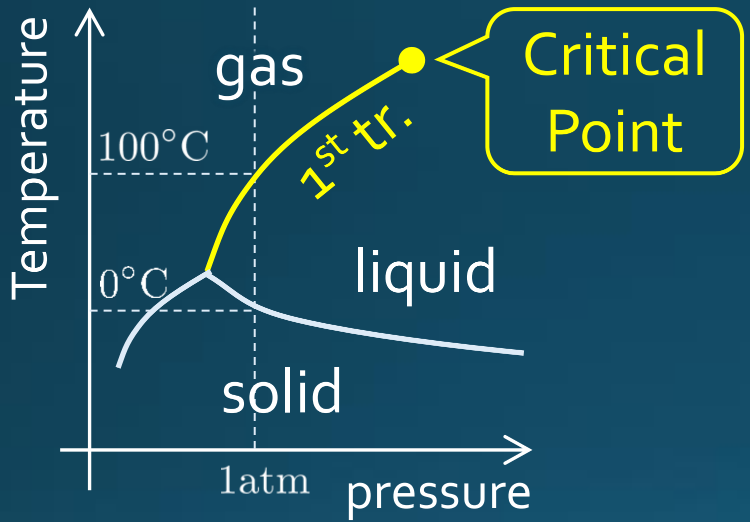
Critical Points in Strongly-Interacting Media

Masakiyo Kitazawa
(Osaka U.)

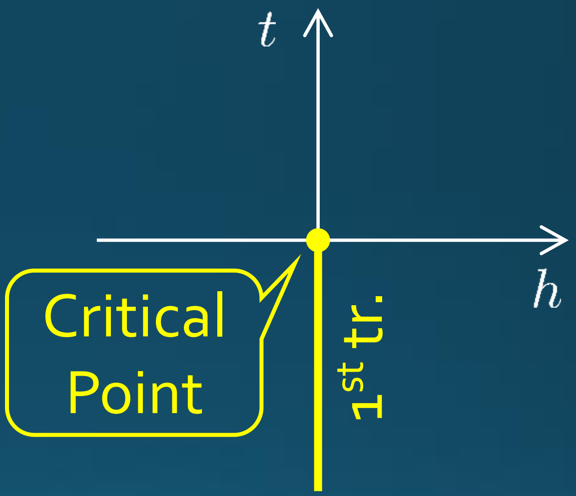
Tsukuba Global Science Week (TGSW), 2021/9/11, online (Tsukuba)

Critical Points

- **Water**



- **Ising Model**



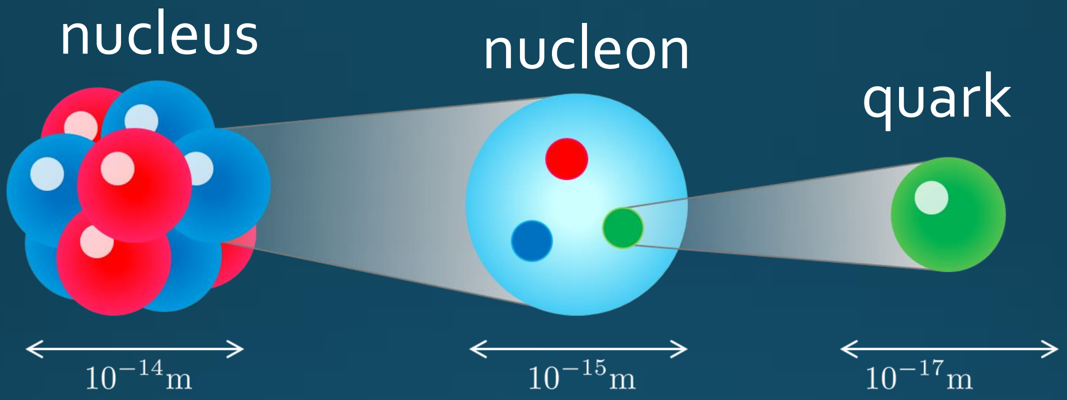
These CPs belong to the same universality class (Z_2).

→ Common critical exponents.

ex. $C \sim (T - T_c)^{-\alpha}$

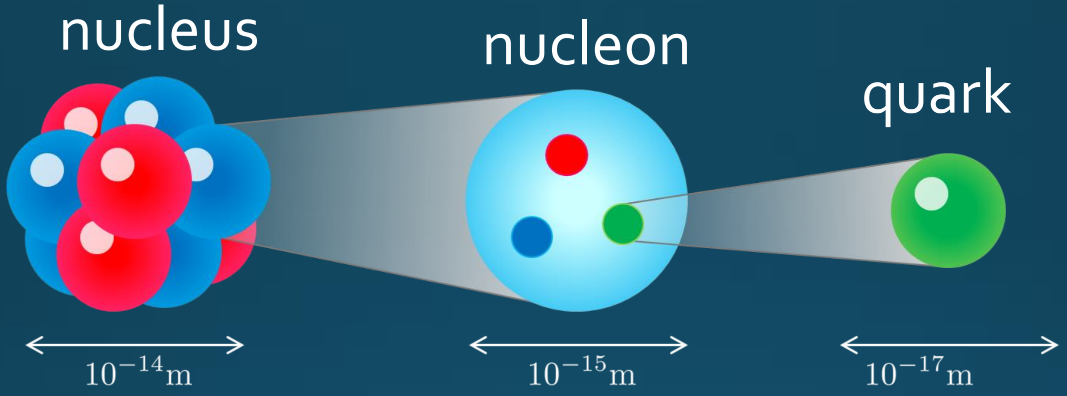
Quantum ChromoDynamics

☐ **Building blocks of matter**



Quantum ChromoDynamics

- **Building blocks of matter**



- **Quantum chromodynamics (QCD)**

=Fundamental theory of strong interaction

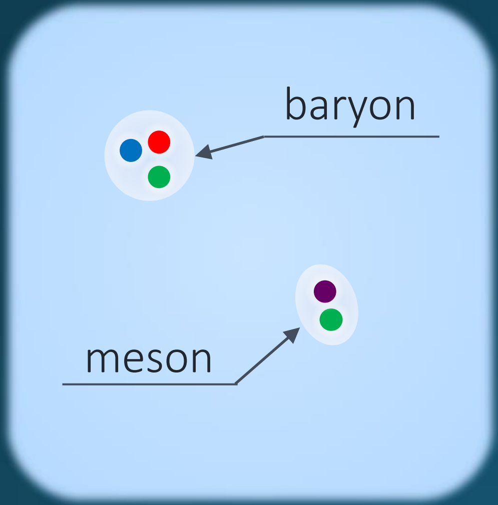
$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}\underline{F_{\mu\nu,a}F_a^{\mu\nu}}$$

quarks

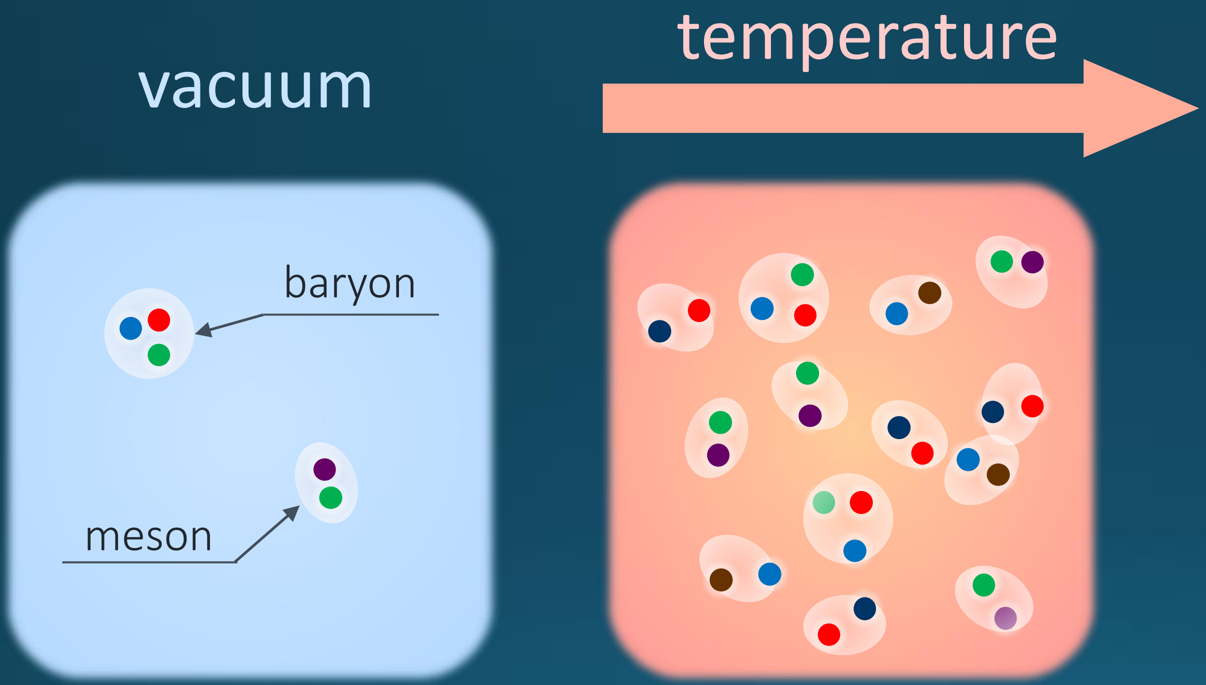
gluons

Quark-Gluon Plasma (QGP)

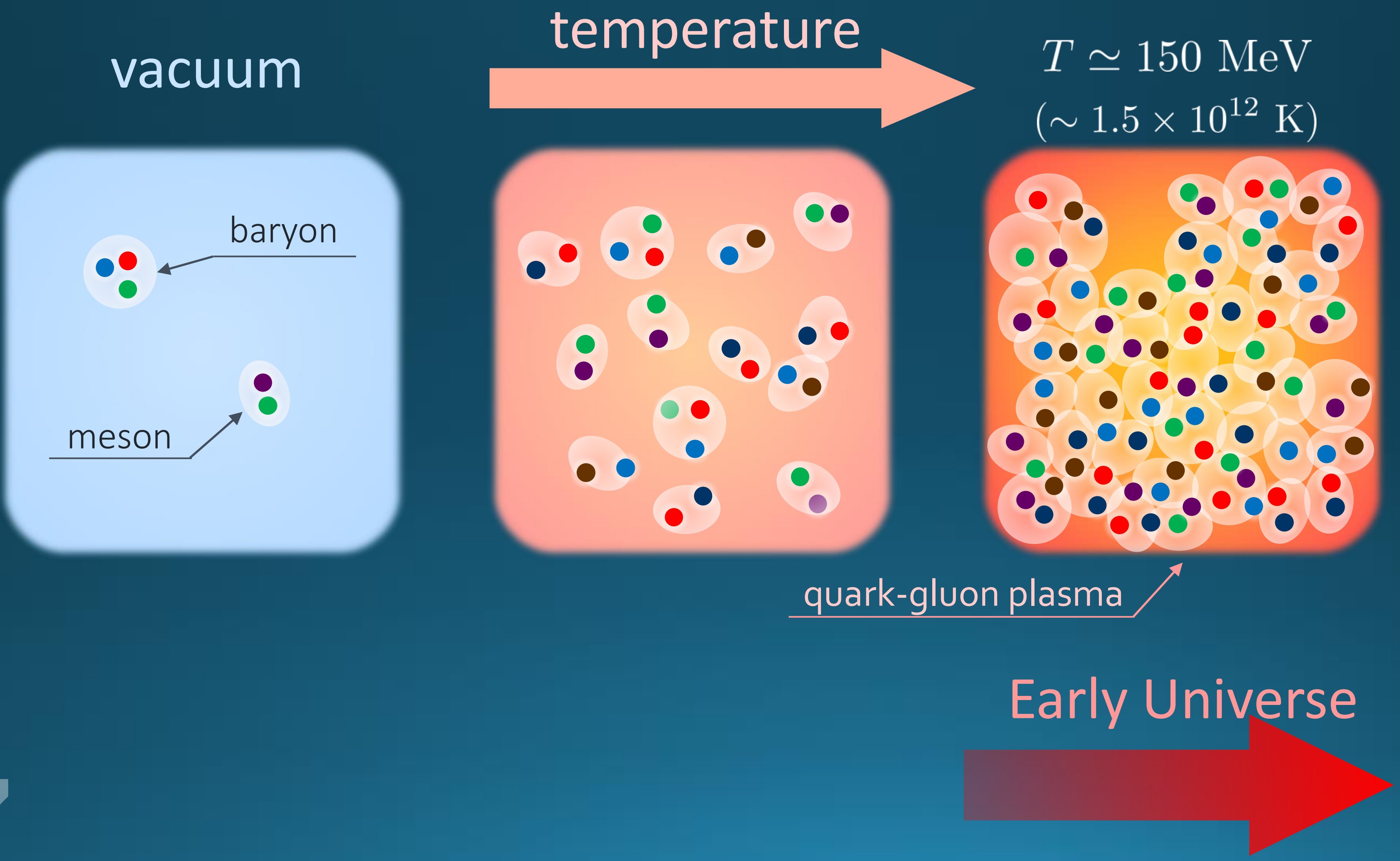
vacuum



Quark-Gluon Plasma (QGP)



Quark-Gluon Plasma (QGP)



vacuum

temperature

$T \simeq 150\ \mathrm{MeV}$

$(\sim 1.5 \times 10^{12}\ \mathrm{K})$

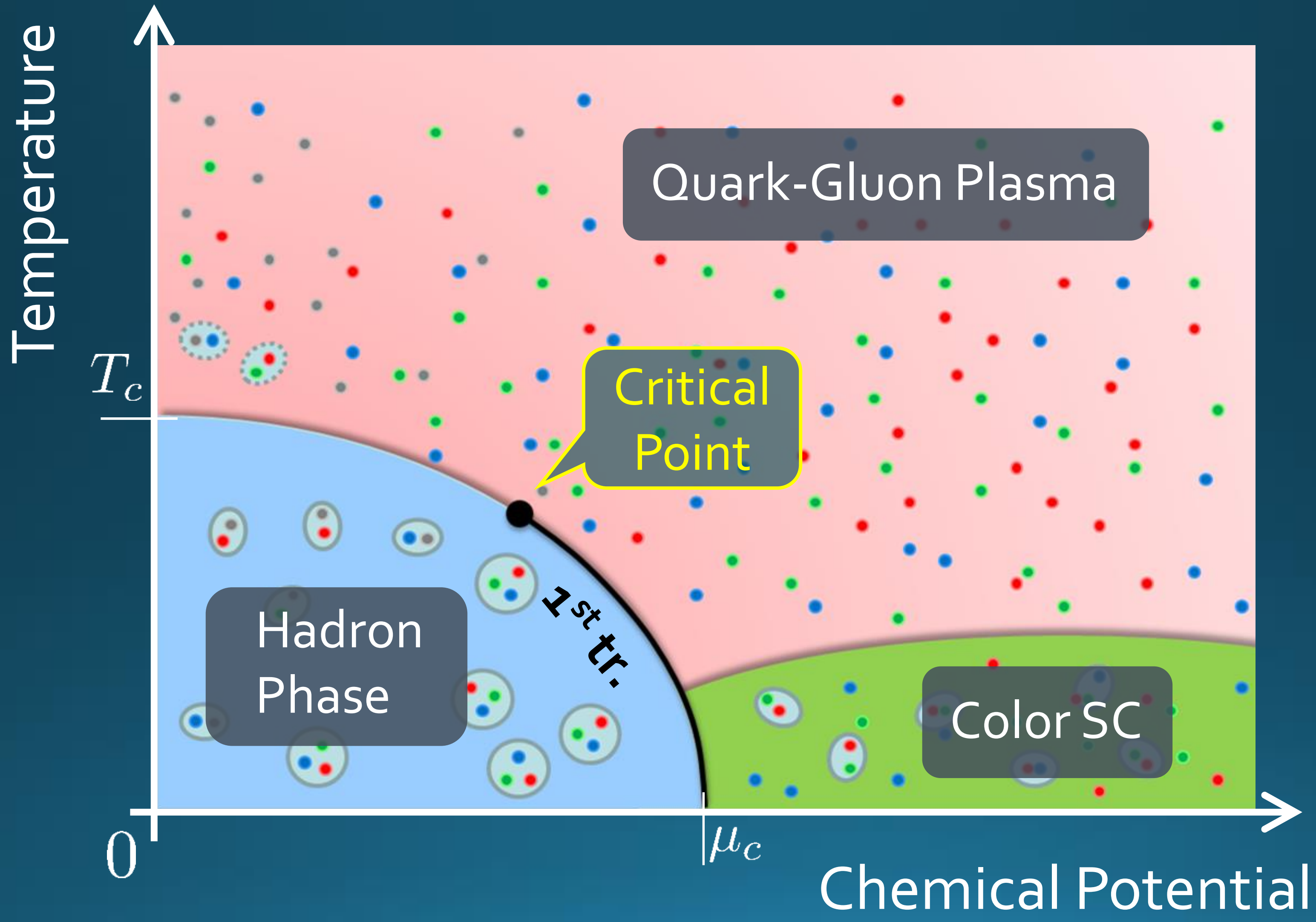
baryon

meson

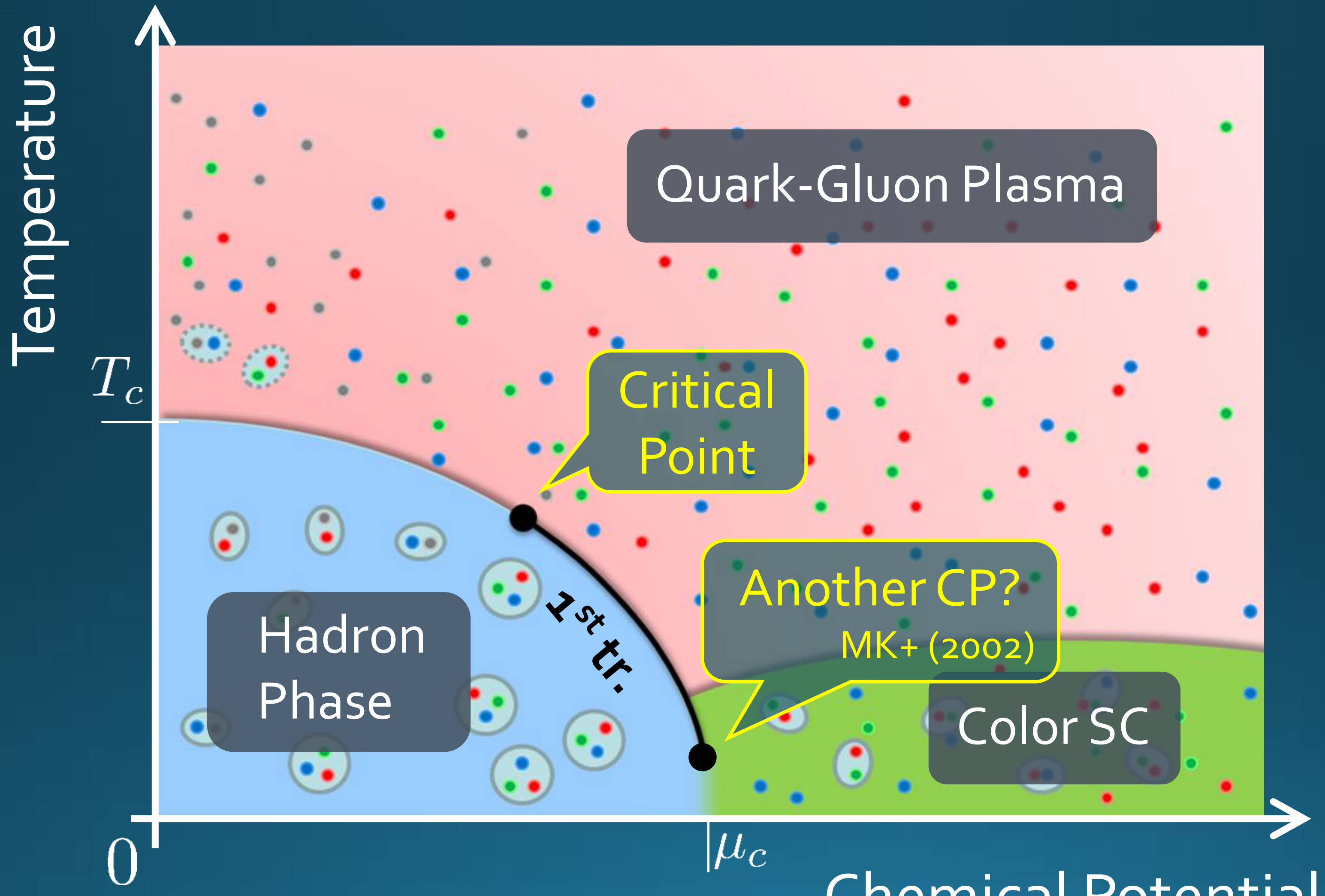
quark-gluon plasma

Early Universe

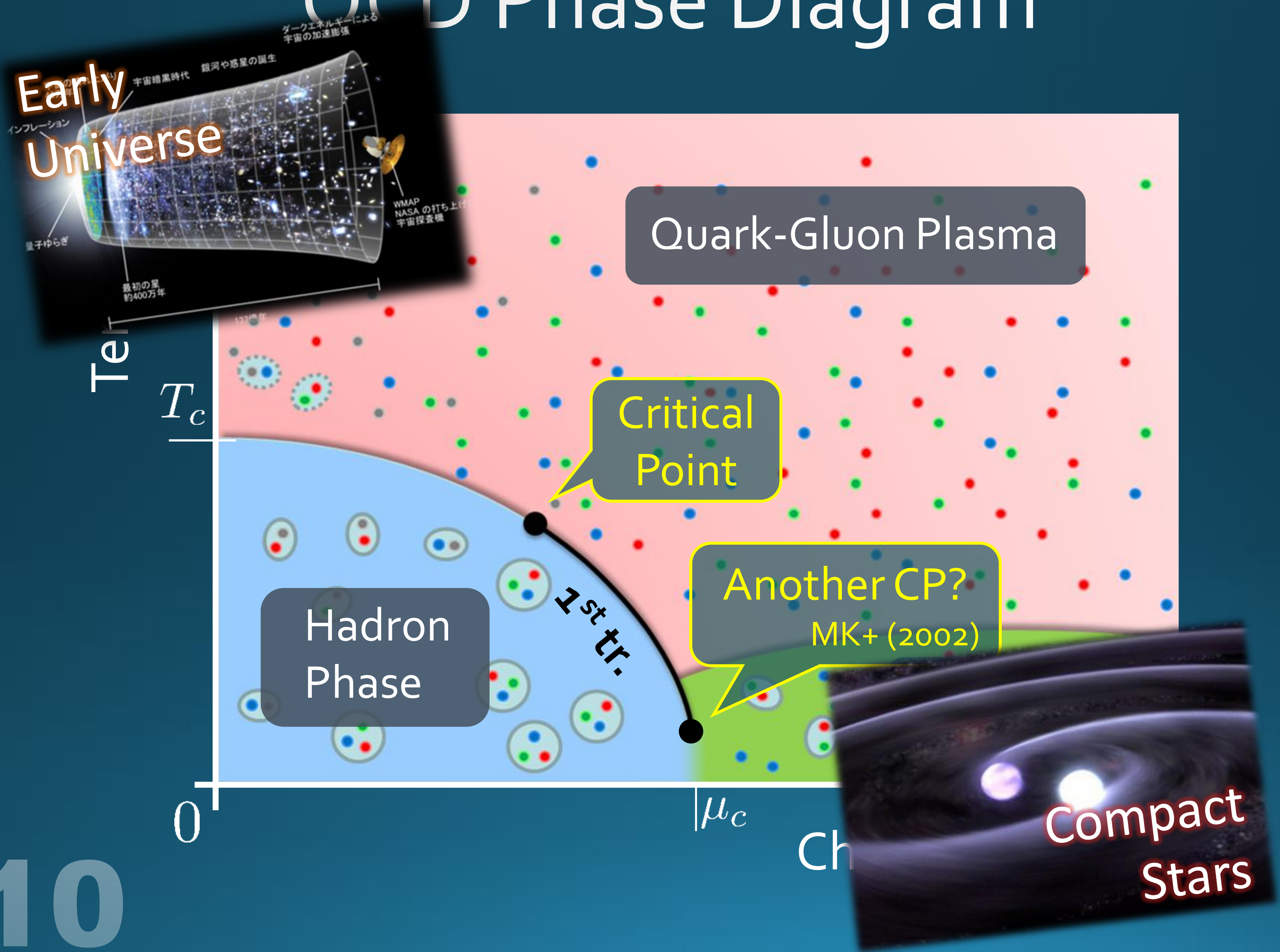
QCD Phase Diagram



QCD Phase Diagram



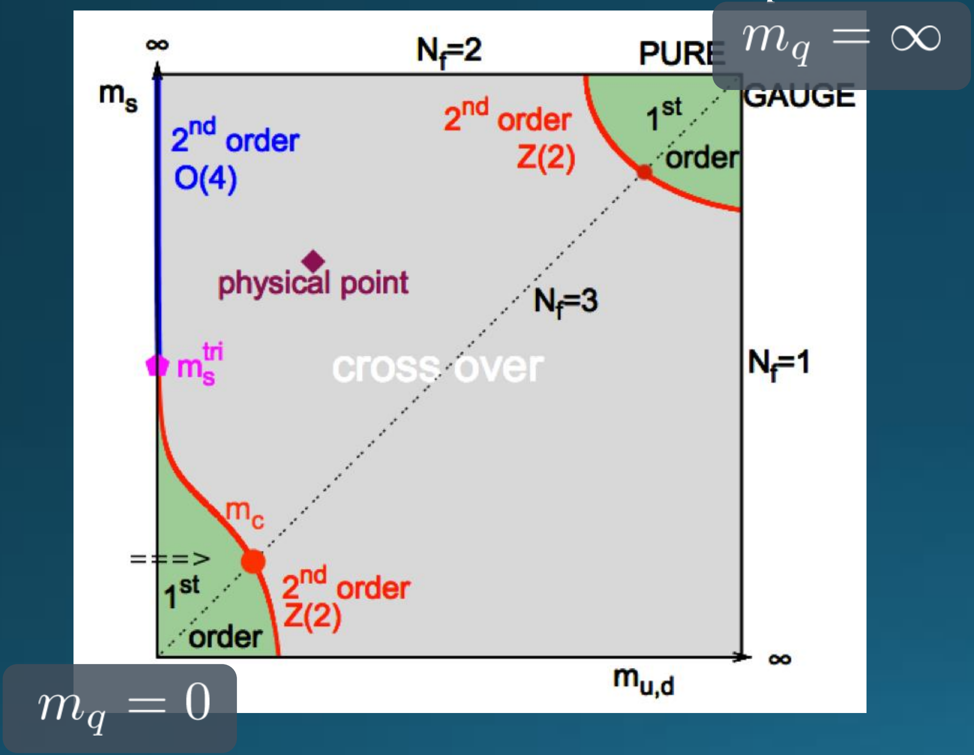
QCD Phase Diagram



Varying Quark Masses

- **Columbia plot**

  = order of phase tr. at $\mu_q = 0$

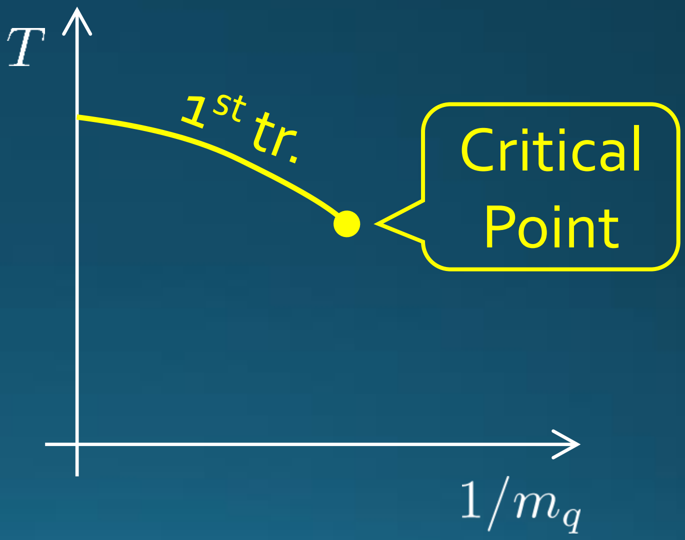


$m_q = \infty$

$m_q = 0$

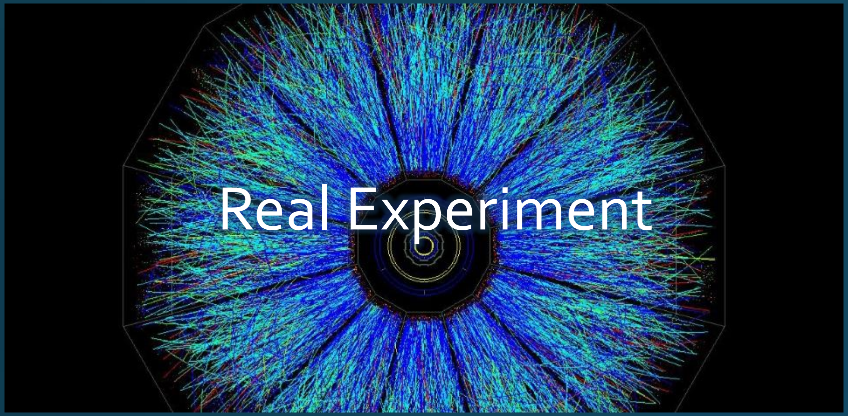
- **Example**

  Phase diagram in heavy-quark region



Various orders of phase transition with variation of m_q.

Two Experimental Tools



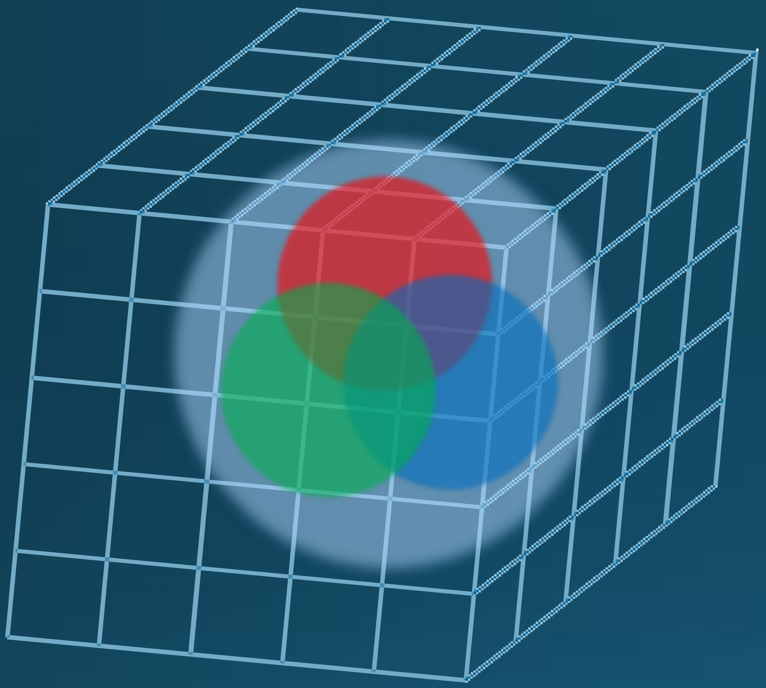
Relativistic Heavy-Ion Collisions

Lattice QCD Numerical Simulations

Colliding two heavy nuclei in accelerators
RHIC (USA), LHC (EU), etc.

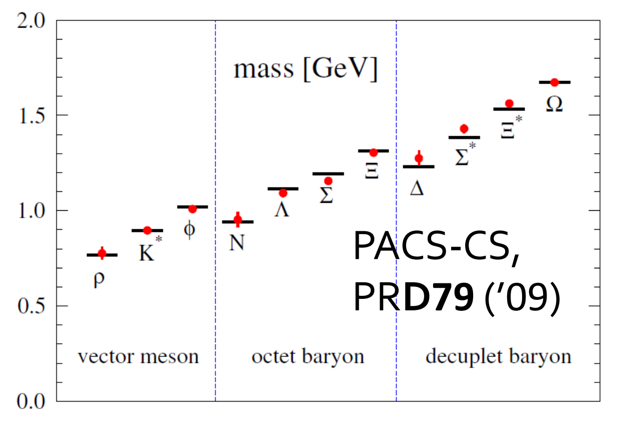
First-principle simulations on supercomputers

Lattice QCD Numerical Simulations



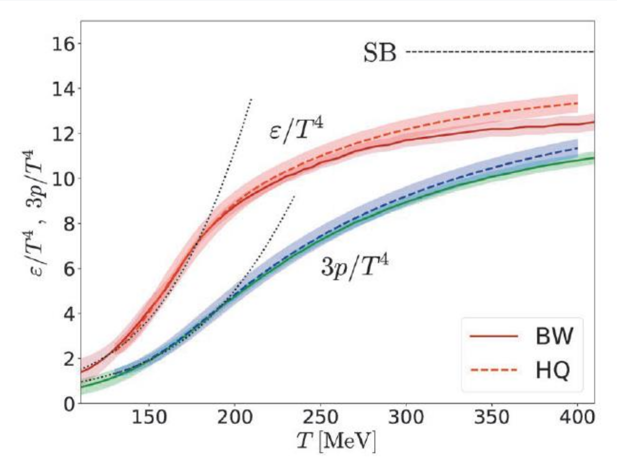
Unique tool to perform **quantitative** analyses of **non-perturbative** QCD aspects

☐ Hadron Spectroscopy

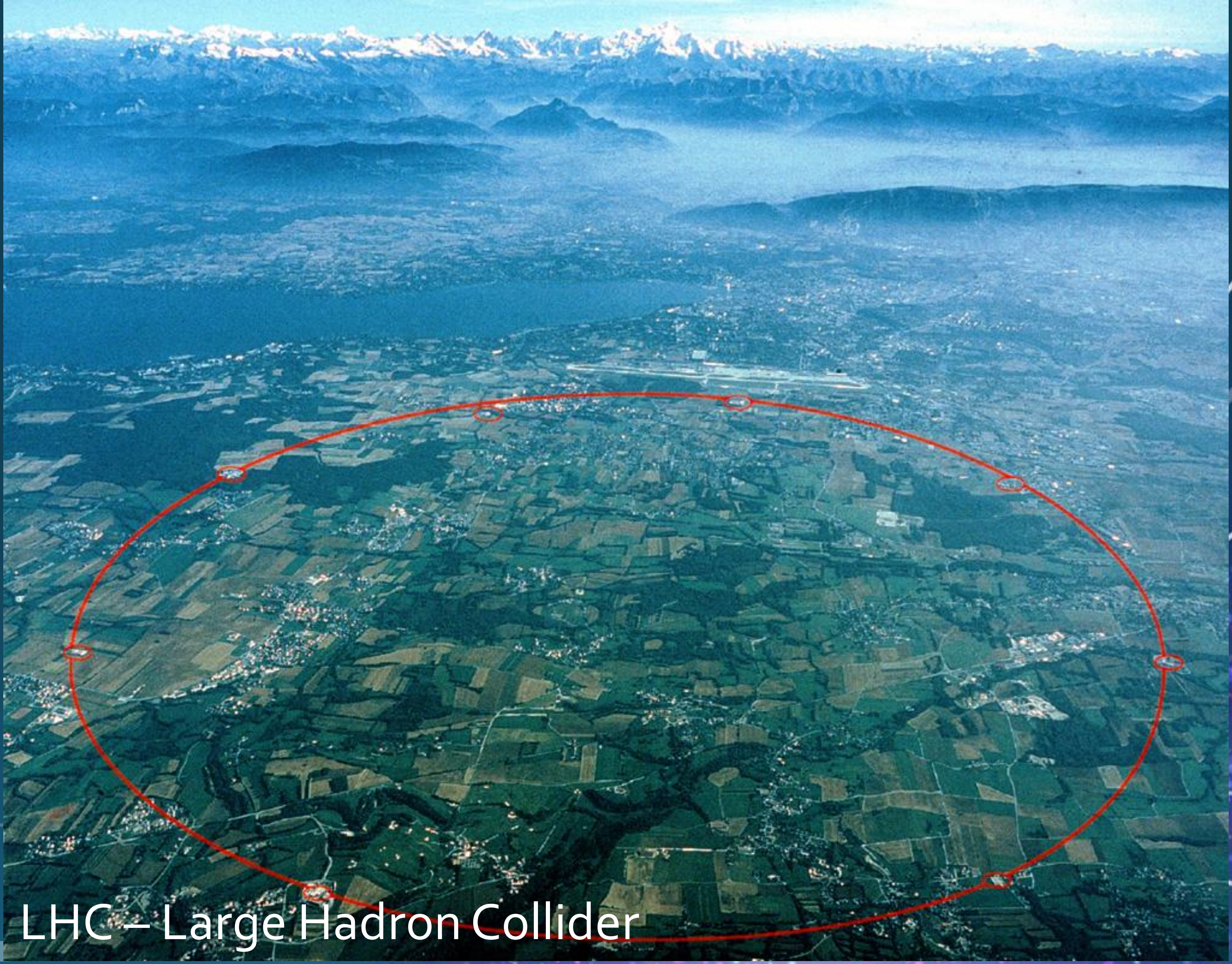


PACS-CS, PR**D79** ('09)

☐ Thermodynamics



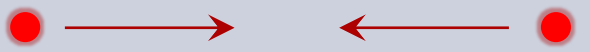
Relativistic Heavy-Ion Collisions



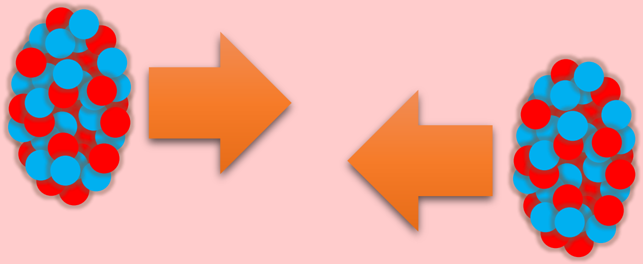
LHC – Large Hadron Collider

Relativistic Heavy-Ion Collisions

Proton-proton



Heavy Ion Collisions

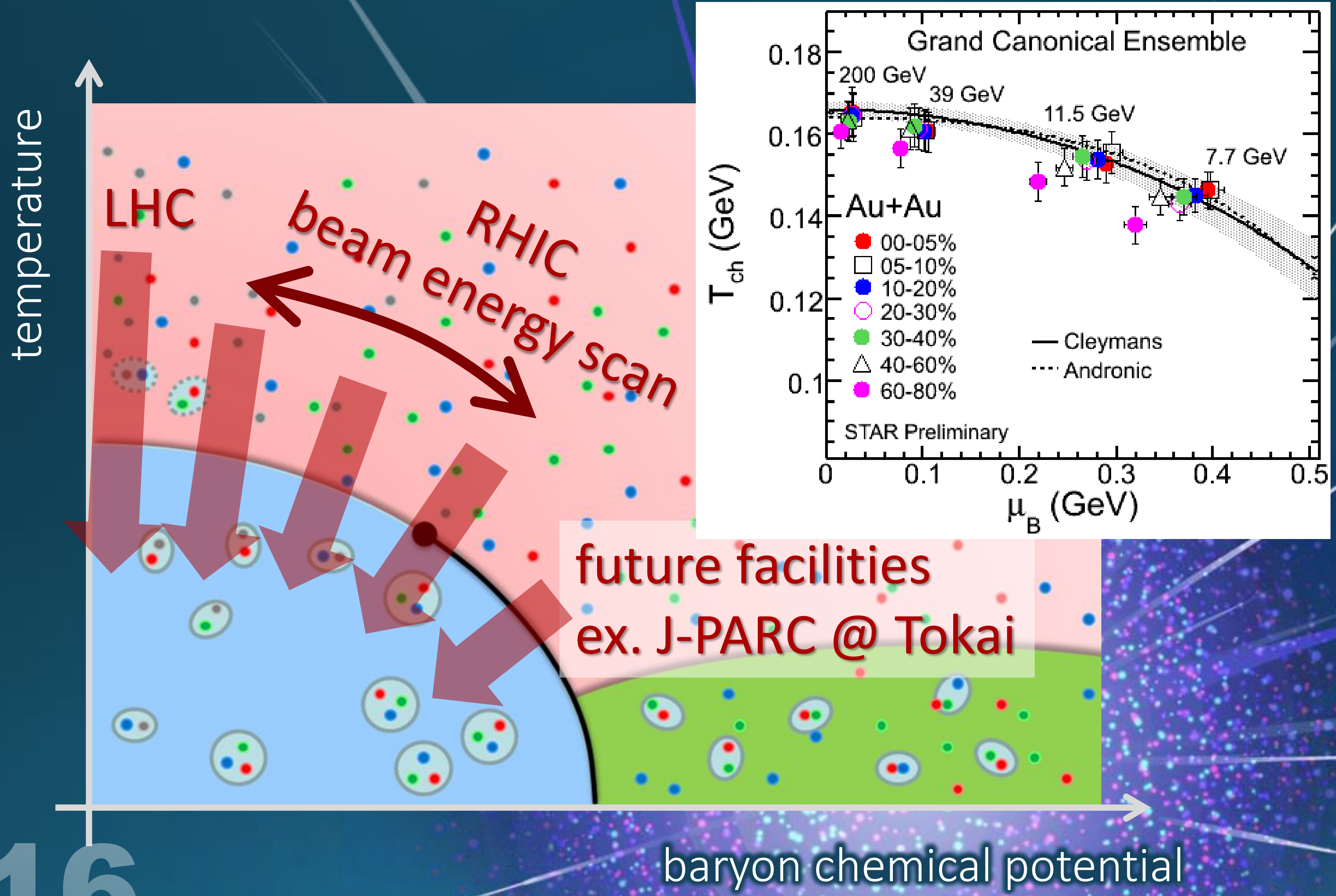


Elementary processes
new particle search
properties of particles

Thermal Medium
hot & dense medium
phase transitions

LHC – Large Hadron Collider

Beam-Energy Scan



temperature

LHC

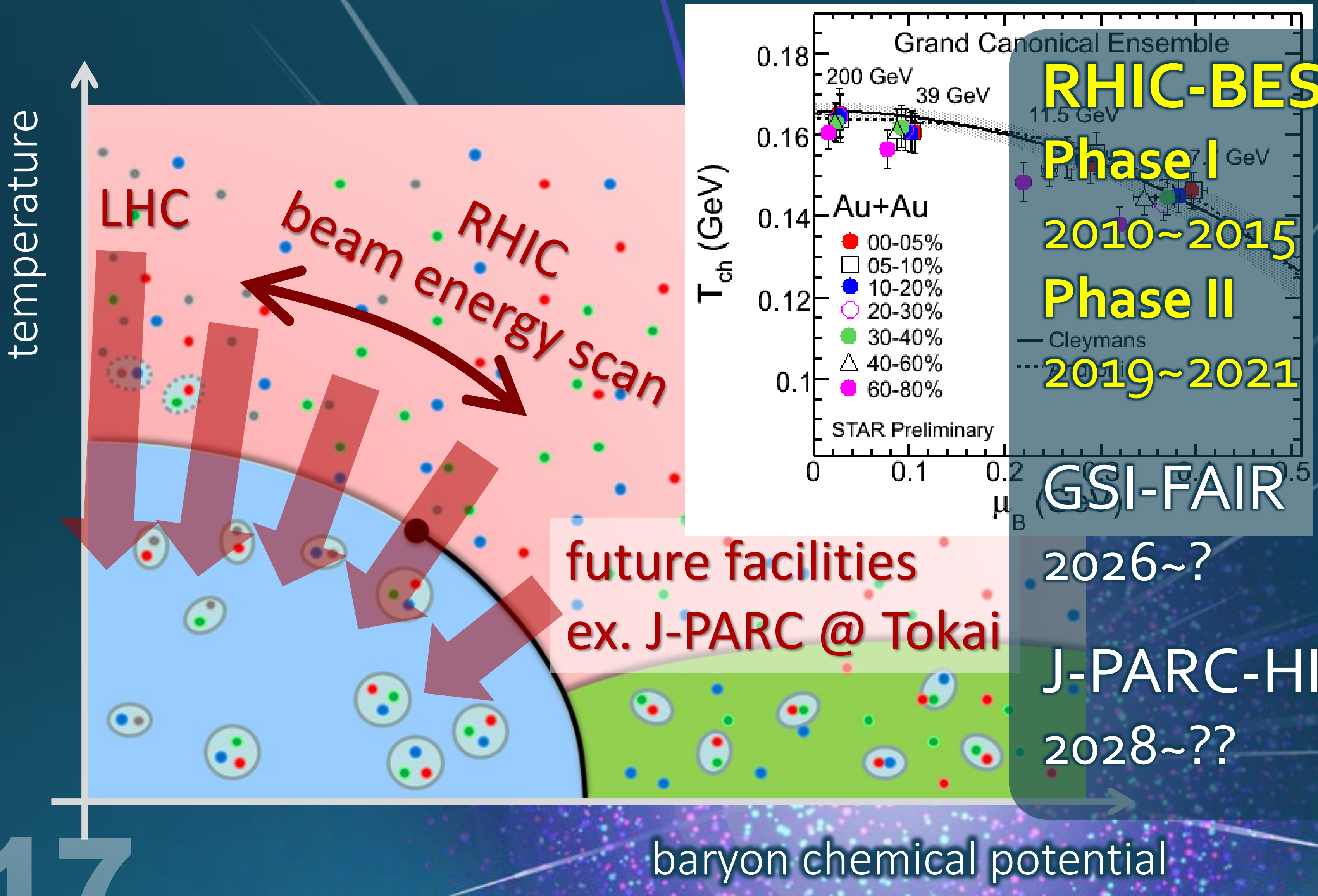
RHIC

beam energy scan

future facilities
ex. J-PARC @ Tokai

baryon chemical potential

Beam-Energy Scan



RHIC-BES

Phase I

2010~2015

Phase II

2019~2021

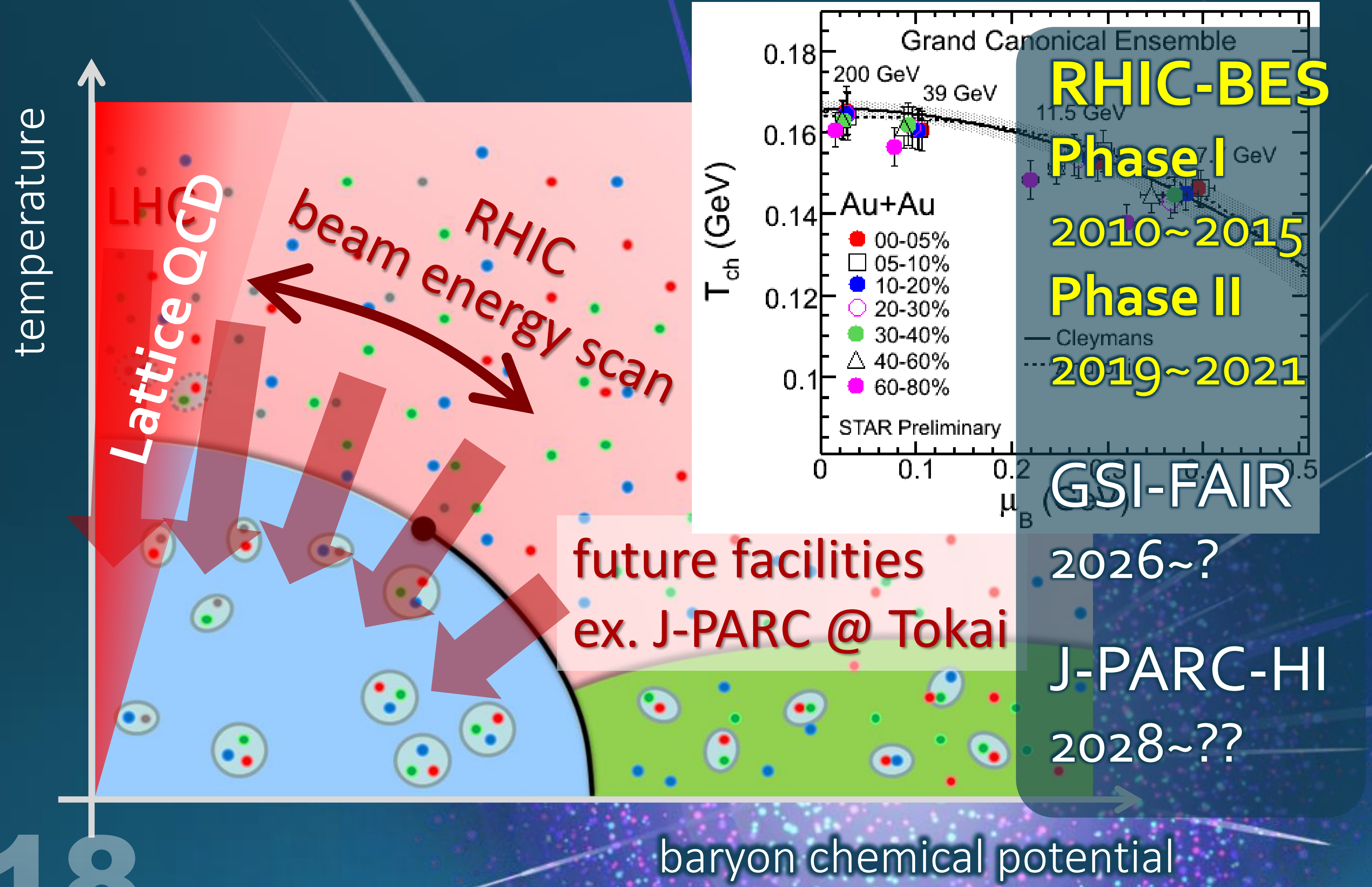
GSI-FAIR

2026~?

J-PARC-HI

2028~??

Beam-Energy Scan



future facilities
ex. J-PARC @ Tokai

RHIC-BES

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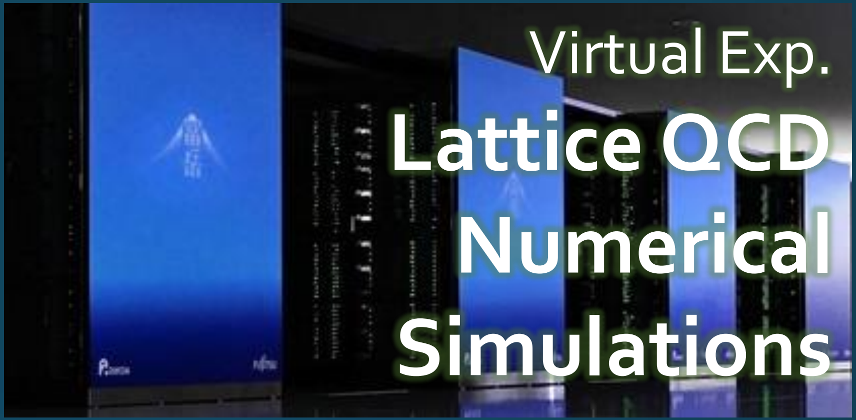
Two Experimental Tools

Real Exp.

Relativistic Heavy-Ion Collisions



Virtual Exp.

Lattice QCD Numerical Simulations

Real experiments ⟺ Virtual, but unphysical params

Various density ⟺ Small baryon density only

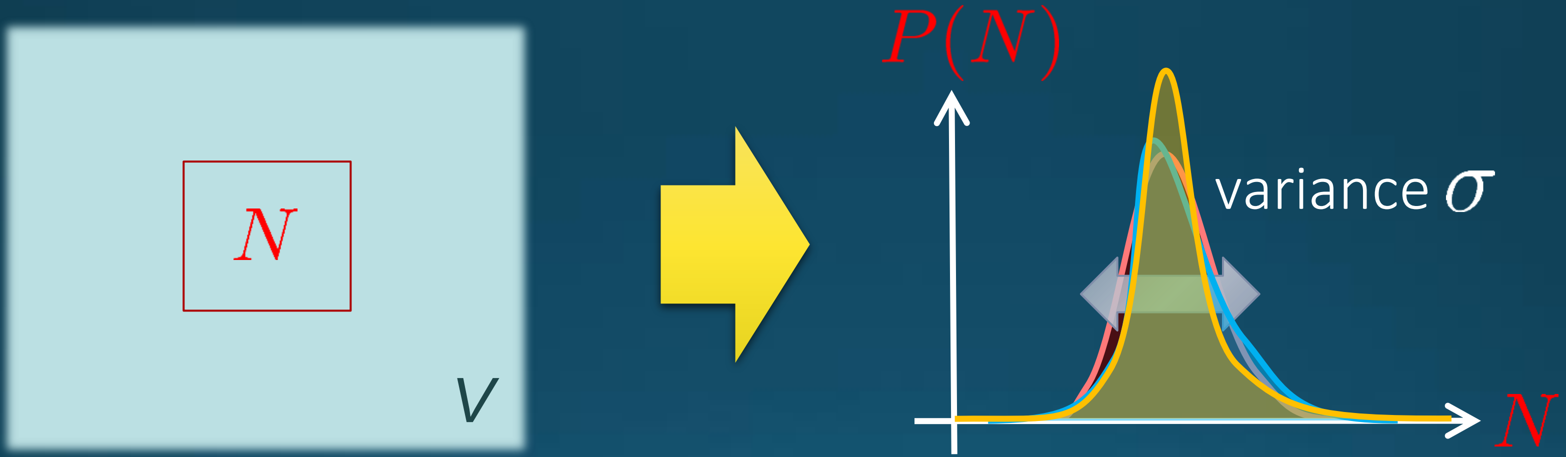
Dynamical evolution ⟺ Ideal thermal system

Final-state observables only   Limited observables

Complementary use of both exps. is important!

Fluctuations

Observables in equilibrium are fluctuating!



Cumulants

Cumulants

$$\begin{cases} \langle N \rangle_c = \langle N \rangle & \text{average} \\ \langle N^2 \rangle_c = \langle \delta N^2 \rangle & \text{variance} \\ \langle N^3 \rangle_c = \langle \delta N^3 \rangle & \\ \langle N^4 \rangle_c = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2 & \end{cases}$$

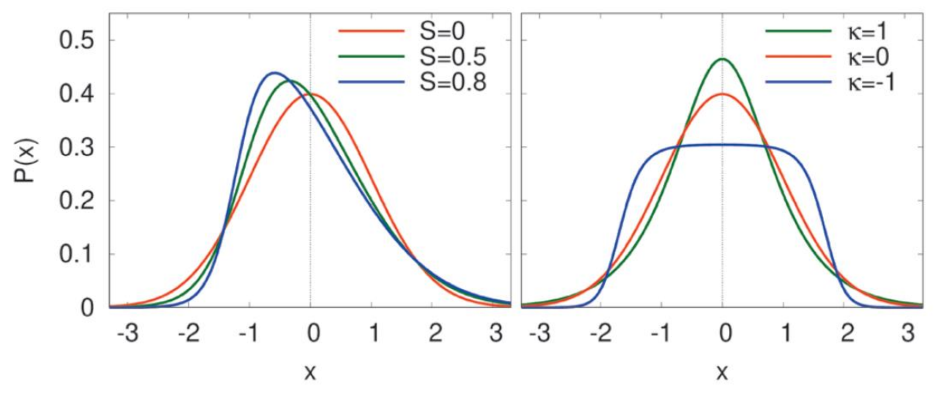
Binder Cumulant

☐ skewness

$$S = \frac{\langle N^3 \rangle_c}{\langle N^2 \rangle_c^{3/2}}$$

☐ kurtosis

$$\kappa = \frac{\langle N^4 \rangle_c}{\langle N^2 \rangle_c^2}$$



☐ NOTE

- Gauss distribution: $\langle N^3 \rangle_c = \langle N^4 \rangle_c = \cdots = 0$
- Poisson distribution: $\langle N^2 \rangle_c = \langle N^3 \rangle_c = \langle N^4 \rangle_c = \cdots = \langle N \rangle$

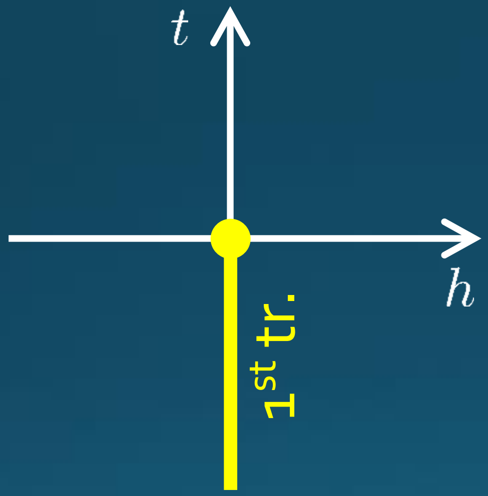
Review: Asakawa, MK, PPNP **90** (2016)

Cumulants around Critical Point

$$P(M) \sim e^{-V(M)}$$

- $P(M)$: probability distr.
- $V(M)$: effective potential
- M : order parameter

Ising Model



- Sign of $\langle N^3 \rangle_c$ is flipped at the CP.

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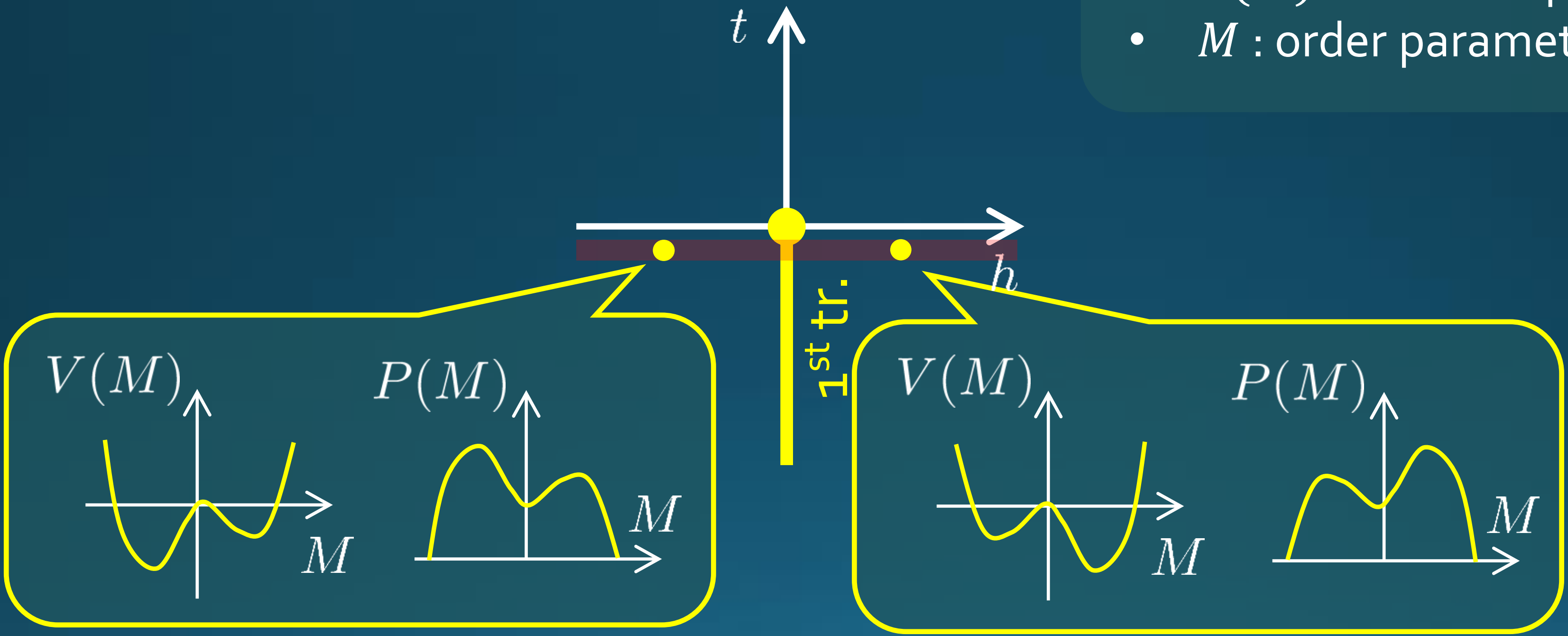

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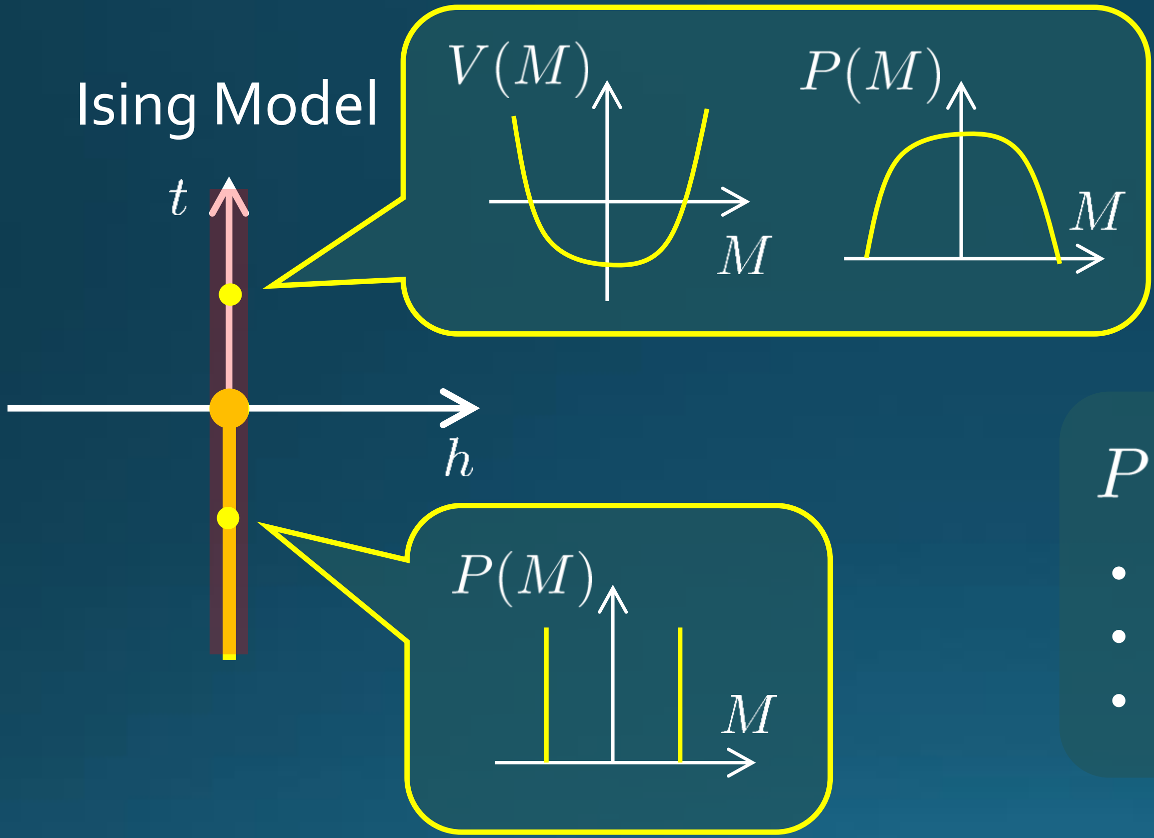
Cumulants around Critical Point

Ising Model



$$P(M) \sim e^{-V(M)}$$

- $P(M)$: probability distr.
- $V(M)$: effective potential
- M : order parameter

- $\langle N^4 \rangle_c$ changes discontinuously at the CP.

Experimental Search for QCD Critical Point

Reviews:

Asakawa, MK, PPNP ('16)

Bluhm, MK+, NPA 1003 ('20)

MK, Esumi, Nonaka, JPS journal, 2021/8

◆◆◆◆ 解説 ◆◆◆◆

非ガウスゆらぎで探る宇宙最高密度の相転移

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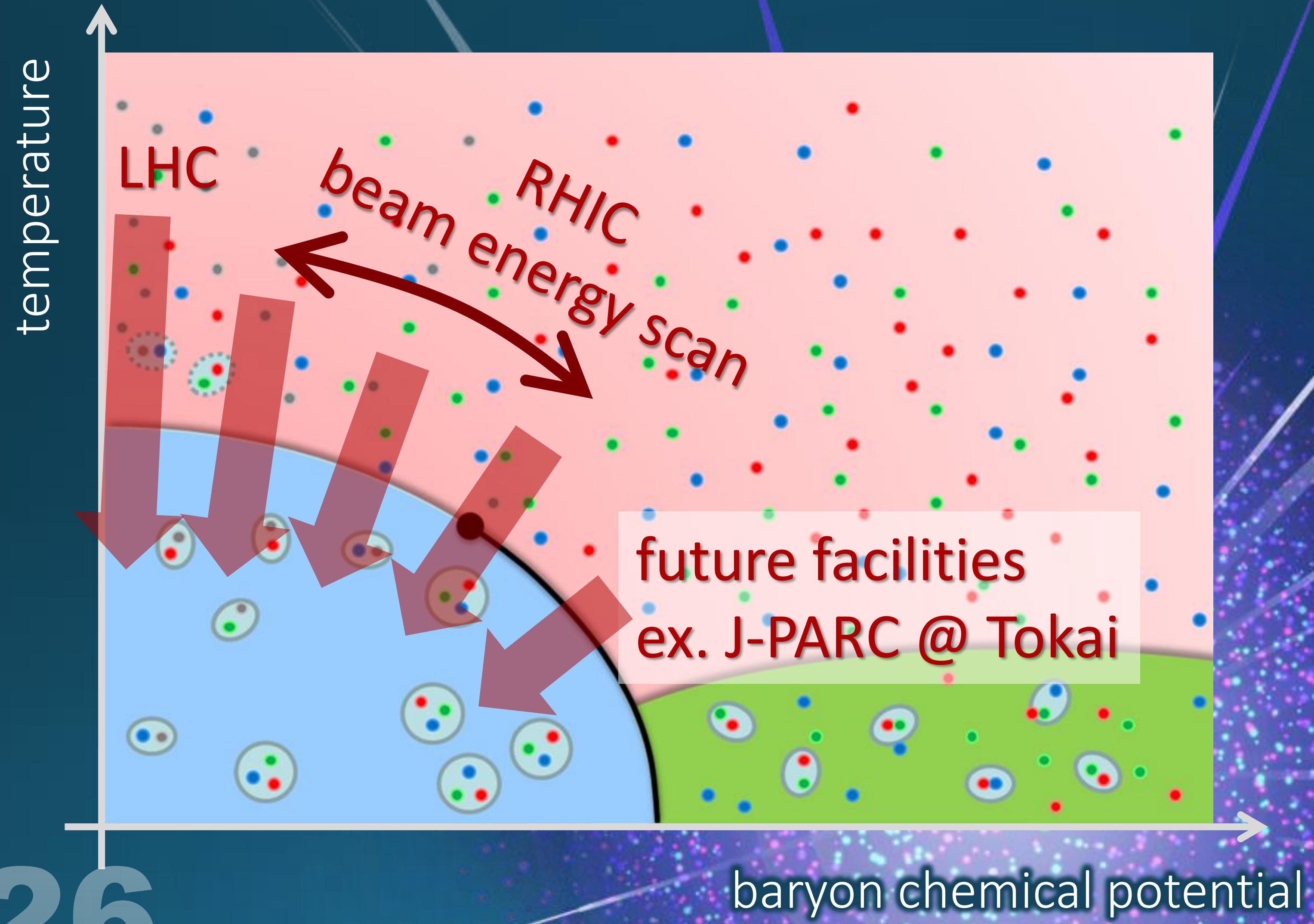
現在，およそ10^{15} g/cm^{3}という超高密度で実現するとされる相転移の実験的探索が世界各地の実験施設で行われているのをご存じだろうか．この相転移とは，強い相互作用の基礎理論である量子色力学（QCD）が低温かつ超高密度の物質中で引き起こす一次相転移と，その一次相転移線の端点であるQCD臨界点のことである．10^{15} g/cm^{3}という密度は，原子核の飽和密度$\rho_0 \simeq 2.5\times10^{14}$ g/cm^{3}を大きく上回り，現在の宇宙における最高密度状態の中性子星中心部に匹敵する．この相転移を，加速した重い原子核を衝突させる実験である高エネルギー重イオン衝突によって地上で実現し，その性質を調べるための実験が進められてこれら一連の実験が目指す最重要課題が，ビームエネルギー走査による高密度領域の相構造探索である．

これら一連の研究の中でも近年特に精力的に調べられてきたのが，非ガウスゆらぎを使ったQCD臨界点の実験的探索である．ゆらぎはキュムラントとよばれる量で特徴づけられるが，QCD臨界点でゆらぎが発散するのに伴い，QCD臨界点周辺では各次数のキュムラントに特徴的な発散や符号変化などの異常が現れることが理論的に指摘されている．一方，重イオン衝突実験では，衝突事象毎解析とよばれる手法で保存電荷数などの観測量のゆらぎが測定でき，10^9をも凌ぐ膨大な衝突事象の解析によっ

—用語解説—

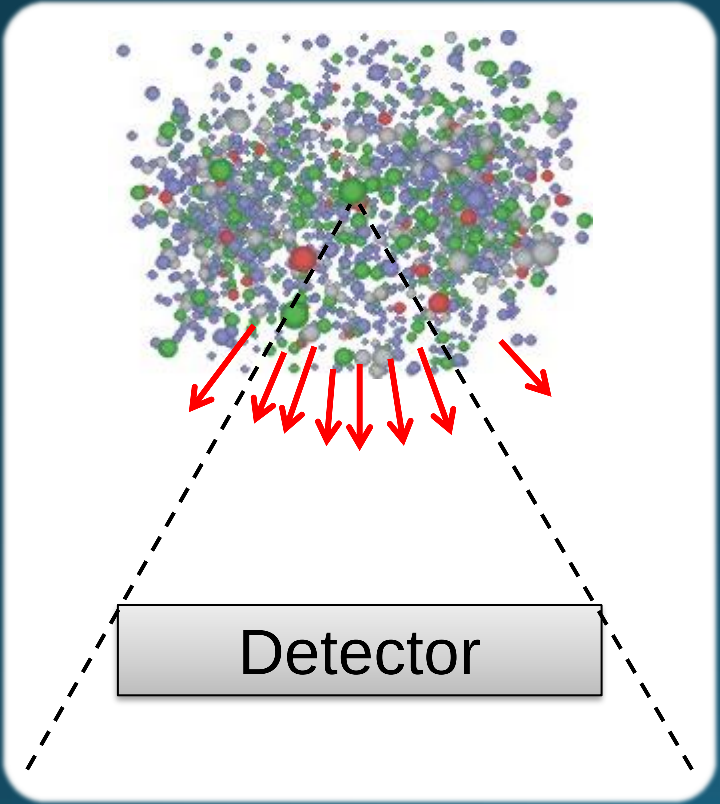
QCD臨界点：
QCD真空にクォーク数密度を印加していくと，10^{15} g/cm^{3}付近で真空構造の変質に伴う一次相転移が起きる可能性が指摘されている．この一次相転移が存在する場合，相転移線は有限温度で端点，すなわち臨界点をもつ．この点をQCD臨界点とよび，現在ビームエネルギー走査によるその実験的探索が注目されている．

Beam-Energy Scan

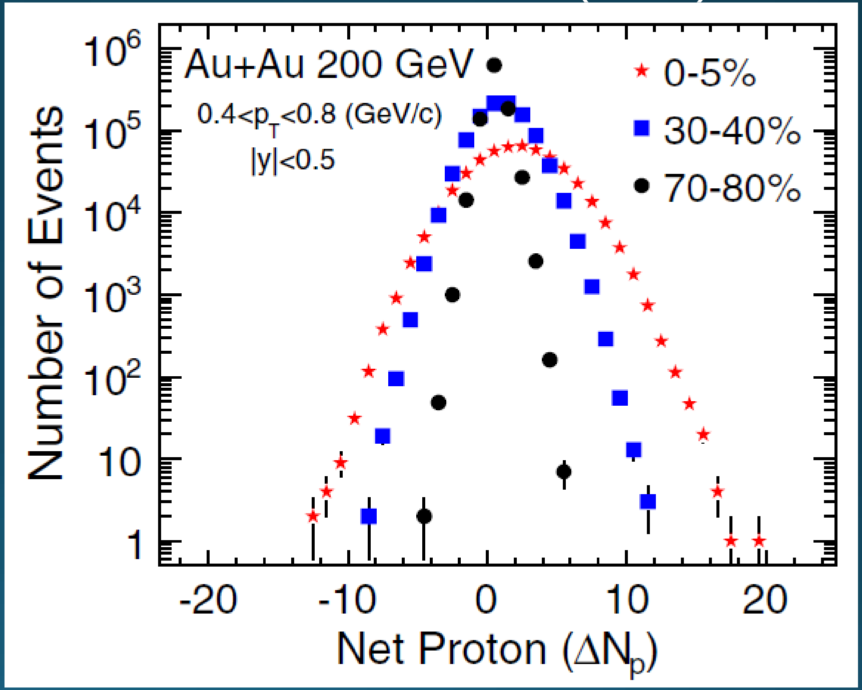


Event-by-event Fluctuations

Review: Asakawa, MK, PPNP **90** (2016)



STAR, PRL**105** (2010)



Cumulants

$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$

Cumulants around Critical Point

$$P(M) \sim e^{-V(M)}$$

- $P(M)$: probability distr.
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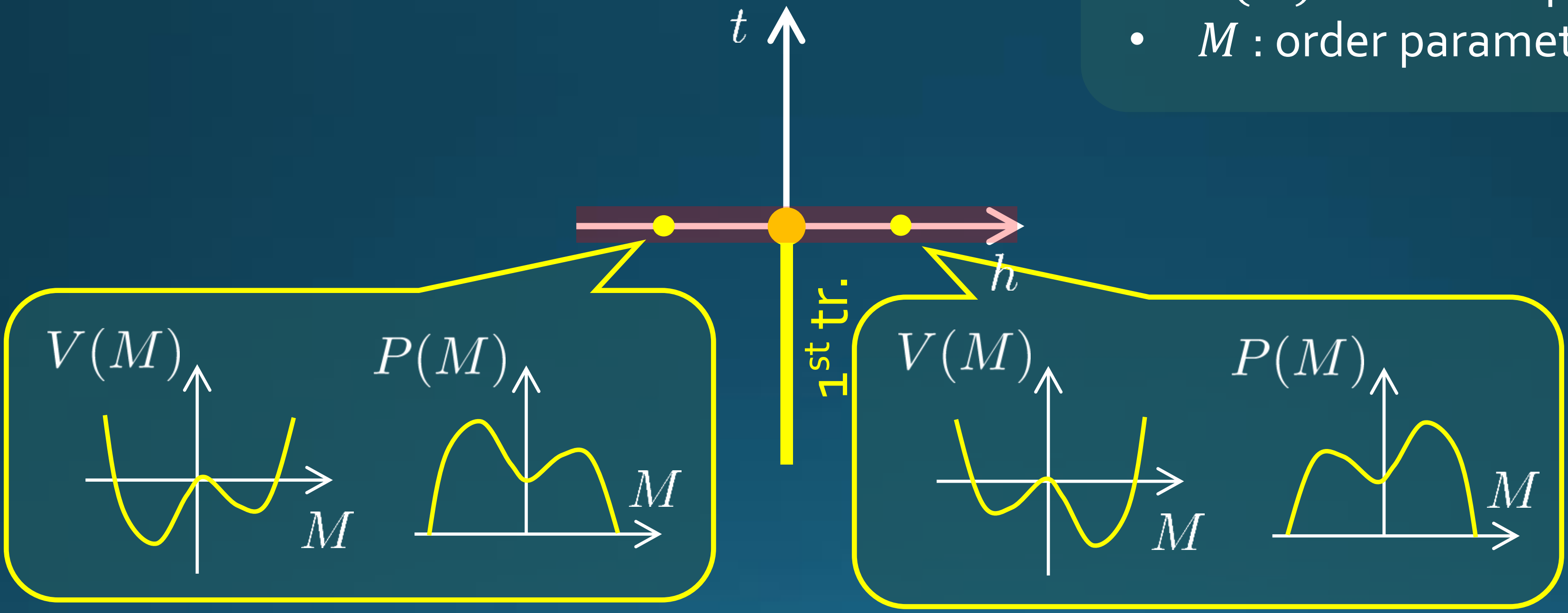
- M : order parameter

Ising Model



- Sign of $\langle N^3 \rangle_c$ is flipped at the CP.

Sign Change of Cumulant

Asakawa, Ejiri, MK, PRL '09

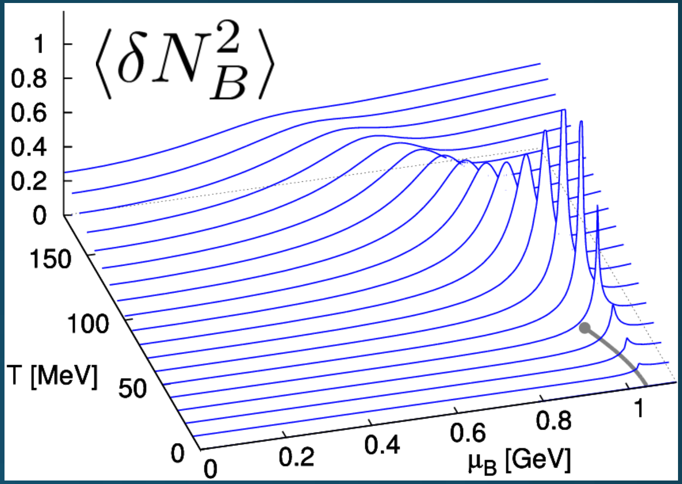
☐ Geometric interpretation on the signs

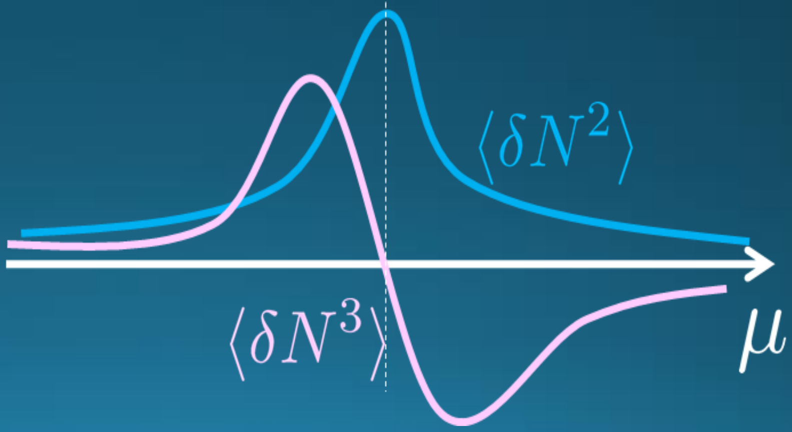
Fluctuations $\langle N_B^2 \rangle_c$ diverge at the QCD-CP.

Themodynamic Relation

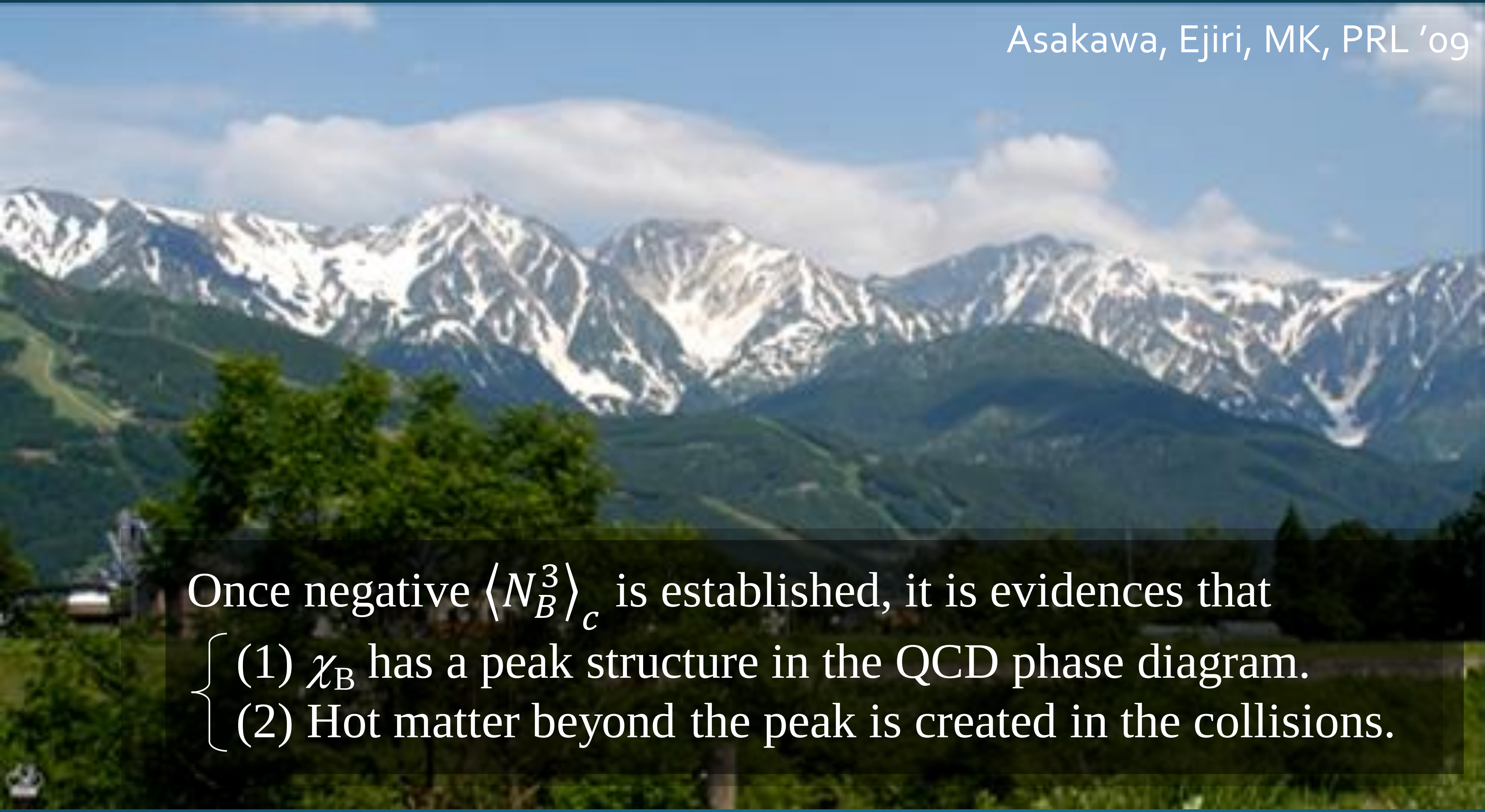
$$\langle N_{\mathrm{B}}^{m+1} \rangle_c = T \frac{\partial \langle N_{\mathrm{B}}^m \rangle_c}{\partial \mu_{\mathrm{B}}}$$

Sign of $\langle N_B^3 \rangle_c$ can distinguish near and away sides!





Impact of Negative Cumulants



Asakawa, Ejiri, MK, PRL '09

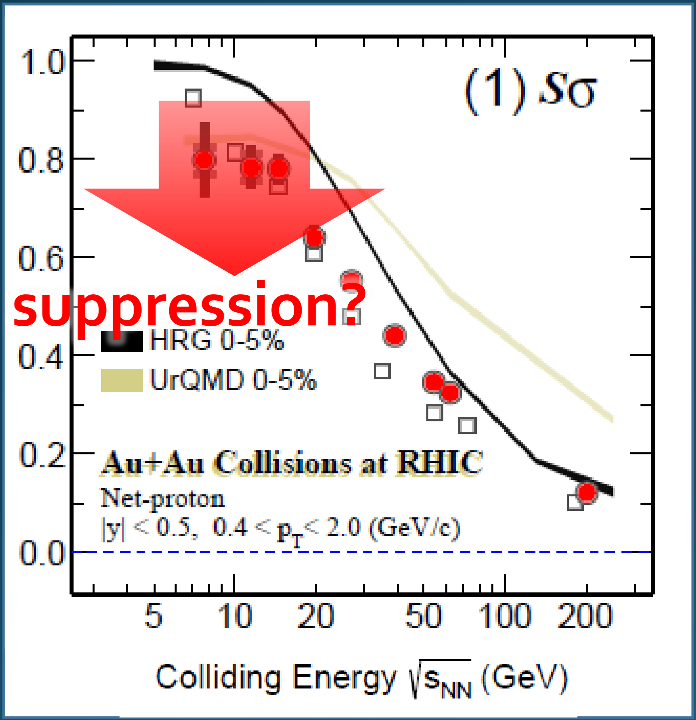
Once negative $\langle N_B^3 \rangle_c$ is established, it is evidences that

$\left\{ \begin{array}{l} \text{(1) } \chi_{\mathrm{B}} \text{ has a peak structure in the QCD phase diagram.} \\ \text{(2) Hot matter beyond the peak is created in the collisions.} \end{array} \right.$

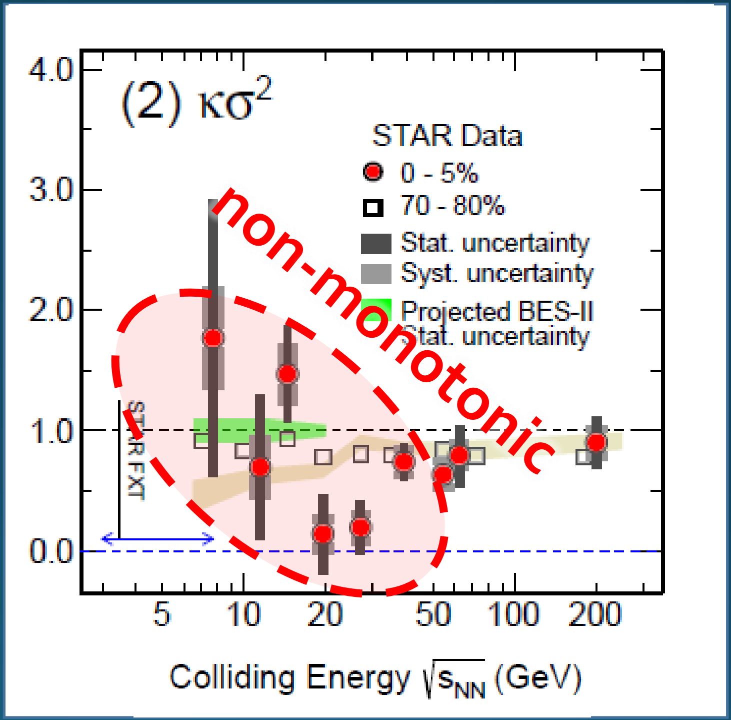
- **No** dependence on any specific models.
- Just the sign! **No** normalization (such as by N_{ch}).

Proton Number Cumulants

$\langle N_p^3 \rangle_c / \langle N_p^2 \rangle_c$



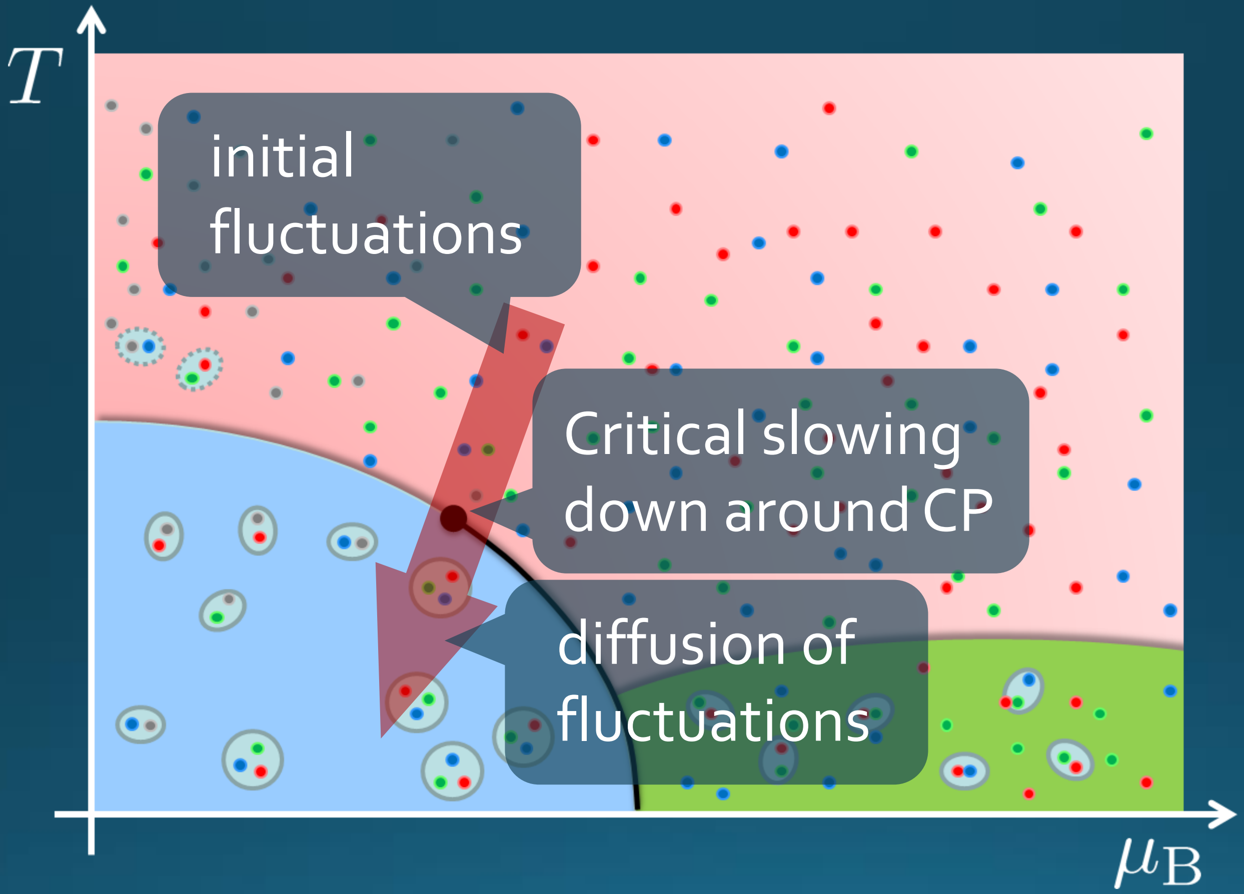
$\langle N_p^4 \rangle_c / \langle N_p^2 \rangle_c$



STAR, 2001.06419

- **Nonzero and non-Poissonian cumulants are experimentally established.**

Time Evolution of Cumulants



initial fluctuations

Critical slowing down around CP

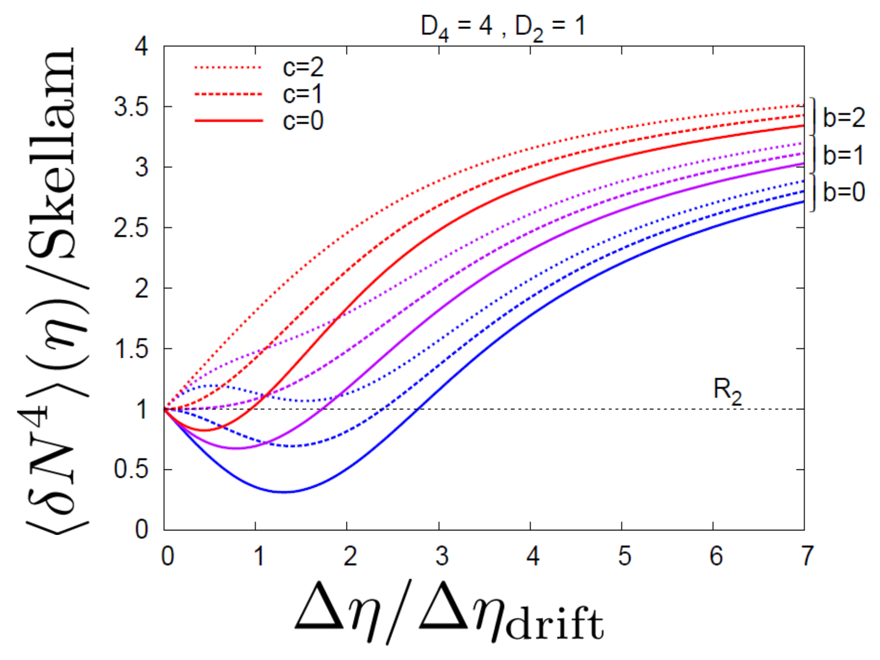
diffusion of fluctuations

Proper understanding of the time evolution of fluctuations is indispensable.

Rapidity Window Dependence
in Diffusion Models

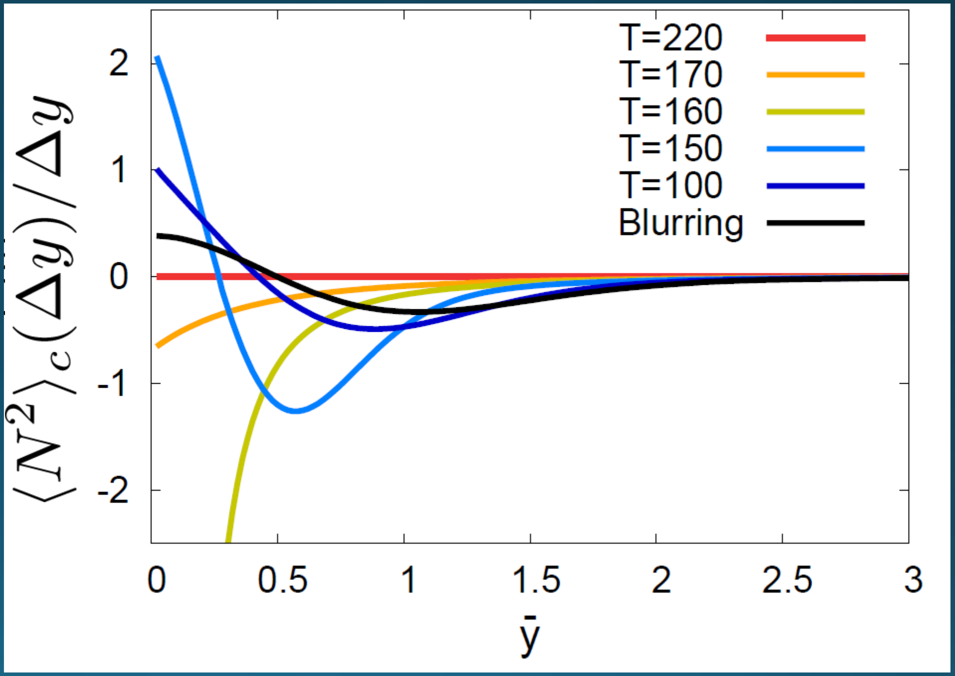
☐ **Higher order cumulants**

in diffusion master equation
MK+ (2014); MK (2015)



☐ **2^{nd} order cumulant near CP**

in stochastic diffusion equation
sakaida, Asakawa, Fujii, MK, 2018



☐ Non-monotonic Δy dependence can emerge reflecting the dynamical evolution.

Issues with Experimental Analysis

☐ Detector-response correction

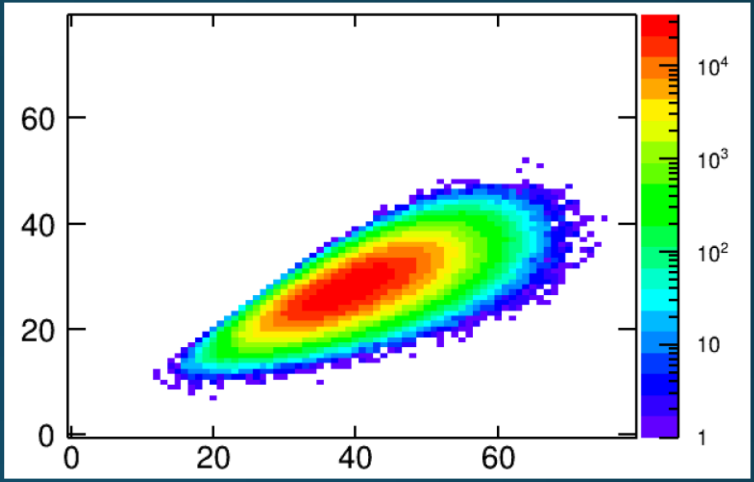
MK, Asakawa, 2012
MK, 2016
MK, Luo, 2017
Nonaka, MK, Esumi, 2017
Nonaka, MK, Esumi, 2018

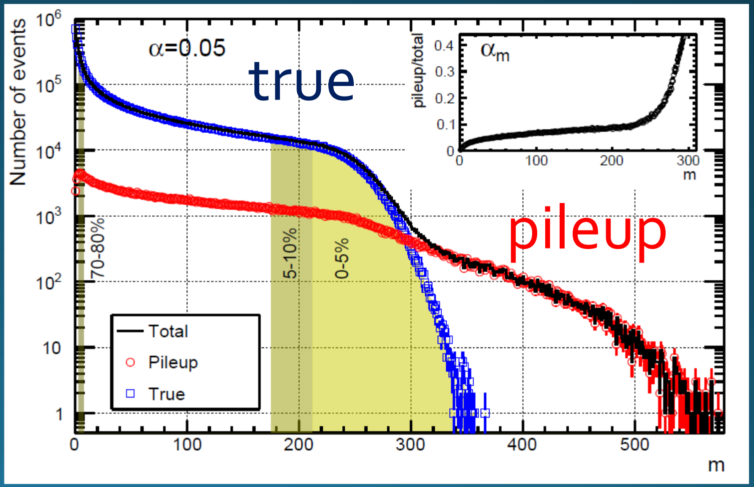
☐ Pileup correction

Nonaka, MK, Esumi, 2020

☐ Closer look at Exp. data

Nonaka, MK, Esumi, in prep.





Lattice Simulation of CP in Heavy-Quark Region

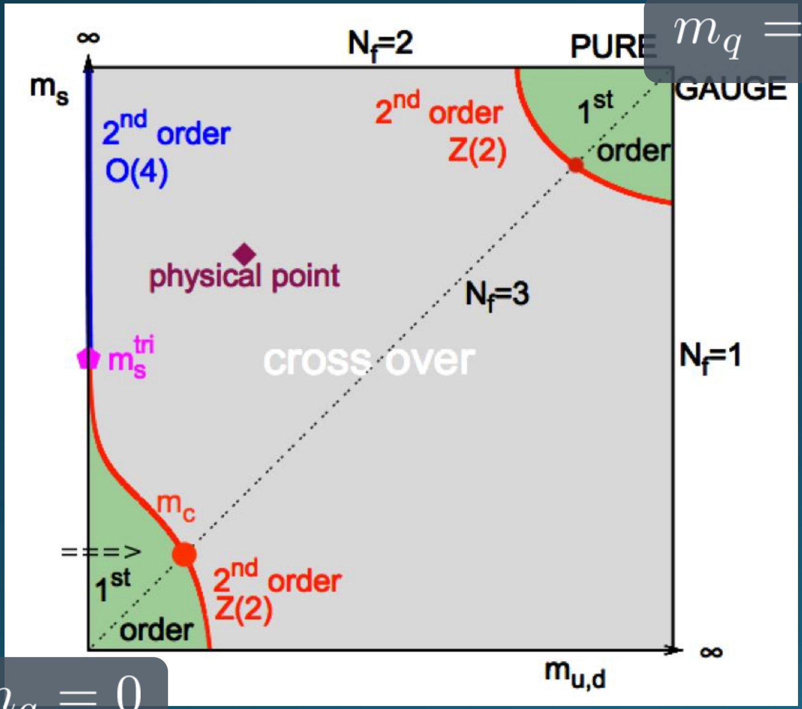
Kiyohara, MK, Ejiri, Kanaya, arXiv:2108.00118

See also
WHOT-QCD Collab.
PTEP 2021 (2021) 013B08; Phys.Rev.D 101 (2020) 05450

Varying Quark Masses

☐ Columbia plot

= order of phase tr. at $\mu = 0$

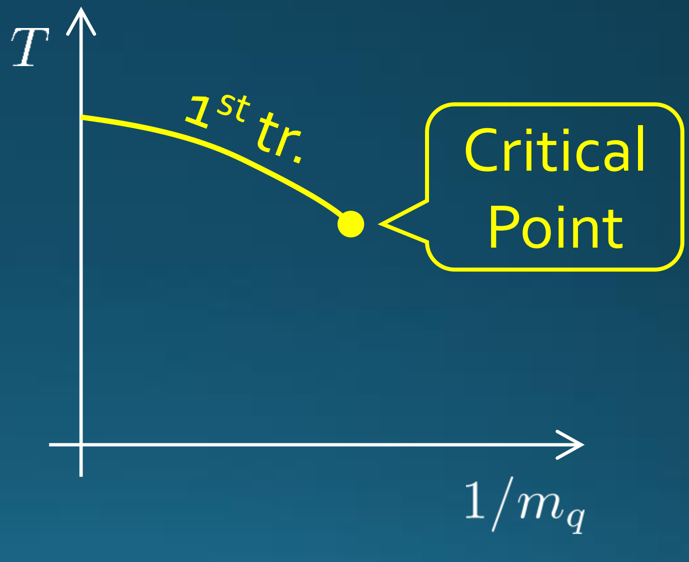


$m_q = \infty$

$m_q = 0$

☐ Example

Phase diagram in heavy-quark region

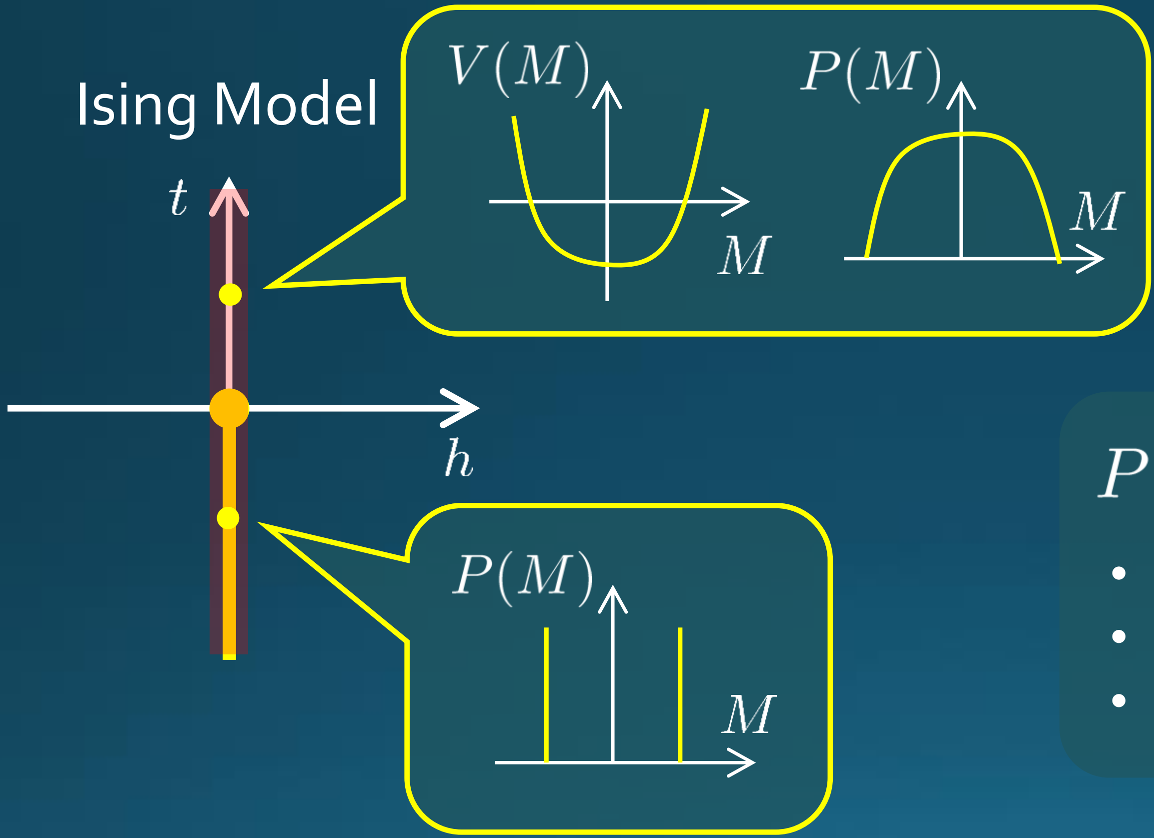


Critical Point

Various orders of phase transition with variation of m_q.

Cumulants around Critical Point

Ising Model



$$P(M) \sim e^{-V(M)}$$

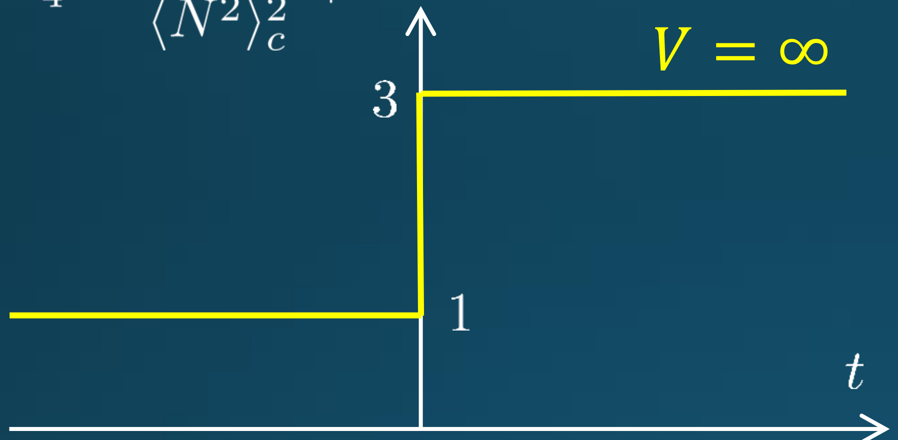
- $P(M)$: probability distr.
- $V(M)$: effective potential
- M : order parameter

- $\langle N^4 \rangle_c$ changes discontinuously at the CP.

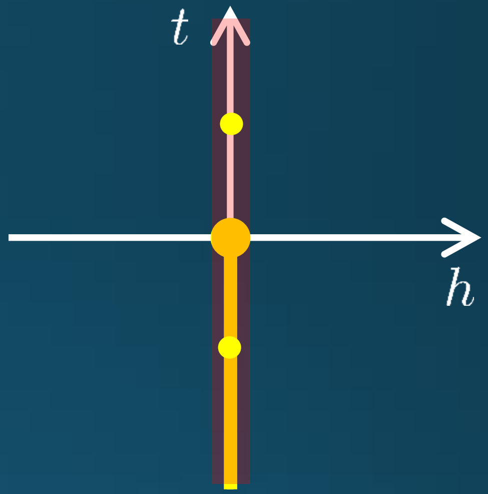
Finite-Volume Effects

Binder Cumulant

$$B_4 = \frac{\langle N^4 \rangle_c}{\langle N^2 \rangle_c^2} + 3$$



Ising Model



- ☐ Sudden change of B_4 at the CP is smeared by finite V effect.
- ☐ B_4 obtained for various V has crossing at $t = 0$.
- ☐ At the crossing point, $B_4 = 1.604$ in Z_2 universality class.

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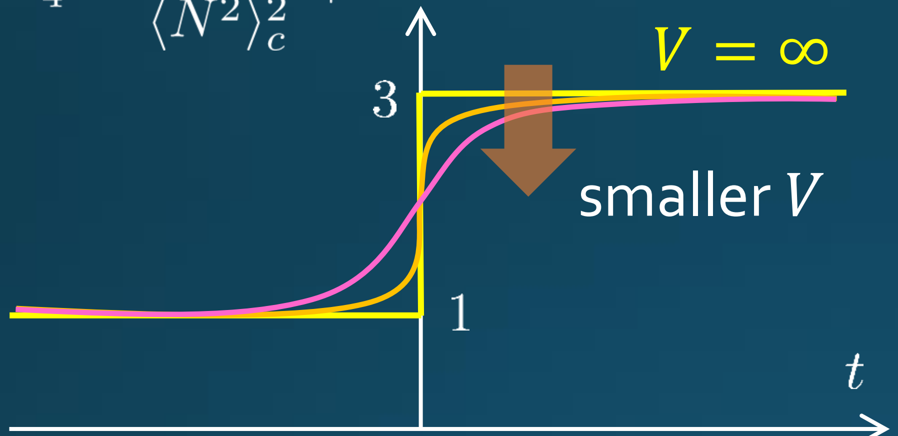
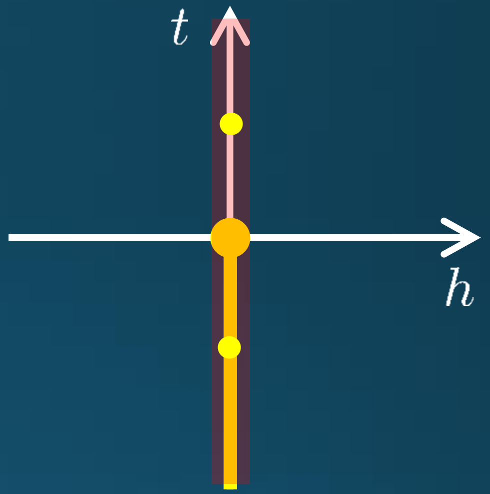


Ising Model



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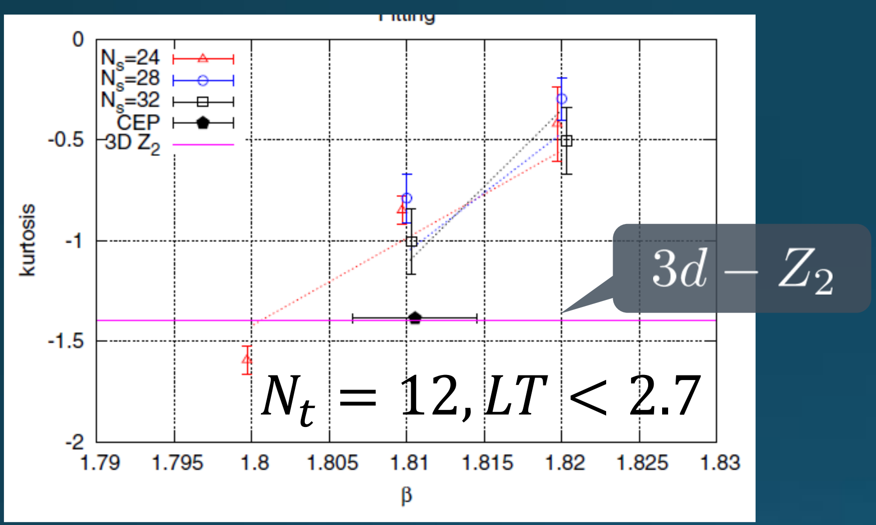
Binder-Cumulant Analysis

☐ Light-quark region

Kuramashi, Nakamura, Ohno, Takeda, ’20

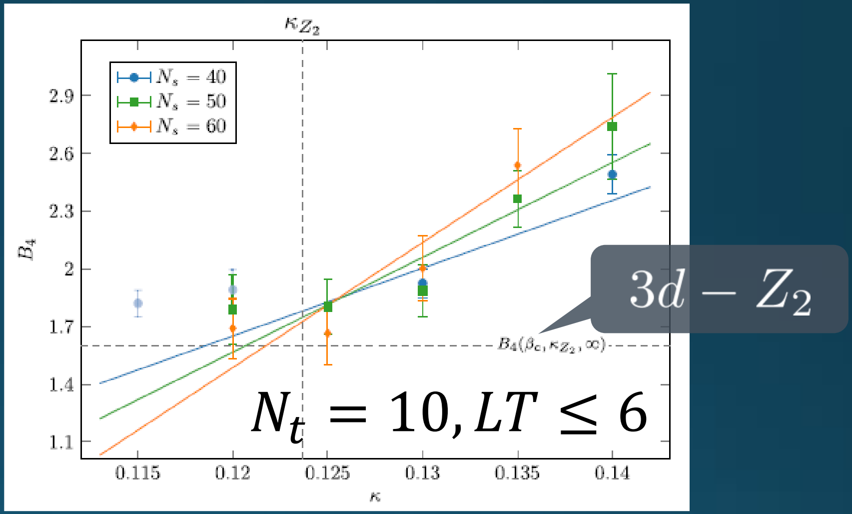


$3d - Z_2$

$N_t = 12, LT < 2.7$

☐ Heavy-quark region

Cuteri , Philipsen , Schön, Sciarra, ’21

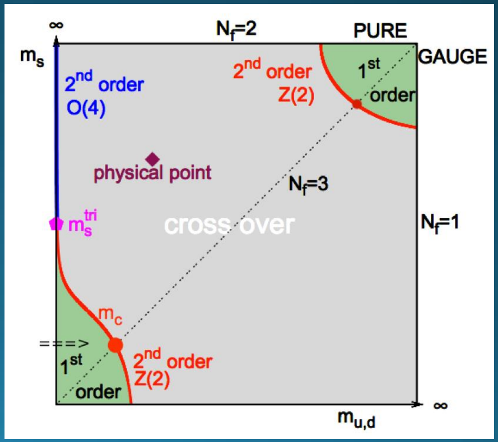


$3d - Z_2$

$N_t = 10, LT \leq 6$

☐ Statistically-significant deviation of the crossing point from the 3d-Ising value.

➡ Too large finite-V effects?



Numerical Simulation

☐ Coarse lattice: $N_t = 4$

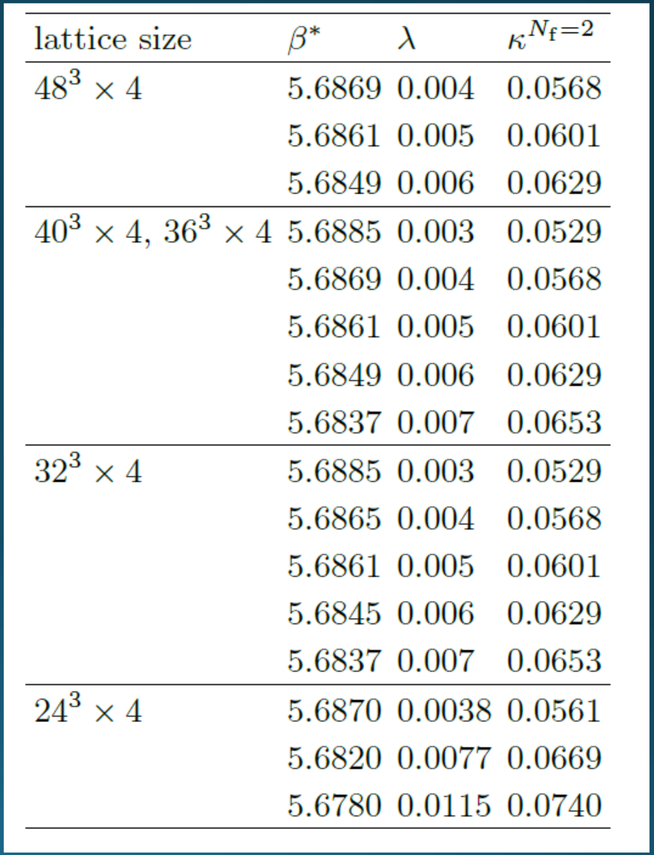
☐ But **large spatial volume**:
$LT = N_s \,/\, N_t \leq 12$

☐ Hopping-param. (~$1/m_q$) expansion

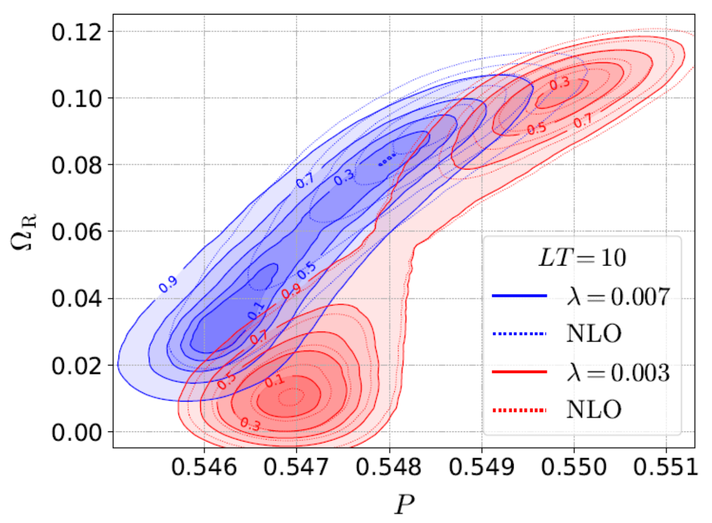
☐ Monte-Calro with LO action

☐ High statistical analysis

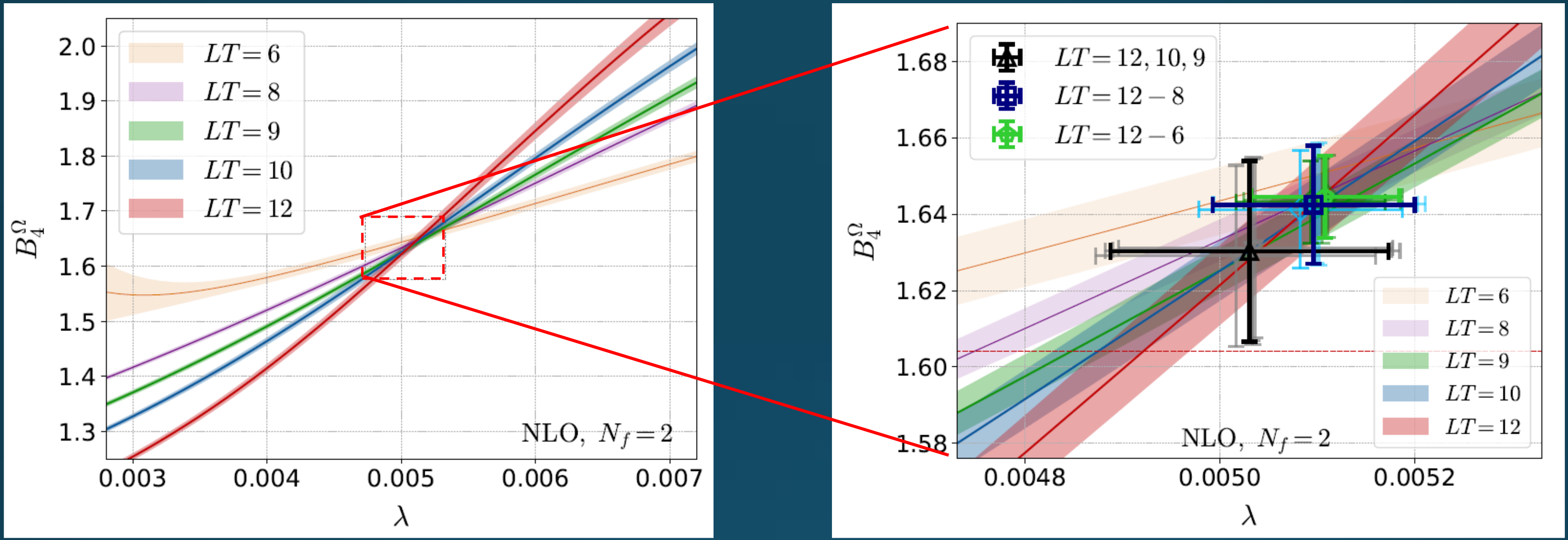
Simulation params.



lattice size	β^*	λ	$\kappa^{N_\mathrm{f}=2}$
$48^3 \times 4$	5.6869	0.004	0.0568
	5.6861	0.005	0.0601
	5.6849	0.006	0.0629
$40^3 \times 4$, $36^3 \times 4$	5.6885	0.003	0.0529
	5.6869	0.004	0.0568
	5.6861	0.005	0.0601
	5.6849	0.006	0.0629
	5.6837	0.007	0.0653
$32^3 \times 4$	5.6885	0.003	0.0529
	5.6865	0.004	0.0568
	5.6861	0.005	0.0601
	5.6845	0.006	0.0629
	5.6837	0.007	0.0653
$24^3 \times 4$	5.6870	0.0038	0.0561
	5.6820	0.0077	0.0669
	5.6780	0.0115	0.0740



Binder-Cumulant Analysis

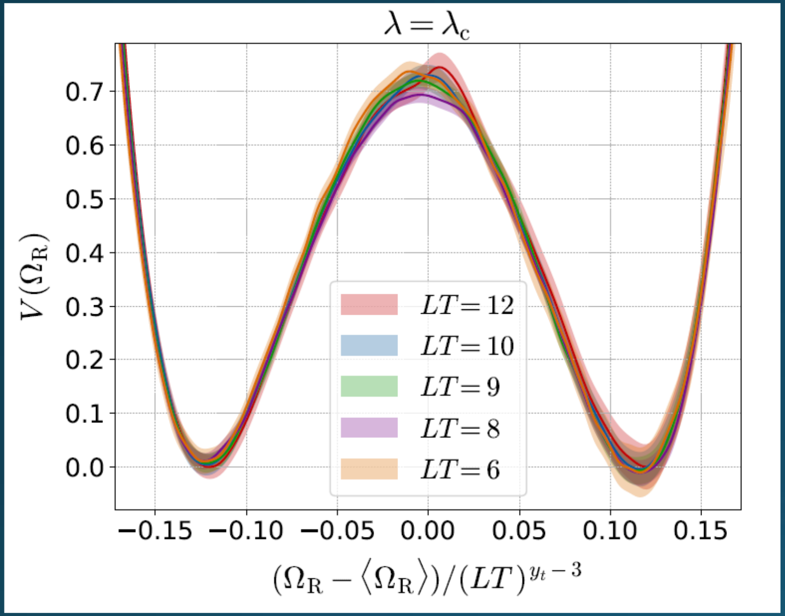


Z2	$B_4 = 1.604$	$\nu = 0.630$
$LT \geq 9$	$B_4 = 1.630(24)(2),$	$\nu = 0.614(48)(3)$
$LT \geq 8$	$B_4 = 1.643(15)(2),$	$\nu = 0.614(29)(3)$

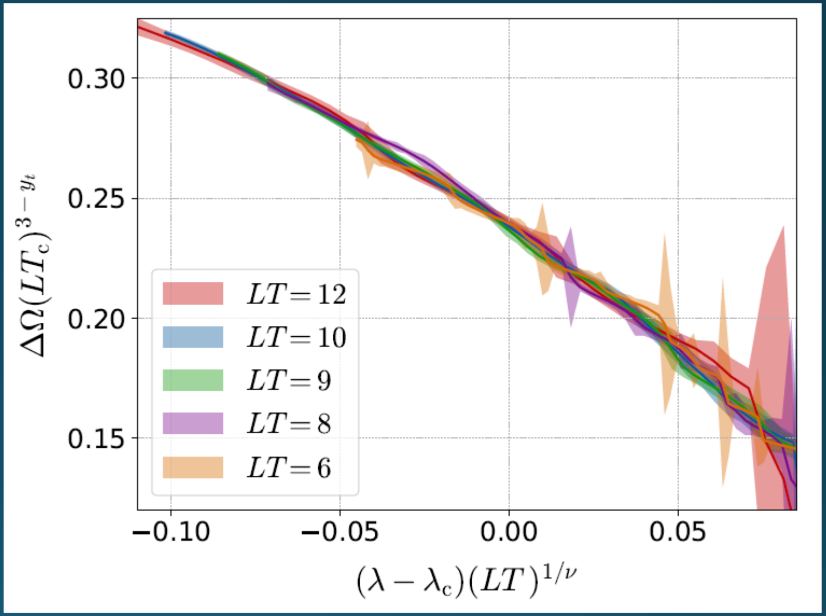
- ☐ B_4 and ν are consistent with Z2 universality class only when $LT \geq 9$ data are used for the analysis.

Further Check of Finite-V Scaling

- Effective potential at the CP



- Scaling of order parameter



Z2 scaling is well established

Summary

- Critical points appear many places in QCD at nonzero temperature.

- Two experimental tools for the search for the CPs:
  - Relativistic heavy-ion collisions
  - Lattice QCD numerical simulations

- Various studies are ongoing using both real and virtual experiments.