From Lattice to Observables: Real & Virtual Experiments for Exploring Hot and Dense QCD

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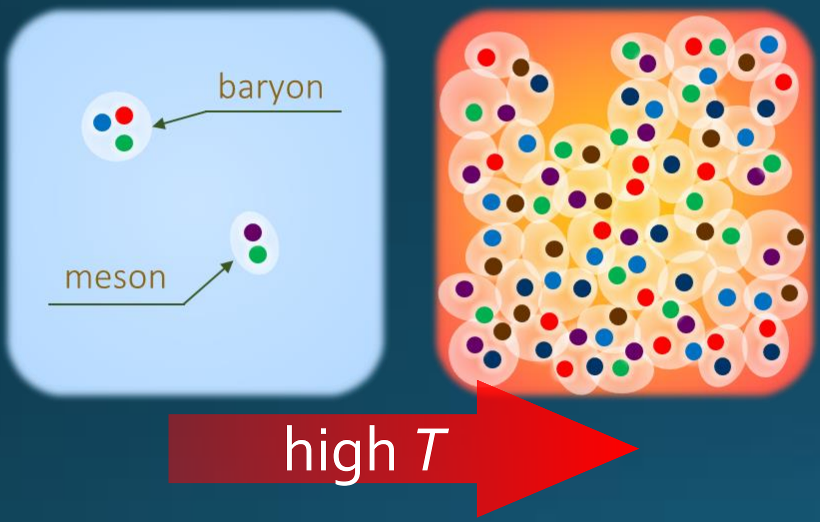
(Osaka U.)

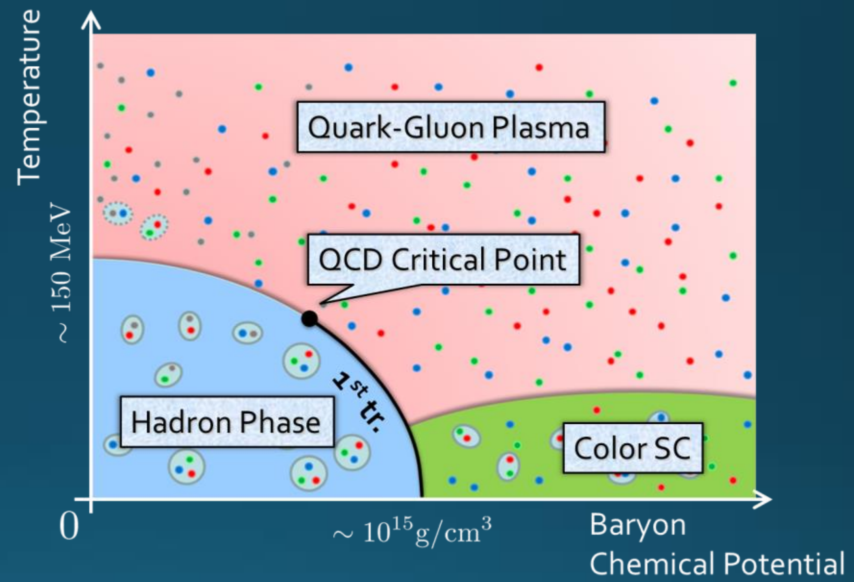
Strangeness in Quark Matter (SQM2022), 2022/6/13, Busan, Korea

Hot & Dense QCD: Motivations

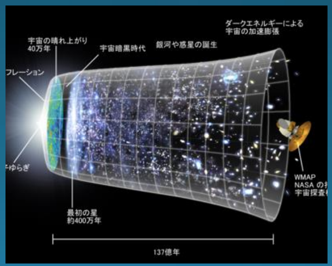
Quark-Gluon Plasma



QCD Phase Diagram

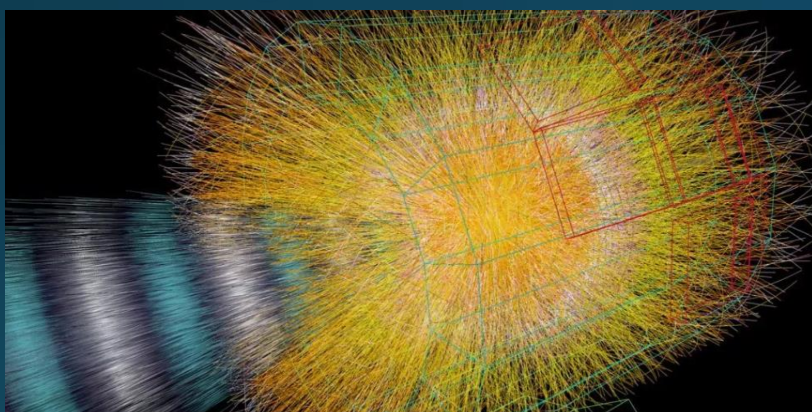


- ☐ Non-perturbative aspects of QCD
- ☐ Early universe, neutron stars, …



Two "Experimental" Tools
to explore hot & dense medium

Relativistic Heavy-Ion Collisions



Real Experiment
"HIC"

Lattice QCD Numerical Simulations

Virtual Experiment
"LAT"

Their complementary use is essential!

HIC vs LAT: Pros & Cons

HIC	LAT
Real	Virtual
Dynamical	Thermal

necessary in physics

simulating unphysical world

need space-time models

ideal environment; unable to simulate real-time evolution

input

motivate

HIC vs LAT: Pros & Cons

HIC	LAT
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necessary in physics

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ideal environment; unable to simulate real-time evolution

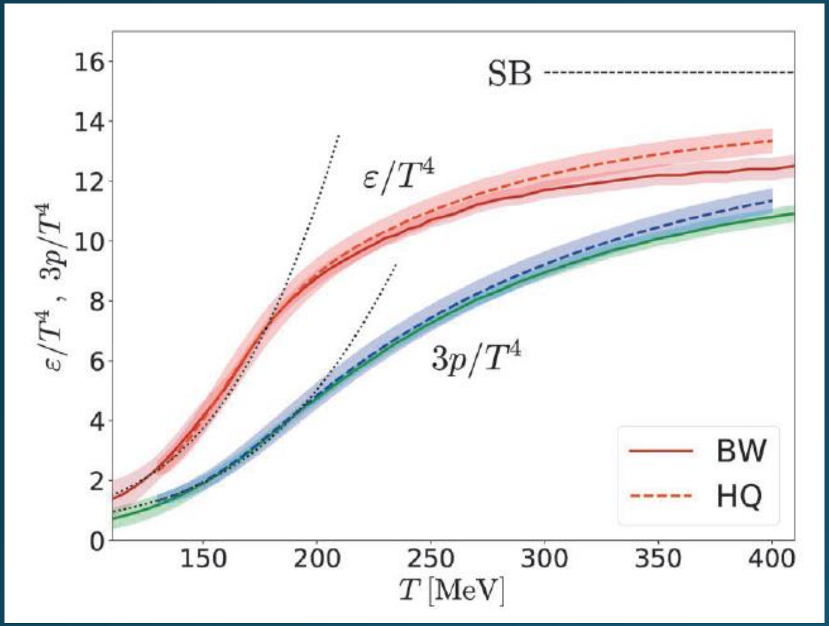
input

motivate

finite density

Accessible	Difficult
beam-energy scan	due to sign problem

Equation of State



Budapest-Wuppertal '14, HotQCD '14
(plot by MK)

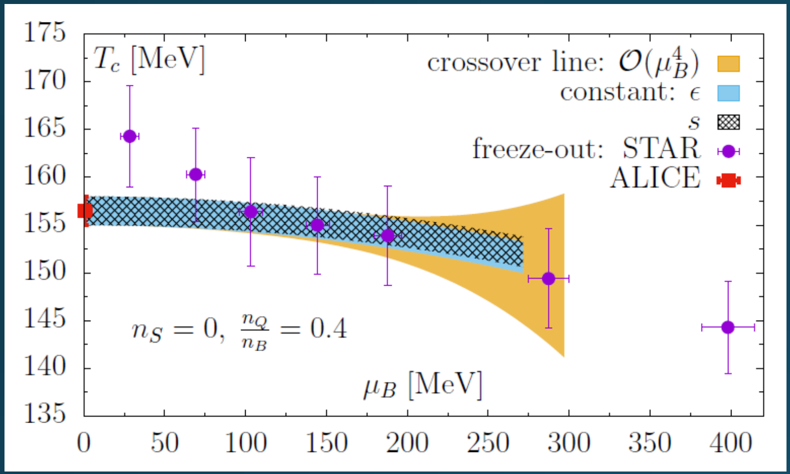
- Crossover transition
- Low *T*: hadron resonance gas (HRG) model
- High *T*: gas of quarks & gluons

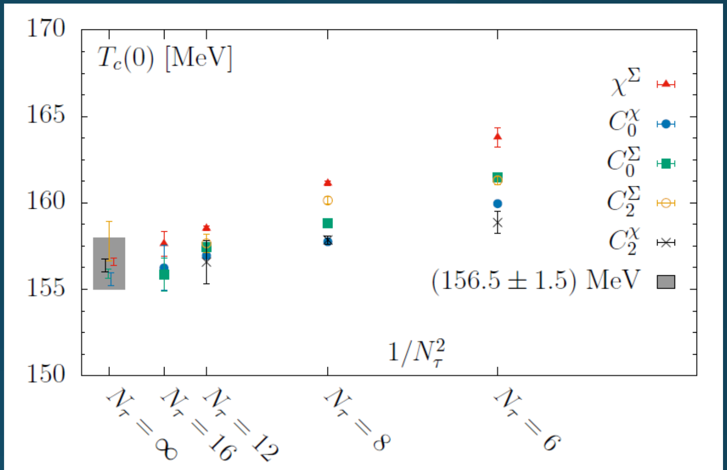
- Input for hydrodynamic models

- Hydro. models need **transport coefficients**.
- Its reliable measurement in LAT is still a challenge.

(Pseudo) Critical Temperature

HotQCD, PLB **795**, 15 (2019)





- ☐ LAT: $T_c^* = 156.5(1.5)$ MeV
- ☐ HIC: thermal model (chemical f.o.)
- ☐ $T_c^* \simeq T_{\text{chem}}$ → Consistent picture of hadronization

- ☐ Taylor expansion method for nonzero μ_{B}

$$p(T,\mu) = p(T,0) + \frac{\chi_2}{2}\left(\frac{\mu}{T}\right)^2 + \frac{\chi_4}{4!}\left(\frac{\mu}{T}\right)^4 + \cdots \qquad \chi_n = \frac{\partial^n p}{\partial \hat{\mu}^n}$$

Non-Gaussian Fluctuations / Higher Order Cumulants

Cumulants

Cumulants

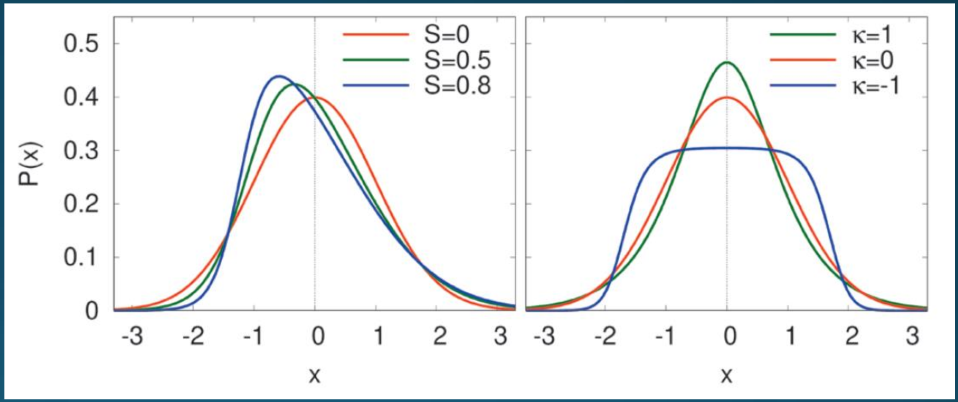
$$\begin{cases} \langle N \rangle_c = \langle N \rangle & \text{average} \\ \langle N^2 \rangle_c = \langle \delta N^2 \rangle & \text{variance} \\ \langle N^3 \rangle_c = \langle \delta N^3 \rangle & \\ \langle N^4 \rangle_c = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2 & \end{cases}$$

□ skewness

$$S = \frac{\langle N^3 \rangle_c}{\langle N^2 \rangle_c^{3/2}}$$

□ kurtosis

$$\kappa = \frac{\langle N^4 \rangle_c}{\langle N^2 \rangle_c^2}$$



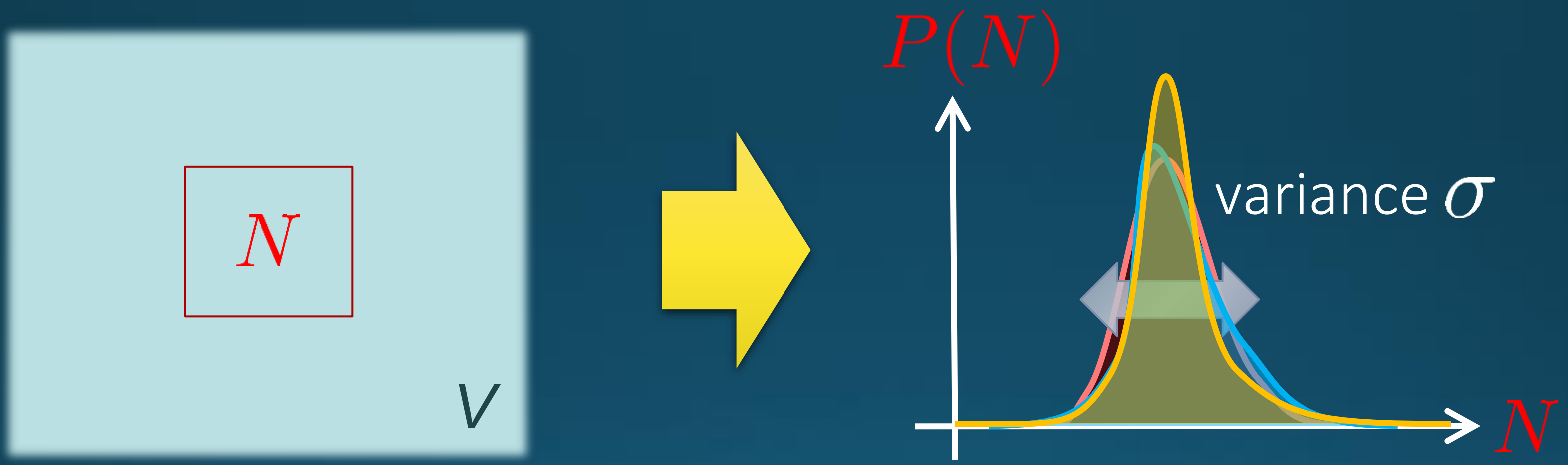
□ NOTE

- Gauss distribution: $\langle N^3 \rangle_c = \langle N^4 \rangle_c = \cdots = 0$
- Poisson distribution: $\langle N^2 \rangle_c = \langle N^3 \rangle_c = \langle N^4 \rangle_c = \cdots = \langle N \rangle$

Review: Asakawa, MK, PPNP **90** (2016) Sec. 2

Thermal Fluctuations

Observables in equilibrium are fluctuating!



Enhancement & sign change of higher order cumulants will be used for the signal of the QCD critical point.

Stephanov, '09; Asakawa, Ejiri, MK, '09

Cumulants of Conserved Charges
=Observable on the Lattice

- Fluctuation-Response Relations

$$\langle N_B^m \rangle_c = V \chi_m^B$$

Thermal Fluctuation — Susceptibility

$$\chi_m^B \sim \frac{\partial^m p}{\partial \mu_B^m}$$

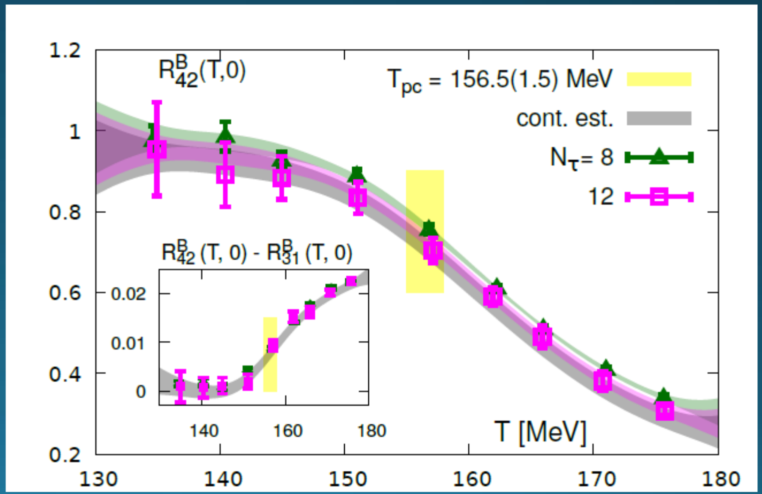
$$p(T,\mu) = p(T,0) + \frac{\chi_2}{2}\left(\frac{\mu}{T}\right)^2 + \cdots$$

- Volume dependence canceled out in ratios

Ejiri, Karsch, Redlich, '05

➡ useful for comparison w/ HIC

$$\langle N_B^4 \rangle_c / \langle N_B^2 \rangle_c$$

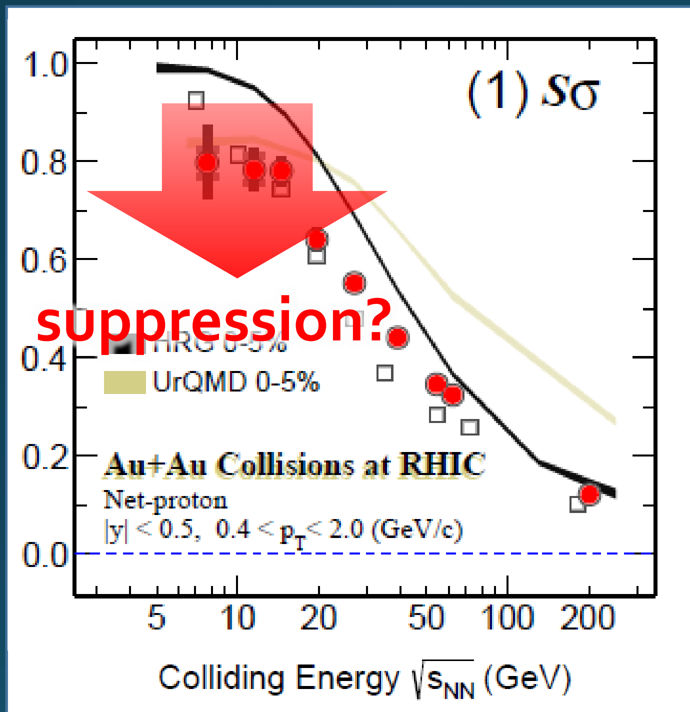


under magnetic field: Ding+, 2104.06843

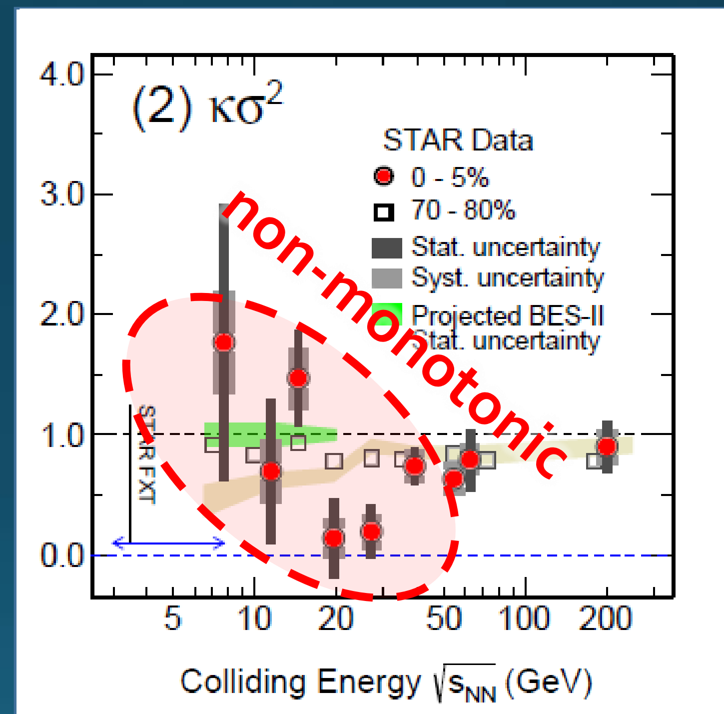
HotQCD, PRD**101** (2020)

Proton Number Cumulants in HIC

$\langle N_p^3 \rangle_c / \langle N_p^2 \rangle_c$



$\langle N_p^4 \rangle_c / \langle N_p^2 \rangle_c$



STAR, PRC 2020 [2001.06419]

- ☐ **Nonzero and non-Poissonian cumulants are experimentally established.**

Issues to be Resolved

- Experiments measure **proton** number cumulants, while lattice calculates **baryon**'s.
- Experiments measure **the final state of the dynamical evolution**, while the lattice measures an equilibrium state.
- And, other issues:
  - Volume fluctuation
  - Efficiency correction / imperfect acceptance
  - Measurement in momentum space
  - Resonance decays, Jets, …

More problematic for higher order cumulants!

See, Asakawa, MK, Mueller, PRC 101 (2020)

Proton vs Baryon Cumulants

MK, Asakawa, 2012; 2012

- $\langle N_p^m \rangle_c \neq \langle N_B^m \rangle_c$
- $\langle N_B^m \rangle_c$ can be obtained from the distribution of N_p thanks to the isospin randomization.

$N_B \to N_p$:

$$\langle N_p^{(\mathrm{net})} \rangle = \frac{1}{2} \langle N_\mathrm{B}^{(\mathrm{net})} \rangle,$$

$$\langle (\delta N_p^{(\mathrm{net})})^2 \rangle = \frac{1}{4} \langle (\delta N_\mathrm{B}^{(\mathrm{net})})^2 \rangle + \frac{1}{4} \langle N_\mathrm{B}^{(\mathrm{tot})} \rangle,$$

$$\langle (\delta N_p^{(\mathrm{net})})^3 \rangle = \frac{1}{8} \langle (\delta N_\mathrm{B}^{(\mathrm{net})})^3 \rangle + \frac{3}{8} \langle \delta N_\mathrm{B}^{(\mathrm{net})} \delta N_\mathrm{B}^{(\mathrm{tot})} \rangle,$$

Information of baryon # cumulants are more suppressed in higher order proton # cumulants!

$N_p \to N_B$:

$$\langle N_\mathrm{B}^{(\mathrm{net})} \rangle = 2 \langle N_p^{(\mathrm{net})} \rangle,$$

$$\langle (\delta N_\mathrm{B}^{(\mathrm{net})})^2 \rangle = 4 \langle (\delta N_p^{(\mathrm{net})})^2 \rangle - 2 \langle N_p^{(\mathrm{tot})} \rangle,$$

$$\langle (\delta N_\mathrm{B}^{(\mathrm{net})})^3 \rangle = 8 \langle (\delta N_p^{(\mathrm{net})})^3 \rangle - 12 \langle \delta N_p^{(\mathrm{net})} \delta N_p^{(\mathrm{tot})} \rangle + 6 \langle N_p^{(\mathrm{net})} \rangle,$$

$$\langle N_B^2 \rangle_c / \langle N_Q^2 \rangle_c$$

MK, Esumi, Nonaka, 2205.10030

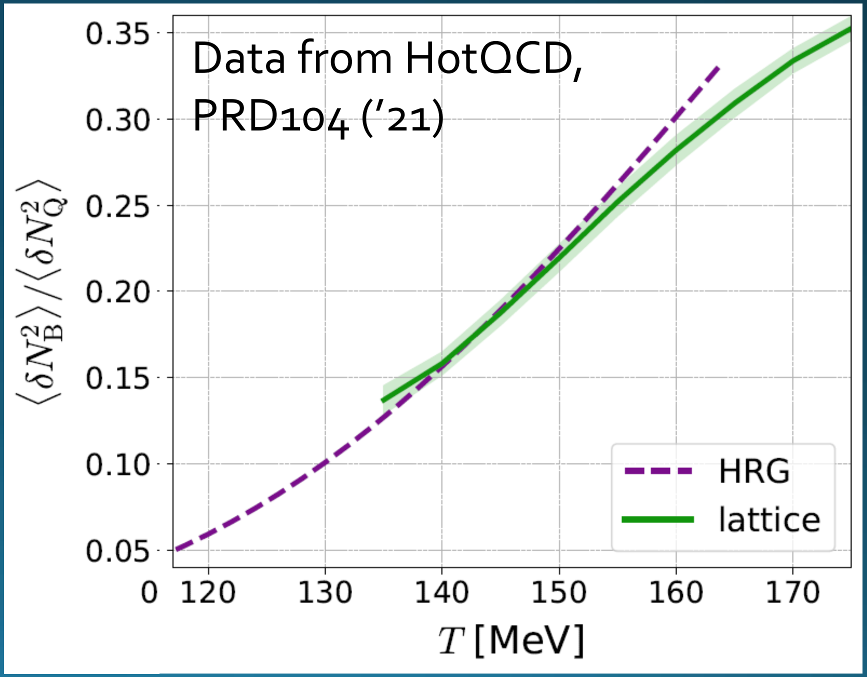
Motivations:

- Ratio of 2nd orders: suppress uncertainties
- Almost linear T dep. around T_c^*

Analysis:

- $\sqrt{s_{NN}} = 200$GeV, 0-5%
- Δy dependence
- Construction of baryon #, p_T-acceptance correction

Data from STAR,
PRC104,024902 (2021)
PRC100,014902 (2019)



p_T-Acceptance Correction

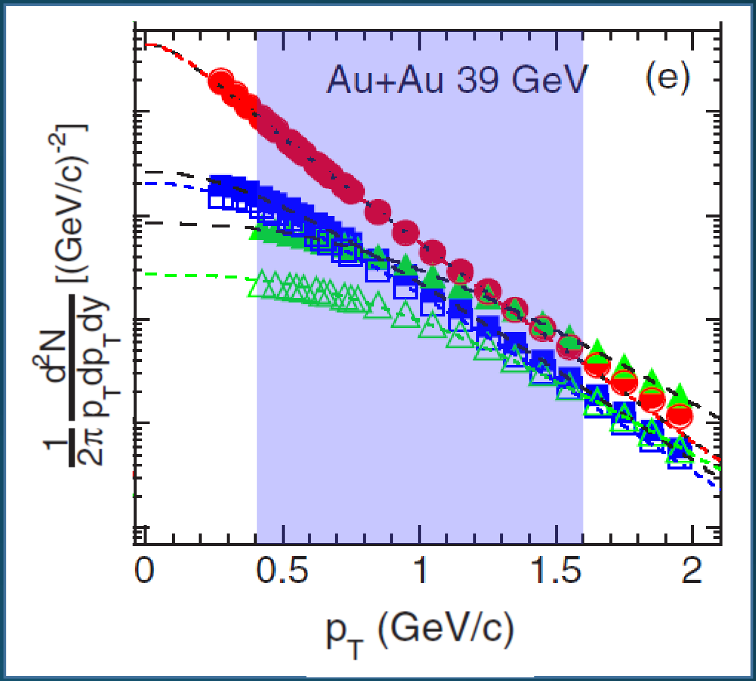
p_T Acceptance

$0.4 < p_T < 1.6$ [GeV/c]

PRC100,014902('19)

$0.4 < p_T < 2.0$ [GeV/c]

PRC104,024902('21)



Particles in p_T acceptance

- Electric charge: **49%**
- Protons: **82%**

blast wave model @ $\sqrt{s_{NN}}$=200 GeV

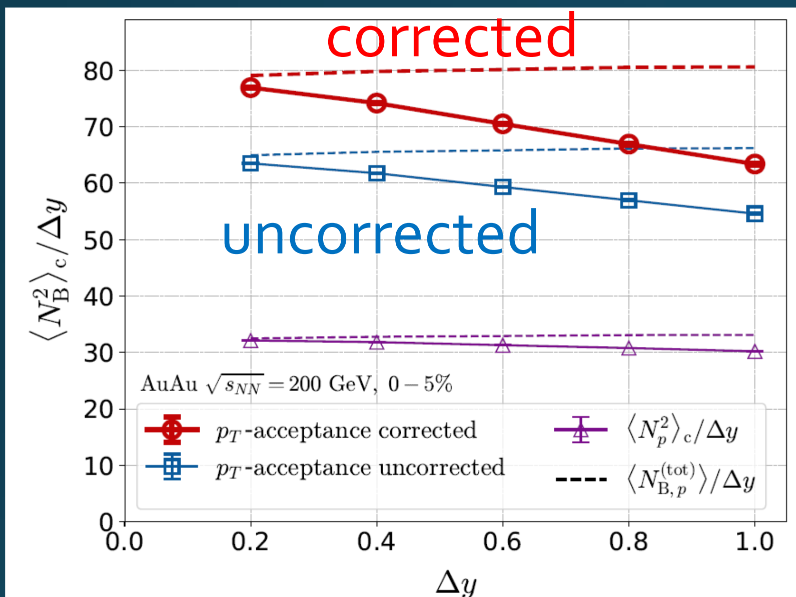
correction assuming binomial distr. model (independent particle emission)

MK, Asakawa, '12; '12

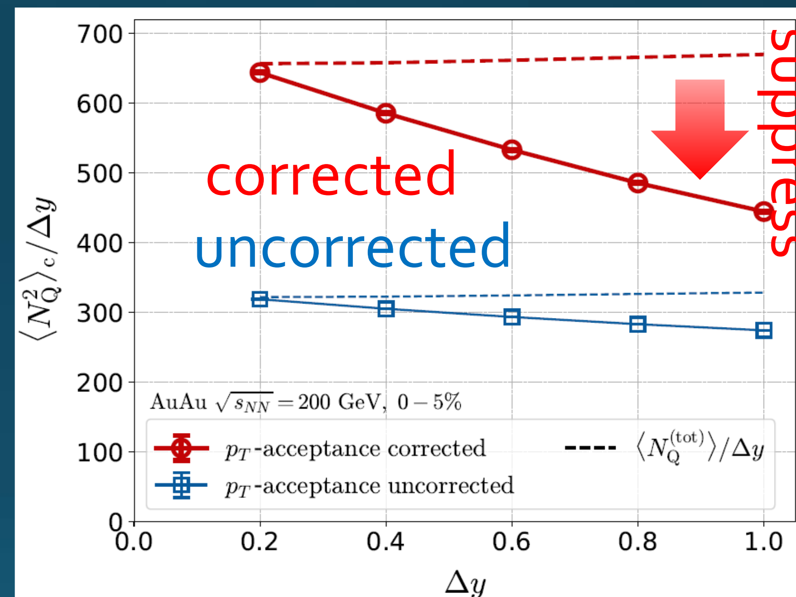
Correction in theoretical models: Alba, Bellwied, '15, ...

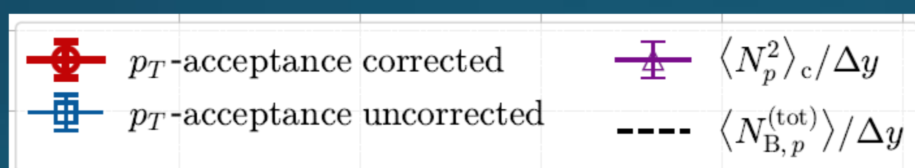
Baryon & Acceptance Corrections

$\langle N_B^2 \rangle_c / \Delta y$



$\langle N_Q^2 \rangle_c / \Delta y$

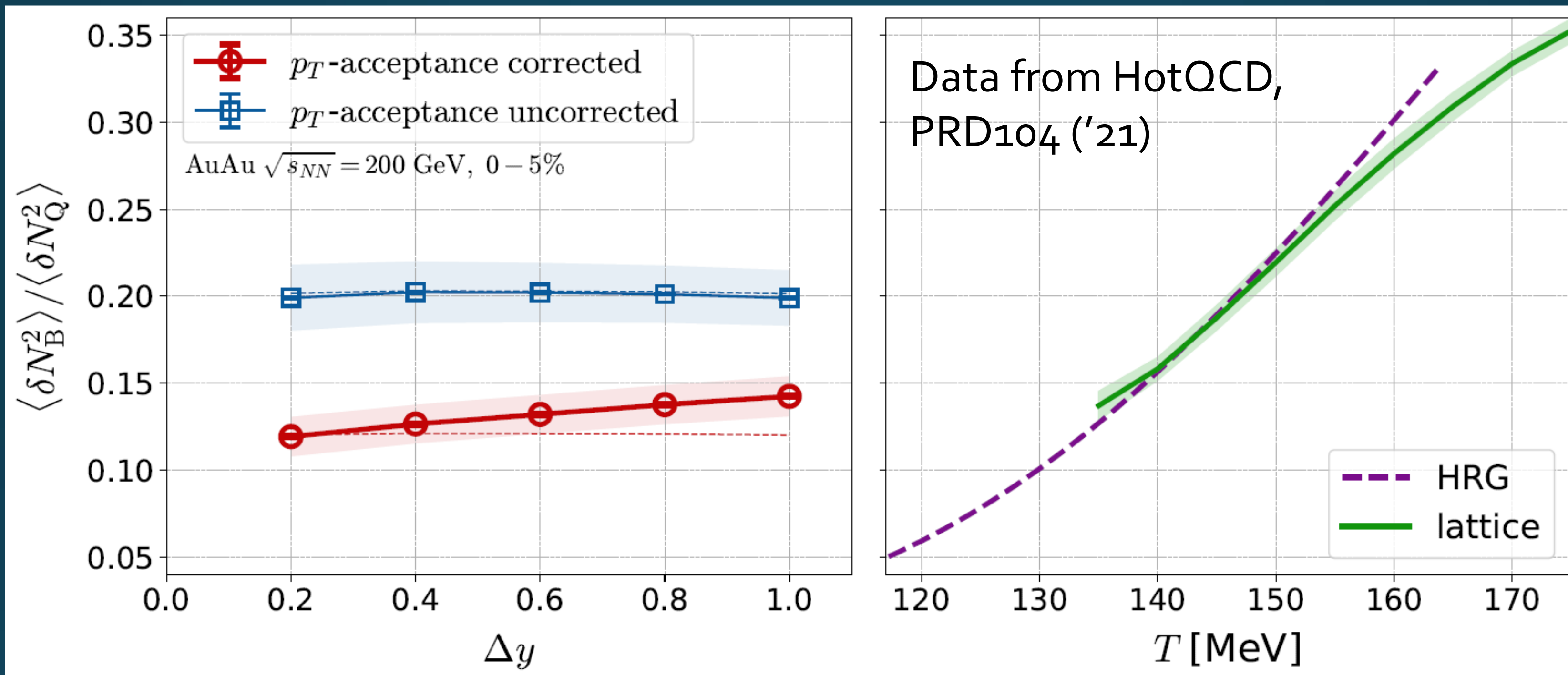




MK, Esumi, Nonaka, 2205.10030

- Deviation from Poisson distribution is more amplified by the correction.

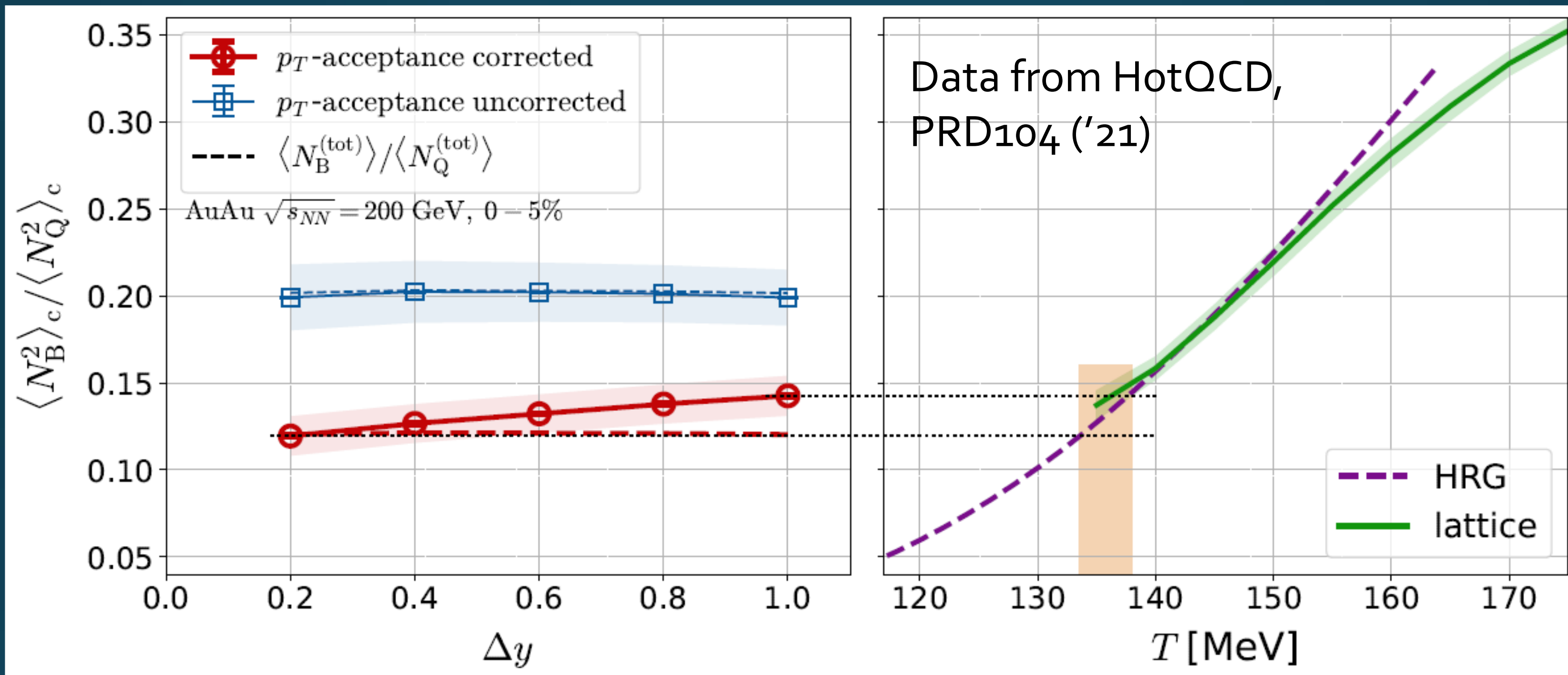
HIC v.s. LAT / HRG



MK, Esumi, Nonaka, 2205.10030

- Rapidity window dependence of $\langle N_B^2 \rangle_c / \langle N_Q^2 \rangle_c$
  - ➡ Non-thermal behavior
- Naïve comparison gives $T = \mathbf{134 \sim 138}$ **MeV**
  - ➡ Significantly lower than T_{chem}

HIC v.s. LAT / HRG



MK, Esumi, Nonaka, 2205.10030

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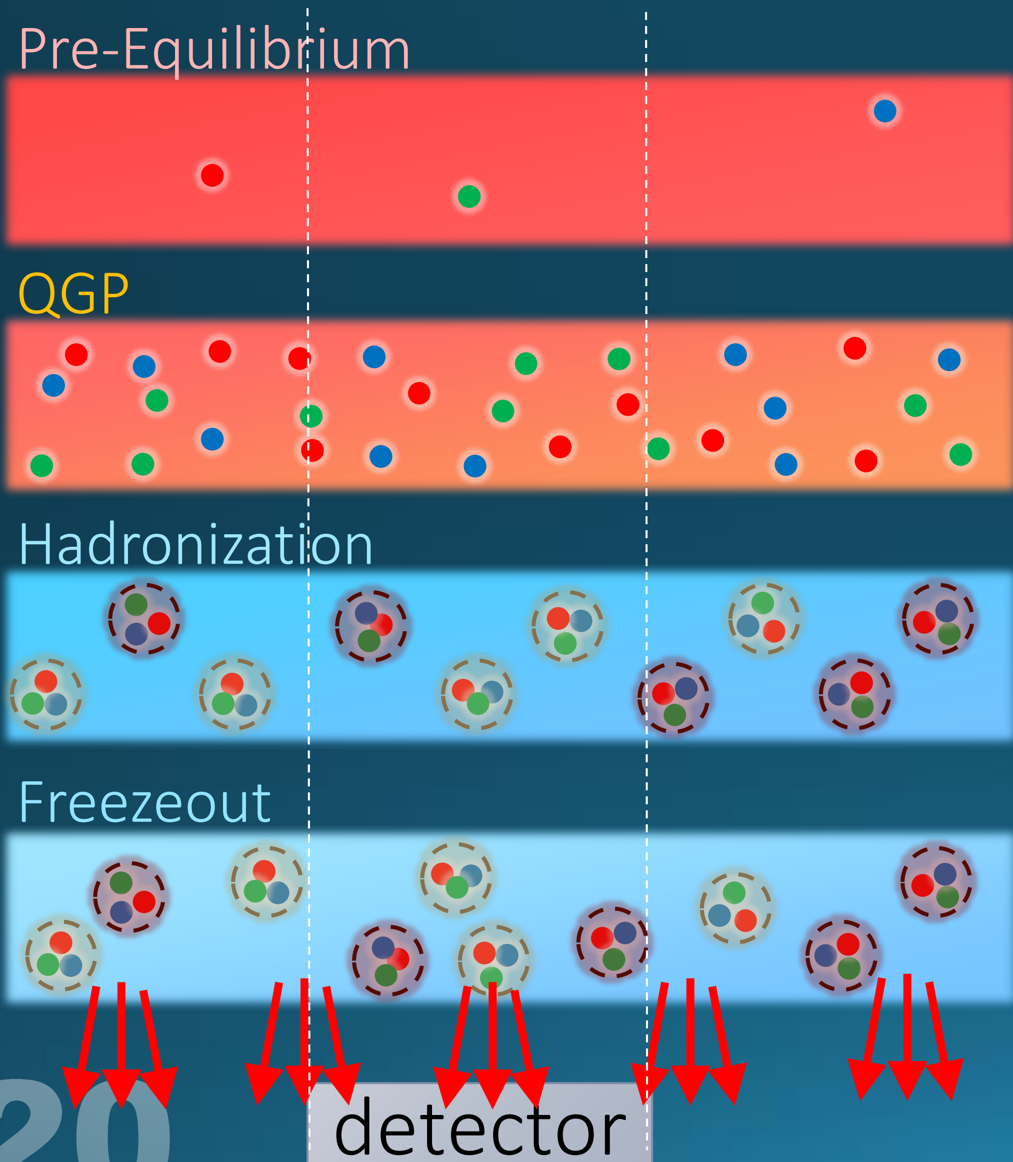
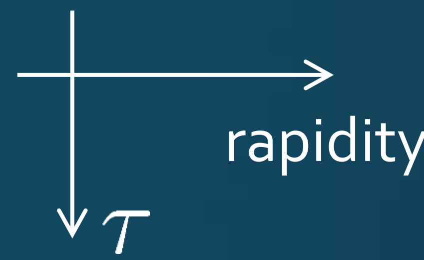
Diffusion of Fluctuations



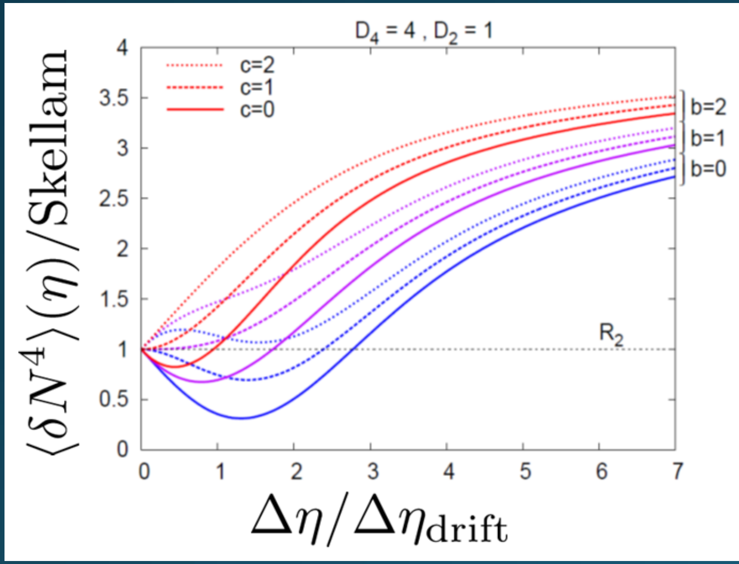


- Fluctuations continue to change even after the chemical FO.
- Measurement in momentum space gives rise to further "blurring" effect.

Ohnishi, MK, Asakawa, PRC 2016

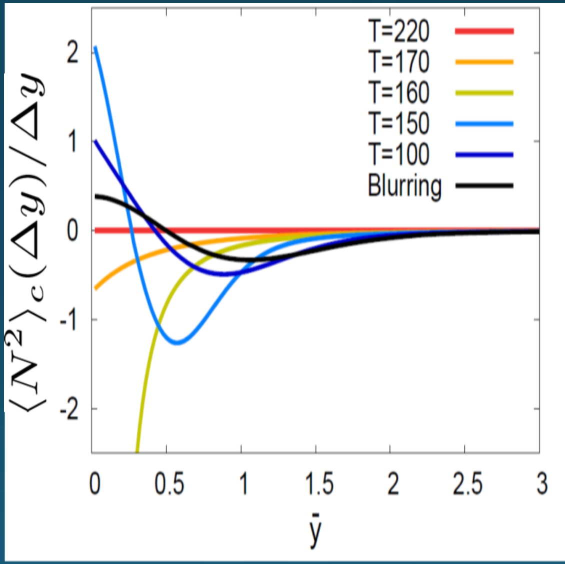
Rapidity Window Dependence
in Diffusion Models

- **Higher order cumulants**
  in diffusion master equation
  MK+, PLB ('14); MK, NPA ('15)

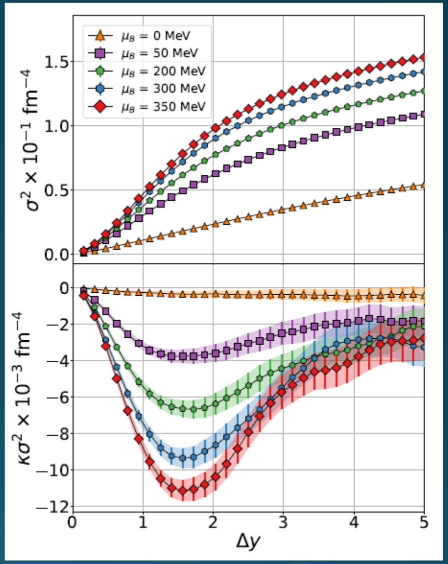


- **Evolution near CP**
  in stochastic diffusion equation

Sakaida+ ('18)



Pihan+, 2205.12834



- Non-monotonic Δy dependence can emerge reflecting the dynamical history.

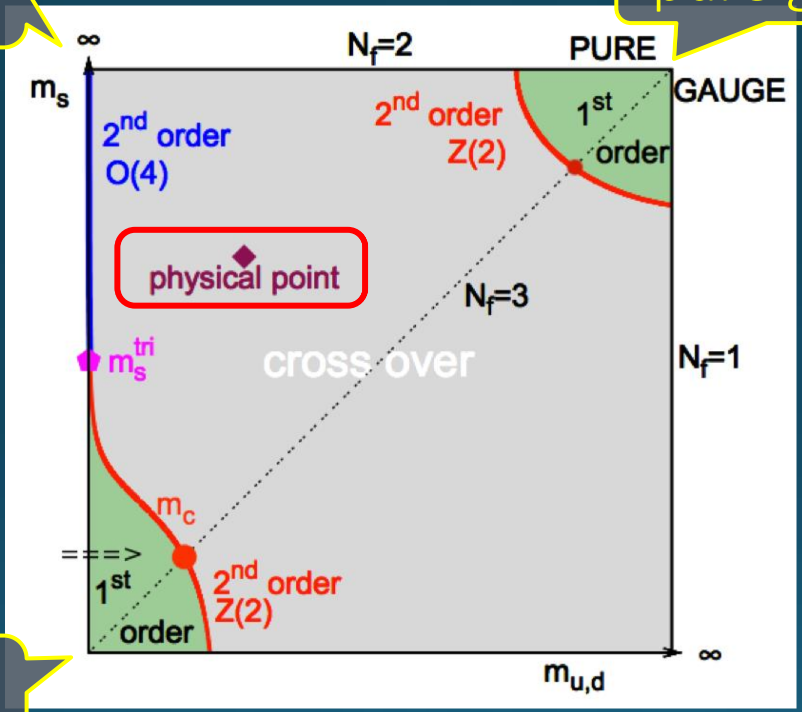
QCD Phase Structure at Unphysical Quark Masses

Columbia Plot

= order of phase tr. at $\mu = 0$

massless 2-flavor

pure gauge



s quark mass

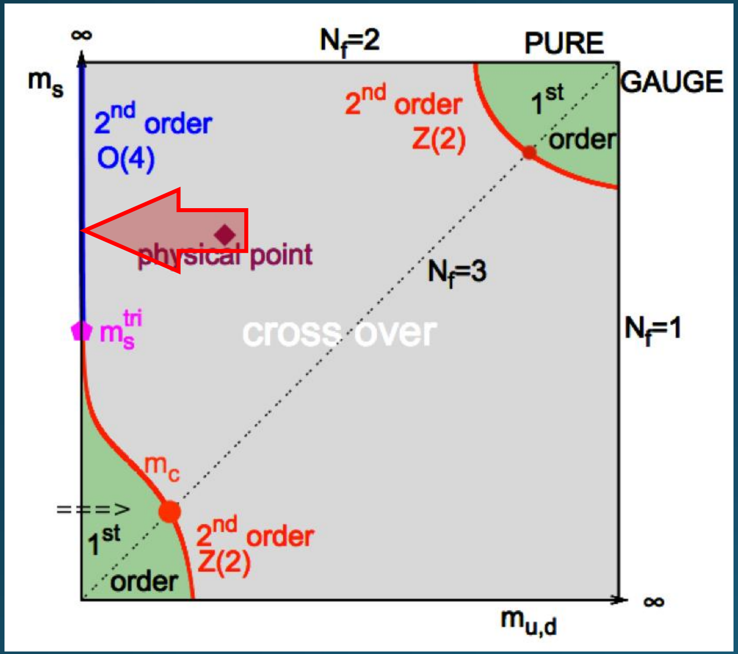
massless 3-flavor

u,d (degenerate) quark mass

Various orders of phase transition with variation of m_q.

Phase Transition in Chiral Limit

HotQCD, PRL123 ('19)



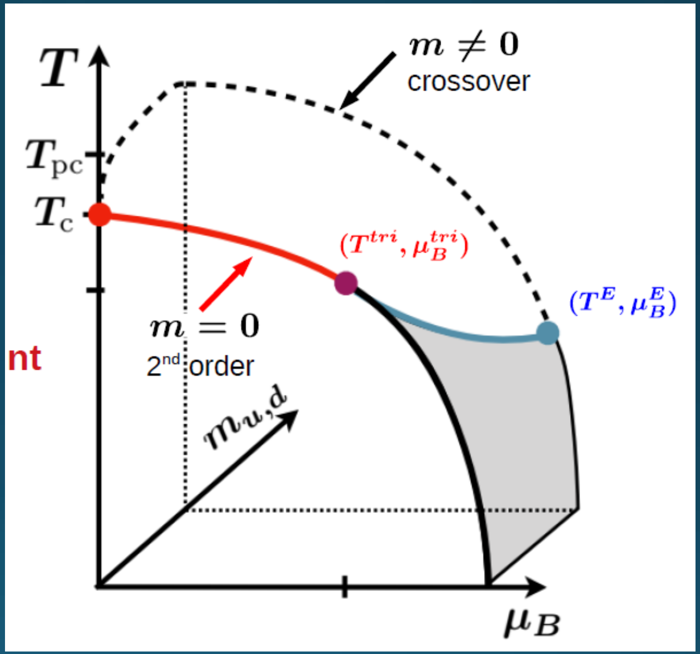


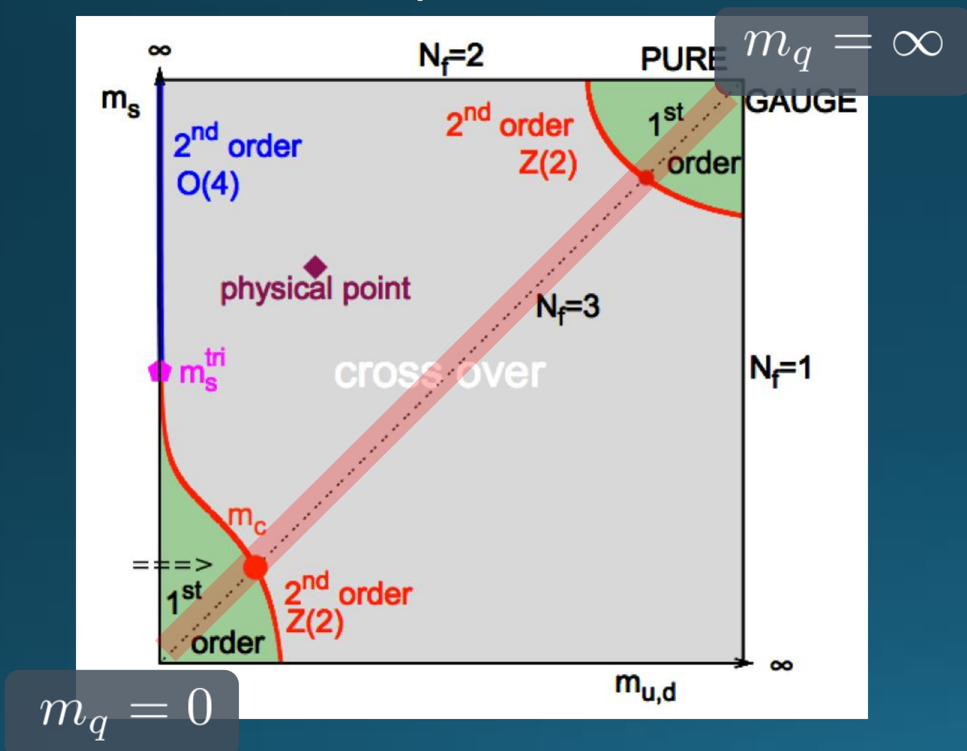
Figure from F. Karsch (GSI, '19)

- For massless 2-flavor: $T_c = 132^{+3}_{-6}$

Varying Quark Masses

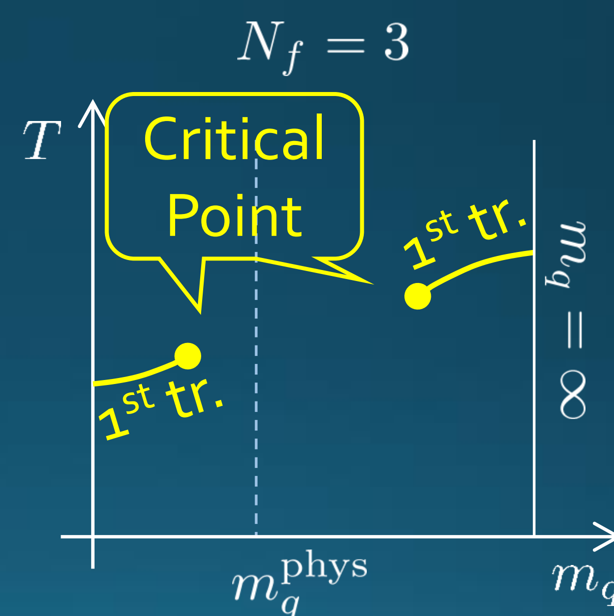
- **Columbia plot**

  = order of phase tr. at $\mu = 0$

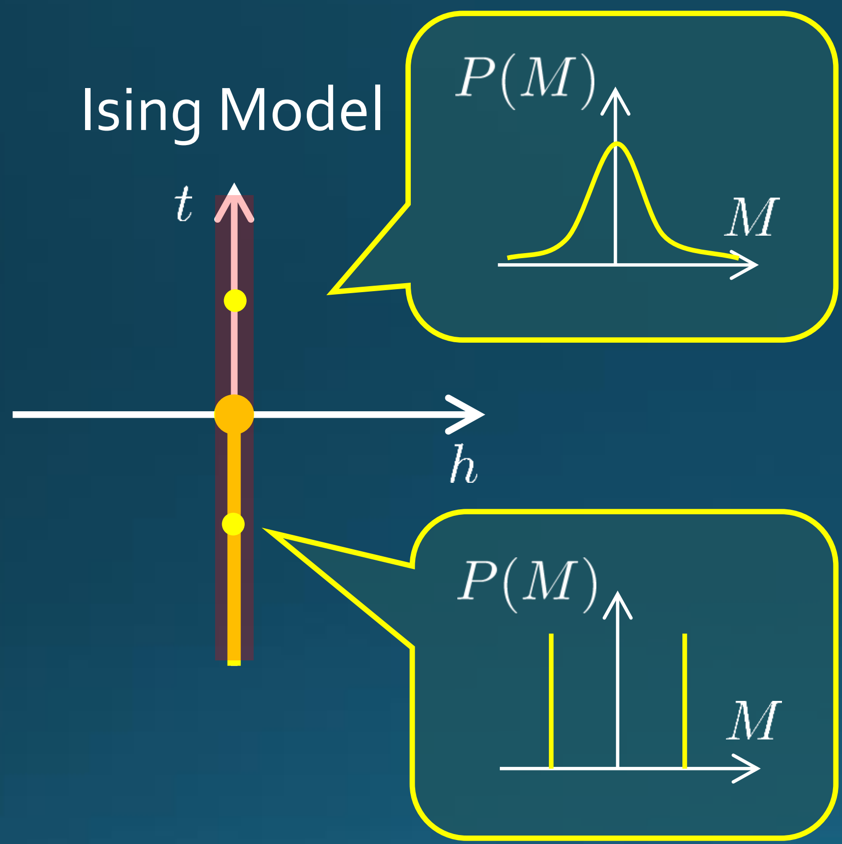


- **Phase Diagram**

  on the $T - m_q$ plane



Cumulants around Critical Point



Gauss distribution

$$\frac{\langle M^4 \rangle_c}{\langle M^2 \rangle_c^2} = 0$$

Two delta functions

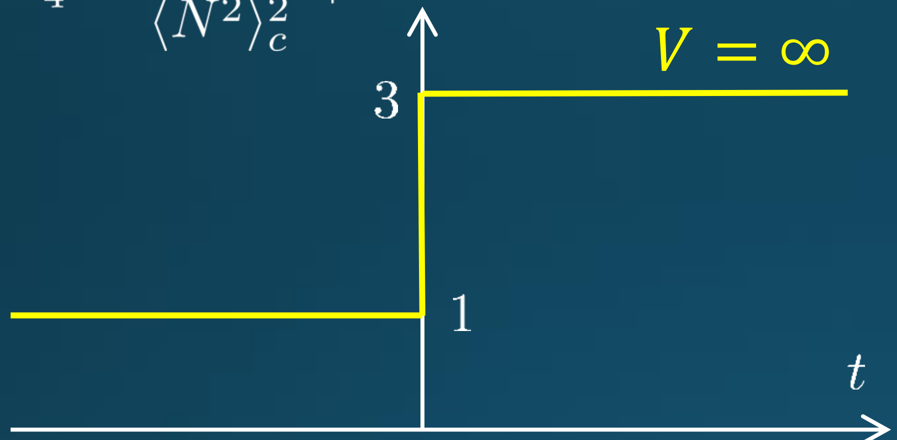
$$\frac{\langle M^4 \rangle_c}{\langle M^2 \rangle_c^2} = -2$$

- Kurtosis $\langle M^4 \rangle_c / \langle M^2 \rangle_c$ changes discontinuously at the CP.

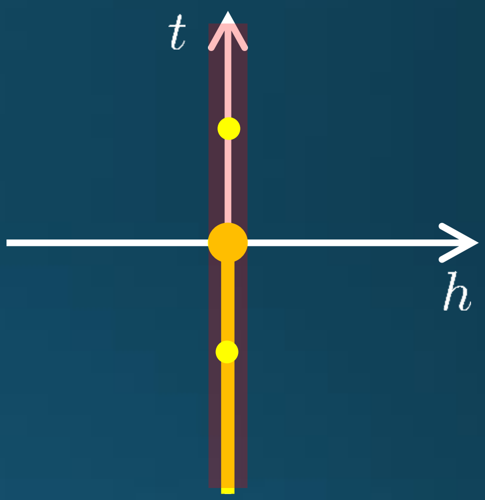
Binder Cumulant

Binder Cumulant

$$B_4 = \frac{\langle N^4 \rangle_c}{\langle N^2 \rangle_c^2} + 3$$



Ising Model

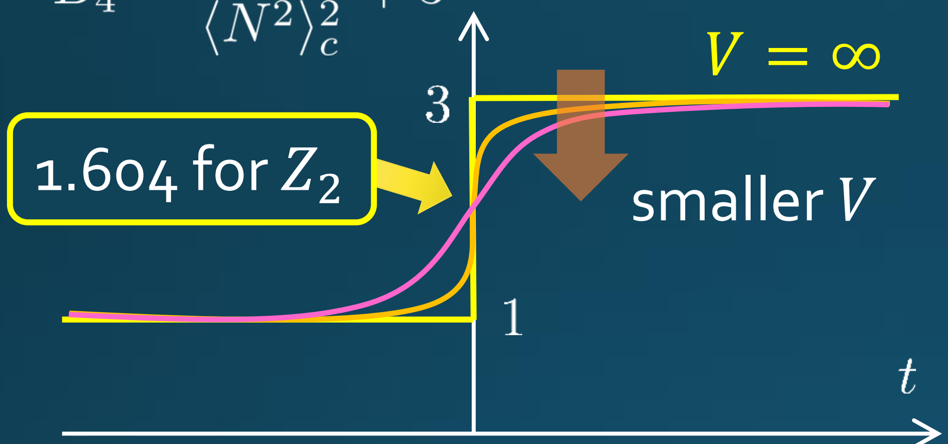


- Sudden change of B_4 is smeared by the finite-size effect.
- B_4 obtained for various V has crossing at $t = 0$.
- At the crossing point, $B_4 = 1.604$ in Z_2 universality class.

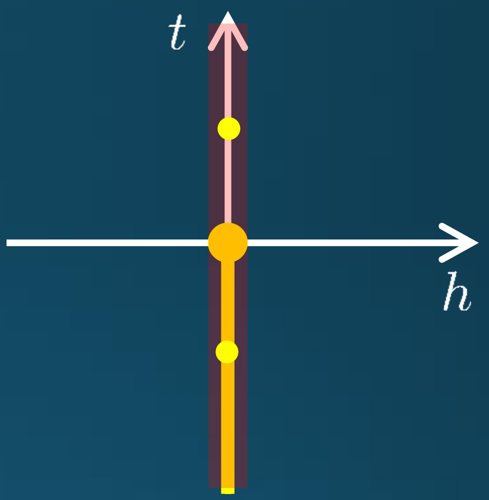
Finite-Size Scaling

Binder Cumulant

$$B_4 = \frac{\langle N^4 \rangle_c}{\langle N^2 \rangle_c^2} + 3$$



Ising Model



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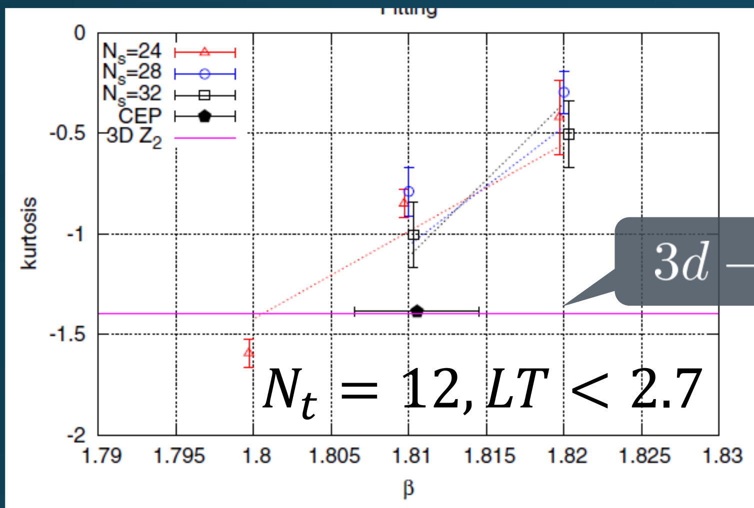
Binder-Cumulant Analysis

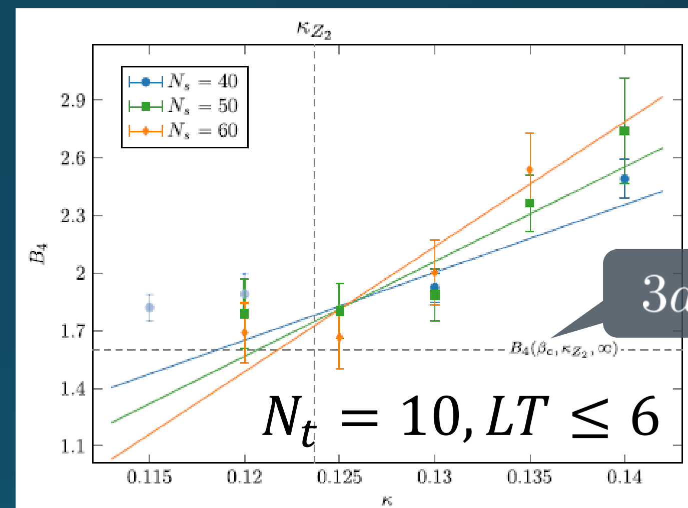
☐ Light-quark region

Kuramashi, Nakamura, Ohno, Takeda, ’20



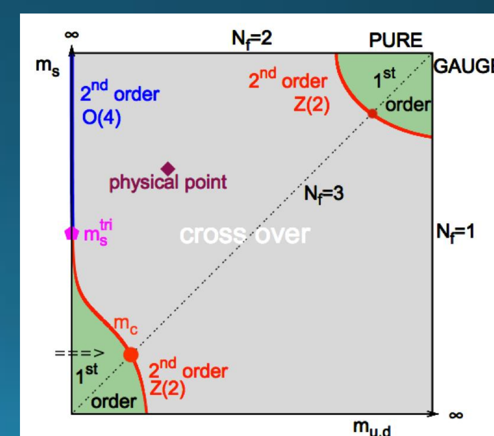
☐ Heavy-quark region

Cuteri , Philipsen , Schön, Sciarra, ’21



☐ Statistically-significant deviation of the crossing point from the 3d-Ising value.

➡ Large non-singular contribution?

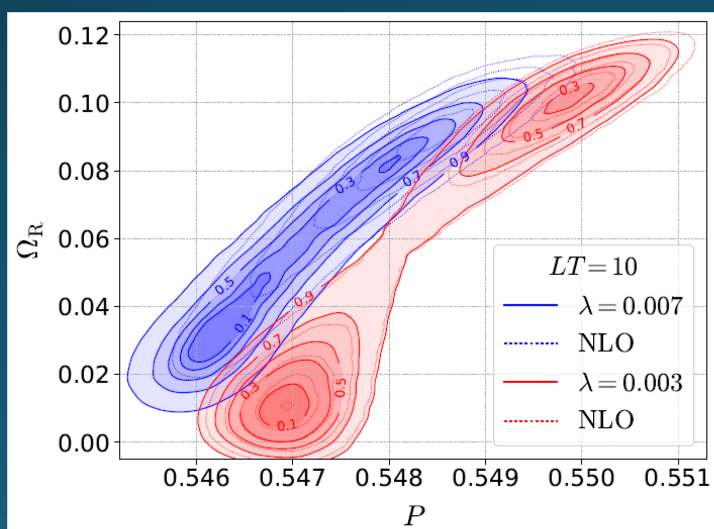


Numerical Simulation

Kiyohara, MK, Ejiri, Kanaya, PRD, 2021

- **Coarse lattice:** $N_t = 4$
- But **large spatial volume**:
  $LT = N_s / N_t \leq 12$

- Hopping-param. (~$1/m_q$) expansion
- Monte-Calro with LO action
- High statistical analysis

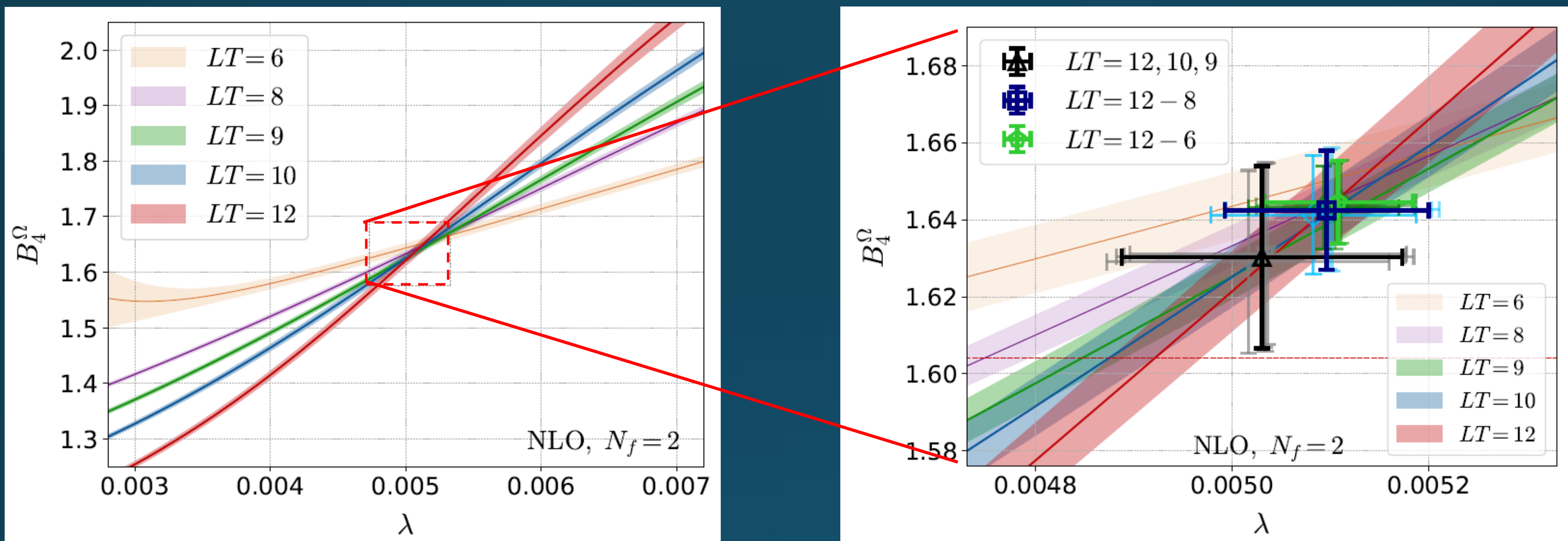


Simulation params.

lattice size	β^*	λ	$\kappa^{N_f=2}$
$48^3 \times 4$	5.6869	0.004	0.0568
	5.6861	0.005	0.0601
	5.6849	0.006	0.0629
$40^3 \times 4$, $36^3 \times 4$	5.6885	0.003	0.0529
	5.6869	0.004	0.0568
	5.6861	0.005	0.0601
	5.6849	0.006	0.0629
	5.6837	0.007	0.0653
$32^3 \times 4$	5.6885	0.003	0.0529
	5.6865	0.004	0.0568
	5.6861	0.005	0.0601
	5.6845	0.006	0.0629
	5.6837	0.007	0.0653
$24^3 \times 4$	5.6870	0.0038	0.0561
	5.6820	0.0077	0.0669
	5.6780	0.0115	0.0740

Binder-Cumulant Analysis

Kiyohara, MK, Ejiri, Kanaya, PRD, 2021



Z2	$B_4 = 1.604$	$\nu = 0.630$
$LT \geq 9$	$B_4 = 1.630(24)(2),$	$\nu = 0.614(48)(3)$
$LT \geq 8$	$B_4 = 1.643(15)(2),$	$\nu = 0.614(29)(3)$

- ☐ B_4 and ν are consistent with Z_2 universality class only when $LT \geq 9$ data are used for the analysis.

HIC vs LAT: Pros & Cons

HIC	LAT
Real	Virtual
Dynamical	Thermal

necessary in physics

simulating unphysical world

need space-time models

ideal environment; unable to simulate real-time evolution

input

motivate

finite density

Accessible	Difficult
beam-energy scan	due to sign problem

Remaining Challenges in LAT

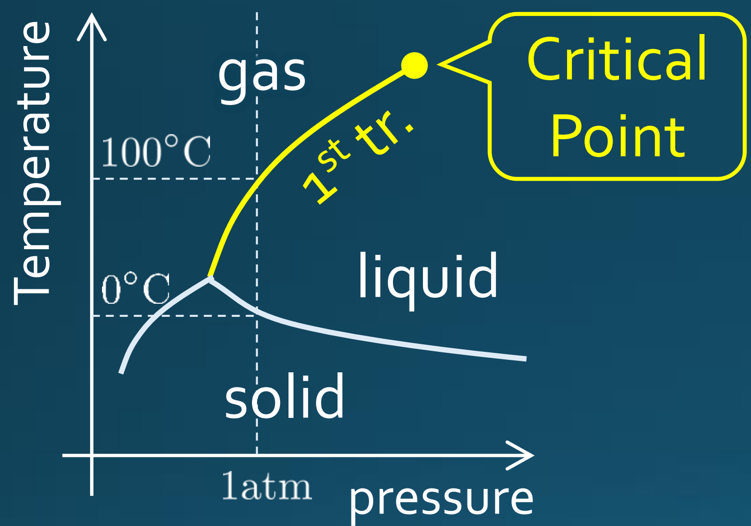
- Transport coefficient
  - shear/bulk viscosity
  - conductivity
- Particles' properties: dissociation, mass shift, etc.
  - charmonia, light hadrons
  - quarks and gluons
- Finite density
  - Taylor expansion, imaginary μ_B
  - Complex Langevin, Thimble, etc.
- Static quantities
  - yet higher order cumulants, screening masses
  - scaling behavior around T_c & CP, etc.

Final Remarks

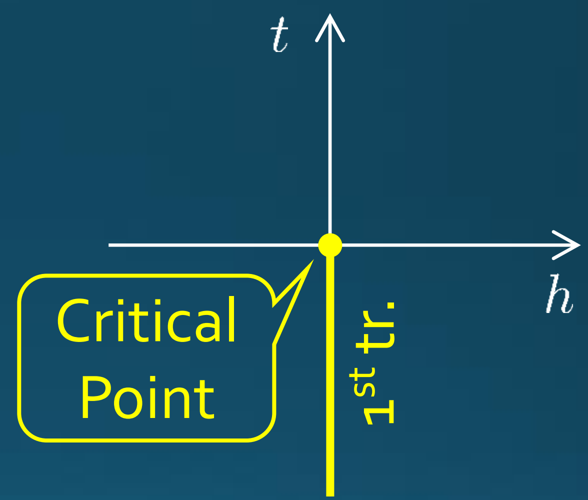
- Relativistic HIC and lattice simulations are useful tools for exploring hot and dense medium. Their complementary use is essential.
- The cumulants measured by the event-by-event analysis in HIC should be interpreted carefully.
- Steady progress in revealing Columbia plot in LAT.
- **Further exchange of ideas between LAT and HIC communities is indispensable for revealing the QCD phase structure!**

Critical Points

□ Water



□ Ising Model

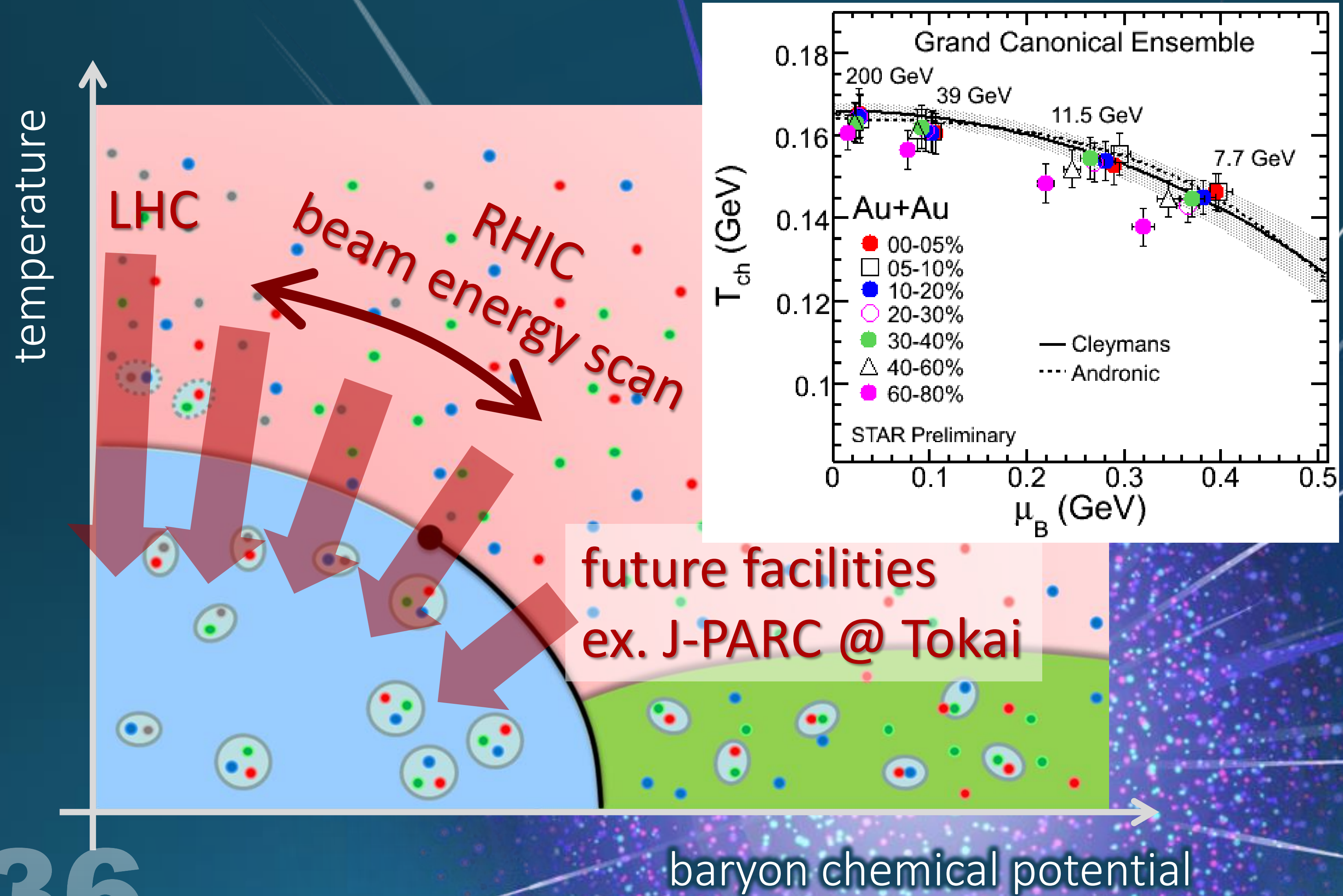


These CPs belong to the same universality class (Z_2).

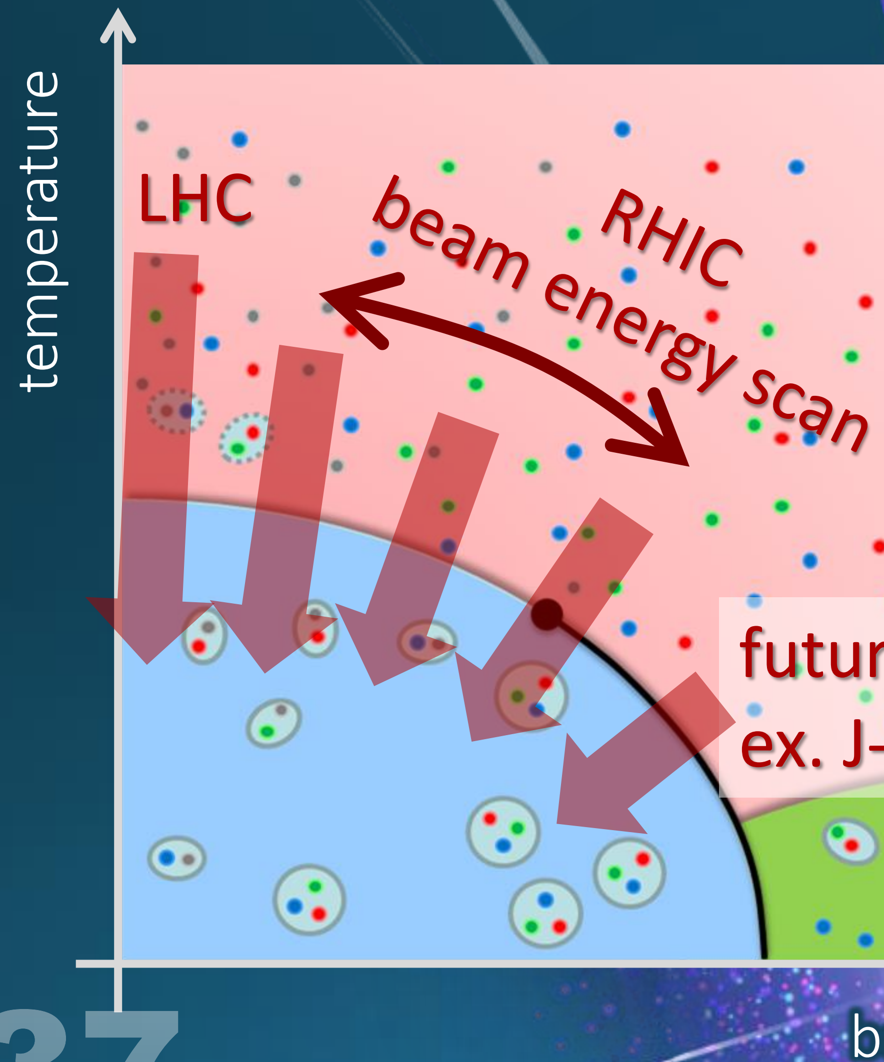
➡ Common critical exponents.

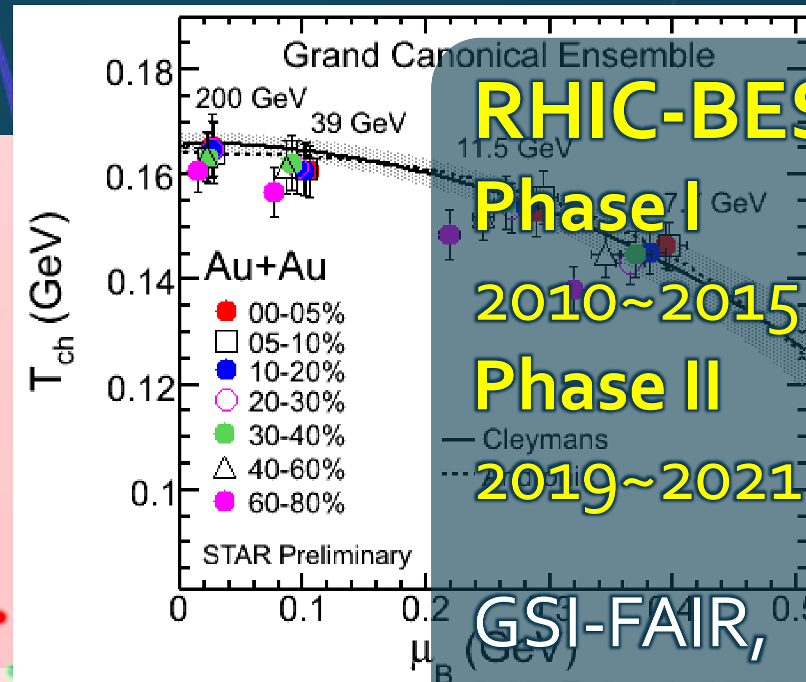
ex. $C \sim (T - T_c)^{-\alpha}$

Beam-Energy Scan



Beam-Energy Scan





RHIC-BES

Phase I

2010~2015

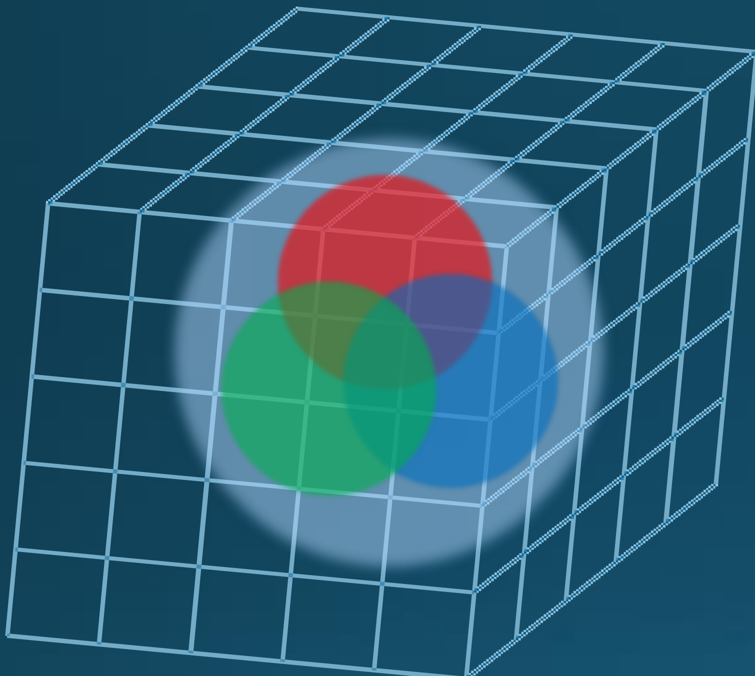
Phase II

2019~2021

GSI-FAIR,
NICA-MPD
2026~?

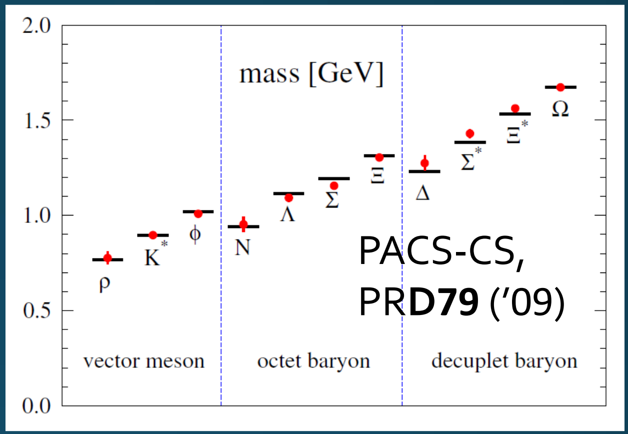
J-PARC-HI
2028~??

Lattice QCD Numerical Simulations

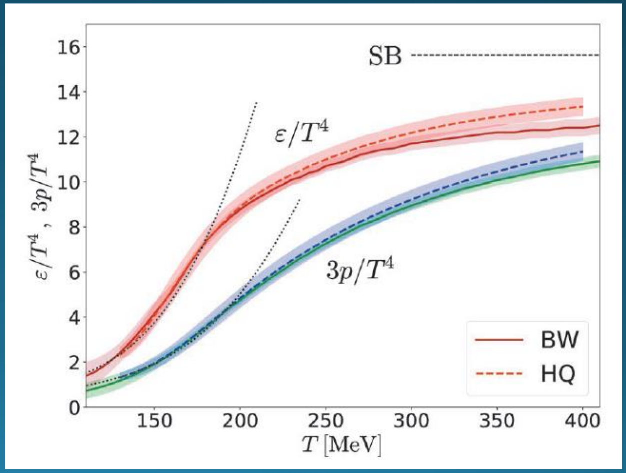


Unique tool to perform **quantitative** analyses of **non-perturbative** QCD aspects

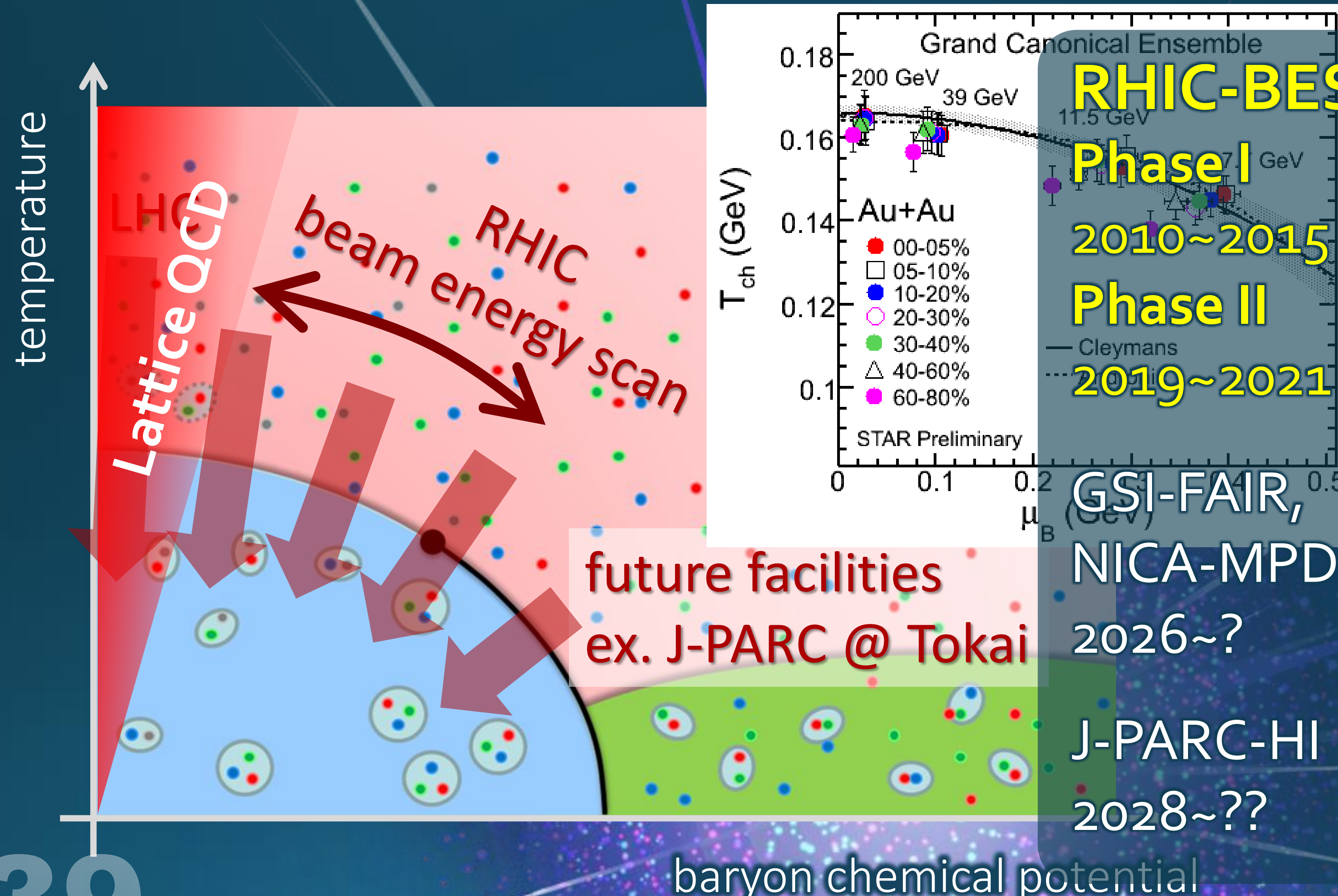
- Hadron Spectroscopy



- Thermodynamics



Beam-Energy Scan



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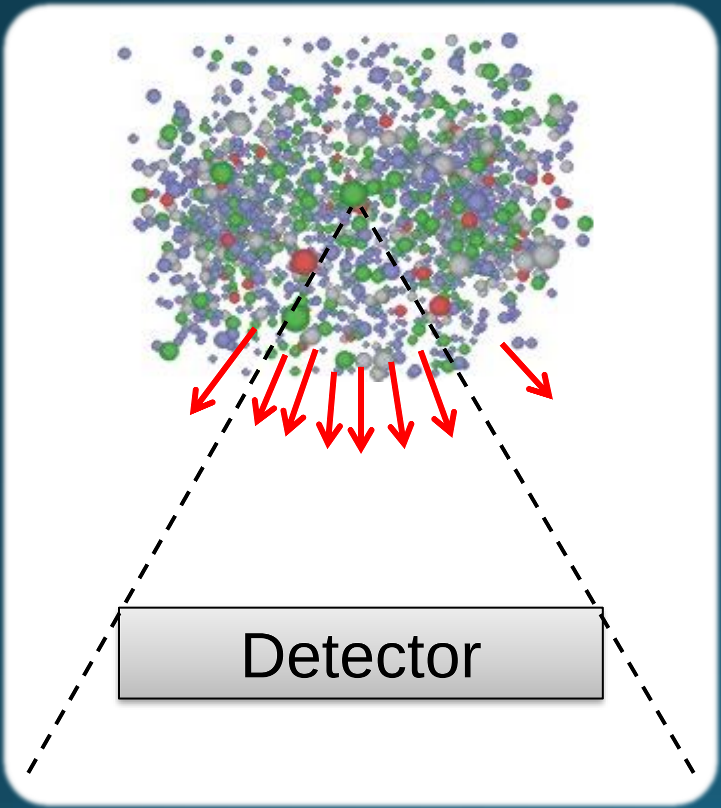
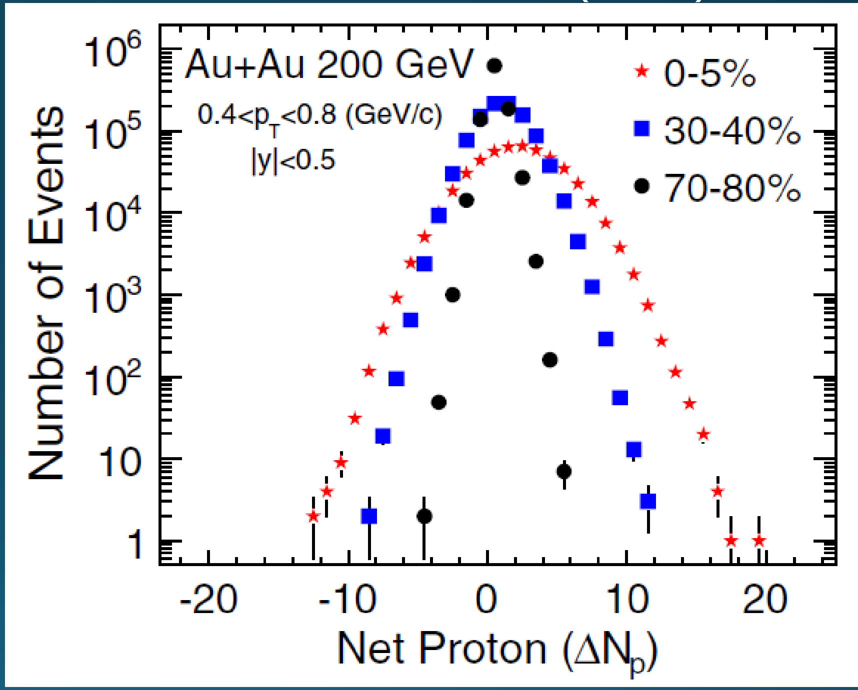
Event-by-event Fluctuations



STAR, PRL**105** (2010)



Cumulants

$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$