Phase Structure of Hot and Dense QCD and its Experimental Search

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GPPU Seminar, Tohoku University, 2022/6/22, (Tohoku U.)

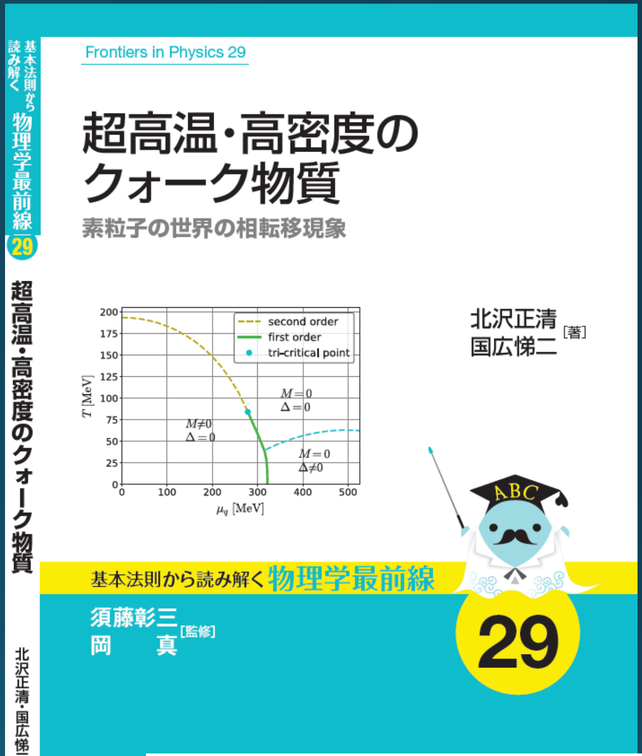
Contents

Advertising

A book **"Quark matter at extreme conditions: Phase transitions in the world of elementary particles"** will come soon (**end of August**)!

- ☐ Intro. to hot and dense QCD
- ☐ Relativistic heavy-ion collisions
- ☐ BCS theory
- ☐ Phase diagram in NJL model
- ☐ Linear response, collective modes
- ☐ Color superconductivity
- ☐ **Numerical codes in Python**

Codes at:
https://github.com/MasakiyoK/Saizensen


Frontiers in Physics 29

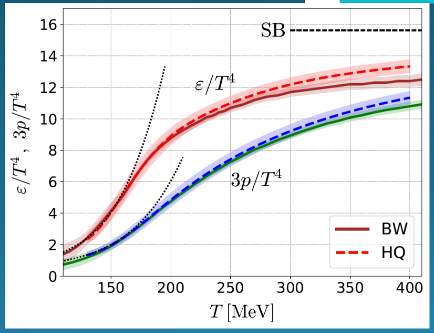
超高温・高密度の
クォーク物質
素粒子の世界の相転移現象

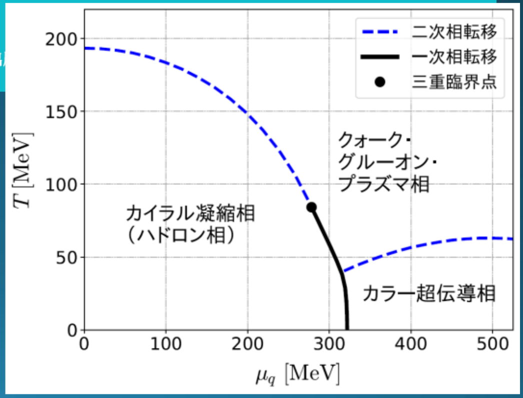
北沢正清
国広悌二 [著]

基本法則から読み解く物理学最前線

須藤彰三
岡　　真 [監修]

29





Contents

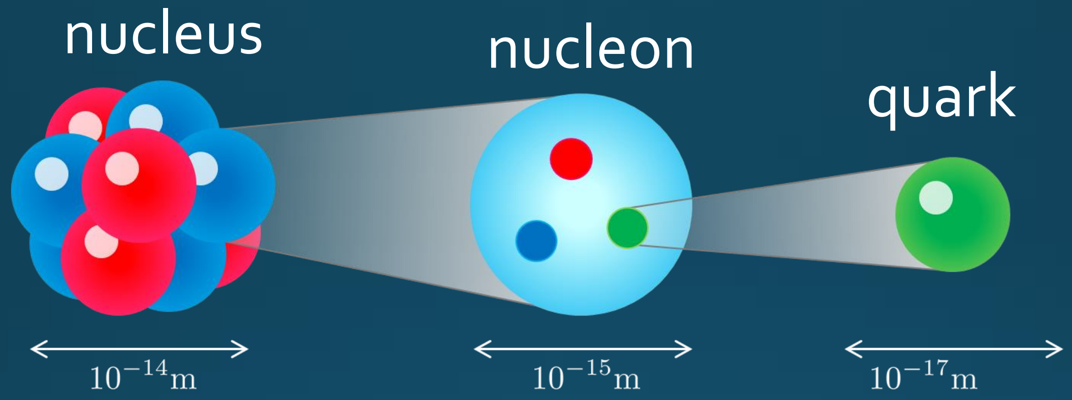
1. Introduction to Hot & Dense QCD
    - phase structure of QCD
    - Relativistic heavy-ion collisions
    - Beam-Energy Scan

2. Search for QCD-CP: Fluctuations

3. Search for CSC: Precursor of CSC

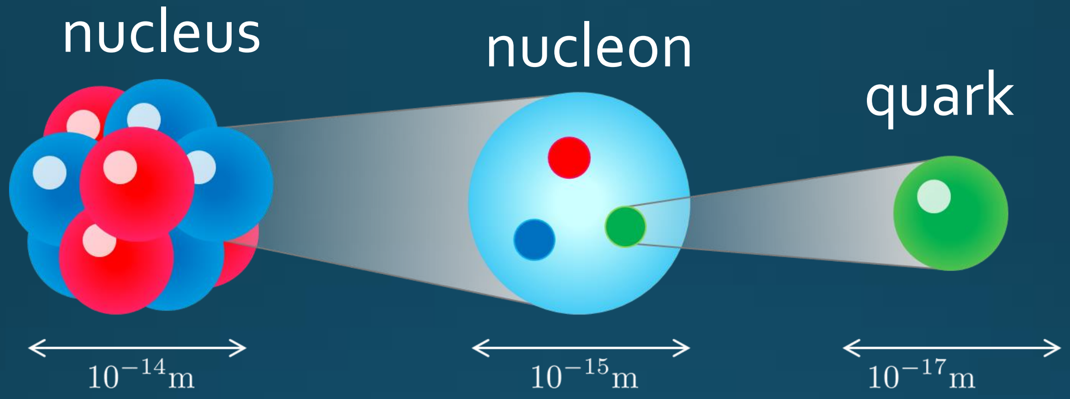
Quantum ChromoDynamics

- ☐ **Building blocks of matter**



Quantum ChromoDynamics

- **Building blocks of matter**



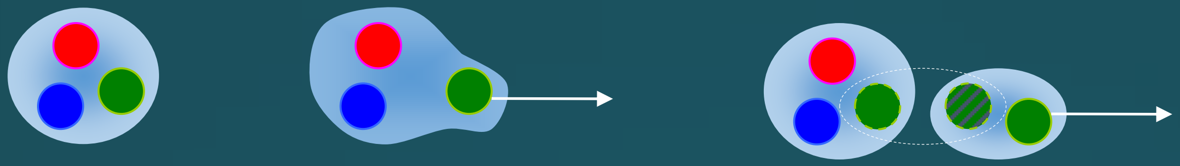
- **Quantum chromodynamics (QCD)**

  =Fundamental theory of strong interaction

$$\mathcal{L} = \underbrace{\bar{\psi}(i\not{D} - m)\psi}_{\text{quarks}} - \frac{1}{4}\underbrace{F_{\mu\nu,a}F_a^{\mu\nu}}_{\text{gluons}}$$

Confinement of Quarks

If one wants to pick up a quark from a nucleon…



A meson is formed via $\bar{q}q$ pair production.

Basic degrees of freedom around us

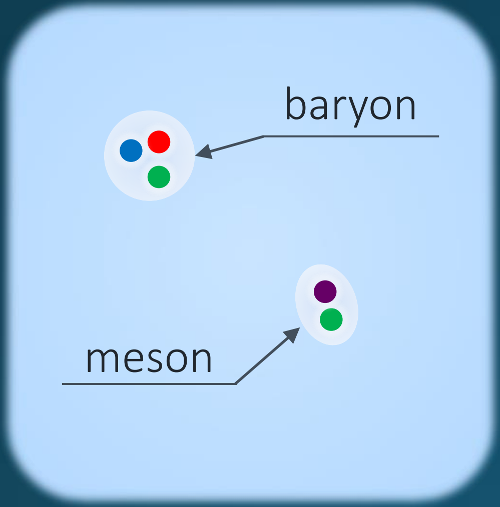
Baryons

Mesons

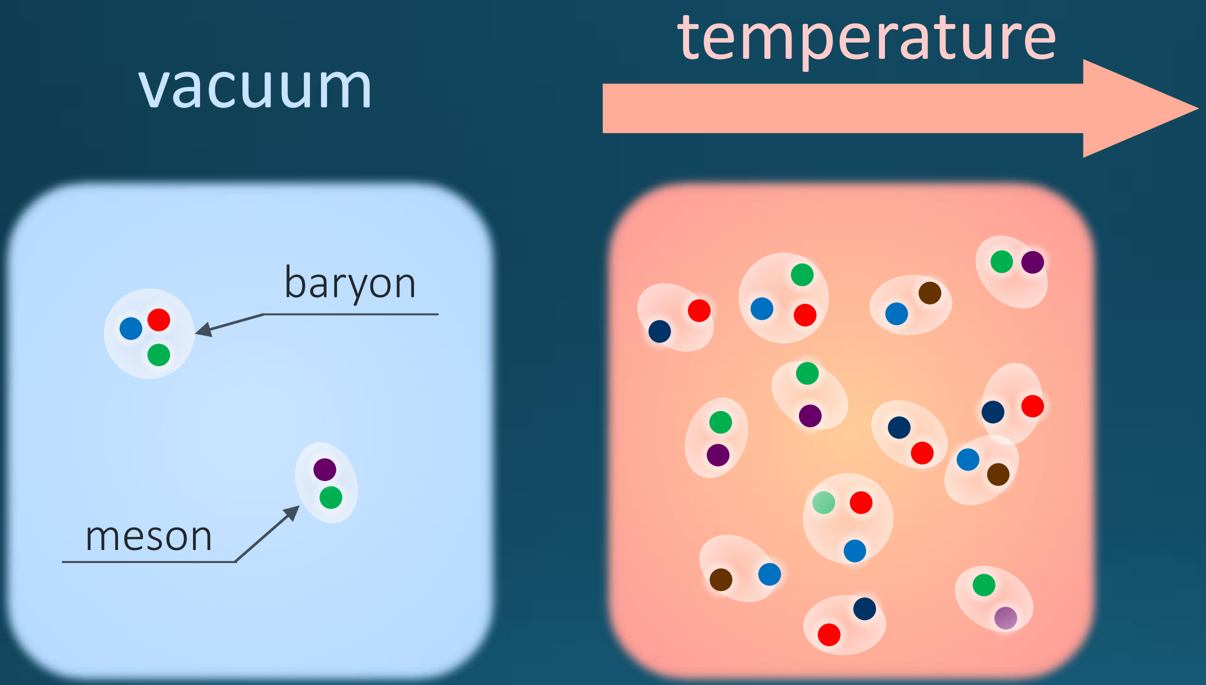
Hadrons

Quark-Gluon Plasma (QGP)

vacuum



Quark-Gluon Plasma (QGP)



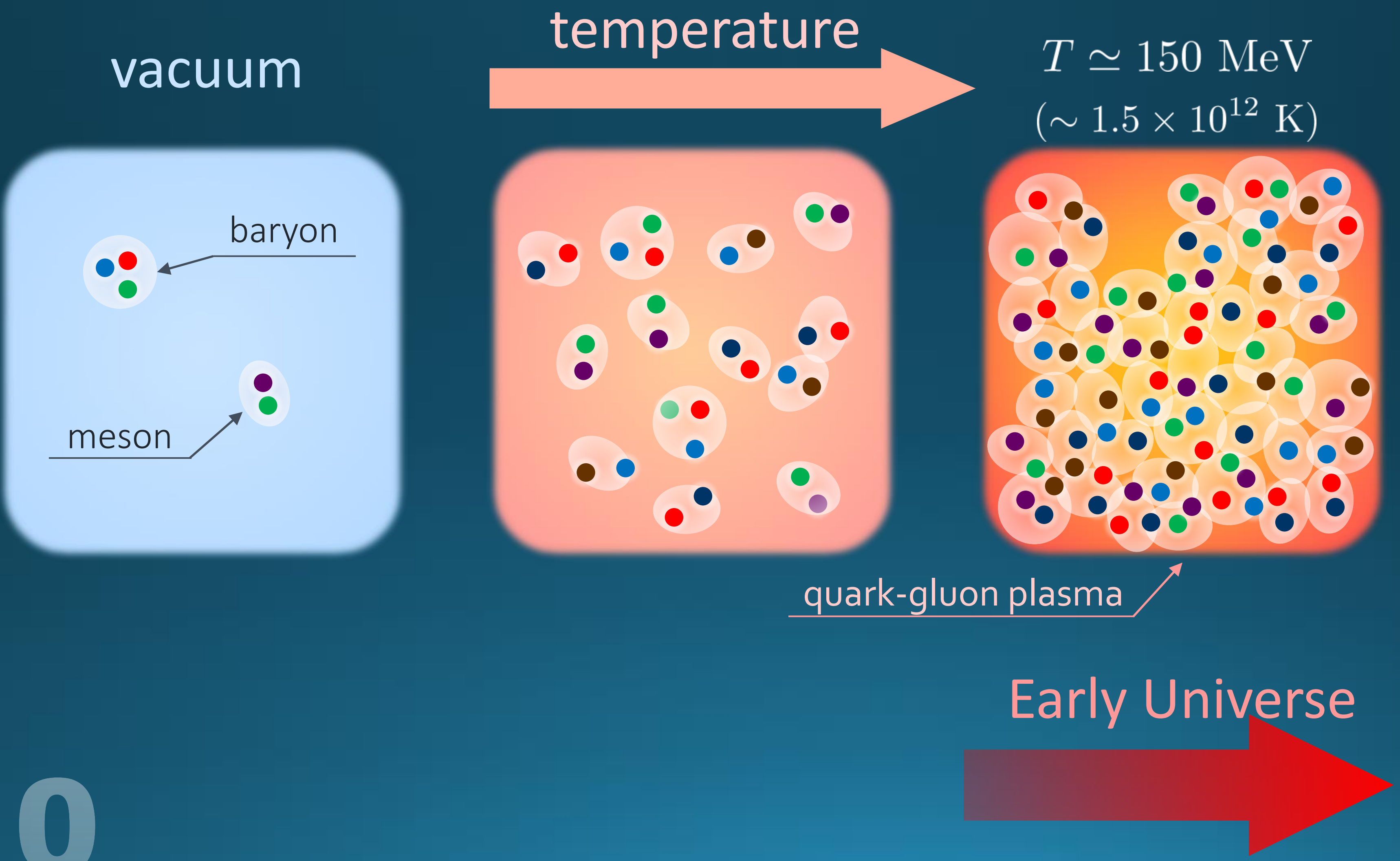
vacuum

temperature

baryon

meson

Quark-Gluon Plasma (QGP)



vacuum

temperature

$T \simeq 150\ \mathrm{MeV}$

$(\sim 1.5 \times 10^{12}\ \mathrm{K})$

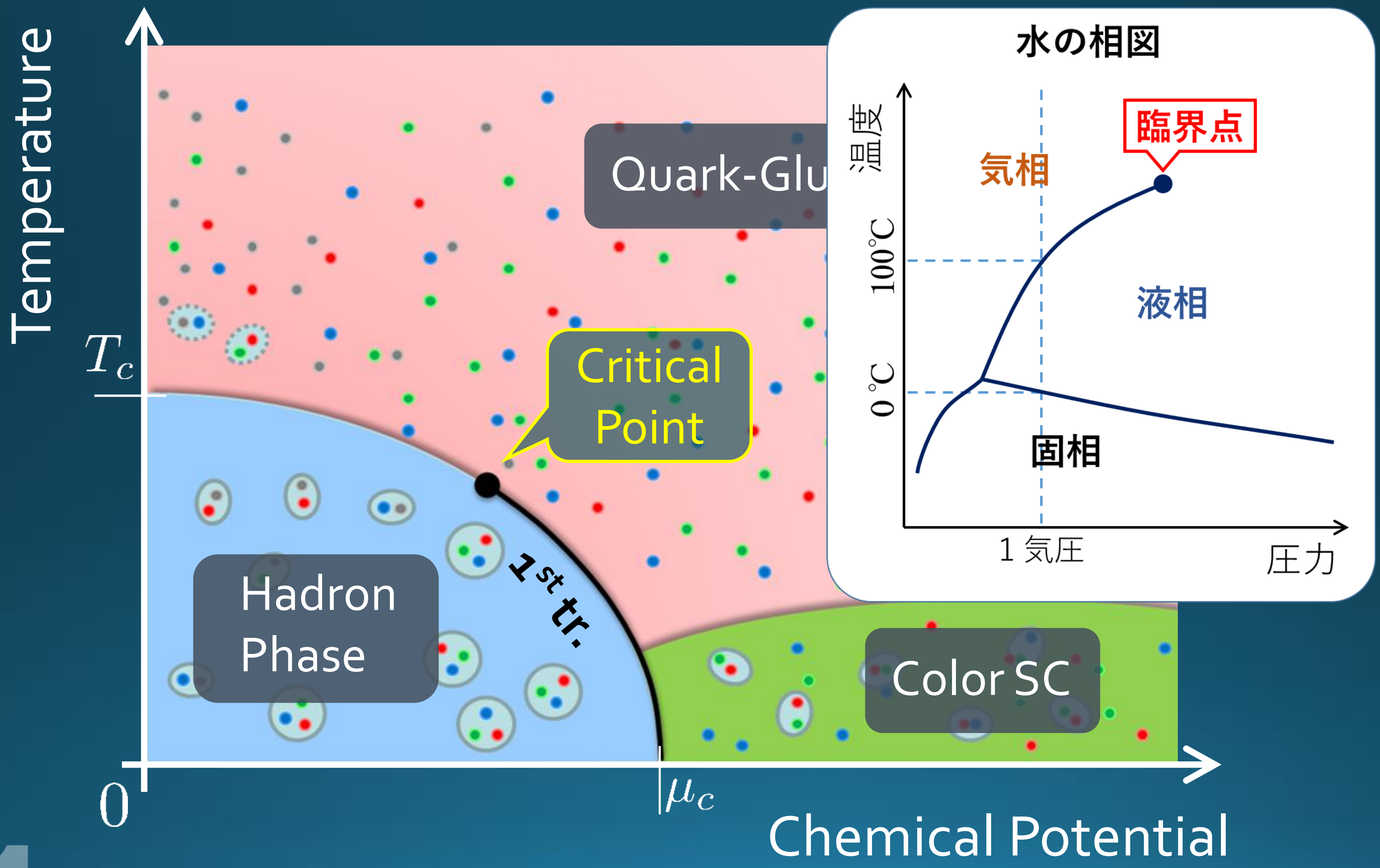
baryon

meson

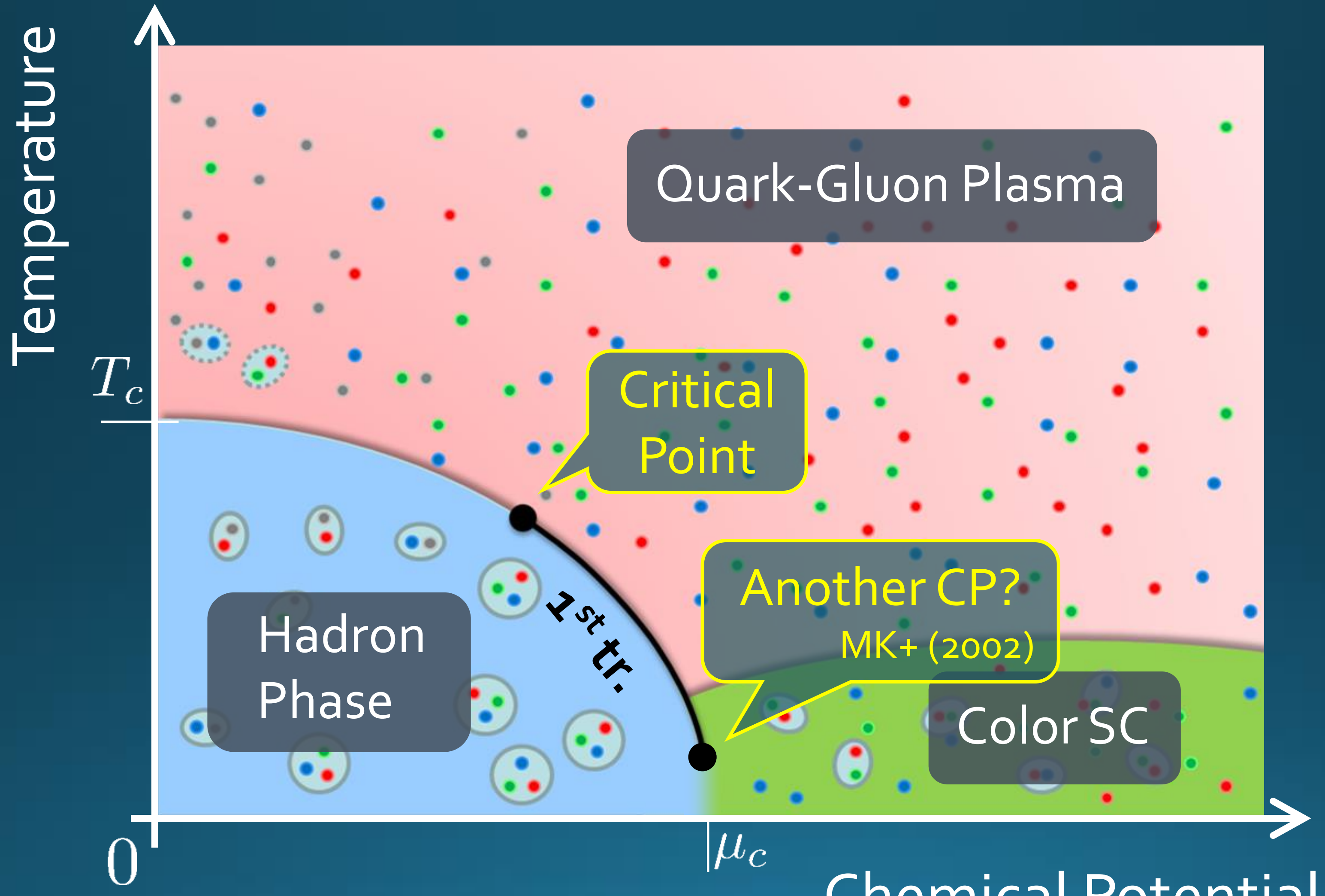
quark-gluon plasma

Early Universe

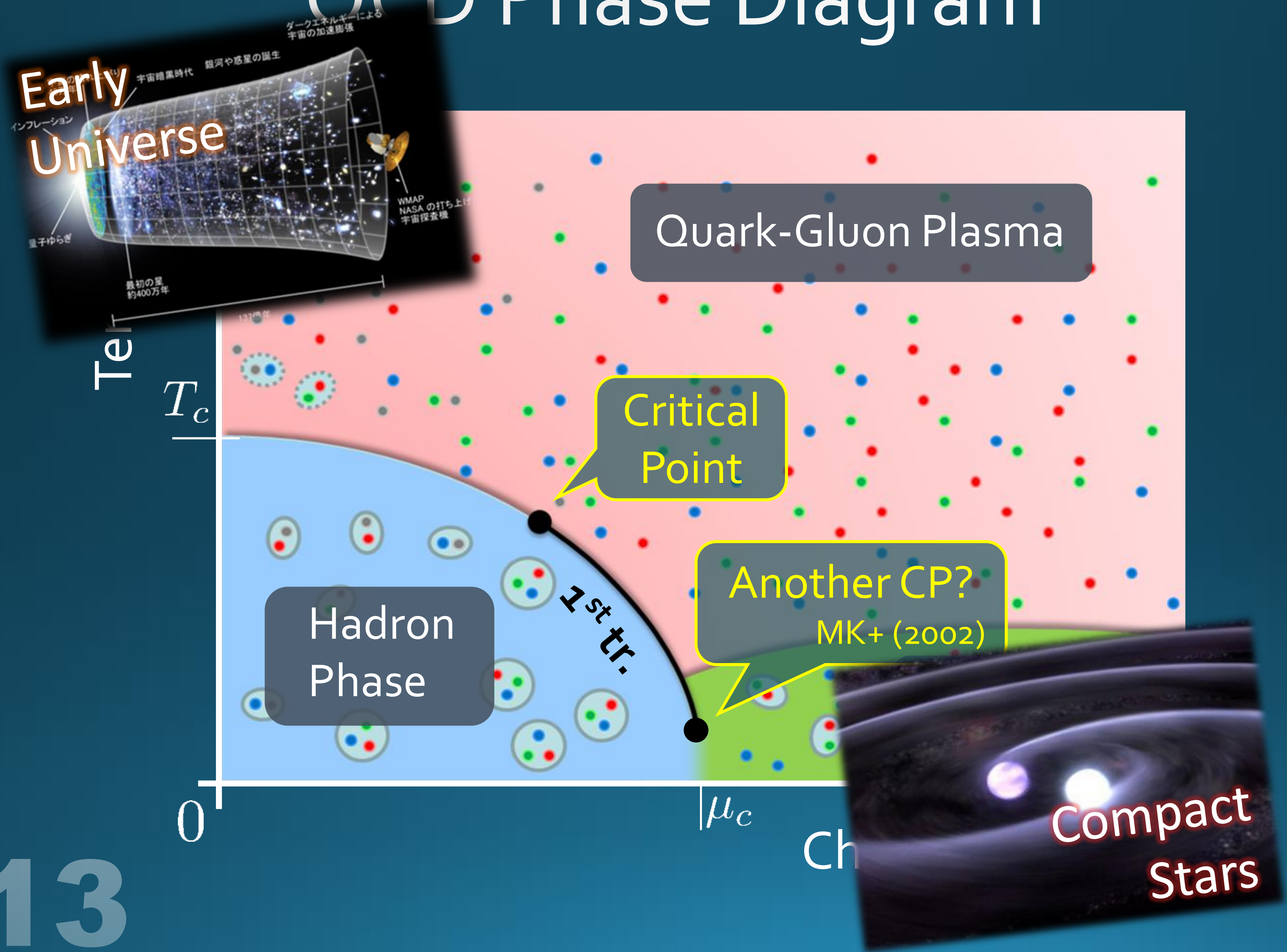
QCD Phase Diagram



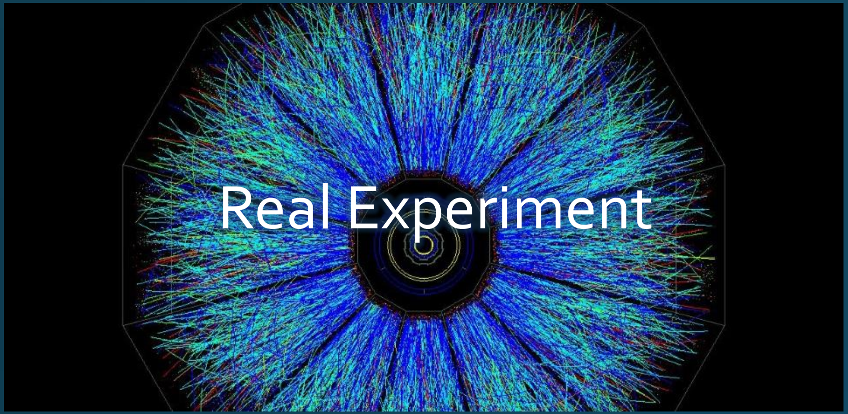
QCD Phase Diagram



QCD Phase Diagram



Two Experimental Tools



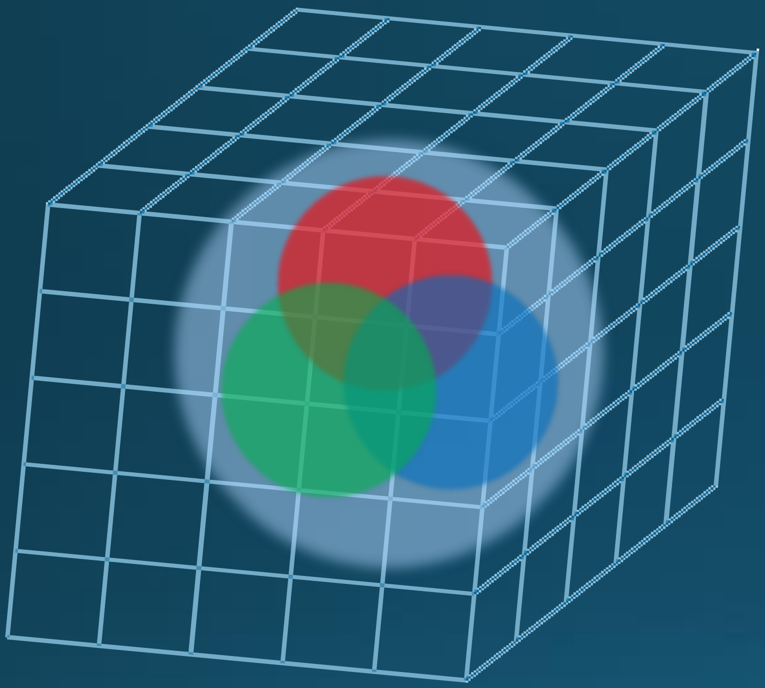
Relativistic Heavy-Ion Collisions

Colliding two heavy nuclei in accelerators
RHIC (USA), LHC (EU), etc.

Lattice QCD Numerical Simulations

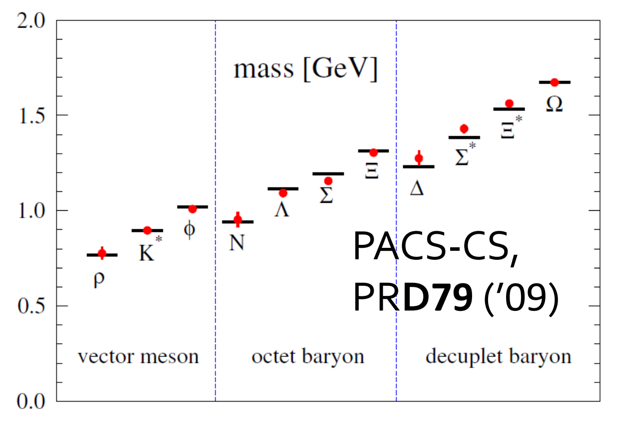
First-principle simulations on supercomputers

Lattice QCD Numerical Simulations



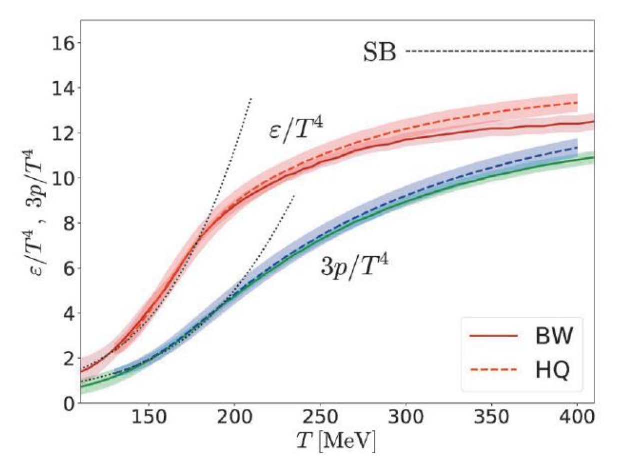
Unique tool to perform **quantitative** analyses of **non-perturbative** QCD aspects

☐ Hadron Spectroscopy



PACS-CS, PR**D79** ('09)

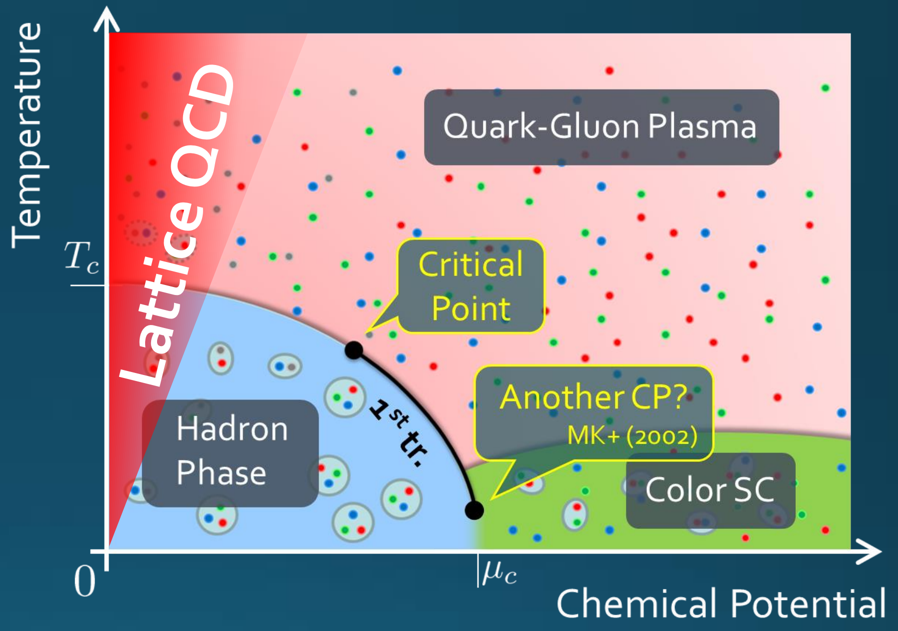
☐ Thermodynamics



Sign Problem

(Complex-Phase Problem)

Lattice simulations are available only at $\mu = 0$.

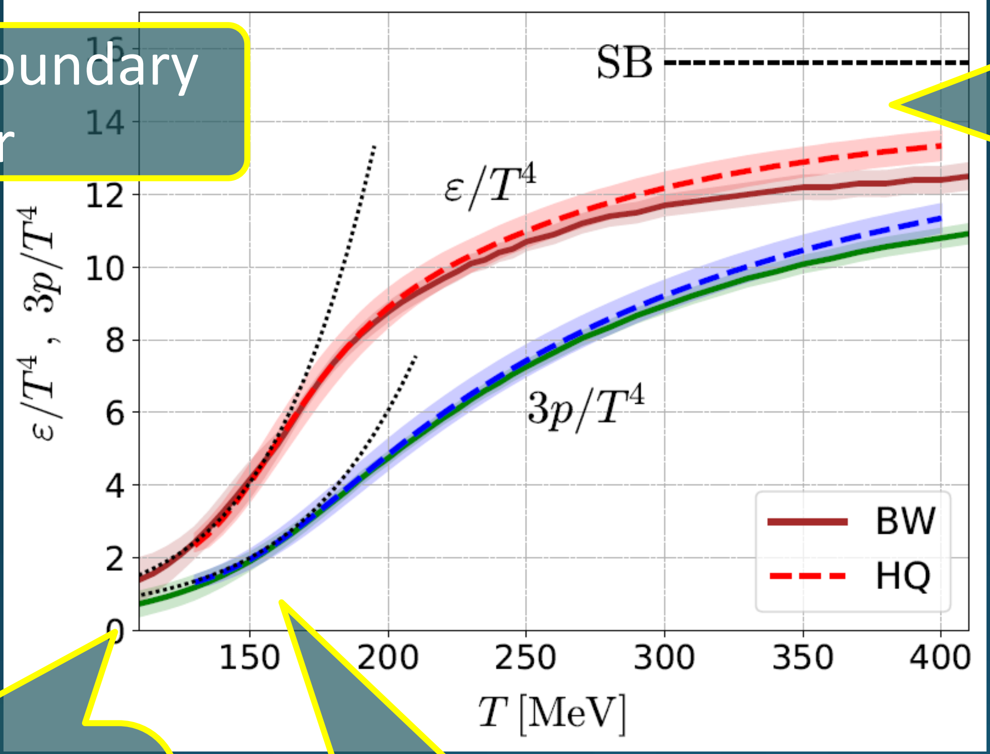


☐ The conventional lattice algorithm (importance sampling) is applicable only to real and positive action.

☐ QCD's action becomes complex at $\mu \neq 0$.

QCD Thermodynamics

No phase boundary
→Crossover



High T:
approach to
Stefan-
Boltzmann limit

Budapest-Wuppertal '14,
HotQCD '14

Low T:
consistent w/
hadron resonance
gas model

Sudden increase
around $T \simeq 160\text{MeV}$

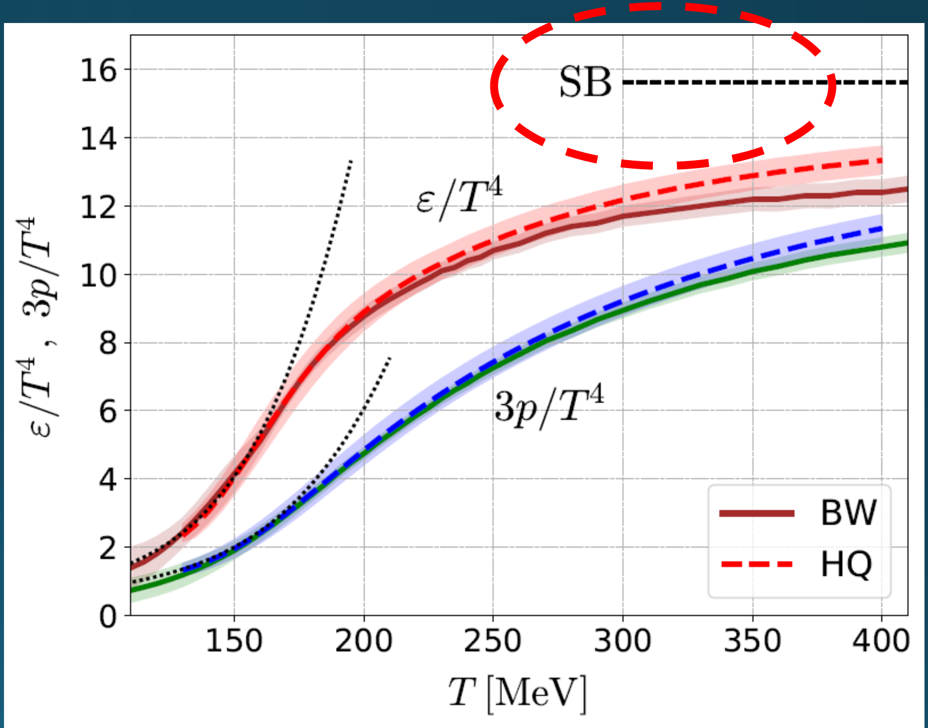
$1\text{eV} \simeq 10^4\text{K}$
$1\text{MeV} \simeq 10^{10}\text{K}$
$100\text{MeV} \simeq 1\text{兆K}$

Stefan-Boltzmann Limit

SB limit = Free gas of massless quarks & gluons

$$\epsilon = \left(16 + \frac{21}{2}N_f\right)\frac{\pi^2}{30}T^4$$

$$\epsilon = 3p$$



$$\epsilon_{\text{free}} = g\int \frac{d^3p}{(2\pi)^3}\frac{p}{e^{p/T} \pm 1} = \frac{\pi^2}{30}T^4$$

Hadron Resonance Gas (HRG) Model

= Free gas composed of all known hadrons

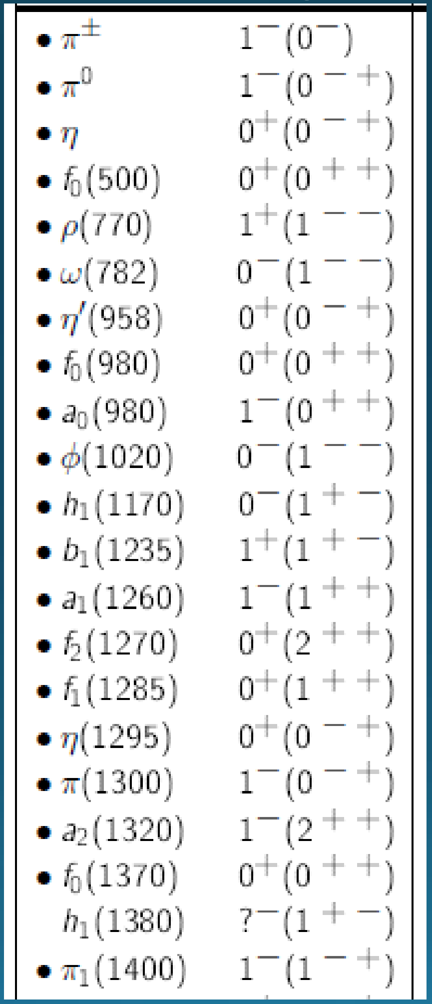
$$\epsilon = \sum_{i=\text{hadrons}} \epsilon_i$$

$$\epsilon_i = \int \frac{d^3p}{(2\pi)^3} \frac{E_p^{(i)}}{e^{E_p^{(i)}/T} \pm 1}$$

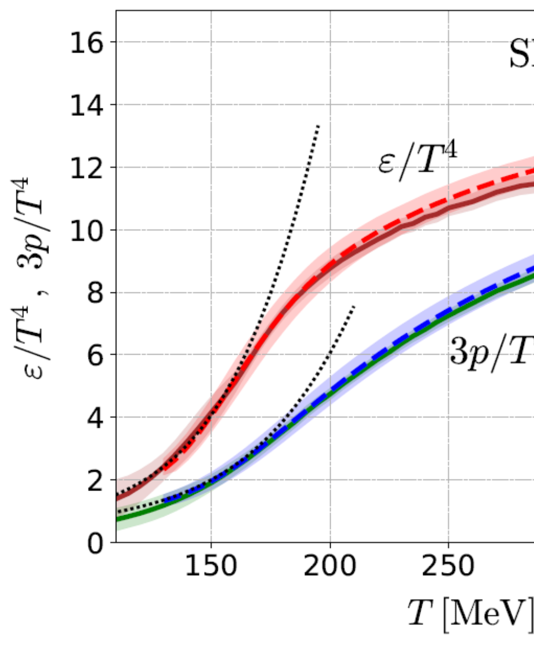
$$E_p = \sqrt{m^2 + p^2}$$

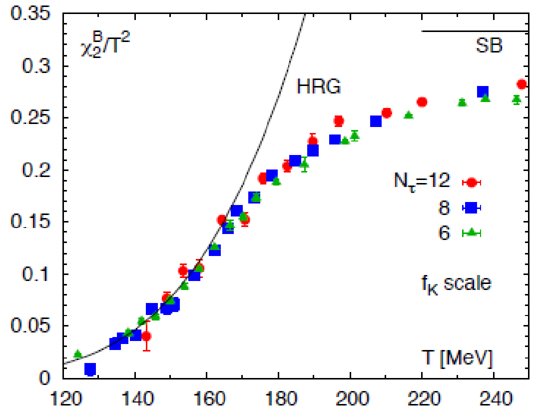
HRG reproduces QCD thermodynamics for $T < 160\text{MeV}$ quite well

Particle data group



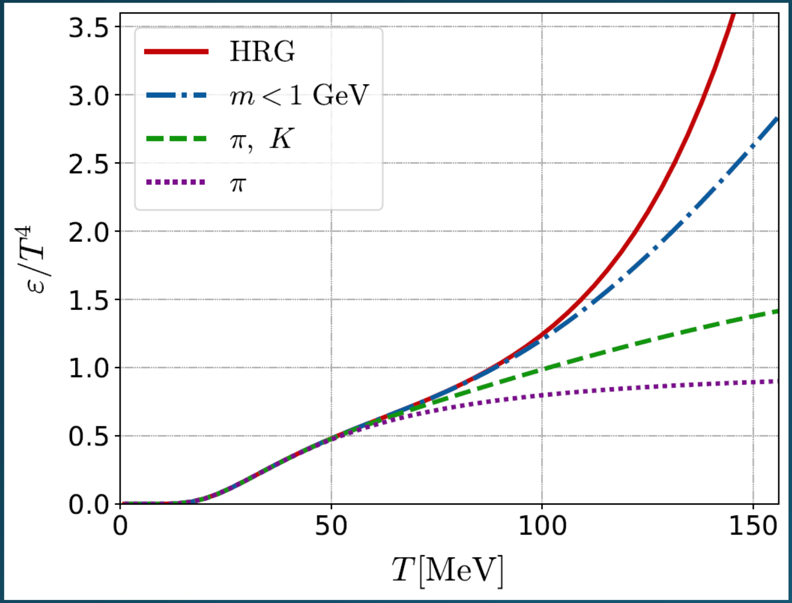
• $\pi^{\pm}$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(500)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$
• $\pi_1(1400)$	$1^-(1^{-+})$



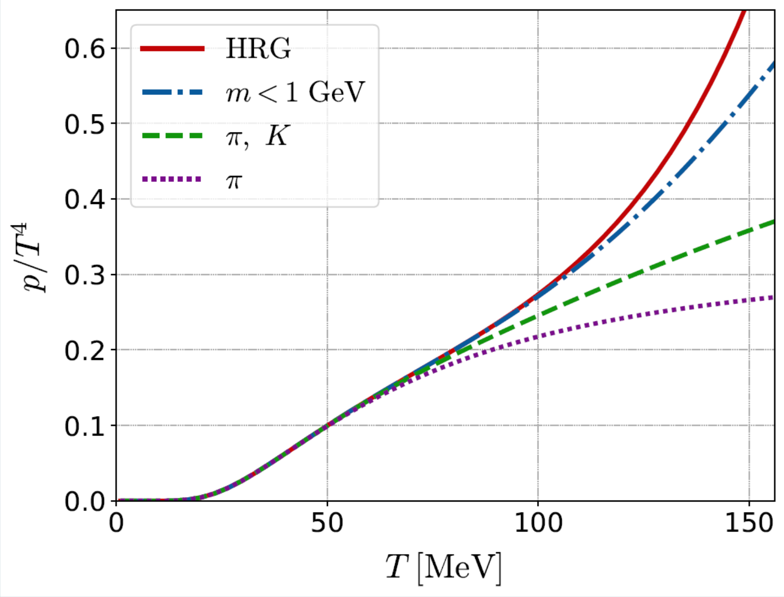


HRG Model 2

ϵ/T^4

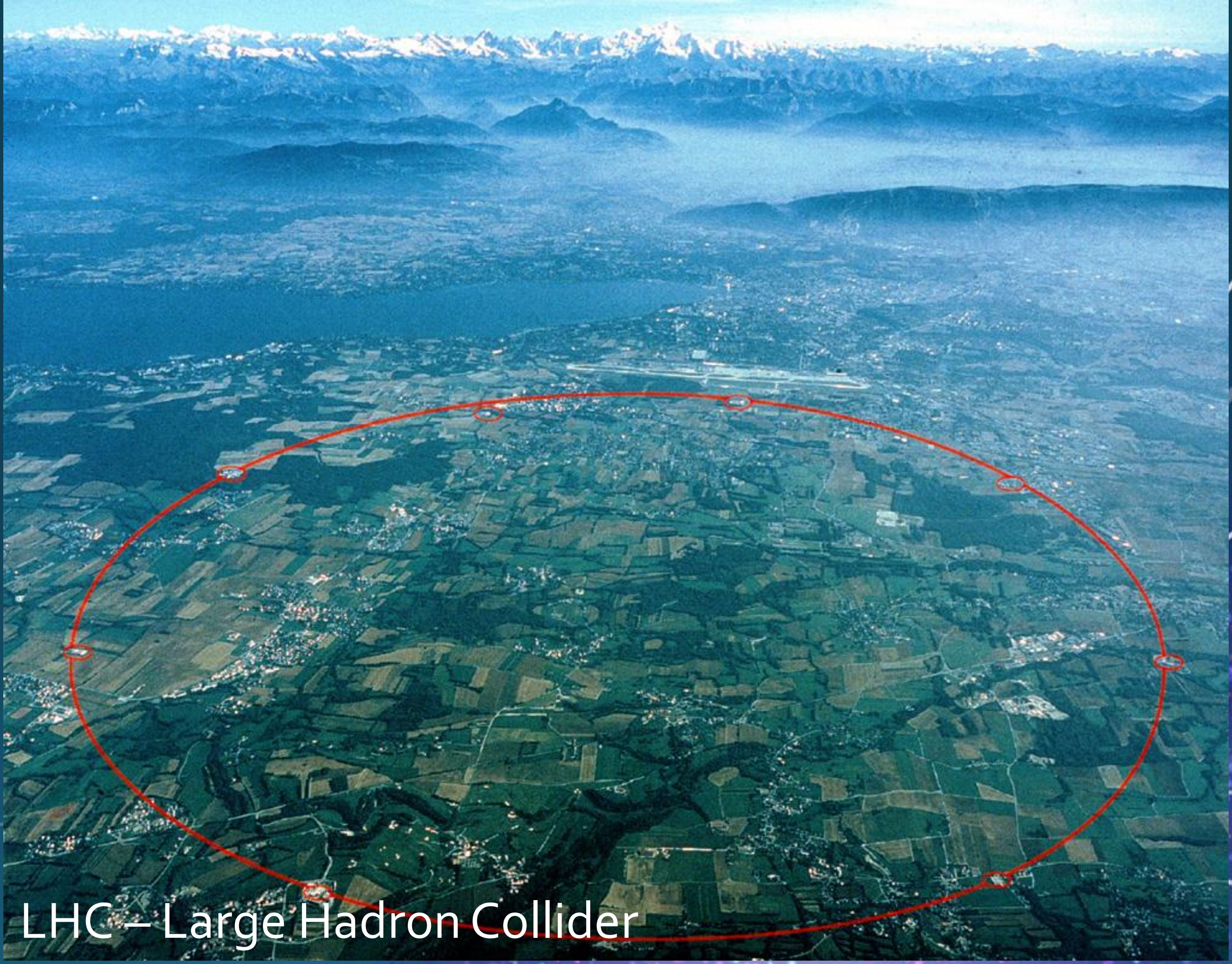


p/T^4



List of hadrons: Bollweg+, PRD104, 7 ('21)
code: https://github.com/MasakiyoK/Saizensen, Fig. 3-1

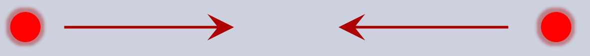
Relativistic Heavy-Ion Collisions



LHC – Large Hadron Collider

Relativistic Heavy-Ion Collisions

Proton-proton



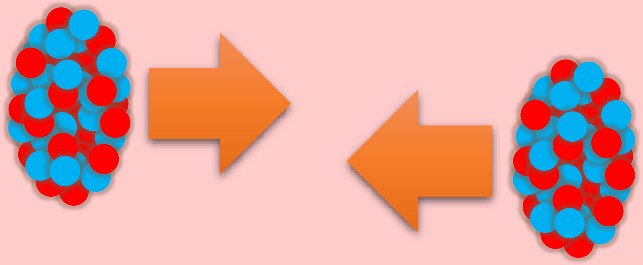
proton proton

Elementary processes

new particle search

properties of particles

Heavy Ion Collisions



Thermal Medium

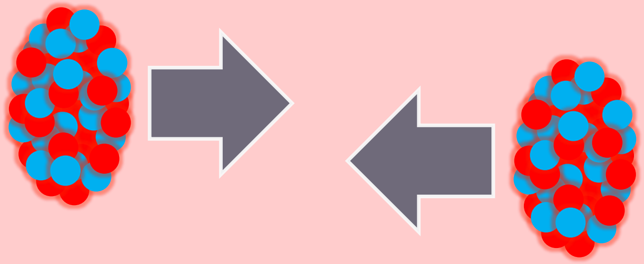
hot & dense medium

phase transitions

LHC – Large Hadron Collider

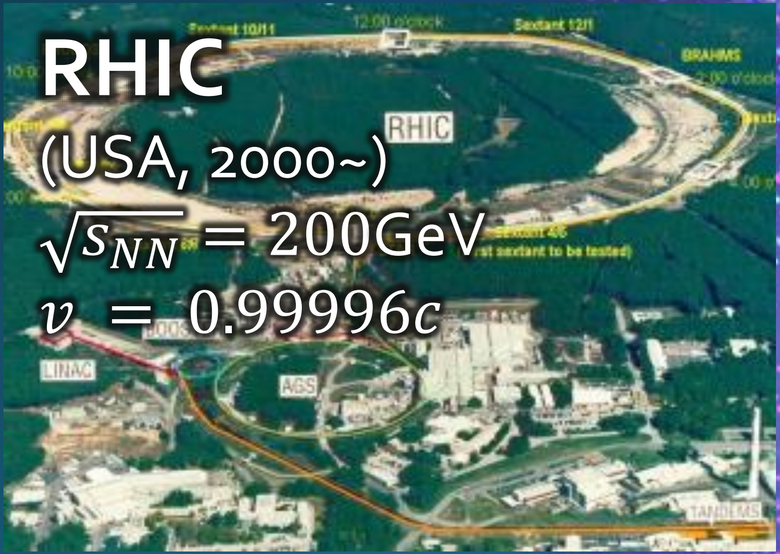
Relativistic Heavy-Ion Collisions

Collide 2 heavy nuclei



Physics

- ☐ Hot & dense medium
- ☐ Early Universe
- ☐ Quark-gluon plasma
- ☐ QCD phase structre

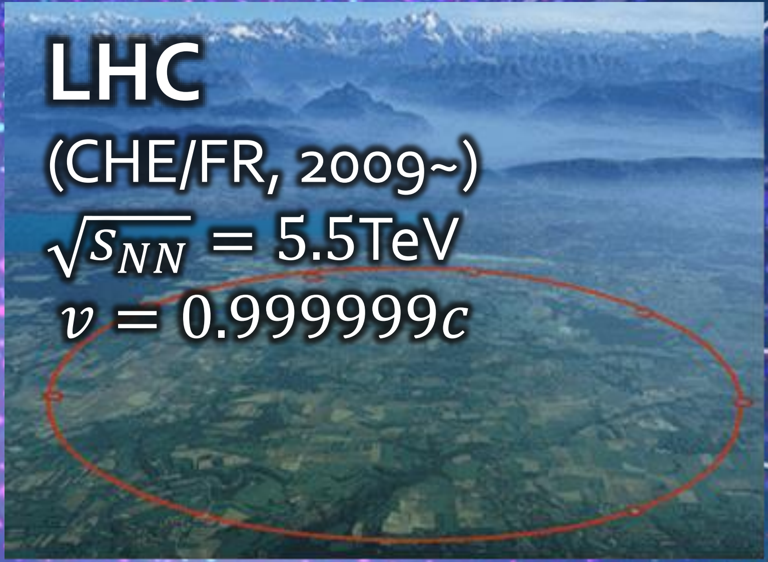


RHIC

(USA, 2000~)

$\sqrt{s_{NN}} = 200\text{GeV}$

$v = 0.99996c$



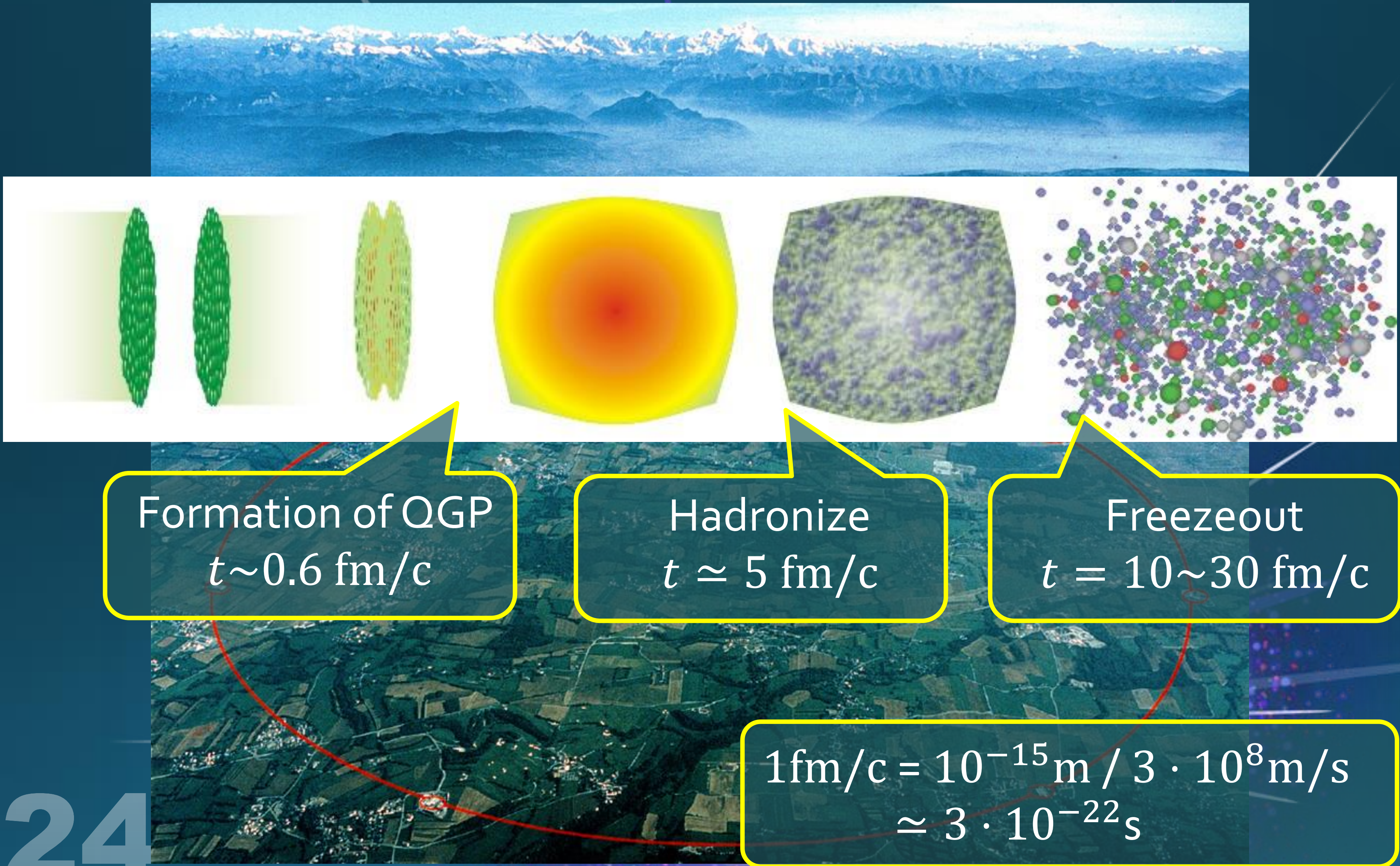
LHC

(CHE/FR, 2009~)

$\sqrt{s_{NN}} = 5.5\text{TeV}$

$v = 0.999999c$

Relativistic Heavy-Ion Collisions



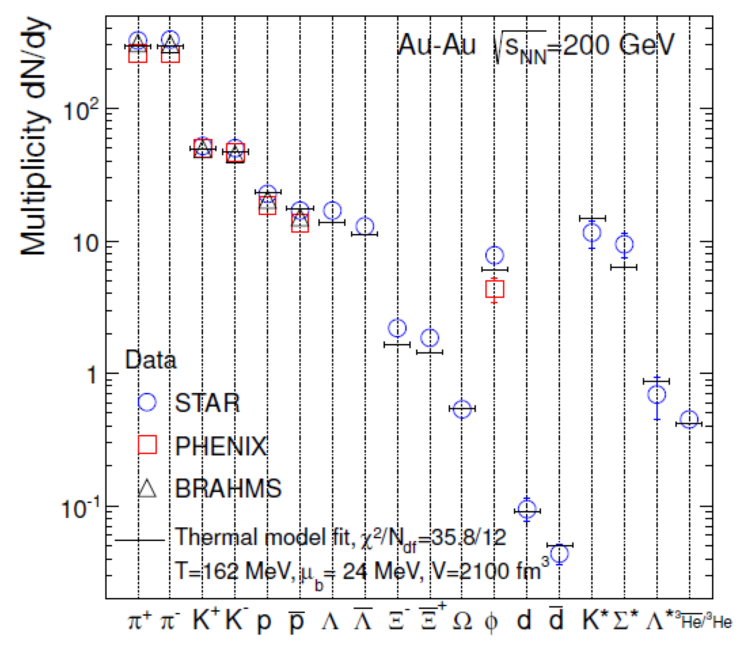
Formation of QGP
$t \sim 0.6\ \mathrm{fm/c}$

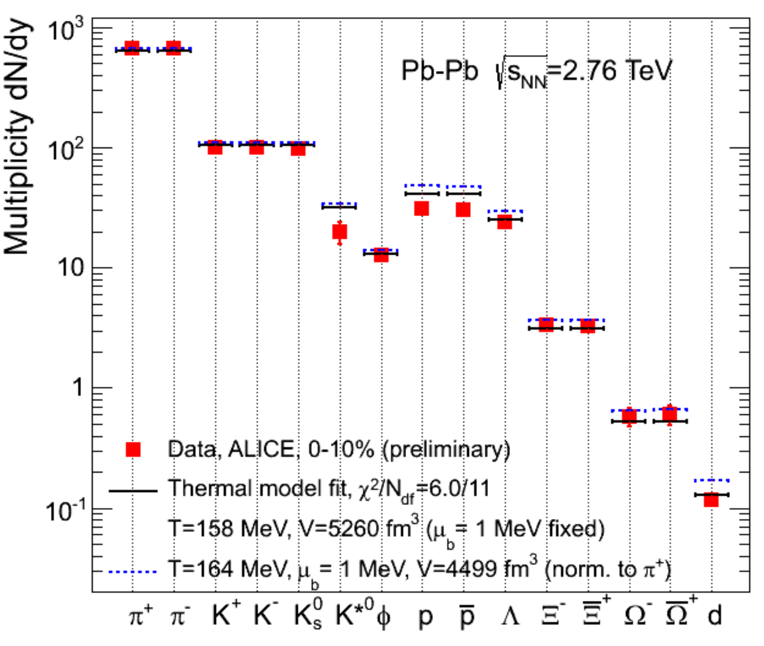
Hadronize
$t \simeq 5\ \mathrm{fm/c}$

Freezeout
$t = 10 \sim 30\ \mathrm{fm/c}$

$$1\mathrm{fm/c} = 10^{-15}\mathrm{m}\ /\ 3 \cdot 10^{8}\mathrm{m/s}$$
$$\simeq 3 \cdot 10^{-22}\mathrm{s}$$

Chemical Freezeout



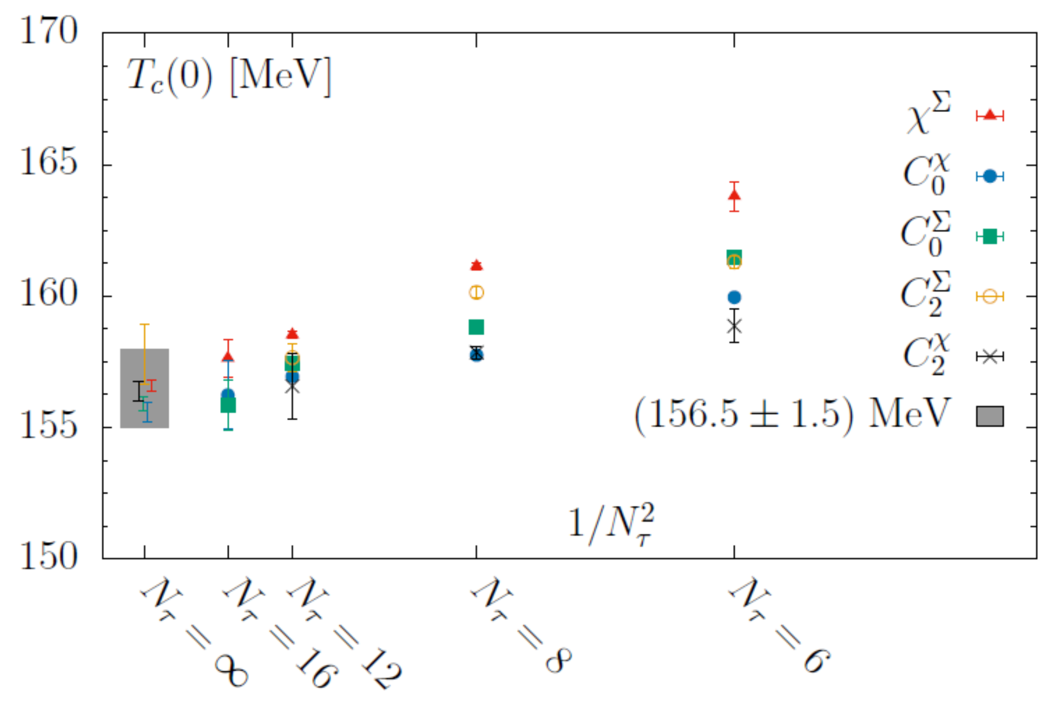


Fit the number of particles observed experimentally by 3 parameters: $T,\ \mu_B, V$

- Fit works quite well!! ➡ Chemical equilibration
- $T_{\rm chem} = 150{\sim}160\mathrm{MeV}$

Pseudo-Critical Temperature

obtained on the lattice



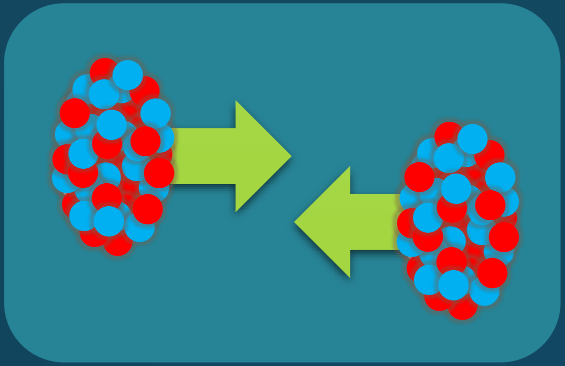
HotQCD, PLB**795**, 15 (2019)

- ☐ From peak of susceptibilities
- ☐ $T_c^* = 156.5(1.5)$ MeV

Susceptibility:

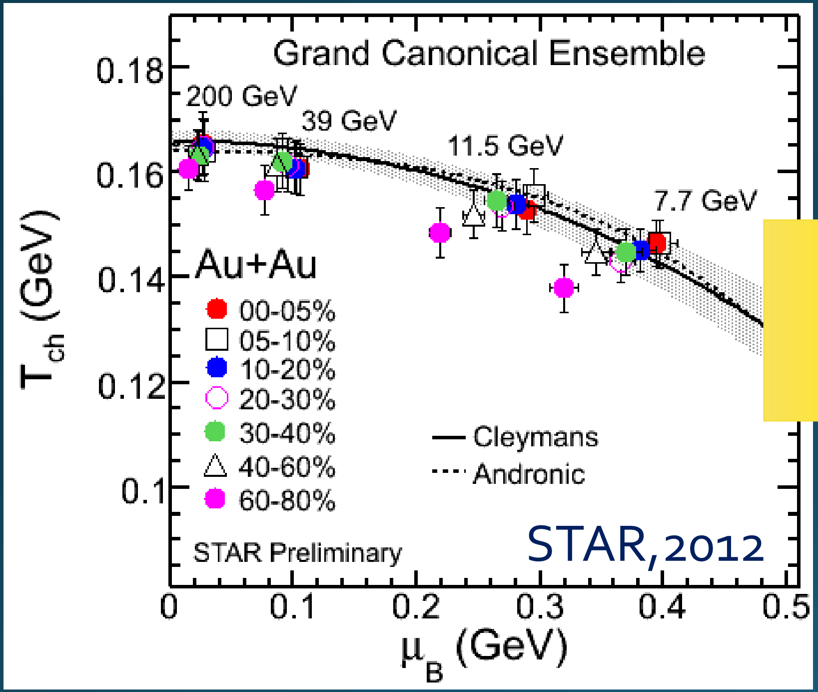
$$\chi_B = \frac{\partial n_B}{\partial \hat{\mu}_B} = \frac{\partial^2 p}{\partial \hat{\mu}_B^2}$$

Beam-Energy Dependence

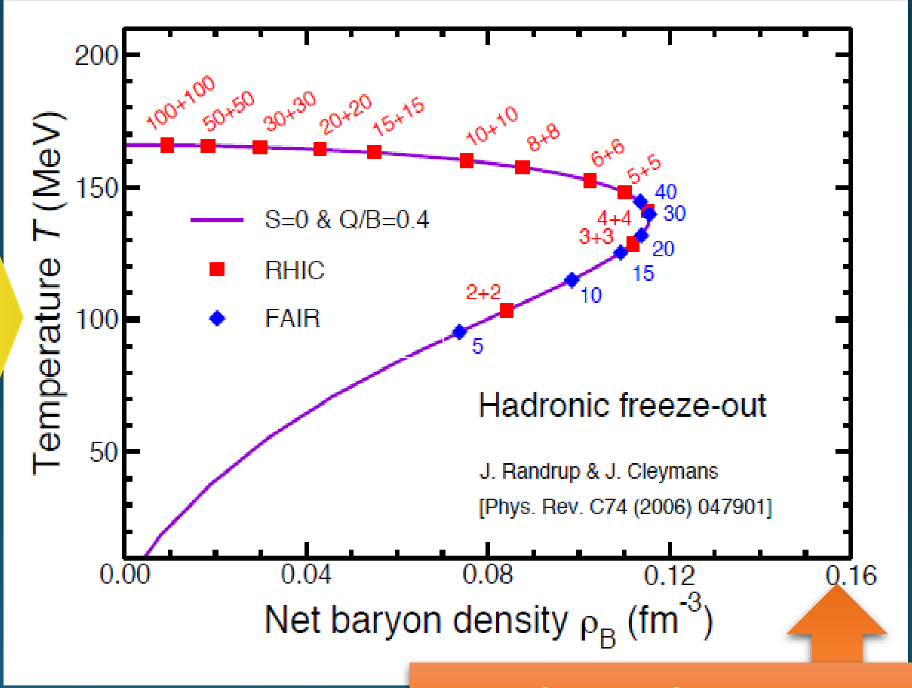


Chemical Freezeout

Chemical Freezeout T, μ



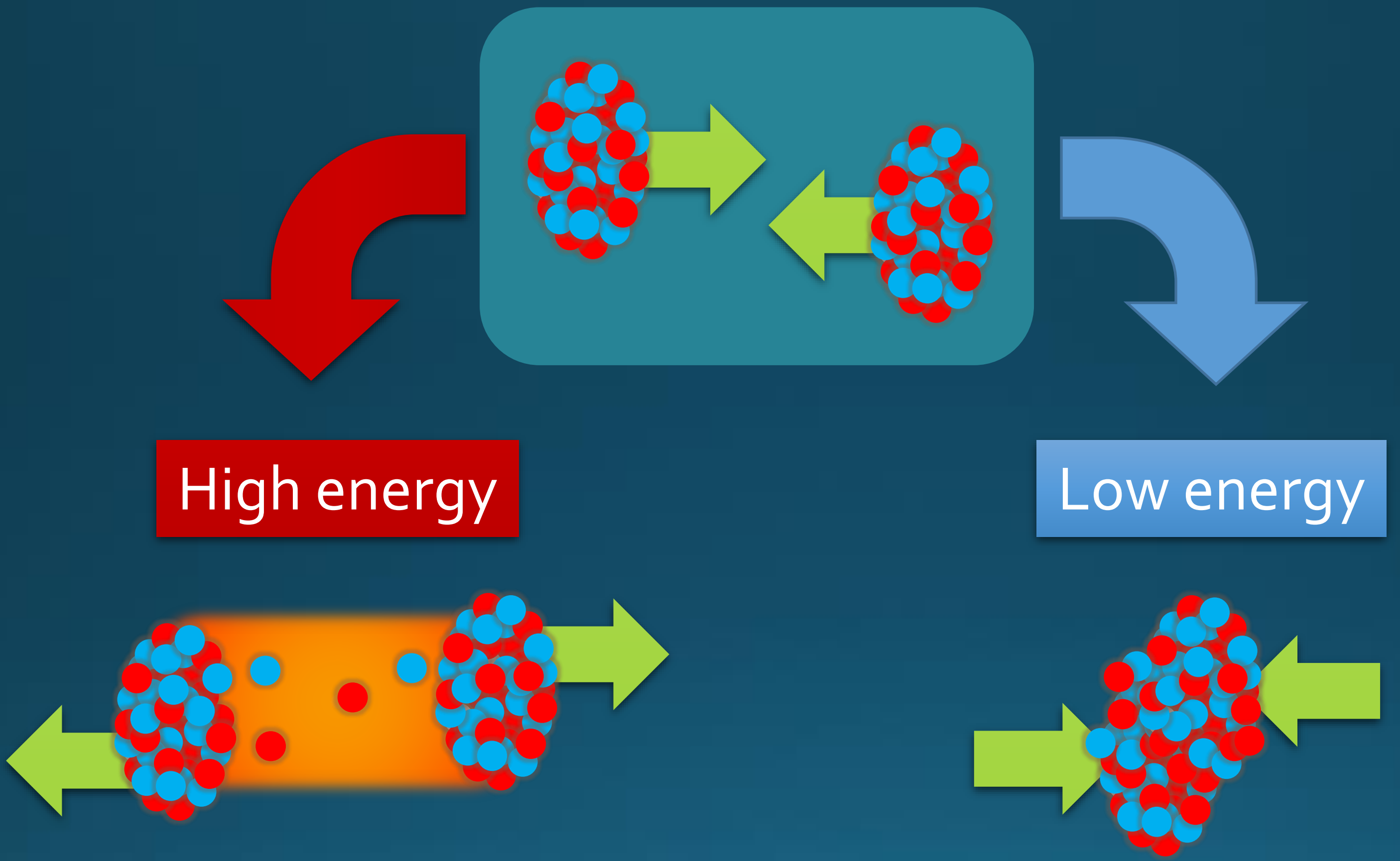
STAR, 2012



J. Randrup & J. Cleymans
[Phys. Rev. C74 (2006) 047901]

Nuclear density ρ_0

Beam-Energy Dependence



High energy

Low energy

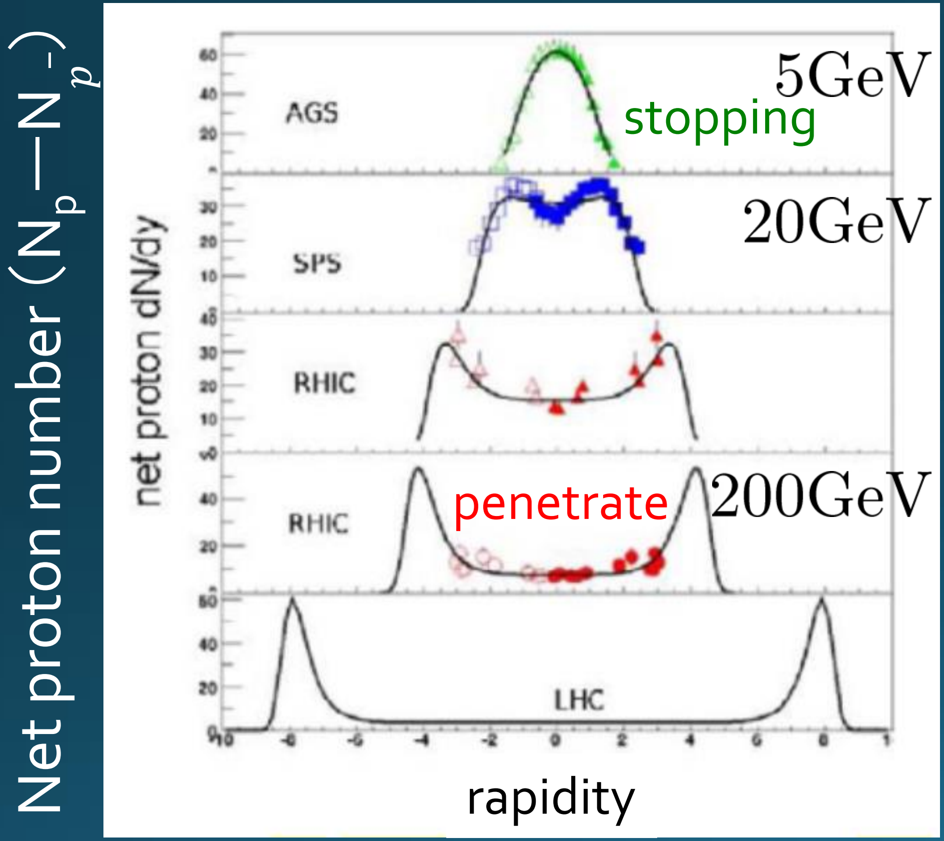
Nuclear transparency

net-baryon #: small

Baryon stopping

net-baryon #: large

Baryon Stopping



$$\sqrt{s_{NN}} \simeq 4 - 6\text{GeV}$$

Baryon stopping

$$\sqrt{s_{NN}} > 10\text{GeV}$$

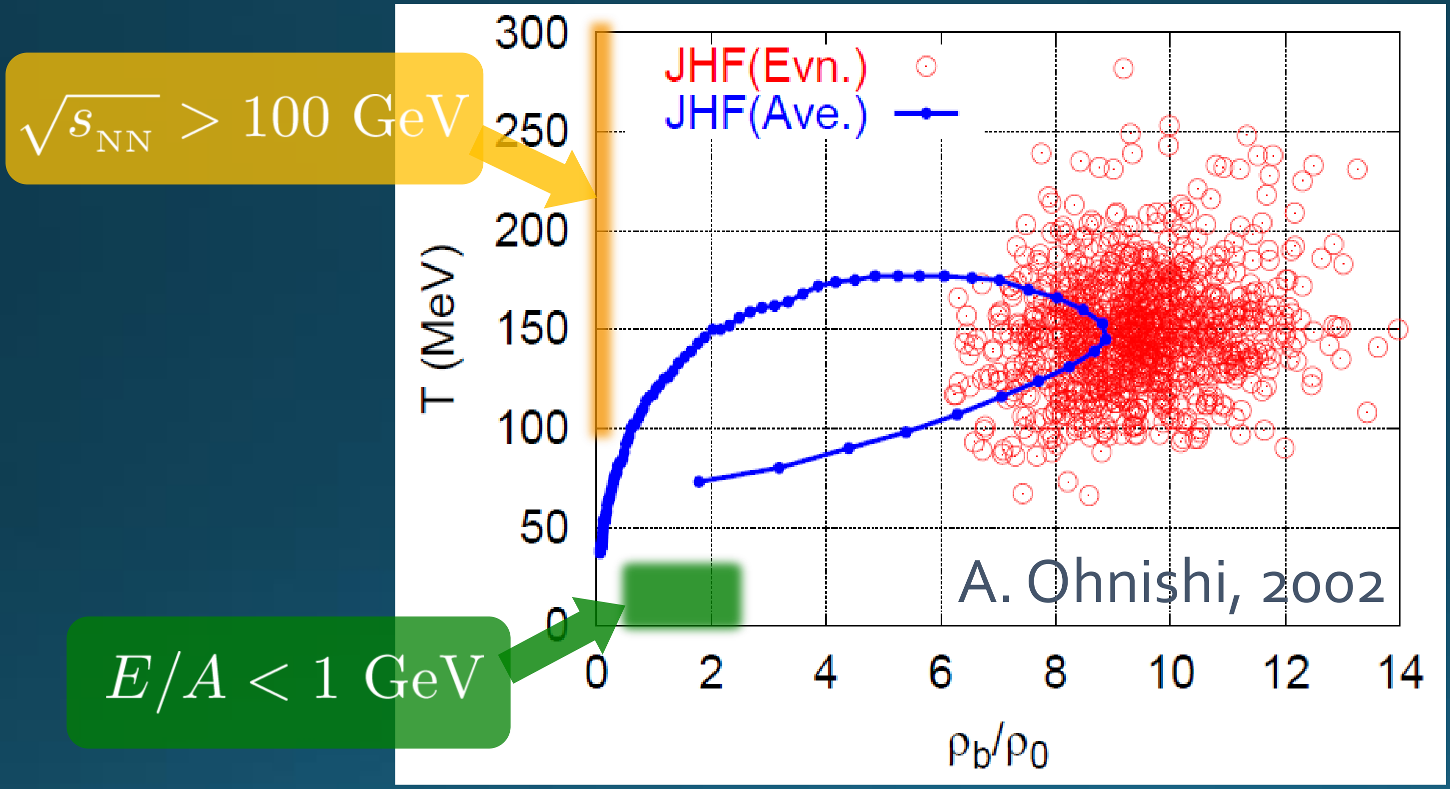
Penetration

⟷

beam axis

Maximum Density

Time evolution in *T*-ρ plane by JAM



$\sqrt{s_{\mathrm{NN}}} > 100\ \mathrm{GeV}$

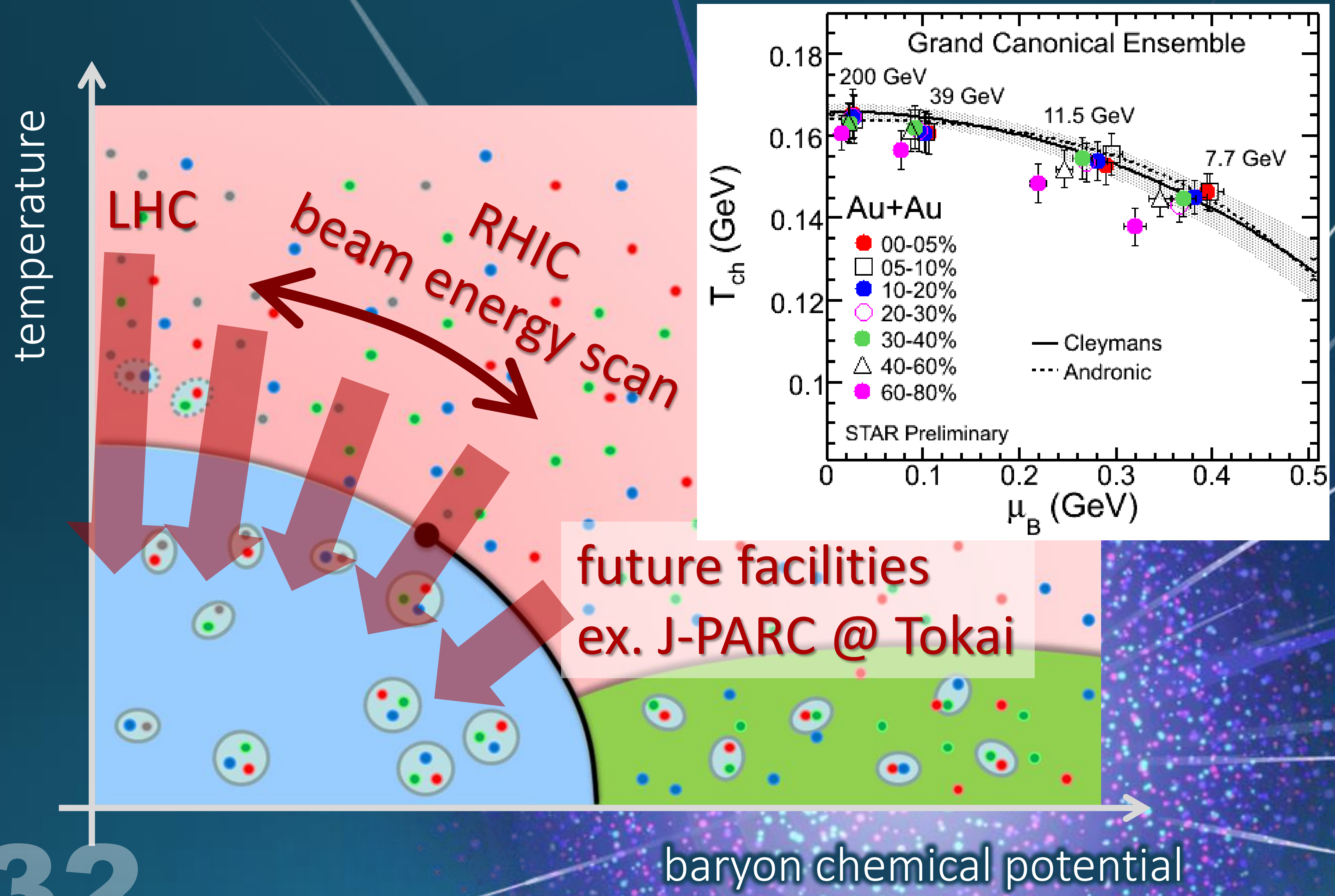
$E/A < 1\ \mathrm{GeV}$

$E/A = 20\mathrm{GeV}$

$\sqrt{s_{NN}} \simeq 6\mathrm{GeV}$

- ☐ Maximum density 5~10ρ_0 @ E/A~20GeV
- ☐ Large event-by-event fluctuations?

Beam-Energy Scan



temperature

LHC

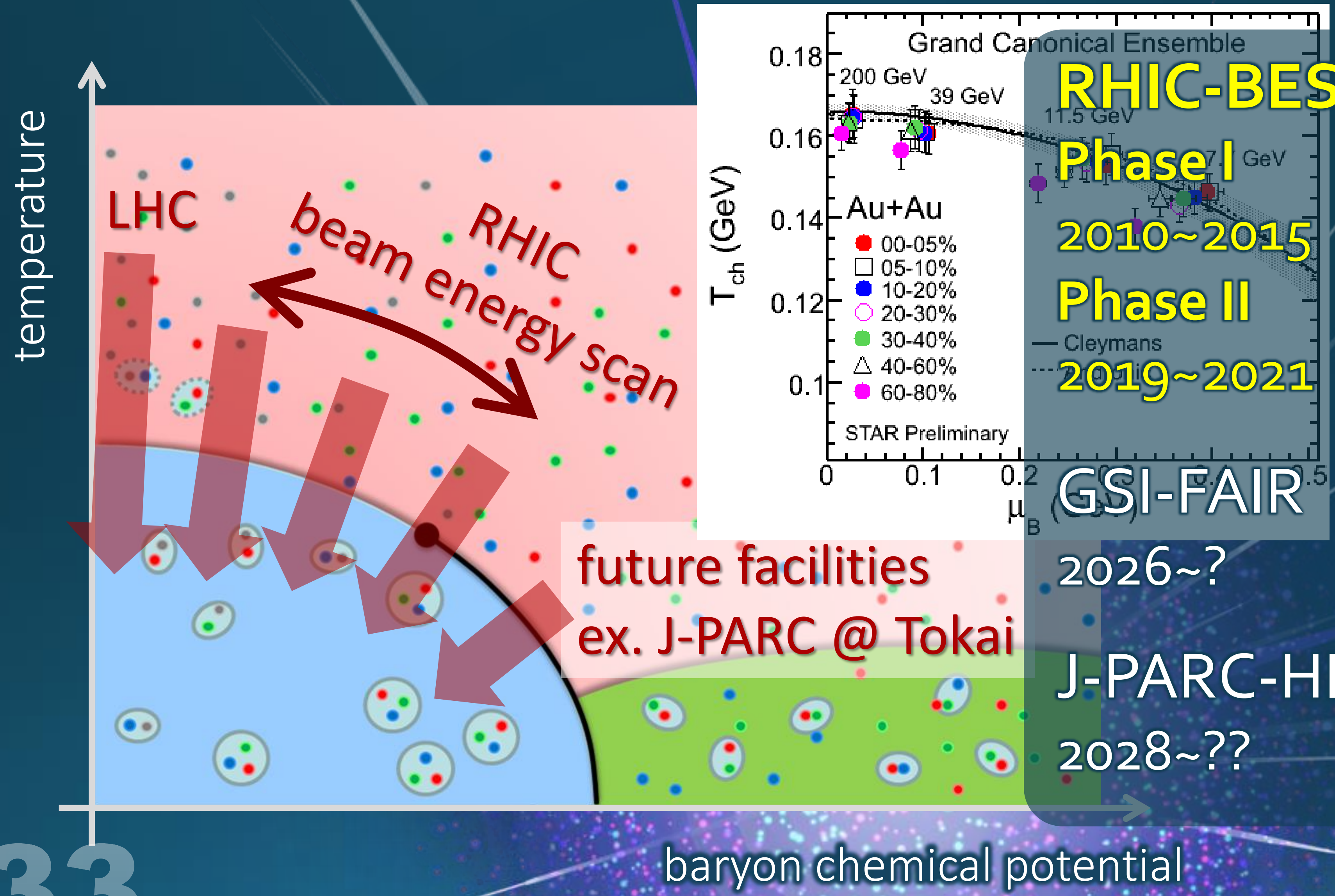
RHIC

beam energy scan

future facilities
ex. J-PARC @ Tokai

baryon chemical potential

Beam-Energy Scan



RHIC-BES

Phase I

2010~2015

Phase II

2019~2021

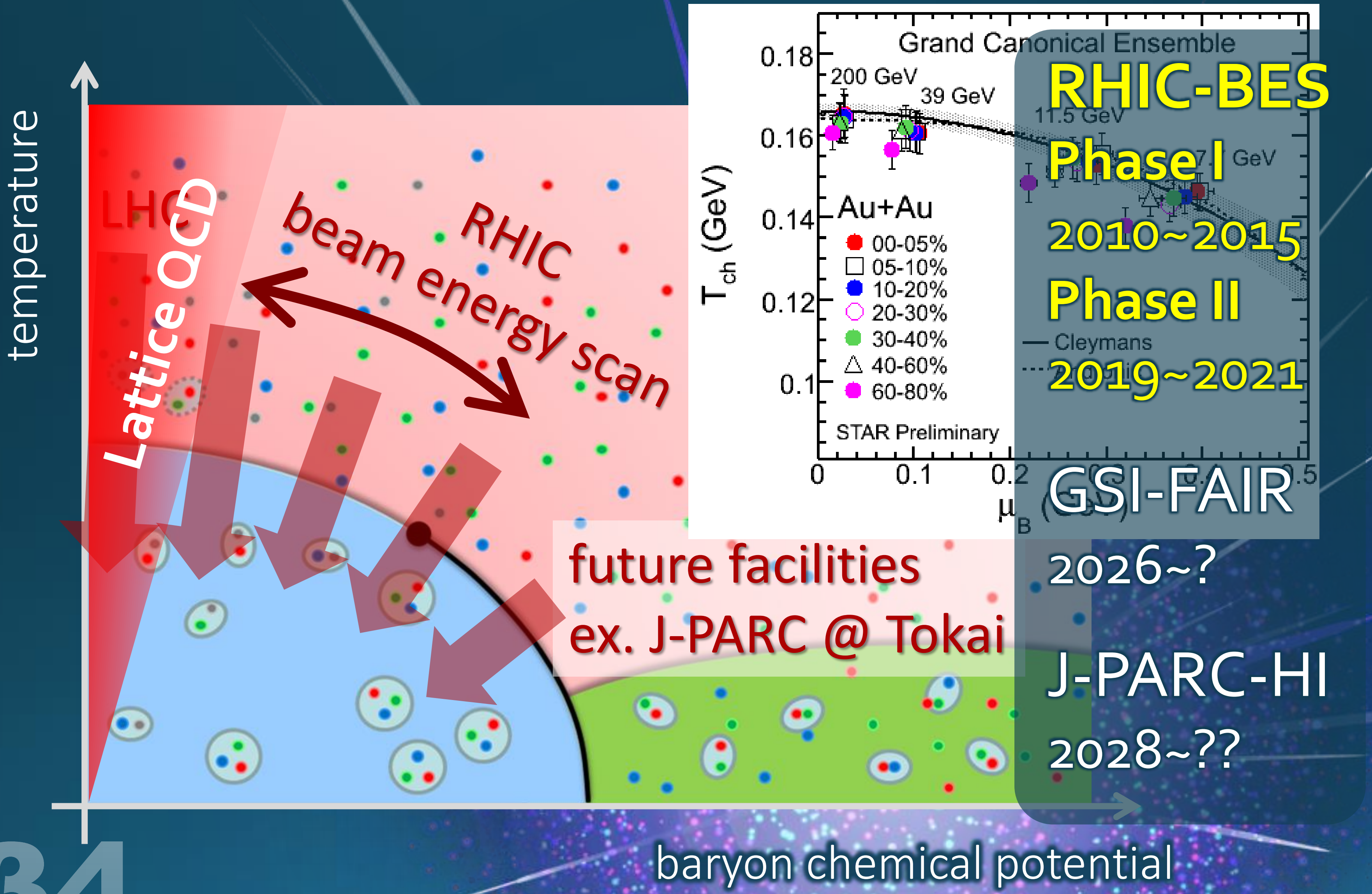
GSI-FAIR

2026~?

J-PARC-HI

2028~??

Beam-Energy Scan



future facilities
ex. J-PARC @ Tokai

RHIC-BES

Phase I

2010~2015

Phase II

2019~2021

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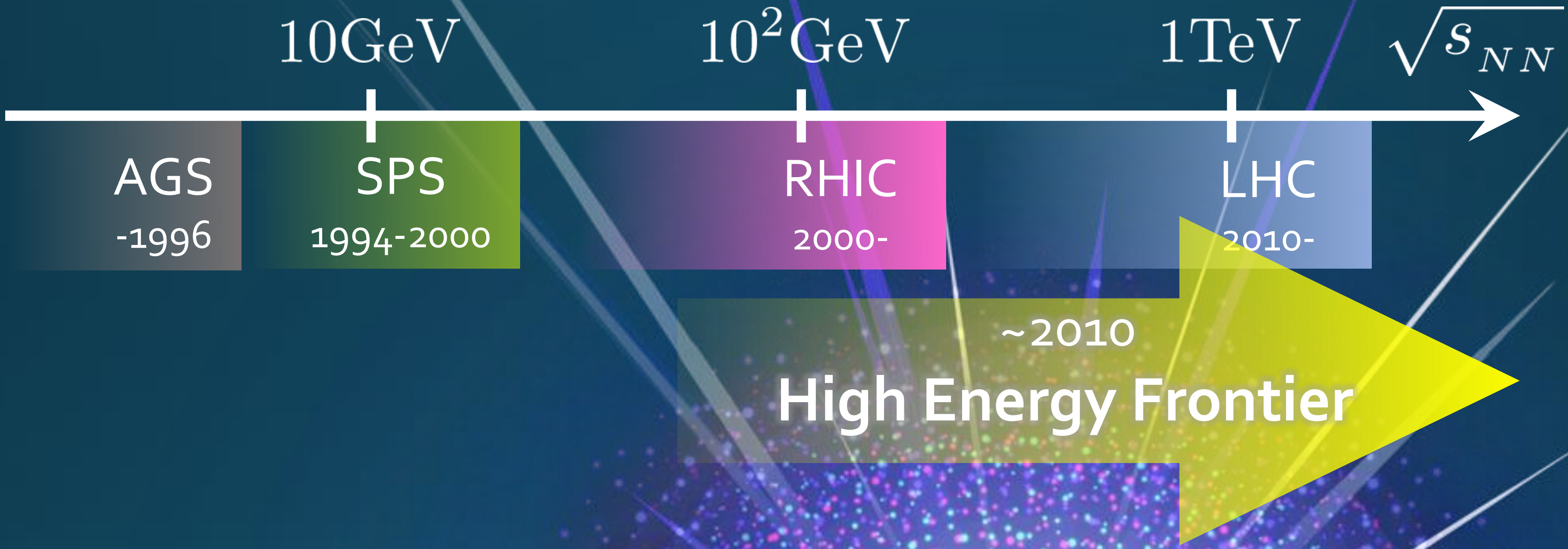
2028~??

Brief History of Relativistic HIC



$10\mathrm{GeV}$ $10^2\mathrm{GeV}$ $1\mathrm{TeV}$ $\sqrt{s_{NN}}$

AGS
-1996

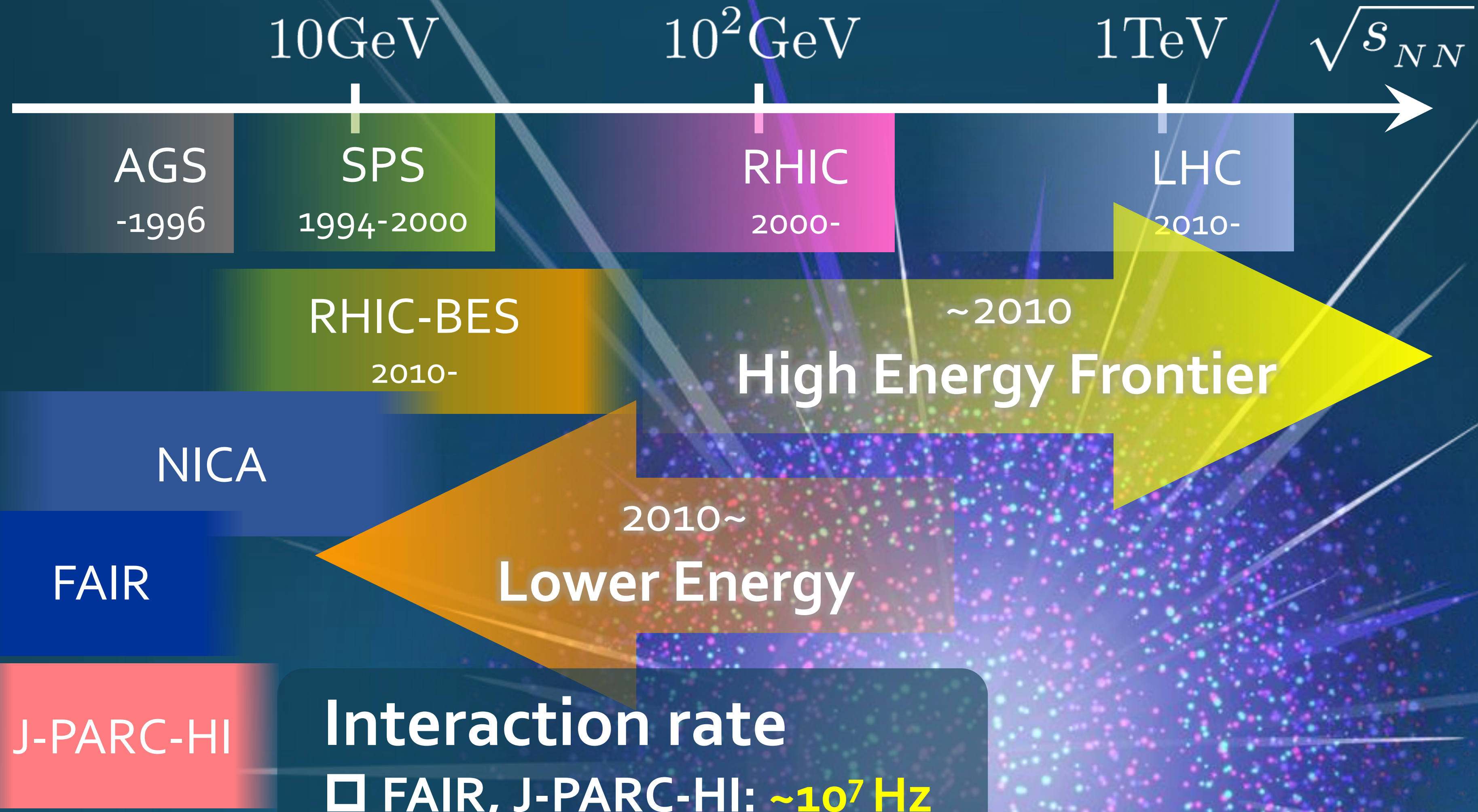
SPS
1994-2000

RHIC
2000-

LHC
2010-

~2010
High Energy Frontier

Brief History of Relativistic HIC



$10\mathrm{GeV}$ $10^2\mathrm{GeV}$ $1\mathrm{TeV}$ $\sqrt{s_{NN}}$

AGS
-1996

SPS
1994-2000

RHIC
2000-

LHC
2010-

RHIC-BES
2010-

~2010
High Energy Frontier

NICA

FAIR

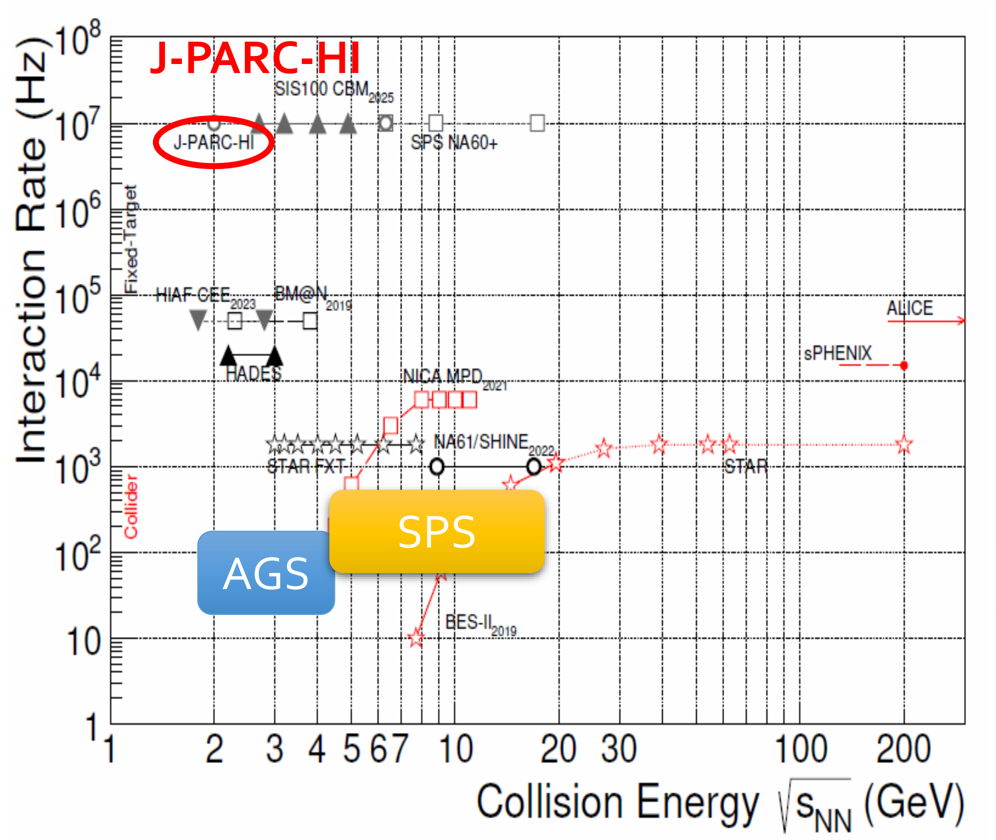
J-PARC-HI

2010~
Lower Energy

Interaction rate

- ☐ **FAIR, J-PARC-HI: ~10^7 Hz**
- ☐ AGS, SPS: ~10^2 Hz

Collision Rate



Galatyuk, NPA982,163 (2019)

J-PARC-HI:

High-luminosity x Fixed target

→World highest rate～10^8Hz

5-order higher than AGS,SPS

AGS, SPS = J-PARC-HI

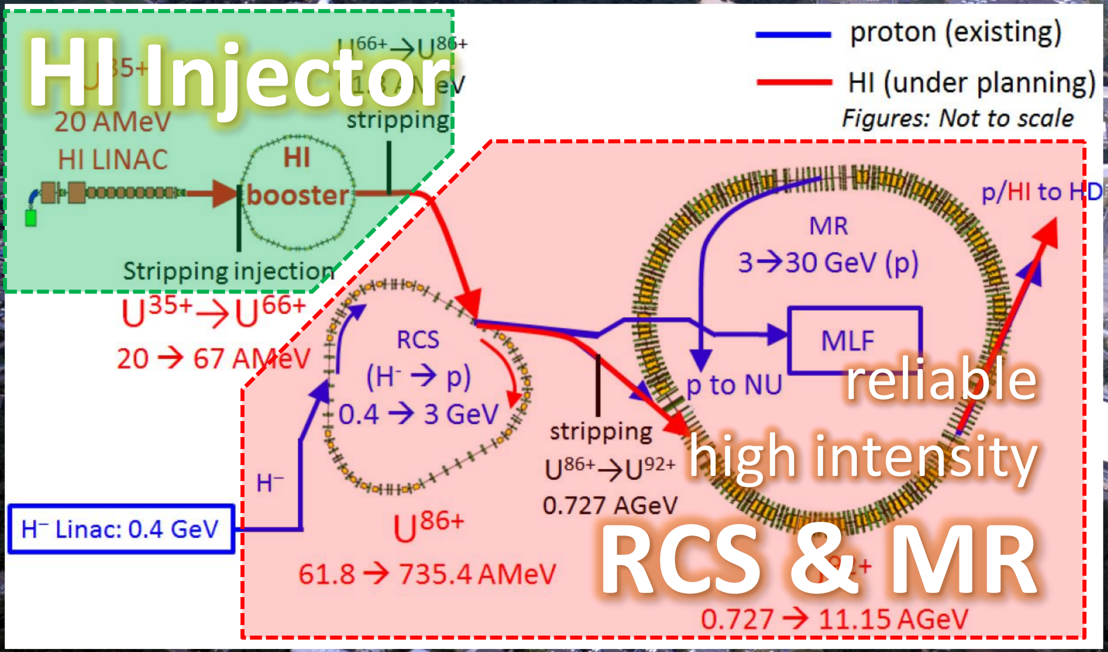
1 year = 5 min.

- ☐ High-statistical exp.
- ☐ various event selections
- ☐ higher order correlations
- ☐ search for rare events

J-PARC-HI

J-PARC Heavy-Ion Program

Heavy-ion collision experiments
using accelerators in **J-PARC (RCS/MR)**
World highest intensity / Low cost

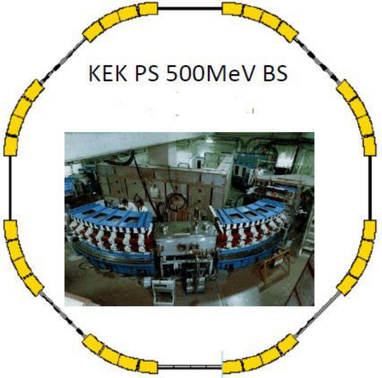


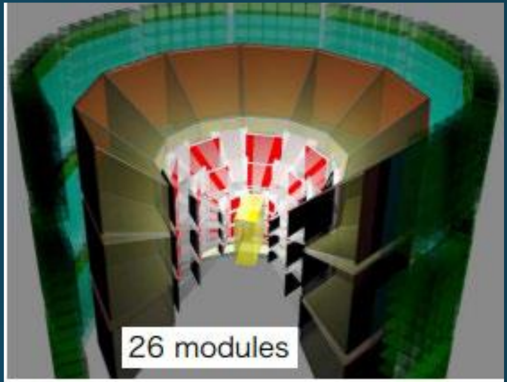
- ☐ E_{lab} ~11→19 AGeV
- ☐ $\sqrt{s_{NN}}$~4.9→6.2 GeV
- ☐ Collision rate: ~10^8Hz
- ☐ 2028~?

J-PARC-HI: Staging Plan

☐ **Phase-I (~2026)**

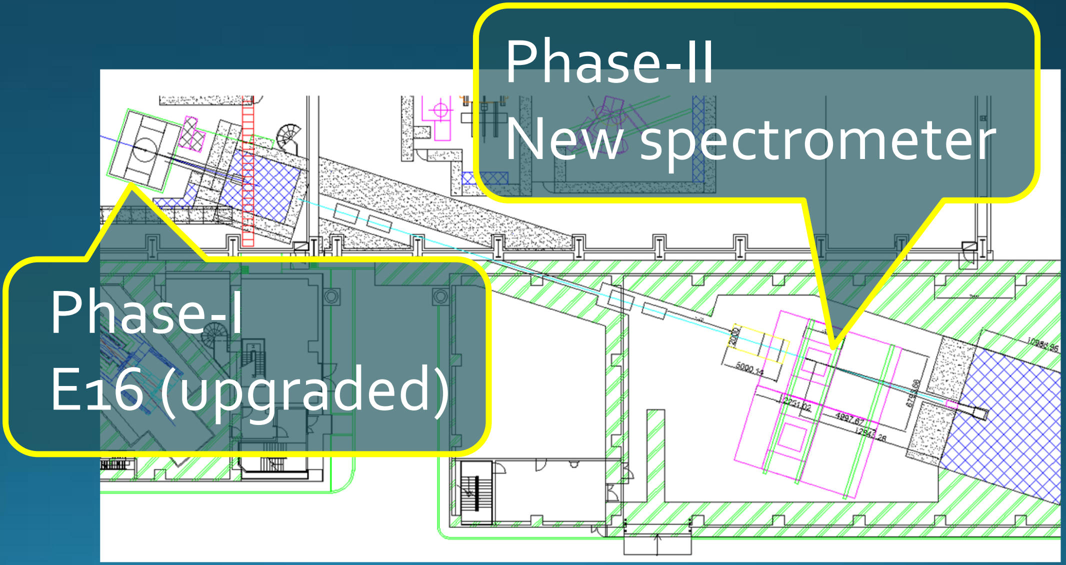
- ☐ SC HI Linac
- ☐ KEK PS booster ~10^8Hz
- ☐ E16 spectrometer (upgrade)
- ☐ Thermal dileptons



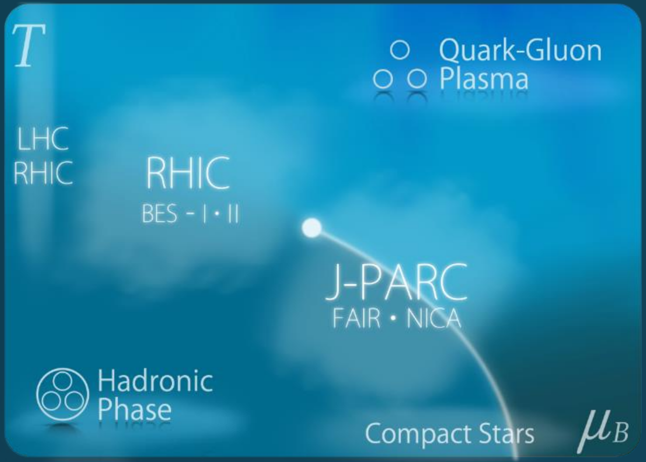


☐ **Phase-II (~2032)**

- ☐ **New** booster ~10^{11}Hz
- ☐ **New** spectrometer
- ☐ Fluctuations
- ☐ Correlations

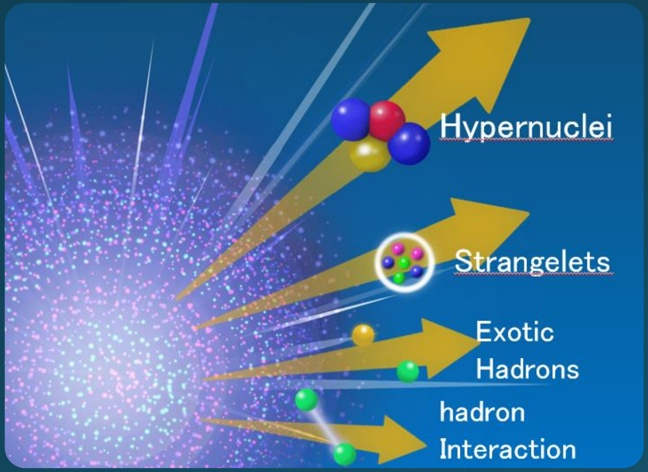


2 Main Goals



Exploring Dense Medium

- ☐ QCD phase diagram
- ☐ 1st order phase transition
- ☐ equation of state

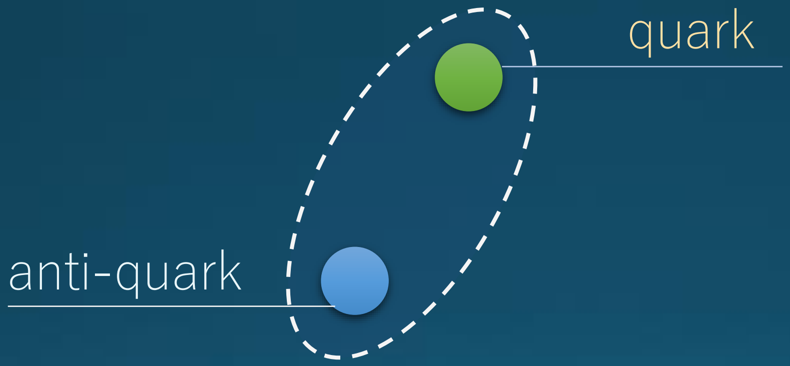


Rare-event Factory

- ☐ hyper nuclei
- ☐ exotic hadrons
- ☐ hadron interaction

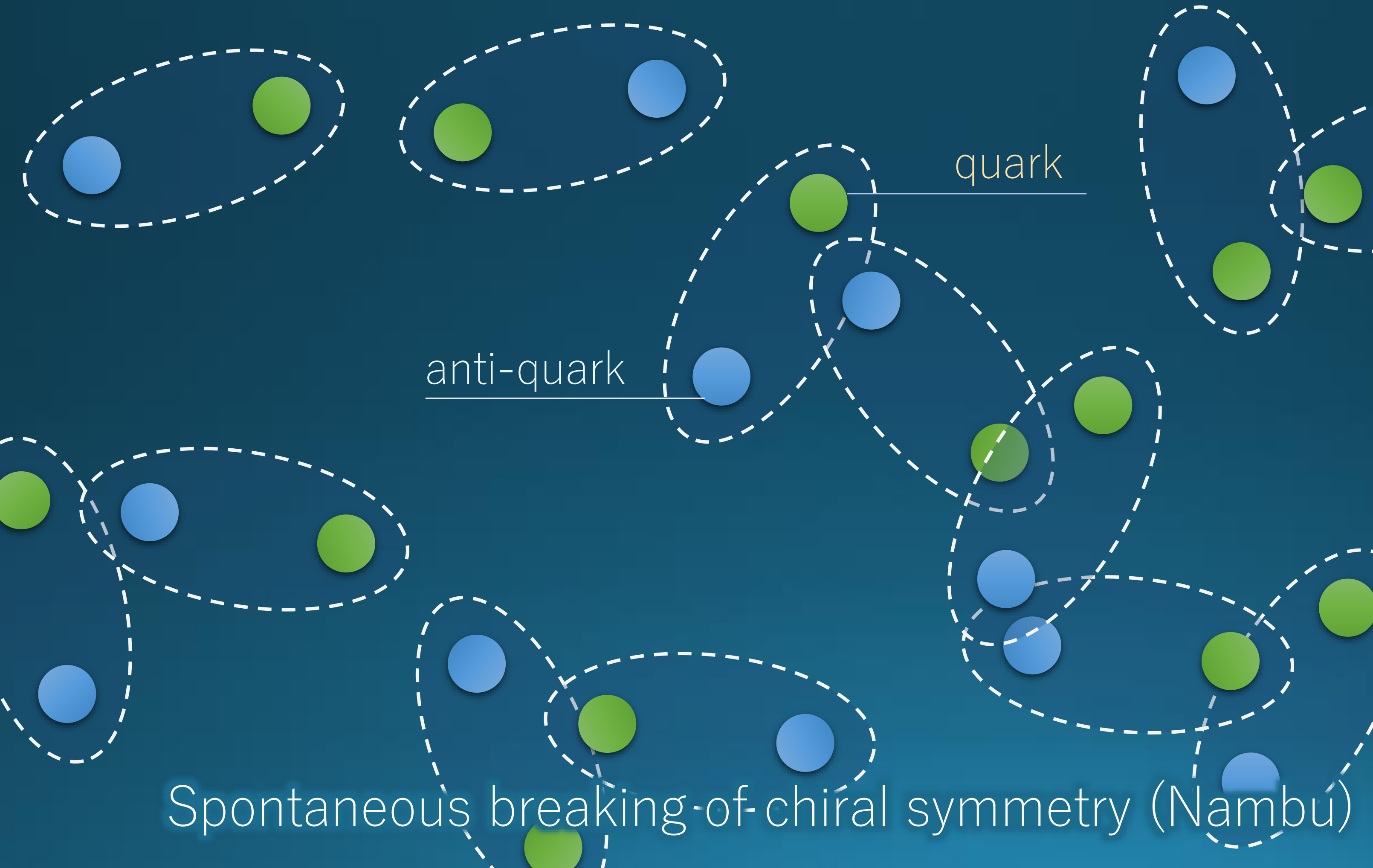
Collapse of Vacuum

Our vacuum is filled with the quark condensate.



Collapse of Vacuum

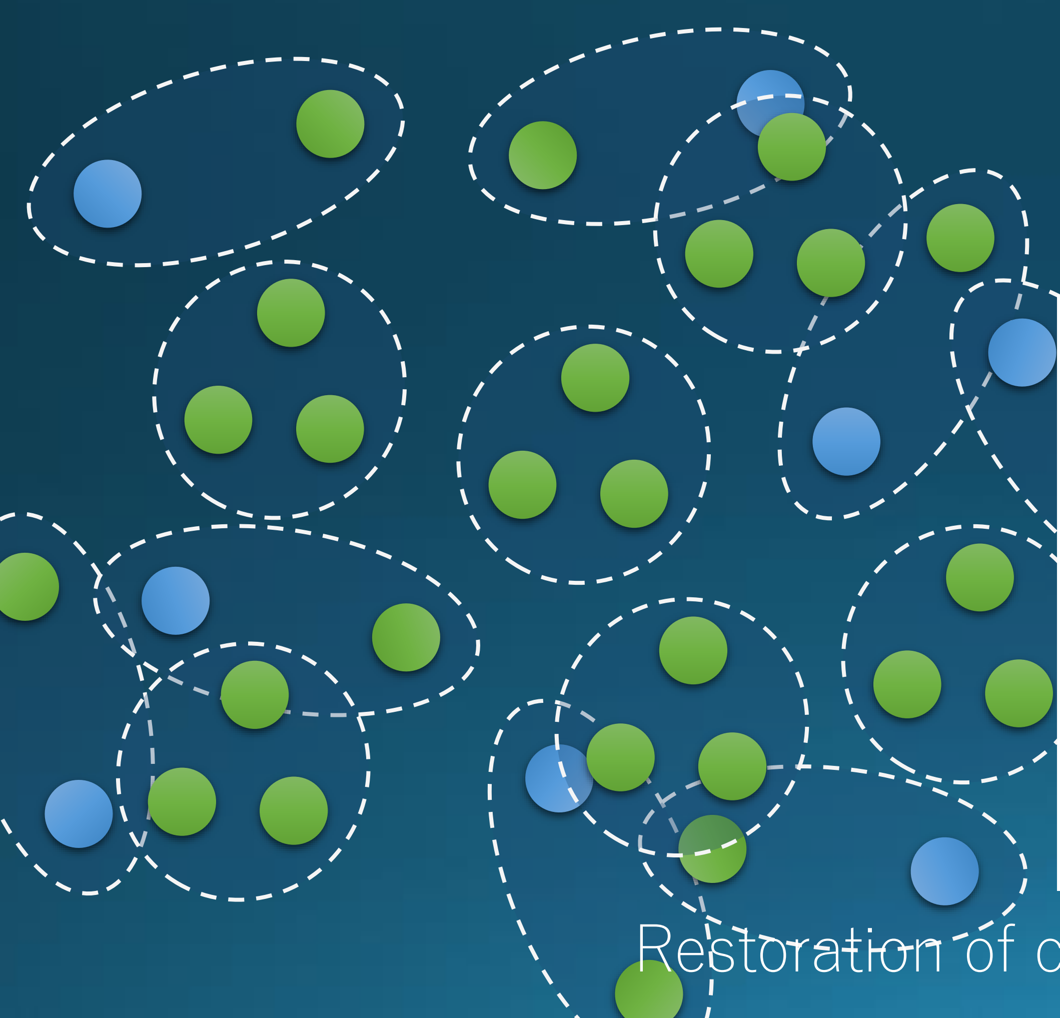
Our vacuum is filled with the quark condensate.

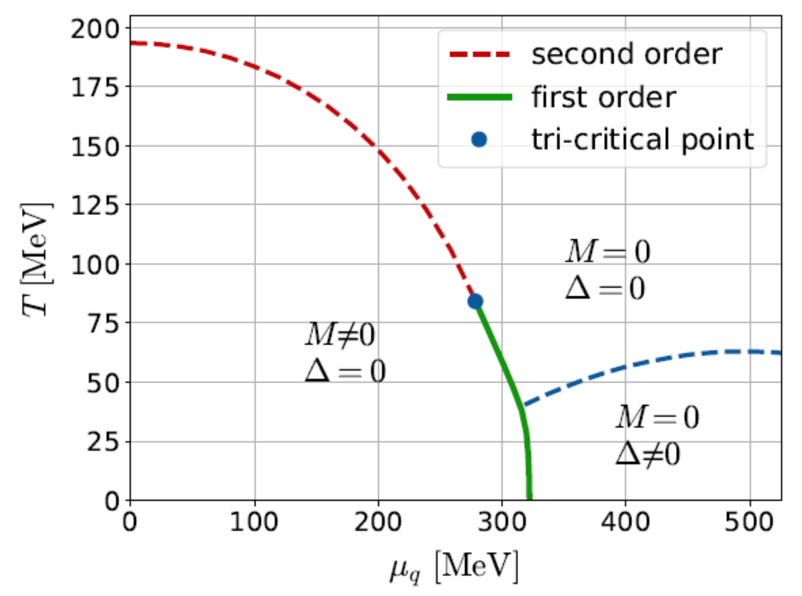


Spontaneous breaking of chiral symmetry (Nambu)

Collapse of Vacuum

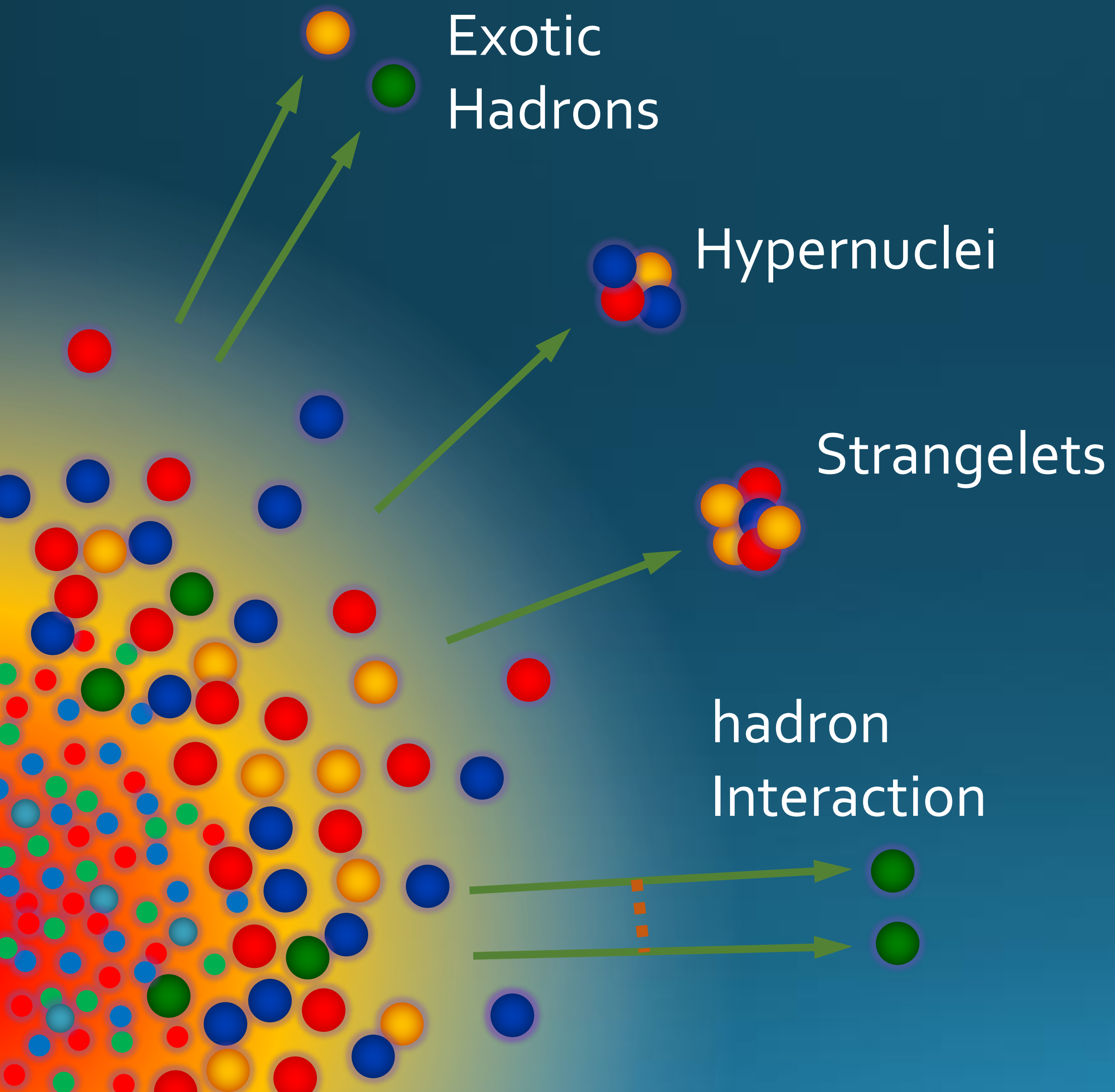
Matter modifies the vacuum.





Restoration of chiral symmetry

Search for Rare Events



Exotic Hadrons

Hypernuclei

Strangelets

hadron Interaction

- High density
- High luminosity
- High strange yield

Rare-event Factory

- creation
- properties
- interaction

Contents

1. Introduction to Hot & Dense QCD
    - phase structure of QCD
    - Relativistic heavy-ion collisions
    - Beam-Energy Scan

2. Search for QCD-CP: Fluctuations

Reviews: Asakawa, MK, PPNP ('16); Bluhm, MK+, NPA 1003 ('20)
MK, Esumi, Nonaka, JPS journal, 2021/8

3. Search for CSC: Precurso

◆◆◆◆解説◆◆◆◆

非ガウスゆらぎで探る宇宙最高密度の相転移

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現在，およそ10^{15} g/cm^{3}という超高密度で実現するとされる相転移の実験的探索が世界各地の実験施設で行われているのをご存じだろうか．この相転移とは，強い相互作用の基礎理論である量子色力学（QCD）が低温かつ超高密度の物質中で引き起こす一次相転移と，その一次相転移線の端点であるQCD臨界点のことである．10^{15} g/cm^{3}という密度は，原子核の飽和密度$\rho_0 \simeq$

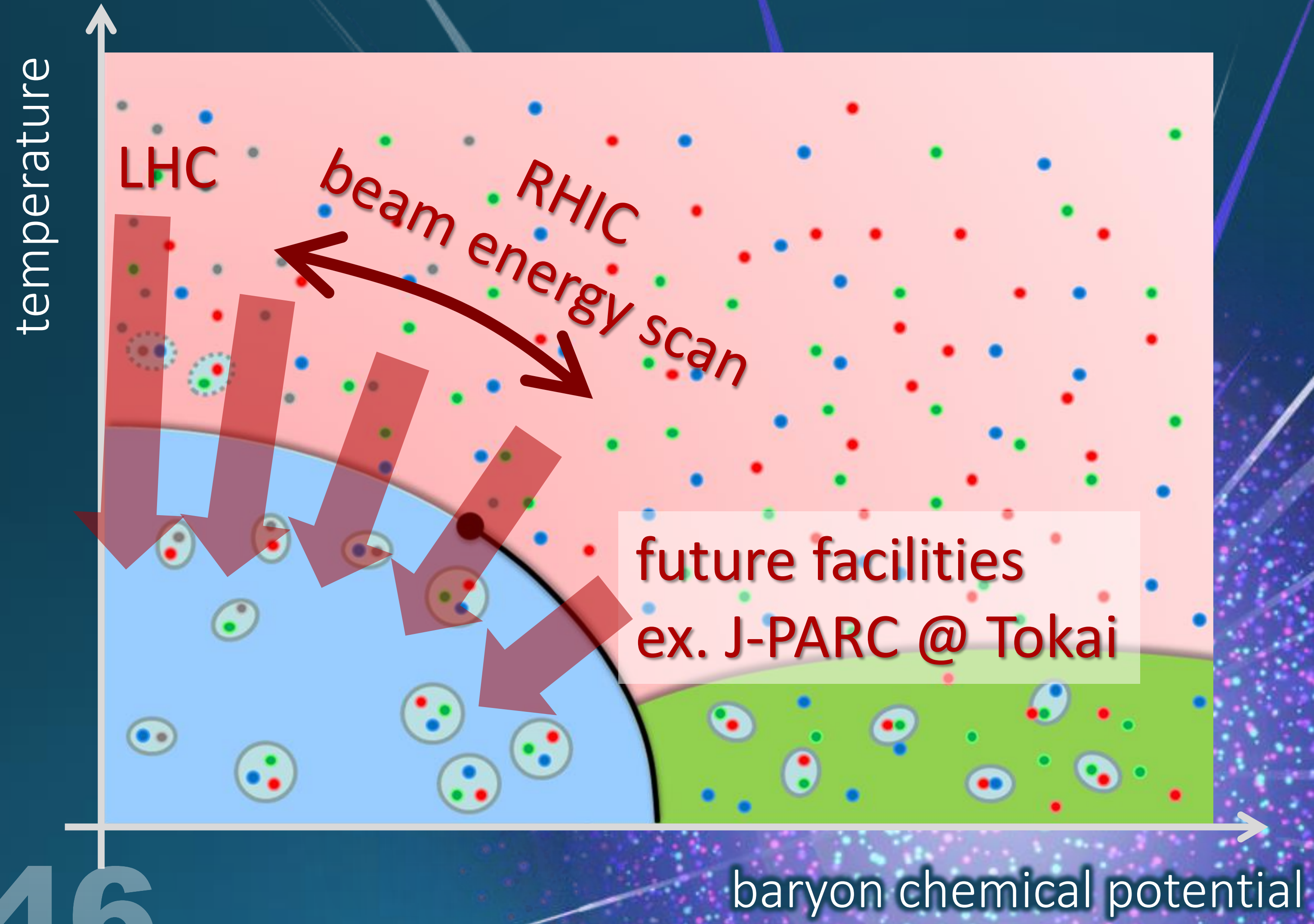
これら一連の実験が目指す最重要課題が，ビームエネルギー走査による高密度領域の相構造探索である．

これら一連の研究の中でも近年特に精力的に調べられてきたのが，非ガウスゆらぎを使ったQCD臨界点の実験的探索である．ゆらぎはキュムラントとよばれる量で特徴づけられるが，QCD臨界点でゆらぎが発散するのに伴い，QCD臨界点周辺では各

—用語解説—

QCD臨界点：
QCD真空にクォーク数密度を印加していくと，10^{15} g/cm^{3}付近で真空構造の変質に伴う一次相転移が起きる可能性が指摘されている．この一次相転移が存在する場合，相転移線は有限温度で端点，すなわち臨界点をもつ．この点をQCD臨界点とよび，現在ビームエネルギー走査によるその実験的探索が注目されている．

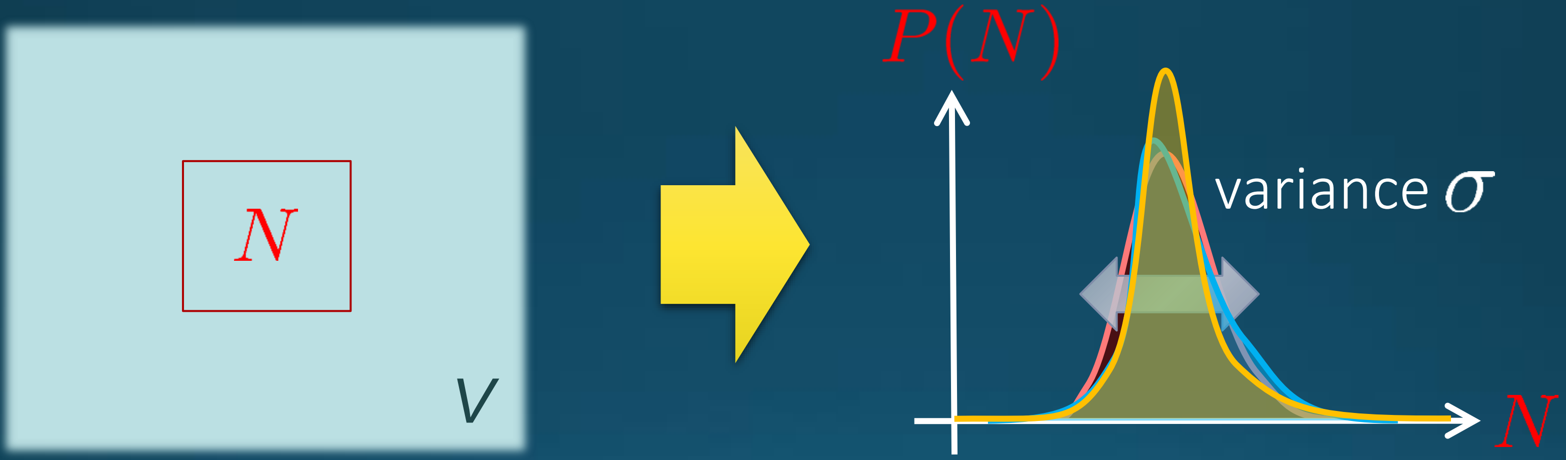
Beam-Energy Scan



future facilities
ex. J-PARC @ Tokai

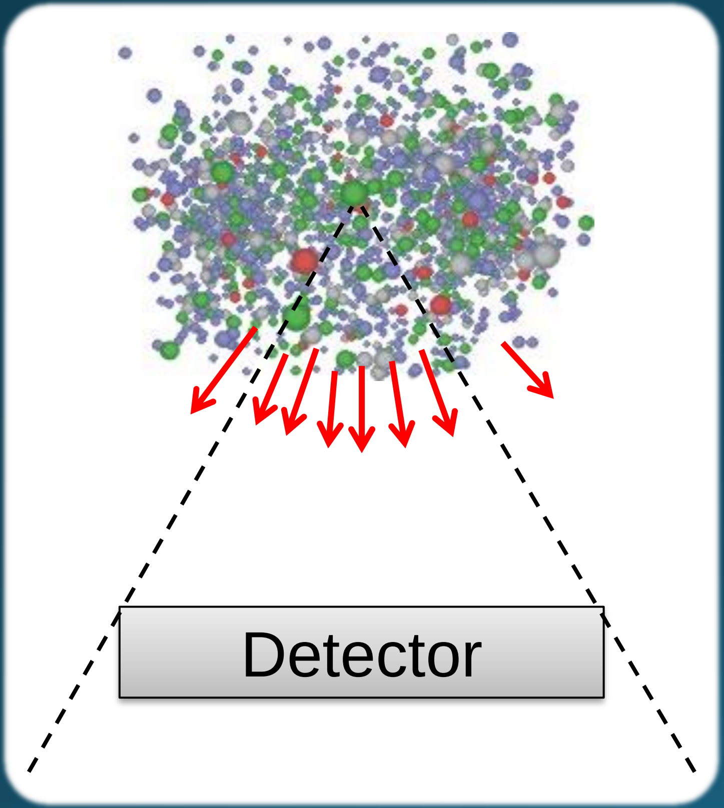
Fluctuations

Observables in equilibrium are fluctuating!

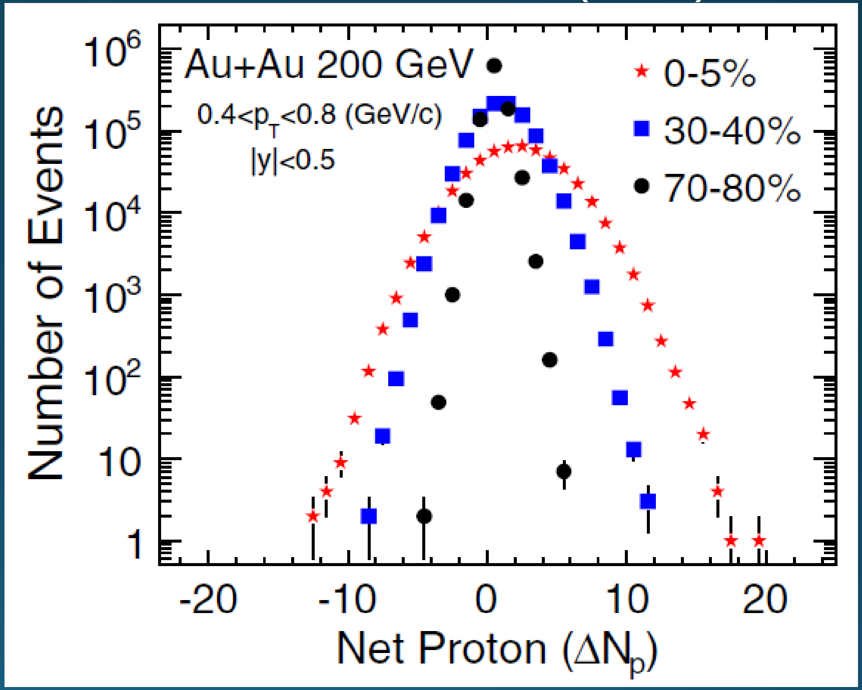


Event-by-event Fluctuations

Review: Asakawa, MK, PPNP **90** (2016)



STAR, PRL**105** (2010)



Cumulants

$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$

Cumulants

Cumulants

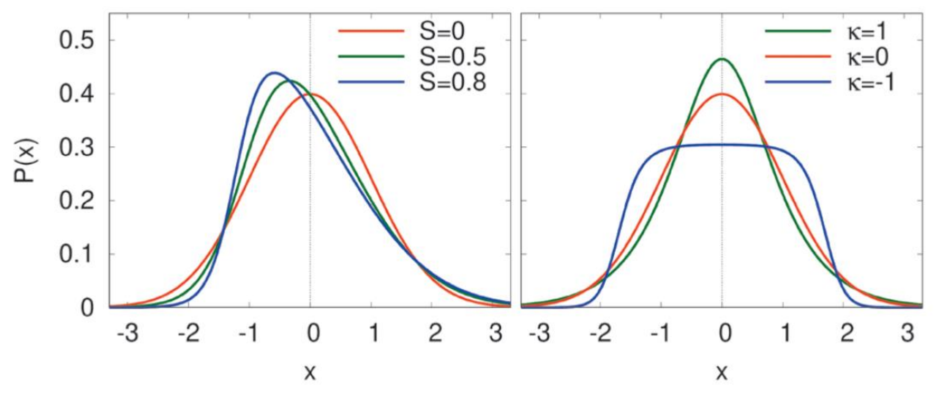
$$\begin{cases} \langle N\rangle_c = \langle N\rangle & \text{average} \\ \langle N^2\rangle_c = \langle \delta N^2\rangle & \text{variance} \\ \langle N^3\rangle_c = \langle \delta N^3\rangle \\ \langle N^4\rangle_c = \langle \delta N^4\rangle - 3\langle \delta N^2\rangle^2 \end{cases}$$

☐ skewness

$$S = \frac{\langle N^3\rangle_c}{\langle N^2\rangle_c^{3/2}}$$

☐ kurtosis

$$\kappa = \frac{\langle N^4\rangle_c}{\langle N^2\rangle_c^2}$$



☐ NOTE

- Gauss distribution: $\langle N^3\rangle_c = \langle N^4\rangle_c = \cdots = 0$
- Poisson distribution: $\langle N^2\rangle_c = \langle N^3\rangle_c = \langle N^4\rangle_c = \cdots = \langle N\rangle$

Review: Asakawa, MK, PPNP **90** (2016)

A Coin Game

① Bet 25 Euro

② You get head coins of

A. 50 x 1 Euro

B. 25 x 2 Euro

Same expectation value.

A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro

B. 25 x 2 Euro

C. 1 x 50 Euro

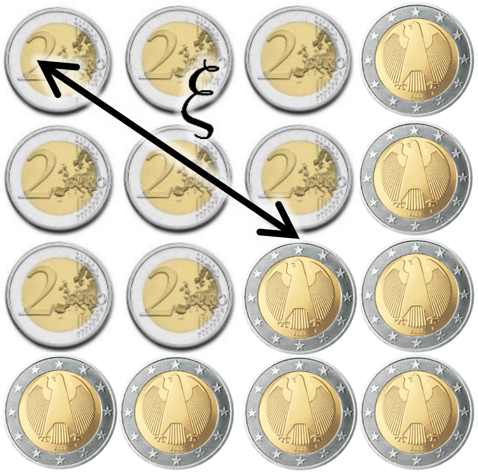
Same expectation value.
But, different fluctuations!

The noise is the signal.

R. Landauer
1998

Fluctuations in QCD Phase Diagram

QCD Critical point



Fluctuations **increase**

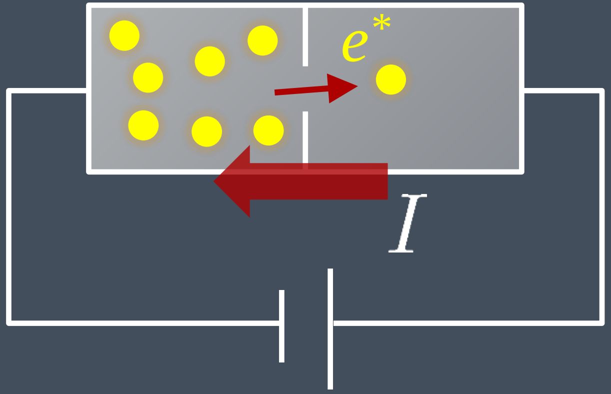
Stephanov, Rajagopal, Shuryak, 1998; 1999

Onset of QGP

Fluctuations **decrease**

Asakawa, Heinz, Muller, 2000;
Jeon, Koch, 2000

Shot Noise（電流雑音 ）



$$S_{\text{shot}} \sim \langle \delta I^2 \rangle$$

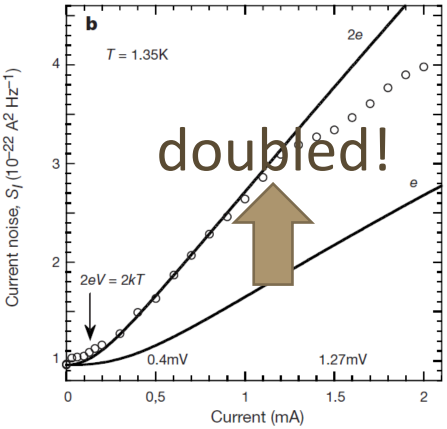
$$S_{\text{shot}} = 2e^* \langle I \rangle$$

charge of quasi-particles

Super-conductivity

$$e^* = 2e$$

Jehl+, Nature **405**,50 (2000)



Quantum Hall Effect

$$e^* = \frac{q}{p} e$$

Saminadayar+, PRL**79**,2526 (1997)

Higher orders:

3rd order: ex. Beenakker+, PRL90,176802(2003)

up to 5th order: Gustavsson+, Surf.Sci.Rep.**64**,191(2009)

Higher Order Cumulants

A. 50 x 1 Euro

B. 25 x 2 Euro

$$2\langle \delta\text{€}^2 \rangle = \langle \delta\text{€}^2 \rangle$$

$$4\langle \delta\text{€}^3 \rangle = \langle \delta\text{€}^3 \rangle$$

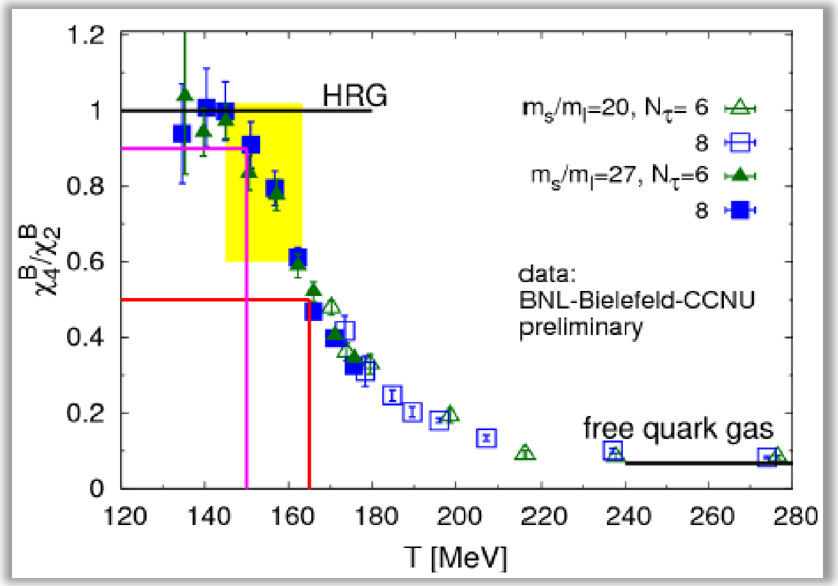
$$8\langle \text{€}^4 \rangle_c = \langle \text{€}^4 \rangle_c$$

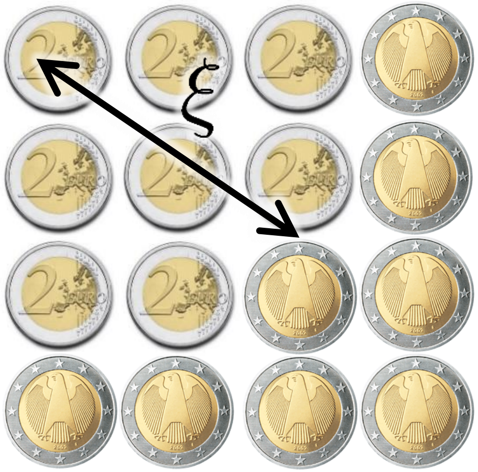
Asakawa, MK,
PPNP 90, 299
(2016)

Non-Gaussian Fluctuations

Onset of QGP

QCD Critical Point





Fluctuations **decrease**

Fluctuations **increase**

Ejiri, Karsch, Redlich, 2006

Stephanov, 2009

More amplified for higher order cumulants

Sign Change of Cumulant

Asakawa, Ejiri, MK, PRL '09

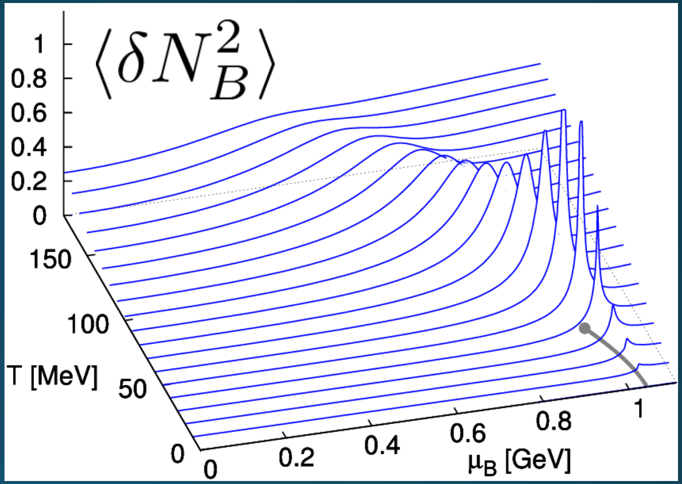
- **Geometric interpretation on the signs**

Fluctuations $\langle N_B^2 \rangle_c$ diverge at the QCD-CP.

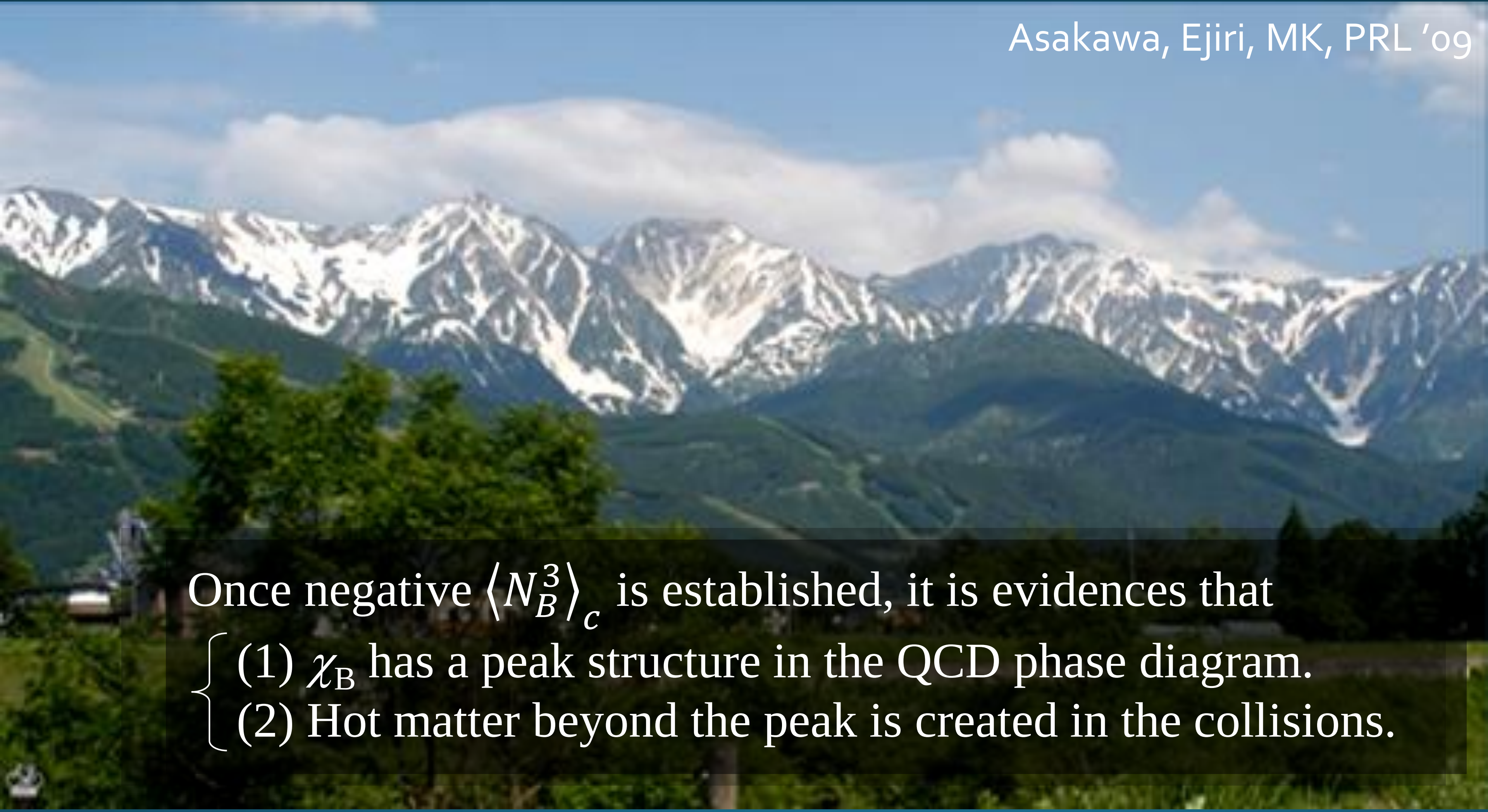
Themodynamic Relation

$$\langle N_{\rm B}^{m+1} \rangle_c = T \frac{\partial \langle N_{\rm B}^m \rangle_c}{\partial \mu_{\rm B}}$$

Sign of $\langle N_B^3 \rangle_c$ can distinguish near and away sides!



Impact of Negative Cumulants



Asakawa, Ejiri, MK, PRL ’09

Once negative $\langle N_B^3 \rangle_c$ is established, it is evidences that

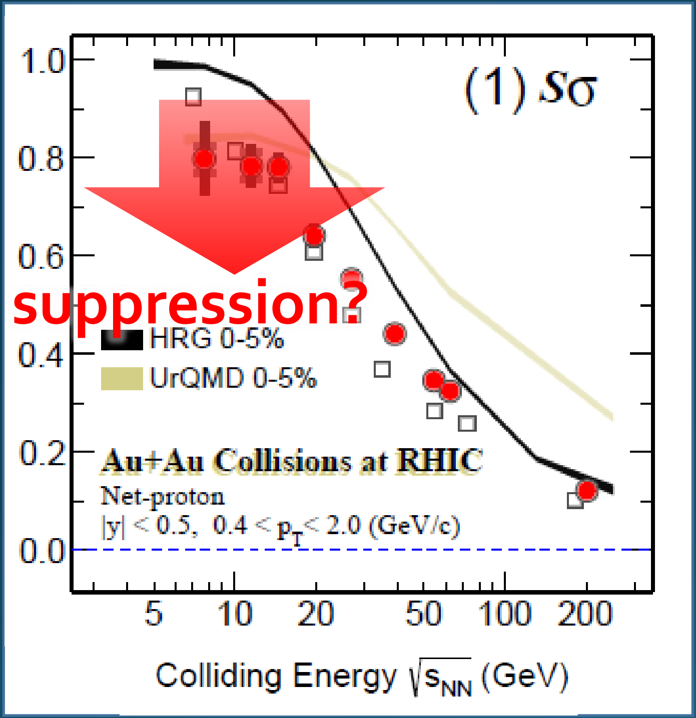
(1) χ_B has a peak structure in the QCD phase diagram.
(2) Hot matter beyond the peak is created in the collisions.

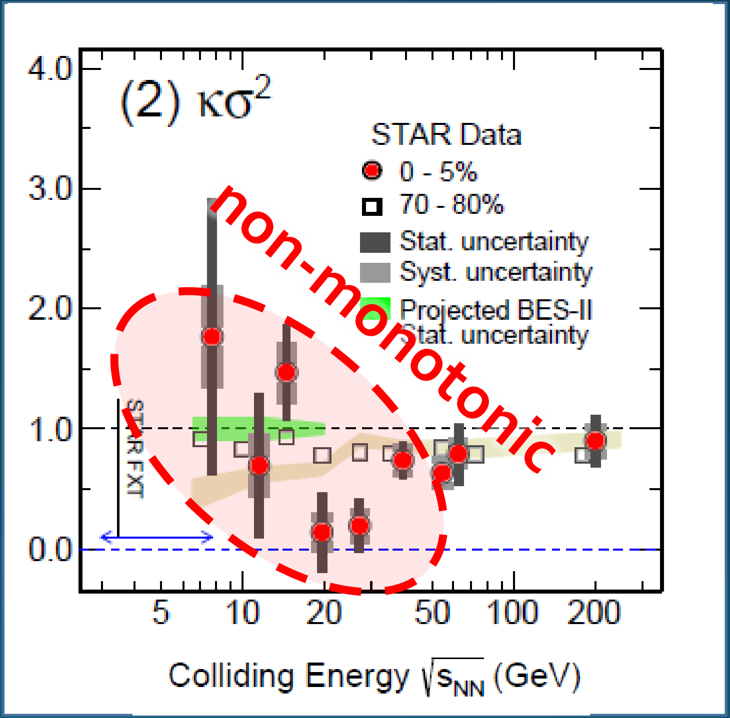
- **No** dependence on any specific models.
- Just the sign! **No** normalization (such as by N_ch).

Proton Number Cumulants

$\langle N_p^3 \rangle_c / \langle N_p^2 \rangle_c$

$\langle N_p^4 \rangle_c / \langle N_p^2 \rangle_c$





STAR, 2001.06419

- ☐ **Nonzero and non-Poissonian cumulants are experimentally established.**

Cumulants of Conserved Charges
=Observable on the Lattice

- Fluctuation-Response Relations

$$\langle N_B^m \rangle_c = V \chi_m^B$$

Thermal Fluctuation

Susceptibility

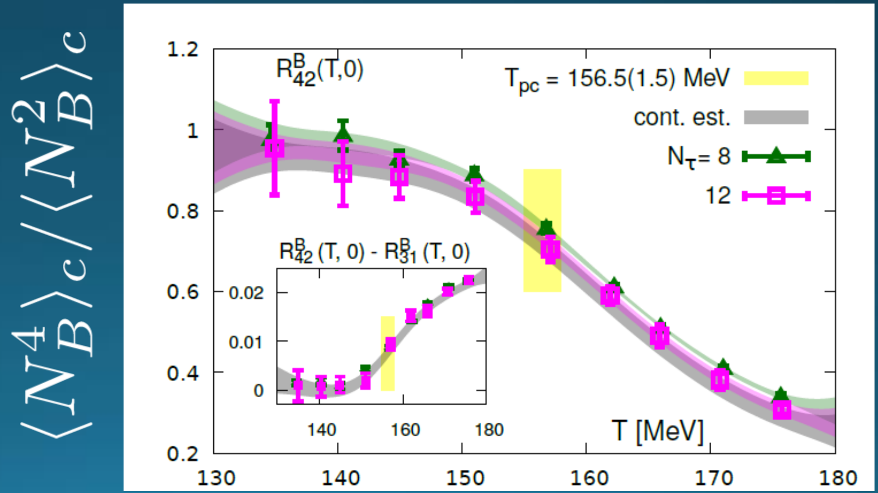
$$\chi_m^B \sim \frac{\partial^m p}{\partial \mu_B^m}$$

$$p(T,\mu) = p(T,0) + \frac{\chi_2}{2}\left(\frac{\mu}{T}\right)^2 + \cdots$$

- Volume dependence canceled out in ratios

Ejiri, Karsch, Redlich, '05

➡ useful for comparison w/ HIC



HotQCD, PRD**101** (2020)

under magnetic field:
Ding+, 2104.06843

Issues to be Resolved

- ☐ Experiments measure **proton** number cumulants, while lattice calculates **baryon**'s.

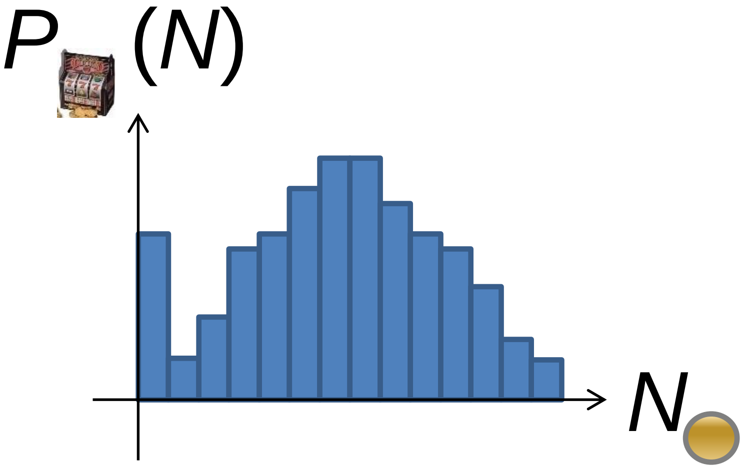
- ☐ Experiments measure **the final state of the dynamical evolution**, while the lattice measures an equilibrium state.

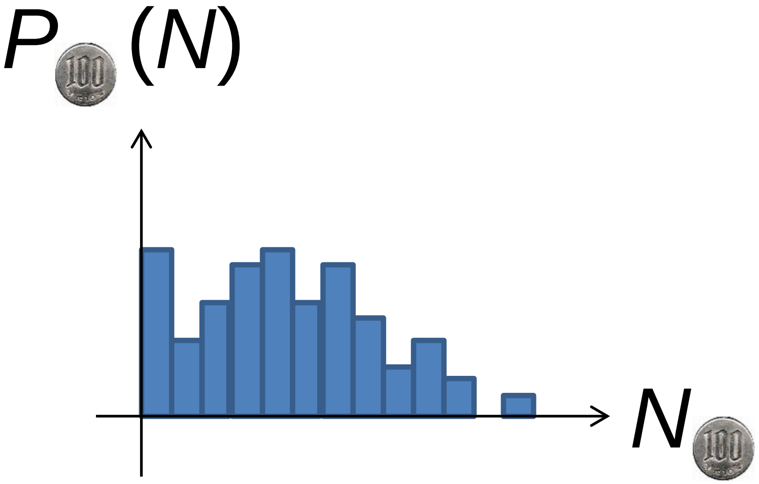
- ☐ And, other issues:
  - ☐ Volume fluctuation
  - ☐ Efficiency correction / imperfect acceptance
  - ☐ Measurement in momentum space
  - ☐ Resonance decays, Jets, …

More problematic for higher order cumulants!

See, Asakawa, MK, Mueller, PRC 101 (2020)

Slot Machine Analogy

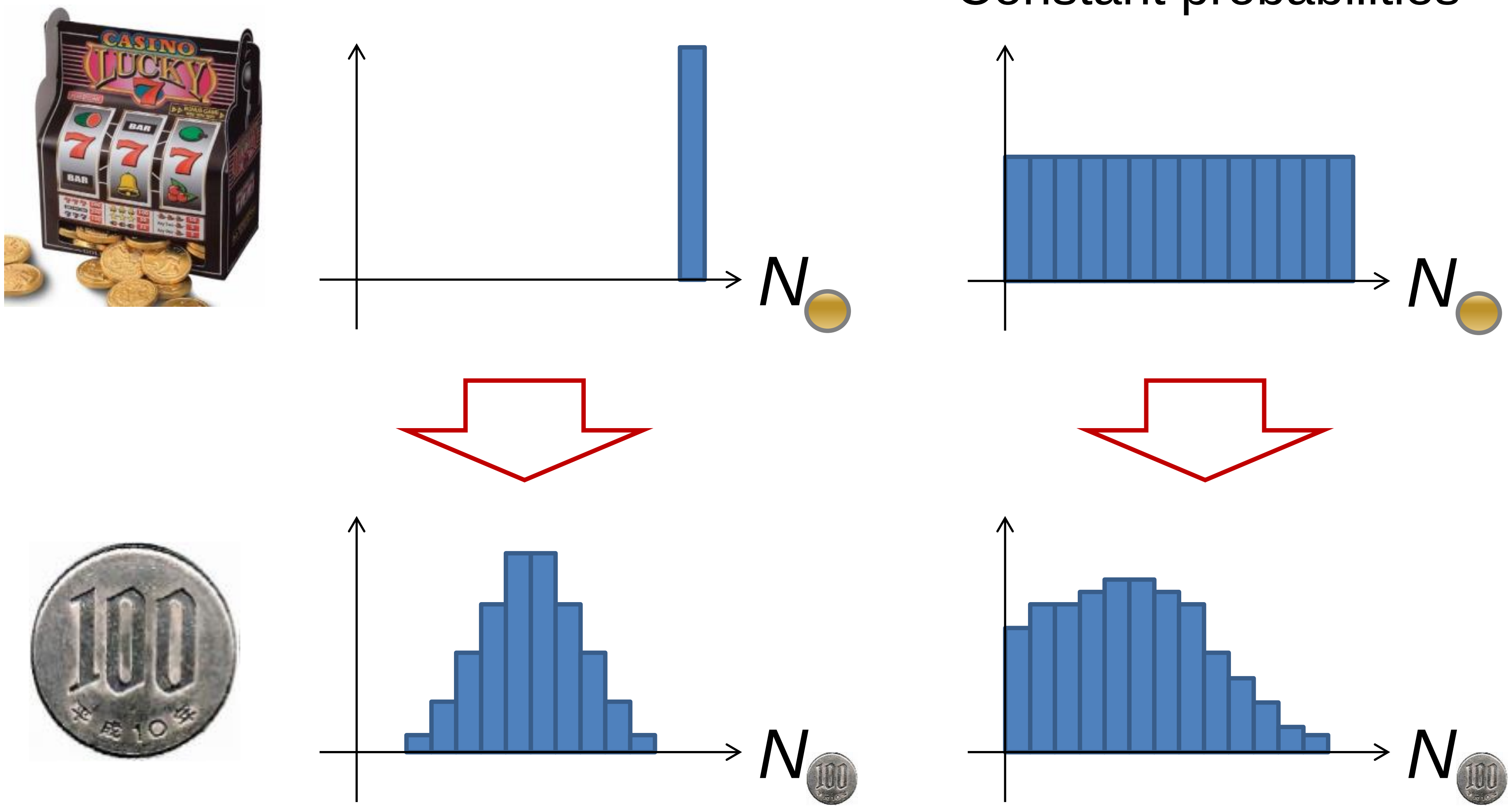




Slot Machine Analogy

Fixed # of coins

Constant probabilities



The Binomial Model

MK, Asakawa, 2012; 2012
Bzdak, Koch, 2012

When efficiency for individual particles are **independent**

$$P_{\rm obs}(n) = \sum_N B_p(n;N)P(N)$$

dist. func. of
observed particle #

binmial
dist. func.

dist. func. of
original particle #

The cumulants connected with each other

$$\langle n^m \rangle_{\rm c} \longleftrightarrow \langle N^m \rangle_{\rm c}$$

Caveat: Effects of nonvanishing correlations: Holtzman+ 2016

Proton vs Baryon Cumulants

MK, Asakawa, 2012; 2012

- ☐ $\langle N_p^m \rangle_c \neq \langle N_B^m \rangle_c$
- ☐ $\langle N_B^m \rangle_c$ can be obtained from the distribution of N_p thanks to the isospin randomization.

$N_B \to N_p$:

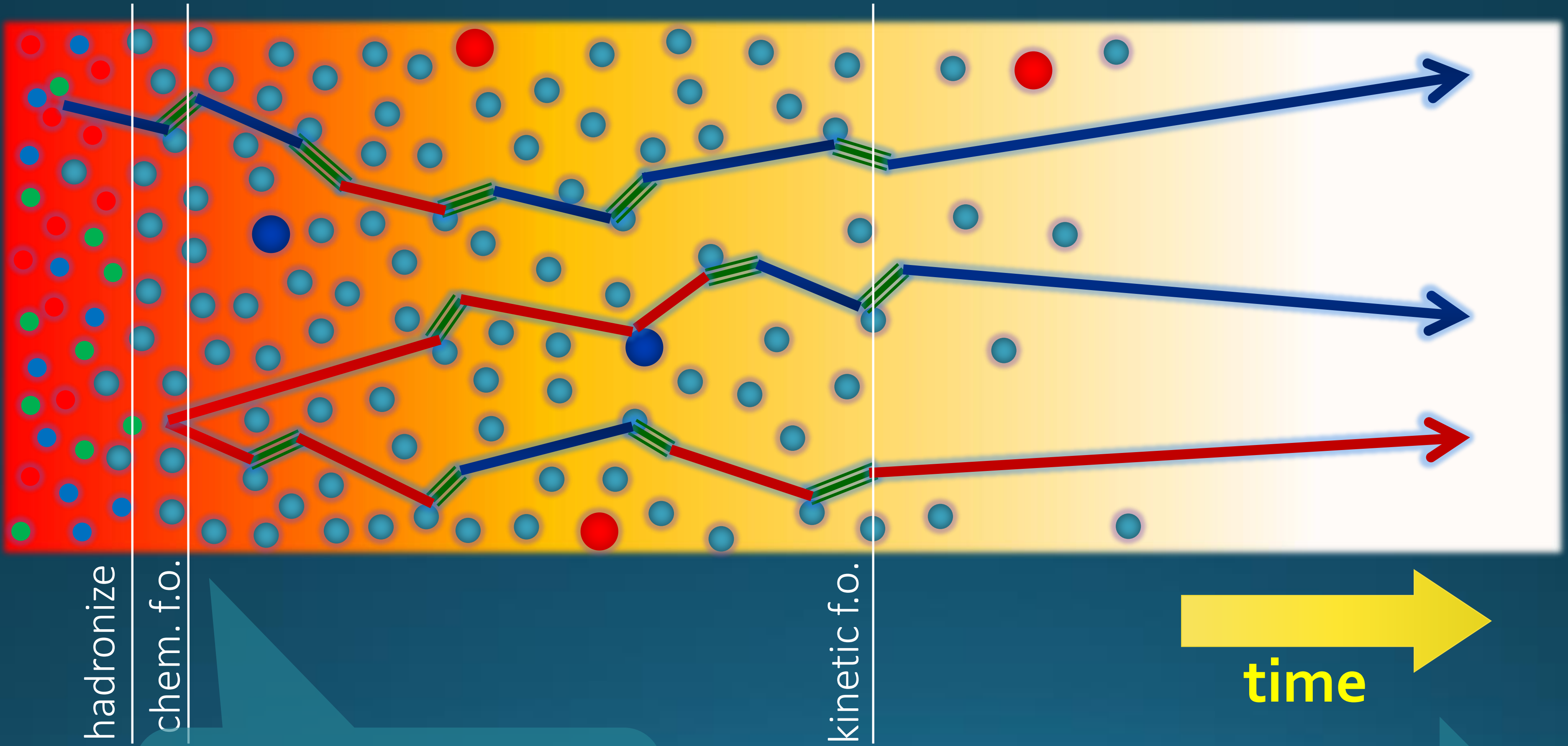
$$\begin{aligned}
\langle N_p^{(\mathrm{net})} \rangle &= \tfrac{1}{2} \langle N_{\mathrm{B}}^{(\mathrm{net})} \rangle, \\
\langle (\delta N_p^{(\mathrm{net})})^2 \rangle &= \tfrac{1}{4} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^2 \rangle + \tfrac{1}{4} \langle N_{\mathrm{B}}^{(\mathrm{tot})} \rangle, \\
\langle (\delta N_p^{(\mathrm{net})})^3 \rangle &= \tfrac{1}{8} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^3 \rangle + \tfrac{3}{8} \langle \delta N_{\mathrm{B}}^{(\mathrm{net})} \delta N_{\mathrm{B}}^{(\mathrm{tot})} \rangle,
\end{aligned}$$

Information of baryon # cumulants are more suppressed in higher order proton # cumulants!

$N_p \to N_B$:

$$\begin{aligned}
\langle N_{\mathrm{B}}^{(\mathrm{net})} \rangle &= 2 \langle N_p^{(\mathrm{net})} \rangle, \\
\langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^2 \rangle &= 4 \langle (\delta N_p^{(\mathrm{net})})^2 \rangle - 2 \langle N_p^{(\mathrm{tot})} \rangle, \\
\langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^3 \rangle &= 8 \langle (\delta N_p^{(\mathrm{net})})^3 \rangle - 12 \langle \delta N_p^{(\mathrm{net})} \delta N_p^{(\mathrm{tot})} \rangle + 6 \langle N_p^{(\mathrm{net})} \rangle,
\end{aligned}$$

Diffusion of Non-Gaussian Fluc.

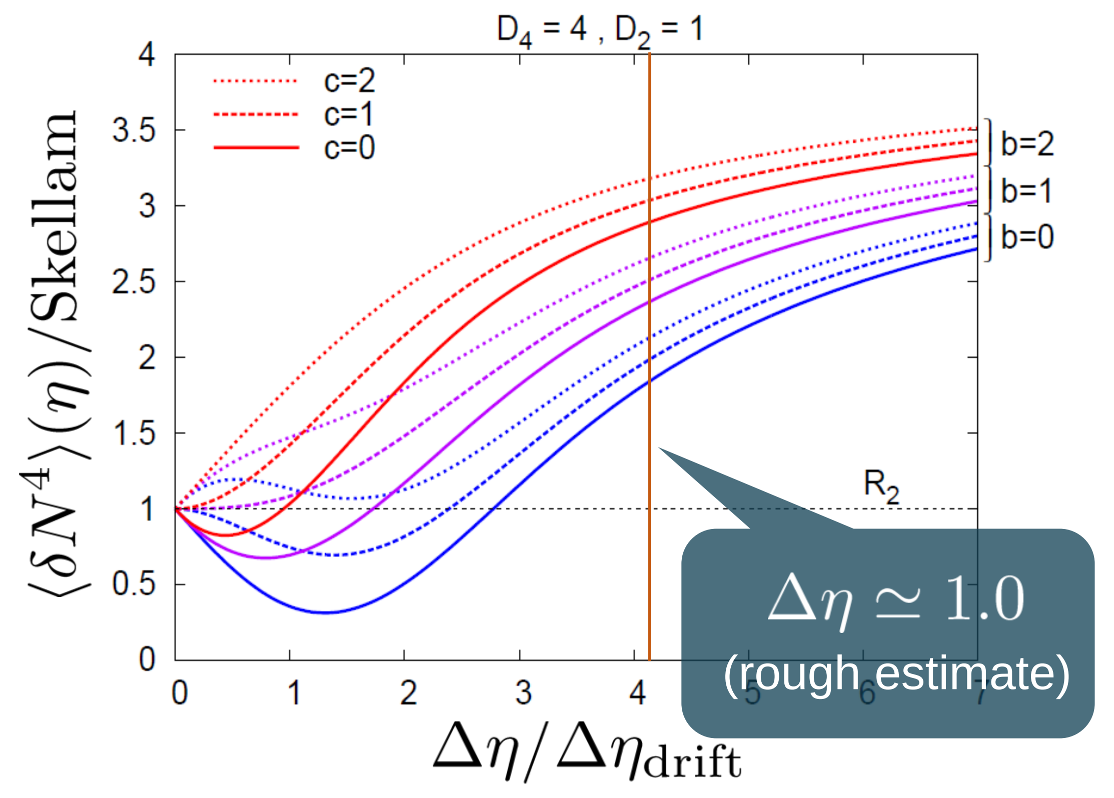


We want to see fluctuations around phase transition

But, fluctuations are modified due to diffusion before observation

Rapidity Window dependence
as a Result of Diffusion

MK+ (2014); MK (2015)



$\Delta\eta \simeq 1.0$

(rough estimate)

Parameters

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

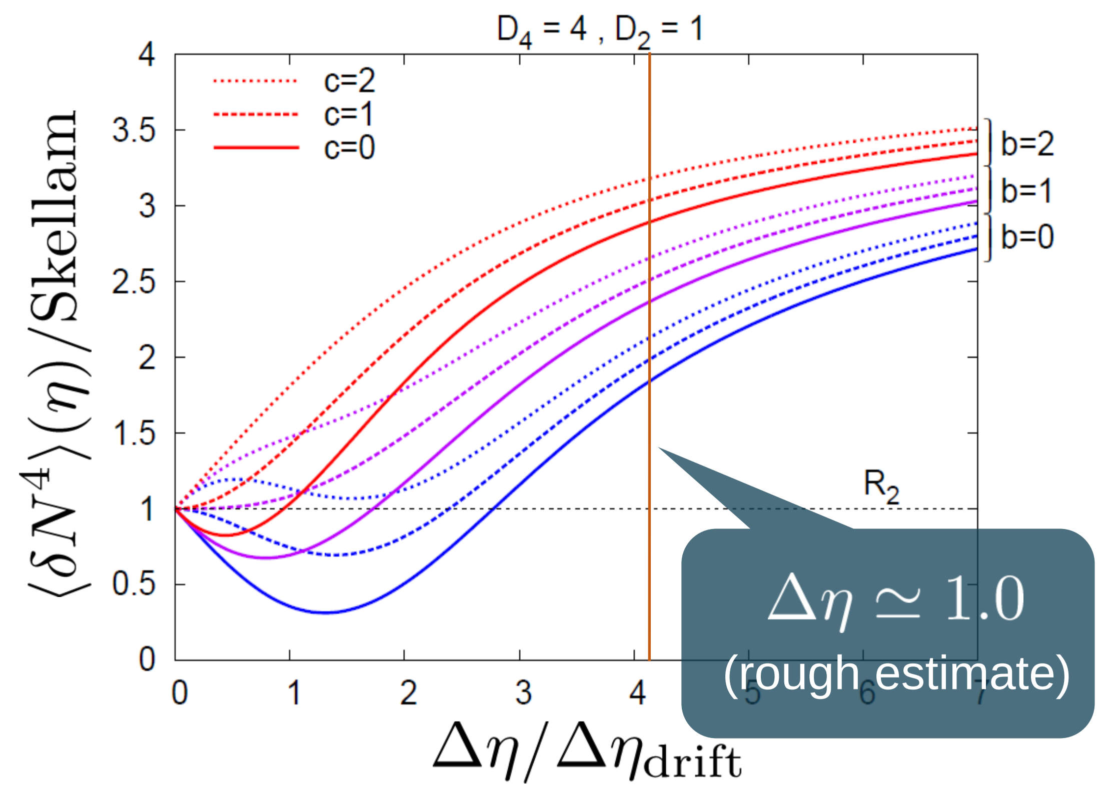
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

☐ Higher order cumulants can behave non-monotonically.

☐ Δη dependence encodes history of time evolution.

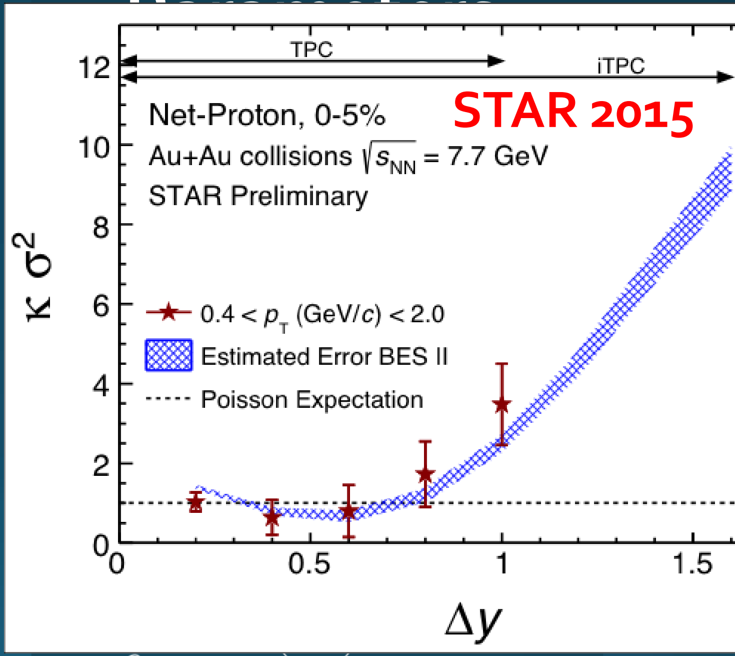
Rapidity Window dependence
as a Result of Diffusion

MK+ (2014); MK (2015)



$\Delta\eta \simeq 1.0$

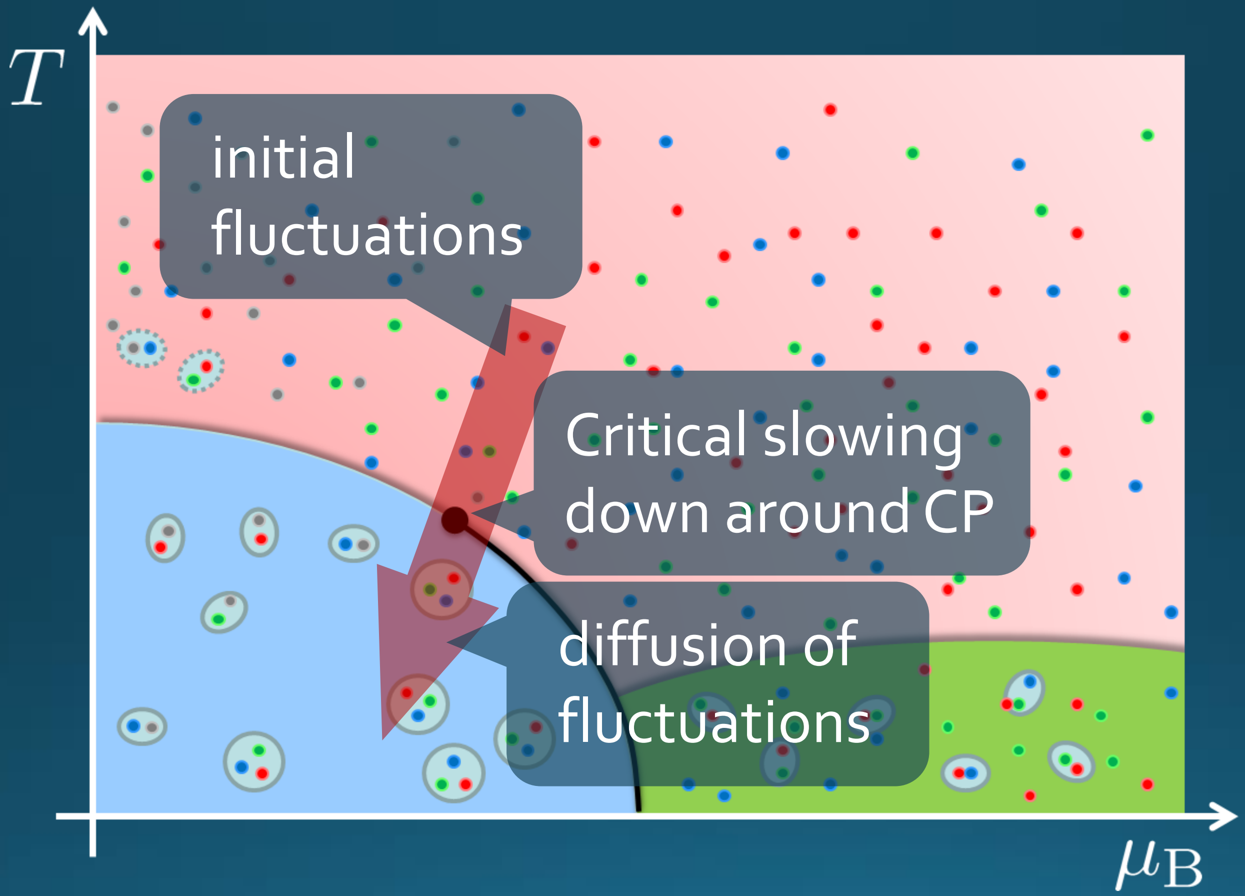
(rough estimate)



$c = \frac{\quad}{\langle Q_{(\mathrm{tot})} \rangle}$

- ☐ Higher order cumulants can behave non-monotonically.
- ☐ Δη dependence encodes history of time evolution.

Time Evolution of Cumulants



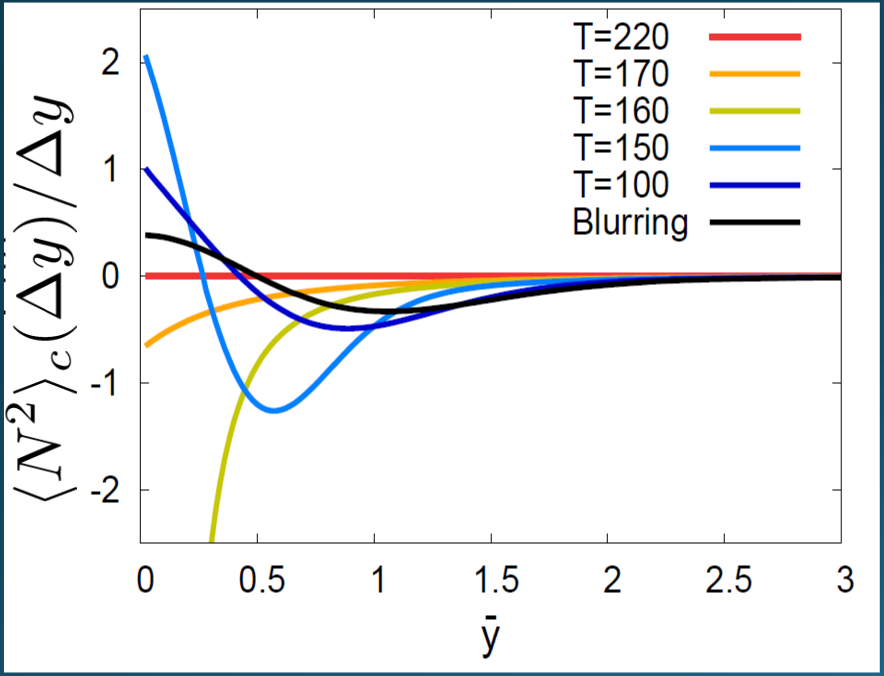
Proper understanding of the time evolution of fluctuations is indispensable.

Rapidity Window Dependence
around QCD-CP in Stochastic Models

☐ **2^{nd} order cumulant**

analytic solution

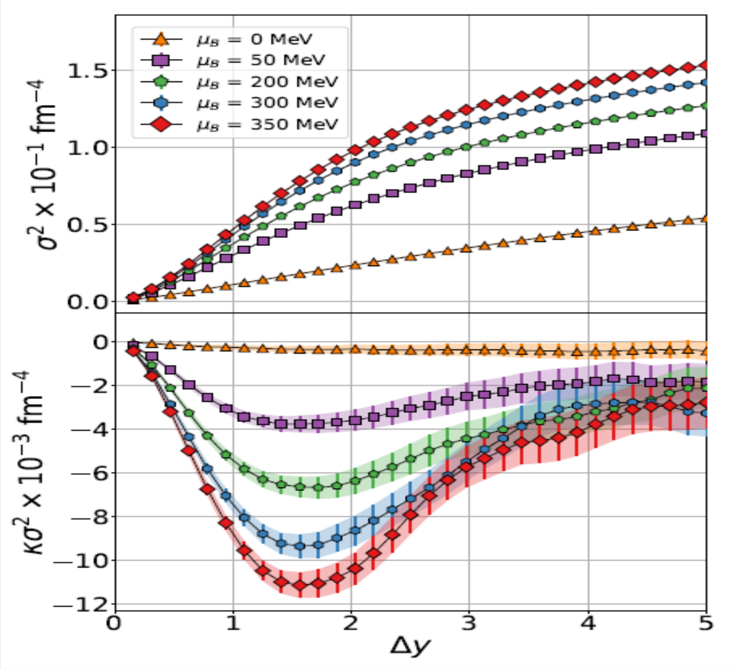
sakaida, Asakawa, Fujii, MK, 2018



☐ **higher order cumulants**

numerical implementation

Pihan+, 2205.12834



☐ Non-monotonic Δy dependence can emerge reflecting the dynamical evolution.

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2. Search for QCD-CP: Fluctuations

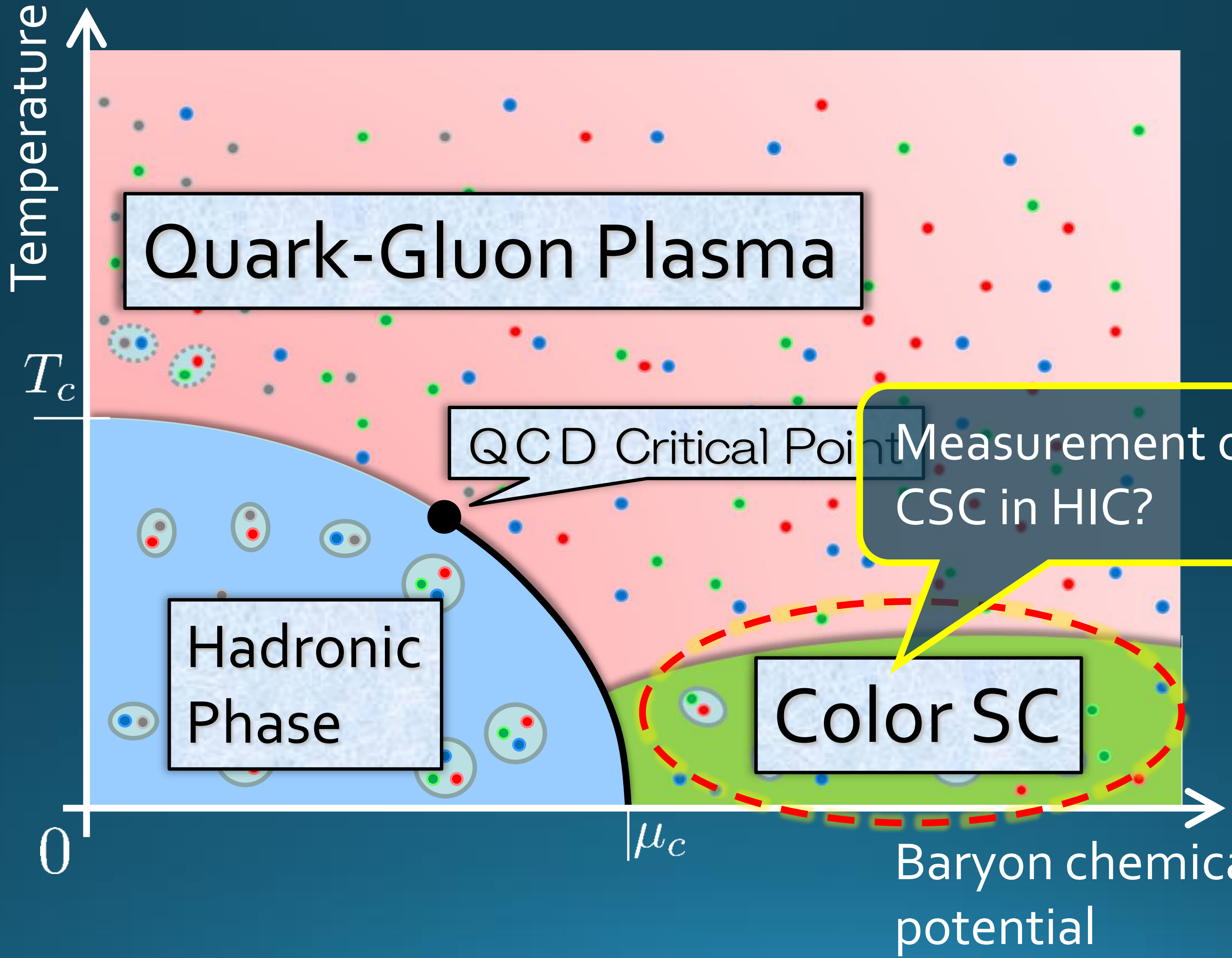
3. Search for CSC: Precursor of CSC

Nishimura, MK, Kunihiro, arXiv:2201.01963

Search for **CSC Phase Transition** using dilepton production rates

Nishimura, MK, Kunihiro, arXiv:2201.01963

Color Superconductivity



Color Superconductivity

☐ Cooper Instability

- Fermi particles ＋ attractive interaction @ Fermi surface

➔**Superconductivity at sufficiently low *T***

☐ Dense Quark Matter

- Quarks = fermions
- Attractive int. by gluon exchange

➔**Cold quark matter is a superconducting state**

1998~1999

☐ Possible large diquark gap (Son,'99; ...)

☐ Various phases: 2SC, CFL, etc. (Alford+, '99; ...)

Variety of Color SC

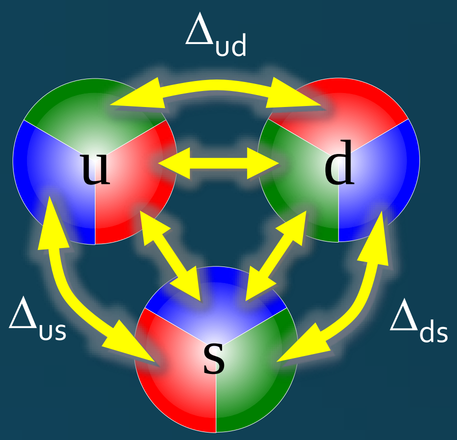
Various Cooper Pairs

☐ anti-symmetric（Pauli principle）

☐ attractive int. in color anti-symm. channel in 1-gluon exchange

☐ Scalar Cooper pairs

➔anti-symm. in flavor channel

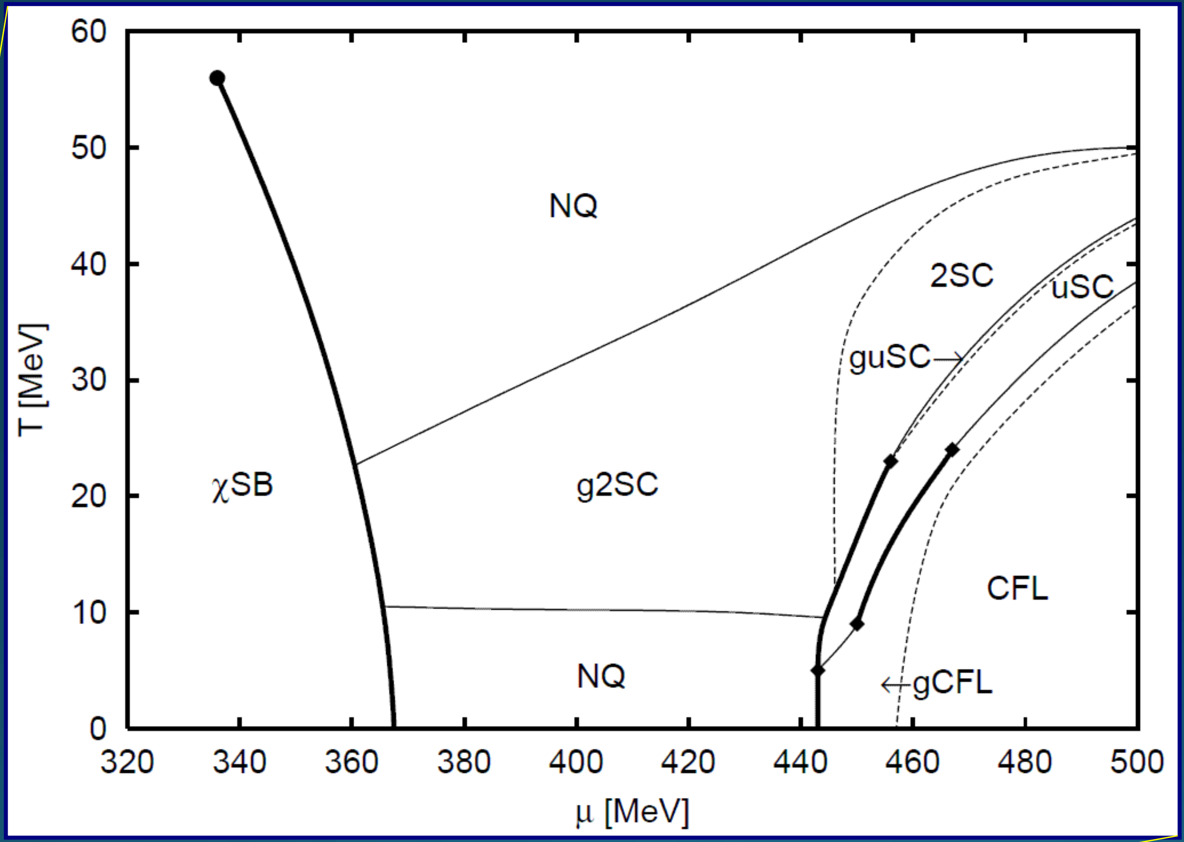


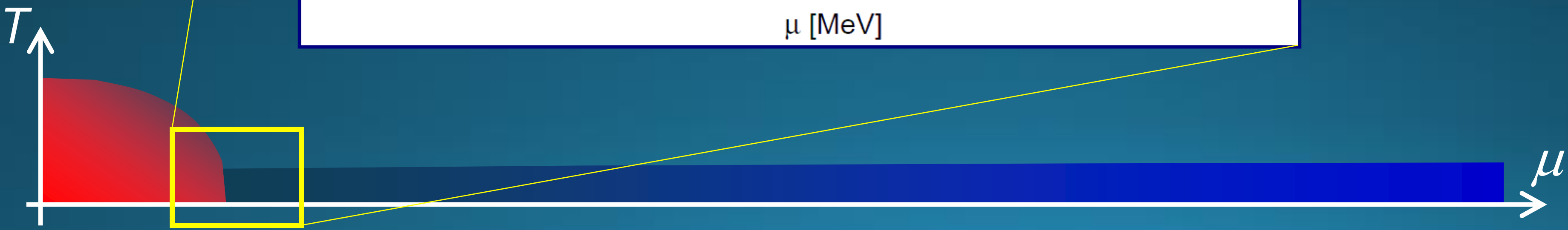
Various Color SC States

$\Delta_{ud}=\Delta_{us}=\Delta_{ds}>0$	CFL	Alford, et al. '98
$\Delta_{ud}>0, \Delta_{us}=\Delta_{ds}=0$	2SC	Bailin, Love '84
$\Delta_{ud}>0, \Delta_{us}>0, \Delta_{ds}=0$	uSC	Ruster, et al. '03
$\Delta_{ud}>0, \Delta_{ds}>0, \Delta_{us}=0$	dSC	Matsuura, et al., '04

Example of Phase Structure

Ruster+ (2005)

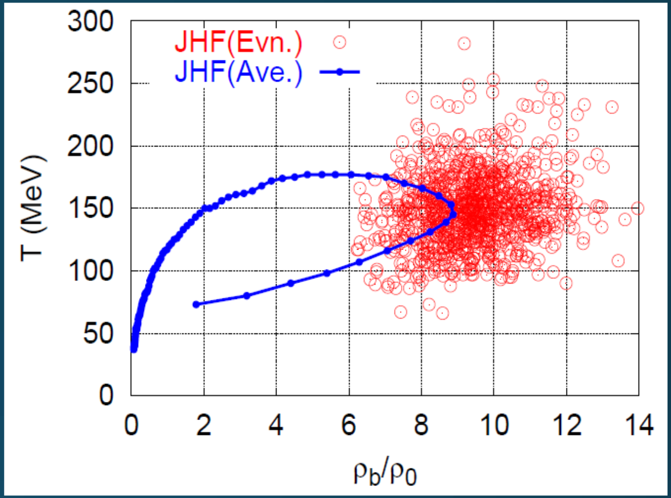




Observing CSC in HIC?

Difficulties

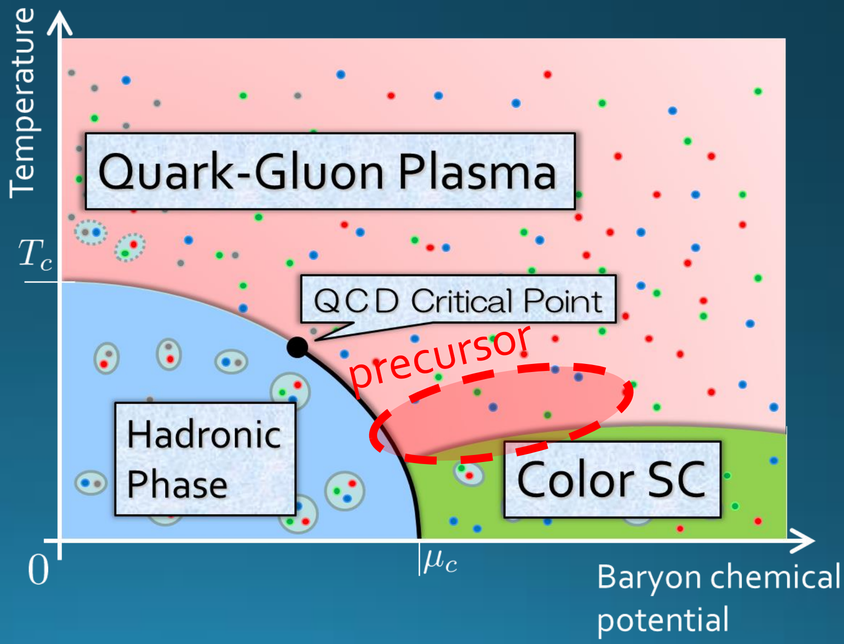
1) Creation of CSC itself would be impossible.
2) CSC would be realized only in the early stage.



Solution

1) Focus on **precursor of CSC**
2) Use **dilepton production**

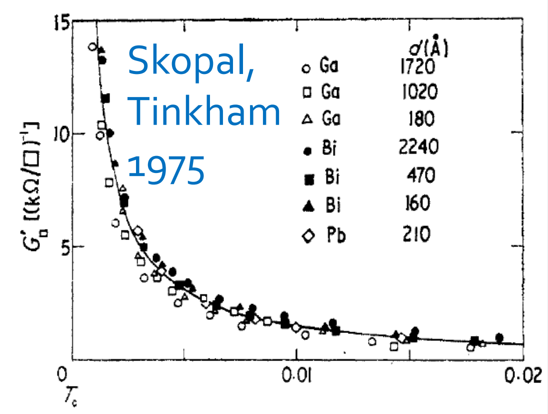
Nishimura, MK, Kunihiro, 2201.01963



Precursory Phenomena

= anomalous behavior of observables near but above T_c

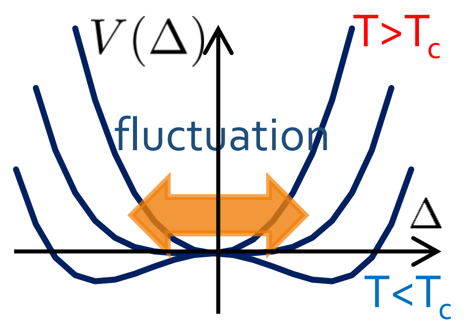
Electric conductivity in metals



Skopal, Tinkham 1975

$$\varepsilon = \frac{T - T_c}{T_c}$$

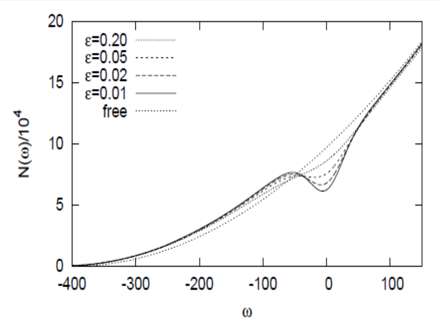
Landau's Free Energy



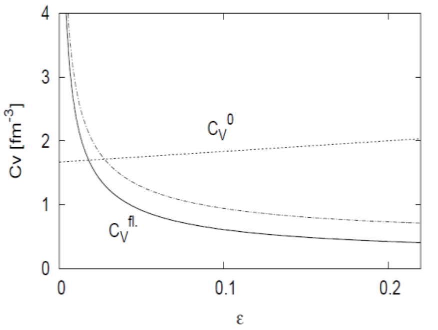
Precursor of CSC

- ☐ **Pseudogap** = Depression in the density of states
- ☐ Specific heat
- ☐ etc.

Density of States

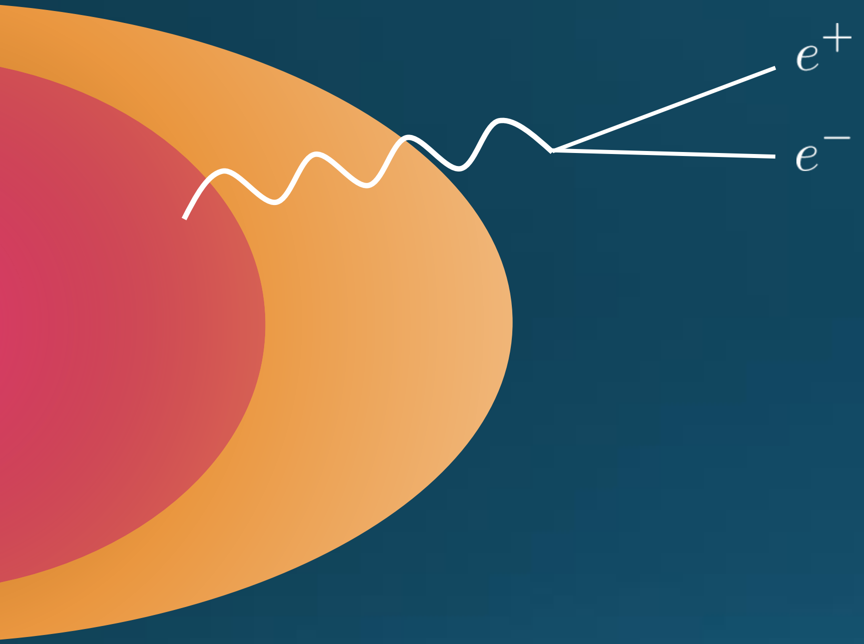


Specific Heat



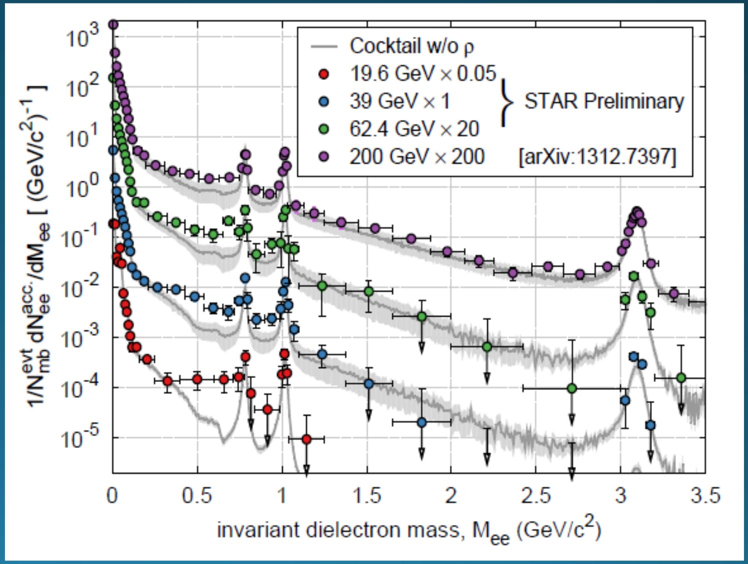
MK, Koide, Kunihiro, Nemoto, '03, '05

Dilepton Production

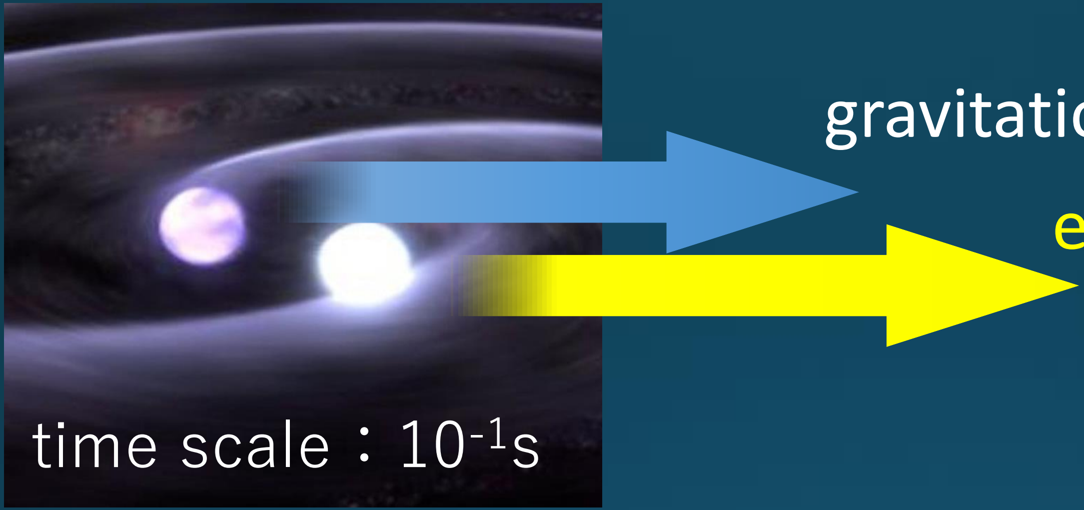


- Production rate of virtual photons
- Once virtual photons are produced, they almost penetrate the hot medium.
- Rate of the primordial medium is accessible.

- Real photons:
  $$\omega = p$$
- Virtual photons:
  $$\omega \neq p$$



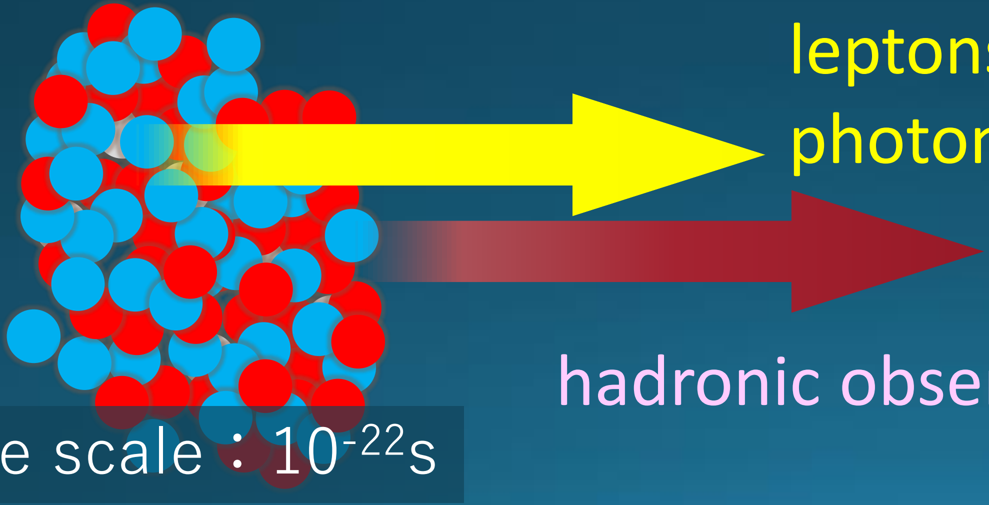
Multi-Messenger Observation



time scale : 10^{-1}s

gravitational waves

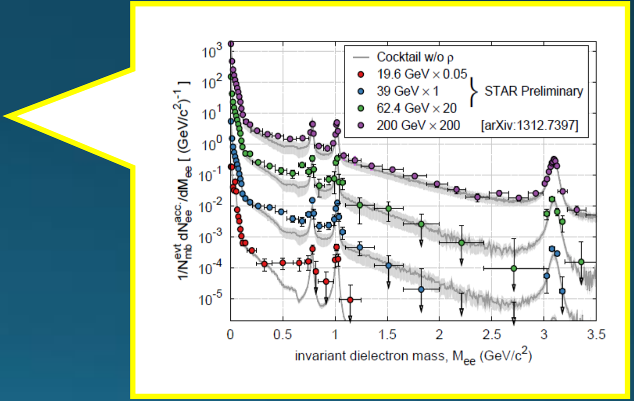
electromagnetic waves



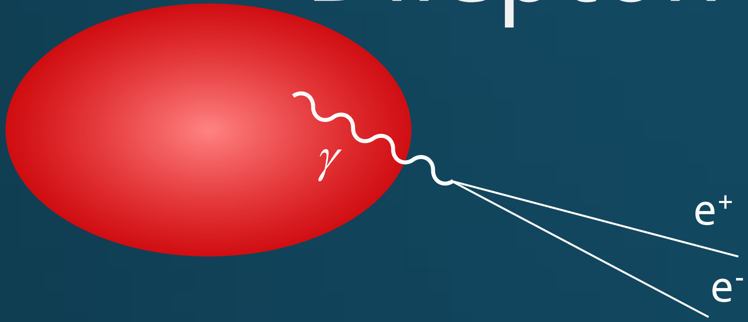
time scale : 10^{-22}s

leptons,
photons

hadronic observables



Dilepton Production 2



Pros:

- ☐ Once produced, they arrive at the detector directly
- ☐ Initial state can be observable

Cons:

- ☐ generation at all stages
- ☐ superposition of various signals

Model

NJL model (massless 2-flavor)

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\mathcal{L}_S = G_S \left((\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \tau \psi)^2 \right)$$

$$\mathcal{L}_C = G_C \left((\bar{\psi} i \gamma_5 \tau_A \lambda_A \psi^C)(\mathrm{h.c.}) \right)$$

diquark interaction

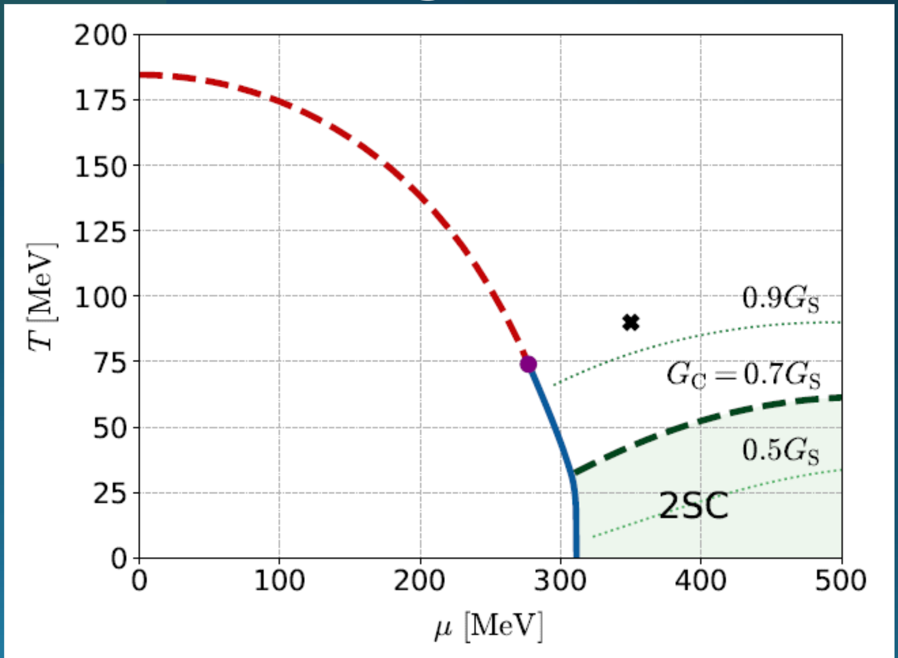
$$G_S = 5.01\ \mathrm{GeV}^{-2}, \quad \Lambda = 650\mathrm{MeV}$$

☐ G_C: free parameter

Order of CSC phase transition

Matsuura+('04), Giannakis+('04)
Noronha+('06), Fejos, Yamamoto('19)

Phase Diagram in MFA

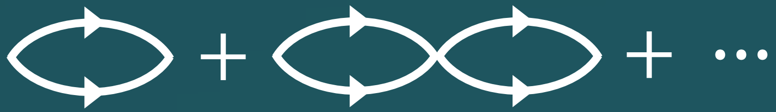


Diquark Mode

Diquark Propagator (T-matrix)

in random-phase approximaion

$$\Xi^R(\boldsymbol{k},\omega) =$$

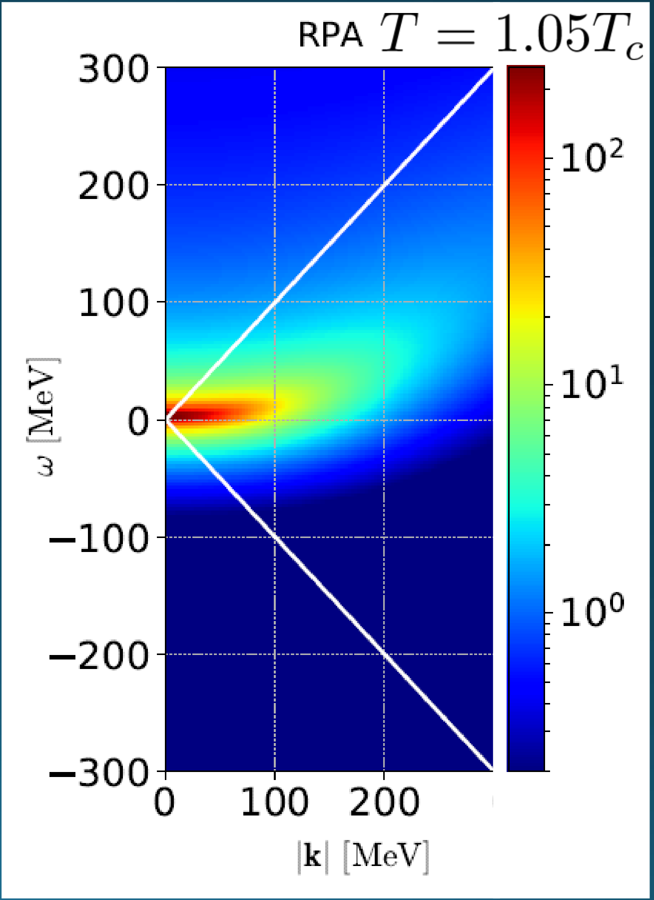


$$= \frac{Q^R(\boldsymbol{k},\omega)}{1+G_C Q^R(\boldsymbol{k},\omega)}$$

- ☐ Massless at $T=T_c$ as a soft mode of CSC transition
- ☐ Strength in the space-like region

MK, Koide, Kunihiro, Nemoto, '01,'05

Dynamical Structure Factor

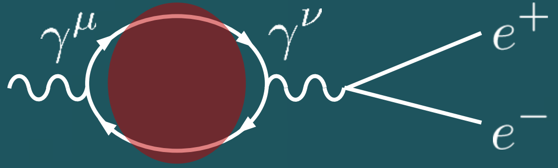


Photon Self-Energy

Dilepton Production Rate

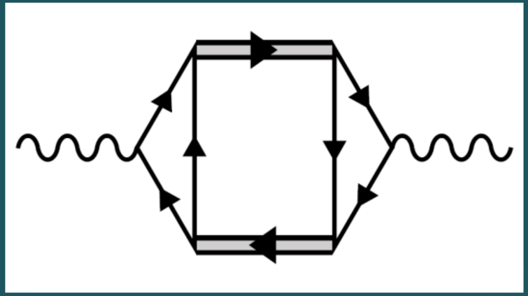
$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\beta\omega-1}}\mathrm{Im}\Pi^{R\mu}_{\mu}(k)$$

$$\Pi^{\mu\nu}(k) =$$

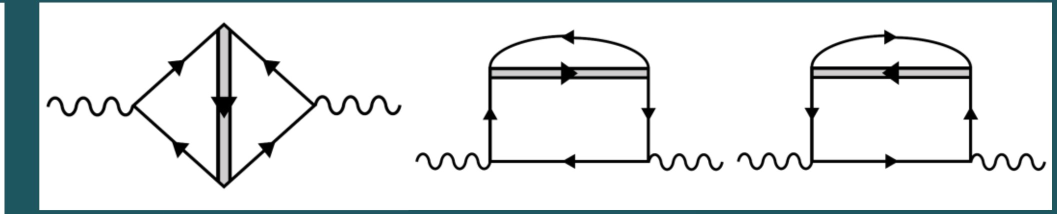


Terms included in $\Pi^{\mu\nu}$

Aslamasov-Larkin



Maki-Thompson, Density of states

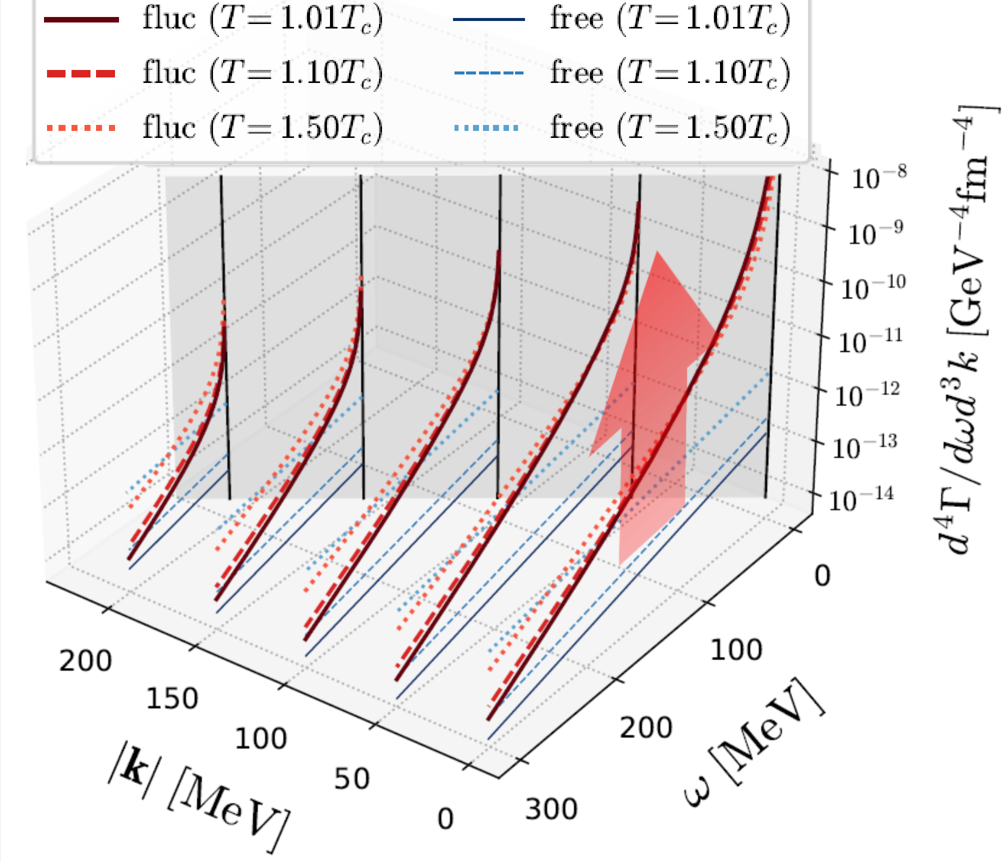


═▶═ = diquark propagator

- ☐ common in metallic superconductors (conductivity)
- ☐ time-dependent GL approx. for diquark field
- ☐ gauge-invariant construction

Nishimura, MK, Kunihiro, 2201.01963

Dilepton Production Rates



$$\frac{d^4\Gamma}{dk^4}(\omega, k)$$

Red: fluctuation contribution
Blue: free quarks

$\mu = 350\ \mathrm{MeV}$

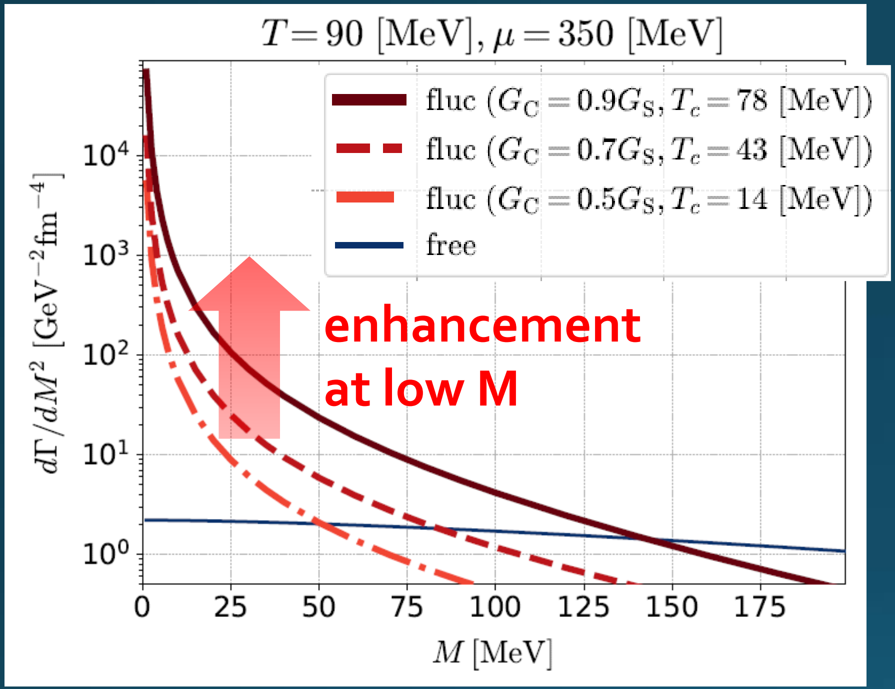
$G_C = 0.7 G_S, \quad (T_c \simeq 43\ \mathrm{MeV})$

Nishimura, MK, Kunihiro,
2201.01963

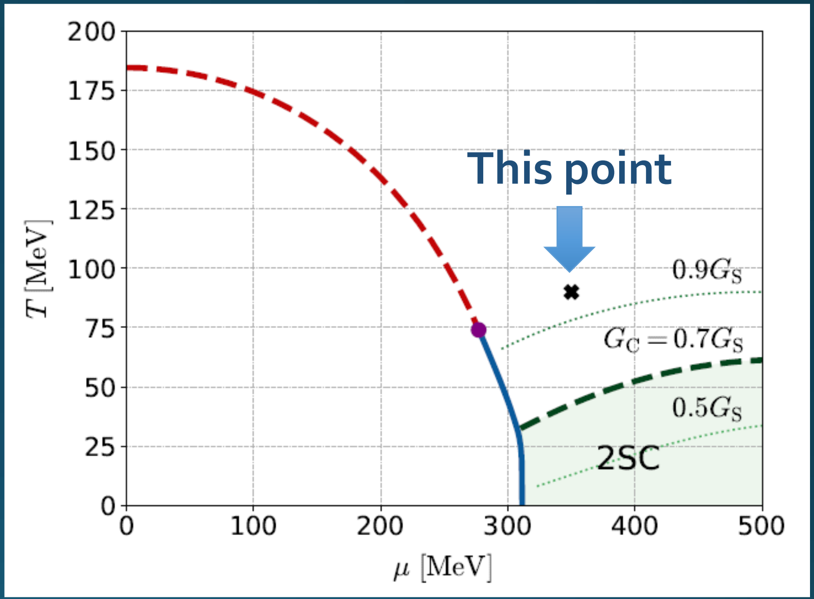
- ☐ Diquark fluctuations give rise to anomalous enhancement in the low energy-momentum region for $T < 1.5T_c$.

Invariant-Mass Spectrum

Fixed Temperature



$$\frac{d\Gamma}{dM^2} = \int \frac{d^3k}{2\omega}\frac{d^4\Gamma}{dk^4}\Big|_{\omega=E_k}$$



☐ Strong enhancement at low invariant mass, though the range of M is narrower than the previous results.

☐ **Observable in the HIC?**

Nishimura, MK, Kunihiro, 2201.01963

Summary

- ☐ Exploring dense medium in relativistic heavy-ion collisions is a hot topic in this field. The beam-energy scan is ongoing, and many new experiments will start in the near future!

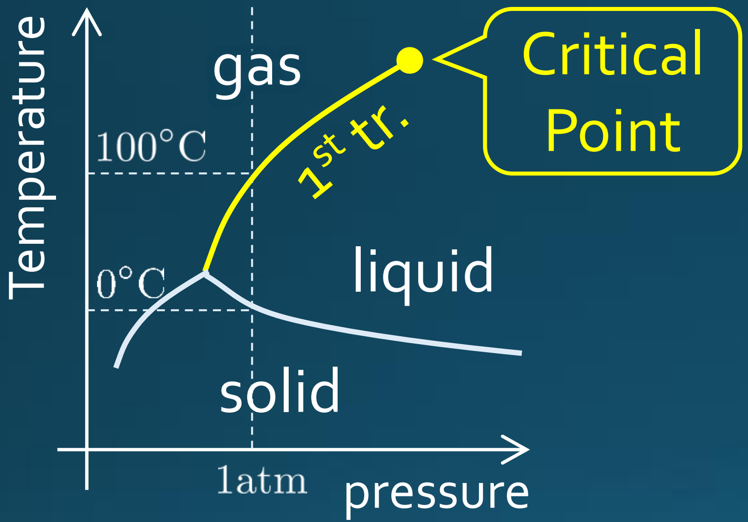
- ☐ Among them, search for
  - ☐ QCD critical point using fluctuation observables
  - ☐ color superconductivity using dileptons

  are especially interesting and important.

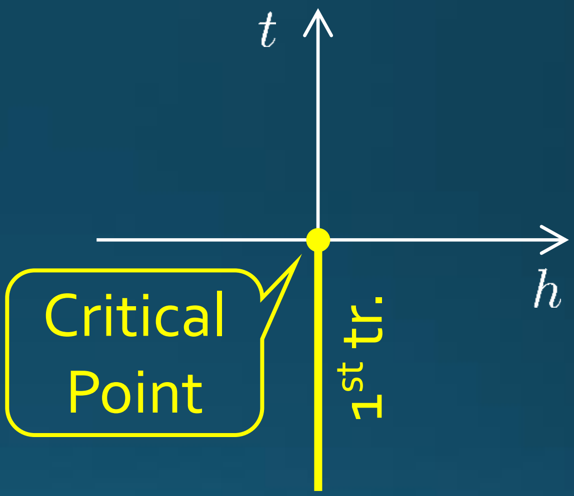
- ☐ Heavy-ion collisions at J-PARC will play important roles in pursuing these subjects.

Critical Points

☐ **Water**



☐ **Ising Model**

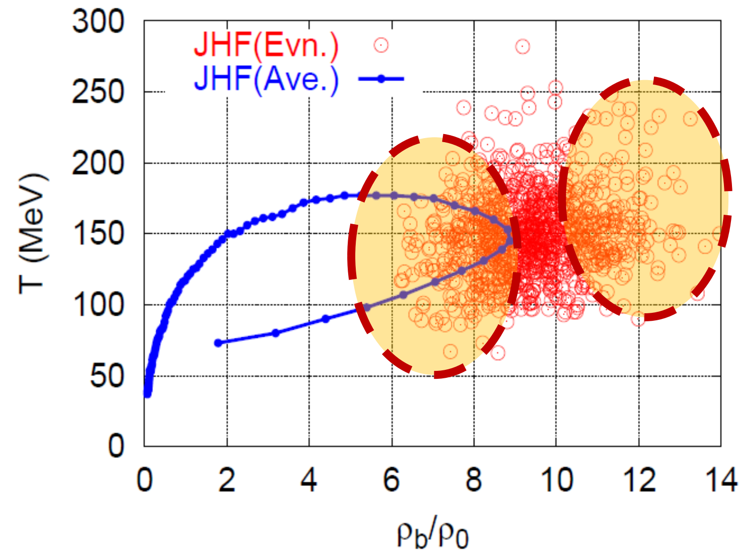


These CPs belong to the same universality class (Z_2).

→ Common critical exponents.

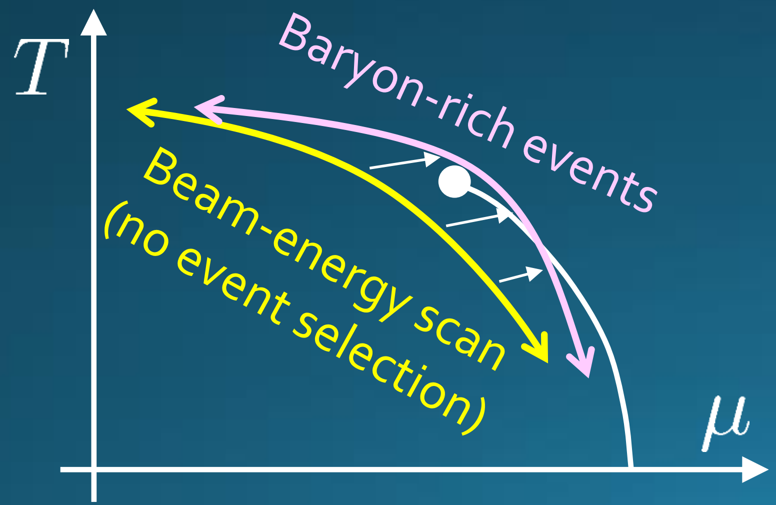
ex. $C \sim (T - T_c)^{-\alpha}$

Maximum Density Scan?



Large event-by-event fluctuations even with fixed centrality.

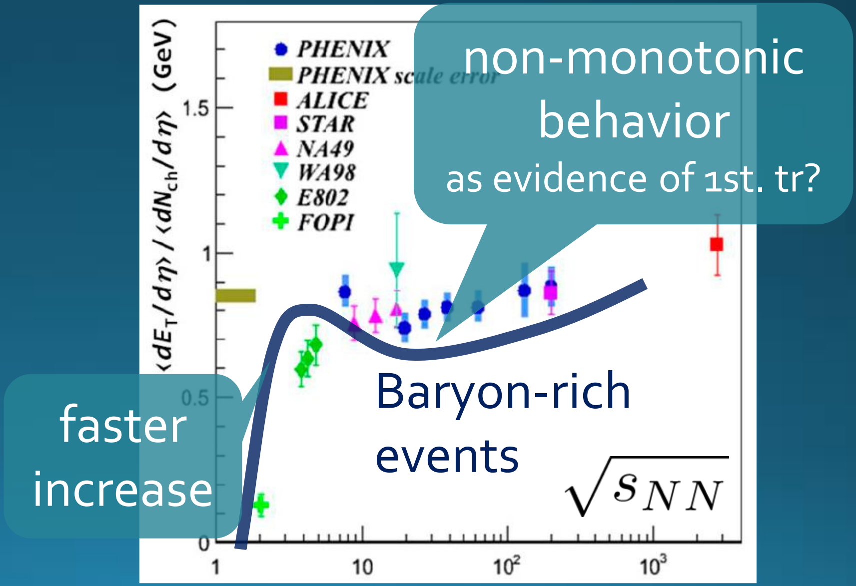
"Maximum density" dependence may be studied experimentally.



Baryon-rich events

Beam-energy scan (no event selection)

average transverse energy



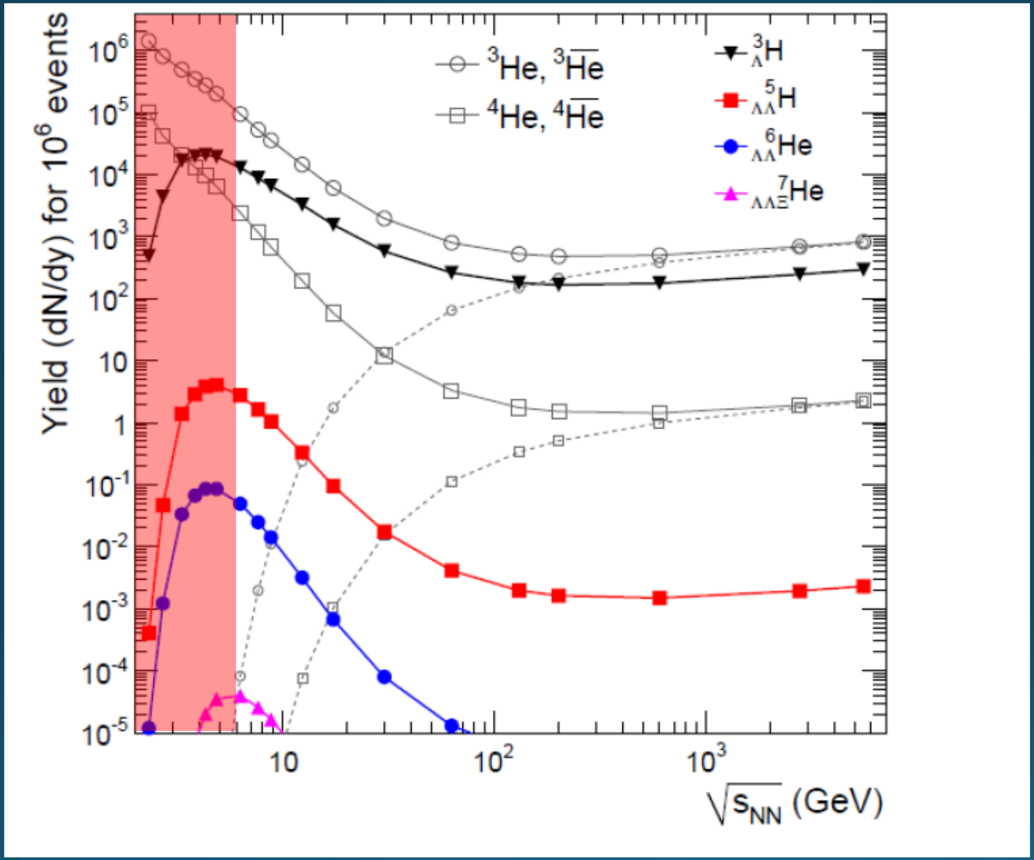
non-monotonic behavior

as evidence of 1st. tr?

faster increase

Baryon-rich events

Strangeness Factory



Particle yields having strangeness have maximum at J-PARC energy

Various Observables

- Flow
- Dilepton / photon
- Fluctuations, higher-order cumulants
- Ξ, Ω, …

- Sophisticated event selections
- Various correlations