Critical Points in Hot and Dense QCD

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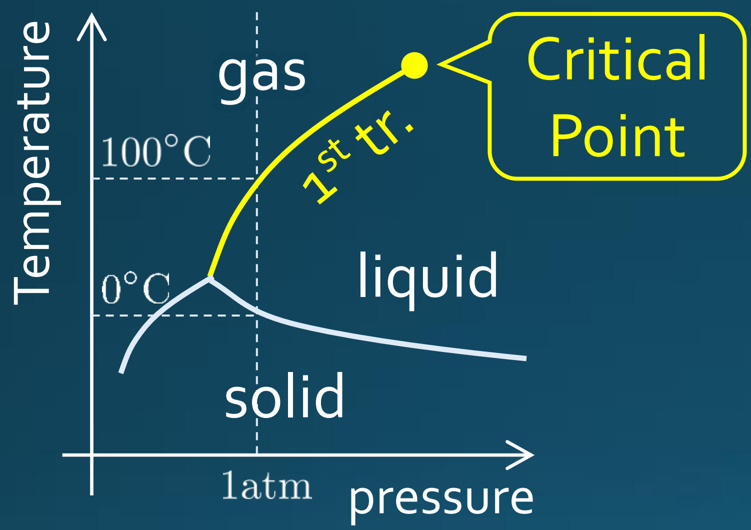
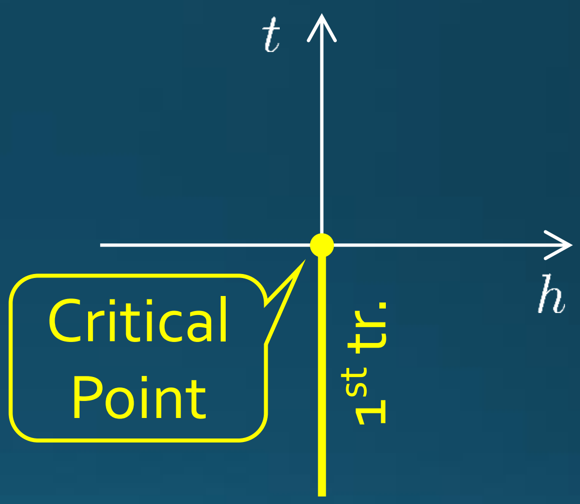
The 15th Asia Pacific Physics Conference, 2022/8/25, online (Korea)

Critical Points

☐ Water



☐ Ising Model

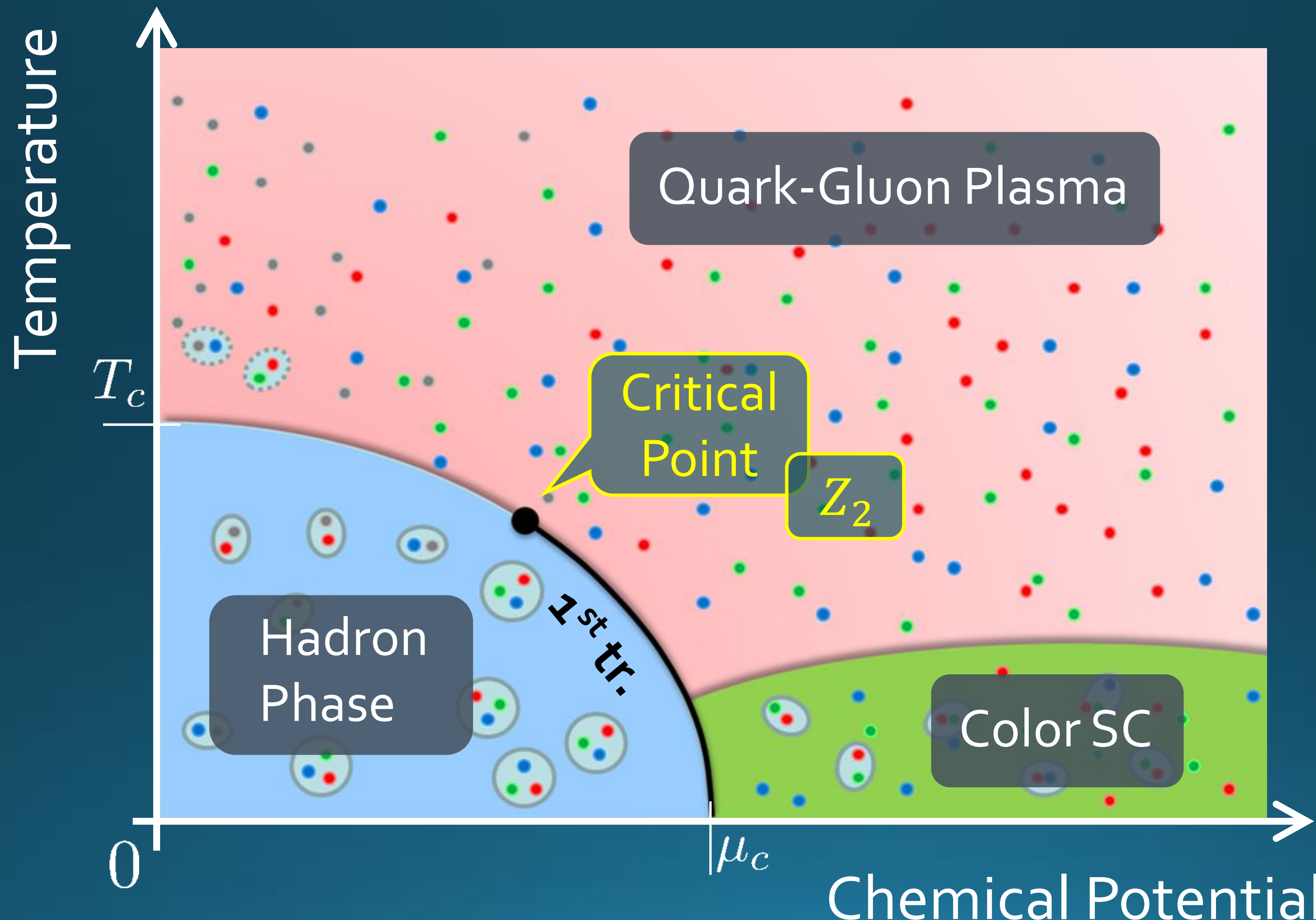


The CP is a singular point.
Divergence of thermodynamic quantities.

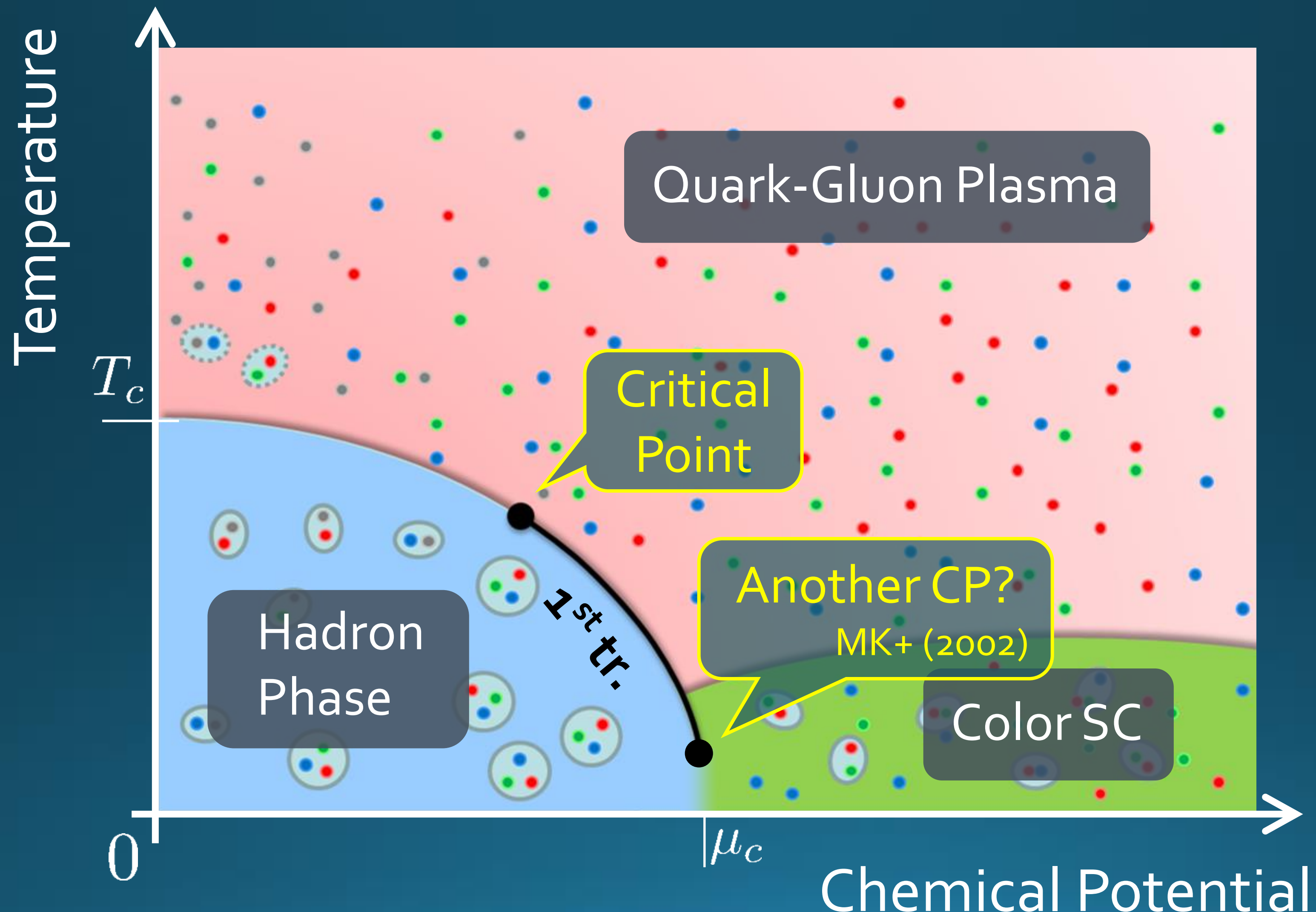
These CPs belong to the same universality class (Z_2).

➡ Common critical exponents. Ex. $C \sim (T - T_c)^{-\alpha}$

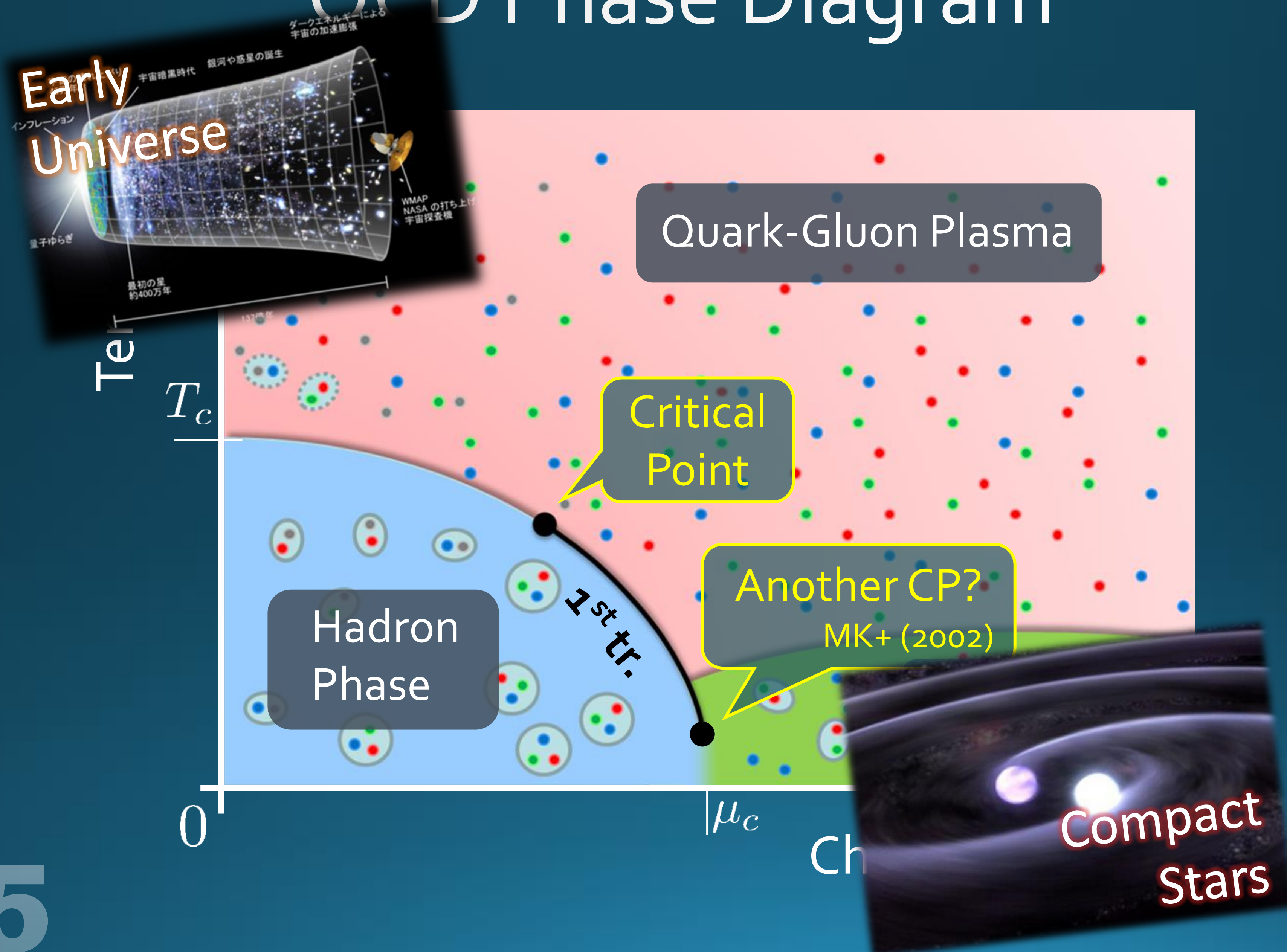
QCD Phase Diagram



QCD Phase Diagram



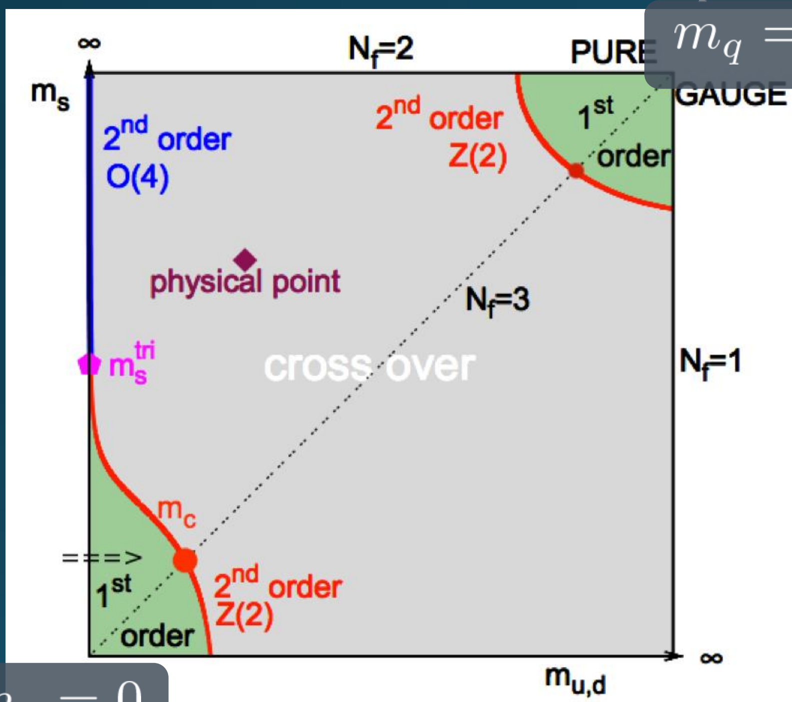
QCD Phase Diagram



Varying Quark Masses

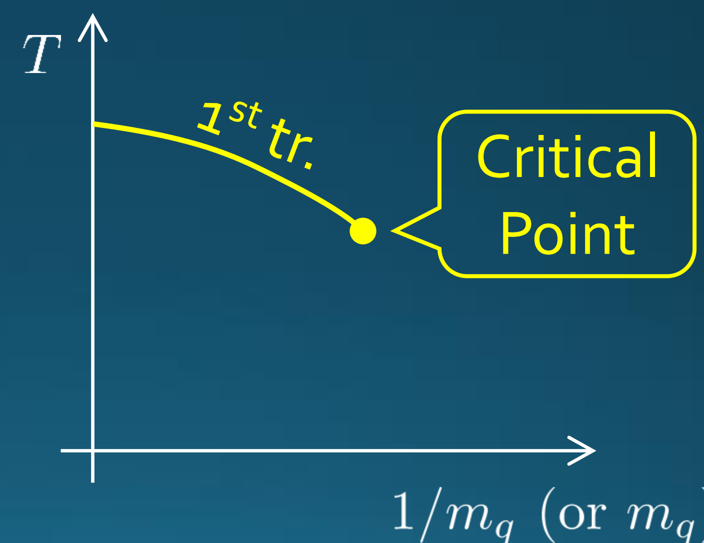
☐ Columbia plot

= order of phase tr. at $\mu_q = 0$



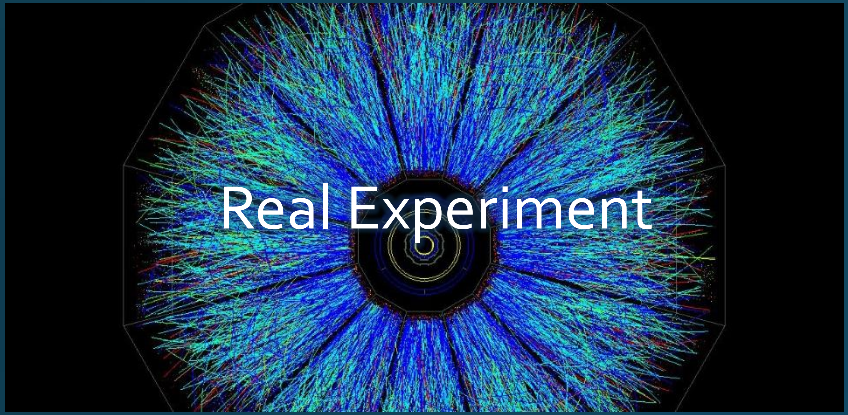
☐ Example

Phase diagram in heavy-quark region



Various orders of phase transition with variation of m_q.

Two Experimental Tools



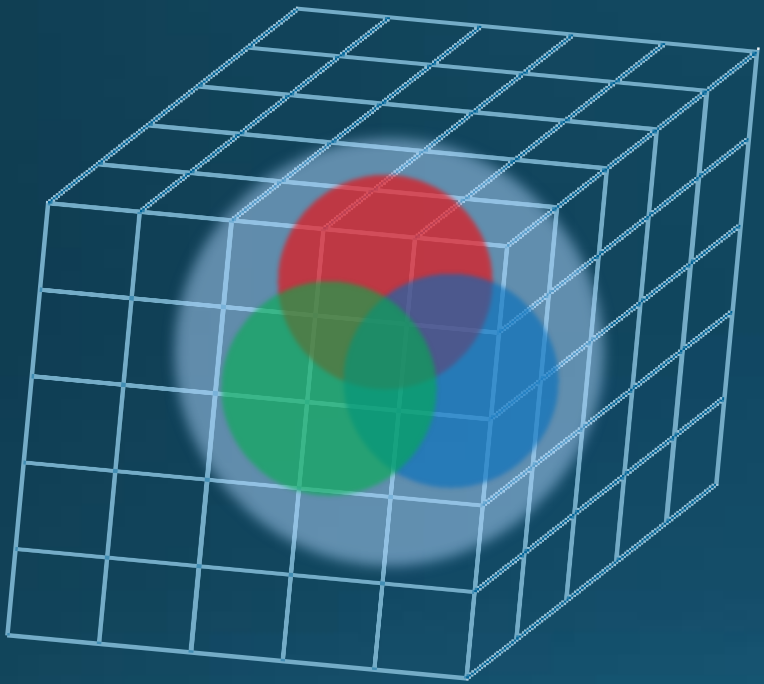
Relativistic Heavy-Ion Collisions

Colliding two heavy nuclei in accelerators
RHIC (USA), LHC (EU), etc.

Lattice QCD Numerical Simulations

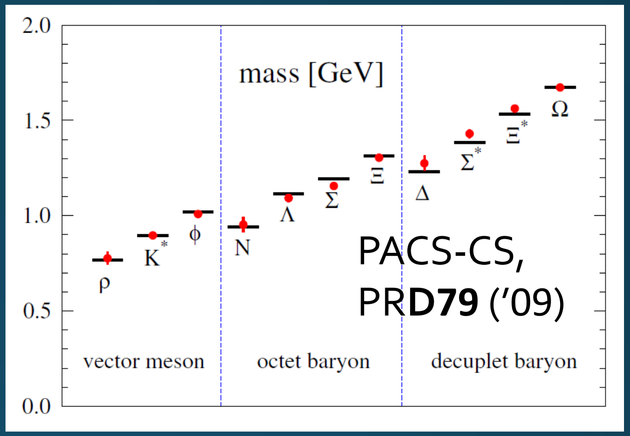
First-principle simulations on supercomputers

Lattice QCD Numerical Simulations

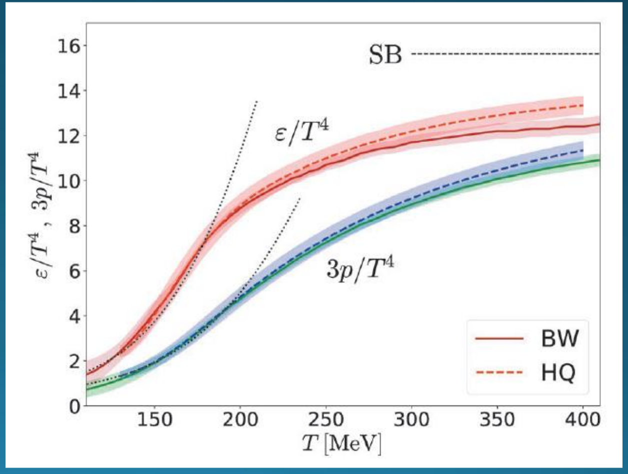


Unique tool to perform **quantitative** analyses of **non-perturbative** QCD aspects

- Hadron Spectroscopy



- Thermodynamics

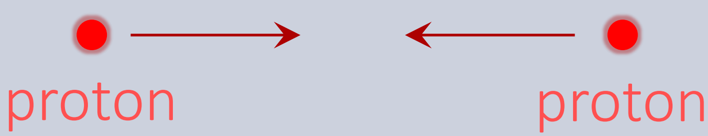


Relativistic Heavy-Ion Collisions

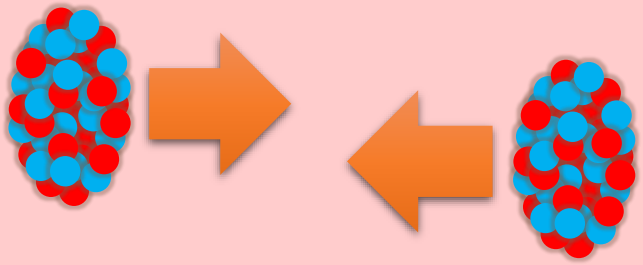
LHC – Large Hadron Collider

Relativistic Heavy-Ion Collisions

Proton-proton



Heavy Ion Collisions

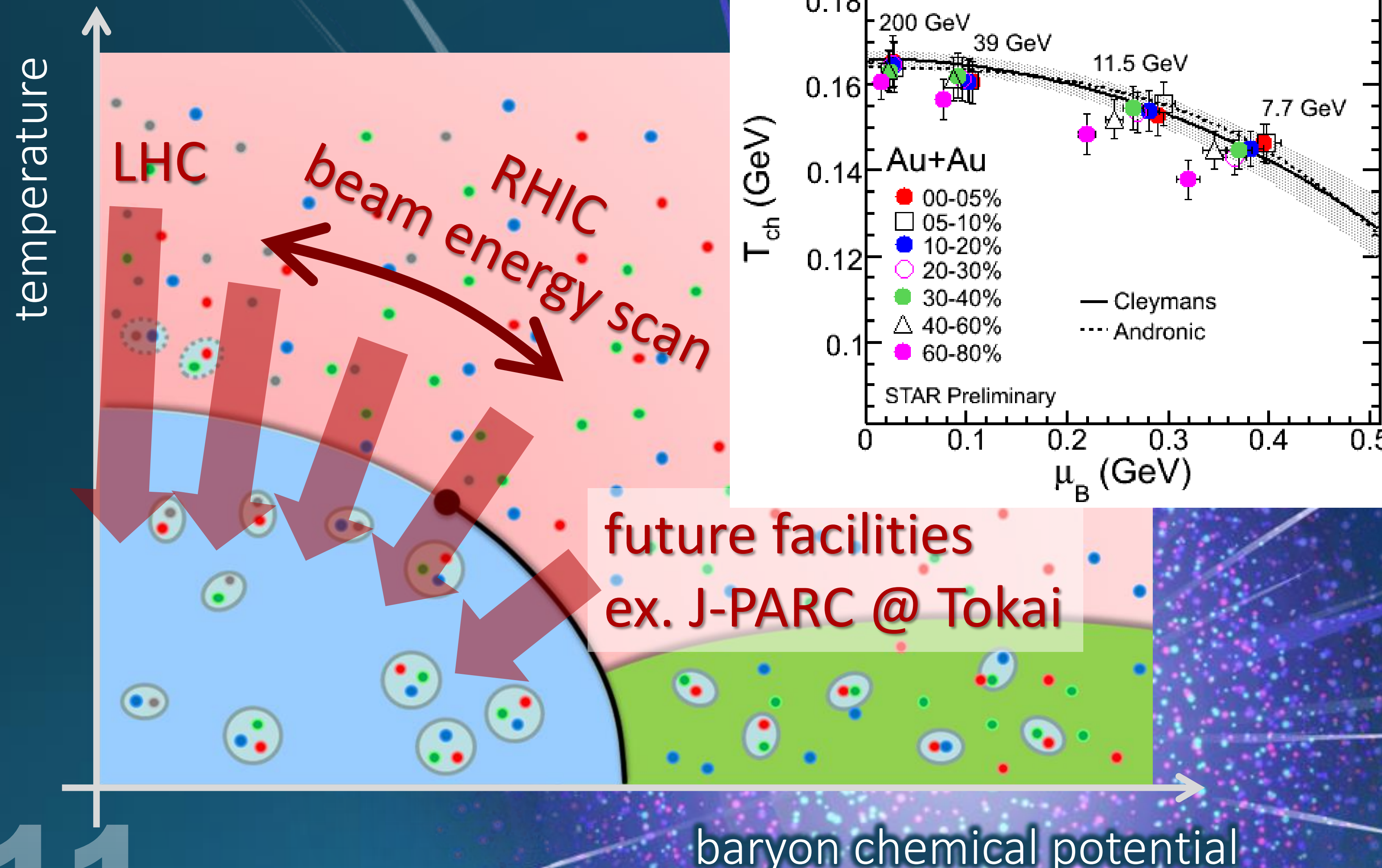


Elementary processes
new particle search
properties of particles

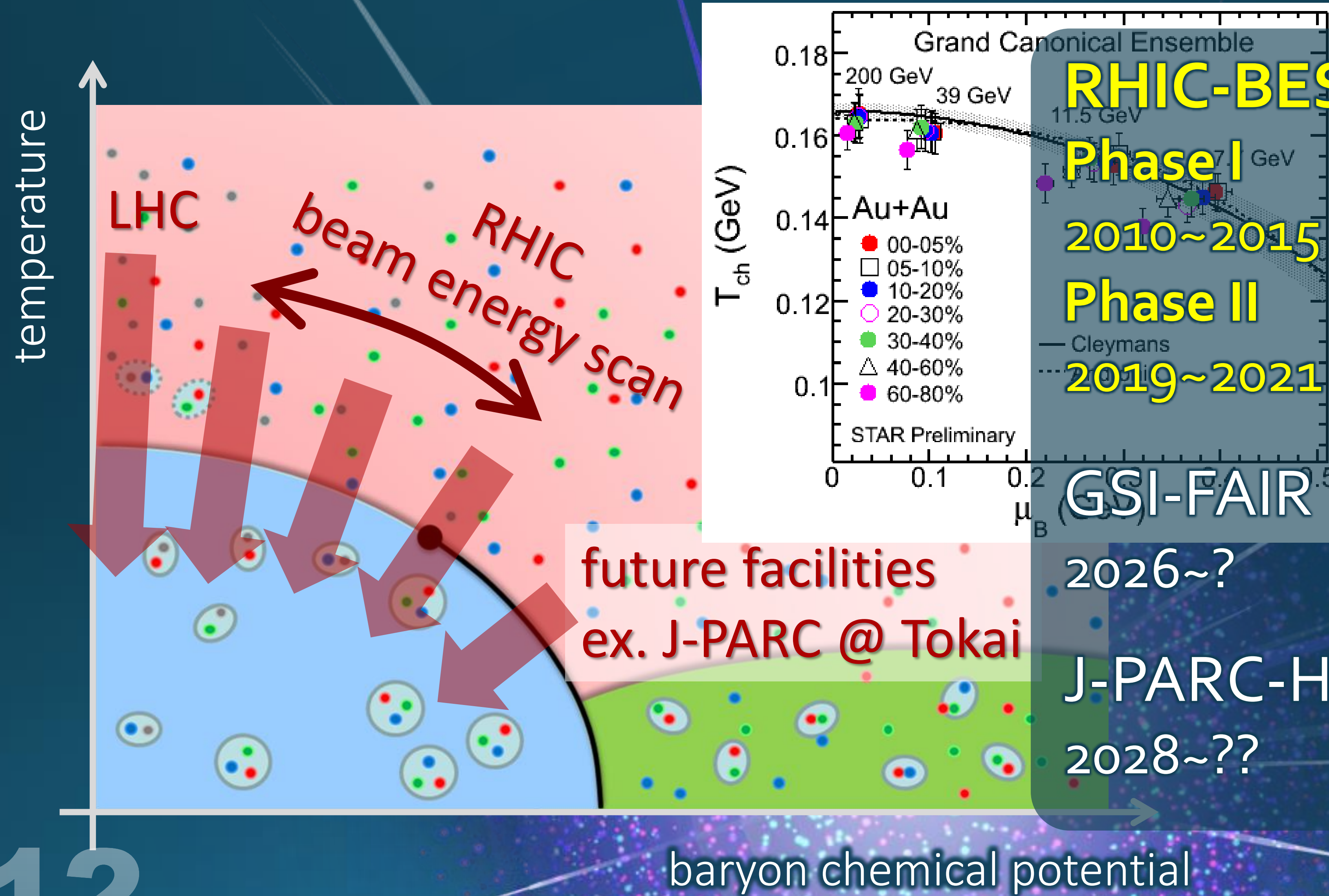
Thermal Medium
hot & dense medium
phase transitions

LHC – Large Hadron Collider

Beam-Energy Scan



Beam-Energy Scan



RHIC-BES

Phase I

2010~2015

Phase II

2019~2021

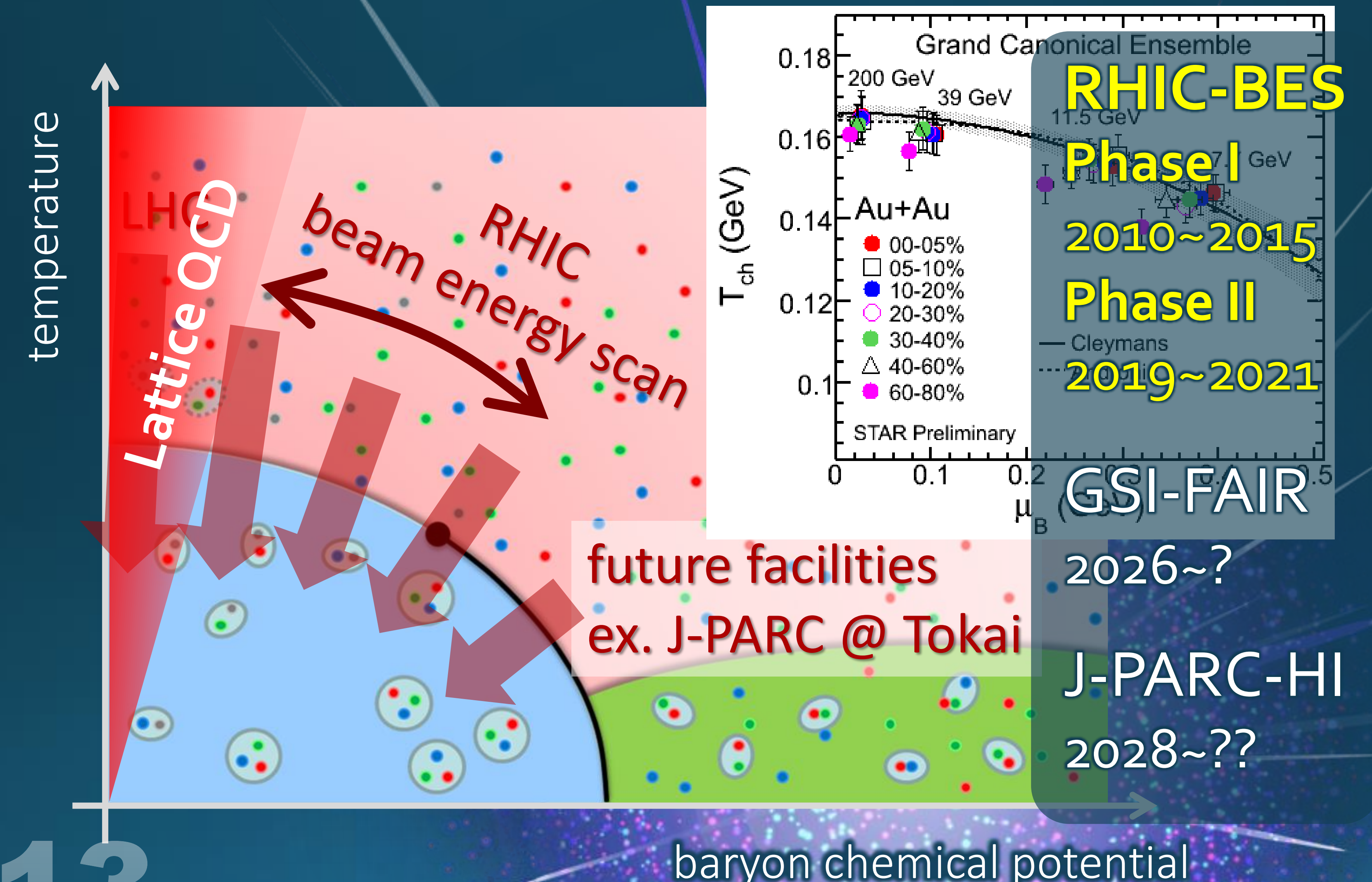
GSI-FAIR

2026~?

J-PARC-HI

2028~??

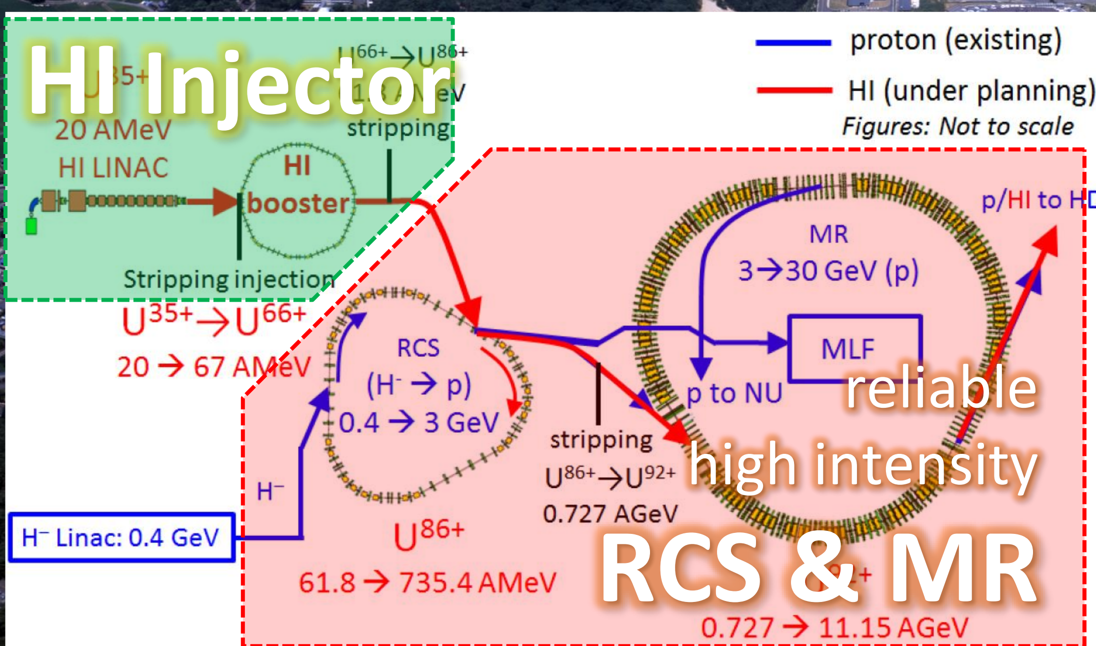
Beam-Energy Scan



J-PARC-HI

J-PARC Heavy-Ion Program

Heavy-ion collision experiments
using accelerators in **J-PARC** **(RCS/MR)**
World highest intensity / Low cost

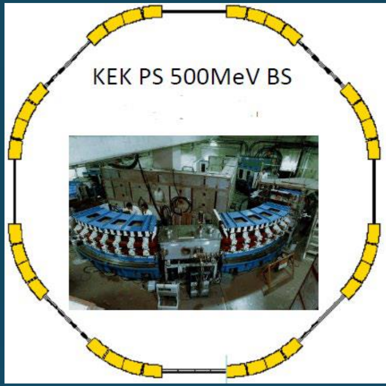


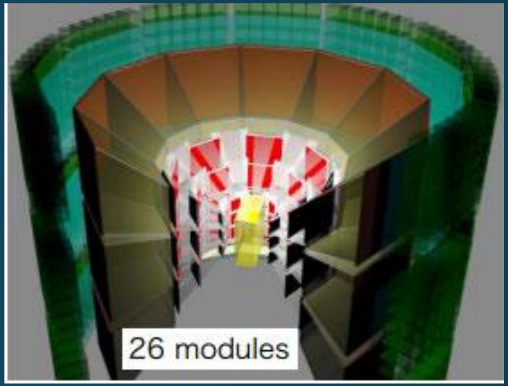
- E_{lab} ~11→19 AGeV
- $\sqrt{s_{NN}}$~4.9→6.2 GeV
- Collision rate: ~10^8Hz
- 2028~?

J-PARC-HI: Staging Plan

☐ Phase-I (~2026)

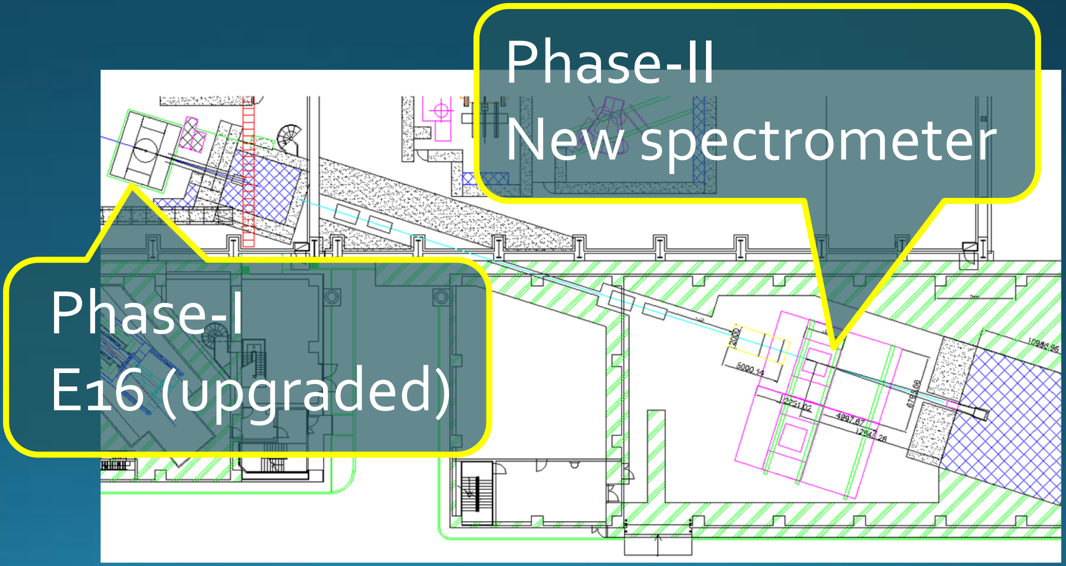
- ☐ SC HI Linac
- ☐ KEK PS booster ~10^8Hz
- ☐ E16 spectrometer (upgrade)
- ☐ Thermal dileptons



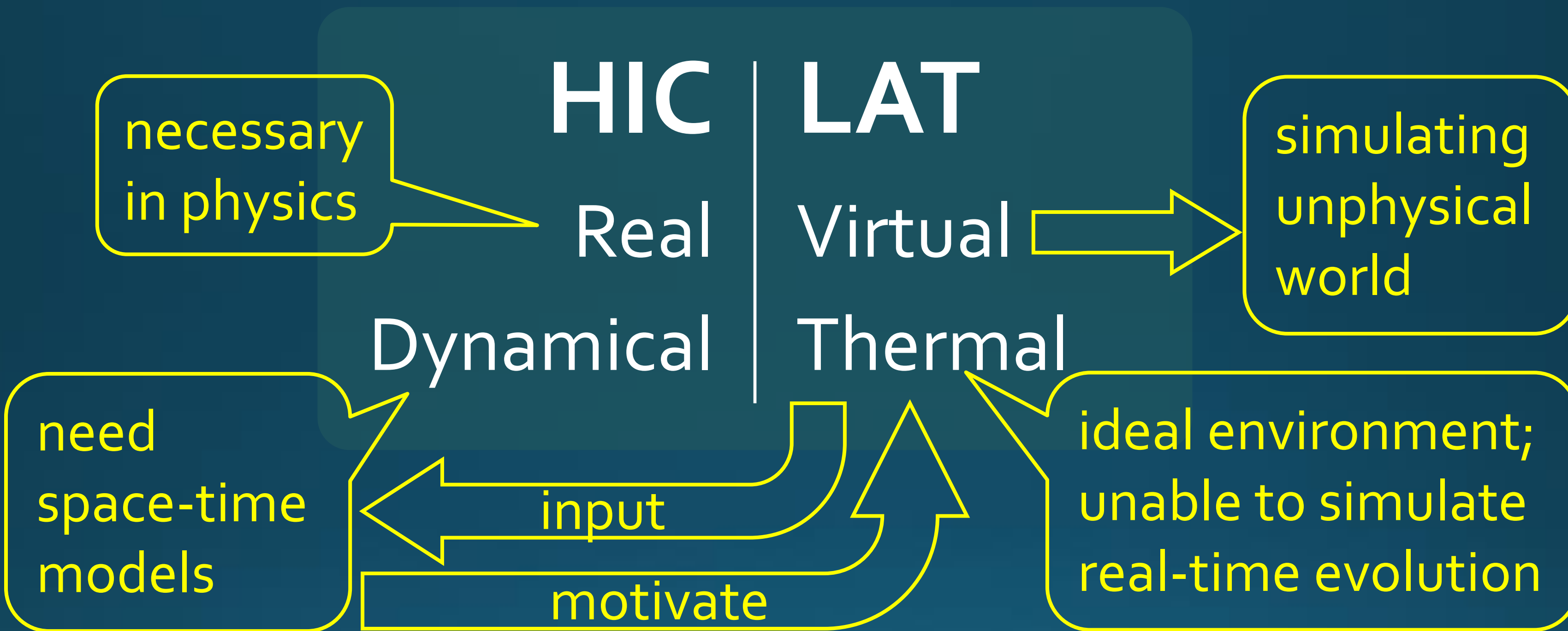


☐ Phase-II (~2032)

- ☐ **New** booster ~10^{11}Hz
- ☐ **New** spectrometer
- ☐ Fluctuations
- ☐ Correlations



HIC vs LAT: Pros & Cons



HIC vs LAT: Pros & Cons

HIC	LAT
Real	Virtual
Dynamical	Thermal

necessary in physics

simulating unphysical world

need space-time models

ideal environment; unable to simulate real-time evolution

input

motivate

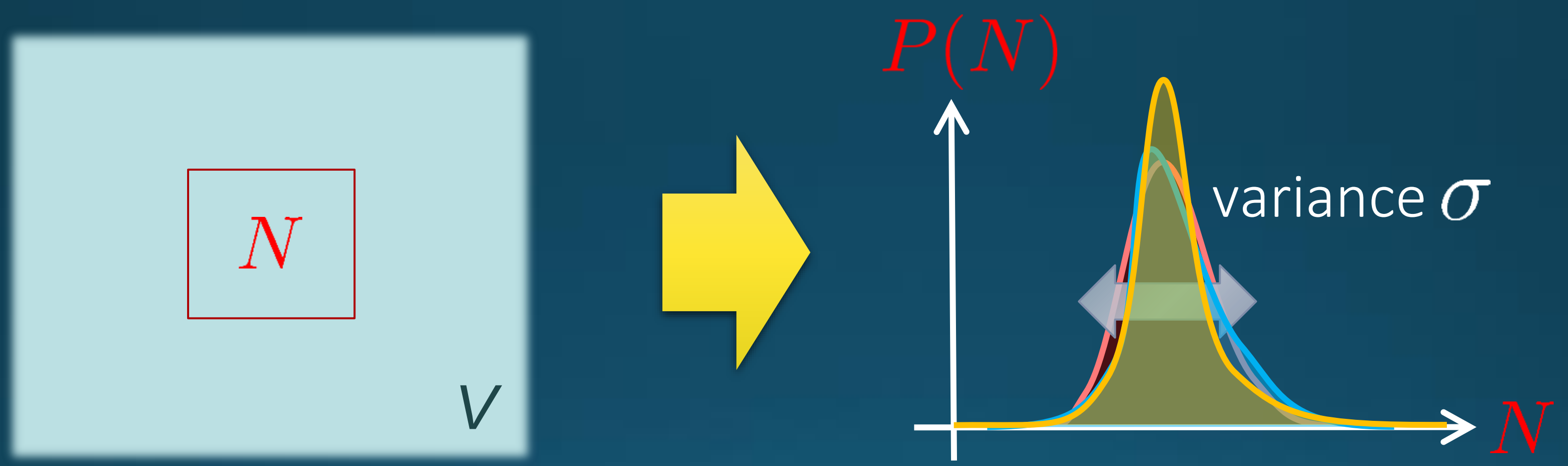
finite density

Accessible	Difficult
beam-energy scan	due to sign problem

Fluctuations & Distribution Functions

Fluctuations

Observables in equilibrium are fluctuating!



Cumulants

Binder Cumulant

Cumulants

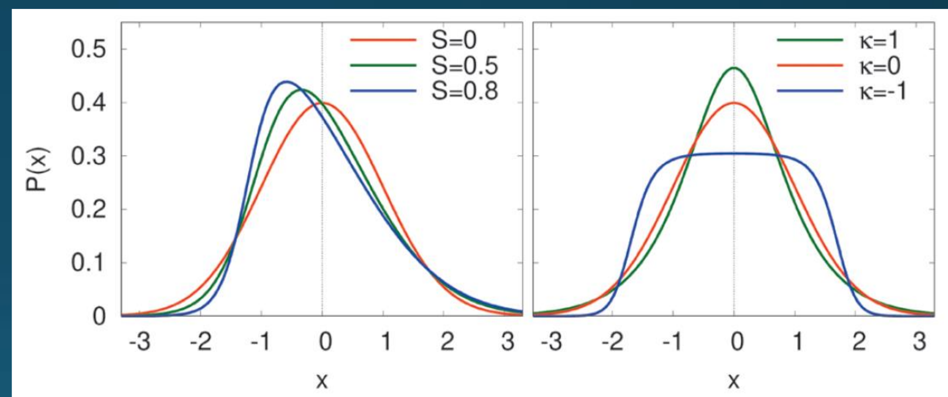
$$\begin{cases} \langle N \rangle_c = \langle N \rangle & \text{average} \\ \langle N^2 \rangle_c = \langle \delta N^2 \rangle & \text{variance} \\ \langle N^3 \rangle_c = \langle \delta N^3 \rangle \\ \langle N^4 \rangle_c = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2 \end{cases}$$

- skewness

$$S = \frac{\langle N^3 \rangle_c}{\langle N^2 \rangle_c^{3/2}}$$

- kurtosis

$$\kappa = \frac{\langle N^4 \rangle_c}{\langle N^2 \rangle_c^2}$$



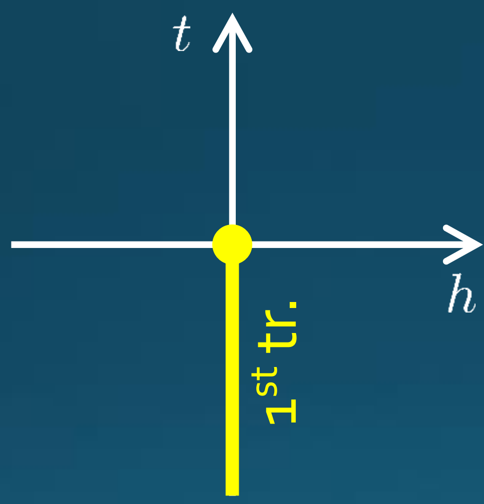
NOTE

- Gauss distribution: $\langle N^3 \rangle_c = \langle N^4 \rangle_c = \cdots = 0$
- Poisson distribution: $\langle N^2 \rangle_c = \langle N^3 \rangle_c = \langle N^4 \rangle_c = \cdots = \langle N \rangle$

Review: Asakawa, MK, PPNP **90** (2016)

Cumulants around Critical Point

Ising Model



$$P(M) \sim e^{-V(M)}$$

- $P(M)$: probability distr.
- $V(M)$: effective potential
- M : order parameter

- Sign of $\langle N^3 \rangle_c$ is flipped at the CP.

Cumulants around Critical Point

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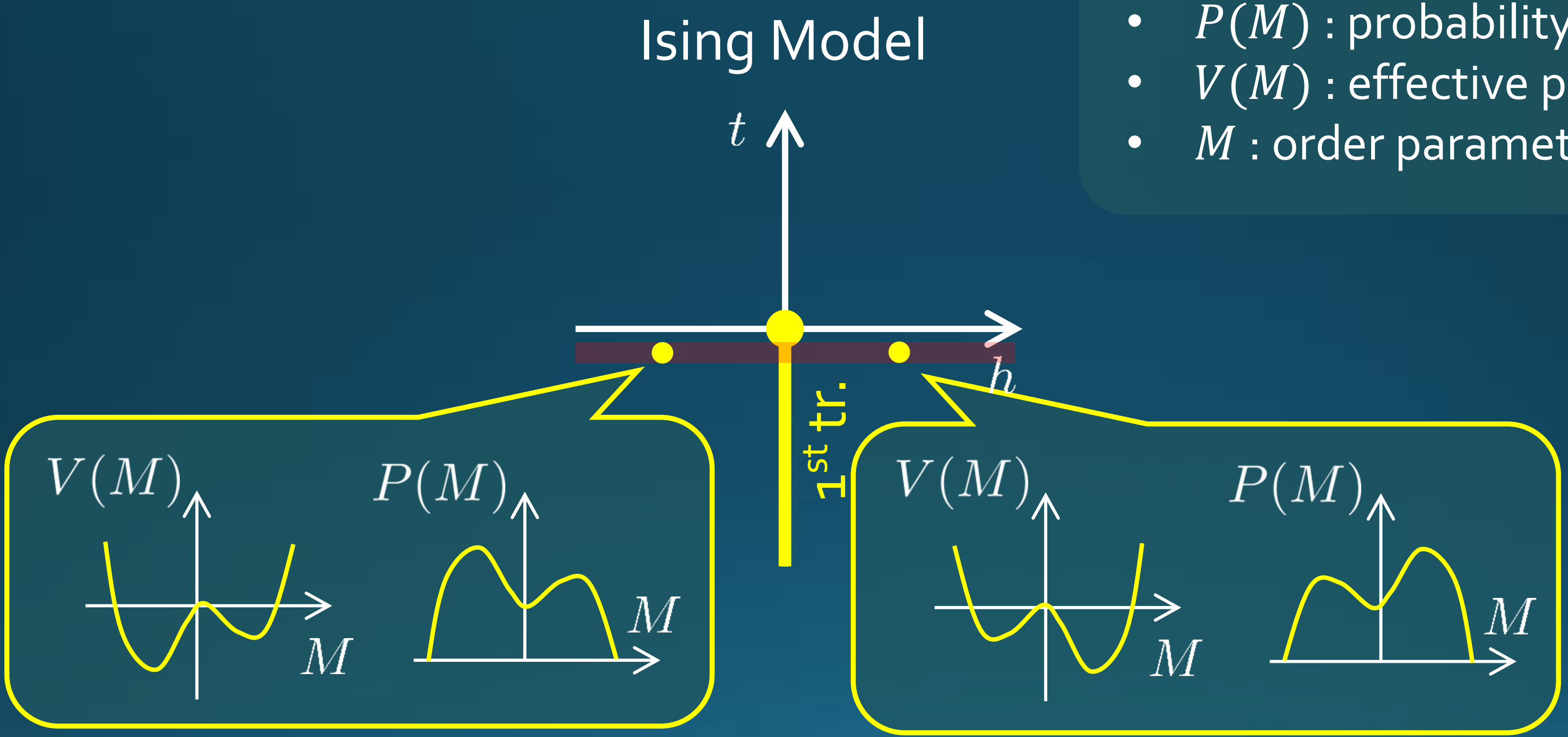

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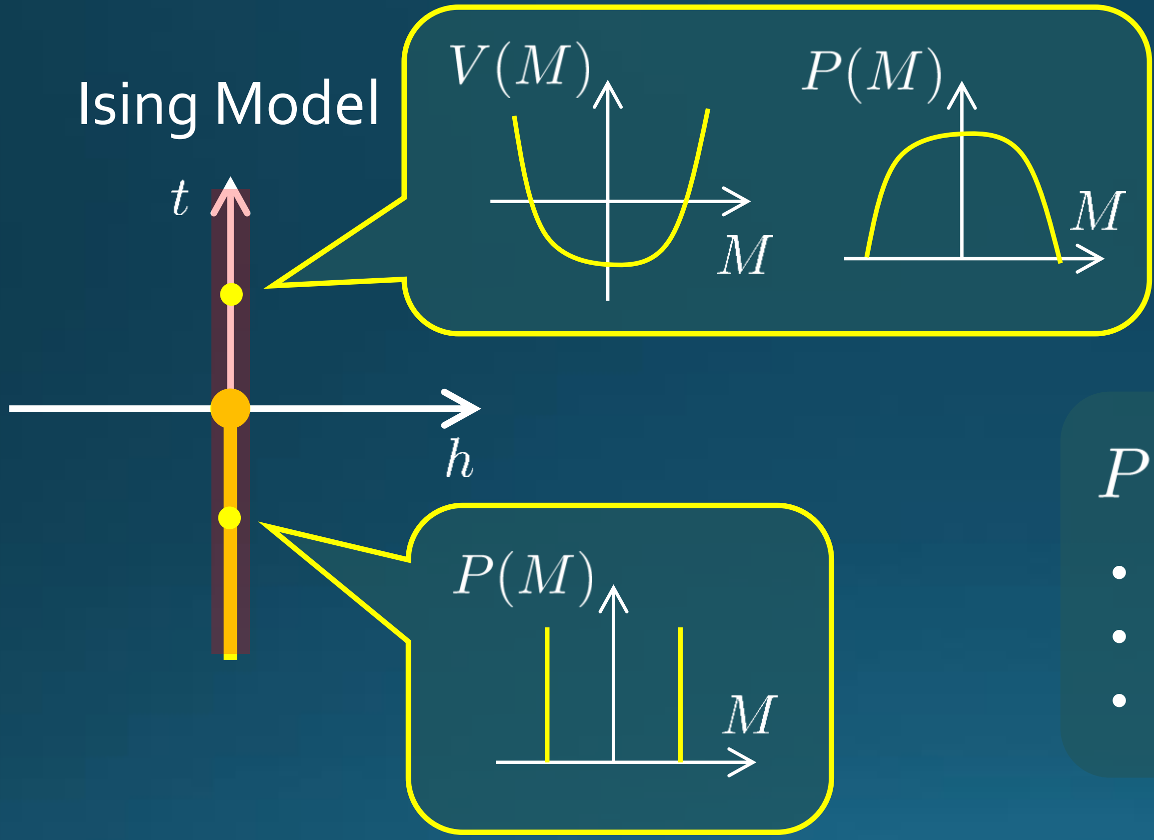
Cumulants around Critical Point



$$P(M) \sim e^{-V(M)}$$

- $P(M)$: probability distr.
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- $\langle N^4 \rangle_c$ changes discontinuously at the CP.

Experimental Search for QCD Critical Point

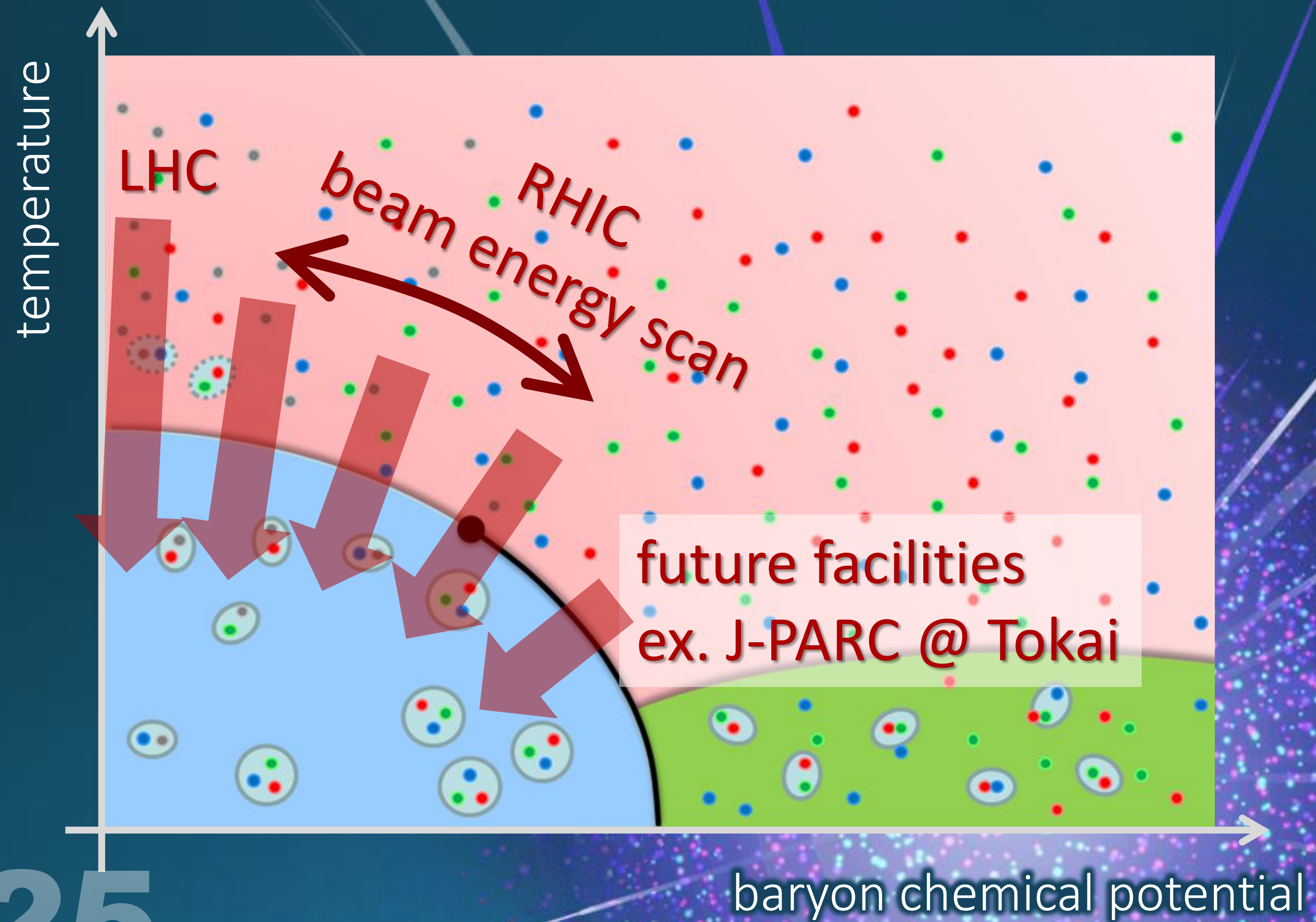
Reviews:

Asakawa, MK, PPNP ('16)

Bluhm, MK+, NPA 1003 ('20)

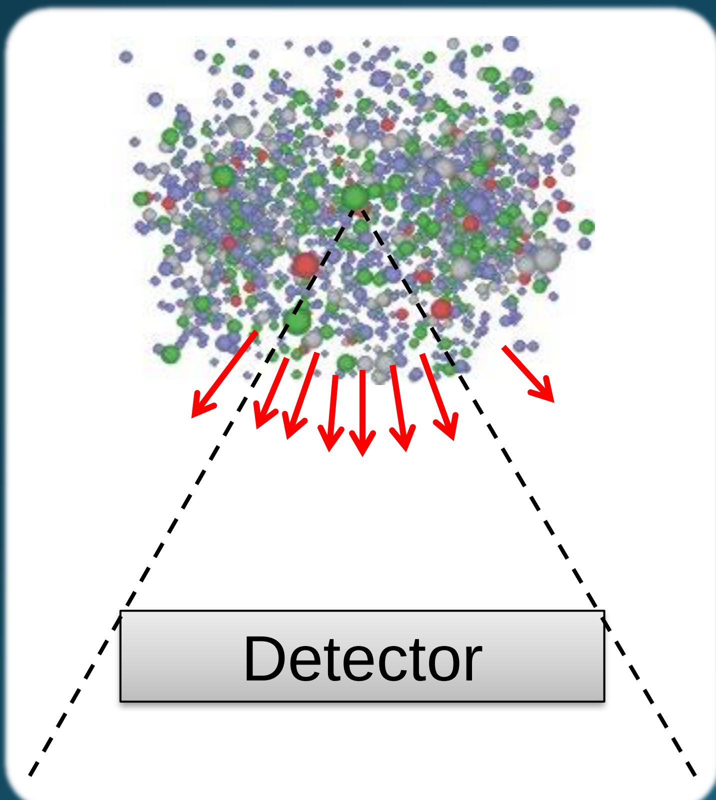
MK, Esumi, Nonaka, JPS journal, 2021/8

Beam-Energy Scan

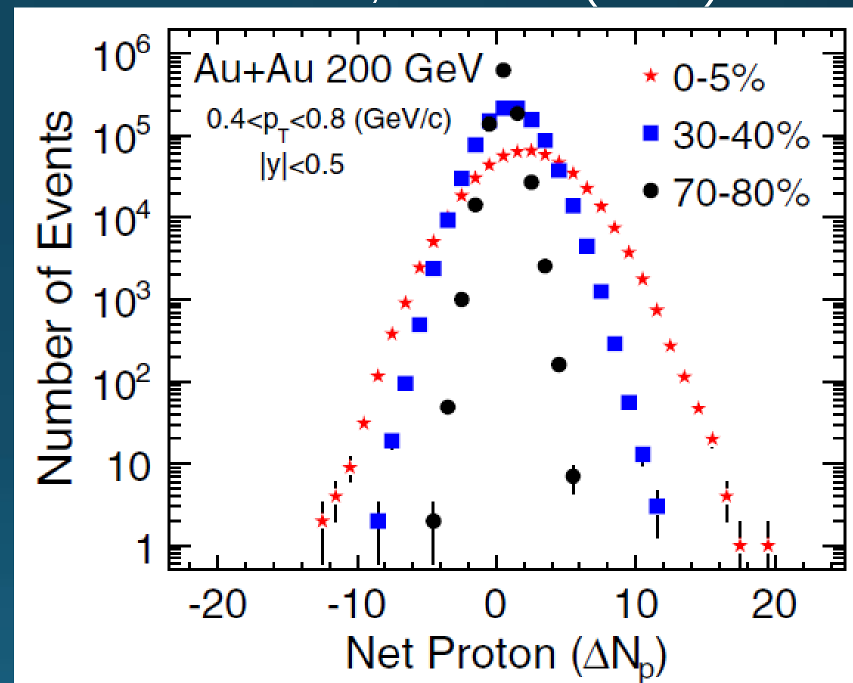


Event-by-event Fluctuations

Review: Asakawa, MK, PPNP **90** (2016)



STAR, PRL**105** (2010)



Cumulants

$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$

Cumulants around Critical Point

$$P(M) \sim e^{-V(M)}$$

- $P(M)$: probability distr.
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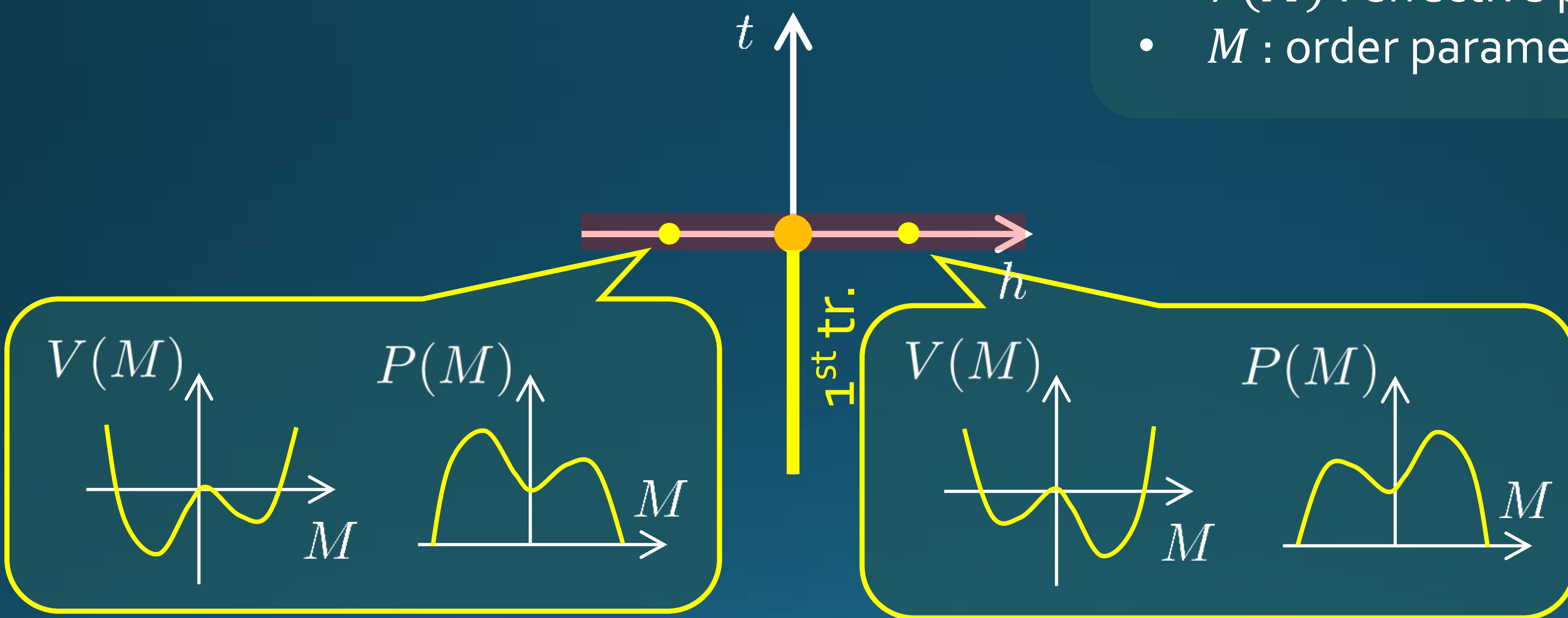
- M : order parameter

Ising Model



- Sign of $\langle N^3 \rangle_c$ is flipped at the CP.

Sign Change of Cumulant

Asakawa, Ejiri, MK, PRL '09

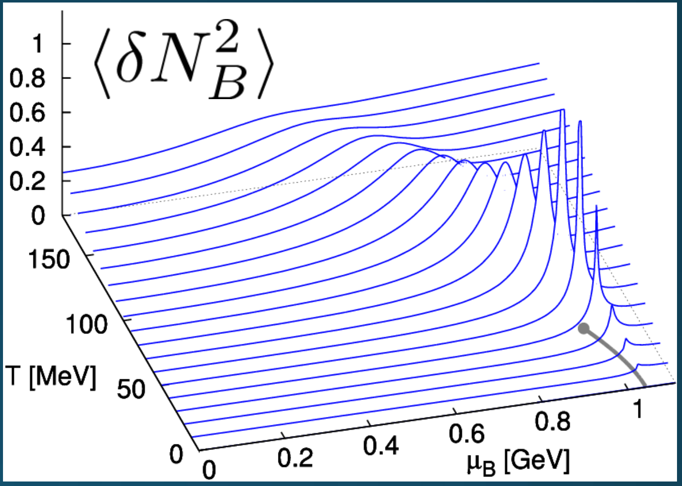
☐ Geometric interpretation on the signs

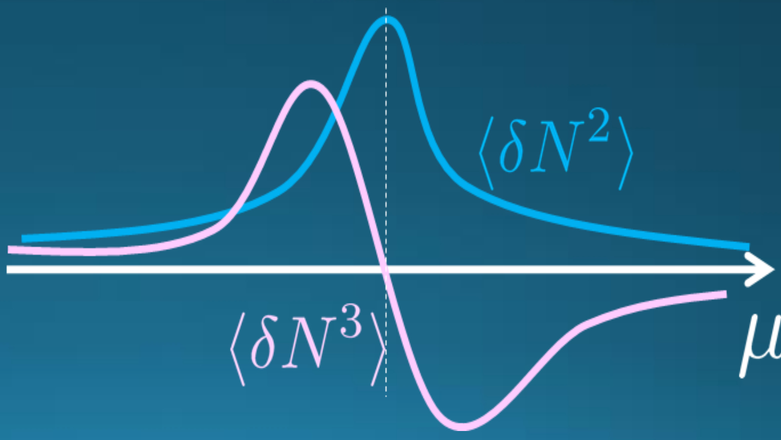
Fluctuations $\langle N_B^2 \rangle_c$ diverge at the QCD-CP.

Themodynamic Relation

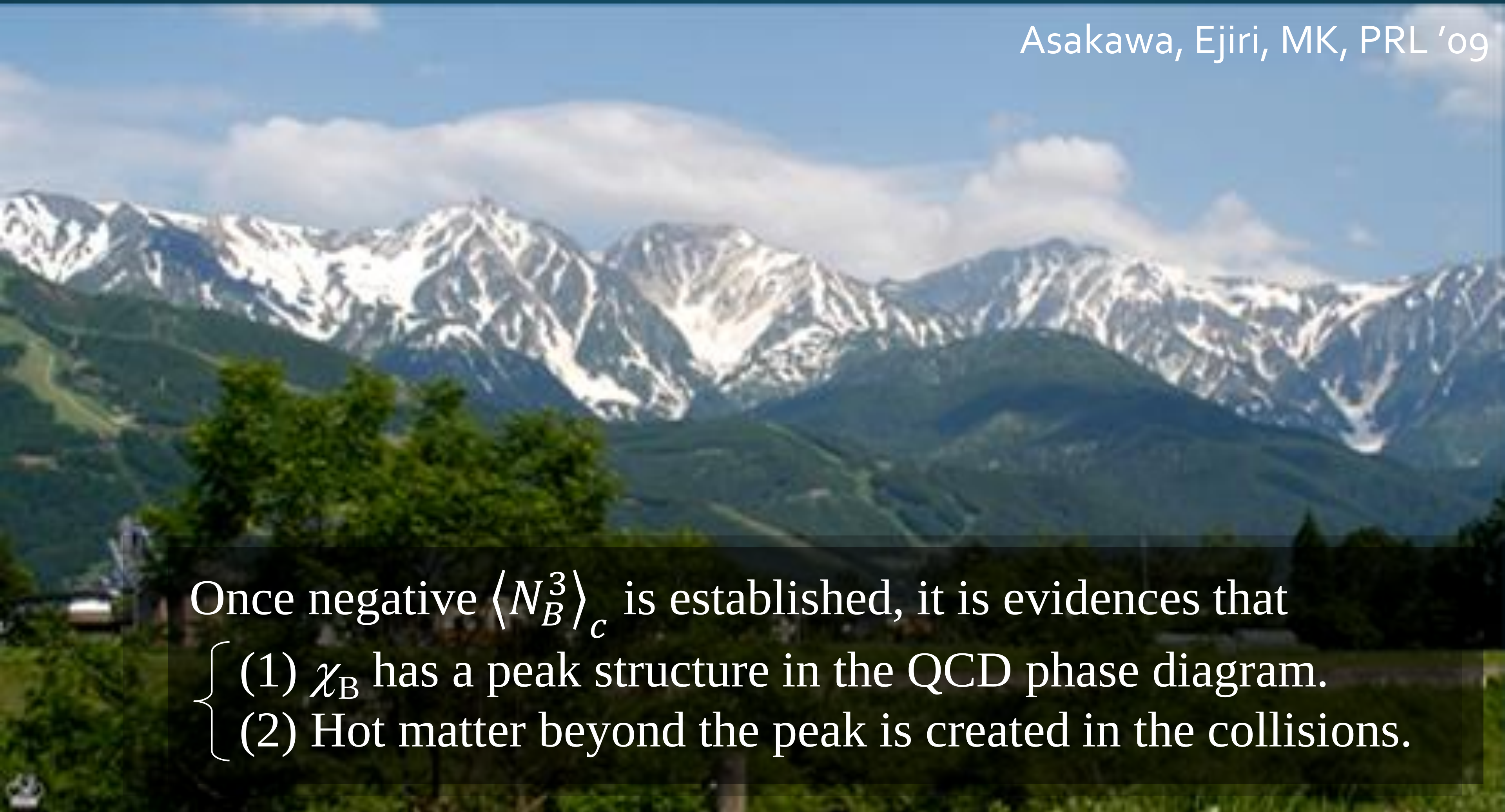
$$\langle N_{\mathrm{B}}^{m+1} \rangle_c = T \frac{\partial \langle N_{\mathrm{B}}^m \rangle_c}{\partial \mu_{\mathrm{B}}}$$

Sign of $\langle N_B^3 \rangle_c$ can distinguish near and away sides!





Impact of Negative Cumulants



Asakawa, Ejiri, MK, PRL '09

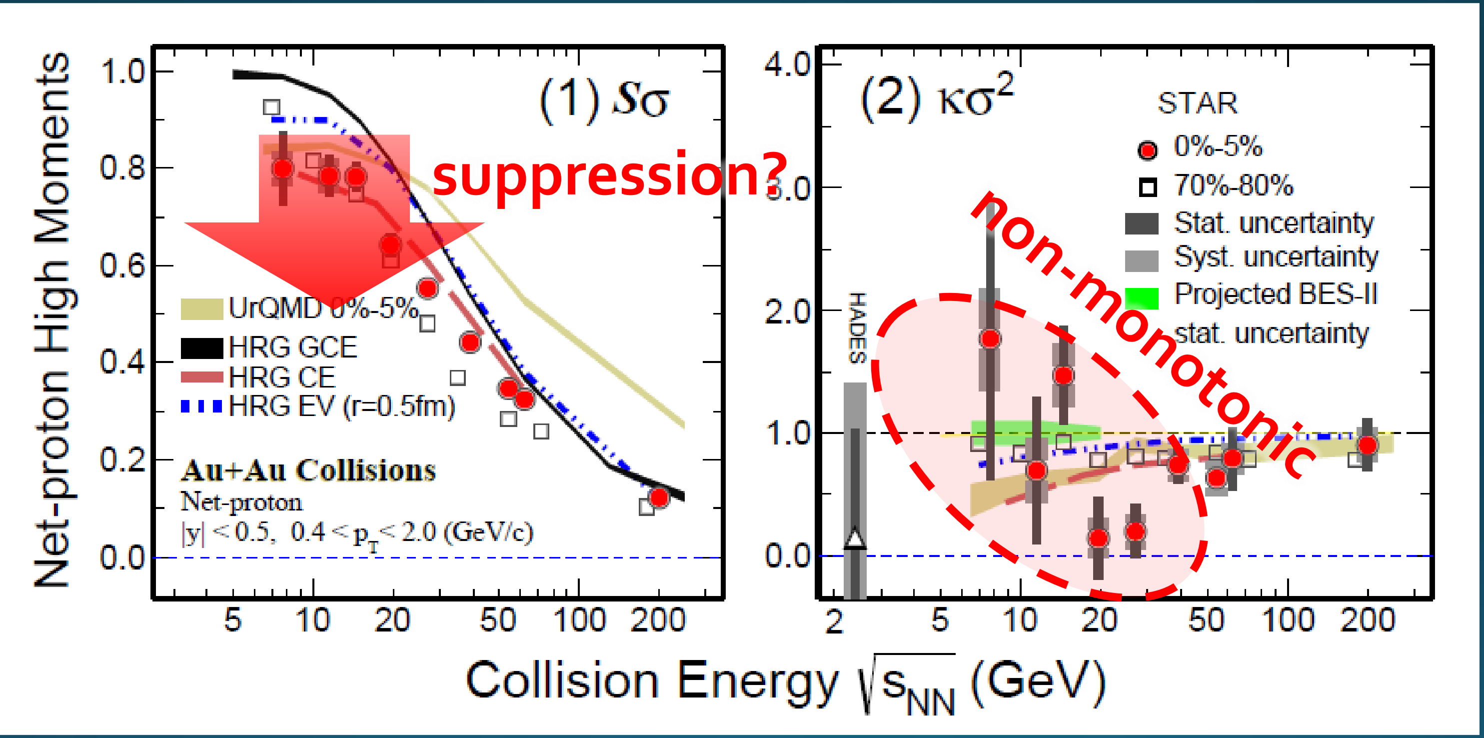
Once negative $\langle N_B^3 \rangle_c$ is established, it is evidences that

(1) χ_B has a peak structure in the QCD phase diagram.
(2) Hot matter beyond the peak is created in the collisions.

- **No** dependence on any specific models.
- Just the sign! **No** normalization (such as by N_{ch}).

Proton Number Cumulants

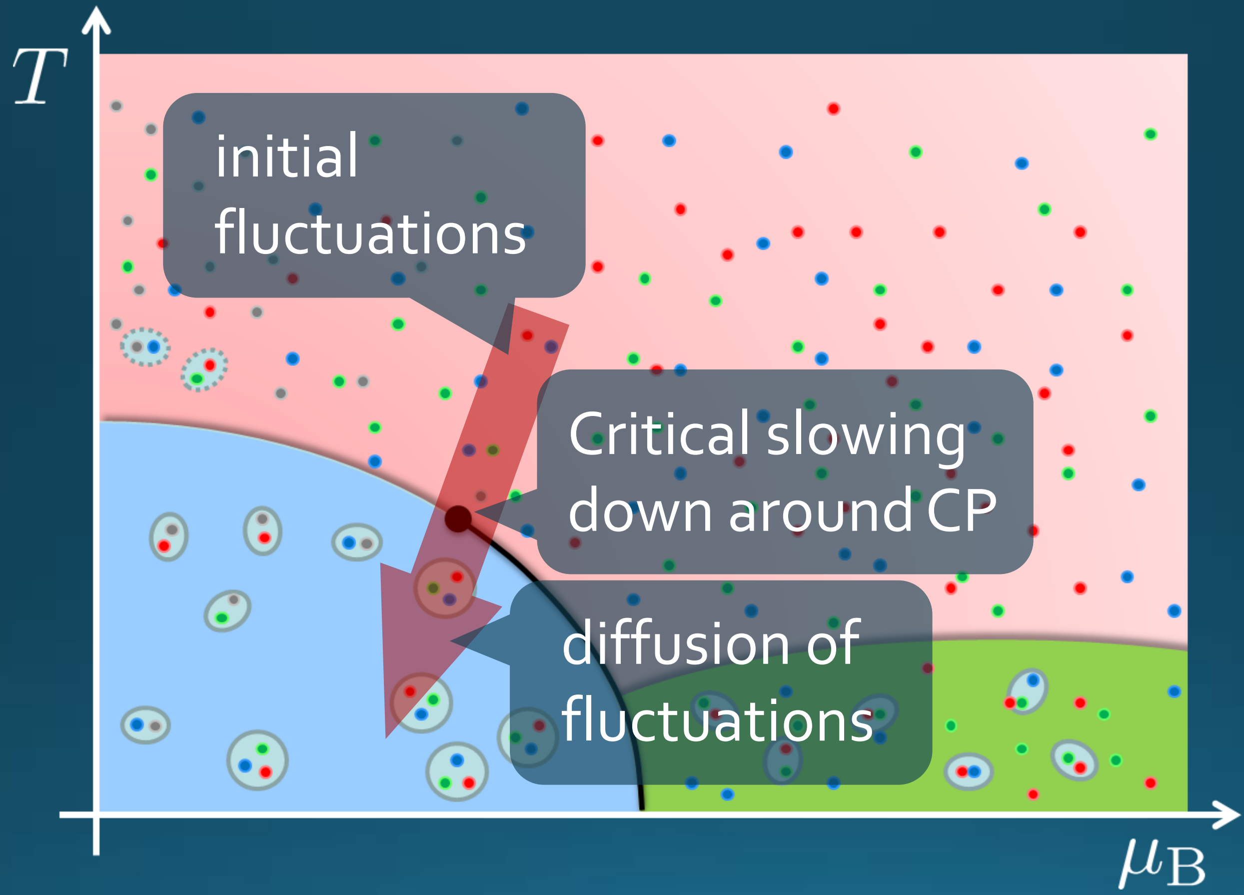
$\langle N_p^3 \rangle_c / \langle N_p^2 \rangle_c$ $\langle N_p^4 \rangle_c / \langle N_p^2 \rangle_c$



STAR, PRL**126** ('21)

- Nonzero and non-Poissonian cumulants are experimentally established.

Time Evolution of Cumulants



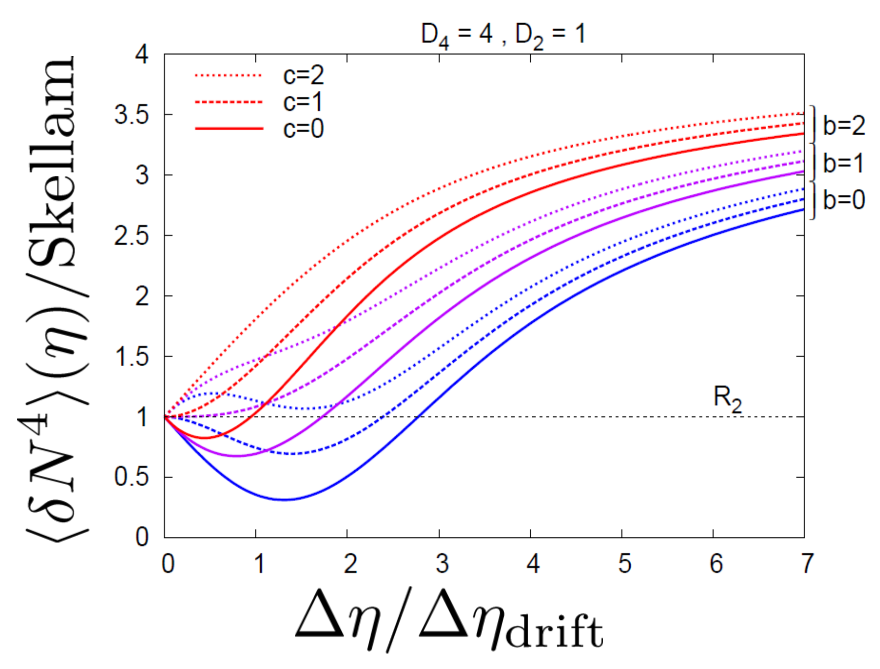
Proper understanding of the time evolution of fluctuations is indispensable.

Rapidity Window Dependence
in Diffusion Models

- **Higher order cumulants**

  in diffusion master equation

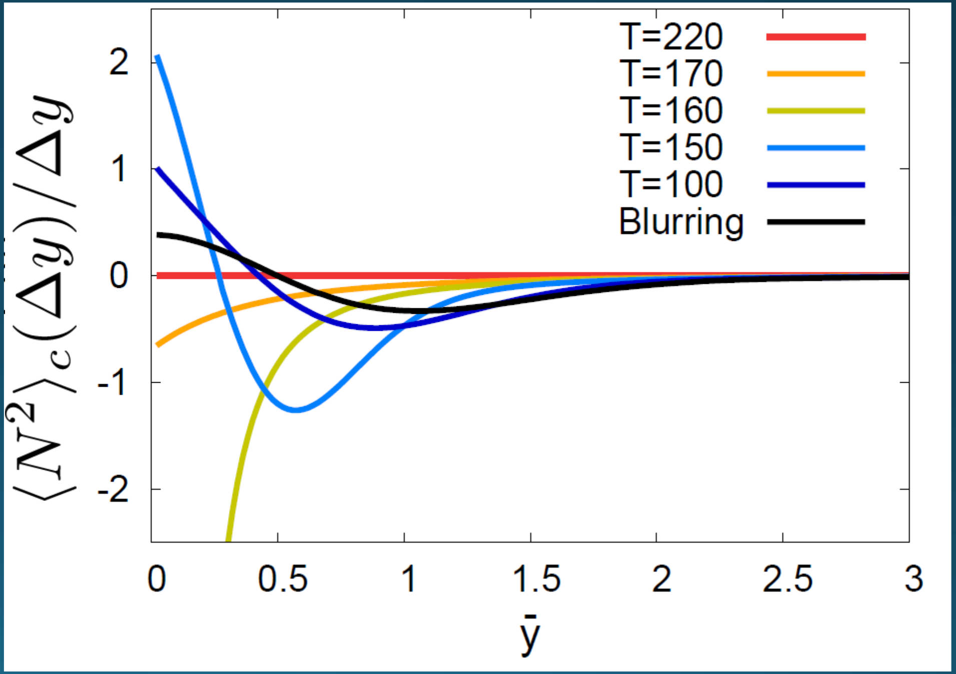
  MK+ (2014); MK (2015)



- **2nd order cumulant near CP**

  in stochastic diffusion equation

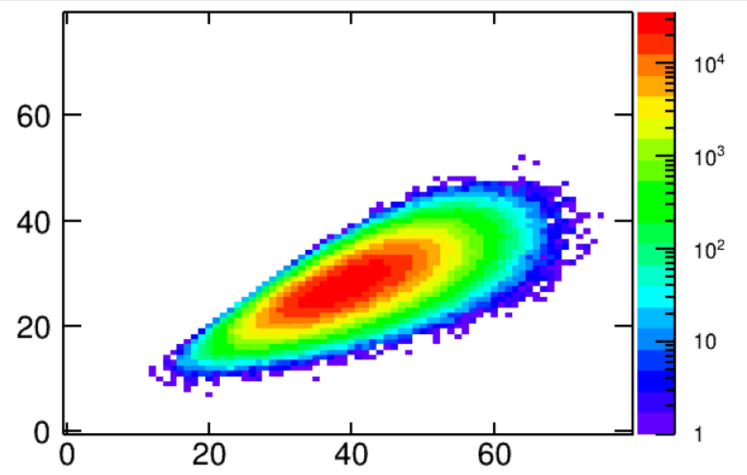
  sakaida, Asakawa, Fujii, MK, 2018

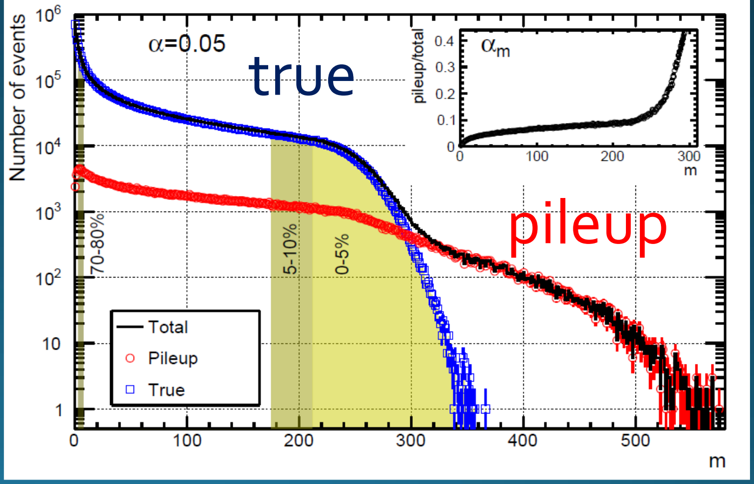


- Non-monotonic Δy dependence can emerge reflecting the dynamical evolution.

Issues with Experimental Analysis

- Detector-response correction
  - MK, Asakawa, 2012
  - MK, 2016
  - MK, Luo, 2017
  - Nonaka, MK, Esumi, 2017
  - Nonaka, MK, Esumi, 2018
- Pileup correction
  - Nonaka, MK, Esumi, 2020
- Acceptance correction
  - Nonaka, MK, Esumi, 2022





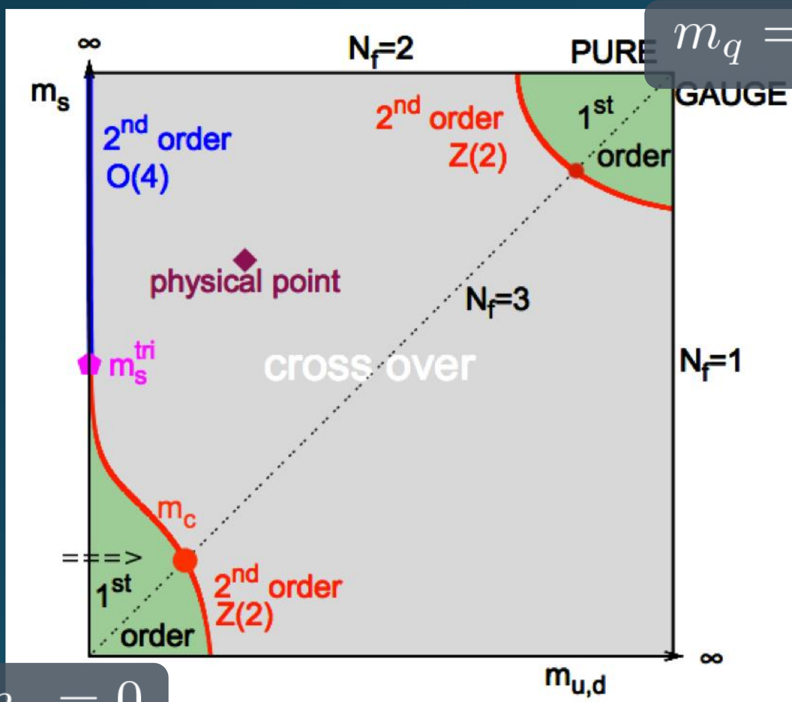
Lattice Simulation of CP in Heavy-Quark Region

WHOT-QCD, Phys. Rev. **D 101** (2020) 05450;
WHOT-QCD, PTEP **2021** (2021) 013B08;
Kiyohara, MK, Ejiri, Kanaya, Phys. Rev. **D 104** (2021) 114509;
WHOT-QCD, PTEP **2022** (2022) 033B05
WHOT-QCD (Ashikawa, et al.), in progress

Varying Quark Masses

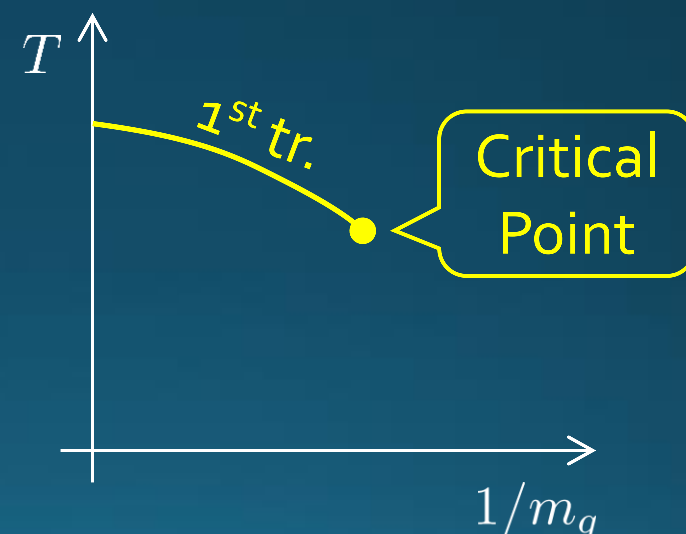
□ Columbia plot

= order of phase tr. at $\mu = 0$



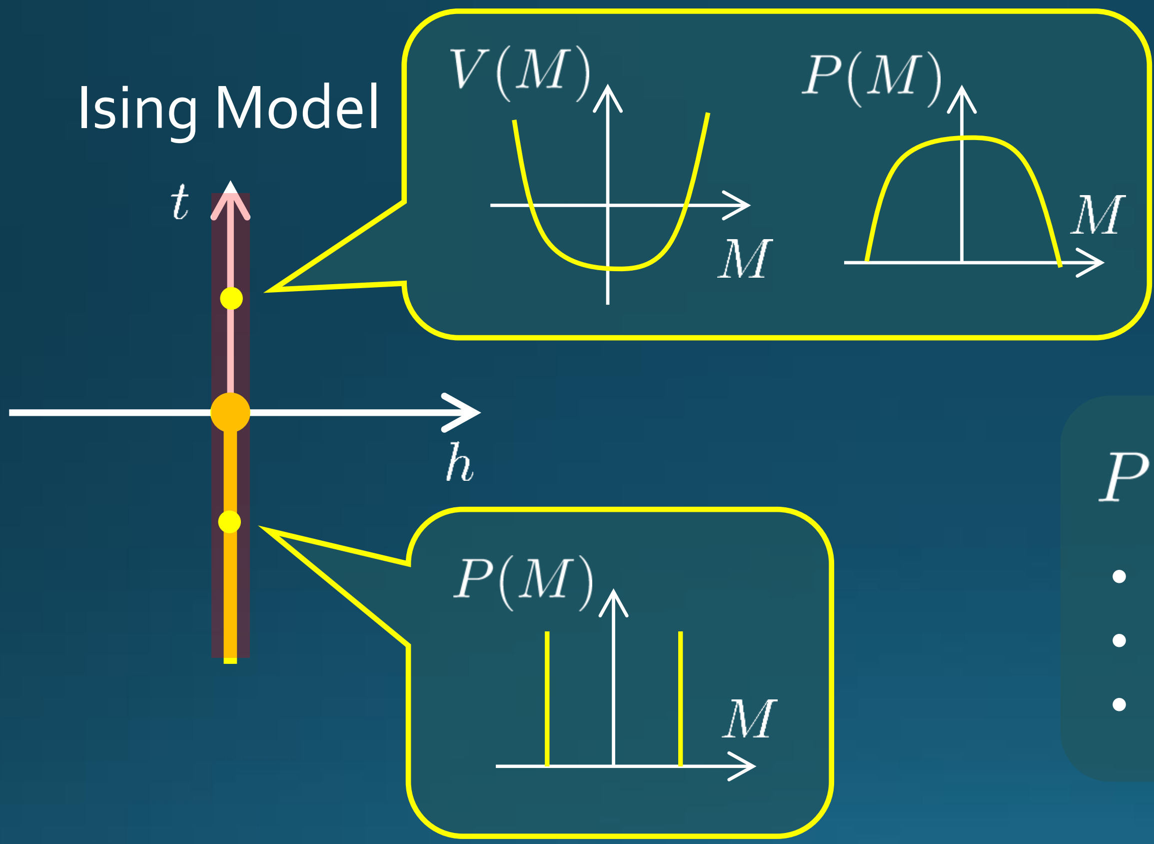
□ Example

Phase diagram in heavy-quark region



Various orders of phase transition with variation of m_q.

Cumulants around Critical Point



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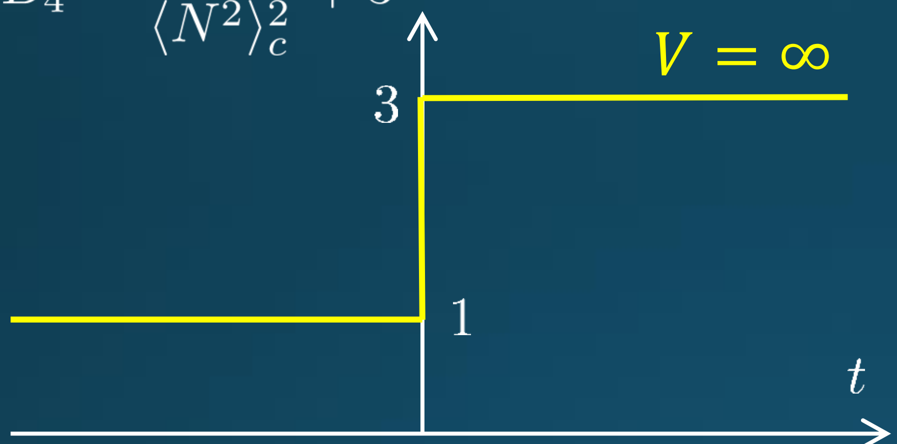
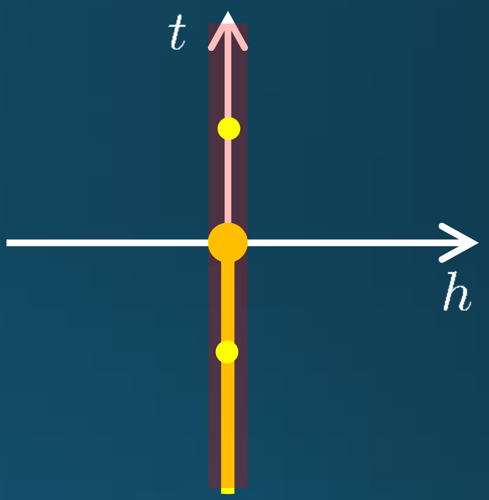
Finite-Volume Effects

Binder Cumulant

$$B_4 = \frac{\langle N^4 \rangle_c}{\langle N^2 \rangle_c^2} + 3$$



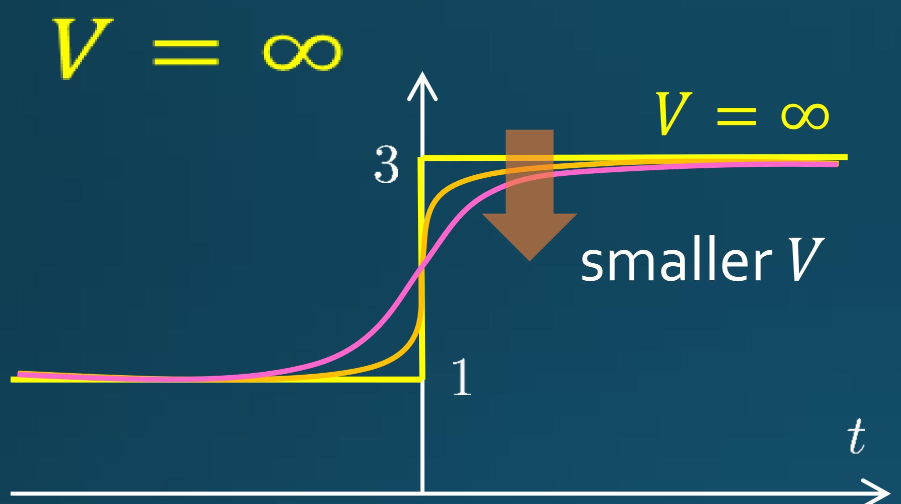
Ising Model



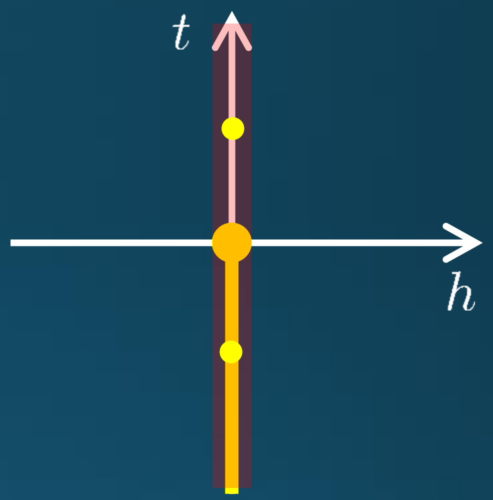
- Sudden change of B_4 at the CP is smeared by finite V effect.
- B_4 obtained for various V has crossing at $t = 0$.
- At the crossing point, $B_4 = 1.604$ in Z_2 universality class.

Finite-Volume Effects

Binder Cumulant



Ising Model



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- B_4 obtained for various V has crossing at $t = 0$.
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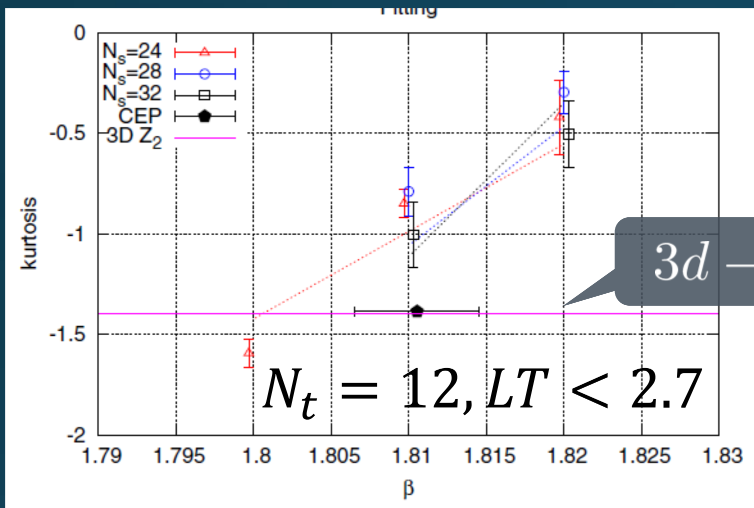
Binder-Cumulant Analysis

☐ Light-quark region

Kuramashi, Nakamura, Ohno, Takeda, '20

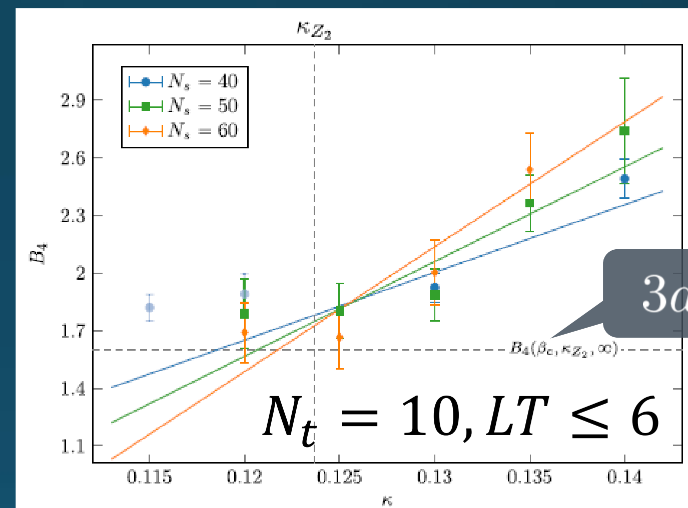


$3d - Z_2$

$N_t = 12, LT < 2.7$

☐ Heavy-quark region

Cuteri , Philipsen , Schön, Sciarra, '21

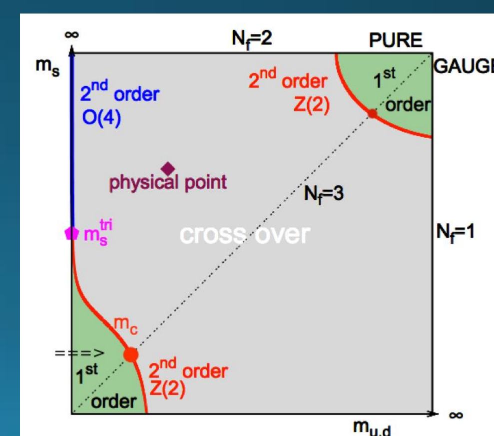


$3d - Z_2$

$N_t = 10, LT \leq 6$

☐ Statistically-significant deviation of the crossing point from the 3d-Ising value.

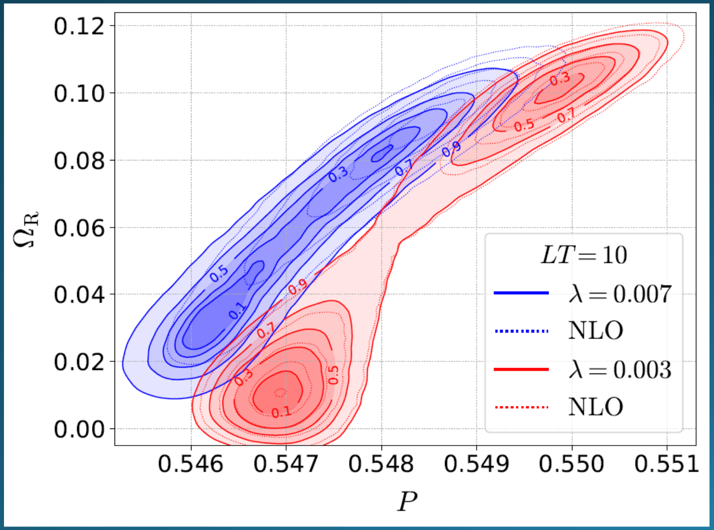
➡ Too large finite-V effects?



Numerical Simulation

- Coarse lattice: $N_t = 4$
- But **large spatial volume**:
  $LT = N_s / N_t \leq 12$

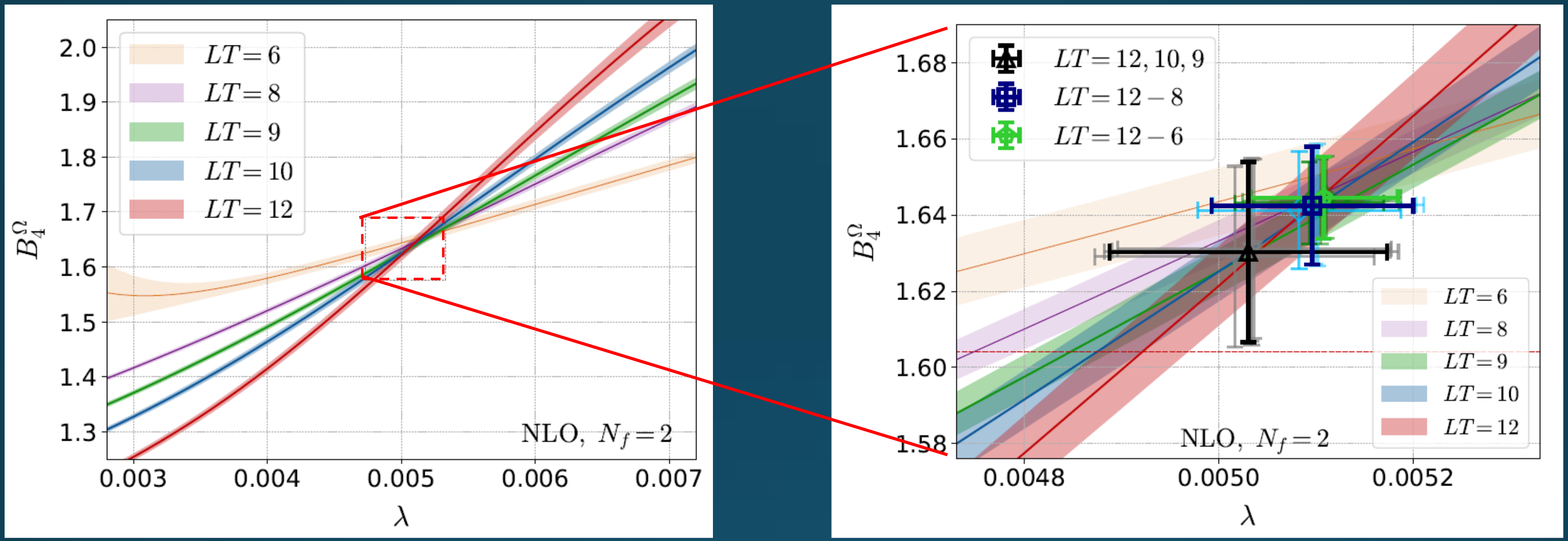
- Hopping-param. ($\sim 1/m_q$) expansion
- Monte-Calro with LO action
- High statistical analysis



Simulation params.

lattice size	β^*	λ	$\kappa^{N_f=2}$
$48^3 \times 4$	5.6869	0.004	0.0568
	5.6861	0.005	0.0601
	5.6849	0.006	0.0629
$40^3 \times 4$, $36^3 \times 4$	5.6885	0.003	0.0529
	5.6869	0.004	0.0568
	5.6861	0.005	0.0601
	5.6849	0.006	0.0629
	5.6837	0.007	0.0653
$32^3 \times 4$	5.6885	0.003	0.0529
	5.6865	0.004	0.0568
	5.6861	0.005	0.0601
	5.6845	0.006	0.0629
	5.6837	0.007	0.0653
$24^3 \times 4$	5.6870	0.0038	0.0561
	5.6820	0.0077	0.0669
	5.6780	0.0115	0.0740

Binder-Cumulant Analysis

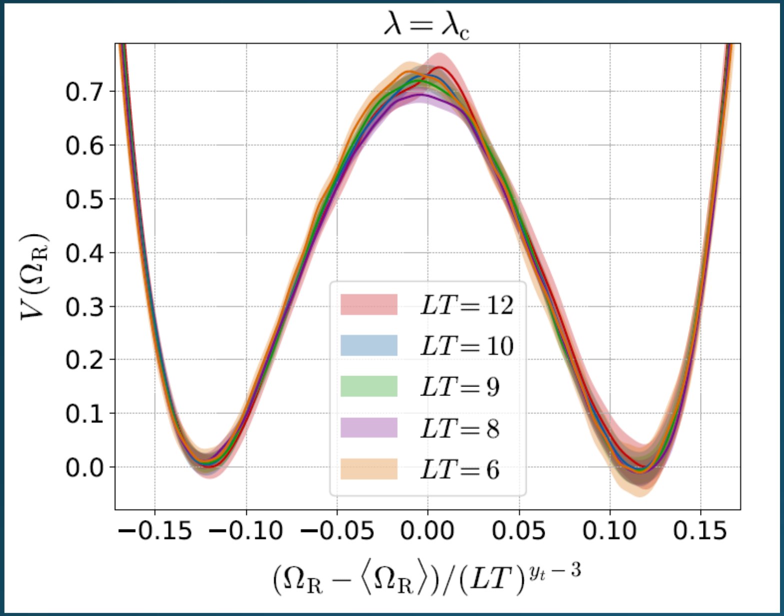


Z2	$B_4 = 1.604$	$\nu = 0.630$
$LT \geq 9$	$B_4 = 1.630(24)(2),$	$\nu = 0.614(48)(3)$
$LT \geq 8$	$B_4 = 1.643(15)(2),$	$\nu = 0.614(29)(3)$

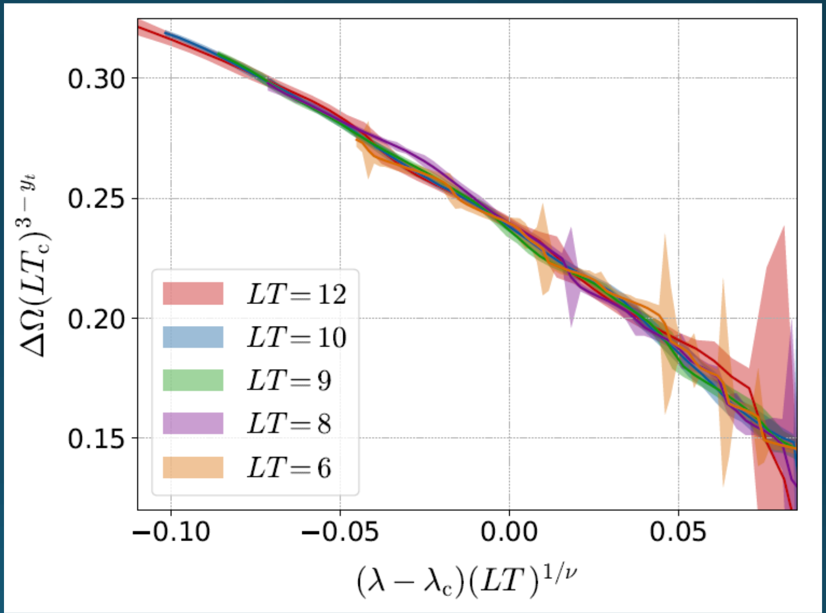
- ☐ B_4 and ν are consistent with Z2 universality class only when $LT \geq 9$ data are used for the analysis.

Further Check of Finite-V Scaling

- Effective potential at the CP



- Scaling of order parameter



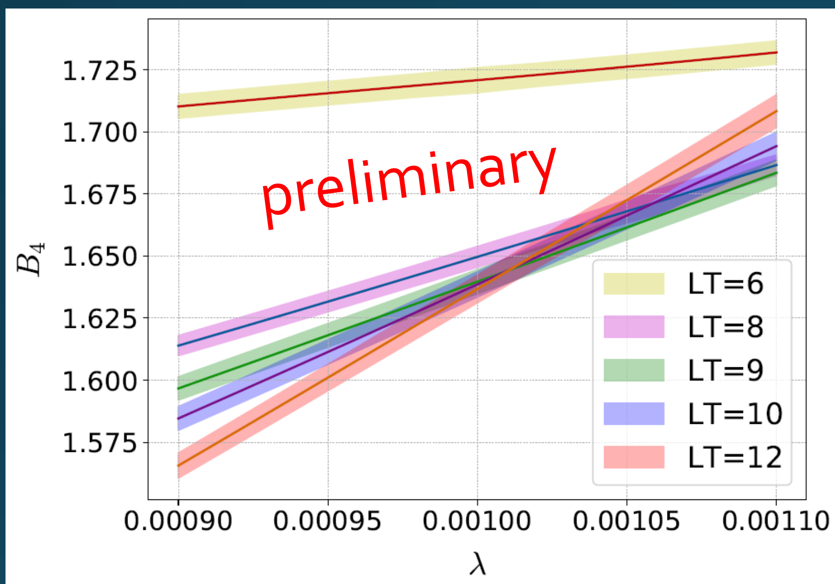
- Z2 scaling is well established
- Deviation from Z2 may come from "non-singular" part that are not negligible when V is not large enough.

Lattice Spacing Dependence

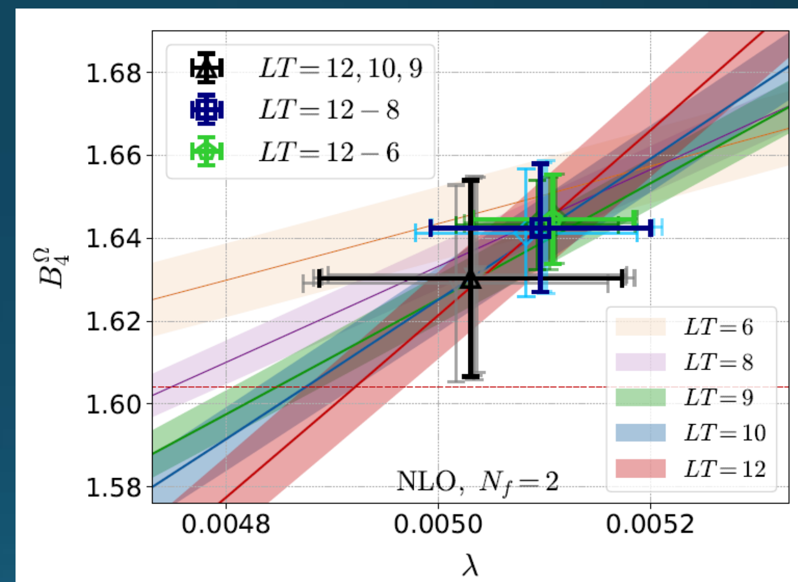
New result for $N_t = 6$ (Ashikawa+, 2022)

$$a = \frac{1}{N_t T}$$

$N_t = 6$



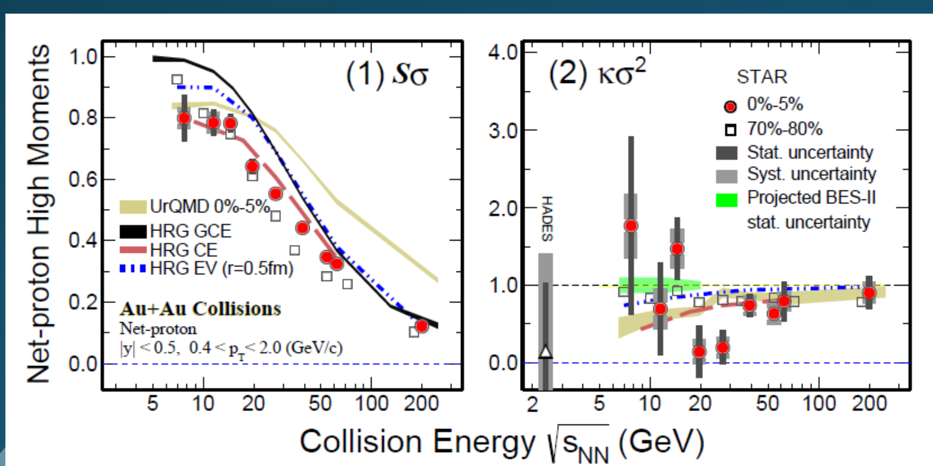
$N_t = 4$

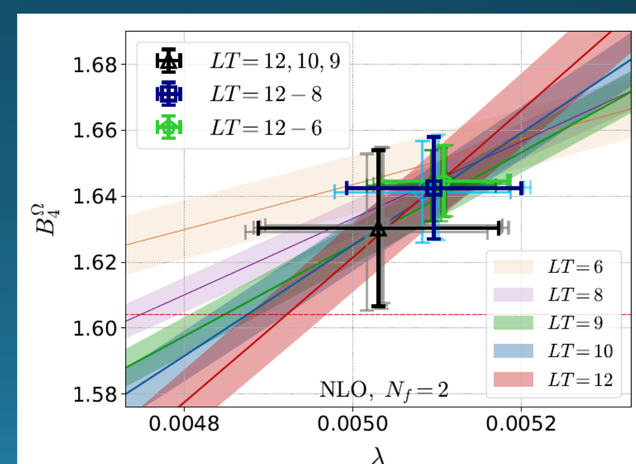


- On the finer lattice, deviation from the finite-size scaling becomes larger with the same V.
- Violation of the HPE → See, WHOT, PTEP2022, 033B05

Summary

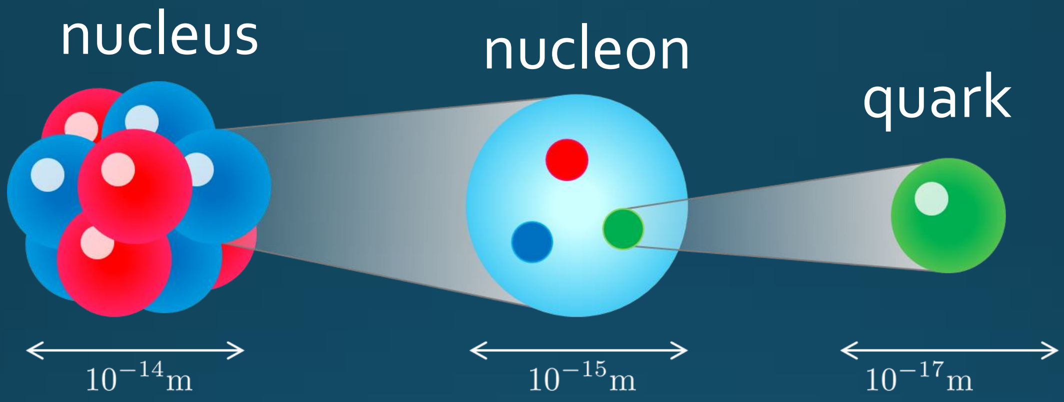
- Critical points appear many places in QCD at nonzero temperature.
- Two experimental tools for the search for the CPs:
  - Relativistic heavy-ion collisions
  - Lattice QCD numerical simulations
- Various studies are ongoing using both real and virtual experiments.





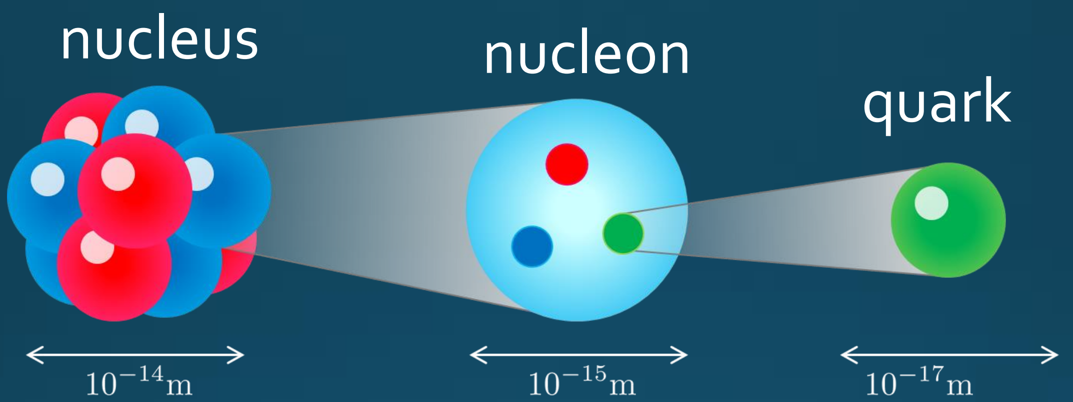
Quantum ChromoDynamics

- **Building blocks of matter**



Quantum ChromoDynamics

- **Building blocks of matter**



- **Quantum chromodynamics (QCD)**

=Fundamental theory of strong interaction

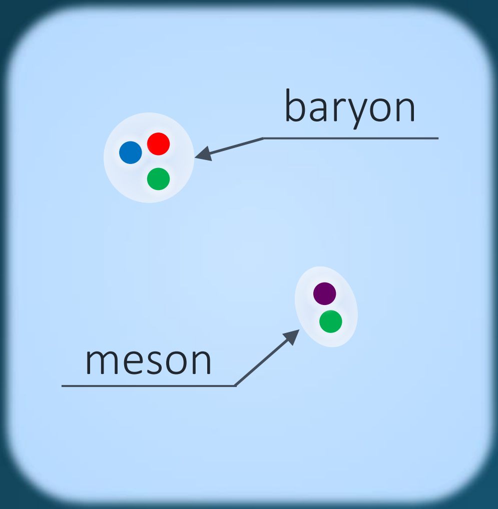
$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu,a}F_a^{\mu\nu}$$

quarks ($\bar{\psi}$, ψ)

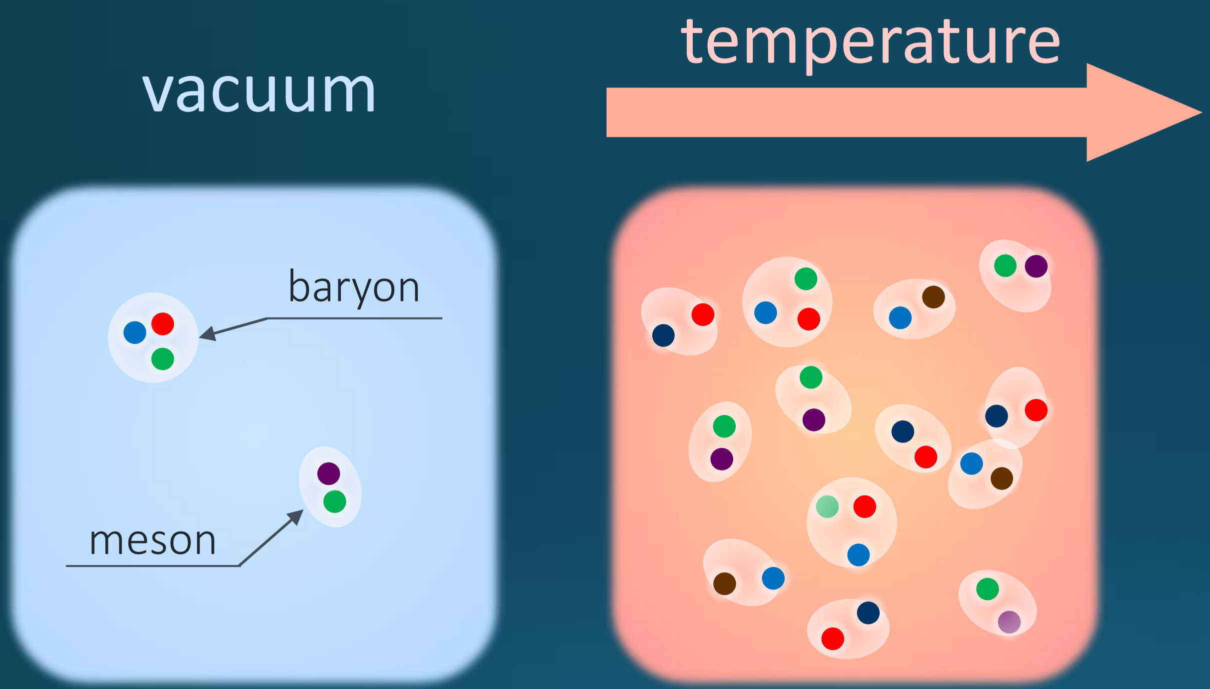
gluons ($F_{\mu\nu,a}F_a^{\mu\nu}$)

Quark-Gluon Plasma (QGP)

vacuum



Quark-Gluon Plasma (QGP)



Quark-Gluon Plasma (QGP)