GPD and Related Topics at J-PARC, 2022/12/22, KEK (online)

Gravitational Form Factors from Lattice QCD

Masakiyo Kitazawa

(Osaka U.)

Thanks to:
H. Ito, R. Yanagihara &
FlowQCD/WHOT-QCD Collaborations

Energy-Momentum Tensor

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

energy momentum pressure stress

All components are important physical observables!

Gravitational Form Factors

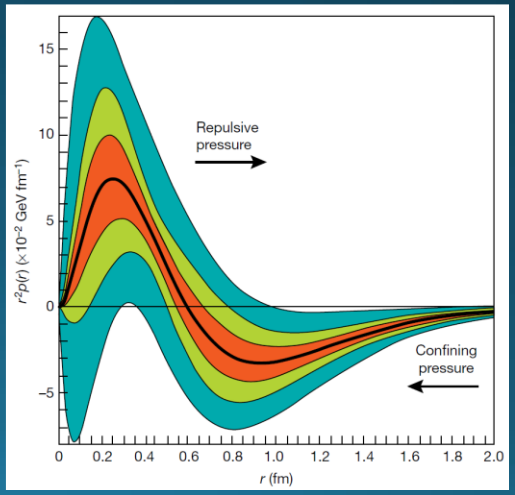
$$\langle p'|T^{a}_{\mu\nu}(0)|p\rangle = \bar{u}'\Big[A^a(t)\frac{P_\mu P_\nu}{M} + J^a(t)\frac{iP_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M} + D^a(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M} + M\bar{c}^a(t)g_{\mu\nu}\Big]$$

- (partially) accessible with hard exclusive processes

- Mechanical structure of hadrons
- D-term: the last global unknown
- Mass decomposition

Polyakov(2003); Kumano, Song, Teryaev (2018); Ji (1995); Locre (2018); Hatta, Rajan, Tanaka (2018);

Pressure distribution inside proton



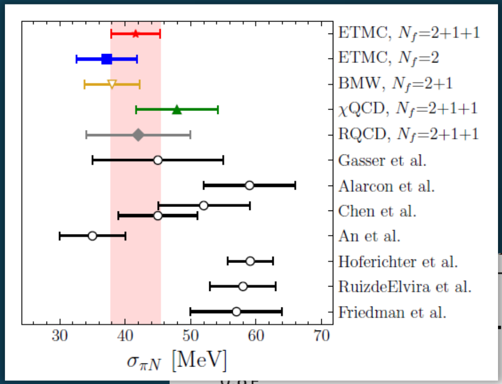
Burkert+, Nature 557, 396 (2018)

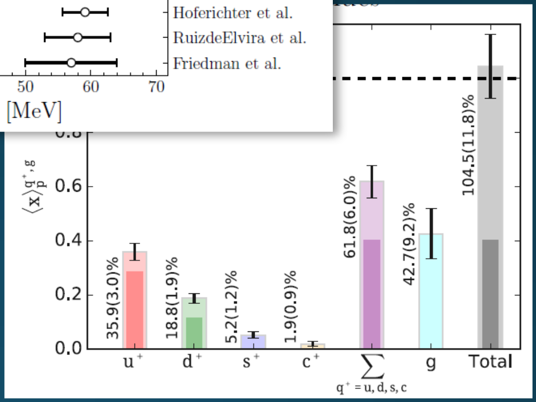
GFF from Lattice

☐ Mass decomposition

- Nucleon σ term, $\langle x_q \rangle, \langle x_g \rangle$, …
- Trace anomaly: difficult to measure

cf: Alexandrou+ (2020); (2020)

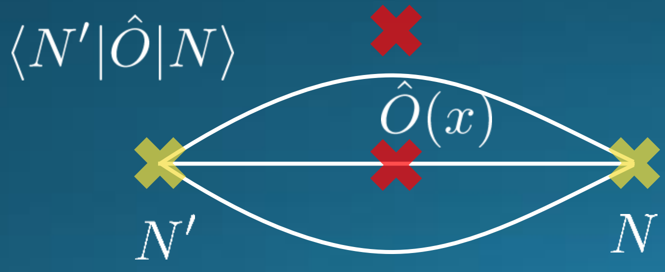


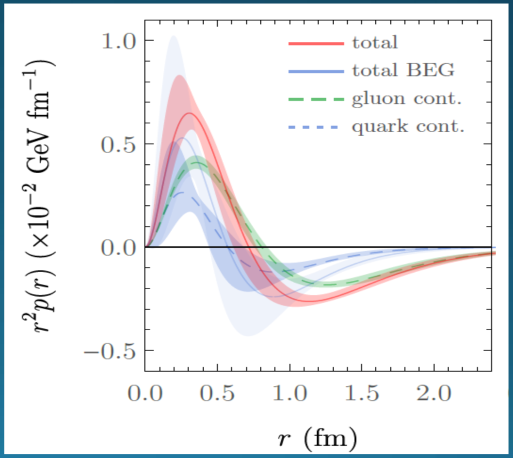


☐ Mechanical structure

- Distribution of EMT
- Momentum space → Coordinate

cf: Shanahan, Detmold (2019)





Contents

1. Constructing EMT
   - SFtX method via gradient flow

2. EMT distribution around Flux Tube
   2-1 Lattice result

   FlowQCD, PL**B789**, 210 (2019)

   2-2 Abelian-Higgs model

   Yanagihara, MK, PTEP**2019**, 093B02 (2019)

3. GFF of soliton in 1+1d ϕ^4 model

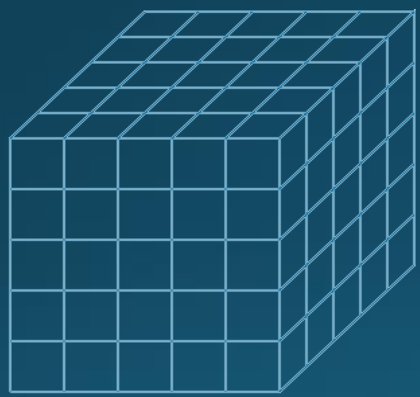
   Ito, MK, in preparation

4. Single-quark system

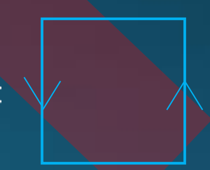
   FlowQCD, PRD102, 114522 (2020)

$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial
because of the explicit breaking of translational invariance



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

② Its measurement is extremely noisy
due to high dimensionality and etc.

Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

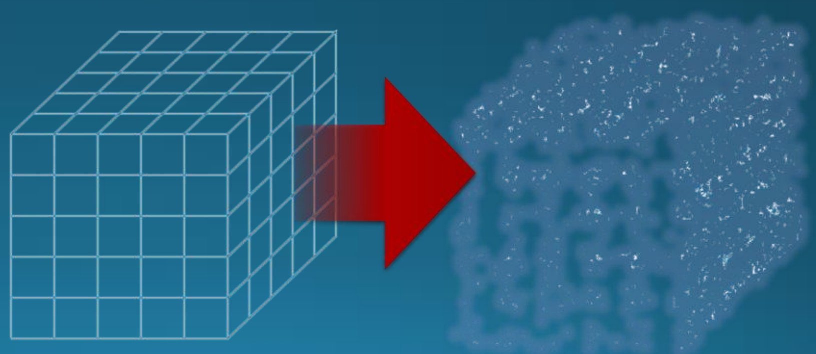
t: "flow time"
dim:[length²]

$$A_\mu(0,x) = A_\mu(x)$$

leading

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at $t > 0$



Gradient Flow for Fermions

$$\partial_t \psi(t,x) = D_\mu D_\mu \psi(t,x)$$

$$\partial_t \bar{\psi}(t,x) = \psi(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu$$

$$D_\mu = \partial_\mu + A_\mu(t,x)$$

Luscher, 2013
Makino, Suzuki, 2014
Taniguchi+ (WHOT) 2016~

- Not "gradient" flow but a "diffusion"-type equation.

- Divergence in field renormalization of fermions.
- All observables are finite at t>0 once Z(t) is fixed.

$$\tilde{\psi}(t,x) = Z(t)\psi(t,x)$$

- Energy-momentum tensor from SF*t*X Makino, Suzuki, 2014

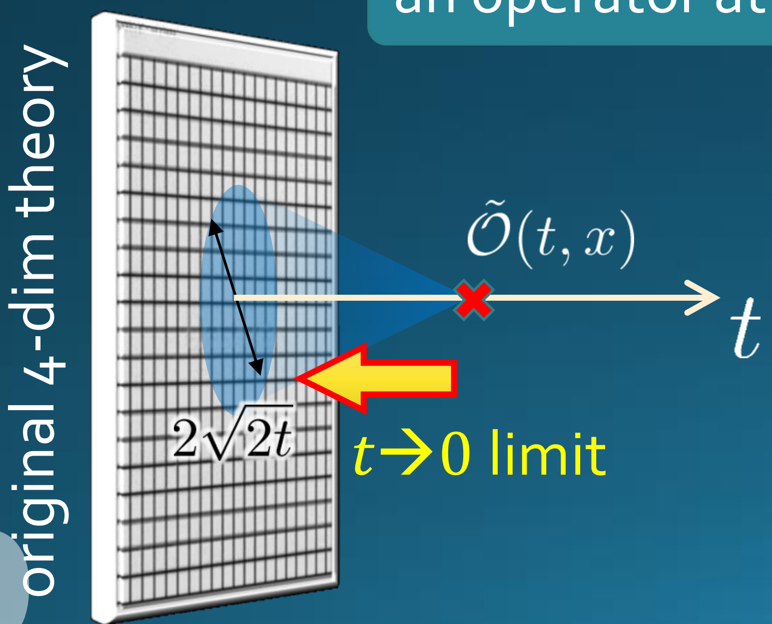
Small Flow-Time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at $t>0$

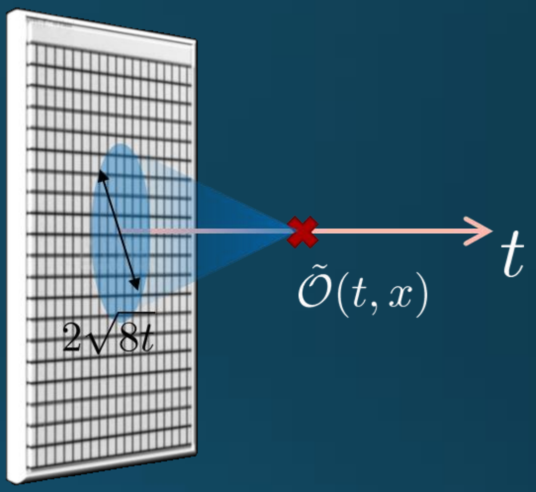
remormalized operators of original theory



Constructing EMT 1

Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$



- **Gauge-invariant dimension 4 operators**

$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\rho\sigma}(t,x)G_{\rho\sigma}(t,x) \\ E(t,x) = \frac{1}{4}G_{\rho\sigma}(t,x)G_{\rho\sigma}(t,x) \end{cases}$$

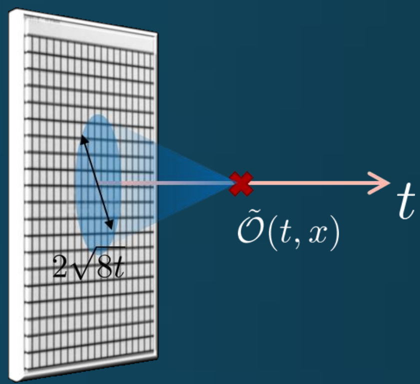
Constructing EMT 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t) T^R_{\rho\rho}(x) + \mathcal{O}(t)$$

vacuum subtr.



Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\rm subt.}\right]$$

"SF*t*X method" (Small Flow *t*ime eXpansion)

EMT in QCD

Makino, Suzuki (2014)
Harlander+ (2018)

$$
\begin{aligned}
T_{\mu\nu}(t,x) =& c_1(t)U_{\mu\nu}(t,x) + c_2(t)\delta_{\mu\nu}\big(E(t,x) - \langle E\rangle_0\big) \\
&+ c_3(t)\big(O_{3\mu\nu}(t,x) - 2O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) \\
&+ c_4(t)\big(O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) + c_5(t)\big(O_{5\mu\nu}(t,x) - \mathrm{VEV}\big)
\end{aligned}
$$

$$
T_{\mu\nu}(x) = \lim_{t\to 0} T_{\mu\nu}(t,x)
$$

$$
\begin{aligned}
\tilde{O}^f_{3\mu\nu}(t,x) &\equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x), \\
\tilde{O}^f_{4\mu\nu}(t,x) &\equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x), \\
\tilde{O}^f_{5\mu\nu}(t,x) &\equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x),
\end{aligned}
$$

$$
\varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.
$$

Perturbative Coefficients

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

$$T_{\mu\nu}(t) = c_1(t)U_{\mu\nu}(t) + \delta_{\mu\nu}c_2(t)E(t)$$

	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○	○	
$c_2(t)$	× zero	○	○	○

Suzuki (2013) Harlander+(2018)

Iritani, MK, Suzuki, Takaura, 2019

☐ Choice of the scale of g^2

$$c_1(t) = c_1\left(g^2(\mu(t))\right)$$

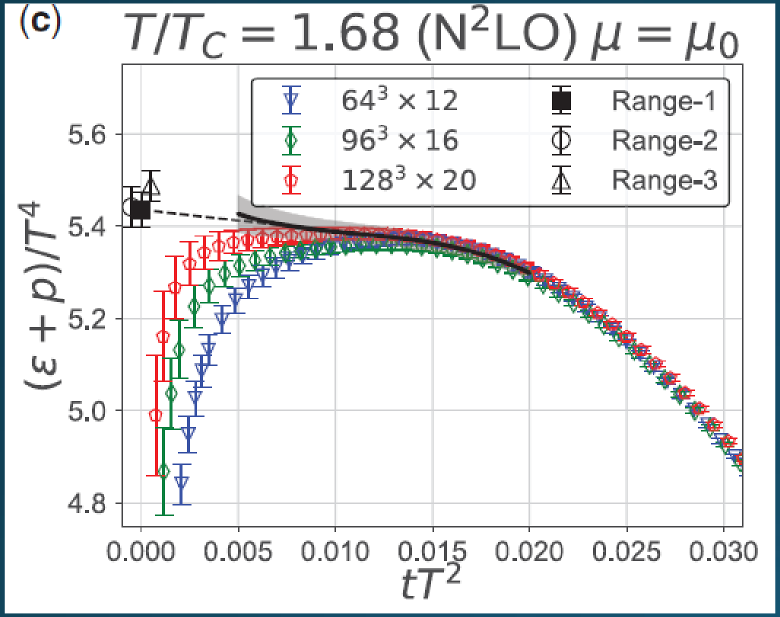
Previous: $\mu_d(t) = 1/\sqrt{8t}$

Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E}t}$

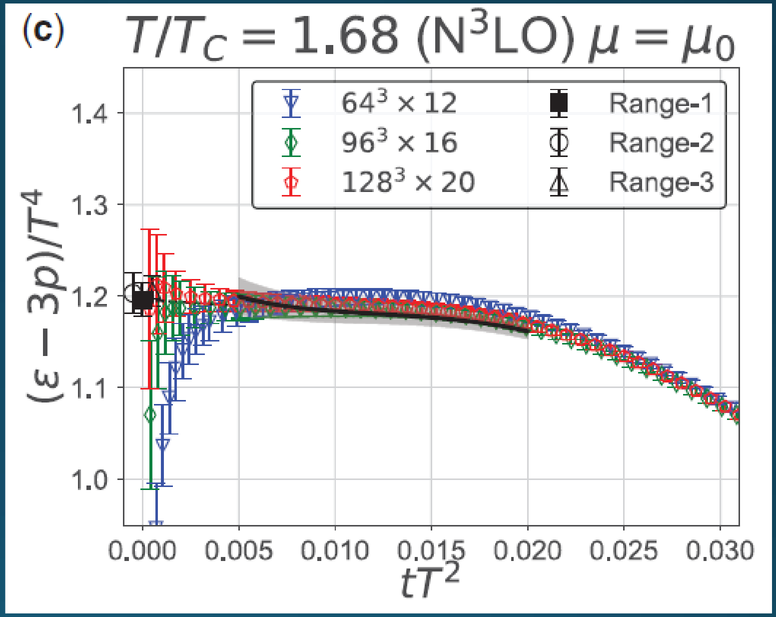
Harlander+ (2018)

t Dependence: SU(3) YM

$\langle T_{00} \rangle + \langle T_{11} \rangle$



$\langle T_{00} \rangle - 3\langle T_{11} \rangle$

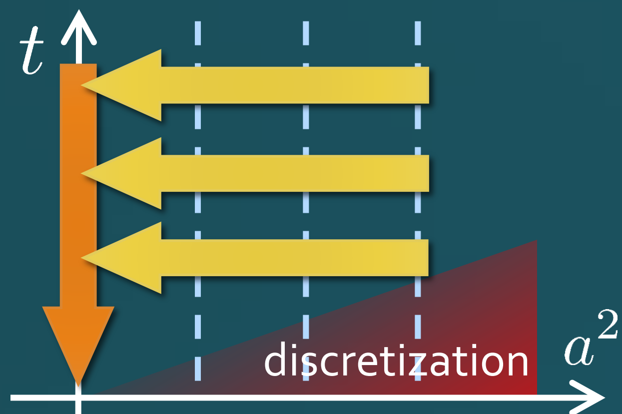


Iritani, MK, Suzuki, Takaura, PTEP 2019

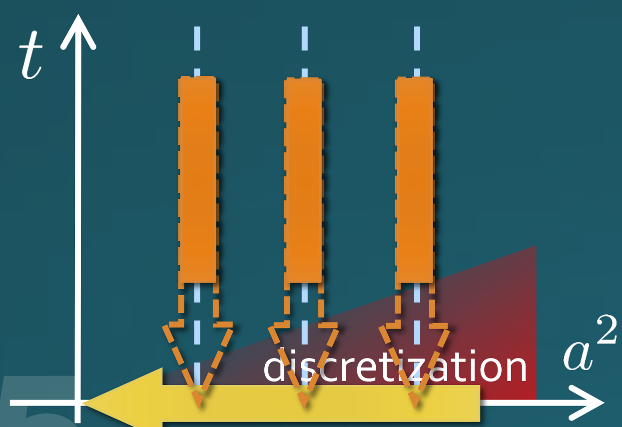
- Existence of "linear window" at intermediate t

Double Extrapolation

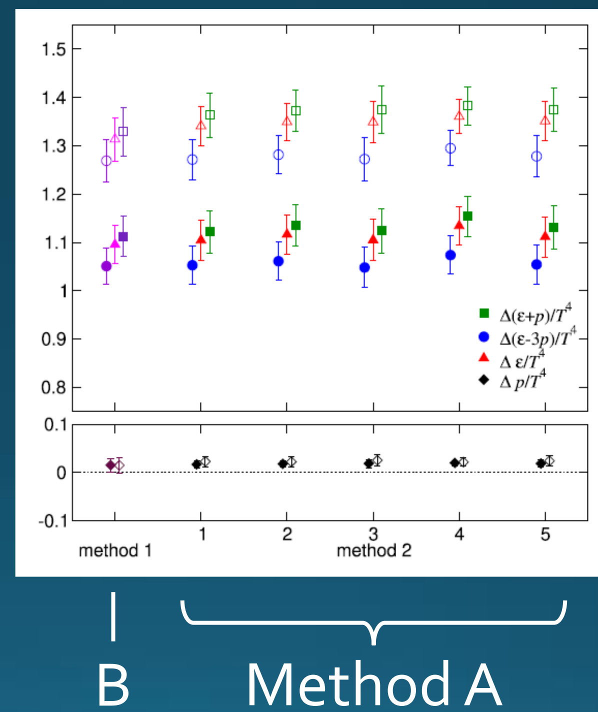
Method A: $a \to 0, \ t \to 0$



Method B: $t \to 0, \ a \to 0$



- **Consistency check**
  - latent heat & pressure gap

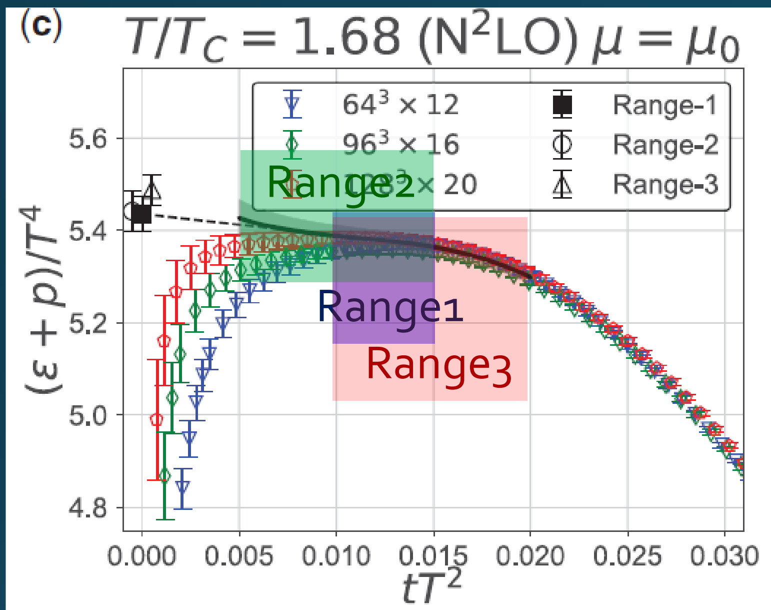


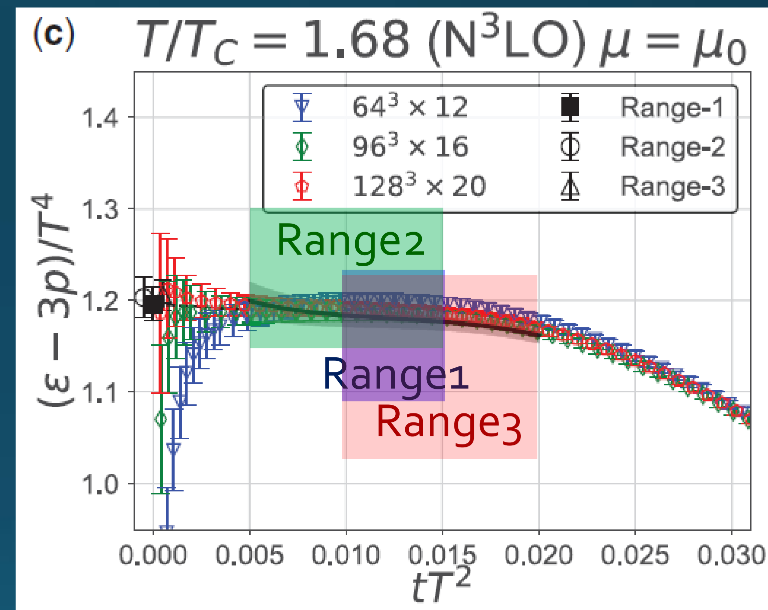
Shirogane+, 2021 (WHOT-QCD)

Consistent result for two methods

t Dependence: SU(3) YM

$\langle T_{00}\rangle + \langle T_{11}\rangle$ $\langle T_{00}\rangle - 3\langle T_{11}\rangle$



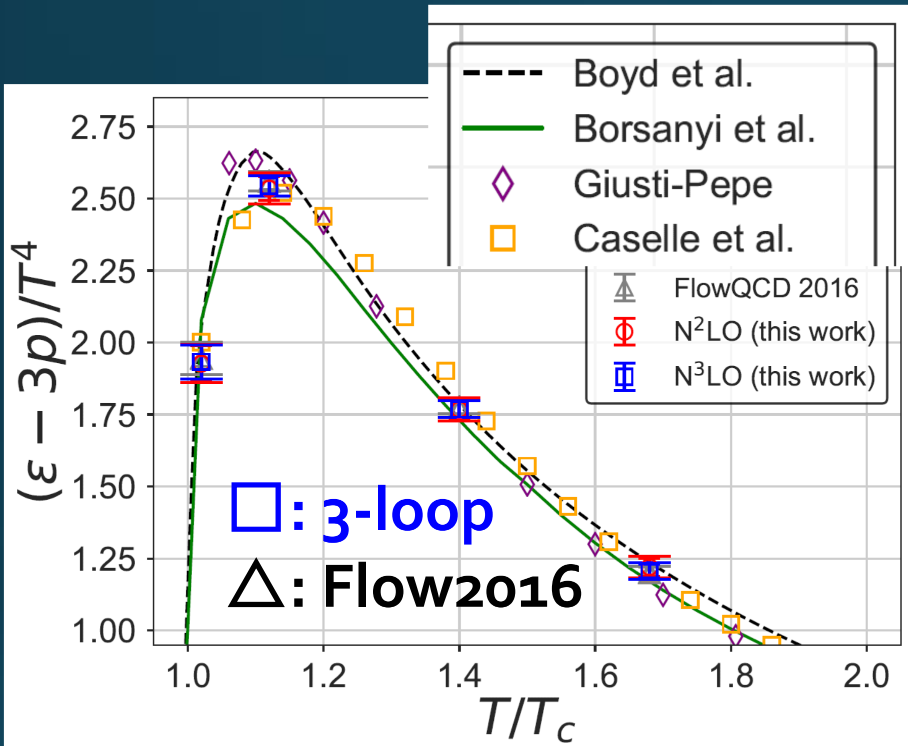


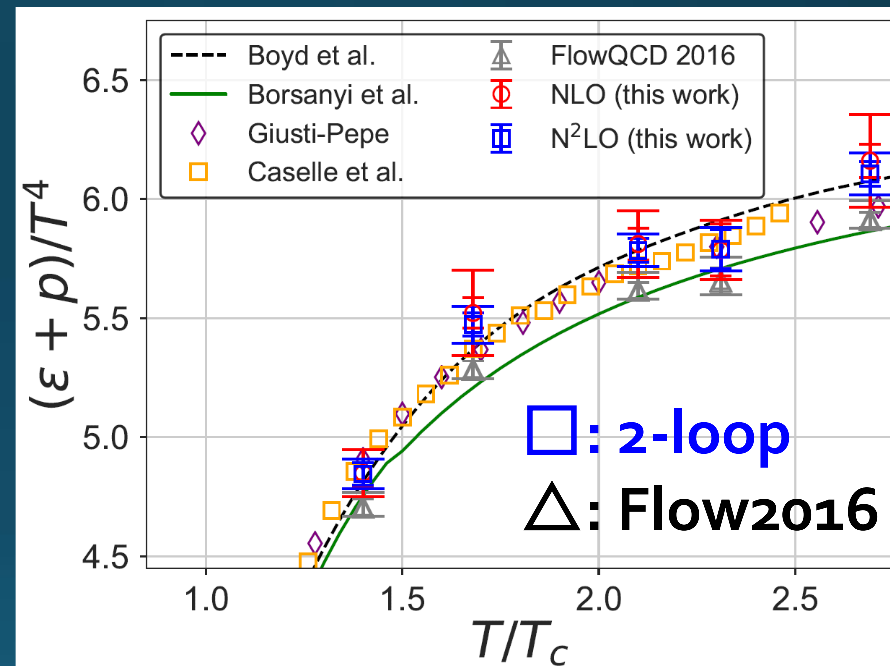
Iritani, MK, Suzuki, Takaura, PTEP 2019
Suzuki, Takaura 2021

- ☐ Existence of “linear window” at intermediate t
- ☐ Stable t⟶0 extrapolation
- ☐ Systematic errors: fit range, uncertaintyof Λ ($\pm$3%), …

T Dependence: Comparison

Iritani, MK, Suzuki, Takaura, 2019



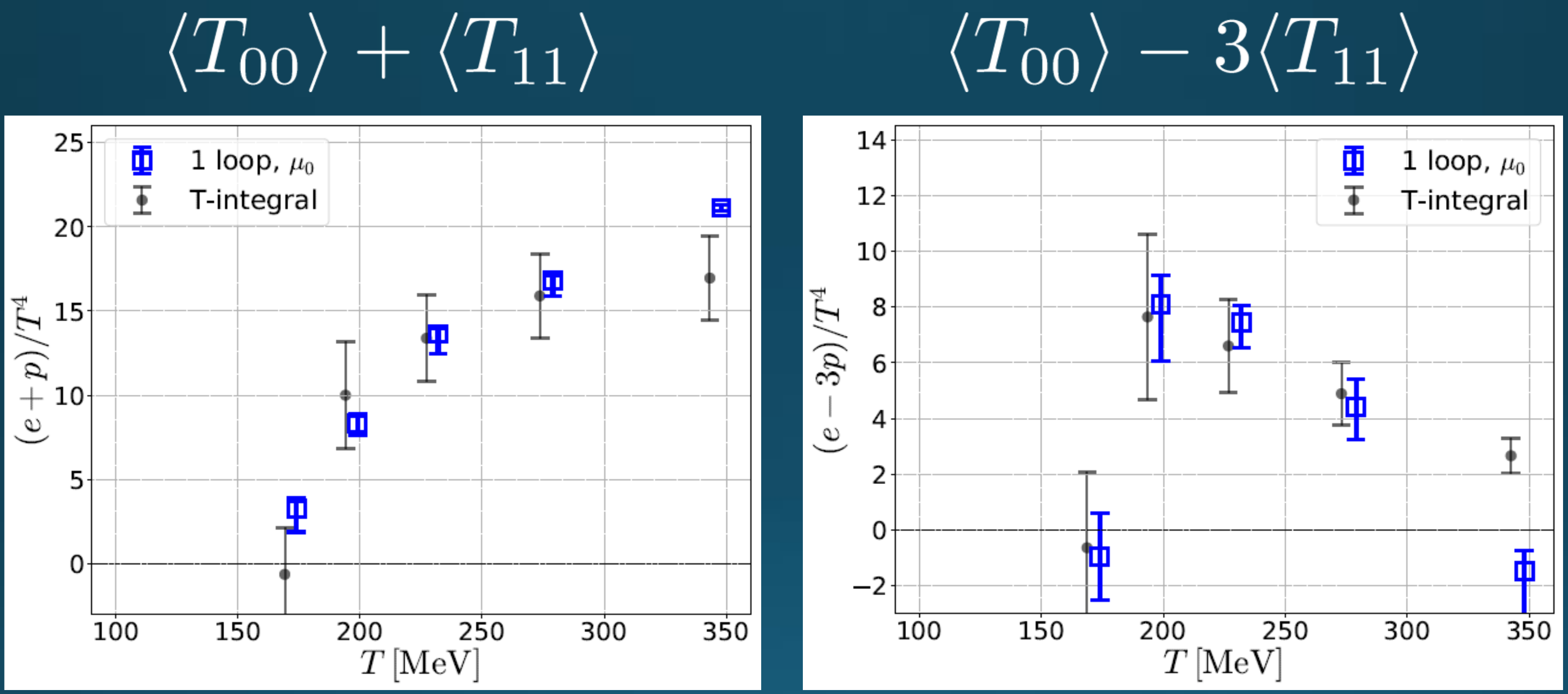


Systematic error: μ_0 or μ_d, Λ, t→0 function, fit range

- ☐ Good agreement with other methods!
- ☐ Smaller statistics thanks to smearing by the flow

2+1 QCD EoS from SF*t*X Method

WHOT-QCD, PRD**96** (2017); PRD**102** (2020)



- ☐ Agreement with integral method
- ☐ Substantial suppression of statistical errors

$m_{PS}/m_V \approx 0.63$

Physical mass: Kanaya+ (WHOT-QCD), 1910.13036

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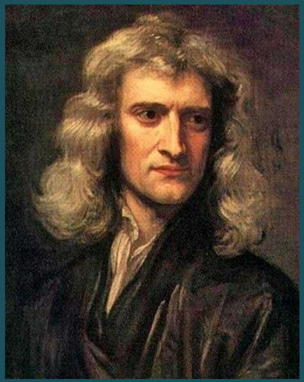
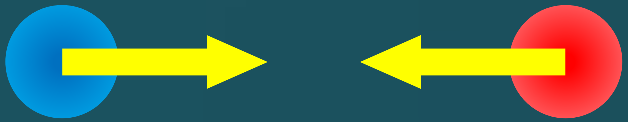
Force

Action-at-a-distance



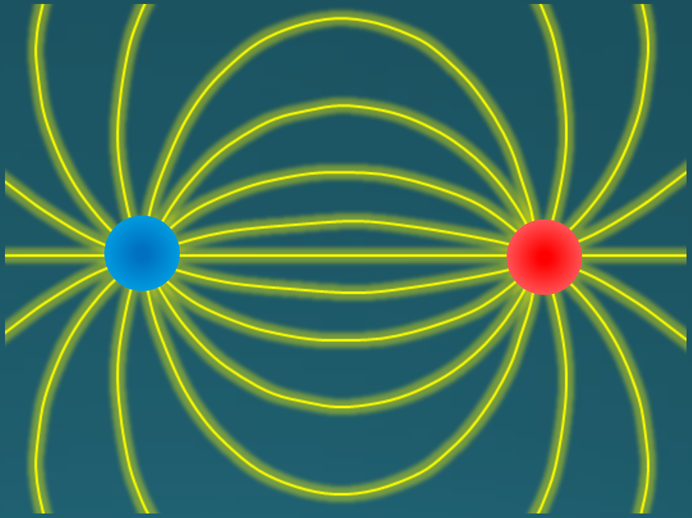
Newton
1687



$$F = -G\frac{m_1 m_2}{r^2} \qquad F = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Local interaction

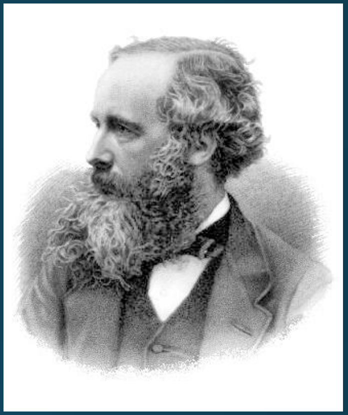
Faraday
1839



Maxwell Stress

(in Maxwell Theory)

$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

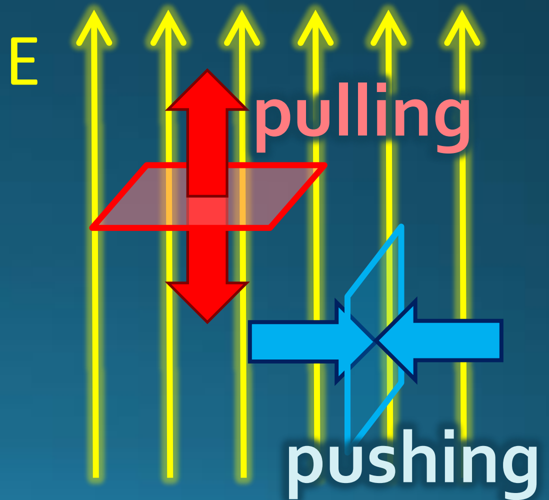


Maxwell

$$\vec{E} = (E, 0, 0)$$

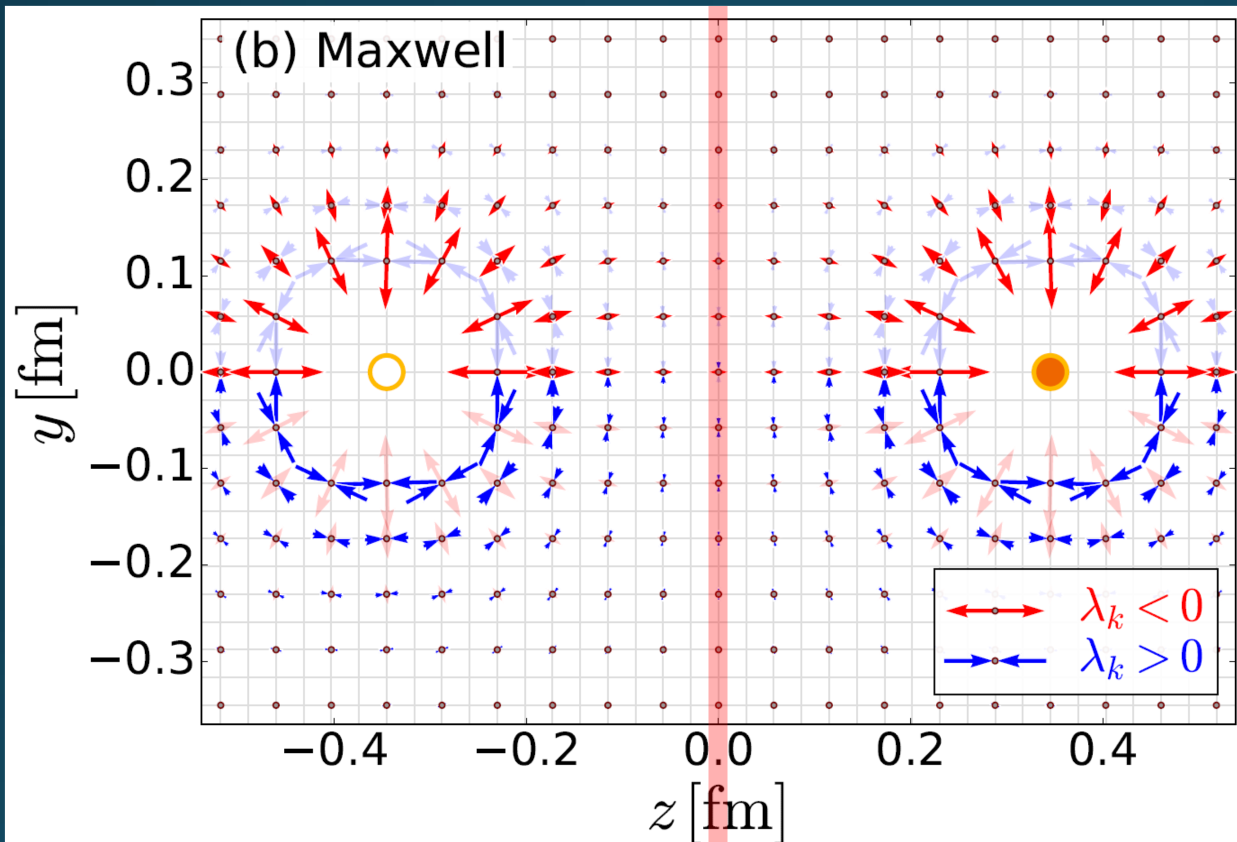
$$T_{ij} = \begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: **Pulling**
- Vertical to field: **Pushing**



Maxwell Stress

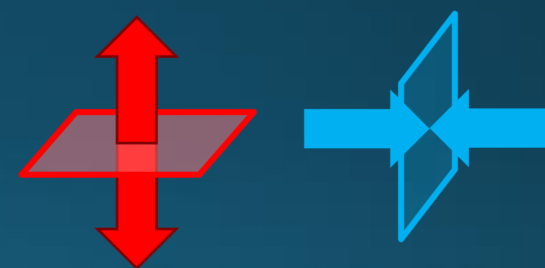
(in Maxwell Theory)



$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$$(k = 1, 2, 3)$$

length: $\sqrt{|\lambda_k|}$



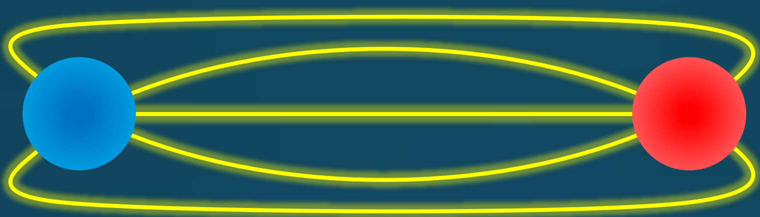
pulling pushing

Definite physical meaning

- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction

Quark-Anti-quark System

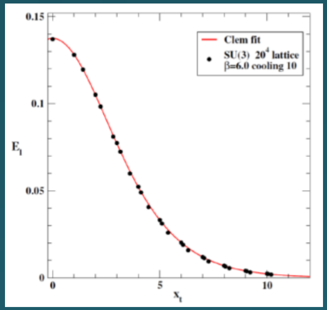
Formation of the flux tube → confinement



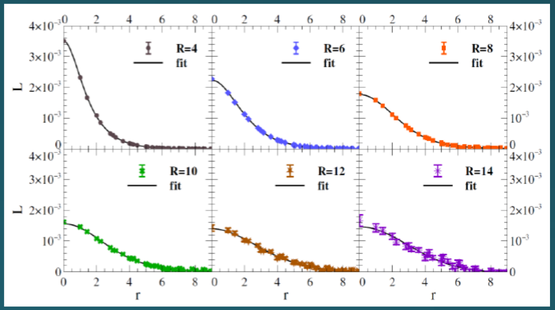
Previous Studies on Flux Tube

- ☐ Potential
- ☐ Action density
- ☐ Color-electric field

so many studies…



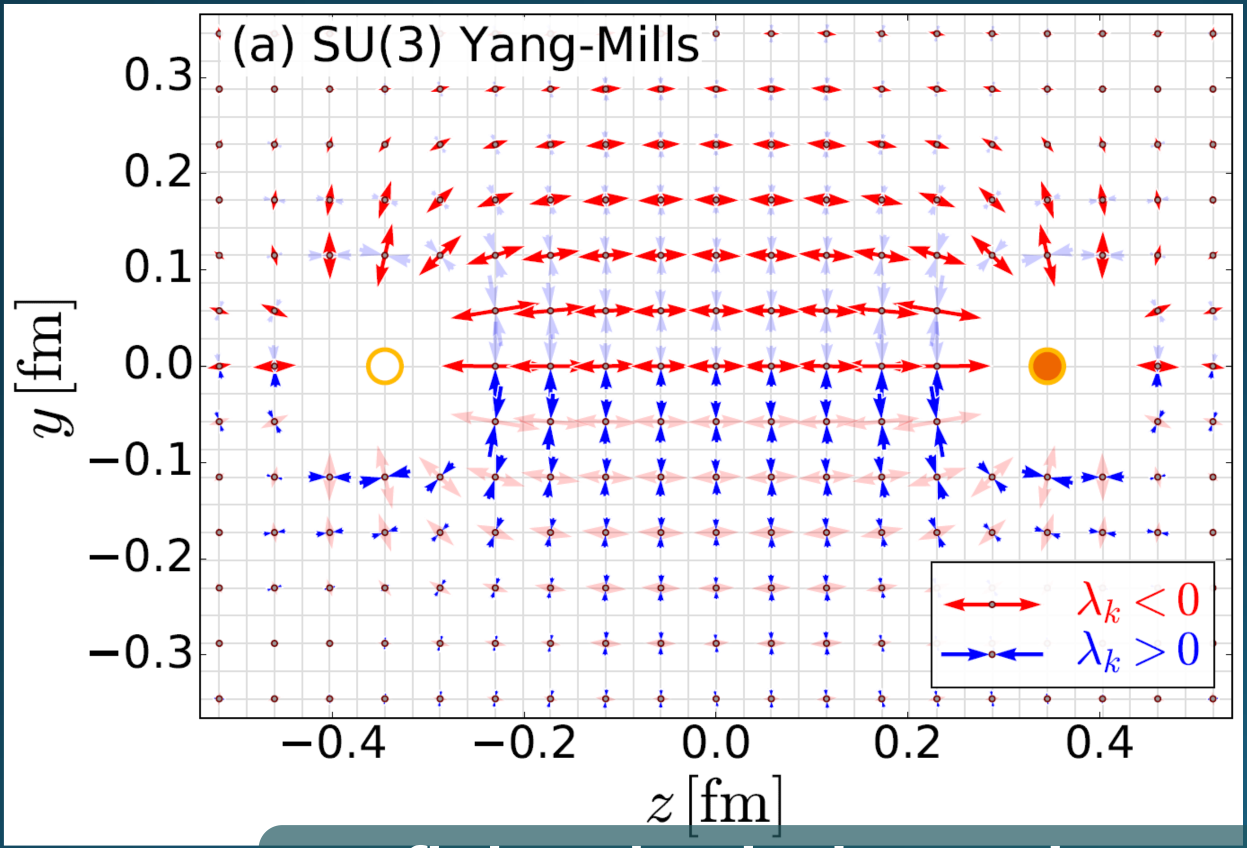
Cea+ (2012)



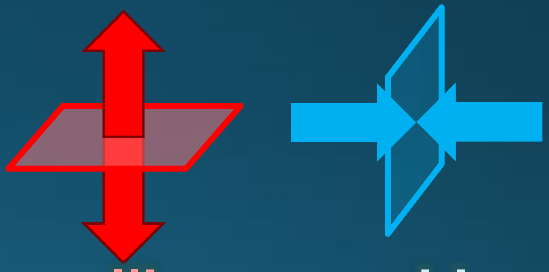
Cardoso+ (2013)

Stress Tensor in $Q\bar{Q}$ System

FlowQCD, PLB (2019)



Lattice simulation
SU(3) Yang-Mills
a=0.029 fm
R=0.69 fm
t/a^2=2.0

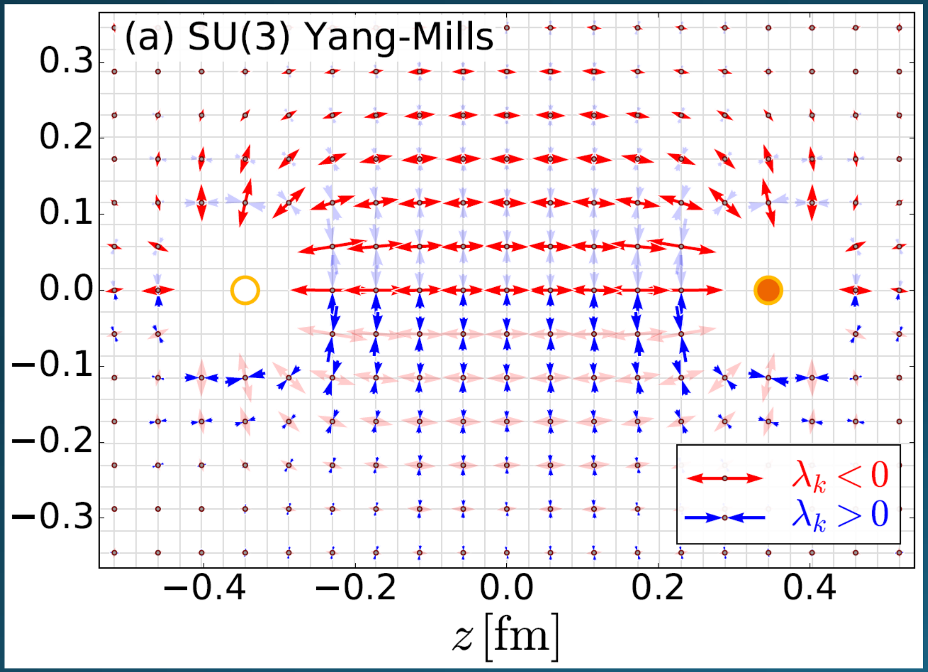


Definite physical meaning

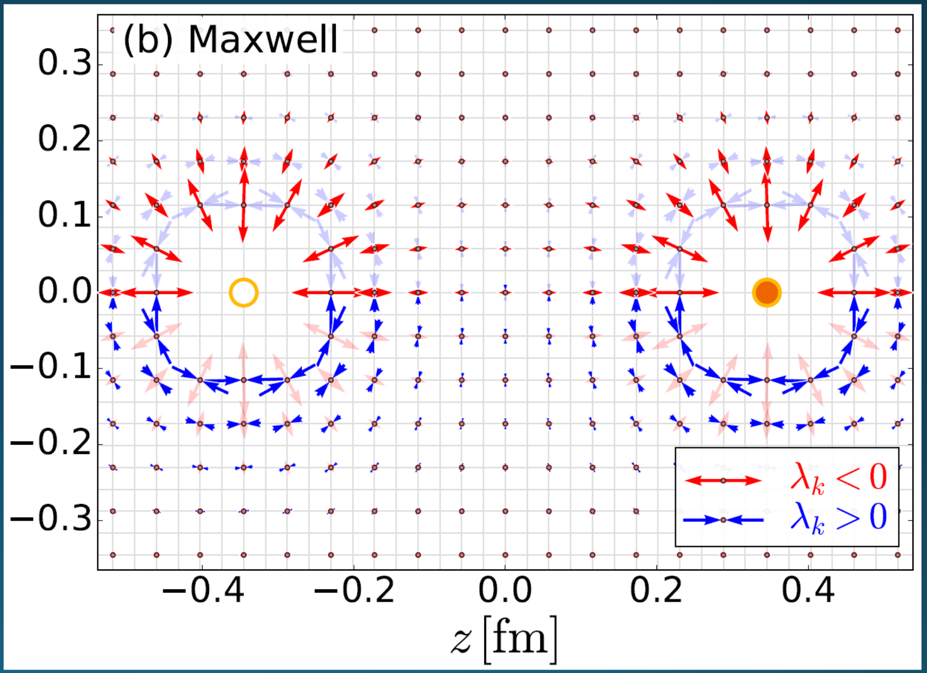
- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction
- ☐ Manifestly gauge invariant

SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)



Maxwell
(classical)



Propagation of the force is clearly different in YM and Maxwell theories!

Stress Distribution on Mid-Plane

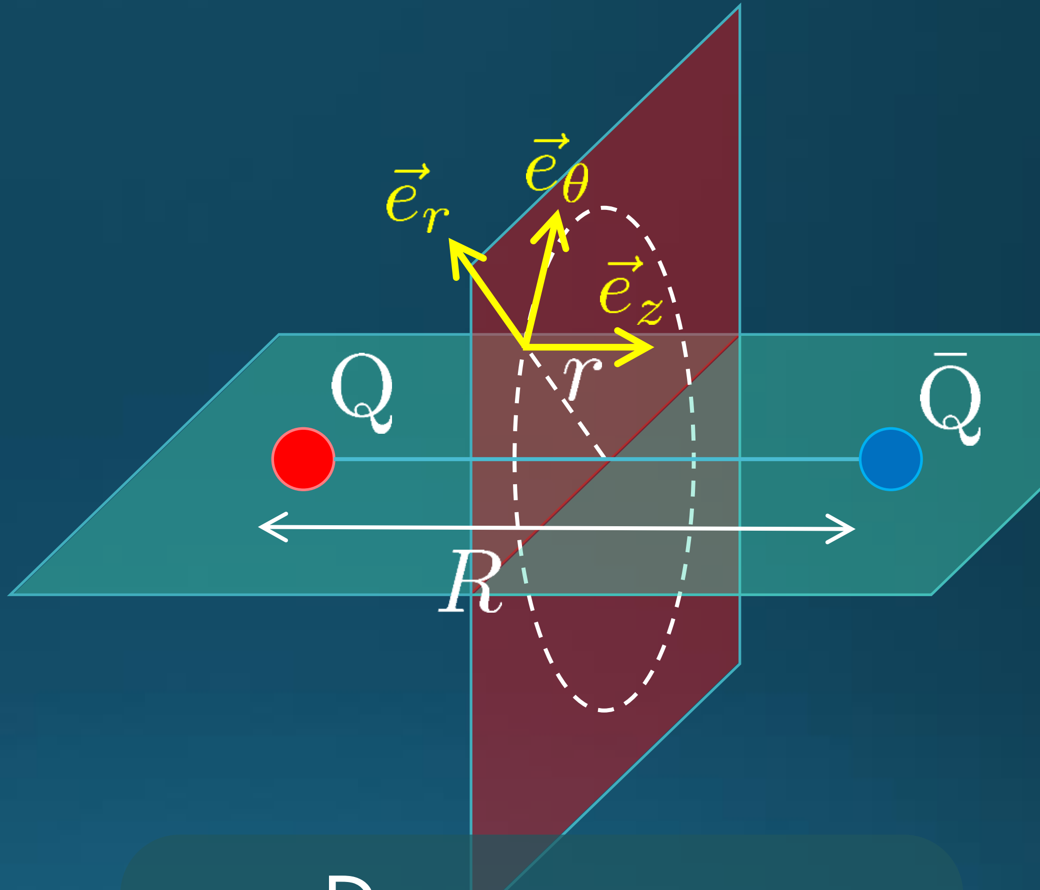
From rotational symm. & parity

EMT is diagonalized
in Cylindrical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{zz} & \\ & & & T_{44} \end{pmatrix}$$

$$T_{rr} = \vec{e}_r^{\,T} T \vec{e}_r$$

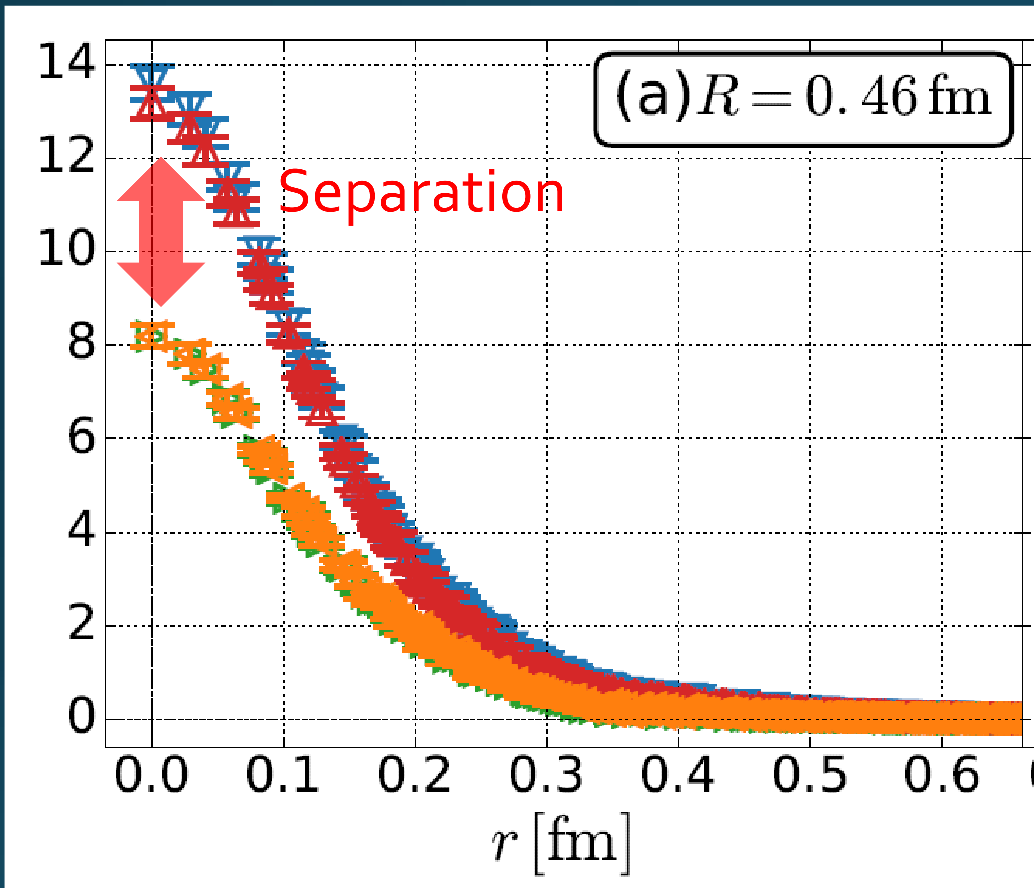
$$T_{\theta\theta} = \vec{e}_\theta^{\,T} T \vec{e}_\theta$$

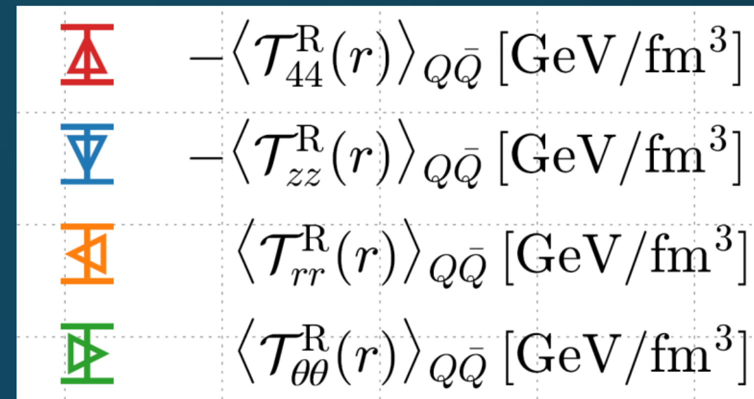


Degeneracy
in Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

Mid-Plane





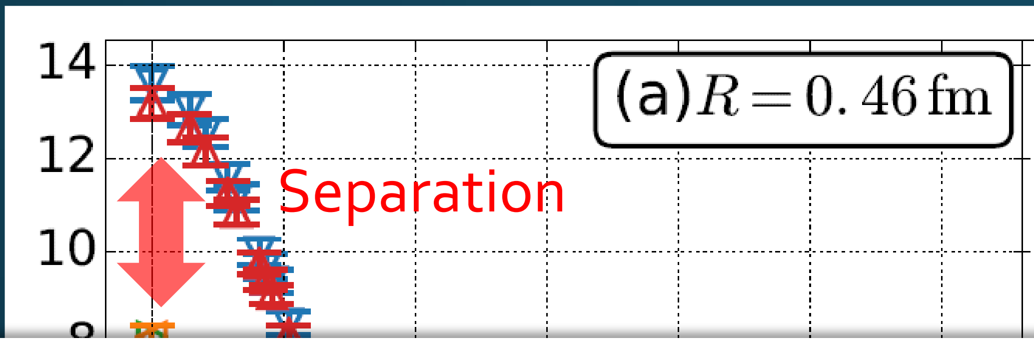
Continuum Extrapolated!

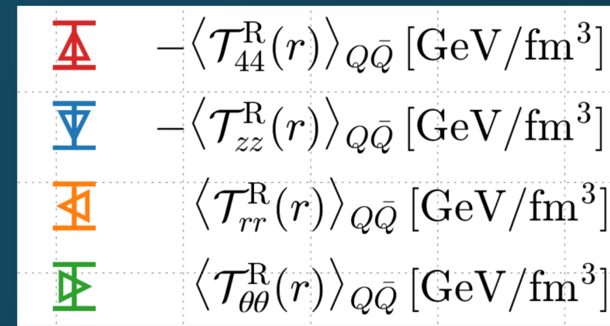
In Maxwell theory

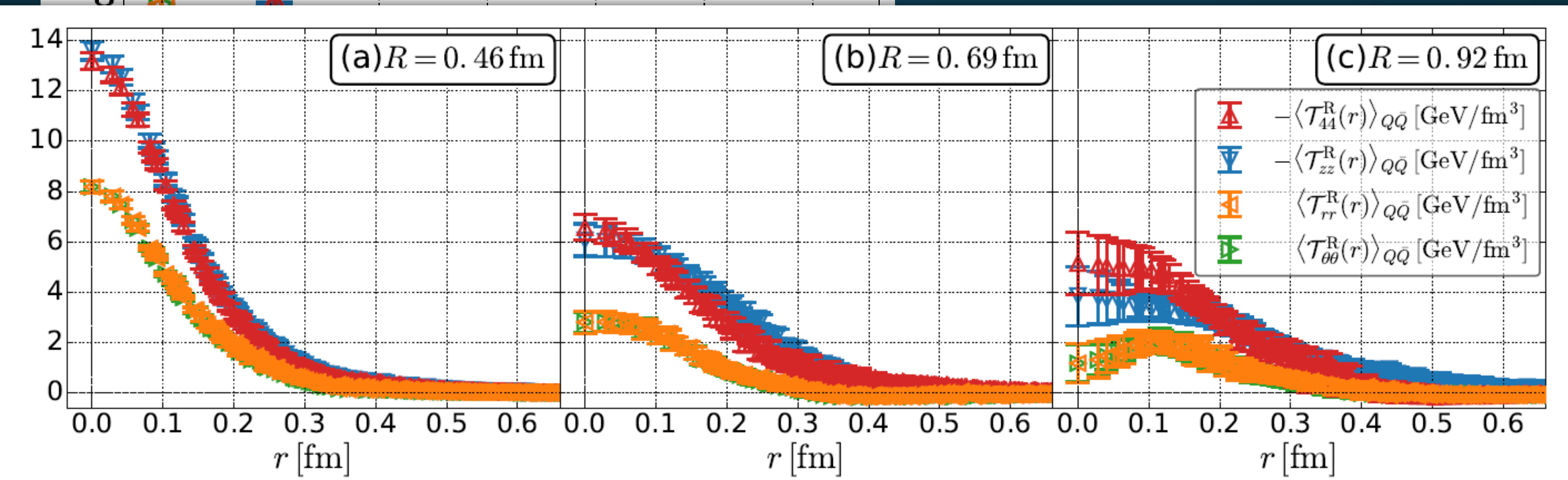
$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

- Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- Separation: $T_{zz} \neq T_{rr}$
- Nonzero trace anomaly $\sum T_{cc} \neq 0$

Mid-Plane







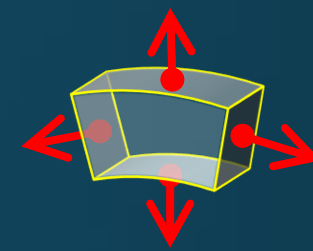
- ☐ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- ☐ Separation: $T_{zz} \neq T_{rr}$
- ☐ Nonzero trace anomaly $\sum T_{cc} \neq 0$

Momentum Conservation

Yanagihara, MK, PTEP2019

- In cylindrical coordinats,

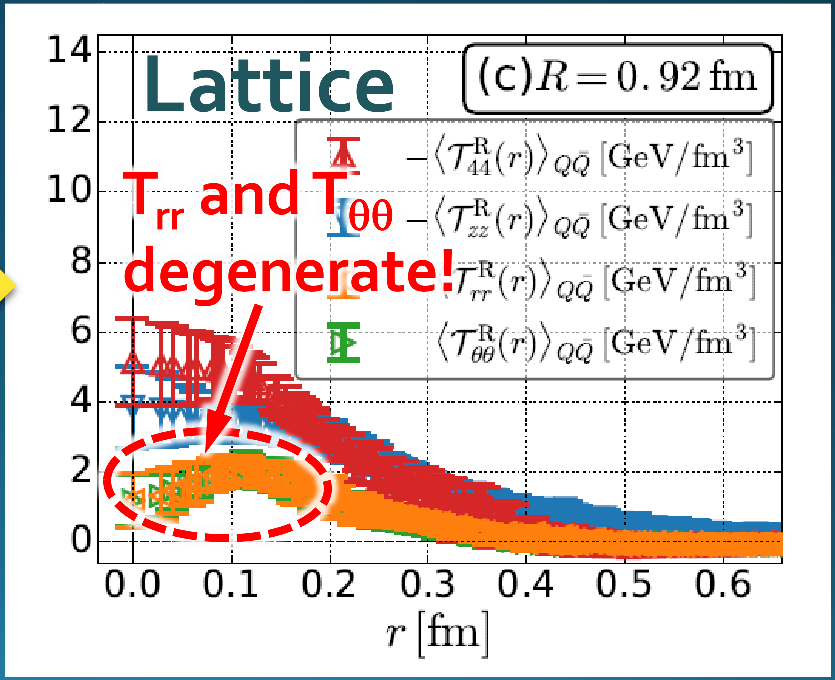
$$\partial_i T_{ij} = 0 \Rightarrow \partial_r (r T_{rr}) = T_{\theta\theta} - r \partial_z T_{rz}$$



- For infinitely-long flux tube

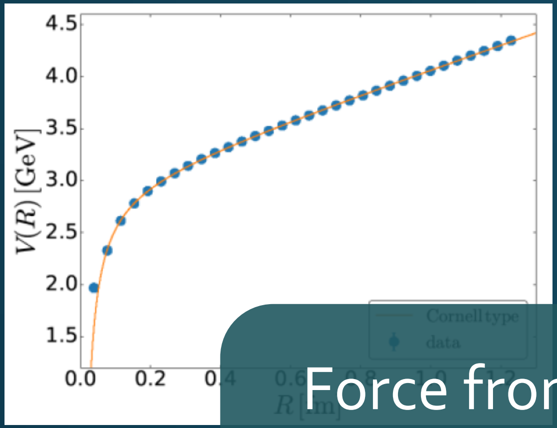
$$\partial_r (r T_{rr}) = T_{\theta\theta}$$

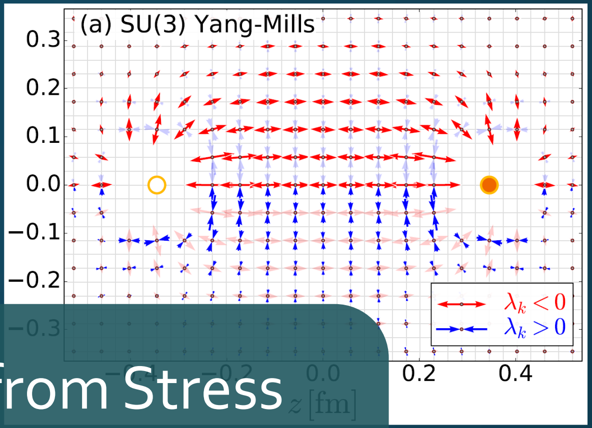
⇒ **T_{rr} and $T_{\theta\theta}$ must separate!** ⇔



Effect of boundaries is not negligible at R=0.92fm

Force





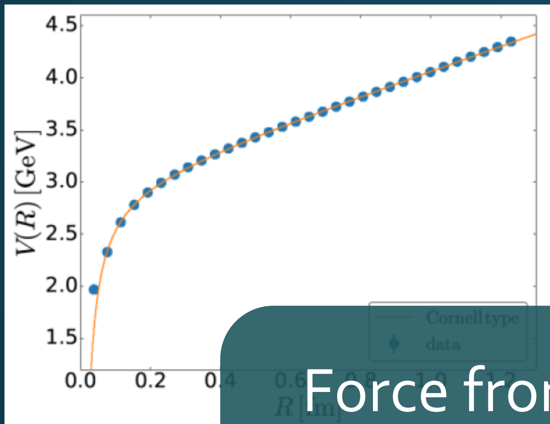
Force from Potential

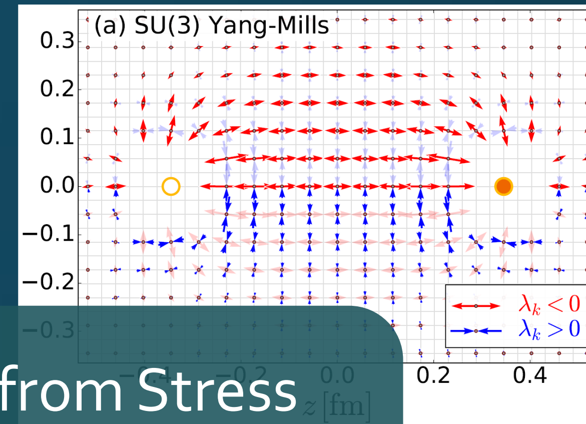
$$F_{\text{pot}} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\text{stress}} = \int_{\text{mid.}} d^2x T_{zz}(x)$$

Force





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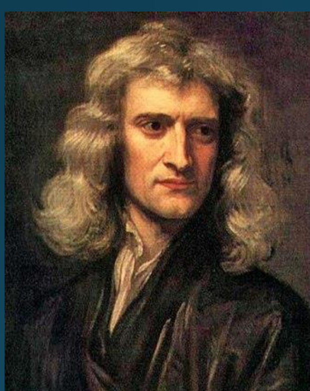
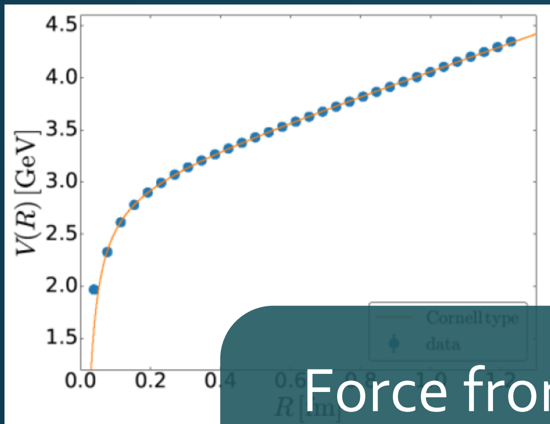
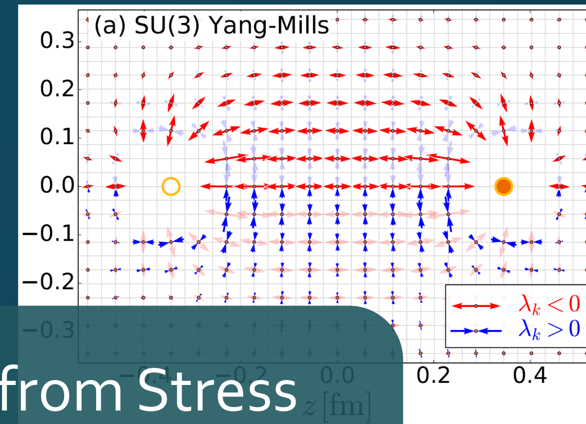


Newton
1687

Faraday
1839

Force





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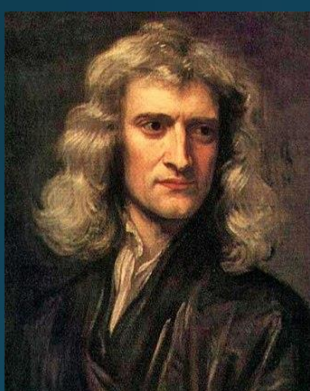
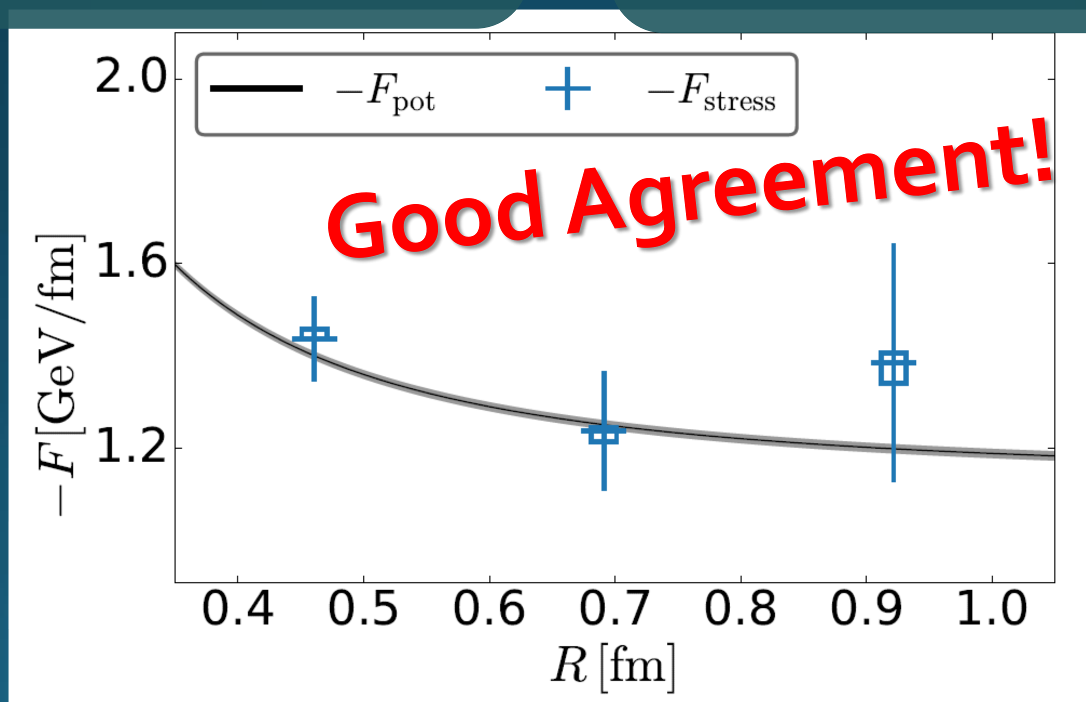


Newton
1687

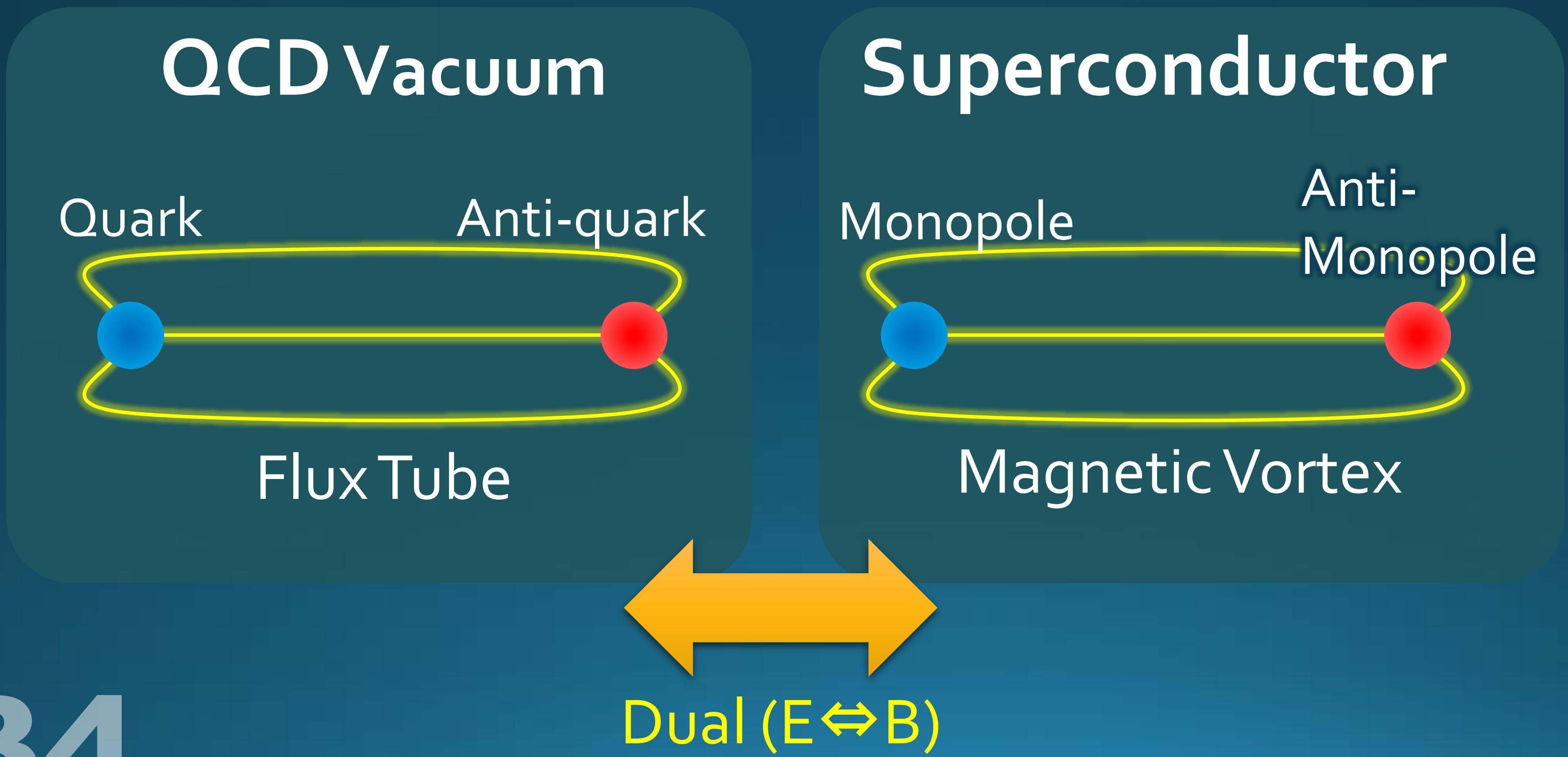


Faraday
1839

Contents

Dual Superconductor Picture

Nambu, 1970
Nielsen, Olesen, 1973
t 'Hooft, 1981
...



Abelian-Higgs Model

Yanagihara, MK, 2019

Abelian-Higgs Model

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

GL parameter: $\kappa = \sqrt{\lambda}/g$

- type-I : $\kappa < 1/\sqrt{2}$
- type-II : $\kappa > 1/\sqrt{2}$
- Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

Infinitely long tube

- degeneracy

  $T_{zz}(r) = T_{44}(r)$ Luscher, 1981
- momentum conservation

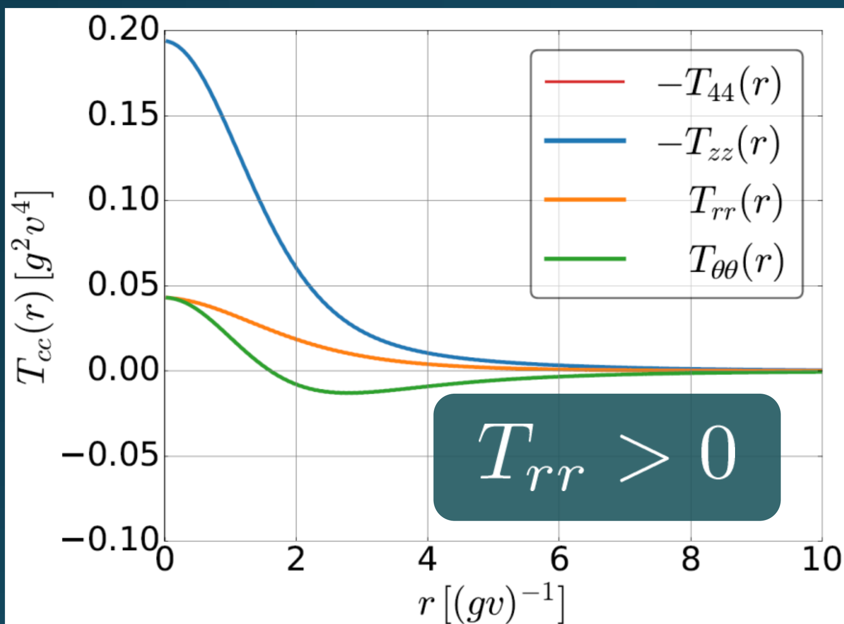
  $$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

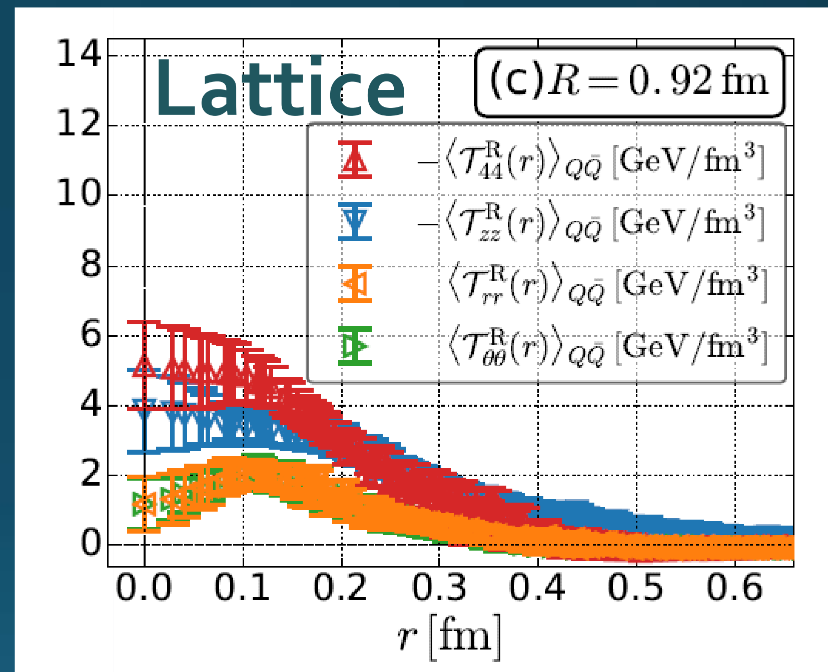
Stress Tensor in AH Model

infinitely-long flux tube

Type-I

$\kappa = 0.1$





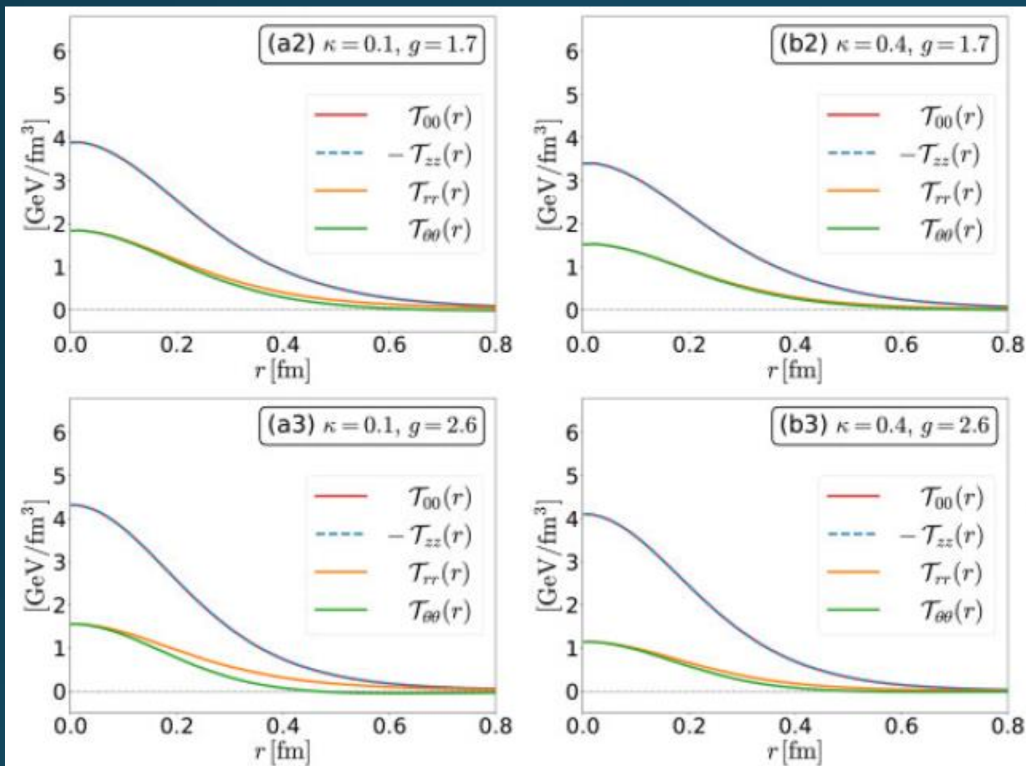
- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

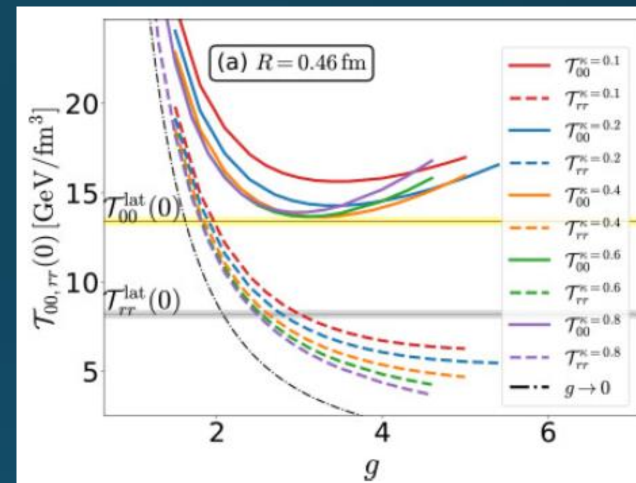
Inconsistent with lattice result

$$T_{rr} \simeq T_{\theta\theta}$$

Flux Tube with Finite Length

Yanagihara, MK (2019)





- ☐ AH model can reproduce lattice results **qualitatively** by tuning parameters.
- ☐ But, **quantitatively** all parameters are rejected.

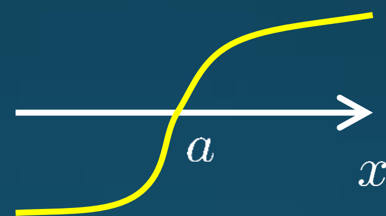
Contents

Soliton in 1+1d

ϕ^4 Theory $\quad \mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi), \quad V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2$

☐ Soliton (kink)

$$\phi_{\rm cl}(x) = \frac{m}{\sqrt{\lambda}} \tanh \frac{m(x-a)}{\sqrt{2}}$$



☐ EMT around a soliton (classical)

$$T_{00}^{\rm cl}(x) = \lambda m \left(\frac{m^2}{\lambda} - \phi_{\rm cl}(x)^2\right)^2 \Rightarrow M_{\rm cl} = \int dx T_{00}^{\rm cl}(x) = \frac{2\sqrt{2}}{3}\frac{m^3}{\lambda}$$

$$T_{01}^{\rm cl}(x) = T_{11}^{\rm cl}(x) = 0$$

How is the EMT distribution modified by the quantum effect?

cf) energy density: Goldhaber+ (2003)

Quantum Correction

Fluctuations around $\phi_{cl}(x)$

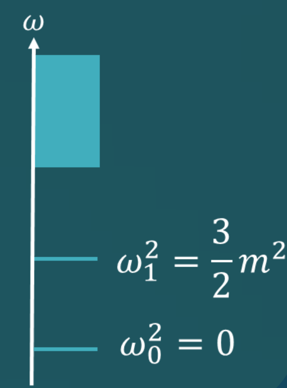
$$\phi(x) = \phi_{\rm cl}(x) + \eta(x)$$

$$V(\phi) = V(\phi_{\rm cl}) + \frac{1}{2}\eta\big(- \partial_x^2 - m^2 + 3m^2 \tanh^2 (mx/\sqrt{2})\big)\eta$$

Eigenmodes: $\psi_0(x) = \partial_x \phi_{\rm cl}(x)$ $\omega_0 = 0$

$\psi_1(x)$ $\omega_1 = \sqrt{3/2}m$

$\psi_k(x)$:continuum



- Total Energy: $E = E_{\rm cl} + \sum_n \omega_n$ Dashen, Hasslacher, Neveu (1974)

Quantum Correction to Total Energy

$$E = E_{\mathrm{cl}} + \sum_n \omega_n$$

Dashen, Hasslacher, Neveu (1974)
Gervais, Gevicki, Sakita (1975)
...
Rebhan, Nieuwenhuizen (1997)
Shifman, Vainshtein, Voloshin (1999)
Goldhaber, Litvinsev, Nieuwenhuizen (2003)
...

- **Vacuum subtraction**
  - finite box with length L
  - fixed mode number
  - $L \to \infty$ at the end

- **Renormalization**

  mass renorm. only $m^2 \to m^2 + \delta m^2$

Final result:

$$E = \underbrace{\frac{2\sqrt{2}}{3}\frac{m^3}{\lambda}}_{\text{classical (order } \lambda^{-1})} + \underbrace{m\left(\frac{1}{6}\frac{\sqrt{3}}{\sqrt{2}} - \frac{3}{\pi\sqrt{2}}\right)}_{\text{quantum (1-loop)}} + \mathcal{O}(\lambda)$$

Collective Coordinate Method

Gervais, Jevicki, Sakita (1975)
Goldstone and Jackiw (1975)
Tomboulis (1975)
Christ, Lee (1975)

Eliminate translational zero mode
in favor of the collective coordinate $\hat{X}$

$$\phi(x) = \sum_{n=0}^{\infty} \hat{c}_n \psi_n(x)$$

$$\Rightarrow \phi(x) = \phi_{\rm cl}(x - \hat{X}) + \sum_{n=1}^{\infty} \hat{c}_n \psi_n(x - \hat{X})$$

- ☐ Elimination of the zero mode
- ☐ Obvious translational symmetry
- ☐ Lorentz symmetry

⇒

EMT Conservation

$$\partial_\mu T^{\mu\nu} = 0$$

For static systems

$$\partial_1 T^{11} = 0$$

- ☐ Matrix element

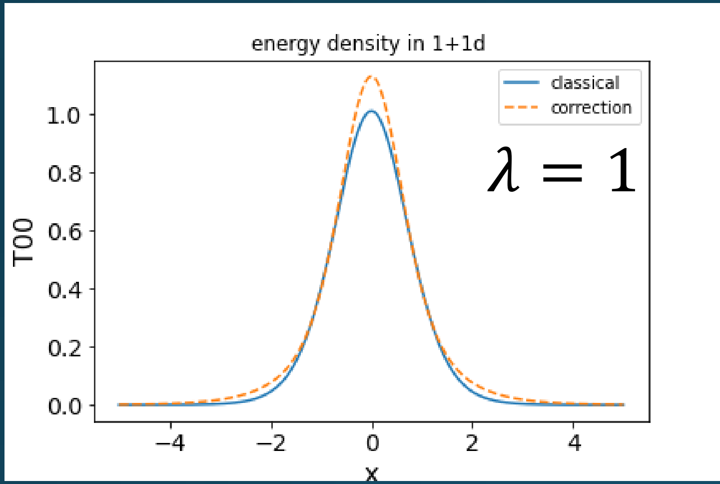
$$\langle P' | \hat{T}_{\mu\nu}(0) | P \rangle = T^{\rm com}_{\mu\nu}(x) e^{-i(P'-P)x}$$

EMT Distr. at 1-loop Order

Ito, MK, in prep.

□ Finite EMT Correction

$T_{00}(x)$



- □ EMT conservation (analytic)

$$\partial_x T_{11}(x) = 0$$

□ **Subtlety**: appearance of spatially uniform component

$$T_{00}(x) = \frac{(\text{const.})}{L} + T_{00}(x)_{L\to\infty}$$

$$T_{11}(x) = \frac{(\text{const.})}{L}$$

L: length of box (anti-pBC)

Contents

1. Constructing EMT
   - SFtX method via gradient flow

2. EMT distribution around Flux Tube
   2-1 Lattice result

FlowQCD, PL**B789**, 210 (2019)

   2-2 Abelian-Higgs model

Yanagihara, MK, PTEP**2019**, 093B02 (2019)

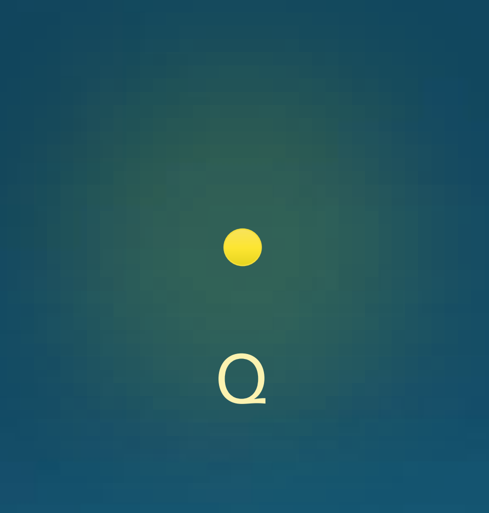
3. GFF of soliton in 1+1d ϕ^4 model

Ito, MK, in preparation

4. Single-quark system

FlowQCD, PRD102, 114522 (2020)

Stress Tensor around a Quark



Stress Tensor around a Quark

- **Deconfined phase** ← **This study**
  - screening property
  - running coupling

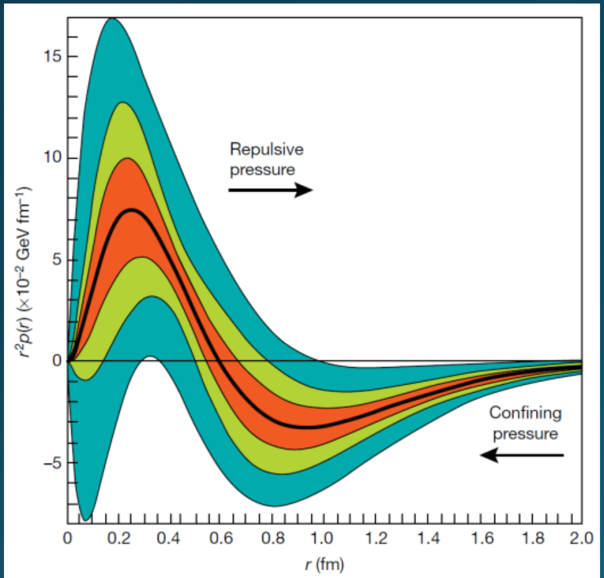
$$V(r) \sim g\frac{e^{-m_D r}}{r}$$

- **Vacuum**
  - heavy-light meson
- **$T \sim T_c$**
  - dissociation of heavy-light meson

Pressure inside Hadrons

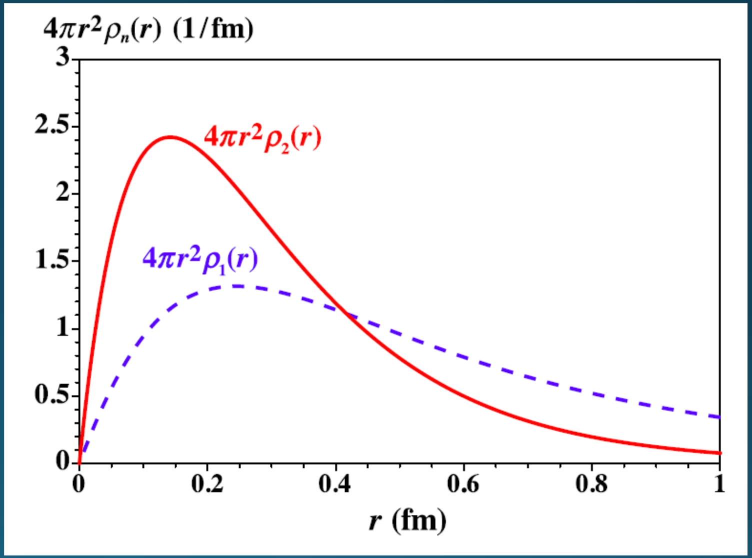
EMT distribution inside hadrons now accessible??

Pressure @ proton



Nature, 557, 396 (2018)
Shanahan, Detmold (2019)

EMT distribution @ pion



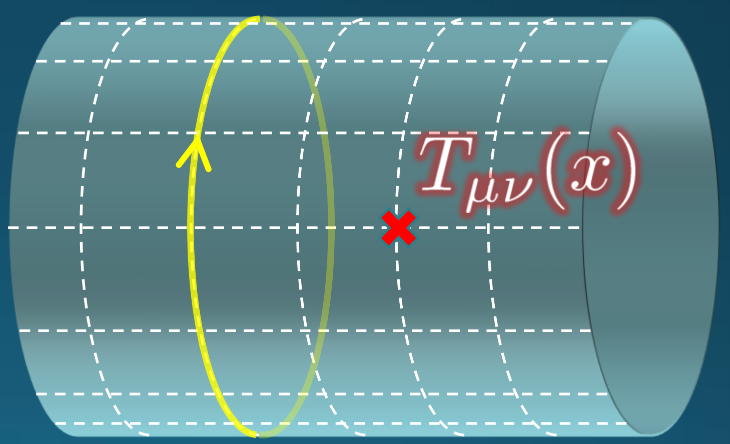
Kumano, Song, Teryaev (2018)

EMT Around a Static Q

$$\langle T_{\mu\nu}(x)\rangle_{\mathrm{Q}} = \frac{\langle T_{\mu\nu}(x)\mathrm{Tr}\Omega(0)\rangle}{\langle \mathrm{Tr}\Omega(0)\rangle} - \langle T_{\mu\nu}(x)\rangle$$

- EMT-Polyakov loop correlation
- Gauge invariant
- Z3 symmetry has to be broken

- EMT by SF*t*X method

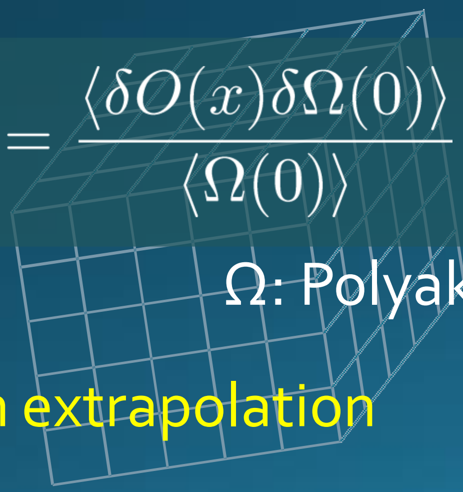


Lattice Setup

- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

- Analysis above Tc
- Simulation on a Z_3 minimum
- EMT around a Polyakov loop

$$\langle O(x)\rangle_{\mathrm{Q}} = \frac{\langle \delta O(x)\delta\Omega(0)\rangle}{\langle \Omega(0)\rangle}$$



Ω: Polyakov loop

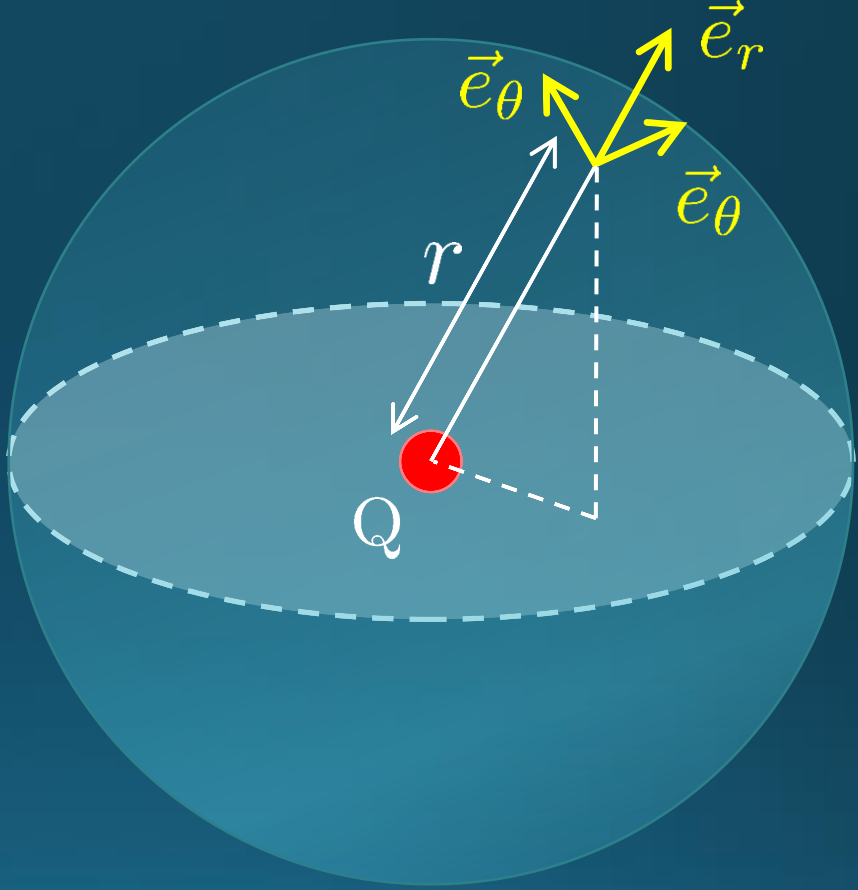
- continuum extrapolation

T/T_c	N_s	N_τ	β	a [fm]	N_{conf}
1.20	40	10	6.336	0.0551	500
	48	12	6.467	0.0460	650
	56	14	6.581	0.0394	840
	64	16	6.682	0.0344	1,000
	72	18	6.771	0.0306	1,000
1.44	40	10	6.465	0.0461	500
	48	12	6.600	0.0384	650
	56	14	6.716	0.0329	840
	64	16	6.819	0.0288	1,000
	72	18	6.910	0.0256	1,000
2.00	40	10	6.712	0.0331	500
	48	12	6.853	0.0275	650
	56	14	6.973	0.0236	840
	64	16	7.079	0.0207	1,000
	72	18	7.173	0.0184	1,000
2.60	40	10	6.914	0.0255	500
	48	12	7.058	0.0212	650
	56	14	7.182	0.0182	840
	64	16	7.290	0.0159	1,000
	72	18	7.387	0.0141	1,000

Spherical Coordinates

EMT is diagonalized
in Spherical Coordinates

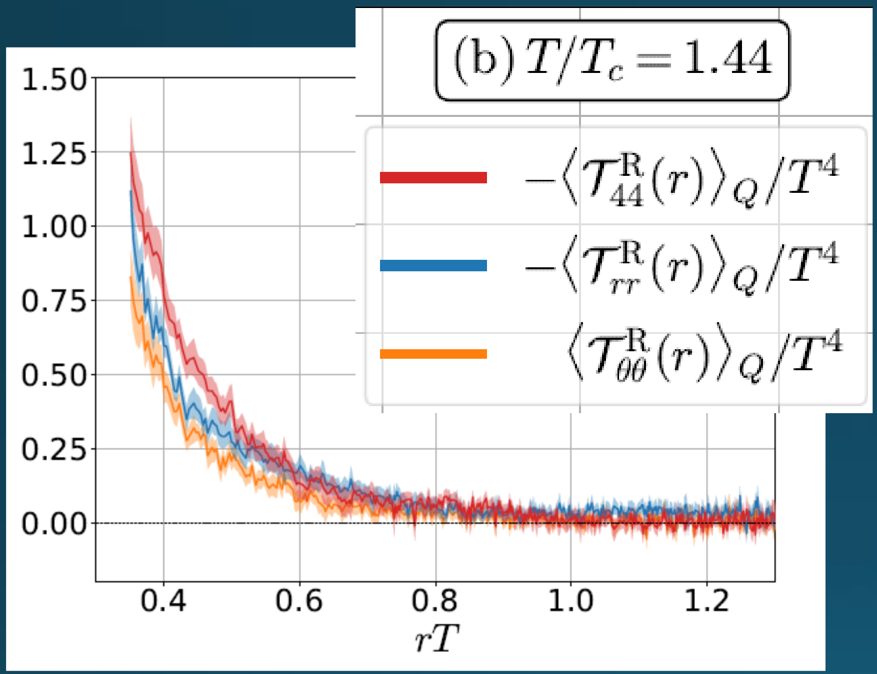
$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{\theta\theta} & \\ & & & T_{44} \end{pmatrix}$$



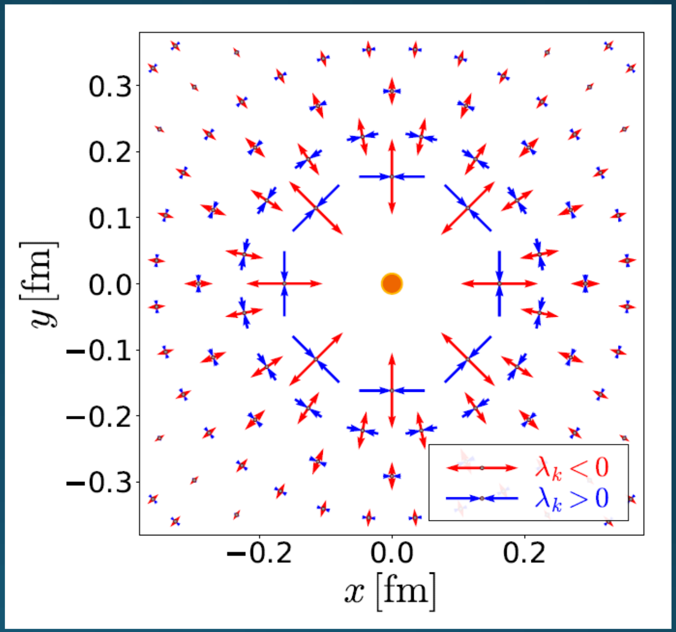
- Maxwell theory

$$T_{44} = T_{rr} = -T_{\theta\theta} = -\frac{|\boldsymbol{E}|^2}{2} = -\frac{\alpha}{8\pi}\frac{1}{r^4}$$

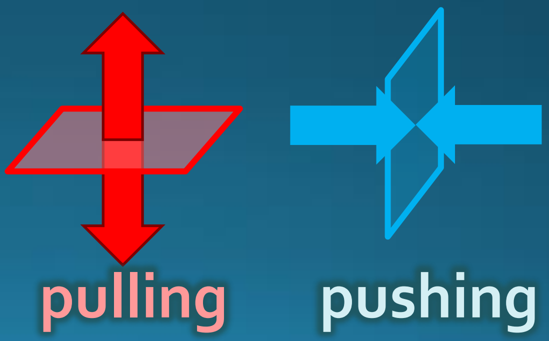
Stress Tensor Around a Quark



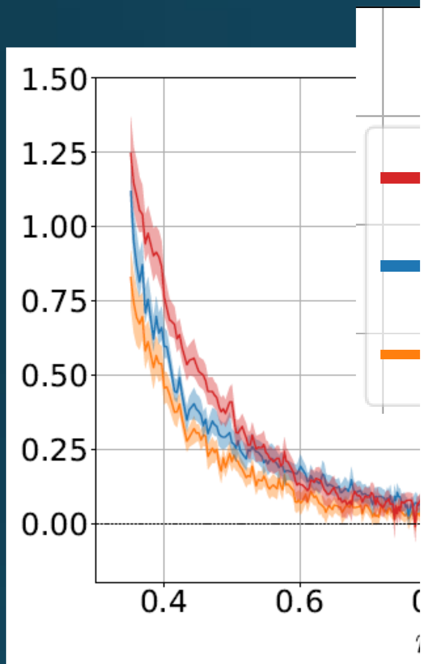
T=1.44T$_c$

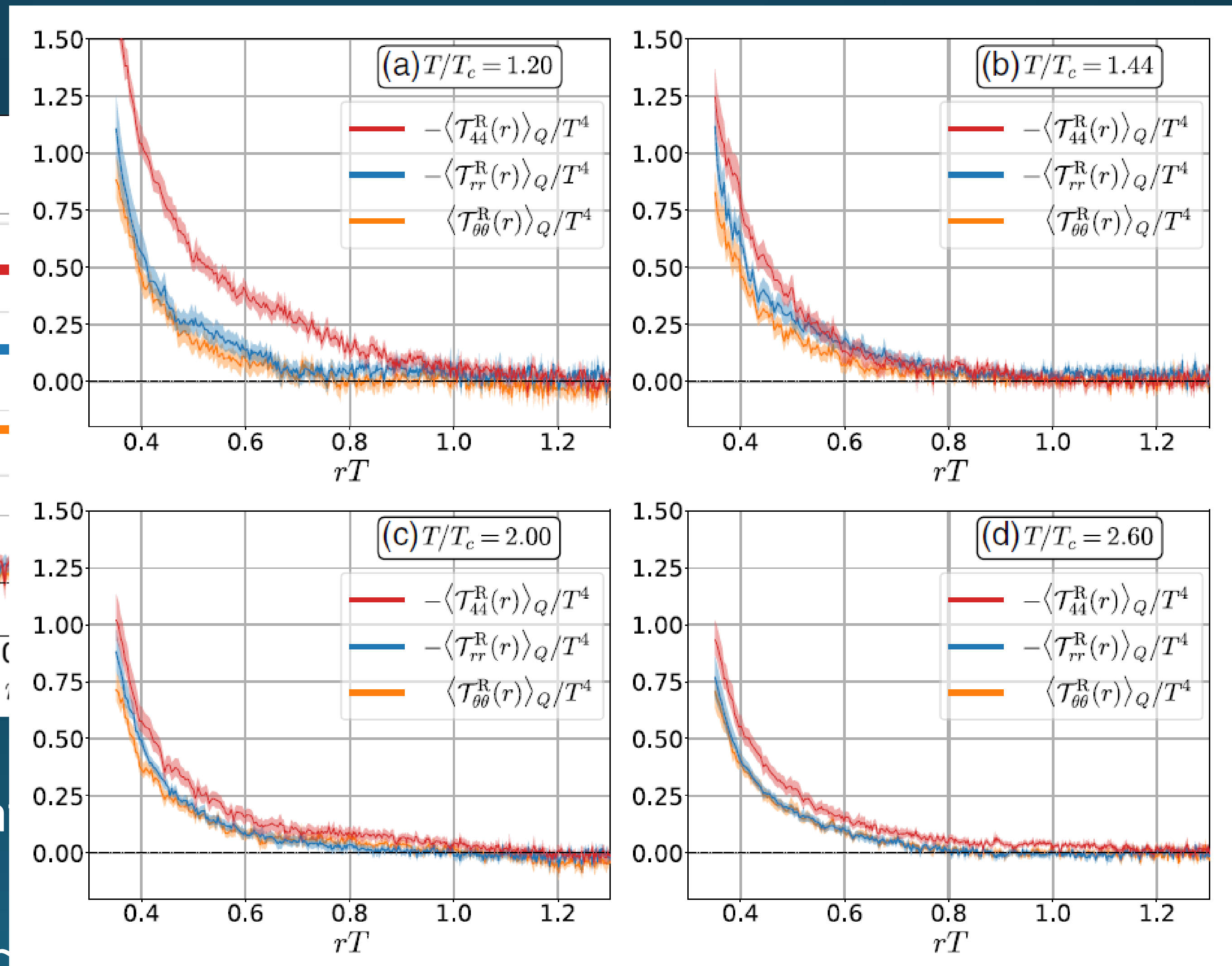


- Suppression at large distance
- Separation of different channels
- $|T_{44}| > |T_{rr}| \sim |T_{\theta\theta}|$



Stress Tensor Around a Quark





- ☐ Suppression a
- ☐ Separation of
- ☐ $|T_{44}| > |T_{rr}| \sim$
- ☐ Clearer separation for lower T

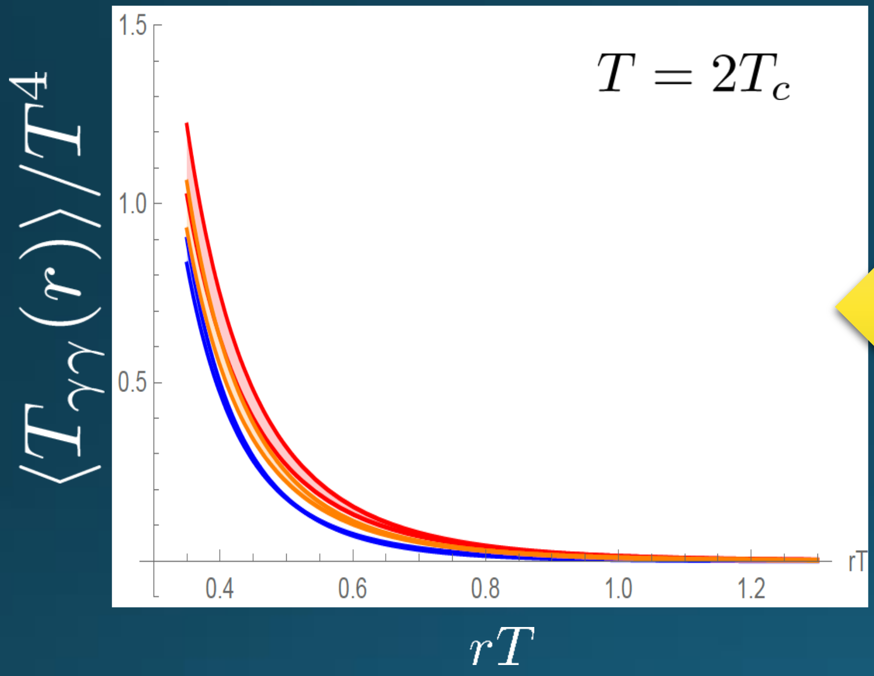
pulling

pushing

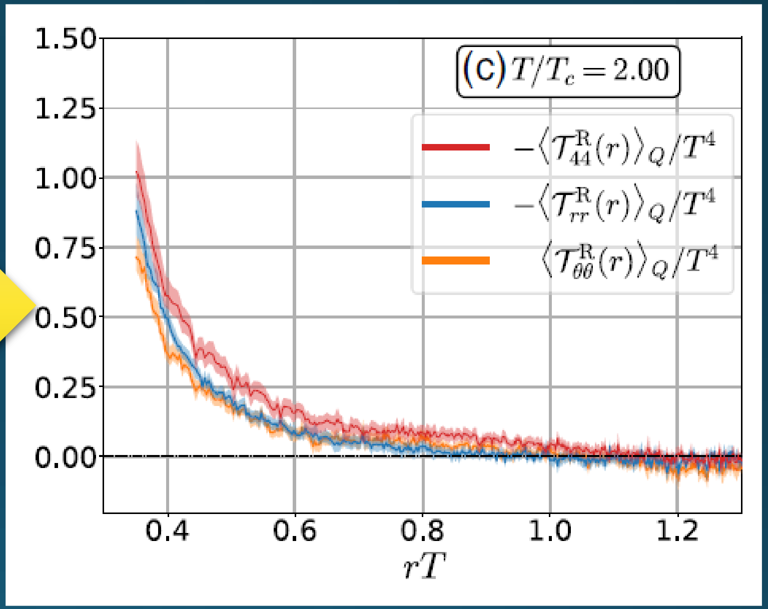
Perturbative Analysis

M. Berwein, private comm.

Perturbation



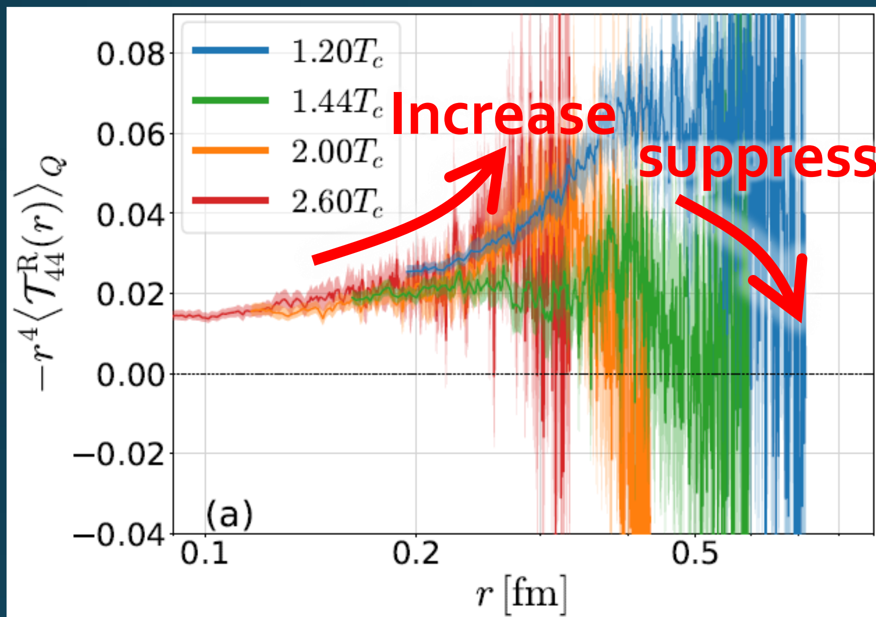
Lattice

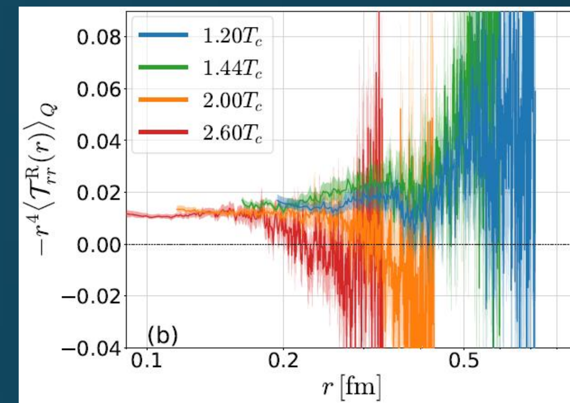


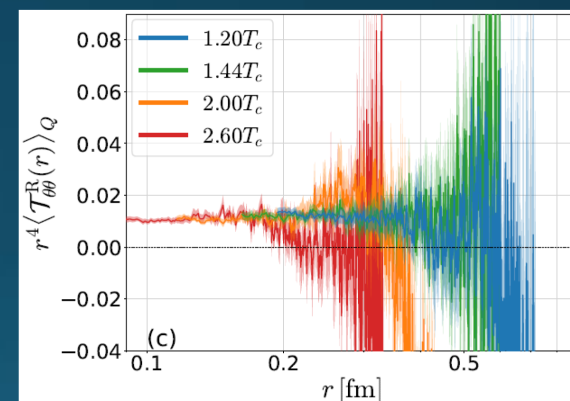
Perturbation:
Combination of
NLO pert. + NLO EQCD

- ☐ $|T_{44}| > |T_{rr}|$ is reproduced by perturbation.
- ☐ Hierarchy of T_{rr} , $T_{\theta\theta}$ does not match?

r Dependence







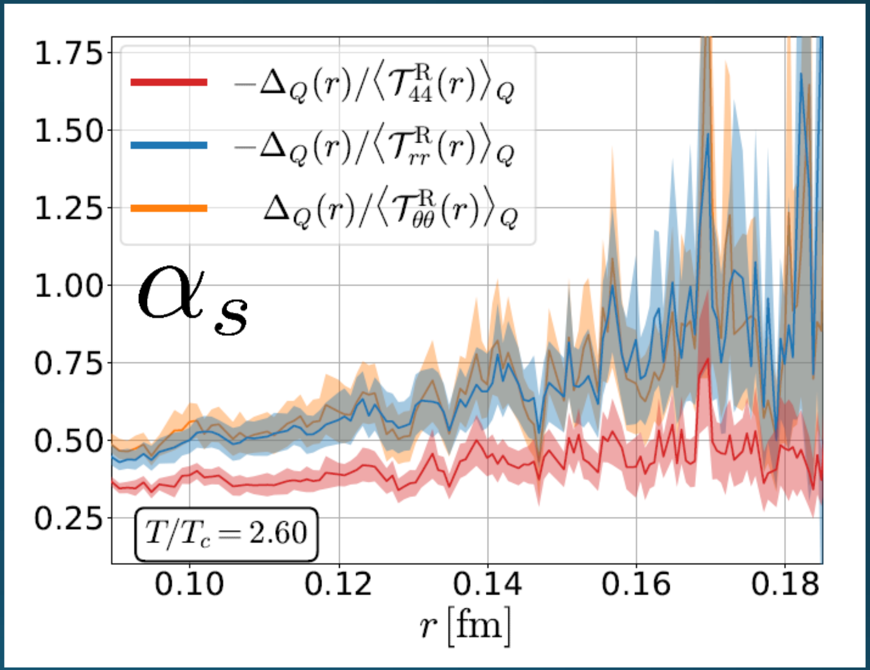
- Increase at short r / suppression at larger r
- T dependence is suppressed at $r < 1/T$
- Too noisy at large r for extracting screening mass m_{D}

Running Coupling

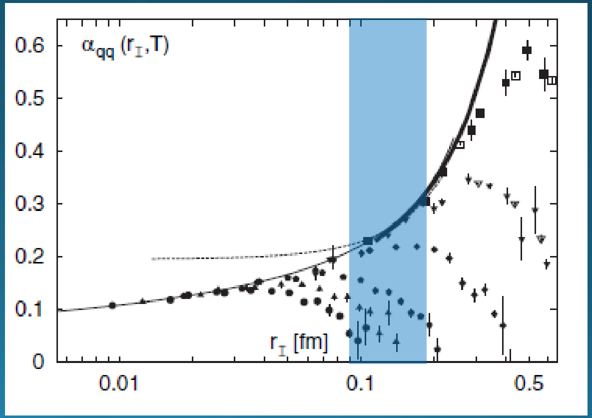
- **Estimate of α_s**

$$\left|\frac{\langle T_{\mu\mu}\rangle}{\langle \mathcal{T}_{44,rr,\theta\theta}(r)\rangle}\right| = \frac{11}{2\pi}\alpha_s + \mathcal{O}(g^3),$$

  - at the leading-order perturbation theory
  - channel dependent



- All results are approximately consistent with the estimate from $Q\bar{Q}$ potential



Kaczmarek, Karsch, Zantow, 2004

Summary

- EMT is an important observable for investigating local systems in QFT. Experimental measurement of GFF of hadrons are ongoing.
- SFtX (small flow-time expansion) method based on the gradient flow provides us with powerful method to carry out the measurement of EMT on the lattice.
  - Flux tube
  - single-Q system
- Corresponding model analyses of EMT are also interesting.
- Numerical analysis of the single-quark system will give us insights into heavy-light mesons.

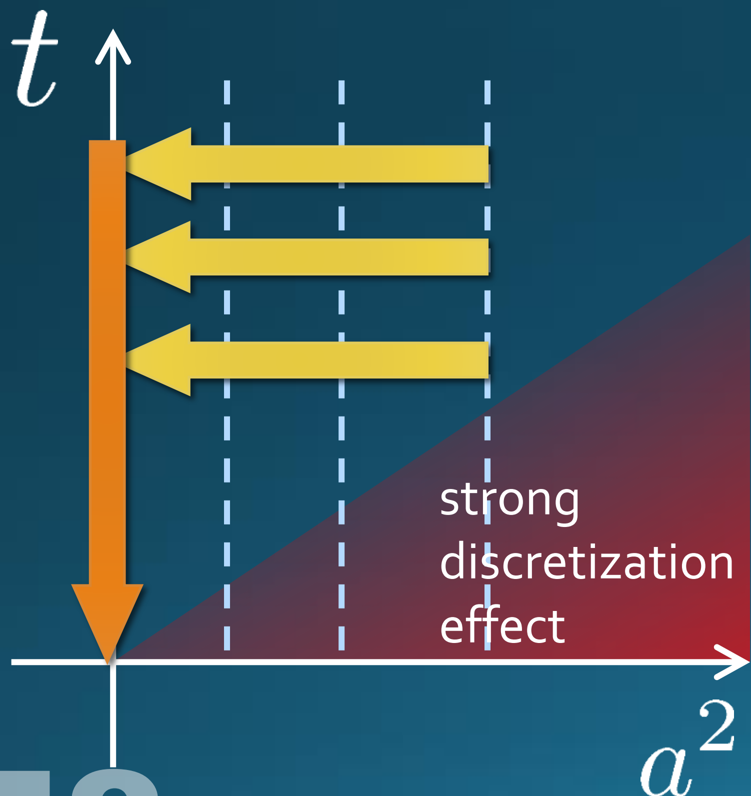
backup

Double Extrapolation

t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE lattice discretization



Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

Small t extrapolation

$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$

EMT on the Lattice: Conventional

Lattice EMT Operator

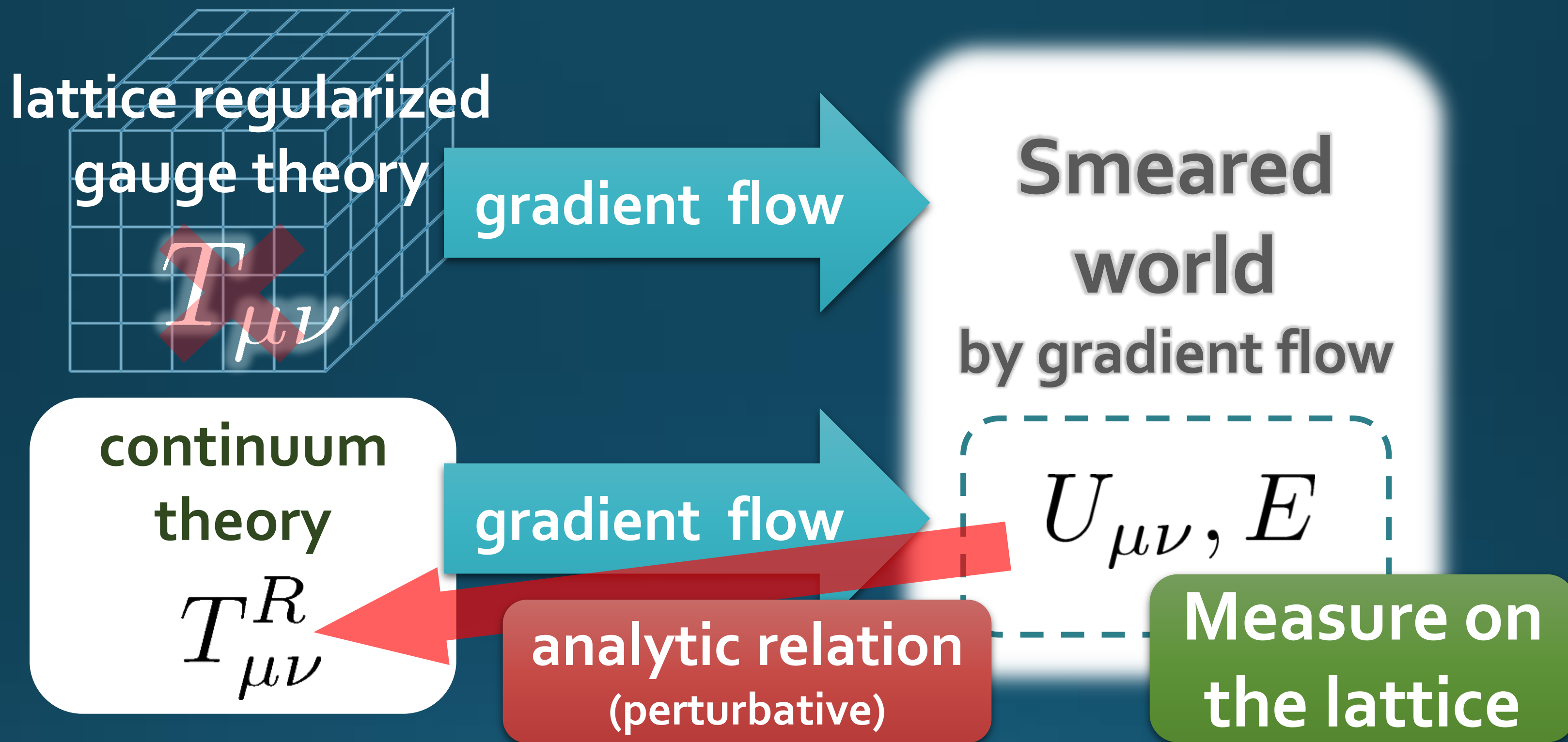
Caracciolo+, 1990

$$T_{\mu\nu} = Z_6 T^{[6]}_{\mu\nu} + Z_3 T^{[3]}_{\mu\nu} + Z_1 \left(T^{[1]}_{\mu\nu} - \langle T^{[1]}_{\mu\nu} \rangle \right)$$

$$T^{[6]}_{\mu\nu} = (1-\delta_{\mu\nu}) F^a_{\mu\rho} F^a_{\nu\rho}, \quad T^{[3]}_{\mu\nu} = \delta_{\mu\nu} \left(F^a_{\mu\rho} F^a_{\nu\rho} - \frac{1}{4} F^a_{\rho\sigma} F^a_{\rho\sigma} \right), \quad T^{[1]}_{\mu\nu} = \delta_{\mu\nu} F^a_{\rho\sigma} F^a_{\rho\sigma}$$

- **Determination of Zs**
  - Fit to thermodynamics: Z_3, Z_1 — Giusti, Meyer, 2011; 2013;
  - Shifted-boundary method: Z_6, Z_3 — Giusti, Pepe, 2014~; Borsanyi+, 2018
  - Full QCD with fermions — Brida, Giusti, Pepe, 2020

SFtX Method



Take Extrapolation (t,a)→(0,0)

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}\frac{a^2}{t} + \cdots$$

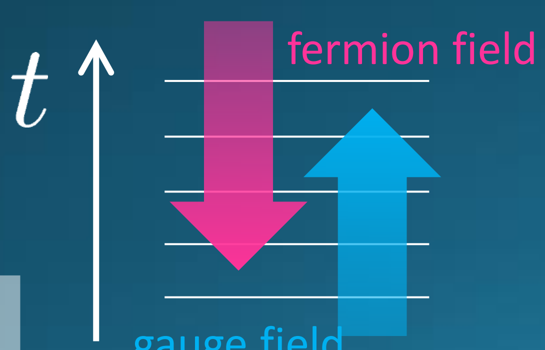
O(t) terms in SFTE lattice discretization

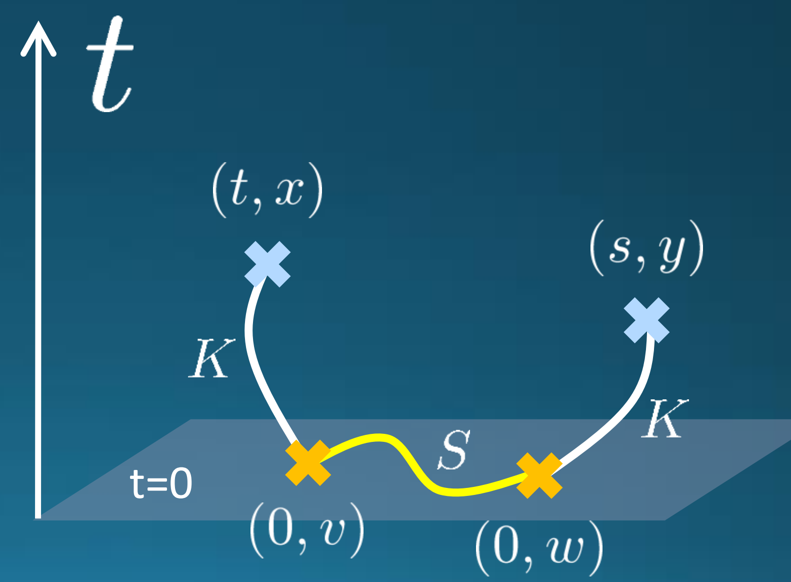
Fermion Propagator

$$S(t,x;s,y) = \langle \chi(t,x)\bar{\chi}(s,y)\rangle$$
$$= \sum_{v,w} K(t,x;0,v)S(v,w)K(s,y;0,w)^\dagger$$

$$\left(\partial_t - D_\mu D_\mu\right)K(t,x) = 0$$

- propagator of flow equation
- Inverse propagator is needed





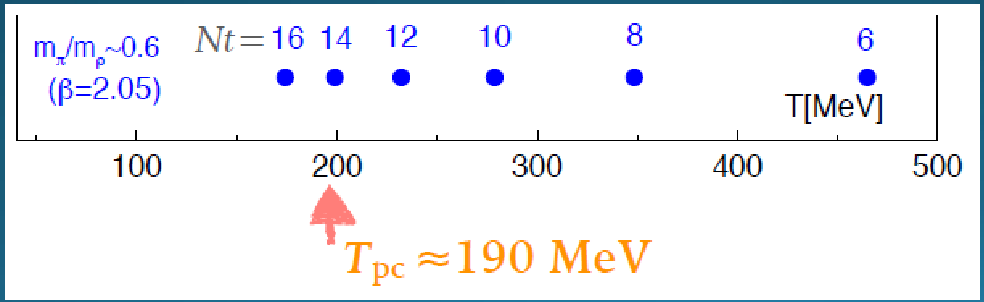
N_f=2+1 QCD Thermodynamics

Taniguchi+ (WHOT-QCD),
PR**D96**, 014509 (2017)

- N_f=2+1 QCD, Iwasaki gauge + NP-clover
- $m_{PS}/m_V \approx 0.63$ / almost physical s quark mass

- T=0: CP-PACS+JLQCD (ß=2.05, 28^3x56, a≈0.07fm)
- T>0: 32^3xN_t, N_t = 4, 6, … , 14, 16):
- T≈174-697MeV

- t→0 extrapolation only (No continuum limit)



Numerical Setup

- SU(3) YM theory
- Wilson gauge action

- N_t = 16, 12
- N_z/N_t=6
- 2000~4000 confs.
- Even N_x

- No Continuum extrap.

T/T_c	β	N_z	N_τ	N_x	$N_{\rm vac}$
1.12	6.418	72	12	12, 14, 16, 18	64
	6.631	96	16	16, 18, 20, 22, 24	96
1.40	6.582	72	12	12, 14, 16, 18	64
	6.800	96	16	16, 18, 20, 22, 24	128
1.68	6.719	72	12	12, 14, 16, 18, 24	64
	6.719	96	12	14, 18	64
	6.941	96	16	16, 18, 20, 22, 24	96
2.10	6.891	72	12	12, 14, 16, 18, 24	72
	7.117	96	16	16, 18, 20, 22, 24	128
2.69	7.086	72	12	12, 14, 16, 18	-
$\simeq$ 8.1	8.0	72	12	12, 14, 16, 18	-
$\simeq$ 25	9.0	72	12	12, 14, 16, 18	-

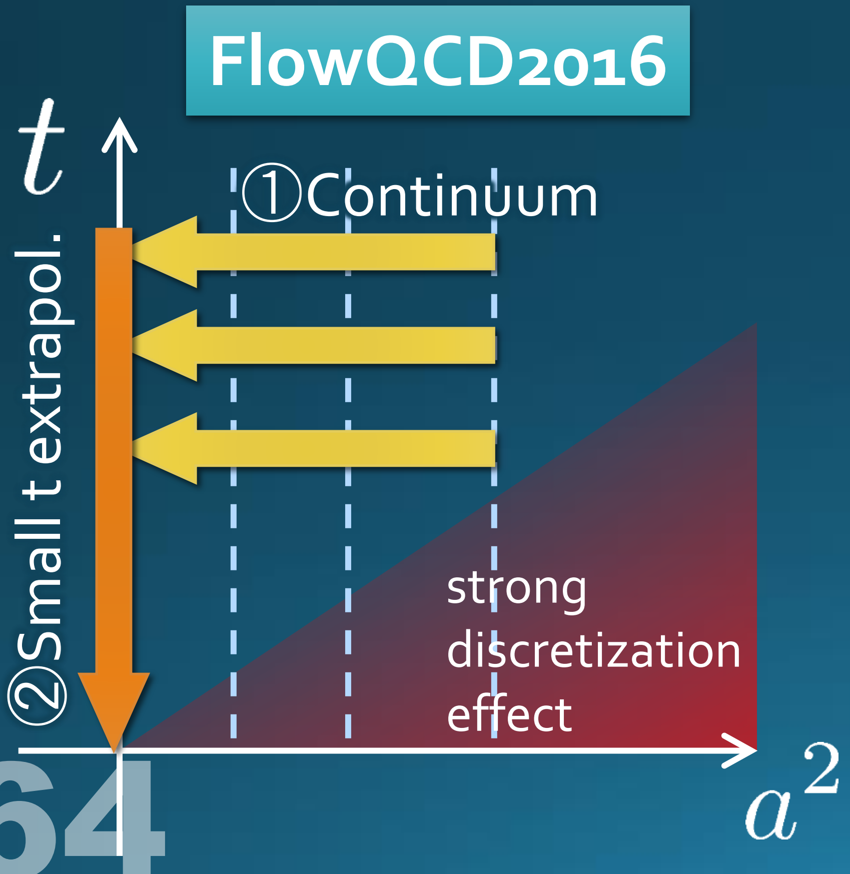
- Same Spatial volume
  - $12 \times 72^2 \times 12 \sim 16 \times 96^2 \times 16$
  - $18 \times 72^2 \times 12 \sim 24 \times 96^2 \times 16$

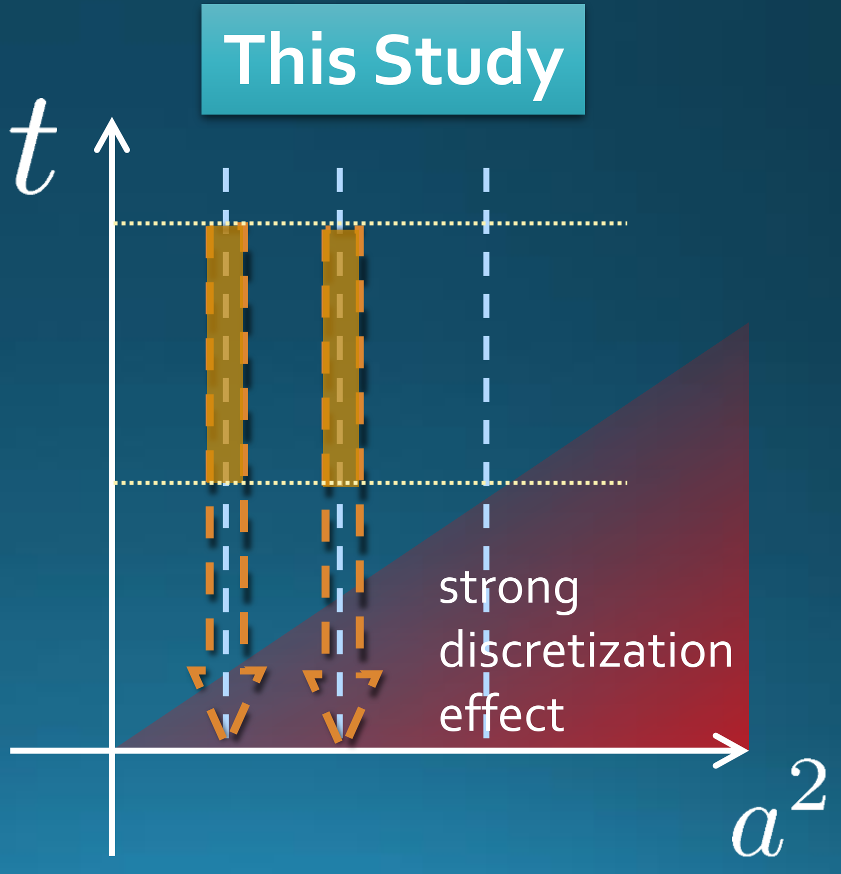
Simulations on
OCTOPUS/Reedbush

Extrapolations t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE lattice discretization





Lattice Setup

FlowQCD, PLB (2019)

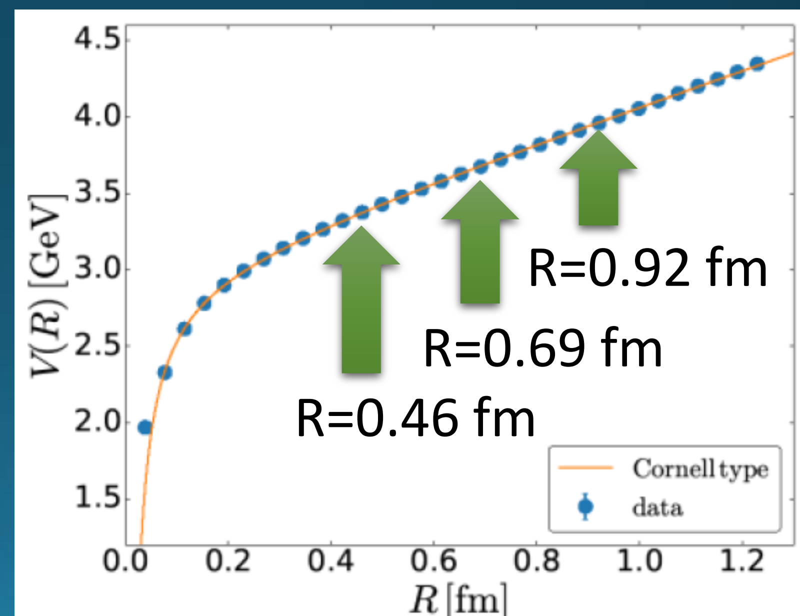
- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

- EMT around Wilson Loop
- APE smearing / multi-hit

- fine lattices (a=0.029-0.06 fm)
- continuum extrapolation

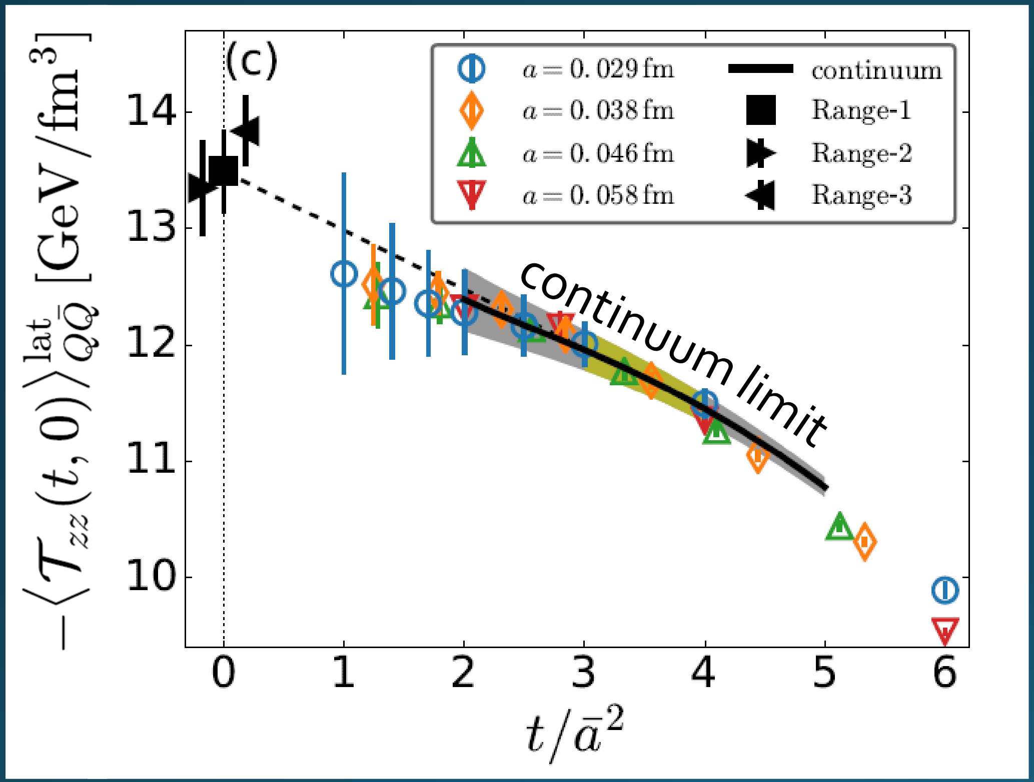
- Simulation: bluegene/Q@KEK

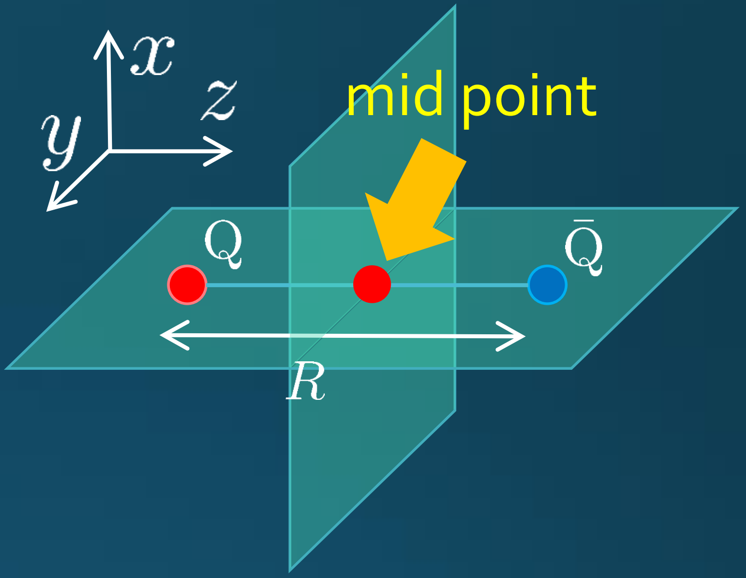
β	a [fm]	$N_{\rm size}^4$	$N_{\rm conf}$	R/a		
6.304	0.058	48^4	140	8	12	16
6.465	0.046	48^4	440	10	–	20
6.513	0.043	48^4	600	–	16	–
6.600	0.038	48^4	1,500	12	18	24
6.819	0.029	64^4	1,000	16	24	32
			R [fm]	0.46	0.69	0.92

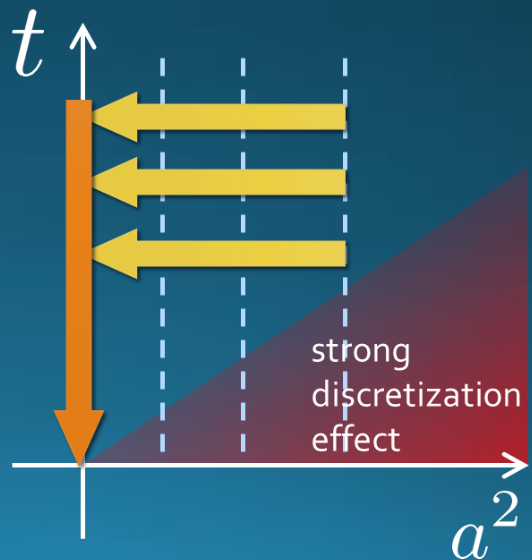


$$\langle O(x)\rangle_{Q\bar{Q}} = \lim_{T\to\infty} \frac{\langle \delta O(x)\delta W(R,T)\rangle}{\langle W(R,T)\rangle}$$

Continuum Extrapolation
at mid-point



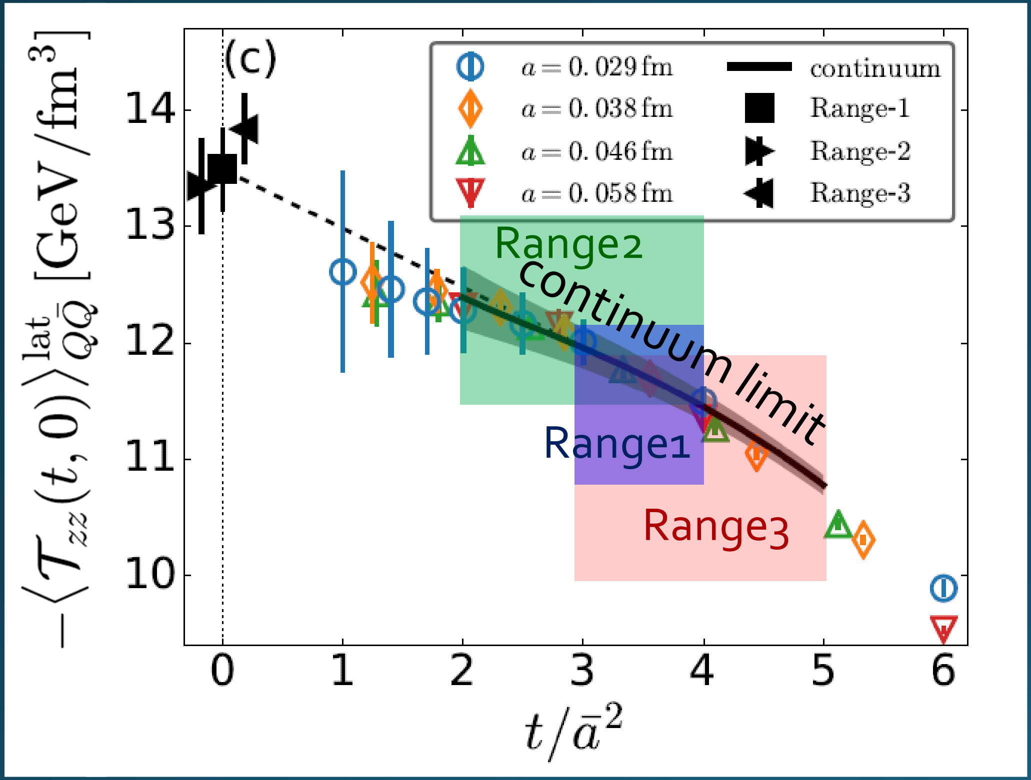


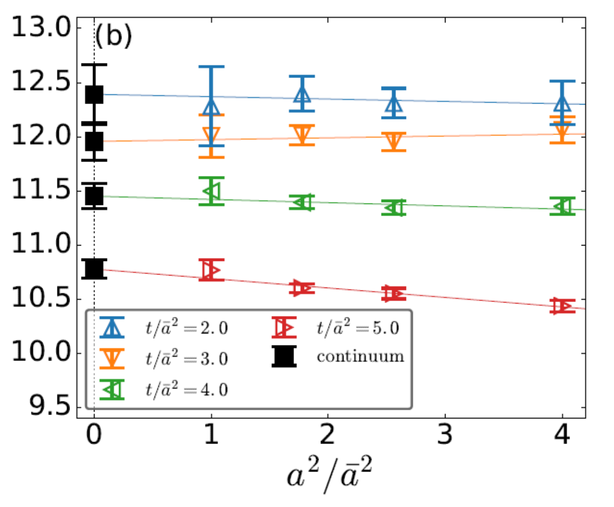


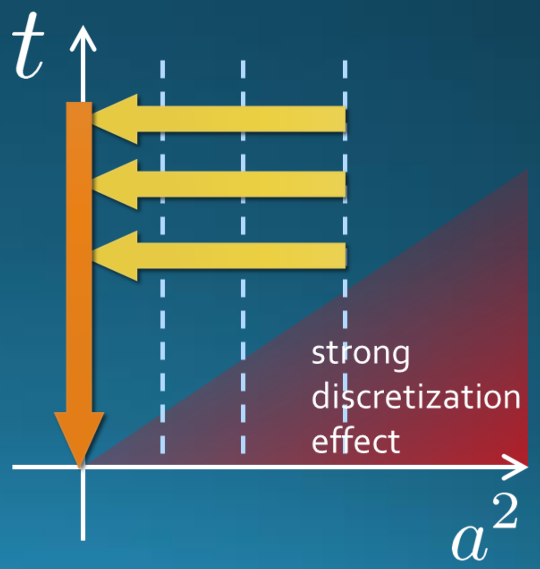
- a→0 extrapolation with fixed t

t→0 Extrapolation

at mid-point







- ☐ a→0 extrapolation with fixed t
- ☐ Then, t→0 with three ranges

Double Extrapolation

$t \to 0, a \to 0$

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

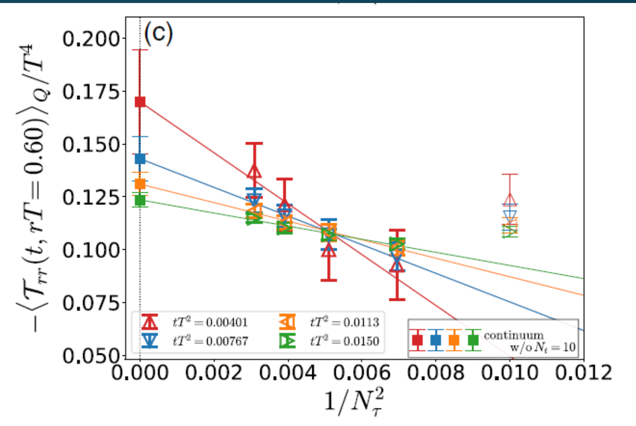
O(t) terms in SFTE lattice discretization

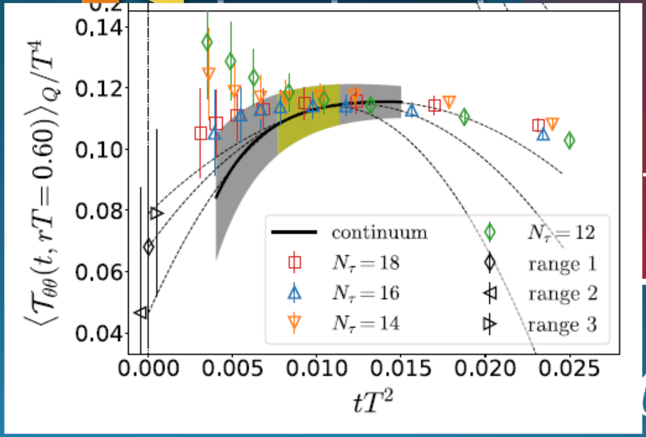
Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

Small t extrapolation

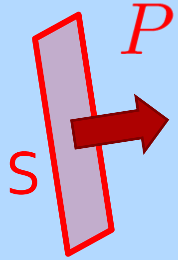
$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$





Stress = Force per Unit Area

Pressure

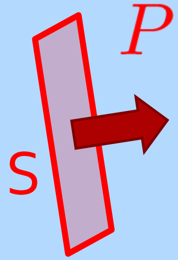


$$\vec{P} = \frac{\vec{F}}{S}$$

$$\vec{P} = P\vec{n}$$

Stress = Force per Unit Area

Pressure



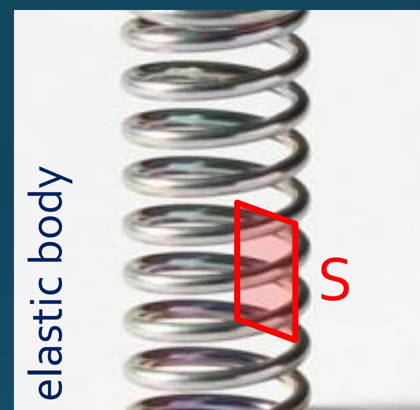
$$\vec{P} = \frac{\vec{F}}{S}$$

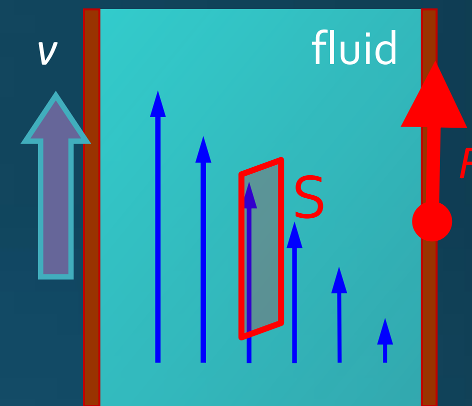
$$\vec{P} = P\vec{n}$$

In thermal medium

$$T_{ij} = P\delta_{ij}$$

Generally, F and n are not parallel





$$\frac{F_i}{S} = \sigma_{ij} n_j$$

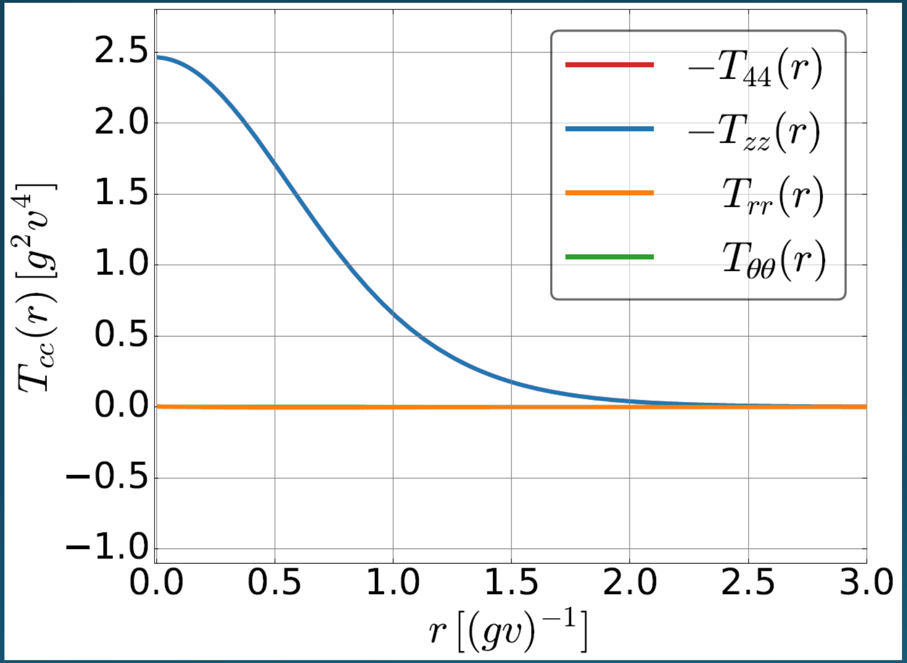
Stress Tensor

$$\sigma_{ij} = -T_{ij}$$

Landau
Lifshitz

Stress Tensor in AH Model
infinitely-long flux tube

Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$



$$T_{rr} = T_{\theta\theta} = 0$$

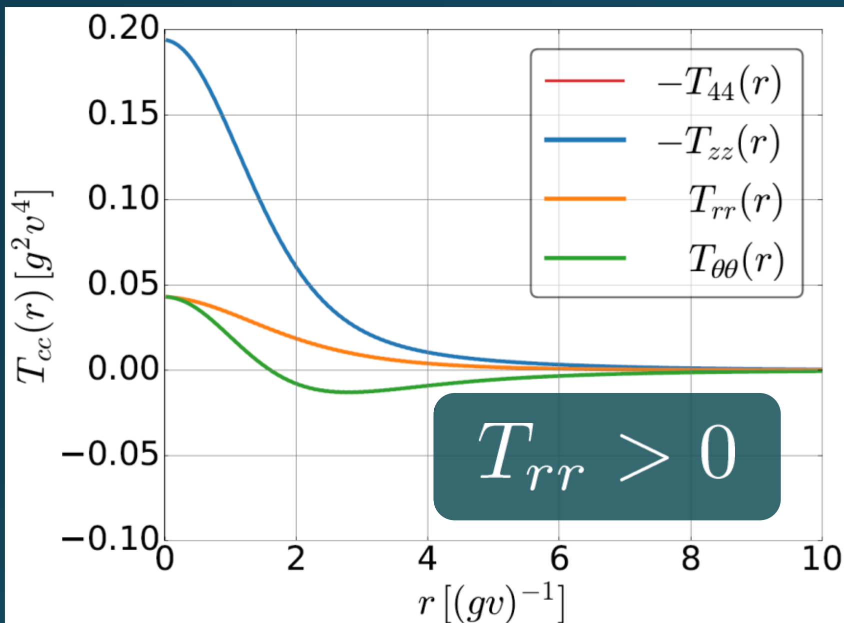
de Vega, Schaposnik, PRD**14**, 1100 (1976).

Stress Tensor in AH Model

infinitely-long flux tube

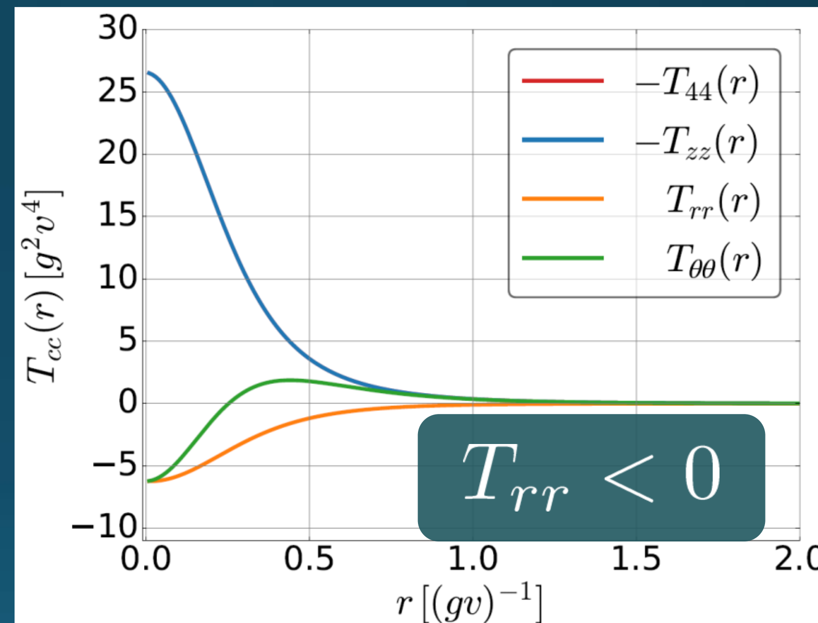
Type-I

$\kappa = 0.1$



Type-II

$\kappa = 3.0$



- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

conservation law

$$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$