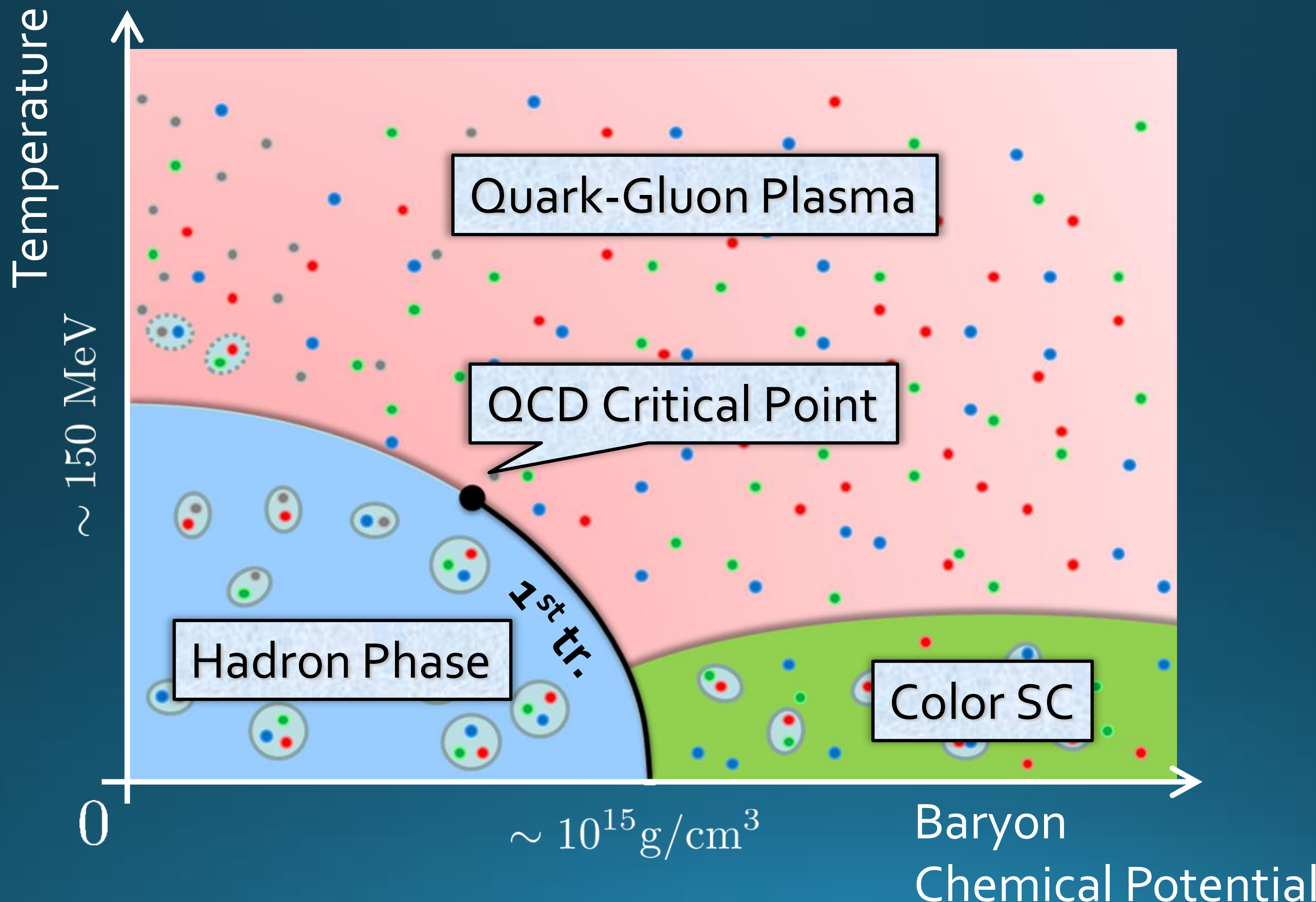
From Lattice to Observables

Masakiyo Kitazawa
(Osaka U.)

The 8th Asian Triangle Heavy-Ion Conference (ATHIC2021), 2021/11/5, Inha U., Incheon (S. Korea)

QCD Phase Diagram



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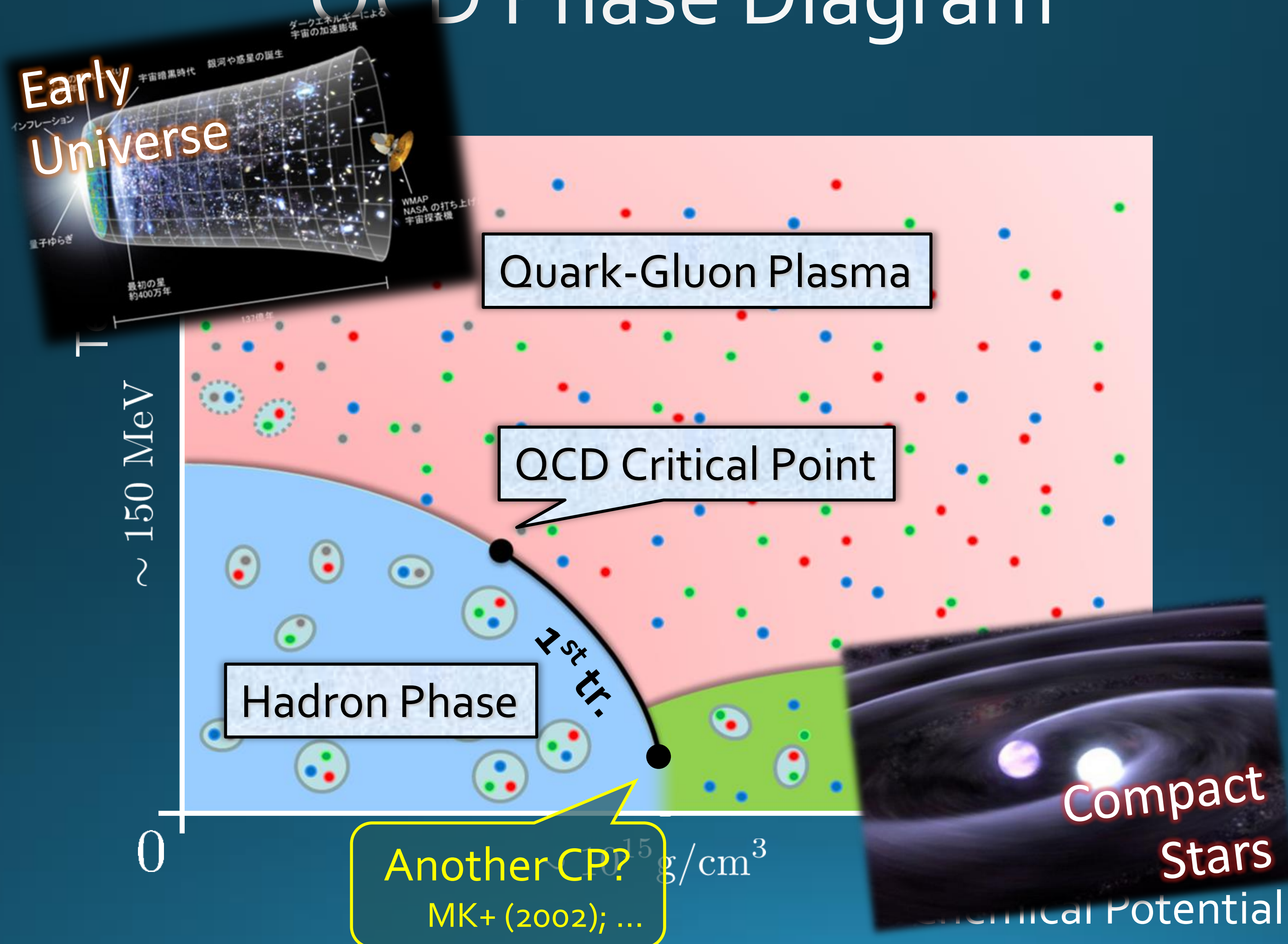
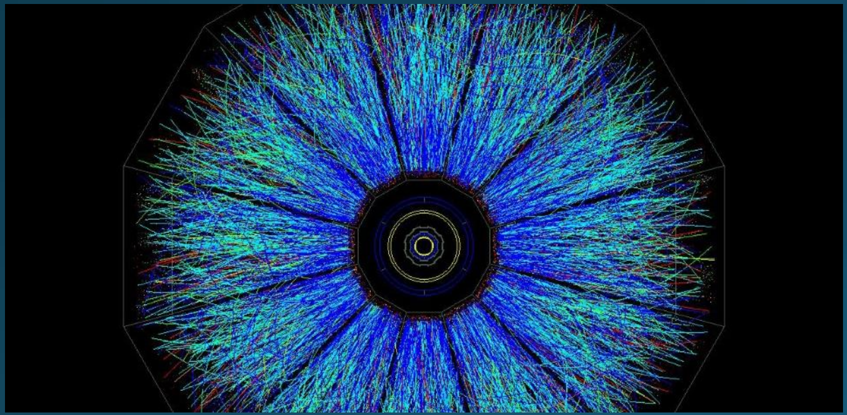


Two "Experimental" Tools
to explore hot & dense medium

Relativistic Heavy-Ion Collisions

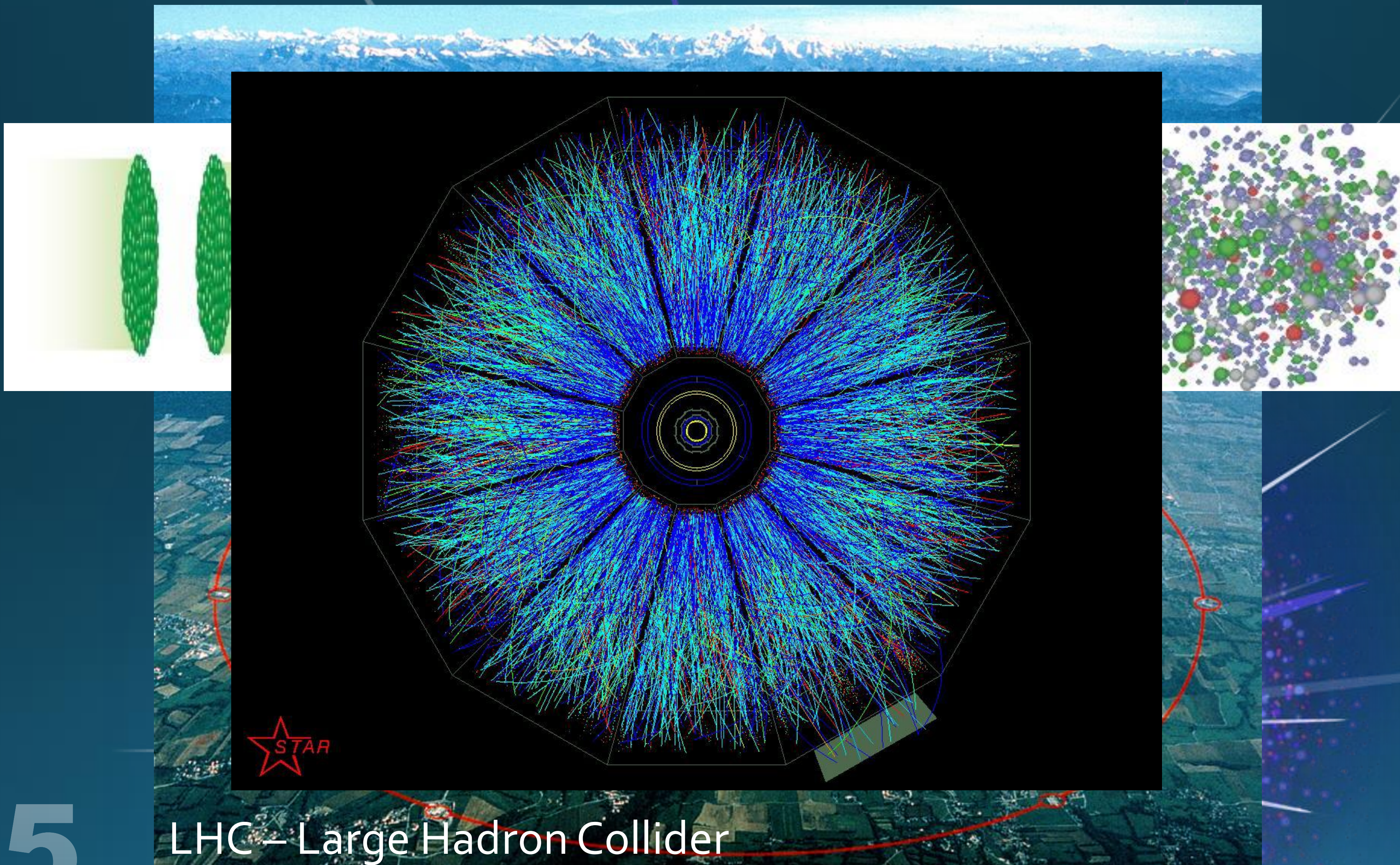


Real Experiment

Lattice QCD Numerical Simulations

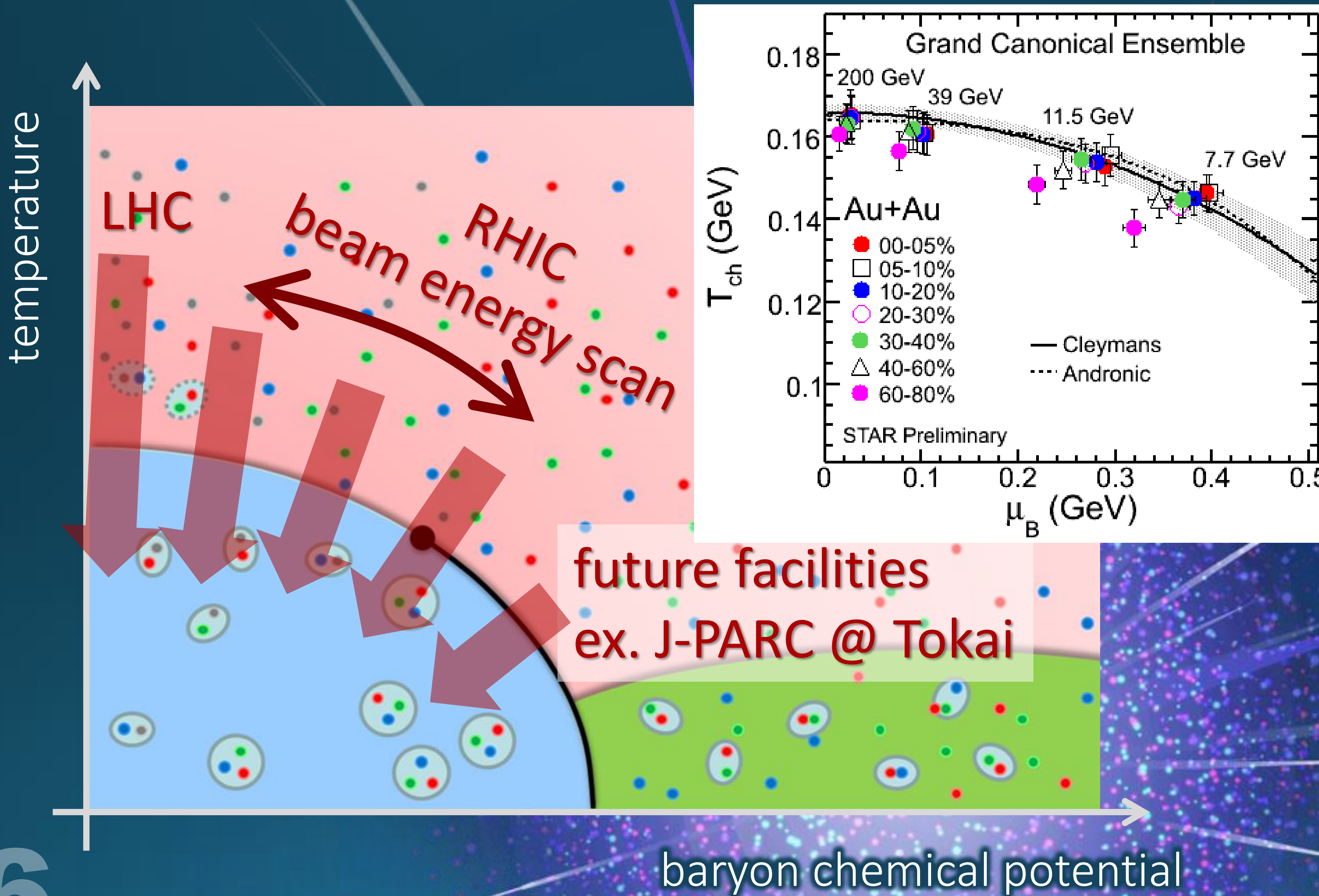
Virtual Experiment

Relativistic Heavy-Ion Collisions

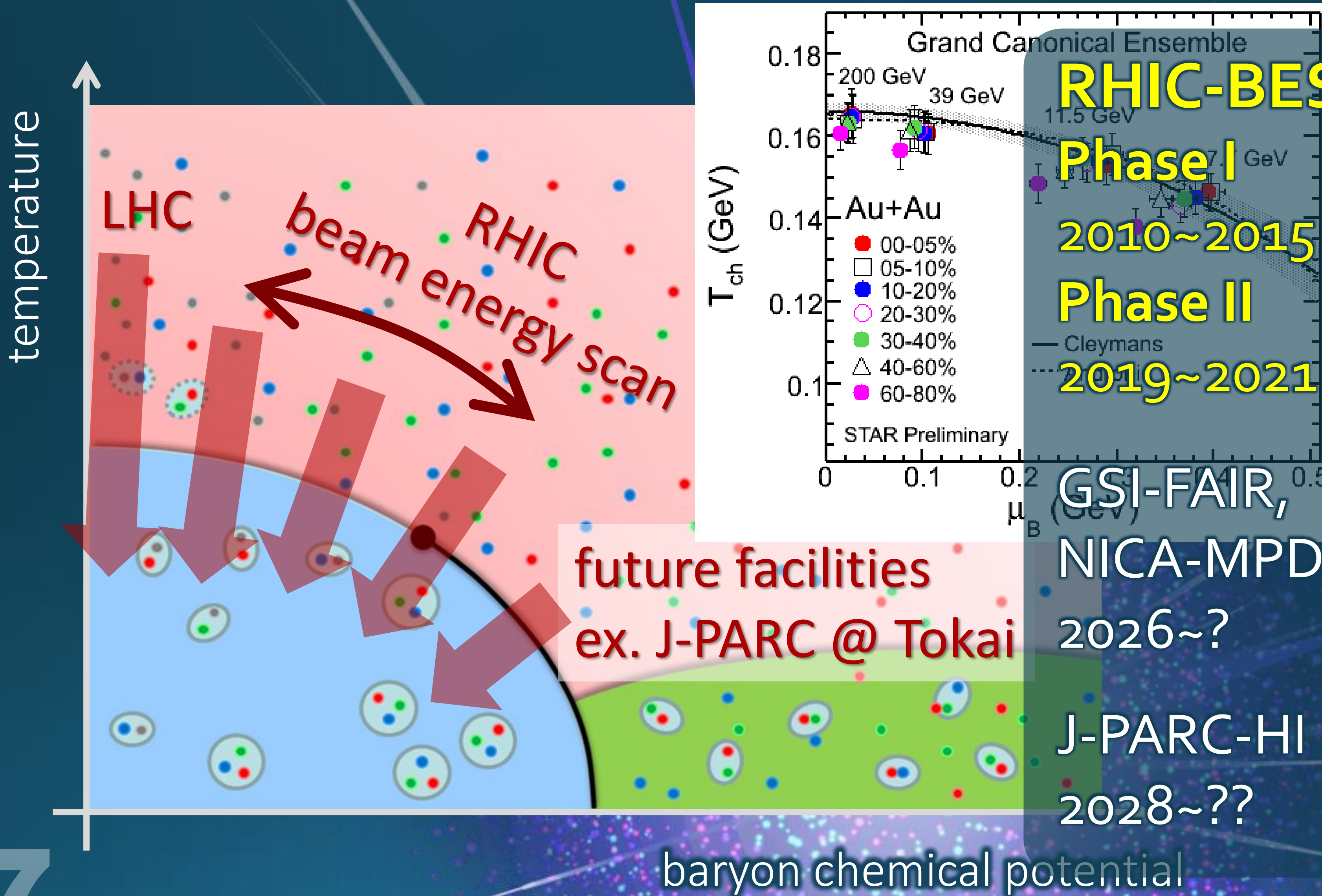


LHC – Large Hadron Collider

Beam-Energy Scan



Beam-Energy Scan



RHIC-BES

Phase I

2010~2015

Phase II

2019~2021

GSI-FAIR,

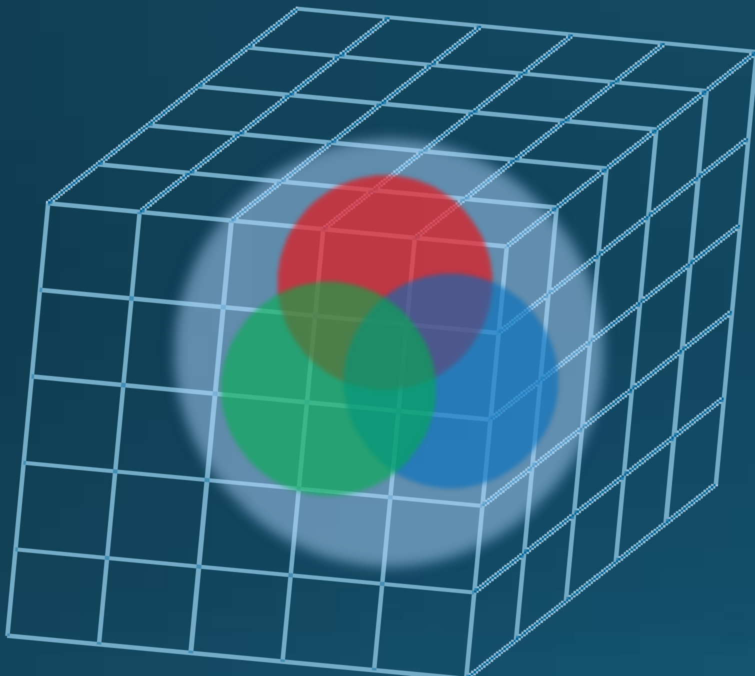
NICA-MPD

2026~?

J-PARC-HI

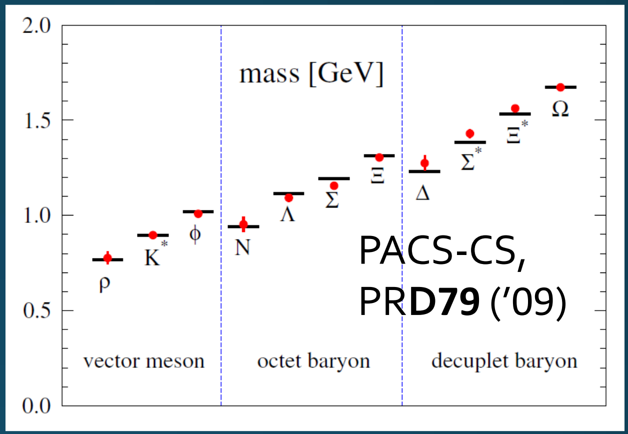
2028~??

Lattice QCD Numerical Simulations

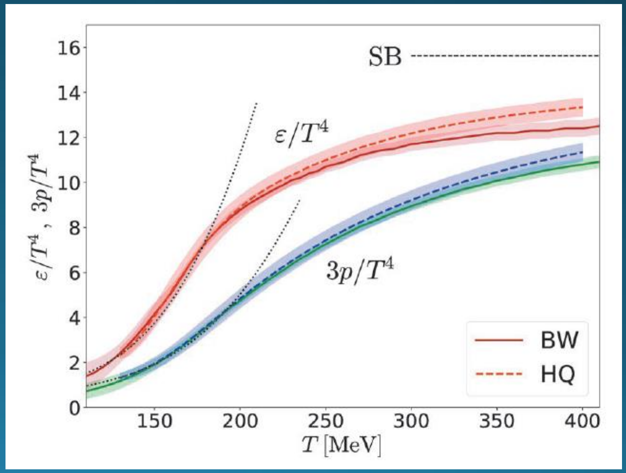


Unique tool to perform **quantitative** analyses of **non-perturbative** QCD aspects

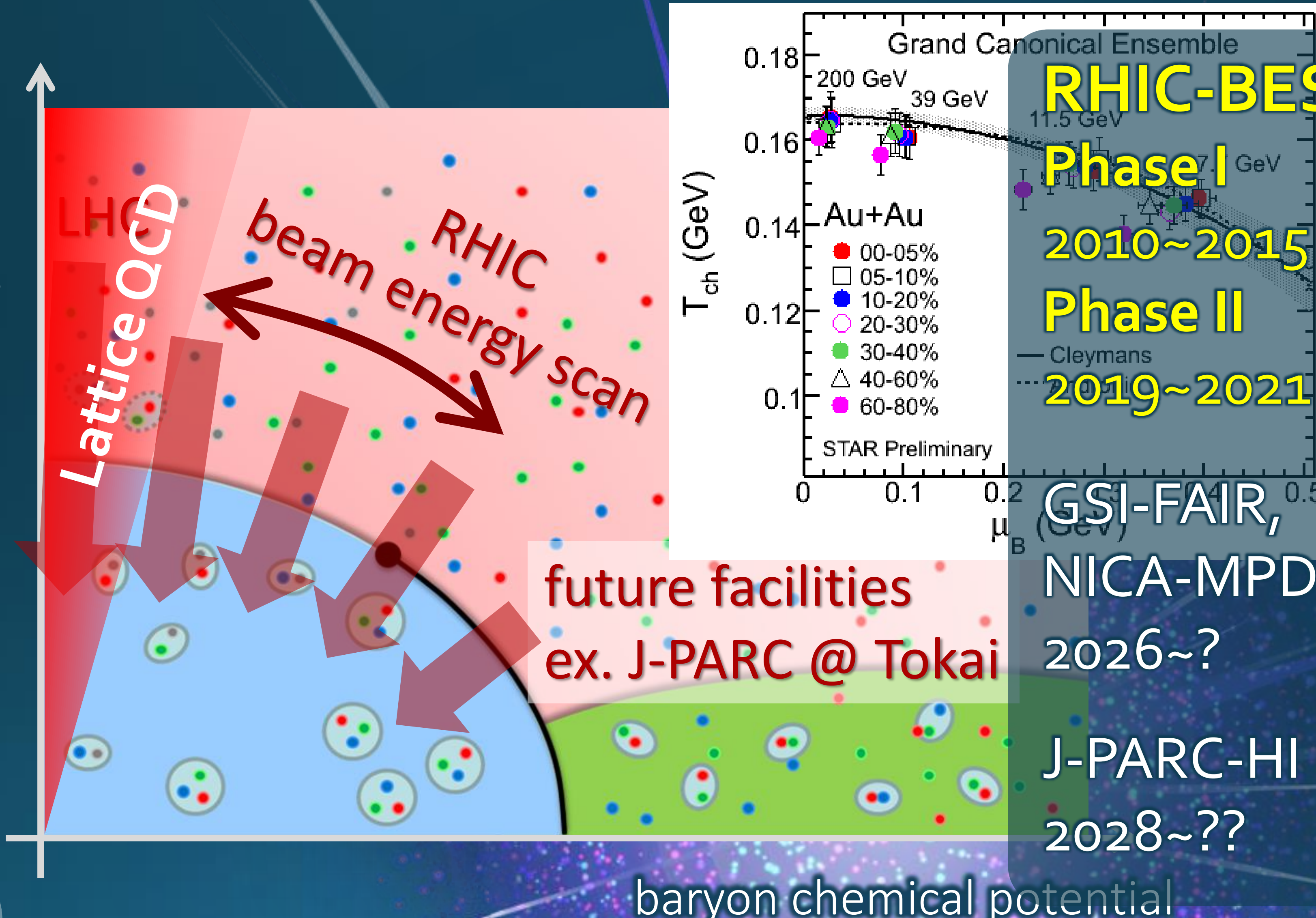
- Hadron Spectroscopy



- Thermodynamics



Beam-Energy Scan



RHIC-BES

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Usage of Lattice for HIC

- ☐ (Pseudo) critical temperature T_c
- ☐ Equations of state
- ☐ Fluctuations of conserved charges
- ☐ Dissociation of quarkonia
- ☐ Transport coefficients

HIC vs Lattice: Pros & Cons

Real experiments	⇔	Virtual, but unphysical params
Zero~high baryon density	⇔	Small baryon density only
Dynamical evolution	⇔	Ideal thermal system
Final-state observables only		Limited observables

Complementary use of both exps. is important!

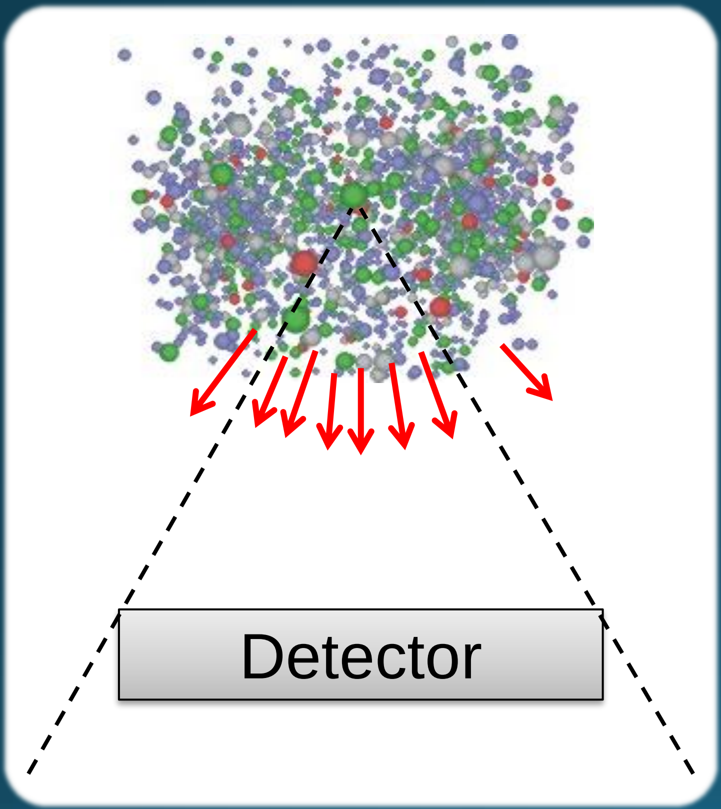
Three Applications

1. Fluctuations of Conserved Charges

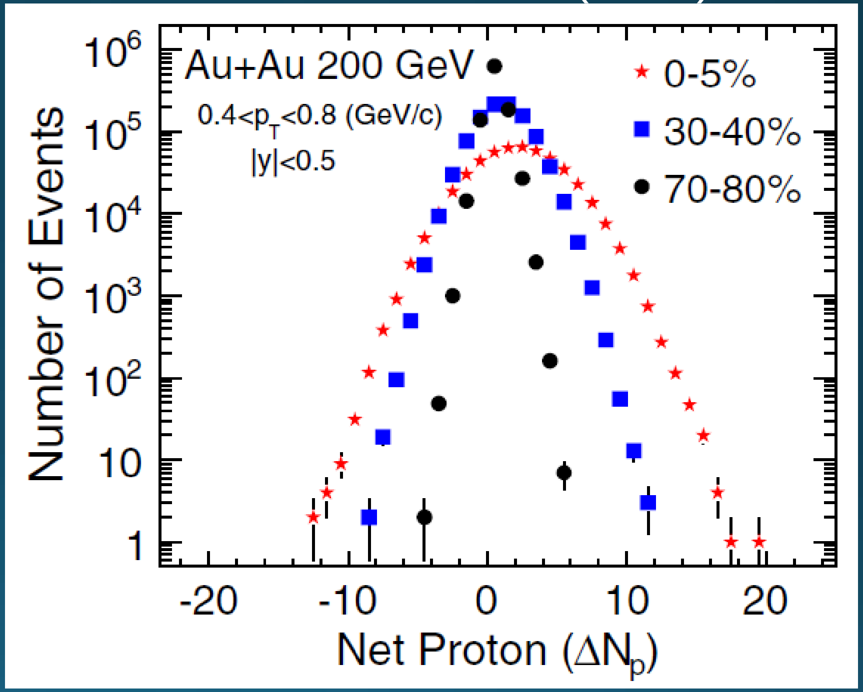
2. Finite-volume Effects
   in anisotropic systems

3. Finite-size Scaling around QCD-CP

Event-by-event Fluctuations



STAR, PRL**105** (2010)



Cumulants

$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$

Cumulants

Cumulants

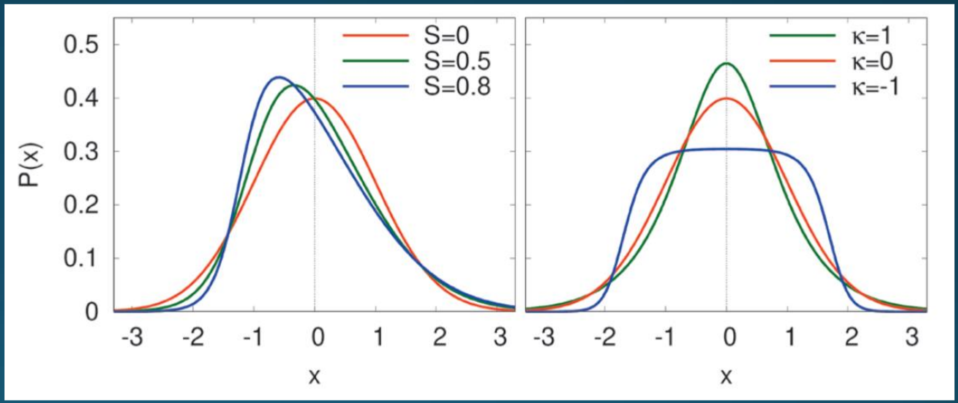
$$\begin{cases} \langle N \rangle_c = \langle N \rangle & \text{average} \\ \langle N^2 \rangle_c = \langle \delta N^2 \rangle & \text{variance} \\ \langle N^3 \rangle_c = \langle \delta N^3 \rangle & \\ \langle N^4 \rangle_c = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2 & \end{cases}$$

☐ skewness

$$S = \frac{\langle N^3 \rangle_c}{\langle N^2 \rangle_c^{3/2}}$$

☐ kurtosis

$$\kappa = \frac{\langle N^4 \rangle_c}{\langle N^2 \rangle_c^2}$$



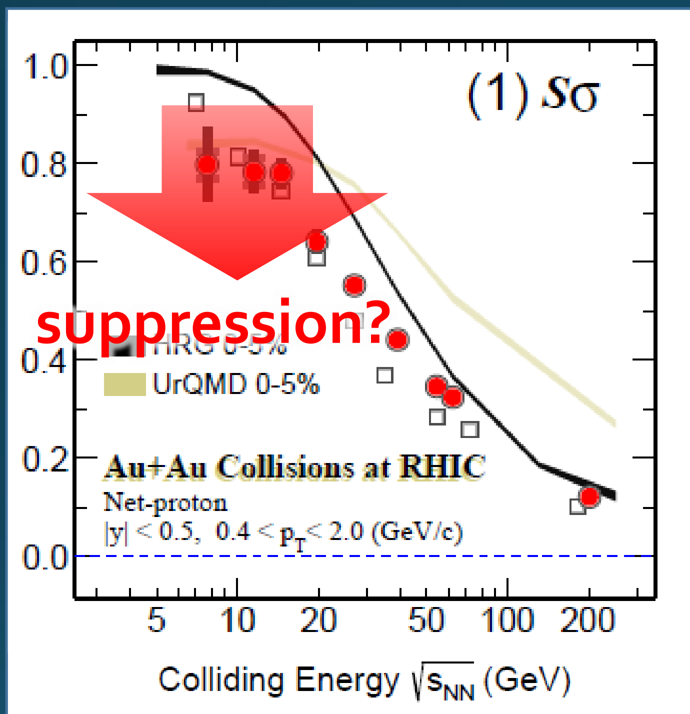
☐ NOTE

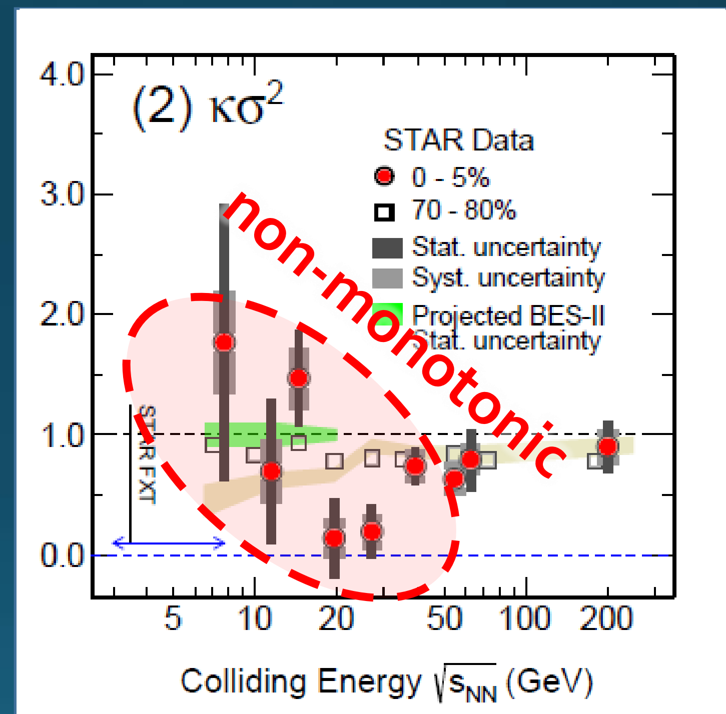
- Gauss distribution: $\langle N^3 \rangle_c = \langle N^4 \rangle_c = \cdots = 0$
- Poisson distribution: $\langle N^2 \rangle_c = \langle N^3 \rangle_c = \langle N^4 \rangle_c = \cdots = \langle N \rangle$

Review: Asakawa, MK, PPNP **90** (2016)

Proton Number Cumulants

$\langle N_p^3 \rangle_c / \langle N_p^2 \rangle_c$ $\langle N_p^4 \rangle_c / \langle N_p^2 \rangle_c$





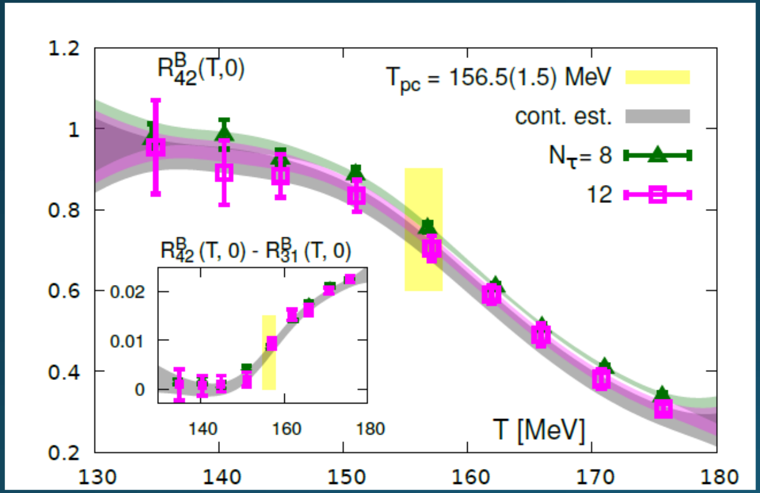
STAR, PRC 2020 [2001.06419]

- ☐ **Nonzero and non-Poissonian cumulants are experimentally established.**

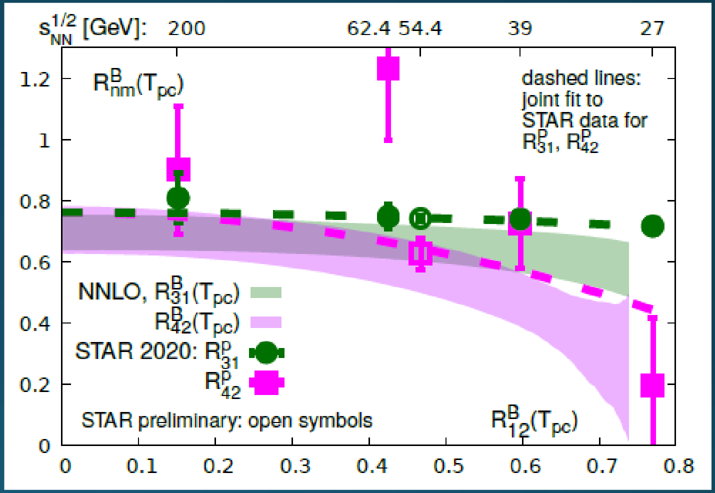
Cumulants of Conserved Charges
= Lattice observable

T dependence @ $\mu = 0$ | **Nonzero density**

$\langle N_B^4\rangle_c / \langle N_B^2\rangle_c$



$\langle N_B^4\rangle_c / \langle N_B^2\rangle_c$



- Lattice observable: $\chi_m^B = \frac{\langle N_B^m\rangle_c}{V} \sim \frac{\partial^m p}{\partial \mu_B^m}$

HotQCD, Phys. Rev. D 101, 074502 (2020)

Issues to be Resolved

- Experiments measure proton number cumulants, while lattice calculates baryon’s.

- Experiments measure the final state of the dynamical evolution, while lattice calculations are performed for equilibrium states with fixed temperature.

- And, other issues:
  - Volume fluctuation
  - Efficiency correction / imperfect acceptance
  - Measurement in momentum space
  - Resonance decays
  - Jets
  - …

See, Asakawa, MK, Mueller, PRC 101 (2020)

Proton vs Baryon Cumulants

MK, Asakawa, 2012; 2012

- $\langle N_p^m \rangle_c \neq \langle N_B^m \rangle_c$
- $\langle N_B^m \rangle_c$ can be obtained from the distribution of N_p thanks to the isospin randomization.

$N_B \to N_p :$

$$\langle N_p^{(\mathrm{net})} \rangle = \frac{1}{2} \langle N_\mathrm{B}^{(\mathrm{net})} \rangle,$$

$$\langle (\delta N_p^{(\mathrm{net})})^2 \rangle = \frac{1}{4} \langle (\delta N_\mathrm{B}^{(\mathrm{net})})^2 \rangle + \frac{1}{4} \langle N_\mathrm{B}^{(\mathrm{tot})} \rangle,$$

$$\langle (\delta N_p^{(\mathrm{net})})^3 \rangle = \frac{1}{8} \langle (\delta N_\mathrm{B}^{(\mathrm{net})})^3 \rangle + \frac{3}{8} \langle \delta N_\mathrm{B}^{(\mathrm{net})} \delta N_\mathrm{B}^{(\mathrm{tot})} \rangle,$$

Information of baryon # cumulants are more suppressed in higher order proton # cumulants!

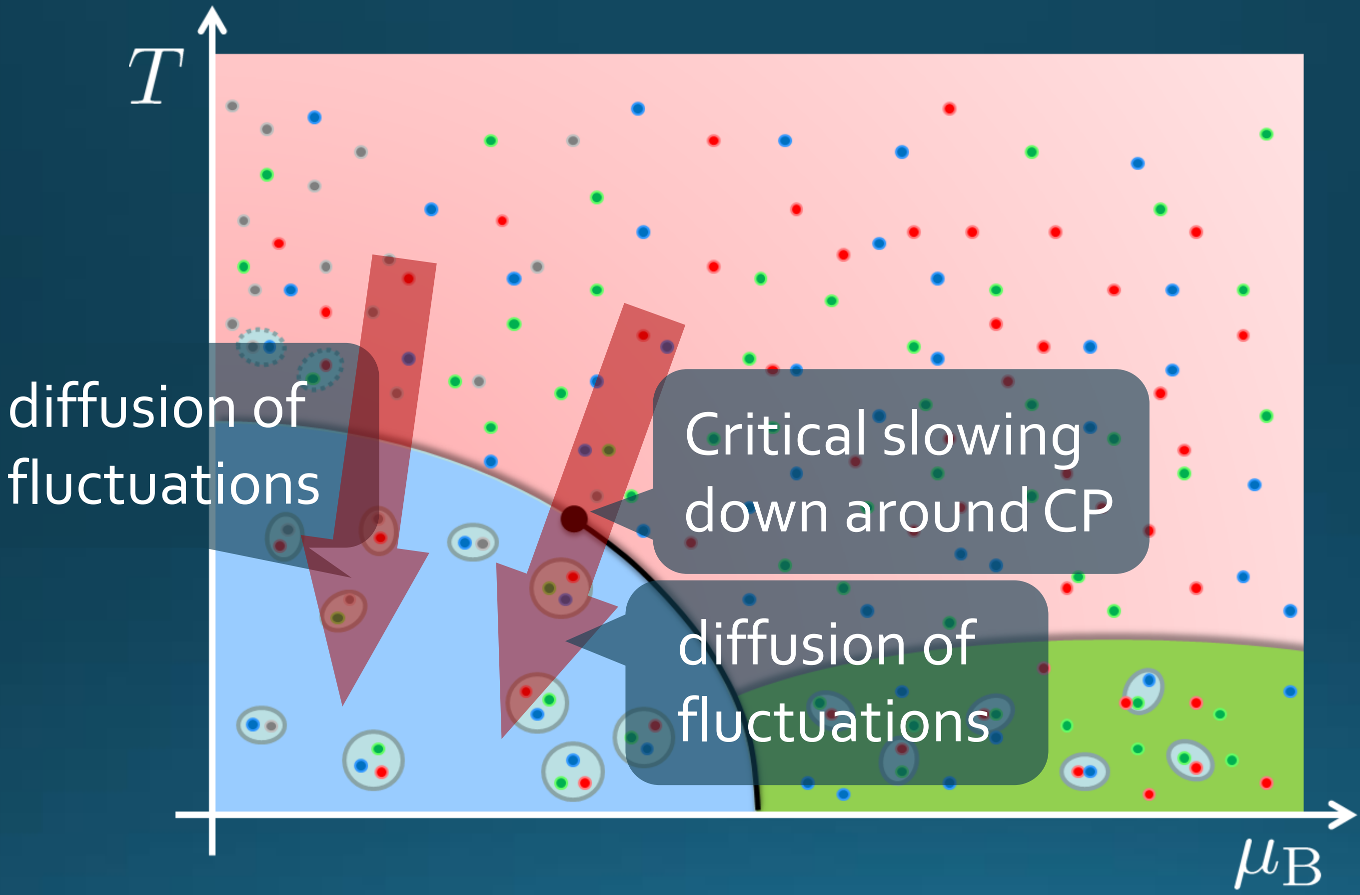
$N_p \to N_B :$

$$\langle N_\mathrm{B}^{(\mathrm{net})} \rangle = 2 \langle N_p^{(\mathrm{net})} \rangle,$$

$$\langle (\delta N_\mathrm{B}^{(\mathrm{net})})^2 \rangle = 4 \langle (\delta N_p^{(\mathrm{net})})^2 \rangle - 2 \langle N_p^{(\mathrm{tot})} \rangle,$$

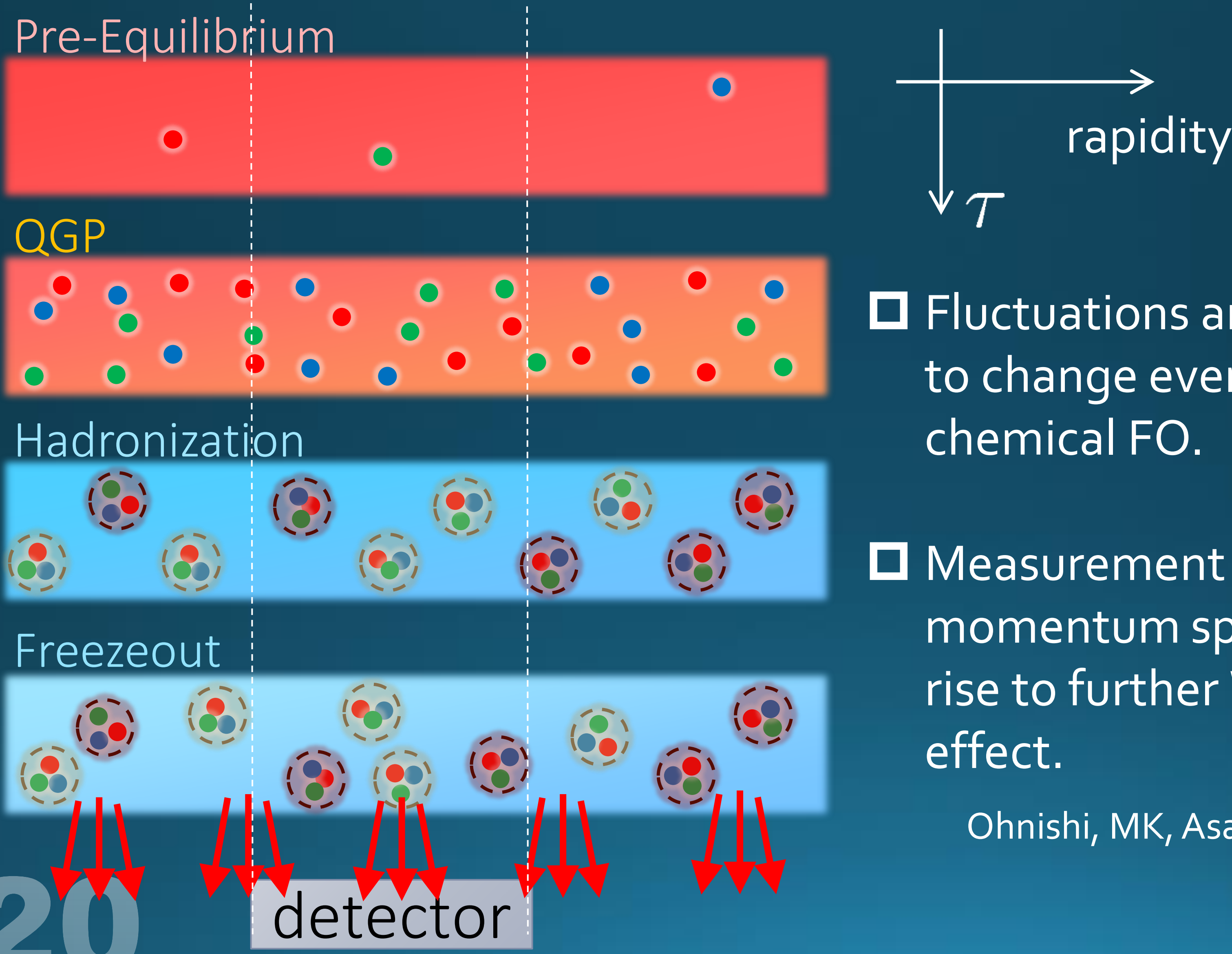
$$\langle (\delta N_\mathrm{B}^{(\mathrm{net})})^3 \rangle = 8 \langle (\delta N_p^{(\mathrm{net})})^3 \rangle - 12 \langle \delta N_p^{(\mathrm{net})} \delta N_p^{(\mathrm{tot})} \rangle + 6 \langle N_p^{(\mathrm{net})} \rangle,$$

Time Evolution of Cumulants



Proper understanding of the time evolution of fluctuations is indispensable.

Diffusion of Fluctuations

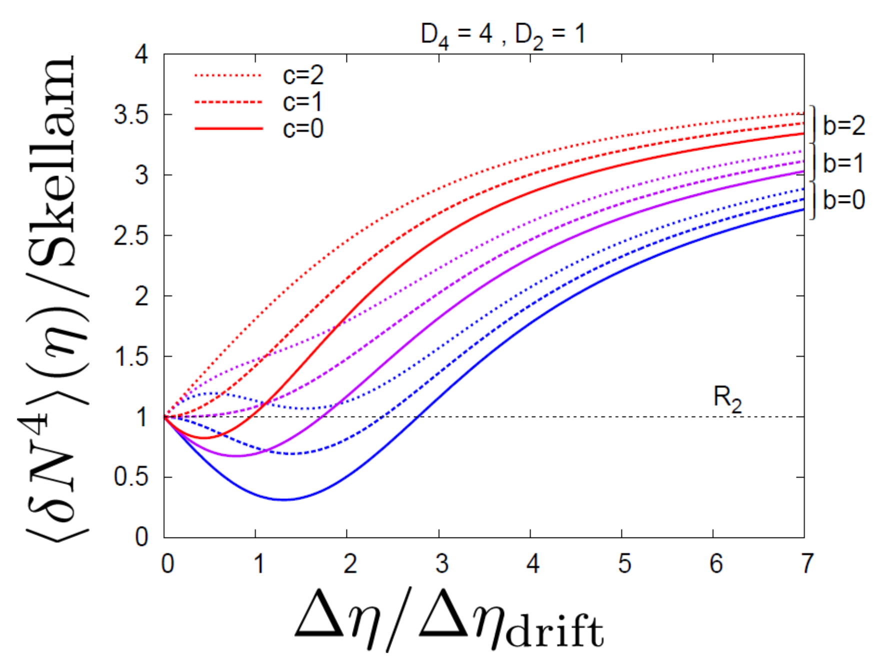


- Fluctuations are continue to change even after the chemical FO.
- Measurement in momentum space gives rise to further “blurring” effect.

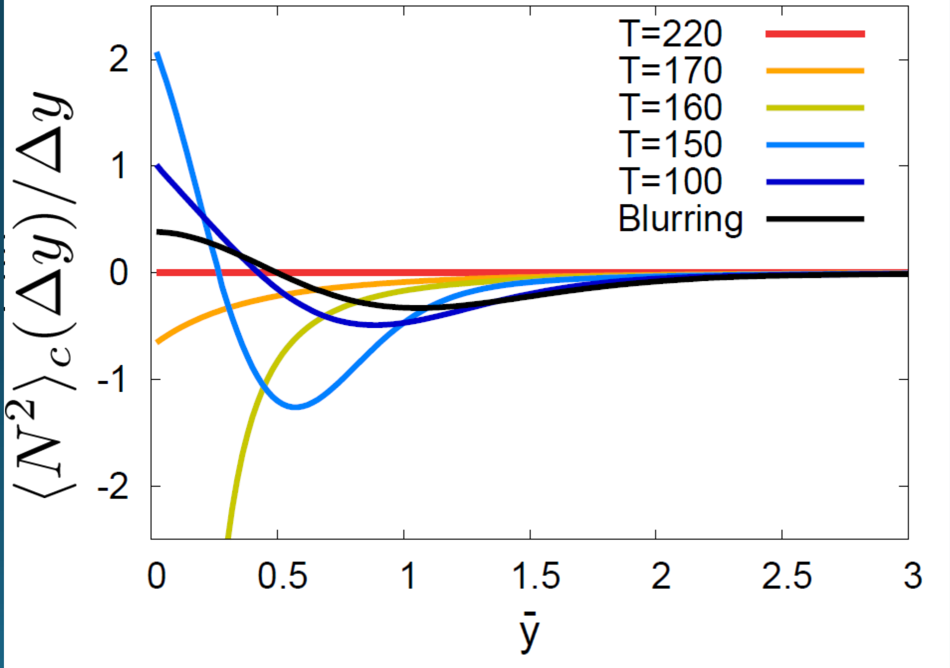
Ohnishi, MK, Asakawa, PRC 2016

Rapidity Window Dependence
in Diffusion Models

- **Higher order cumulants**
  in diffusion master equation
  MK+, PLB (2014); MK, NPA (2015)



- **2nd order cumulant near CP**
  in stochastic diffusion equation
  sakaida, Asakawa, Fujii, MK, (2018)



- Non-monotonic Δy dependence can emerge reflecting the dynamical evolution.

Three Applications

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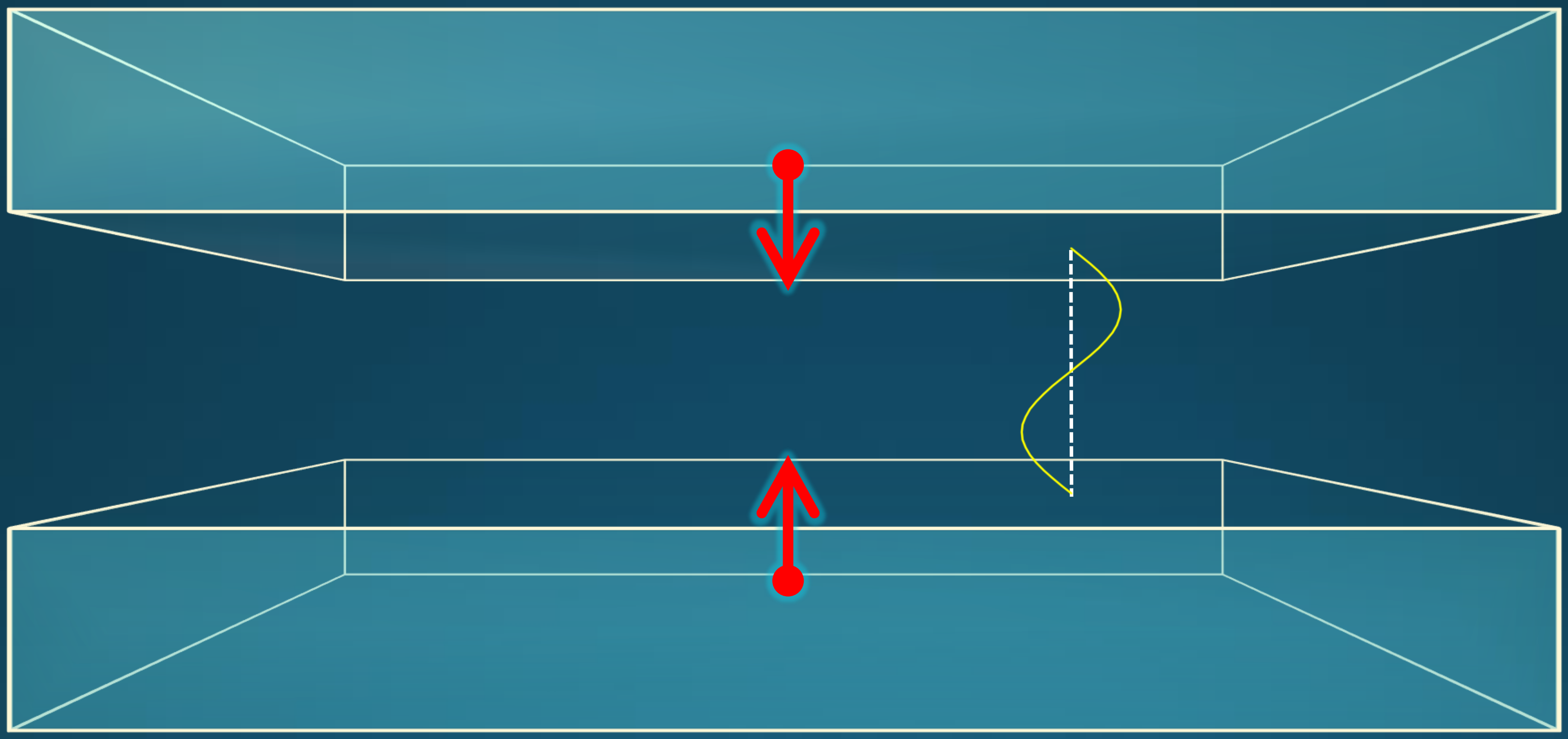
2. Finite-volume Effects
   in anisotropic systems

3. Finite-size Scaling around QCD-CP

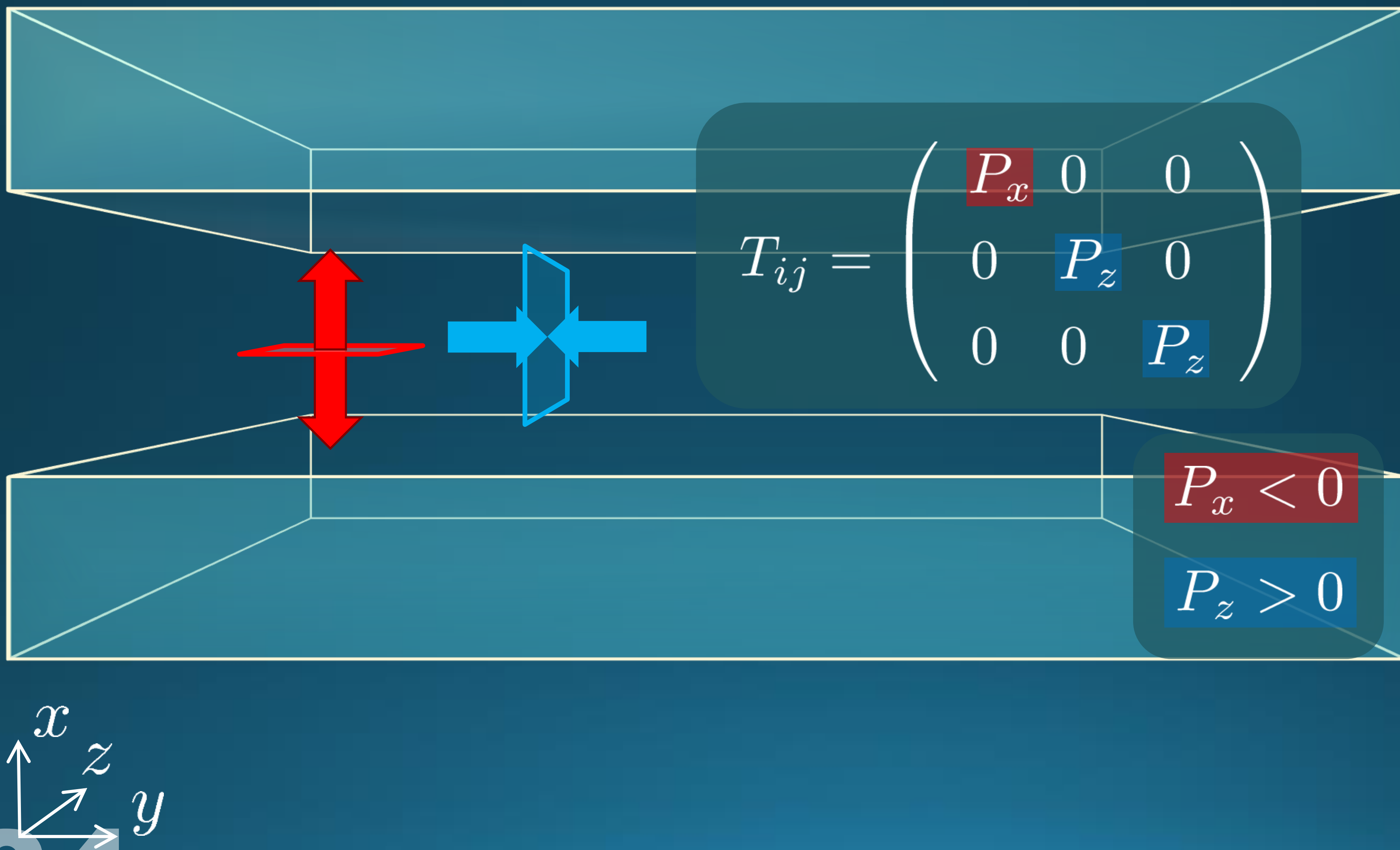
Casimir Effect



attractive force between two conductive plates

Casimir Effect

Brown, Maclay
1969



$$T_{ij} = \begin{pmatrix} P_x & 0 & 0 \\ 0 & P_z & 0 \\ 0 & 0 & P_z \end{pmatrix}$$

$P_x < 0$

$P_z > 0$

Thermodynamics on the Lattice

Various Methods

- **Integral**, differential, moving frame, non-equilibrium, …
- rely on thermodynamic relations valid in V→∞

$$P = \frac{T}{V} \ln Z \qquad sT = \varepsilon + P$$

→ **Not applicable to anisotropic systems**

- We employ **SF*t*X Method**

$$\varepsilon = \langle T_{00} \rangle \quad P = \langle T_{11} \rangle$$

Components of EMT are directly accessible!

SF*t*X = **S**mall **F**low ***t***ime e**X**pansion

Casimir Effect

Brown, Maclay
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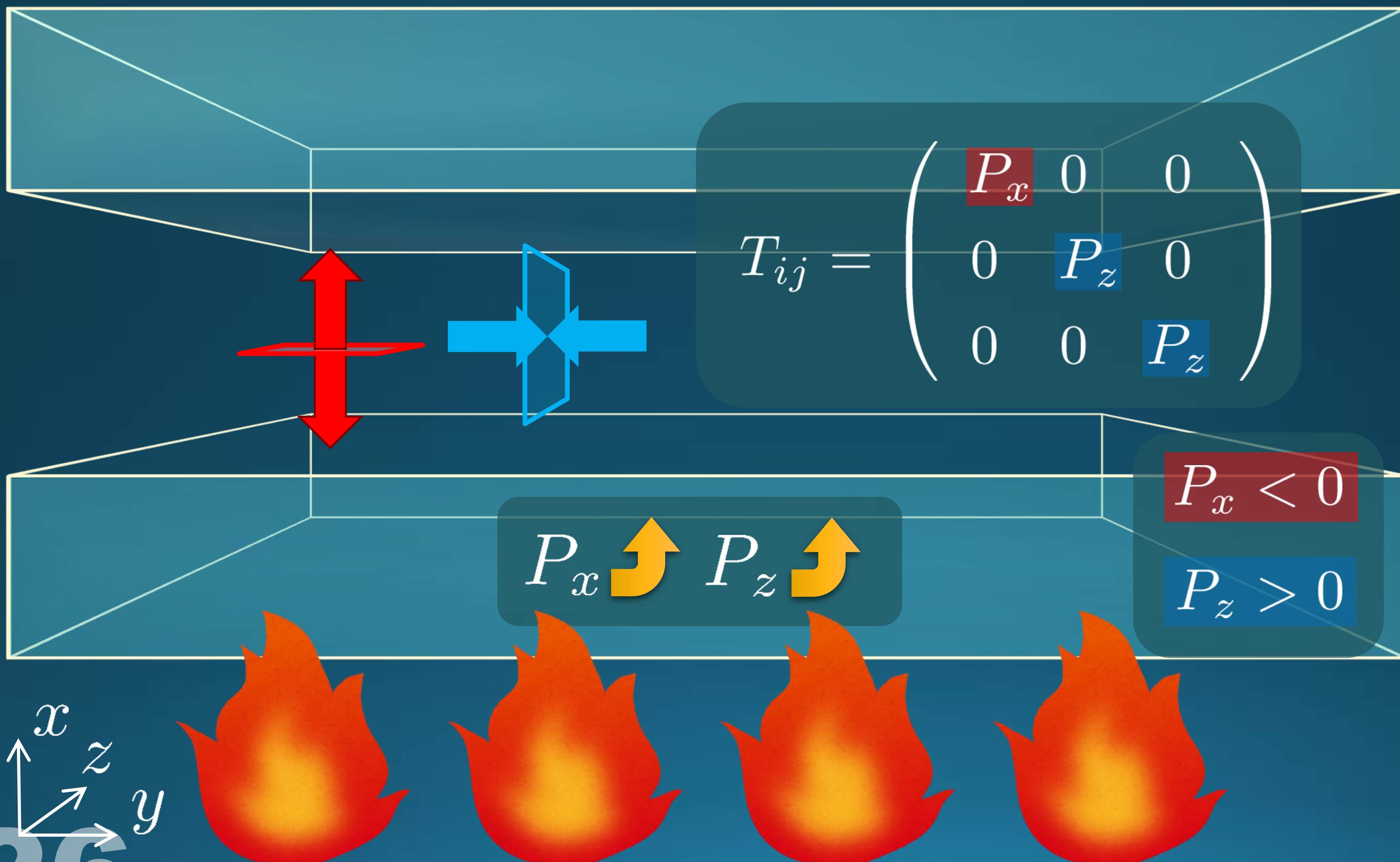

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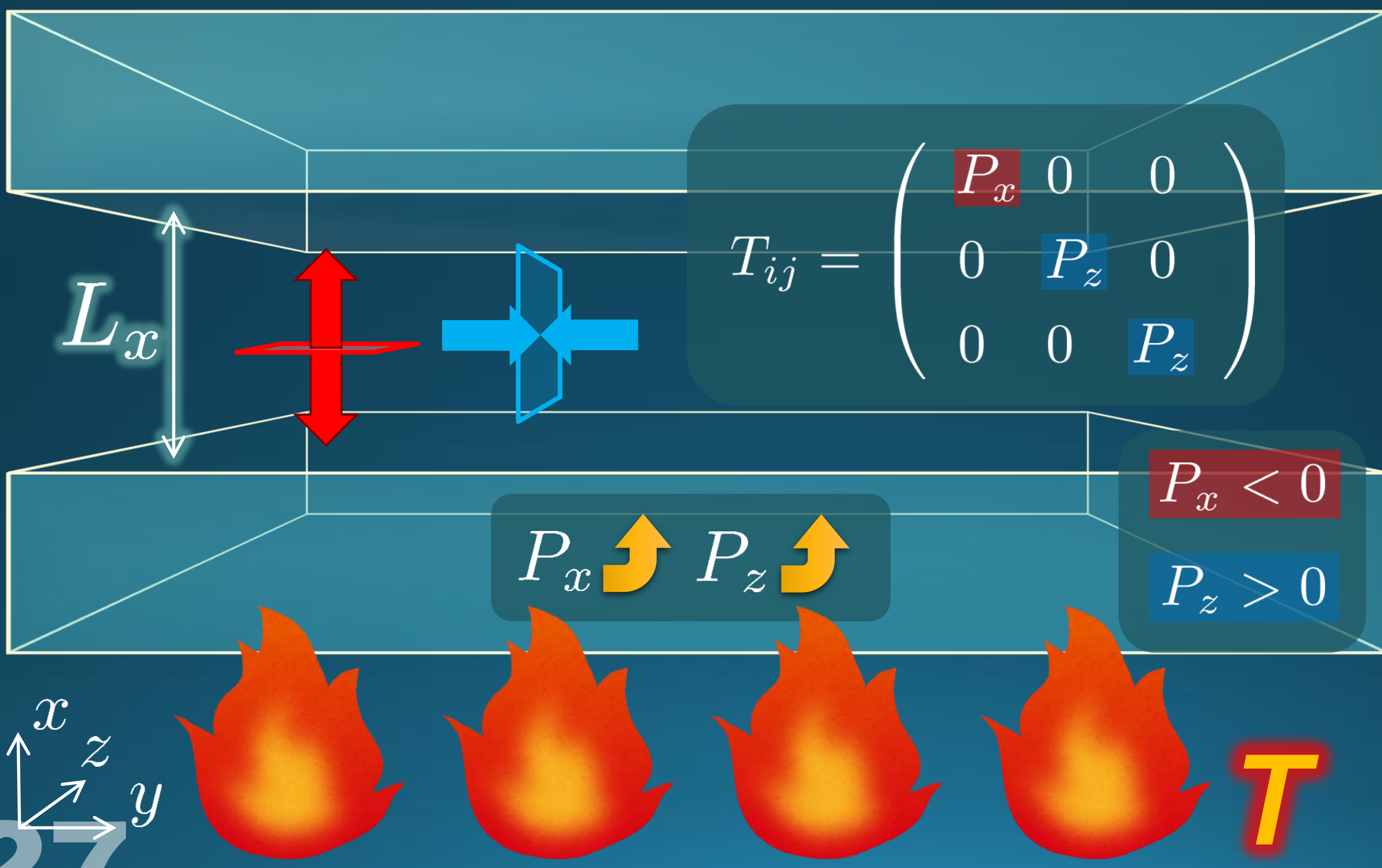
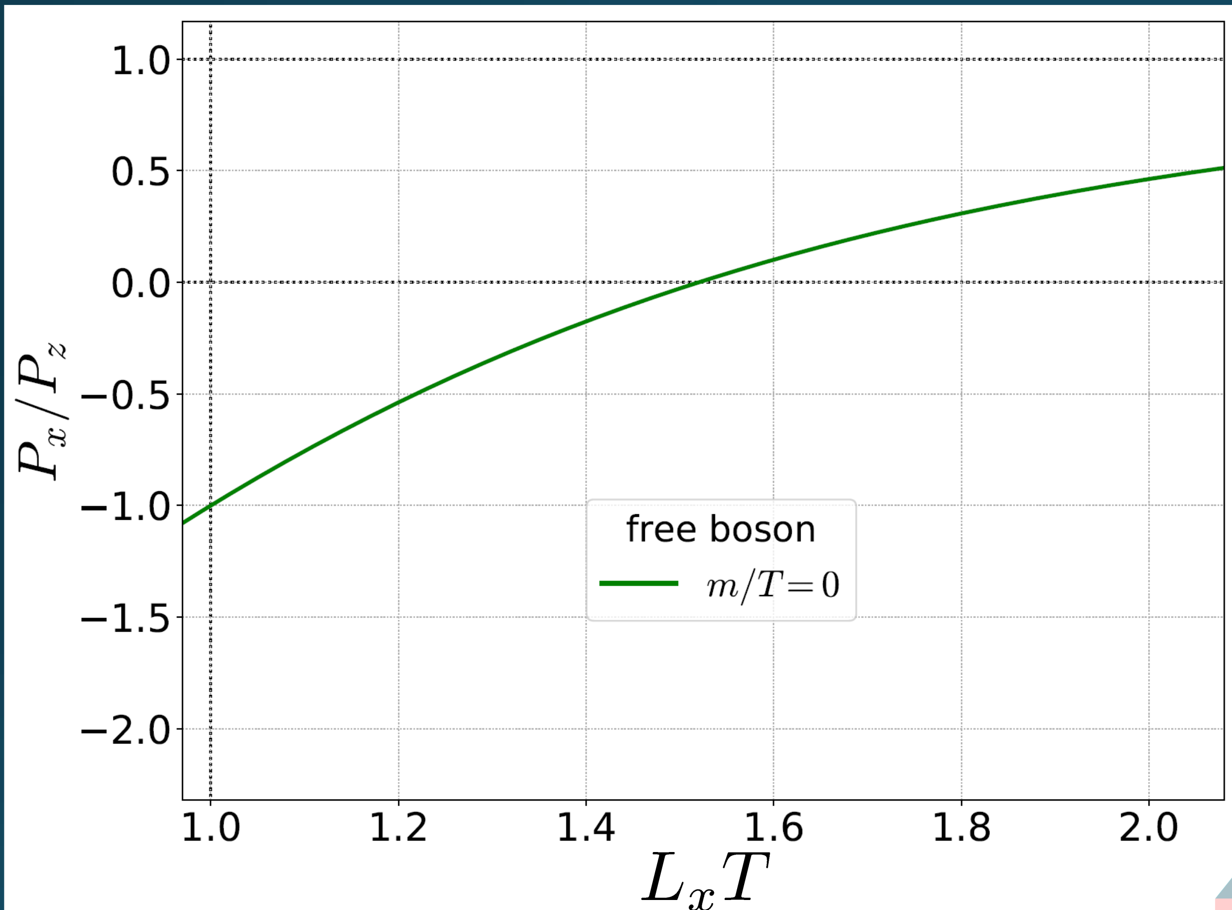


Pressure Anisotropy @ T≠o

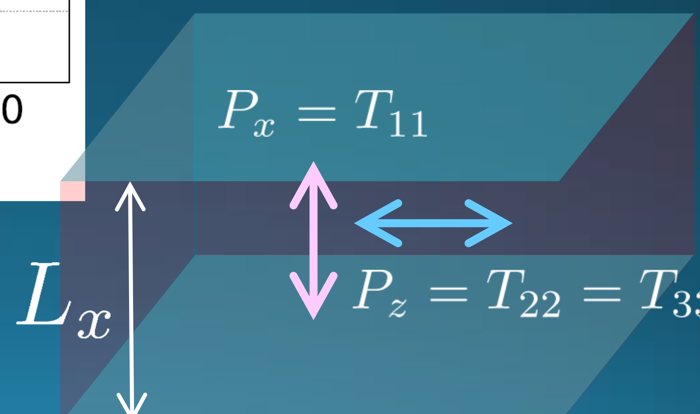
MK, Mogliacci, Kolbe, Horowitz, PRD (2019)



Free scalar field

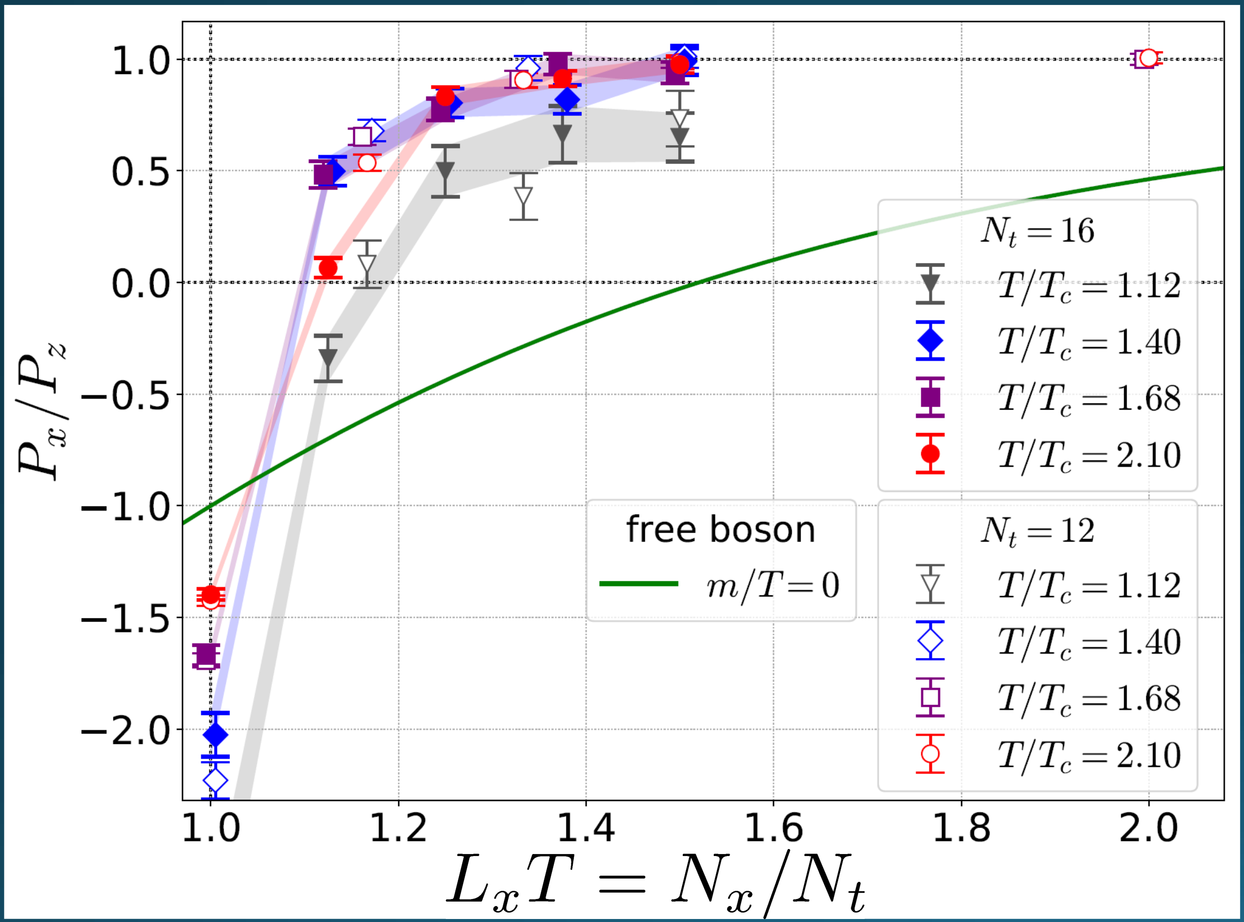
- $L_2=L_3=\infty$
- Periodic BC

Mogliacci+, 1807.07871



Pressure Anisotropy @ T≠0

MK, Mogliacci, Kolbe, Horowitz, PRD (2019)



Free scalar field

- $L_2=L_3=\infty$
- Periodic BC

Mogliacci+, 1807.07871

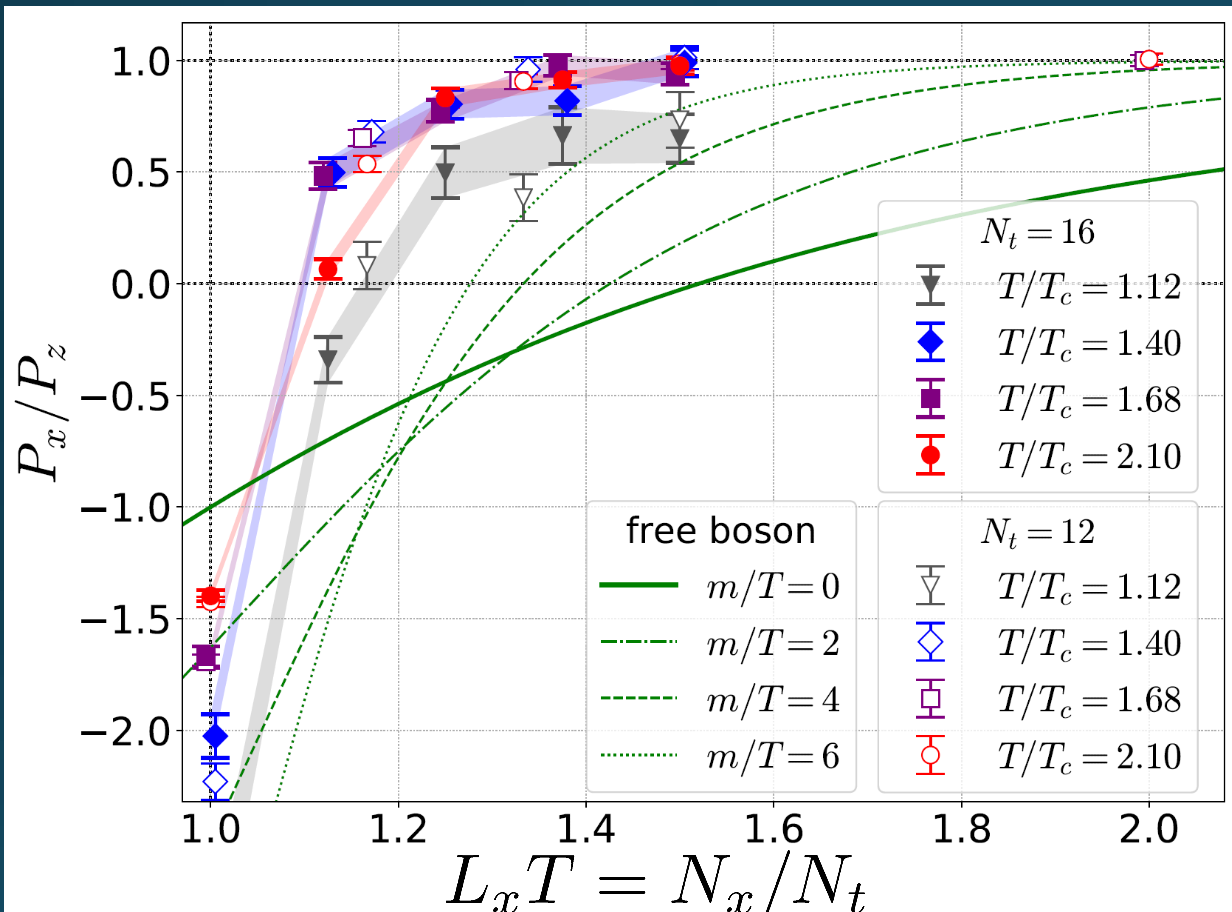
Lattice result

- Periodic BC
- Only t→0 limit
- Error: stat.+sys.

Medium near T_c is remarkably insensitive to finite size!

Pressure Anisotropy @ T≠0

MK, Mogliacci, Kolbe, Horowitz, PRD (2019)



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Three Applications

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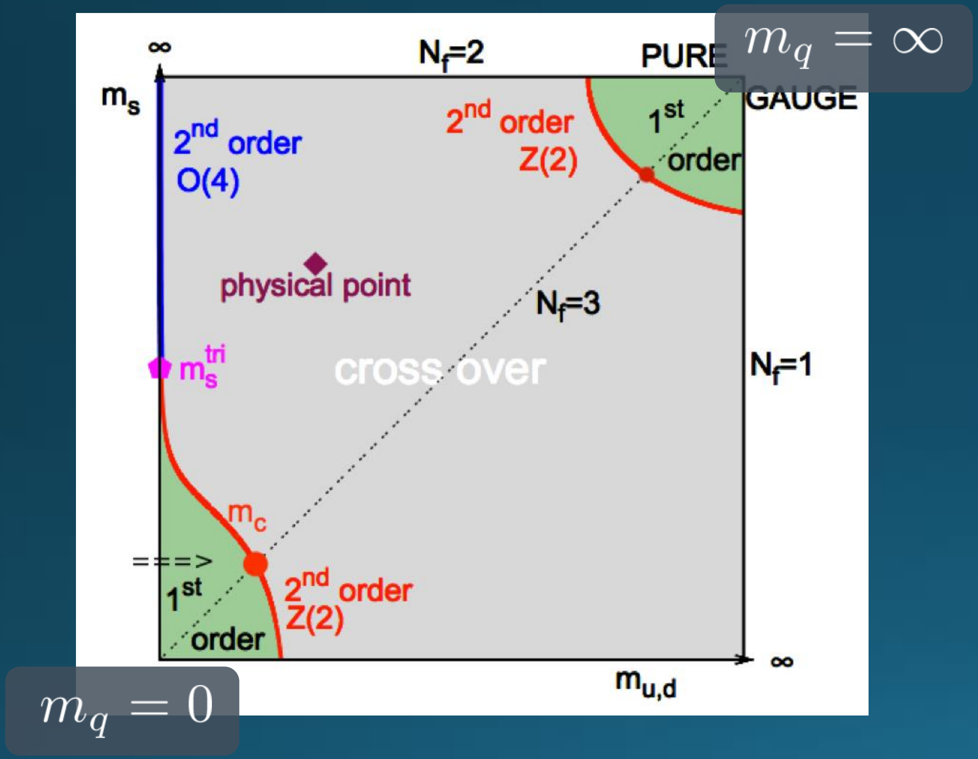
Varying Quark Masses

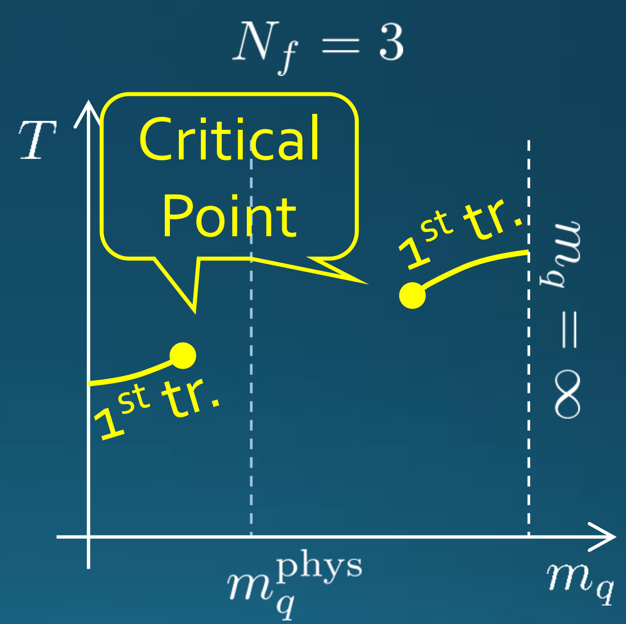
- **Columbia plot**

  = order of phase tr. at $\mu = 0$



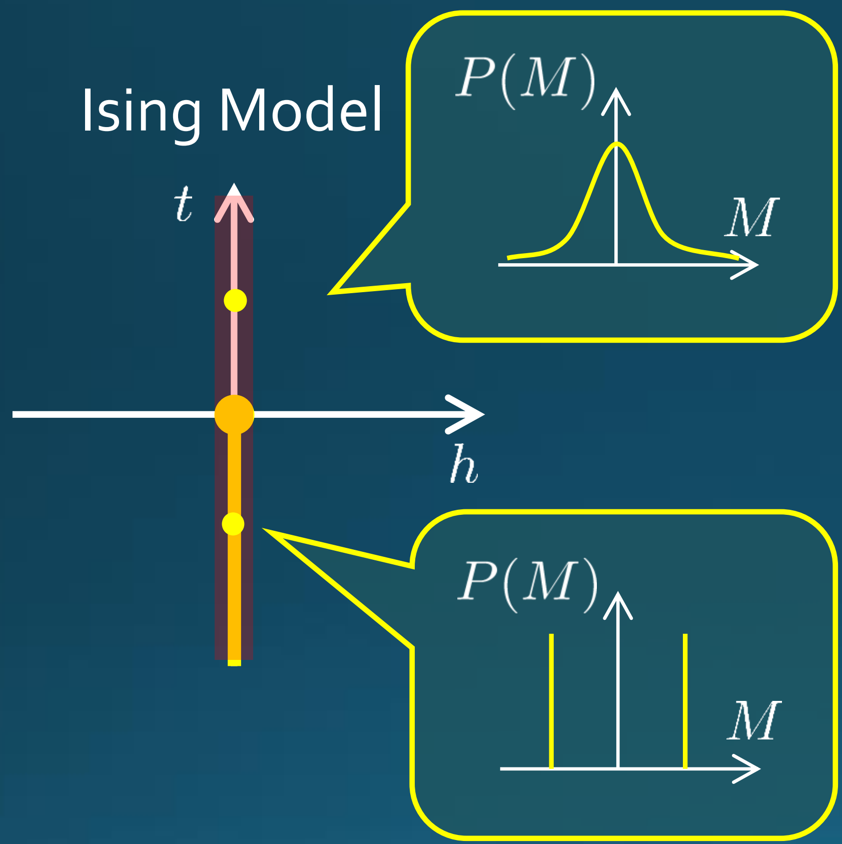
- **Phase Diagram**

  on the $T - m_q$ plane



Various orders of phase transition with variation of m_q.

Cumulants around Critical Point



Gauss distribution

$$\frac{\langle M^4 \rangle_c}{\langle M^2 \rangle_c^2} = 0$$

Two delta functions

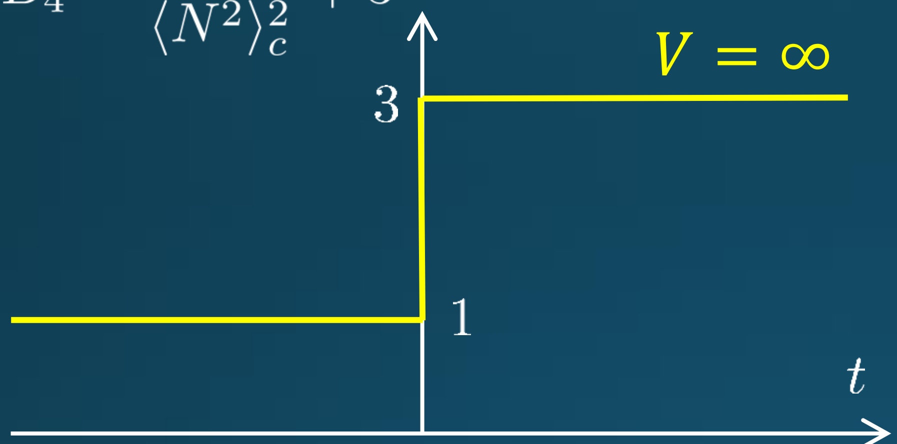
$$\frac{\langle M^4 \rangle_c}{\langle M^2 \rangle_c^2} = -2$$

- Kurtosis $\langle M^4 \rangle_c / \langle M^2 \rangle_c$ changes discontinuously at the CP.

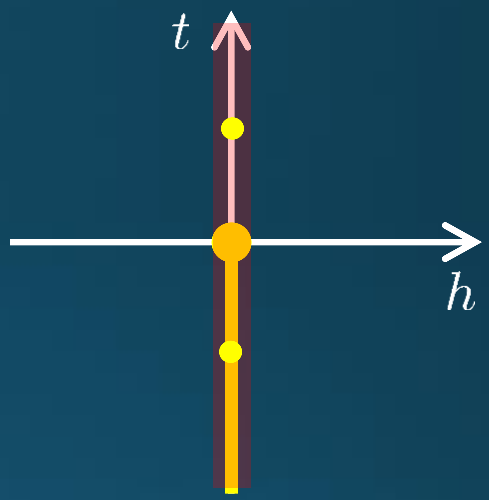
Finite-Size Effects

Binder Cumulant

$$B_4 = \frac{\langle N^4 \rangle_c}{\langle N^2 \rangle_c^2} + 3$$



Ising Model

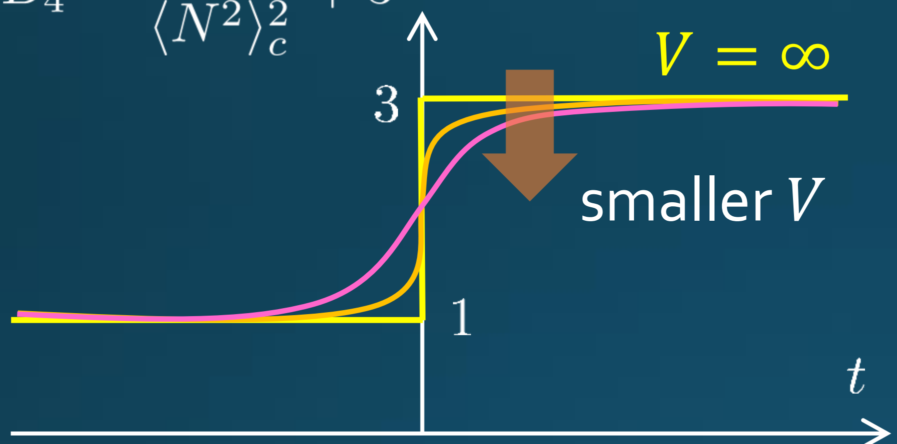


- Sudden change of B_4 at the CP is smeared by finite-size effect.
- B_4 obtained for various size have crossing at $t = 0$.
- At the crossing point, $B_4 = 1.604$ in Z_2 universality class.

Finite-Volume Effects

Binder Cumulant

$$B_4 = \frac{\langle N^4 \rangle_c}{\langle N^2 \rangle_c^2} + 3$$



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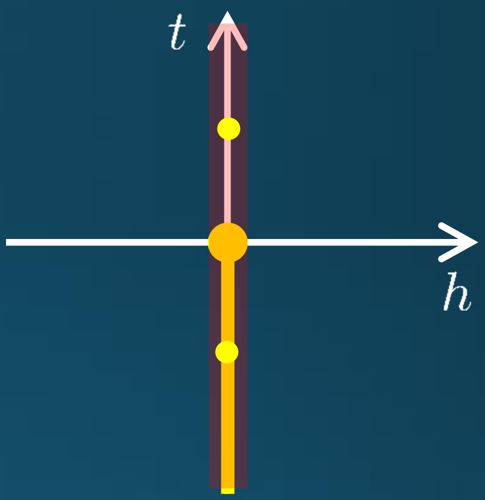

- Sudden change of B_4 at the CP is smeared by finite V effect.
- B_4 obtained for various V has crossing at $t = 0$.
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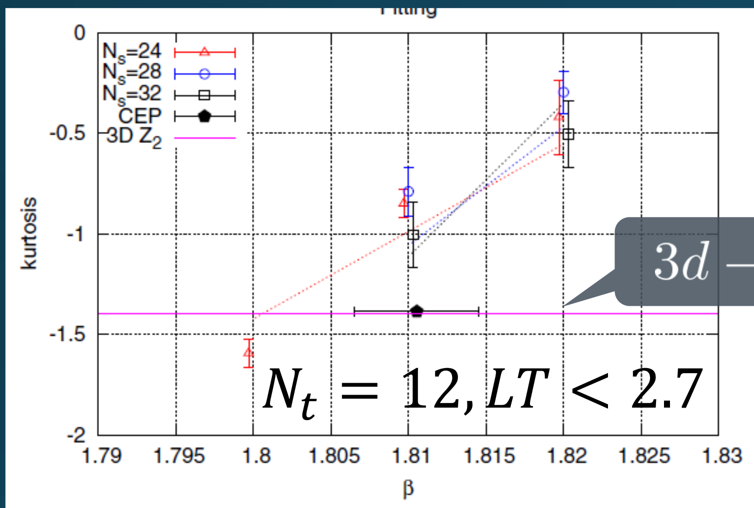
Binder-Cumulant Analysis

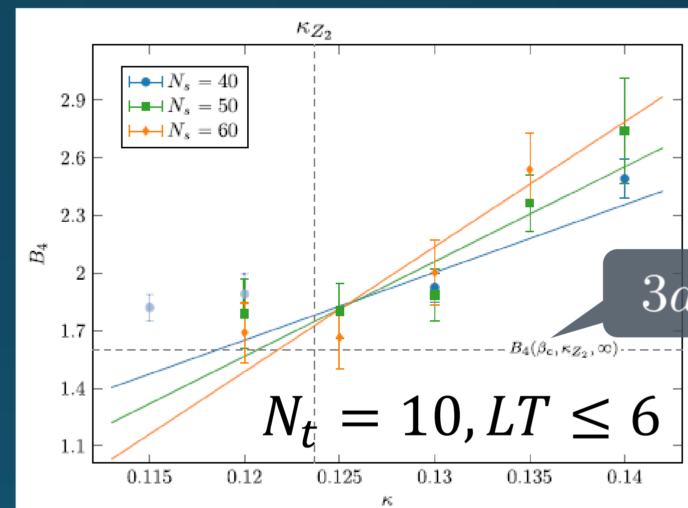
☐ Light-quark region

Kuramashi, Nakamura, Ohno, Takeda, '20



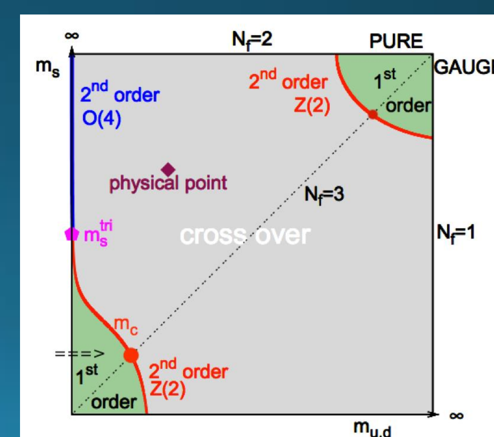
☐ Heavy-quark region

Cuteri , Philipsen , Schön, Sciarra, '21



☐ Statistically-significant deviation of the crossing point from the 3d-Ising value.

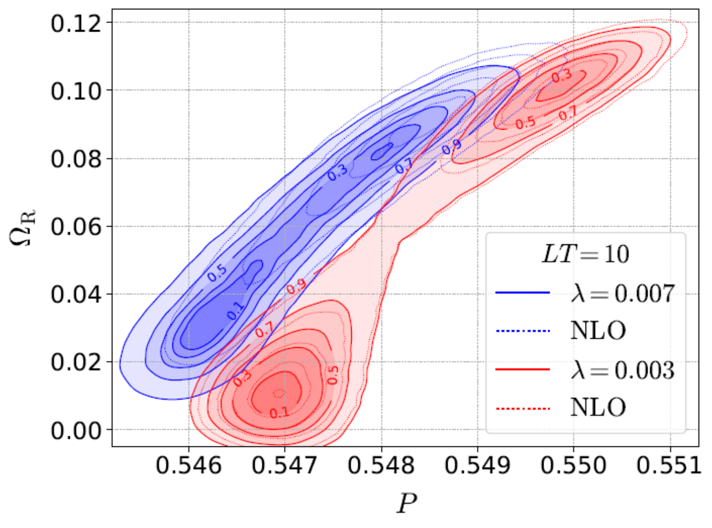
➡ Too large finite-S effects?



Numerical Simulation

- Coarse lattice: $N_t = 4$
- But **large spatial volume**:
  $LT = N_s / N_t \leq 12$

- Hopping-param. ($\sim 1/m_q$) expansion
- Monte-Calro with LO action
- High statistical analysis

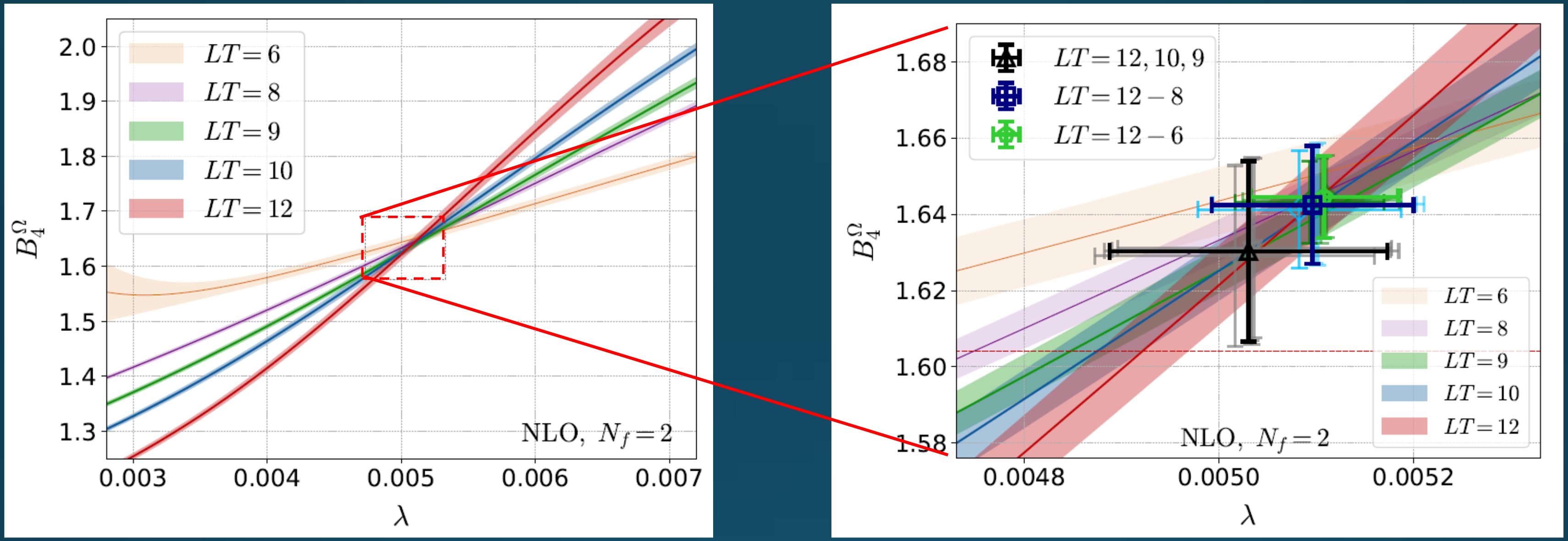


Simulation params.

lattice size	β^*	λ	$\kappa^{N_f=2}$
$48^3 \times 4$	5.6869	0.004	0.0568
	5.6861	0.005	0.0601
	5.6849	0.006	0.0629
$40^3 \times 4$, $36^3 \times 4$	5.6885	0.003	0.0529
	5.6869	0.004	0.0568
	5.6861	0.005	0.0601
	5.6849	0.006	0.0629
	5.6837	0.007	0.0653
$32^3 \times 4$	5.6885	0.003	0.0529
	5.6865	0.004	0.0568
	5.6861	0.005	0.0601
	5.6845	0.006	0.0629
	5.6837	0.007	0.0653
$24^3 \times 4$	5.6870	0.0038	0.0561
	5.6820	0.0077	0.0669
	5.6780	0.0115	0.0740

Kiyohara, MK, Ejiri, Kanaya
2108.00118, PRD, in press

Binder-Cumulant Analysis



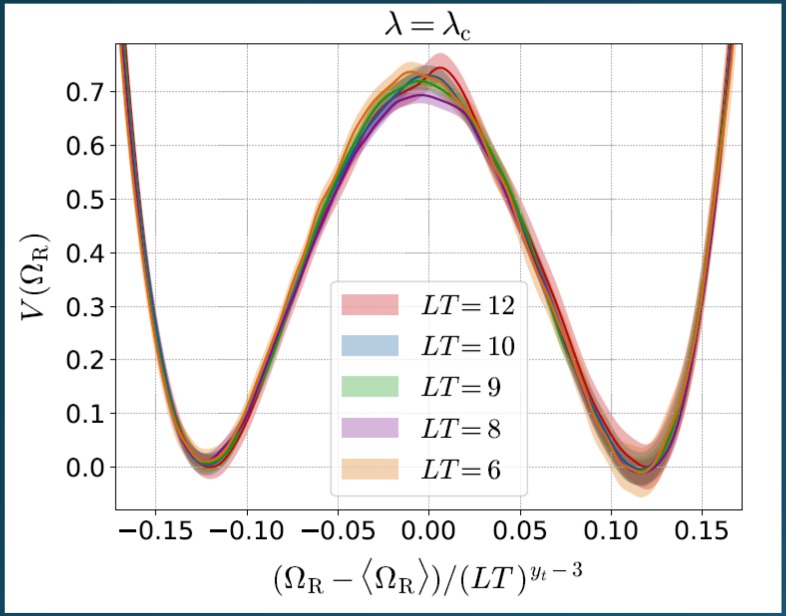
Z2	$B_4 = 1.604$	$\nu = 0.630$
$LT \geq 9$	$B_4 = 1.630(24)(2),$	$\nu = 0.614(48)(3)$
$LT \geq 8$	$B_4 = 1.643(15)(2),$	$\nu = 0.614(29)(3)$

- B_4 and ν are consistent with Z2 universality class only when $LT \geq 9$ data are used for the analysis.

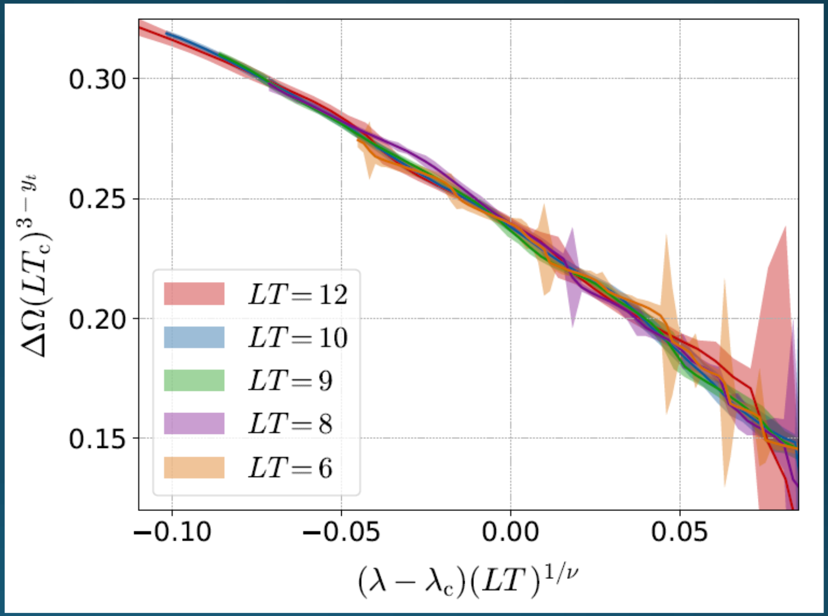
Kiyohara, MK, Ejiri, Kanaya, 2108.00118

Further Check of Z_2 Scaling

- ☐ Effective potential at the CP



- ☐ Scaling of order parameter



Z2 scaling is well established

Kiyohara, MK, Ejiri, Kanaya, 2108.00118

HIC vs Lattice: Pros & Cons

Real experiments	⟺	Virtual, but unphysical params
Zero~high baryon density	⟺	Small baryon density only
Dynamical evolution	⟺	Ideal thermal system
Final-state observables only		Limited observables

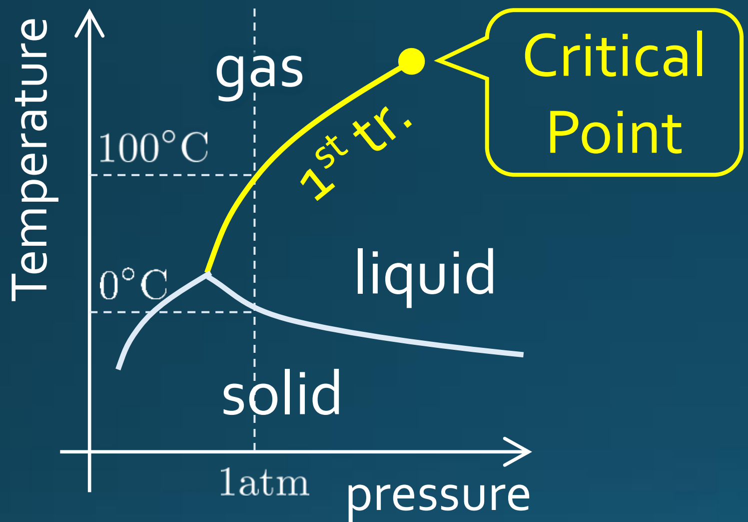
Complementary use of both exps. is important!

Final Remarks

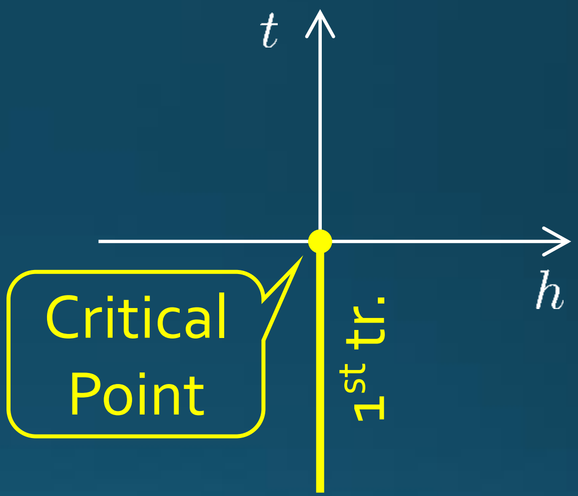
- Relativistic HIC and lattice simulations are useful "experimental" tools for exploring hot and dense medium.
- Both experiments have pros and cons. Their complementary use is important.
- Further exchanges of ideas between lattice and experimental communities will be especially effective for the search for the QCD phase structure!

Critical Points

☐ Water



☐ Ising Model

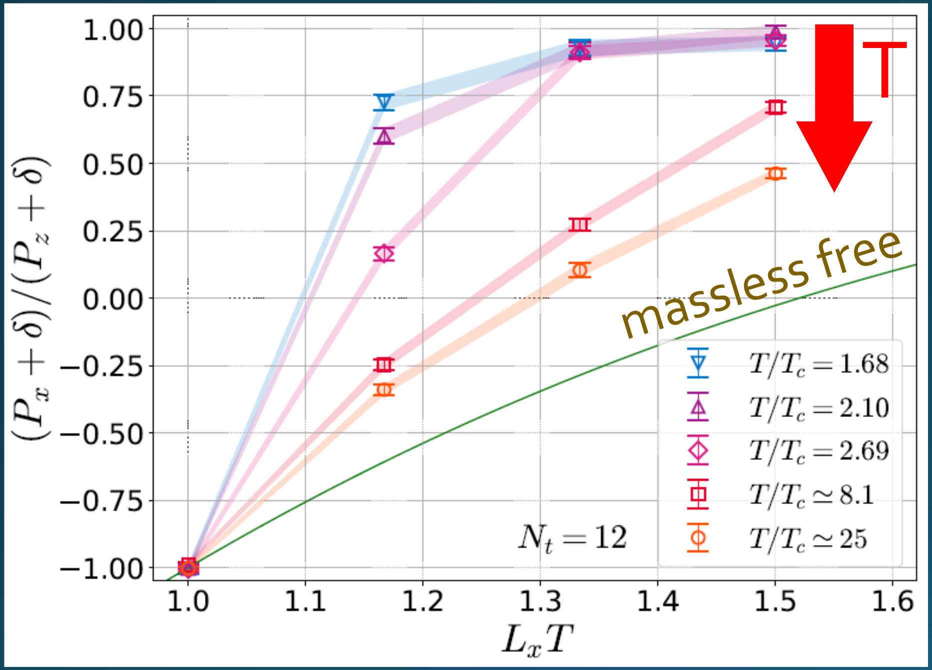


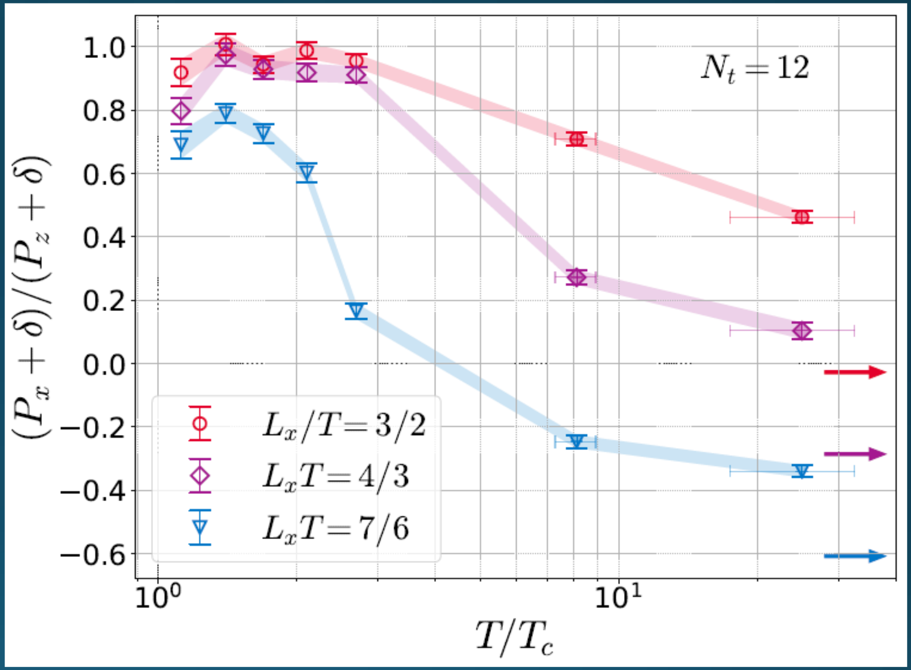
These CPs belong to the same universality class (Z_2).

→ Common critical exponents.

ex. $C \sim (T - T_c)^{-\alpha}$

$$\frac{P_x + \delta}{P_z + \delta}$$





$T/T_c \cong 8.1$ (β=8.0) / $T/T_c \cong 25$ (β=9.0)

- ☐ Ratio slowly approaches the asymptotic value.
- ☐ But, large deviation still exists even at $T/T_c \sim 25$.