Critical Fluctuations in Heavy-Ion Collisions

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Beam-Energy Scan Program in Heavy-Ion Collisions

Our Universe

Color SC

Quark-Gluon Plasma

QCD Critical Point

$T_{\text{high}}$

$T_{\text{low}}$

$150\text{MeV}$

$\sim 10^{15}\text{g/cm}^3$

Hadron Phase (confined)
Event-by-Event Fluctuations

Detector

Structure of distribution reflects microscopic properties

Cumulants: $\langle \delta N_p^2 \rangle$, $\langle \delta N_p^3 \rangle$, $\langle \delta N_p^4 \rangle_c$

Review: Asakawa, MK, PPNP 90 (2016)
A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro
B. 25 x 2 Euro

Same expectation value.
A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro
B. 25 x 2 Euro
C. 1 x 50 Euro

Same expectation value.
But, different fluctuation.
Fluctuations in HIC: 2\textsuperscript{nd} Order

Search for QCD CP

Fluctuation increases

Onset of QGP

Fluctuation decreases

Stephanov, Rajagopal, Shuryak, 1998; 1999

Asakawa, Heinz, Muller, 2000; Jeon, Koch, 2000
Higher-order Cumulants

A. 50 x 1 Euro

\[ 2 \langle \delta \mathbb{E}^2 \rangle = \langle \delta \mathbb{E}^2 \rangle \]

\[ 4 \langle \delta \mathbb{E}^3 \rangle = \langle \delta \mathbb{E}^3 \rangle \]

B. 25 x 2 Euro

\[ 8 \langle \mathbb{E}^4 \rangle_c = \langle \mathbb{E}^4 \rangle_c \]

Asakawa, MK, PPNP 90, 299 (2016)
Non-Gaussian Fluctuations

Onset of QGP

Search for QCD CP

Fluctuation decreases

Fluctuation increases

Ejiri, Karsch, Redlich, 2006

Stephanov, 2009
Sign of Higher-order Cumulants

Higher order cumulants can change sign near CP.

\[ \langle \delta N^3 \rangle = T \frac{\partial \langle \delta N^2 \rangle}{\partial \mu} \]

Asakawa, Ejiri, MK, 2009

Stephanov, 2011; Friman, Karsch, Redlich, Skokov, 2011; ...
Higher-Order Cumulants

Non-zero non-Gaussian cumulants have been established!

Net charge fluctuation

\[ D \approx 4 \frac{\langle \delta N_Q^2 \rangle}{\langle \delta N_Q^2 \rangle_{HRG}} \]

\[ \Delta \eta \]

ALICE, PRL2013
Net-charge fluctuation has a suppression, but net-proton fluctuation does not. Why??
$<\delta N_B^2>$ and $<\delta N_p^2>$ at LHC?

$\langle \delta N_Q^2 \rangle$, $\langle \delta N_B^2 \rangle$, $\langle \delta N_p^2 \rangle$

should have different $\Delta \eta$ dependence.

Baryon # cumulants are experimentally observable! MK, Asakawa, 2012

MK, presentations
GSI, Jan. 2013
Berkeley, Sep. 2014
FIAS, Jul. 2015
GSI, Jan. 2016
...
1. Problems in experimental analysis
   • proper correction of detector’s property

1. Dynamics of non-Gaussian fluctuations

2. A suggestion: chiB/chiQ
Detector-Response Correction

- Correction assuming a binomial response
  - Bialas, Peschanski (1986);
  - MK, Asakawa (2012); Bzdak, Koch (2012); ....
  - But, the response of the detector is not binomial...

![Diagram showing true distribution, efficiency loss, and observed distribution.](image-url)
Slot Machine Analogy

\[ P(N) = N \]

\[ P_1(N) = N \]

\[ P_2(N) = N \]
Extreme Examples

Fixed # of coins

Constant probabilities

N

N

N

N

N

N
Reconstructing Total Coin Number

\[ P(B; N) = \sum P(N) B_{1/2}(N; N) \]

\[ B_p(k; N) = p^k (1 - p)^{N-k} \binom{N}{k} \quad \text{binomial distr. func.} \]
Proton vs Baryon Cumulants

MK, Asakawa, 2012; 2012

Experiments
- proton number cumulants
  \( \langle N_p^n \rangle_c \)

- measurement with 50% efficiency loss

Many theories
- baryon number cumulants
  \( \langle N_B^n \rangle_c \)

- Clear difference b/w these cumulants.

- **Isospin randomization** justifies the reconstruction of \( \langle N_B^n \rangle_c \) via the binomial model.

- Similar problem on the **momentum cut**...
Fragile Higher Orders

Ex.: Relation b/w baryon & proton # cumulants
(with approximations)
MK, Asakawa, 2012

\[
\begin{align*}
2\langle (\delta N_p^{\text{net}})^2 \rangle &= \frac{1}{2} \langle (\delta N_B^{\text{net}})^2 \rangle + \frac{1}{2} \langle (\delta N_B^{\text{net}})^2 \rangle_{\text{free}} \\
2\langle (\delta N_p^{\text{net}})^3 \rangle &= \frac{1}{4} \langle (\delta N_B^{\text{net}})^3 \rangle + \frac{3}{4} \langle (\delta N_B^{\text{net}})^3 \rangle_{\text{free}} \\
2\langle (\delta N_p^{\text{net}})^4 \rangle_c &= \frac{1}{8} \langle (\delta N_B^{\text{net}})^4 \rangle + \cdots
\end{align*}
\]

Higher orders are more seriously affected by efficiency loss.

Genuine info. Poisson noise
Non-Binomial Correction

- **Response matrix**
  \[ \tilde{P}(n) = \sum_{N} R(n; N)P(N) \]
  Reconstruction for any \( R(n; N) \)
  with moments of \( R(n; N) \)
  \[ \langle n^m \rangle_R = \sum_{n} n^m R(n; N) \]

- **Caveats:**
  - \( R(n; N) \) describes the property of the detector.
  - Detailed properties of the detector have to be known.
  - Multi-distribution function can be handled.
  - Huge numerical cost would be required.
  - Truncation is required in general: another systematics?
Result in a Toy-Model

Binomial w/ multiplicity-dependent efficiency

$$\epsilon(N) = \epsilon_0 + (N - N_{ave})\epsilon'$$

Holtzman, Bzdak, Koch (16)

Input $P(N)$: Poisson($\lambda=40$)

$$\epsilon_0 = 0.7$$

Red: true cumulant

Reconstructed cumulants

True cumulants are reproduced within statistics!
Message

Understand 2\textsuperscript{nd}-order fluctuations @ LHC & top-RHIC

1. Problems in experimental analysis
   • proper correction of detector’s property

2. Dynamics of non-Gaussian fluctuations

2. A suggestion: \chi_B/\chi_Q
Why Conserved Charges?

- Direct comparison with theory / lattice
  - Strong constraint from lattice
  - Ignorance on spatial volume of medium
- Slow time evolution
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AHM-JK (2000)

\[ D \sim \frac{\langle \delta N_Q^2 \rangle}{S} \]

S is model dependent

Ejiri-Karsch-Redlich

Ratio of cumulants

\[ \frac{\langle N_Q^4 \rangle_c}{\langle N_Q^2 \rangle_c}, \quad \frac{\langle N_B^4 \rangle_c}{\langle N_B^2 \rangle_c} \]

Experimentally difficult
Time Evolution of Fluctuations

Distributions in $\Delta Y$ and $\Delta y$ are different due to "thermal blurring". 
Ohnishi, MK, Asakawa, PRC(2016)

Fluctuations in $\Delta Y$ continue to change until kinetic f.o.
Variation of a conserved charge is achieved only through diffusion. The larger $\Delta \eta$, the slower diffusion.
Thermal distribution in $y$ space

Blast wave squeezes the distribution in rapidity space

- Assume Bjorken picture
- Blast wave
- Flat freezeout surface

Ohnishi, MK, Asakawa, PRC (2016)
$\Delta \eta$ Dependence

Initial condition (before blurring)
no e-v-e fluctuations

Cumulants after blurring

can take nonzero values

At $\Delta y=1$, the effect is not well suppressed

Cumulants after blurring

$w = \frac{m}{T}$

- pions $w \approx 1.5$
- nucleons $w \approx 9$

Ohnishi, MK, Asakawa, PRC (2016)
Very Low Energy Collisions

- Large contribution of global charge conservation
- Violation of Bjorken scaling

Careful treatment is required to interpret fluctuations at low beam energies!
Many information should be encoded in $\Delta \eta$ dep.
Evolution of Conserved-Charge Fluctuations

Equations describing transport of $n$:

- **Diffusion Equation**
  \[
  \frac{\partial n}{\partial t} = D \nabla^2 n
  \]

- **Stochastic Diffusion Equation (SDE)**
  \[
  \frac{\partial n}{\partial t} = D \nabla^2 n + \nabla \xi(x, t)
  \]

- **SDE with non-linear terms**
  \[
  \frac{\partial n}{\partial t} = \kappa \nabla^2 \frac{\delta F}{\delta n} + \frac{\partial}{\partial x} \xi(x, t)
  \]

\[
\langle \xi(1) \xi(2) \rangle = 2D \chi_2 \delta(1 - 2)
\]

\[
\mathcal{F} = \int dx \left( a \Delta n^2 + c (\nabla n)^2 + \lambda_3 \Delta n^3 + \cdots \right)
\]
Evolution of baryon number density

**Stochastic Diffusion Equation**

\[
\partial_t n = D(t) \partial_x^2 n + \partial_x \xi
\]

\[
\langle \xi(x_1, t_1)\xi(x_2, t_2) \rangle = 2D\chi_2 \delta^{(2)}(1 - 2)
\]

- Analytic solution is obtained.
- Study 2nd order cumulant & correlation function.
Parametrizing $D(\tau)$ and $\chi(\tau)$

- Critical behavior
  - 3D Ising $(r,H)$
  - model H

Berdnikov, Rajagopal (2000)
Stephanov (2011); Mukherjee+(2015)

- Temperature dep.
Crossover / Cumulant

\[ K(\Delta y) = \frac{\langle \delta Q^2 \rangle}{\langle \delta Q^2 \rangle_{eq.}} \]

- Monotonically decreasing

Analytic result

\[ \chi(\tau) \text{ monotonically increasing} \]

\[ K(\Delta y) \text{ monotonically decreasing} \]

ALICE PRL 2013
\[ K(\Delta y) = \frac{\langle \delta Q^2 \rangle}{\langle \delta Q^2 \rangle_{\text{eq.}}} \]

- non-monotonic \( \Delta y \) dep.

Analytic result: \[ K(\Delta y) \] non-monotonic \( \chi(\tau) \) non-monotonic

See also,
Wu, Song
arXiv: 1903.06075
Criticap Point / Correlation Func.

\[ C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}} \]

- non-monotonic \( \Delta y \) dep.

See also, Wu, Song
arXiv: 1903.06075
Away from the CP

\[ K(\Delta y) = \frac{\langle \delta Q^2 \rangle}{\langle \delta Q^2 \rangle_{eq}}. \]

- Signal of the critical enhancement can be clearer on a path away from the CP.

Away from the CP → Weaker critical slowing down
Extension to Higher-order Cumulants

Analyses with
1. Stochastic diffusion equation
2. Diffusion master equation
Baryons in Hadronic Phase

hadronize
chem. f.o.

10~20fm

kinetic f.o.

Baryons behave like Brownian pollens in water

\( p, \bar{p} \)
\( n, \bar{n} \)
\( \Delta(1232) \)

mesons
baryons
(Non-Interacting) Brownian Particle Model

Initial condition (uniform)

Cumulants: $\langle \tilde{Q}^2 \rangle_c$, $\langle \tilde{Q}^3 \rangle_c$, $\langle \tilde{Q}^4 \rangle_c$

Random walk

diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)
(Non-Interacting) Brownian Particle Model

Initial condition (uniform)

$\Delta Y_{\text{drift}}$

diffusion distance

t $\rightarrow \infty$

Poisson distribution

$\Delta Y$

Study $\Delta Y$ dependence

cumulants: $\langle \tilde{Q}^2 \rangle_c$, $\langle \tilde{Q}^3 \rangle_c$, $\langle \tilde{Q}^4 \rangle_c$

random walk

diffusion master equation: MK+, PLB(2014)

probabilistic argument: Ohnishi+, PRC(2016)
Before the diffusion

\[ D_4 = 4, \ D_2 = 1 \]

\[ \langle \delta N^4 \rangle_{\text{Skellam}} \]

\[ \Delta \eta / \Delta \eta_{\text{drift}} \]

Initial Condition

\[ D_4 = \frac{\langle Q_{\text{net}}^4 \rangle_c}{\langle Q_{\text{tot}} \rangle} = 4 \]

\[ b = \frac{\langle Q_{\text{net}}^2 Q_{\text{tot}} \rangle_c}{\langle Q_{\text{net}} \rangle} \]

\[ c = \frac{\langle Q_{\text{tot}}^2 \rangle_c}{\langle Q_{\text{tot}} \rangle} \]

\[ D_2 = \frac{\langle Q_{\text{net}}^2 \rangle_c}{\langle Q_{\text{tot}} \rangle} = 1 \]
4th Order Cumulant

After the diffusion

$D_4 = 4, D_2 = 1$

- Cumulant at small $\Delta \eta$ is modified toward a Poisson value.
- Non-monotonic behavior can appear.
Time Evolution of Fluctuations

As a result of a simple random walk...

After the diffusion

Before the diffusion

$\langle \delta N^4 \rangle / \text{Skellam}$

$\Delta \eta / \Delta \eta_{\text{drift}}$
Rapidity Window Dep.

4th-order cumulant

Initial Conditions

\[ D_4 = \frac{\langle Q_{\text{net}}^4 \rangle}{\langle Q_{\text{tot}} \rangle} \quad b = \frac{\langle Q_{\text{net}}^2 Q_{\text{tot}} \rangle}{\langle Q_{\text{net}} \rangle} \]

\[ D_2 = \frac{\langle Q_{\text{net}}^2 \rangle}{\langle Q_{\text{tot}} \rangle} \quad c = \frac{\langle Q_{\text{tot}}^2 \rangle}{\langle Q_{\text{tot}} \rangle} \]

Is non-monotonic \( \Delta \eta \) dependence already observed?

Different initial conditions give rise to different characteristic \( \Delta \eta \) dependence. \( \rightarrow \) Study initial condition

Finite volume effects: Sakaida+., PRC90 (2015)
SDE with Non-Linear Terms

Higher order cumulants

Nahrgang, Bluhm, Schaefer, Bass, PRD (2019); Pihan, Touroux, Nahrgang, Bluhm, Sami, MK, in prep.

Time evolution of 4th cumulant can be described.

1st order transition

Domain formation and peak structure in the correlation function are found.
Message

Understand 2^{nd}-order fluctuations @ LHC & top-RHIC

1. Problems in experimental analysis
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1. Dynamics of non-Gaussian fluctuations

2. A suggestion: chiB/chiQ
Net-charge fluctuation has a suppression, but net-proton fluctuation does not. Why??
<$$\delta N_B^2$$> and <$$\delta N_p^2$$> @ LHC?

$$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$$ should have different $$\Delta \eta$$ dependence.

Baryon # cumulants are experimentally observable! MK, Asakawa, 2012
A Suggestion

Net charge fluctuation

Construct $\langle \delta N_{B}^{2} \rangle / \langle \delta N_{N}^{2} \rangle$, $\langle \delta N_{Q}^{2} \rangle$

Then, take ratio $\langle \delta N_{B}^{2} \rangle / \langle \delta N_{Q}^{2} \rangle$

Compare it with lattice

Net proton fluctuation

HotQCD preliminary

✓ linear T dependence near $T_c$ !!
✓ only 2nd order: reliable !!
Prediction

LATTICE

\[ \frac{\langle \delta N^2_B \rangle}{\langle \delta N^2_Q \rangle} \]

ALICE

Primordial Fluctuation

\[ \sqrt{s_{NN}} = 5.02 \text{ TeV} \]

2.76 TeV

200 GeV

Resonance decays

HotQCD preliminary before continuum limit

HotQCD preliminary for tracing back the history!

\[ \Delta \eta \]

1.6

\[ \Delta \eta \] dependence for tracing back the history!
Summary

- Large ambiguity in the experimental analysis of higher-order cumulants.
- Fluctuations observed in HIC are not in equilibrium.
- Plenty of information encoded in rapidity window dependences
- 2nd-order cumulant (correlation function) already contains interesting information.

Future
- Evolution of higher-order cumulants around the critical point / 1st transition
- Combination to momentum (model-H)
- More realistic model (dimension, Y dependence, ...)
Resonance Decay

Neutral Particles

Decay into charged particles

\[ \langle \Delta N^2 \rangle \]
Resonance Decay

The larger $D_h$, the slower diffusion.

Neutral Particles

Decay into charged particles

$\langle \Delta N^2 \rangle$

$\Delta \eta$

The larger $\Delta \eta$, the slower diffusion.