Critical Fluctuation in a Dynamically Expanding Heavy-Ion Collisions

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Search for QCD Phase Structure

- Possible existence of
  - 1st order transition
  - QCD critical point

Diagram:
- Quark-Gluon Plasma
- Hadron Phase (confined)
- Color SC
- QCD Critical Point

Axes:
- T (temperature)
- μ (chemical potential)
- ~10^{15} g/cm^3
Search for QCD Phase Structure

Possible existence of
- 1st order transition
- QCD critical point

Beam-energy scan
- RHIC-BES-I 2010～
- RHIC-BES-II 2019～
- Future: FAIR, NICA, J-PARC-HI, HADES, ...
Event-by-Event Fluctuations

Cumulants

\[
\langle N^2 \rangle_c = \langle \delta N^2 \rangle = \sigma^2
\]

\[
\langle N^3 \rangle_c = S \sigma^3
\]

\[
\langle N^4 \rangle_c = \kappa \sigma^4
\]

General Review:
Asakawa, MK, PPNP (2016)
Event-by-Event Fluctuations

Net-proton number cumulants

- Non-zero non-Gaussian cumulants have been established experimentally!
- Are they the signal of the QCD-CP?
- Note: Baryon number cumulants are actually needed! MK, Asakawa, 2012;2012
Time Evolution of Fluctuations

Distributions in $\Delta Y$ and $\Delta y$ are different due to “thermal blurring”.

Ohnishi, MK, Asakawa, PRC(2016)

Fluctuations in $\Delta Y$ continue to change until kinetic f.o.
Evolution of Conserved-Charge Fluct.

Equations describing the transport of $n$:
Evolution of Conserved-Charge Fluct.

**Equations describing the transport of** $n$:

- **Diffusion Equation**
  \[
  \frac{\partial n}{\partial t} = D \nabla^2 n
  \]

- **Stochastic Diffusion Equation (SDE)**
  \[
  \frac{\partial n}{\partial t} = D \nabla^2 n + \nabla \xi(x, t) \quad \langle \xi(1) \xi(2) \rangle = 2D \chi_2 \delta(1 - 2)
  \]

- **SDE with non-linear terms**
  \[
  \frac{\partial n}{\partial t} = \kappa \nabla^2 \frac{\delta \mathcal{F}}{\delta n} + \frac{\partial}{\partial x} \xi(x, t)
  \]

  \[
  \mathcal{F} = \int dx \left( a \Delta n^2 + c (\nabla n)^2 + \lambda_3 \Delta n^3 + \cdots \right)
  \]
Fluctuations of \( \sigma \) and \( n_B \) are coupled around the CP!

\[
\delta \sigma \simeq \delta n_B
\]

To a first approximation, SDE describes the soft mode of the CP.

Coupling to \( \sigma \) & \( T_{\mu\nu} \) has to be included for more accurate description.
1. **CrossOver with SDE (Gaussian)**
   - 2nd cumulant/correlation func.
     Sakaida, Asakawa, Fujii, MK, PRC95 (2017)

2. **CrossOver with Non-Linear SDE**
   - higher-order cumulants
     Nahrgang, Bluhm, Schaefer, Bass PRD99 (2019);
     Pihan, Touroux, Nahrgang, Bluhm, Sami, MK, in prep.

3. **1st-Order with Non-Linear SDE**
   Nonaka, MK, et al., in prep.
Stochastic Diffusion Equation (Gaussian)

\[ \partial_t n = D(t) \partial_x^2 n + \partial_x \xi \]

\[ \langle \xi(x_1, t_1) \xi(x_2, t_2) \rangle = 2D \chi_2 \delta^{(2)}(1 - 2) \]

- \( D(t), \chi_2(t) \): parameters characterizing evolution of the medium

☐ Analytic solution is obtained.
☐ Study 2\textsuperscript{nd} order cumulant & correlation function.
Evolution of Parameters

Critical behavior
- 3D Ising ($r,H$)
- model $H$

Berdnikov, Rajagopal (2000)
Stephanov (2011); Mukherjee+ (2015)

Temperature dependence
- $D(T)$
  - critical slowing down
- $\chi(T)$
  - critical enhancement

$T > 0, r = 0$ (critical point)
$T_0 = 220 \text{ [MeV]}$
$T_c = 160 \text{ [MeV]}$
$T_r = 100 \text{ [MeV]}$
STAR (2014)
Time Evolution

Cumulant: $K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{eq}$

Corr. Func.: $C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_H$

Non-monotonic rapidity dependence

Analytic property

$C(\Delta y)$ non-monotonic $\Rightarrow$ $\chi(\tau)$ non-monotonic

See also, Kapusta, Torres-Rincon, PRC86 (2012); Wu, Song, Chin. Phys. C43 (2019)
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Introducing Non-Linear Terms

\[
\frac{\partial n}{\partial t} = \kappa \nabla^2 \frac{\delta F}{\delta n} + \frac{\partial}{\partial x} \xi(x, t)
\]

\[
F[n] = T \int d^3r \left( \frac{m^2}{2n_c^2} \Delta n^2 + \frac{K}{2n_c^2} (\nabla n)^2 + \frac{\lambda_3}{3n_c^3} \Delta n^3 + \frac{\lambda_4}{4n_c^4} \Delta n^4 + \frac{\lambda_6}{6n_c^6} \Delta n^6 \right)
\]

- Diffusive dynamics of **higher order** cumulants can be described.
- No analytic solution. Need numerical analysis.
- Parameters: \( \kappa, m, K, \lambda_3, \lambda_4, \lambda_6 \)
  - Hubble expansion, Ising universality
Cumulants in Equilibrium

2nd: $\sigma^2$

3rd: $S_\sigma$

4th: $\kappa \sigma^2$

- Simulation with fixed $T$.
- Spatial length $L=20$fm
- Weaker criticality due to the finite volume effects
- Shape of $S_\sigma$ can be explained by the finite volume effects

M. Agah Nouhou+, arXiv:1906.02647; Bluhm, SQM2019
Evolution with Bjorken Expansion

Milne coordinates

\[ \partial_{\tau} n = \frac{\kappa(t)}{\tau^2} \partial_y^2 \frac{\delta F}{\delta n} + \frac{1}{\tau} \partial_y \xi - \frac{n}{\tau} \]

- Critical Point: T=150 MeV, \( \mu = 390 \) MeV
- Initial temperature: T=200 MeV
- \( \mu = 50, 200, 300, 350 \) MeV
- Cumulants on a single cell
- Compare results with & without NL terms

Evolution of \( C(y, \tau) \)

(Gaussian)
Numerical Result

- Gaussian (without nonlinear terms)
  - $\langle N_B^2 \rangle_c$

- Full Non-linear model
  - $\langle N_B^2 \rangle_c$
  - $\langle N_B^3 \rangle_c$
  - $\langle N_B^4 \rangle_c$

- 2nd cumulant in Gaussian model has a peak at the CP.
- But, this behavior is washed out by the effect of the non-linear terms.
- Need further investigation.

Pihan, Touroux, Nahrgang, Bluhm, Sami, MK, in prep.
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1st-Order Transition

- Domain formation
- Non-uniform system

Herold, Nahrgang, et al. (2011~); Steinheimer, Randrup (2012; 2013)
Free Energy

- **At 1st transition point**

\[ f(n) = \frac{1}{2}a(n - n_s)^2 + \frac{1}{4}b(n - n_s)^4 + c(\tau)n + k(\partial_Y n)^2 \]

- **Large and small n**

\[ \chi(n) = \frac{\partial^2 f}{\partial n^2} \rightarrow \chi_{\text{QGP}} \ (n \rightarrow \infty) \]
\[ \rightarrow \chi_{\text{hadron}} \ (n \rightarrow 0) \]

\[ \text{Poisson} \]

- **κ**: positive
- adjust κ and A to reproduce the behavior of D at small and large n

\[ \tilde{D} = \Gamma\left(\frac{\partial^2 f}{\partial n^2} + X\right) \quad A = 2D\chi_2 \]
Dynamical domain formation
- Domains survive even after 1\textsuperscript{st} transition
Correlation Function

\[ C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}} \]

- Domain leads to a peak structure in \( C(y) \).
- The peak can survive even in the final state.
Diffusive dynamics is important in describing fluctuations in heavy-ion collisions. We studied dynamical evolution near the QCD-CP and at the 1st transition in stochastic diffusion equation with and without non-linear terms.

Future: coupling with sigma & momentum / more realistic space-time evolution