Asia-Pacific Symposium for Lattice Field Theory (APLAT2020) 2020/Aug./4

Anisotropic pressure induced by finite-size effects at nonzero temperature in SU(3) YM theory

Masakiyo Kitazawa

(Osaka U.)

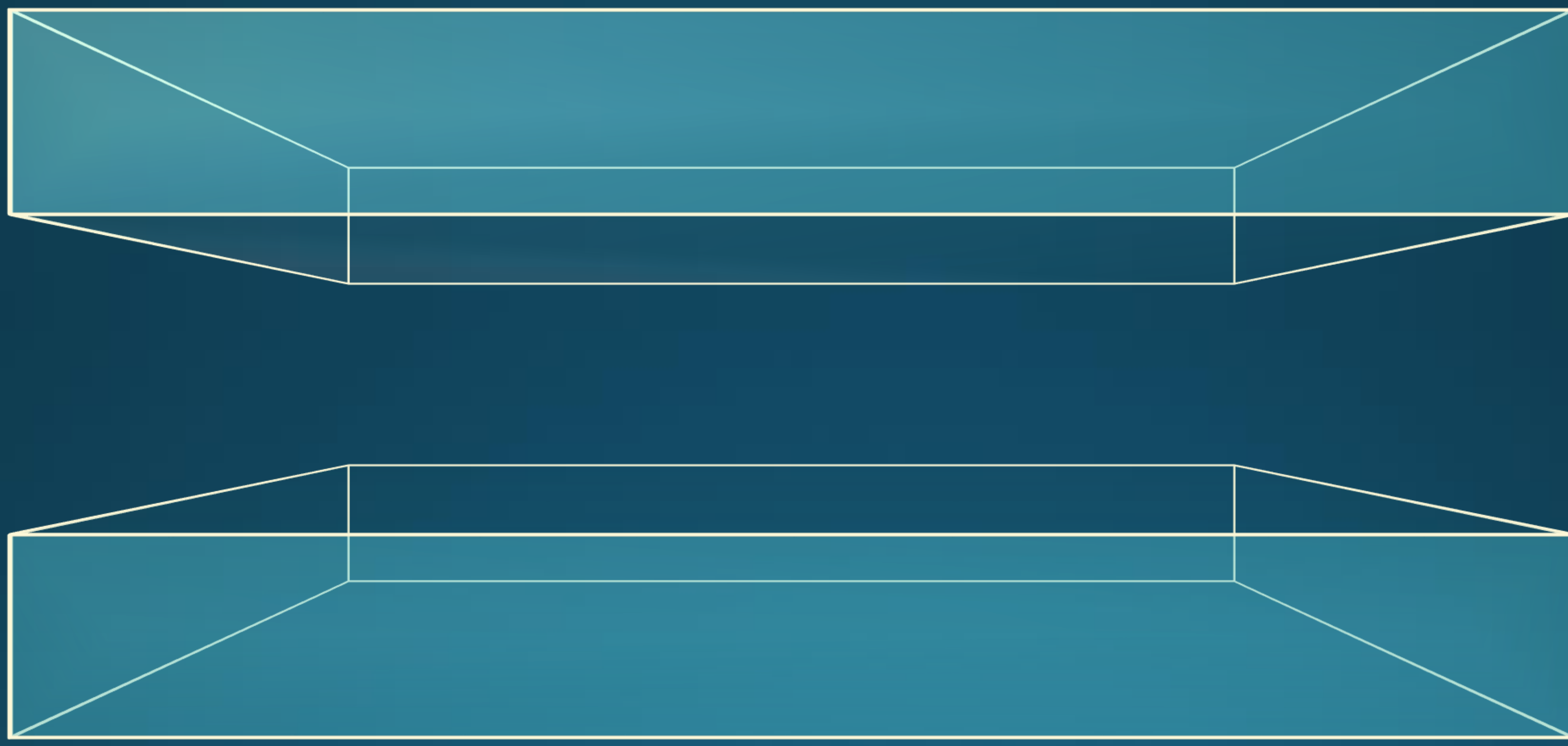
with S. Mogliacci, I. Kolbe, W.A. Horowitz

MK, Mogliacci, Kolbe, Horowitz, Phys.Rev.D 99 (2019) 094507

[arXiv:1904.00241[hep-lat]]

Casimir Effect

Casimir Effect



attractive force between two conductive plates

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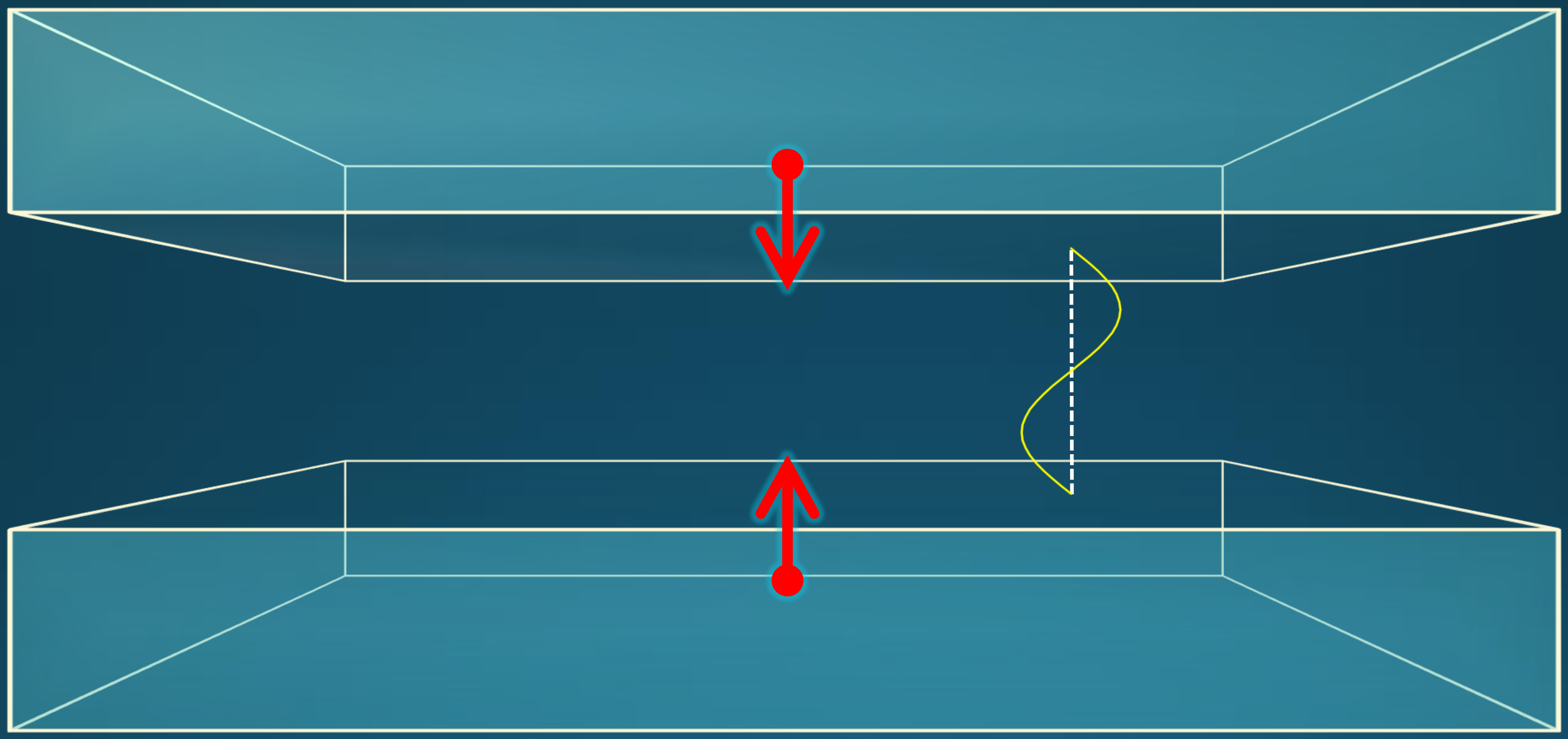

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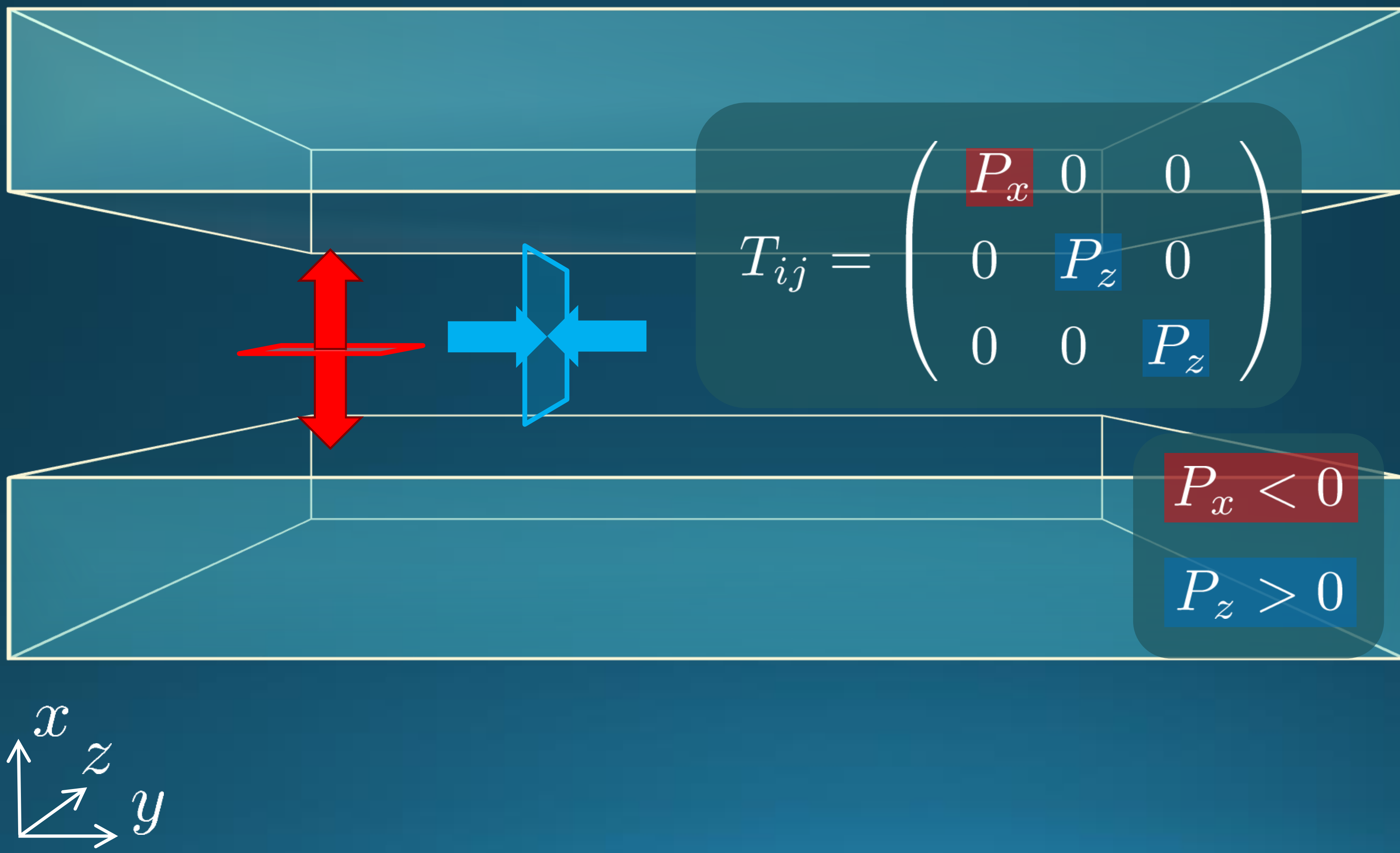
Brown, Maclay
1969



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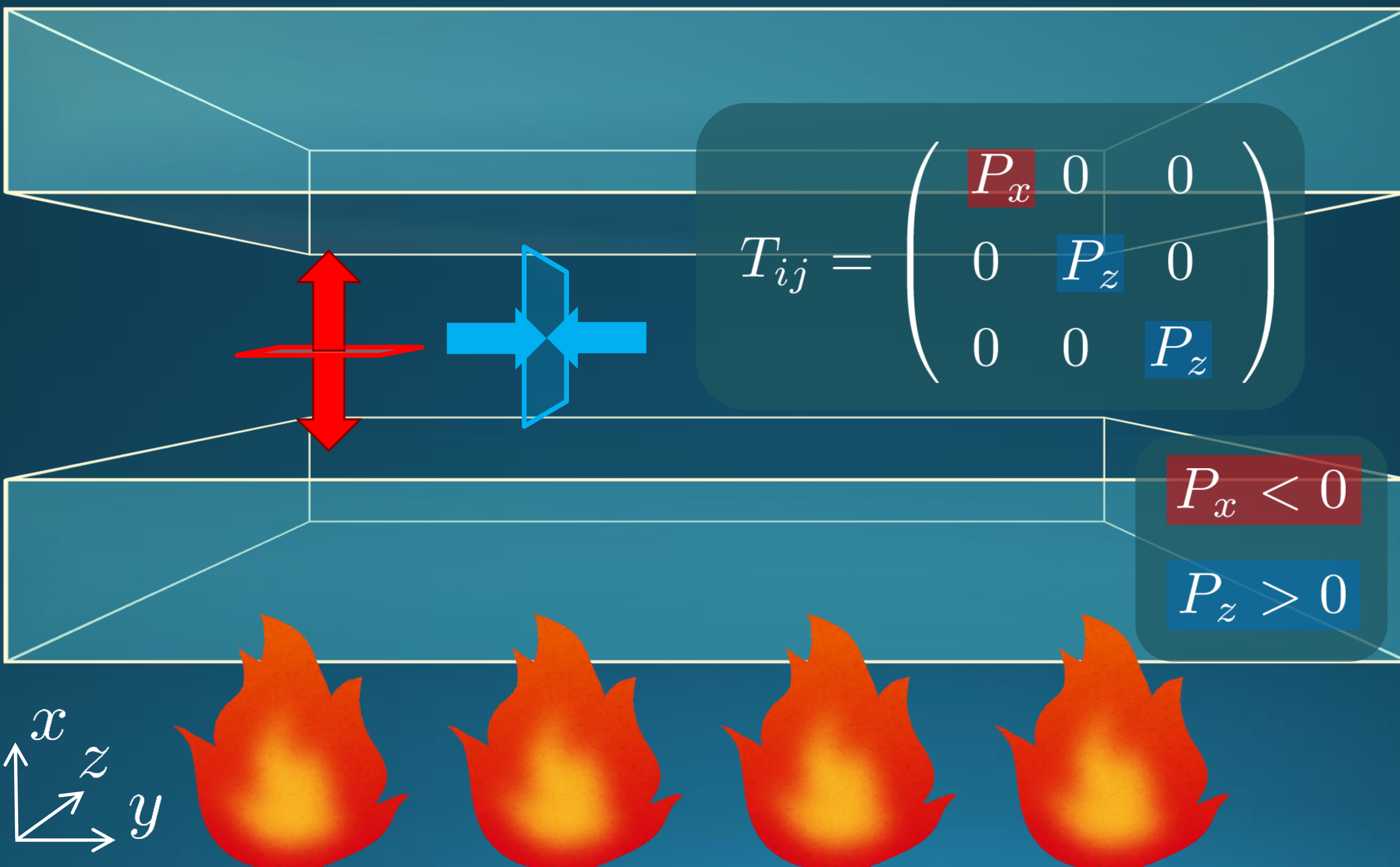

$$T_{ij} = \begin{pmatrix} P_x & 0 & 0 \\ 0 & P_z & 0 \\ 0 & 0 & P_z \end{pmatrix}$$

$$P_x < 0$$

$$P_z > 0$$

Casimir Effect

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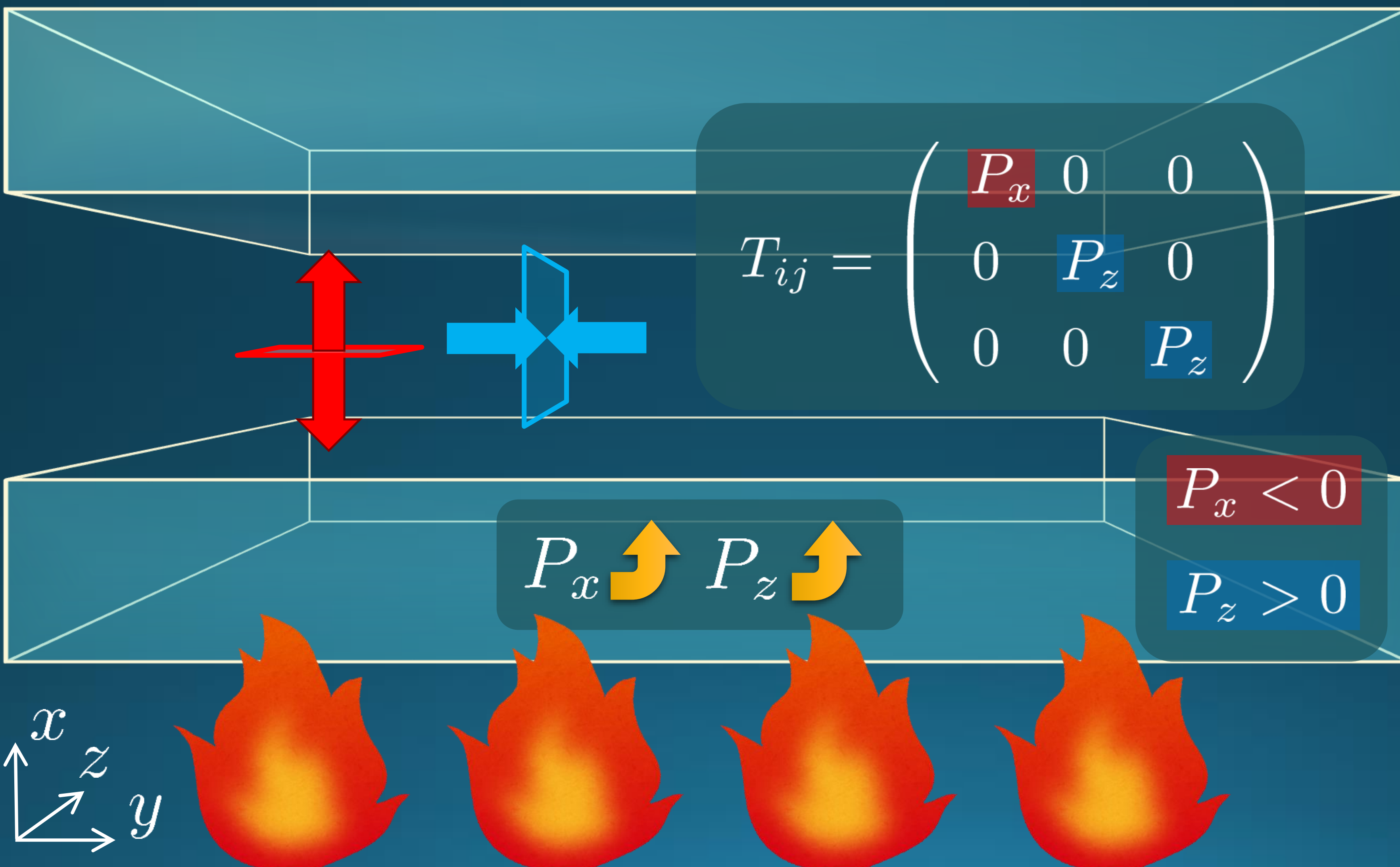

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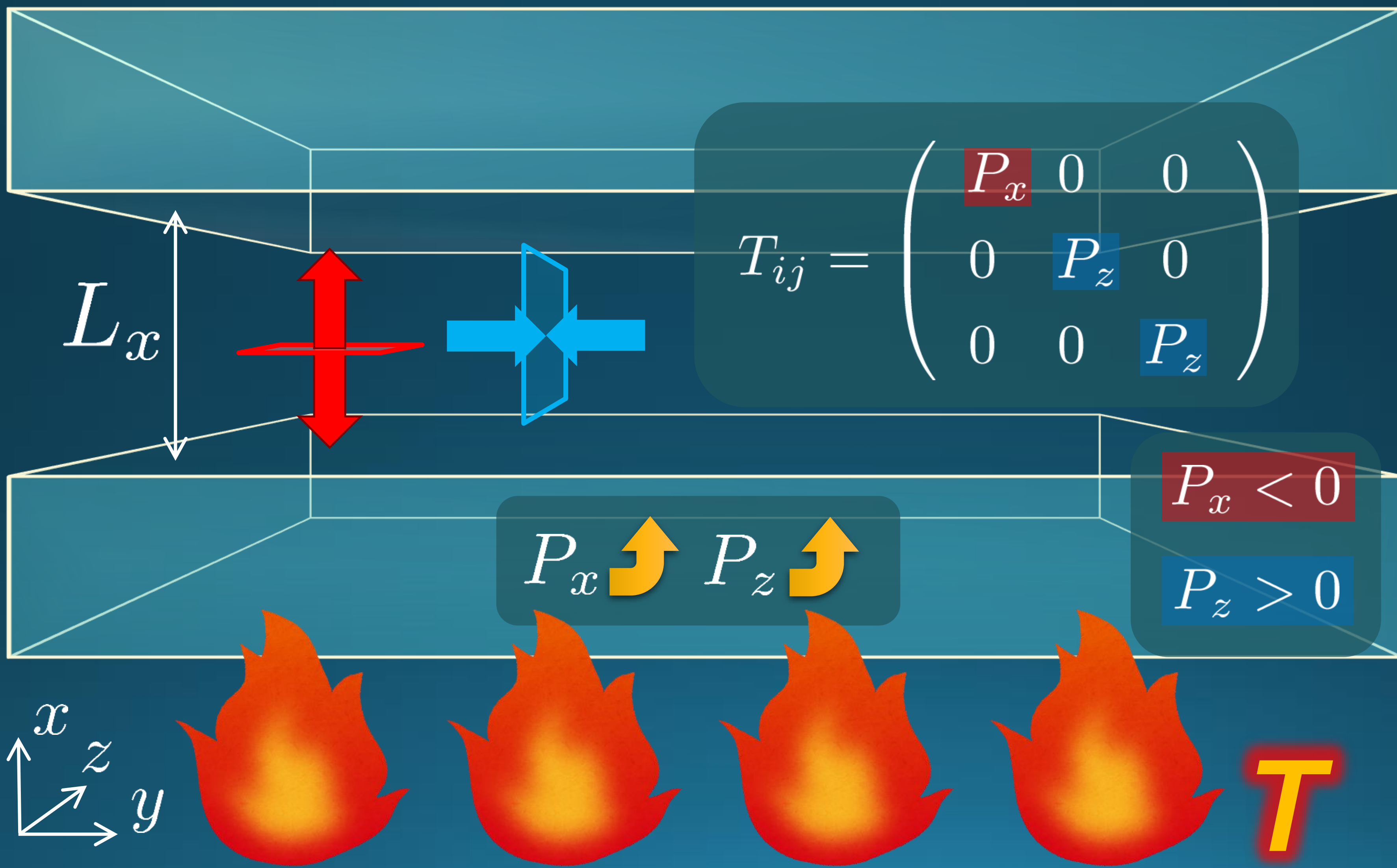
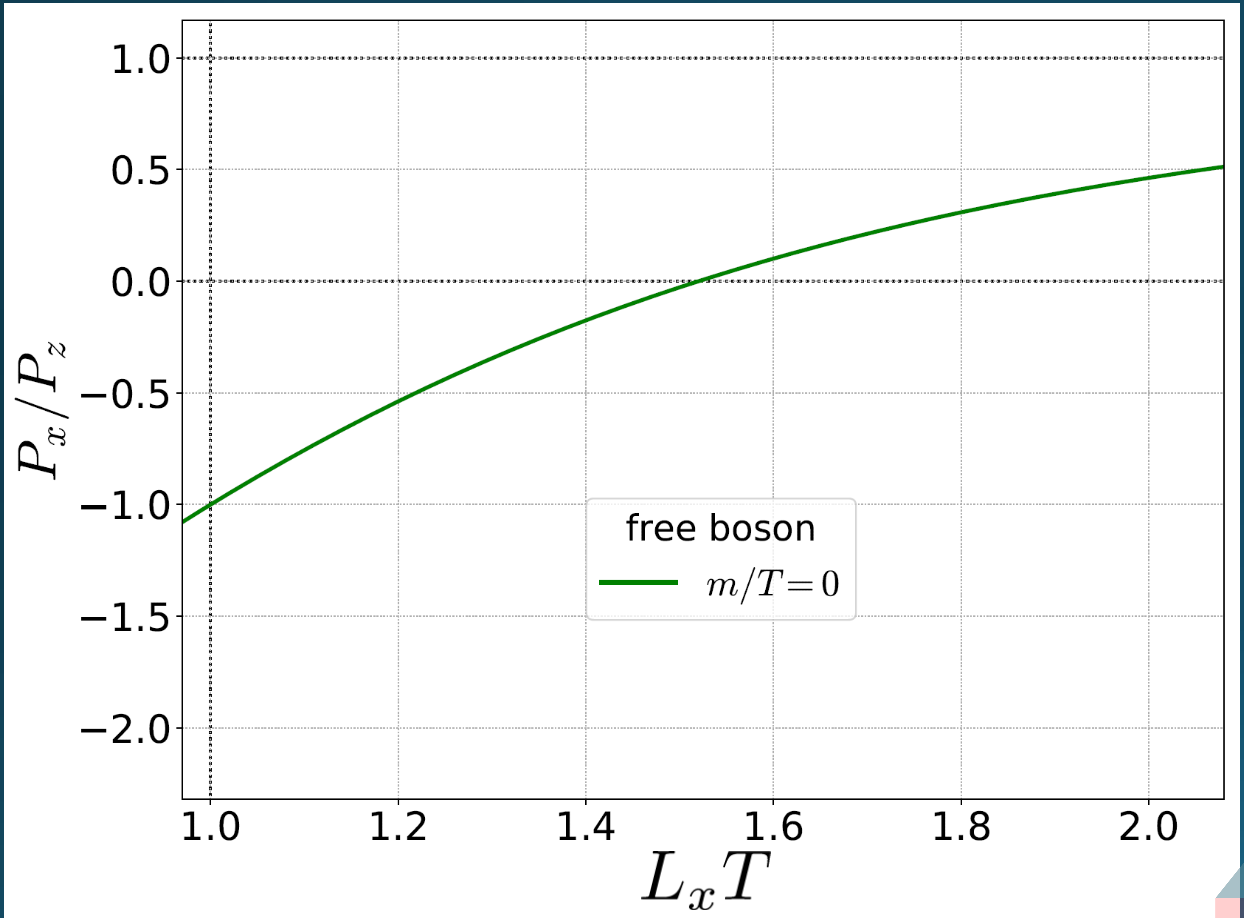


Pressure Anisotropy @ T≠0

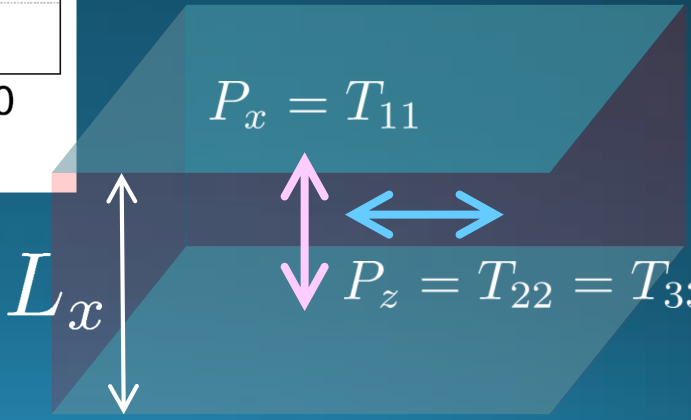
MK, Mogliacci, Kolbe, Horowitz, PRD(2019)



Free scalar field

- ☐ $L_2=L_3=\infty$
- ☐ Periodic BC

Mogliacci+, 1807.07871



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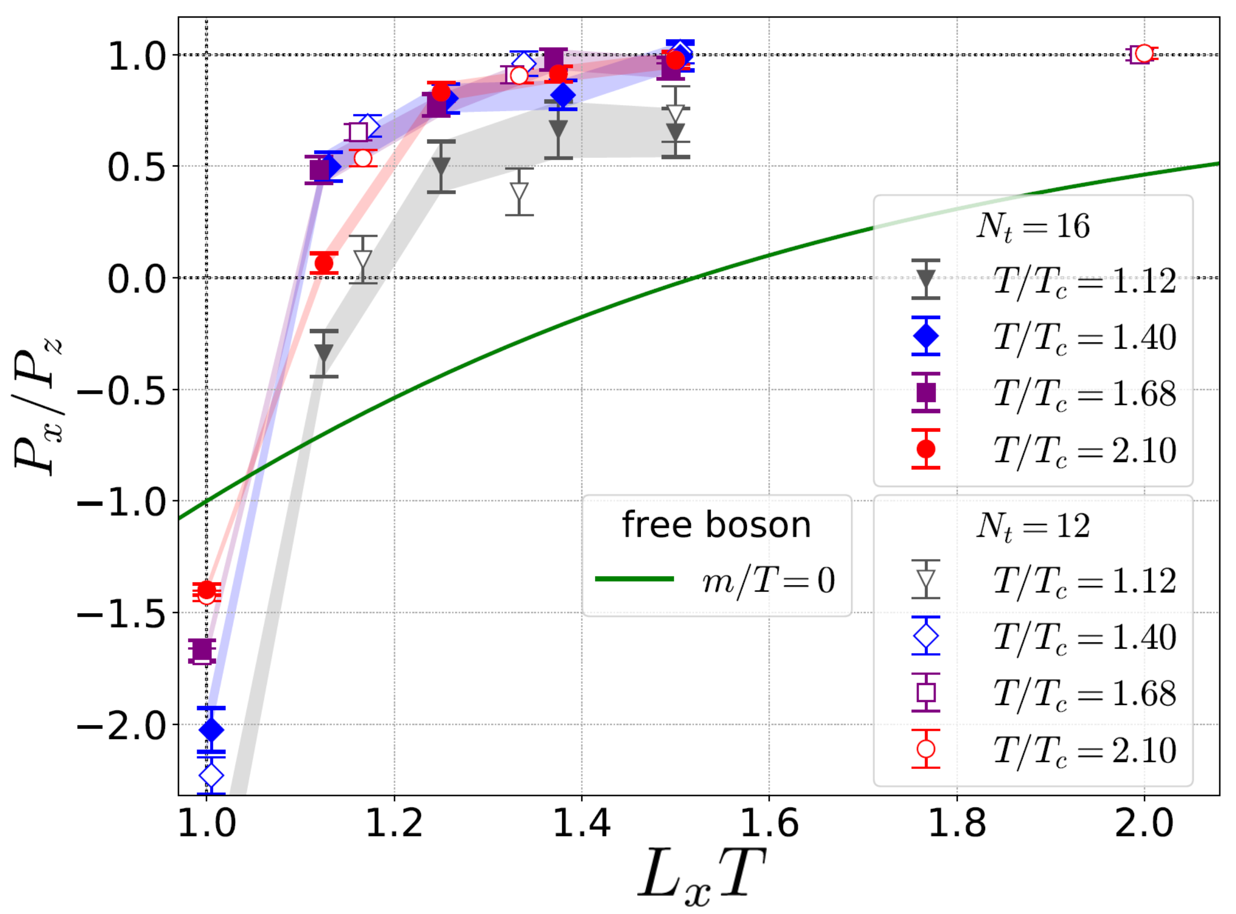

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Lattice result

- Periodic BC
- Only t→0 limit
- Error: stat.+sys.

Medium near T_c is remarkably insensitive to finite size!

Thermodynamics on the Lattice

Various Methods

- **Integral**, differential, moving frame, non-equilibrium, ...
- rely on thermodynamic relations valid in $V \rightarrow \infty$

$$P = \frac{T}{V} \ln Z$$

$$sT = \varepsilon + P$$

➡ **Not applicable to anisotropic systems**

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- We employ **Gradient Flow Method**

$$\varepsilon = \langle T_{00} \rangle \quad P = \langle T_{11} \rangle$$

Components of EMT are directly accessible!

Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

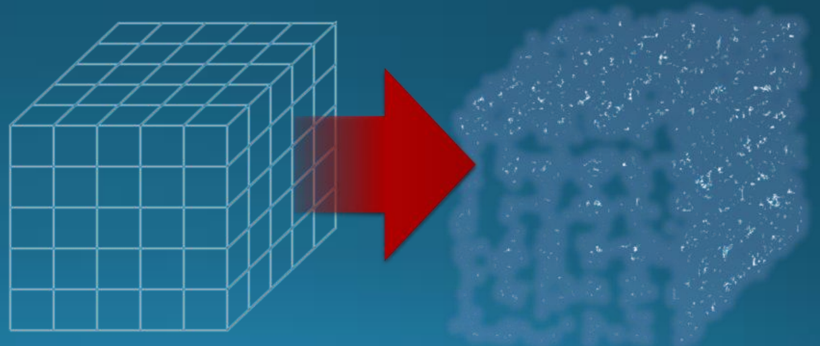
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length^2]

leading

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at t>0



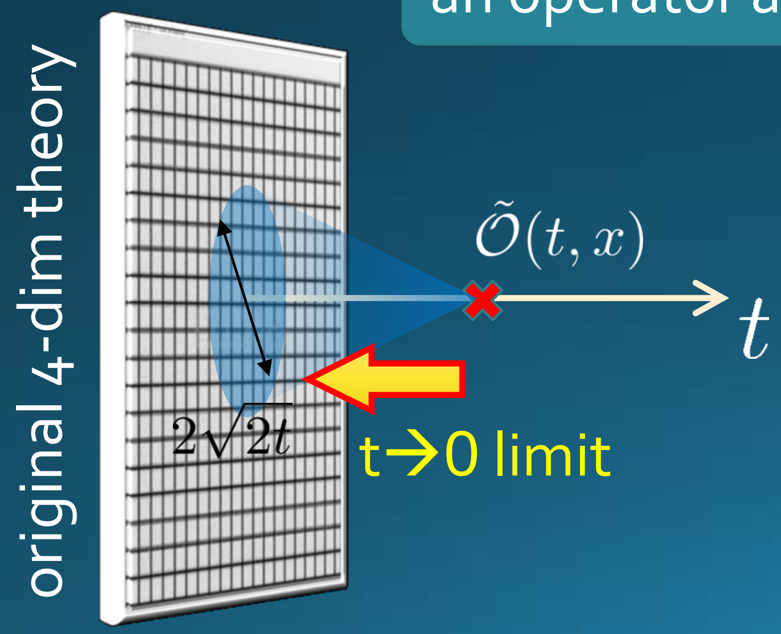
Small Flow-Time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at t>0

remormalized operators of original theory

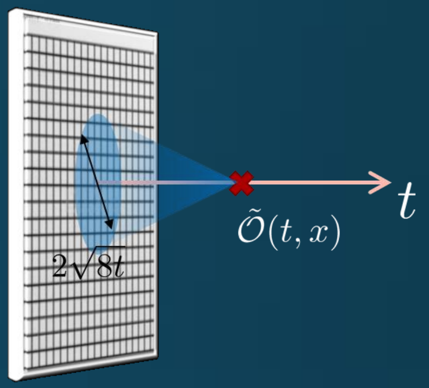


Constructing EMT

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t) T^R_{\rho\rho}(x) + \mathcal{O}(t)$$



vacuum subtr.

Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\text{subt.}}\right]$$

Perturbative coefficient:
Suzuki (2013); Makino, Suzuki (2014); Harlander+ (2018); Iritani, MK, Suzuki, Takaura (2019)

Perturbative Coefficients

$$T_{\mu\nu}(t) = c_1(t)U_{\mu\nu}(t) + \delta_{\mu\nu}c_2(t)E(t)$$

	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○	○	
$c_2(t)$	× zero	○	○	○

Suzuki (2013) Harlander+(2018)

Iritani, MK, Suzuki, Takaura, 2019

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

□ Choice of the scale of g²

$$c_1(t) = c_1\left(g^2(\mu(t))\right)$$

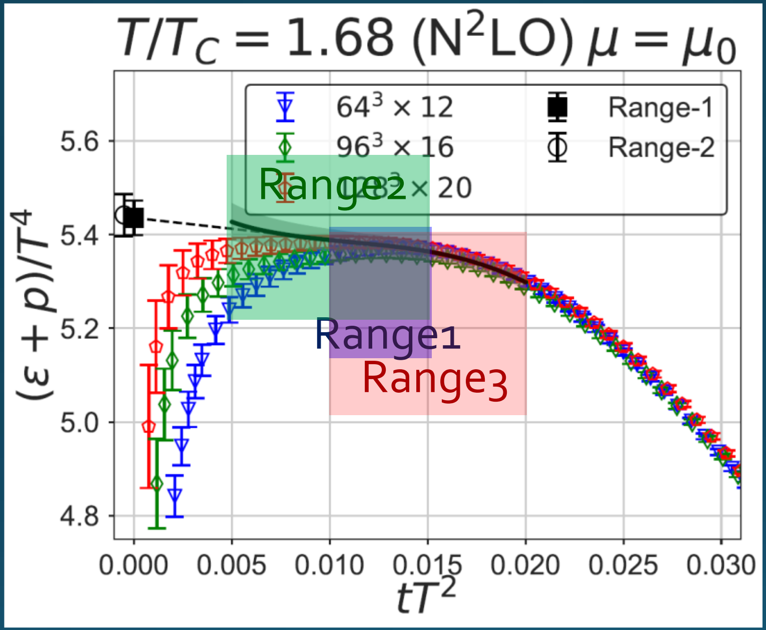
Previous: $\mu_d(t) = 1/\sqrt{8t}$

Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E}t}$

Harlander+ (2018)

t→0 Extrapolation: ε+p

N²LO (2-loop)

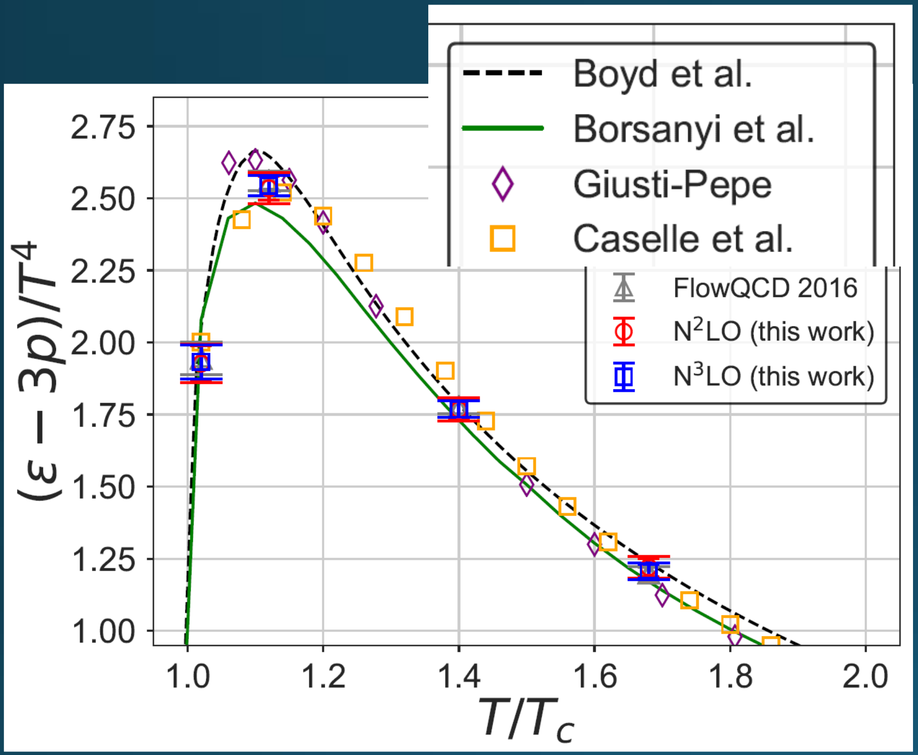


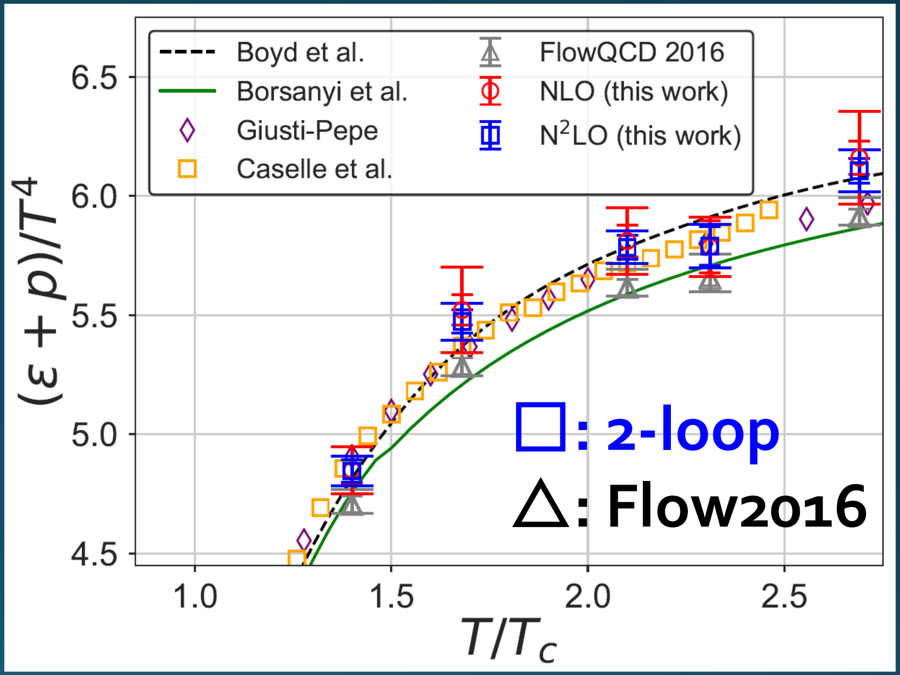
Iritani, MK, Suzuki, Takaura, PTEP 2019

- Stable t→0 extrapolation with higher order coeff.
- Systematic error: fit range, μ_0 or μ_d, uncertaintyof Λ (±3%)
- Extrapolation func: linear, higher order term in c_1 (~g^6)

Effect of Higher-Order Coeffs.

Iritani, MK, Suzuki, Takaura, 2019





Systematic error: μ_0 or μ_d, Λ, t→0 function, fit range

More stable extrapolation with higher order c_1 & c_2 (pure gauge)

Numerical Setup

- SU(3) YM theory
- Wilson gauge action

- N_t = 16, 12
- N_z/N_t=6
- 2000~4000 confs.
- Even N_x

- No Continuum extrap.

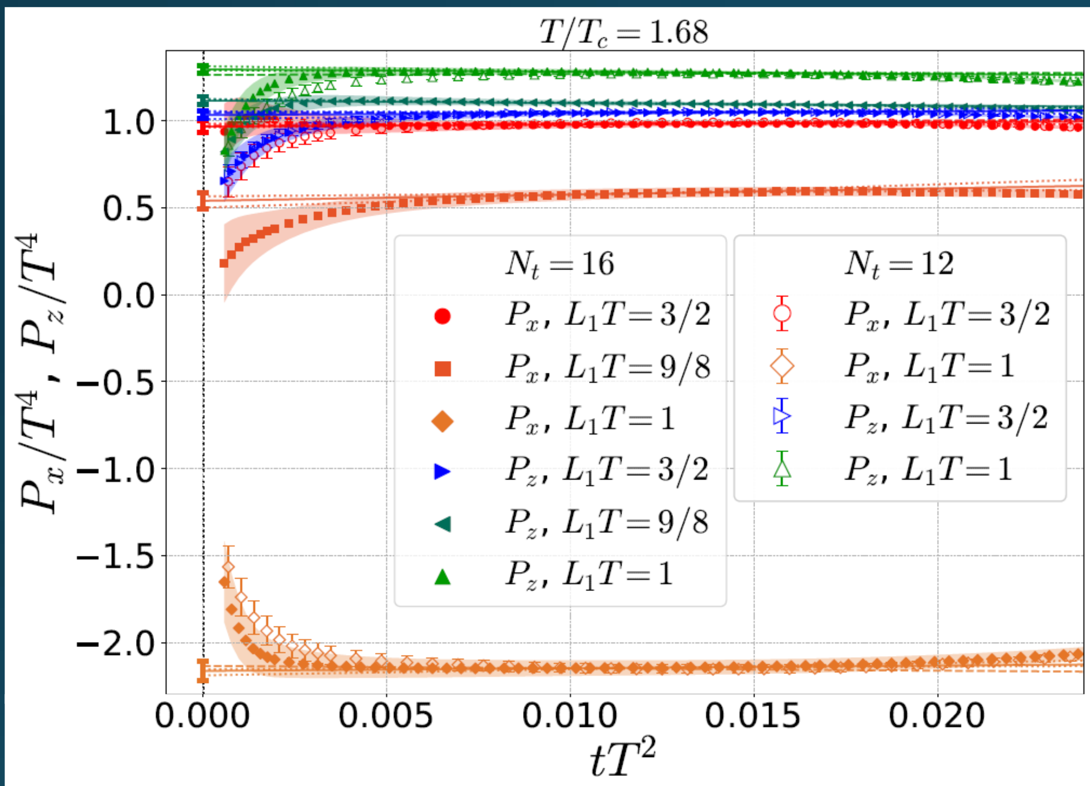
T/T_c	β	N_z	N_τ	N_x	$N_{\rm vac}$
1.12	6.418	72	12	12, 14, 16, 18	64
	6.631	96	16	16, 18, 20, 22, 24	96
1.40	6.582	72	12	12, 14, 16, 18	64
	6.800	96	16	16, 18, 20, 22, 24	128
1.68	6.719	72	12	12, 14, 16, 18, 24	64
	6.719	96	12	14, 18	64
	6.941	96	16	16, 18, 20, 22, 24	96
2.10	6.891	72	12	12, 14, 16, 18, 24	72
	7.117	96	16	16, 18, 20, 22, 24	128
2.69	7.086	72	12	12, 14, 16, 18	-
$\simeq$ 8.1	8.0	72	12	12, 14, 16, 18	-
$\simeq$ 25	9.0	72	12	12, 14, 16, 18	-

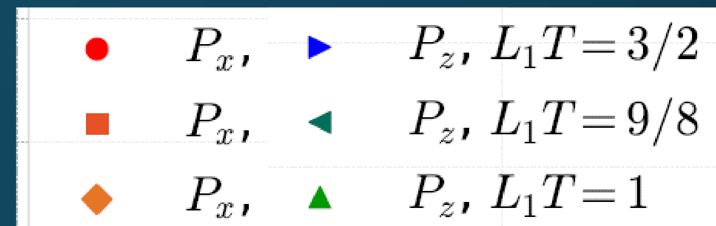
- Same Spatial volume
  - $12 \times 72^2 \times 12$ ~ $16 \times 96^2 \times 16$
  - $18 \times 72^2 \times 12$ ~ $24 \times 96^2 \times 16$

Simulations on
OCTOPUS/Reedbush

Small-t Extrapolation

$T/T_c = 1.68$





Filled: N_t=16 / Open: N_t=12

Small-t extrapolation

- Solid: N_t=16, Range-1
- Dotted: N_t=16, Range-2,3
- Dashed: N_t=12, Range-1

- ☐ Stable small-t extrapolation
- ☐ No N_t dependence within statistics for L_xT=1, 1.5

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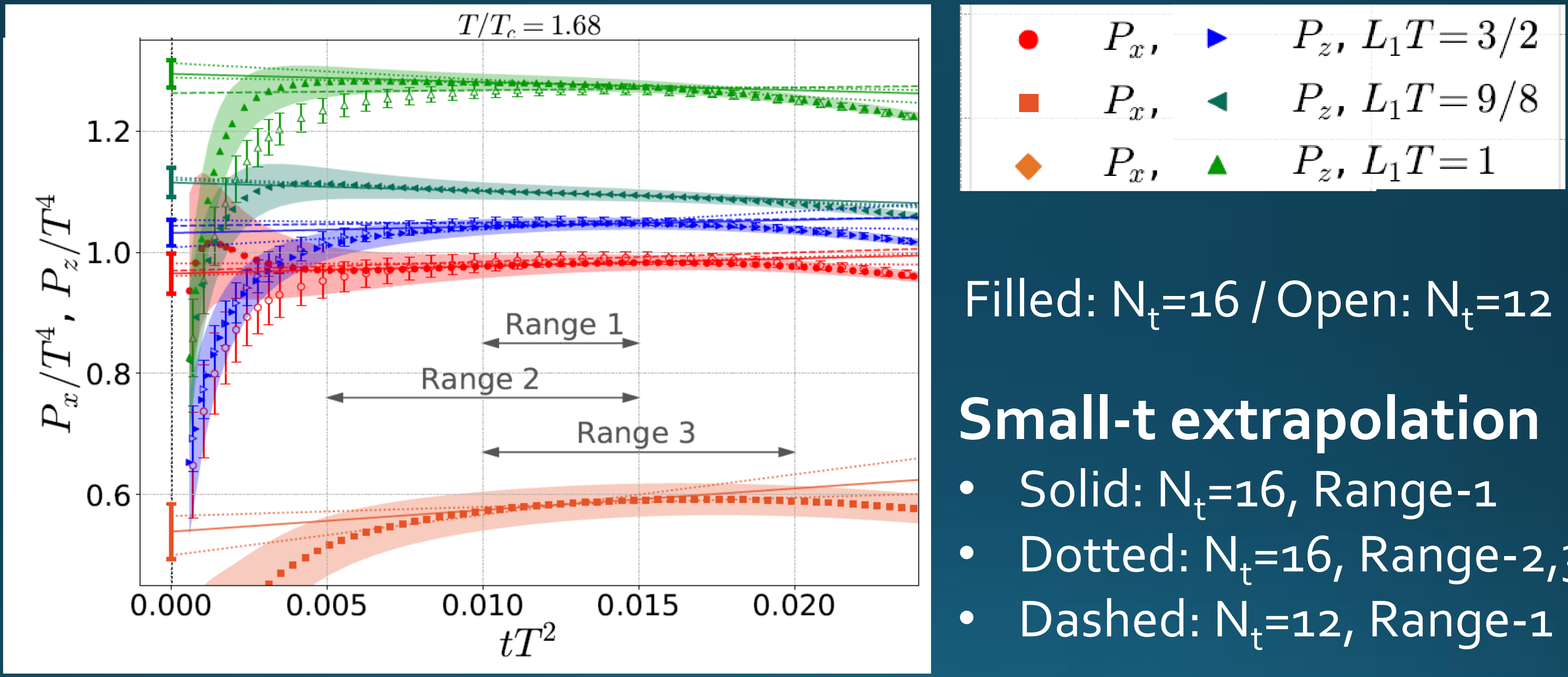

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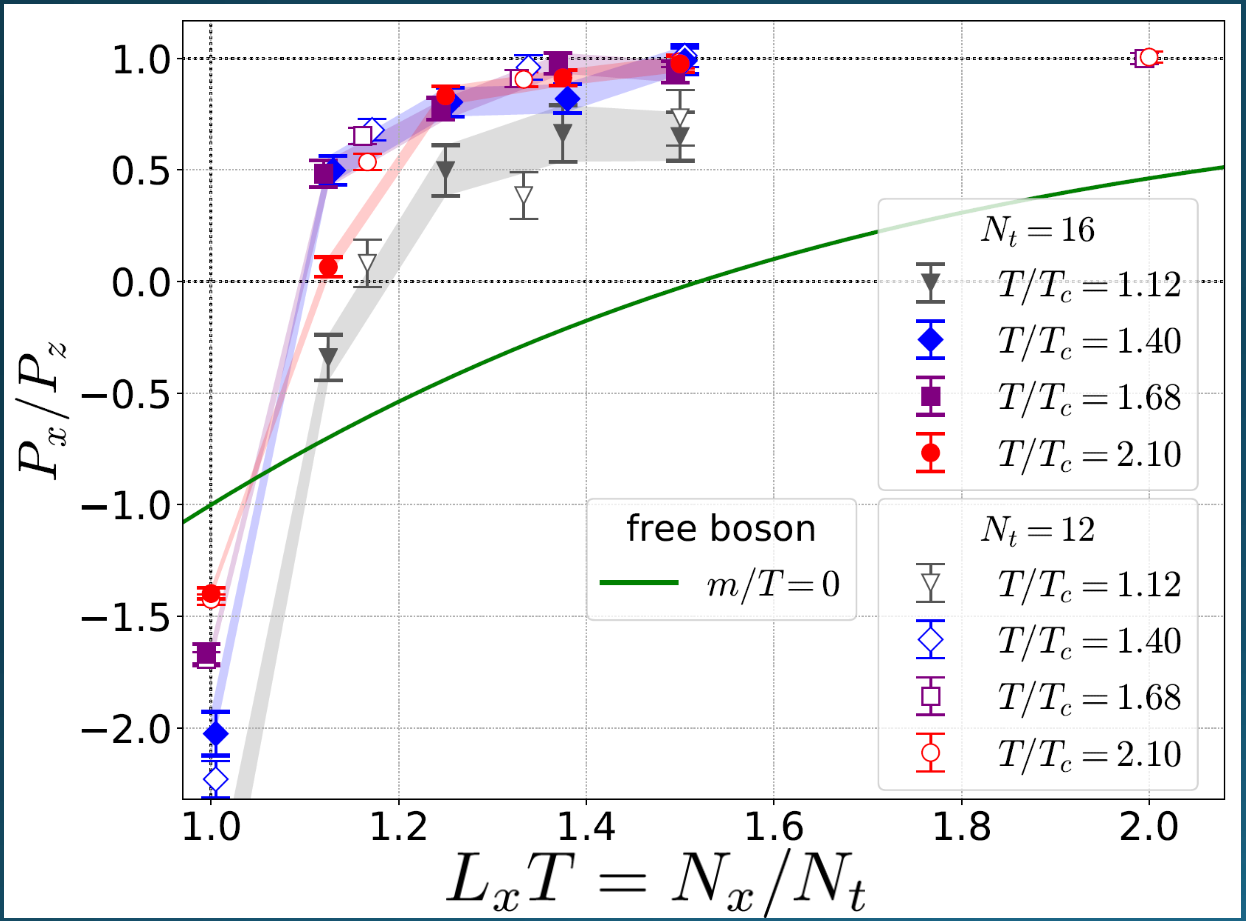
Pressure Anisotropy @ T≠0

MK, Mogliacci, Kolbe, Horowitz, PRD(2019)



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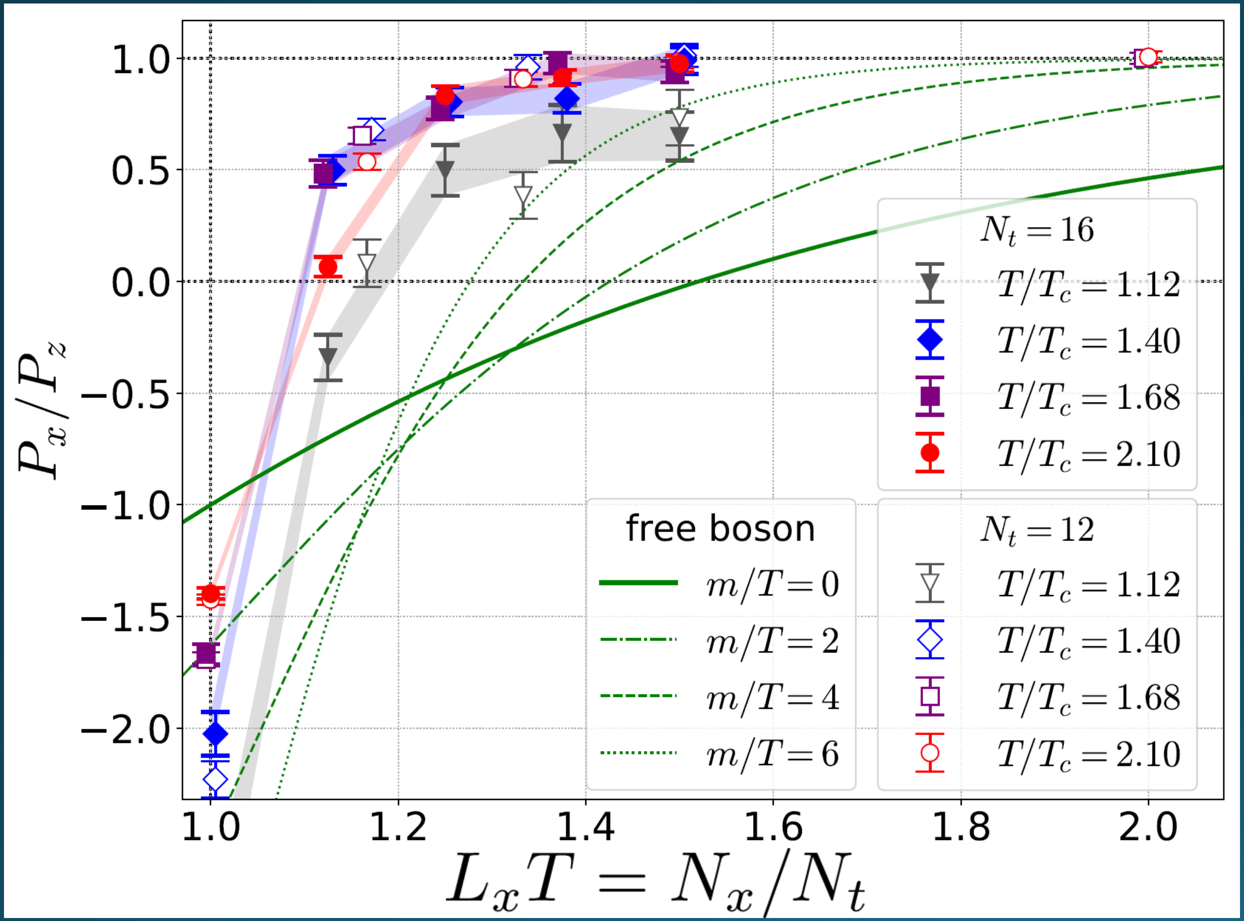

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Higher T

High-T limit: massless free gluons

How does the anisotropy approach this limit?

Difficulties

- ☐ Vacuum subtraction requires large-volume simulations.
- ☐ Lattice spacing not available → $c_1(t)$, $c_2(t)$ are not determined.

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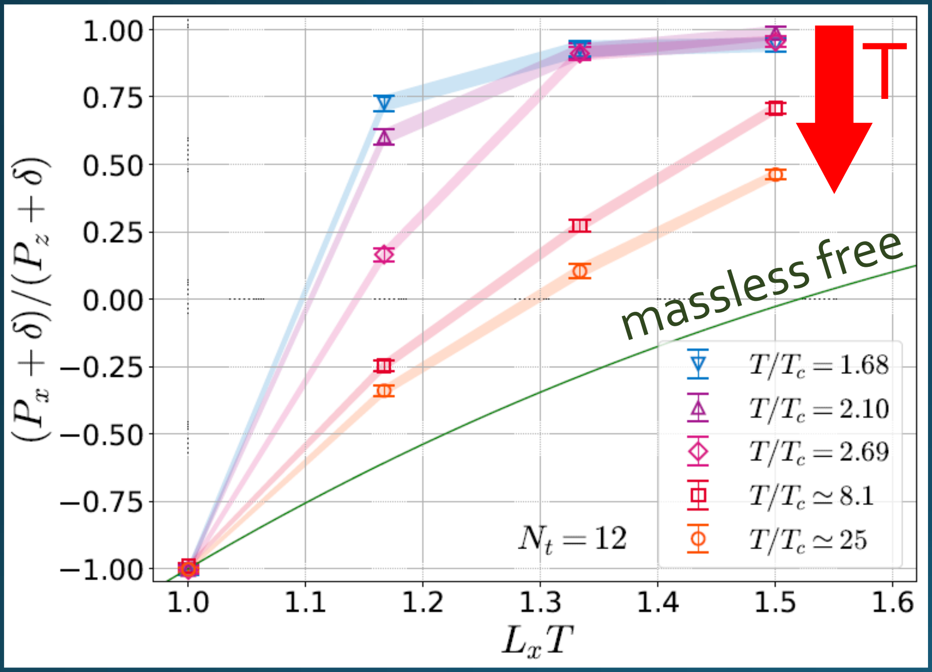
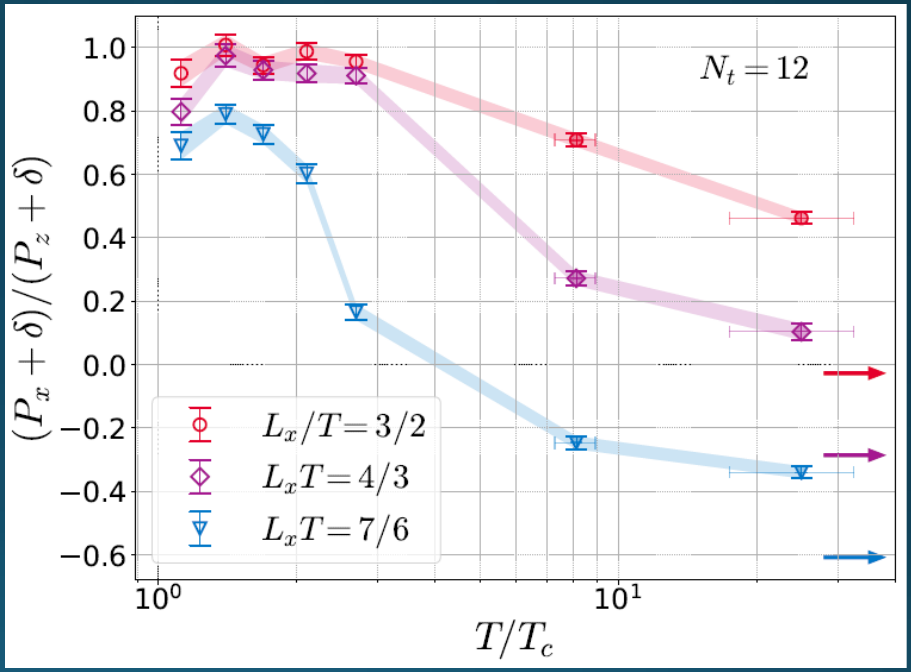
We study

$$\frac{P_x + \delta}{P_z + \delta} \qquad \delta = -\frac{1}{4}\sum_{\mu} T^{\mathrm{E}}_{\mu\mu}$$

No vacuum subtr. nor Suzuki coeffs. necessary!

$$\frac{P_x + \delta}{P_z + \delta}$$

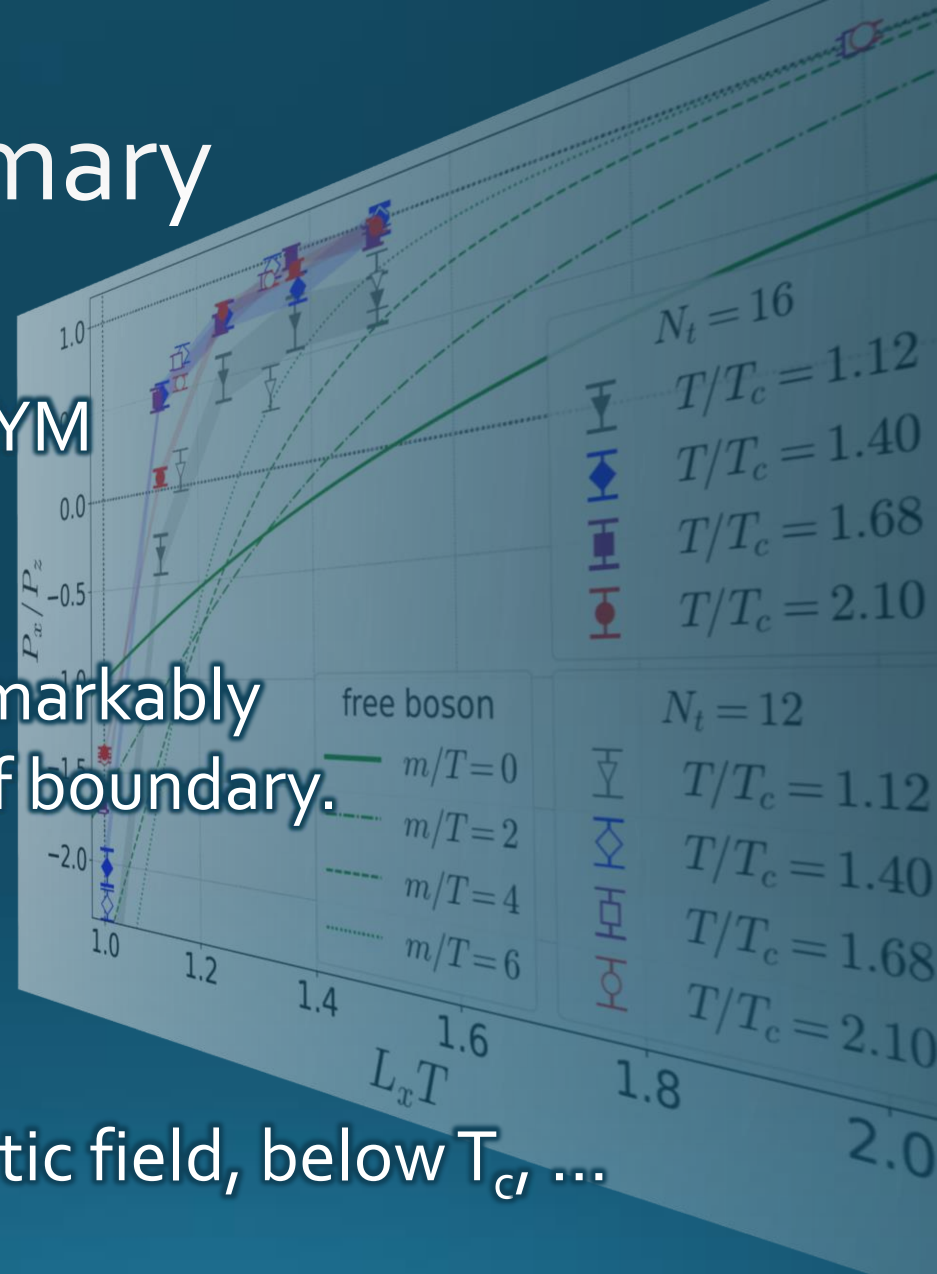




$T/T_c \cong 8.1$ (β=8.0) / $T/T_c \cong 25$ (β=9.0)

- ☐ Ratio approaches the asymptotic value.
- ☐ But, large deviation exists even at $T/T_c \sim 25$.

Summary



First numerical simulation of anisotropic pressure in SU(3) YM with periodic BC.

Medium at $1.4 < T/T_c < 2.1$ is remarkably insensitive to the existence of boundary.

Future

Anti-periodic / Dirichlet BCs

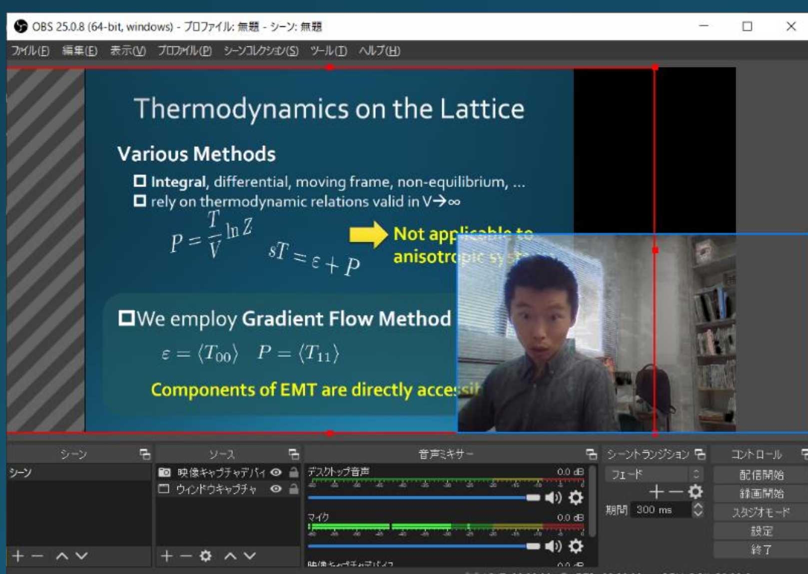
BC for two directions, magnetic field, below T_c, …

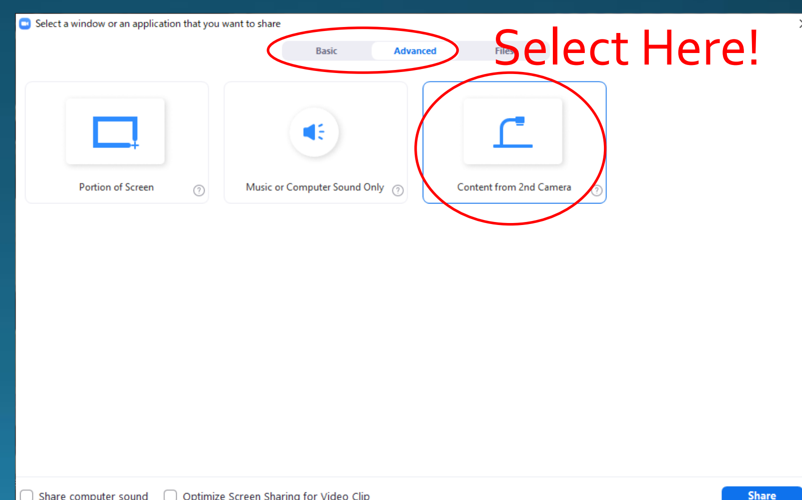
And, many other problems related to EMT!!

backup

Tips for Presentation

- Install
  - OBS Studio https://obsproject.com/
  - OBS VirtualCam https://github.com/CatxFish/obs-virtual-cam/
- Setup
  - tools – VirtualCam – Start
  - Chroma key composition: Need a green sheet behind you!
- Zoom: Share Screen – Advanced – Content from 2nd camera

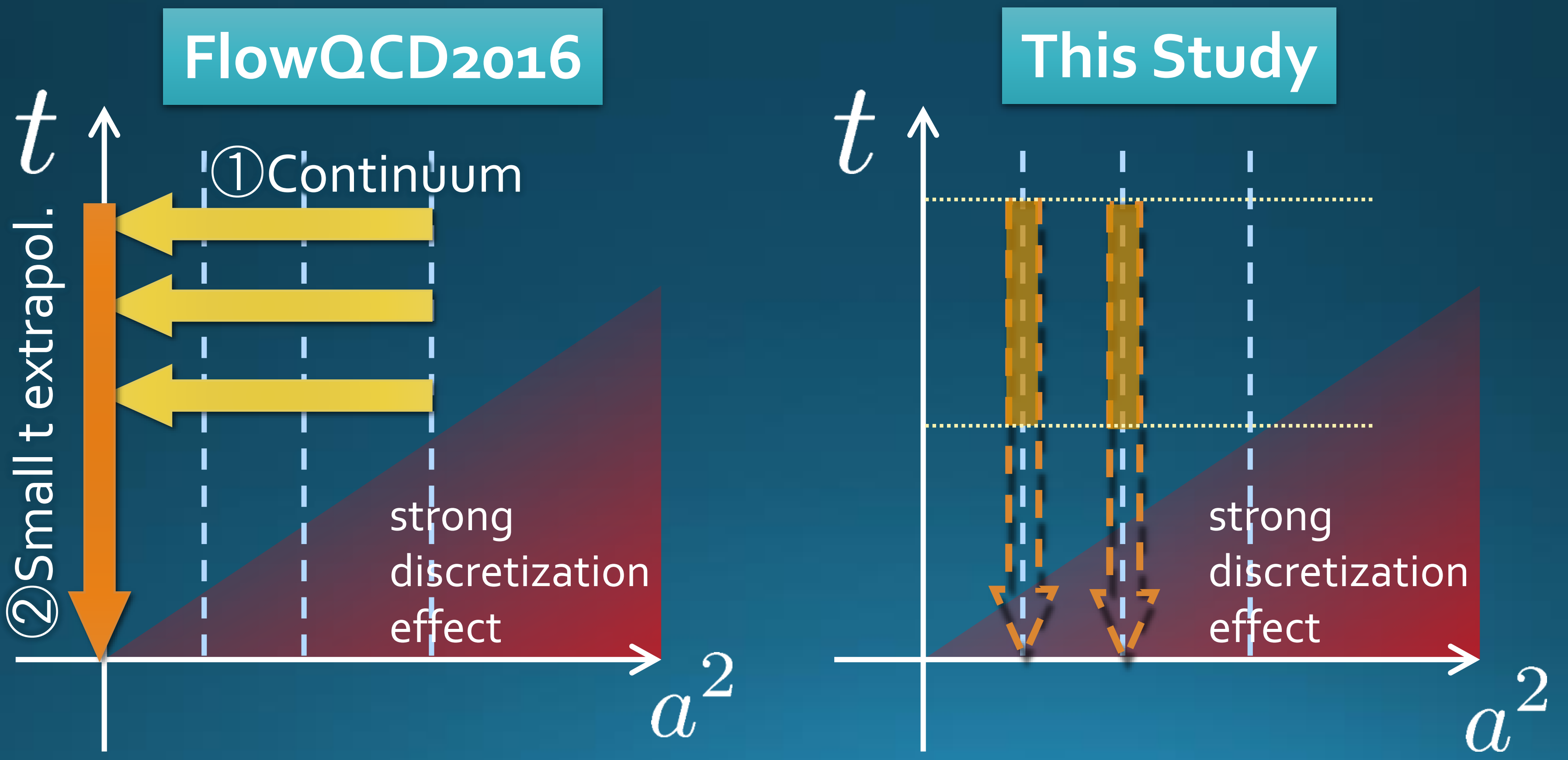




Extrapolations t→0, a→0

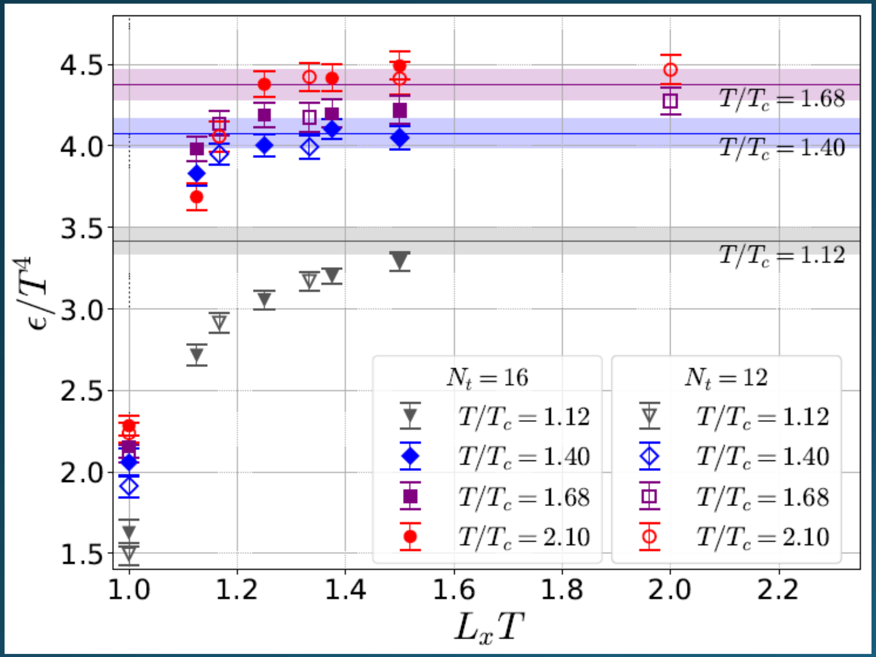
$$\langle T_{\mu\nu}(t)\rangle_{\mathrm{latt}} = \langle T_{\mu\nu}(t)\rangle_{\mathrm{phys}} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE lattice discretization

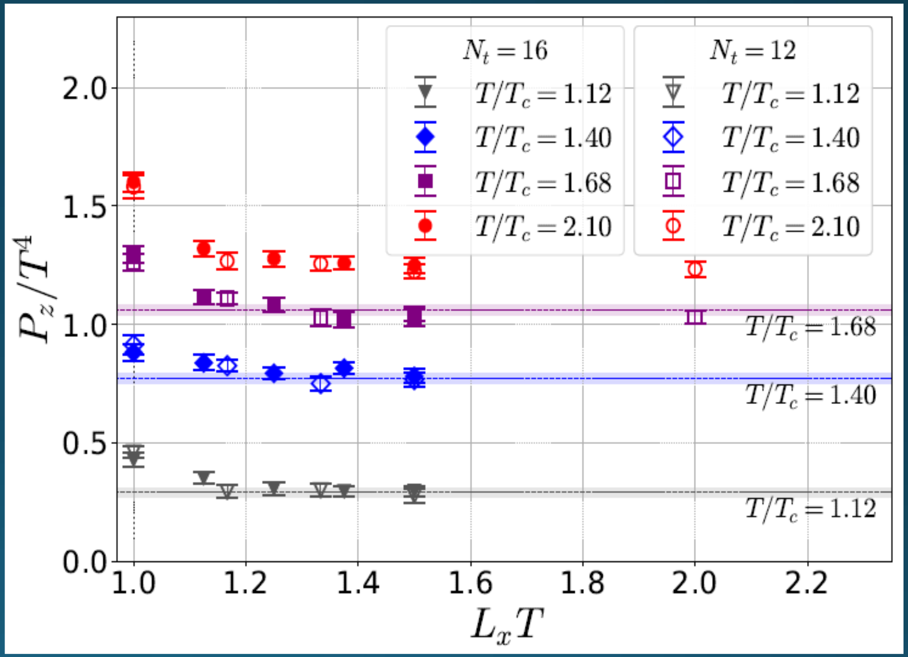


energy densty / transverse P

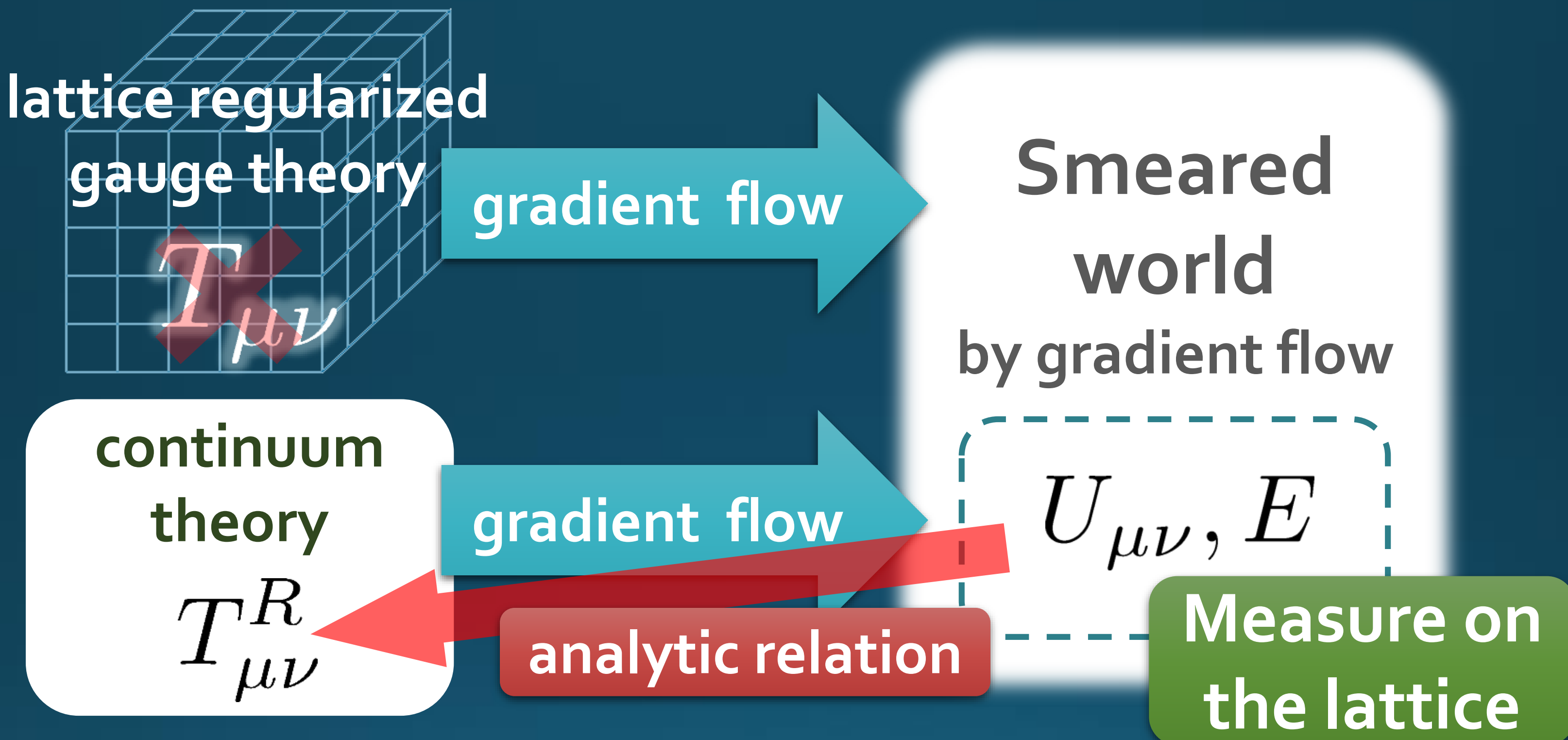
Energy Density



Transverse Pressure P_z



Gradient Flow Method



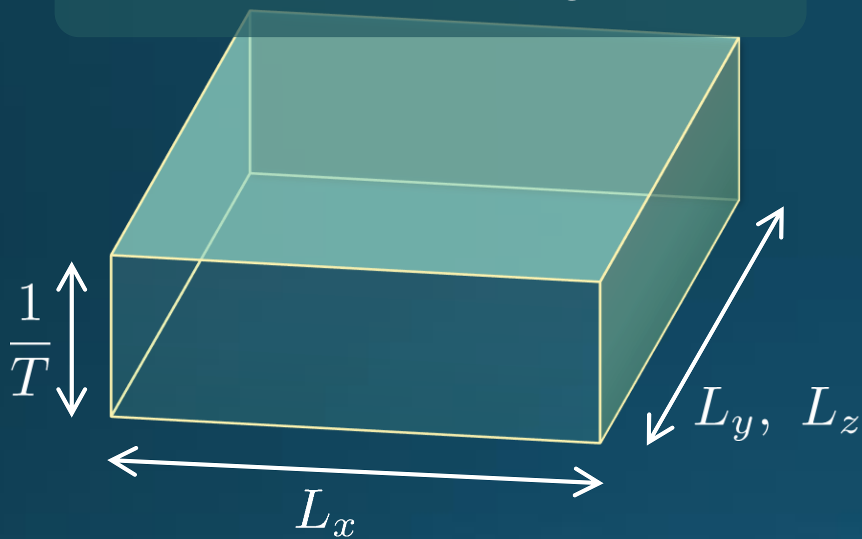
Take Extrapolation (t,a)→(0,0)

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}\frac{a^2}{t} + \cdots$$

O(t) terms in SFTE lattice discretization

Two Special Cases with PBC

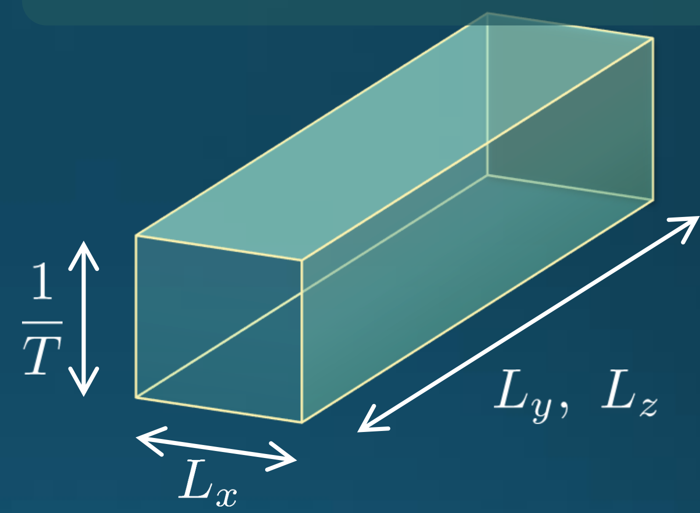
$$1/T \ll L_x = L_y = L_z$$



$$T_{11} = T_{22} = T_{33}$$

$$\frac{p_1}{p_2} = 1$$

$$1/T = L_x,\ L_y = L_z$$



$$T_{44} = T_{11},\ T_{22} = T_{33}$$

In conformal ($\Sigma_\mu T_{\mu\mu}=0$)

$$\frac{p_1}{p_2} = -1$$

EMT on the Lattice: Conventional

Lattice EMT Operator

Caracciolo+, 1990

$$T_{\mu\nu} = Z_6 T_{\mu\nu}^{[6]} + Z_3 T_{\mu\nu}^{[3]} + Z_1 \left(T_{\mu\nu}^{[1]} - \langle T_{\mu\nu}^{[1]} \rangle \right)$$

$$T_{\mu\nu}^{[6]} = (1-\delta_{\mu\nu}) F_{\mu\rho}^a F_{\nu\rho}^a, \quad T_{\mu\nu}^{[3]} = \delta_{\mu\nu} \left(F_{\mu\rho}^a F_{\nu\rho}^a - \frac{1}{4} F_{\rho\sigma}^a F_{\rho\sigma}^a \right), \quad T_{\mu\nu}^{[1]} = \delta_{\mu\nu} F_{\rho\sigma}^a F_{\rho\sigma}^a$$

- Fit to thermodynamics: Z_3, Z_1
- Shifted-boundary method: Z_6, Z_3

Giusti, Meyer, 2011; 2013;
Giusti, Pepe, 2014~; Borsanyi+, 2018

Multi-level algorithm

- effective in reducing statistical error of correlator

Meyer, 2007;
Borsanyi, 2018;
Astrakhantsev+, 2018