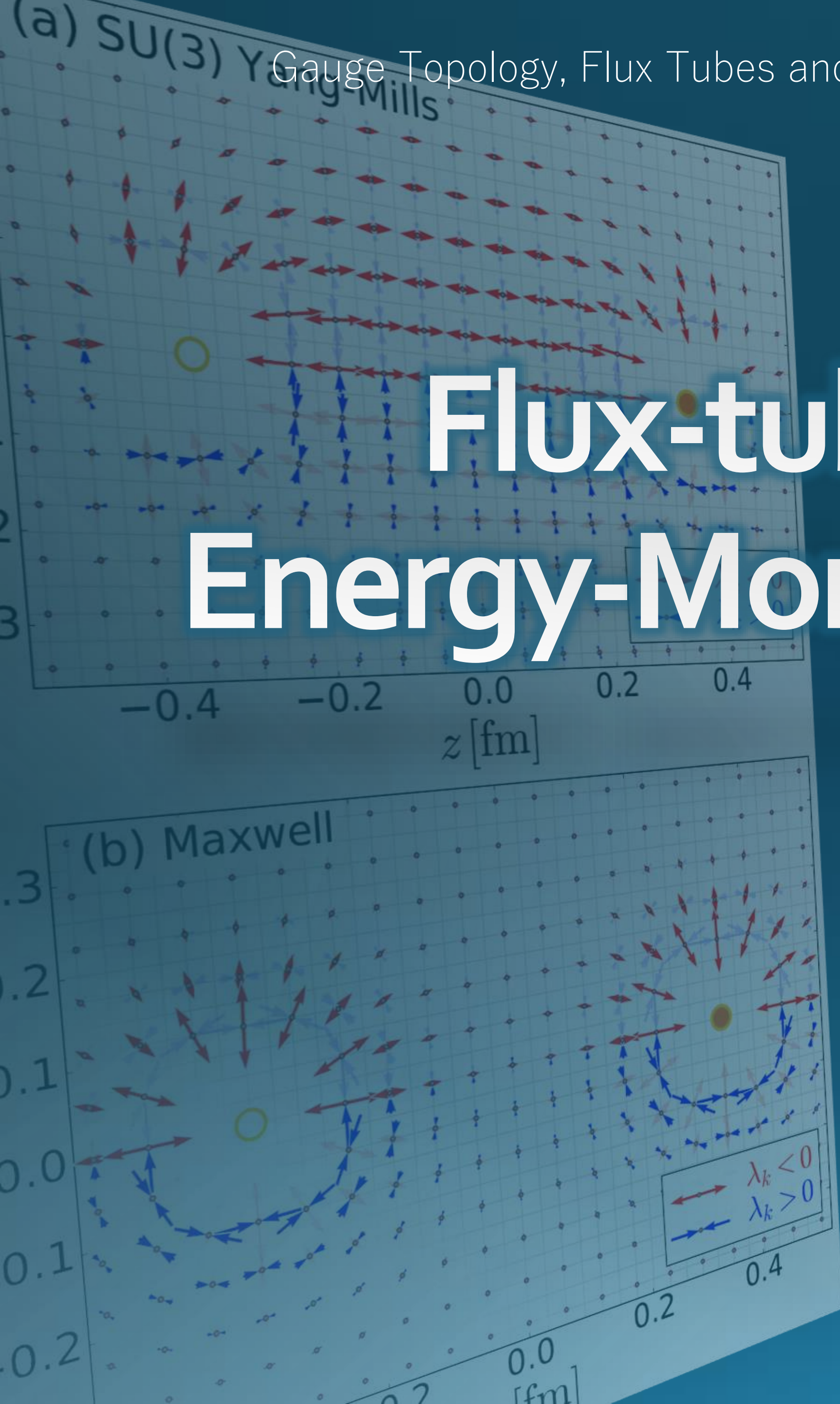
Gauge Topology, Flux Tubes and Holographic Models, ECT*, May 24, 2022



Flux-tube Structure via Energy-Momentum Tensor

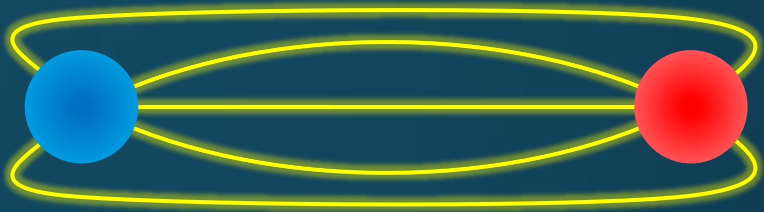
Masakiyo Kitazawa

(Osaka U.)

FlowQCD, PLB **789**, 210 (2019)
Yanagihara, MK, PTEP**2019**, 093B02 (2019)
FlowQCD, PRD **102**, 114522 (2020)
Ito, MK, in prep.

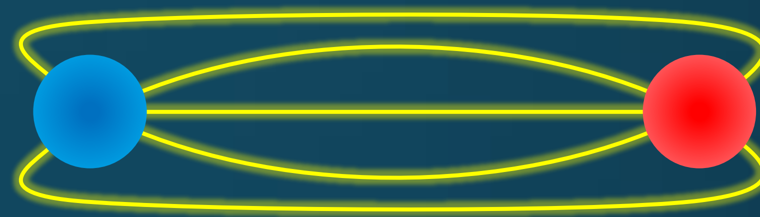
Flux Tube

- Quark confinement
- Non-pert. dynamics
- Linear potential
- String theory



Flux Tube

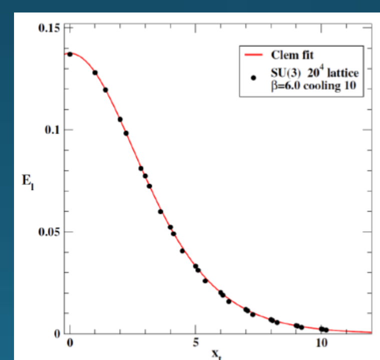
- ☐ Quark confinement
- ☐ Non-pert. dynamics
- ☐ Linear potential
- ☐ String theory



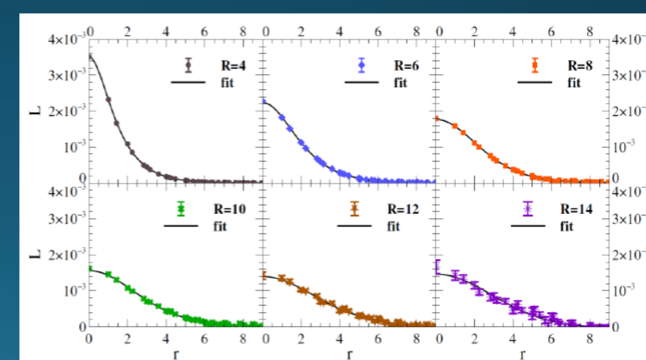
Many Studies on Flux Tube

- ☐ Potential
- ☐ Color-electric field
- ☐ Action density

so many studies…



Cea+ (2012)



Cardoso+ (2013)

Energy-Momentum Tensor

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

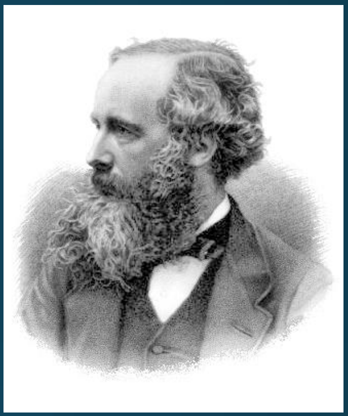
energy, momentum, pressure, stress

- The most fundamental quantity in physics
- Gauge invariant observable
- All components have important physical meaning.
- stress tensor = mechanical distortion in static systems

Maxwell Stress

(in Maxwell Theory)

$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

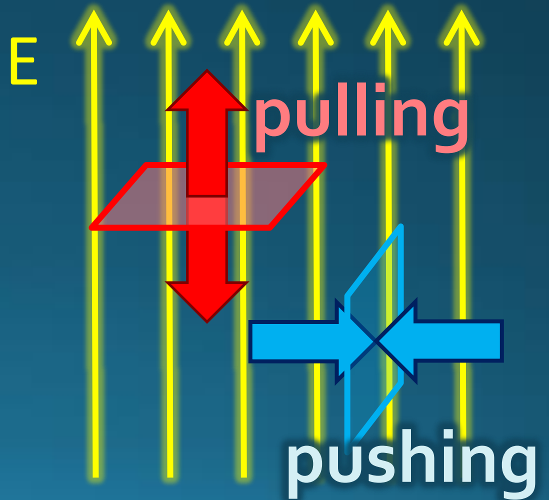


Maxwell

$$\vec{E} = (E, 0, 0)$$

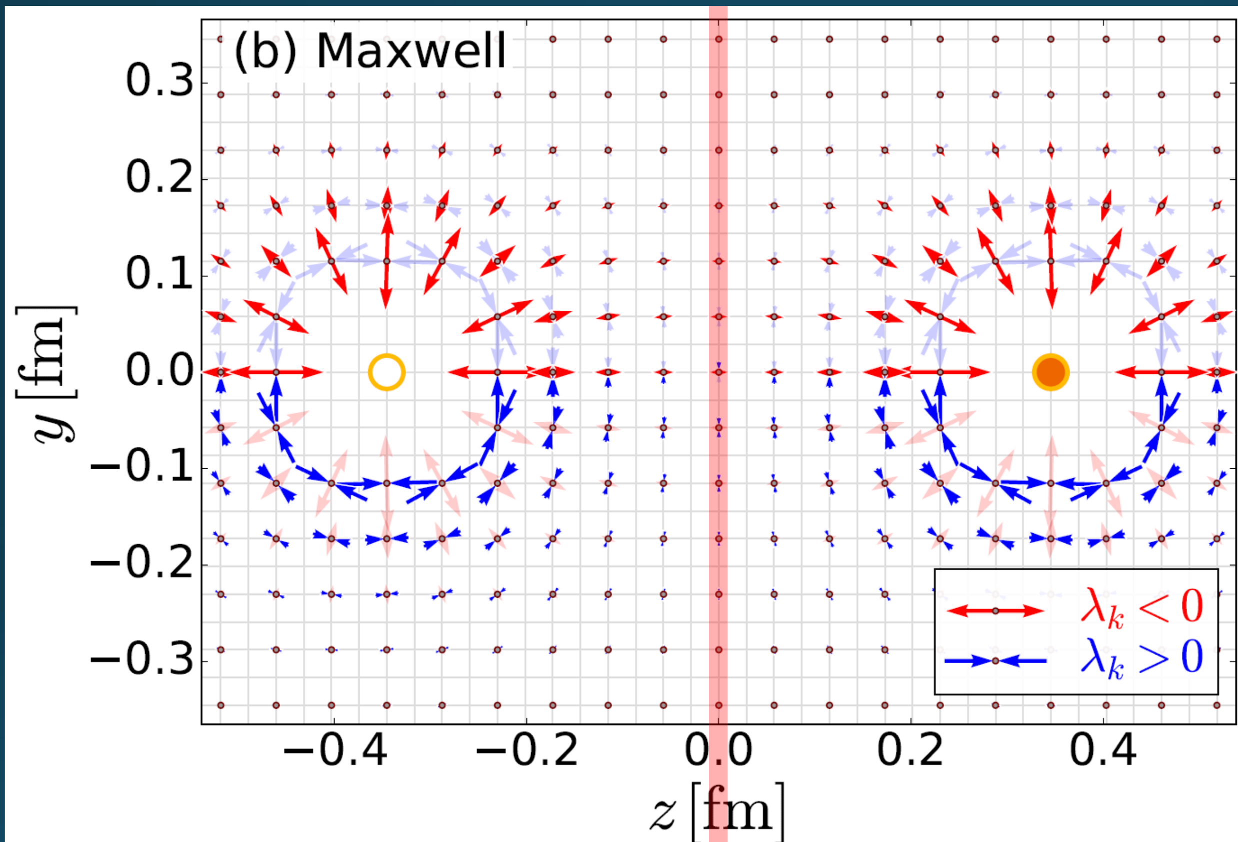
$$T_{ij} = \begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: **Pulling**
- Vertical to field: **Pushing**



Maxwell Stress

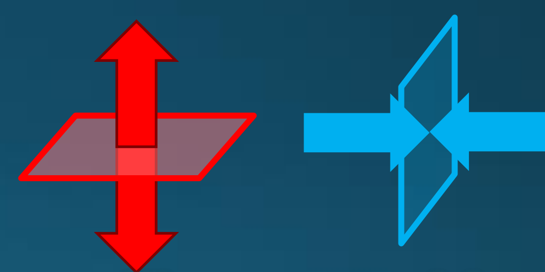
(in Maxwell Theory)



$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$$(k = 1, 2, 3)$$

length: $\sqrt{|\lambda_k|}$

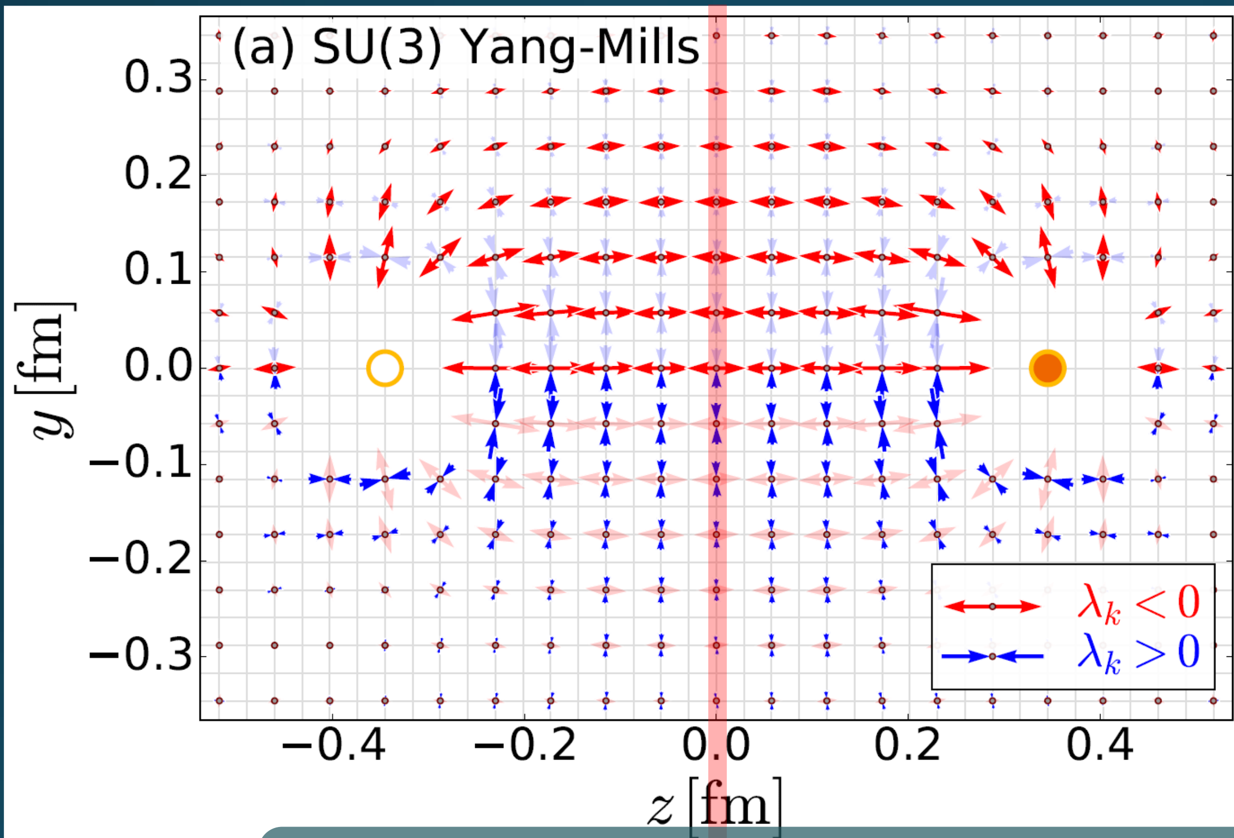


Definite physical meaning

- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction

Stress Tensor in $Q\bar{Q}$ System

FlowQCD, PLB (2019)



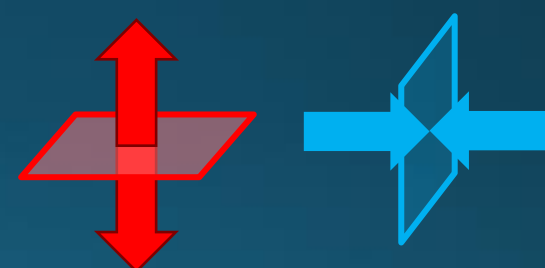
Lattice simulation

SU(3) Yang-Mills

a=0.029 fm

R=0.69 fm

$t/a^2=2.0$

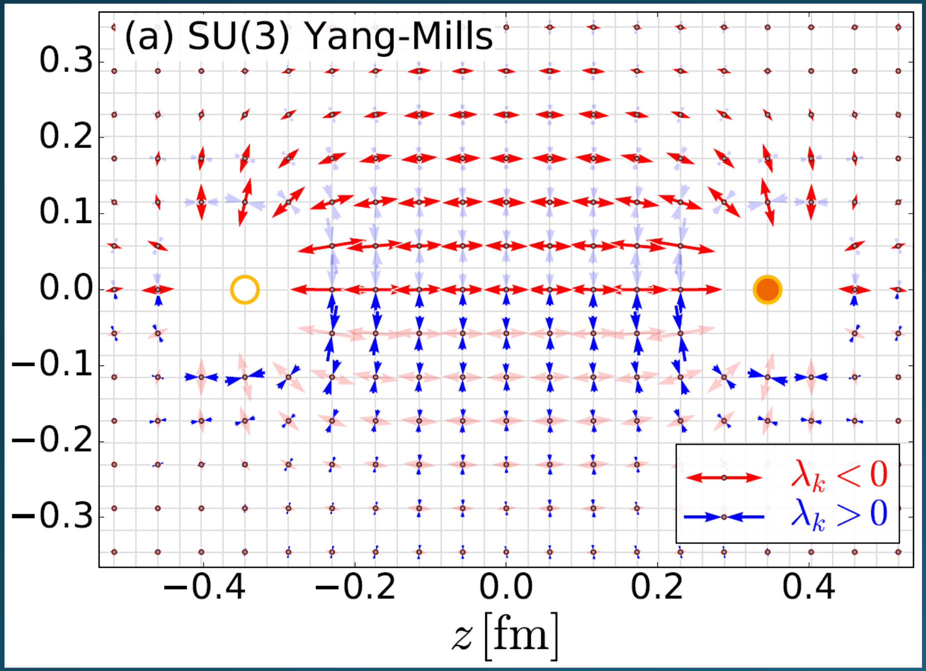


Definite physical meaning

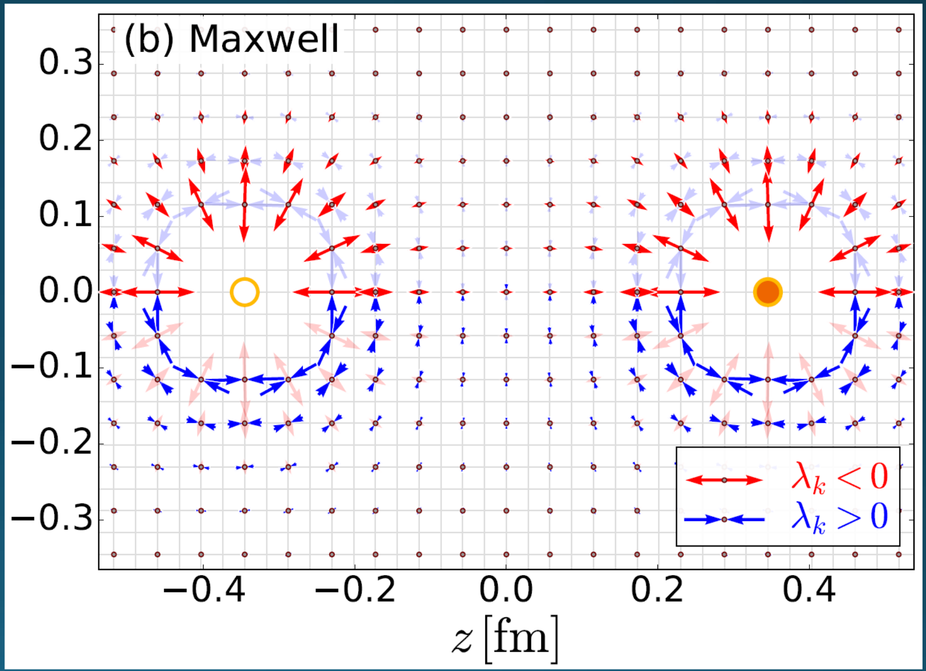
- ☐ Distortion of field, line of field
- ☐ Propagation of the force as local interaction
- ☐ Manifestly gauge invariant

SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)



Maxwell
(classical)



Propagation of the force is clearly different in YM and Maxwell theories!

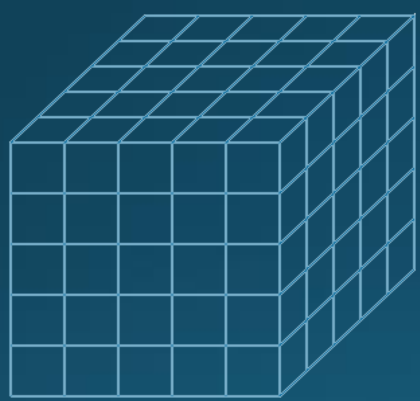
Flux-Tube Structure

FlowQCD, PLB **789**, 210 (2019)

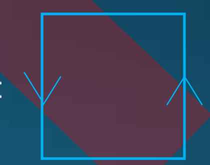
$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial

because of the explicit breaking of translational invariance



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

② Its measurement is noisy

due to high dimensionality and etc.

Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

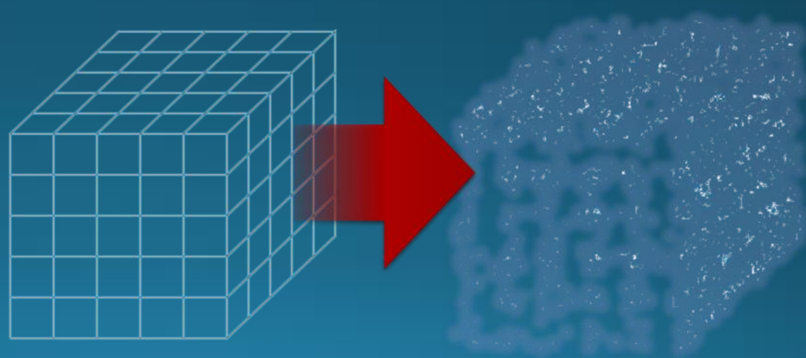
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length²]

leading

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at $t > 0$



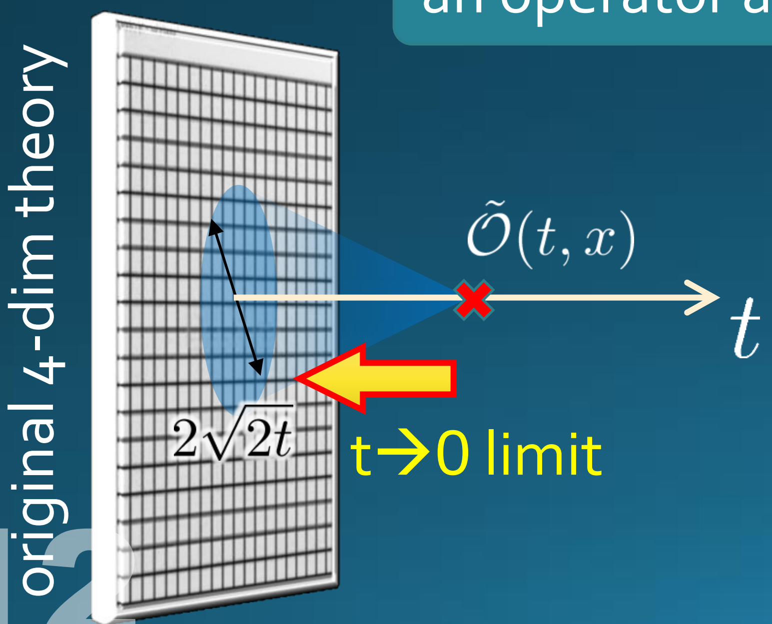
Small Flow-Time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at t>0

remormalized operators of original theory



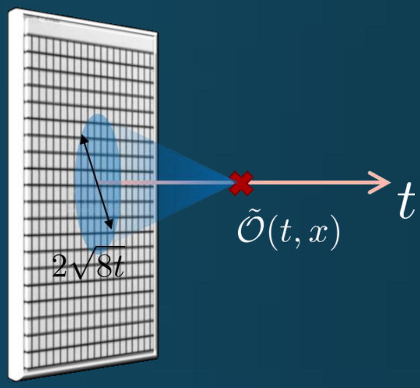
Constructing EMT

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t) T^R_{\rho\rho}(x) + \mathcal{O}(t)$$

vacuum subtr.



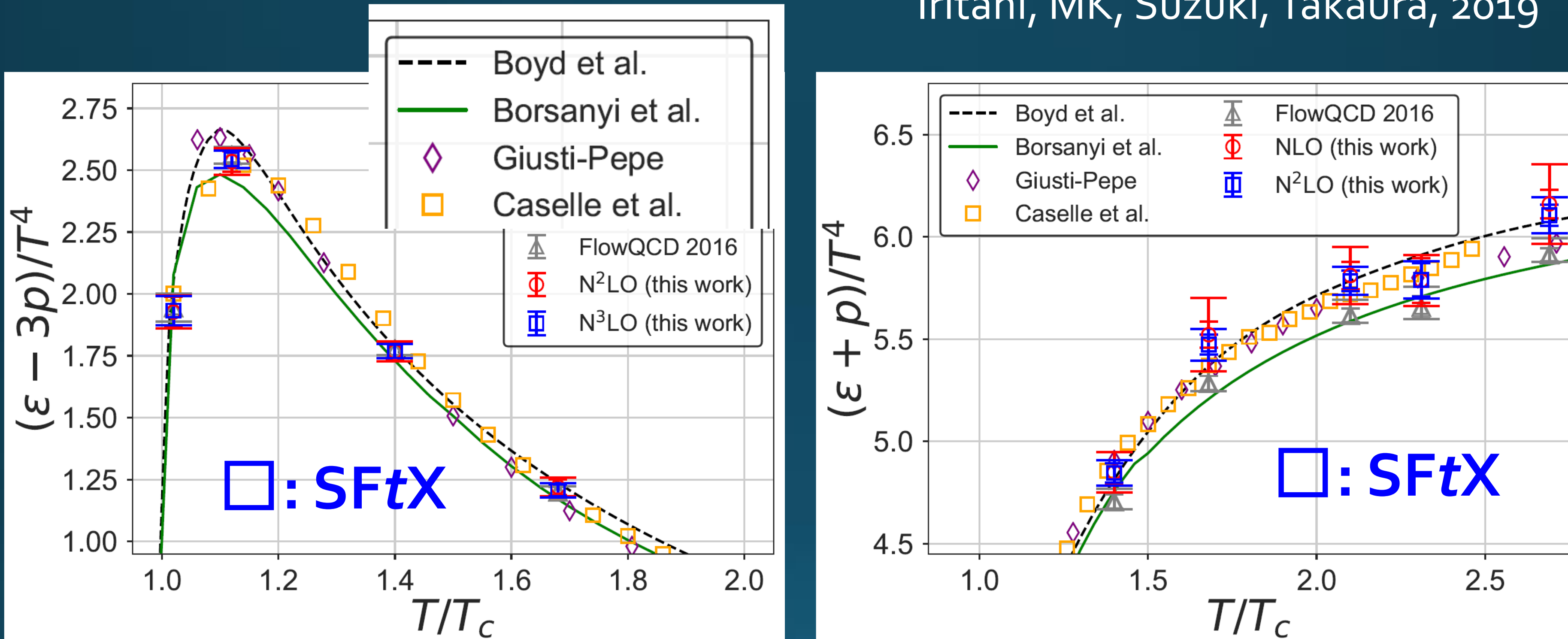
Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\text{subt.}}\right]$$

"SF*t*X method" (Small Flow *t*ime eXpansion)

Thermodynamics: $\varepsilon = \langle T_{00} \rangle, \ p = \langle T_{11} \rangle$

Iritani, MK, Suzuki, Takaura, 2019

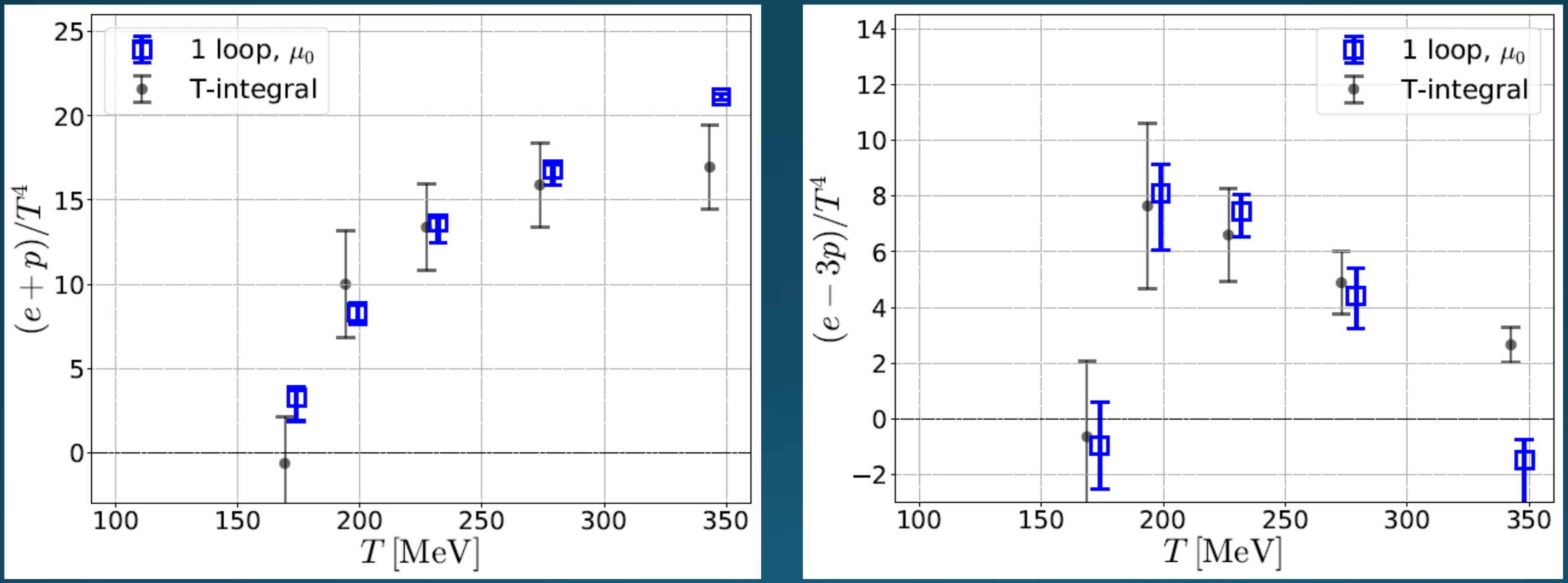


- ☐ Agreement with other methods within 1% level!
- ☐ Smaller statistics thanks to smearing by the flow

2+1 QCD EoS from Gradient Flow

WHOT-QCD, PR**D96** (2017); PR**D102** (2020)

$\langle T_{00}\rangle + \langle T_{11}\rangle$ $\langle T_{00}\rangle - 3\langle T_{11}\rangle$



- ☐ Agreement with integral method
- ☐ Substantial suppression of statistical errors

$m_{PS}/m_V \approx 0.63$

Physical mass: Kanaya+ (WHOT-QCD), 1910.13036

Lattice Setup

FlowQCD, PLB (2019)

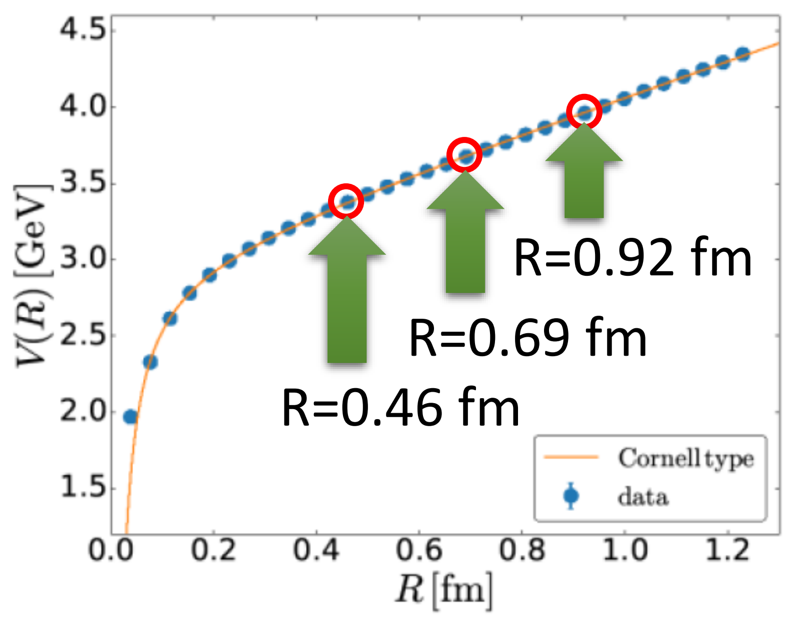
- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

- EMT around Wilson Loop
- APE smearing / multi-hit

- fine lattices (a=0.029-0.06 fm)
- **continuum extrapolation**

- Simulation: bluegene/Q@KEK

β	a [fm]	$N_{\rm size}^4$	$N_{\rm conf}$	R/a		
6.304	0.058	48^4	140	8	12	16
6.465	0.046	48^4	440	10	–	20
6.513	0.043	48^4	600	–	16	–
6.600	0.038	48^4	1,500	12	18	24
6.819	0.029	64^4	1,000	16	24	32
			R [fm]	0.46	0.69	0.92



$$\langle O(x)\rangle_{Q\bar{Q}} = \lim_{T\to\infty} \frac{\langle \delta O(x)\delta W(R,T)\rangle}{\langle W(R,T)\rangle}$$

Stress Distribution on Mid-Plane

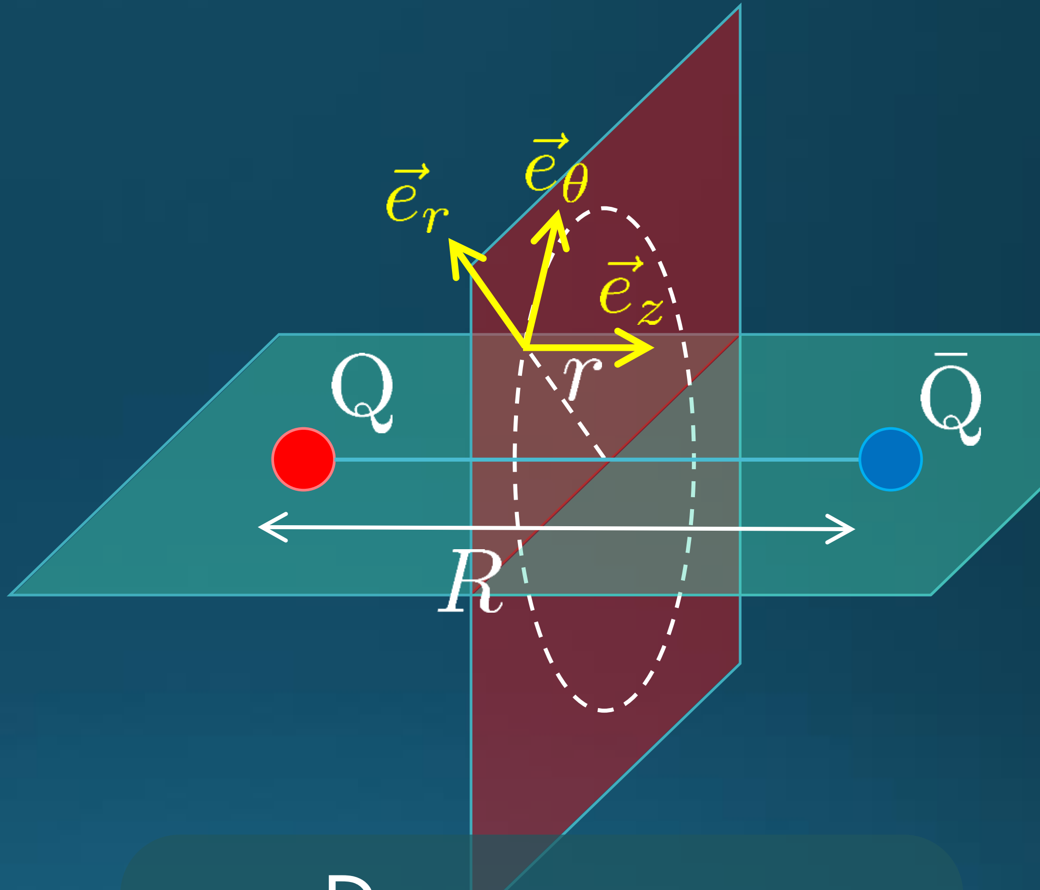
From rotational symm. & parity

EMT is diagonalized
in Cylindrical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{zz} & \\ & & & T_{44} \end{pmatrix}$$

$$T_{rr} = \vec{e}_r^{\,T} T \vec{e}_r$$

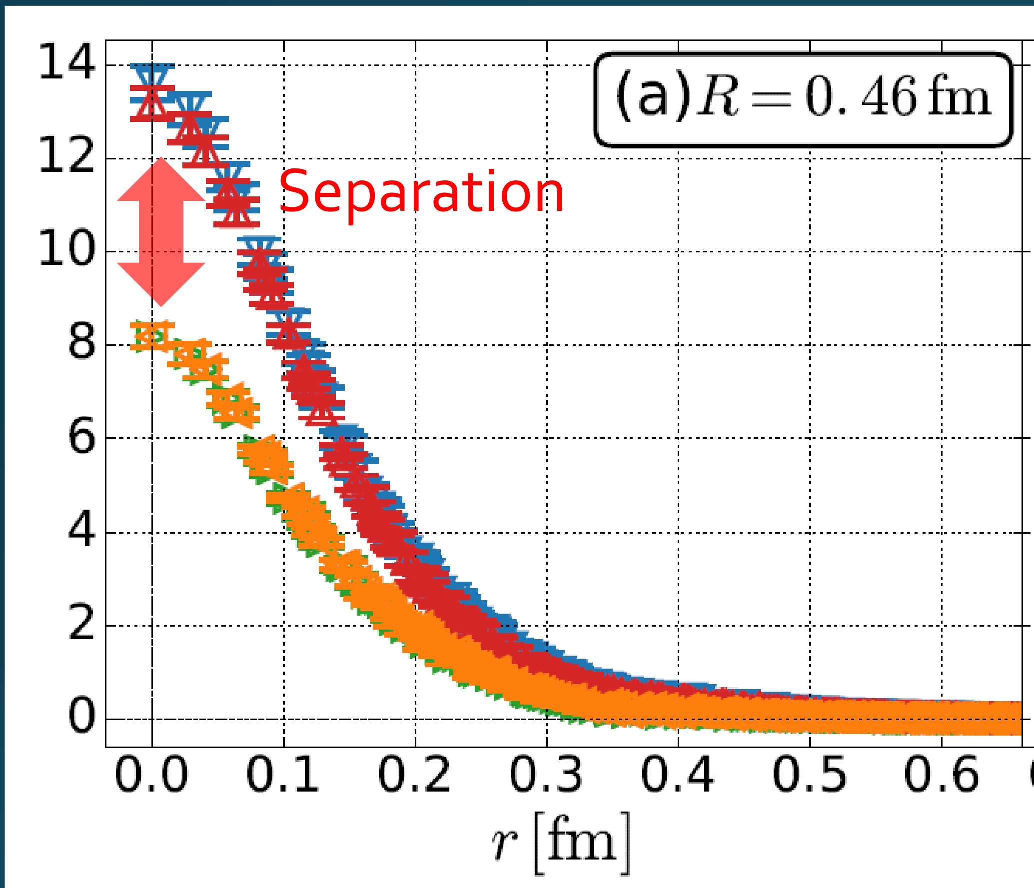
$$T_{\theta\theta} = \vec{e}_\theta^{\,T} T \vec{e}_\theta$$

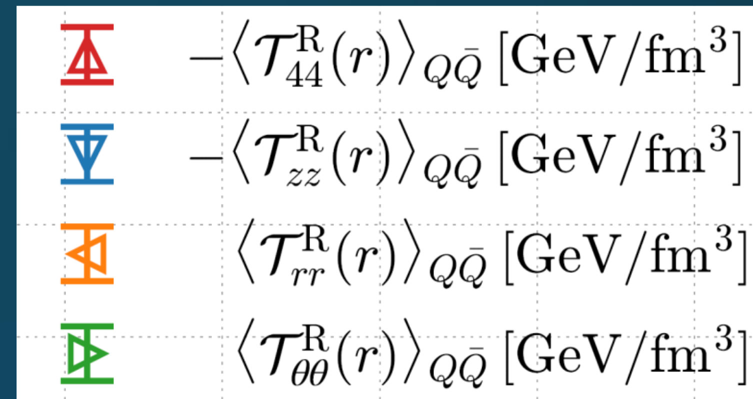


Degeneracy
in Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

Mid-Plane





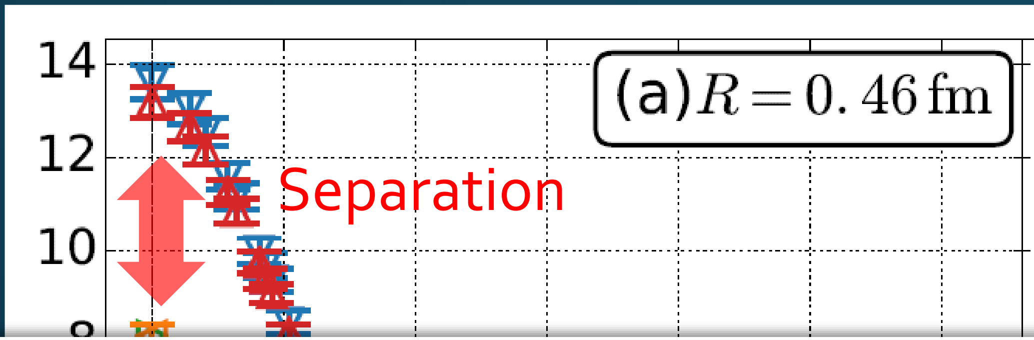
Continuum Extrapolated!

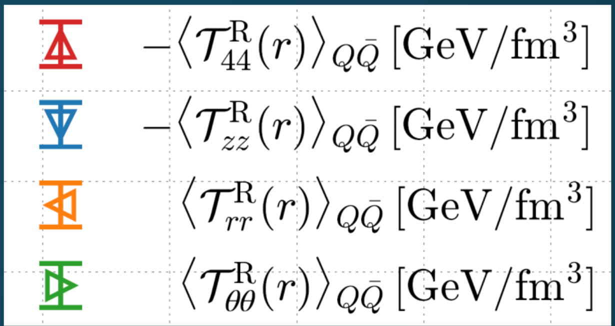
In Maxwell theory

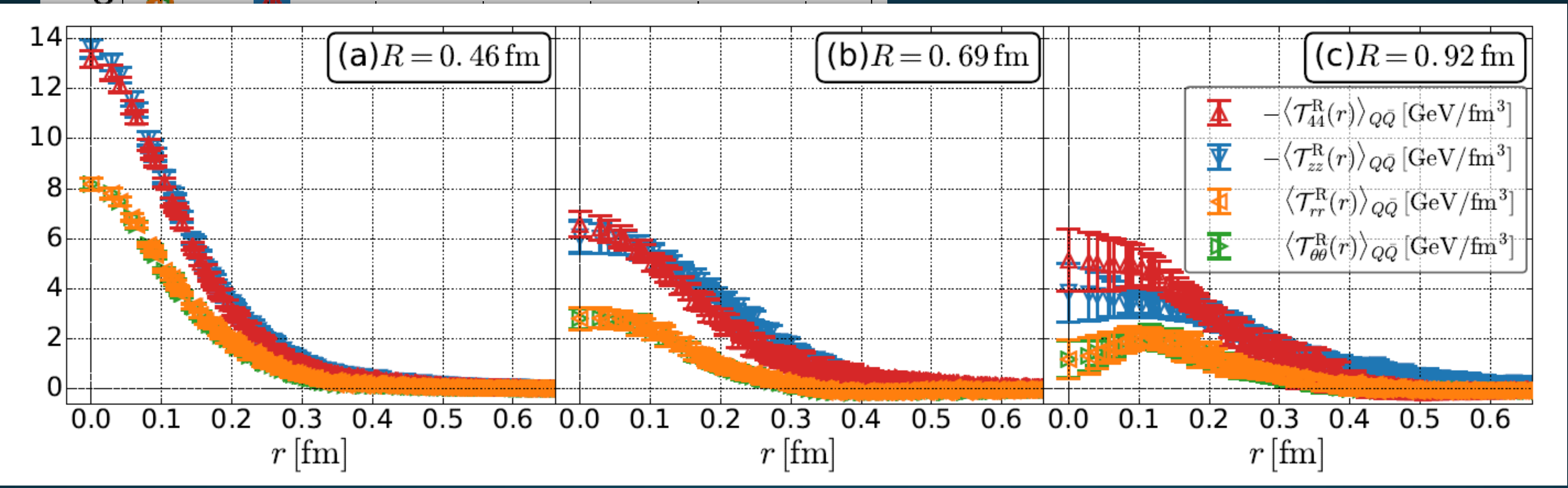
$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

- ☐ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- ☐ Separation: $T_{zz} \neq T_{rr}$
- ☐ Nonzero trace anomaly $\sum T_{cc} \neq 0$

Mid-Plane

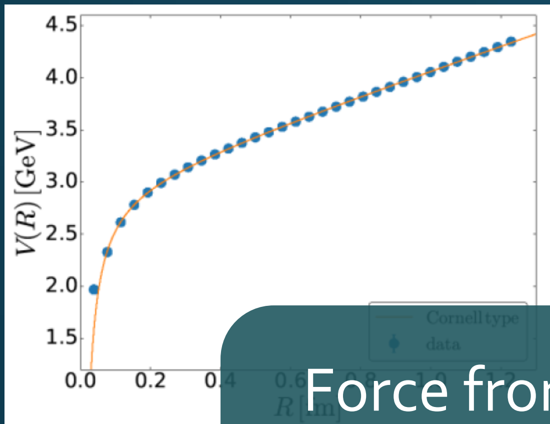


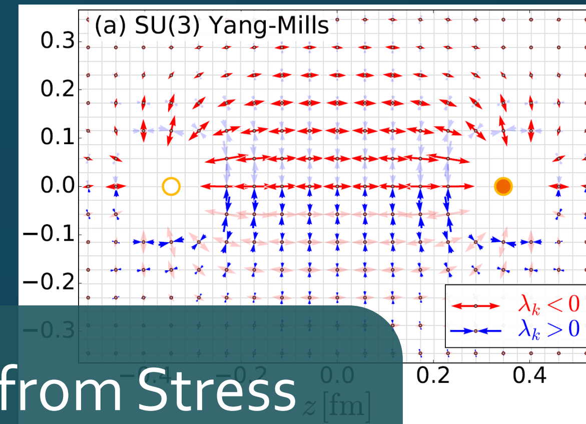




- ☐ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- ☐ Separation: $T_{zz} \neq T_{rr}$
- ☐ Nonzero trace anomaly $\sum T_{cc} \neq 0$

Force





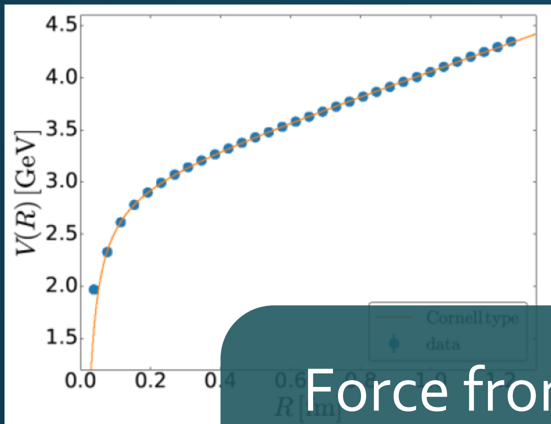
Force from Potential

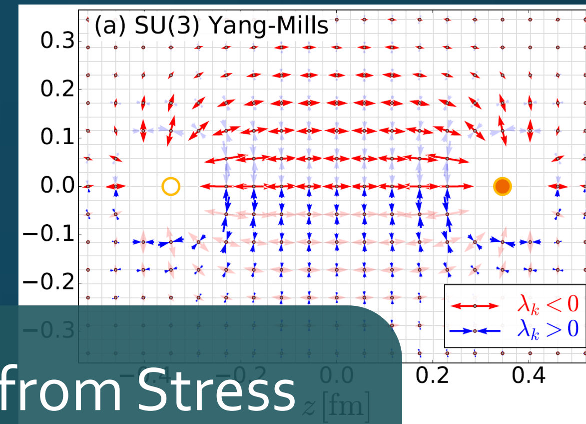
$$F_{\rm pot} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\rm stress} = \int_{\rm mid.} d^2x T_{zz}(x)$$

Force



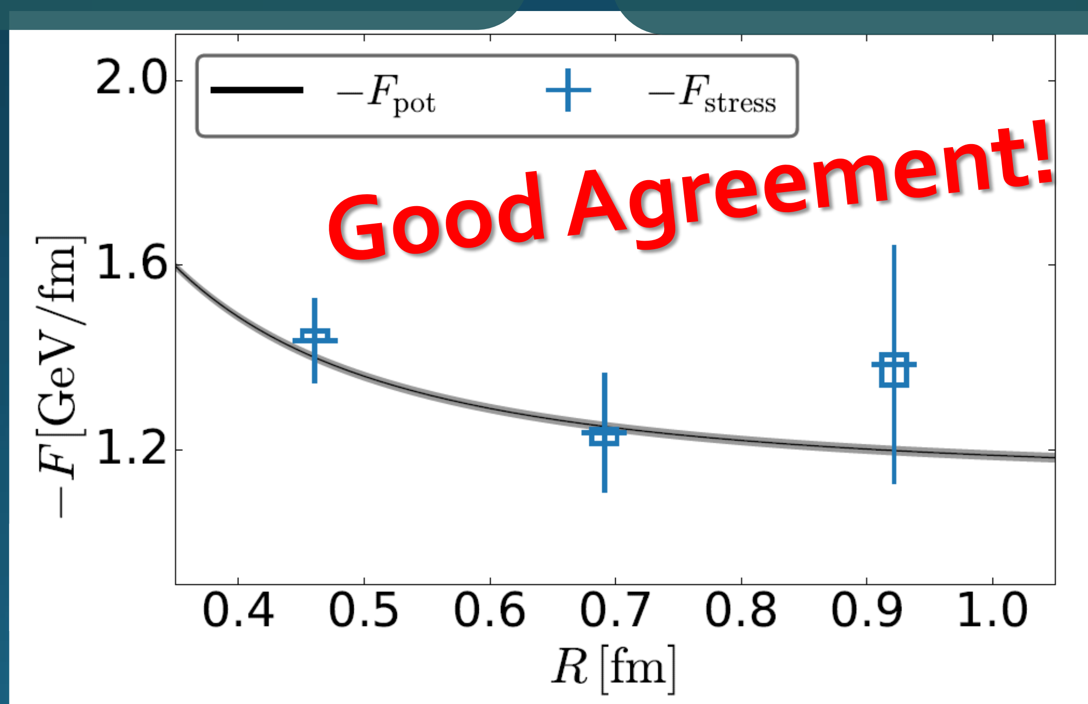


Force from Potential

$$F_{\mathrm{pot}} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\mathrm{stress}} = \int_{\mathrm{mid.}} d^2x T_{zz}(x)$$



Good Agreement!

EM Conservation,

Dual Superconductor Picture

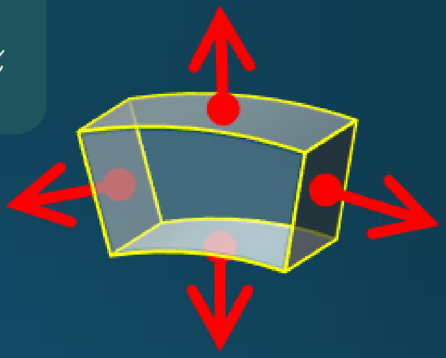
Yanagihara, MK, PTEP**2019**, 093B02 (2019)

Momentum Conservation

Yanagihara, MK, PTEP2019

- **In cylindrical coordinats,**

$$\partial_i T_{ij} = 0 \Rightarrow \partial_r (r T_{rr}) = T_{\theta\theta} - r \partial_z T_{rz}$$



For infinitely-long tube

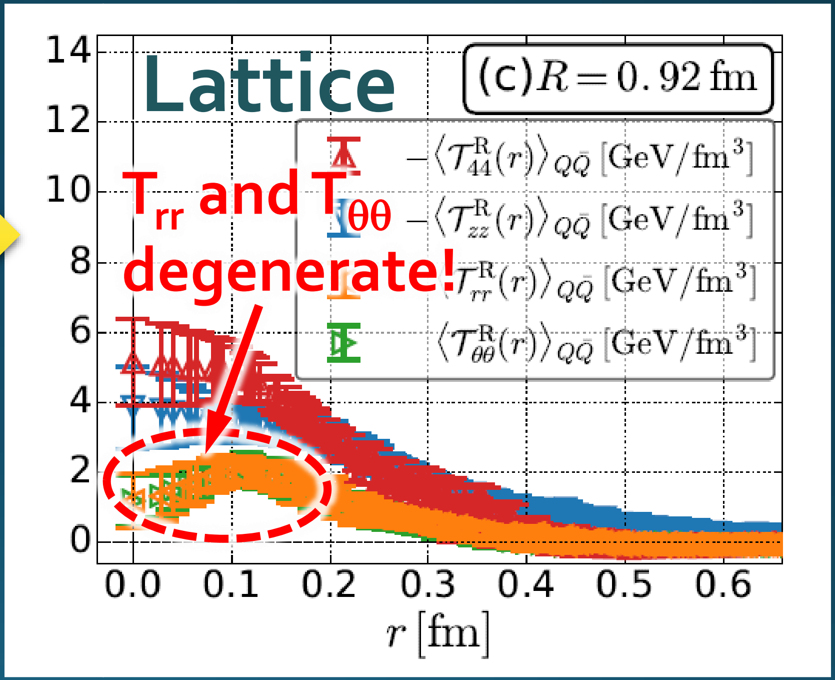
$$\partial_r (r T_{rr}) = T_{\theta\theta} \Rightarrow \int_0^\infty dr T_{\theta\theta}(r) = \left[r T_{rr} \right]_0^\infty = 0$$

- T_{rr} and $T_{\theta\theta}$ must separate!
- $T_{\theta\theta}$ must change sign!

Momentum Conservation

Yanagihara, MK, PTEP2019

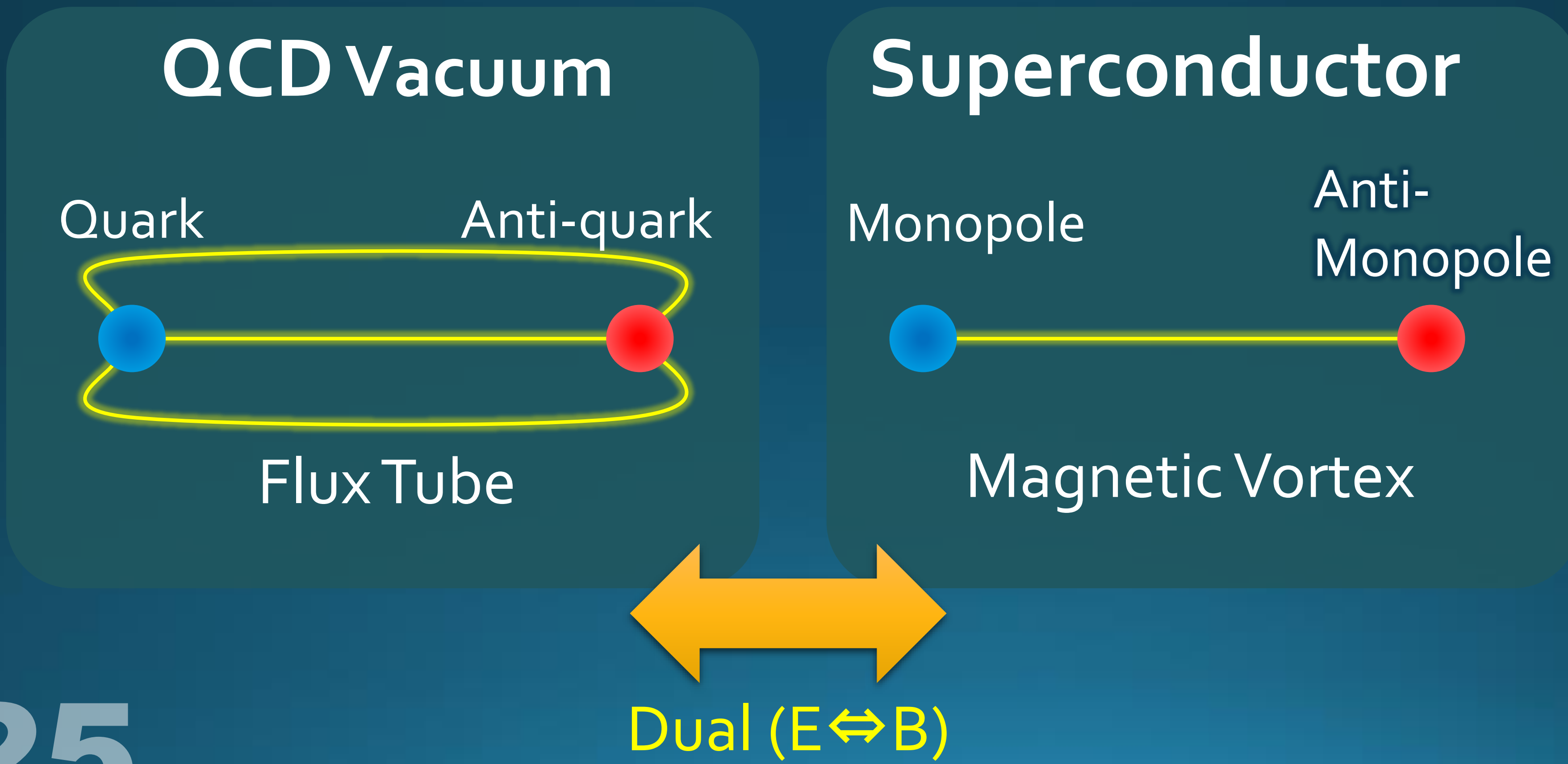
- **Infinitely-long system**
  - T_{rr} and $T_{\theta\theta}$ must separate
  - $T_{\theta\theta}$ must change sign



Effect of boundaries is important for the flux tube at R=0.92fm

Dual Superconductor Picture

Nambu, 1970
Nielsen, Olesen, 1973
t 'Hooft, 1981
...



Abelian-Higgs Model

Yanagihara, MK, 2019

Abelian-Higgs Model

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

GL parameter: $\kappa = \sqrt{\lambda}/g$

- ☐ type-I : $\kappa < 1/\sqrt{2}$
- ☐ type-II : $\kappa > 1/\sqrt{2}$
- ☐ Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

Infinitely long tube

- ☐ degeneracy

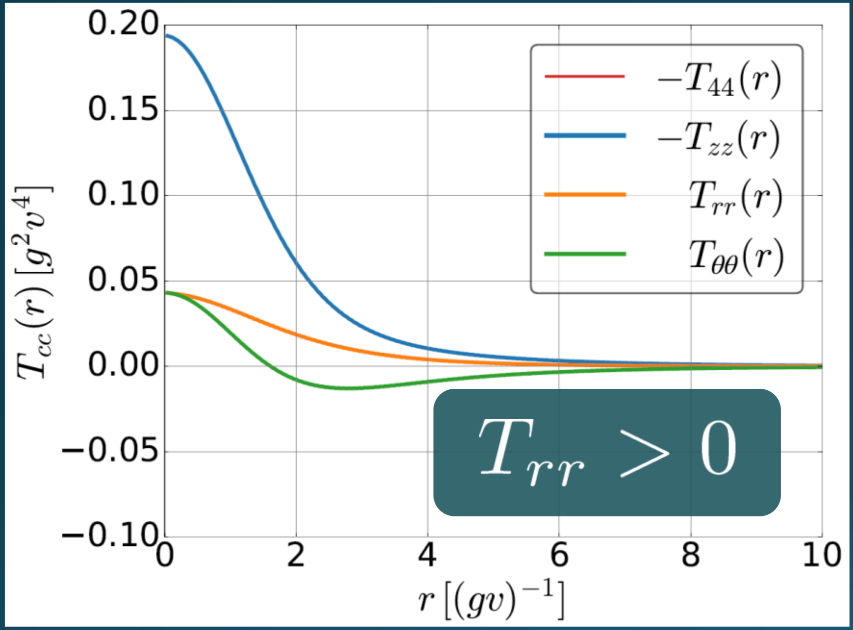
  $T_{zz}(r) = T_{44}(r)$ Luscher, 1981

- ☐ momentum conservation

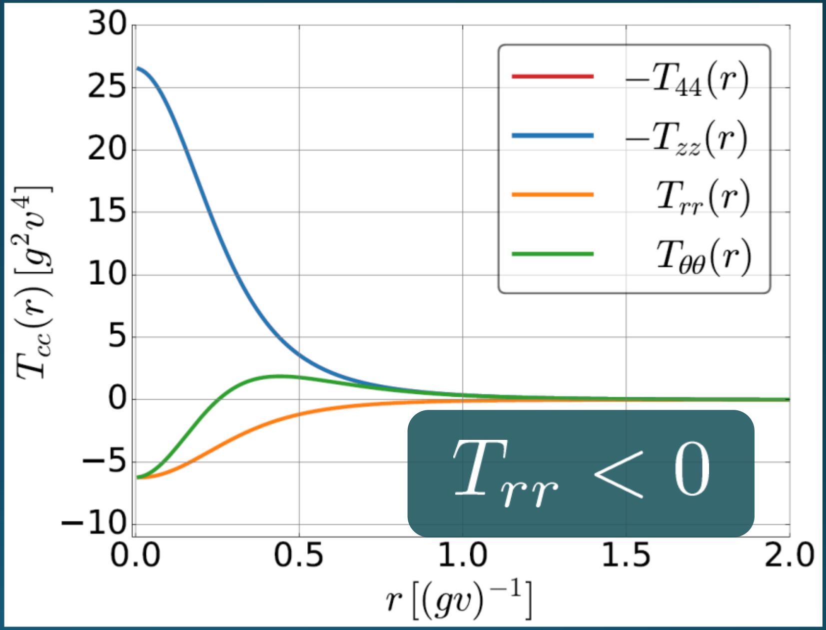
  $$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

Stress Tensor in AH Model
infinitely-long flux tube

Type-I $\kappa = 0.1$



$\kappa = 3.0$ **Type-II**



- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

conservation law

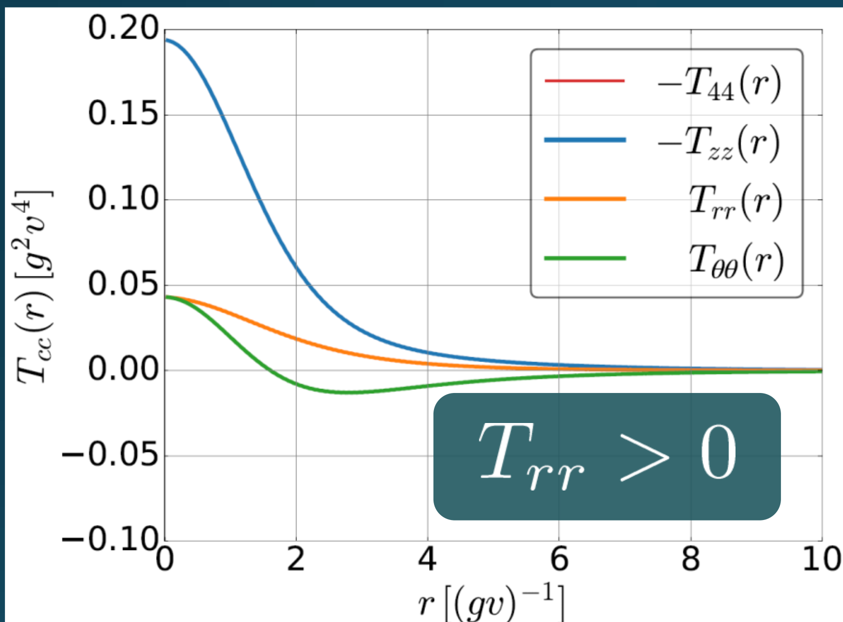
$$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

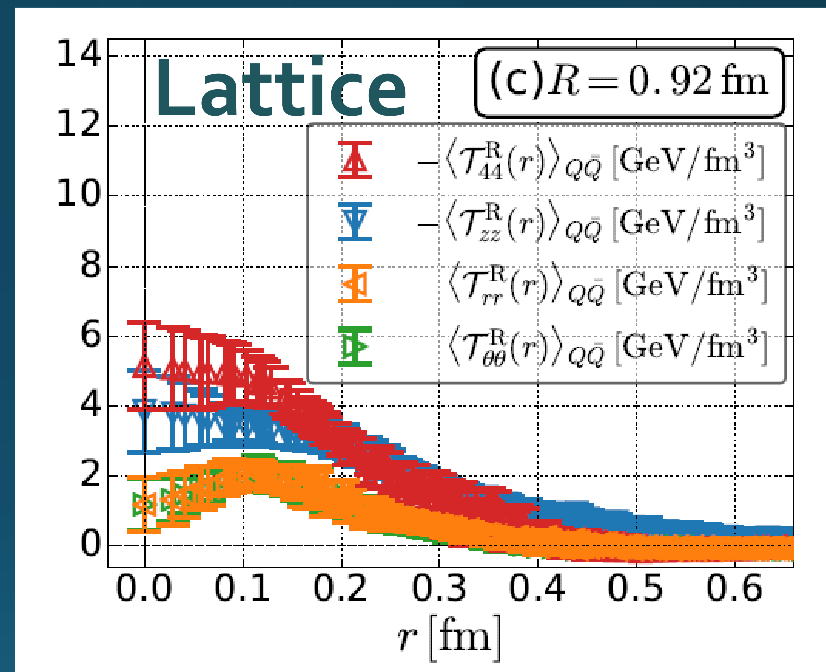
Stress Tensor in AH Model

infinitely-long flux tube

Type-I

$\kappa = 0.1$





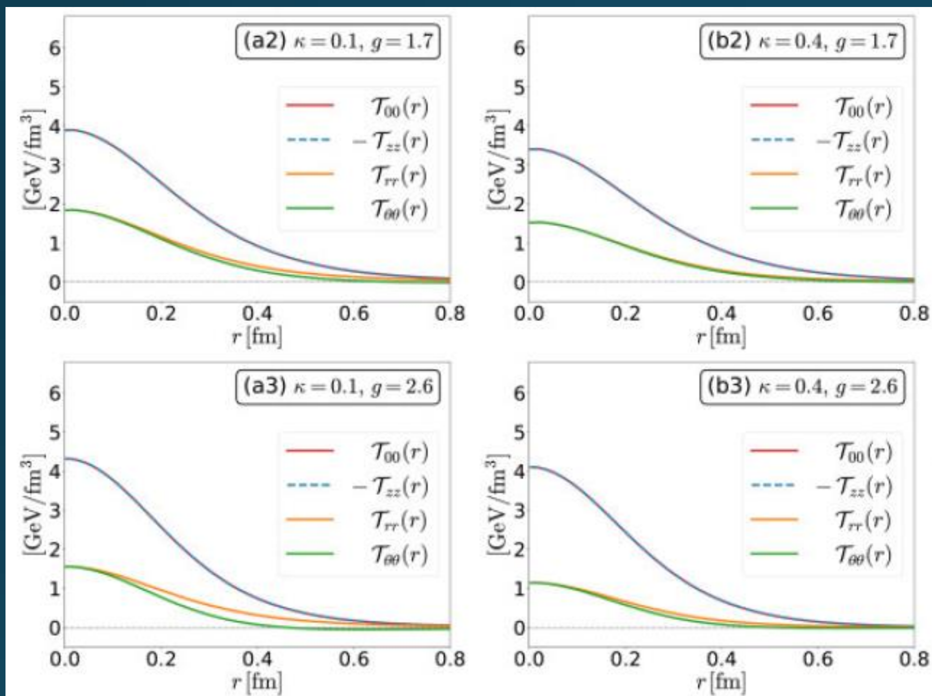
- No degeneracy bw T_{rr} & $T_{\theta\theta}$
- $T_{\theta\theta}$ changes sign

Inconsistent with lattice result

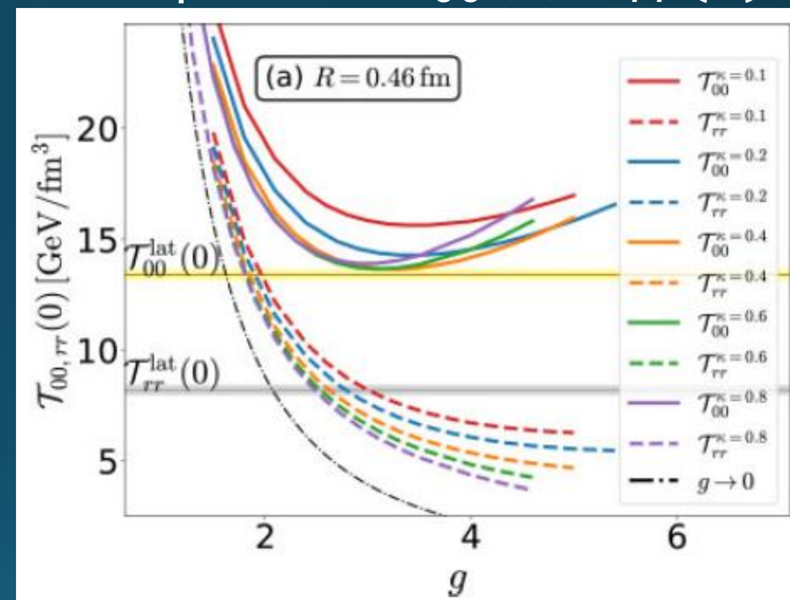
$$T_{rr} \simeq T_{\theta\theta}$$

Flux Tube with Finite Length

Yanagihara, MK (2019)



Comparison: $T_{00}(0), T_{rr}(0)$



- ☐ AH model can reproduce lattice results **qualitatively** by tuning parameters.
- ☐ But, **quantitatively** all parameters are rejected.

Quantum Effects?

- Classical vortex is unstable against quantum fluctuations
- Quantum effects give rise to
  - Luscher term in potential Luscher (1981)
  - Fattening of the tube Luscher, Munster, Weisz (1981)

$w^2 \sim \log r$

How do these effects modify EMT distribution?

EMT Distr. in Simple Systems

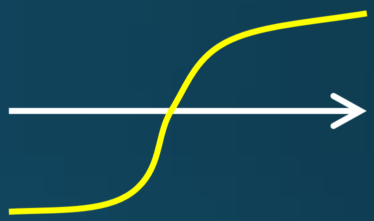
Talk by H. Ito Wednesday

ϕ^4 Theory in 1+1d

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2$$

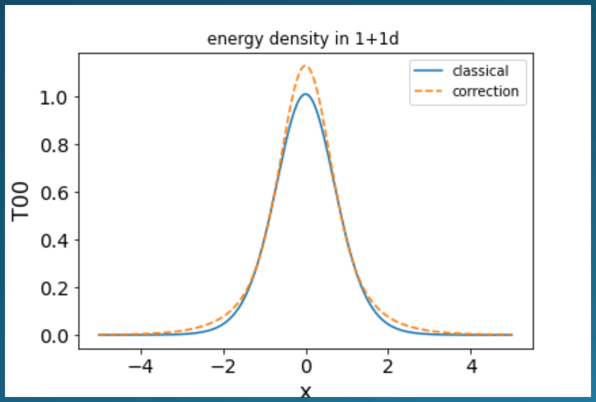
- **Soliton (kink)**

$$\phi(x) = \frac{m}{\sqrt{\lambda}} \tanh \frac{mx}{\sqrt{2}}$$



- **Quantum effect on EMT at 1-loop order**

$T_{00}(x)$



Confirmation of EMT conservation

$$\partial_x T_{11}(x) = 0$$

Flux Tube @ Nonzero T, Single Q System

FlowQCD, PRD **102**, 114522 (2020)

Motivations

☐ $T < T_c$: Heavy-light meson
- EMT distribution in the meson

☐ $T > T_c$: Single charge
- Screening
- Running coupling

☐ $T \approx T_c$
- Confinement transition

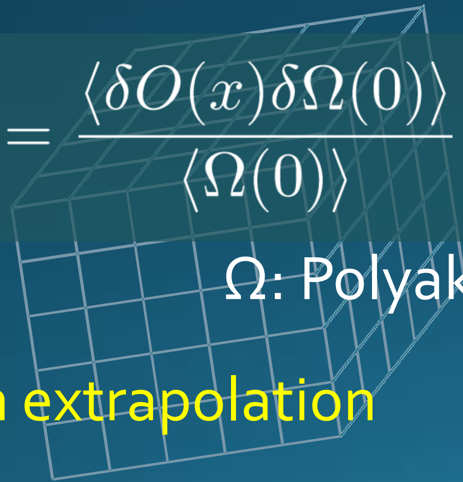
This study:
$T > T_c$ in pure YM

Lattice Setup

- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

- Analysis above Tc
- Simulation on a Z_3 minimum
- EMT around a Polyakov loop

$$\langle O(x) \rangle_{\mathrm{Q}} = \frac{\langle \delta O(x) \delta\Omega(0) \rangle}{\langle \Omega(0) \rangle}$$



Ω: Polyakov loop

- continuum extrapolation

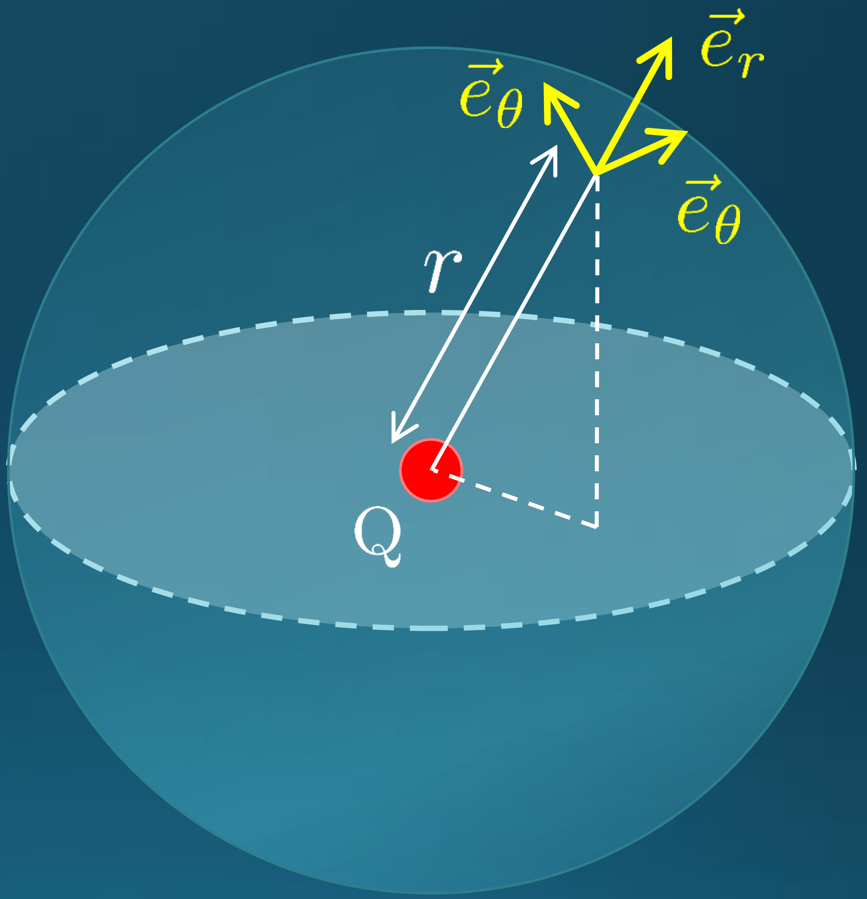
T/T_c	N_s	N_τ	β	a [fm]	N_{conf}
1.20	40	10	6.336	0.0551	500
	48	12	6.467	0.0460	650
	56	14	6.581	0.0394	840
	64	16	6.682	0.0344	1,000
	72	18	6.771	0.0306	1,000
1.44	40	10	6.465	0.0461	500
	48	12	6.600	0.0384	650
	56	14	6.716	0.0329	840
	64	16	6.819	0.0288	1,000
	72	18	6.910	0.0256	1,000
2.00	40	10	6.712	0.0331	500
	48	12	6.853	0.0275	650
	56	14	6.973	0.0236	840
	64	16	7.079	0.0207	1,000
	72	18	7.173	0.0184	1,000
2.60	40	10	6.914	0.0255	500
	48	12	7.058	0.0212	650
	56	14	7.182	0.0182	840
	64	16	7.290	0.0159	1,000
	72	18	7.387	0.0141	1,000

FlowQCD, PRD **102**, 114522 (2020)

Spherical Coordinates

EMT is diagonalized
in Spherical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{\theta\theta} & \\ & & & T_{44} \end{pmatrix}$$

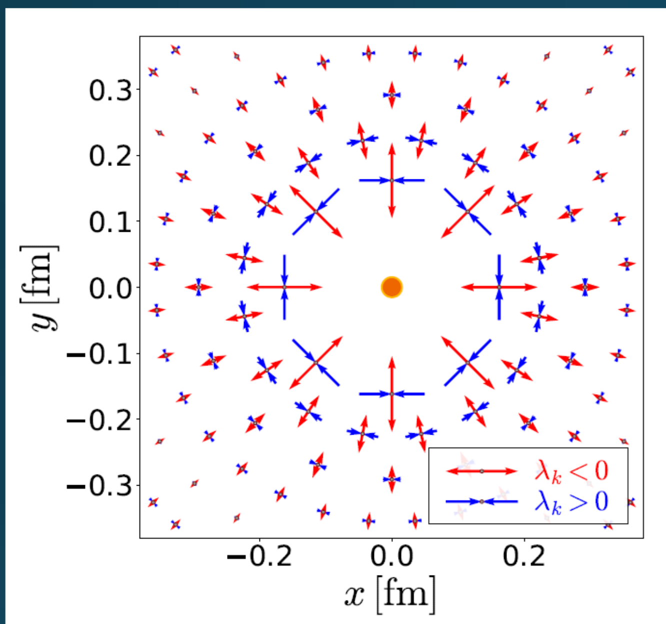


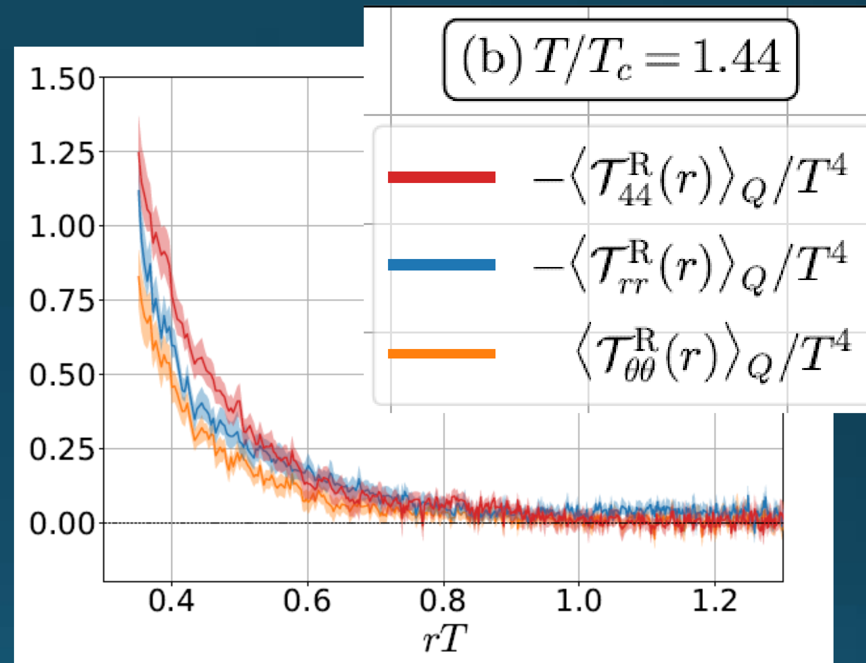
- Maxwell theory

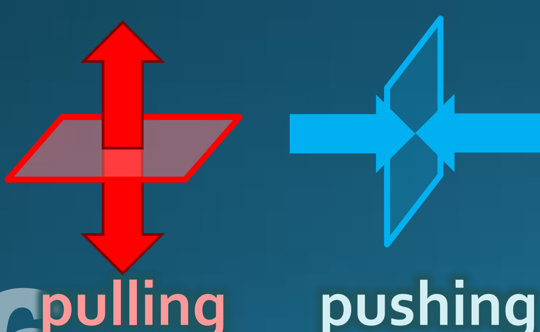
$$T_{44} = T_{rr} = -T_{\theta\theta} = -\frac{|\boldsymbol{E}|^2}{2} = -\frac{\alpha}{8\pi}\frac{1}{r^4}$$

Stress Tensor Around a Quark

$T=1.44T_c$



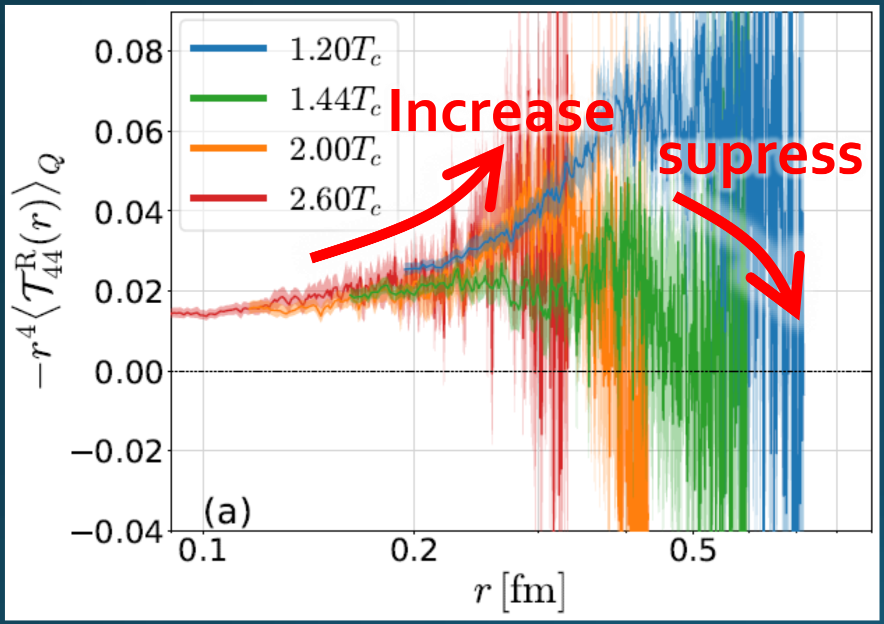




- ☐ Suppression at large distance
- ☐ Separation of different channels

r Dependence

$r^4\langle T_{00}(r)\rangle$



Leading order perturbation

$$\langle \mathcal{T}_{44}(r)\rangle = \langle \mathcal{T}_{rr}(r)\rangle = -\langle \mathcal{T}_{\theta\theta}(r)\rangle$$

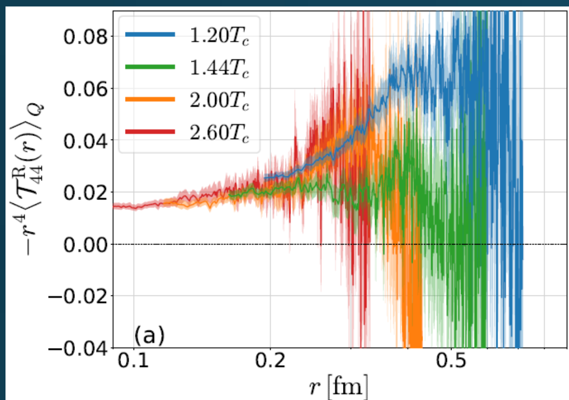
$$= -\frac{C_F}{8\pi}\alpha_s \frac{(m_{\mathrm{D}}r+1)^2}{r^4} e^{-2m_{\mathrm{D}}r}$$

Higher order terms:
M. Berwein, in progress

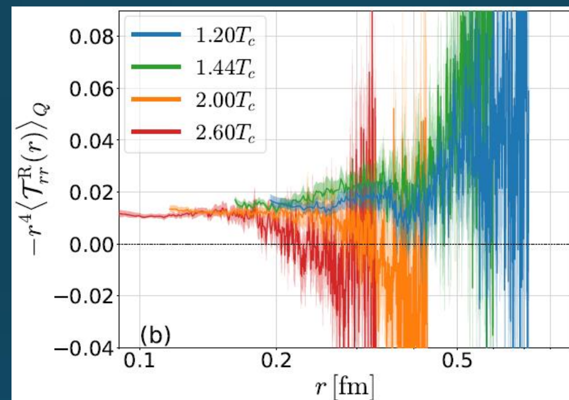
- ☐ Increase at short r / suppression at larger r
- ☐ T dependence is suppressed at $r < 1/T$
- ☐ Too noisy at large r for extracting screening mass m_{D}

Channel Dependence

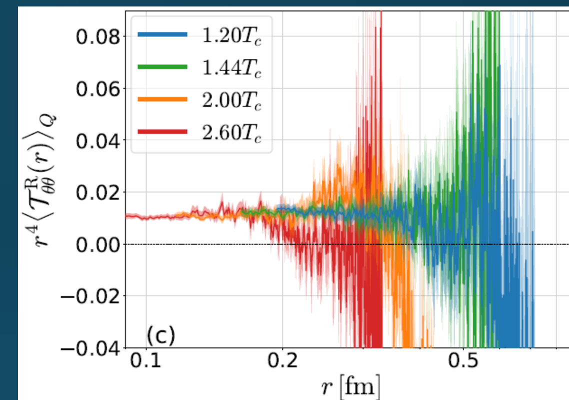
$$r^4 \langle T_{00}(r) \rangle$$



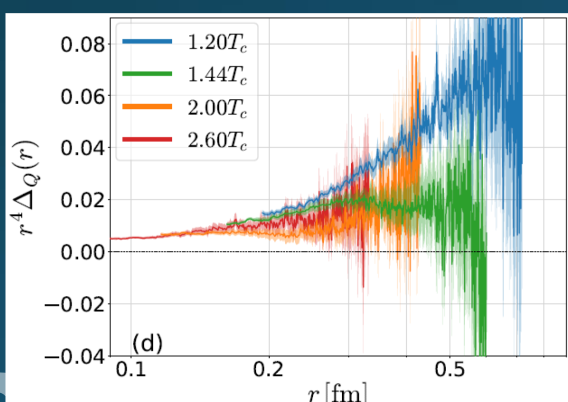
$$-r^4 \langle T_{rr}(r) \rangle$$



$$r^4 \langle T_{\theta\theta}(r) \rangle$$



$$r^4 \Delta(r) = -r^4 \langle T_{\mu\mu} \rangle$$



- Separation b/w channels becomes clearer for smaller *T*

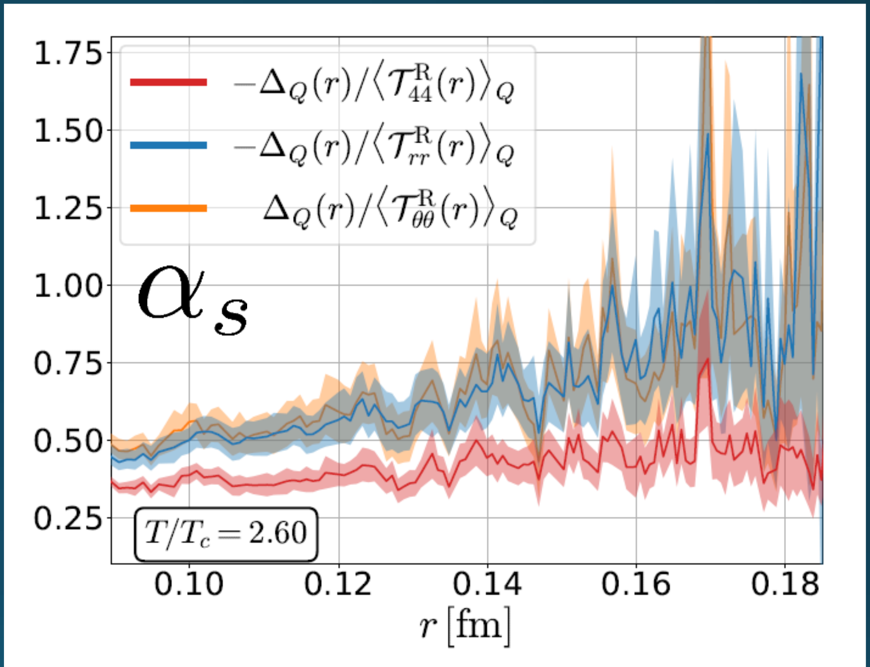
FlowQCD, PRD **102**, 114522 (2020)

Running Coupling

- **Estimate of α_s**

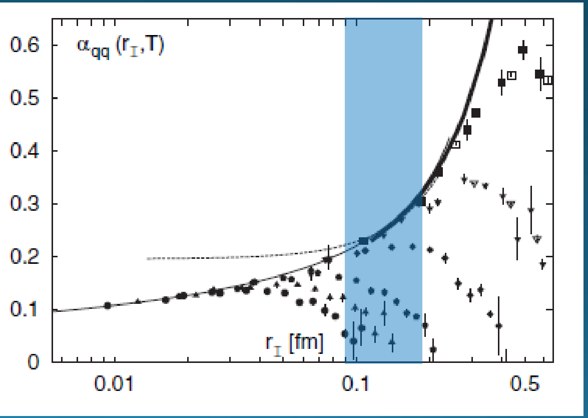
$$\left|\frac{\langle T_{\mu\mu}\rangle}{\langle \mathcal{T}_{44,rr,\theta\theta}(r)\rangle}\right| = \frac{11}{2\pi}\alpha_s + \mathcal{O}(g^3),$$

  - by the formula at the leading-order perturbation theory
  - channel dependent



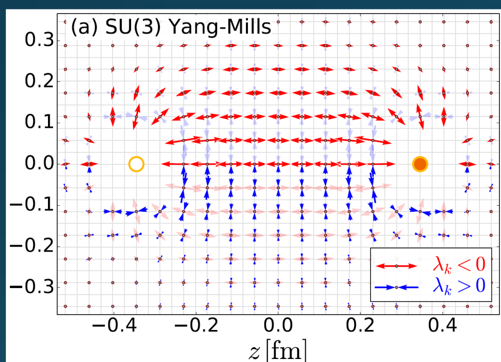
- Consistent with the estimate from $Q\bar{Q}$ potential

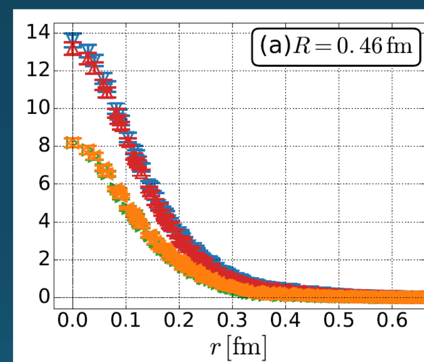
Kaczmarek, Karsch, Zantow, 2004

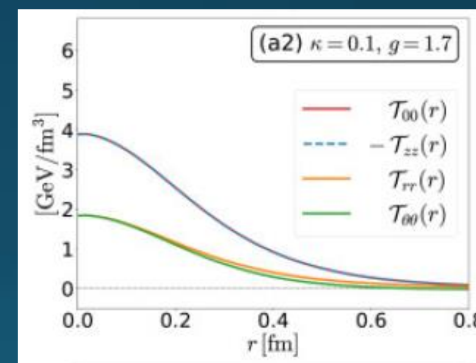


Summary

- Now, lattice simulation of EMT around the flux tube is available thanks to SF*t*X (gradient flow) method.
- EMT distribution of the flux tube tells us many interesting features of this system.







Future studies

- Theoretical understanding of the lattice results
- QQQ, QQ, etc. / *T* dependence / hadrons

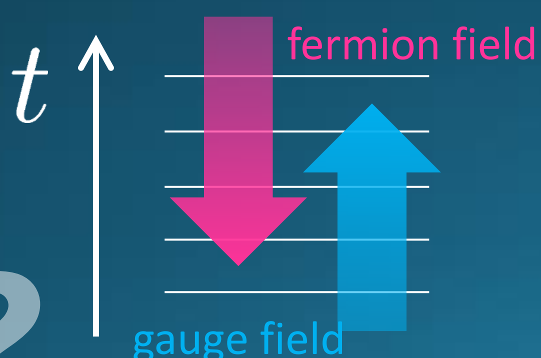
backup

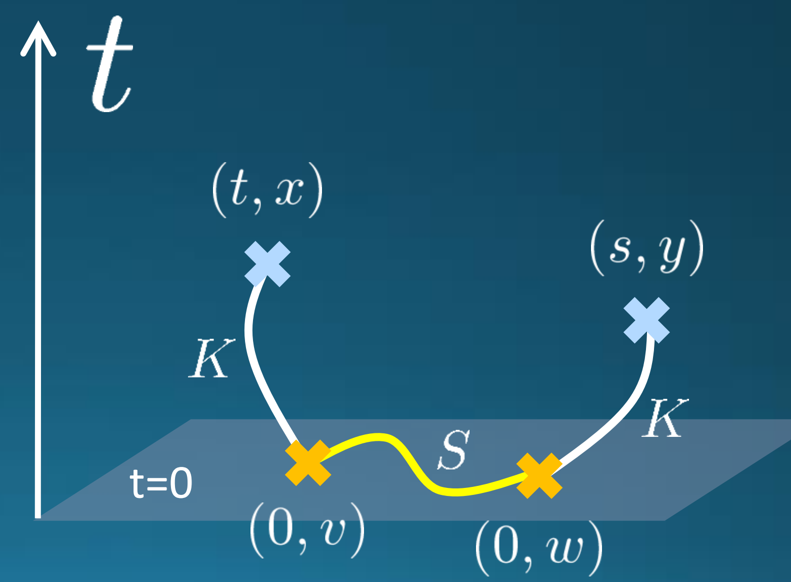
Fermion Propagator

$$S(t,x;s,y) = \langle \chi(t,x)\bar{\chi}(s,y)\rangle$$
$$= \sum_{v,w} K(t,x;0,v)S(v,w)K(s,y;0,w)^{\dagger}$$

$$\Big(\partial_t - D_\mu D_\mu\Big)K(t,x) = 0$$

- propagator of flow equation
- Inverse propagator is needed





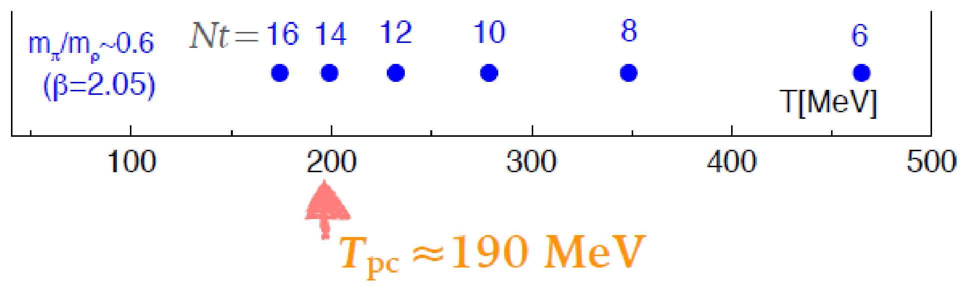
N_f=2+1 QCD Thermodynamics

Taniguchi+ (WHOT-QCD),
PRD**96**, 014509 (2017)

- N_f=2+1 QCD, Iwasaki gauge + NP-clover
- $m_{PS}/m_V \approx 0.63$ / almost physical s quark mass

- T=0: CP-PACS+JLQCD (ß=2.05, 28^3x56, a≈0.07fm)
- T>0: 32^3xN_t, N_t = 4, 6, … , 14, 16):
- T≈174-697MeV

- t→0 extrapolation only (No continuum limit)



Perturbative Coefficients

$$T_{\mu\nu}(t) = c_1(t)U_{\mu\nu}(t) + \delta_{\mu\nu}c_2(t)E(t)$$

	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○	○	
$c_2(t)$	× zero	○	○	○

Suzuki (2013) Harlander+(2018)

Iritani, MK, Suzuki, Takaura, 2019

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

□ Choice of the scale of g²

$$c_1(t) = c_1\left(g^2(\mu(t))\right)$$

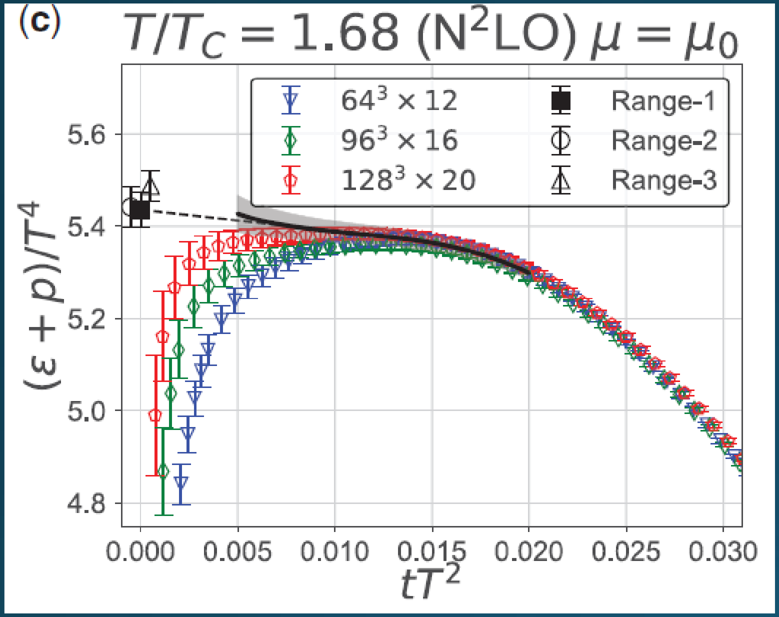
Previous: $\mu_d(t) = 1/\sqrt{8t}$

Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E}t}$

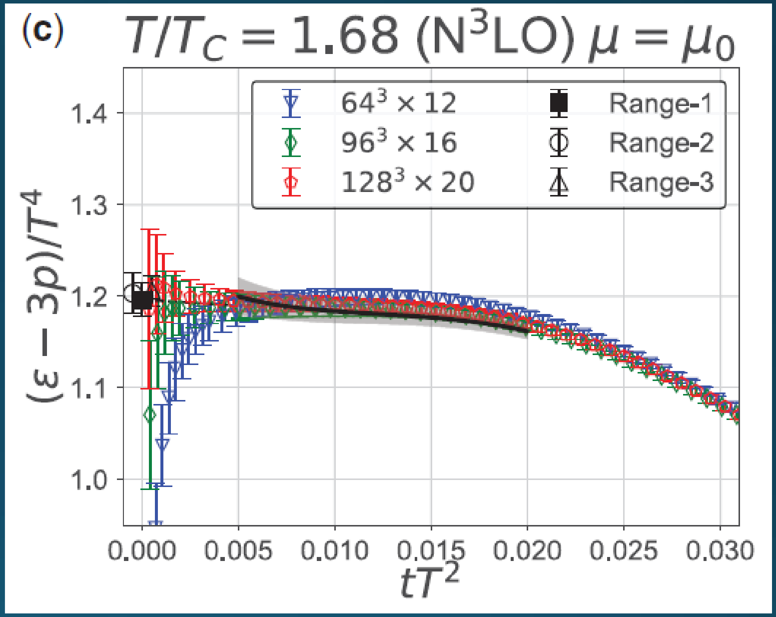
Harlander+ (2018)

t Dependence

$\langle T_{00} \rangle + \langle T_{11} \rangle$



$\langle T_{00} \rangle - 3\langle T_{11} \rangle$



Iritani, MK, Suzuki, Takaura, PTEP 2019

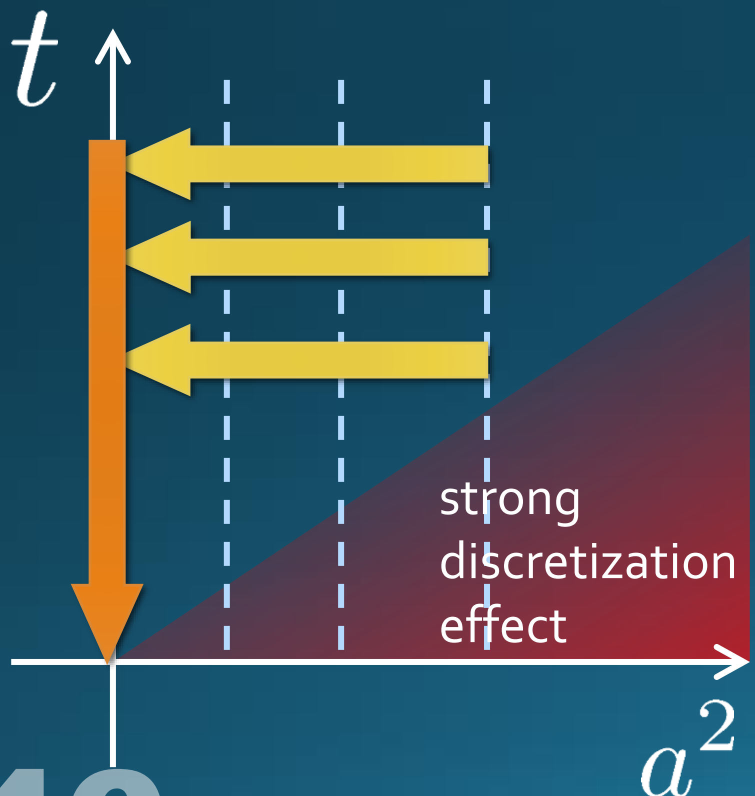
- Existence of "linear window" at intermediate t

Double Extrapolation

t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE   lattice discretization



Continuum extrapolation

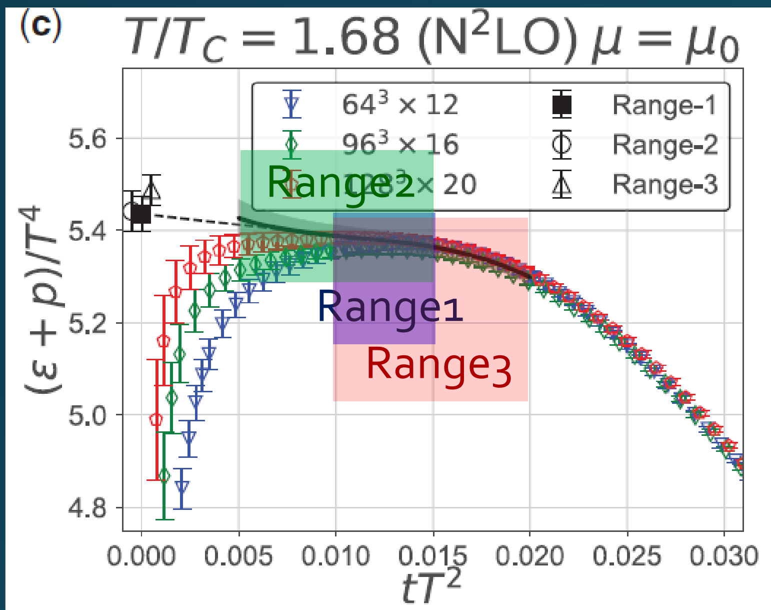
$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

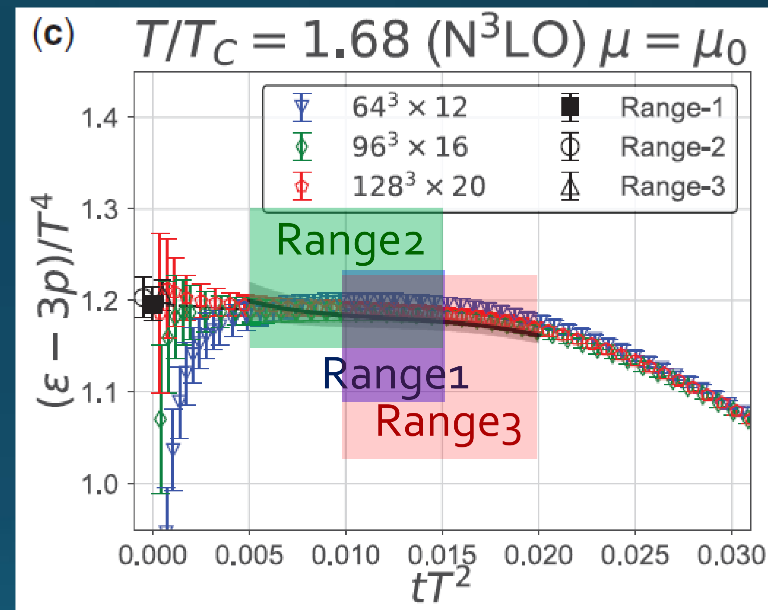
Small t extrapolation

$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$

Thermodynamics: ε+p & ε-3p

$\langle T_{00}\rangle + \langle T_{11}\rangle$ $\qquad$ $\langle T_{00}\rangle - 3\langle T_{11}\rangle$





Iritani, MK, Suzuki, Takaura, PTEP 2019

- Existence of "linear window" at intermediate t
- Stable t⟶0 extrapolation
- Systematic errors: fit range, uncertaintyof Λ (±3%), ...

EMT on the Lattice: Conventional

Lattice EMT Operator Caracciolo+, 1990

$$T_{\mu\nu} = Z_6 T^{[6]}_{\mu\nu} + Z_3 T^{[3]}_{\mu\nu} + Z_1 \left(T^{[1]}_{\mu\nu} - \langle T^{[1]}_{\mu\nu} \rangle \right)$$

$$T^{[6]}_{\mu\nu} = (1 - \delta_{\mu\nu}) F^a_{\mu\rho} F^a_{\nu\rho}, \quad T^{[3]}_{\mu\nu} = \delta_{\mu\nu} \left(F^a_{\mu\rho} F^a_{\nu\rho} - \frac{1}{4} F^a_{\rho\sigma} F^a_{\rho\sigma} \right), \quad T^{[1]}_{\mu\nu} = \delta_{\mu\nu} F^a_{\rho\sigma} F^a_{\rho\sigma}$$

- Determination of Zs are necessary.
- Non-pert. Determination of Zs
  - Shifted-boundary method
  - Full QCD with fermions

Giusti, Pepe, 2014~; Borsanyi+, 2018
Brida, Giusti, Pepe, 2020