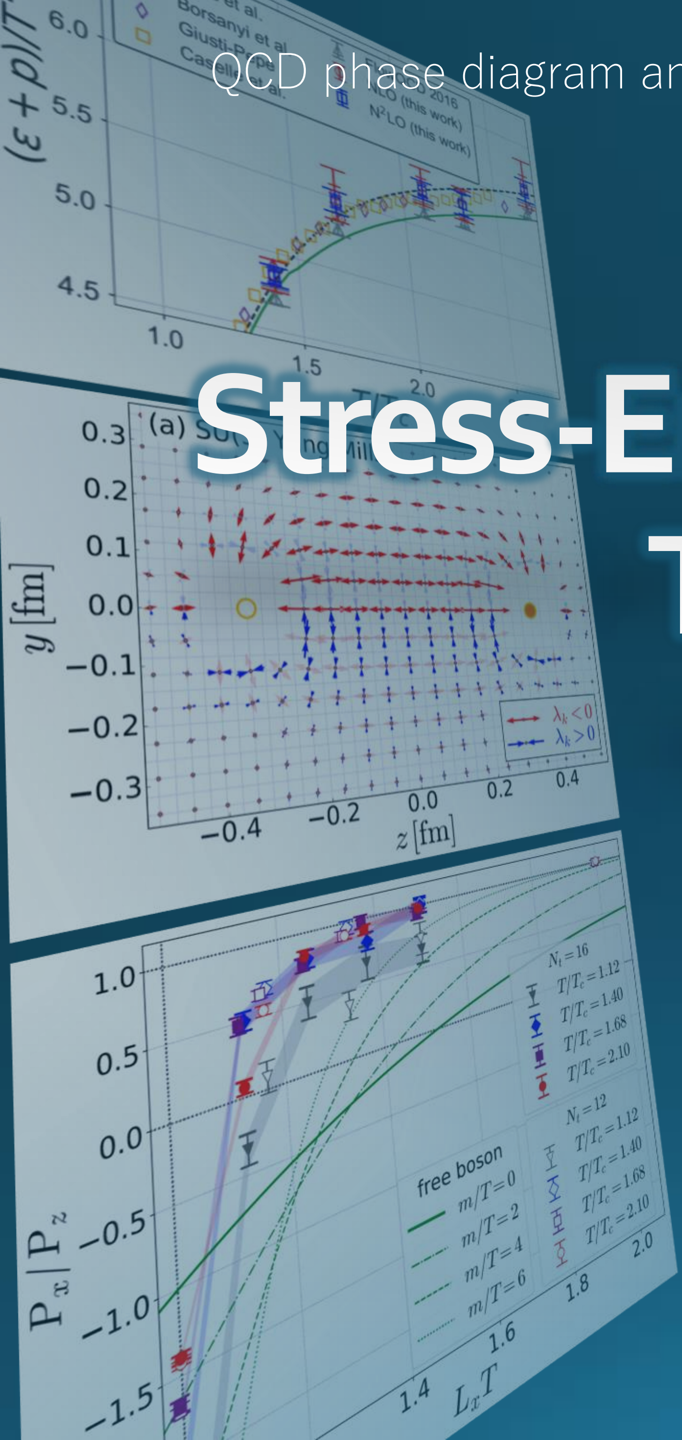


Stress-Energy-Momentum Tensor on the Lattice

Masakiyo Kitazawa

(Osaka U.)

FlowQCD/WHOT-QCD Collaborations

Energy-Momentum Tensor

energy momentum

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

pressure

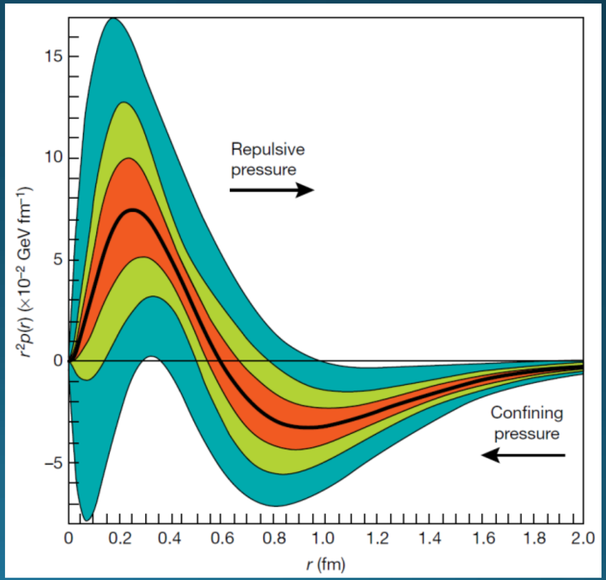
stress

All components are important physical observables!

Applications

- Thermodynamics
- Fluctuations
- Viscosity
- Hadron Structure
- …

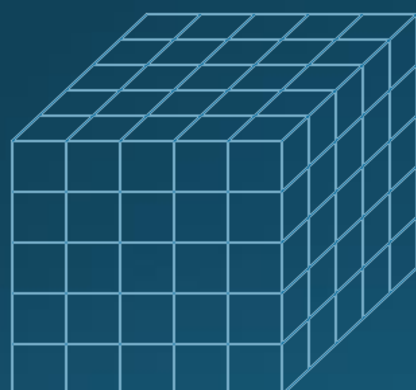
Pressure distribution inside proton



Burkert+, Nature 557, 396 (2018)

$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial
because of the explicit breaking of translational invariance



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

② Its measurement is extremely noisy
due to high dimensionality and etc.

Contents

1. Constructing EMT through gradient flow

2. Thermodynamics

FlowQCD, PRD**90**,011501 (2014); PRD**94**, 114512 (2016)
WHOT-QCD, PRD**96**, 014509 (2017); PRD**102**, 014510 (2020)
Iritani+, PTEP **2019**, 023B02 (2019)

3. Casimir Effect & Pressure Anisotropy

MK, Mogliacci, Horowitz, Kolbe, PRD**99**, 094507 (2019)

4. EMT Correlation Functions

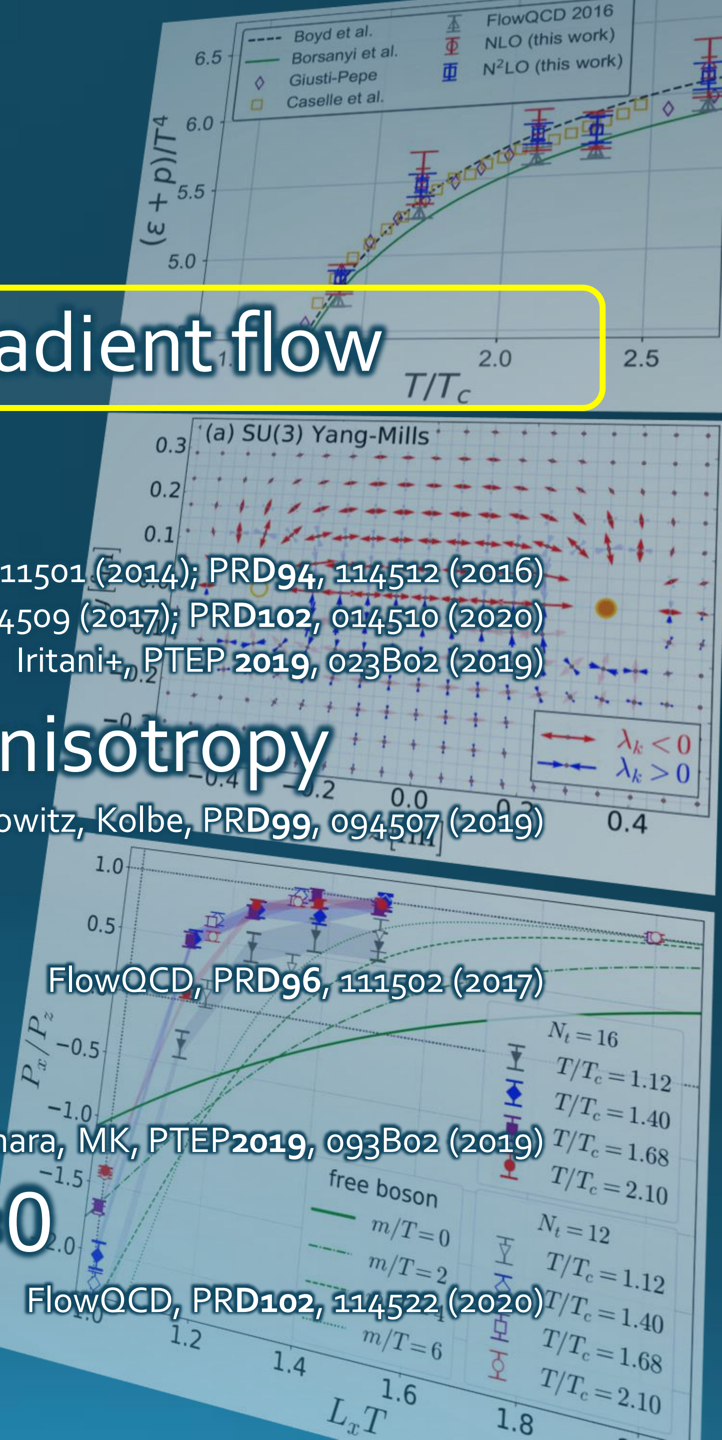
FlowQCD, PRD**96**, 111502 (2017)

5. Flux Tube

FlowQCD, PLB**789**, 210 (2019); Yanagihara, MK, PTEP**2019**, 093B02 (2019)

6. Single-Quark System at T≠0

FlowQCD, PRD**102**, 114522 (2020)



Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

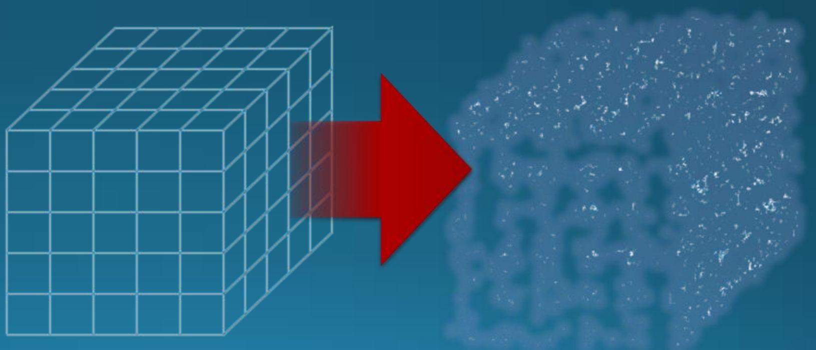
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length²]

leading

$$\boxed{\partial_t A_\mu} = D_\nu G_{\mu\nu} = \boxed{\partial_\nu \partial_\nu A_\mu} + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at $t > 0$



Small Flow-Time Expansion

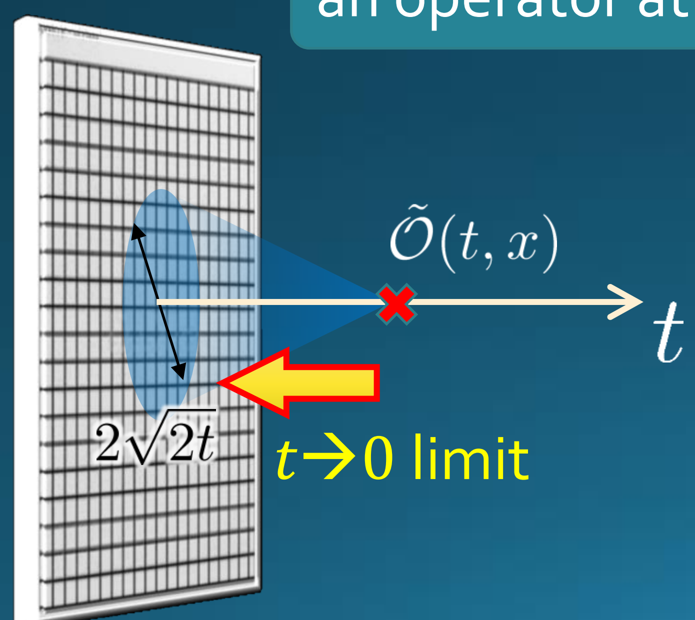
Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at $t > 0$

remormalized operators of original theory

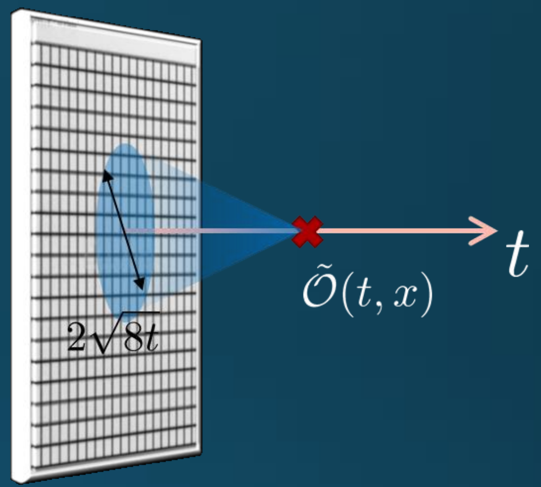
original 4-dim theory



Constructing EMT 1

Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$



- **Gauge-invariant dimension 4 operators**

$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\rho\sigma}(t,x)G_{\rho\sigma}(t,x) \\ E(t,x) = \frac{1}{4}G_{\rho\sigma}(t,x)G_{\rho\sigma}(t,x) \end{cases}$$

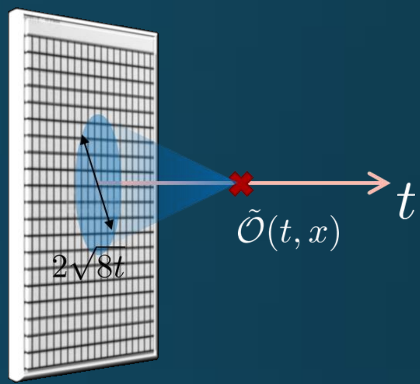
Constructing EMT

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)\,T^R_{\rho\rho}(x) + \mathcal{O}(t)$$

vacuum subtr.



Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\mathrm{subt.}}\right]$$

"SFtX method" (Small Flow time eXpansion)

Perturbative Coefficients

$$T_{\mu\nu}(t) = c_1(t)U_{\mu\nu}(t) + \delta_{\mu\nu}c_2(t)E(t)$$

	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○	○	
$c_2(t)$	× zero	○	○	○

Suzuki (2013) Harlander+(2018)

Iritani, MK, Suzuki, Takaura, 2019

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

□ Choice of the scale of g^2

$$c_1(t) = c_1\left(g^2(\mu(t))\right)$$

Previous: $\mu_d(t) = 1/\sqrt{8t}$

Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E}t}$

Harlander+ (2018)

Gradient Flow for Fermions

$$\partial_t \psi(t,x) = D_\mu D_\mu \psi(t,x)$$

$$\partial_t \bar{\psi}(t,x) = \psi(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu$$

$$D_\mu = \partial_\mu + A_\mu(t,x)$$

Luscher, 2013
Makino, Suzuki, 2014
Taniguchi+ (WHOT) 2016~

- Not "gradient" flow but a "diffusion"-type equation.

- Divergence in field renormalization of fermions.
- All observables are finite at t>0 once Z(t) is fixed.

$$\tilde{\psi}(t,x) = Z(t)\psi(t,x)$$

- Energy-momentum tensor from SF*t*X Makino, Suzuki, 2014

EMT in QCD

Makino, Suzuki (2014)
Harlander+ (2018)

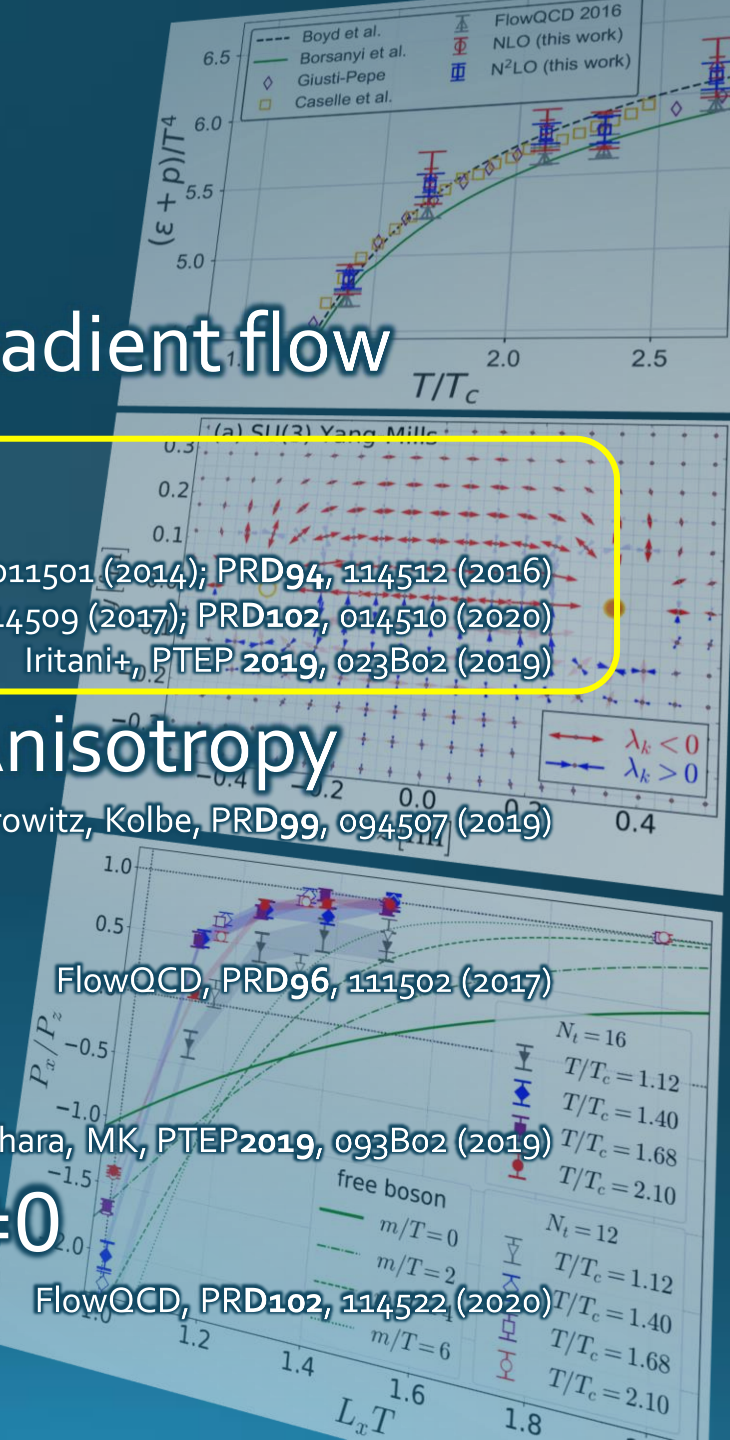
$$
\begin{aligned}
T_{\mu\nu}(t,x) = & c_1(t)U_{\mu\nu}(t,x) + c_2(t)\delta_{\mu\nu}\big(E(t,x) - \langle E\rangle_0\big) \\
& + c_3(t)\big(O_{3\mu\nu}(t,x) - 2O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) \\
& + c_4(t)\big(O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) + c_5(t)\big(O_{5\mu\nu}(t,x) - \mathrm{VEV}\big)
\end{aligned}
$$

$$
T_{\mu\nu}(x) = \lim_{t\to 0} T_{\mu\mu}(t,x)
$$

$$
\begin{aligned}
\tilde{O}^f_{3\mu\nu}(t,x) &\equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x), \\
\tilde{O}^f_{4\mu\nu}(t,x) &\equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x), \\
\tilde{O}^f_{5\mu\nu}(t,x) &\equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x),
\end{aligned}
$$

$$
\varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.
$$

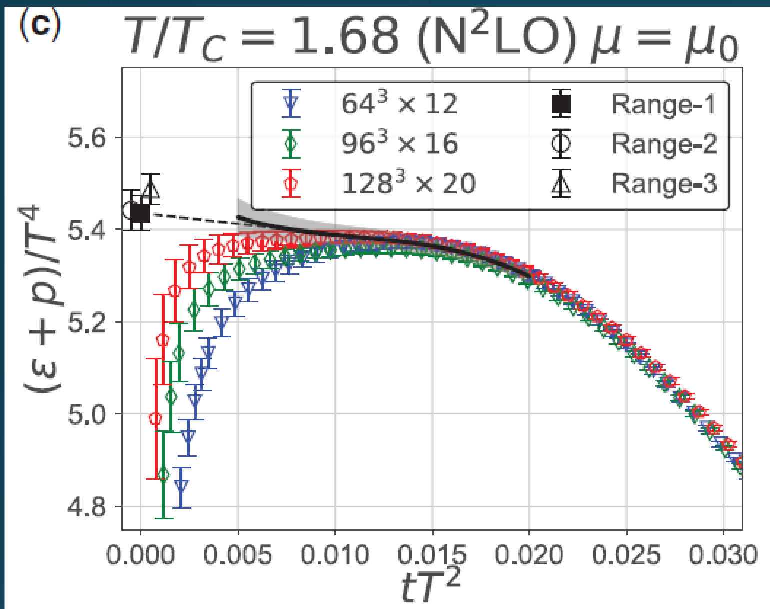
Contents



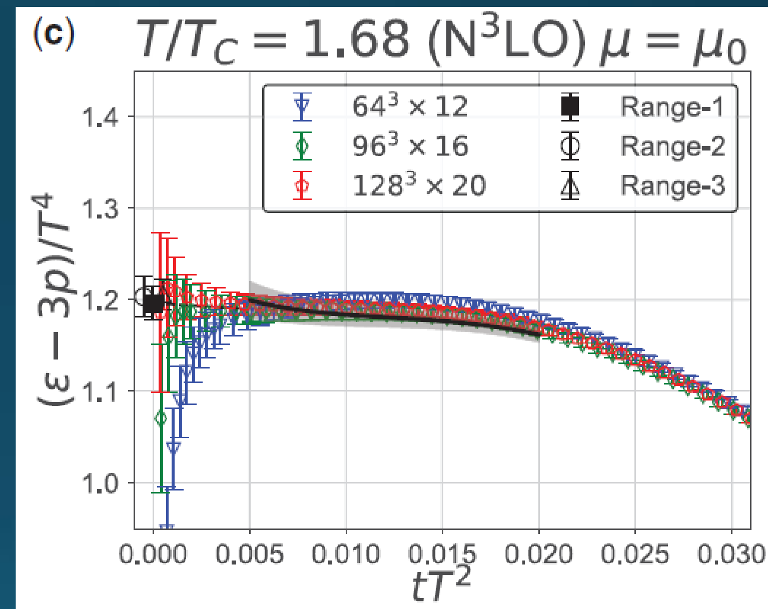
Boyd et al.
Borsanyi et al.
Giusti-Pepe
Caselle et al.
FlowQCD 2016
NLO (this work)
N^2LO (this work)
$(\varepsilon+p)/T^4$
T/T_c
(a) SU(3) Yang Mills
$\lambda_k<0$
$\lambda_k>0$
P_x/P_z
free boson
L_xT

t Dependence

$\langle T_{00}\rangle + \langle T_{11}\rangle$



$\langle T_{00}\rangle - 3\langle T_{11}\rangle$



Iritani, MK, Suzuki, Takaura, PTEP 2019

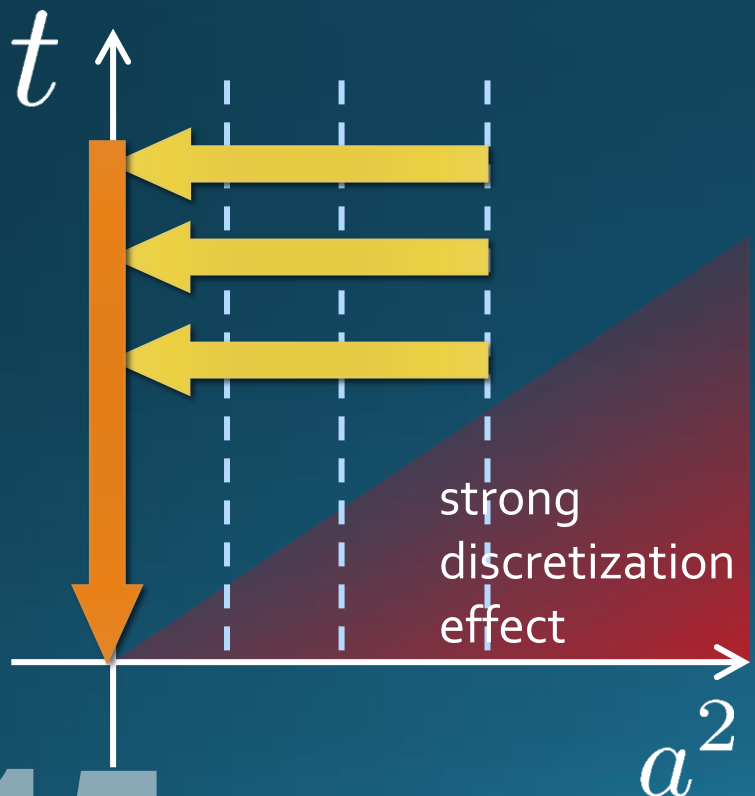
- Existence of "linear window" at intermediate t

Double Extrapolation

t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + \boxed{C_{\mu\nu}t} + \boxed{D_{\mu\nu}(t)\frac{a^2}{t}}$$

O(t) terms in SFTE lattice discretization



Continuum extrapolation

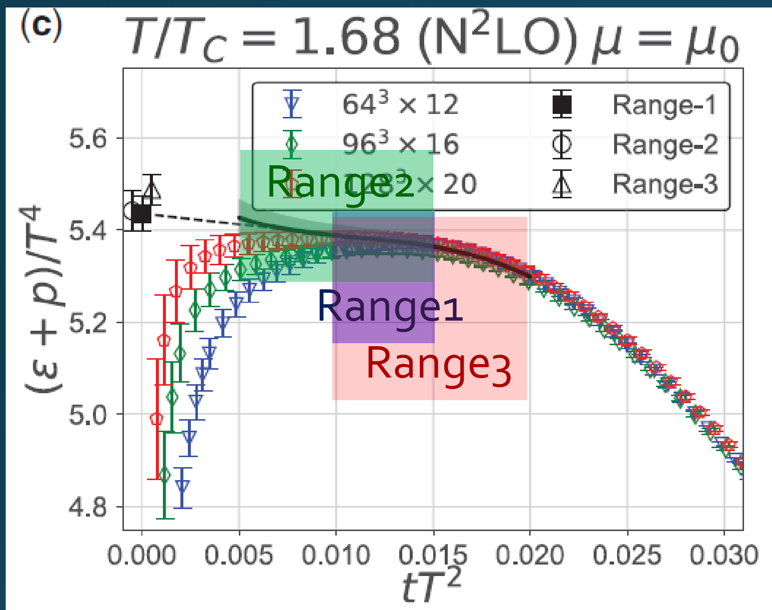
$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

Small t extrapolation

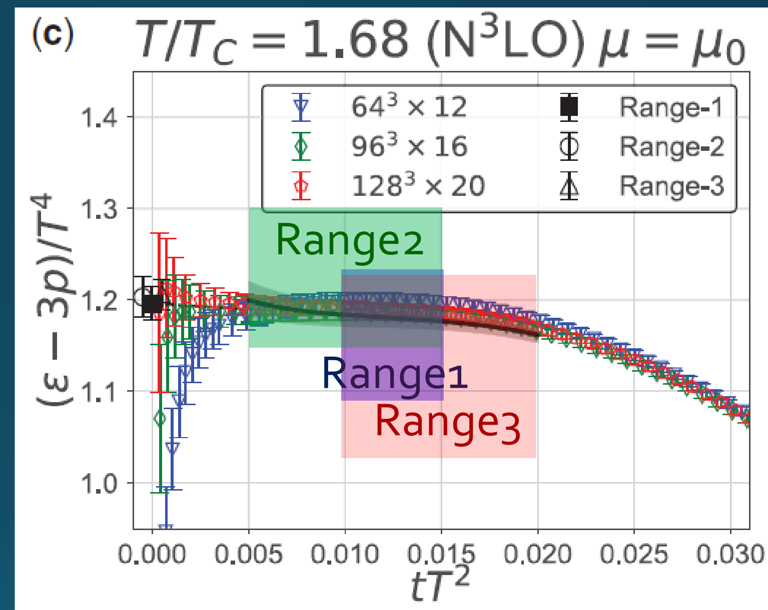
$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$

Thermodynamics: ε+p & ε-3p

$$\langle T_{00}\rangle + \langle T_{11}\rangle$$



$$\langle T_{00}\rangle - 3\langle T_{11}\rangle$$

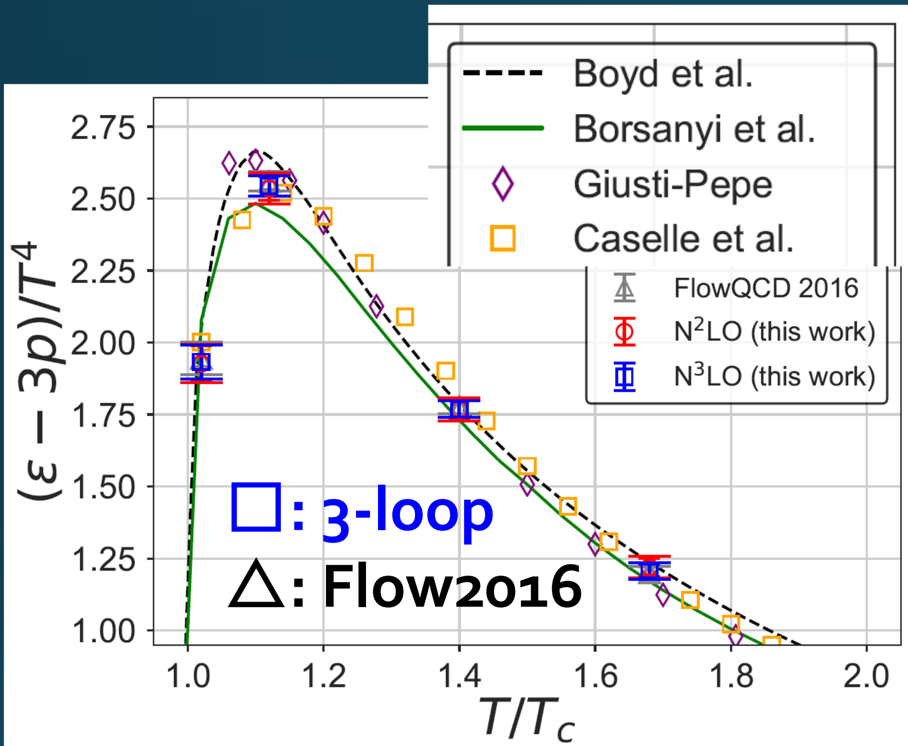


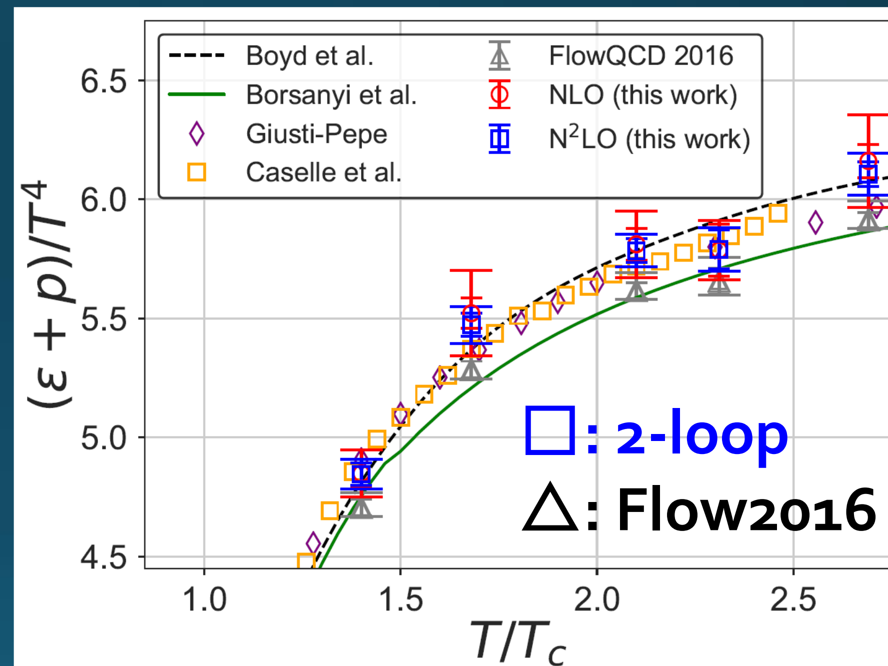
Iritani, MK, Suzuki, Takaura, PTEP 2019

- Existence of "linear window" at intermediate t
- Stable t→0 extrapolation
- Systematic errors: fit range, uncertaintyof Λ (±3%), …

T Dependence: Comparison

Iritani, MK, Suzuki, Takaura, 2019



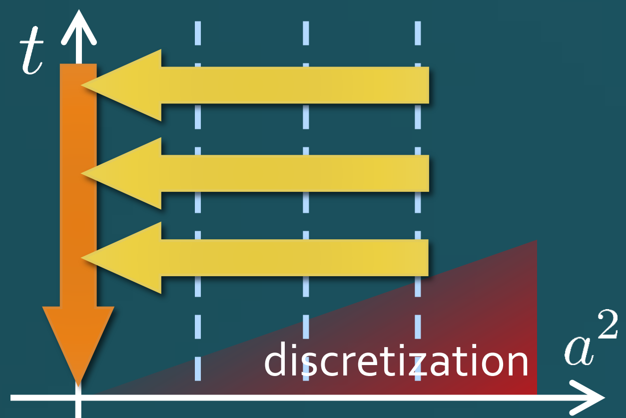


Systematic error: μ_0 or μ_d, Λ, t→0 function, fit range

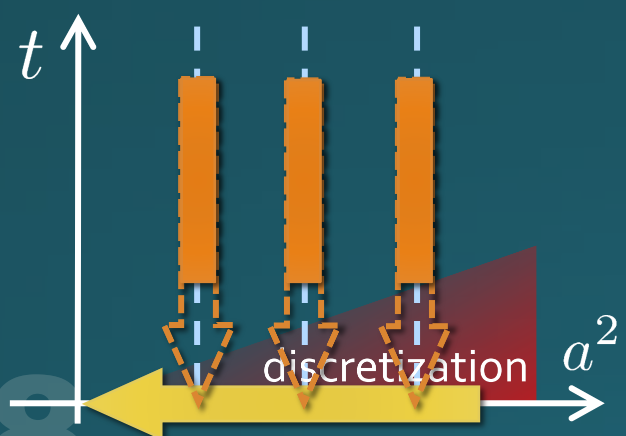
- Good agreement with other methods!
- Smaller statistics thanks to smearing by the flow

Alternative Extrapolation

Method A: $a \to 0,\ t \to 0$

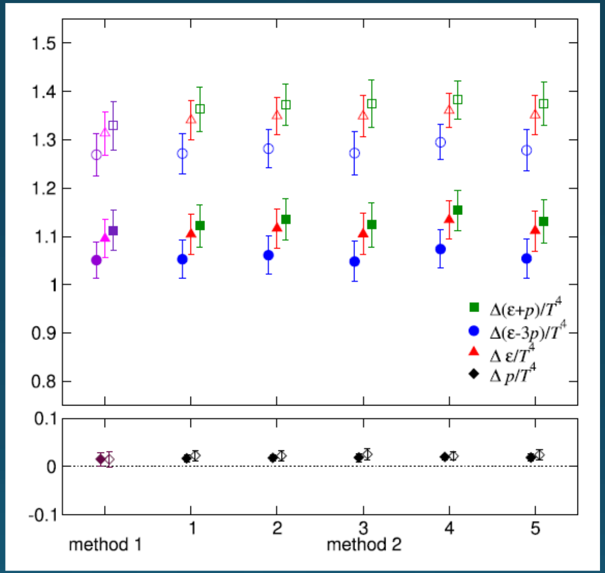


Method B: $t \to 0,\ a \to 0$



☐ Consistency check

- ☐ latent heat & pressure gap



Shirogane+, 2021 (WHOT-QCD)

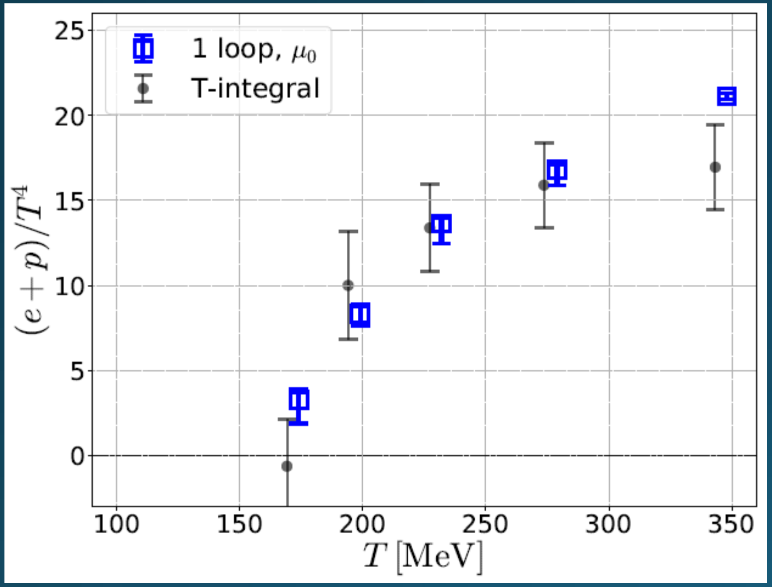
B   Method A

➡ Consistent result for two methods

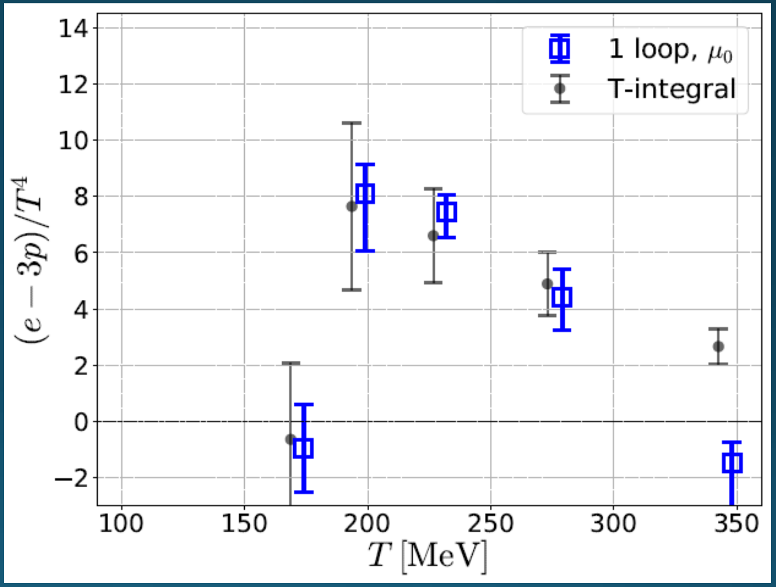
2+1 QCD EoS from SF*t*X Method

WHOT-QCD, PRD**96** (2017); PRD**102** (2020)

$\langle T_{00} \rangle + \langle T_{11} \rangle$



$\langle T_{00} \rangle - 3\langle T_{11} \rangle$

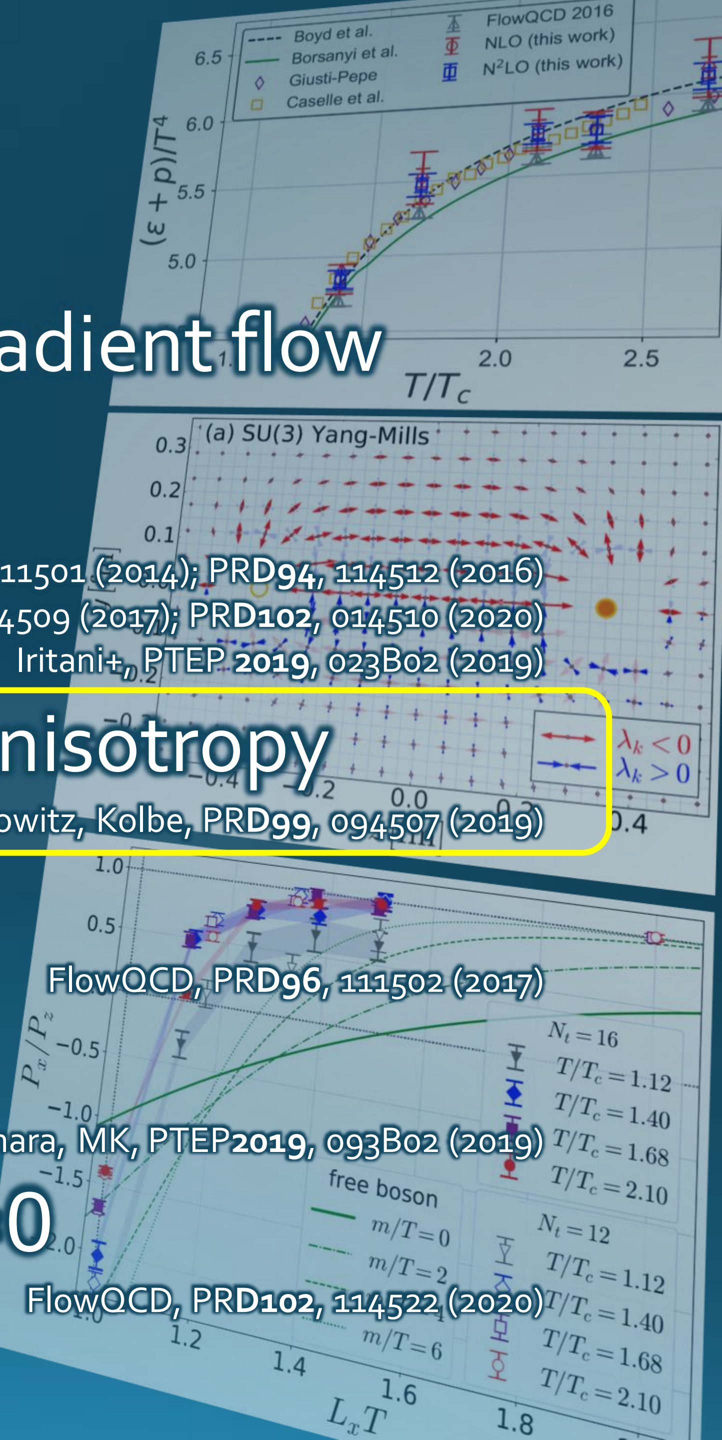


- ☐ Agreement with integral method
- ☐ Substantial suppression of statistical errors

$m_{PS}/m_V \approx 0.63$

Physical mass: Kanaya+ (WHOT-QCD), 1910.13036

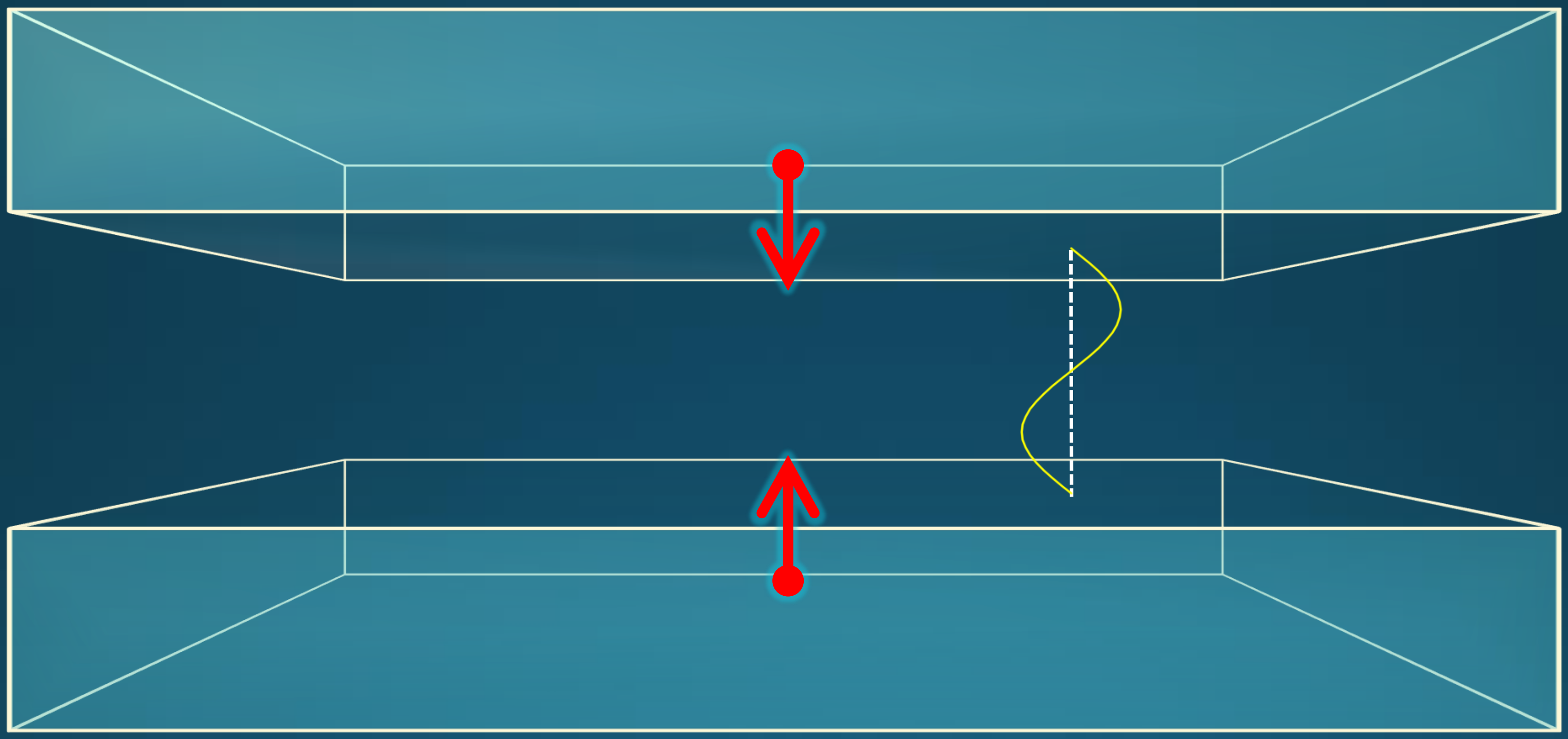
Contents



Boyd et al.
Borsanyi et al.
Giusti-Pepe
Caselle et al.
FlowQCD 2016
NLO (this work)
N^2LO (this work)
$(\varepsilon+p)/T^4$
T/T_c
(a) SU(3) Yang-Mills
$\lambda_k<0$
$\lambda_k>0$
P_x/P_z
L_xT
free boson
$m/T=0$
$m/T=2$
$m/T=6$
$N_t=16$
$N_t=12$
$T/T_c=1.12$
$T/T_c=1.40$
$T/T_c=1.68$
$T/T_c=2.10$

Casimir Effect

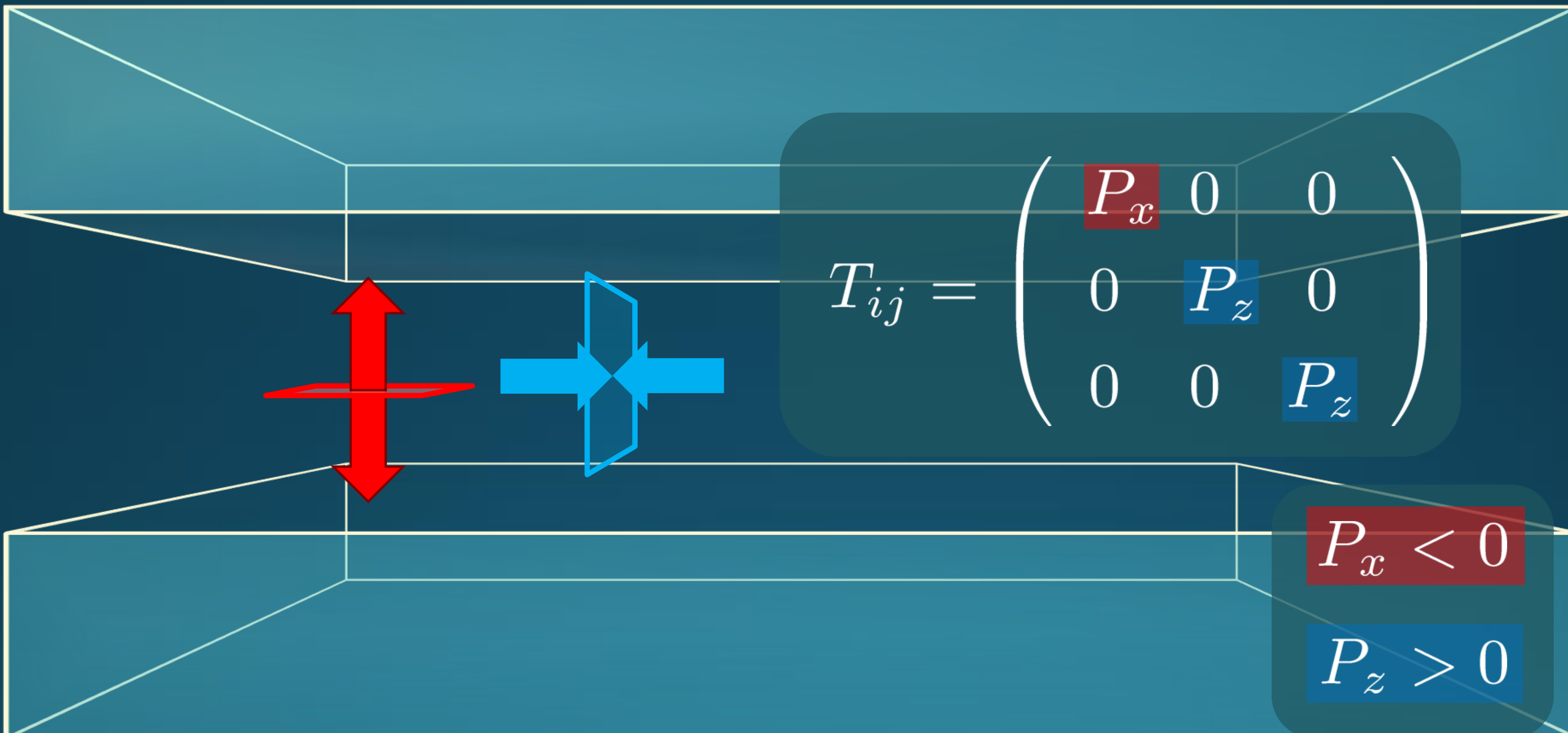
Casimir Effect



attractive force between two conductive plates

Casimir Effect

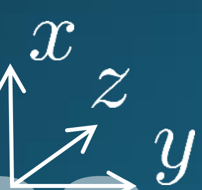
Brown, Maclay
1969



$$T_{ij} = \begin{pmatrix} P_x & 0 & 0 \\ 0 & P_z & 0 \\ 0 & 0 & P_z \end{pmatrix}$$

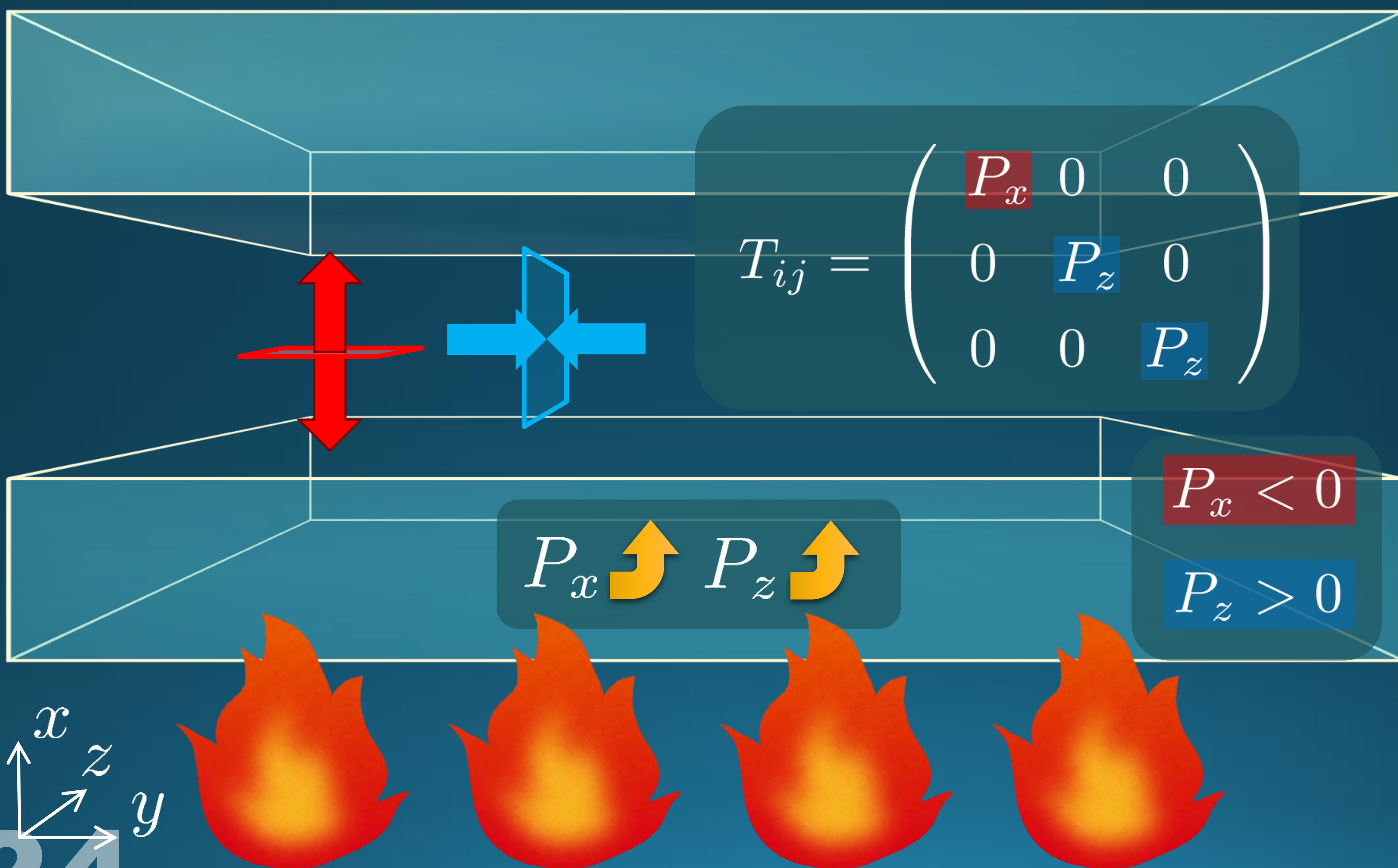
$$P_x < 0$$

$$P_z > 0$$



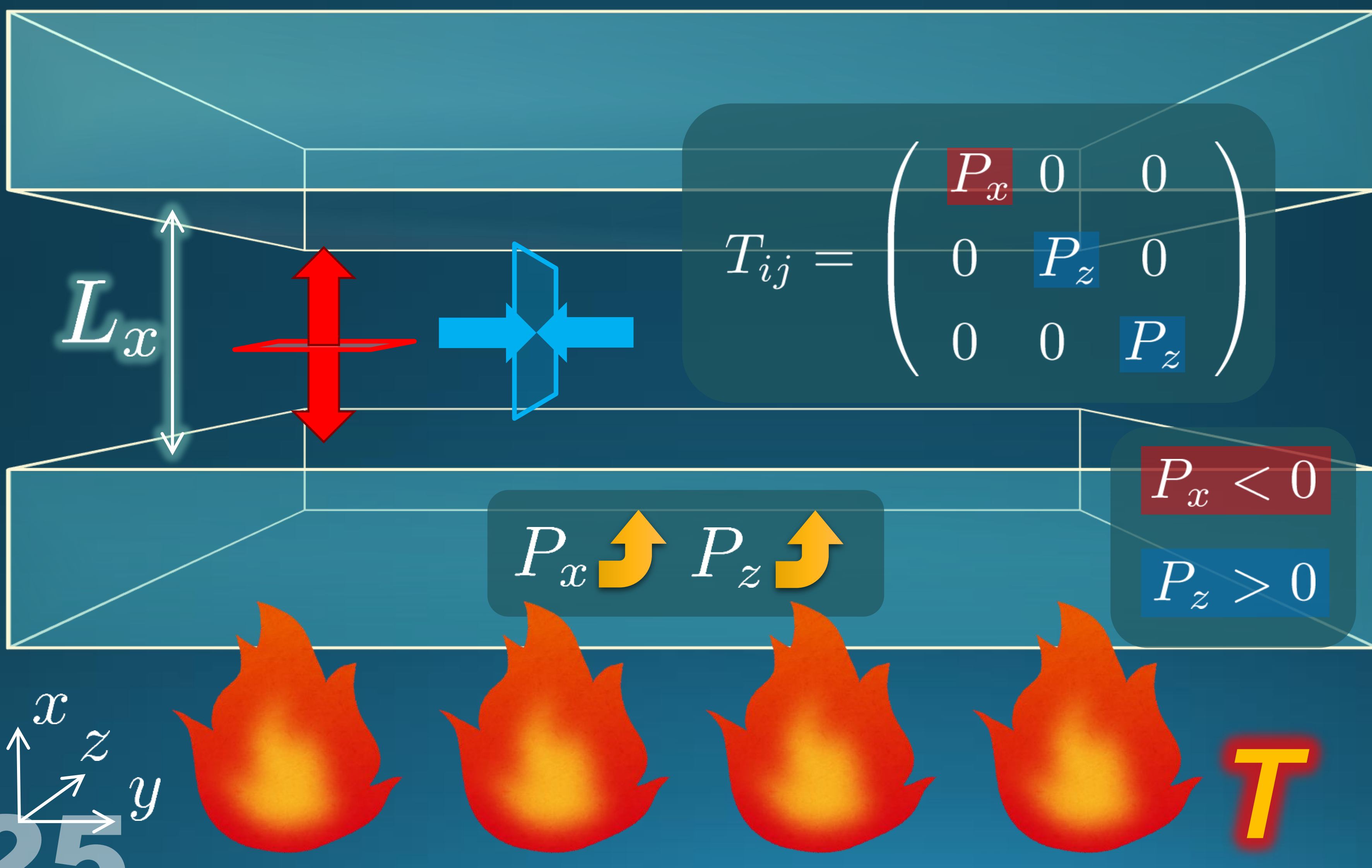
Casimir Effect

Brown, Maclay
1969



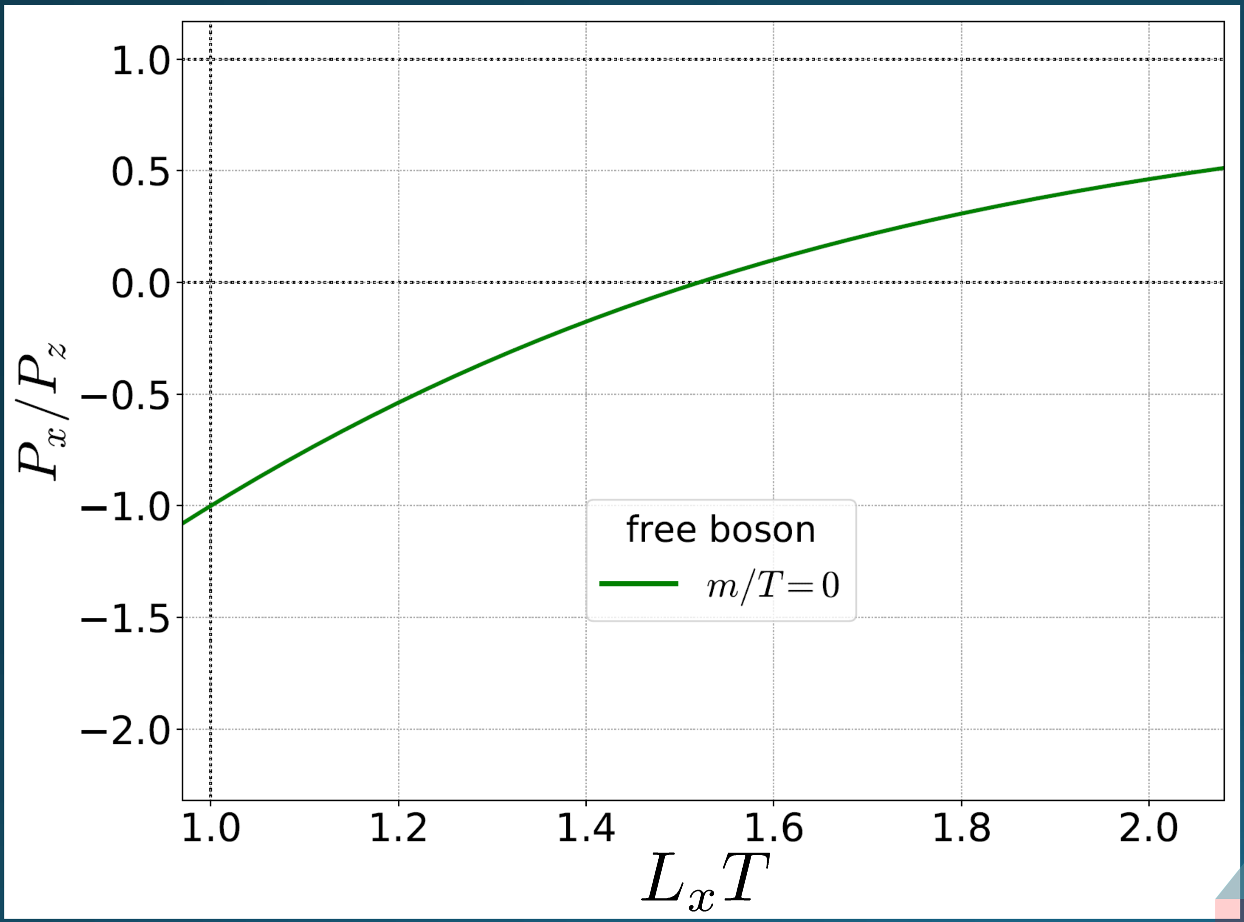
Casimir Effect

Brown, Maclay
1969



Pressure Anisotropy @ T≠0

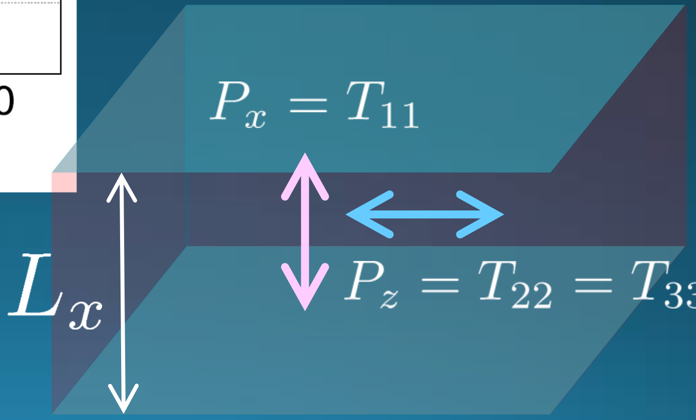
MK, Mogliacci, Kolbe, Horowitz, PRD (2019)



Free scalar field

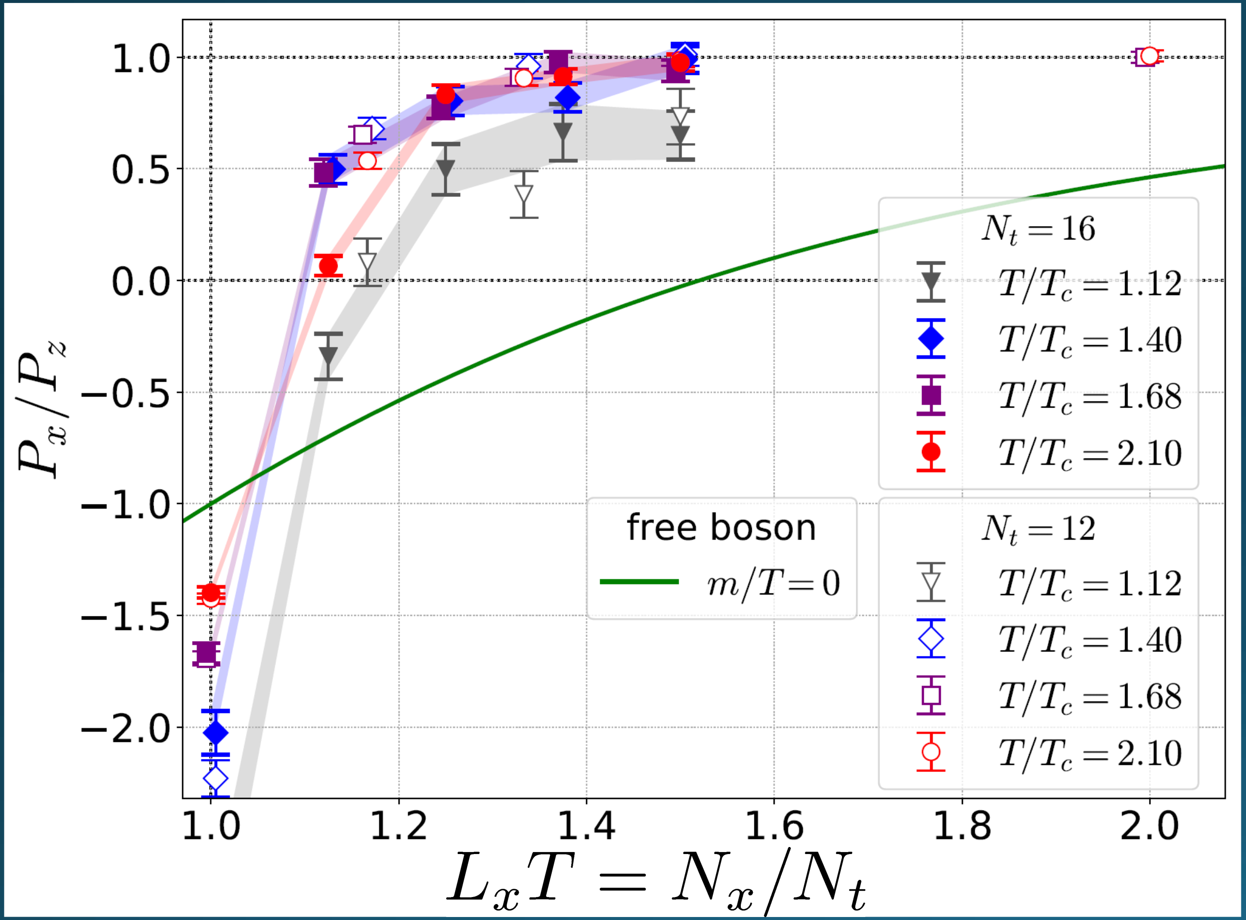
- $L_2=L_3=\infty$
- Periodic BC

Mogliacci+, 1807.07871



Pressure Anisotropy @ T≠0

MK, Mogliacci, Kolbe, Horowitz, PRD (2019)



Free scalar field

- $L_2=L_3=\infty$
- Periodic BC

Mogliacci+, 1807.07871

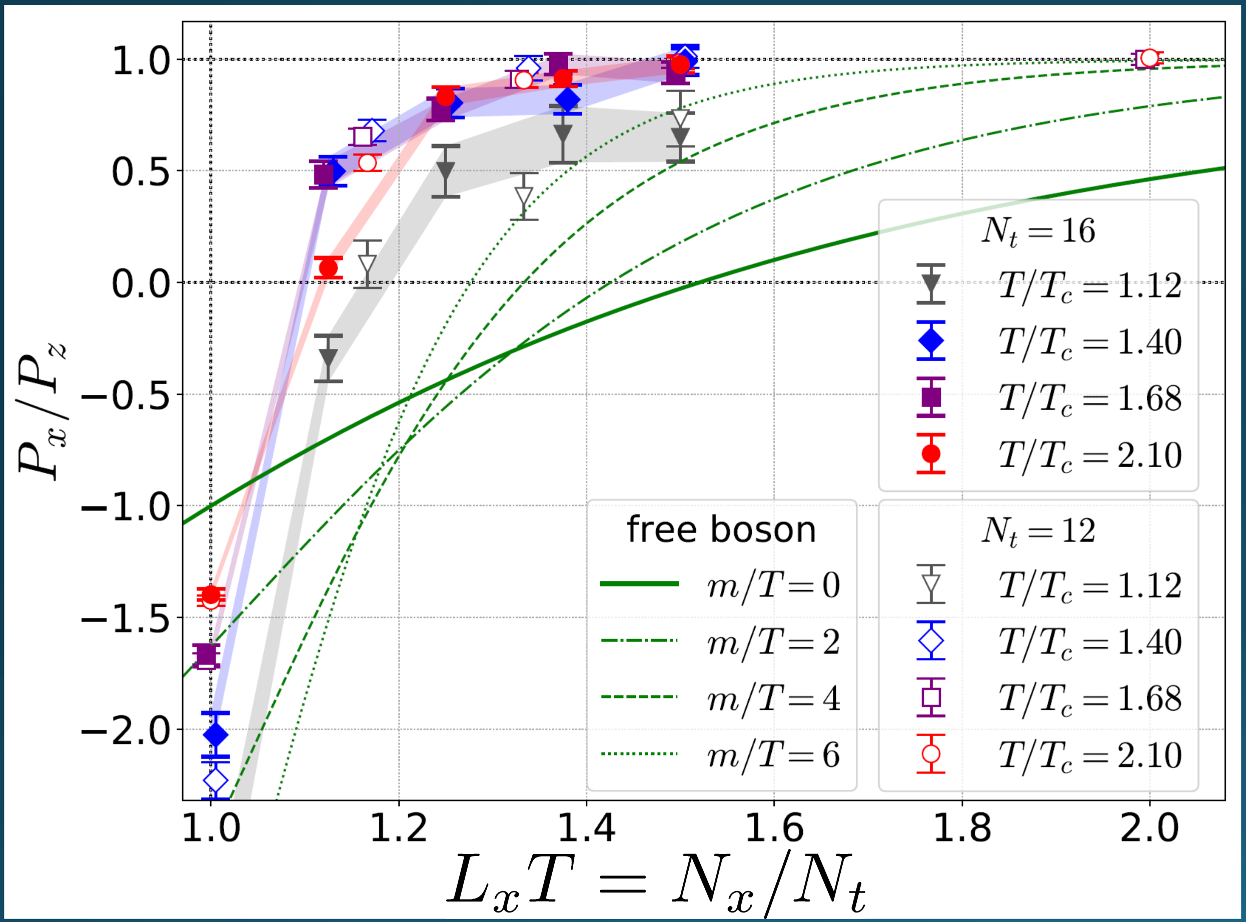
Lattice result

- Periodic BC
- Only $t\to 0$ limit
- Error: stat.+sys.

Medium near T_c is remarkably insensitive to finite size!

Pressure Anisotropy @ T≠o

MK, Mogliacci, Kolbe, Horowitz, PRD (2019)



Free scalar field

- $L_2=L_3=\infty$
- Periodic BC

Mogliacci+, 1807.07871

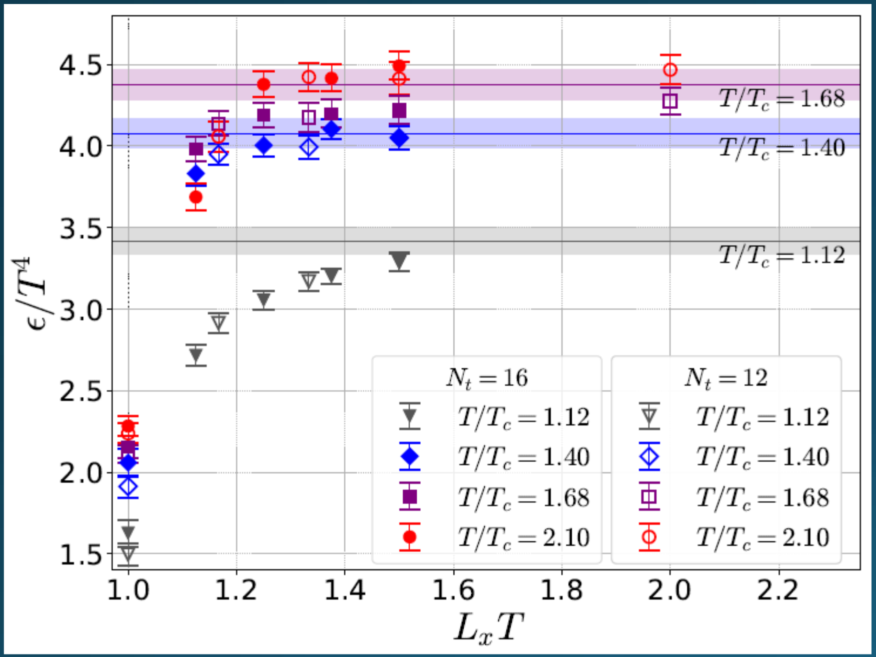
Lattice result

- Periodic BC
- Only t→0 limit
- Error: stat.+sys.

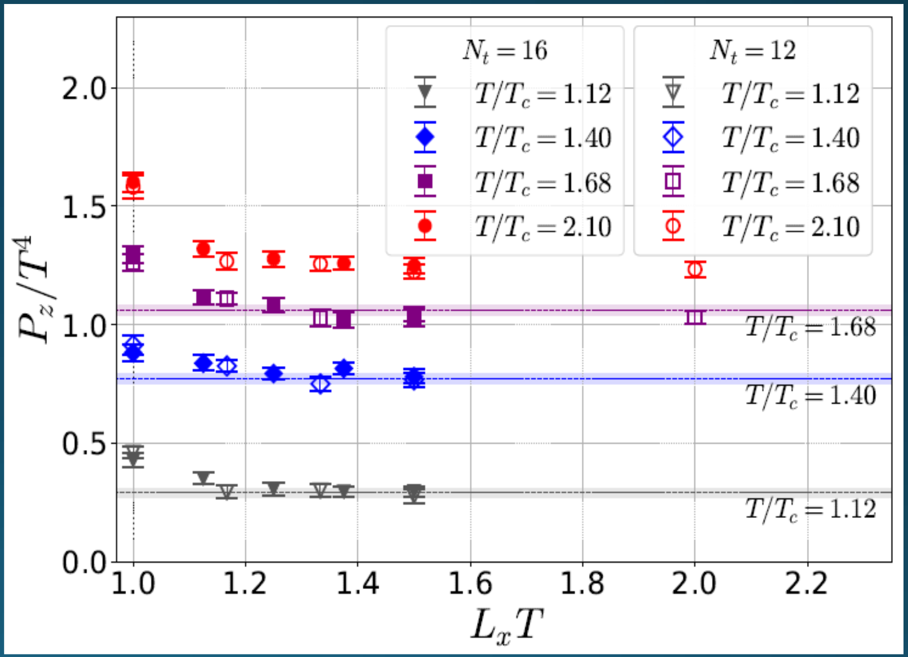
Medium near T_c is remarkably insensitive to finite size!

Energy densty / transverse P

Energy Density



Transverse Pressure P_z



Thermodynamics on the Lattice

Various Methods

- **Integral**, differential, moving frame, non-equilibrium, ...
- rely on thermodynamic relations valid in V→∞

$$P = \frac{T}{V} \ln Z \qquad sT = \varepsilon + P$$

➡ **Not applicable to anisotropic systems**

- We employ **SF*t*X Method**

$$\varepsilon = \langle T_{00} \rangle \quad P = \langle T_{11} \rangle$$

Components of EMT are directly accessible!

Higher T

High-T limit: massless free gluons
How does the anisotropy approach this limit?

Difficulties

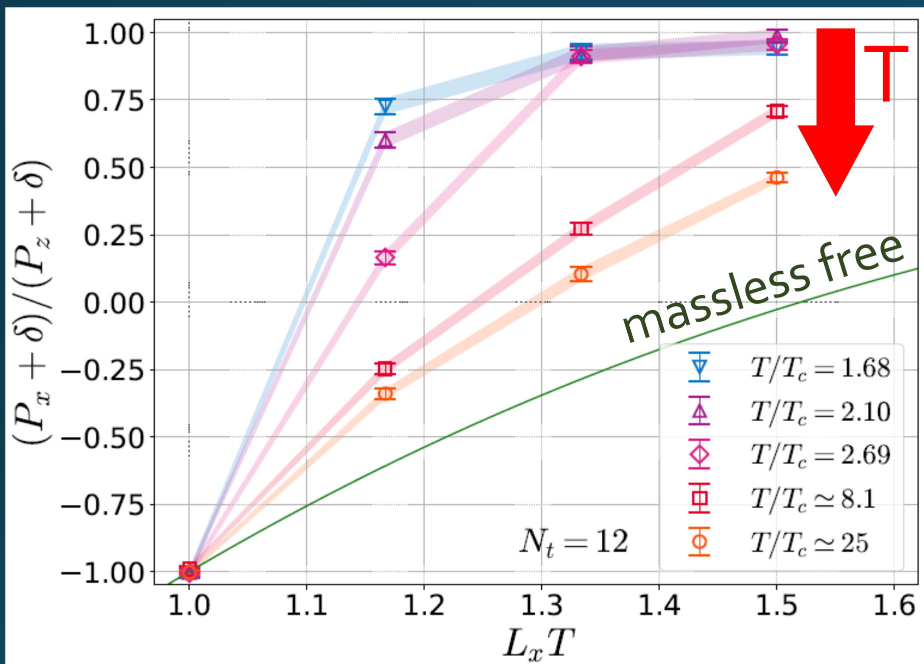
- ☐ Vacuum subtraction requires large-volume simulations.
- ☐ Lattice spacing not available → $c_1(t)$, $c_2(t)$ are not determined.

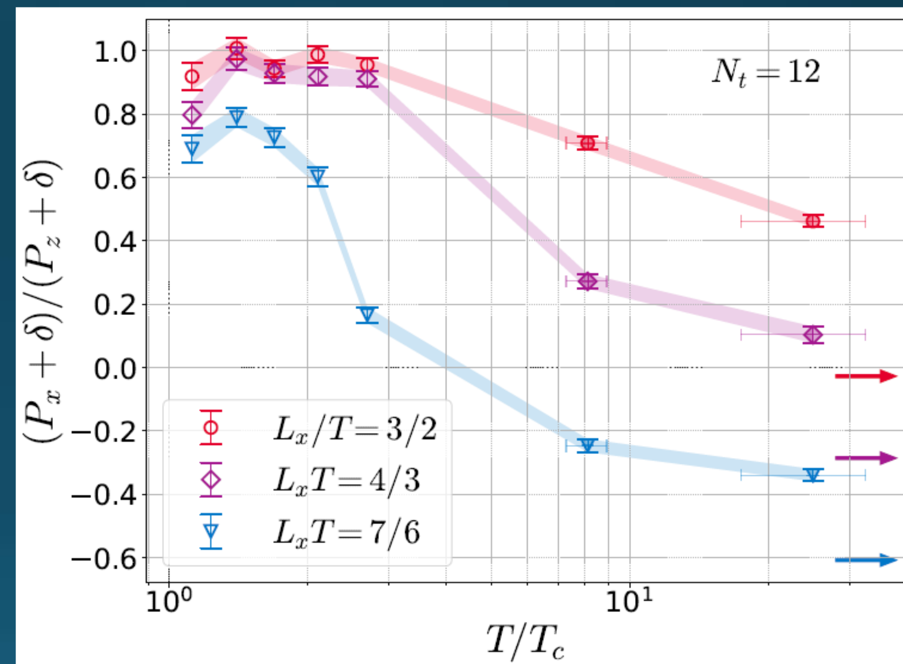
Ratio of Traceless Parts

$$\frac{P_x + \delta}{P_z + \delta} \qquad \delta = -\frac{1}{4}\sum_{\mu} T^{\mathrm{E}}_{\mu\mu}$$

Neither vacuum subtr. nor Suzuki coeffs. necessary!

$$\frac{P_x + \delta}{P_z + \delta}$$





$T/T_c \cong 8.1$ (β=8.0) / $T/T_c \cong 25$ (β=9.0)

- Ratio slowly approaches the asymptotic value.
- But, large deviation still exists even at $T/T_c \sim 25$.

Contents

1. Constructing EMT through gradient flow

2. Thermodynamics

FlowQCD, PRD**90**,011501 (2014); PRD**94**, 114512 (2016)
WHOT-QCD, PRD**96**, 014509 (2017); PRD**102**, 014510 (2020)
Iritani+, PTEP **2019**, 023B02 (2019)

3. Casimir Effect & Pressure Anisotropy

MK, Mogliacci, Horowitz, Kolbe, PRD**99**, 094507 (2019)

4. EMT Correlation Functions

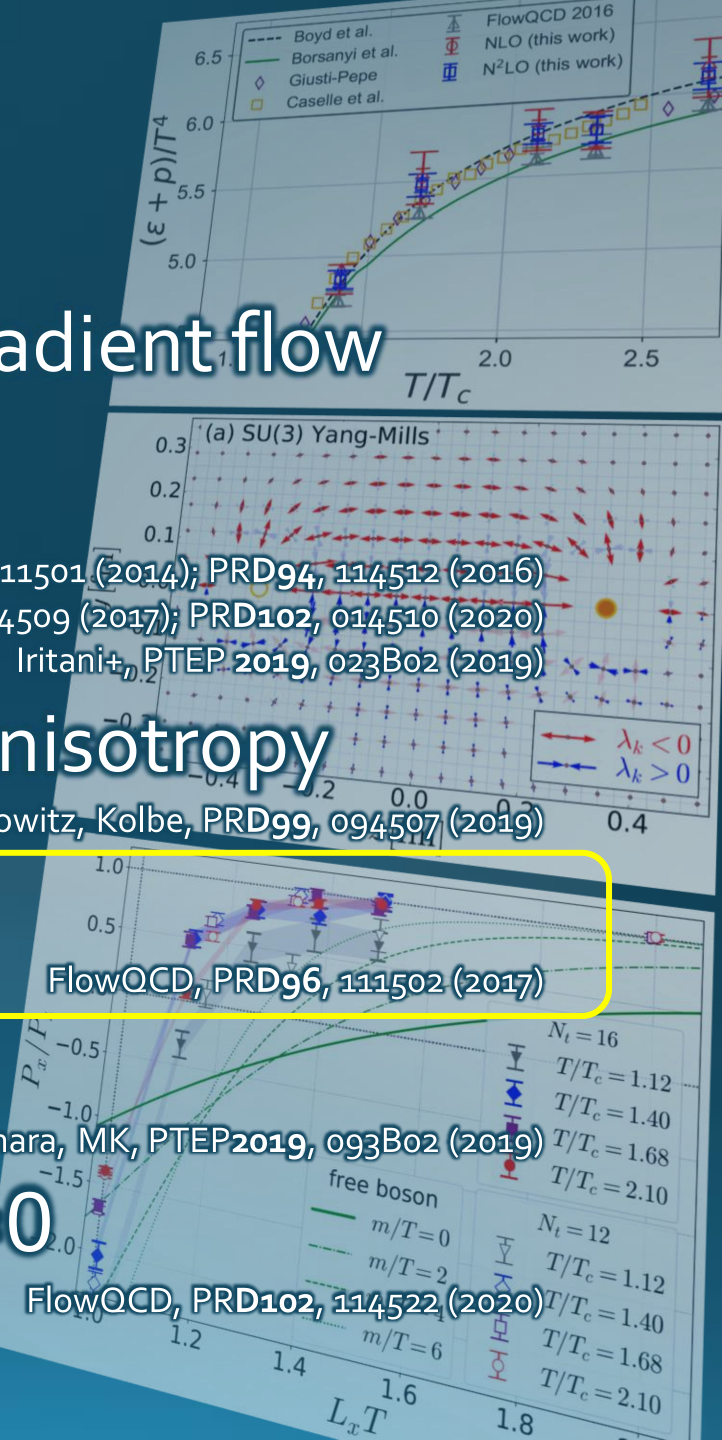
FlowQCD, PRD**96**, 111502 (2017)

5. Flux Tube

FlowQCD, PLB**789**, 210 (2019); Yanagihara, MK, PTEP**2019**, 093B02 (2019)

6. Single-Quark System at T≠0

FlowQCD, PRD**102**, 114522 (2020)



EMT Correlator: Motivation

- Viscosity

Kubo formula → viscosity

$$\eta = \int_0^\infty dt \int_0^{1/T} d\tau \int d^3x \langle T_{12}(x, -i\tau) T_{12}(0, t) \rangle$$

Karsch, Wyld, 1987
Nakamura, Sakai, 2005
Meyer, 2007; 2008
...
Borsanyi+, 2018
Astrakhantsev+, 2018

- Energy/Momentum Conservation

$\langle \bar{T}_{0\mu}(\tau) \bar{T}_{\rho\sigma}(0) \rangle$: τ-independent constant

- Fluctuation-Response Relations

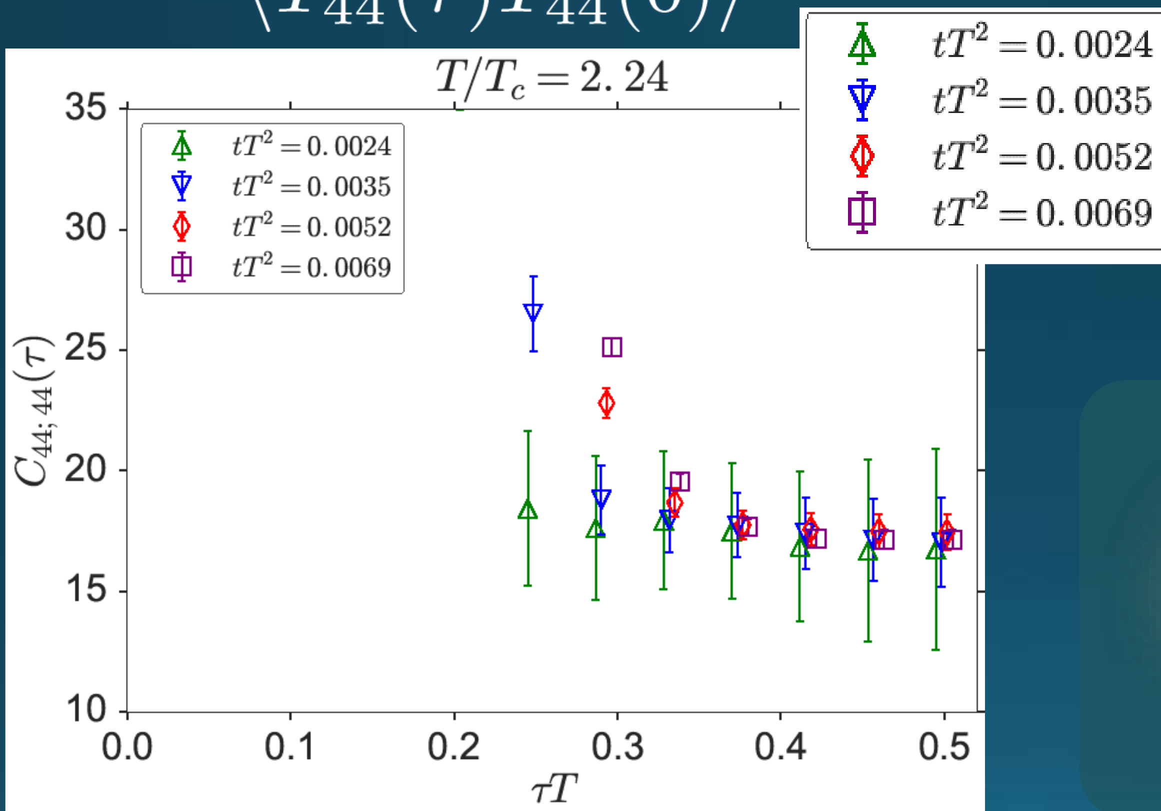
$$c_V = \frac{\langle \delta E^2 \rangle}{VT^2} \qquad \varepsilon + p = \frac{\langle \bar{T}_{01}^2 \rangle}{VT} = \frac{\langle \bar{T}_{11} \bar{T}_{00} \rangle}{VT}$$

EMT Euclidean Correlator

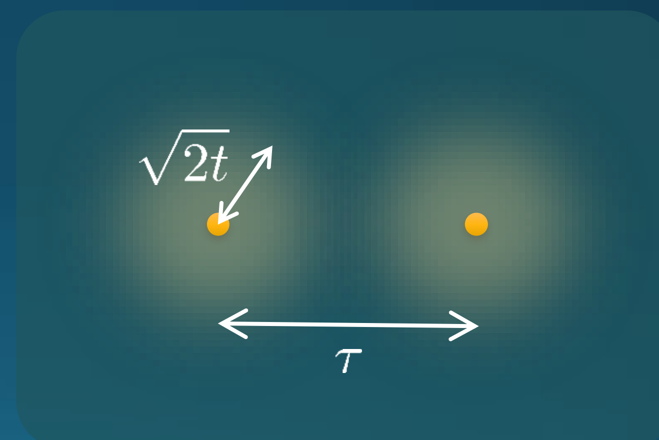
FlowQCD (2017)

See also: Eller, Moore (2018)

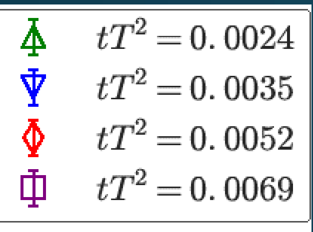
$$\langle \bar{T}_{44}(\tau)\bar{T}_{44}(0)\rangle$$



$$\bar{T}_{\mu\nu} = \int d^3x T_{\mu\nu}$$



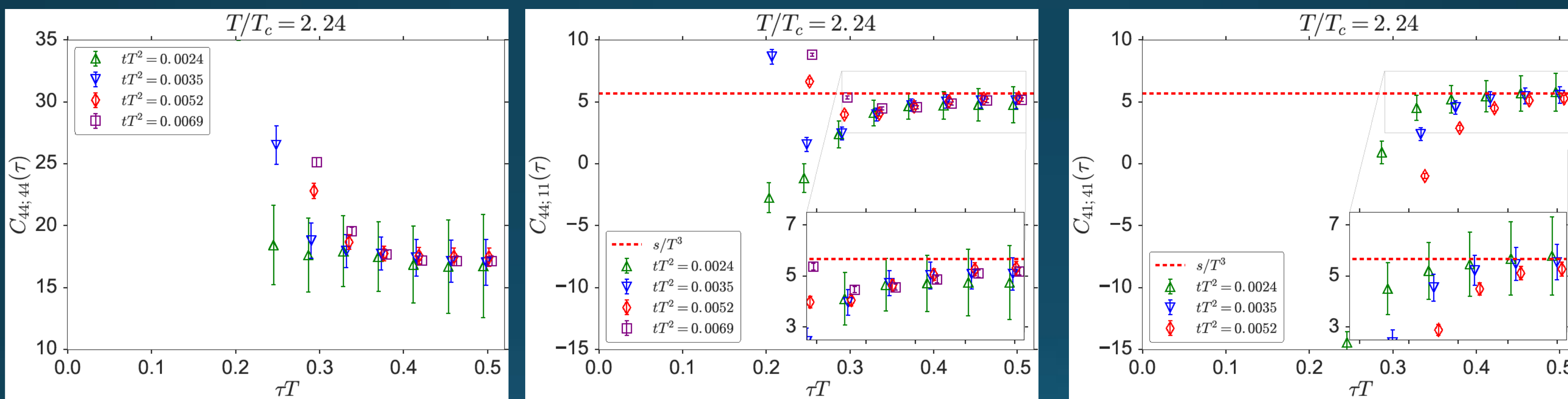
- ☐ Suppression of errors at large τ
- ☐ τ-independent plateau 🡐 energy conservation
- ☐ small τ region: not reliable due to smearing $\tau > 2\sqrt{2t}$



EMT Euclidean Correlator

FlowQCD, PR **D96**, 111502 (2017)

$\langle \bar{T}_{44}(\tau)\bar{T}_{44}(0)\rangle$ $\langle \bar{T}_{44}(\tau)\bar{T}_{11}(0)\rangle$ $\langle \bar{T}_{41}(\tau)\bar{T}_{41}(0)\rangle$



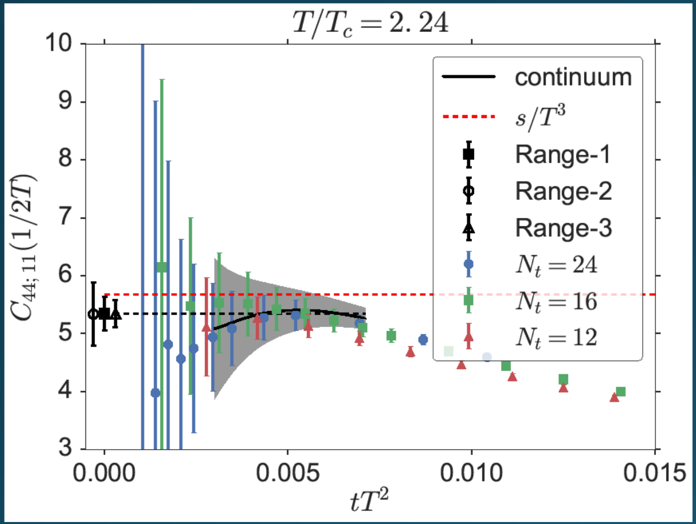
- ☐ τ-independent plateau in all channels ➔ conservation law
- ☐ Confirmation of fluctuation-response relations
- ☐ New method to measure c_V

- ☐ Similar result for (41;41) channel: Borsanyi+, 2018
- ☐ Perturbative analysis: Eller, Moore, 2018

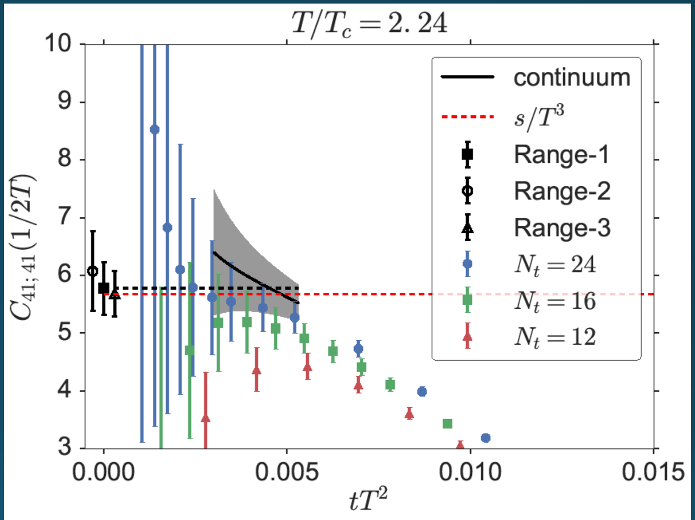
Fluctuation-Response Relations

- Correlators at mid-point

$$\langle T_{44}(\beta/2)T_{11}(0)\rangle$$



$$\langle T_{41}(\beta/2)T_{41}(0)\rangle$$



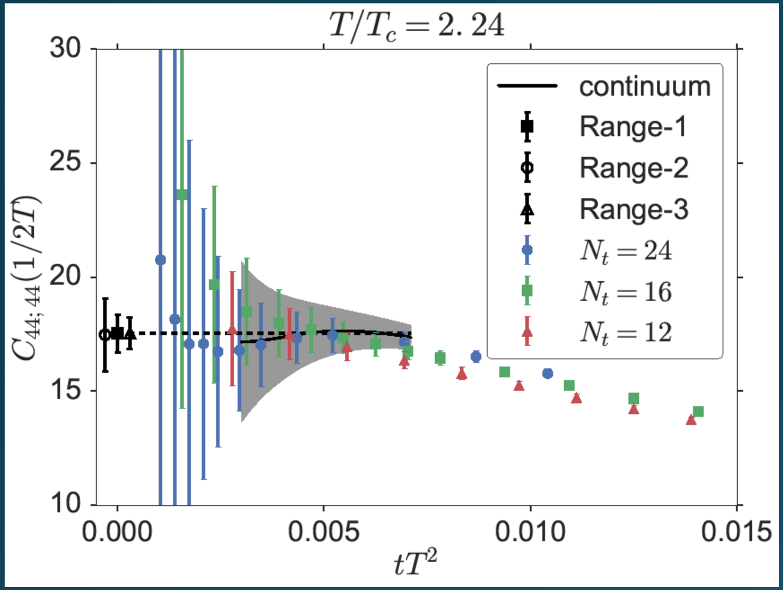
Confirmation of FRR

$$\varepsilon + p = \frac{\langle \bar{T}_{01}^2 \rangle}{VT} = \frac{\langle \bar{T}_{11}\bar{T}_{00} \rangle}{VT}$$

2+1 QCD: WHOT-QCD, 1711.02262

Fluctuation-Response Relations

$$\langle T_{44}(\tau) T_{44}(0) \rangle$$



fluctuation of energy
~ specific heat

$$c_V = \frac{\langle \delta E^2 \rangle}{V T^2}$$

New measurement of c_V

c_V/T^3

T/T_c	$C_{44;44}(\tau_m)$	Ref.[19]	Ref.[11]	ideal gas
1.68	$17.7(8)(^{+2.1}_{-0.4})$	$22.8(7)^*$	17.7	21.06
2.24	$17.5(0.8)(^{+0}_{-0.1})$	$17.9(7)^{**}$	18.2	21.06

[11] Borsanyi+ (2012)
[19] Gavai+ (2005)

Contents

1. Constructing EMT through gradient flow

2. Thermodynamics

FlowQCD, PR**D90**,011501 (2014); PR**D94**, 114512 (2016)
WHOT-QCD, PR**D96**, 014509 (2017); PR**D102**, 014510 (2020)
Iritani+, PTEP **2019**, 023B02 (2019)

3. Casimir Effect & Pressure Anisotropy

MK, Mogliacci, Horowitz, Kolbe, PR**D99**, 094507 (2019)

4. EMT Correlation Functions

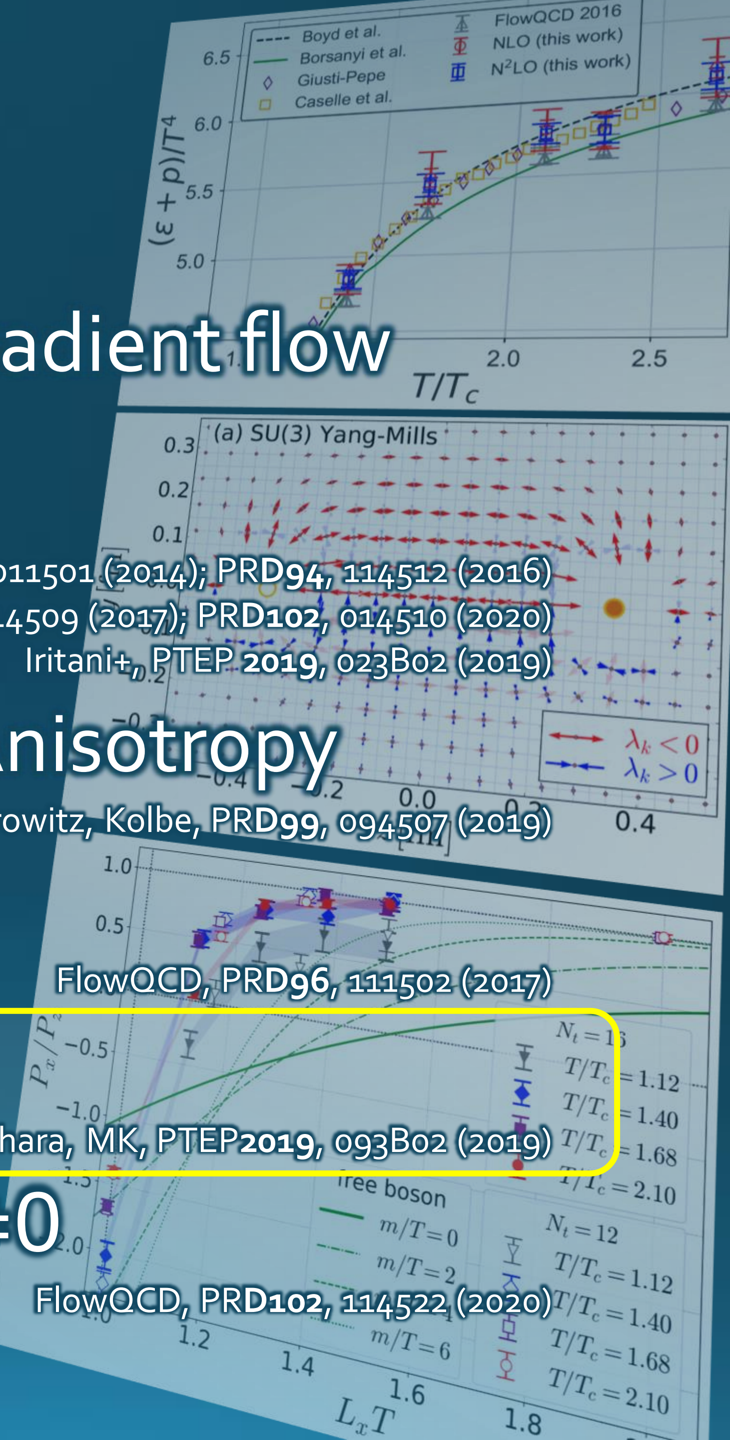
FlowQCD, PR**D96**, 111502 (2017)

5. Flux Tube

FlowQCD, PL**B789**, 210 (2019); Yanagihara, MK, PTEP**2019**, 093B02 (2019)

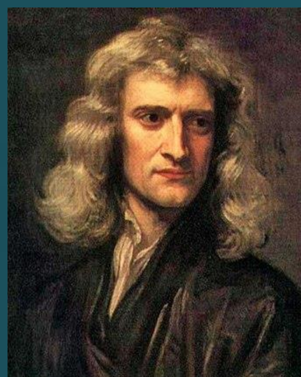
6. Single-Quark System at T≠0

FlowQCD, PR**D102**, 114522 (2020)

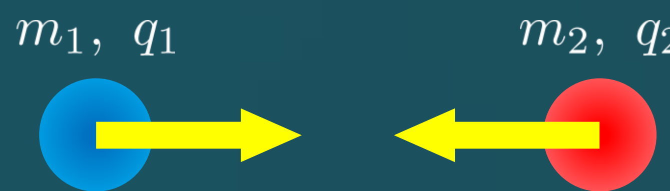


Force

Action-at-a-distance



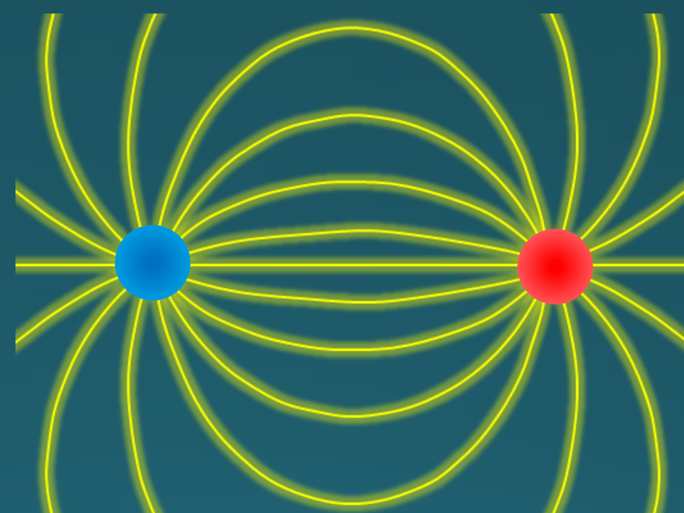
Newton
1687



$$F = -G\frac{m_1 m_2}{r^2} \qquad F = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

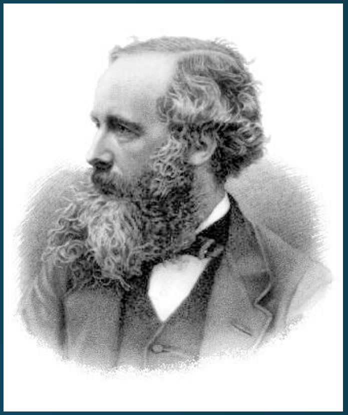
Local interaction

Faraday
1839



Maxwell Stress
(in Maxwell Theory)

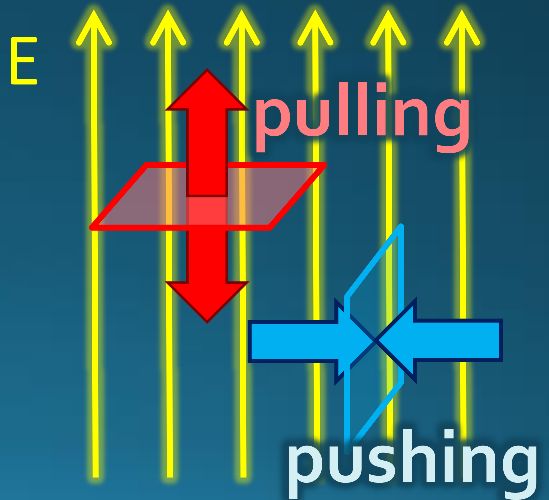
$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$



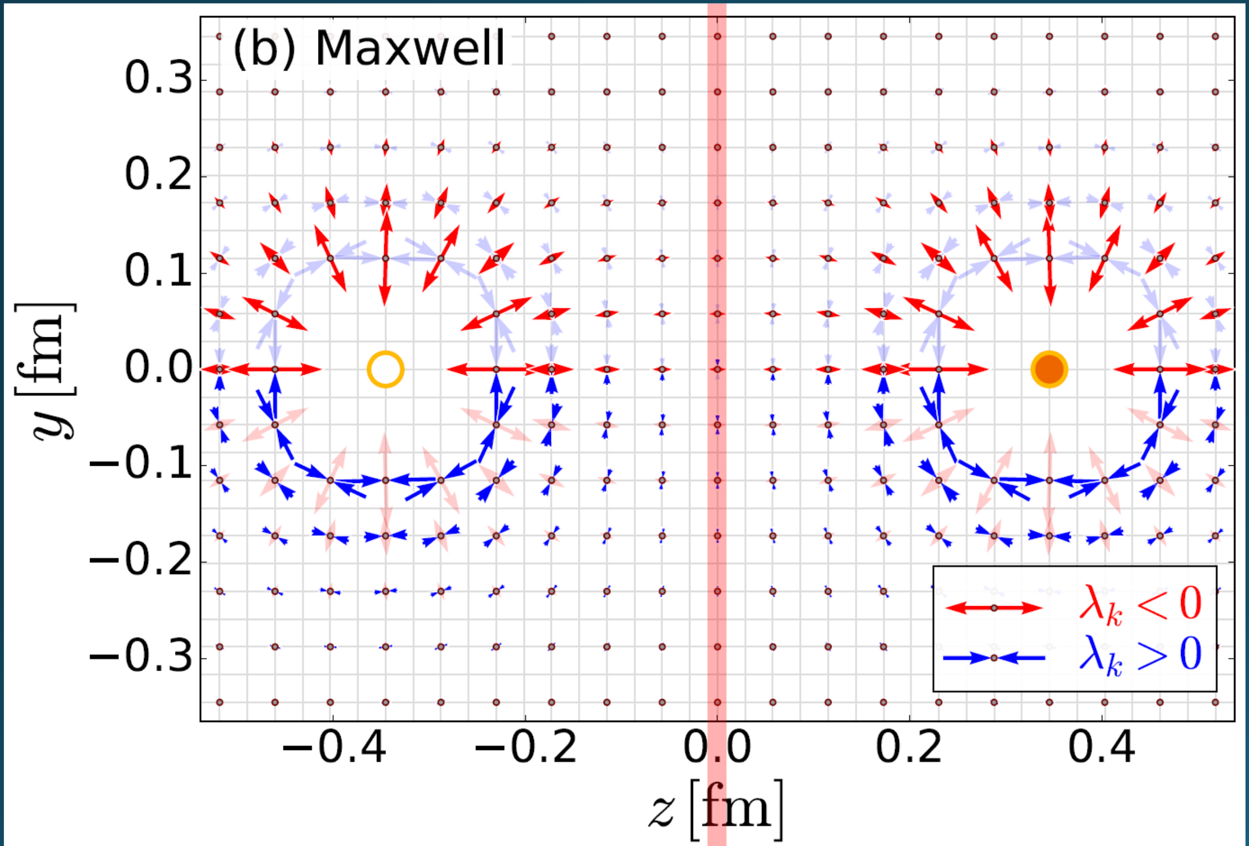
$$\vec{E} = (E, 0, 0)$$

$$T_{ij} = \begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: **Pulling**
- Vertical to field: **Pushing**



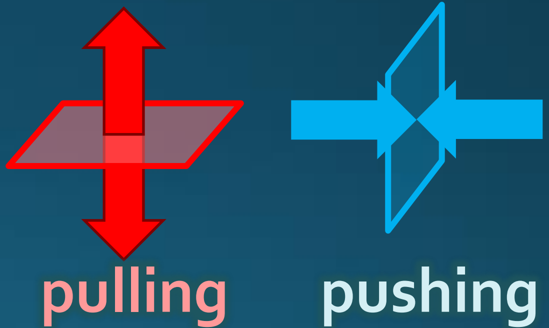
Maxwell Stress
(in Maxwell Theory)



$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$$(k = 1, 2, 3)$$

length: $\sqrt{|\lambda_k|}$

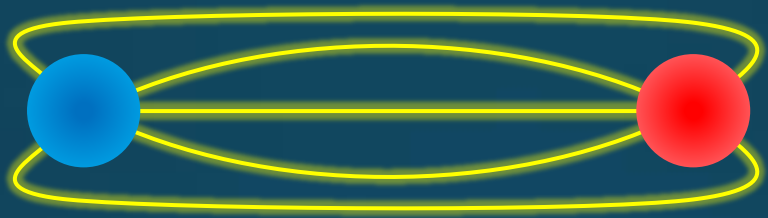


Definite physical meaning

- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction

Quark-Anti-quark System

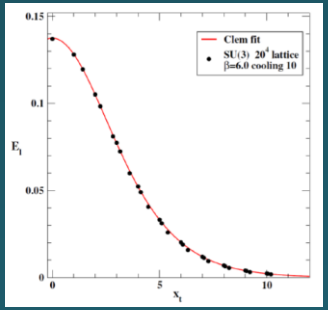
Formation of the flux tube → confinement



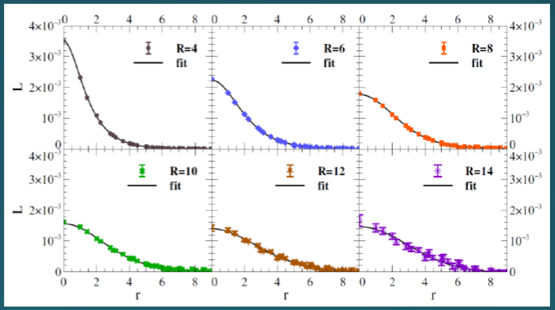
Previous Studies on Flux Tube

- ☐ Potential
- ☐ Action density
- ☐ Color-electric field

so many studies…



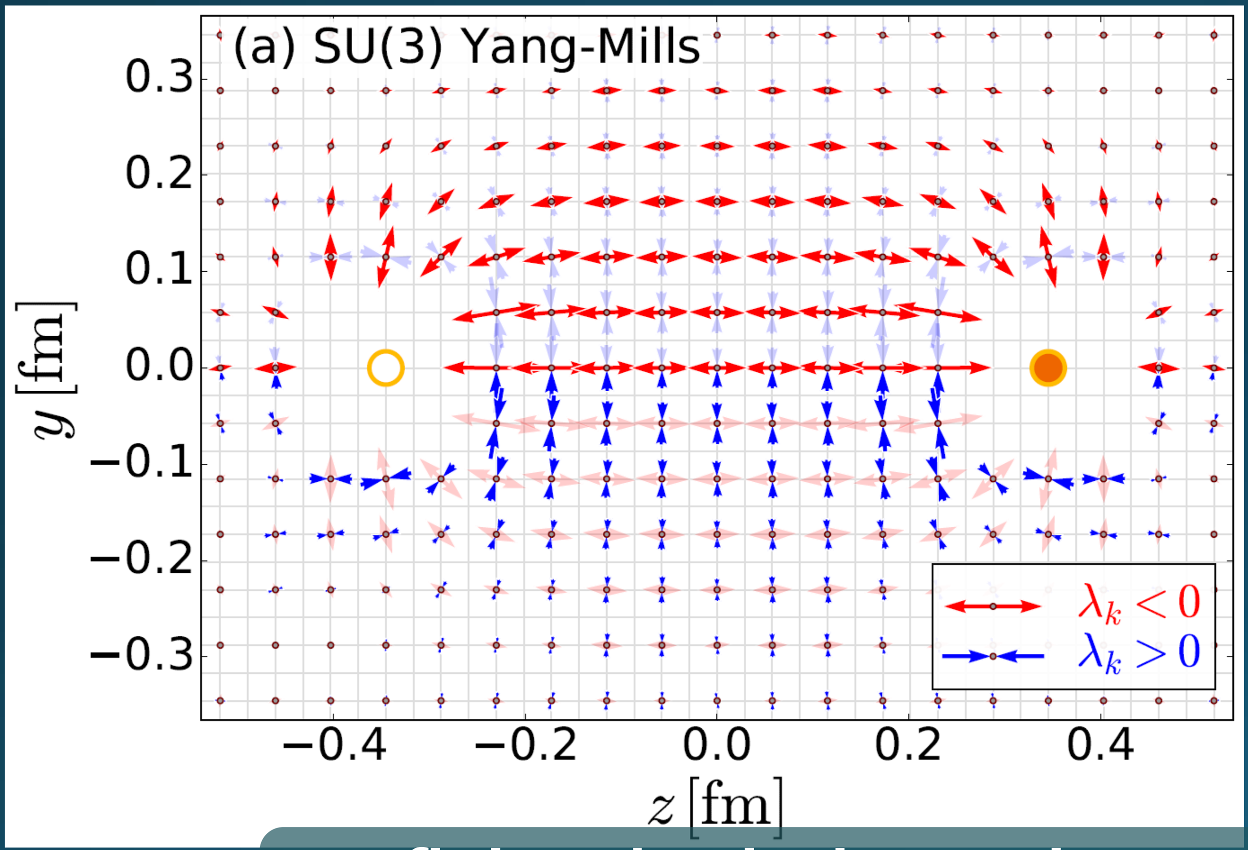
Cea+ (2012)



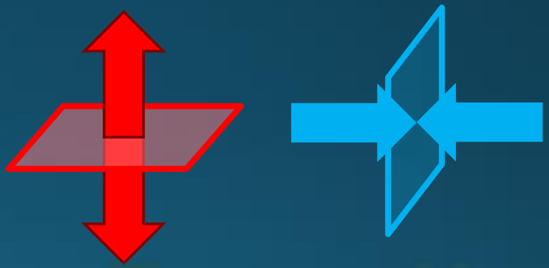
Cardoso+ (2013)

Stress Tensor in $Q\bar{Q}$ System

FlowQCD, PLB (2019)



Lattice simulation
SU(3) Yang-Mills
a=0.029 fm
R=0.69 fm
t/a^2=2.0

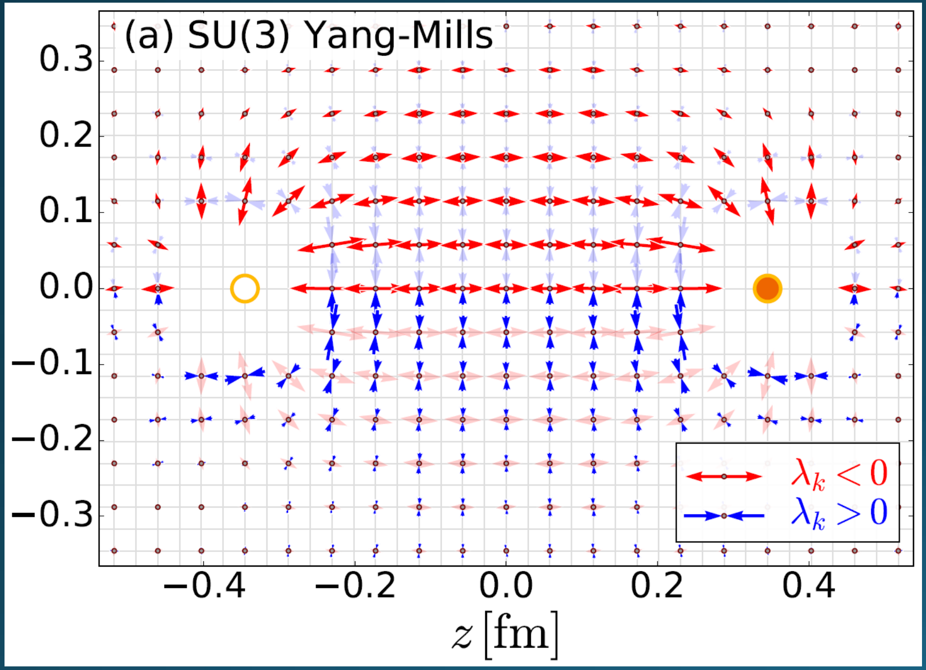


Definite physical meaning

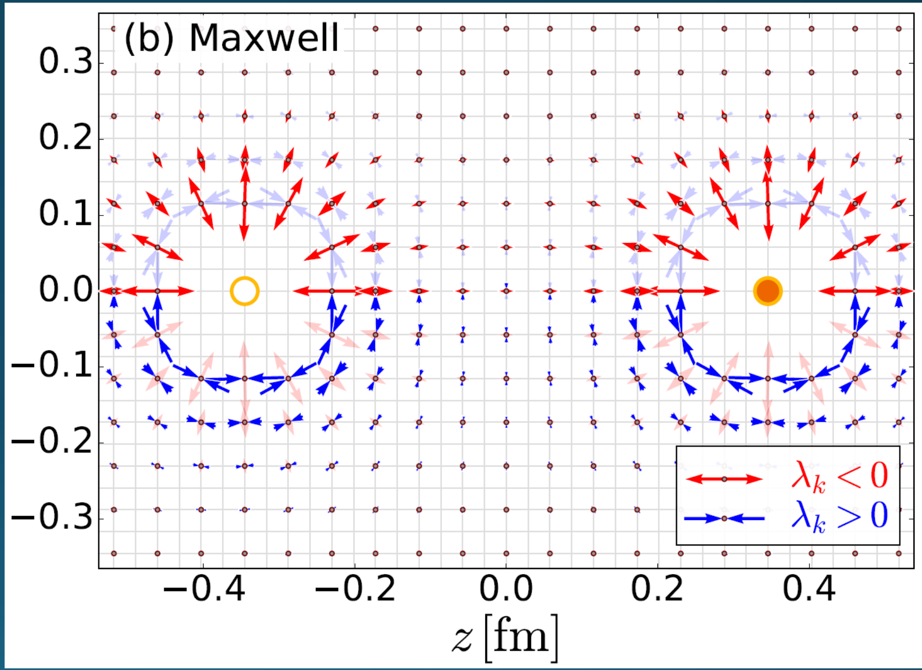
- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction
- ☐ Manifestly gauge invariant

SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)



Maxwell
(classical)



Propagation of the force is clearly different in YM and Maxwell theories!

Stress Distribution on Mid-Plane

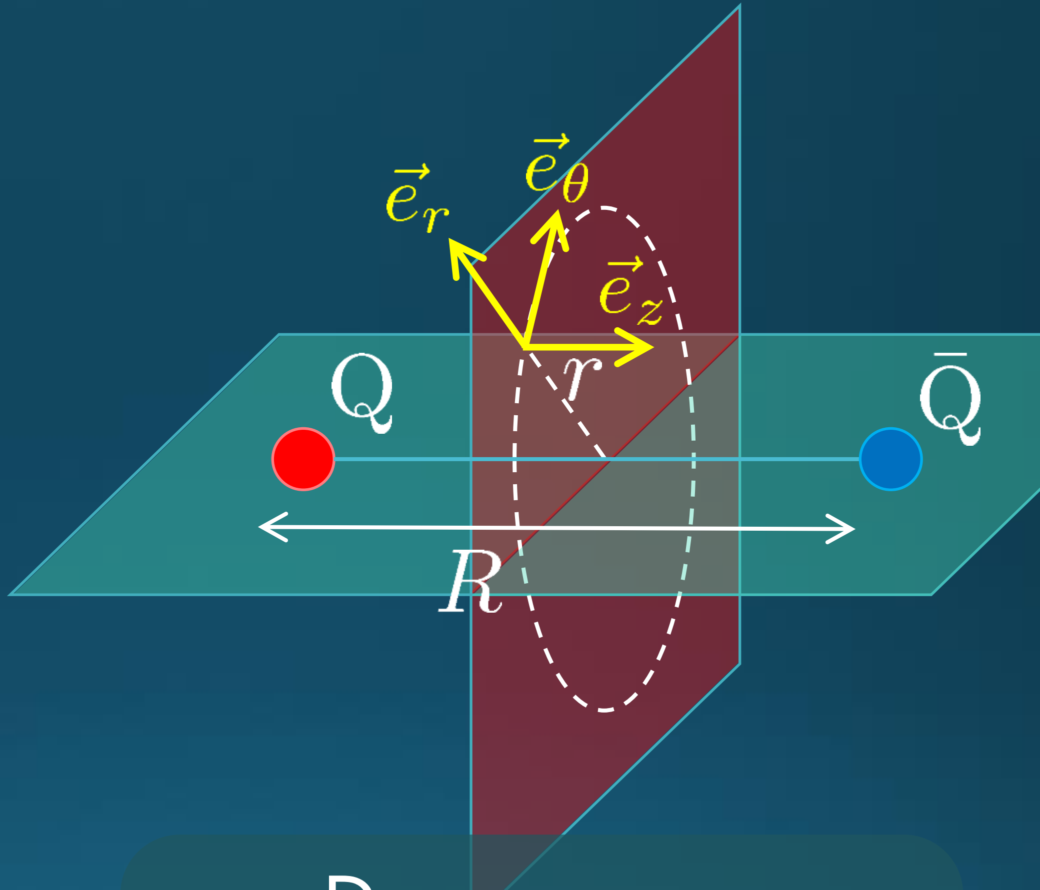
From rotational symm. & parity

EMT is diagonalized
in Cylindrical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{zz} & \\ & & & T_{44} \end{pmatrix}$$

$$T_{rr} = \vec{e}_r^{\,T} T \vec{e}_r$$

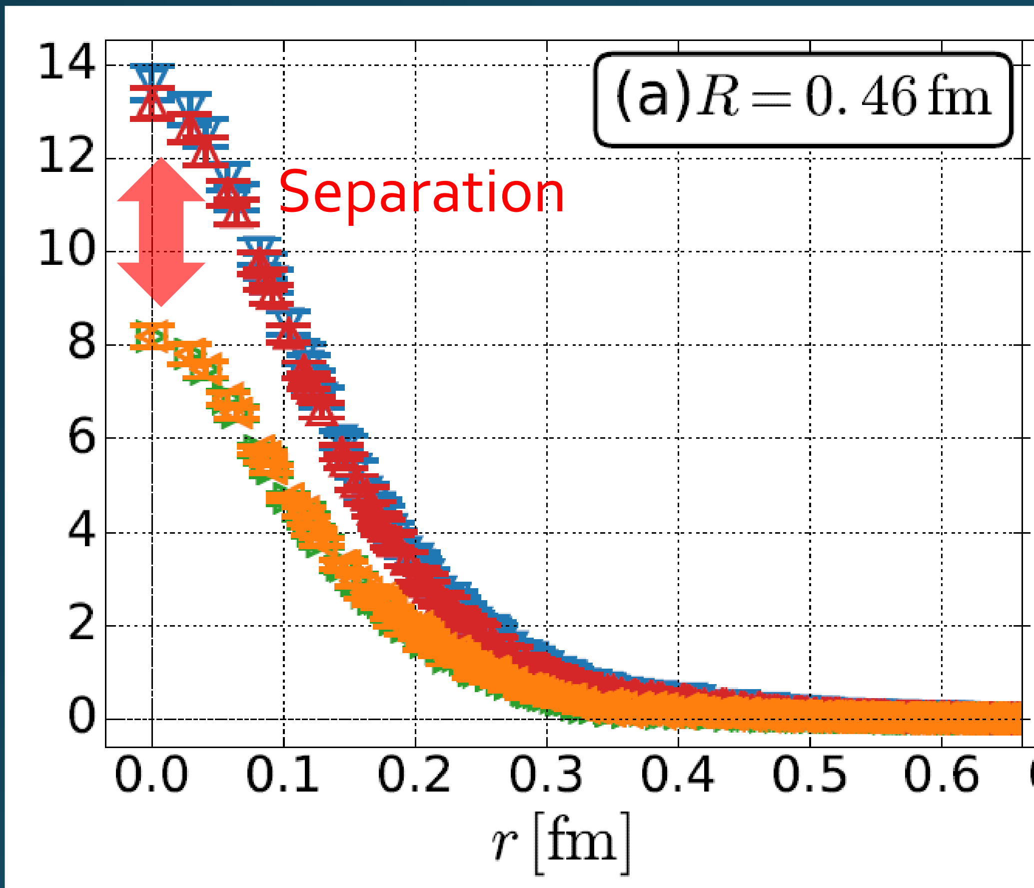
$$T_{\theta\theta} = \vec{e}_\theta^{\,T} T \vec{e}_\theta$$

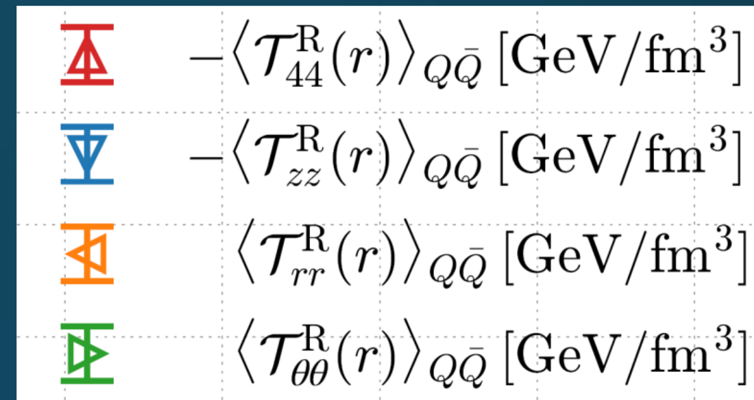


Degeneracy
in Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

Mid-Plane





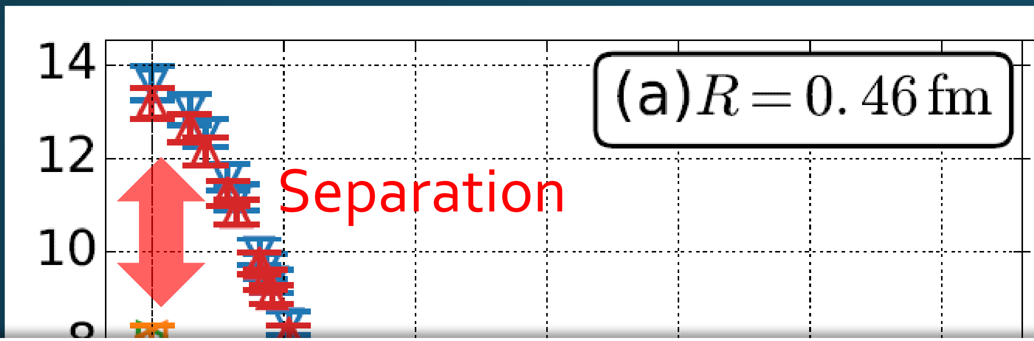
Continuum Extrapolated!

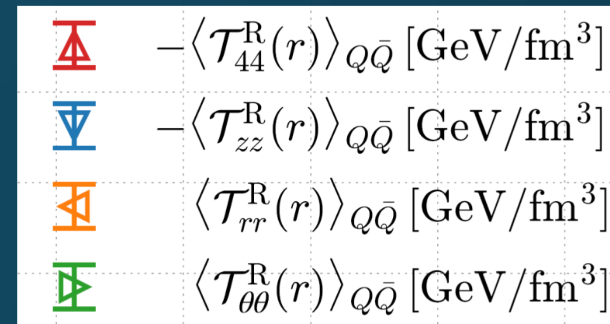
In Maxwell theory

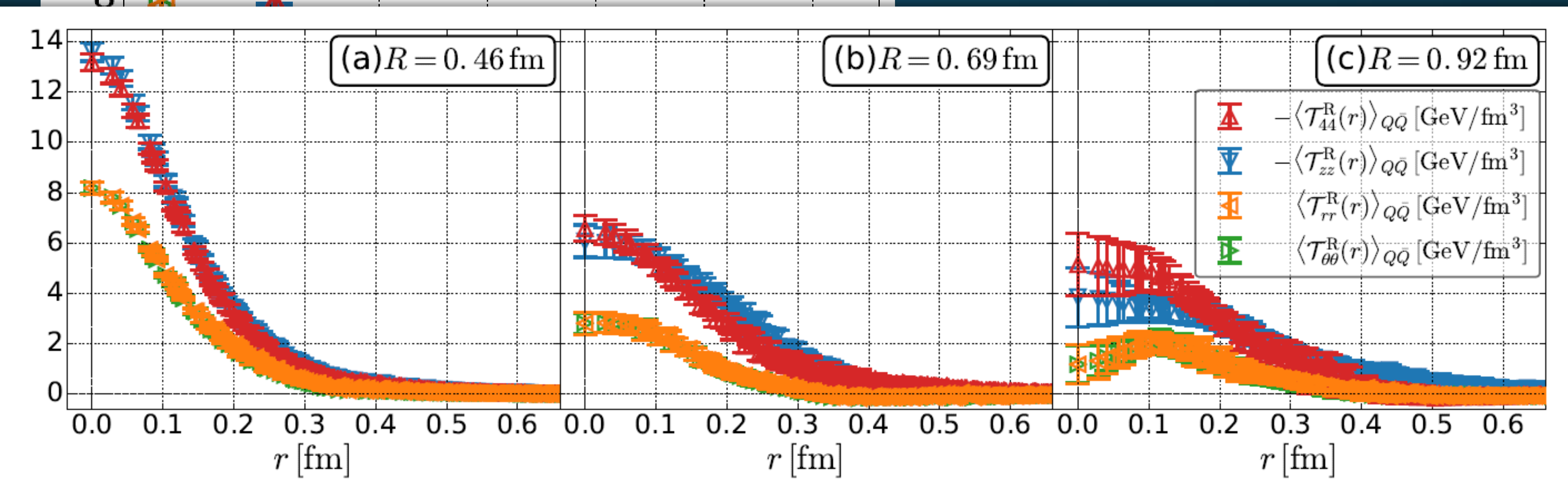
$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

- ☐ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- ☐ Separation: $T_{zz} \neq T_{rr}$
- ☐ Nonzero trace anomaly $\sum T_{cc} \neq 0$

Mid-Plane







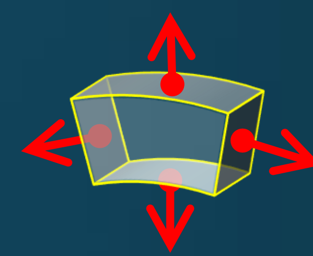
- Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- Separation: $T_{zz} \neq T_{rr}$
- Nonzero trace anomaly $\sum T_{cc} \neq 0$

Momentum Conservation

Yanagihara, MK, PTEP2019

- In cylindrical coordinats,

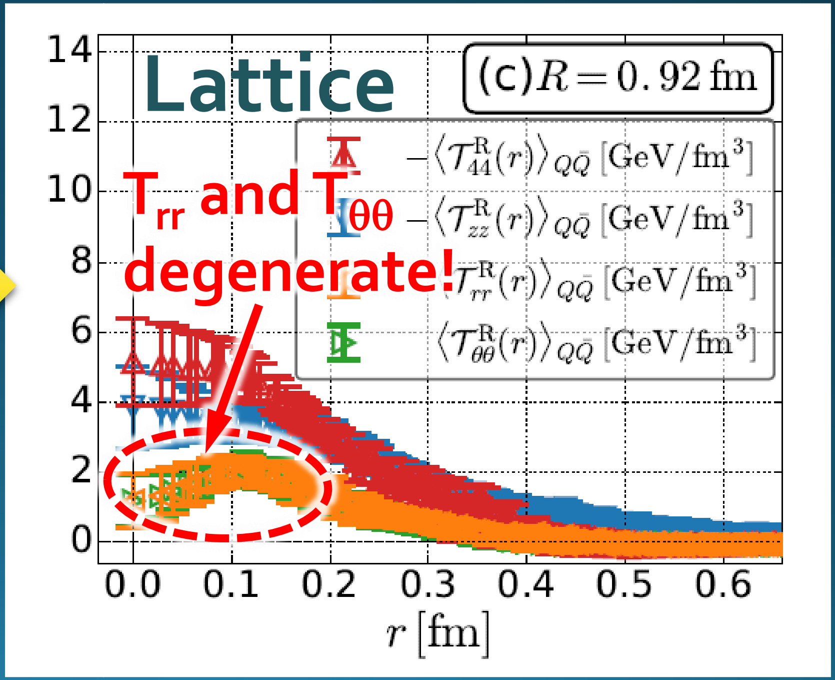
$$\partial_i T_{ij} = 0 \;\Rightarrow\; \partial_r (r T_{rr}) = T_{\theta\theta} - r\partial_z T_{rz}$$



- For infinitely-long flux tube

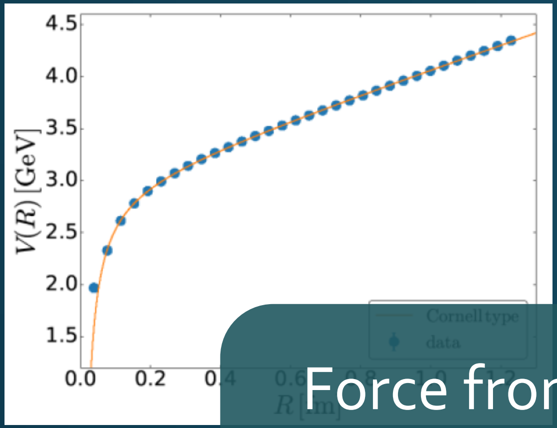
$$\partial_r (r T_{rr}) = T_{\theta\theta}$$

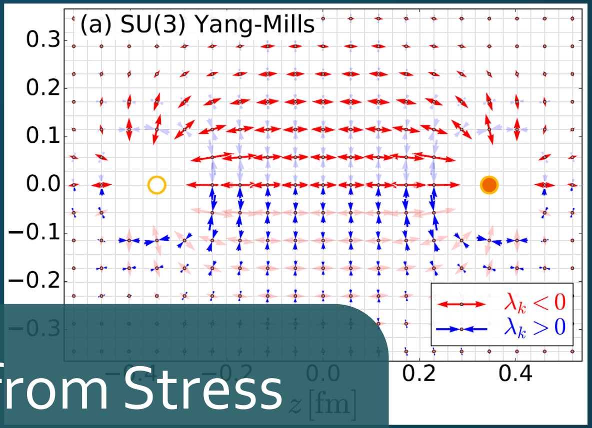
⇒ **T_{rr} and $T_{\theta\theta}$ must separate!** ⇔



Effect of boundaries is not negligible at R=0.92fm

Force





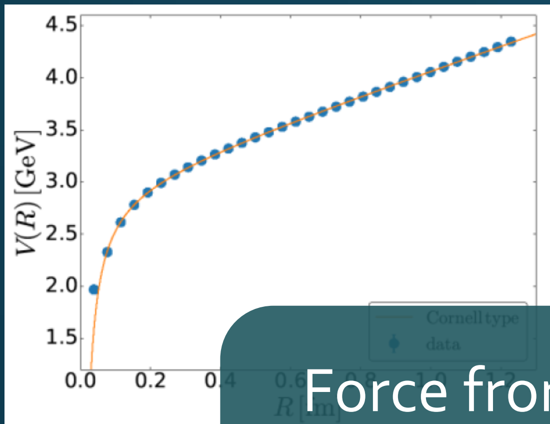
Force from Potential

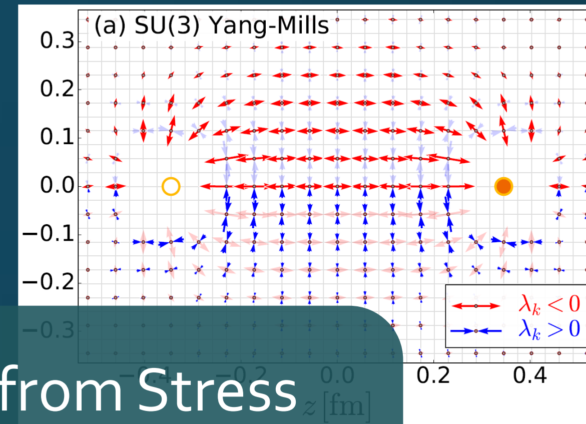
$$F_{\mathrm{pot}} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\mathrm{stress}} = \int_{\mathrm{mid.}} d^2x T_{zz}(x)$$

Force



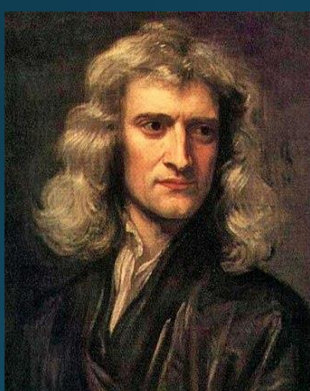


Force from Potential

$$F_{\text{pot}} = -\frac{dV}{dR}$$

Force from Stress

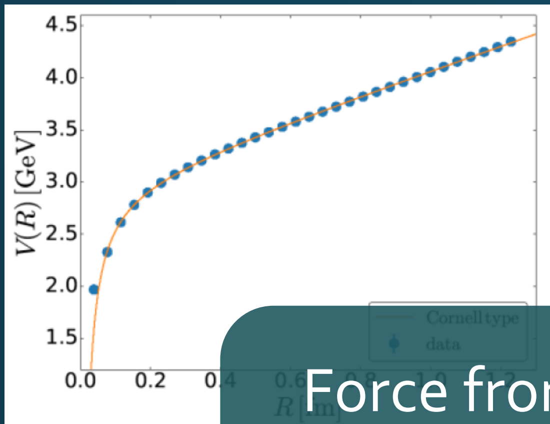
$$F_{\text{stress}} = \int_{\text{mid.}} d^2x T_{zz}(x)$$

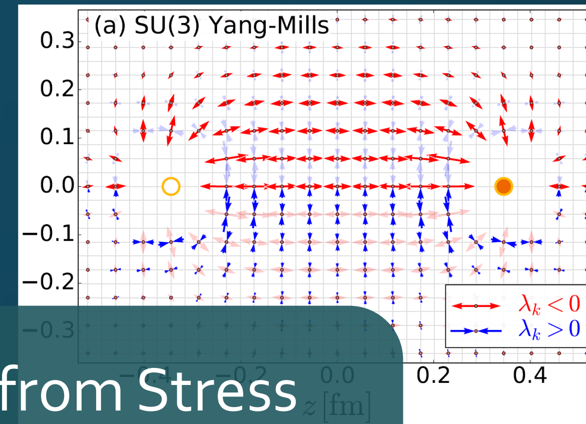


Newton
1687

Faraday
1839

Force



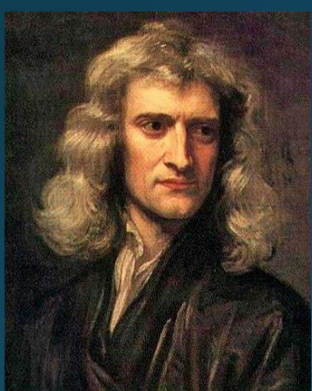


Force from Potential

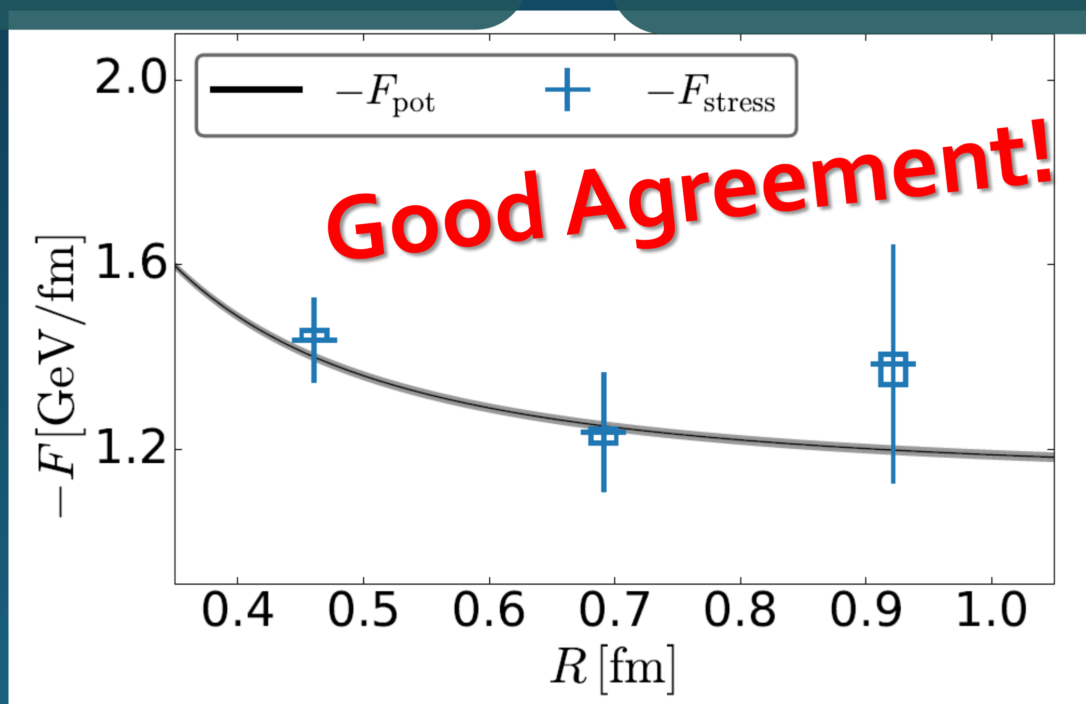
$$F_{\mathrm{pot}} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\mathrm{stress}} = \int_{\mathrm{mid.}} d^2x T_{zz}(x)$$



Newton
1687



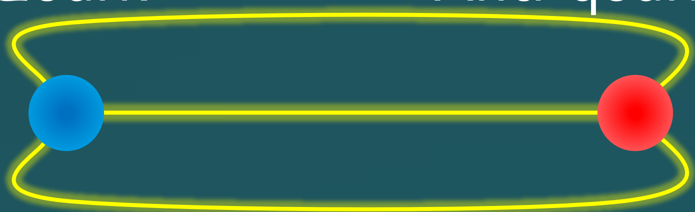
Faraday
1839

Dual Superconductor Picture

Nambu, 1970
Nielsen, Olesen, 1973
t 'Hooft, 1981
...

QCD Vacuum

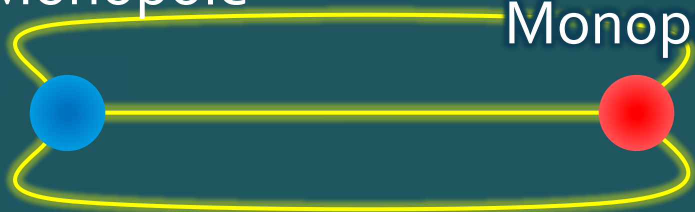
Quark Anti-quark



Flux Tube

Superconductor

Monopole Anti-Monopole



Magnetic Vortex

Dual (E⇔B)

Abelian-Higgs Model

Yanagihara, MK, 2019

Abelian-Higgs Model

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

GL parameter: $\kappa = \sqrt{\lambda}/g$

- ☐ type-I : $\kappa < 1/\sqrt{2}$
- ☐ type-II : $\kappa > 1/\sqrt{2}$
- ☐ Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

Infinitely long tube

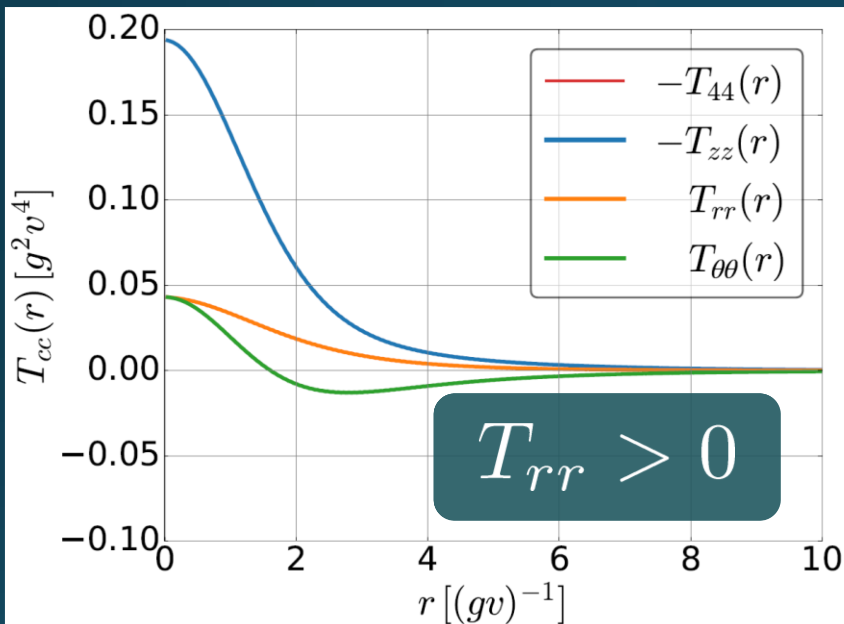
- ☐ degeneracy
  $T_{zz}(r) = T_{44}(r)$ Luscher, 1981
- ☐ momentum conservation
  $$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

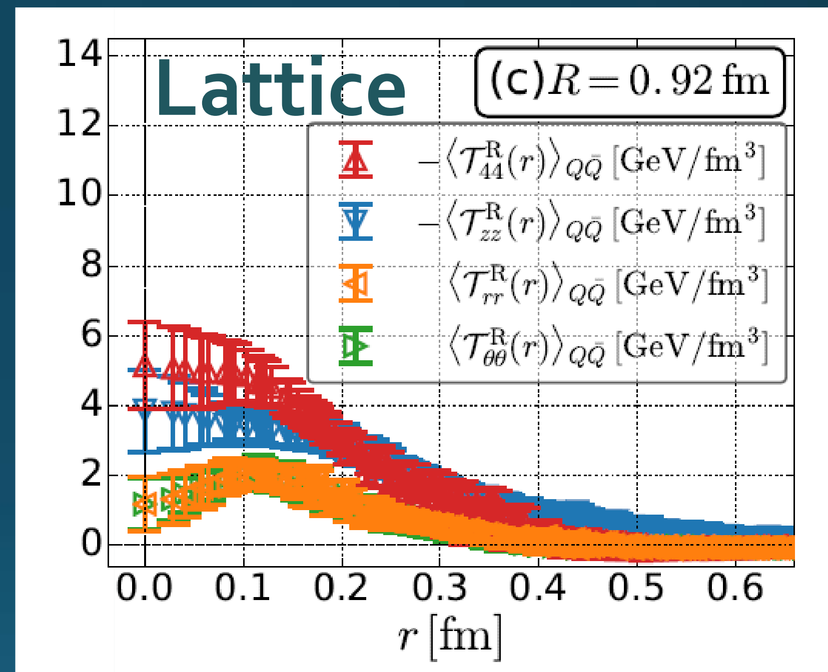
Stress Tensor in AH Model

infinitely-long flux tube

Type-I

$\kappa = 0.1$





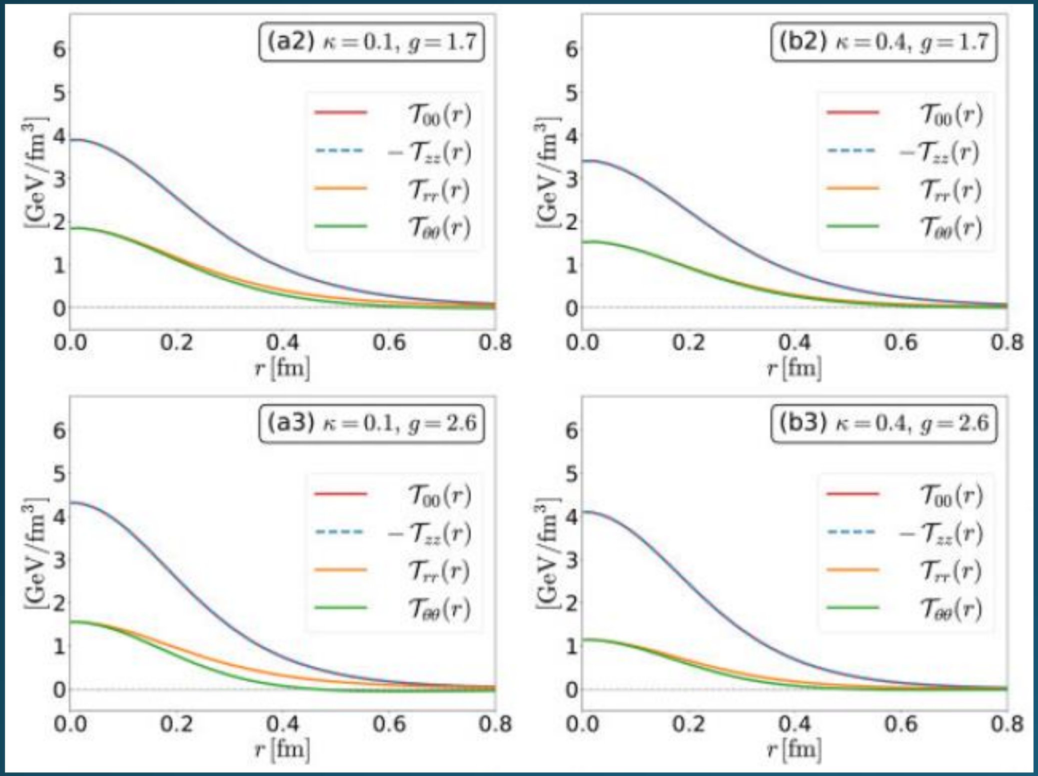
- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

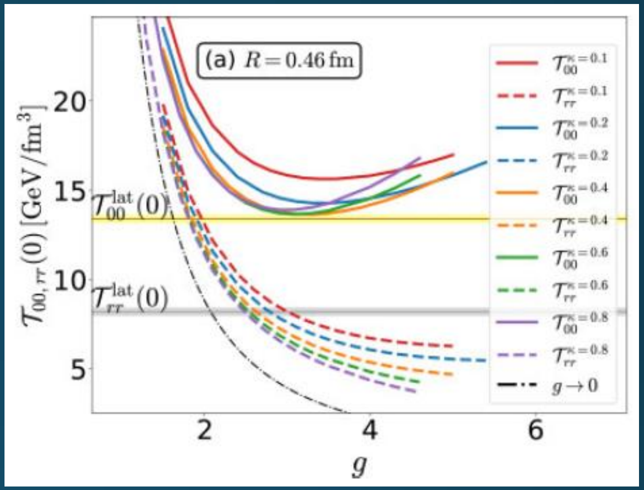
Inconsistent with lattice result

$$T_{rr} \simeq T_{\theta\theta}$$

Flux Tube with Finite Length

Yanagihara, MK (2019)





- ☐ AH model can reproduce lattice results **qualitatively** by tuning parameters.
- ☐ But, **quantitatively** all parameters are rejected.

EMT Distr. in Simple Systems

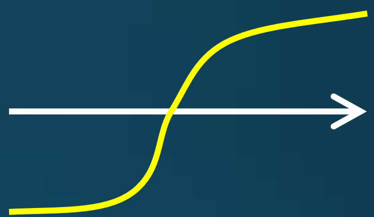
Talk by H. Ito soon later

ϕ^4 Theory in 1+1d

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2$$

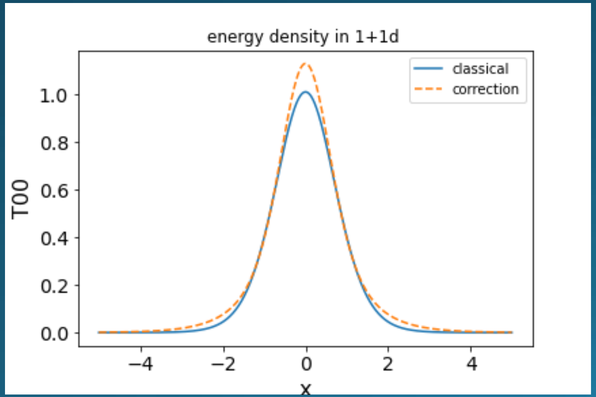
- **Soliton (kink)**

$$\phi(x) = \frac{m}{\sqrt{\lambda}} \tanh \frac{mx}{\sqrt{2}}$$



- **Quantum effect on EMT at 1-loop order**

$T_{00}(x)$



Confirmation of EMT conservation

$$\partial_x T_{11}(x) = 0$$

Contents

1. Constructing EMT through gradient flow

2. Thermodynamics

FlowQCD, PRD**90**,011501 (2014); PRD**94**, 114512 (2016)
WHOT-QCD, PRD**96**, 014509 (2017); PRD**102**, 014510 (2020)
Iritani+, PTEP **2019**, 023B02 (2019)

3. Casimir Effect & Pressure Anisotropy

MK, Mogliacci, Horowitz, Kolbe, PRD**99**, 094507 (2019)

4. EMT Correlation Functions

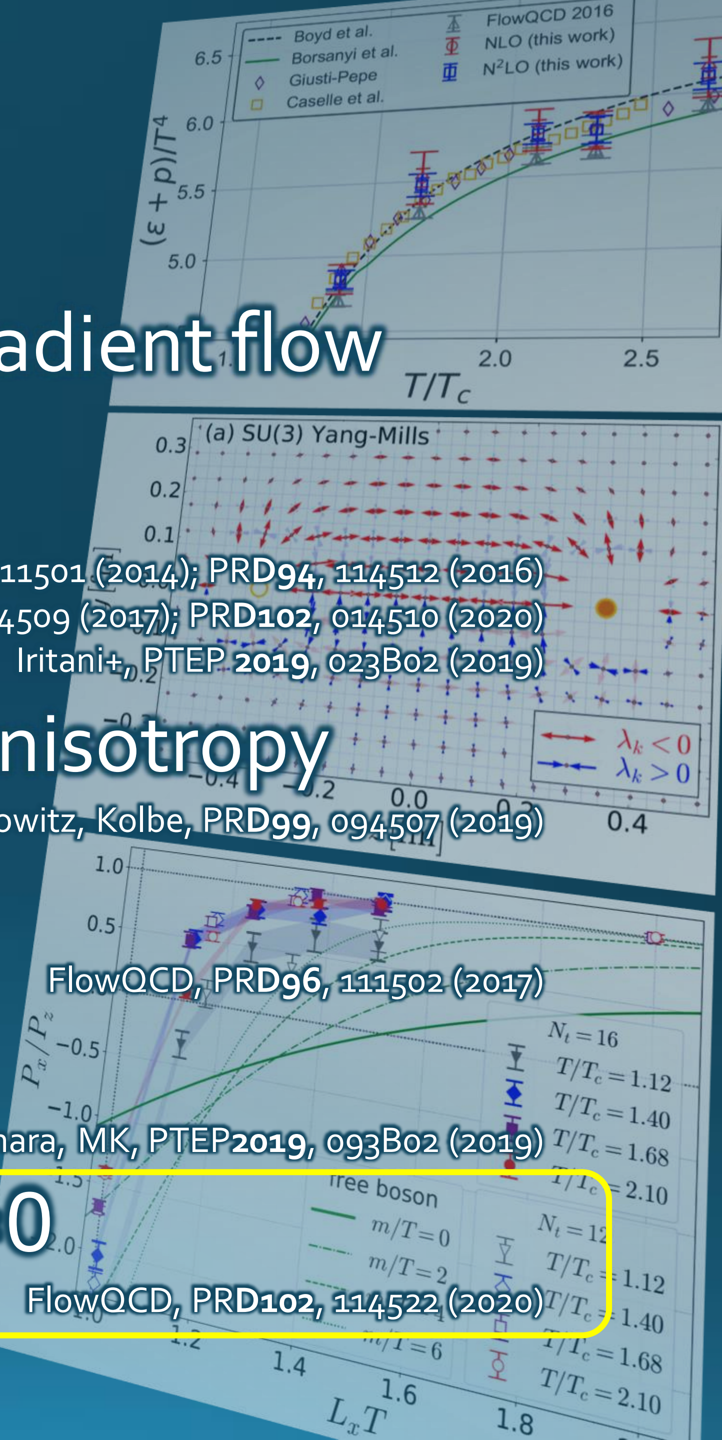
FlowQCD, PRD**96**, 111502 (2017)

5. Flux Tube

FlowQCD, PLB**789**, 210 (2019); Yanagihara, MK, PTEP**2019**, 093B02 (2019)

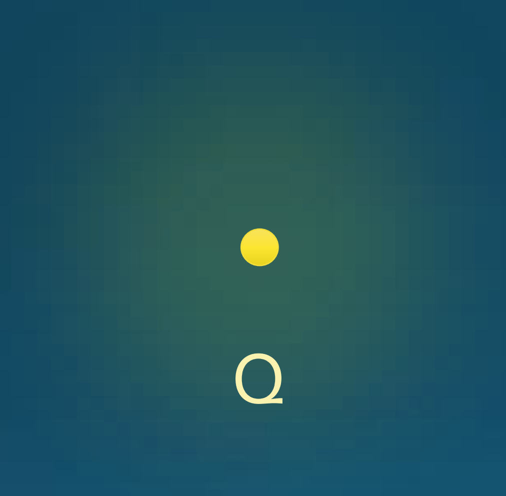
6. Single-Quark System at T≠0

FlowQCD, PRD**102**, 114522 (2020)



Stress Tensor around A Quark

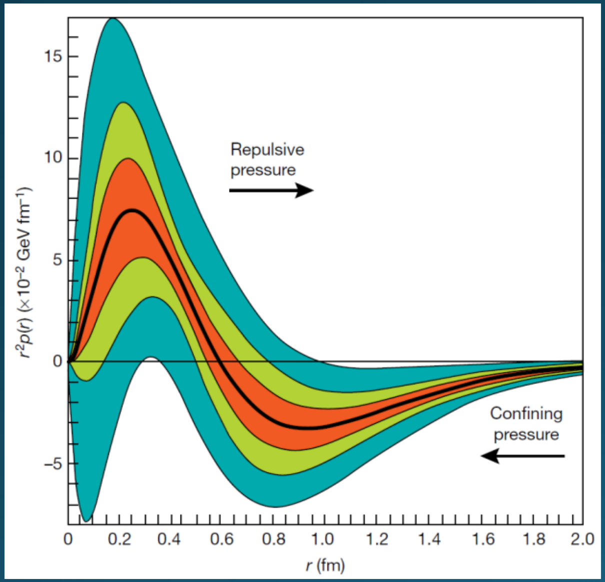
in a deconfined phase



Pressure inside Hadrons

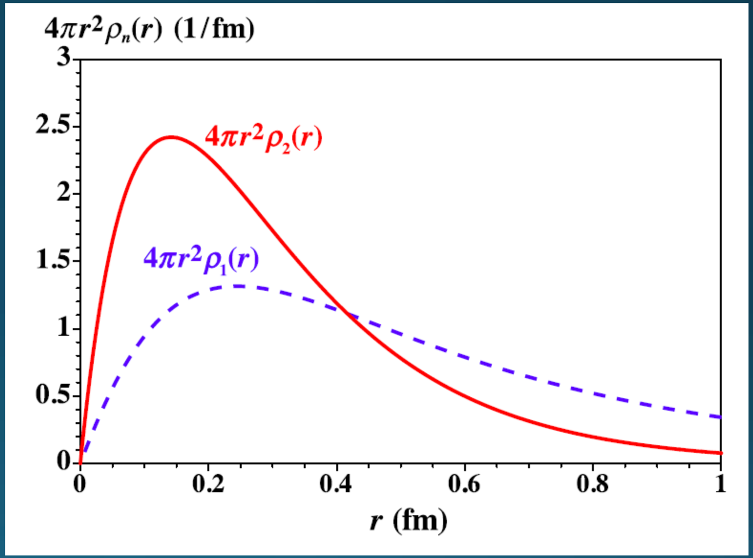
EMT distribution inside hadrons now accessible??

Pressure @ proton



Nature, 557, 396 (2018)
Shanahan, Detmold (2019)

EMT distribution @ pion

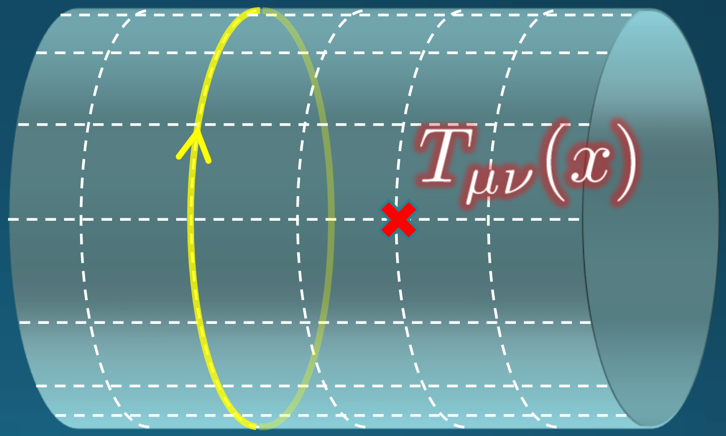


Kumano, Song, Teryaev (2018)

EMT Around a Static Q

$$\langle T_{\mu\nu}(x)\rangle_{\mathrm{Q}} = \frac{\langle T_{\mu\nu}(x)\mathrm{Tr}\Omega(0)\rangle}{\langle \mathrm{Tr}\Omega(0)\rangle} - \langle T_{\mu\nu}(x)\rangle$$

Ω: Polyakov loop



- EMT-Polyakov loop correlation
- Gauge invariant
- Z3 symmetry has to be broken

- EMT by SF*t*X method

Lattice Setup

- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

- Analysis above Tc
- Simulation on a Z_3 minimum
- EMT around a Polyakov loop

$$\langle O(x)\rangle_{\mathrm{Q}} = \frac{\langle \delta O(x)\delta\Omega(0)\rangle}{\langle \Omega(0)\rangle}$$

Ω: Polyakov loop

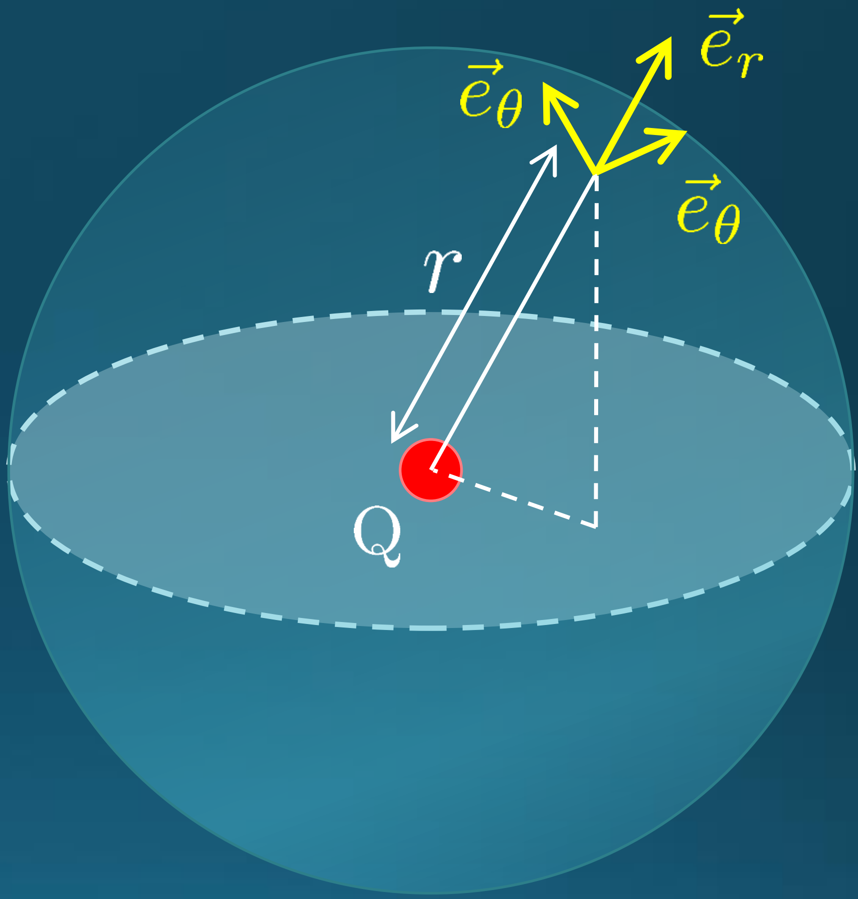
- continuum extrapolation

T/T_c	N_s	N_τ	β	a [fm]	N_{conf}
1.20	40	10	6.336	0.0551	500
	48	12	6.467	0.0460	650
	56	14	6.581	0.0394	840
	64	16	6.682	0.0344	1,000
	72	18	6.771	0.0306	1,000
1.44	40	10	6.465	0.0461	500
	48	12	6.600	0.0384	650
	56	14	6.716	0.0329	840
	64	16	6.819	0.0288	1,000
	72	18	6.910	0.0256	1,000
2.00	40	10	6.712	0.0331	500
	48	12	6.853	0.0275	650
	56	14	6.973	0.0236	840
	64	16	7.079	0.0207	1,000
	72	18	7.173	0.0184	1,000
2.60	40	10	6.914	0.0255	500
	48	12	7.058	0.0212	650
	56	14	7.182	0.0182	840
	64	16	7.290	0.0159	1,000
	72	18	7.387	0.0141	1,000

Spherical Coordinates

EMT is diagonalized
in Spherical Coordinates

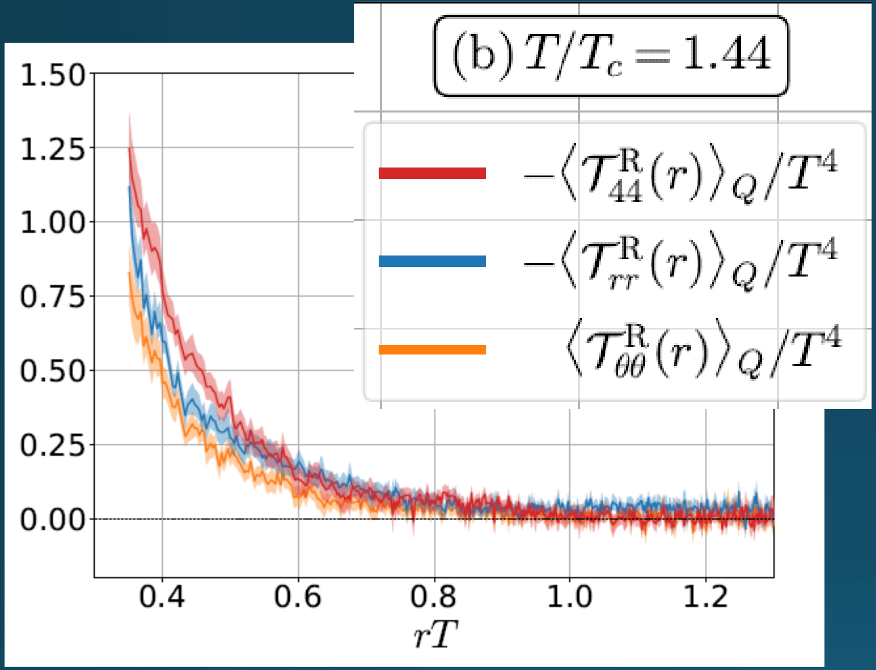
$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{\theta\theta} & \\ & & & T_{44} \end{pmatrix}$$



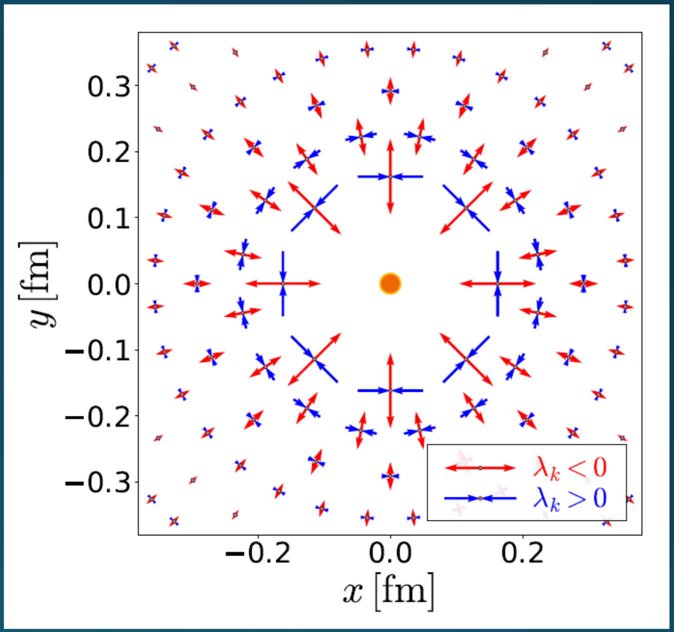
- Maxwell theory

$$T_{44} = T_{rr} = -T_{\theta\theta} = -\frac{|\boldsymbol{E}|^2}{2} = -\frac{\alpha}{8\pi}\frac{1}{r^4}$$

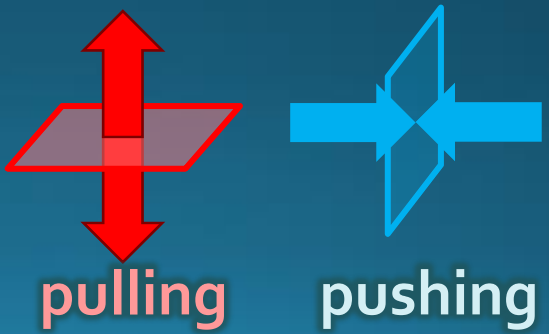
Stress Tensor Around a Quark



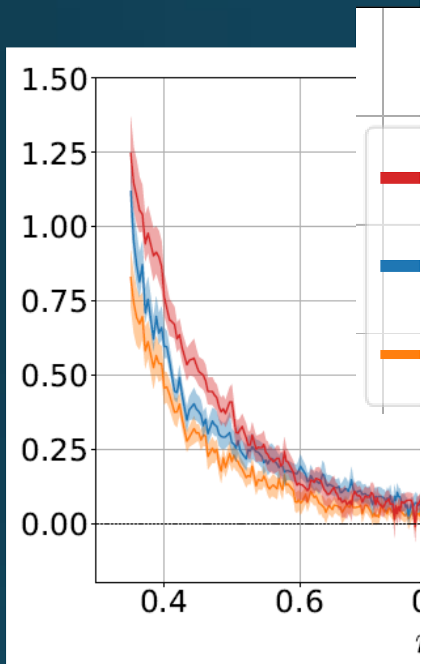
T=1.44T_c

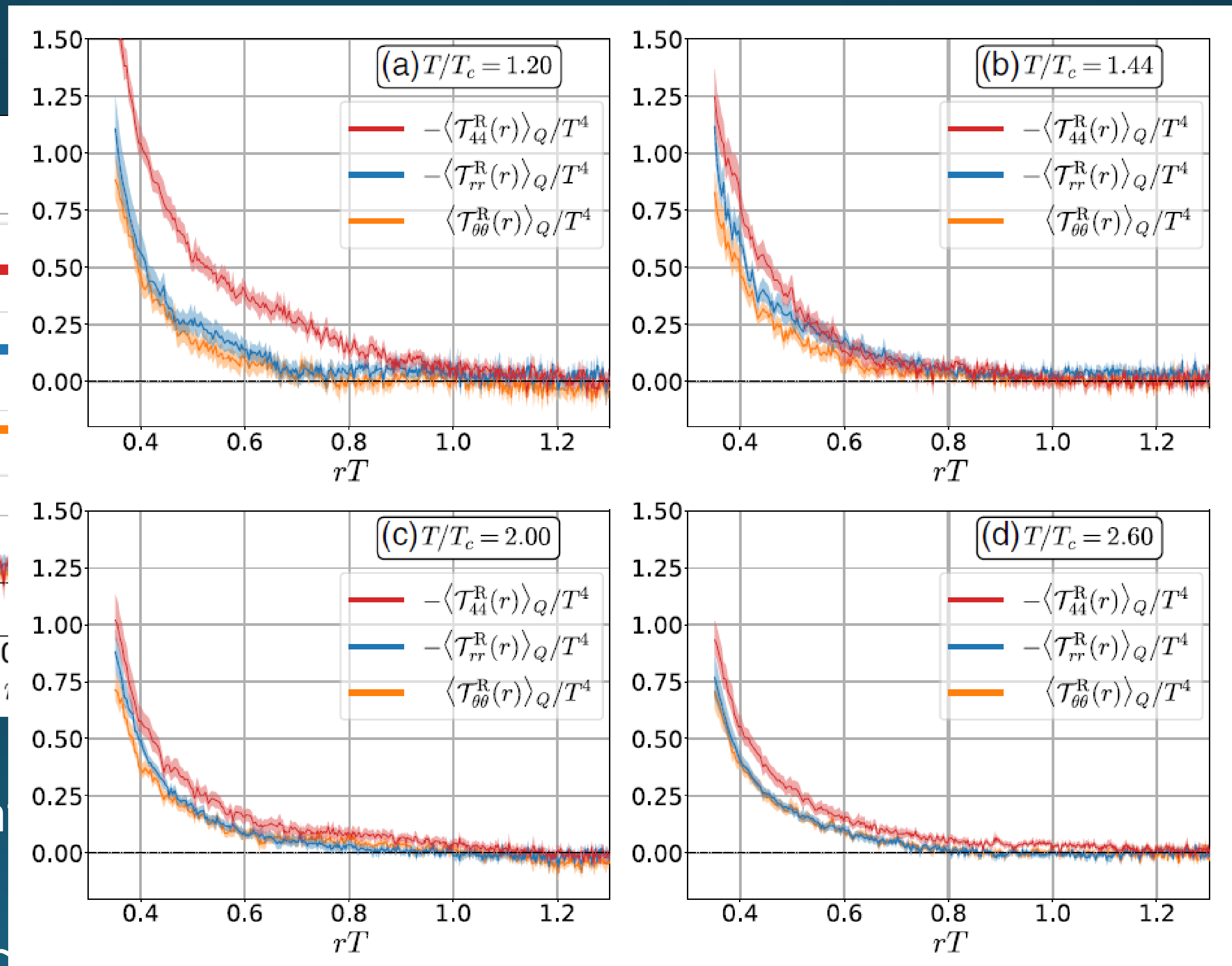


- Suppression at large distance
- Separation of different channels
- $|T_{44}| > |T_{rr}| \sim |T_{\theta\theta}|$



Stress Tensor Around a Quark





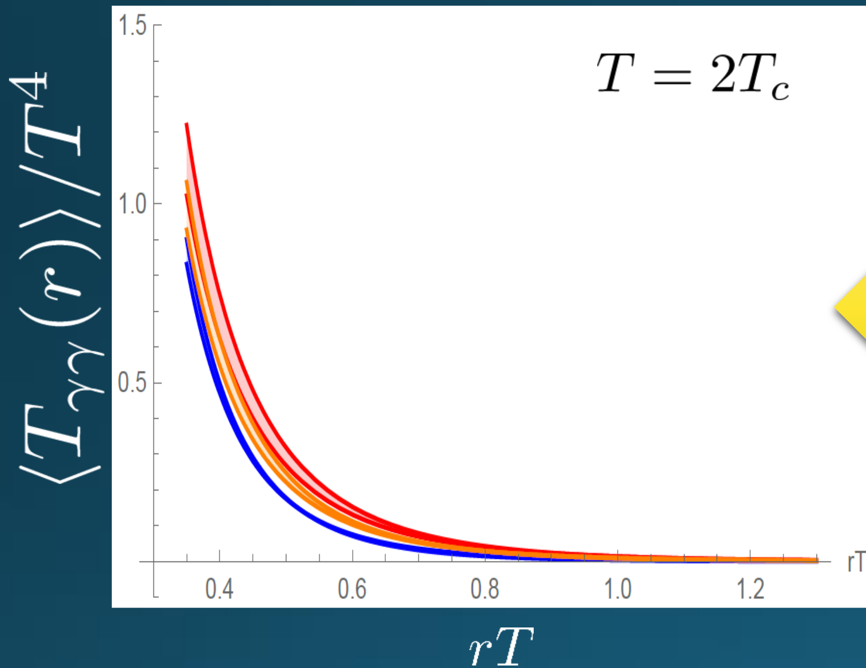
- Suppression a
- Separation of
- $|T_{44}| > |T_{rr}| \sim$
- Clearer separation for lower T

pulling pushing

Perturbative Analysis

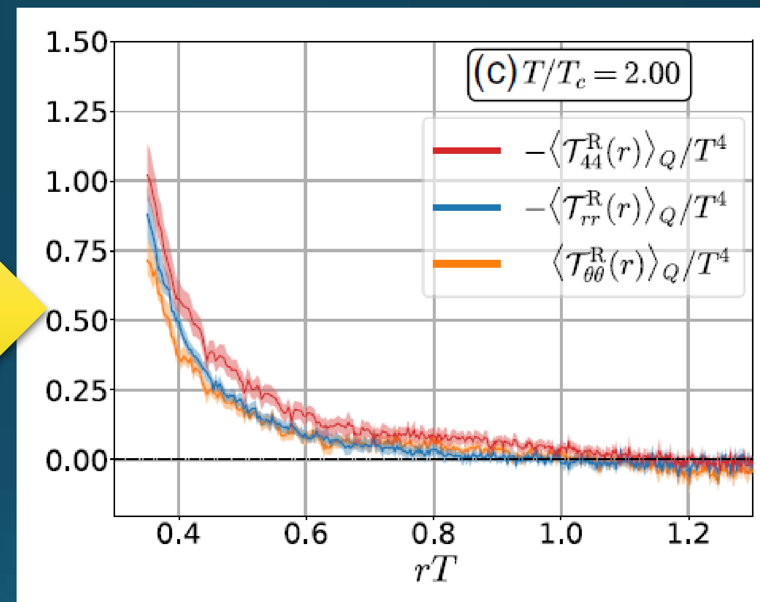
M. **Berwein**, private comm.

Perturbation



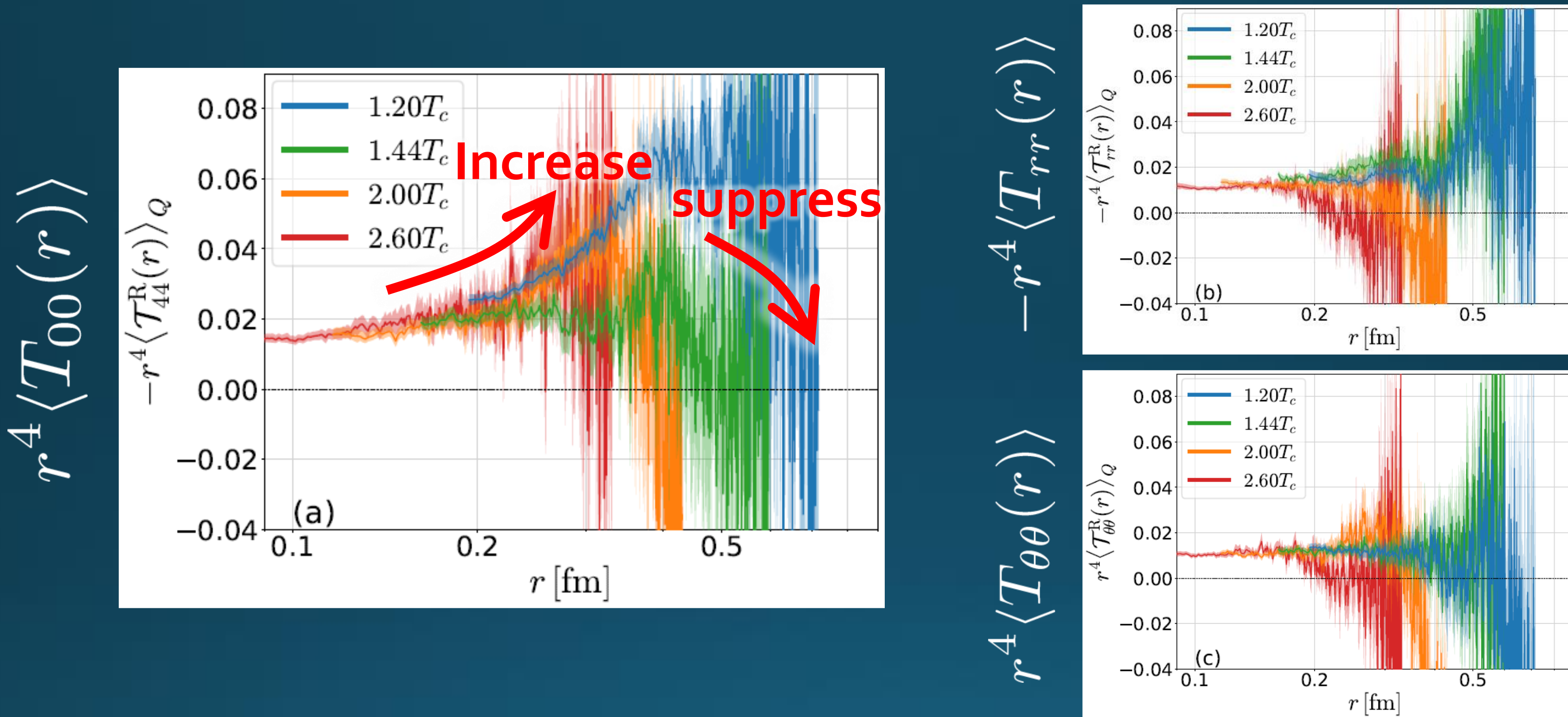
Perturbation:
Combination of
NLO pert. + NLO EQCD

Lattice



- ☐ $|T_{44}| > |T_{rr}|$ is reproduced by perturbation.
- ☐ Hierarchy of T_{rr} , $T_{\theta\theta}$ does not match?

r Dependence



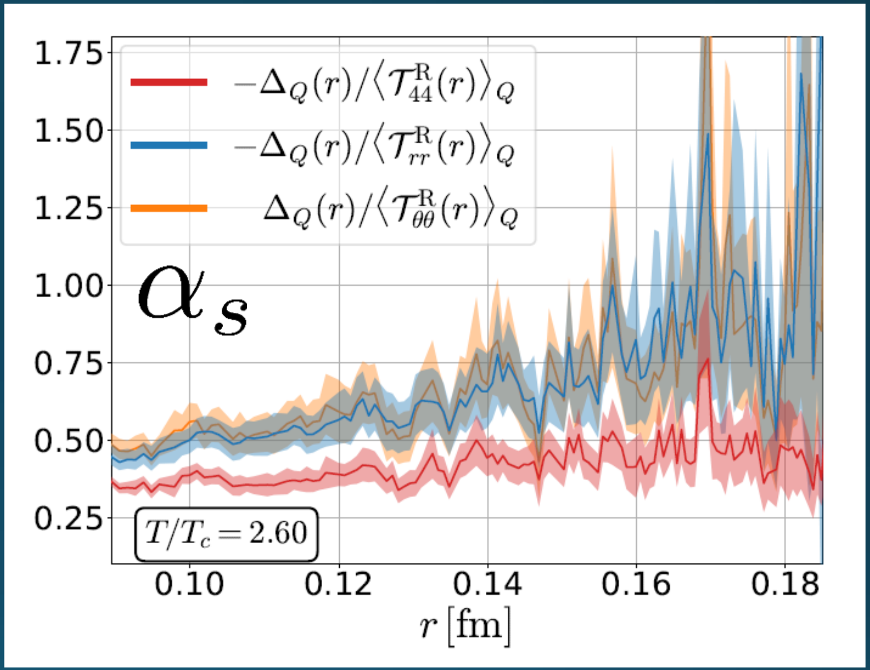
- Increase at short r / suppression at larger r
- T dependence is suppressed at $r < 1/T$
- Too noisy at large r for extracting screening mass m_{D}

Running Coupling

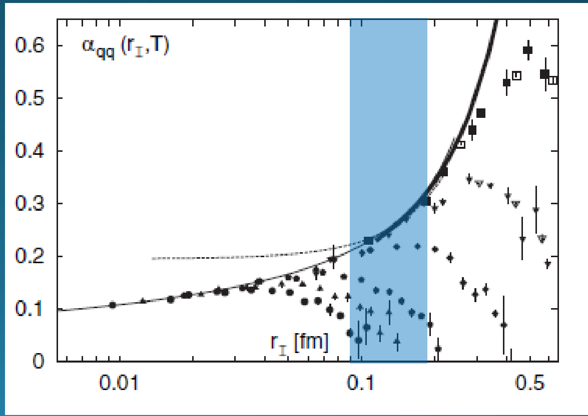
- **Estimate of α_s**

$$\left|\frac{\langle T_{\mu\mu}\rangle}{\langle \mathcal{T}_{44,rr,\theta\theta}(r)\rangle}\right| = \frac{11}{2\pi}\alpha_s + \mathcal{O}(g^3),$$

  - at the leading-order perturbation theory
  - channel dependent



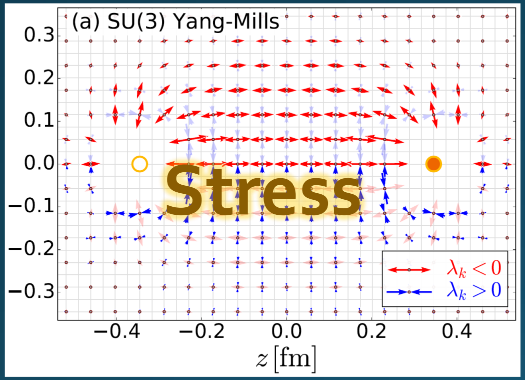
- All results are approximately consistent with the estimate from $Q\bar{Q}$ potential

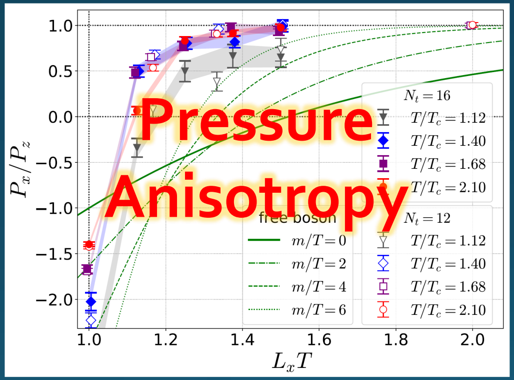


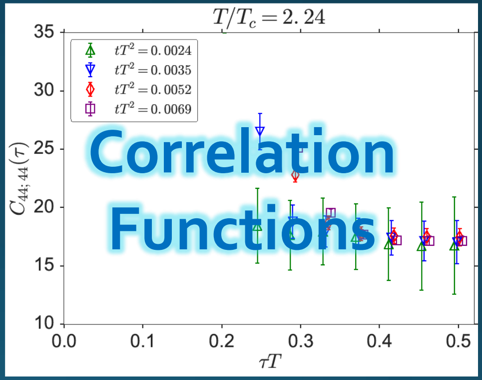
Kaczmarek, Karsch, Zantow, 2004

Summary

- ☐ Successful analysis of energy-momentum tensor on the lattice is now available, and various studies are ongoing!
  - ☐ SF*t*X (gradient flow) method
  - ☐ significant error suppression







- ☐ **So many future studies**
  - ☐ Flux tube at **nonzero temperature**
  - ☐ EMT distribution **inside hadrons**
  - ☐ viscosity / other operators / topology / **QCD**

backup

EMT on the Lattice: Conventional

Lattice EMT Operator

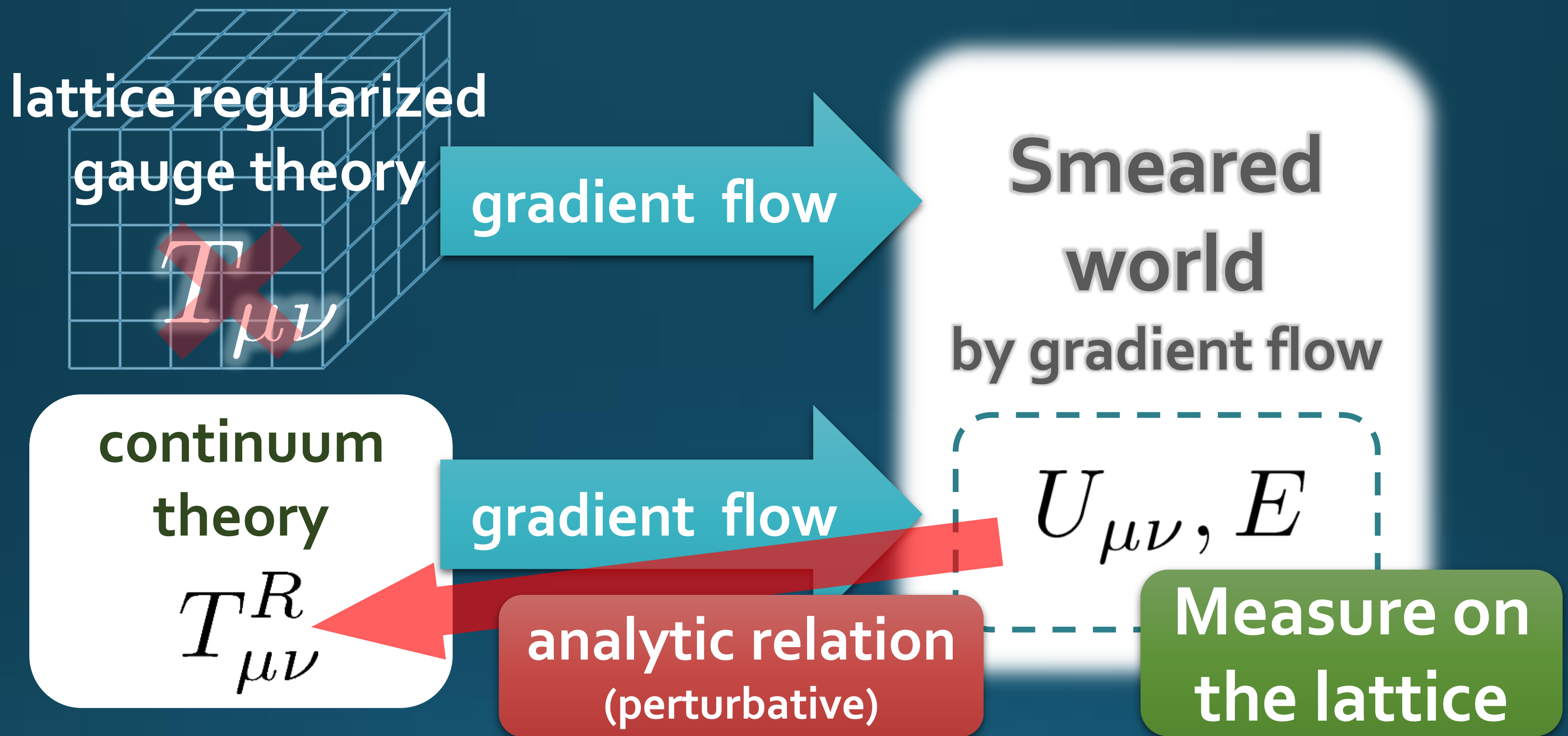
Caracciolo+, 1990

$$T_{\mu\nu} = Z_6 T^{[6]}_{\mu\nu} + Z_3 T^{[3]}_{\mu\nu} + Z_1 \left(T^{[1]}_{\mu\nu} - \langle T^{[1]}_{\mu\nu} \rangle \right)$$

$$T^{[6]}_{\mu\nu} = (1-\delta_{\mu\nu}) F^a_{\mu\rho} F^a_{\nu\rho}, \quad T^{[3]}_{\mu\nu} = \delta_{\mu\nu} \left(F^a_{\mu\rho} F^a_{\nu\rho} - \frac{1}{4} F^a_{\rho\sigma} F^a_{\rho\sigma} \right), \quad T^{[1]}_{\mu\nu} = \delta_{\mu\nu} F^a_{\rho\sigma} F^a_{\rho\sigma}$$

- **Determination of Zs**
  - Fit to thermodynamics: Z_3, Z_1 — Giusti, Meyer, 2011; 2013;
  - Shifted-boundary method: Z_6, Z_3 — Giusti, Pepe, 2014~; Borsanyi+, 2018
  - Full QCD with fermions — Brida, Giusti, Pepe, 2020

SF*t*X Method



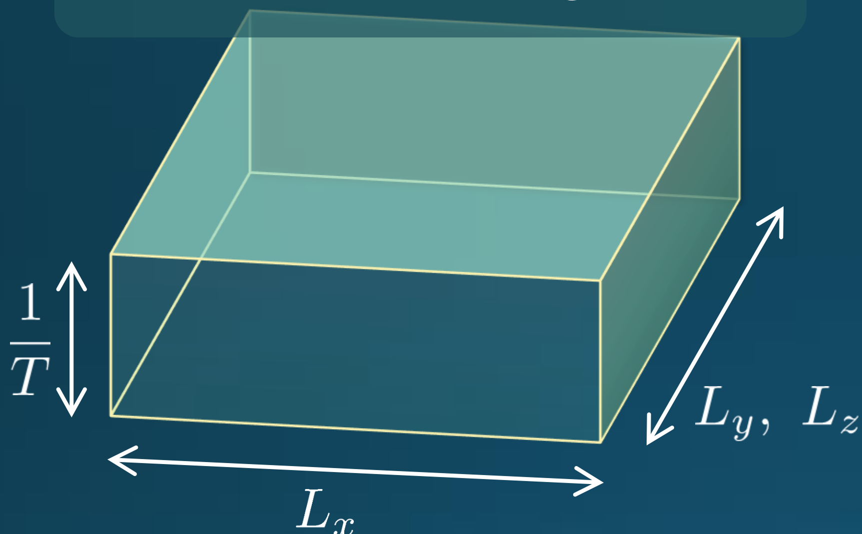
Take Extrapolation (t,a)→(0,0)

$$\langle T_{\mu\nu}(t)\rangle_{\mathrm{latt}} = \langle T_{\mu\nu}(t)\rangle_{\mathrm{phys}} + \boxed{C_{\mu\nu}t} + \boxed{D_{\mu\nu}\frac{a^2}{t}} + \cdots$$

O(t) terms in SFTE lattice discretization

Two Special Cases with PBC

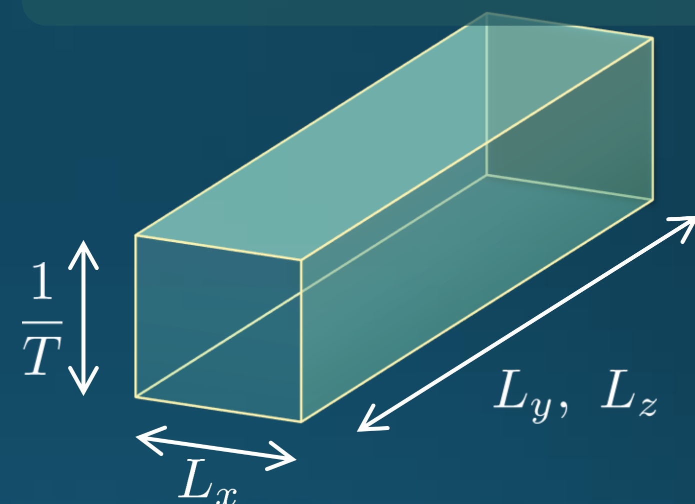
$$1/T \ll L_x = L_y = L_z$$



$$T_{11} = T_{22} = T_{33}$$

$$\frac{p_1}{p_2} = 1$$

$$1/T = L_x,\ L_y = L_z$$



$$T_{44} = T_{11},\ T_{22} = T_{33}$$

In conformal ($\Sigma_\mu T_{\mu\mu}=0$)

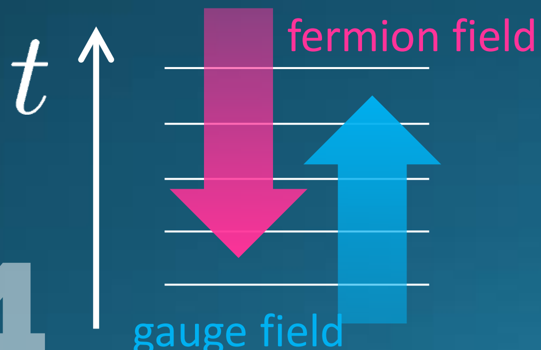
$$\frac{p_1}{p_2} = -1$$

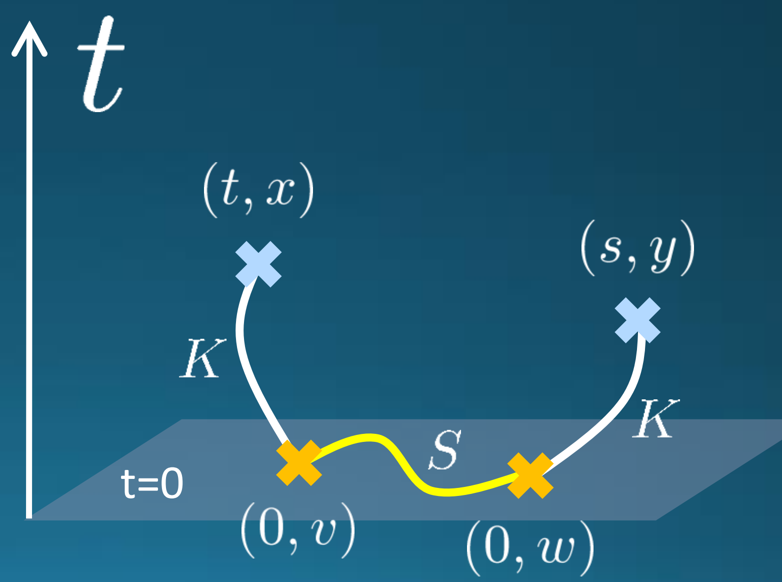
Fermion Propagator

$$S(t,x;s,y) = \langle \chi(t,x)\bar{\chi}(s,y)\rangle$$
$$= \sum_{v,w} K(t,x;0,v)S(v,w)K(s,y;0,w)^\dagger$$

$$\left(\partial_t - D_\mu D_\mu\right)K(t,x) = 0$$

- propagator of flow equation
- Inverse propagator is needed





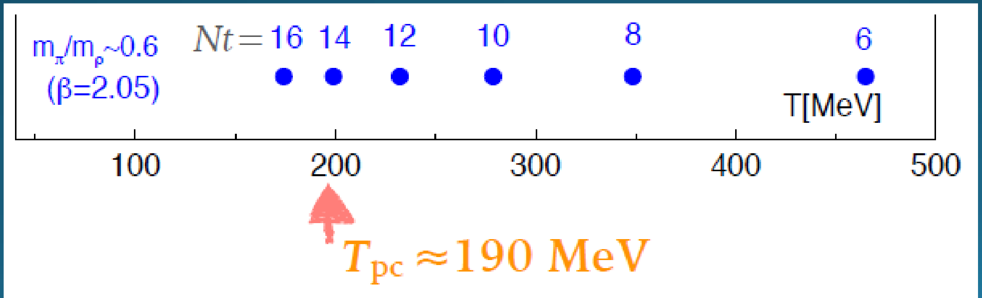
N_f=2+1 QCD Thermodynamics

Taniguchi+ (WHOT-QCD),
PRD**96**, 014509 (2017)

- N_f=2+1 QCD, Iwasaki gauge + NP-clover
- m_{PS}/m_V ≈0.63 / almost physical s quark mass

- T=0: CP-PACS+JLQCD (ß=2.05, 28^3x56, a≈0.07fm)
- T>0: 32^3xN_t, N_t = 4, 6, … , 14, 16):
- T≈174-697MeV

- t→0 extrapolation only (No continuum limit)



Numerical Setup

- SU(3) YM theory
- Wilson gauge action

- N_t = 16, 12
- N_z/N_t=6
- 2000~4000 confs.
- Even N_x

- No Continuum extrap.

T/T_c	β	N_z	N_τ	N_x	$N_{\rm vac}$
1.12	6.418	72	12	12, 14, 16, 18	64
	6.631	96	16	16, 18, 20, 22, 24	96
1.40	6.582	72	12	12, 14, 16, 18	64
	6.800	96	16	16, 18, 20, 22, 24	128
1.68	6.719	72	12	12, 14, 16, 18, 24	64
	6.719	96	12	14, 18	64
	6.941	96	16	16, 18, 20, 22, 24	96
2.10	6.891	72	12	12, 14, 16, 18, 24	72
	7.117	96	16	16, 18, 20, 22, 24	128
2.69	7.086	72	12	12, 14, 16, 18	-
$\simeq 8.1$	8.0	72	12	12, 14, 16, 18	-
$\simeq 25$	9.0	72	12	12, 14, 16, 18	-

- Same Spatial volume
  - $12 \times 72^2 \times 12 \sim 16 \times 96^2 \times 16$
  - $18 \times 72^2 \times 12 \sim 24 \times 96^2 \times 16$

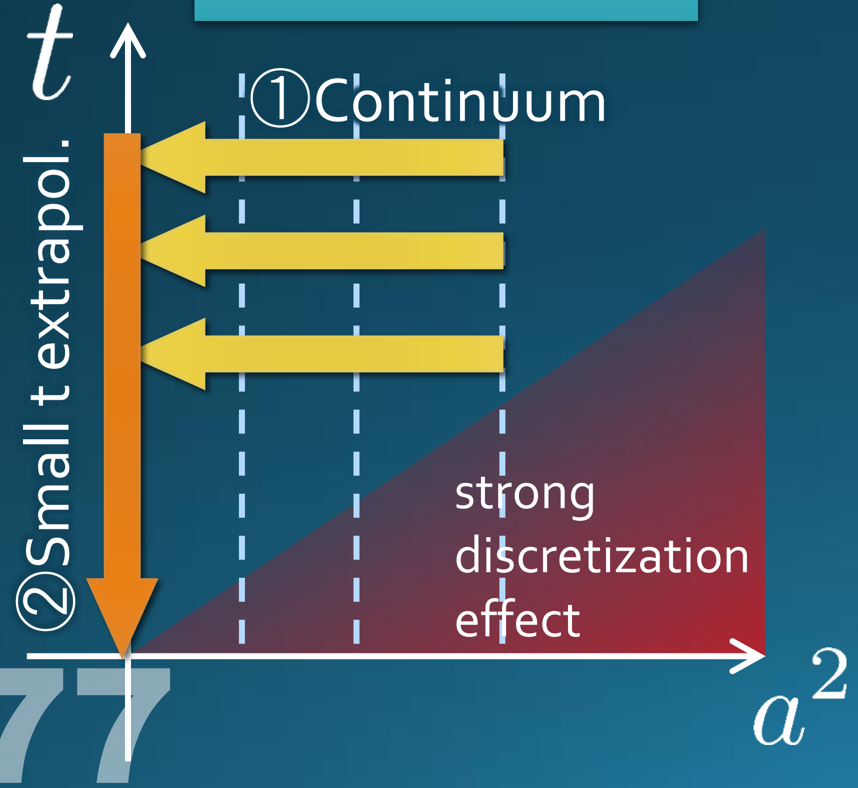
Simulations on
OCTOPUS/Reedbush

Extrapolations t→0, a→0

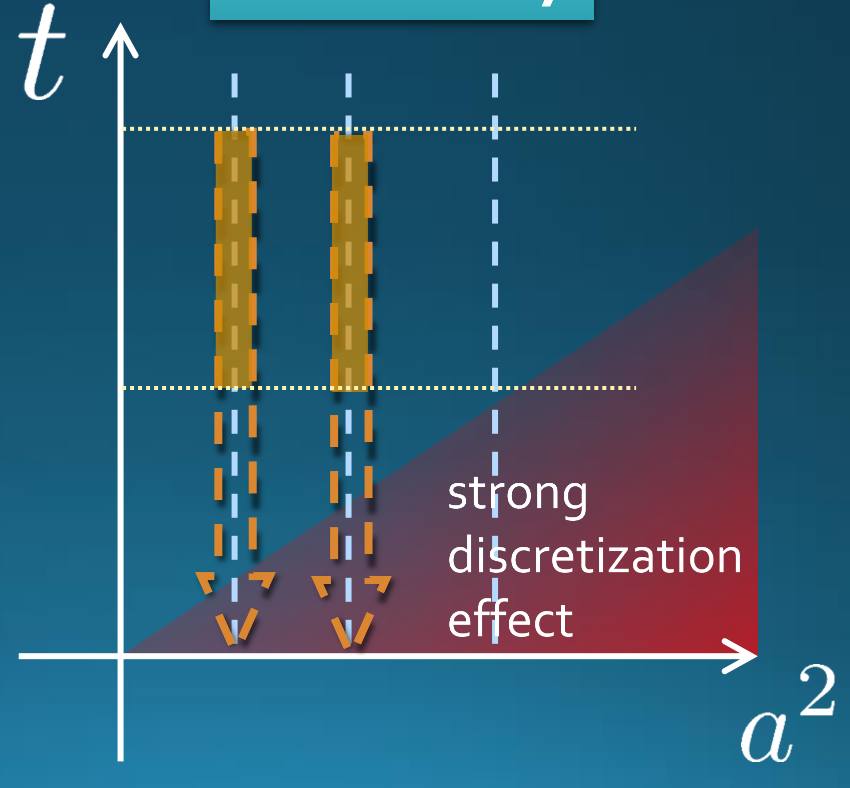
$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + \boxed{C_{\mu\nu}t} + \boxed{D_{\mu\nu}(t)\frac{a^2}{t}}$$

O(t) terms in SFTE lattice discretization

FlowQCD2016

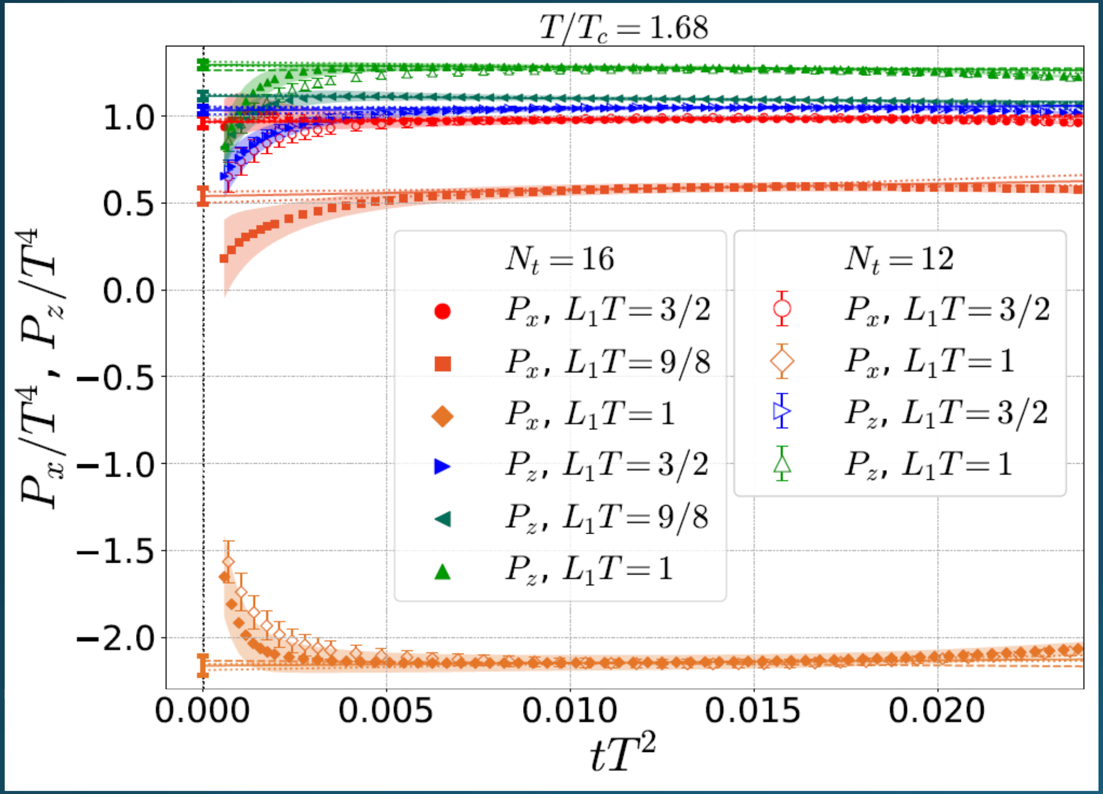


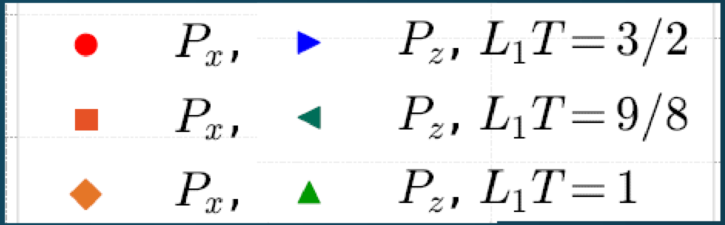
This Study



Small-t Extrapolation

$T/T_c = 1.68$





Filled: N_t=16 / Open: N_t=12

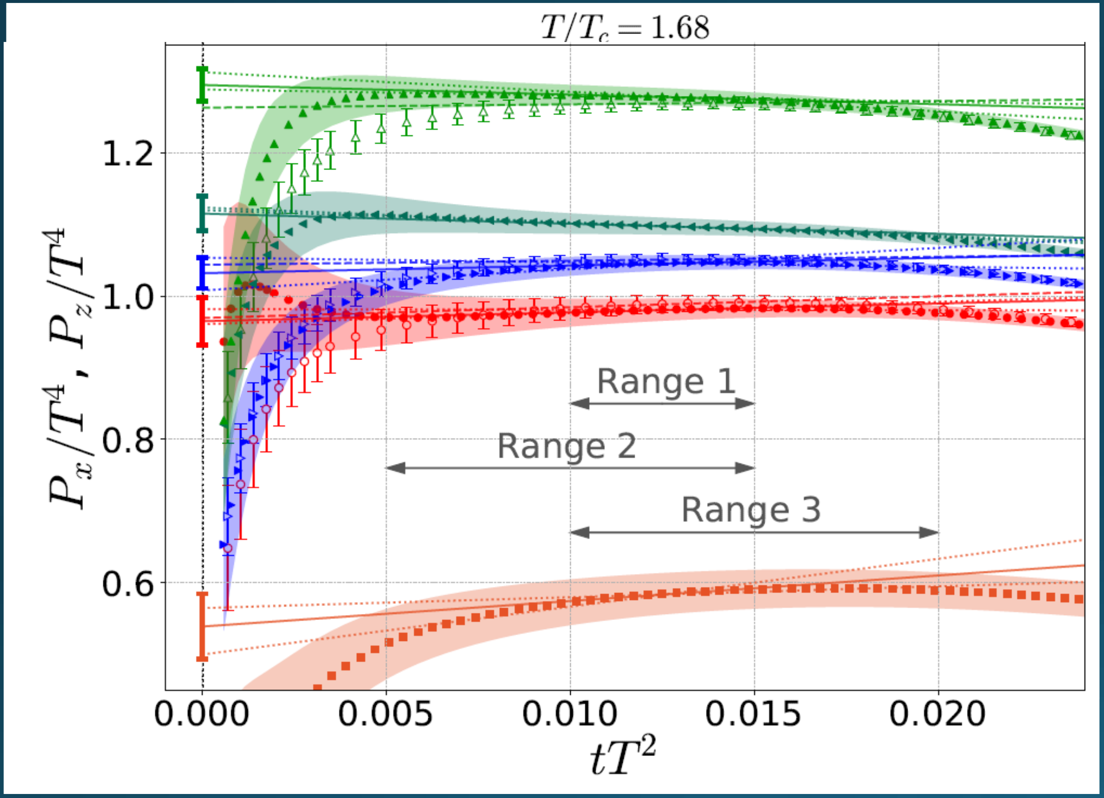
Small-t extrapolation

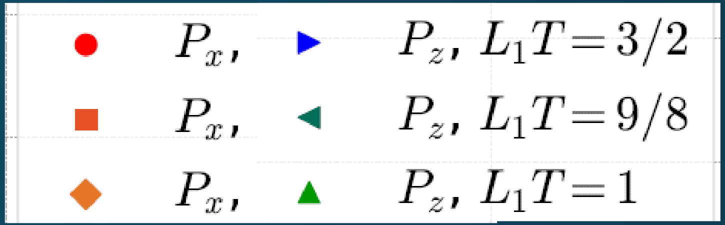
- Solid: N_t=16, Range-1
- Dotted: N_t=16, Range-2,3
- Dashed: N_t=12, Range-1

- ☐ Stable small-t extrapolation
- ☐ No N_t dependence within statistics for L_xT=1, 1.5

Small-t Extrapolation

$T/T_c = 1.68$





Filled: N_t=16 / Open: N_t=12

Small-t extrapolation

- Solid: N_t=16, Range-1
- Dotted: N_t=16, Range-2,3
- Dashed: N_t=12, Range-1

☐ Stable small-t extrapolation

☐ No N_t dependence within statistics for L_xT=1, 1.5

Lattice Setup

FlowQCD, PLB (2019)

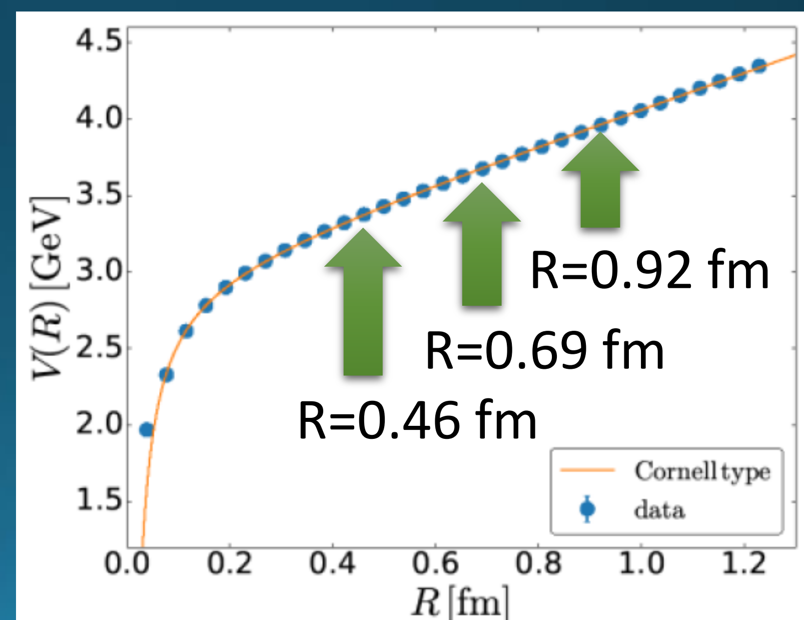
- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

- EMT around Wilson Loop
- APE smearing / multi-hit

- fine lattices (a=0.029-0.06 fm)
- continuum extrapolation

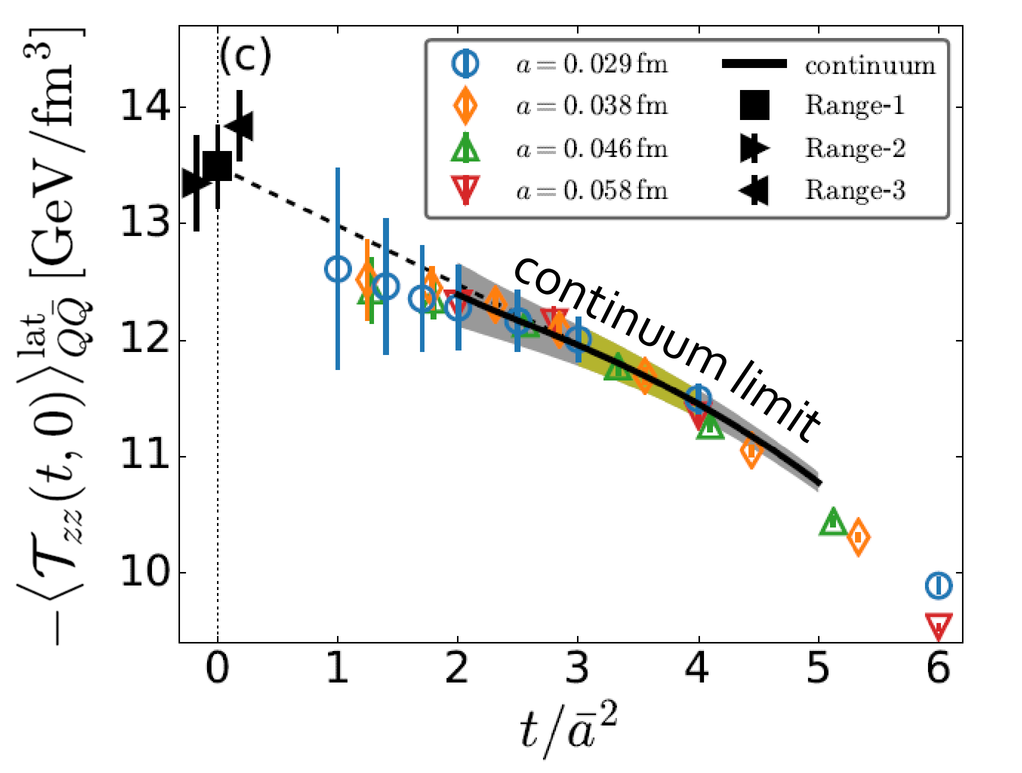
- Simulation: bluegene/Q@KEK

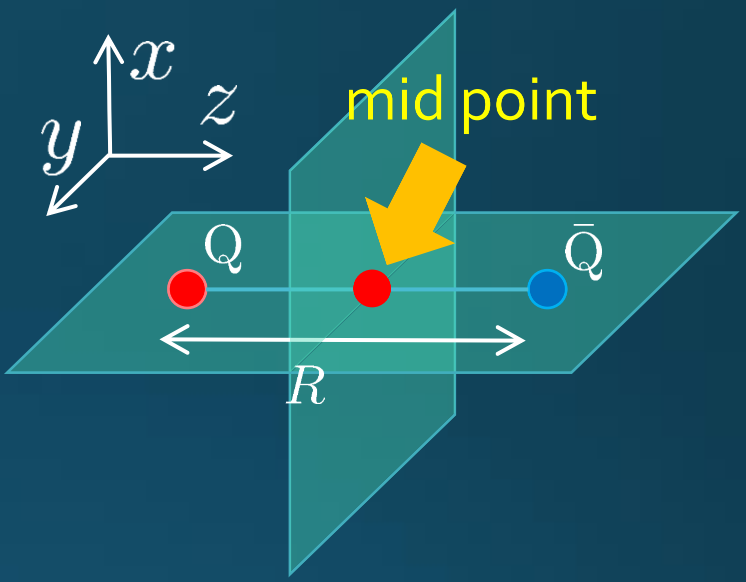
β	a [fm]	$N_{\rm size}^4$	$N_{\rm conf}$	R/a		
6.304	0.058	48^4	140	8	12	16
6.465	0.046	48^4	440	10	–	20
6.513	0.043	48^4	600	–	16	–
6.600	0.038	48^4	1,500	12	18	24
6.819	0.029	64^4	1,000	16	24	32
			R [fm]	0.46	0.69	0.92

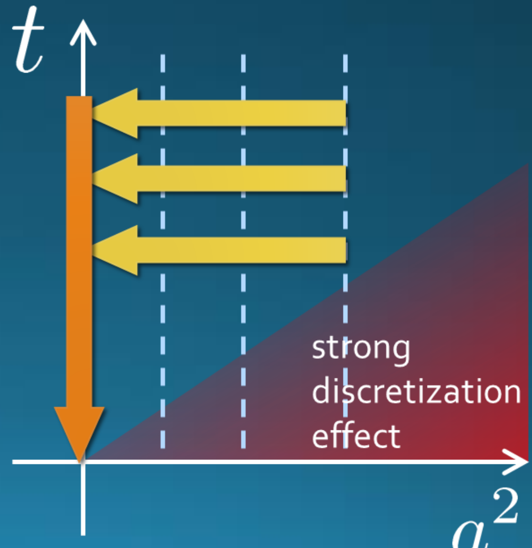


$$\langle O(x)\rangle_{Q\bar{Q}} = \lim_{T\to\infty} \frac{\langle \delta O(x)\delta W(R,T)\rangle}{\langle W(R,T)\rangle}$$

Continuum Extrapolation
at mid-point

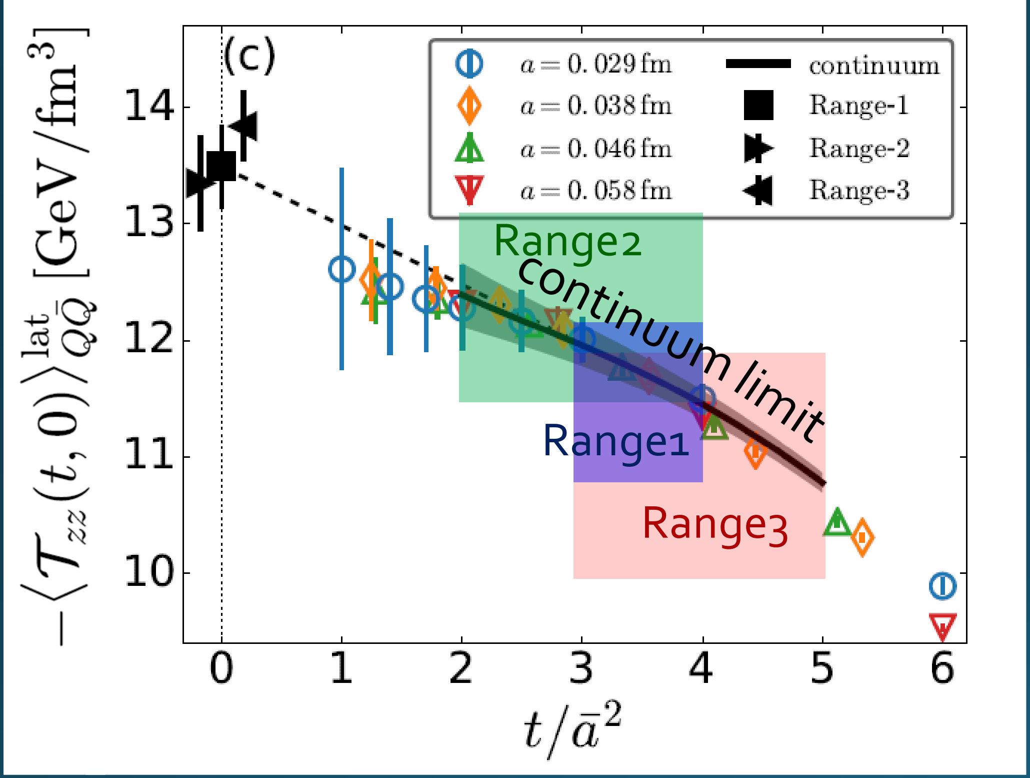


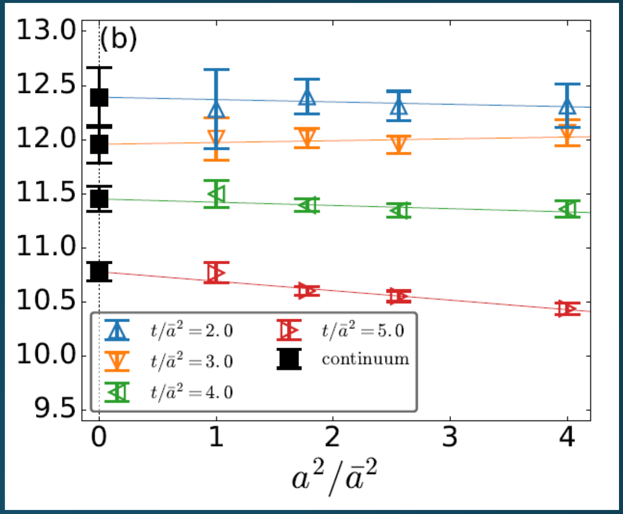


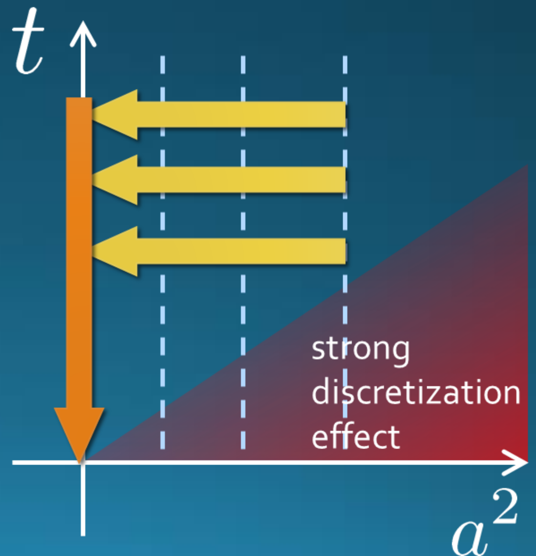


- ☐ a→0 extrapolation with fixed t

t→0 Extrapolation
at mid-point







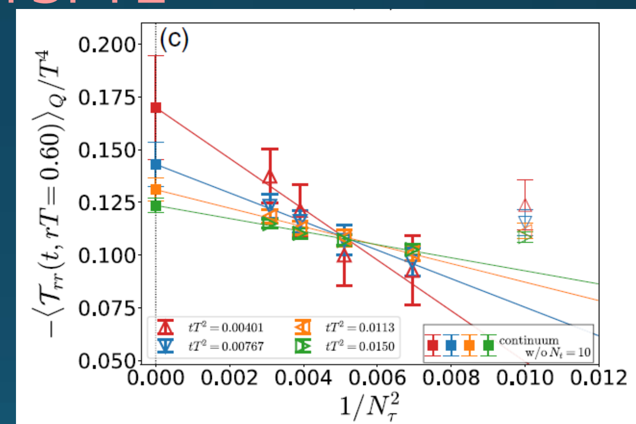
- ☐ a→0 extrapolation with fixed t
- ☐ Then, t→0 with three ranges

Double Extrapolation

$$t \to 0, a \to 0$$

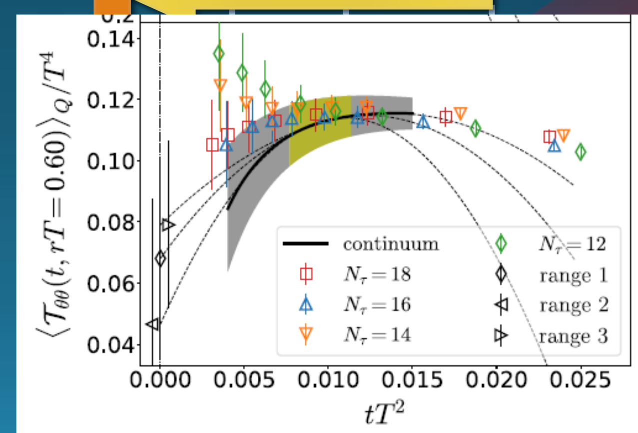
$$\langle T_{\mu\nu}(t)\rangle_{\text{latt}} = \langle T_{\mu\nu}(t)\rangle_{\text{phys}} + \boxed{C_{\mu\nu}t} + \boxed{D_{\mu\nu}(t)\frac{a^2}{t}}$$

O(t) terms in SFTE lattice discretization



Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\text{cont}} = \langle T_{\mu\nu}(t)\rangle_{\text{lat}} + C(t)a^2$$



Small t extrapolation

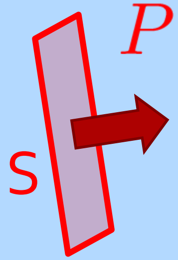
$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$

tion

a^2

Stress = Force per Unit Area

Pressure

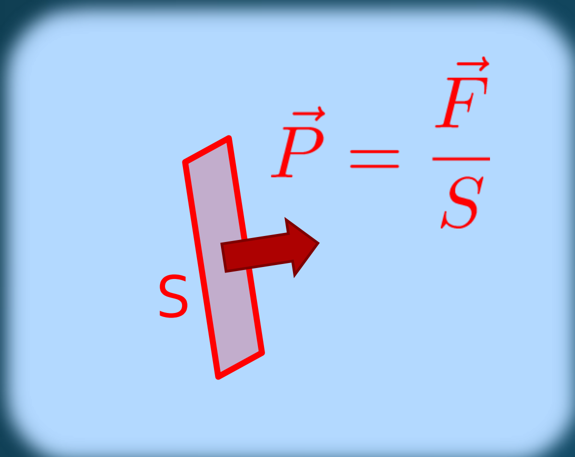


$$\vec{P} = \frac{\vec{F}}{S}$$

$$\vec{P} = P\vec{n}$$

Stress = Force per Unit Area

Pressure

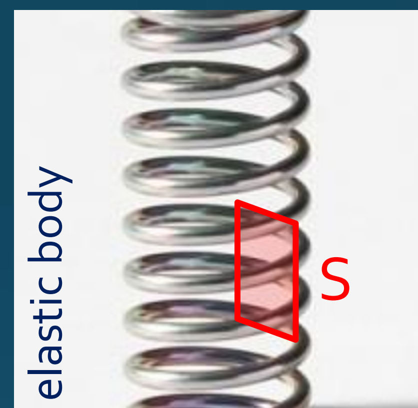


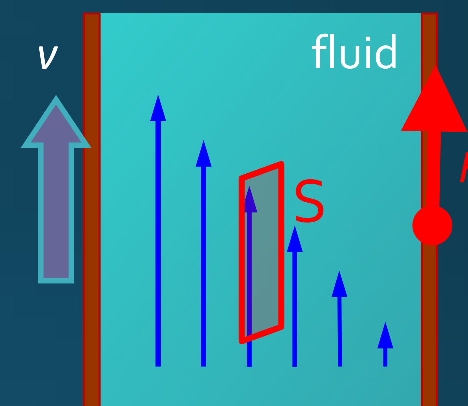
$$\vec{P} = P\vec{n}$$

In thermal medium

$$T_{ij} = P\delta_{ij}$$

Generally, F and n are not parallel





$$\frac{F_i}{S} = \boxed{\sigma_{ij}} n_j$$

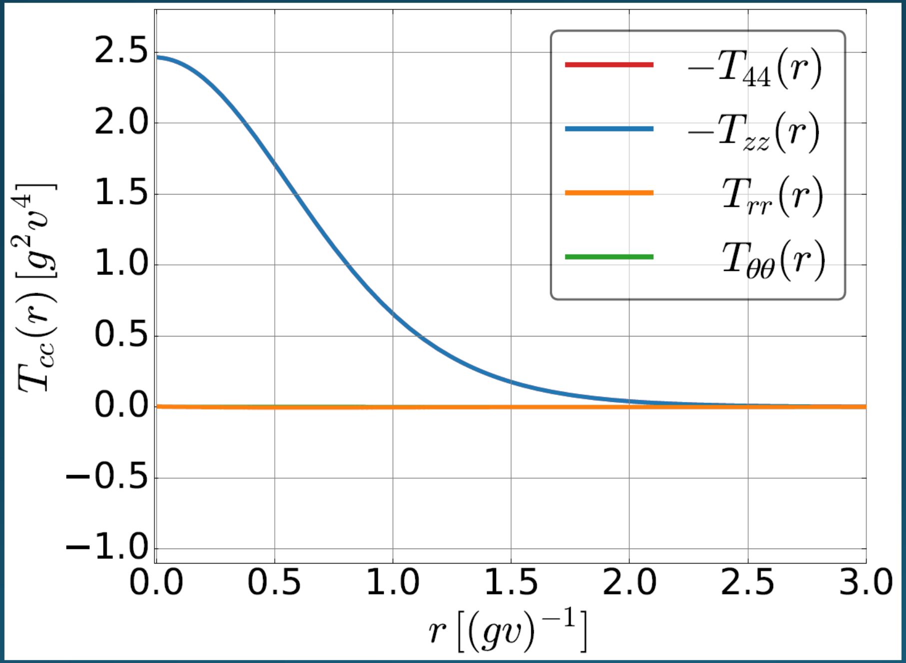
Stress Tensor

$$\sigma_{ij} = -T_{ij}$$

Landau
Lifshitz

Stress Tensor in AH Model
infinitely-long flux tube

Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$



$$T_{rr} = T_{\theta\theta} = 0$$

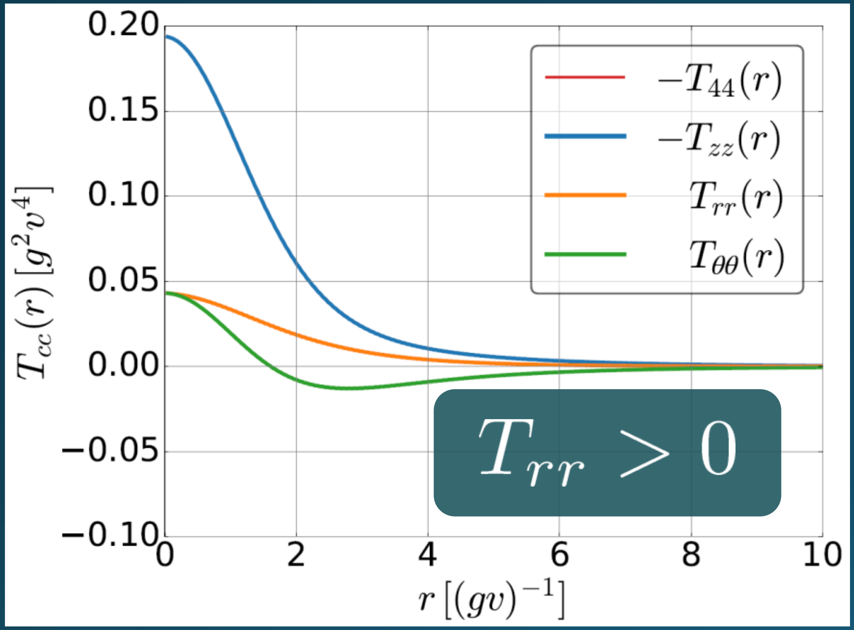
de Vega, Schaposnik, PR**D14**, 1100 (1976).

Stress Tensor in AH Model

infinitely-long flux tube

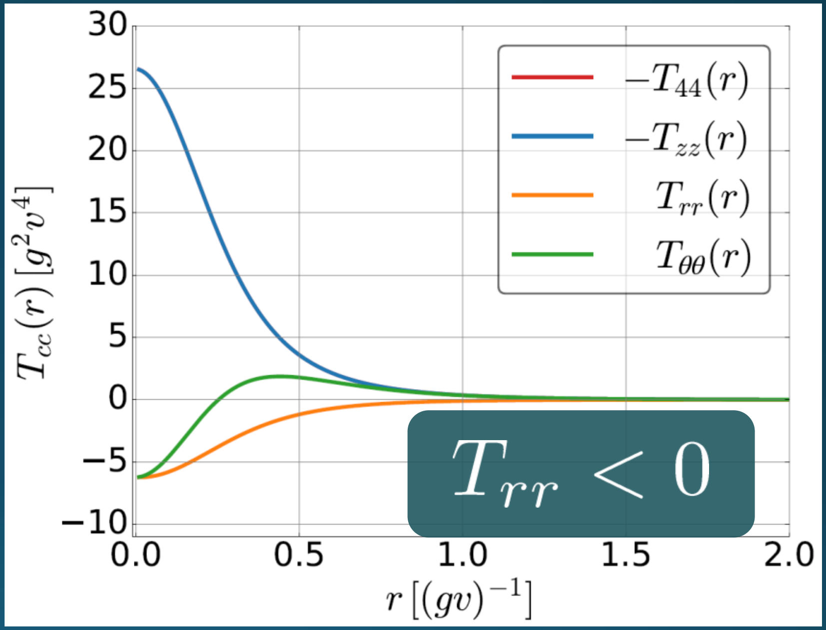
Type-I

$\kappa = 0.1$



Type-II

$\kappa = 3.0$



- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

conservation law

$$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$