Deep Learning Topological Sector in Lattice Gauge Theory

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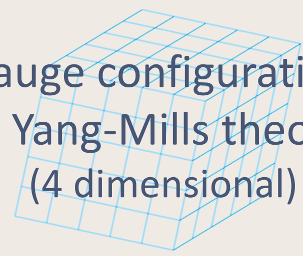
Topological Charge in Yang-Mills theory



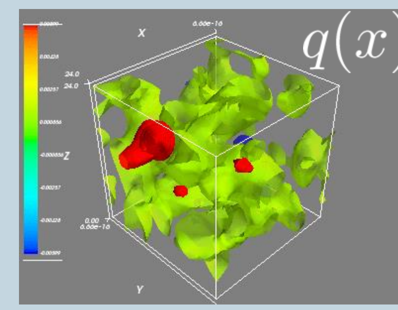
gauge configuration in Yang-Mills theory (4 dimensional) ⇨ Q integer number

- ☐ Classical picture: **instantons** (local objects in 4-dim. space)
- ☐ topology: $S_3 \to S_3$
- ☐ Various lattice definitions of Q
  - Fermionic: Atiyar-Singer index theorem
  - Gluonic: smearing, **gradient flow**

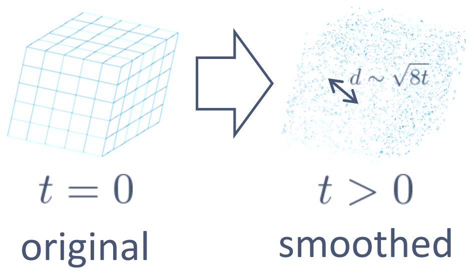
⇨ **Faster algorithm is desirable!**

$$Q = \int_V d^4x q(x)$$

⇧ topological charge density

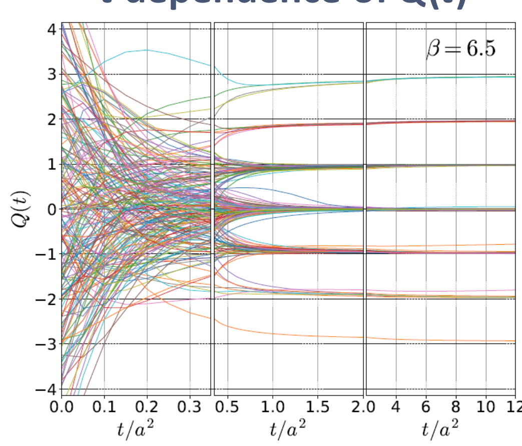


Gradient Flow =smoothing of gauge field



Suppression of quantum fluctuations at t>0.

t dependence of Q(t)



Questions

- ☐ **Can ML learn features in 4-dim space?**
- ☐ **Acceleration of the analysis of *Q*?**

Results

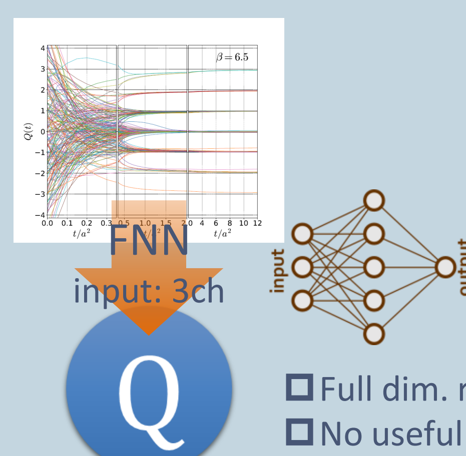
1. Learn $q_t(x)$ in 4D by 4-dim. CNN

Result (P: accuracy / R_Q: recall)

N_{ch}	input t/a^2	P	R_Q								
			-4	-3	-2	-1	0	1	2	3	4
1	0	0.388	0	0	0	0	1.000	0	0	0	0
1	0.1	0.396	0	0	0	0.086	0.889	0.129	0	0	0
1	0.2	0.479	0	0	0.108	0.445	0.641	0.459	0.150	0	0
1	0.3	0.698	0	0.170	0.585	0.730	0.727	0.701	0.624	0.395	0.071
3	0.3,0.2,0.1	0.953	0	0.830	0.951	0.956	0.952	0.962	0.968	0.953	0.286

- ☐ Better accuracy as t→large
- ☐ Combination of several t improves P

2. Learn Q(t) in 0D by simple FNN

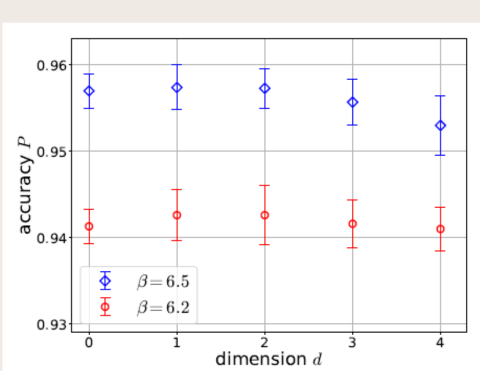


Result (accuracy)

input t/a^2	$\beta = 6.2$	$\beta = 6.5$
0.45, 0.4, 0.35	0.974(2)	0.998(1)
0.4, 0.35, 0.3	0.975(2)	0.997(1)
0.35, 0.3, 0.25	0.967(2)	0.996(1)
0.3, 0.25, 0.2	**0.959(2)**	**0.990(2)**
0.25, 0.2, 0.15	0.939(3)	0.951(2)
0.2, 0.15, 0.1	0.864(3)	0.831(5)
0.15, 0.1, 0.05	0.692(4)	0.647(8)
0.1, 0.05, 0	0.538(5)	0.499(6)

- ☐ Full dim. reduction gives better P.
- ☐ No useful feature in 4D found by CNN.
- ☐ **CNN in trained to look at Q(t).**

3. Dimensional Reduction to d=3, 2, 1



No d dep.

⇩

All cases may focus only on $Q(t)$

Summary

- ☐ By feeding $Q(t)$, ML can obtain a good accuracy.
- ☐ Data at t/a^2<0.3 gives 99% accuracy.
  - ➔ML can accelerate the analysis of Q.
- ☐ Analysis of 4D data does not improve the accuracy.
  - ➔ML may fail in finding features in 4-dim space.

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