Critical Fluctuations in Heavy-Ion Collisions

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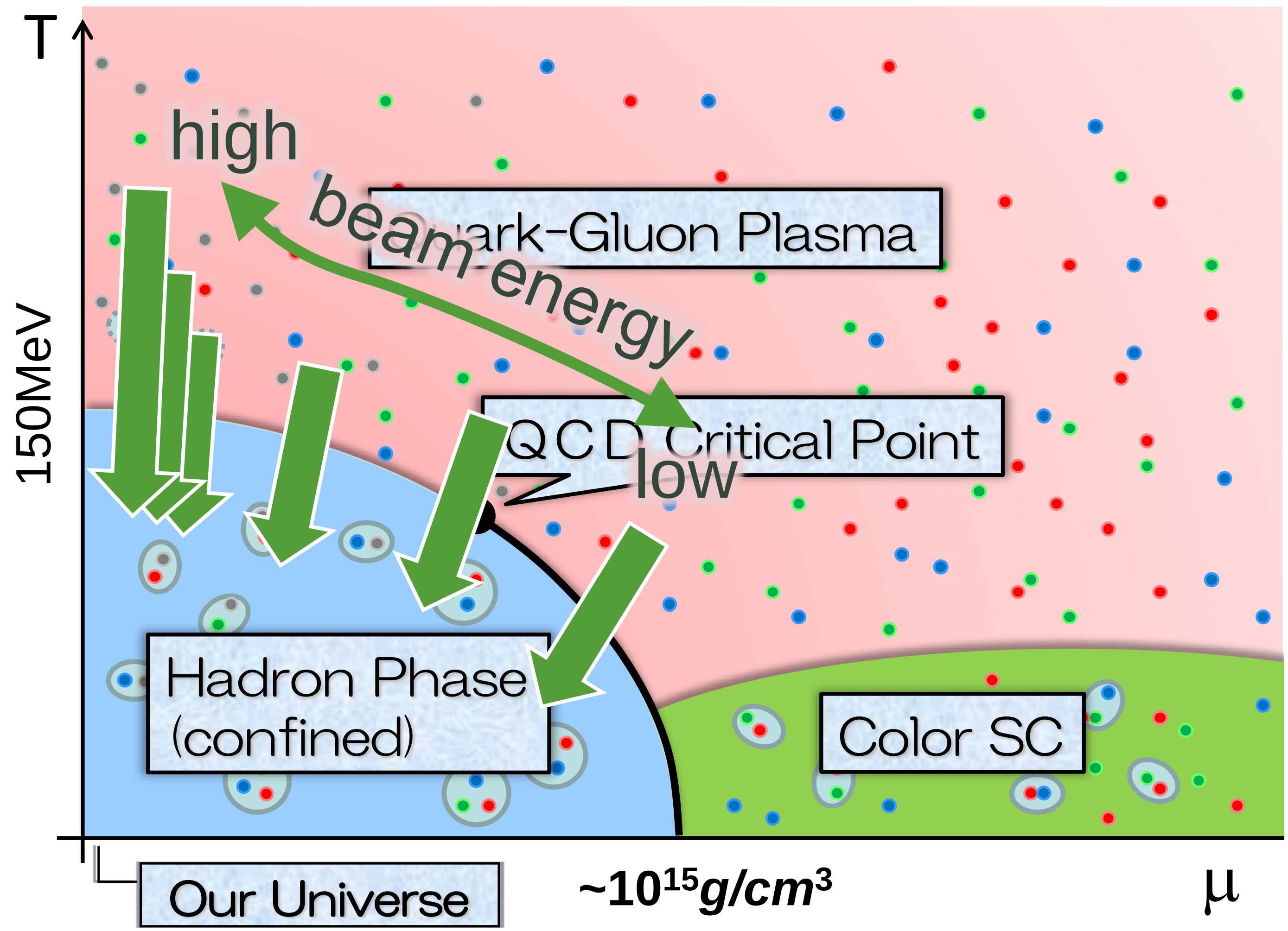
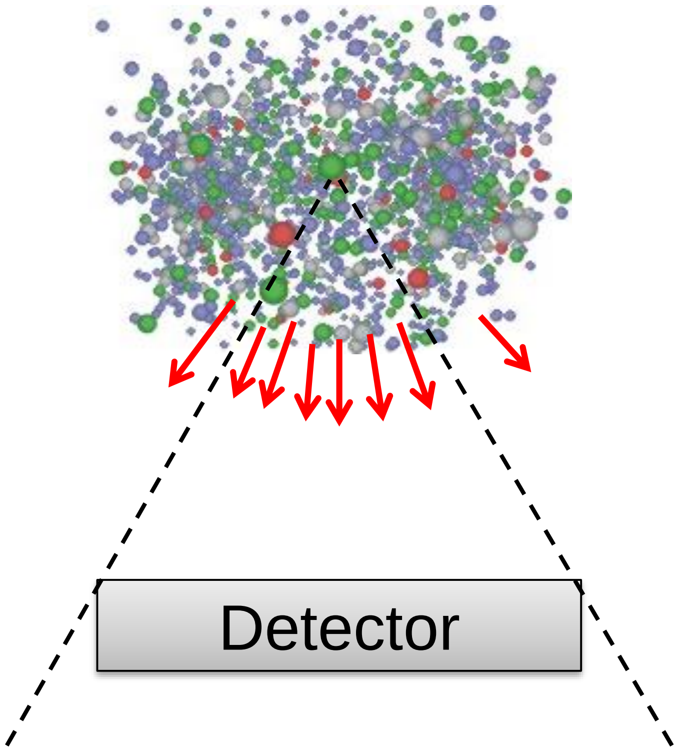
Workshop on QCD in the Nonperturbative Regime
TIFR, Mumbai, India, 18/Nov./2019

Beam-Energy Scan Program in Heavy-Ion Collisions

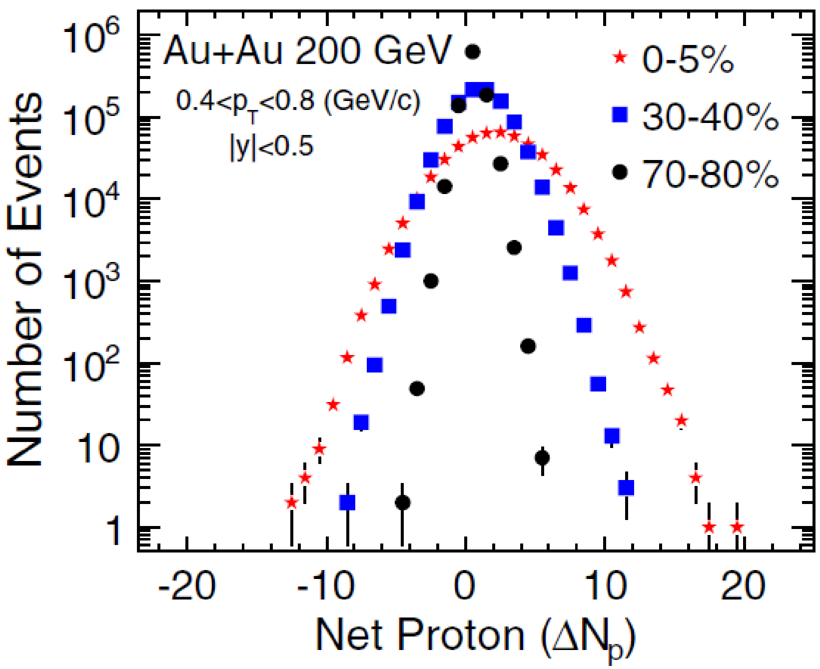


Event-by-Event Fluctuations

Review: Asakawa, MK, PPNP **90** (2016)



STAR, PRL**105** (2010)



Structure of distribution reflects microscopic properties

Cumulants: $\langle\delta N_p^2\rangle, \langle\delta N_p^3\rangle, \langle\delta N_p^4\rangle_c$

A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro

B. 25 x 2 Euro

Same expectation value.

A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro

B. 25 x 2 Euro

C. 1 x 50 Euro

Same expectation value.
But, different fluctuation.

Fluctuations in HIC: 2nd Order

Search for QCD CP

Fluctuation
increases

Stephanov, Rajagopal, Shuryak, 1998; 1999

Onset of QGP

Fluctuation
decreases

Asakawa, Heinz, Muller, 2000;
Jeon, Koch, 2000

Higher-order Cumulants

A. 50 x 1 Euro

B. 25 x 2 Euro

$2\langle \delta€^2 \rangle$  $=$ $\langle \delta€^2 \rangle$ 

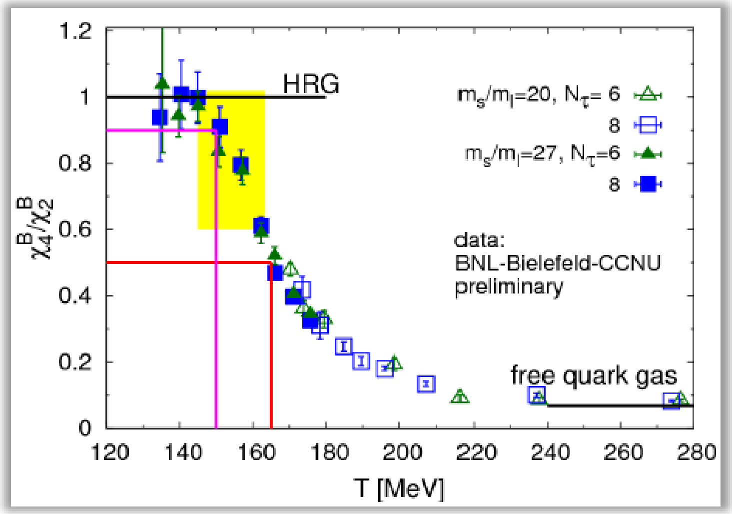
$4\langle \delta€^3 \rangle$  $=$ $\langle \delta€^3 \rangle$ 

$8\langle €^4 \rangle_c$  $=$ $\langle €^4 \rangle_c$ 

Asakawa, MK,
PPNP 90, 299
(2016)

Non-Gaussian Fluctuations

Onset of QGP



Fluctuation
decreases

Ejiri, Karsch, Redlich, 2006

Search for QCD CP

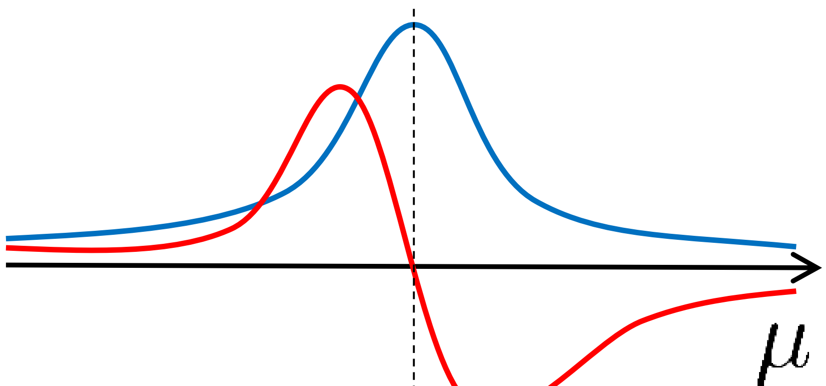
Fluctuation
increases

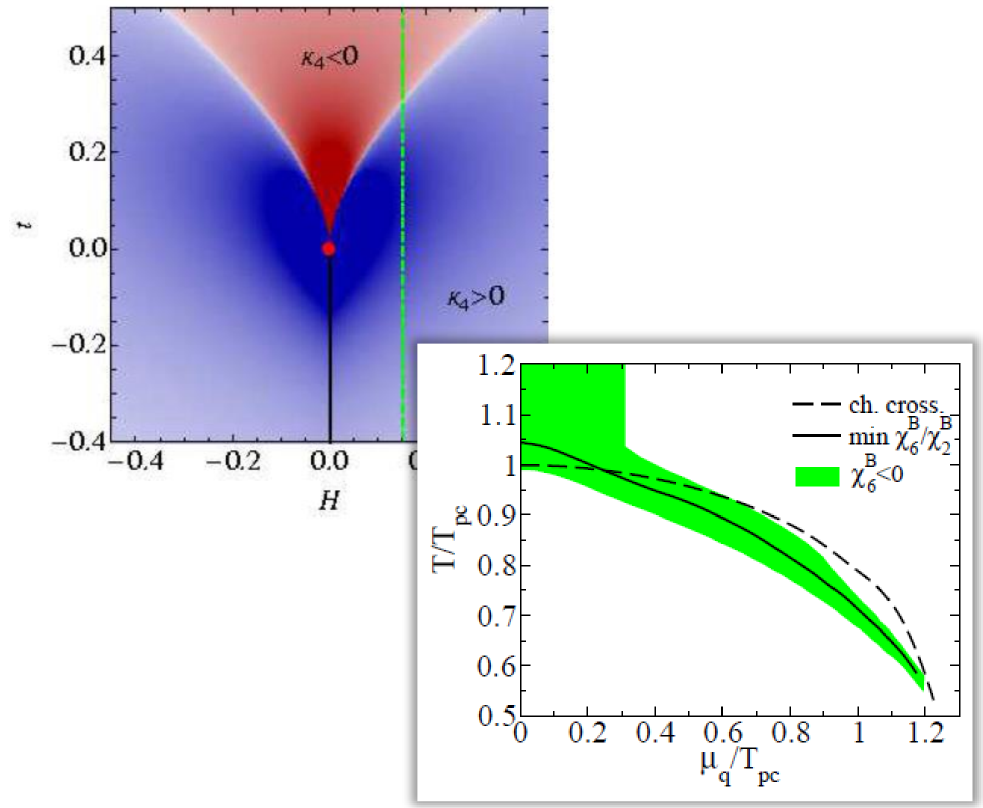
Stephanov, 2009

Sign of Higher-order Cumulants

Higher order cumulants can change sign near CP.

$$\langle \delta N^3 \rangle = T \frac{\partial \langle \delta N^2 \rangle}{\partial \mu}$$





Asakawa, Ejiri, MK, 2009

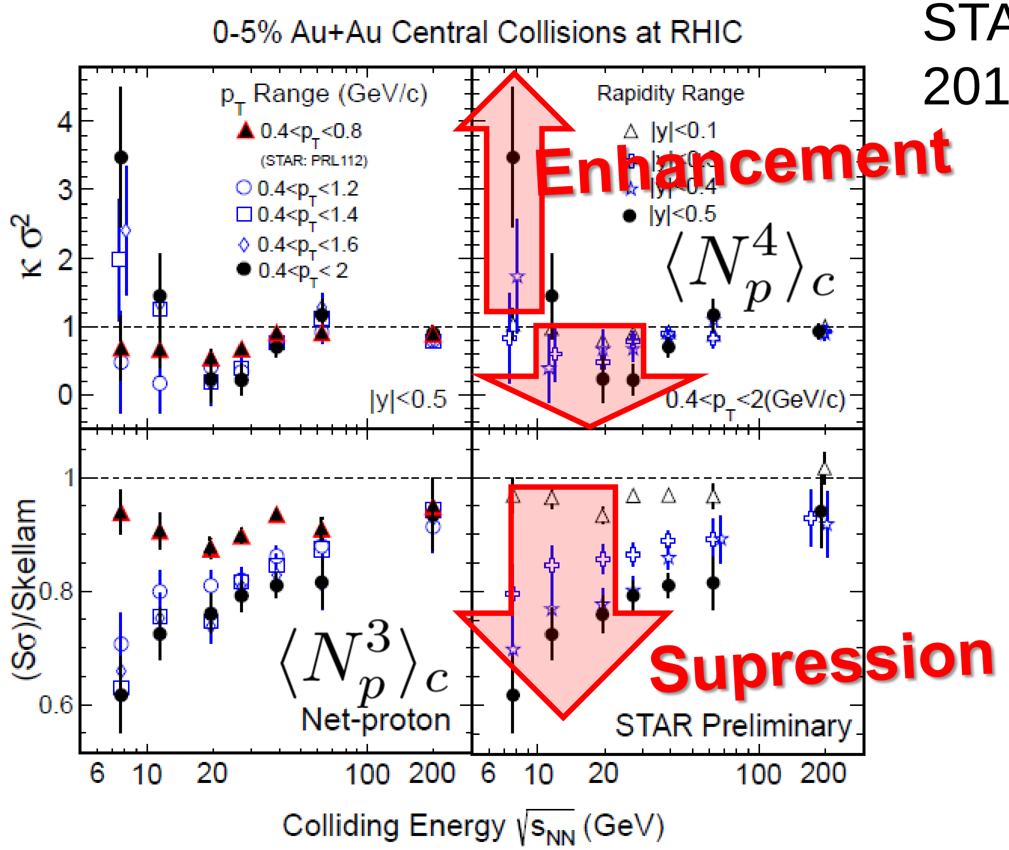
Stephanov, 2011;
Friman, Karsch, Redlich, Skokov, 2011; …

Higher-Order Cumulants

STAR
2010~

$$\frac{\langle N_p^4 \rangle_c}{\langle N_p^2 \rangle_c}$$

$$\frac{\langle N_p^3 \rangle_c}{\langle N_p^2 \rangle_c}$$

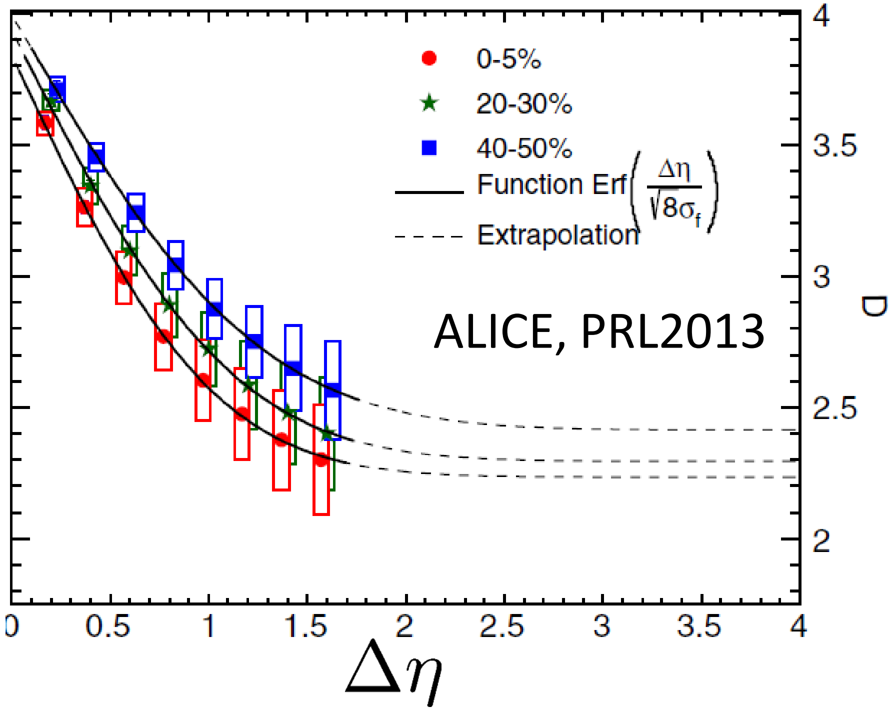


Non-zero **non**-Gaussian cumulants have been established!

General Review:
Asakawa, MK, PPNP (2016)

2nd Order @ ALICE

Net charge fluctuation

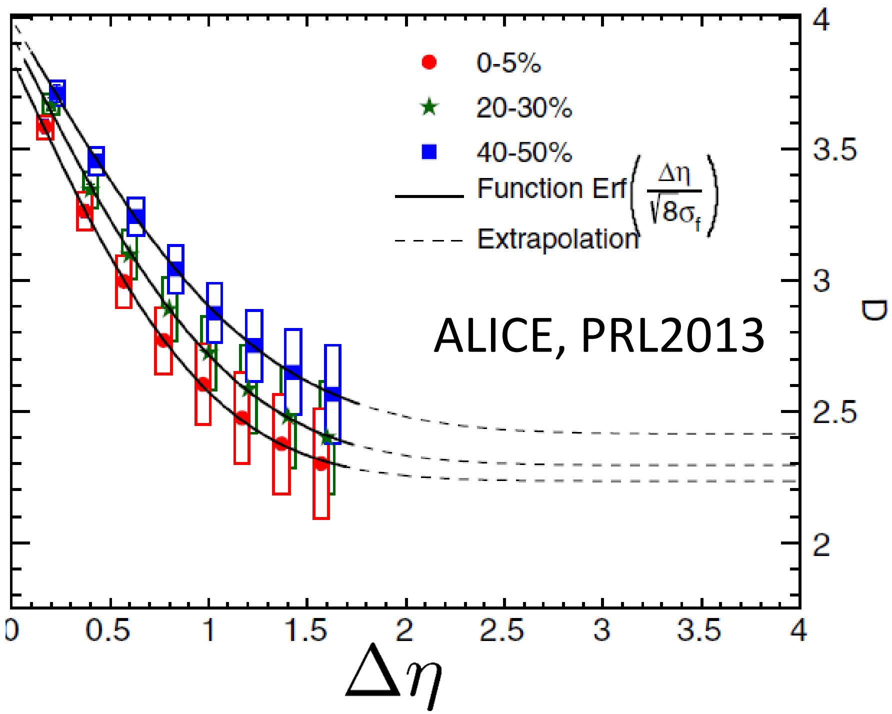


D-measure

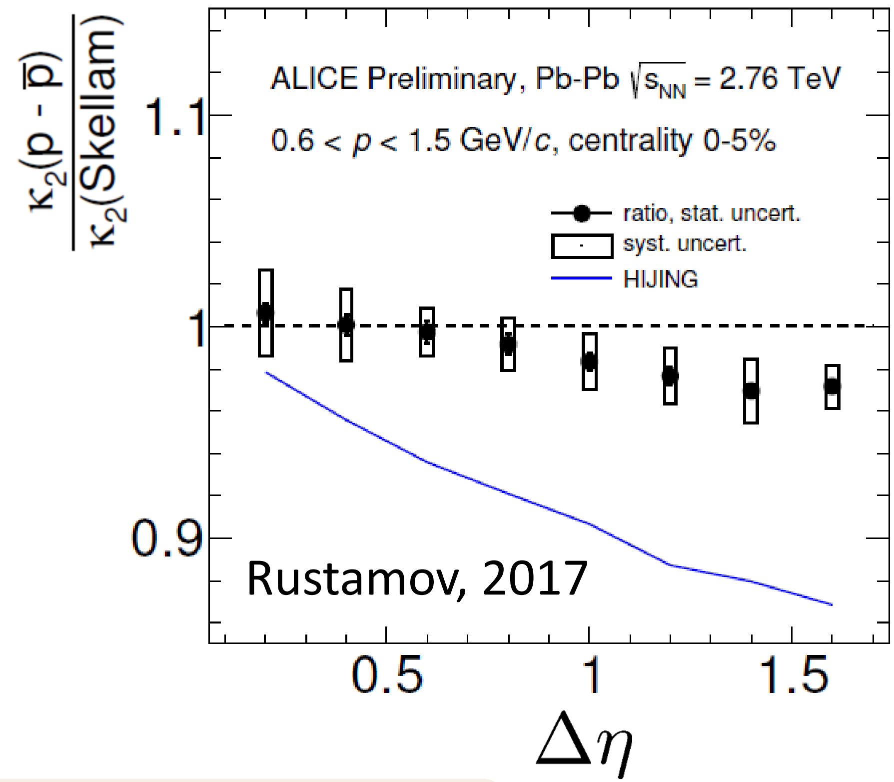
$$D \simeq 4\frac{\langle \delta N_Q^2 \rangle}{\langle \delta N_Q^2 \rangle_{\mathrm{HRG}}}$$

2nd Order @ ALICE

Net charge fluctuation



Net proton fluctuation

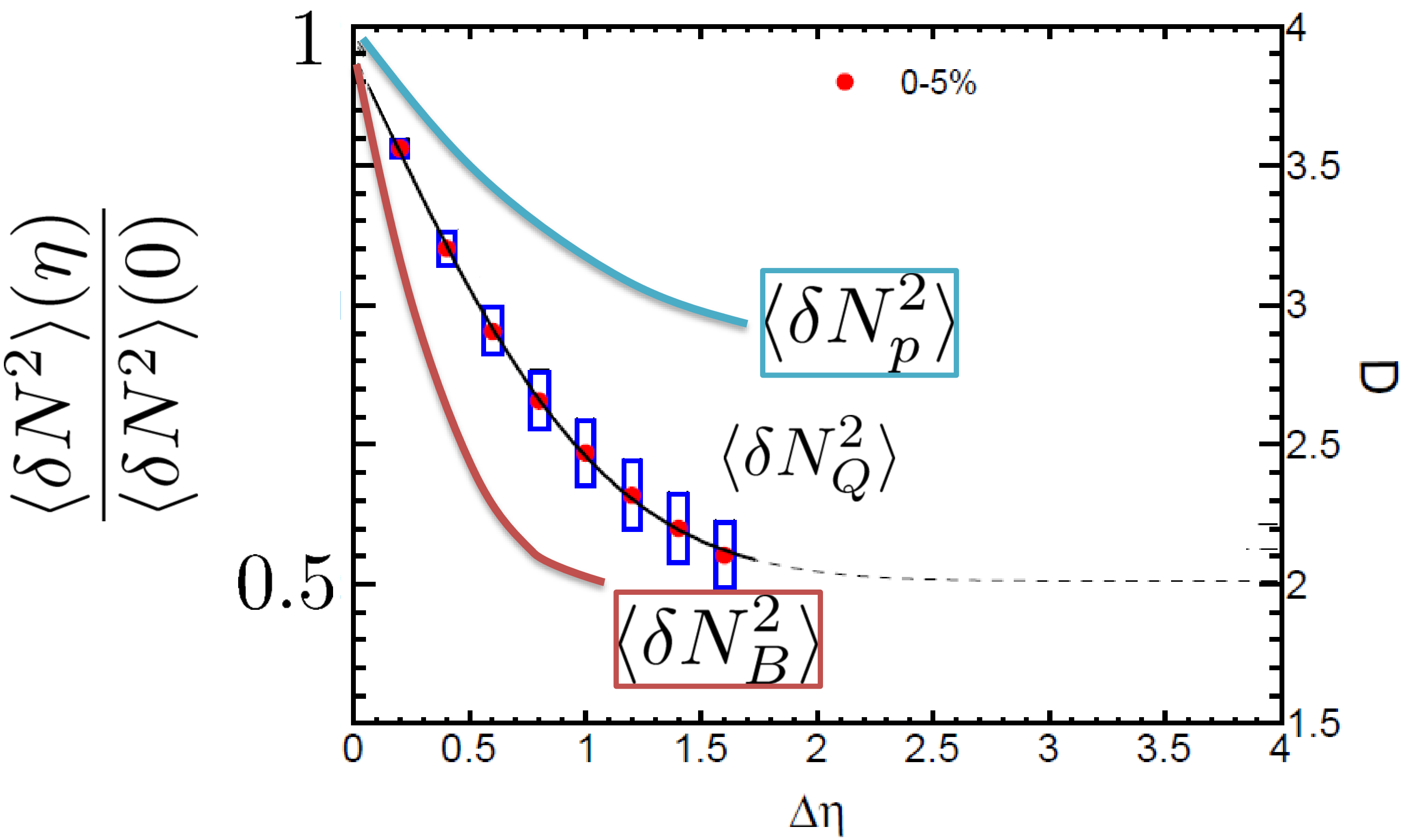


- Net-charge fluctuation has a suppression,
- but net-proton fluctuation does not. Why??

$\langle\delta N_B^2\rangle$ and $\langle\delta N_p^2\rangle$ @ LHC ?

MK, presentations
GSI, Jan. 2013
Berkeley, Sep. 2014
FIAS, Jul. 2015
GSI, Jan. 2016
…

$\langle\delta N_Q^2\rangle, \langle\delta N_B^2\rangle, \langle\delta N_p^2\rangle$
should have different $\Delta\eta$ dependence.



Baryon # cumulants are experimentally observable! MK, Asakawa, 2012

Message

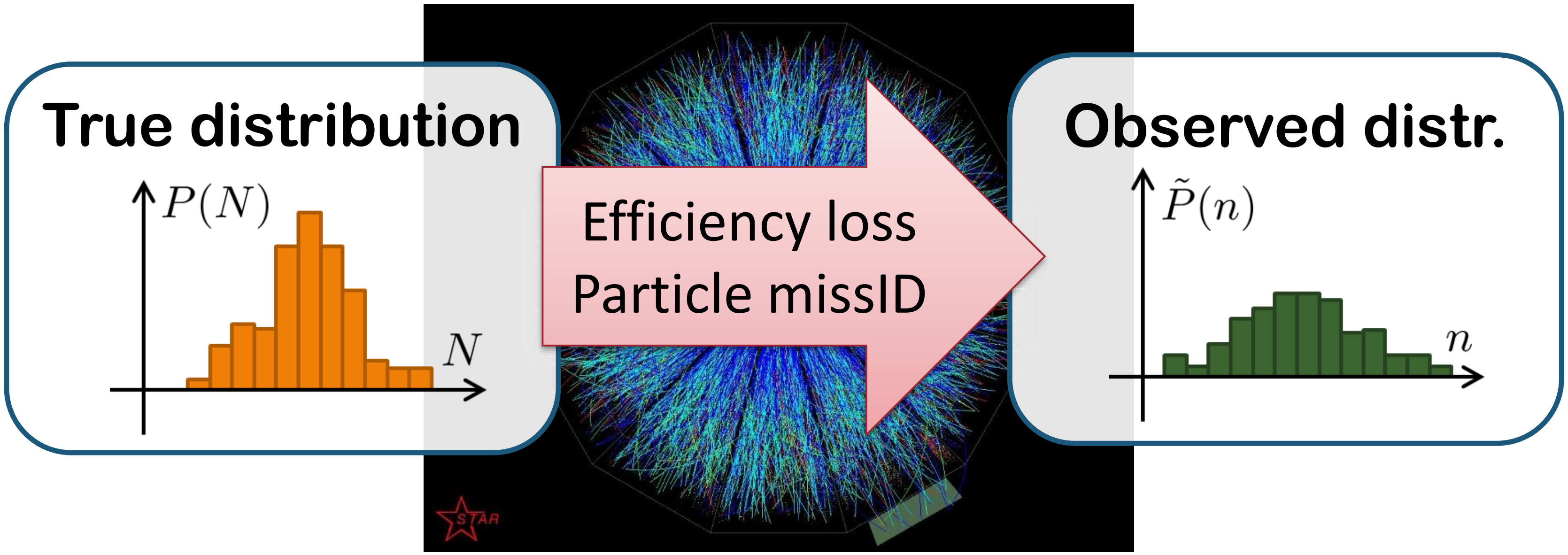
Understand 2^{nd}-order fluctuations
@ LHC & top-RHIC

1. Problems in experimental analysis
   - proper correction of detector's property

1. Dynamics of non-Gaussian fluctuations

2. A suggestion: chiB/chiQ

Detector-Response Correction

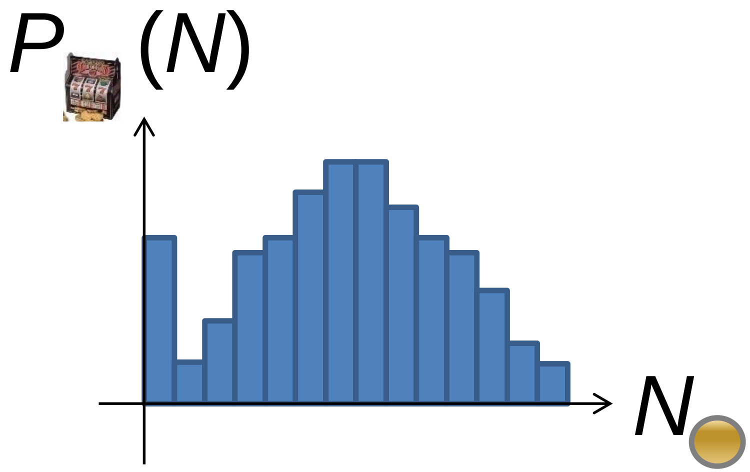


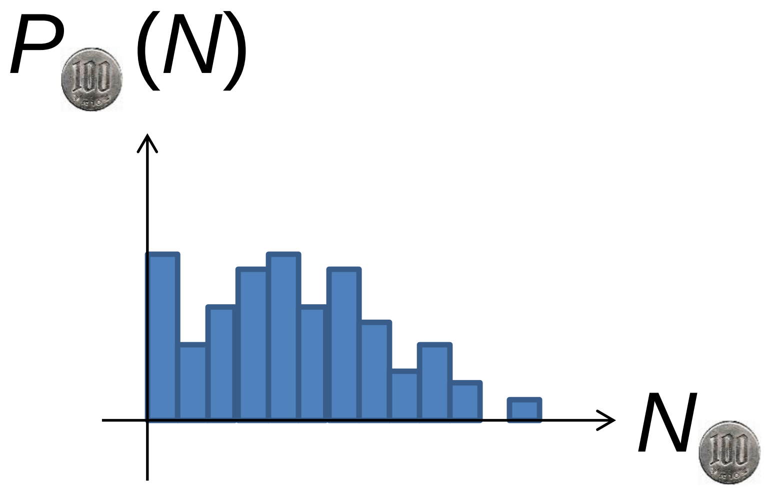
- **Correction assuming a binomial response**

  Bialas, Peschanski (1986);
  MK, Asakawa (2012); Bzdak, Koch (2012); ....

  But, the response of the detector is not binomial...

Slot Machine Analogy

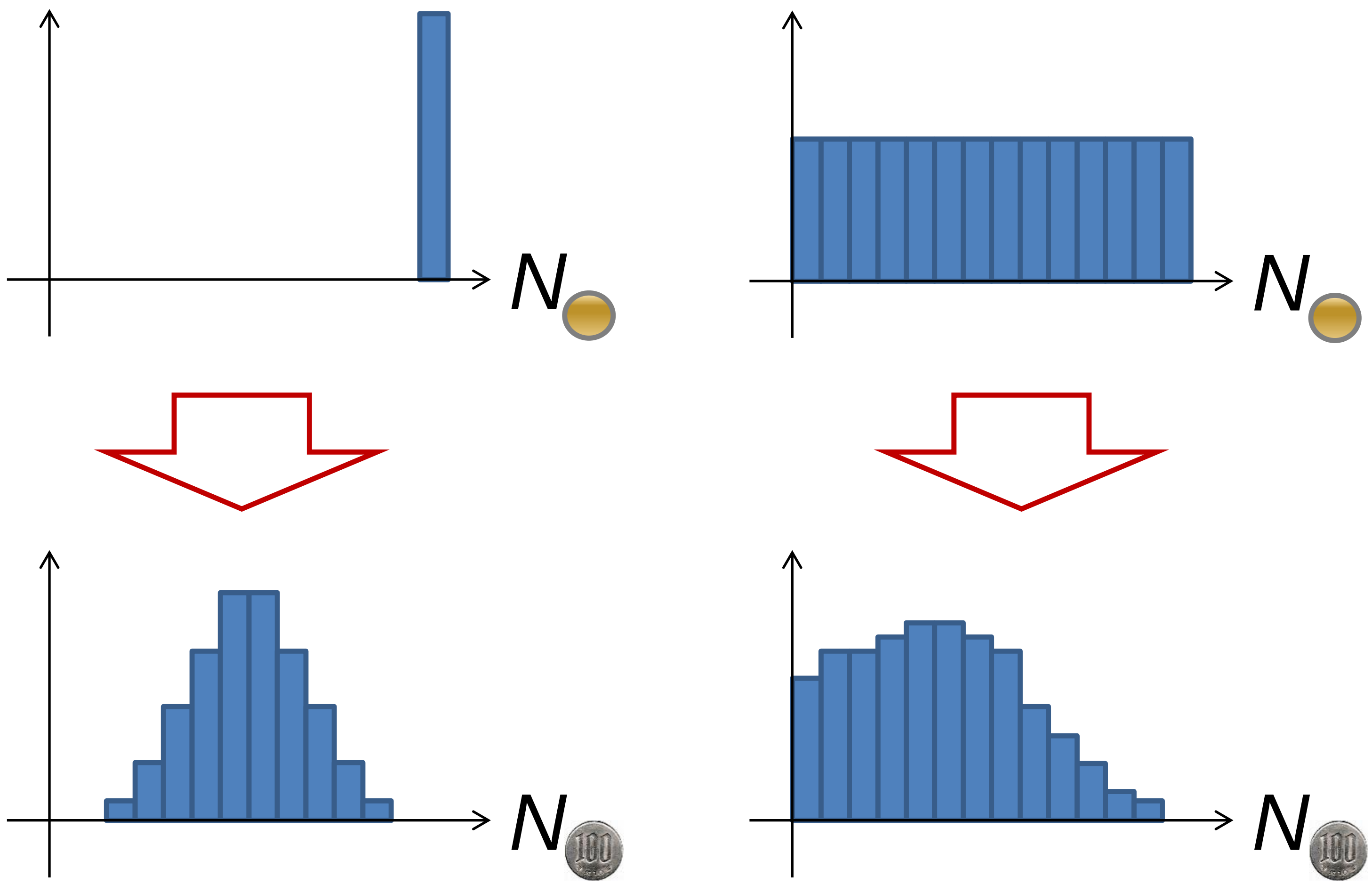




Extreme Examples

Fixed # of coins

Constant probabilities



Reconstructing Total Coin Number

$$P_{\,}(N_{\,}) = \sum_{\,} P_{\,}(N_{\,})B_{1/2}(N_{\,};N_{\,})$$

$$B_p(k;N) = p^k(1-p)^{N-k}\ {}_kC_N$$:binomial distr. func.

Proton vs Baryon Cumulants

MK, Asakawa, 2012; 2012

Experiments
proton number cumulants
$$\langle N_p^n \rangle_c$$

Many theories
baryon number cumulants
$$\langle N_{\rm B}^n \rangle_c$$

measurement with
50% efficiency loss

- Clear difference b/w these cumulants.
- **Isospin randomization** justifies the reconstruction of $\langle N_B^n \rangle_c$ via the binomial model.
- Similar problem on the **momentum cut**…

Fragile Higher Orders

Ex.: Relation b/w baryon & proton # cumulants

(with approximations)

MK, Asakawa, 2012

$$
\left\{
\begin{aligned}
2\langle(\delta N_p^{(\mathrm{net})})^2\rangle &= \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^3\rangle &= \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c &= \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots
\end{aligned}
\right.
$$

genuine info. Poisson noise

Higher orders are more seriously affected by efficiency loss.

Non-Binomial Correction

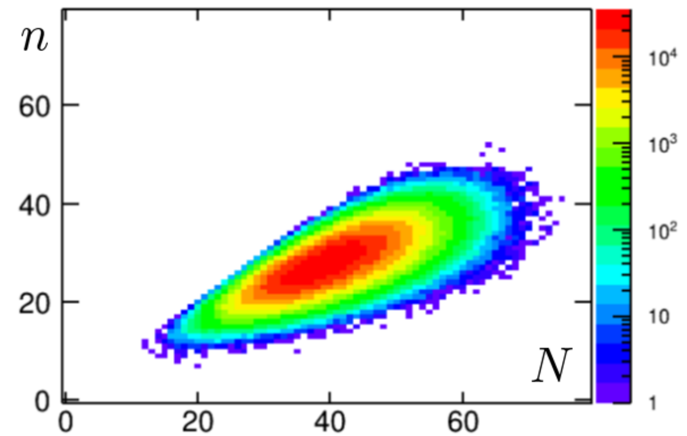
Nonaka, MK, Esumi (2018)

- Response matrix

$$\tilde{P}(n) = \sum_N \mathcal{R}(n;N)P(N)$$

Reconstruction for any R(n;N)
with moments of R(n;N)

$$\langle n^m \rangle_R = \sum_n n^m \mathcal{R}(n;N)$$



- Caveats:
  - R(n;N) describes the property of the detector.
  - Detailed properties of the detector have to be known.
  - Multi-distribution function can be handled.
  - Huge numerical cost would be required.
  - Truncation is required in general: another systematics?

Result in a Toy-Model

Binomial w/ multiplicity-dependent efficiency

$$\epsilon(N) = \epsilon_0 + (N - N_{\text{ave}})\epsilon'$$

Holtzman, Bzdak, Koch (16)

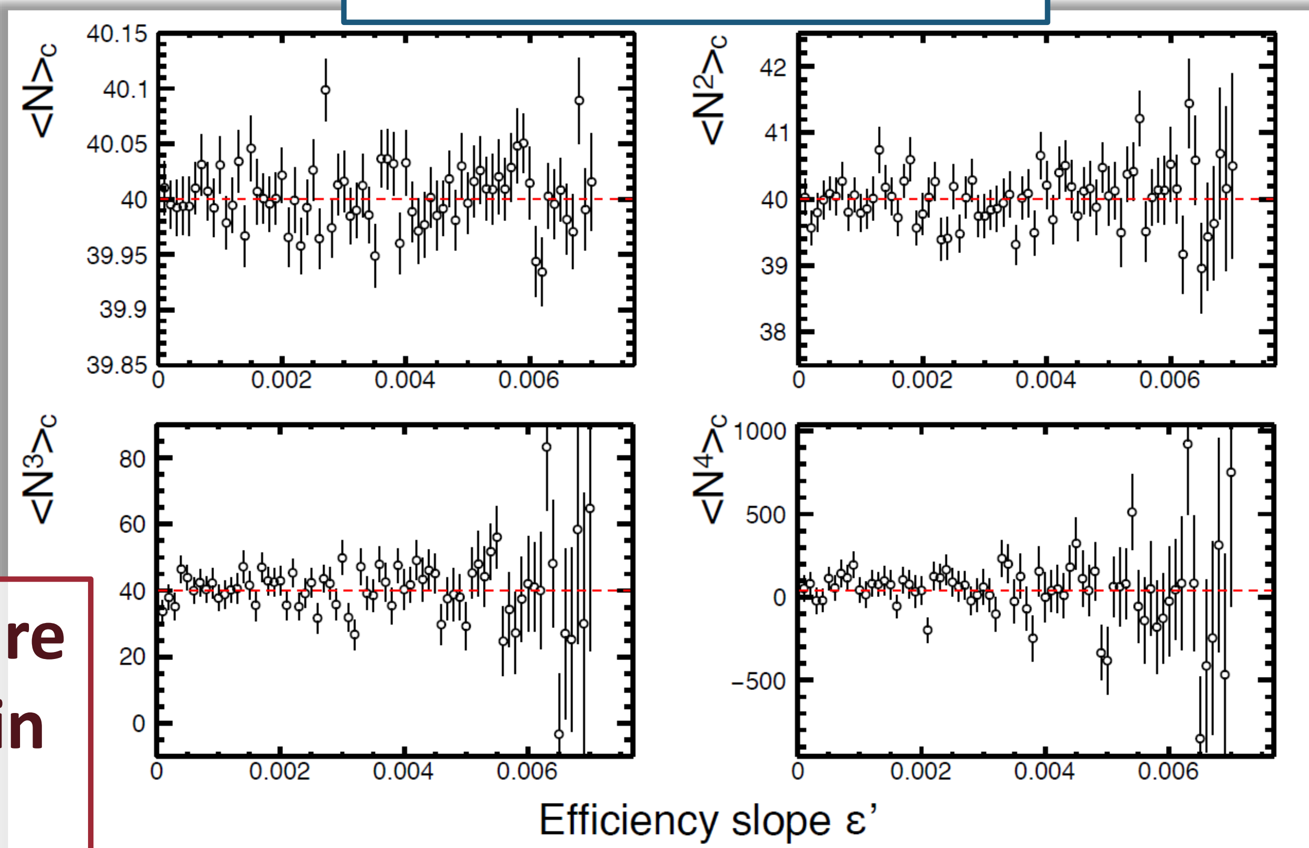
Nonaka, MK, Esumi (2018)

Input P(N): Poisson(λ=40)

$\epsilon_0 = 0.7$

Red: true cumulant

Reconstructed cumulants



True cumulants are reproduced within statistics!

Message

Understand 2nd-order fluctuations
@ LHC & top-RHIC

1. Problems in experimental analysis
   - proper correction of detector's property

1. Dynamics of non-Gaussian fluctuations

2. A suggestion: chiB/chiQ

Why Conserved Charges?

- Direct comparison with theory / lattice
  - Strong constraint from lattice
  - Ignorance on spatial volume of medium
- Slow time evolution

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- Direct comparison with theory / lattice
  - Strong constraint from lattice
  - Ignorance on spatial volume of medium
- Slow time evolution

AHM-JK (2000)

D-measure

$$D \sim \frac{\langle \delta N_{\mathrm{Q}}^2 \rangle}{S}$$

S is model dependent

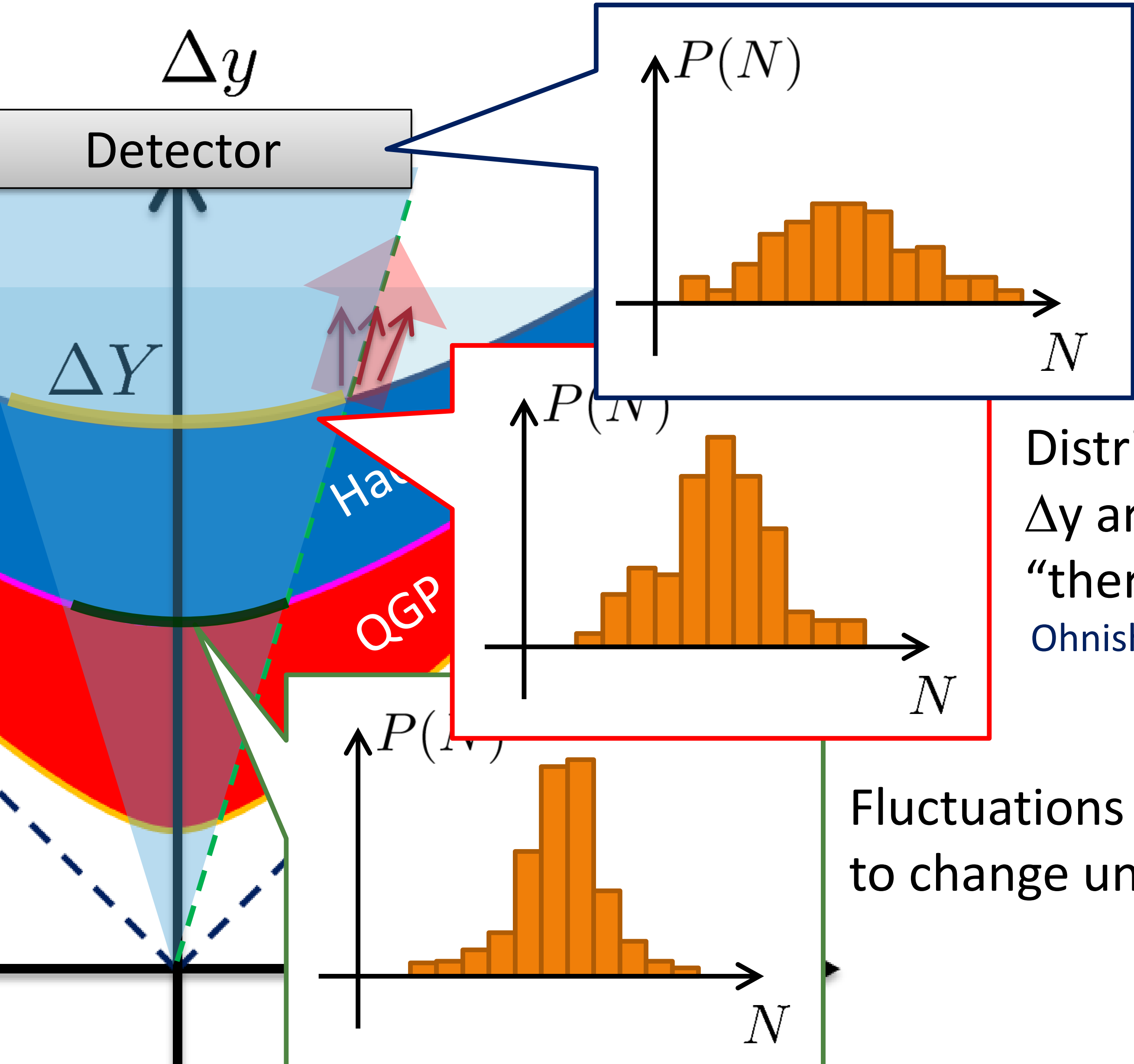
Ejiri-Karsch-Redlich

Ratio of cumulants

$$\frac{\langle N_{\mathrm{Q}}^4 \rangle_c}{\langle N_{\mathrm{Q}}^2 \rangle_c}, \quad \frac{\langle N_{\mathrm{B}}^4 \rangle_c}{\langle N_{\mathrm{B}}^2 \rangle_c}$$

Experimentally difficult

Time Evolution of Fluctuations

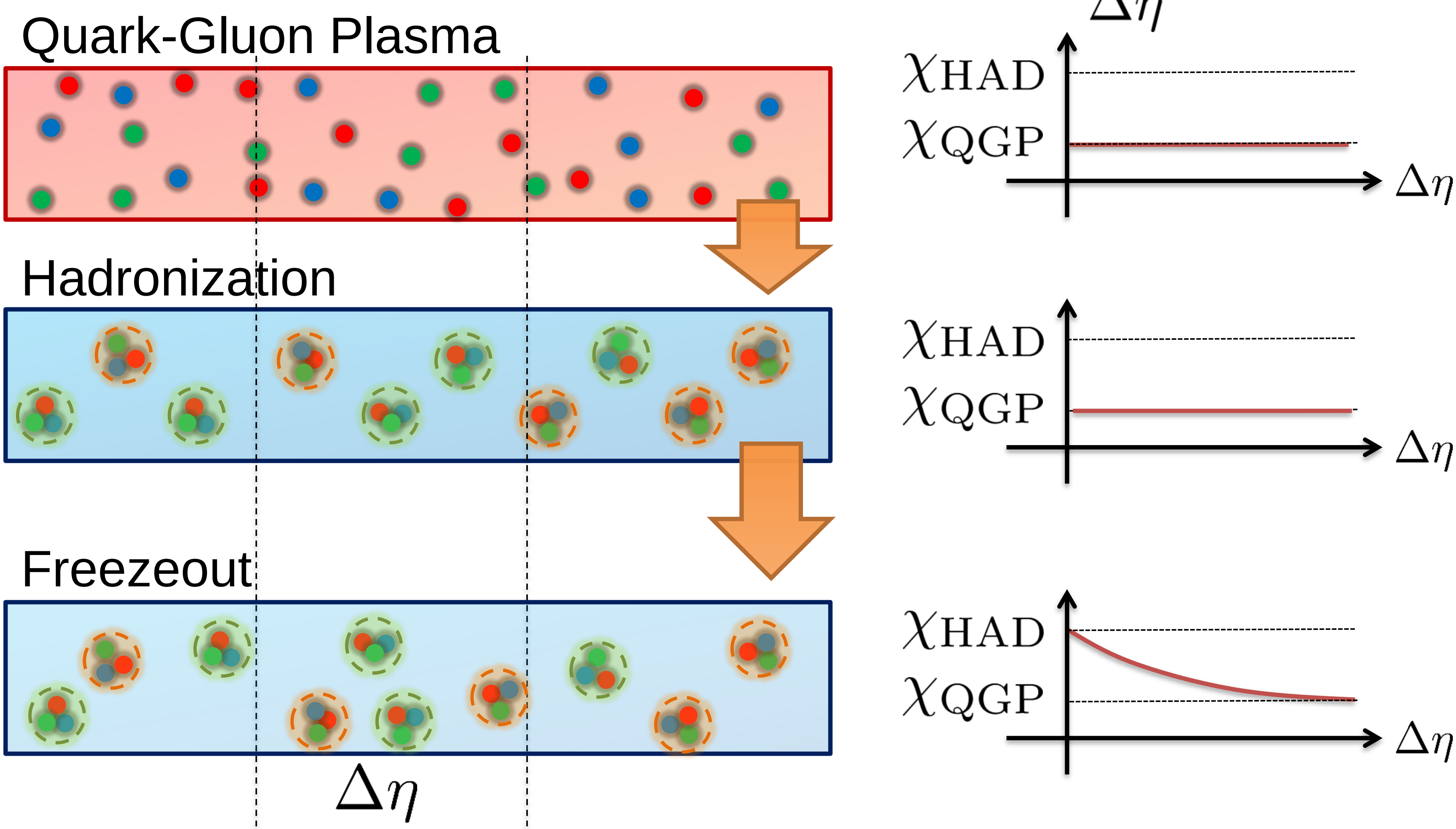


Distributions in ΔY and Δy are different due to “thermal blurring”.

Ohnishi, MK, Asakawa, PRC(2016)

Fluctuations in ΔY continue to change until kinetic f.o.

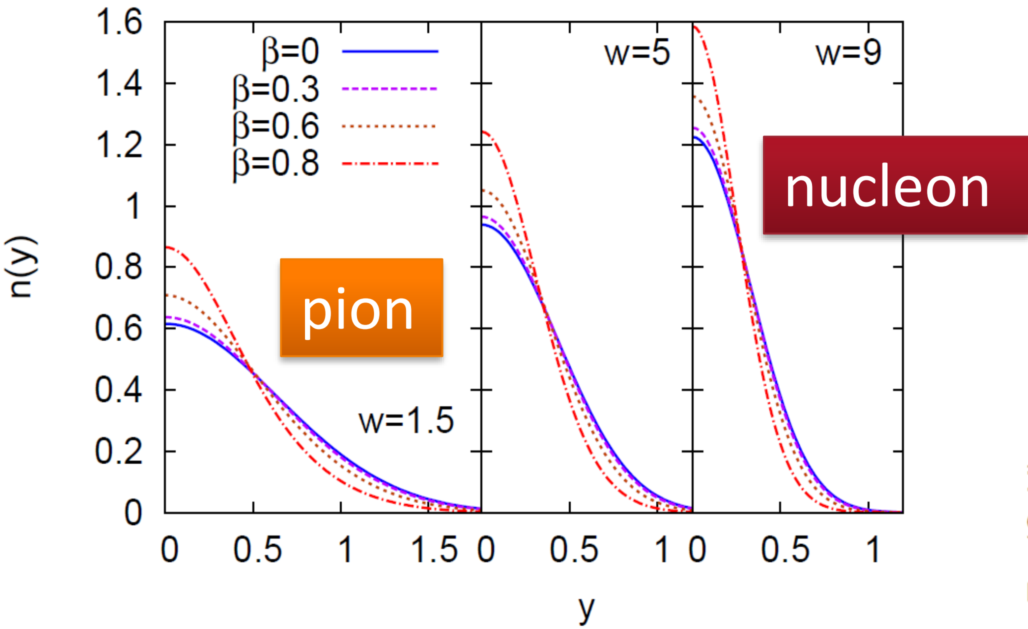
Time Evolution of Fluctuations



Variation of a conserved charge is achieved only through diffusion. ⇨ The larger $\Delta\eta$, the slower diffusion

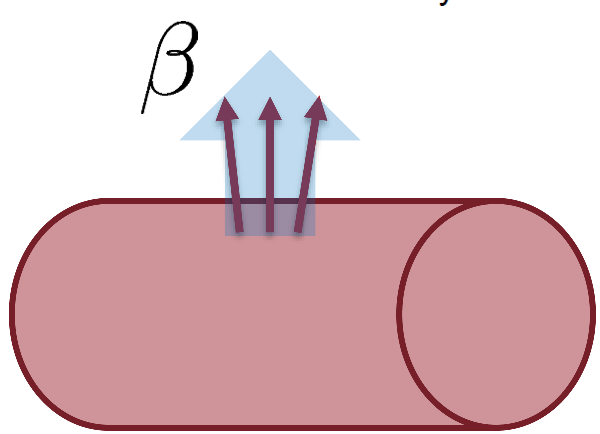
Thermal distribution in y space

Ohnishi, MK, Asakawa, PRC (2016)

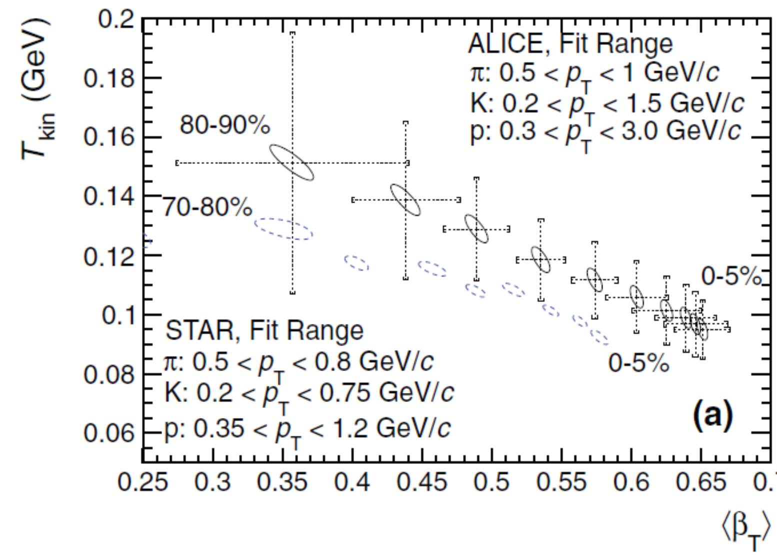


$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$



Blast wave squeezes the distribution in rapidity space



- assume Bjoroken picture
- blast wave
- flat freezeout surface

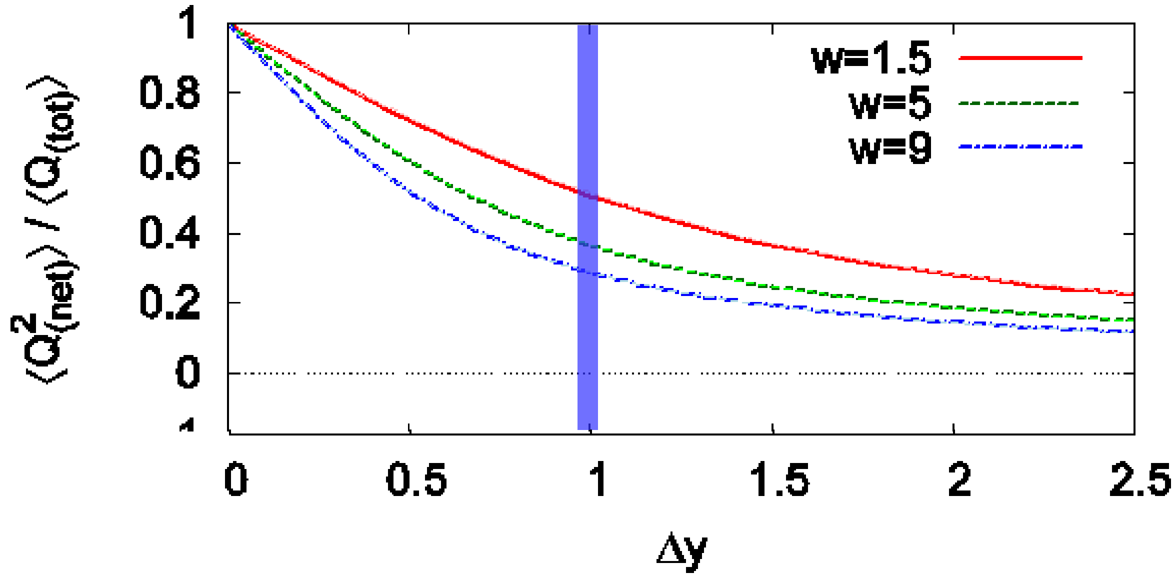
$\Delta\eta$ Dependence

Ohnishi, MK, Asakawa, PRC (2016)

Initial condition (before blurring) no e-v-e fluctuations

Cumulants after blurring can take nonzero values

Cumulants after blurring



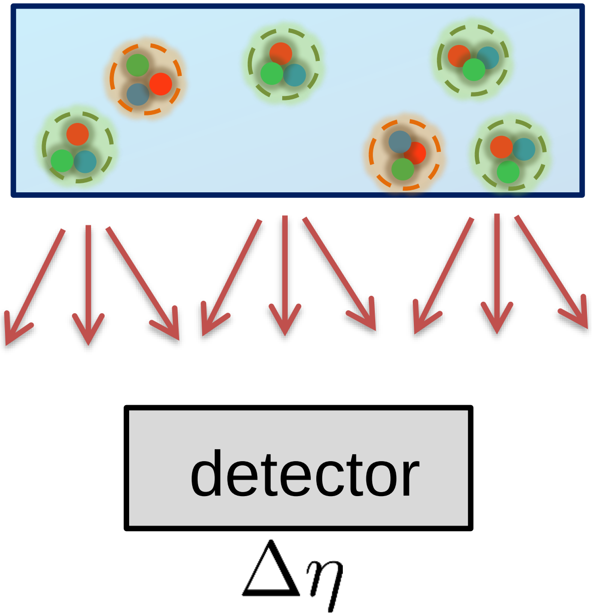
At Δy=1, the effect is **not** well suppressed

$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$

Very Low Energy Collisions

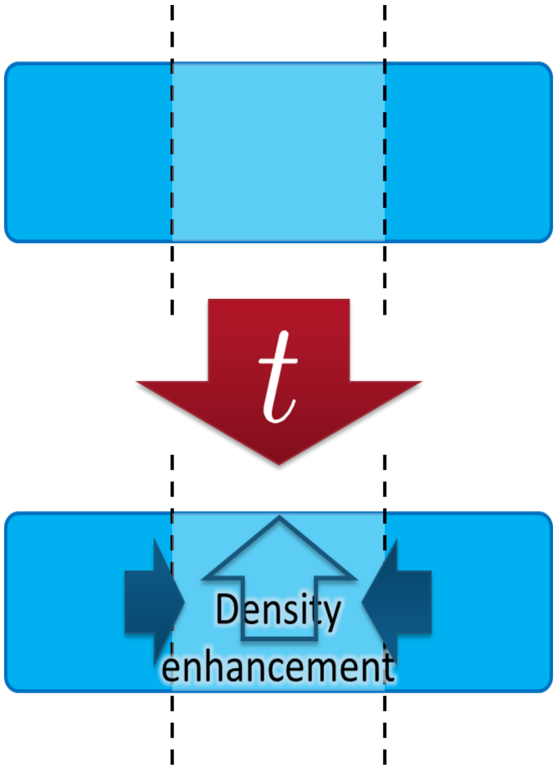
- Large contribution of global charge conservation
- Violation of Bjorken scaling



Careful treatment is required to interpret fluctuations at low beam energies!

Many information should be encoded in $\Delta\eta$ dep.

Evolution of Conserved-Charge Fluctuations



$$\langle \xi(1)\xi(2) \rangle = 2D\chi_2\delta(1-2)$$

Equations describing transport of *n*:

- Diffusion Equation

$$\frac{\partial n}{\partial t} = D\nabla^2 n$$

- Stochastic Diffusion Equation (SDE)

$$\frac{\partial n}{\partial t} = D\nabla^2 n + \nabla\xi(x,t)$$

- SDE with non-linear terms

$$\frac{\partial n}{\partial t} = \kappa\nabla^2 \frac{\delta\mathcal{F}}{\delta n} + \frac{\partial}{\partial x}\xi(x,t)$$

$$\mathcal{F} = \int dx \left(a\Delta n^2 + c(\nabla n)^2 + \lambda_3 \Delta n^3 + \cdots \right)$$

Analysis of 2nd-order Cumulant

Sakaida+ (2017)

Evolution of baryon number density

Stochastic Diffusion Equation

$$\partial_t n = D(t)\partial_x^2 n + \partial_x \xi$$

$$\langle \xi(x_1,t_1)\xi(x_2,t_2)\rangle = 2D\chi_2\delta^{(2)}(1-2)$$

$D(t),\ \chi_2(t)$:parameters characterizing criticality

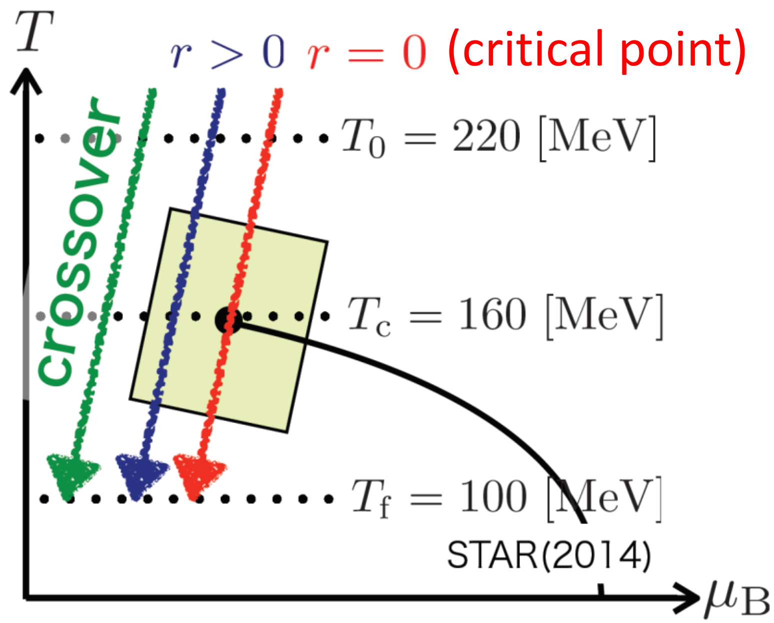
- Analytic solution is obtained.
- Study 2nd order cumulant & correlation function.

Parametrizing D(τ) and χ(τ)

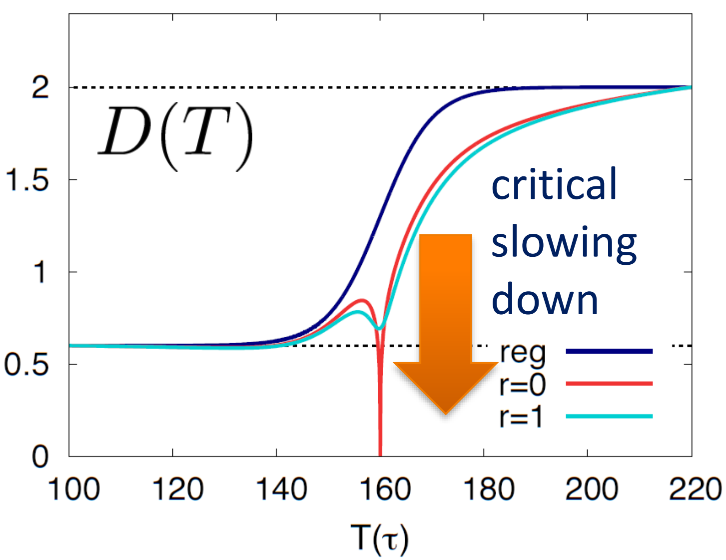
Critical behavior

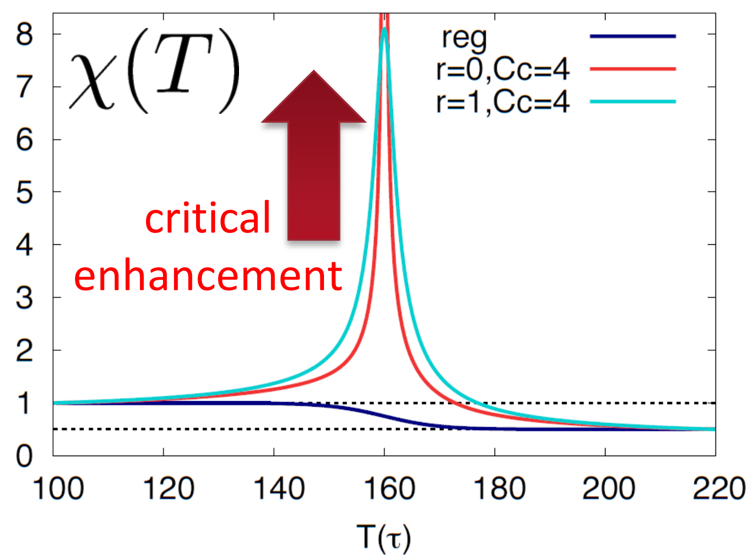
- 3D Ising (r,H)
- model H

Berdnikov, Rajagopal (2000)
Stephanov (2011); Mukherjee+(2015)



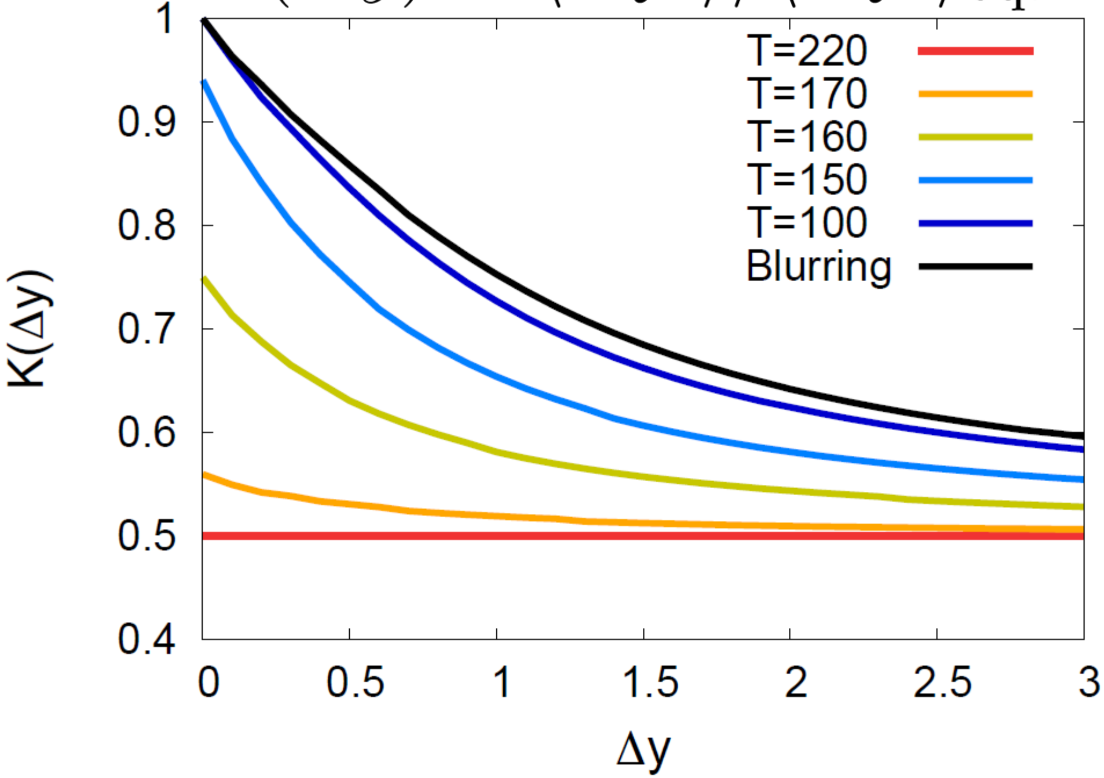
Temperature dep.

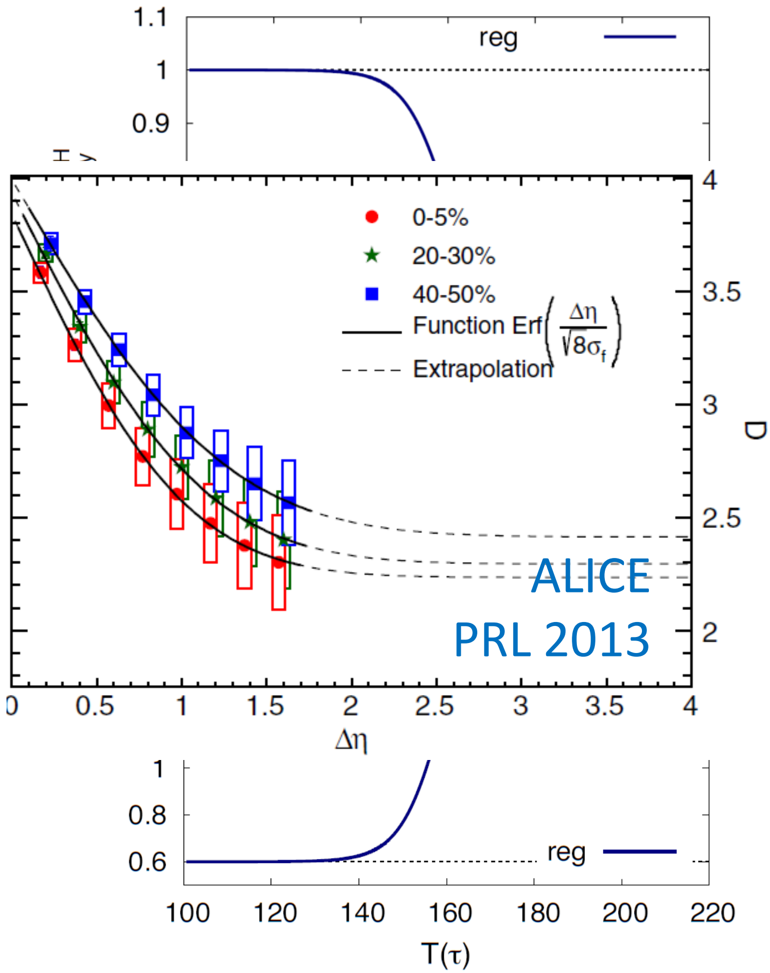




Crossover / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$





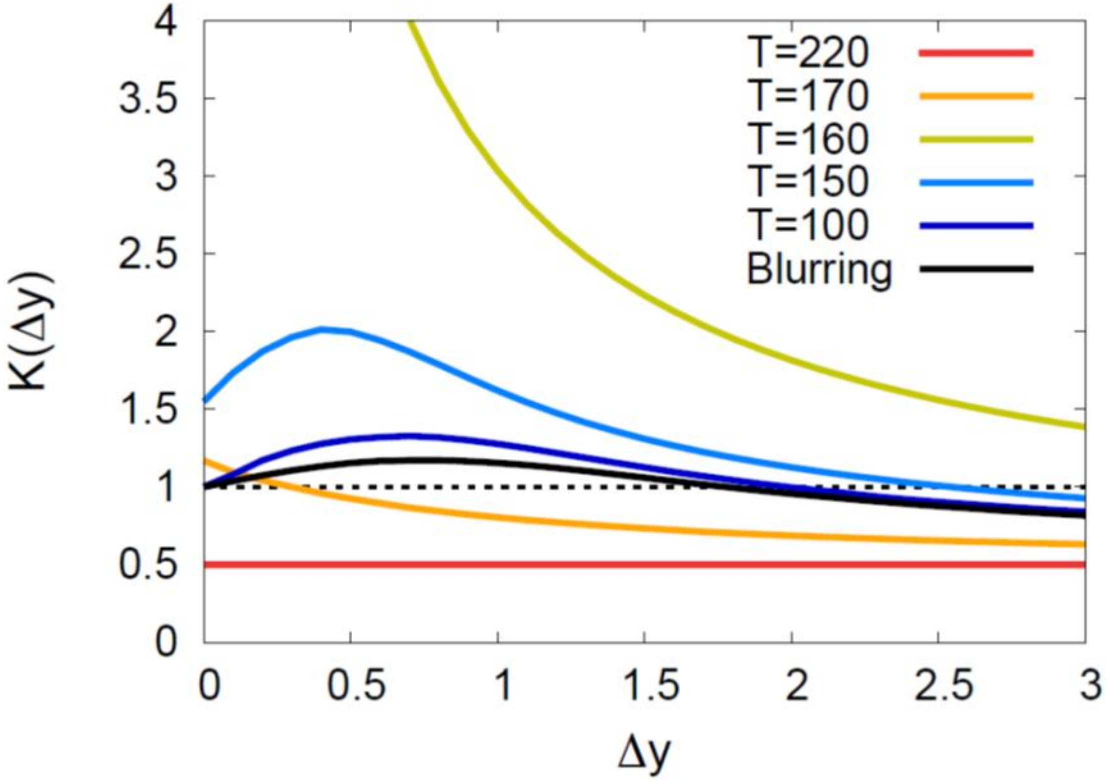
□ monotonically decresing

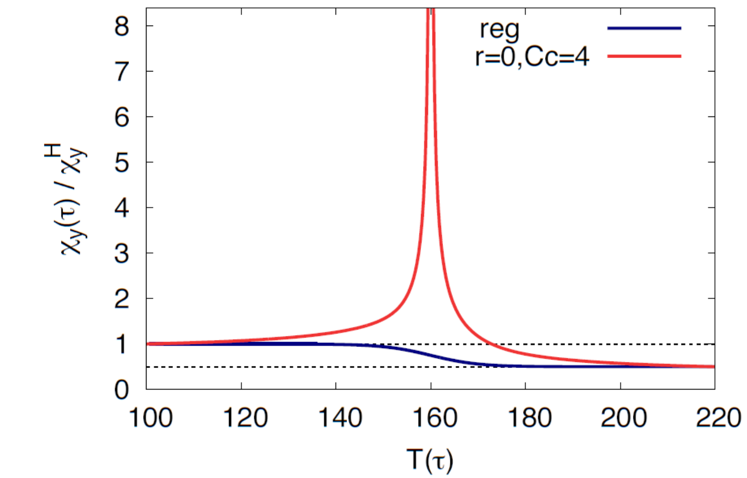
Analytic result

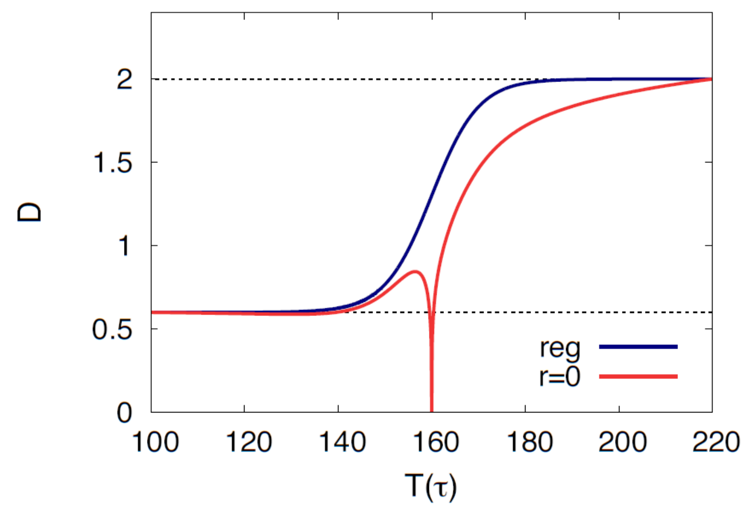
$\chi(\tau)$ monotonically increasing → $K(\Delta y)$ monotonically decreasing

Critical Point / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$







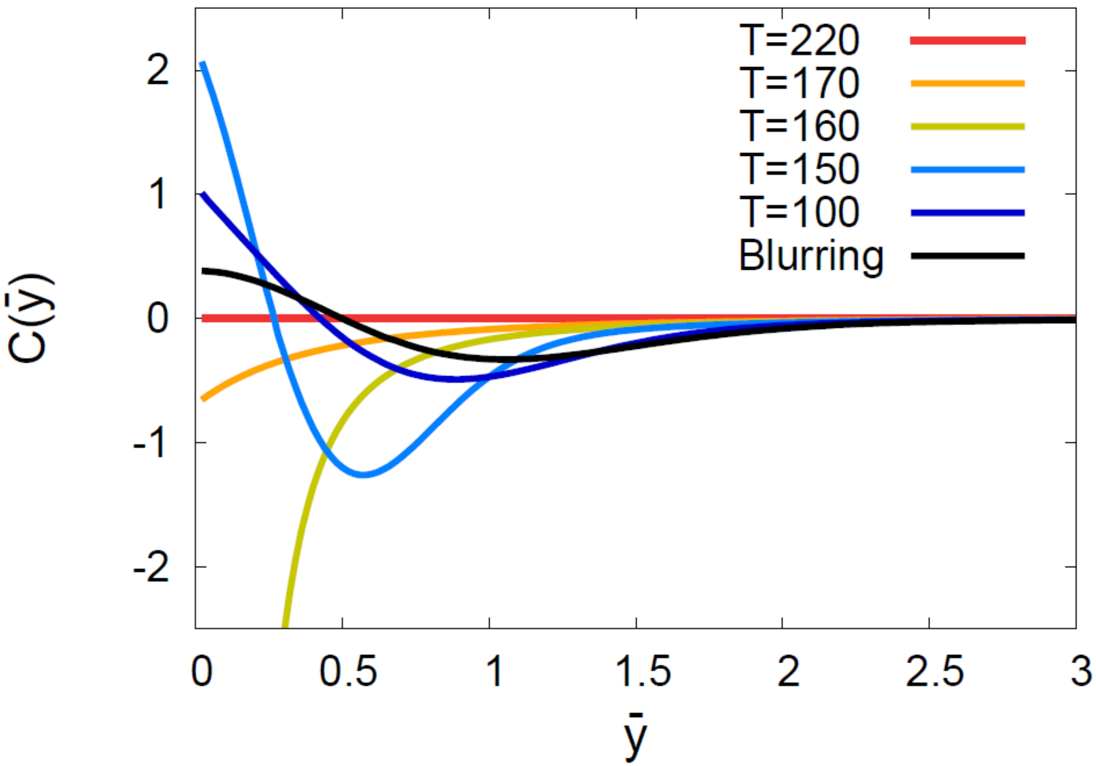
- non-monotonic Δy dep.

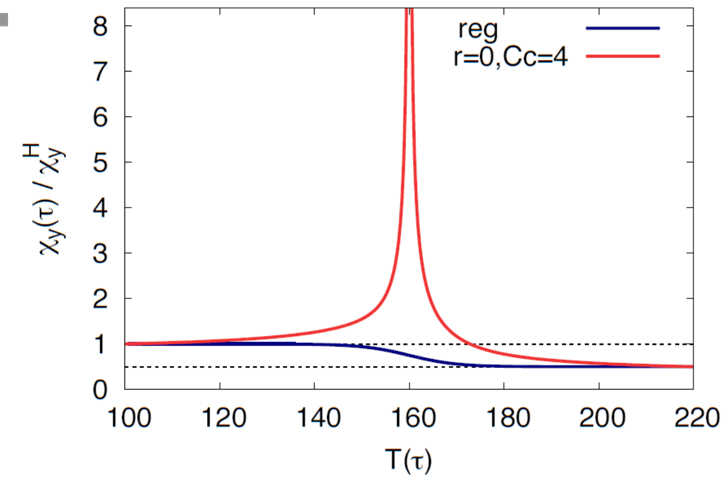
Analytic result: $K(\Delta y)$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

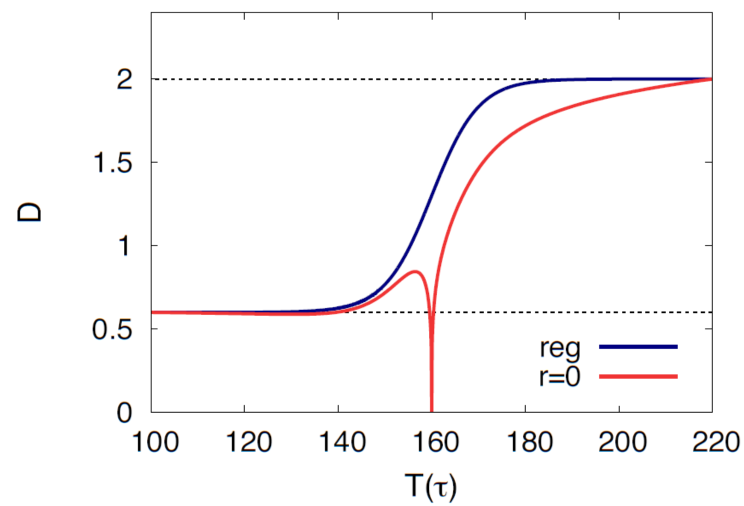
See also,
Wu, Song
arXiv: 1903.06075

Criticap Point / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\mathrm{hadron}}$$







- ☐ non-monotonic Δy dep.

Analytic result: $C(\Delta y)$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

See also,
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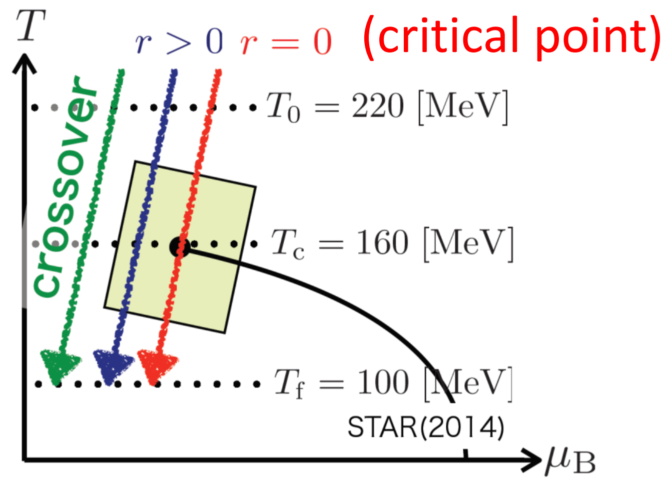
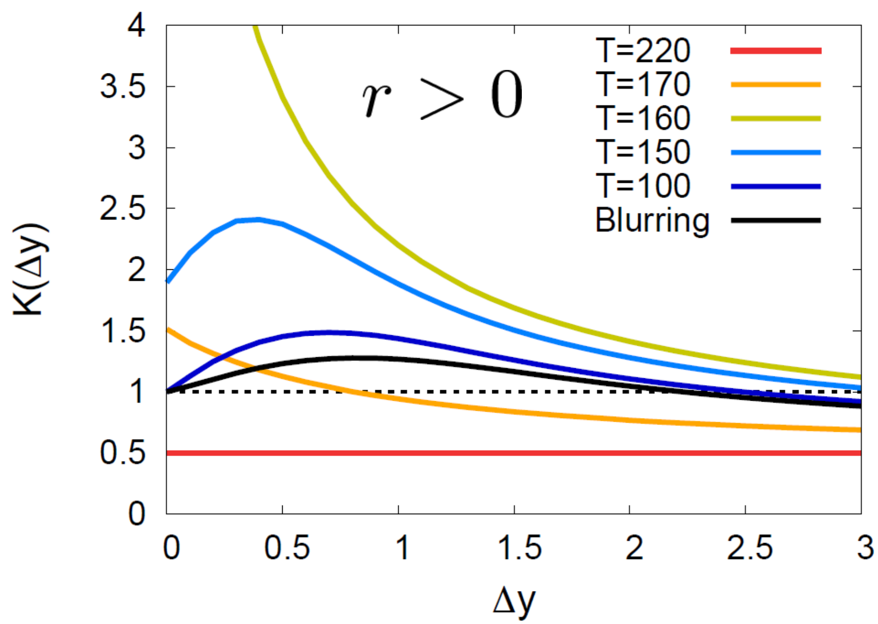
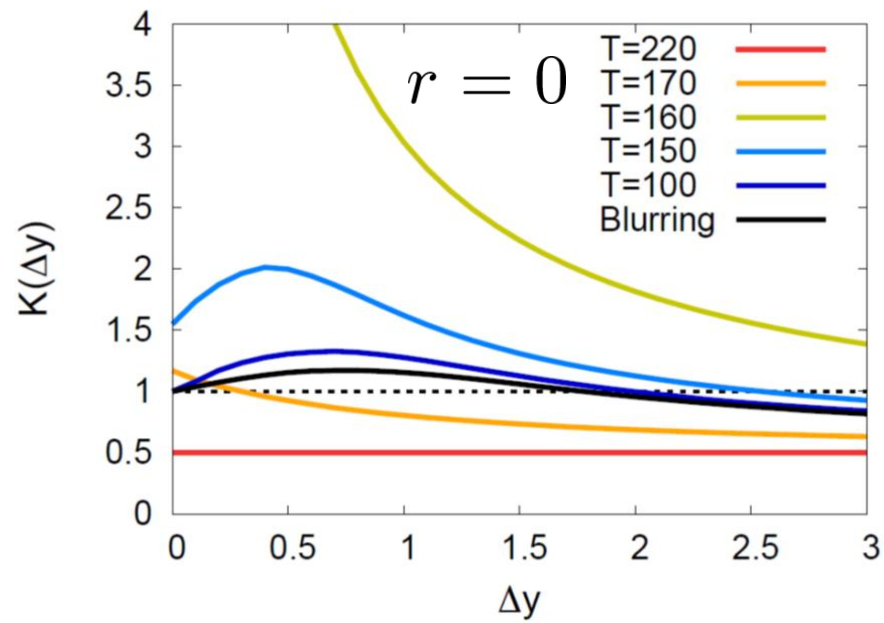
arXiv: 1903.06075

Away from the CP

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.}$$







- Signal of the critical enhancement can be clearer on a path away from the CP.

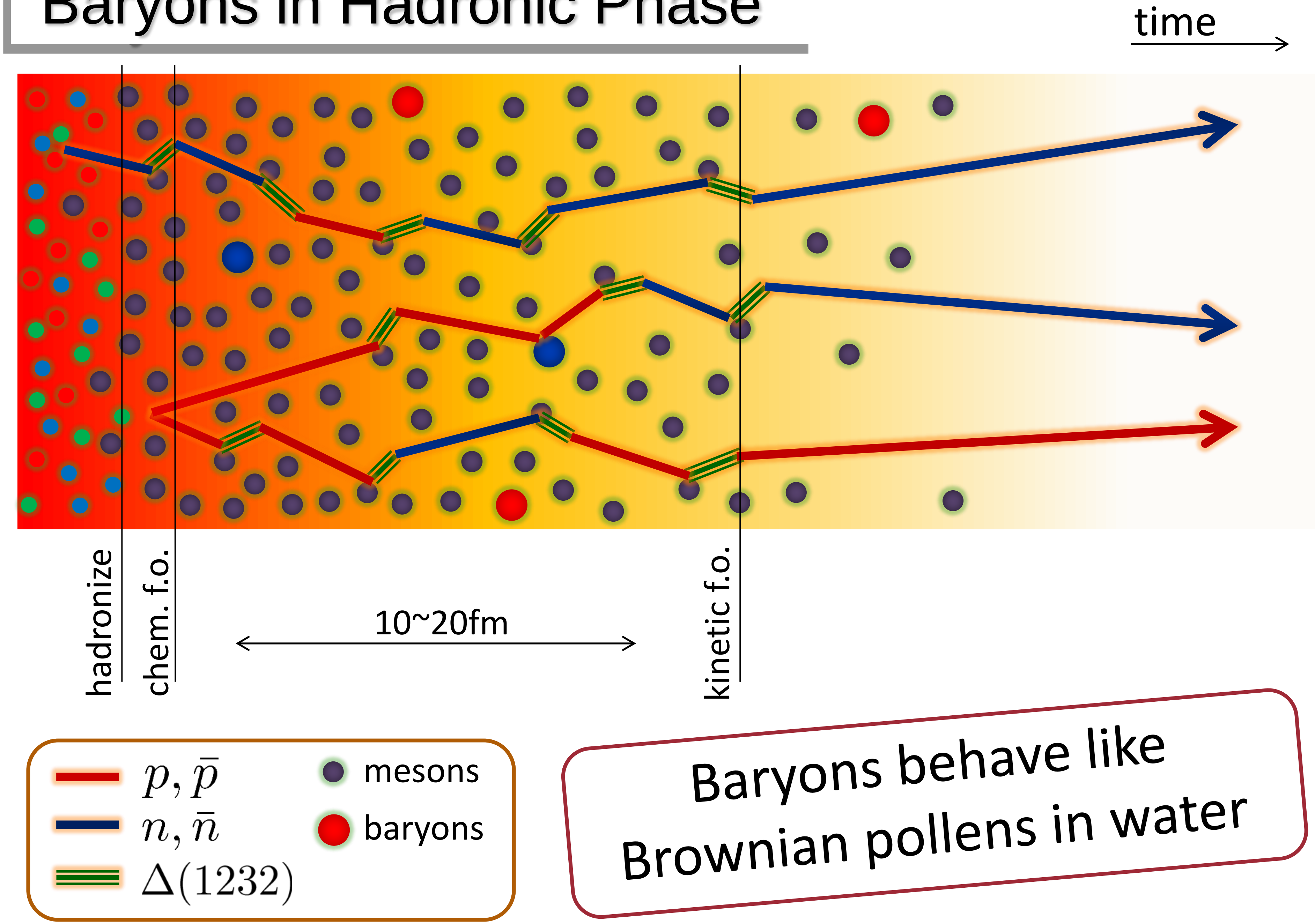
Away from the CP → Weaker critical slowing down

Extension to Higher-order Cumulants

Analyses with

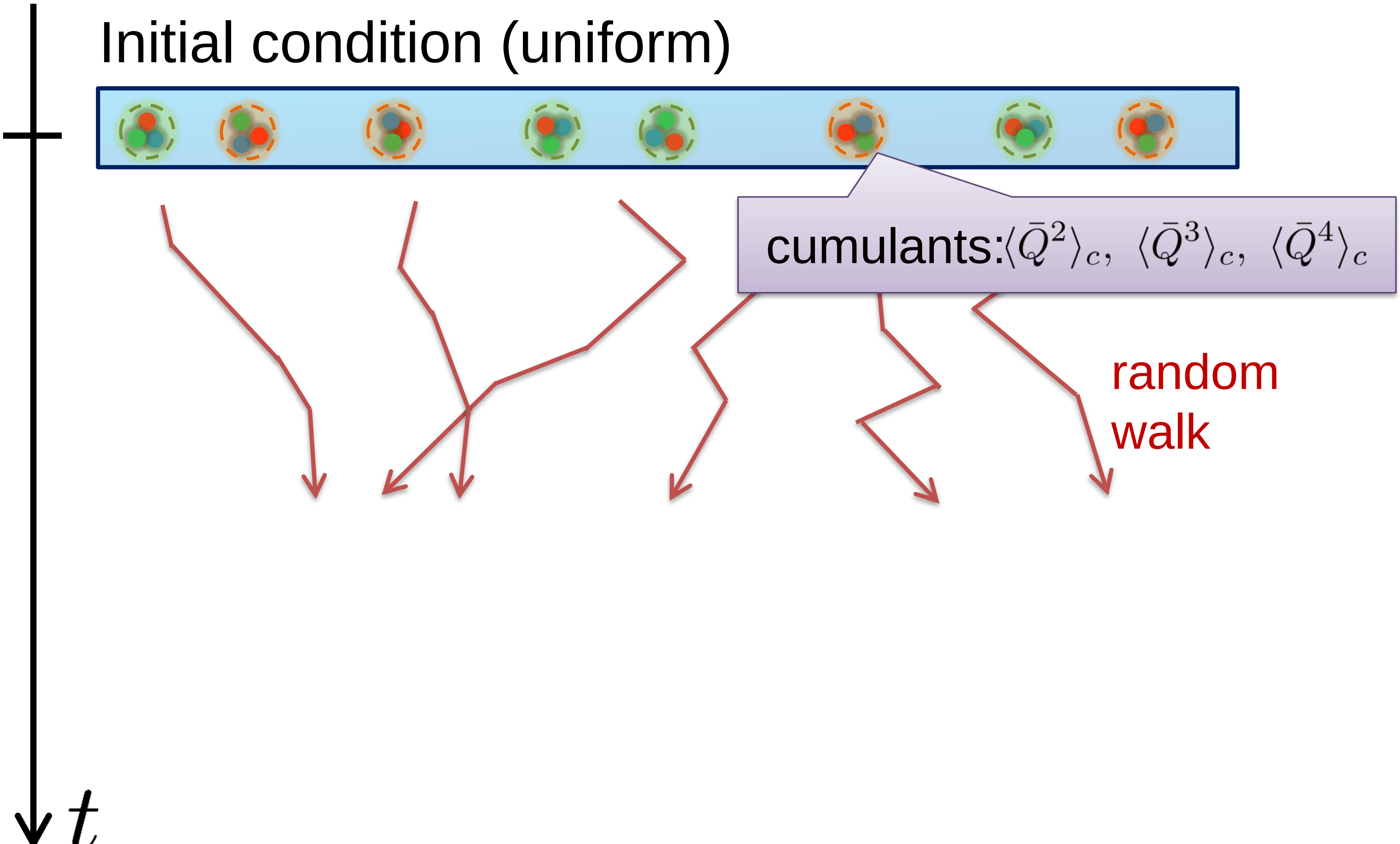
1. Stochastic diffusion equation
2. Diffusion master equation

Baryons in Hadronic Phase



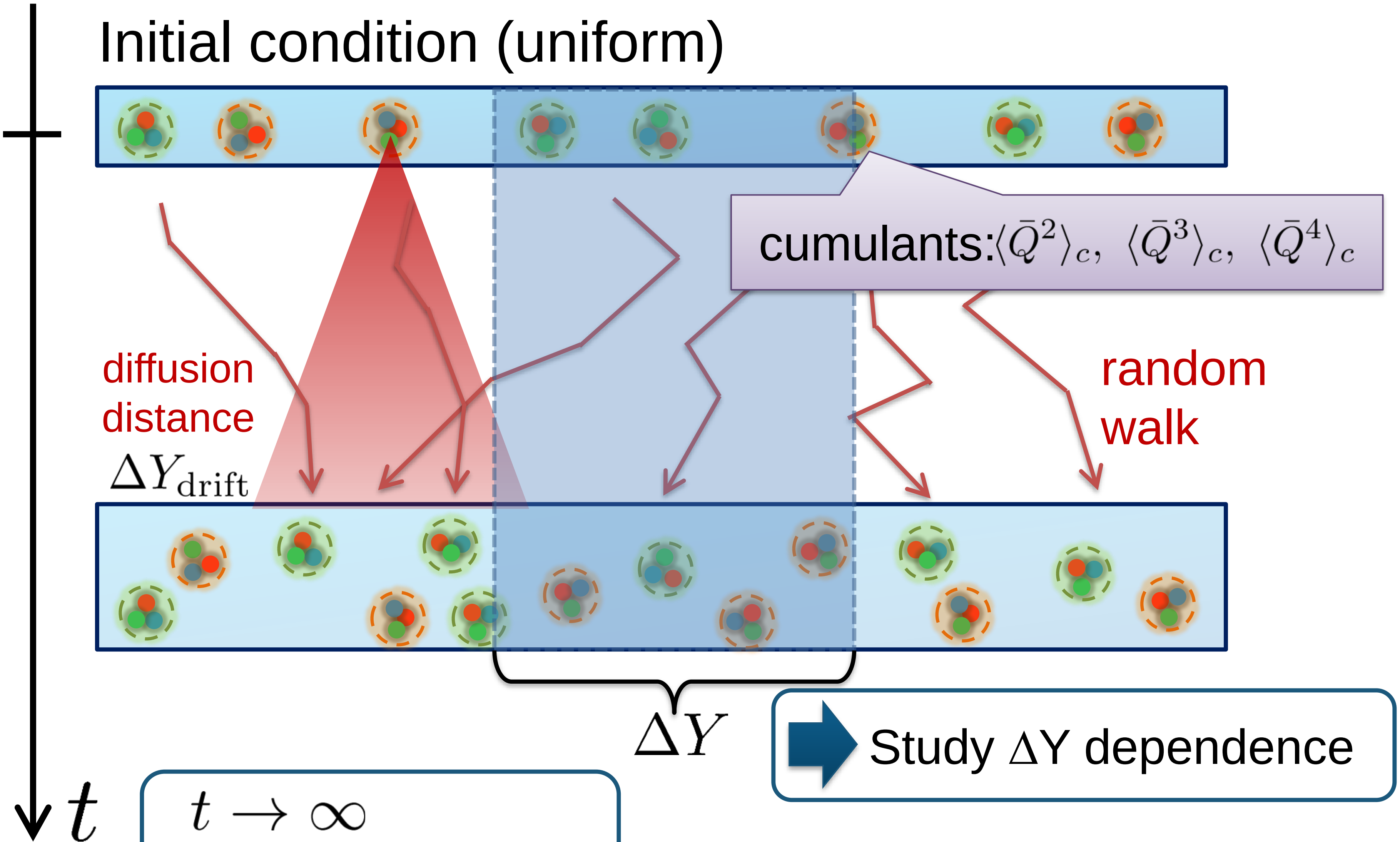
Baryons behave like
Brownian pollens in water

(Non-Interacting) Brownian Particle Model



diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

(Non-Interacting) Brownian Particle Model



$t \to \infty$
Poisson distribution

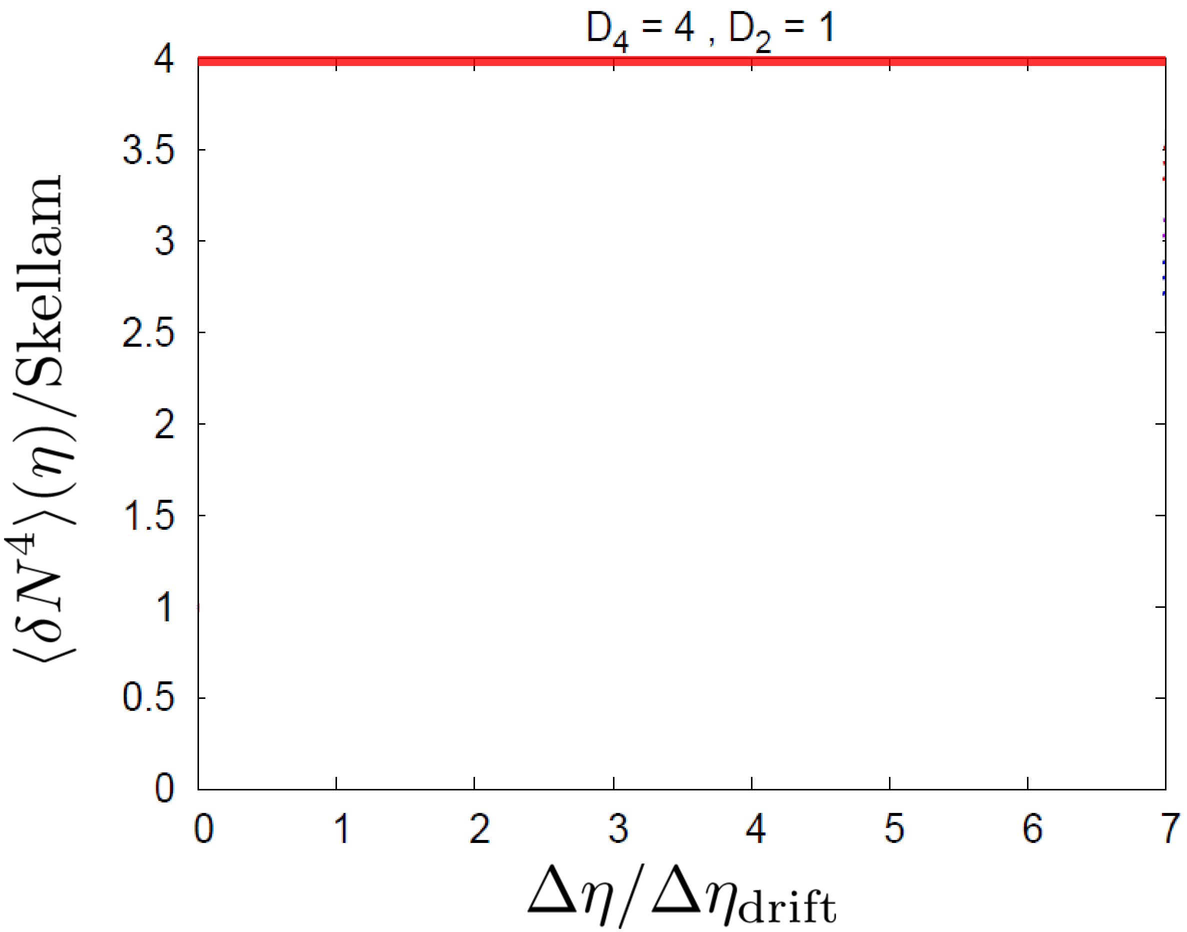
Study ΔY dependence

diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

4^{th} Order Cumulant

MK+ (2014)
MK (2015)

Before the diffusion



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

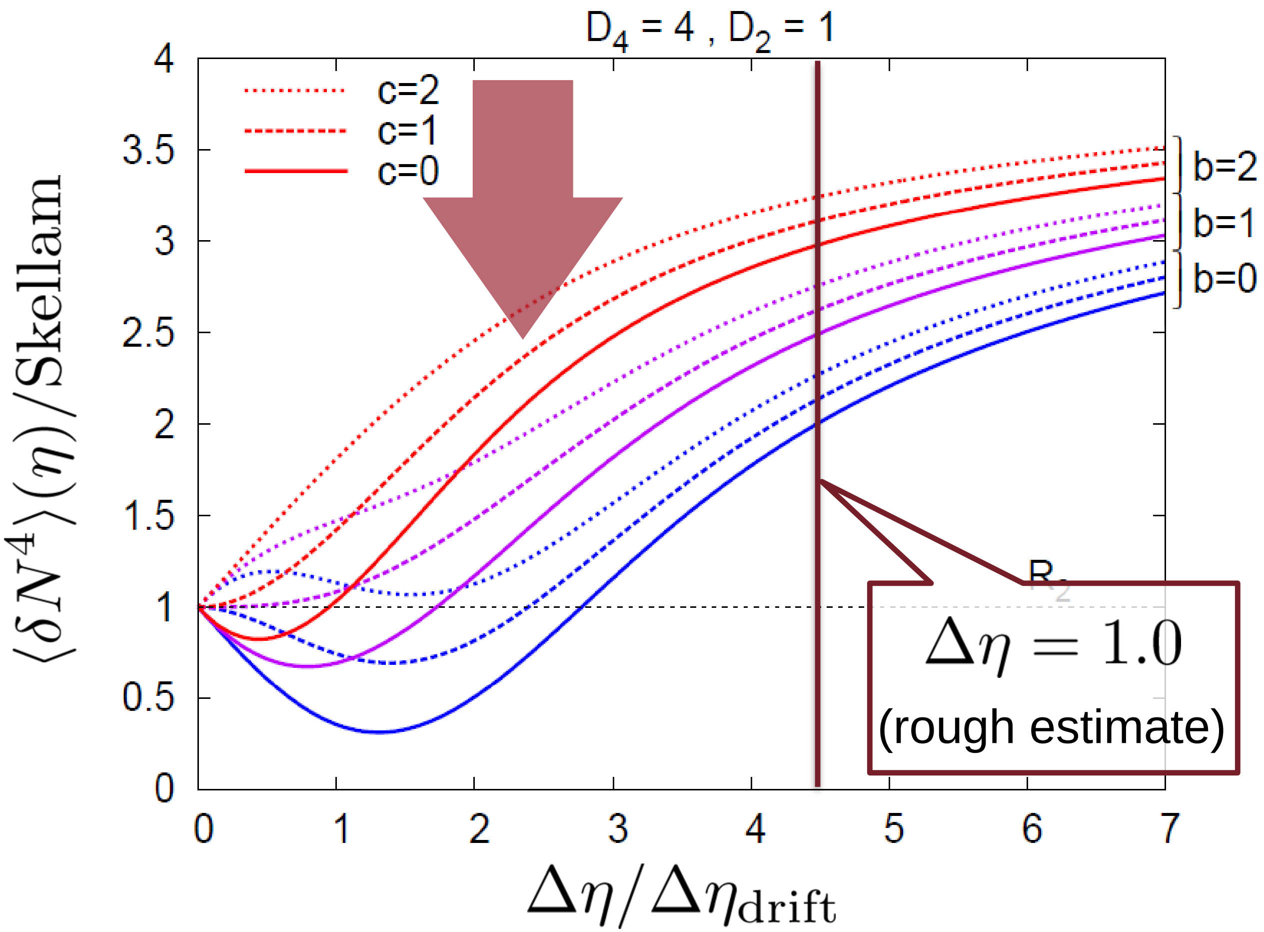
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

4^{th} Order Cumulant

MK+ (2014)
MK (2015)

After the diffusion



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

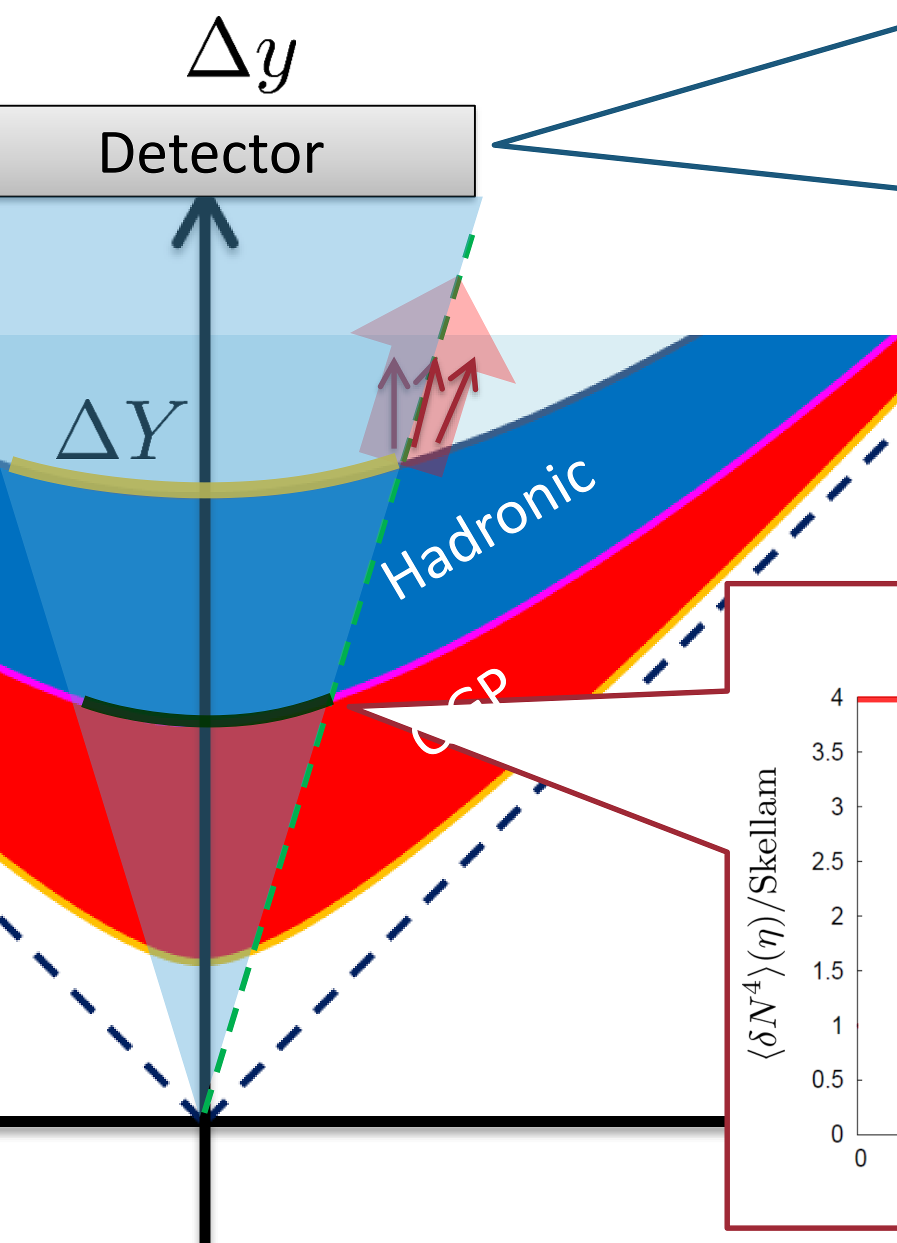
$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

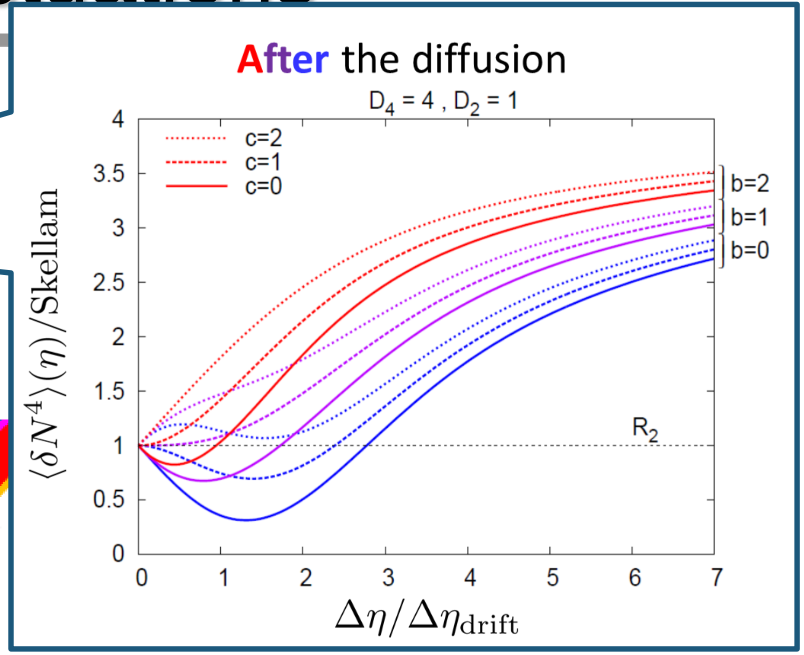
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

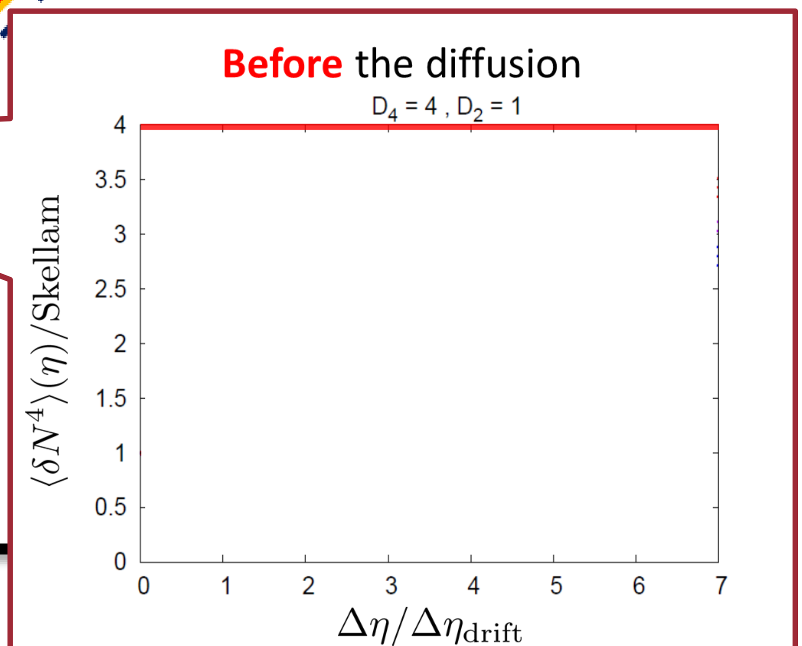
$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

- Cumulant at small $\Delta\eta$ is modified toward a Poisson value.
- Non-monotonic behavior can appear.

Time Evolution of Fluctuations







As a result of a simple random walk...

Rapidity Window Dep.

4th-order cumulant

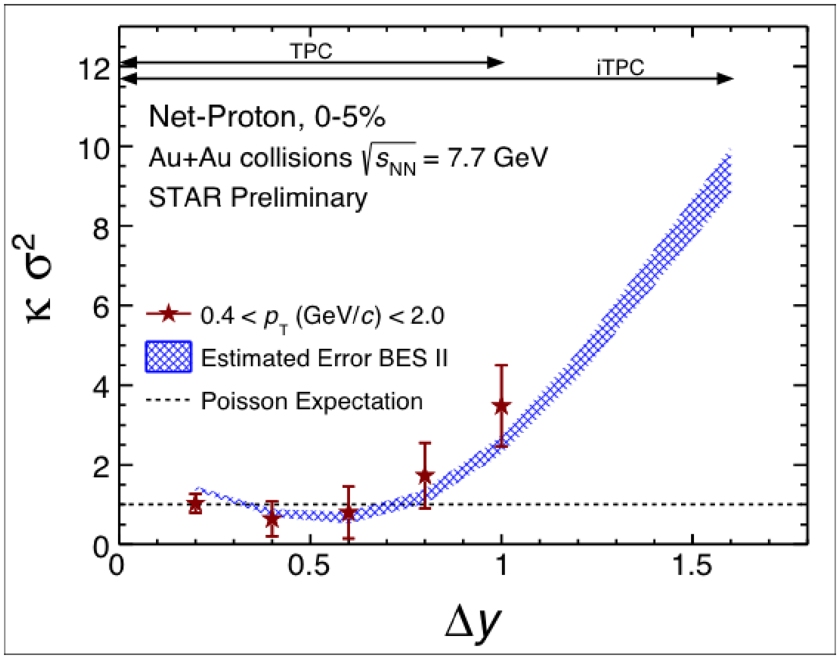
MK+, 2014
MK, 2015

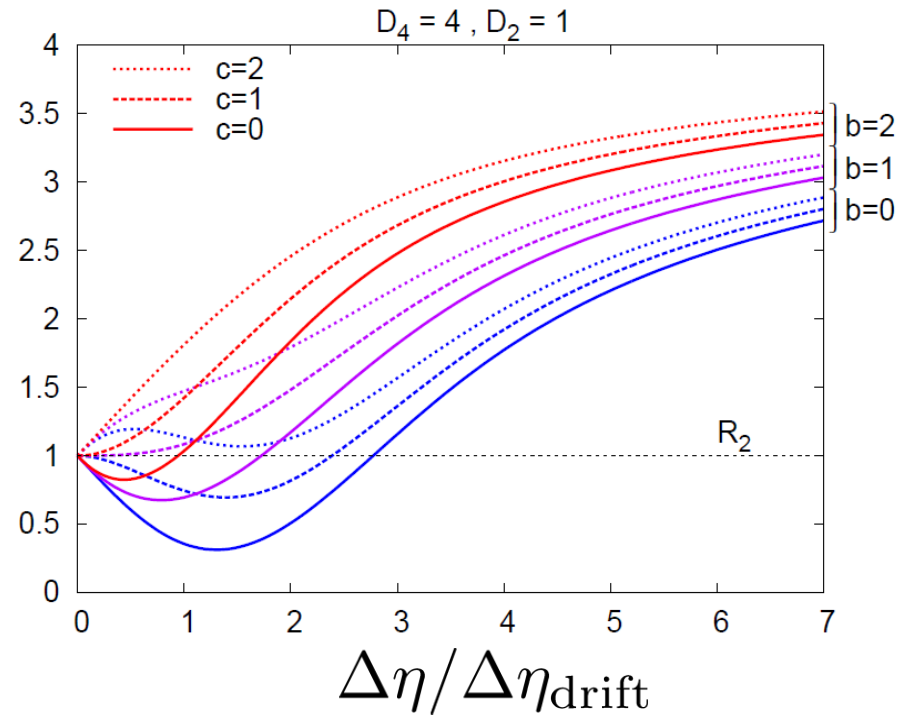
Initial Conditions

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad c = \frac{\langle Q^2_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

STAR Collab. (X. Luo, CPOD2014)





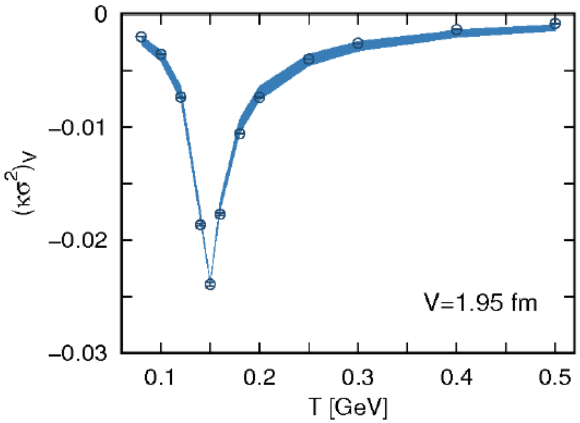
- ☐ Is non-monotonic Δη dependence already observed?
- ☐ Different initial conditions give rise to different characteristic Δη dependence. → Study initial condition

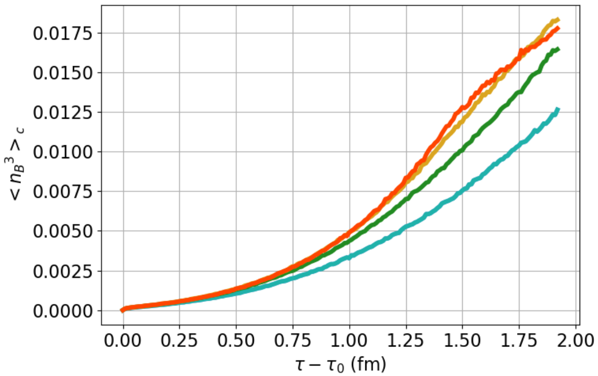
Finite volume effects: Sakaida+, PRC90 (2015)

SDE with Non-Linear Terms

☐ Higher order cumulants

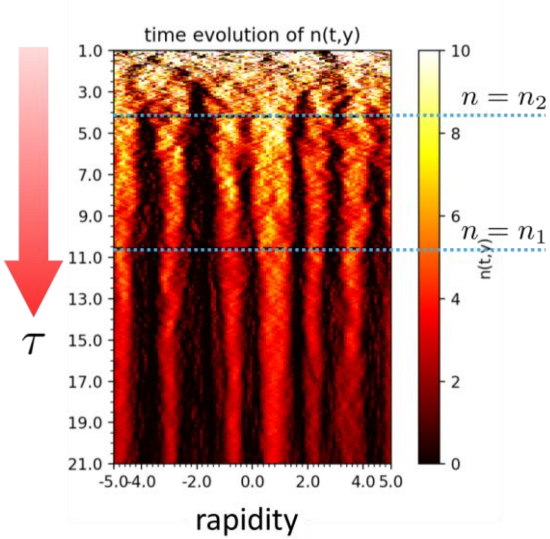
Nahrgang, Bluhm, Schaefer, Bass, PRD (2019);
Pihan, Touroux, Nahrgang, Bluhm, Sami, MK, in prep.

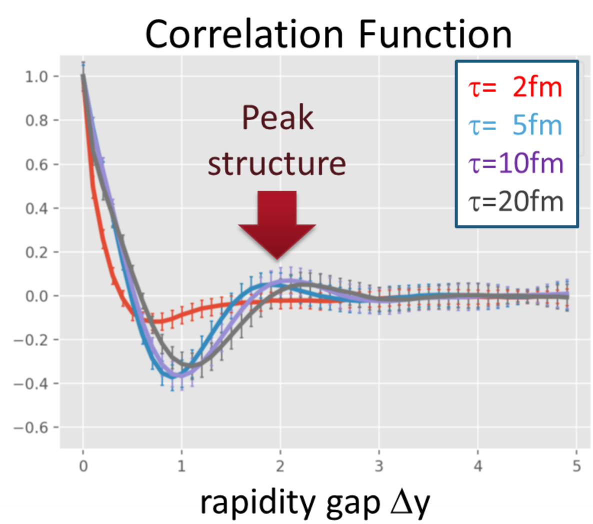




Time evolution of 4th cumulant can be described.

☐ 1st order transition





Domain formation and peak structure in the correlation function are found.

Message

Understand 2^{nd}-order fluctuations
@ LHC & top-RHIC

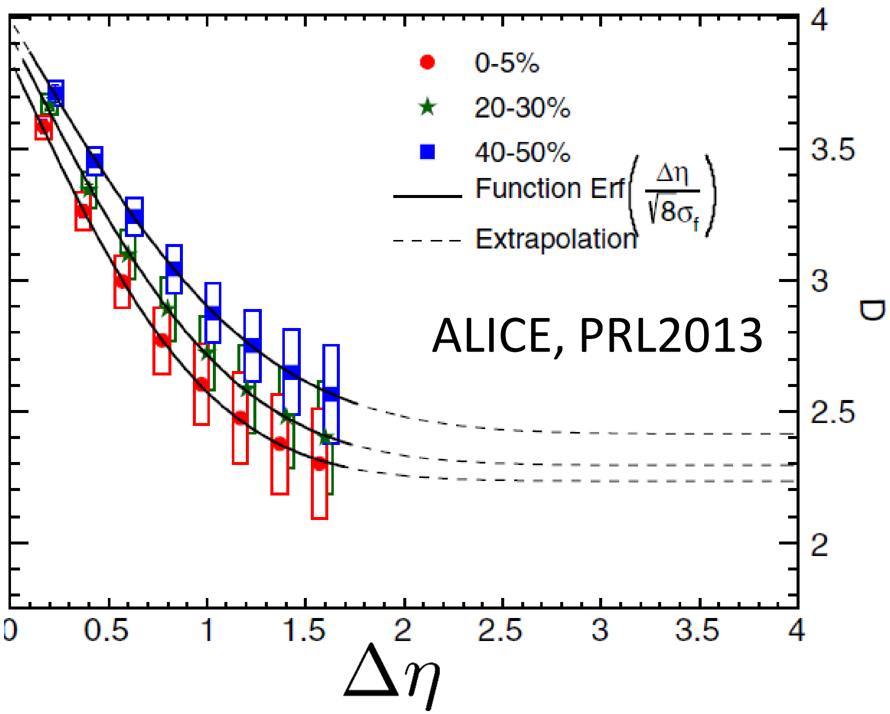
1. Problems in experimental analysis
   - proper correction of detector's property

1. Dynamics of non-Gaussian fluctuations

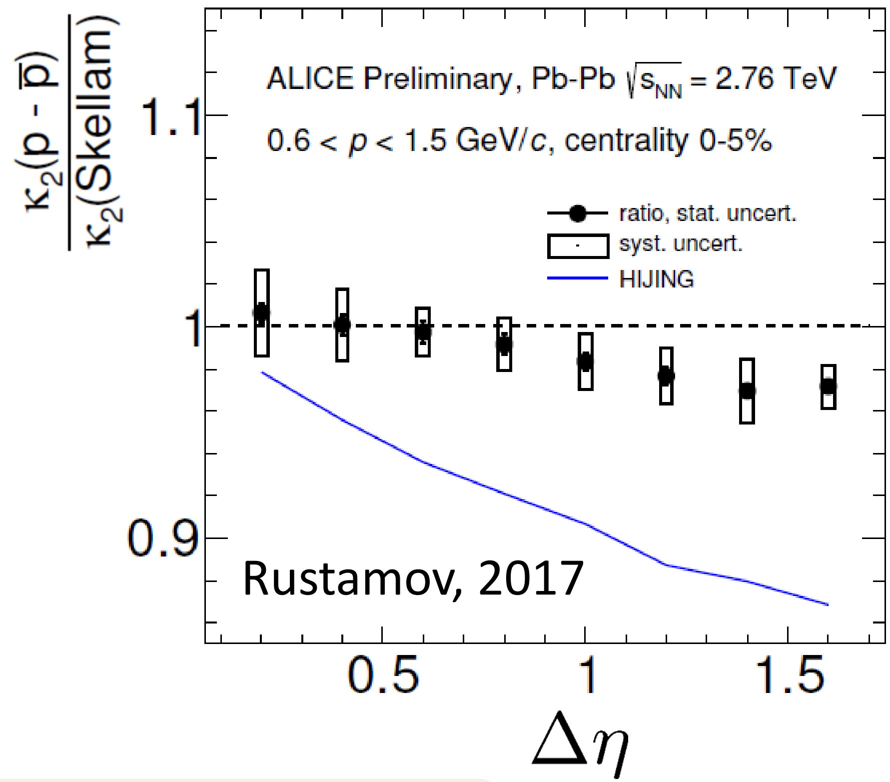
2. A suggestion: chiB/chiQ

2nd Order @ ALICE

Net charge fluctuation



Net proton fluctuation

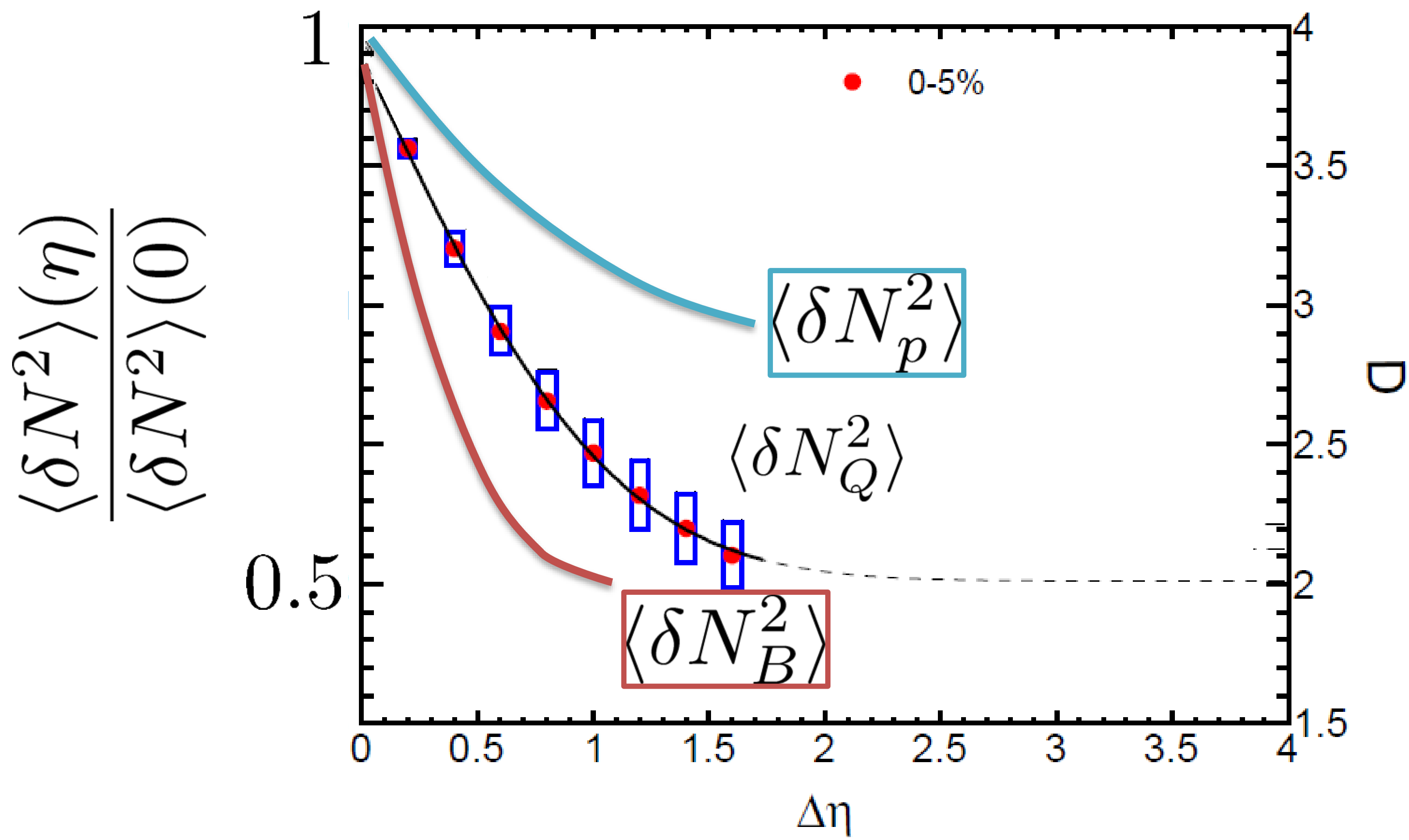


- Net-charge fluctuation has a suppression,
- but net-proton fluctuation does not. Why??

$\langle \delta N_B^2 \rangle$ and $\langle \delta N_p^2 \rangle$ @ LHC ?

MK, presentations
GSI, Jan. 2013
Berkeley, Sep. 2014
FIAS, Jul. 2015
GSI, Jan. 2016
…

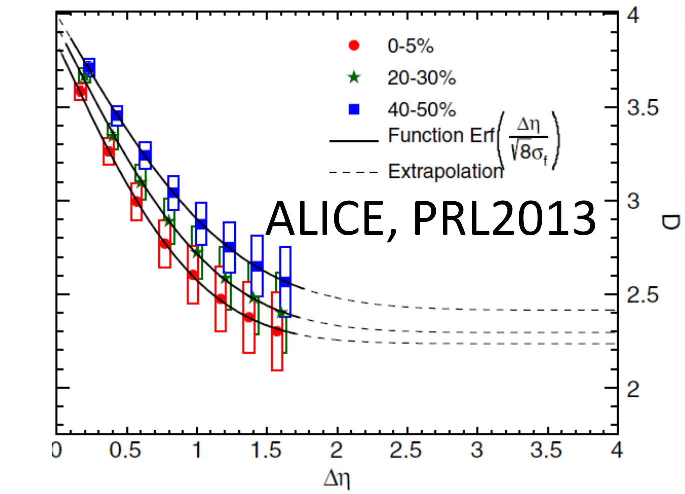
$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
should have different $\Delta\eta$ dependence.



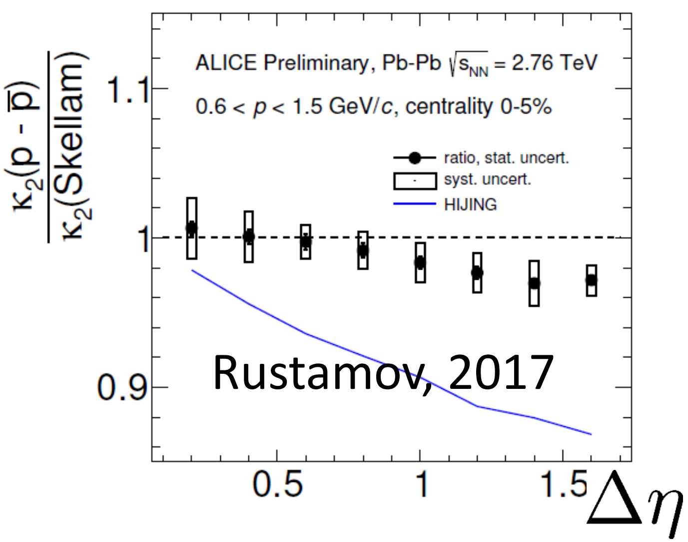
Baryon # cumulants are experimentally observable! MK, Asakawa, 2012

A Suggestion

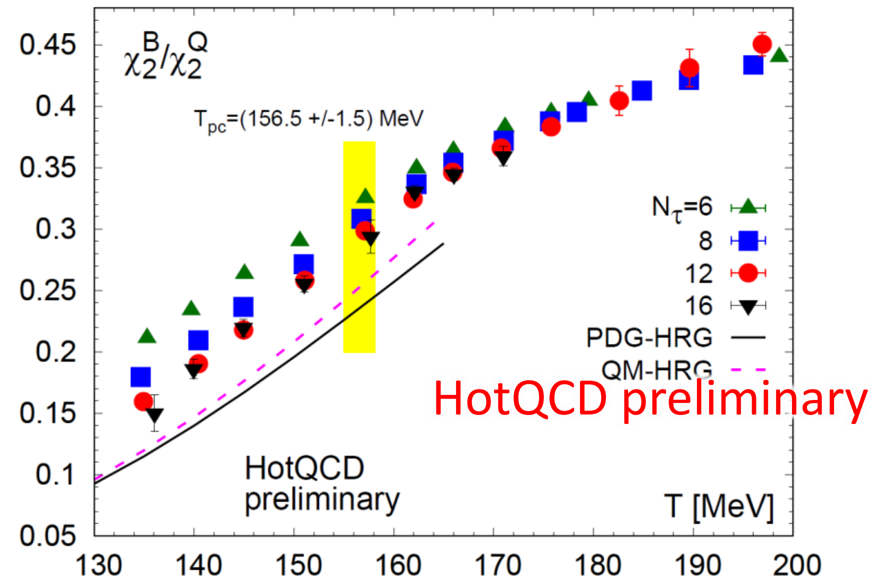
Net charge fluctuation



Net proton fluctuation



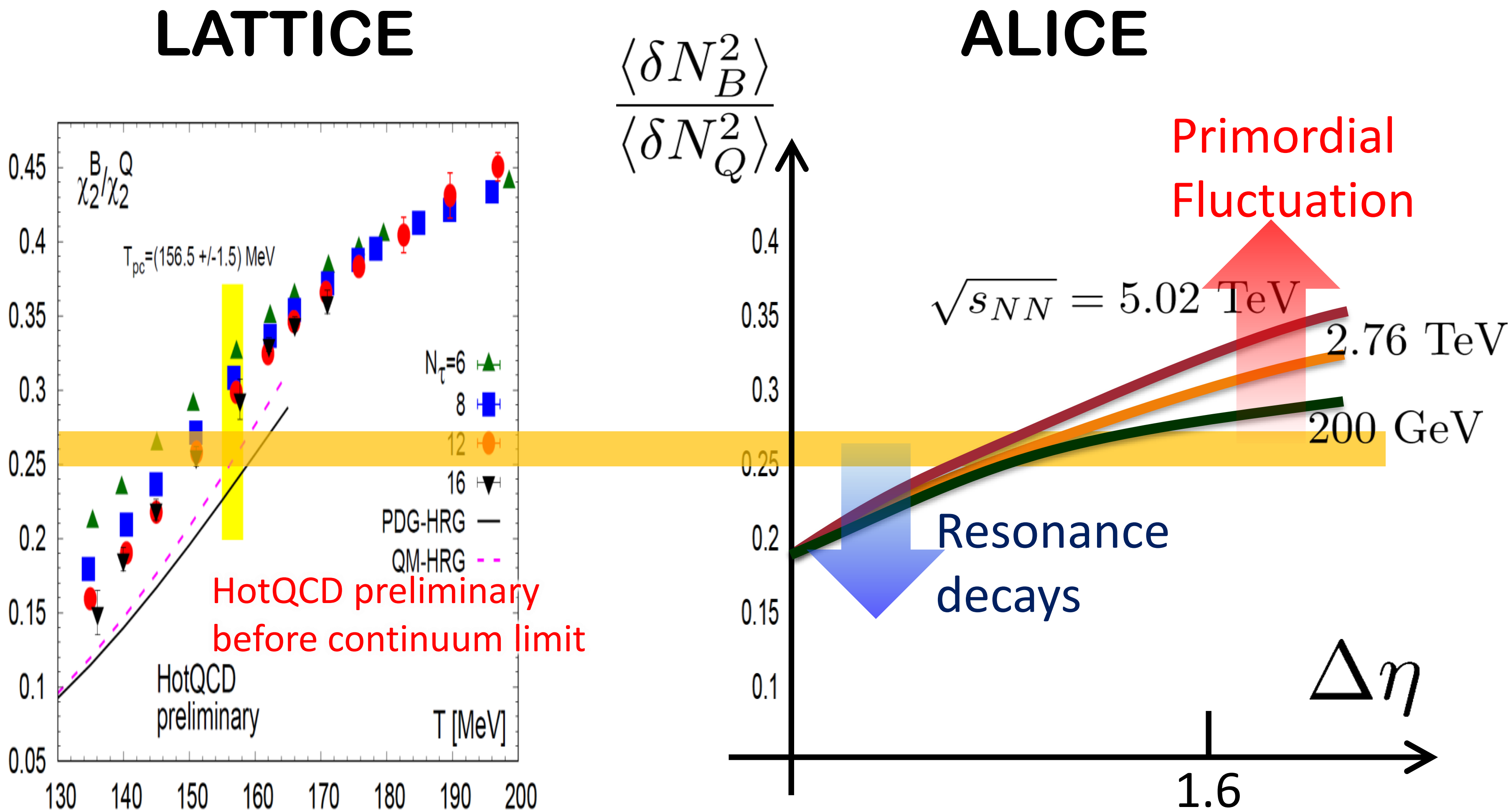
- ☐ Construct $\langle\delta N_B^2\rangle$ $(\langle\delta N_N^2\rangle)$, $\langle\delta N_Q^2\rangle$
- ☐ Then, take ratio $\langle\delta N_B^2\rangle/\langle\delta N_Q^2\rangle$
- ☐ Compare it with lattice



✓ **linear T dependence near Tc !!**

✓ **only 2nd order: reliable !!**

Prediction



Δη dependence for tracing back the history!

Summary

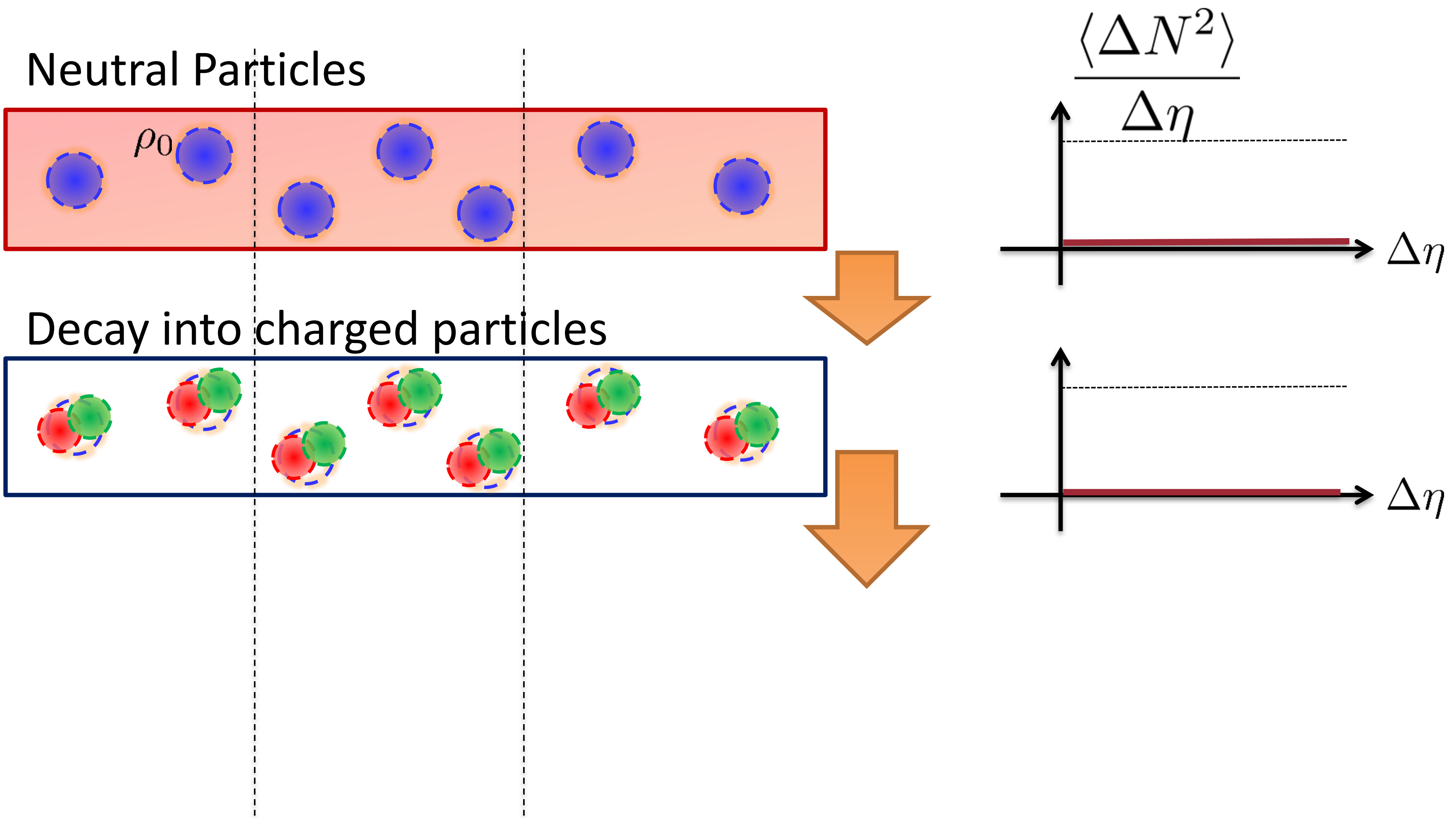
- Large ambiguity in the experimental analysis of higher-order cumulants.

- Fluctuations observed in HIC are not in equilibrium.
- Plenty of information encoded in rapidity window dependences

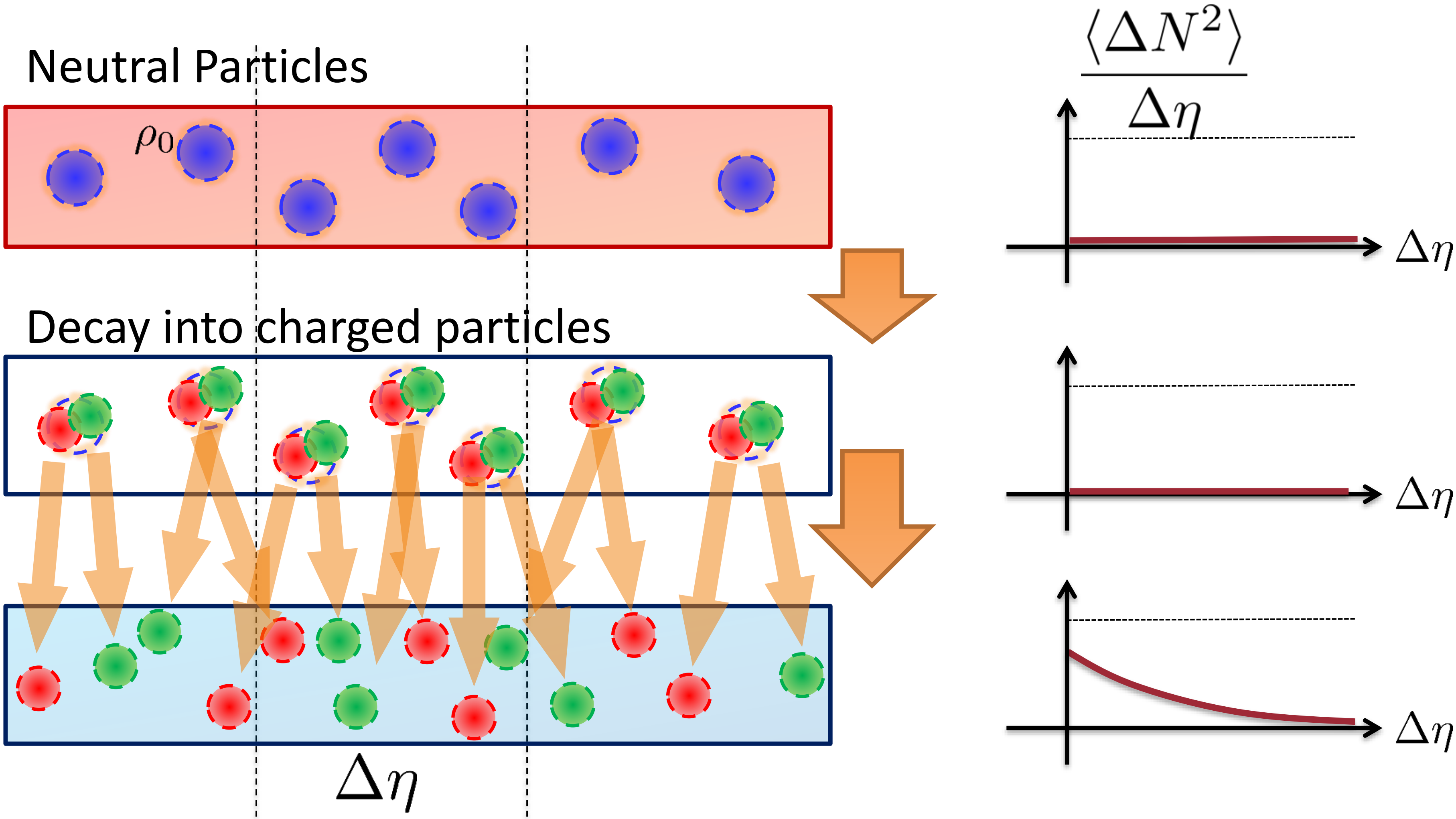
- 2nd-order cumulant (correlation function) already contains interesting information.

- Future
  - Evolution of higher-order cumulants around the critical point / 1st transition
  - combination to momentum (model-H)
  - more realistic model (dimension, Y dependence, ...)

Resonance Decay



Resonance Decay



The larger Δη, the slower diffusion.