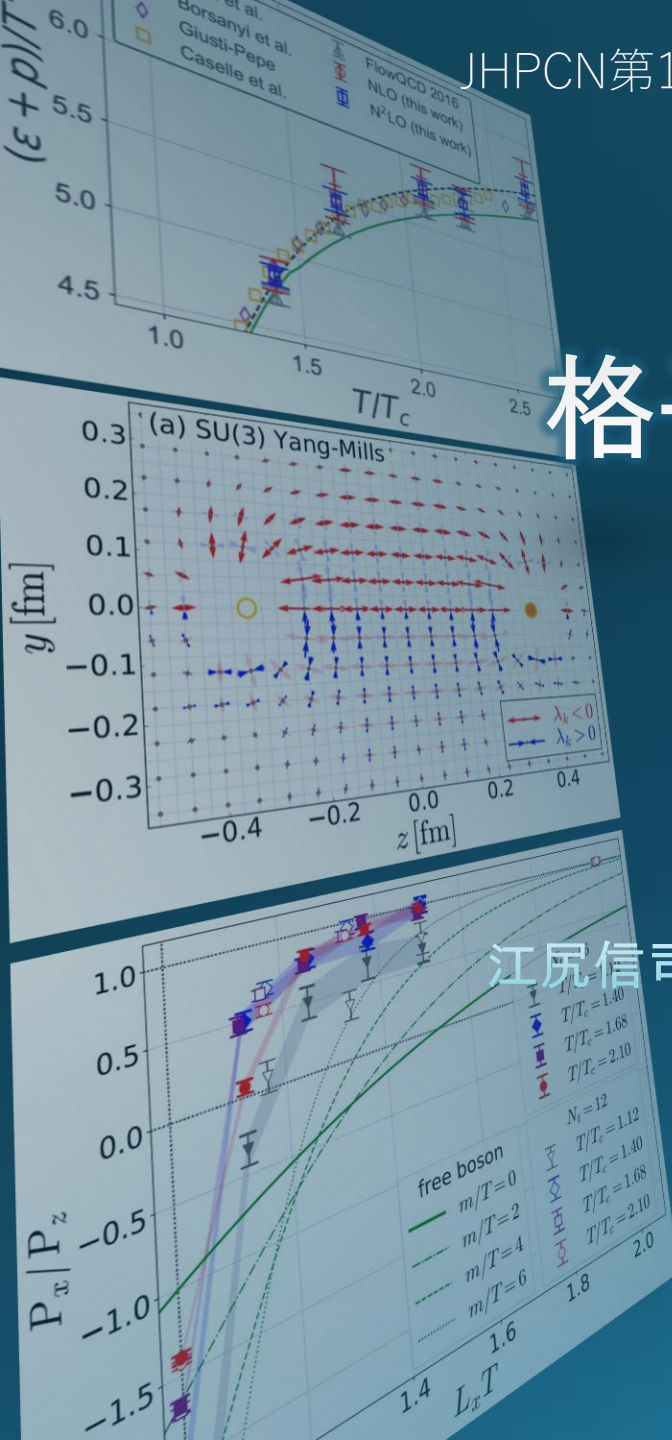


格子量子色力学に基づく 初期宇宙の諸性質の 精密数値解析

北澤正清 (大阪大学)

江尻信司、金谷和至、柳原良亮、白銀瑞樹、若林直輝
jh200049-NAH



素粒子階層の相転移現象

真空



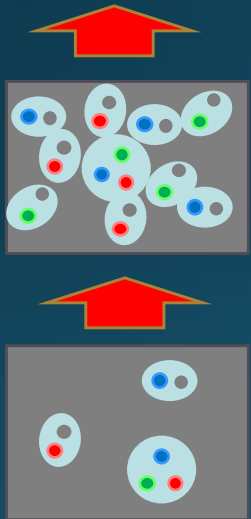
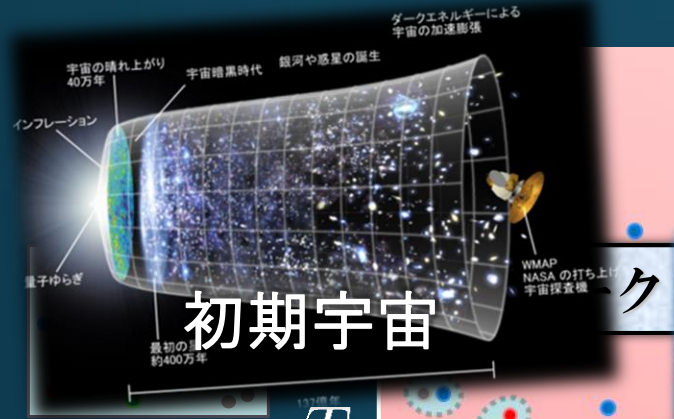
温度を上げていくと ...



クォーク・グルーオン・
プラズマ (QGP)

ハドロンを構成するクォークが溶け出す相転移
量子色力学 (QCD) が記述する物性現象

QCDの相図



T_c

ハドロン相
(閉じこめ相)

日本物理学会学会誌 8月号 ◆◆◆ 解説

非ガウスゆらぎで探る宇宙最高密度の相転移



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nonaka.toshihiro.ge@u.tsukuba.ac.jp

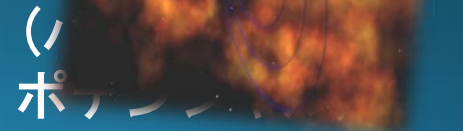
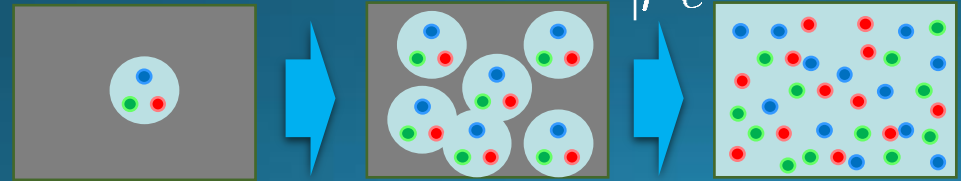


江角晋一
筑波大学数理物質科学研究所
esumi.shinichi.gn@u.tsukuba.ac.jp

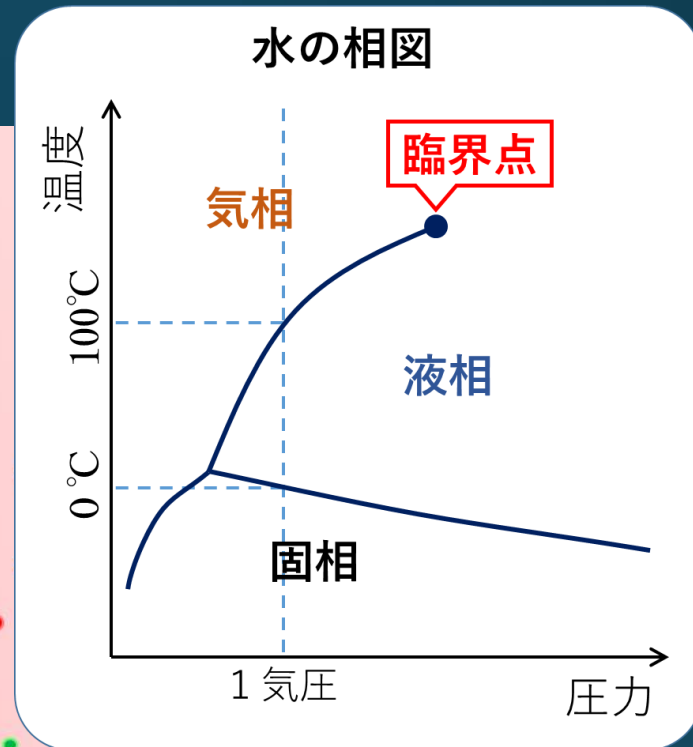
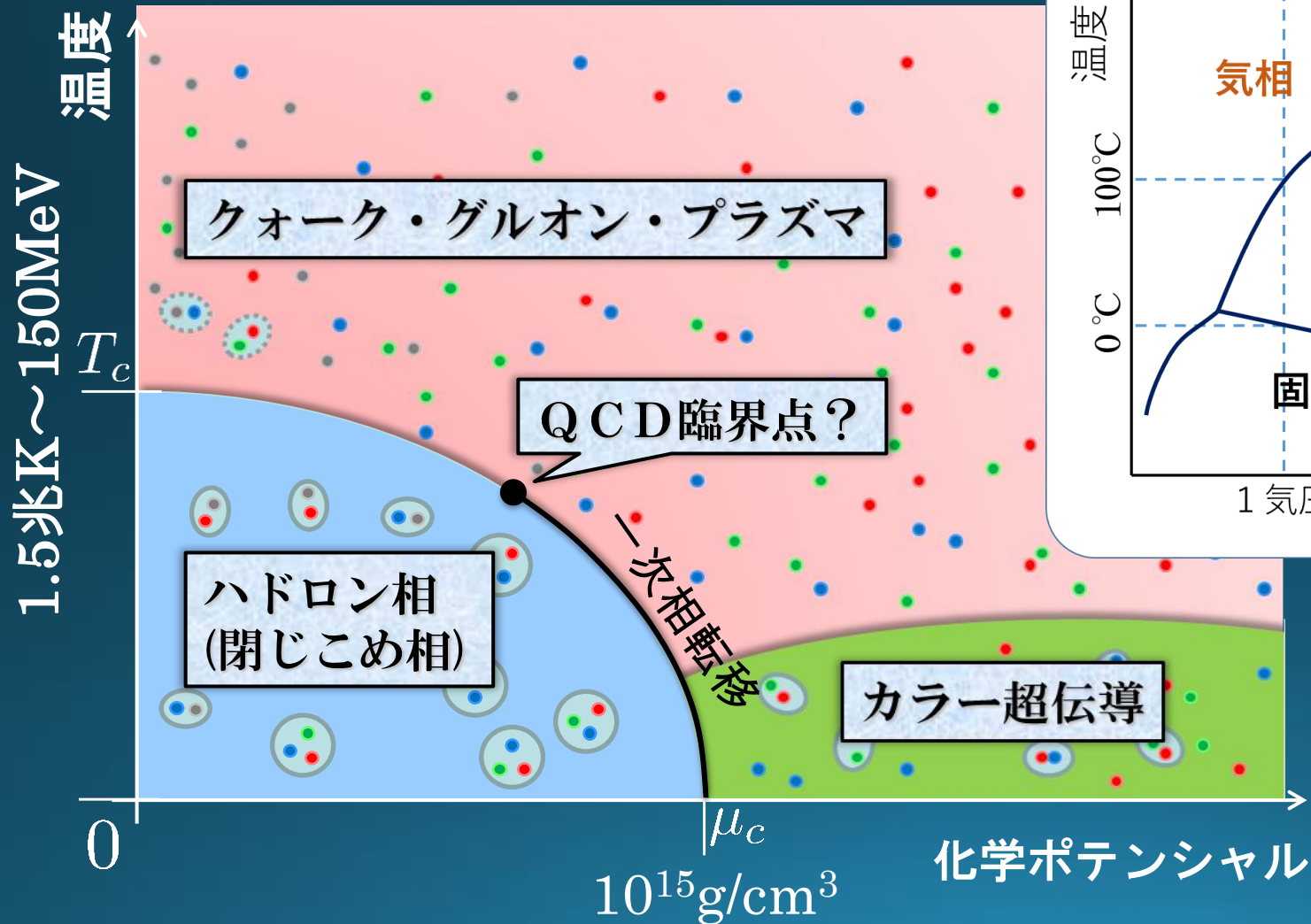
カラー超伝導 中性子星

0

μ_c

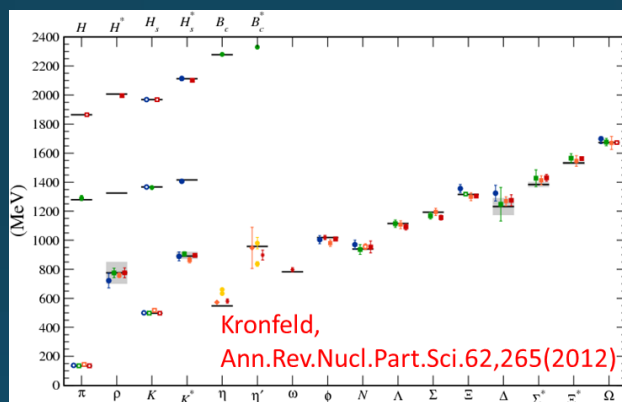
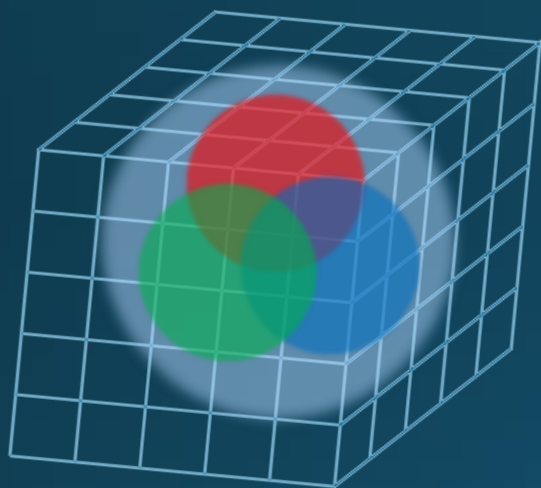


相転移の次数

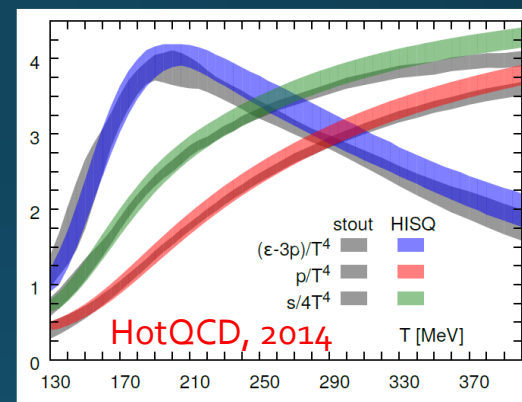


格子QCD第一原理数値計算

QCDの非摂動的性質を探る現状唯一の手段



ハドロンスペクトルの計算例



熱力学量の計算例

本研究の目的

格子QCD大規模数値計算により、初期宇宙を満たしていた超高密度物質の性質を調べる。

WHOT-QCD, FlowQCD共同研究の連携による、ユニークな研究

三つの研究課題

① 相転移温度近傍におけるクォーク間相互作用変質の解析

Yanagihara+, PLB789 (2019);
Yanagihara, MK, PTEP2020 (2020); Yanagihara+, PRD102 (2020)

② 重クォーク領域における QCD 相構造の精密解析

Ejiri+, PRD101 (2020);
Shirogane+, PTEP2021 (2021); Kiyohara+, in prep.

③ 境界条件を課した系における非等方圧力の測定

MK+, PRD99 (2019)

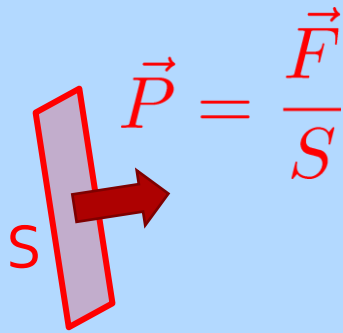
JHPCN:

2019年度 格子量子色力学に基づく初期宇宙の諸性質の数値解析

2020年度 格子量子色力学に基づく初期宇宙の諸性質の精密数値解析

Stress = Force per Unit Area

Pressure

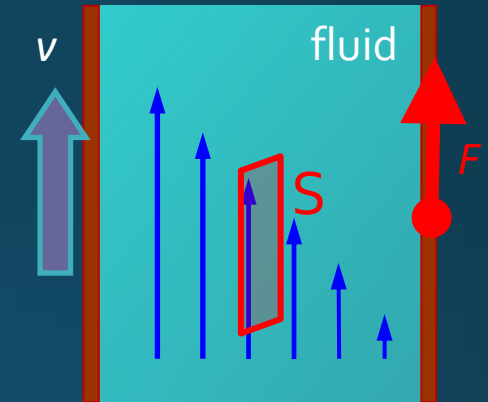
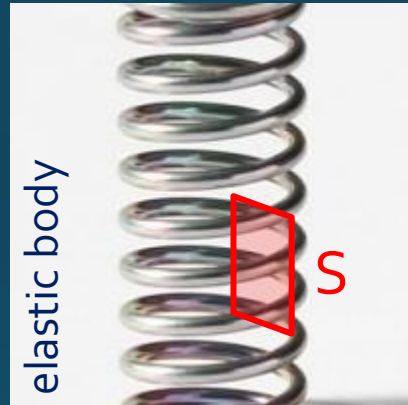


$$\vec{P} = P\vec{n}$$

In thermal medium

$$T_{ij} = P\delta_{ij}$$

Generally, F and n are not parallel



$$\frac{F_i}{S} = \sigma_{ij}n_j$$

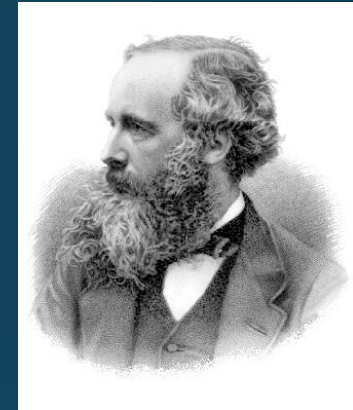
Stress Tensor

$$\sigma_{ij} = -T_{ij}$$

Landau
Lifshitz

Maxwell Stress

(in Maxwell Theory)



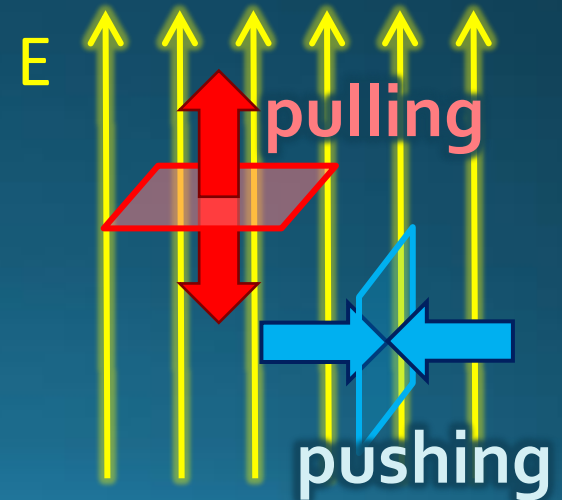
Maxwell

$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2} \delta_{ij} \left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right)$$

$$\vec{E} = (E, 0, 0)$$

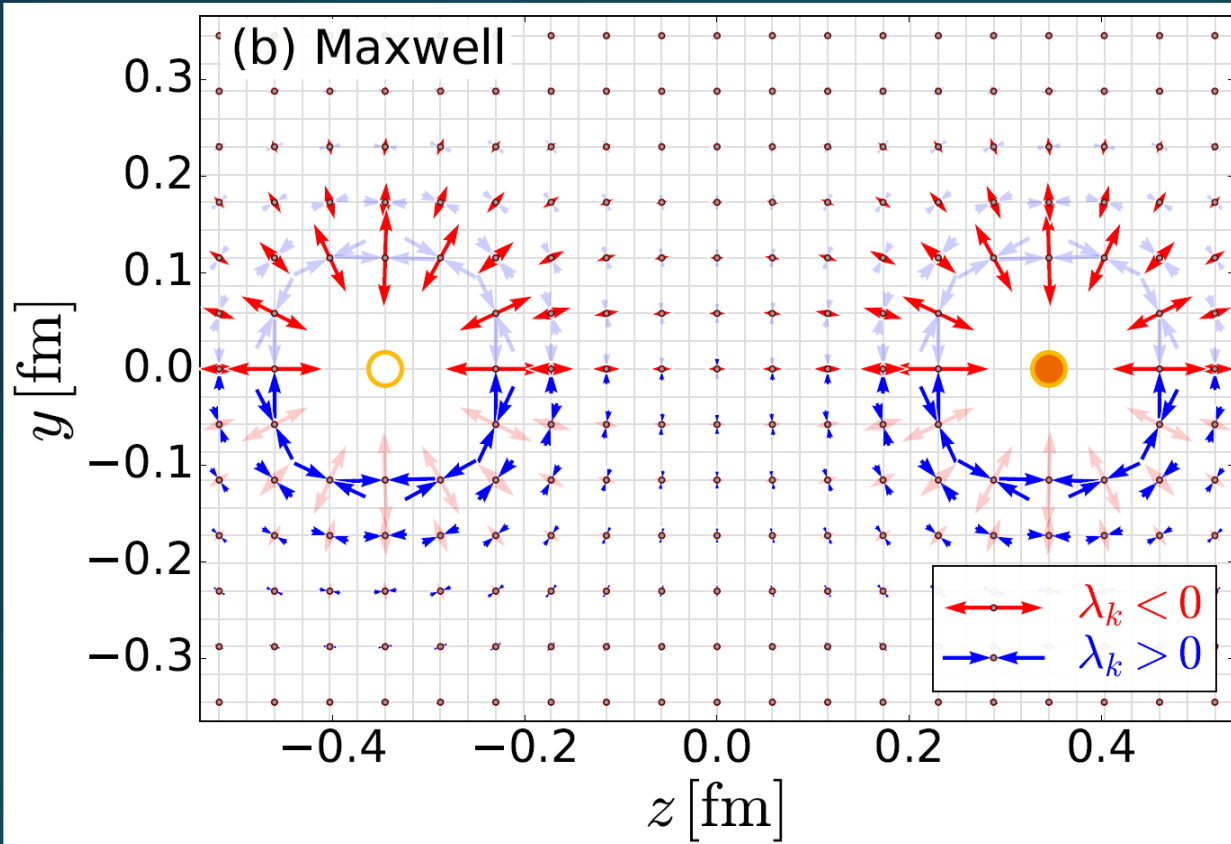
$$T_{ij} = \begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: **Pulling**
- Vertical to field: **Pushing**



Maxwell Stress

(in Maxwell Theory)



$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$(k = 1, 2, 3)$

length: $\sqrt{|\lambda_k|}$

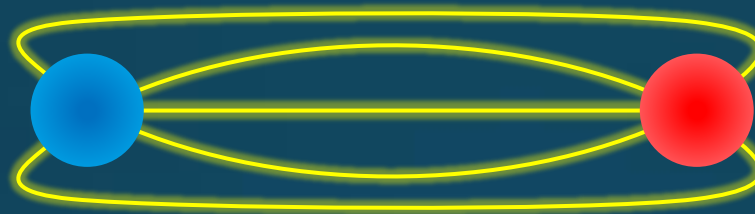


Definite physical meaning

- Distortion of field, line of the field
- Propagation of the force as local interaction

Quark-Anti-quark System

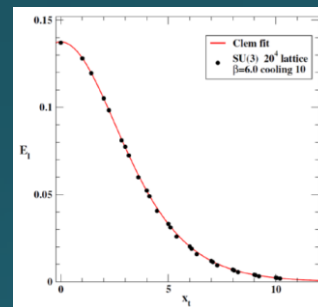
Formation of the flux tube \rightarrow confinement



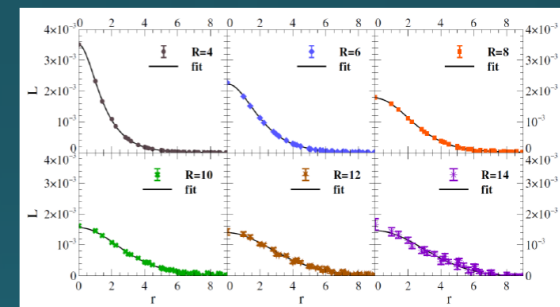
Previous Studies on Flux Tube

- Potential
- Action density
- Color-electric field

so many studies...



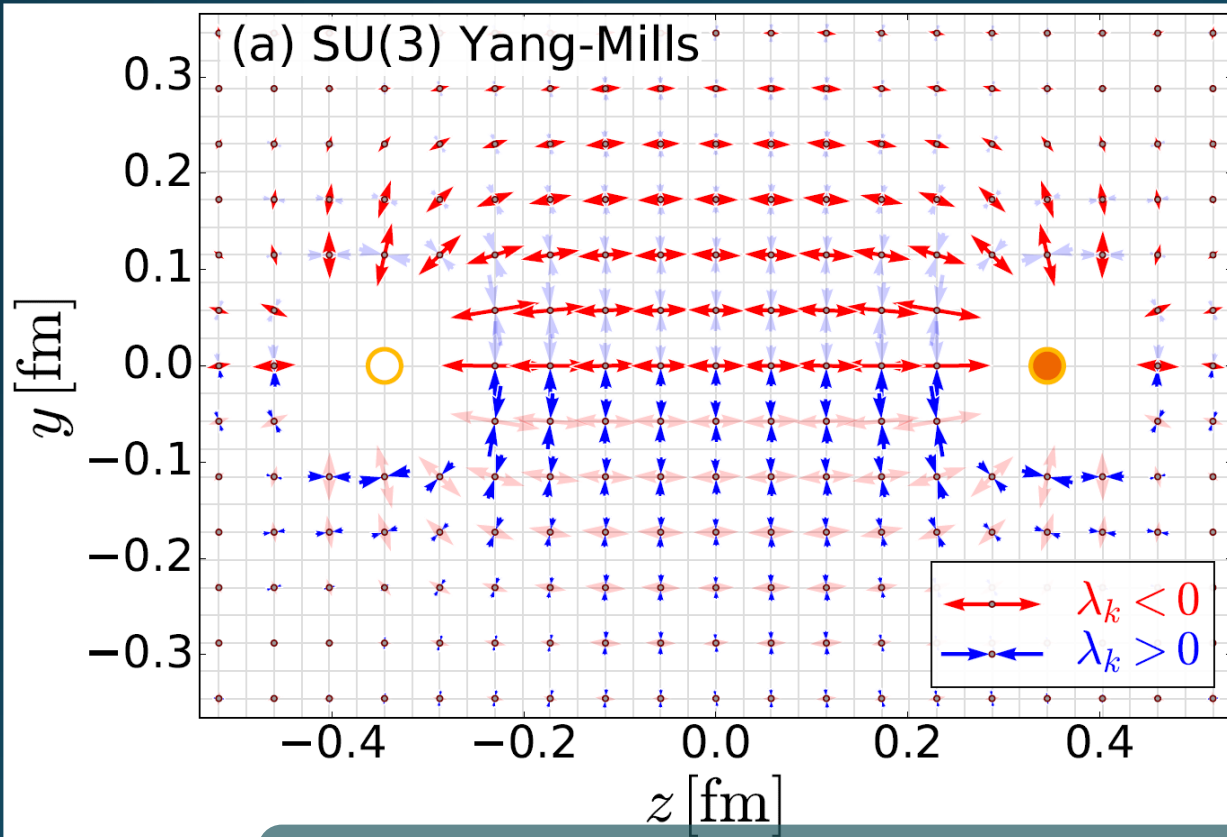
Cea+ (2012)



Cardoso+ (2013)

Stress Tensor in $Q\bar{Q}$ System

FlowQCD, PLB (2019)

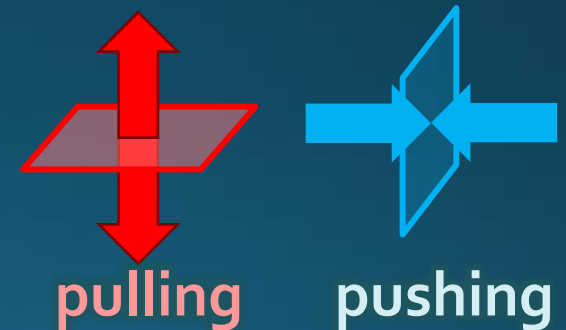


Lattice simulation
SU(3) Yang-Mills

$a=0.029$ fm

$R=0.69$ fm

$t/a^2=2.0$



Definite physical meaning

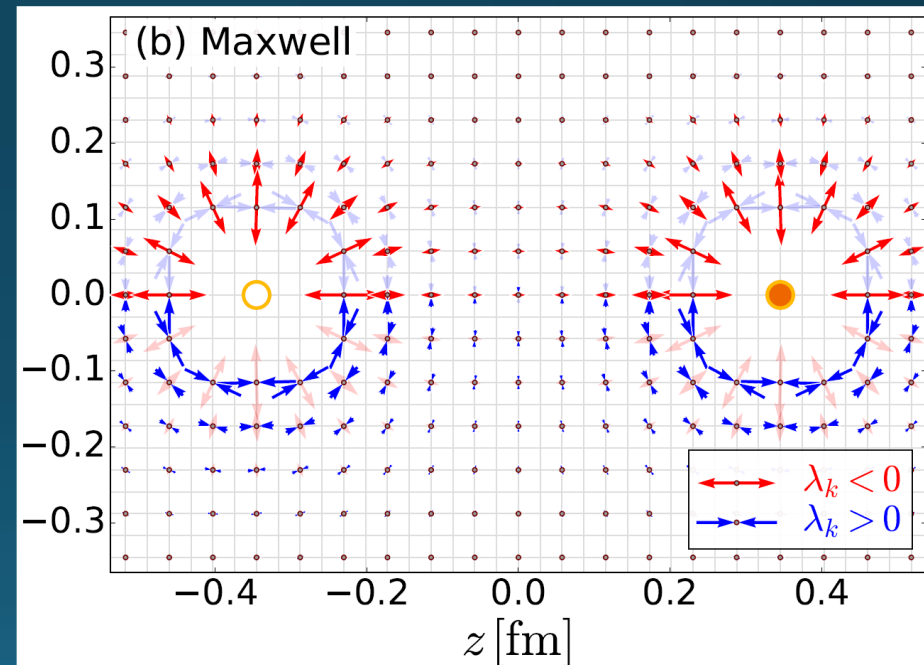
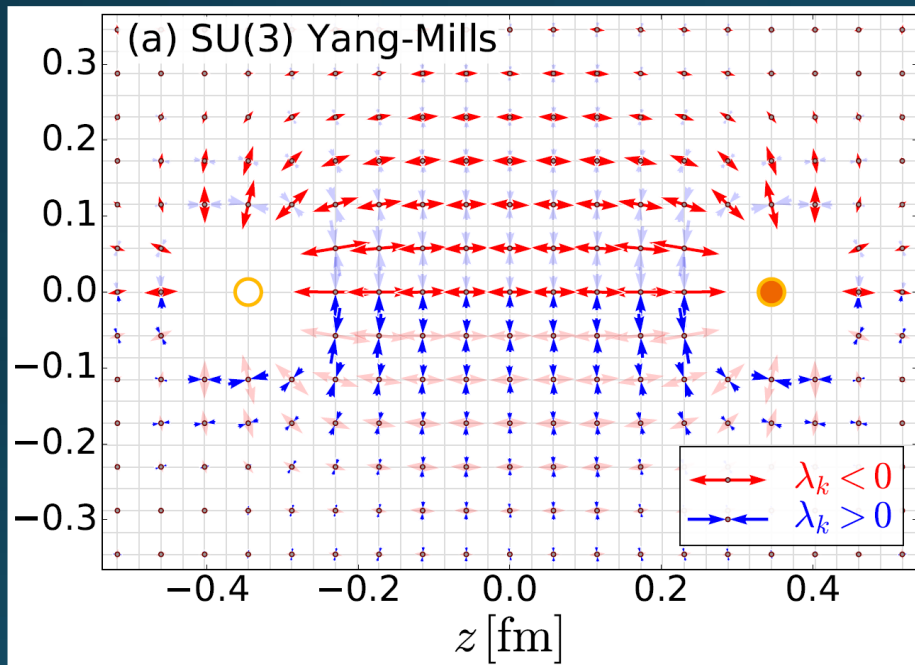
- Distortion of field, line of the field
- Propagation of the force as local interaction
- Manifestly gauge invariant



SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)

Maxwell
(classical)



Propagation of the force is clearly different
in YM and Maxwell theories!

研究成果

Yanagihara+, PLB (2019)

- ゲージ不変な物理量によるフラックスチューブ形成の確認
- チューブ内部の応力構造の可視化
 - Maxwell理論には存在しない各チャンネル縮退の破れの確認
 - チューブ端の定量的効果
 - 保存則の確認
- 有効モデルによる応力構造解析 Yanagihara, MK (2019)
- 今後の課題：有限温度への拡張など

A Static Quark

□ $T < T_c$: Heavy-light meson

- EMT distribution in the meson

□ $T \approx T_c$

- Confinement transition

Q

□ $T > T_c$: Single charge

- Screening
- Running coupling

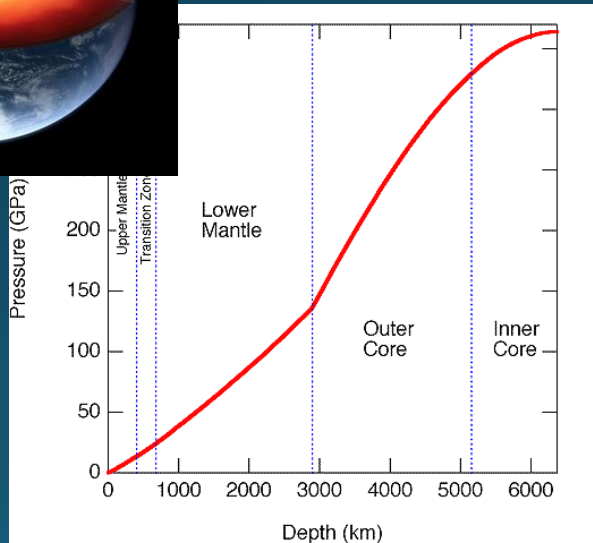
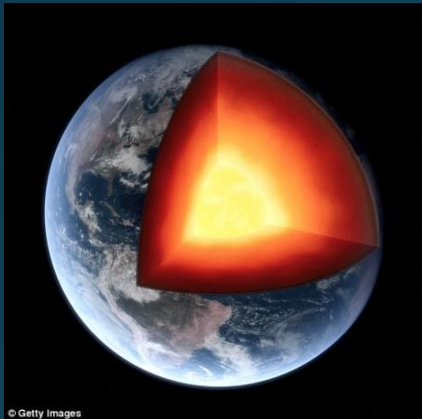
This study:

$T > T_c$ in pure YM

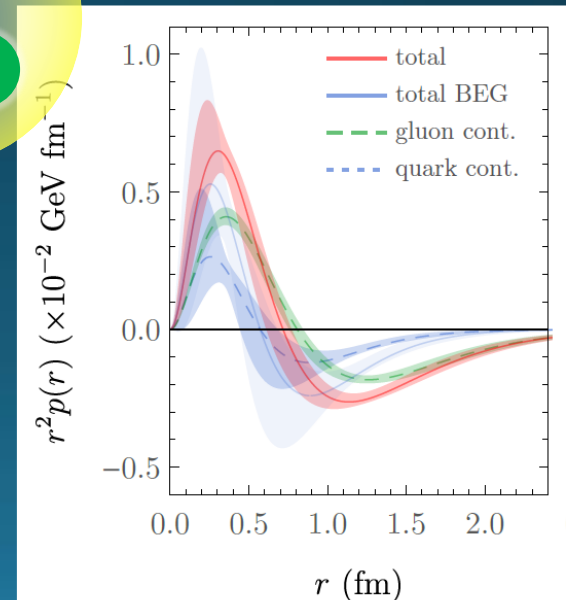
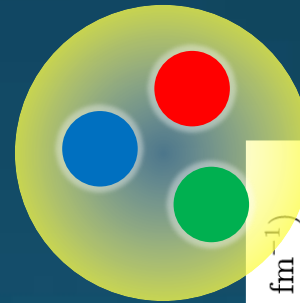
Pressure inside Hadrons

EMT distribution inside hadrons now accessible??

Pressure @ Earth



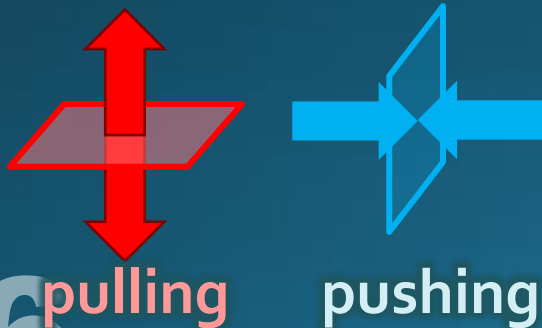
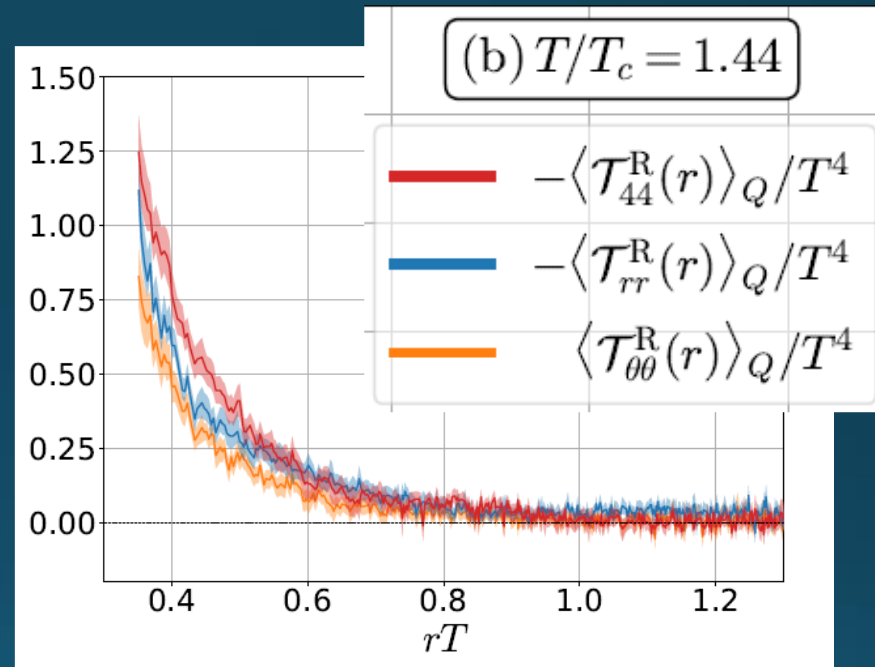
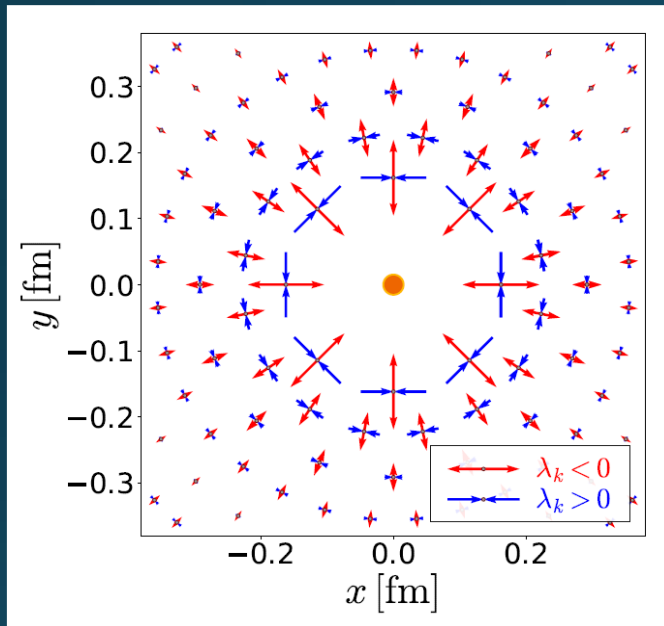
Pressure @ proton



Nature, 557, 396 (2018)

Stress Tensor Around a Quark

$$T = 1.44 T_c$$



- Suppression at large distance
- Separation of different channels

研究成果

Yanagihara+, PRD (2020)

- 非閉じ込め相($T > T_c$)における静的クォーク周辺のエネルギー密度・応力構造の測定
 - Running couplingの測定
 - 遮蔽質量の測定
 - 摂動論との比較
- 今後の課題
 - $T < T_c$ での測定
 - ハドロン構造の研究

三つの研究課題

① 相転移温度近傍におけるクォーク間相互作用変質の解析

Yanagihara+, PLB789 (2019);
Yanagihara, MK, PTEP2020 (2020); Yanagihara+, PRD102 (2020)

② 重クォーク領域における QCD 相構造の精密解析

Ejiri+, PRD101 (2020);
Shirogane+, PTEP2021 (2021); Kiyohara+, in prep.

③ 境界条件を課した系における非等方圧力の測定

MK+, PRD99 (2019)

JHPCN:

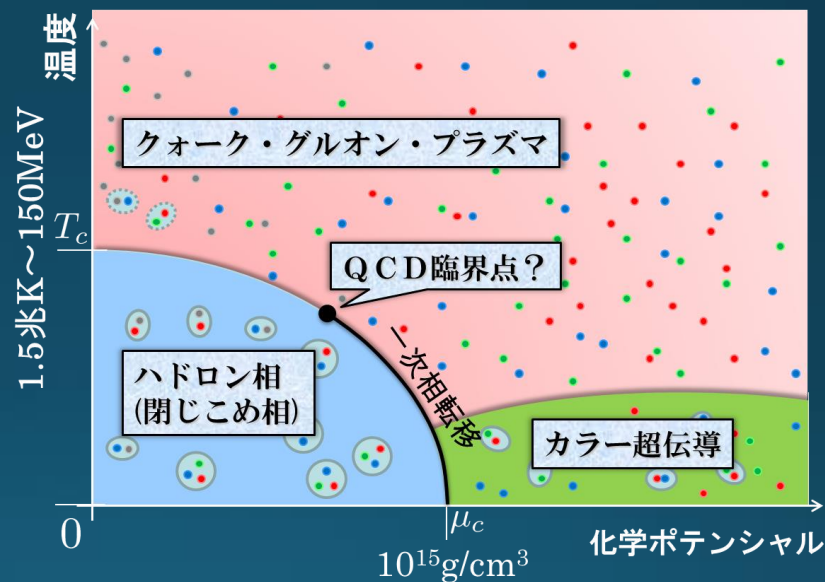
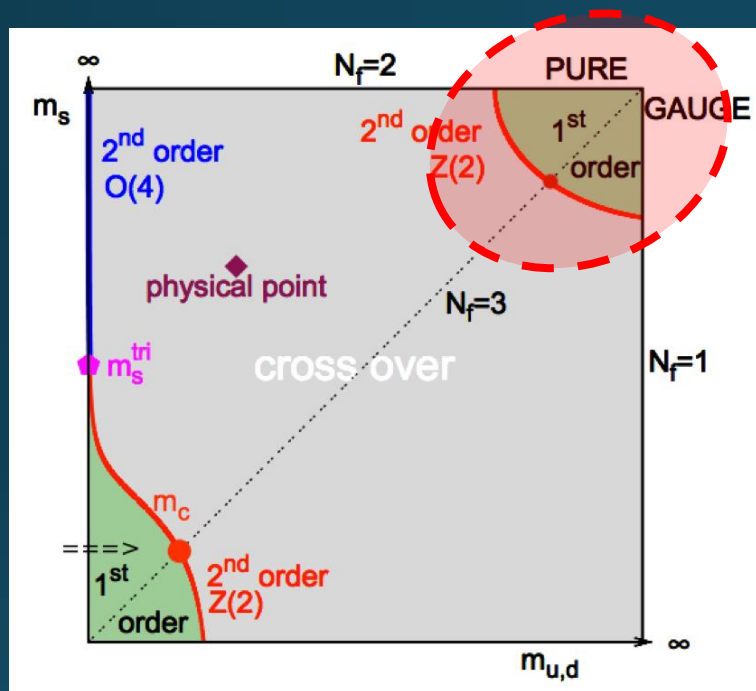
2019年度 格子量子色力学に基づく初期宇宙の諸性質の数値解析

2020年度 格子量子色力学に基づく初期宇宙の諸性質の精密数値解析

QCDの一次相転移と臨界点

クォーク質量平面

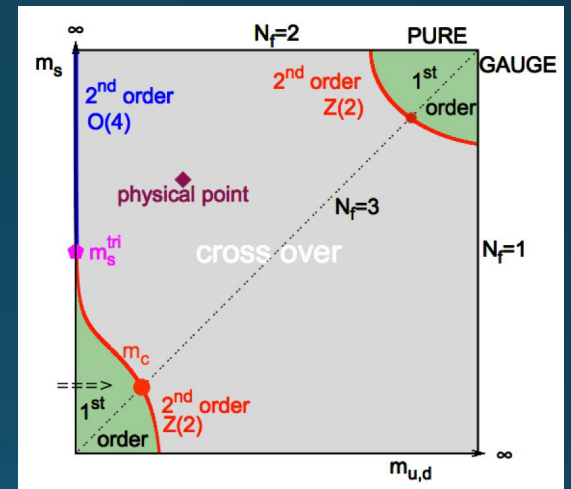
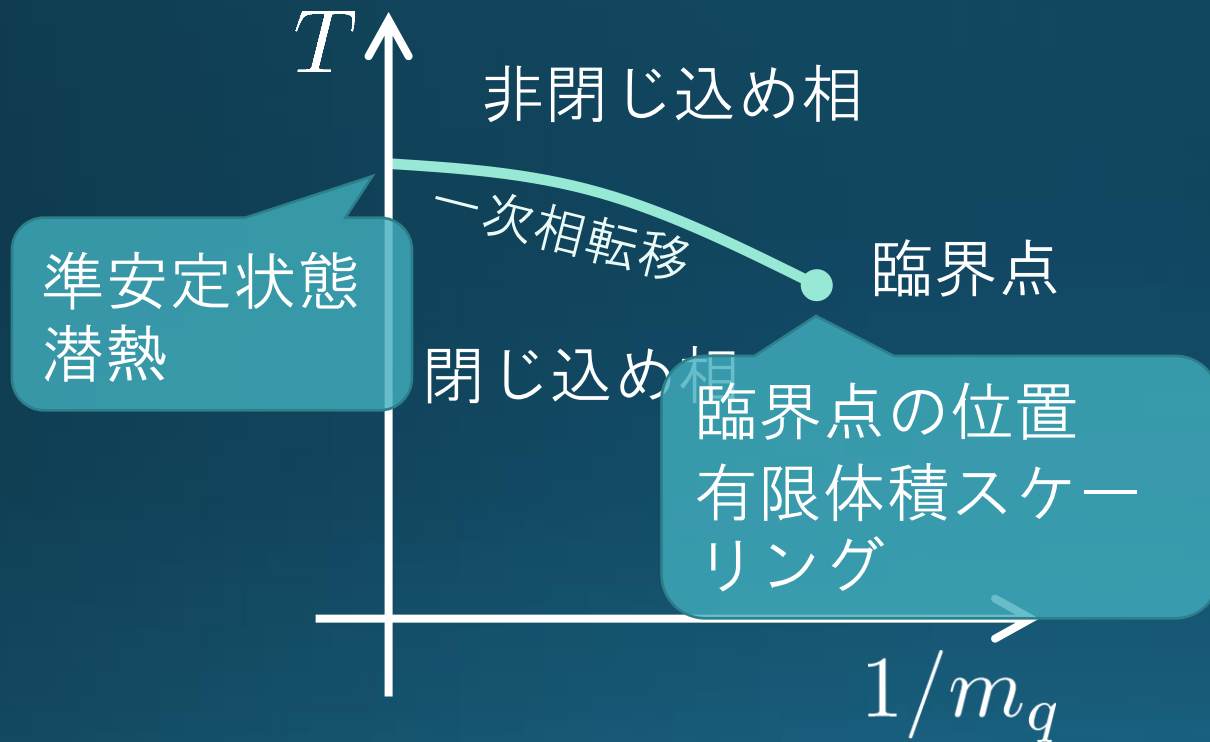
温度・密度平面



本研究の目的

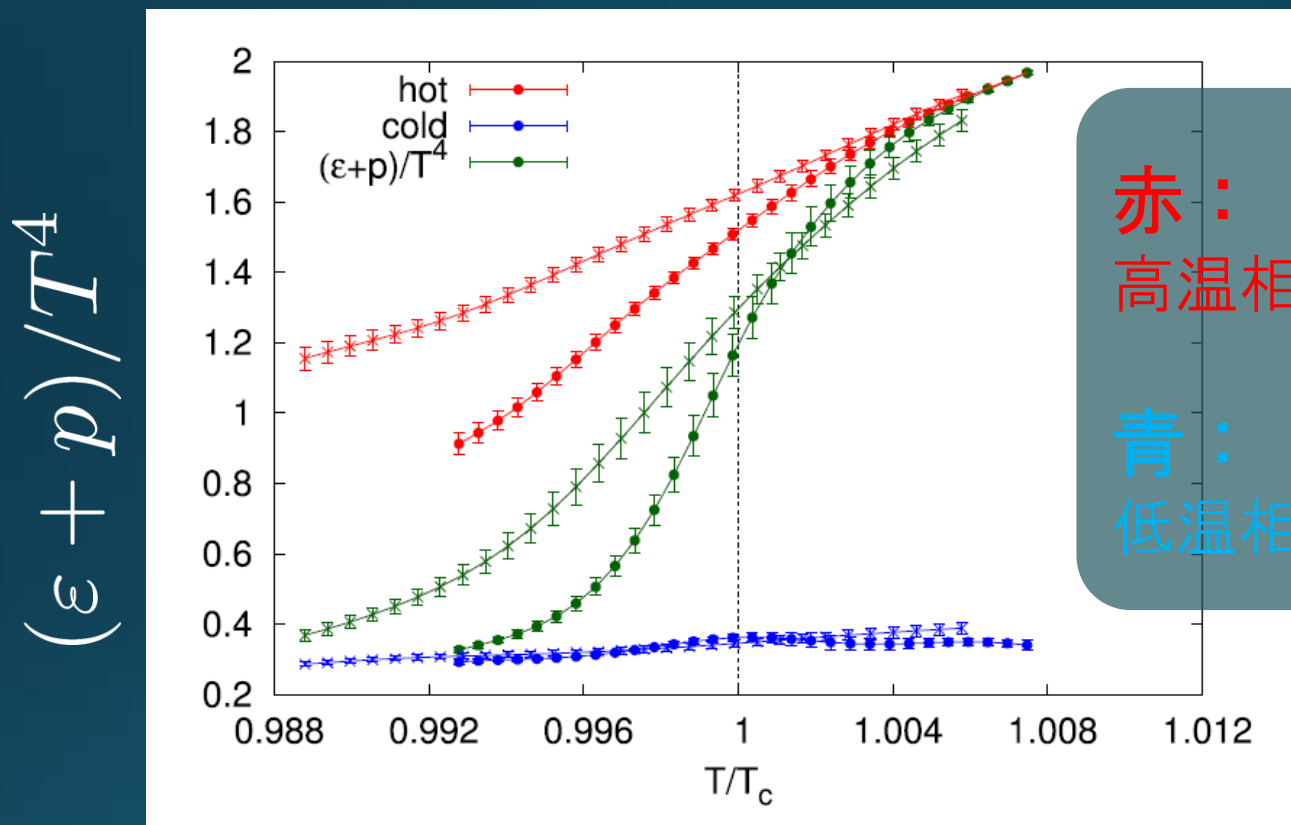
重クォーク領域の一次相転移・臨界点の性質を精密測定する

重クォークQCD相図



ヒステリシス曲線

Ejiri+, 2021



赤：
高温相の熱力学量

青：
低温相の熱力学量

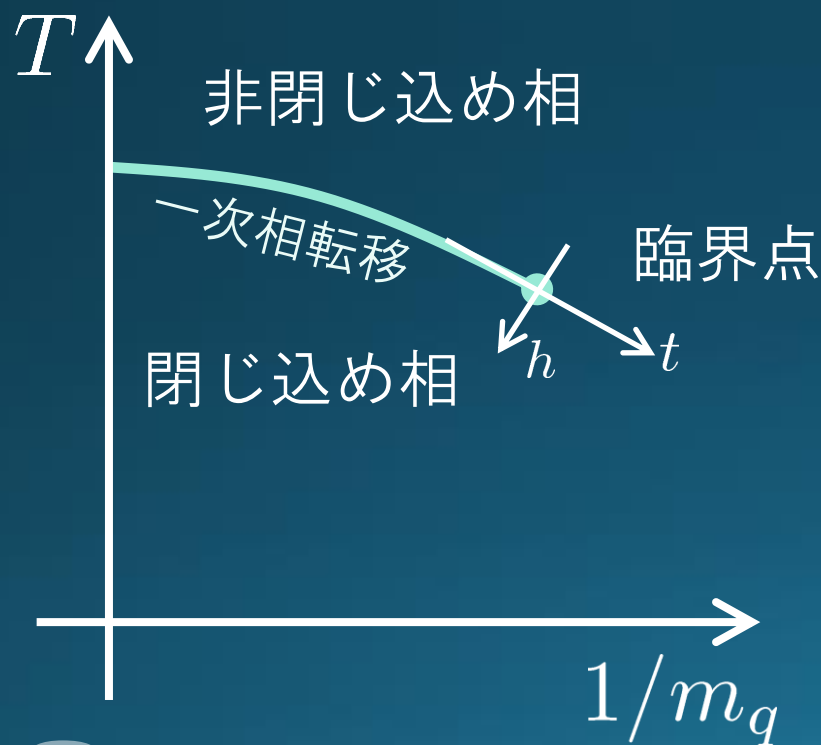
- 準安定状態の熱力学量測定
 - 高温相に強い体積依存性
- 潜熱の測定

$$\Delta\epsilon/T^4 = 1.117(40),$$

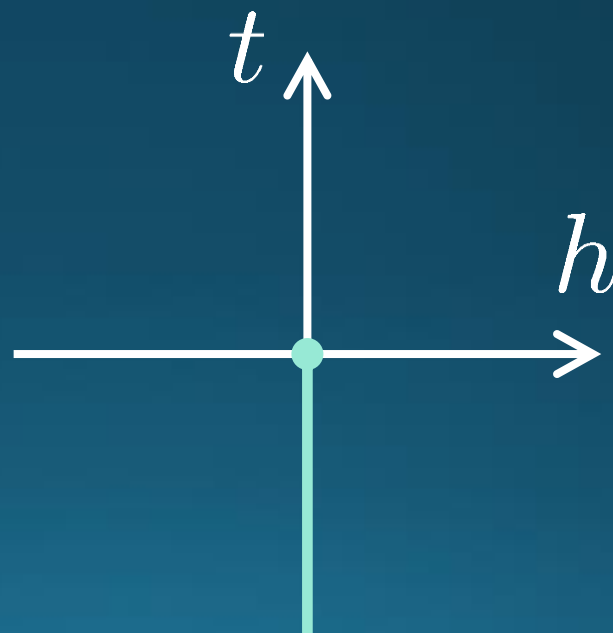
重クォーク QCD相図

QCD臨界点は3次元Z(2)普遍類に属する

重クォーク相図

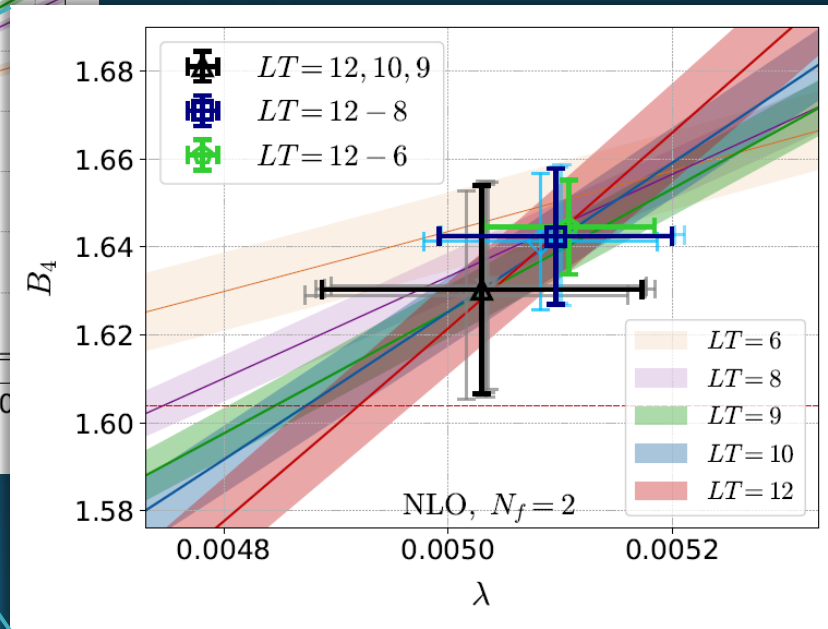
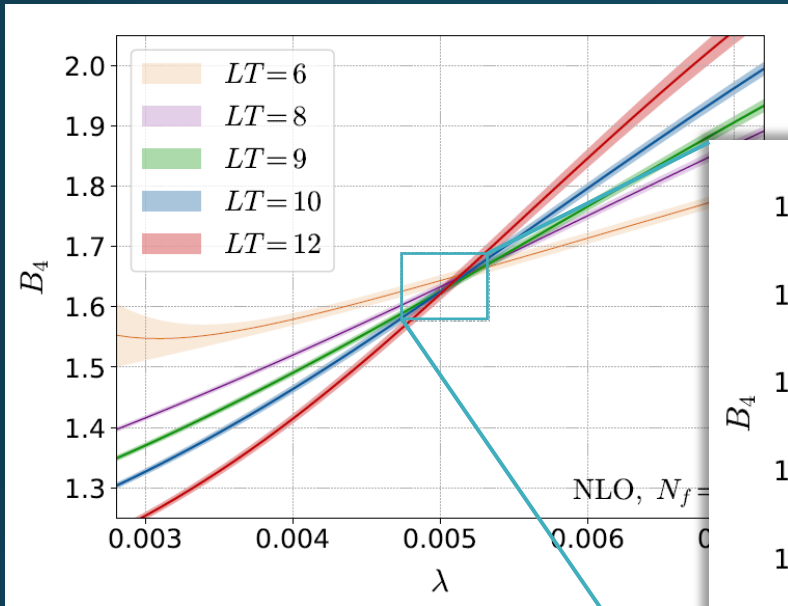


Ising模型相図



Finite-size Scaling 1: Binder Cumulant analysis

Kiyohara+, in prep.



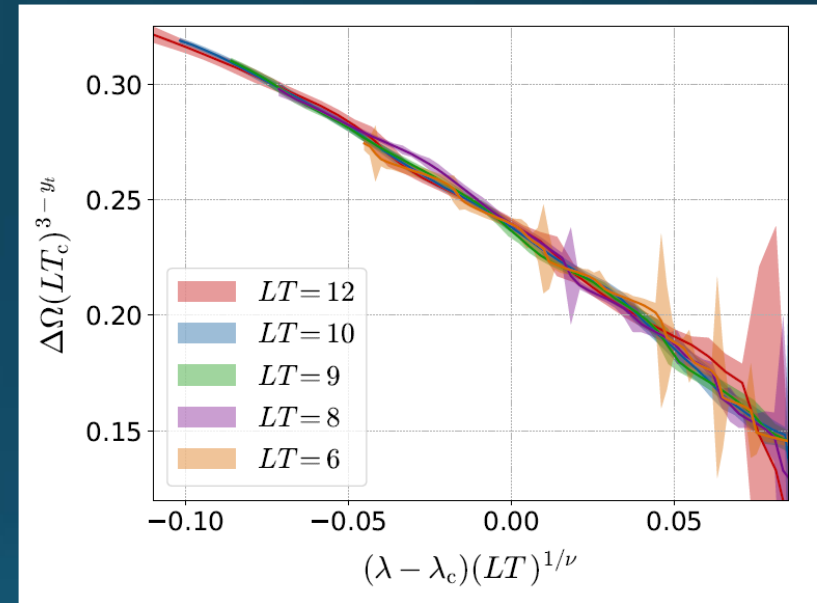
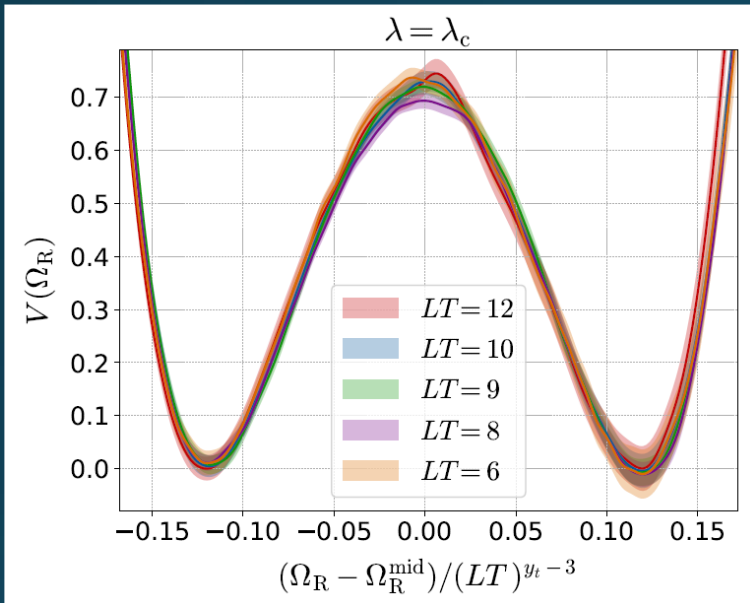
	B_4	ν
Heavy QCD	1.630(26)	0.614(51)
3d-Ising	1.604	0.630

**Z_2 Universality
confirmed!**

Finite-size Scaling 2: Scaling of Distribution Func.

$$V(\Omega) = -\ln p(\Omega)$$

Gap bw/ two minima



FSS on the CP

$$V(\Omega, L) = \tilde{V}((\Omega - \Omega_0)(LT)^{3 - y_h})$$

**Z₂ Universality
confirmed!**

研究成果

Ejiri+, PRD101 (2020);

Shirogane+, PTEP2021 (2021); Kiyohara+, in prep.

□ 重クォーク領域の一次相転移、臨界点に関する多様な研究成果

□ ヒステリシス曲線

□ 潜熱の測定

□ 臨界点の位置精密測定

- $N_t = 4$: (Kiyohara+, in prep.)

- $N_t = 6$: (Ejiri+, 2020)

□ $Z(2)$ 普遍類有限サイズスケーリングの確認

□ 今後の課題

□ 格子間隔依存性の精査、連続極限

まとめ

① 相転移温度近傍におけるクォーク間相互作用変質の解析

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③ 境界条件を課した系における非等方圧力の測定

MK+, PRD99 (2019)

- 三つの研究課題で着実な研究成果
- JHPCNの計算資源が有効活用できた

backup

Lattice Setup

FlowQCD, PLB (2019)

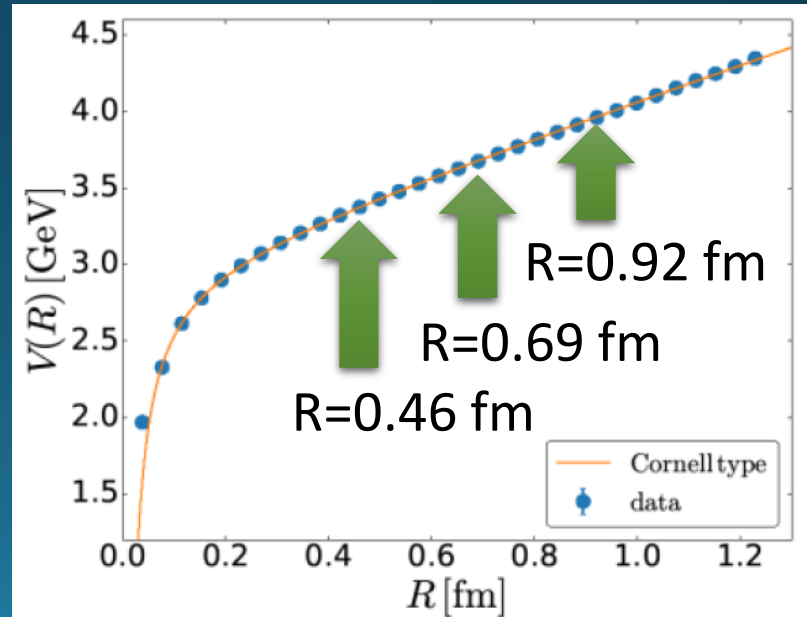
- ❑ SU(3) Yang-Mills (Quenched)
- ❑ Wilson gauge action
- ❑ Clover operator

- ❑ EMT around Wilson Loop
- ❑ APE smearing / multi-hit

- ❑ fine lattices ($a=0.029-0.06$ fm)
- ❑ continuum extrapolation

- ❑ Simulation: bluegene/Q@KEK

β	a [fm]	N_{size}^4	N_{conf}	R/a		
6.304	0.058	48^4	140	8	12	16
6.465	0.046	48^4	440	10	–	20
6.513	0.043	48^4	600	–	16	–
6.600	0.038	48^4	1,500	12	18	24
6.819	0.029	64^4	1,000	16	24	32
R [fm]				0.46	0.69	0.92



$$\langle O(x) \rangle_{\text{Q}\bar{\text{Q}}} = \lim_{T \rightarrow \infty} \frac{\langle \delta O(x) \delta W(R, T) \rangle}{\langle W(R, T) \rangle}$$

Lattice Setup

- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

- Analysis above T_c
- Simulation on a Z_3 minimum
- EMT around a Polyakov loop

$$\langle O(x) \rangle_Q = \frac{\langle \delta O(x) \delta \Omega(0) \rangle}{\langle \Omega(0) \rangle}$$

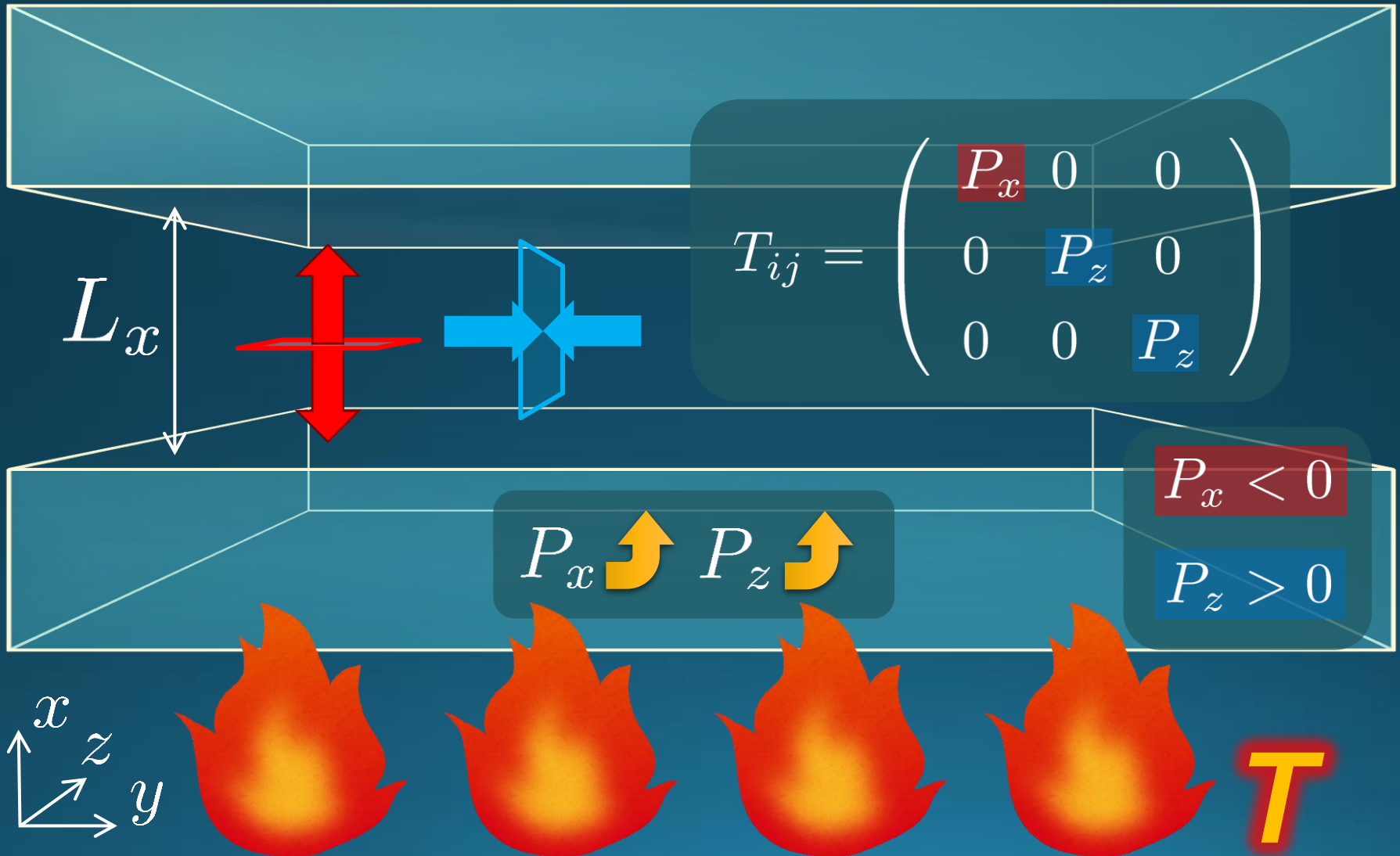
Ω : Polyakov loop

- continuum extrapolation

T/T_c	N_s	N_τ	β	a [fm]	N_{conf}
1.20	40	10	6.336	0.0551	500
	48	12	6.467	0.0460	650
	56	14	6.581	0.0394	840
	64	16	6.682	0.0344	1,000
	72	18	6.771	0.0306	1,000
1.44	40	10	6.465	0.0461	500
	48	12	6.600	0.0384	650
	56	14	6.716	0.0329	840
	64	16	6.819	0.0288	1,000
	72	18	6.910	0.0256	1,000
2.00	40	10	6.712	0.0331	500
	48	12	6.853	0.0275	650
	56	14	6.973	0.0236	840
	64	16	7.079	0.0207	1,000
	72	18	7.173	0.0184	1,000
2.60	40	10	6.914	0.0255	500
	48	12	7.058	0.0212	650
	56	14	7.182	0.0182	840
	64	16	7.290	0.0159	1,000
	72	18	7.387	0.0141	1,000

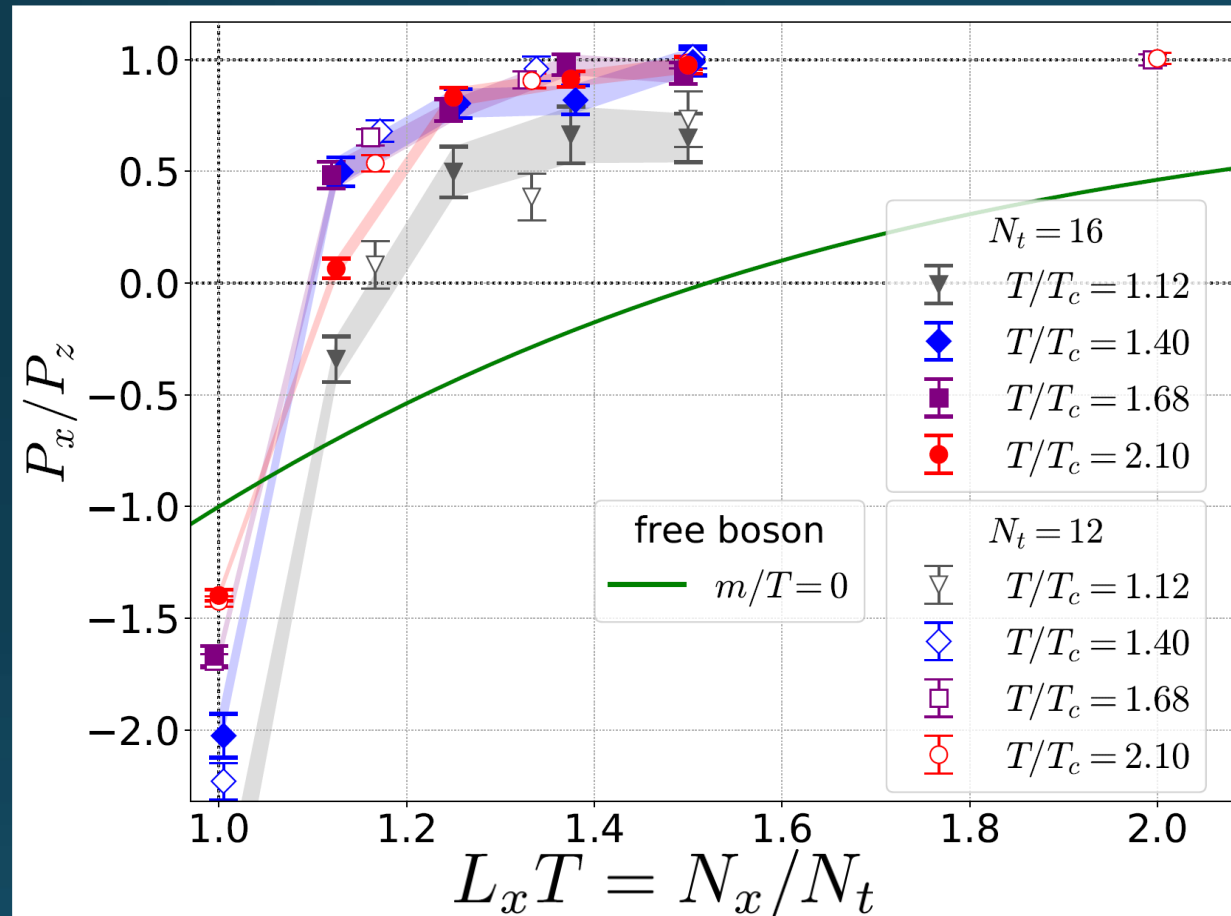
Casimir Effect

Brown, Maclay
1969



Pressure Anisotropy @ $T \neq 0$

MK, Mogliacci, Kolbe,
Horowitz, PRD(2019)



Free scalar field

□ $L_2=L_3=\infty$

□ Periodic BC

Mogliacci+, 1807.07871

Lattice result

□ Periodic BC

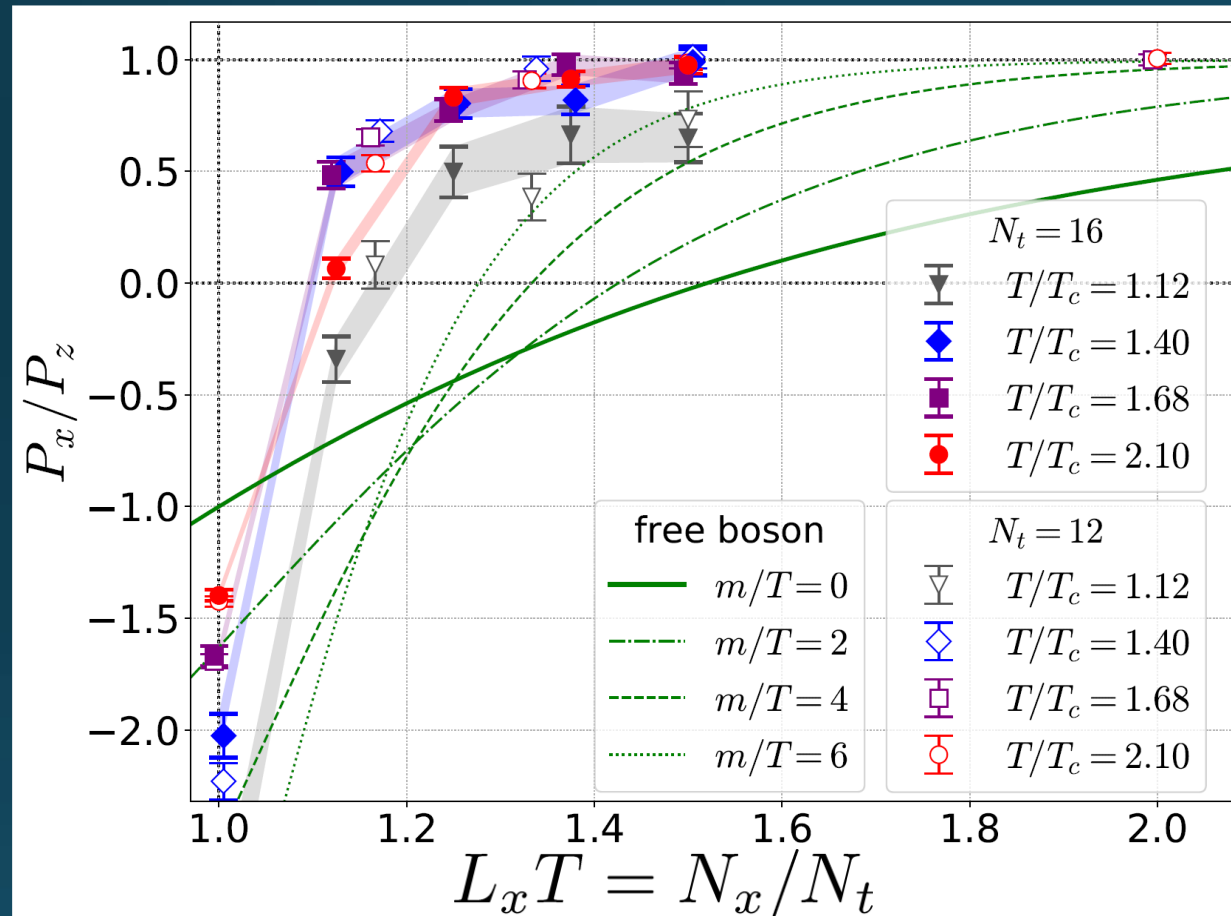
□ Only $t \rightarrow 0$ limit

□ Error: stat.+sys.

Medium near T_c is remarkably insensitive to finite size!

Pressure Anisotropy @ $T \neq 0$

MK, Mogliacci, Kolbe,
Horowitz, PRD(2019)



Free scalar field

\square $L_2=L_3=\infty$

\square Periodic BC

Mogliacci+, 1807.07871

Lattice result

\square Periodic BC

\square Only $t \rightarrow 0$ limit

\square Error: stat.+sys.

Medium near T_c is remarkably insensitive to finite size!

Lattice Setup

FlowQCD, PLB (2019)

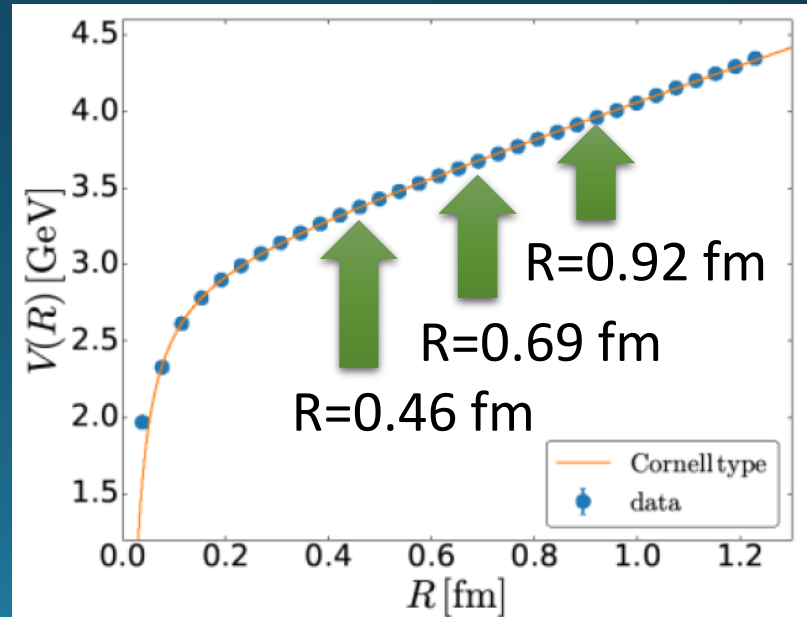
- ❑ SU(3) Yang-Mills (Quenched)
- ❑ Wilson gauge action
- ❑ Clover operator

- ❑ EMT around Wilson Loop
- ❑ APE smearing / multi-hit

- ❑ fine lattices ($a=0.029-0.06$ fm)
- ❑ continuum extrapolation

- ❑ Simulation: bluegene/Q@KEK

β	a [fm]	N_{size}^4	N_{conf}	R/a		
6.304	0.058	48^4	140	8	12	16
6.465	0.046	48^4	440	10	–	20
6.513	0.043	48^4	600	–	16	–
6.600	0.038	48^4	1,500	12	18	24
6.819	0.029	64^4	1,000	16	24	32
R [fm]				0.46	0.69	0.92



$$\langle O(x) \rangle_{\text{Q}\bar{\text{Q}}} = \lim_{T \rightarrow \infty} \frac{\langle \delta O(x) \delta W(R, T) \rangle}{\langle W(R, T) \rangle}$$

Stress Distribution on Mid-Plane

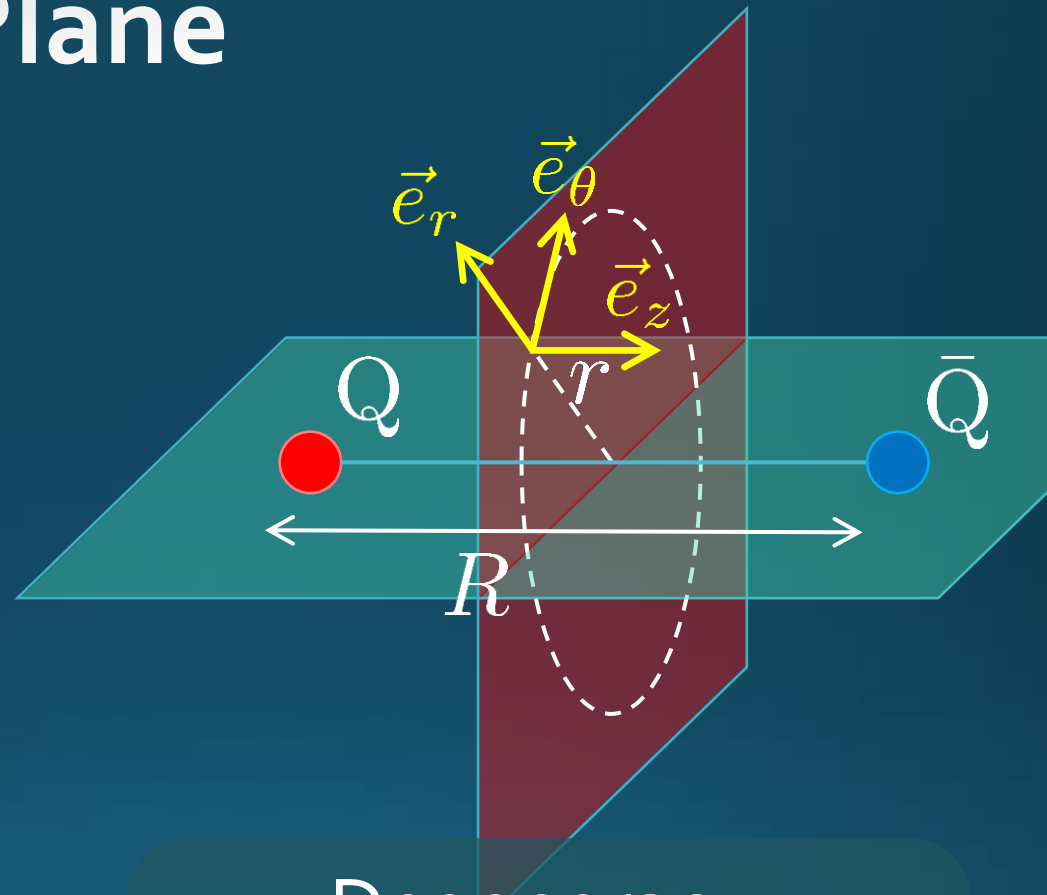
From rotational symm. & parity

EMT is diagonalized
in Cylindrical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{zz} & \\ & & & T_{44} \end{pmatrix}$$

$$T_{rr} = \vec{e}_r^T T \vec{e}_r$$

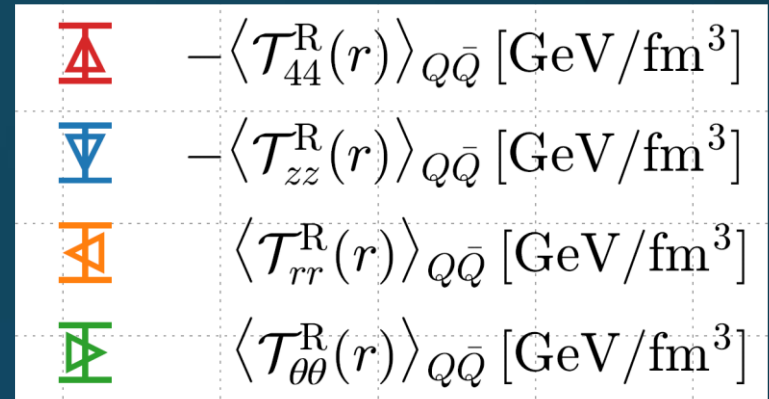
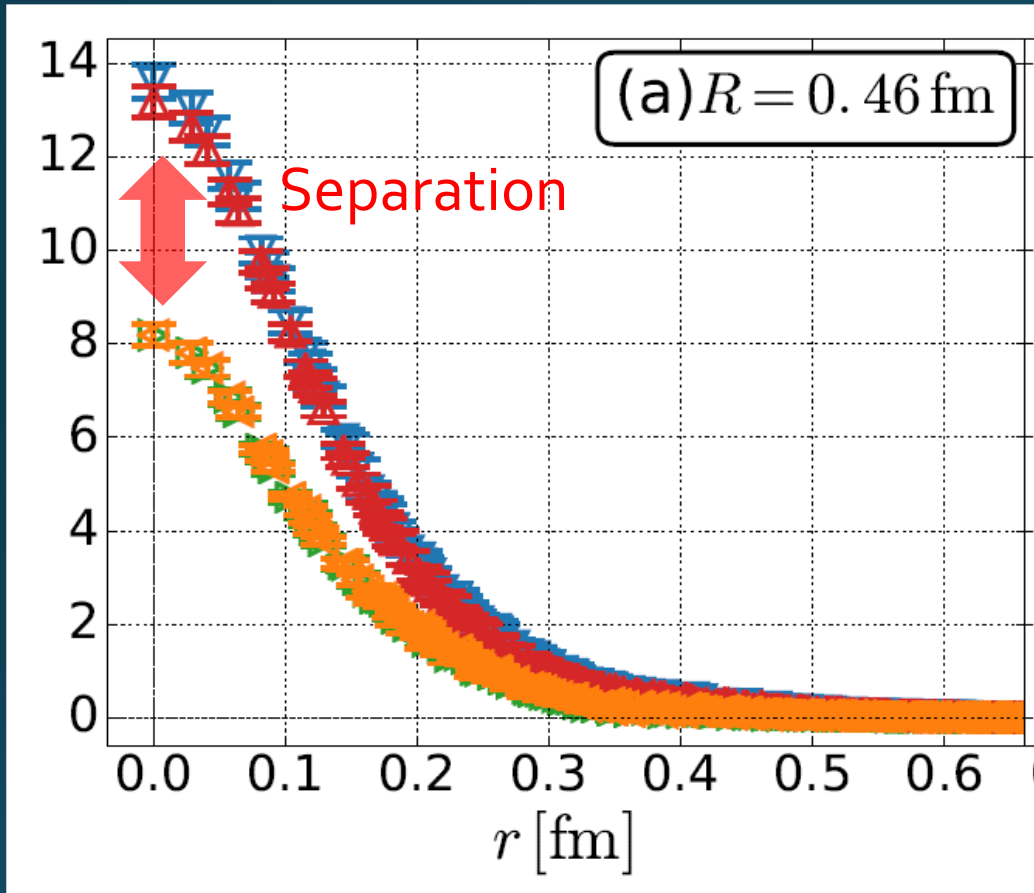
$$T_{\theta\theta} = \vec{e}_\theta^T T \vec{e}_\theta$$



Degeneracy
in Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

Mid-Plane



**Continuum
Extrapolated!**

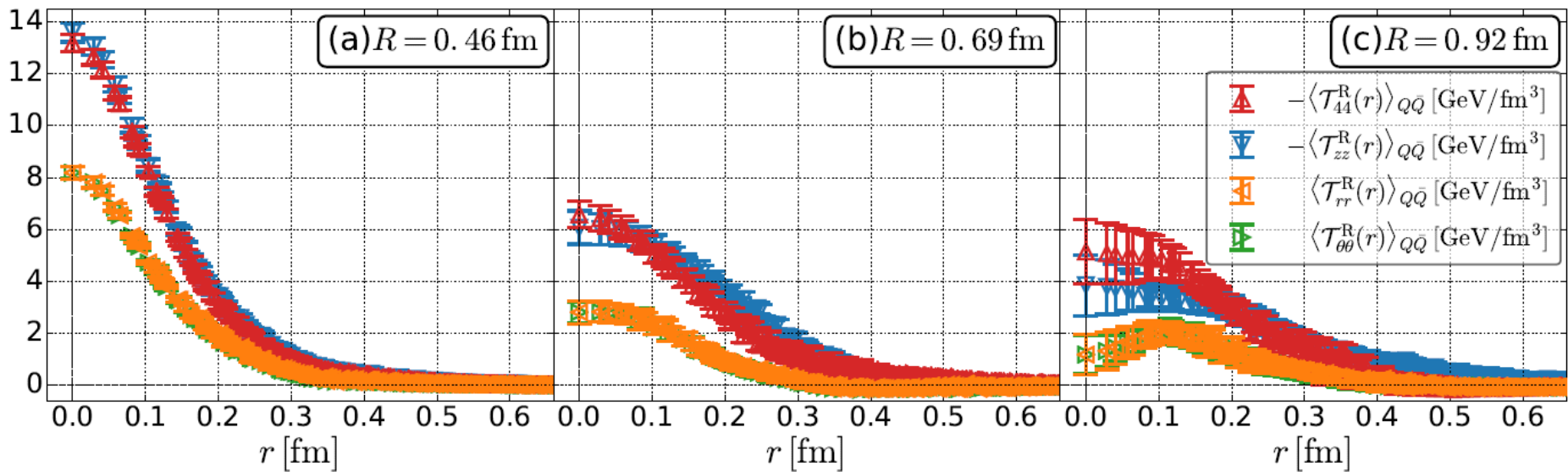
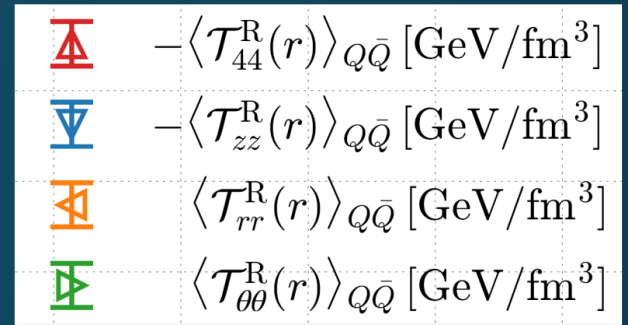
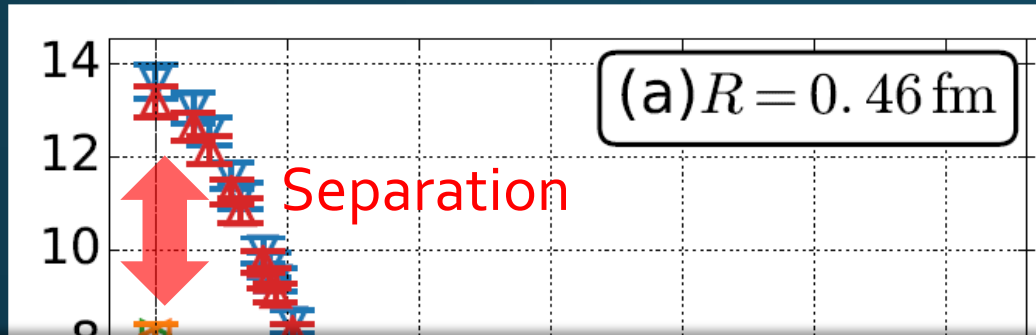
In Maxwell theory
 $T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$

□ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$

□ Separation: $T_{zz} \neq T_{rr}$

□ Nonzero trace anomaly $\sum T_{cc} \neq 0$

Mid-Plane



□ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$

□ Separation: $T_{zz} \neq T_{rr}$

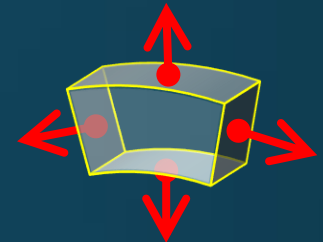
□ Nonzero trace anomaly $\sum T_{cc} \neq 0$

Momentum Conservation

Yanagihara, MK, PTEP2019

- In cylindrical coordinates,

$$\partial_i T_{ij} = 0 \quad \Rightarrow \quad \partial_r(rT_{rr}) = T_{\theta\theta} - r\partial_z T_{rz}$$

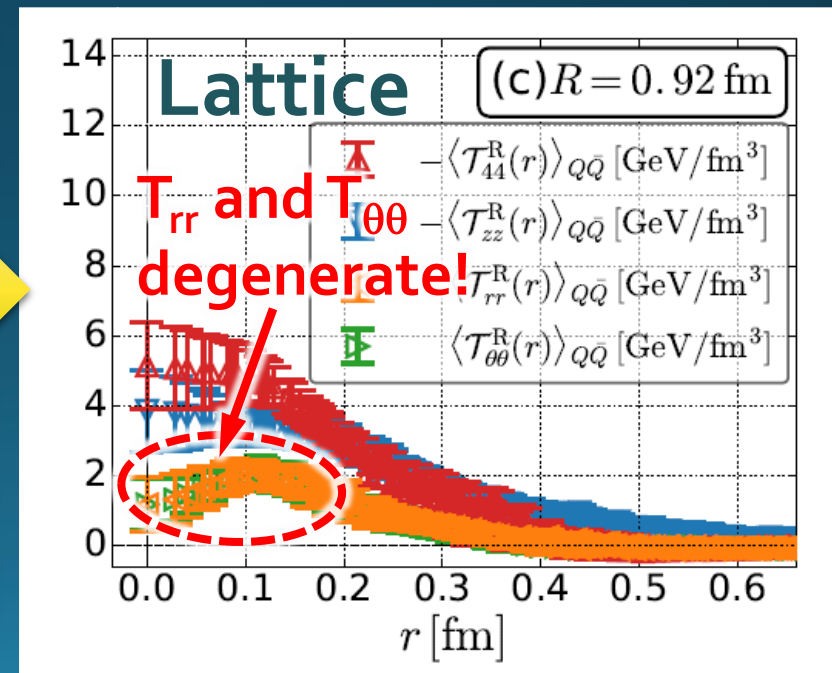


- For infinitely-long flux tube

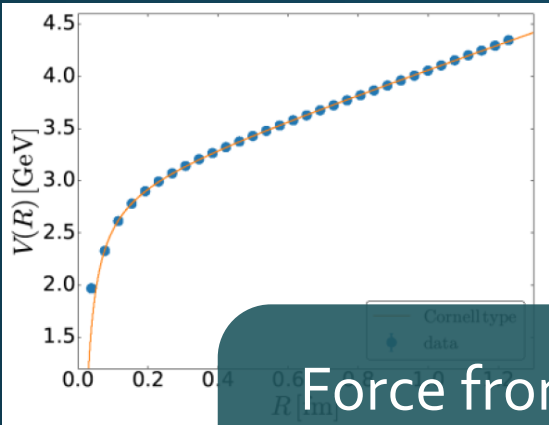
$$\partial_r(rT_{rr}) = T_{\theta\theta}$$

→ T_{rr} and $T_{\theta\theta}$ must separate! ←

Effect of boundaries is important for the flux tube at $R=0.92\text{fm}$

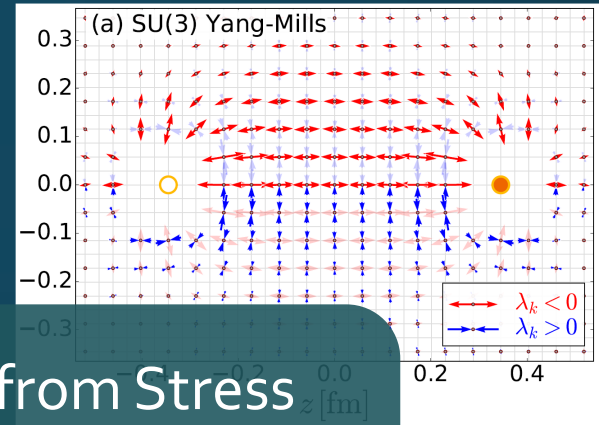


Force



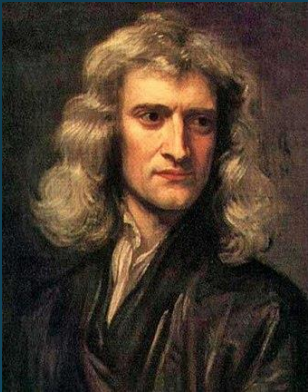
Force from Potential

$$F_{\text{pot}} = -\frac{dV}{dR}$$

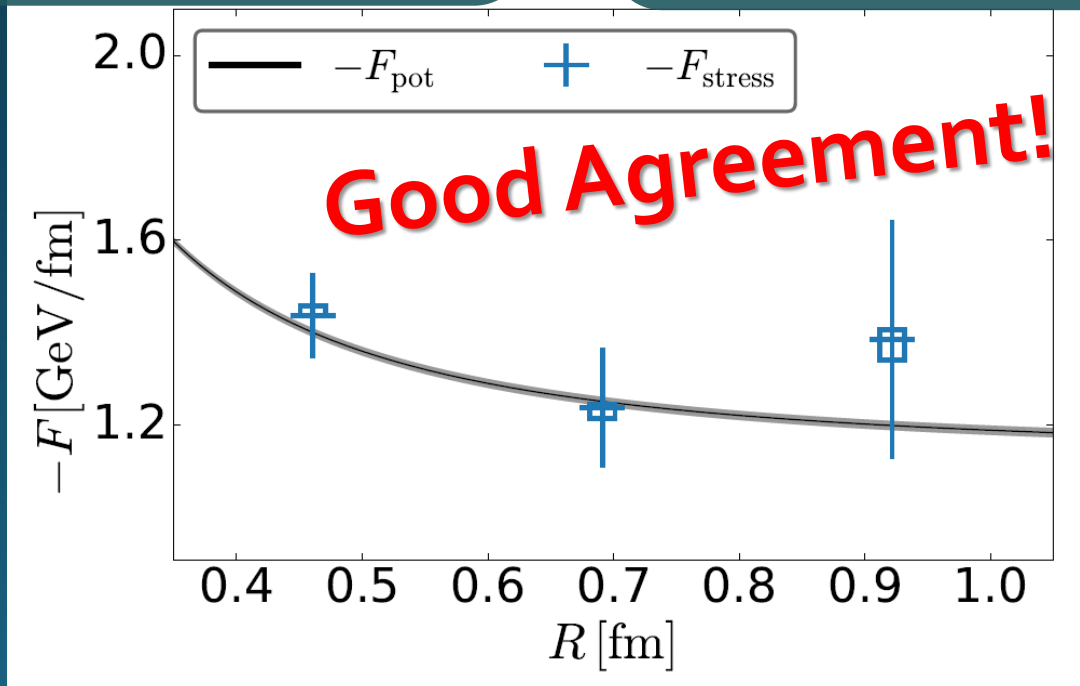


Force from Stress

$$F_{\text{stress}} = \int_{\text{mid.}} d^2x T_{zz}(x)$$



Newton
1687



Faraday
1839