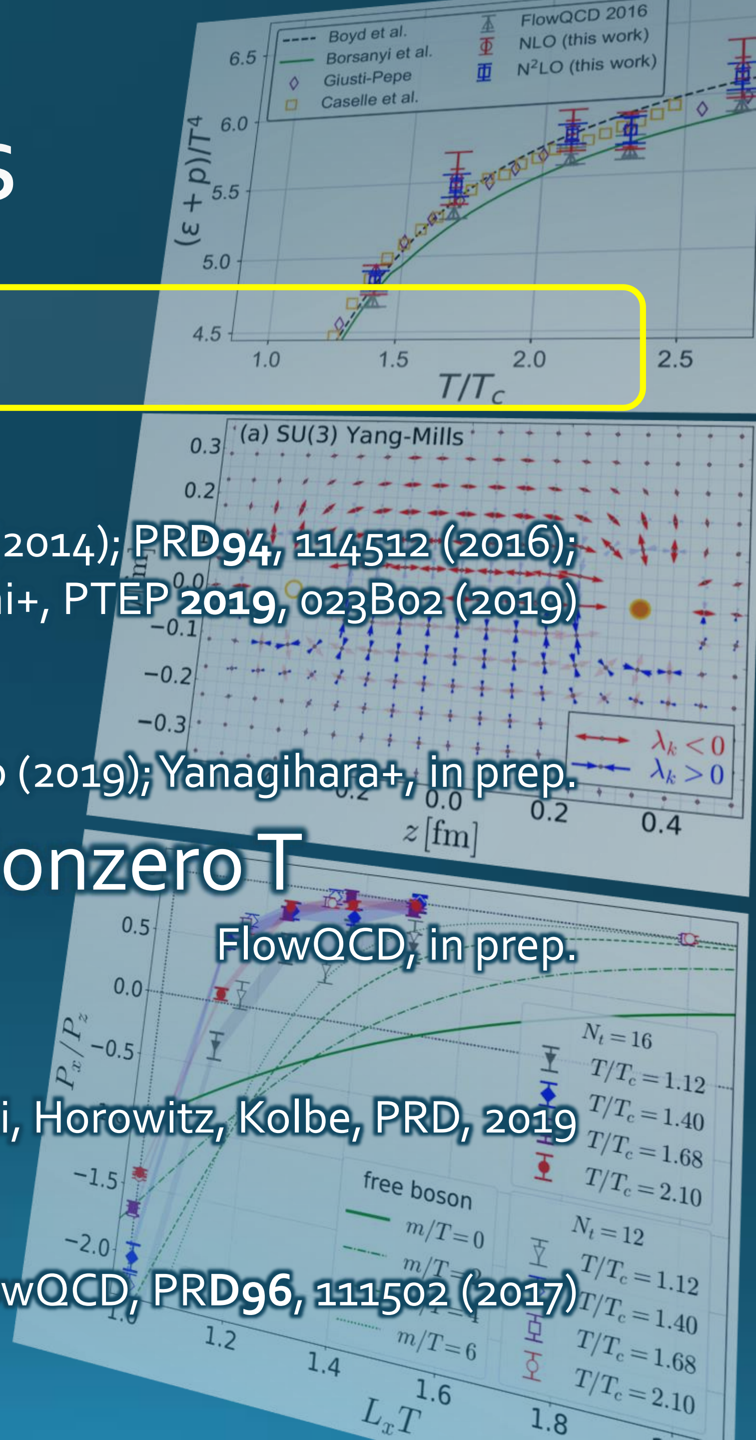
FlowQCD, in prep.

5. Casimir Effect

MK, Mogliacci, Horowitz, Kolbe, PRD, 2019

6. Correlation Function

FlowQCD, PR**D96**, 111502 (2017)



# Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

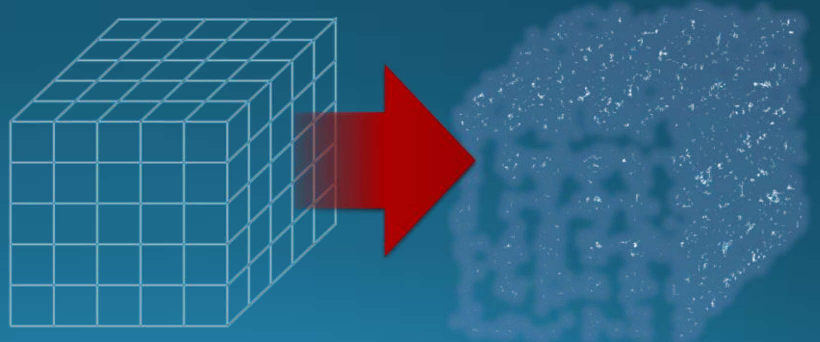
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length²]

leading

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at t>0



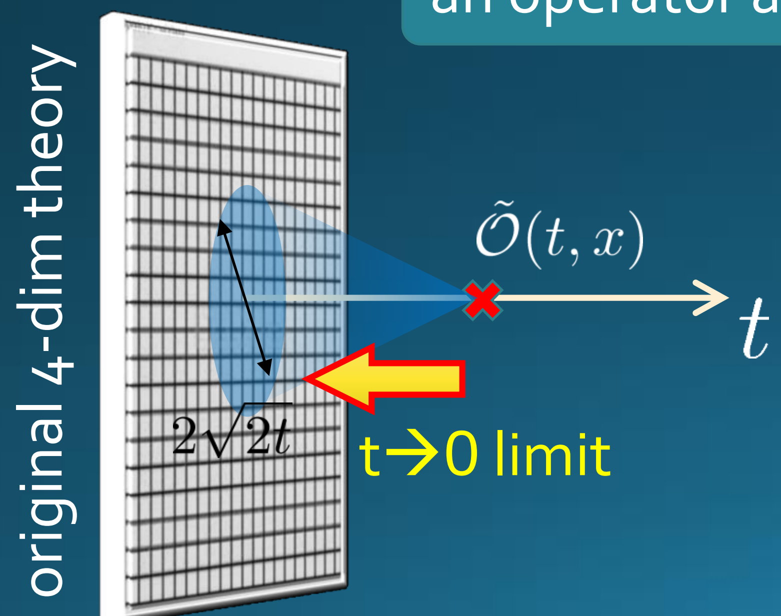
# Small Flow-Time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at t>0

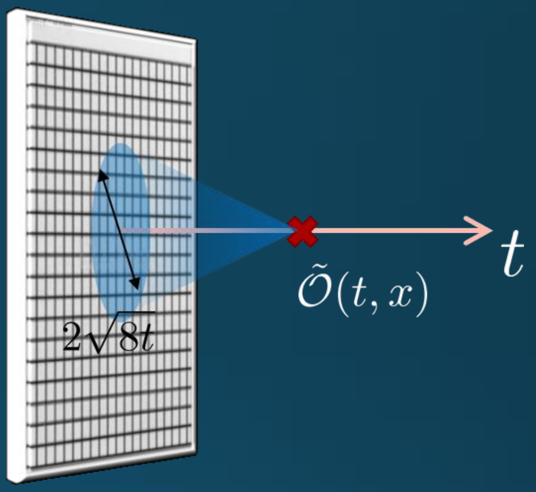
remormalized operators of original theory



# Constructing EMT 1

Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \underset{t\to 0}{\longrightarrow} \sum_i c_i(t)\mathcal{O}_i^R(x)$$




- **Gauge-invariant dimension 4 operators**

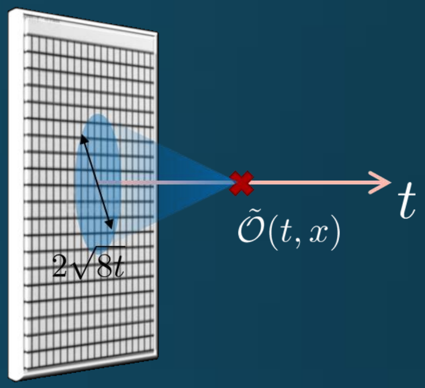
$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

# Constructing EMT

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$




vacuum subtr.

## Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\mathrm{subt.}}\right]$$

Perturbative coefficient:
Suzuki (2013); Makino, Suzuki (2014); Harlander+ (2018); Iritani, MK, Suzuki, Takaura (2019)

# Perturbative Coefficients

$$T_{\mu\nu}(t) = c_1(t) U_{\mu\nu}(t) + \delta_{\mu\nu} c_2(t) E(t)$$

| | LO | 1-loop | 2-loop | 3-loop |
|---|---|---|---|---|
| $c_1(t)$ | ○ | ○ | ○ | |
| $c_2(t)$ | × zero | ○ | ○ | ○ |

Suzuki (2013) Harlander+(2018)

Iritani, MK, Suzuki, Takaura, 2019

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

## □ Choice of the scale of g²

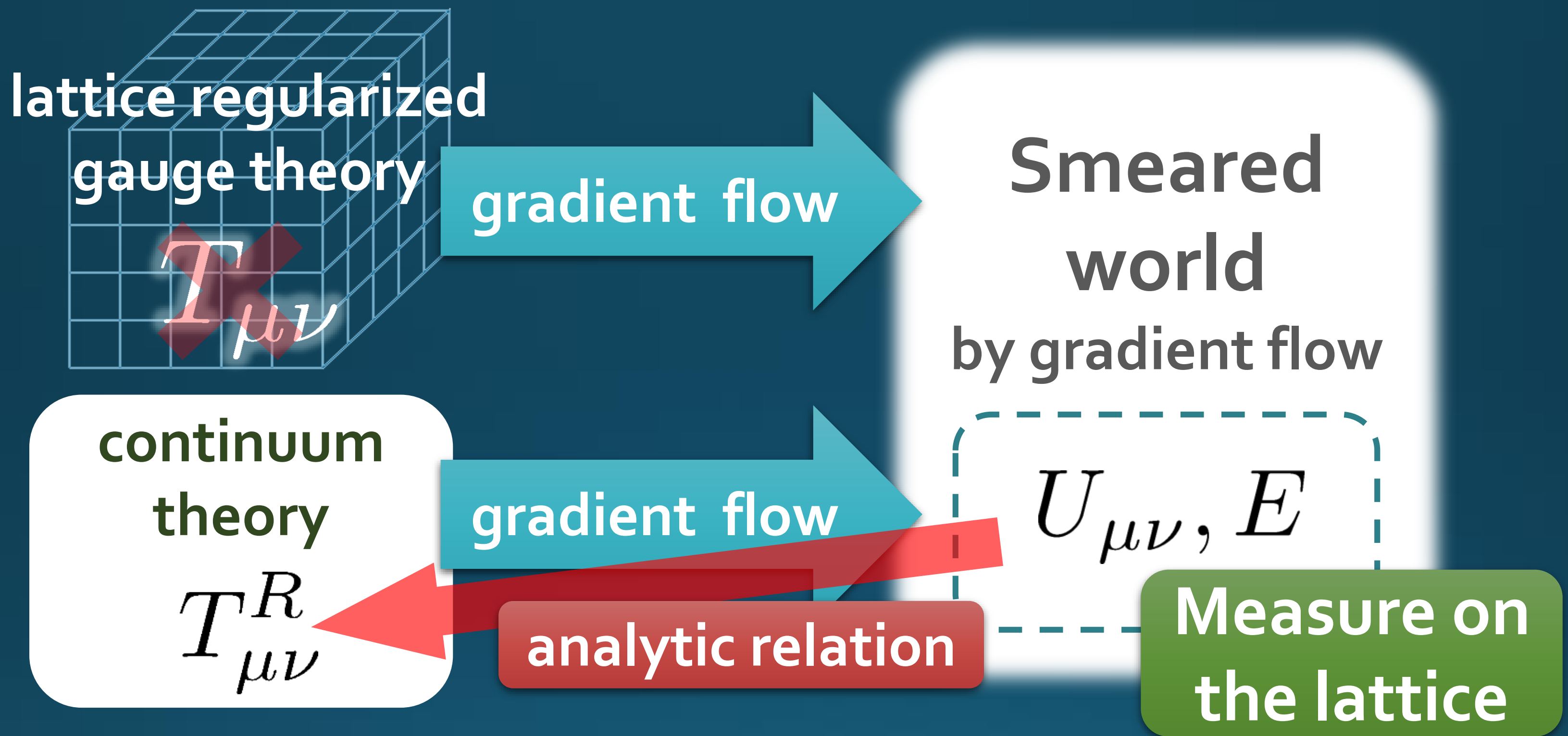
$$c_1(t) = c_1\left(g^2(\mu(t))\right)$$

Previous: $\mu_d(t) = 1/\sqrt{8t}$

Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E} t}$

Harlander+ (2018)

# Gradient Flow Method




**Take Extrapolation (t,a)→(0,0)**

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}\frac{a^2}{t} + \cdots$$

O(t) terms in SFTE   lattice discretization

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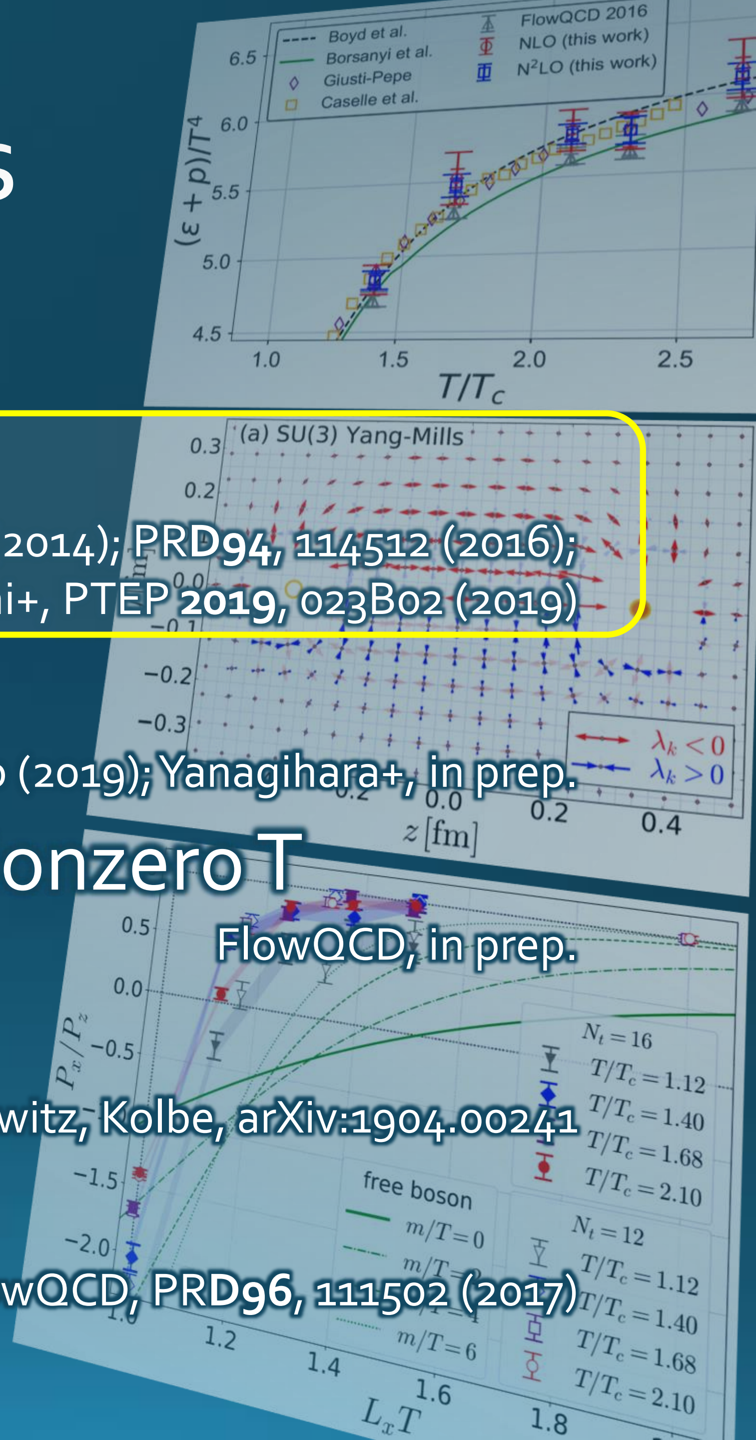
FlowQCD, in prep.

5. Casimir Effect

MK, Mogliacci, Horowitz, Kolbe, arXiv:1904.00241

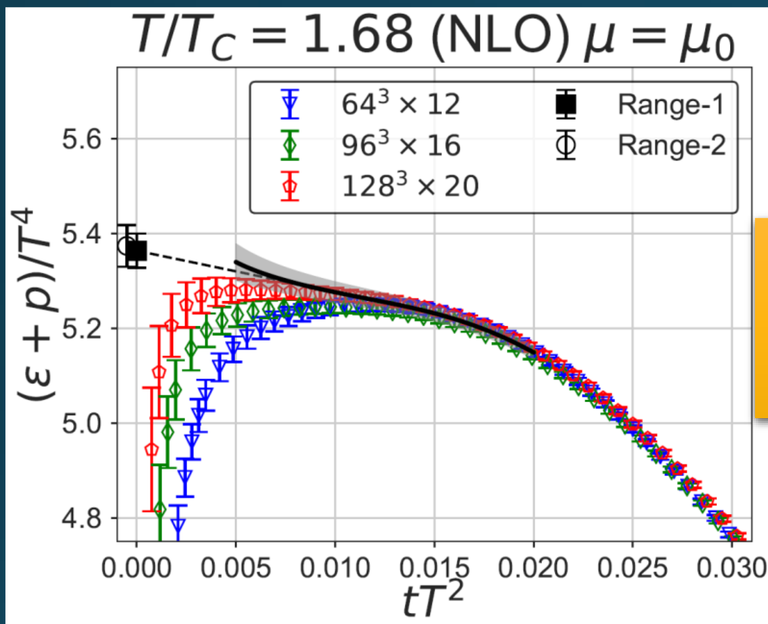
6. Correlation Function

FlowQCD, PRD**96**, 111502 (2017)

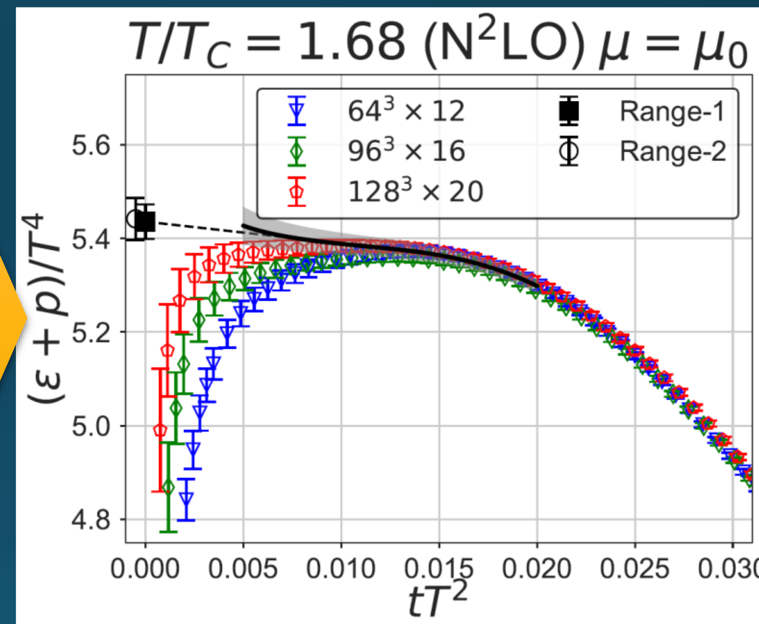


# Higher Order Coefficient: ε+p

## NLO (1-loop)




## N²LO (2-loop)




Iritani, MK, Suzuki, Takaura, PTEP 2019

- ☐ t dependence becomes milder with higher order coeff.
- ☐ Better t→0 extrapolation

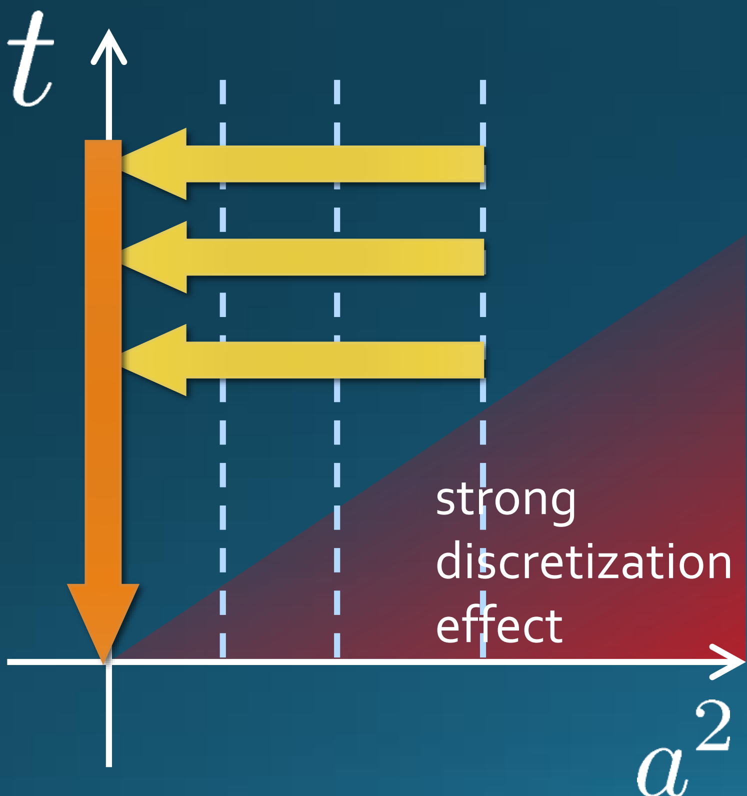
- ☐ Systematic error: $\mu_0$ or $\mu_d$, uncertaintyof Λ (±3%), fit range
- ☐ Extrapolation func: linear, higher order term in $c_1$ ($\sim g^6$)

# Double Extrapolation

t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE    lattice discretization




Continuum extrapolation

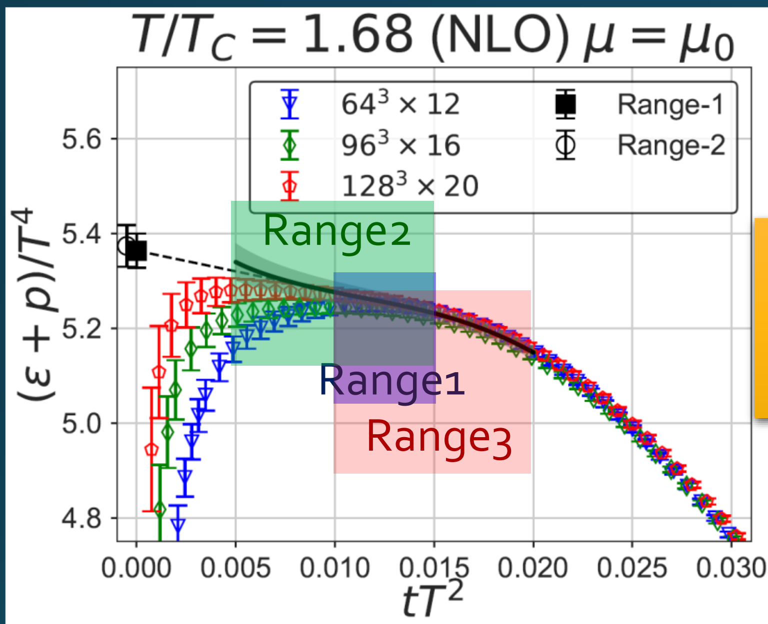
$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

Small t extrapolation

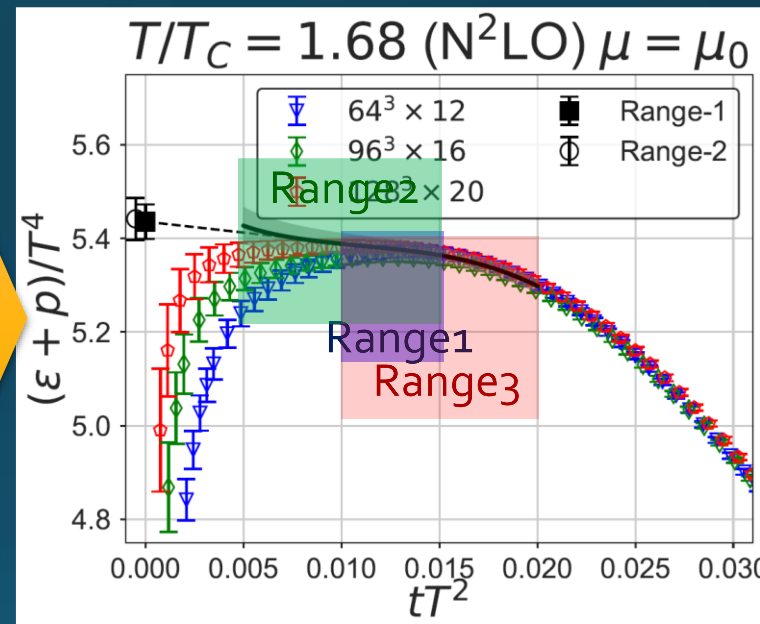
$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$

# Higher Order Coefficient: ε+p

## NLO (1-loop)




## N²LO (2-loop)



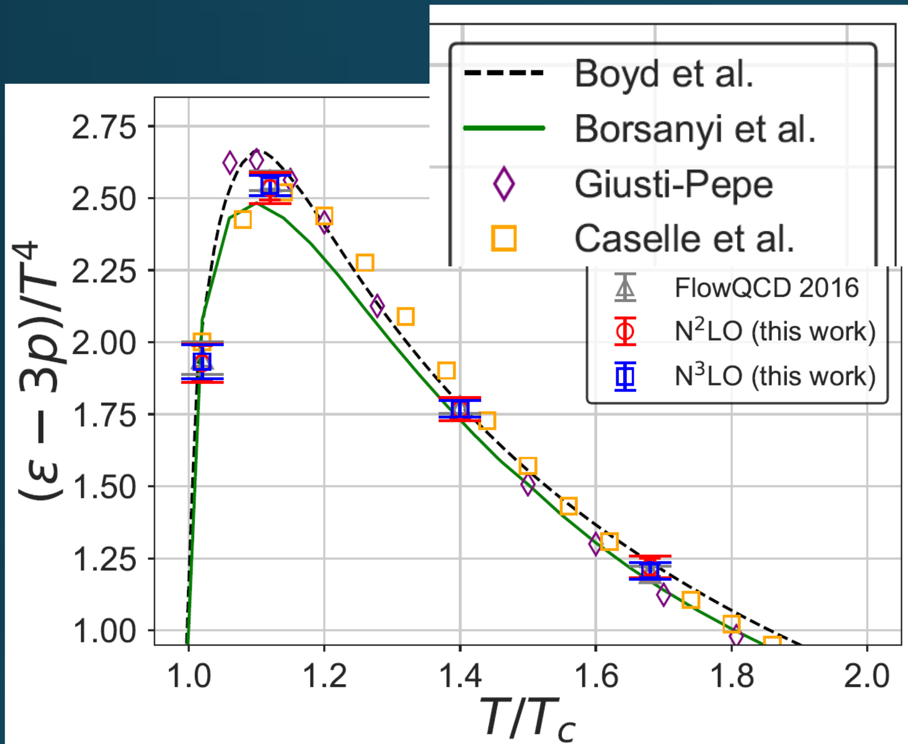

Iritani, MK, Suzuki, Takaura, PTEP 2019

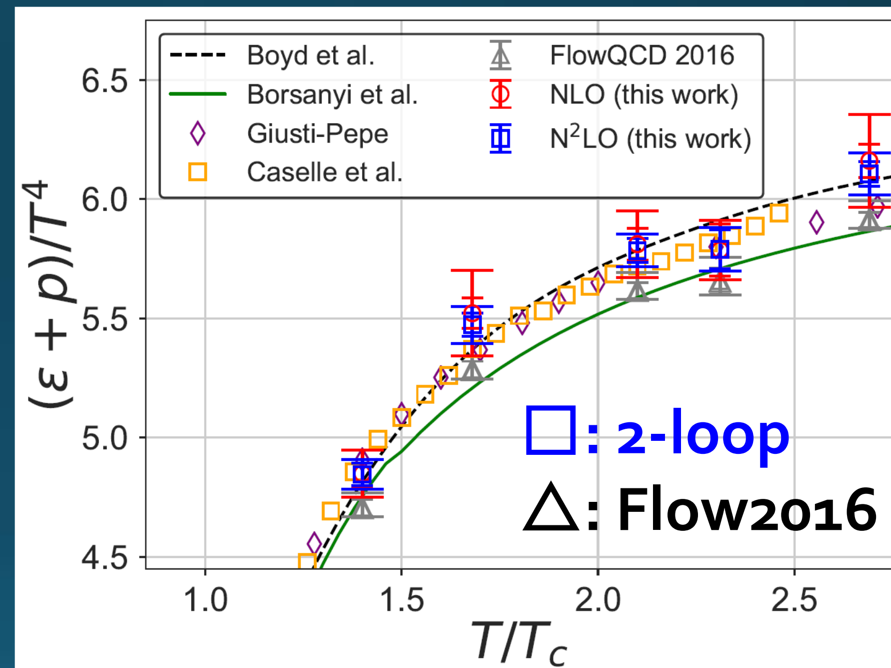
- ☐ t dependence becomes milder with higher order coeff.
- ☐ Better t→0 extrapolation

- ☐ Systematic error: $\mu_0$ or $\mu_d$, uncertaintyof $\Lambda$ (±3%), fit range
- ☐ Extrapolation func: linear, higher order term in $c_1$ ($\sim g^6$)

# Effect of Higher-Order Coeffs.

Iritani, MK, Suzuki, Takaura, 2019




Systematic error: $\mu_0$ or $\mu_d$, $\Lambda$, t→0 function, fit range

More stable extrapolation with higher order $c_1$ & $c_2$ (pure gauge)

# Gradient Flow for Fermions

$$\partial_t \psi(t,x) = D_\mu D_\mu \psi(t,x)$$

$$\partial_t \bar{\psi}(t,x) = \psi(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu$$

$$D_\mu = \partial_\mu + A_\mu(t,x)$$

Luscher, 2013
Makino, Suzuki, 2014
Taniguchi+ (WHOT) 2016; 2017

- Not "gradient" flow but a "diffusion" equation.

- Divergence in field renormalization of fermions.
- All observables are finite at t>0 once Z(t) is fixed.

$$\tilde{\psi}(t,x) = Z(t)\psi(t,x)$$

- Energy-momentum tensor from SFTE Makino, Suzuki, 2014

# EMT in QCD

$$T_{\mu\nu}(t,x) = c_1(t)U_{\mu\nu}(t,x) + c_2(t)\delta_{\mu\nu}\big(E(t,x) - \langle E\rangle_0\big)$$
$$+ c_3(t)\big(O_{3\mu\nu}(t,x) - 2O_{4\mu\nu}(t,x) - \mathrm{VEV}\big)$$
$$+ c_4(t)\big(O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) + c_5(t)\big(O_{5\mu\nu}(t,x) - \mathrm{VEV}\big)$$

$$T_{\mu\nu}(x) = \lim_{t\to 0} T_{\mu\mu}(t,x)$$

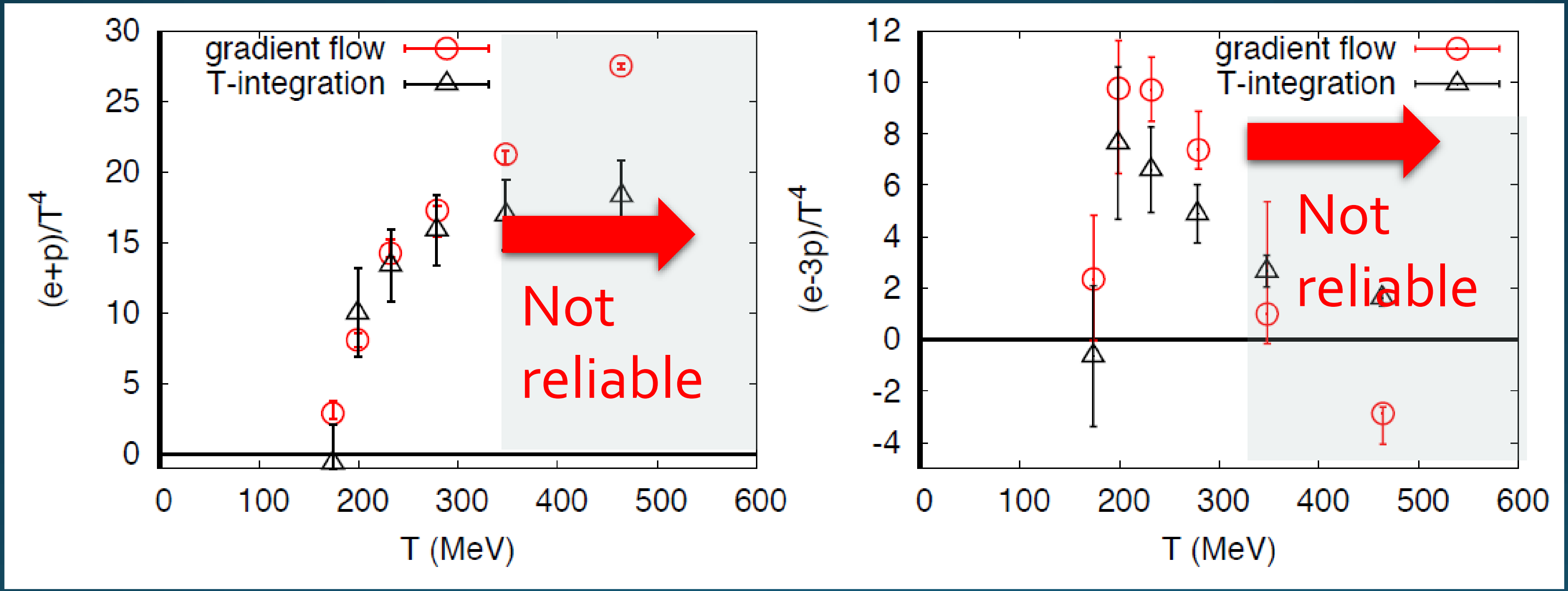
$$\tilde{O}^f_{3\mu\nu}(t,x) \equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x),$$
$$\tilde{O}^f_{4\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x),$$
$$\tilde{O}^f_{5\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x),$$

$$\varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.$$

# 2+1 QCD EoS from Gradient Flow

Taniguchi+ (WHOT-QCD), PR**D96**, 014509 (2017)

$m_{PS}/m_V \approx 0.63$




- Agreement with integral method except for $N_t$=4, 6
- $N_t$=4, 6: No stable extrapolation is possible
- Statistical error is substantially suppressed!

Physical mass: Kanaya+ (WHOT-QCD), 1710.10015

# Contents

1. Constructing EMT
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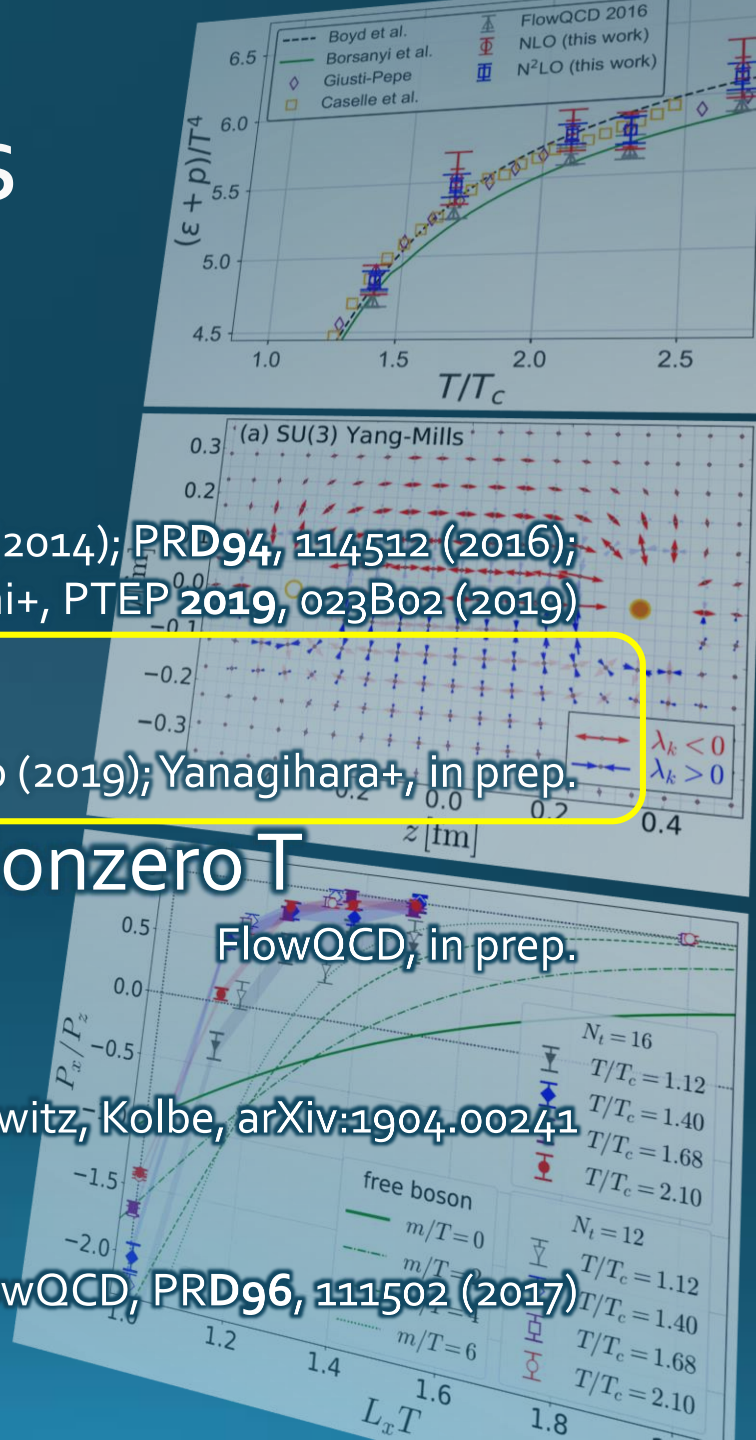
5. Casimir Effect

MK, Mogliacci, Horowitz, Kolbe, arXiv:1904.00241

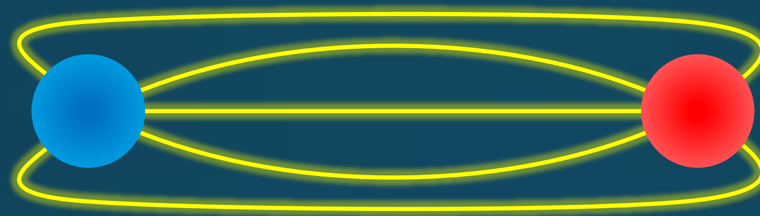
6. Correlation Function

FlowQCD, PRD**96**, 111502 (2017)



# Quark-Anti-quark system

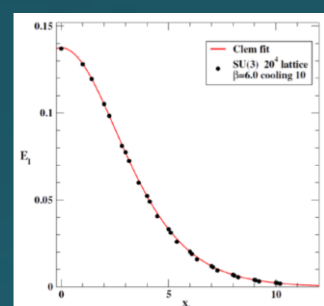
Formation of the flux tube → confinement



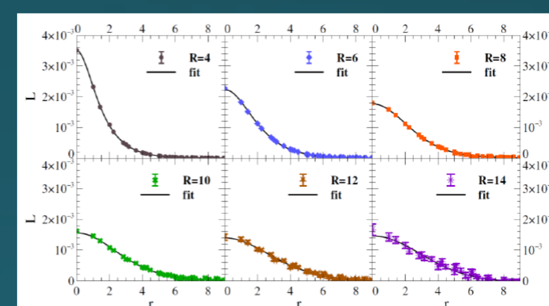
## Previous Studies on Flux Tube

- ☐ Potential
- ☐ Action density
- ☐ Color-electric field

so many studies…

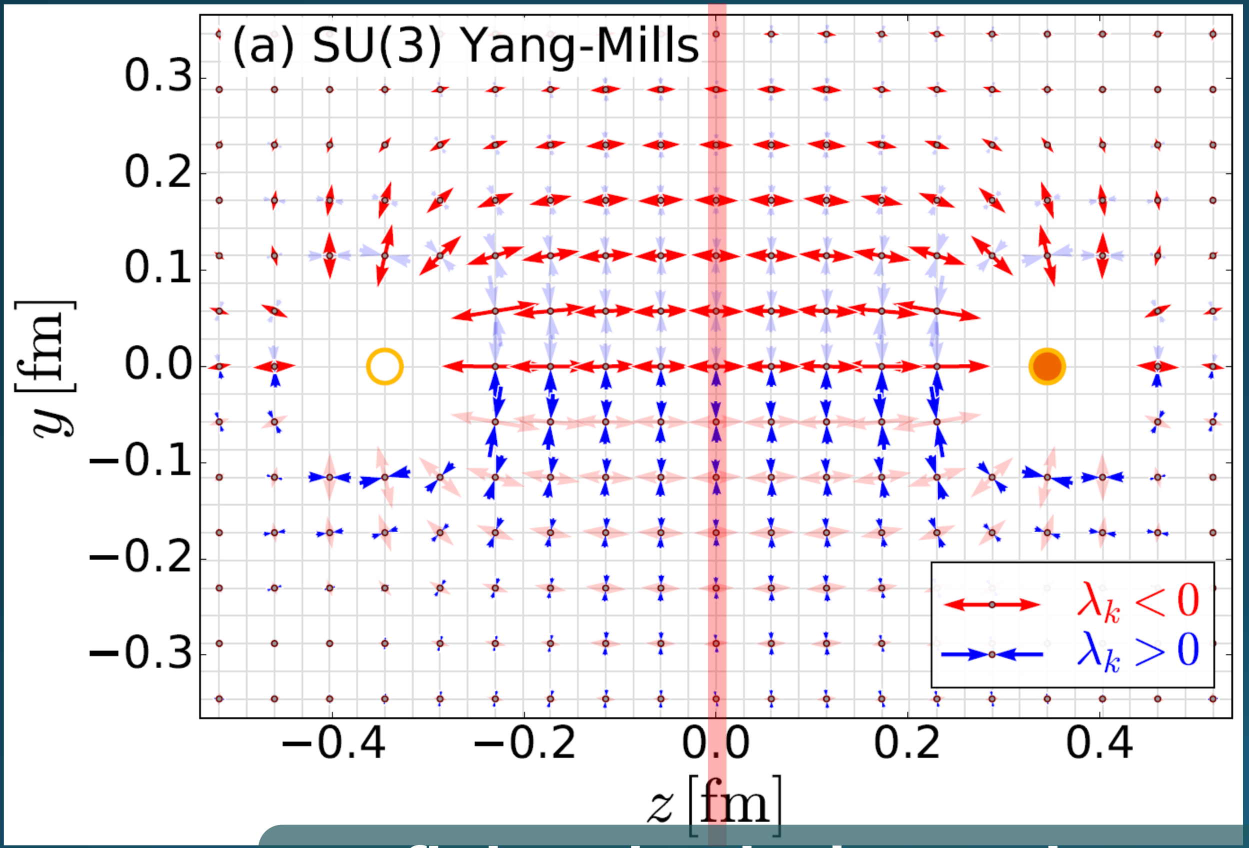



Cea+ (2012)



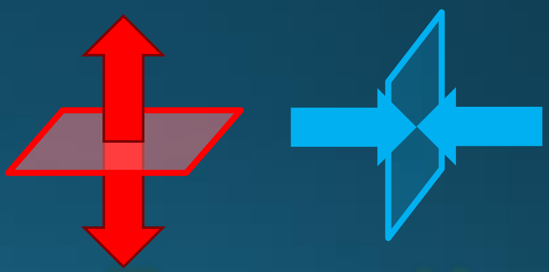

Cardoso+ (2013)

# Stress Tensor in $Q\bar{Q}$ System




Yanagihara+, 1803.05656
PLB, in press

Lattice simulation
SU(3) Yang-Mills
a=0.029 fm
R=0.69 fm
$t/a^2$=2.0




**Definite physical meaning**

- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction
- ☐ Manifestly gauge invariant

# Lattice Setup

Yanagihara+, PLB, 2019

- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

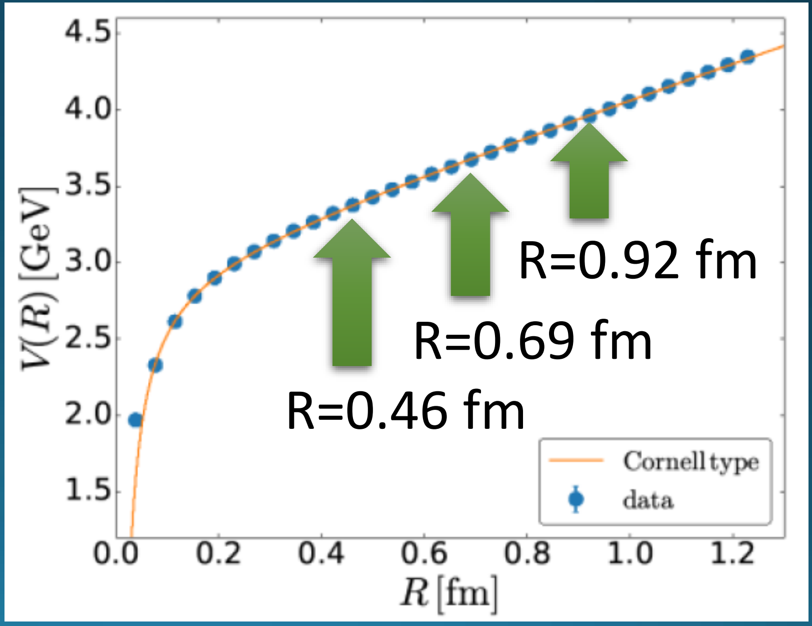
- APE smearing / multi-hit

- fine lattices (a=0.029-0.06 fm)
- continuum extrapolation

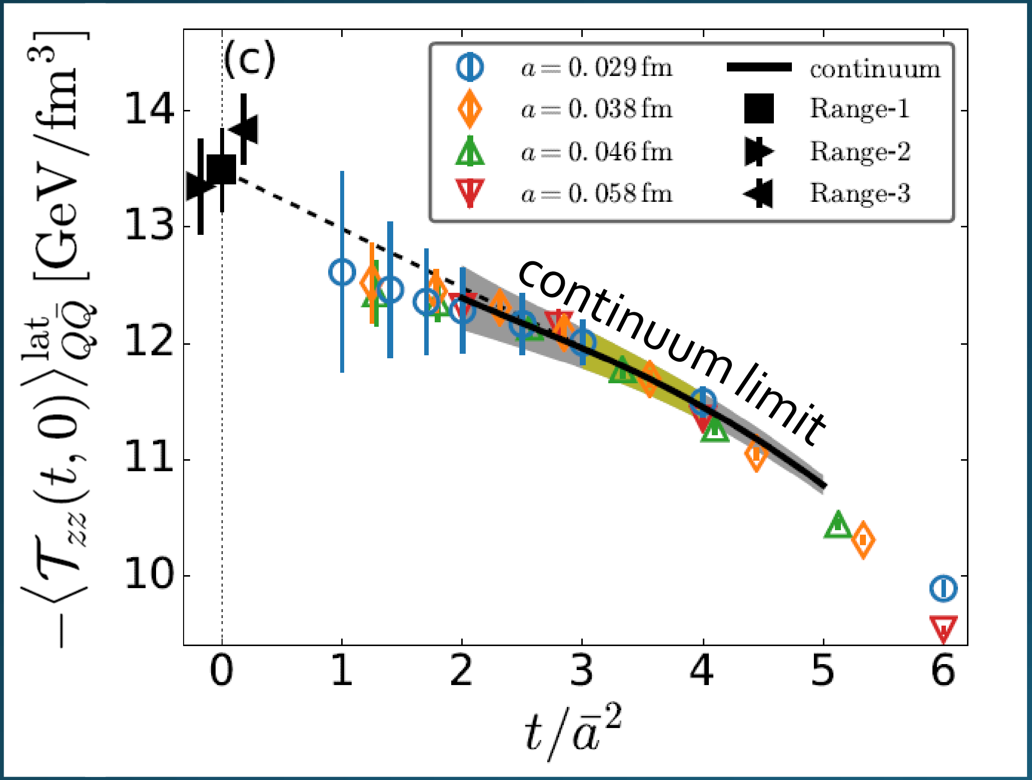
- Simulation: bluegene/Q@KEK

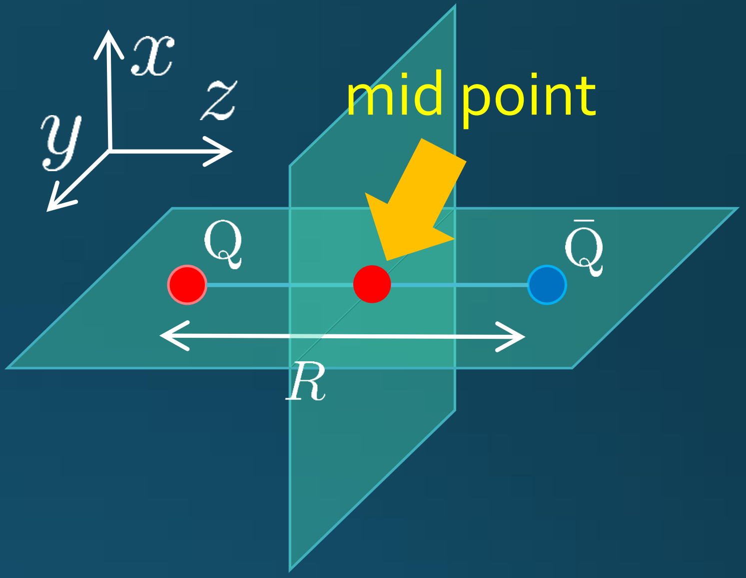
$$\langle O(x)\rangle_{Q\bar{Q}} = \lim_{T\to\infty} \frac{\langle \delta O(x)\delta W(R,T)\rangle}{\langle W(R,T)\rangle}$$

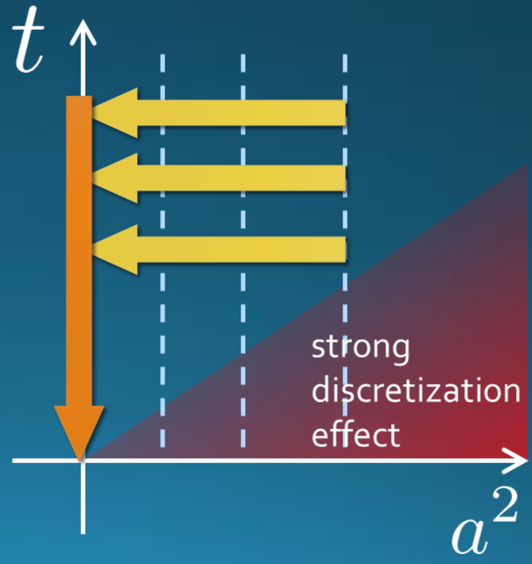
| $\beta$ | $a$ [fm] | $N_{\rm size}^4$ | $N_{\rm conf}$ | $R/a$ | | |
|---|---|---|---|---|---|---|
| 6.304 | 0.058 | $48^4$ | 140 | 8 | 12 | 16 |
| 6.465 | 0.046 | $48^4$ | 440 | 10 | – | 20 |
| 6.513 | 0.043 | $48^4$ | 600 | – | 16 | – |
| 6.600 | 0.038 | $48^4$ | 1,500 | 12 | 18 | 24 |
| 6.819 | 0.029 | $64^4$ | 1,000 | 16 | 24 | 32 |
| | | | $R$ [fm] | 0.46 | 0.69 | 0.92 |



# Continuum Extrapolation
## at mid-point

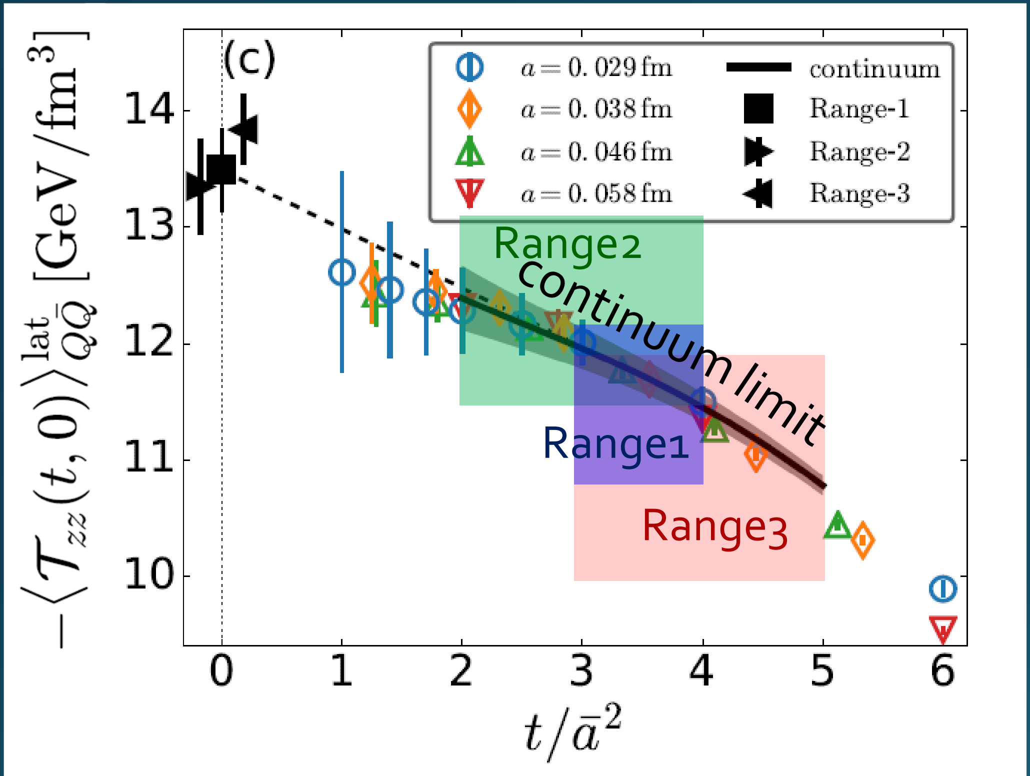


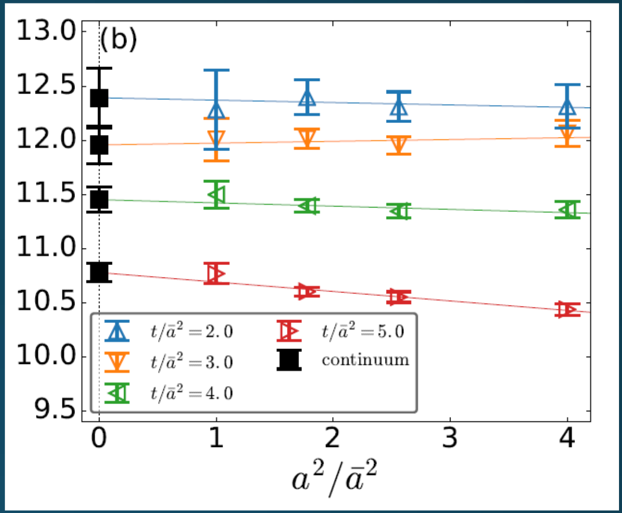



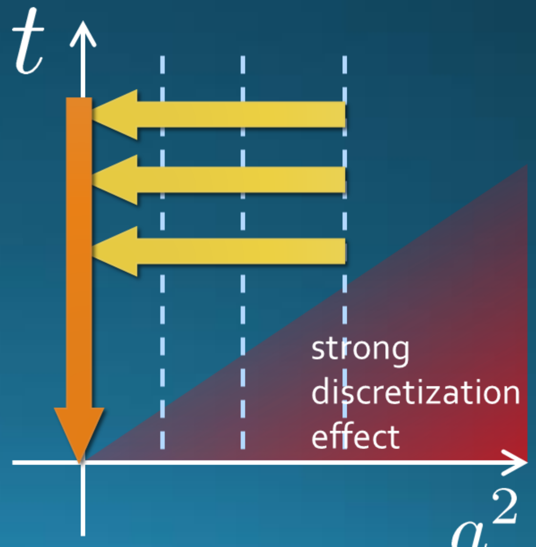




- a→0 extrapolation with fixed t

# t→0 Extrapolation
## at mid-point




- ☐ a→0 extrapolation with fixed t
- ☐ Then, t→0 with three ranges

# Stress Distribution on Mid-Plane

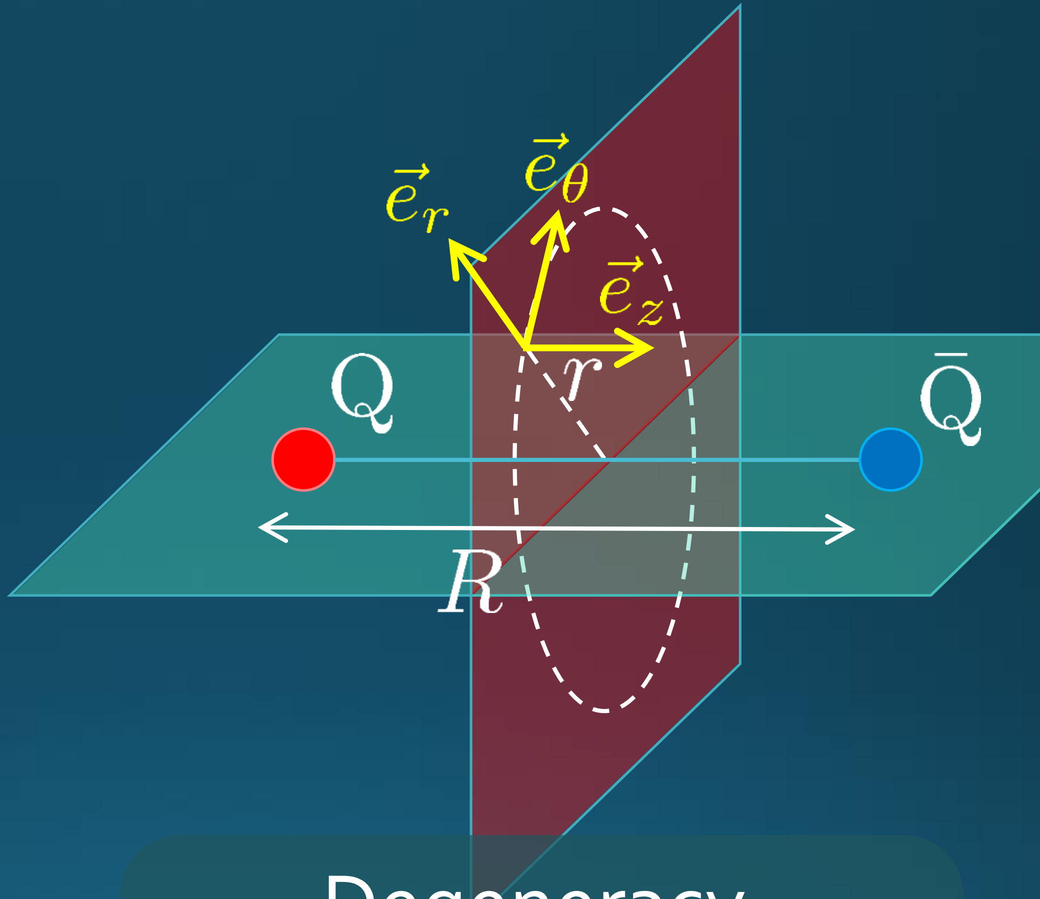
From rotational symm. & parity

EMT is diagonalized
in Cylindrical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{zz} & \\ & & & T_{44} \end{pmatrix}$$

$$T_{rr} = \vec{e}_r^{\,T} T \vec{e}_r$$

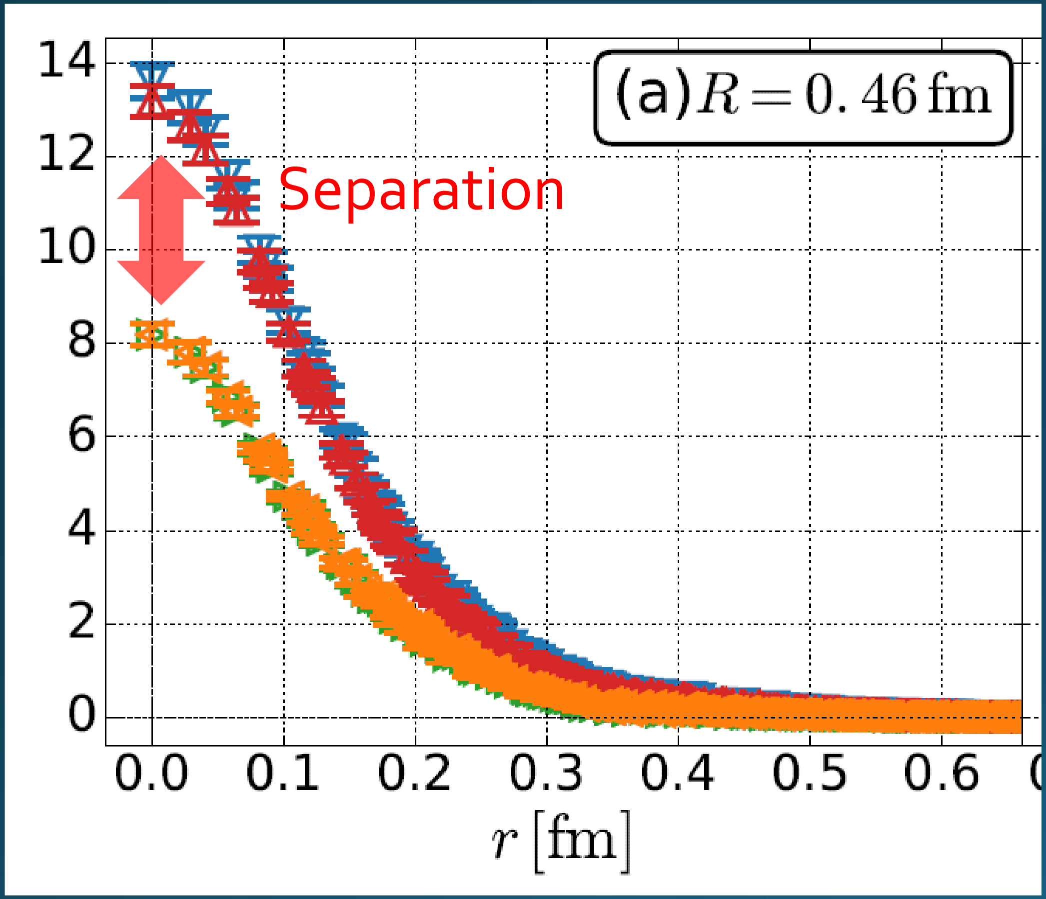
$$T_{\theta\theta} = \vec{e}_\theta^{\,T} T \vec{e}_\theta$$

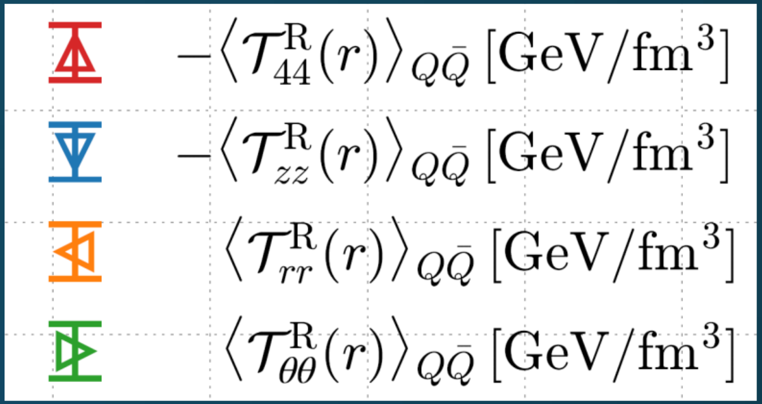



Degeneracy
in Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

# Mid-Plane



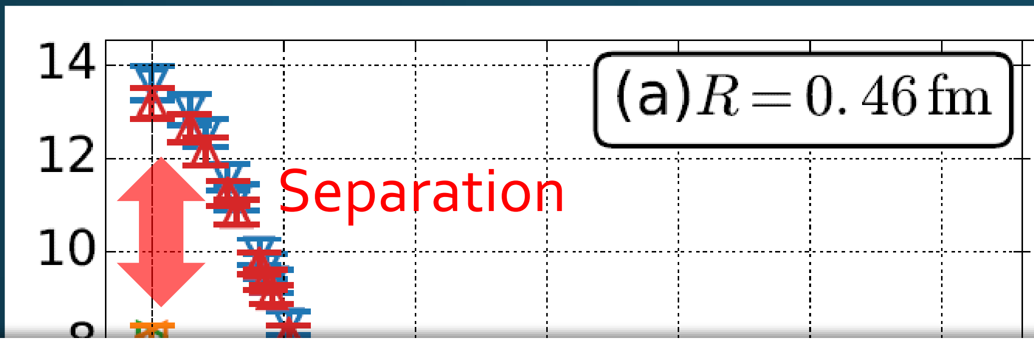

**Continuum Extrapolated!**

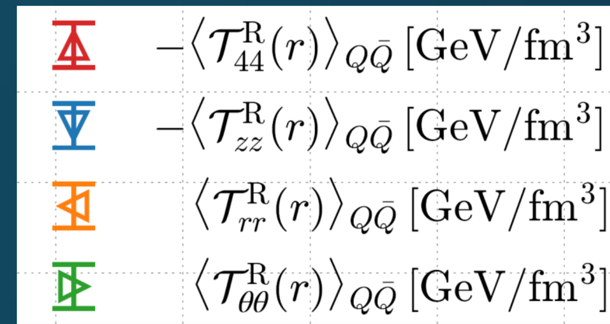
In Maxwell theory

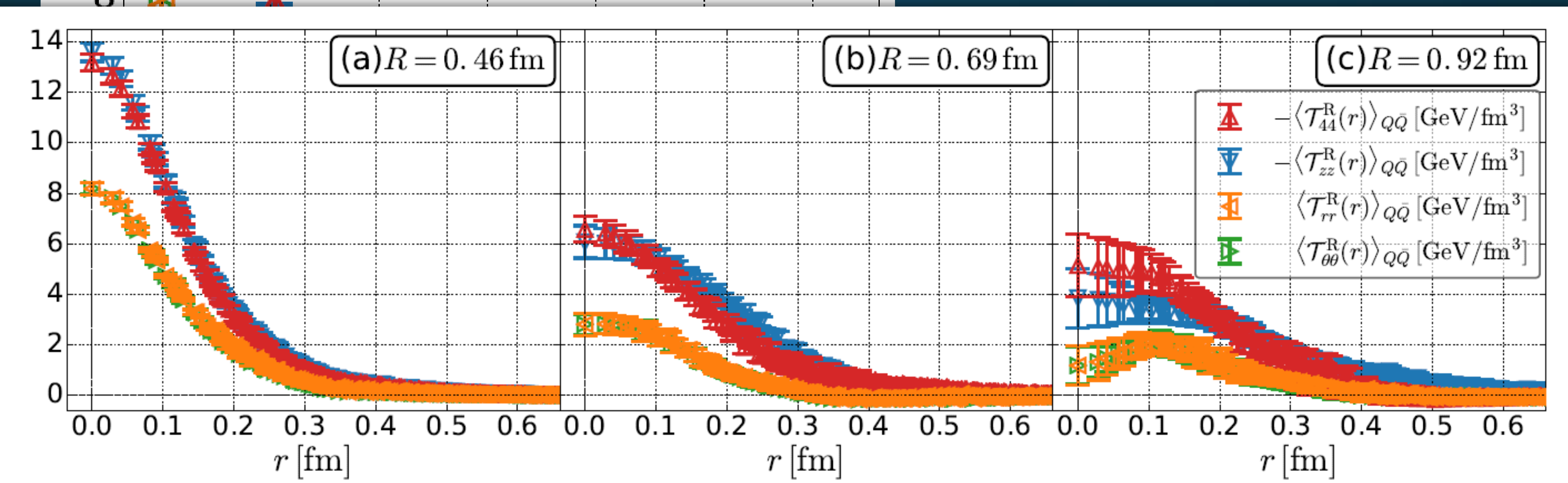
$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

- Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- Separation: $T_{zz} \neq T_{rr}$
- Nonzero trace anomaly $\sum T_{cc} \neq 0$

# Mid-Plane



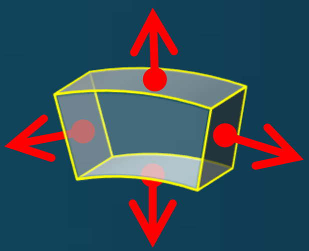

- Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- Separation: $T_{zz} \neq T_{rr}$
- Nonzero trace anomaly $\sum T_{cc} \neq 0$

# Momentum Conservation

Yanagihara+, in prep.

- In cylindrical coordinats,

$$\partial_i T_{ij} = 0 \Rightarrow \partial_r (r T_{rr}) = T_{\theta\theta} - r \partial_z T_{rz}$$

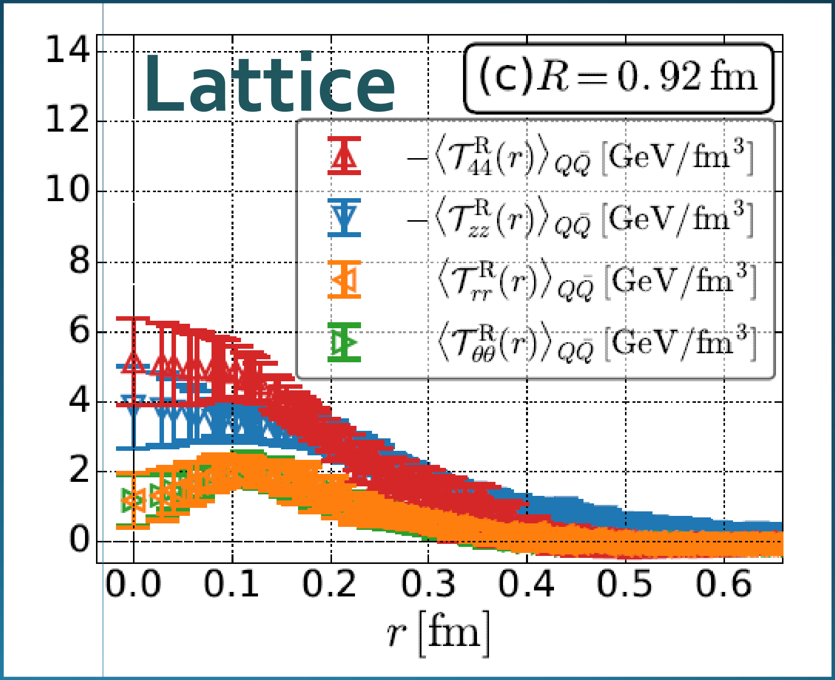


- For infinitely-long flux tube

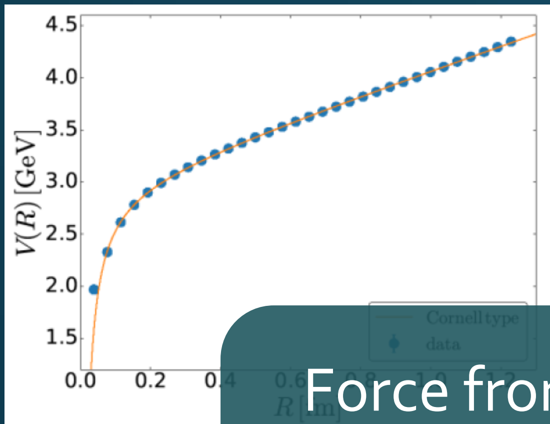
$$\partial_r (r T_{rr}) = T_{\theta\theta}$$

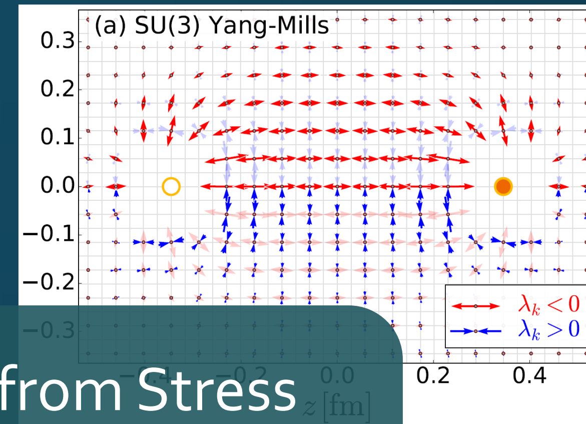
⇒ **$T_{rr}$ and $T_{\theta\theta}$ must separate!**

**Effect of boundaries is important for the flux tube at R=0.92fm**



# Force



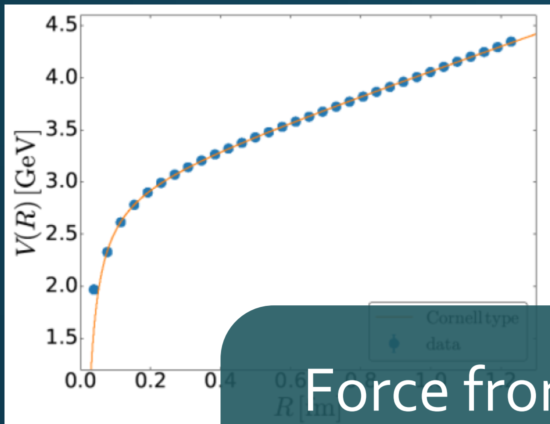

## Force from Potential

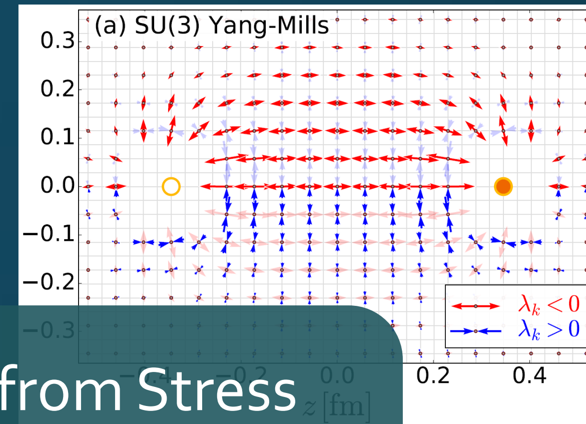
$$F_{\text{pot}} = -\frac{dV}{dR}$$

## Force from Stress

$$F_{\text{stress}} = \int_{\text{mid.}} d^2x T_{zz}(x)$$

# Force

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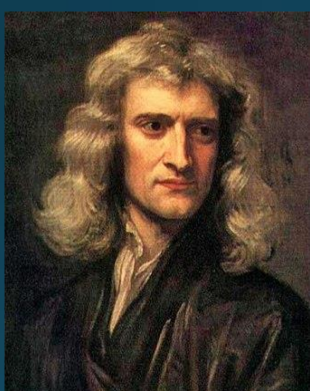
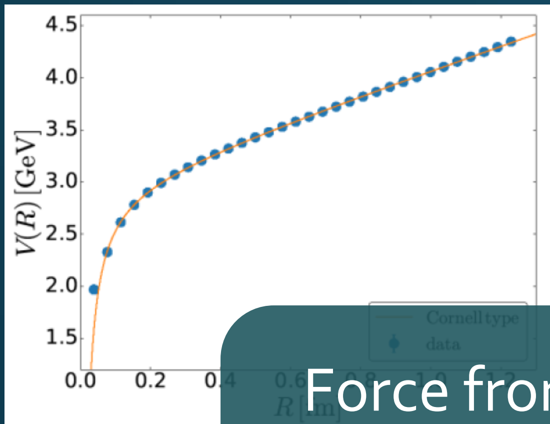
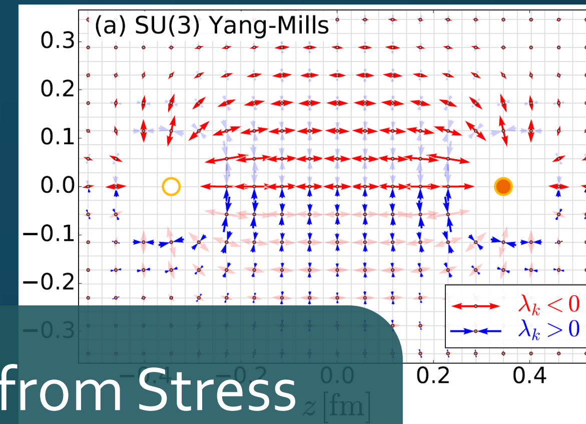


Newton
1687

Faraday
1839

# Force

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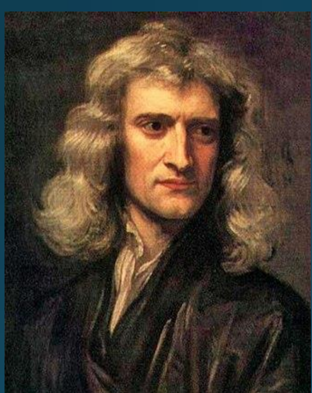
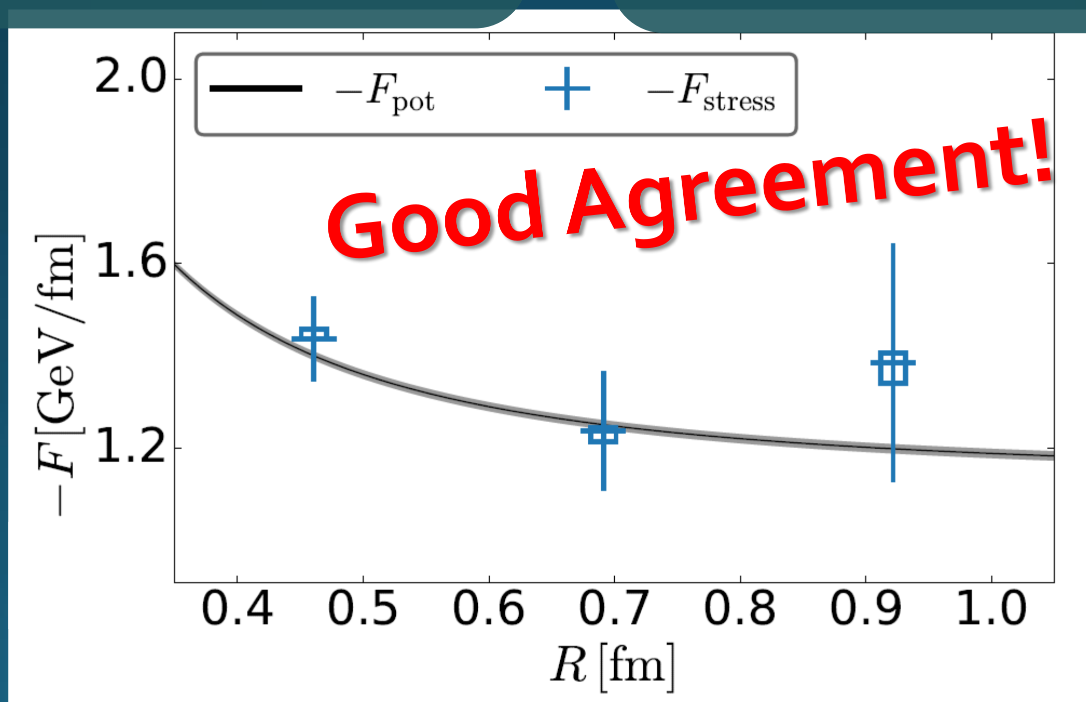


Newton
1687




Faraday
1839

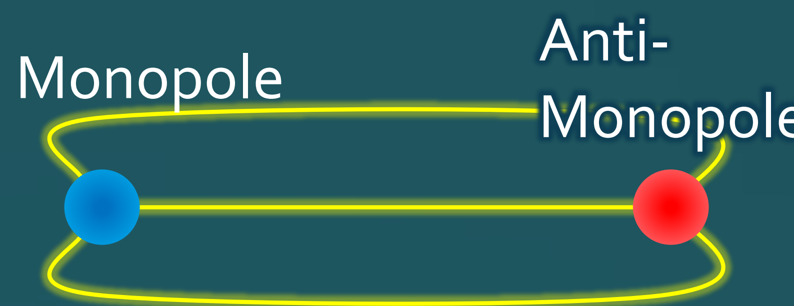
# Dual Superconductor Picture

Nambu, 1970
Nielsen, Olesen, 1973
t 'Hooft, 1981
...

## QCD Vacuum


Flux Tube

## Superconductor




Magnetic Vortex

Dual (E⇔B)

# Abelian-Higgs Model

Yanagihara, MK, PTEP 2019

**Abelian-Higgs Model**

$$\mathcal{L}_{\rm AH} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

**GL parameter:** $\kappa = \sqrt{\lambda}/g$

- ☐ type-I : $\kappa < 1/\sqrt{2}$
- ☐ type-II : $\kappa > 1/\sqrt{2}$
- ☐ Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

**Infinitely long tube**

- ☐ degeneracy

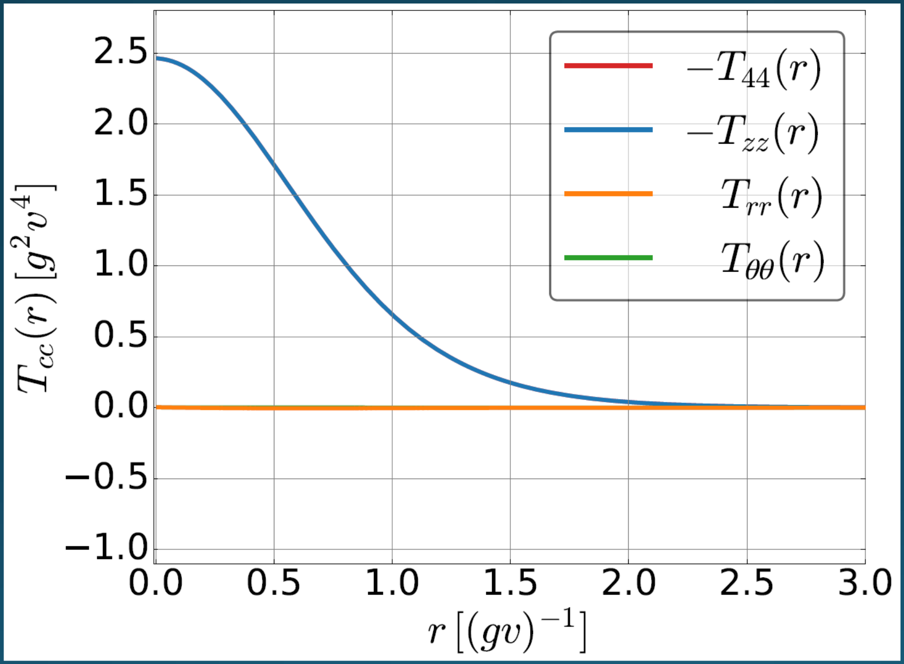
  $T_{zz}(r) = T_{44}(r)$ Luscher, 1981
- ☐ momentum conservation

  $$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

# Stress Tensor in AH Model

## infinitely-long flux tube

Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$




$$T_{rr} = T_{\theta\theta} = 0$$

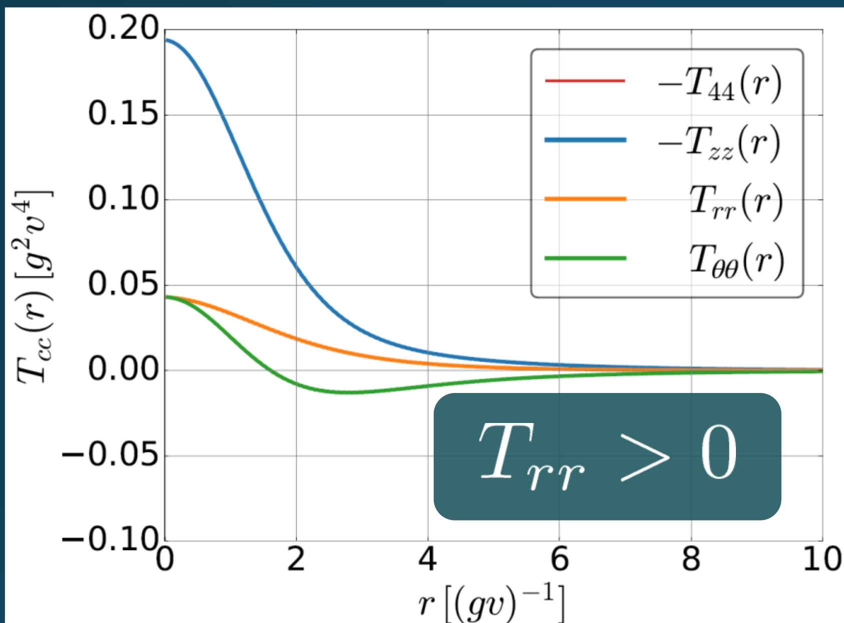
de Vega, Schaposnik, PR**D14**, 1100 (1976).

# Stress Tensor in AH Model

## infinitely-long flux tube

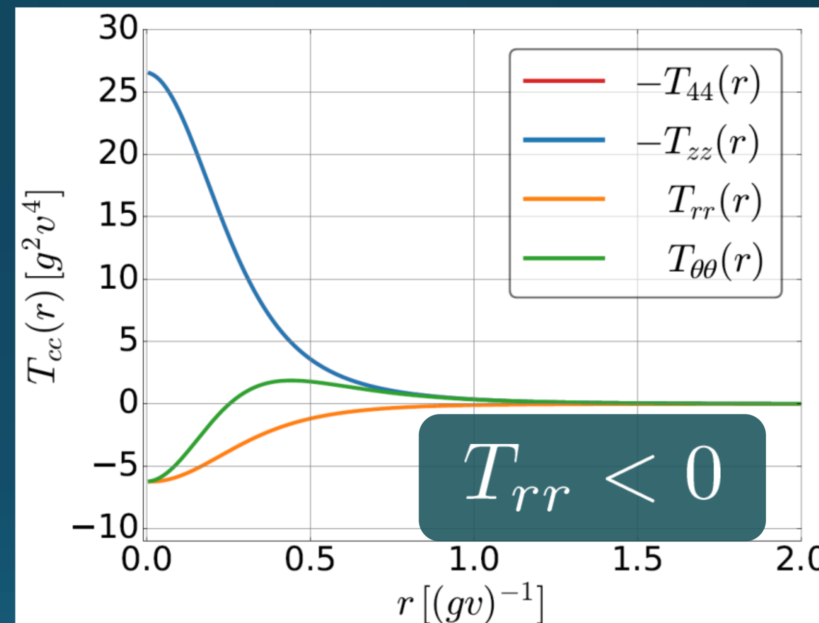
**Type-I**

$\kappa = 0.1$




**Type-II**

$\kappa = 3.0$




- ☐ No degeneracy bw $T_{rr}$ & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

conservation law

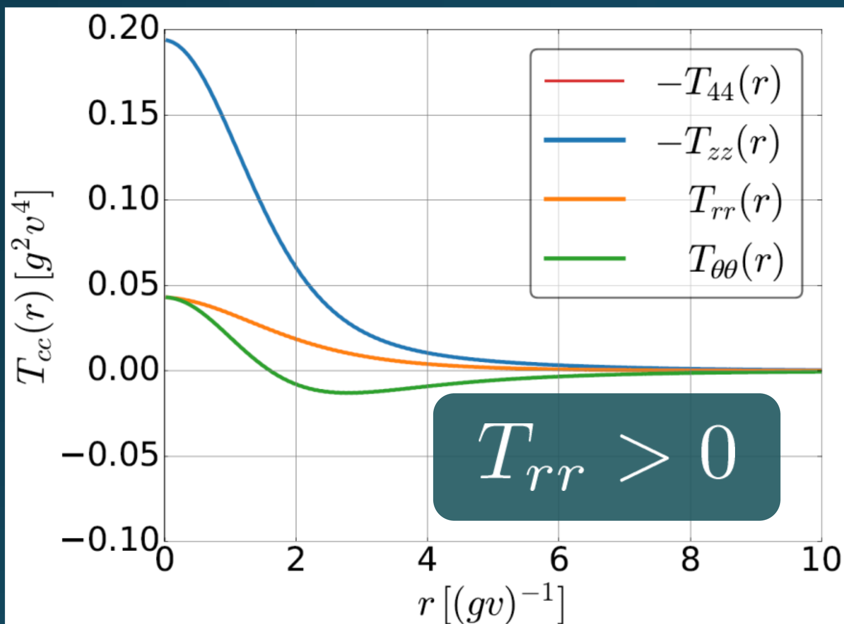
$$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

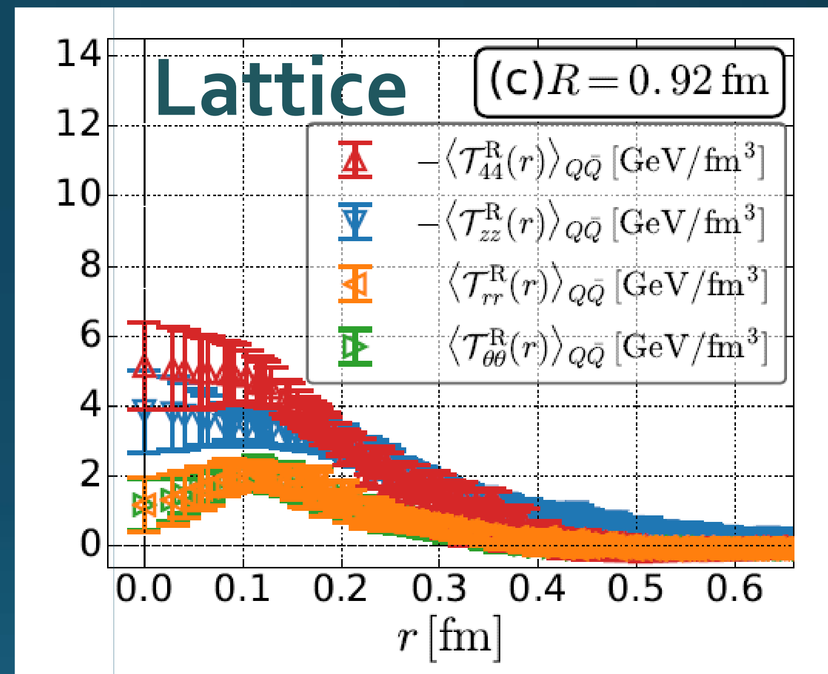
# Stress Tensor in AH Model

## infinitely-long flux tube

**Type-I**

$\kappa = 0.1$



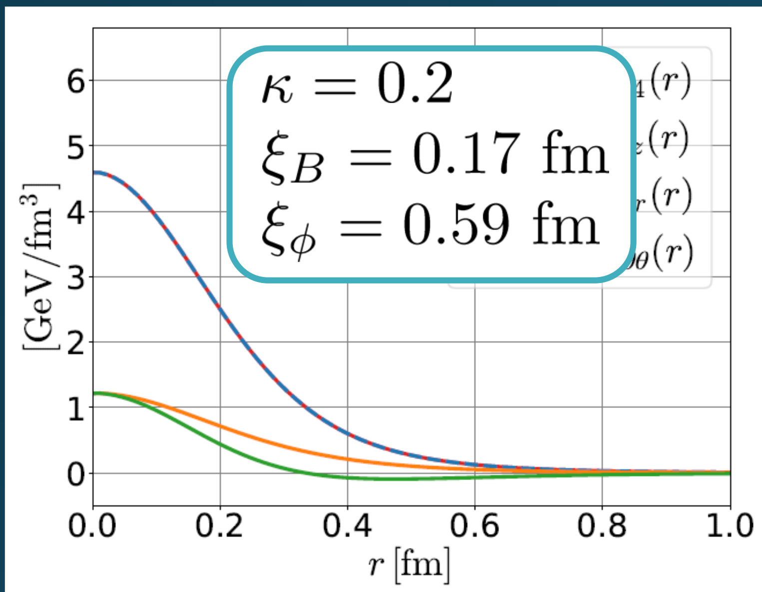

- ☐ No degeneracy bw $T_{rr}$ & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

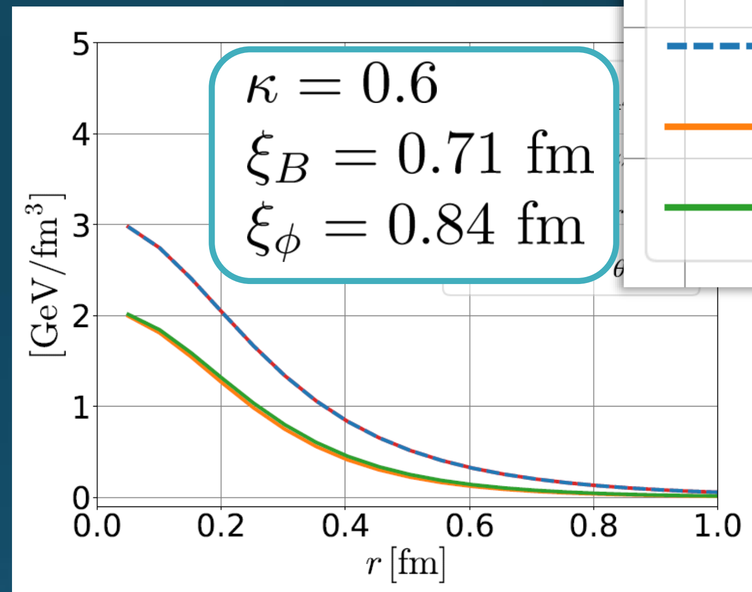
Inconsistent with lattice result

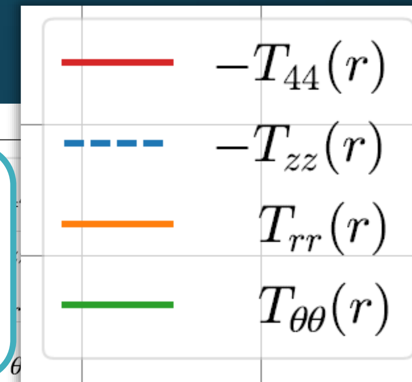
$$T_{rr} \simeq T_{\theta\theta}$$

# Flux Tube with Finite Length

## R=0.92 fm

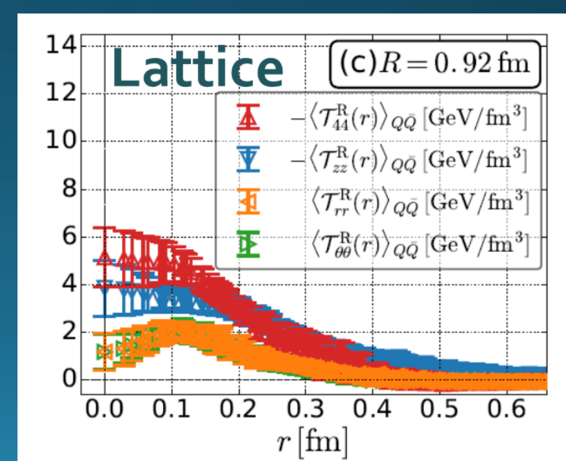



**Left:** $T_{zz}(0)$, $T_{rr}(0)$ reproduce lattice result
**Right:** A parameter satisfying $T_{rr} \approx T_{\theta\theta}$

No parameters to reproduce lattice data at R=0.92fm.



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MK, Mogliacci, Horowitz, Kolbe, arXiv:1904.00241

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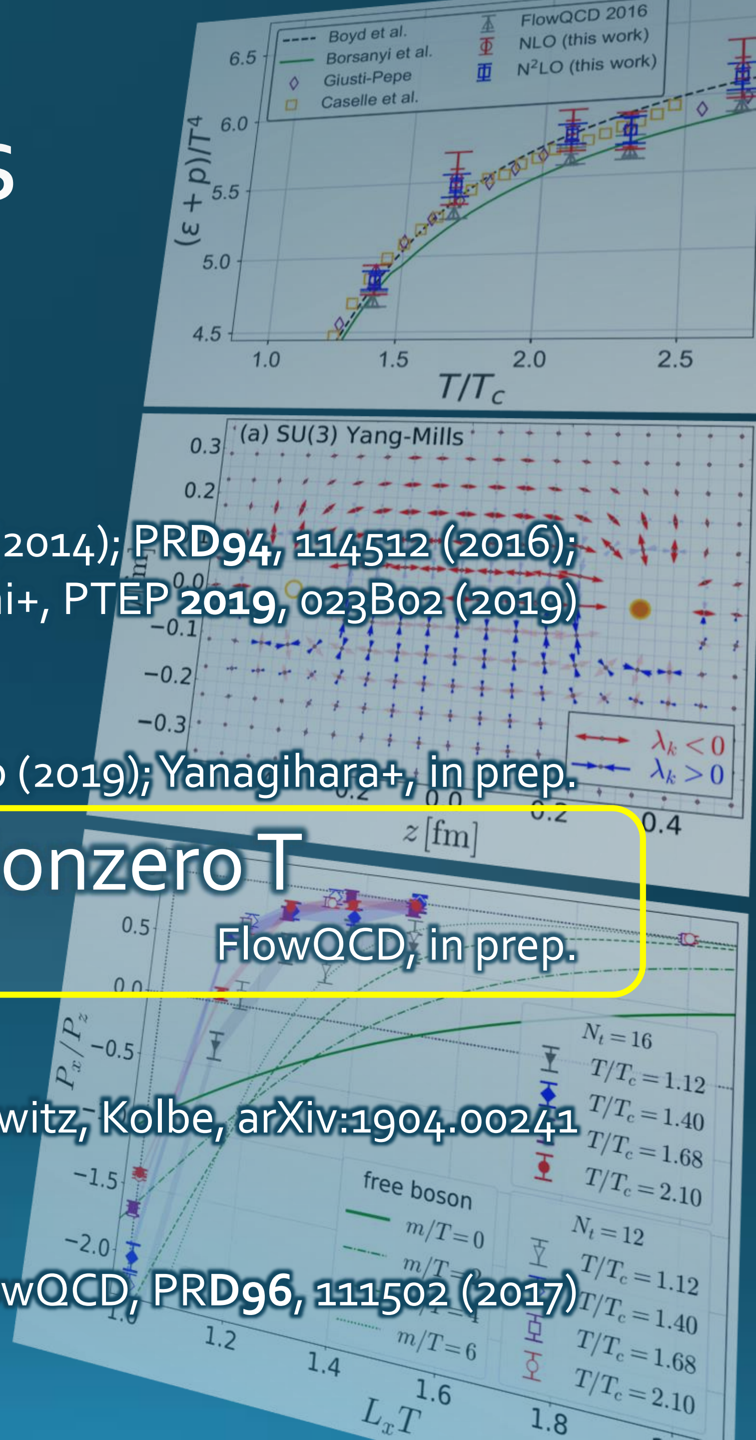
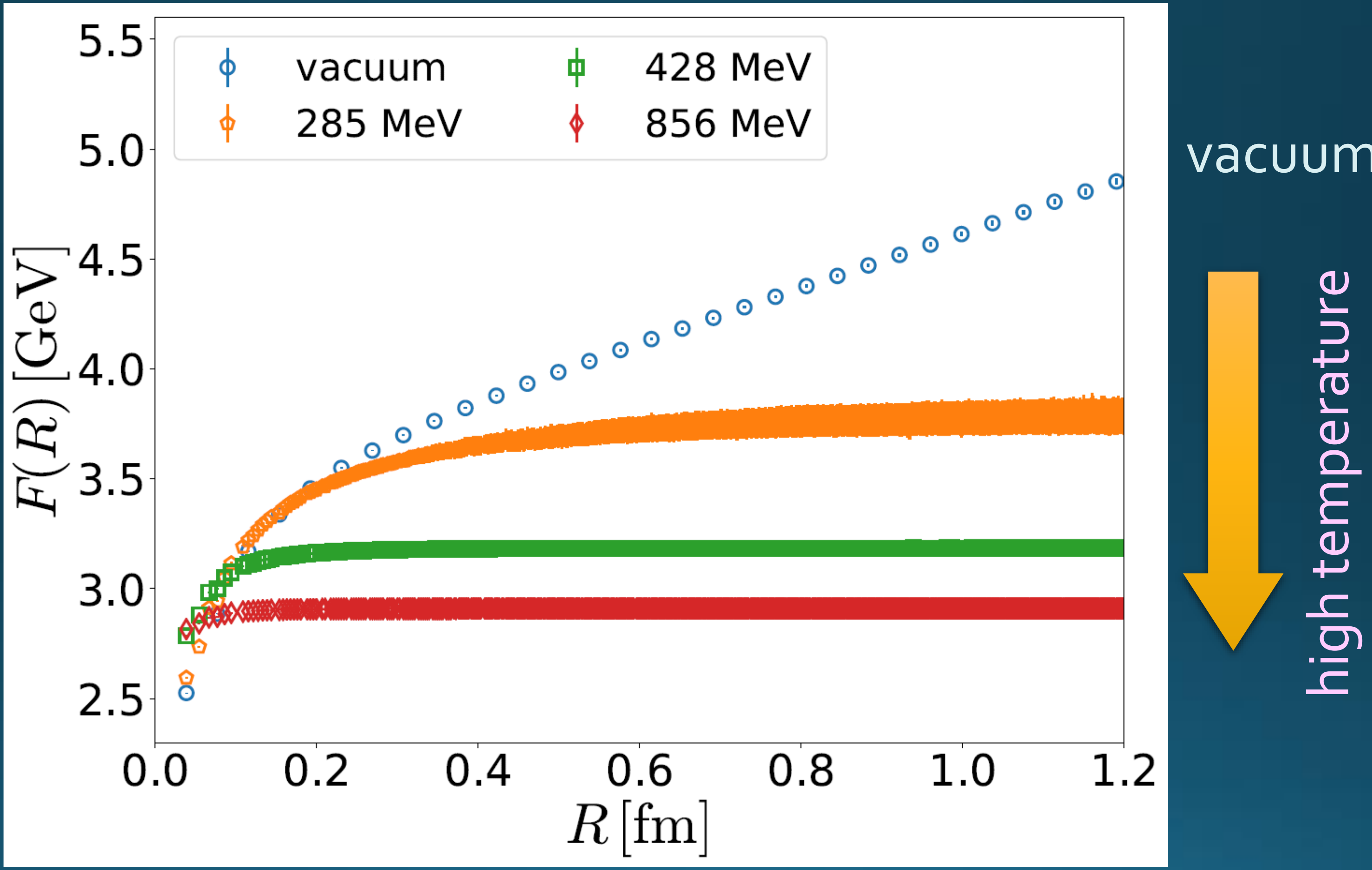


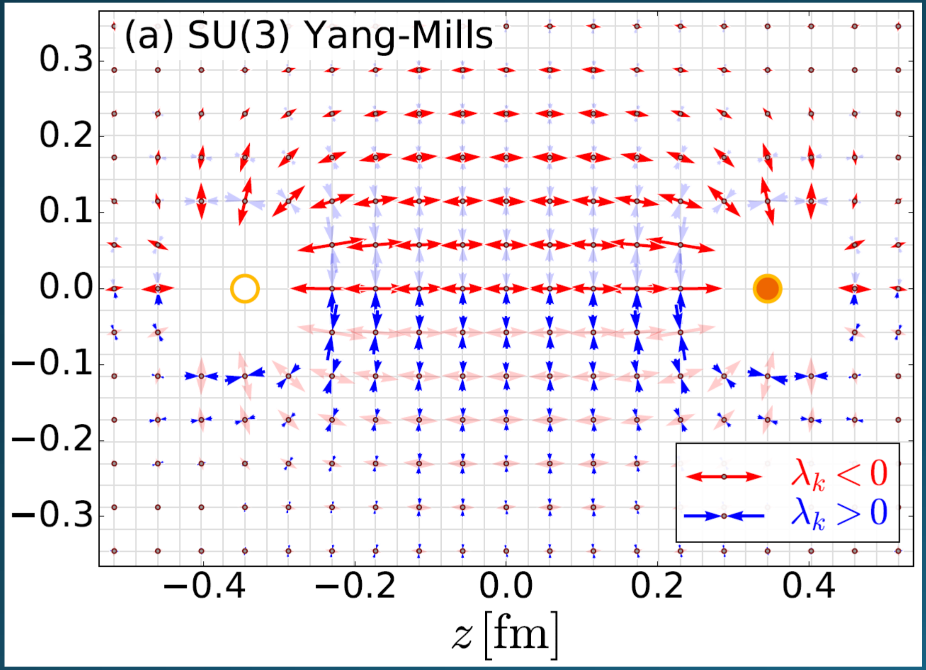
# Screening of $Q\bar{Q}$ Force above $T_c$




Q-Qbar force is screened in the deconfined phase.

# Temperature Dependence

**Vacuum**
(Current Universe)



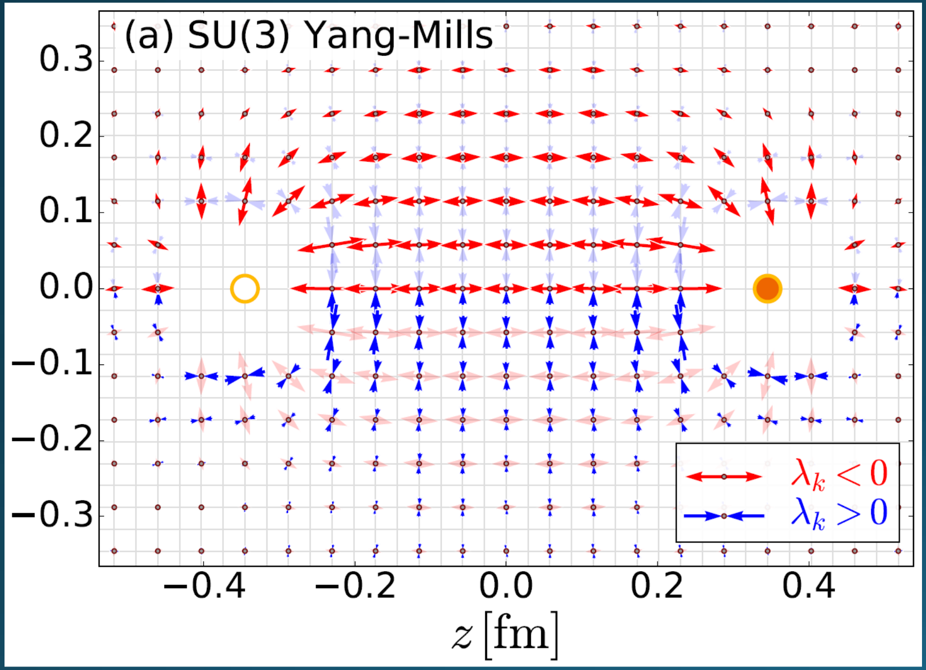

**High Temperature**
(Early Universe)

?

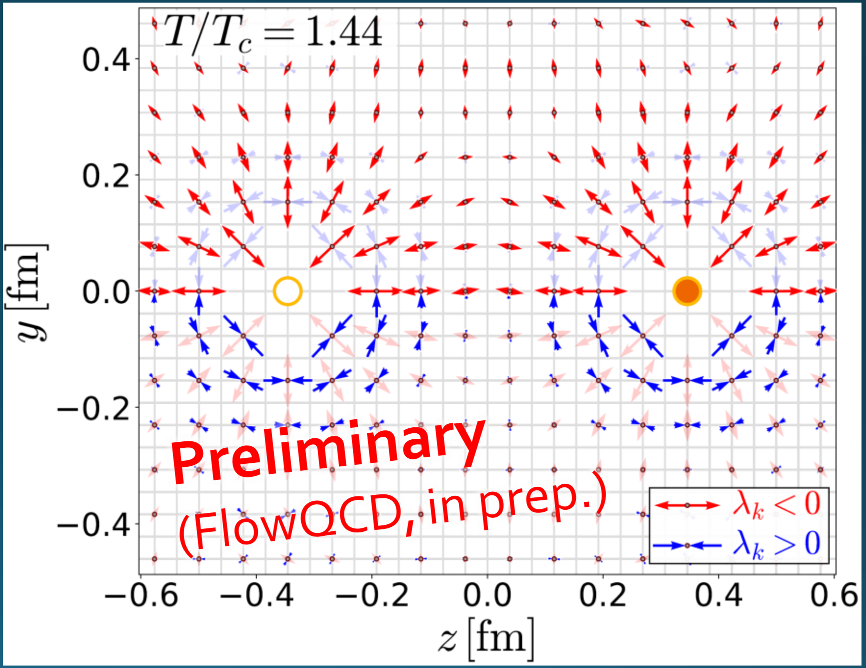
$$\langle T_{\mu\nu}(x)\rangle_{\mathrm{Q\bar{Q}}} = \frac{\langle \delta T_{\mu\nu}(x)\delta\Omega(y)\Omega^\dagger(z)\rangle}{\langle \Omega(y)\Omega^\dagger(z)\rangle}$$

# Temperature Dependence

**Vacuum**
(Current Universe)




**High Temperature**
(Early Universe)

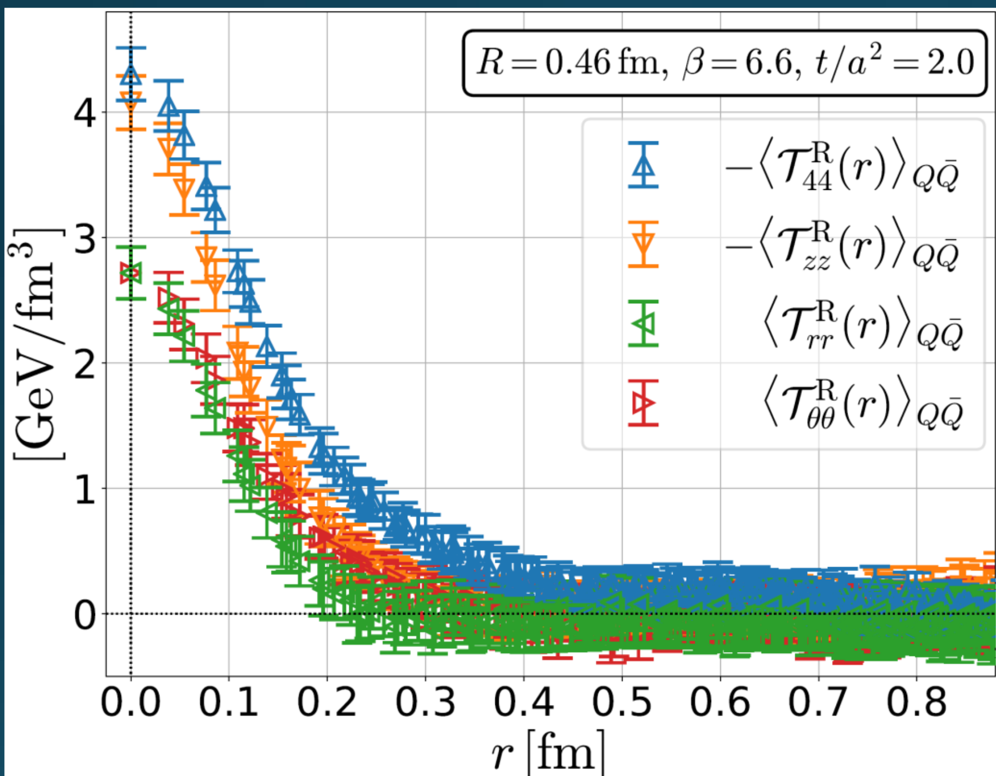



$T=1.44T_c$

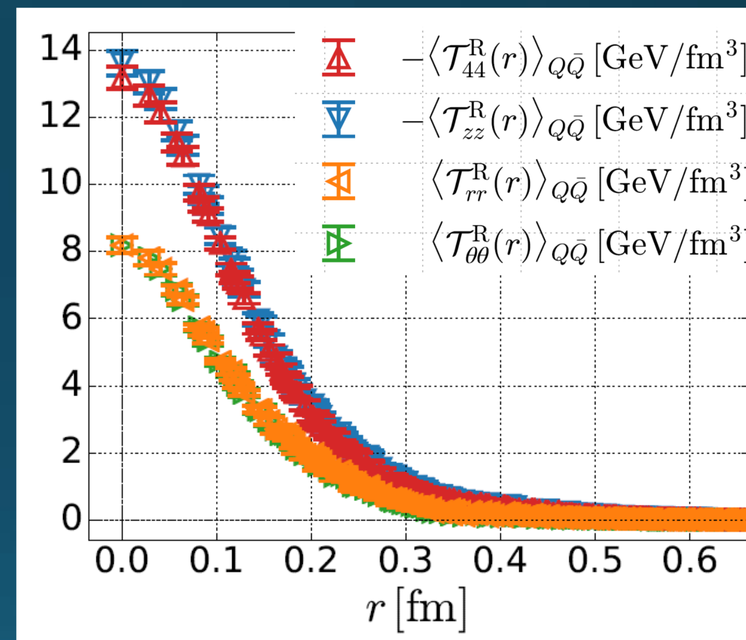
- ☐ Singlet projection for $T=1.44T_c$
- ☐ Flux-tube structure is screened above $T_c$.

# Mid Plane

T=1.44Tc, R=0.46 fm



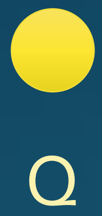

Vacuum, R=0.46 fm



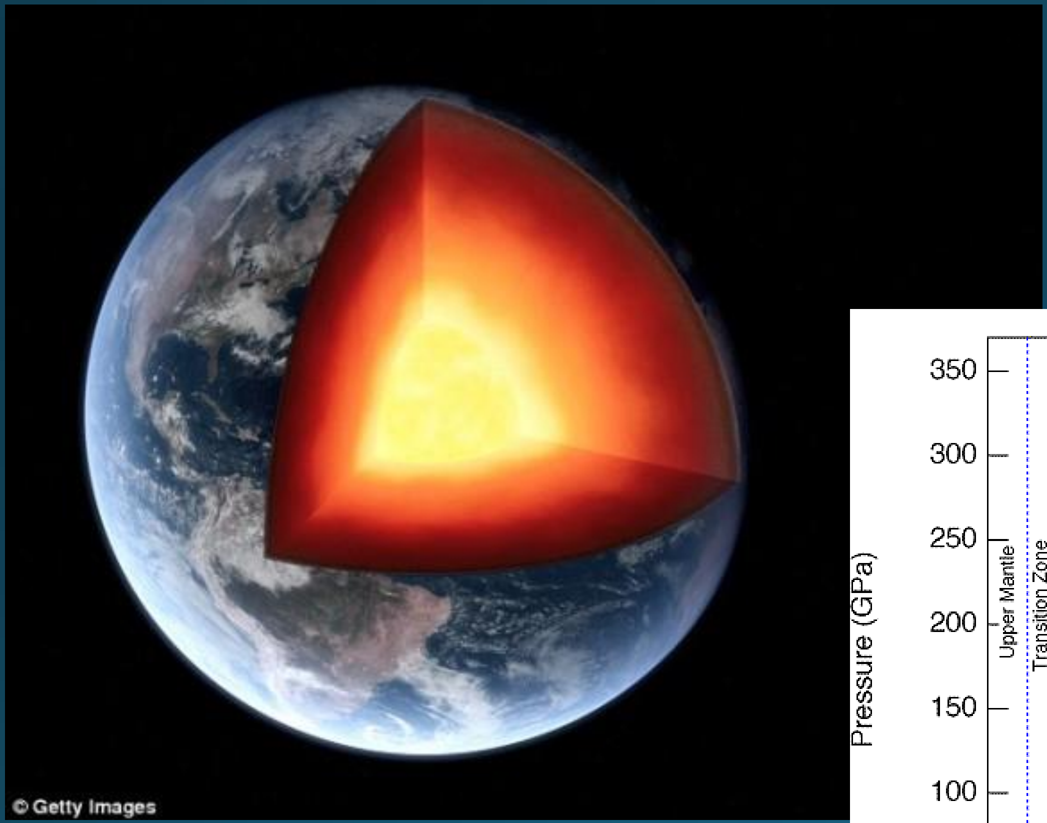

- Separation b/w $T_{44}$ & $T_{zz}$?

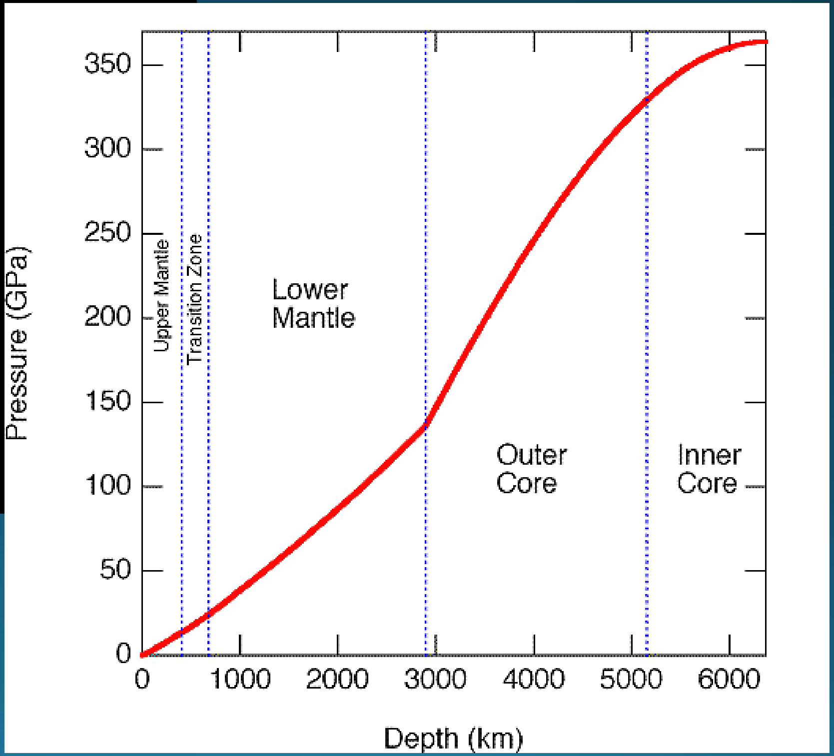
# Stress Tensor around A Quark

## in a deconfined phase



# Pressure inside the Earth

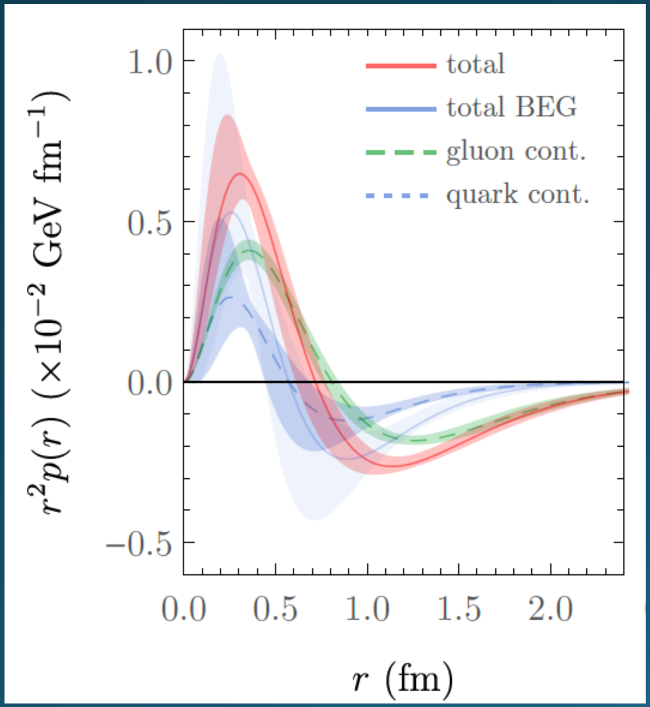


# Pressure inside Hadrons

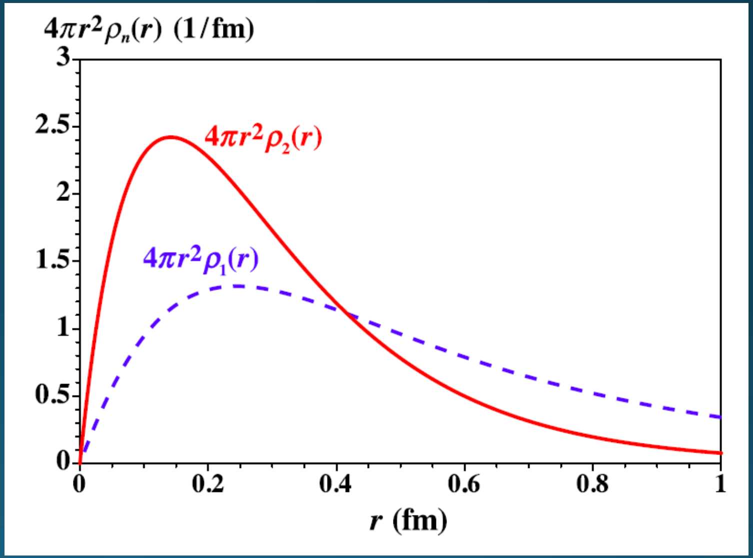
EMT distribution inside hadrons now accessible??

**Pressure @ proton**




arXiv:1810.07589
Nature, 557, 396 (2018)

**EMT distribution @ pion**

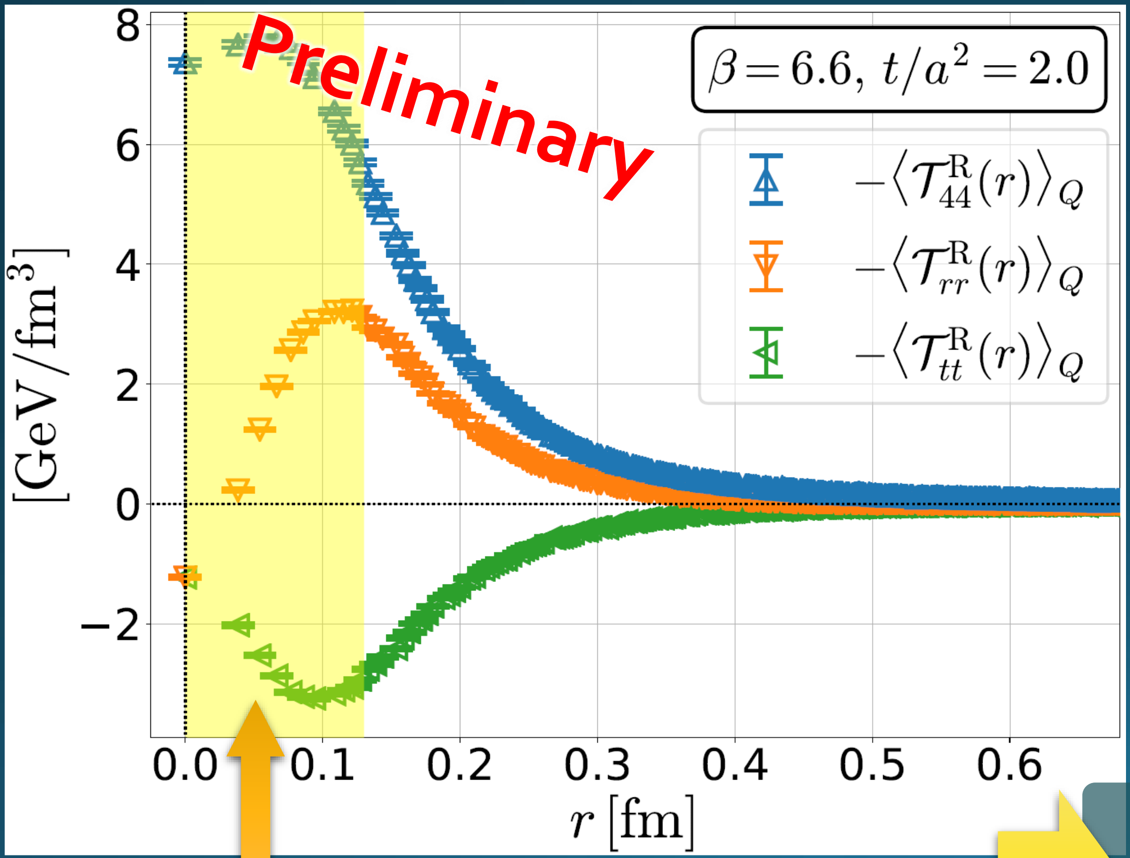



Kumano, Song, Teryaev
Phys. Rev. D 97, 014020 (2018)

# Stress Tensor around A Quark

## in a deconfined phase

$$\langle T_{\mu\nu}(x)\rangle_{\rm Q} = \frac{\langle \delta T_{\mu\nu}(x)\delta\Omega(0)\rangle}{\langle\Omega\rangle}$$




Not reliable

Yanagihara+, in prep.
Quenched QCD
$48^3$x12 (T≈1.4$T_c$)
fixed t , a

**Spherical Coordinates**

- Energy density
  $- \langle T_{44}\rangle = \varepsilon$
- Longitudinal pressure
  $- \langle T_{rr}\rangle = -p(r)$
- Transverse pressure
  $- \langle T_{tt}\rangle$

☐ Screening mass
☐ Strong coupling const.

# Contents

1. Constructing EMT
2. Thermodynamics

FlowQCD, PR**D90**,011501 (2014); PR**D94**, 114512 (2016);
WHOT-QCD, PR**D96**, 014509 (2017); Iritani+, PTEP **2019**, 023B02 (2019)

3. Flux Tube

FlowQCD, PLB789, 210 (2019); Yanagihara+, in prep.

4. Static Quark Systems at Nonzero T

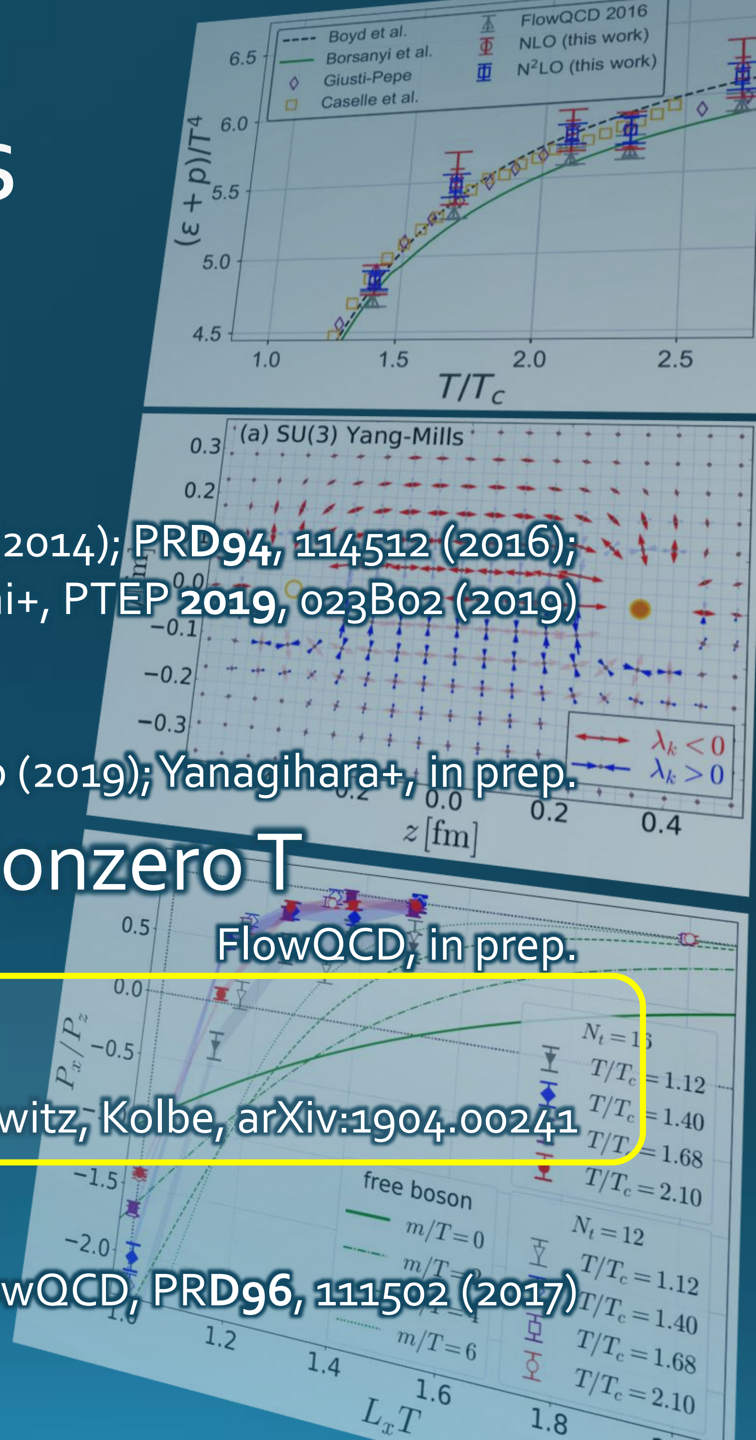
FlowQCD, in prep.

5. Casimir Effect

MK, Mogliacci, Horowitz, Kolbe, arXiv:1904.00241

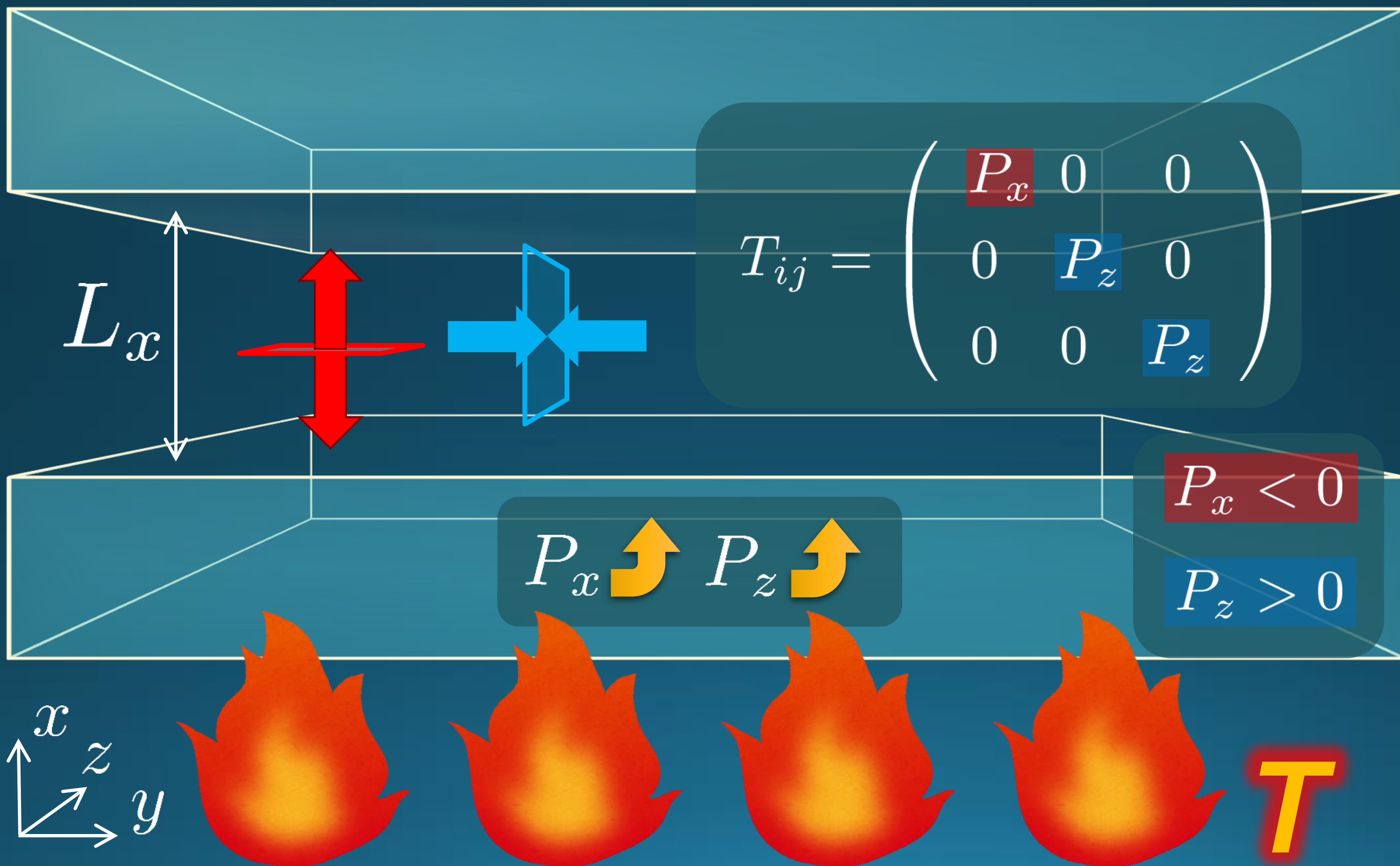
6. Correlation Function

FlowQCD, PR**D96**, 111502 (2017)



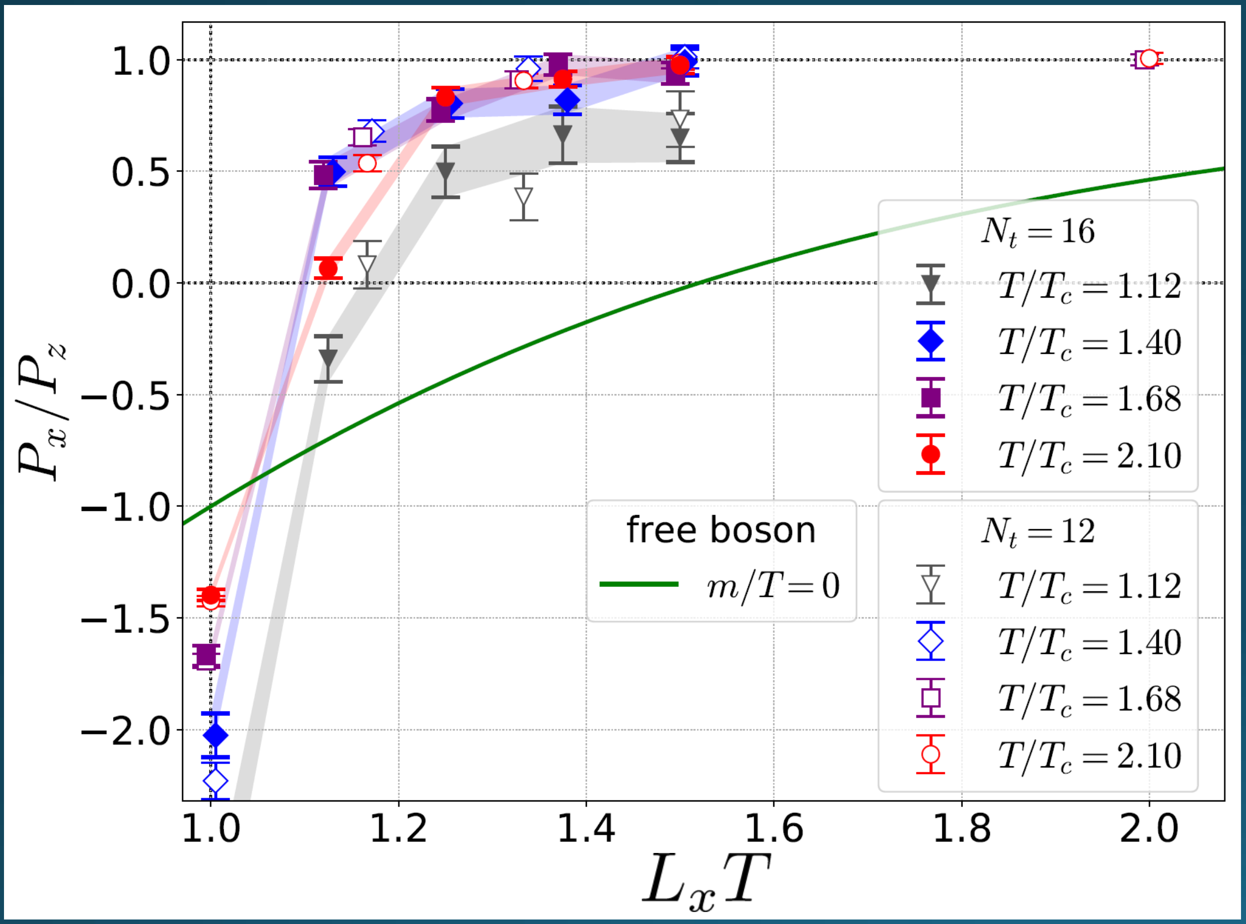
# Casimir Effect

Brown, Maclay
1969



# Pressure Anisotropy @ T≠0

MK, Mogliacci, Kolbe, Horowitz, 1904.00241




## Free scalar field

- ☐ $L_2=L_3=\infty$
- ☐ Periodic BC

Mogliacci+, 1807.07871

## Lattice result

- ☐ Periodic BC
- ☐ Only t→0 limit
- ☐ Error: stat.+sys.

**Medium near $T_c$ is remarkably insensitive to finite size!**

# Thermodynamics on the Lattice

## Various Methods

- ☐ **Integral**, differential, moving frame, non-equilibrium, ...
- ☐ rely on thermodynamic relations valid in V→∞

$$P = \frac{T}{V} \ln Z \qquad sT = \varepsilon + P$$

→ **Not applicable to anisotropic systems**

☐We employ **Gradient Flow Method**

$$\varepsilon = \langle T_{00} \rangle \quad P = \langle T_{11} \rangle$$

**Components of EMT are directly accessible!**

# Numerical Setup

- SU(3) YM theory
- Wilson gauge action

- $N_t$ = 16, 12
- $N_z/N_t$=6
- 2000~4000 confs.
- Even $N_x$

- No Continuum extrap.

| $T/T_c$ | $\beta$ | $N_z$ | $N_\tau$ | $N_x$ | $N_{\rm vac}$ |
|---|---|---|---|---|---|
| 1.12 | 6.418 | 72 | 12 | 12, 14, 16, 18 | 64 |
| | 6.631 | 96 | 16 | 16, 18, 20, 22, 24 | 96 |
| 1.40 | 6.582 | 72 | 12 | 12, 14, 16, 18 | 64 |
| | 6.800 | 96 | 16 | 16, 18, 20, 22, 24 | 128 |
| 1.68 | 6.719 | 72 | 12 | 12, 14, 16, 18, 24 | 64 |
| | 6.719 | 96 | 12 | 14, 18 | 64 |
| | 6.941 | 96 | 16 | 16, 18, 20, 22, 24 | 96 |
| 2.10 | 6.891 | 72 | 12 | 12, 14, 16, 18, 24 | 72 |
| | 7.117 | 96 | 16 | 16, 18, 20, 22, 24 | 128 |
| 2.69 | 7.086 | 72 | 12 | 12, 14, 16, 18 | - |
| $\simeq 8.1$ | 8.0 | 72 | 12 | 12, 14, 16, 18 | - |
| $\simeq 25$ | 9.0 | 72 | 12 | 12, 14, 16, 18 | - |

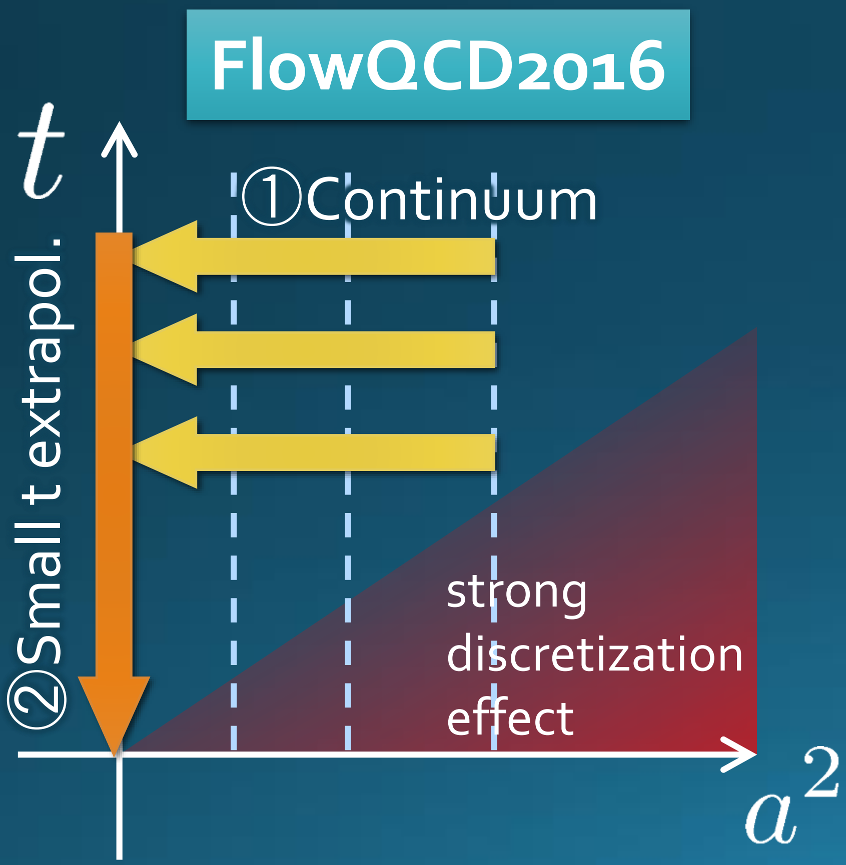
- Same Spatial volume
  - $12 \times 72^2 \times 12 \sim 16 \times 96^2 \times 16$
  - $18 \times 72^2 \times 12 \sim 24 \times 96^2 \times 16$

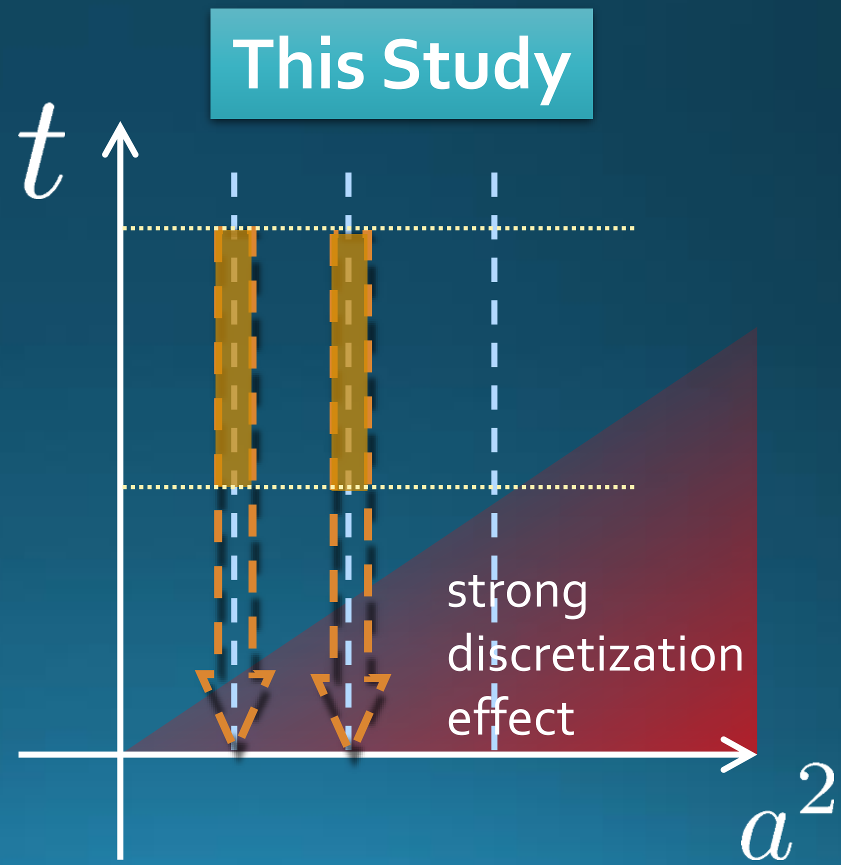
Simulations on
OCTOPUS/Reedbush

# Extrapolations t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

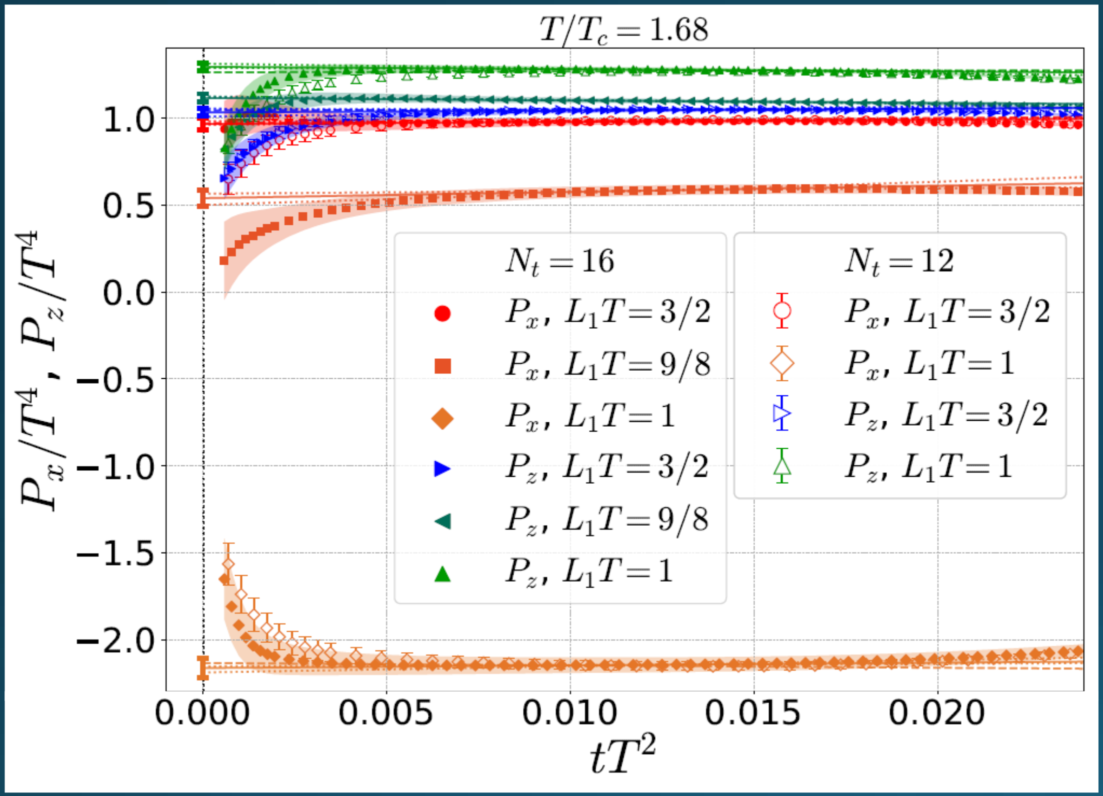
O(t) terms in SFTE    lattice discretization

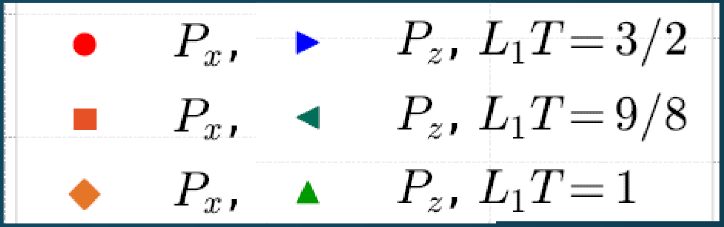


# Small-t Extrapolation

$T/T_c = 1.68$




Filled: $N_t$=16 / Open: $N_t$=12

## Small-t extrapolation

- Solid: $N_t$=16, Range-1
- Dotted: $N_t$=16, Range-2,3
- Dashed: $N_t$=12, Range-1

- ☐ Stable small-t extrapolation
- ☐ No $N_t$ dependence within statistics for $L_xT$=1, 1.5

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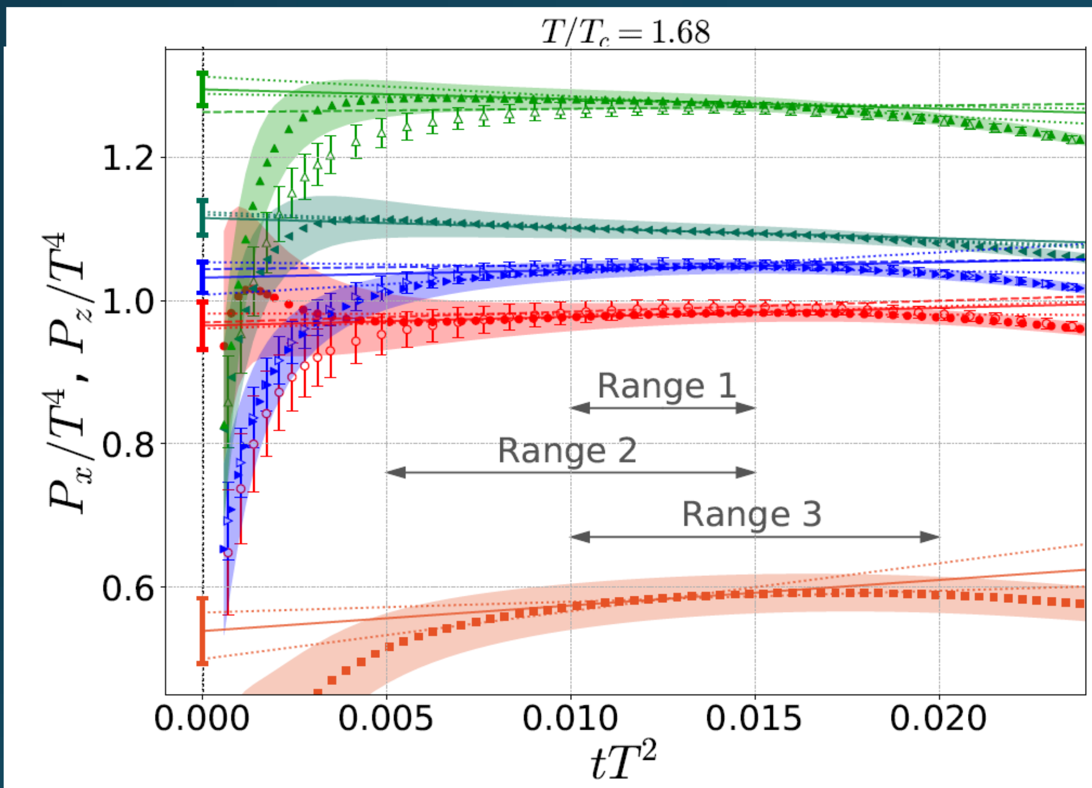


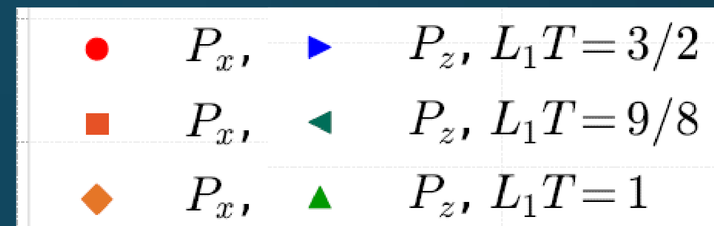


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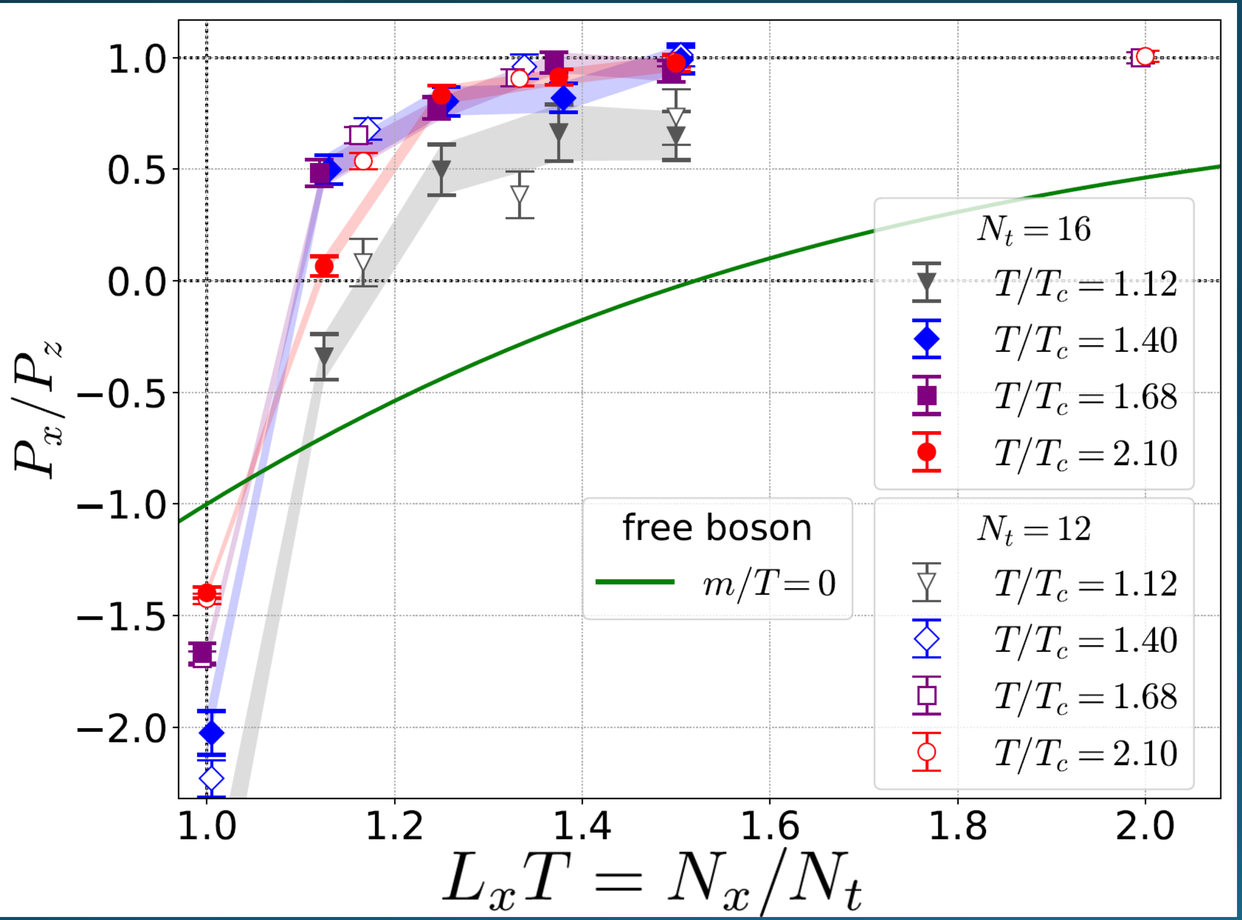
# Pressure Anisotropy @ T≠0

MK, Mogliacci, Kolbe, Horowitz, 1904.00241

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Mogliacci+, 1807.07871

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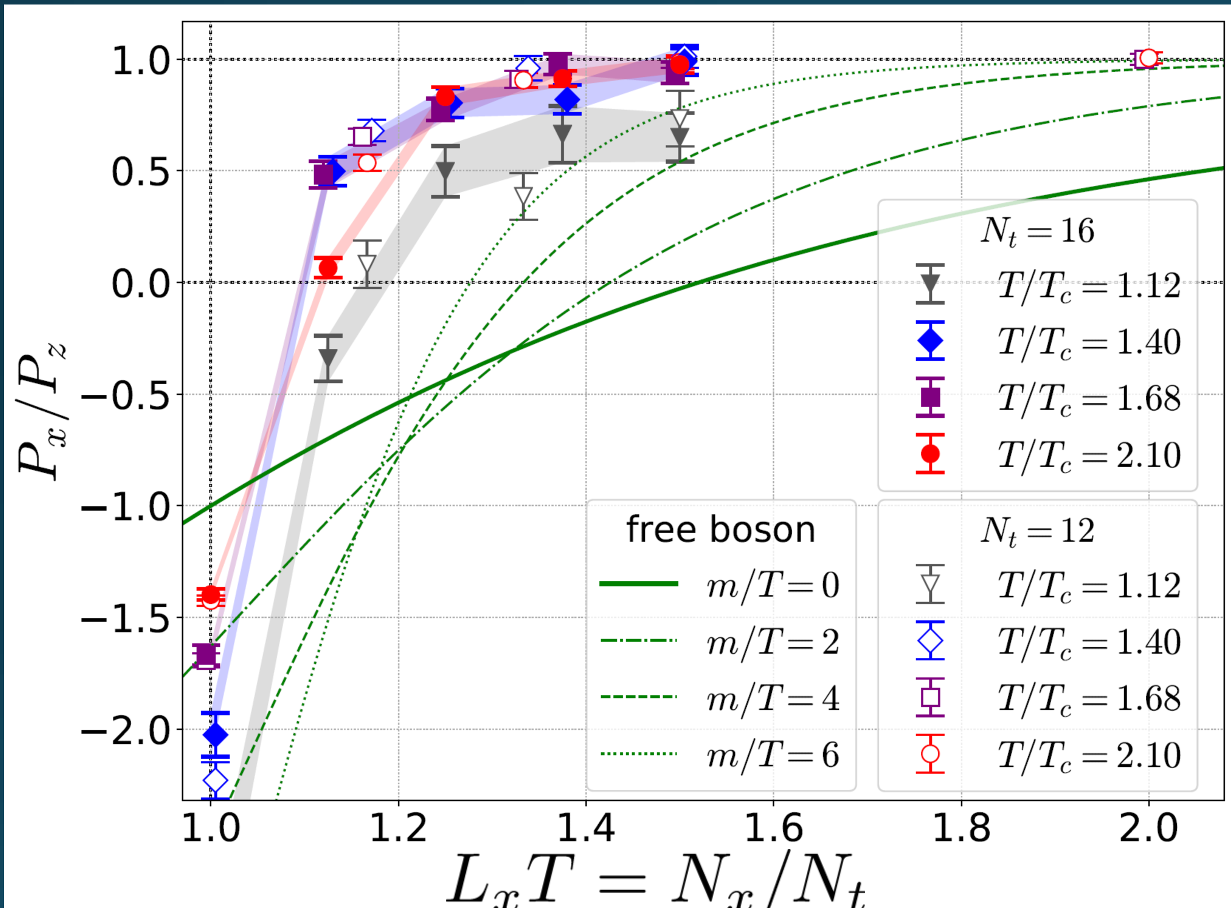


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Mogliacci+, 1807.07871

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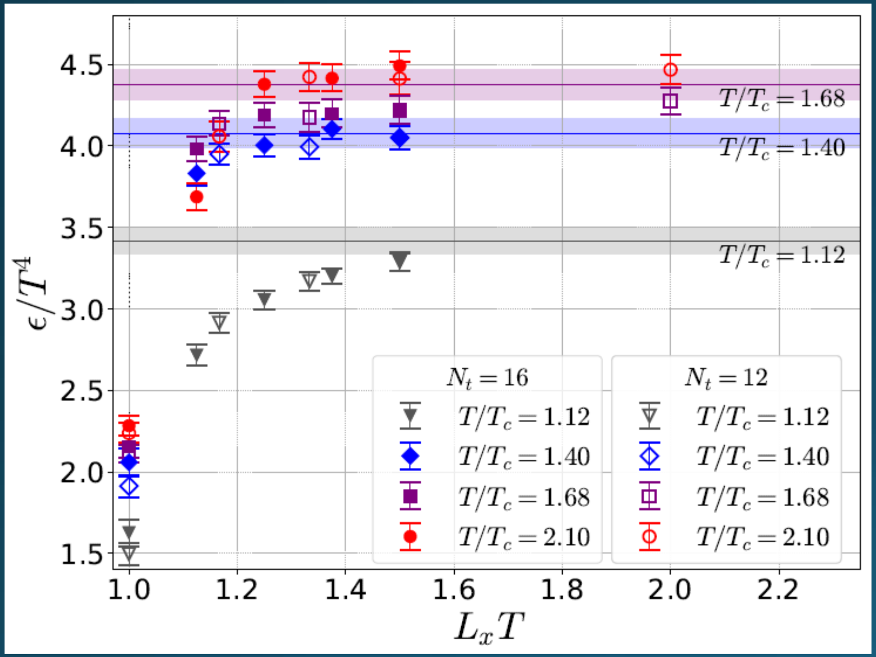
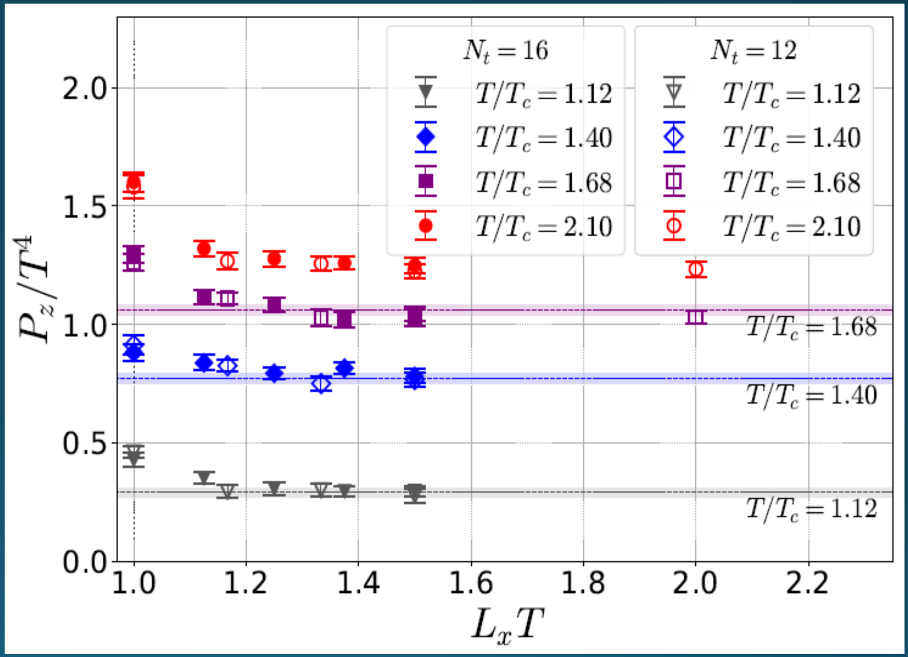
# Energy densty / transverse P

## Energy Density




## Transverse Pressure $P_z$



# Higher T

**High-T limit: massless free gluons**

How does the anisotropy approach this limit?

## Difficulties

- ☐ Vacuum subtraction requires large-volume simulations.
- ☐ Lattice spacing not available → $c_1(t)$, $c_2(t)$ are not determined.

# Higher T

**High-T limit: massless free gluons**
How does the anisotropy approach this limit?

## Difficulties

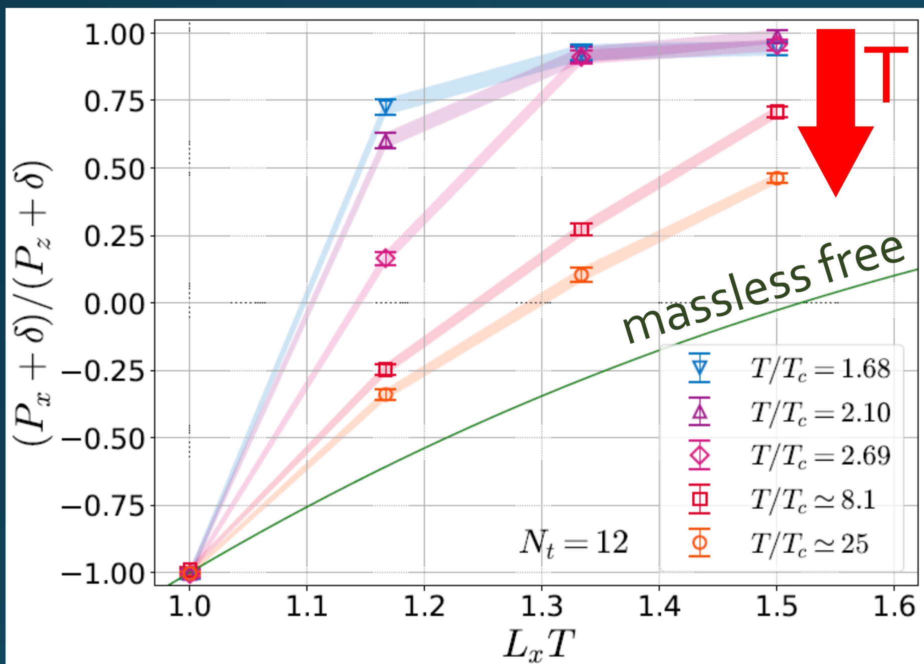
- ☐ Vacuum subtraction requires large-volume simulations.
- ☐ Lattice spacing not available → $c_1(t)$, $c_2(t)$ are not determined.

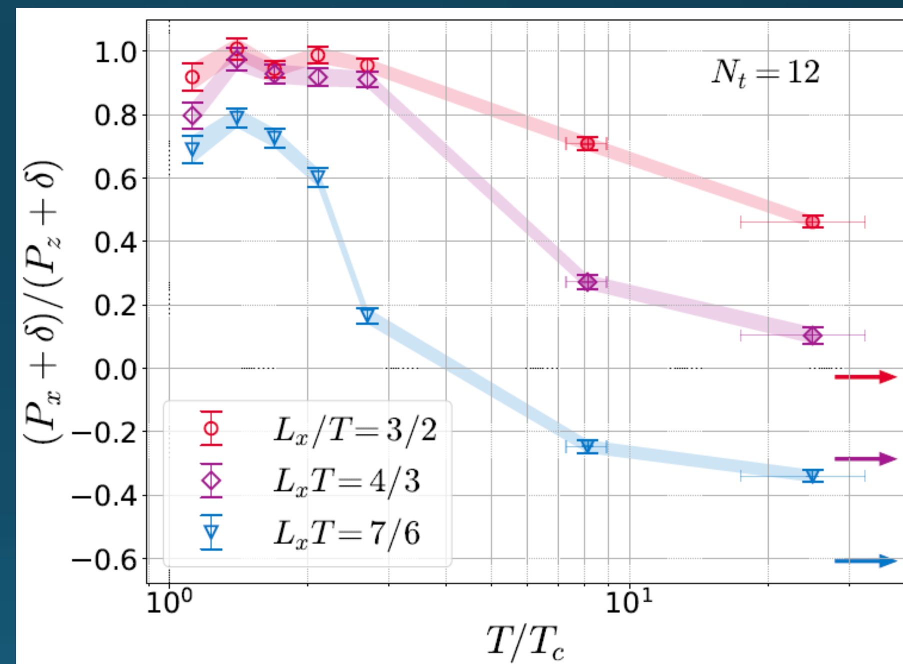
## We study

$$\frac{P_x + \delta}{P_z + \delta} \qquad \delta = -\frac{1}{4}\sum_\mu T^{\mathrm{E}}_{\mu\mu}$$

No vacuum subtr. nor Suzuki coeffs. necessary!

$$\frac{P_x + \delta}{P_z + \delta}$$




$T/T_c$≅8.1 (β=8.0) / $T/T_c$≅25 (β=9.0)

- ☐ Ratio approaches the asymptotic value.
- ☐ But, large deviation exists even at $T/T_c$~25.

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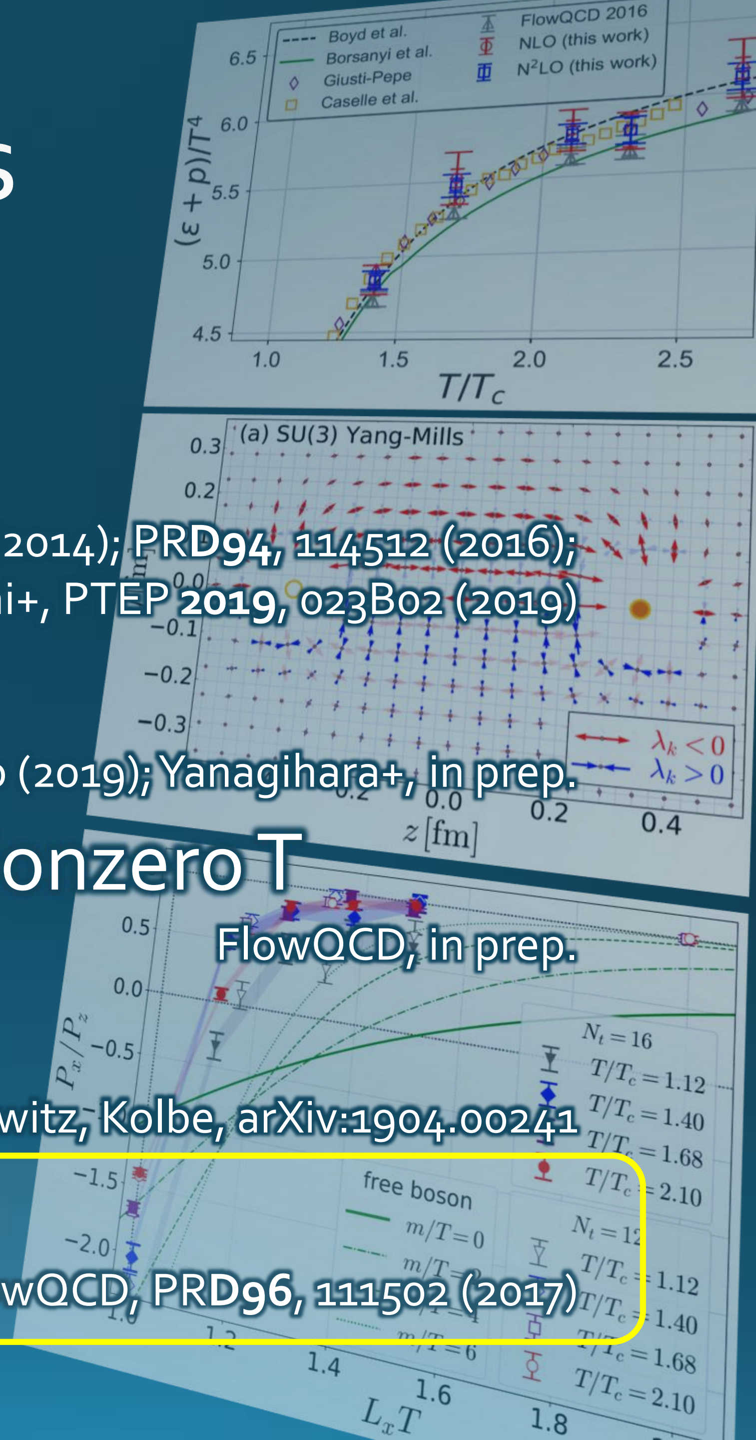


# EMT Correlator: Motivation

- Transport Coefficient

  Kubo formula → viscosity

$$\eta = \int_0^\infty dt \int_0^{1/T} d\tau \int d^3x \langle T_{12}(x, -i\tau) T_{12}(0,t) \rangle$$

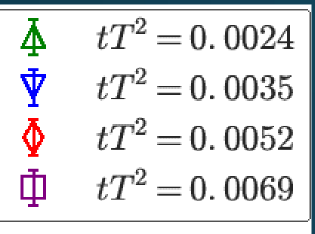
Karsch, Wyld, 1987
Nakamura, Sakai, 2005
Meyer; 2007, 2008
...
Borsanyi+, 2018
Astrakhantsev+, 2018

- Energy/Momentum Conservation

$$\langle \bar{T}_{0\mu}(\tau) \bar{T}_{\rho\sigma}(0) \rangle$$ : τ-independent constant

- Fluctuation-Response Relations

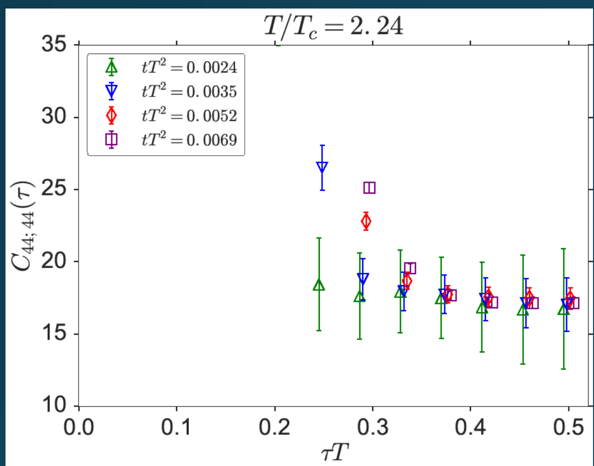
$$c_V = \frac{\langle \delta E^2 \rangle}{VT^2} \qquad E + p = \frac{\langle \bar{T}_{01}^2 \rangle}{VT} = \frac{\langle \bar{T}_{11} \bar{T}_{00} \rangle}{VT}$$

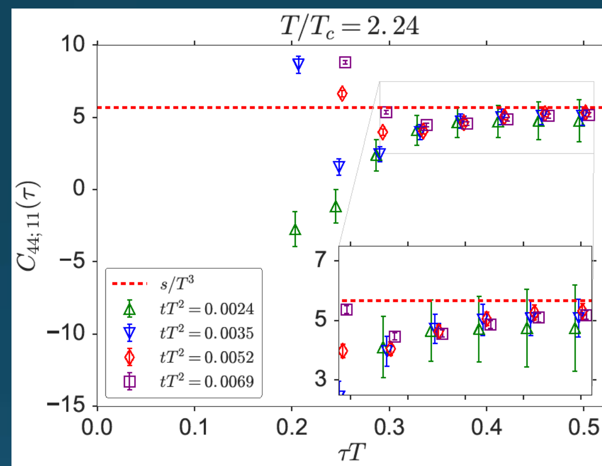


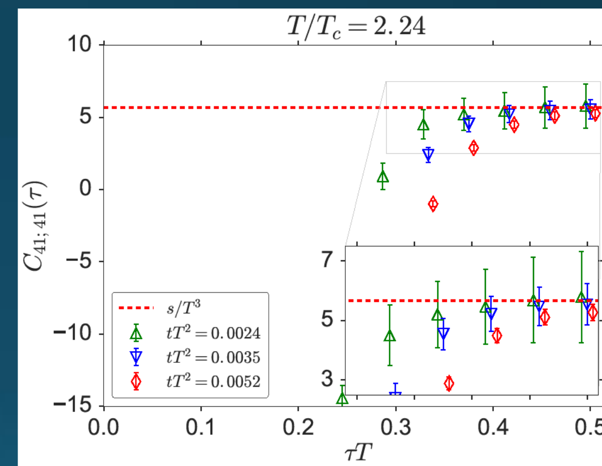

# EMT Euclidean Correlator

FlowQCD, PR **D96**, 111502 (2017)

$\langle \bar{T}_{44}(\tau)\bar{T}_{44}(0)\rangle$ $\langle \bar{T}_{44}(\tau)\bar{T}_{11}(0)\rangle$ $\langle \bar{T}_{41}(\tau)\bar{T}_{41}(0)\rangle$



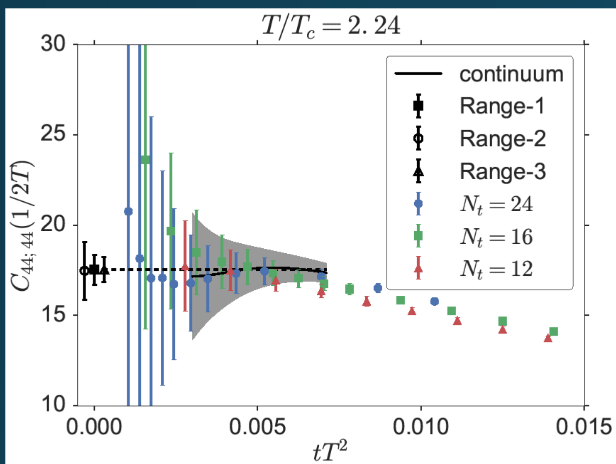




- ❑ τ-independent plateau in all channels ➔ conservation law
- ❑ Confirmation of fluctuation-response relations
- ❑ New method to measure $c_V$

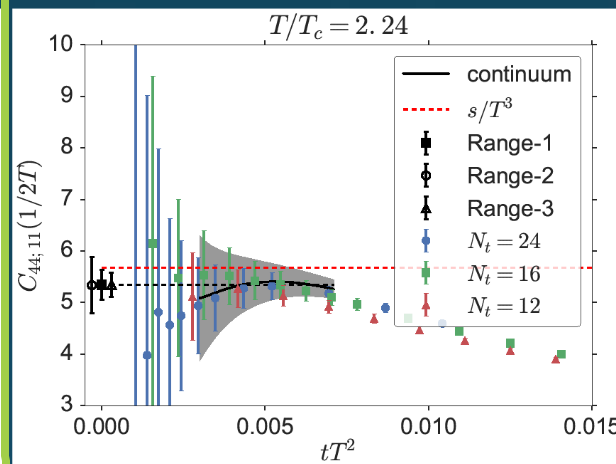
- ❑ Similar result for (41;41) channel: Borsanyi+, 2018
- ❑ Perturbative analysis: Eller, Moore, 2018

# Fluctuation-Response Relations

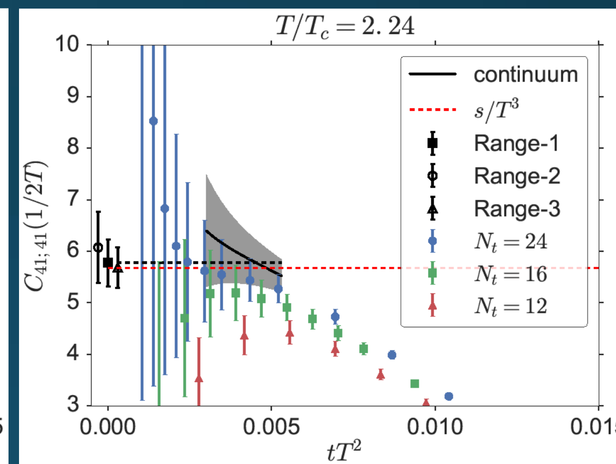
$\langle T_{44}(\tau) T_{44}(0) \rangle$




$\langle T_{44}(\tau) T_{11}(0) \rangle$




$\langle T_{41}(\tau) T_{41}(0) \rangle$




**New measurement of $c_V$**

$c_V/T^3$

| $T/T_c$ | $C_{44;44}(\tau_m)$ | Ref.[19] | Ref.[11] | ideal gas |
|---|---|---|---|---|
| 1.68 | $17.7(8)(^{+2.1}_{-0.4})$ | $22.8(7)^{*}$ | 17.7 | 21.06 |
| 2.24 | $17.5(0.8)(^{+0}_{-0.1})$ | $17.9(7)^{**}$ | 18.2 | 21.06 |

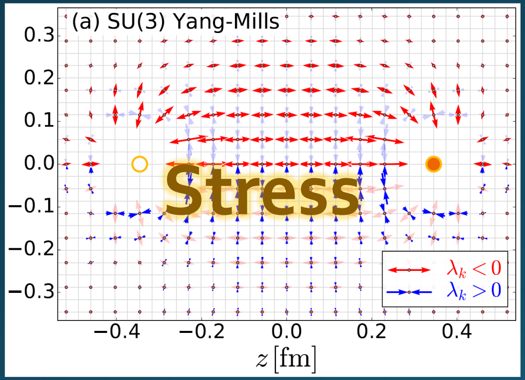
**Confirmation of FRR**

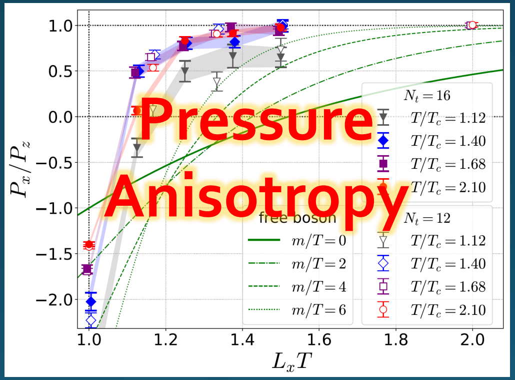
$$E + p = \frac{\langle \bar{T}_{01}^2 \rangle}{VT} = \frac{\langle \bar{T}_{11} \bar{T}_{00} \rangle}{VT}$$

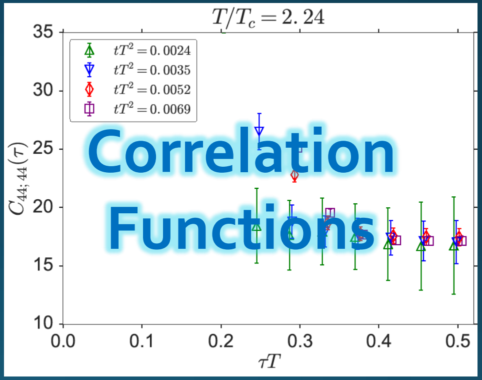
2+1 QCD:
Taniguchi+ (WHOT-QCD), 1711.02262

# Summary

- Successful analysis of energy-momentum tensor on the lattice is now available, and various stuides are ongoing!
  - gradient flow method
  - higher-order perturbative coefficients



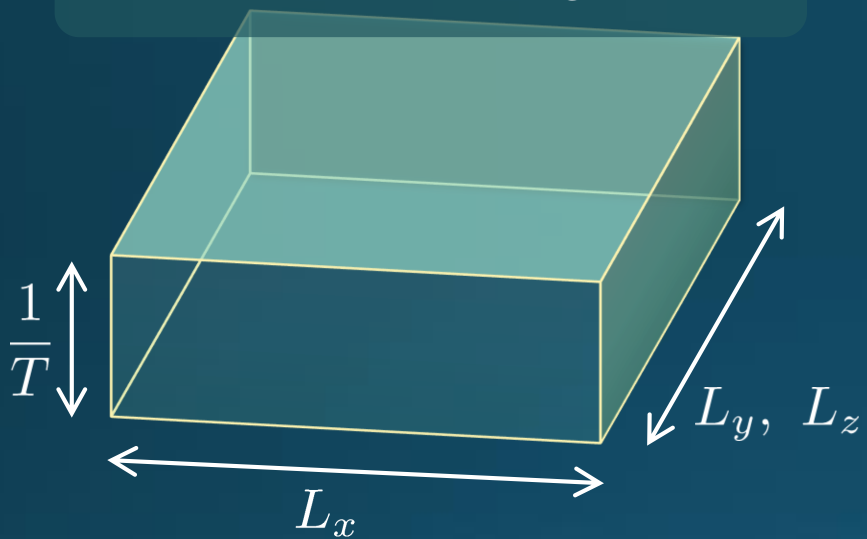




- **So many future studies**
  - Flux tube at **nonzero temperature**
  - EMT distribution **inside hadrons**
  - viscosity / other operators / instantons / **full QCD**

# backup

# Two Special Cases with PBC

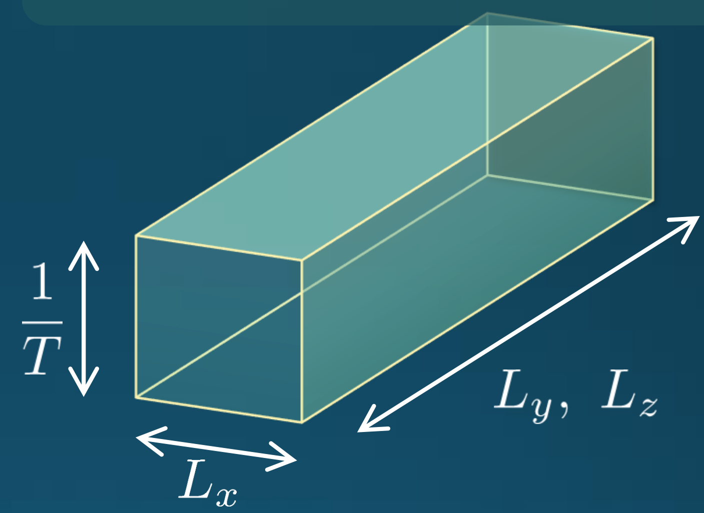
$$1/T \ll L_x = L_y = L_z$$




$$T_{11} = T_{22} = T_{33}$$

$$\frac{p_1}{p_2} = 1$$

$$1/T = L_x,\ L_y = L_z$$




$$T_{44} = T_{11},\ T_{22} = T_{33}$$

In conformal ($\Sigma_\mu T_{\mu\mu}=0$)

$$\frac{p_1}{p_2} = -1$$

# EMT on the Lattice: Conventional

**Lattice EMT Operator** Caracciolo+, 1990

$$T_{\mu\nu} = Z_6 T_{\mu\nu}^{[6]} + Z_3 T_{\mu\nu}^{[3]} + Z_1 \left( T_{\mu\nu}^{[1]} - \langle T_{\mu\nu}^{[1]} \rangle \right)$$

$$T_{\mu\nu}^{[6]} = (1-\delta_{\mu\nu}) F_{\mu\rho}^a F_{\nu\rho}^a, \;\; T_{\mu\nu}^{[3]} = \delta_{\mu\nu} \left( F_{\mu\rho}^a F_{\nu\rho}^a - \frac{1}{4} F_{\rho\sigma}^a F_{\rho\sigma}^a \right), \;\; T_{\mu\nu}^{[1]} = \delta_{\mu\nu} F_{\rho\sigma}^a F_{\rho\sigma}^a$$

- ☐ Fit to thermodynamics: $Z_3$, $Z_1$
- ☐ Shifted-boundary method: $Z_6$, $Z_3$ Giusti, Meyer, 2011; 2013; Giusti, Pepe, 2014~; Borsanyi+, 2018

**Multi-level algorithm**

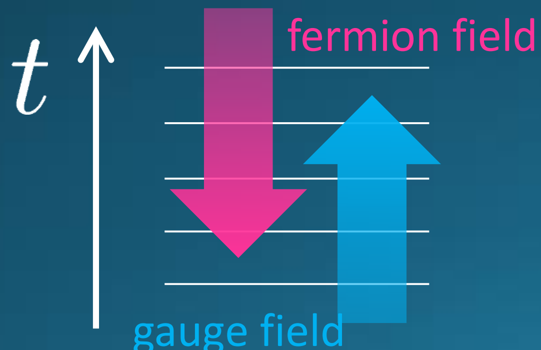
- ☐ effective in reducing statistical error of correlator Meyer, 2007; Borsanyi, 2018; Astrakhantsev+, 2018

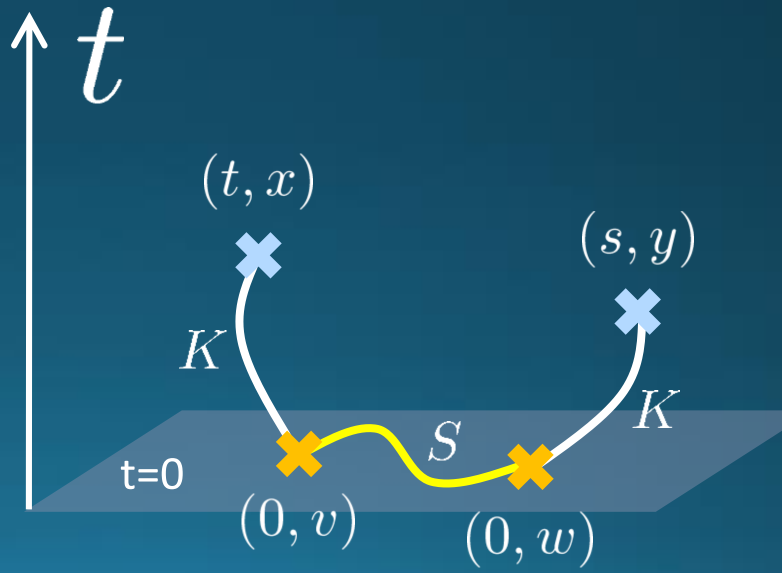
# Fermion Propagator

$$S(t,x;s,y) = \langle \chi(t,x)\bar{\chi}(s,y)\rangle$$
$$= \sum_{v,w} K(t,x;0,v)S(v,w)K(s,y;0,w)^\dagger$$

$$\Big(\partial_t - D_\mu D_\mu\Big)K(t,x) = 0$$

- propagator of flow equation
- Inverse propagator is needed



# $N_f$=2+1 QCD Thermodynamics

Taniguchi+ (WHOT-QCD), PRD**96**, 014509 (2017)

- $N_f$=2+1 QCD, Iwasaki gauge + NP-clover
- $m_{PS}/m_V \approx 0.63$ / almost physical s quark mass

- T=0: CP-PACS+JLQCD (ß=2.05, $28^3$x56, a≈0.07fm)
- T>0: $32^3$x$N_t$, $N_t$ = 4, 6, … , 14, 16):
- T≈174-697MeV

- t→0 extrapolation only (No continuum limit)