Real & Virtual Experiments to Explore Non-Perturbative Aspects of QCD

Masakiyo Kitazawa

(from Osaka U., Aug. '22)

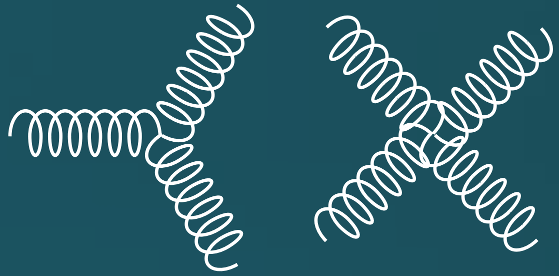
YITP Lunch Seminar, 2022/10/19

QCD

Fundamental Theory of Strong Interaction

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu,a}F_a^{\mu\nu}$$

Gluon self-interaction

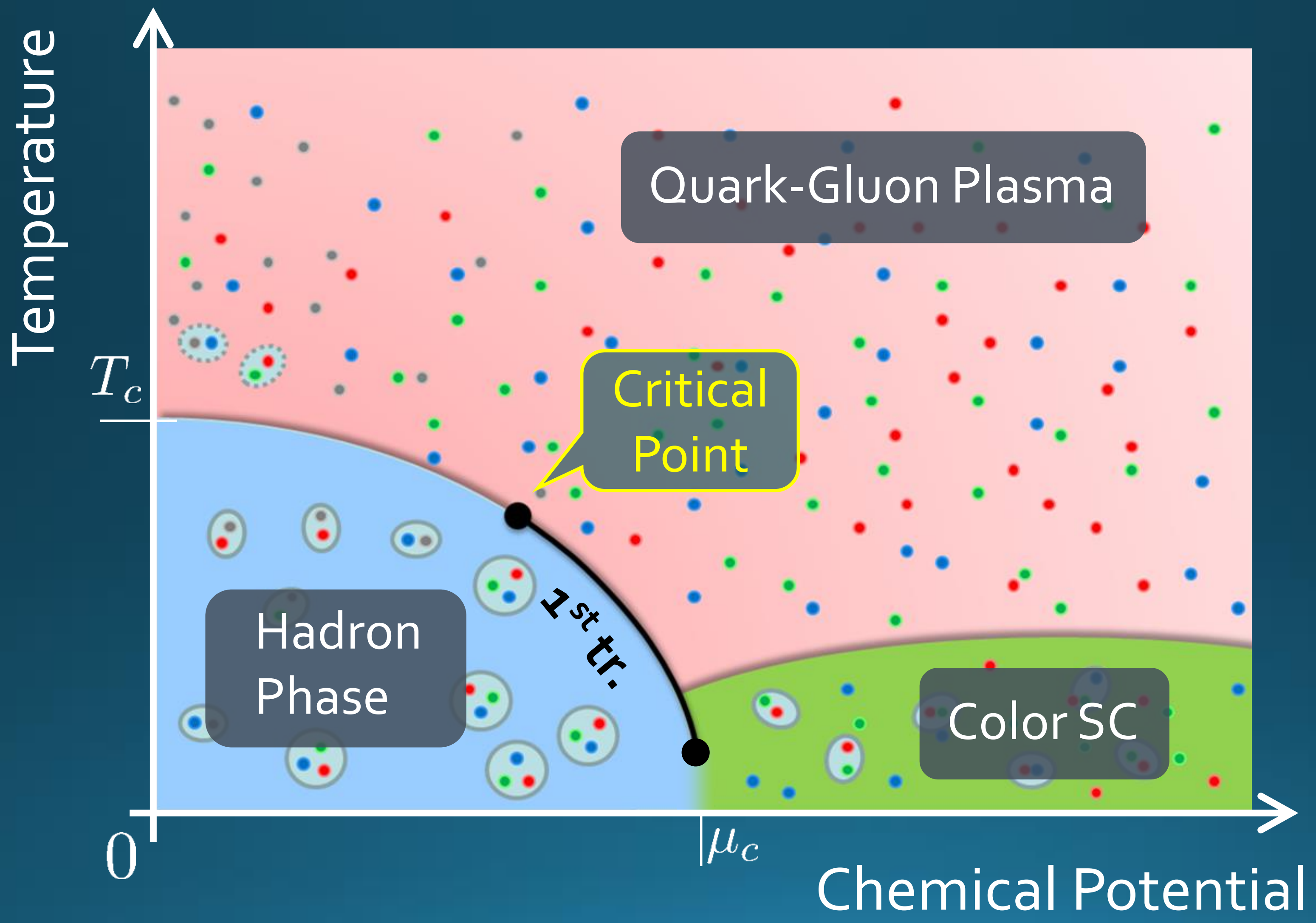


- ☐ asymptotic freedom
- ☐ non-perturvative vacuum
- ☐ color confinement
- ☐ chiral symmetry breaking
- ☐ phase transition at finite T, μ
- ☐ …

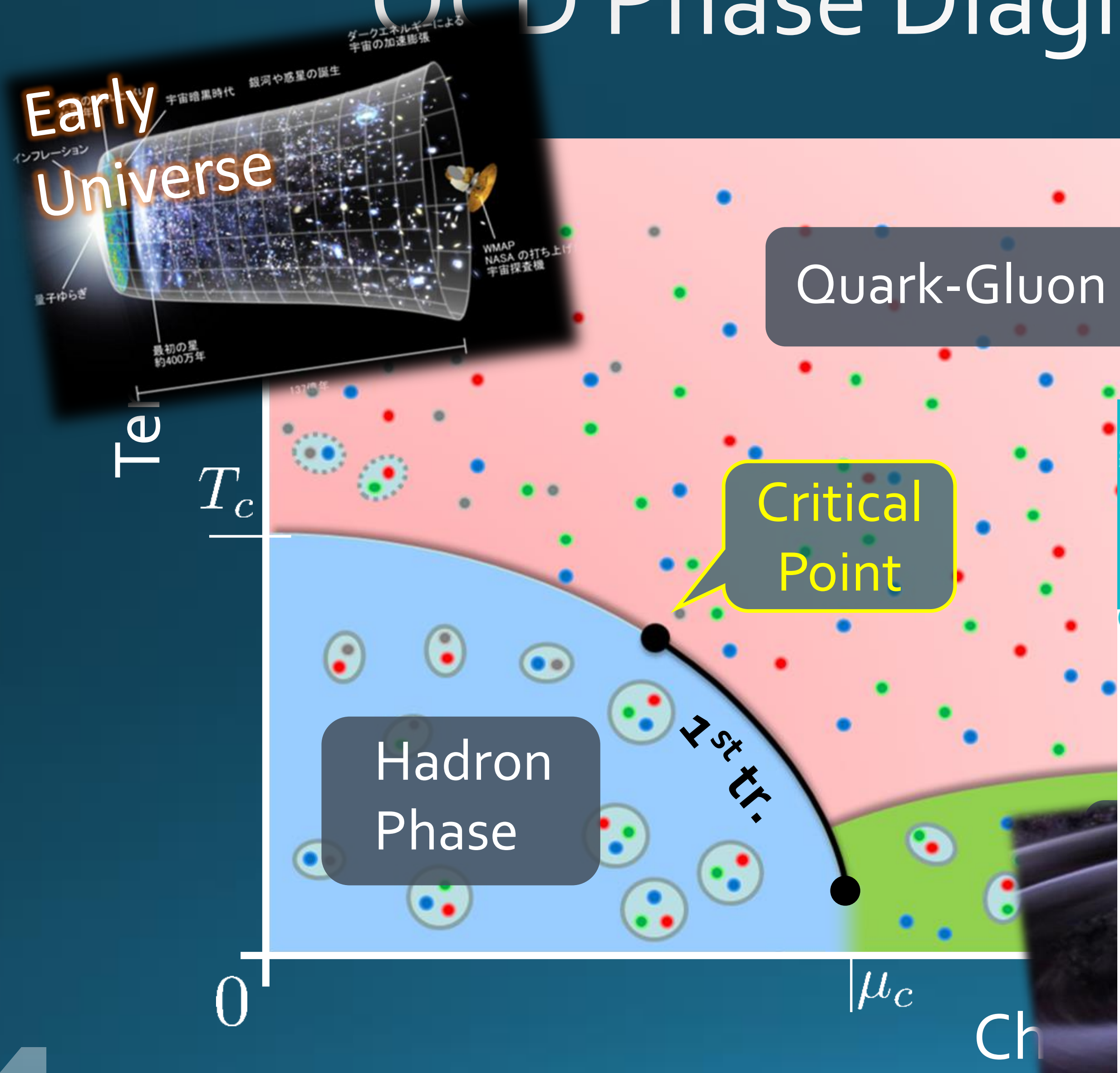
- ☐ Hadron structure
- ☐ Neutron stars
- ☐ early Universe
- ☐ …

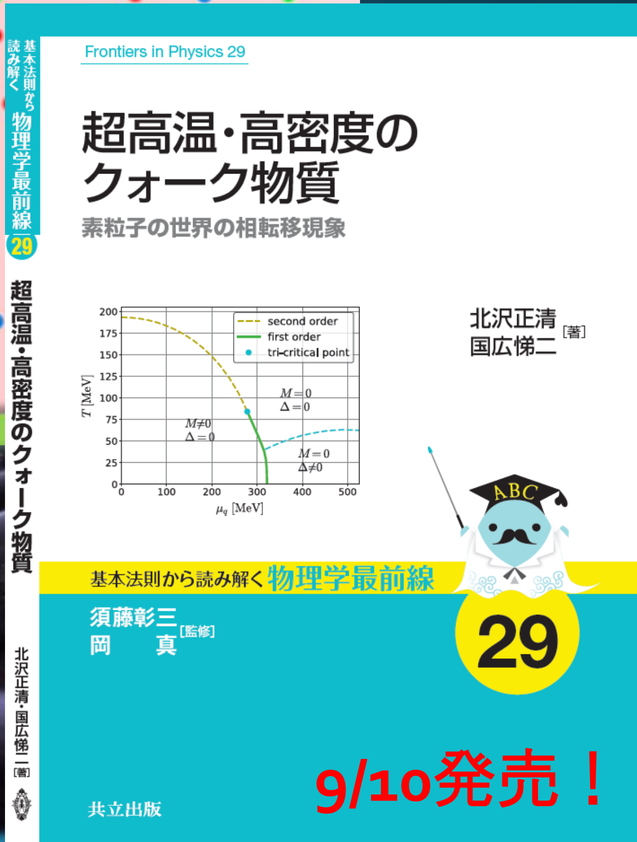
Complemental use of experiments, simulations, as well as theories, is indispensable!

QCD Phase Diagram



QCD Phase Diagram




Frontiers in Physics 29

超高温・高密度の
クォーク物質

素粒子の世界の相転移現象

北沢正清
国広悌二［著］

基本法則から読み解く物理学最前線

須藤彰三
岡　　真［監修］

29

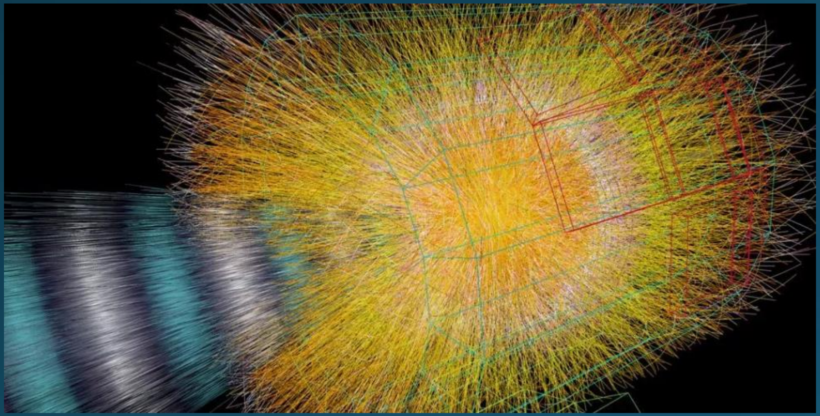
9/10発売！

共立出版

基本法則から読み解く物理学最前線 29 超高温・高密度のクォーク物質 北沢正清・国広悌二［著］

Two "Experimental" Tools
to explore hot & dense medium

Relativistic Heavy-Ion Collisions



Real Experiment
"HIC"

Lattice QCD Numerical Simulations

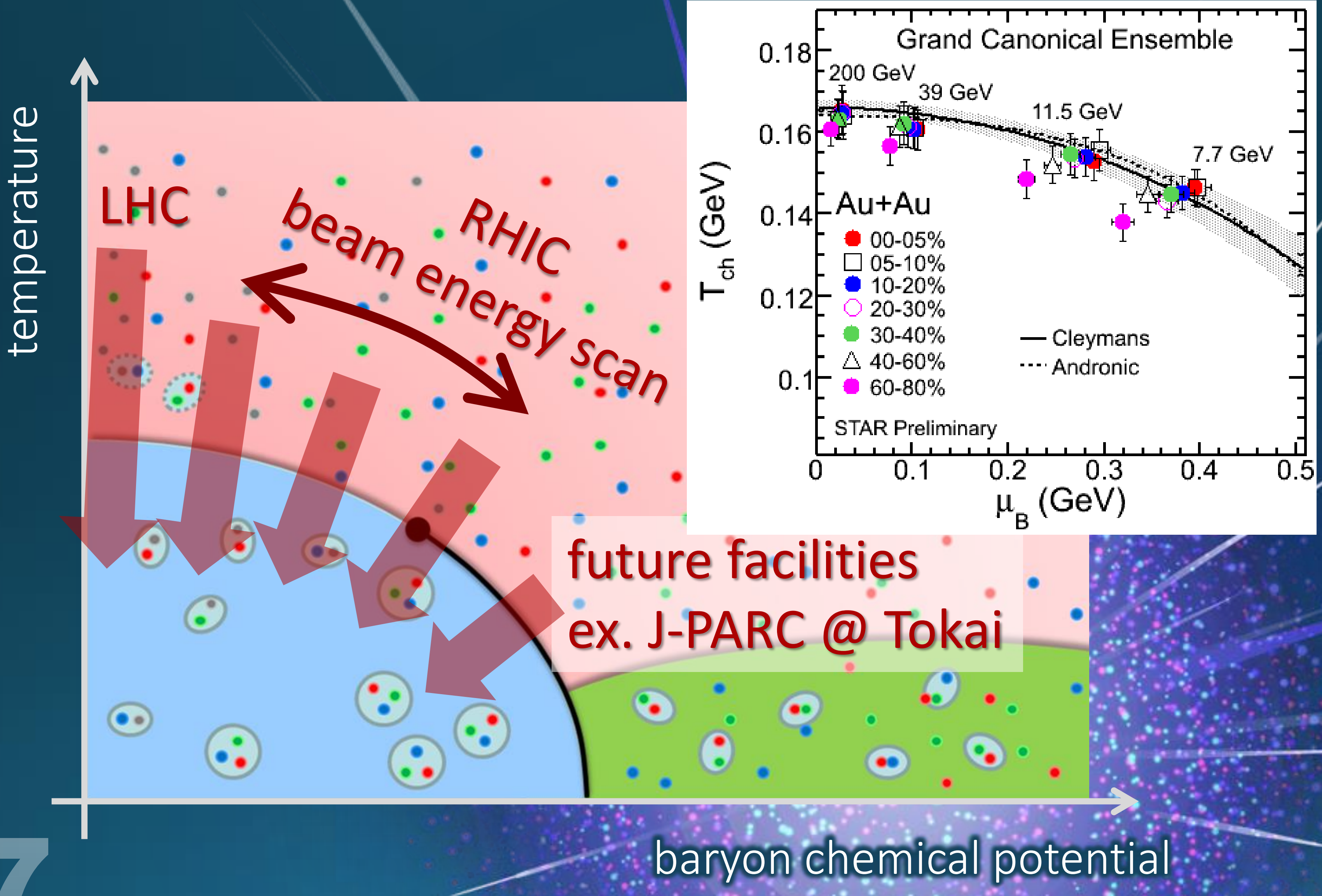
Virtual Experiment
"LAT"

Their complementary use is crucial!

Topic 1

Experimental Search for QCD Phase Structure

Beam-Energy Scan



temperature

LHC

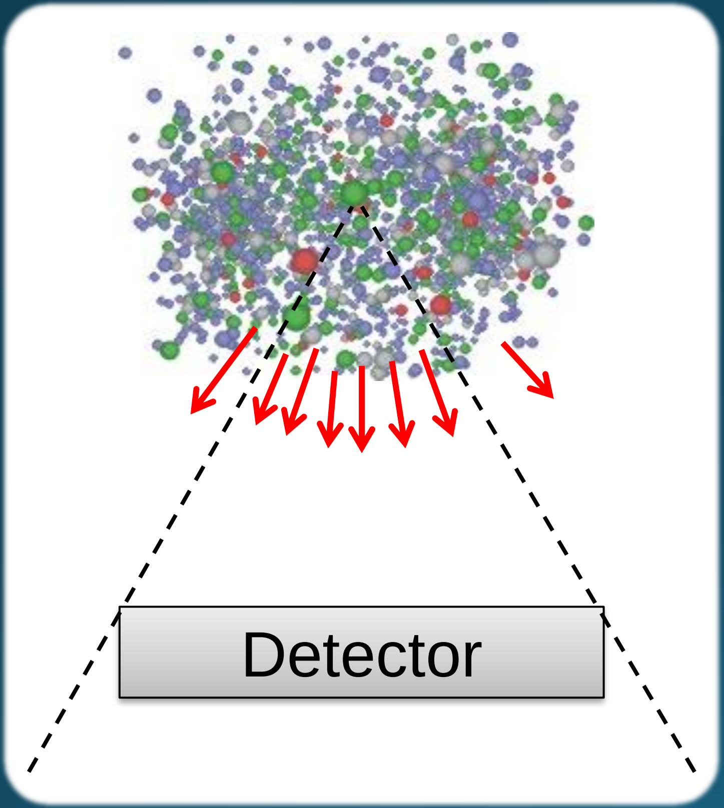
RHIC

beam energy scan

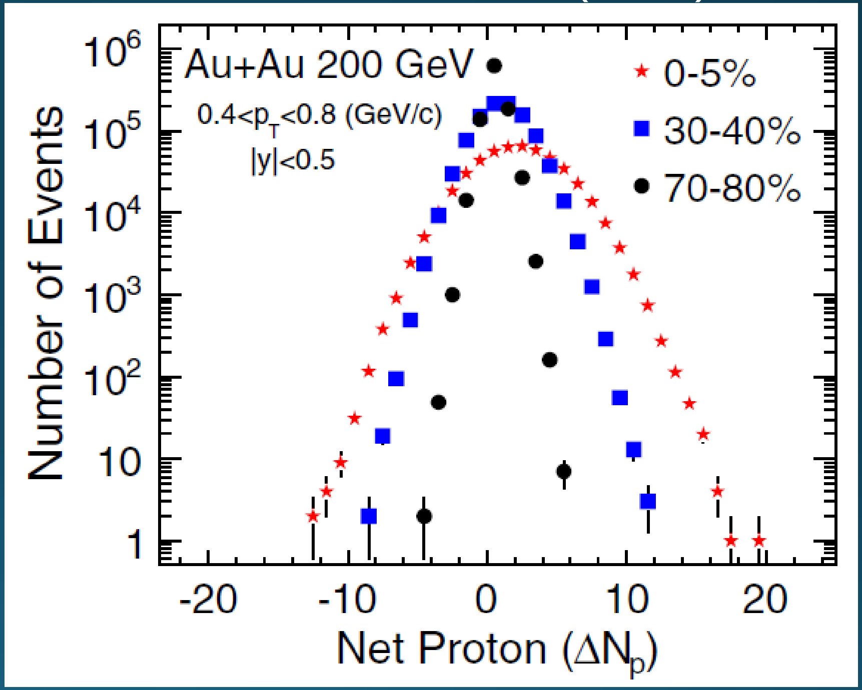
future facilities
ex. J-PARC @ Tokai

baryon chemical potential

Event-by-event Fluctuations



STAR, PRL**105** (2010)



Cumulants

$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$

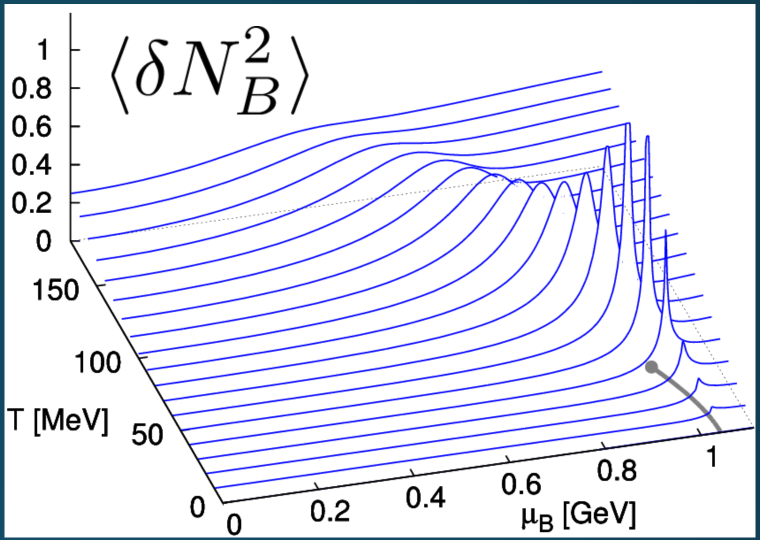
Non-Gaussian Cumulants

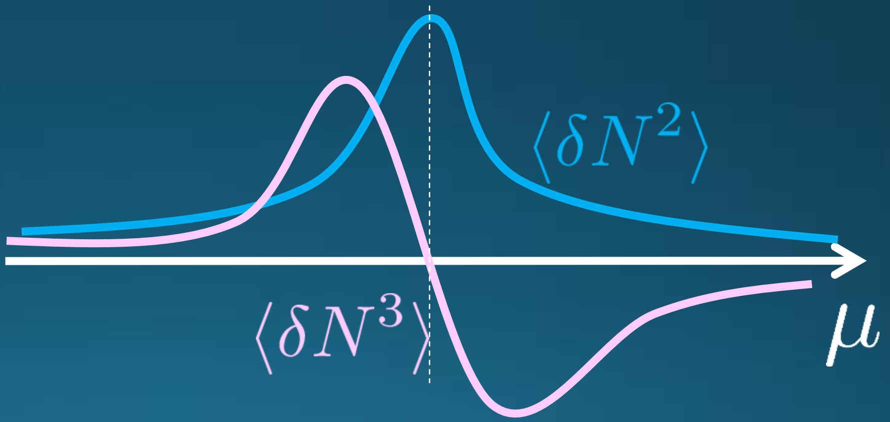
Gaussian fluctuations diverge at the QCD-CP

- Higher order cumulants change sign at the phase boundary

$$\langle \delta N^3 \rangle = T \frac{\partial \langle \delta N^2 \rangle}{\partial \mu}$$

Asakawa, Ejiri, MK, 2009



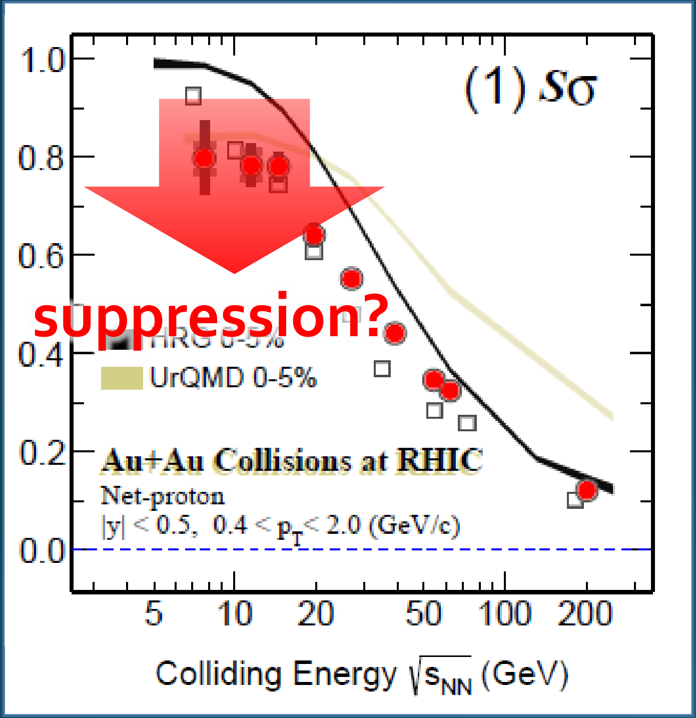


- Steeper divergence for higher-order cumulants

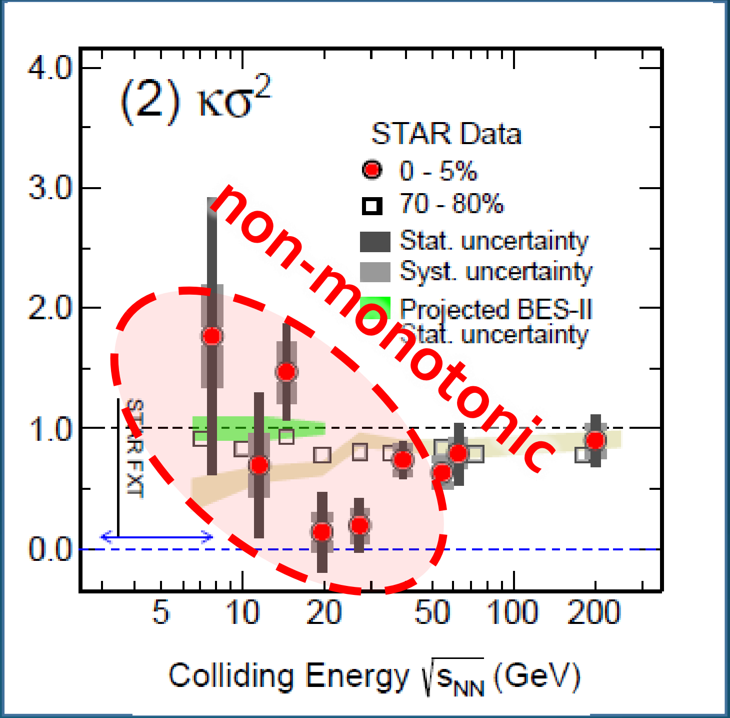
Stephanov, 2009

Proton Number Cumulants in HIC

$$\langle N_p^3 \rangle_c / \langle N_p^2 \rangle_c$$



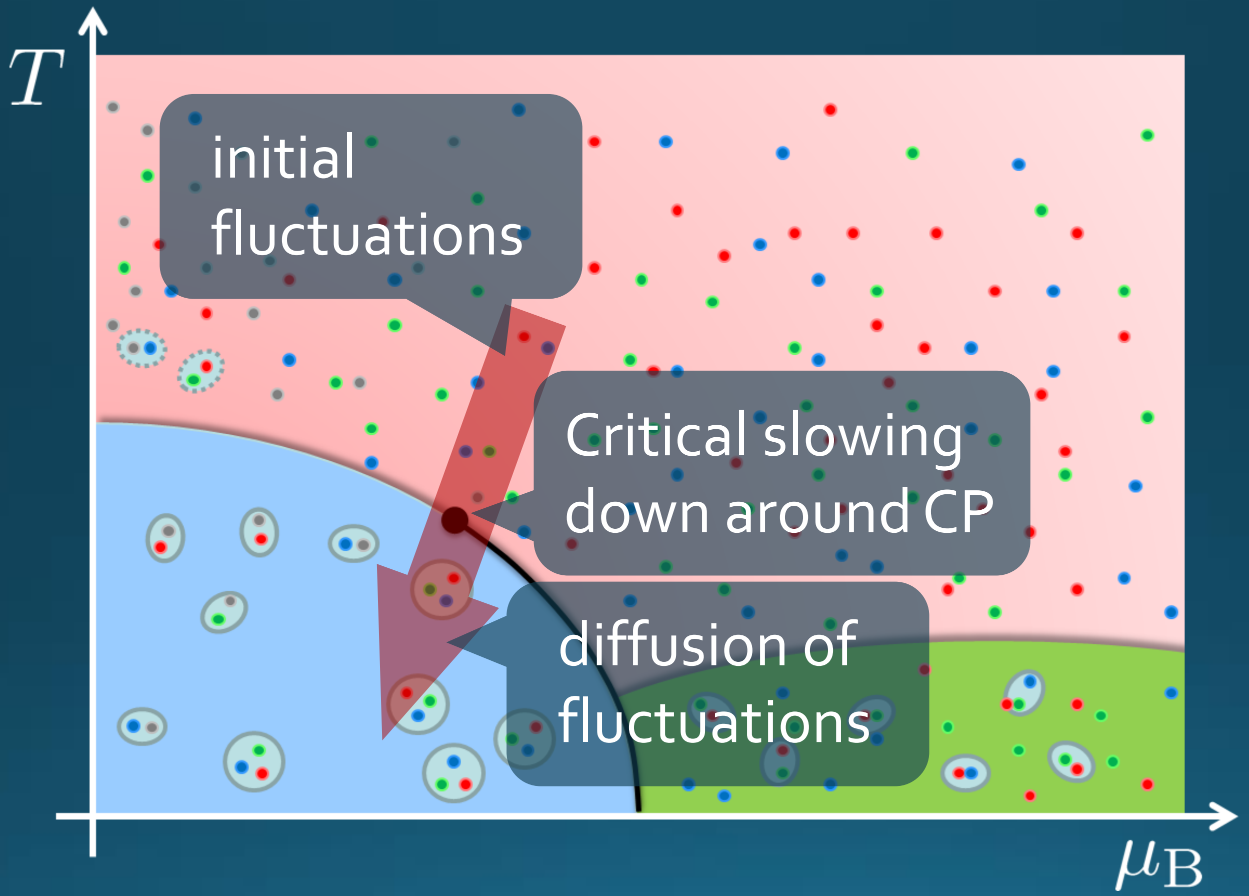
$$\langle N_p^4 \rangle_c / \langle N_p^2 \rangle_c$$



STAR, PRC 2020 [2001.06419]

☐ **Nonzero and non-Poissonian cumulants are experimentally established.**

Time Evolution of Cumulants



initial fluctuations

Critical slowing down around CP

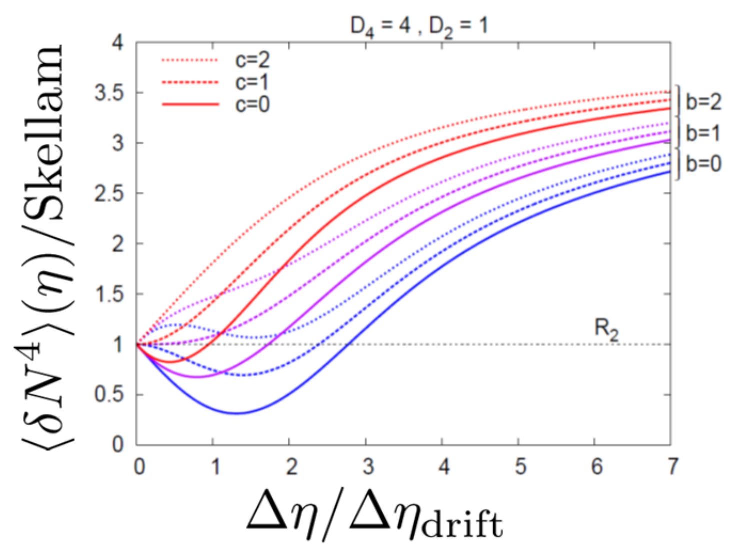
diffusion of fluctuations

Proper understanding of the time evolution of fluctuations is indispensable.

Rapidity Window Dependence
in Diffusion Models

☐ **Higher order cumulants**

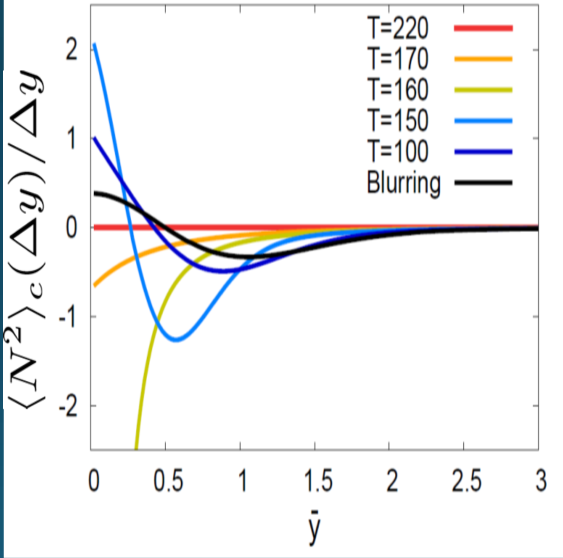
in diffusion master equation
MK+, PLB ('14); MK, NPA ('15)



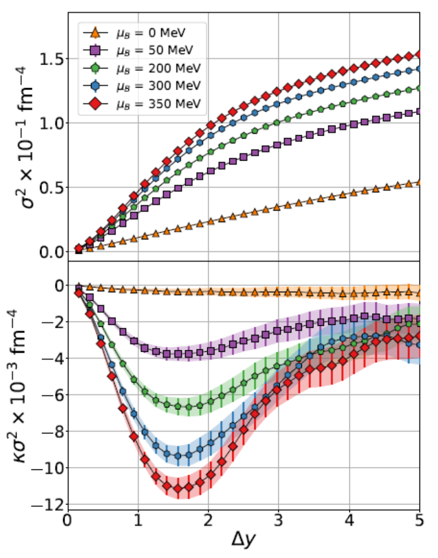
☐ **Evolution near CP**

in stochastic diffusion equation

Sakaida+ ('18)

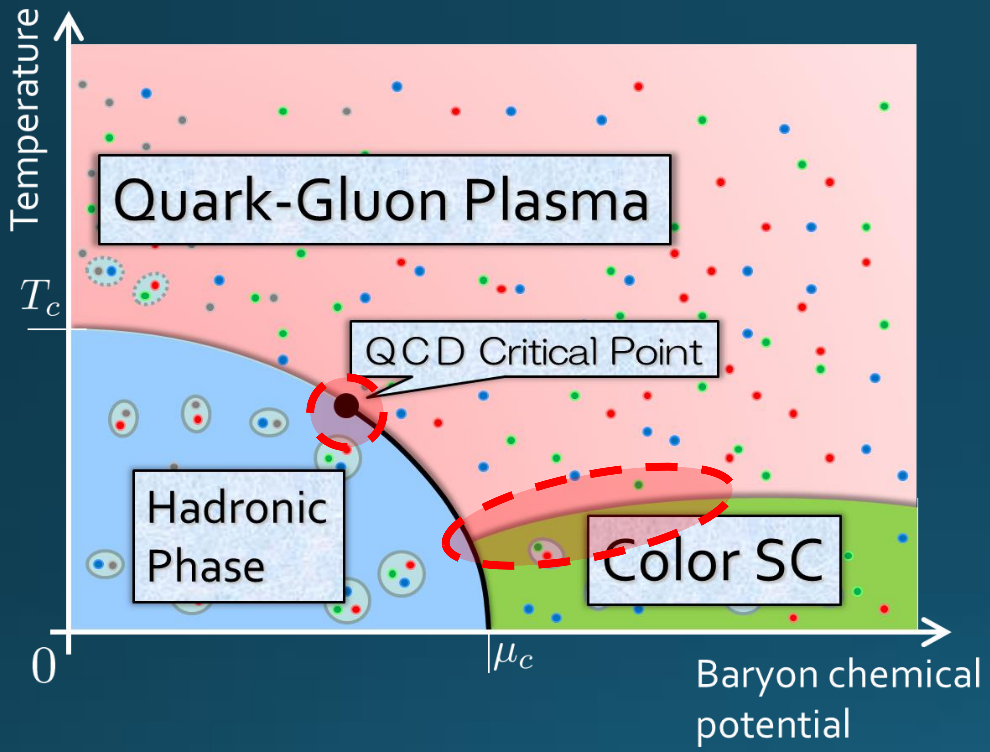


Pihan+, 2205.12834



☐ Non-monotonic Δy dependence can emerge reflecting the dynamical history.

Measurement of Soft Modes using (virtual) Photon Emission



Nishimura, MK, Kunihiro, PTEP '22; in prep.

2nd-order PT at

☐ QCD-CP

☐ Color superconductivity

Formation of soft modes

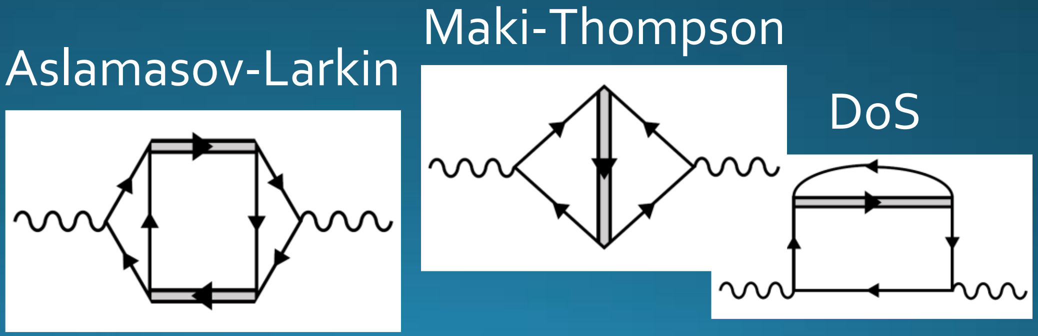
fluctuation of order parameter

Photon emission?

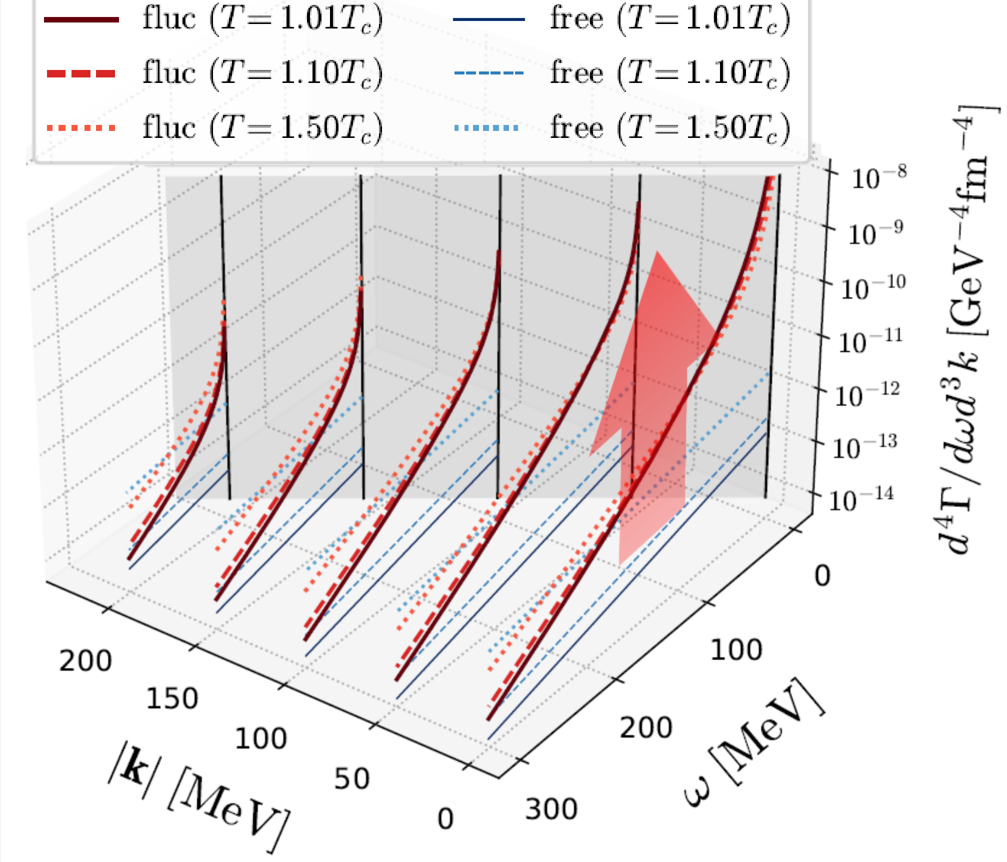
☐ Photon self-energy

→Photon (dilepton) emission

$$\Pi^{\mu\nu}(k) =$$



Dilepton Production Rates



$$\frac{d^4\Gamma}{dk^4}(\omega, k)$$

Red: fluctuation contribution

Blue: free quarks

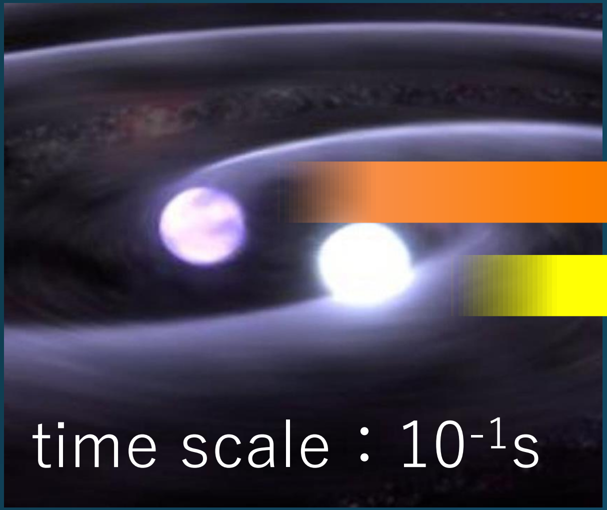
$$\mu = 350 \text{ MeV}$$

$$G_C = 0.7G_S, \quad (T_c \simeq 43 \text{ MeV})$$

Nishimura, MK, Kunihiro,
PTEP, 2201.01963

- Diquark fluctuations give rise to anomalous enhancement in the low energy-momentum region for $T < 1.5T_c$.

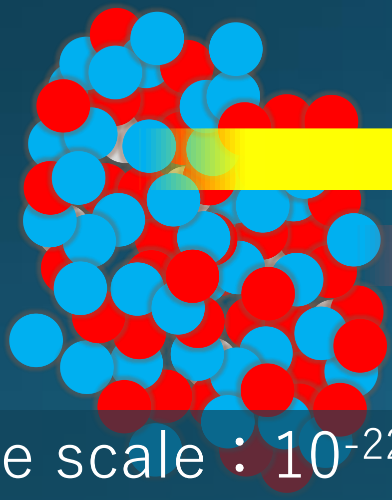
Multi-Messenger Observation



time scale : 10^{-1}s

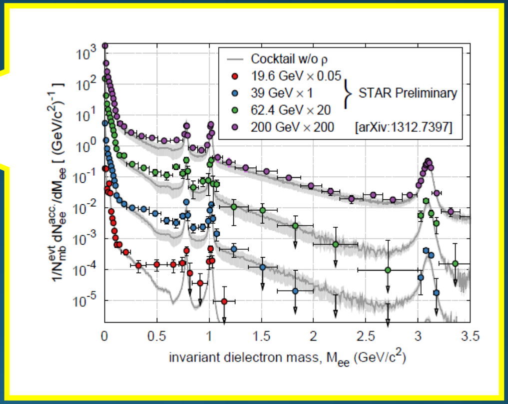
gravitational waves

electromagnetic waves



time scale : 10^{-22}s

leptons,
photons



hadronic observables

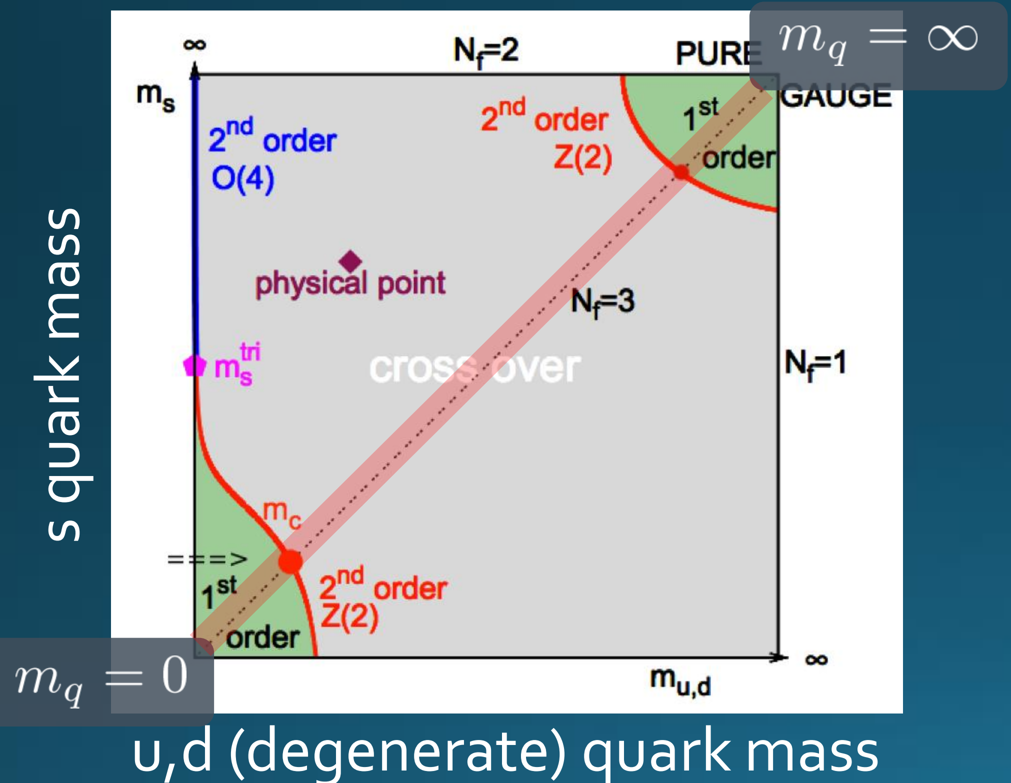
Topic 2

QCD Phase Structure at Unphysical Quark Masses

Varying Quark Masses at $\mu_q = 0$

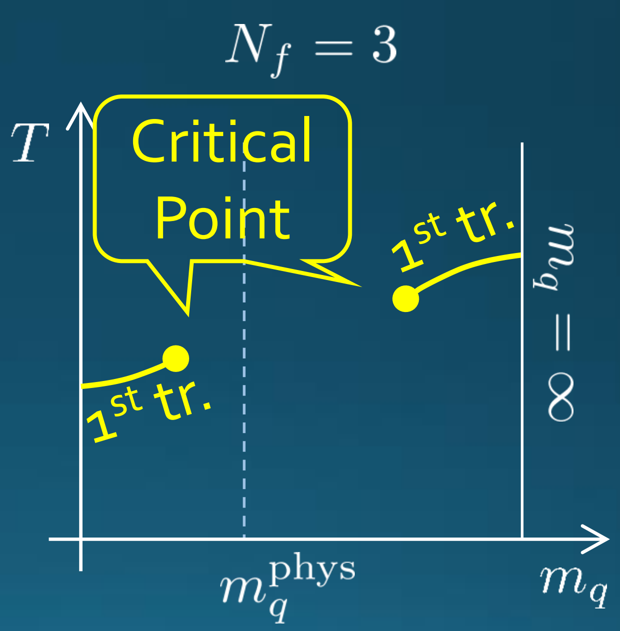
- **Columbia plot**

  = order of phase tr. at $\mu = 0$



- **Phase Diagram**

  on the $T - m_q$ plane

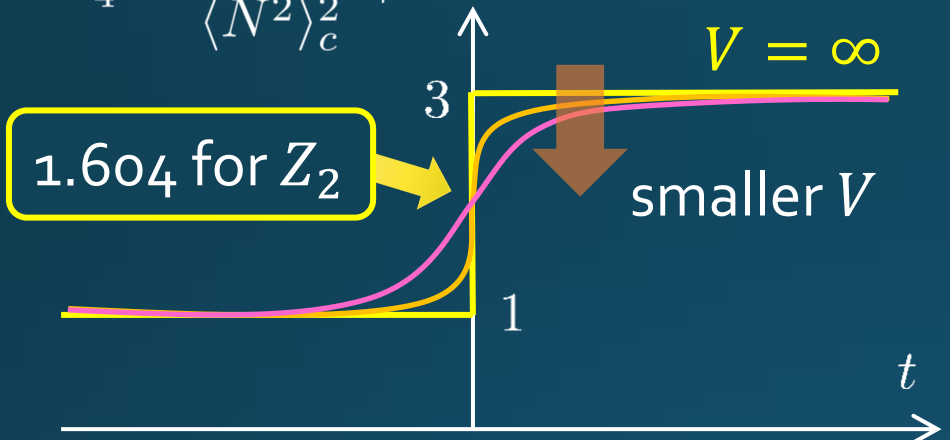


Various orders of phase transition with variation of m_q.

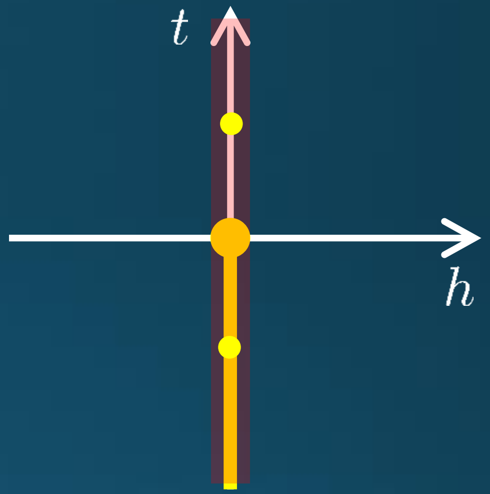
Finite-Size Scaling / Binder Cumulant

Binder Cumulant

$$B_4 = \frac{\langle N^4 \rangle_c}{\langle N^2 \rangle_c^2} + 3$$



Ising Model



- ☐ Sudden change of B_4 is smeared by the finite-size effect.
- ☐ B_4 obtained for various V has crossing at $t = 0$.
- ☐ At the crossing point, $B_4 = 1.604$ in Z_2 universality class.

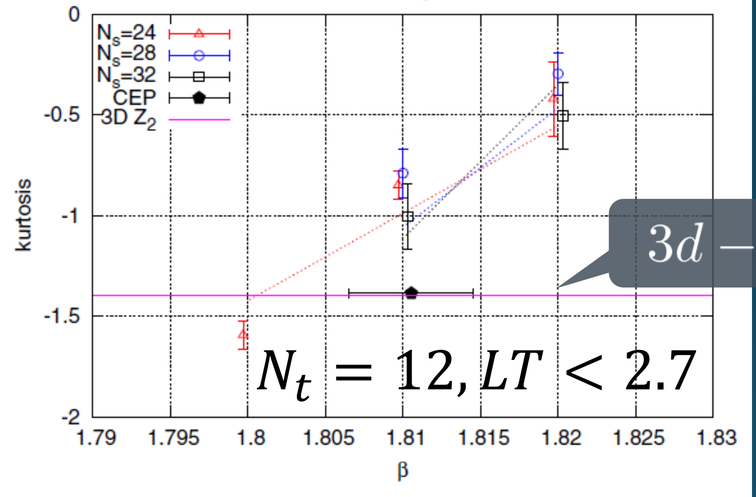
Binder-Cumulant Analysis

- ☐ **Light-quark region**

  Kuramashi, Nakamura, Ohno, Takeda, ’20

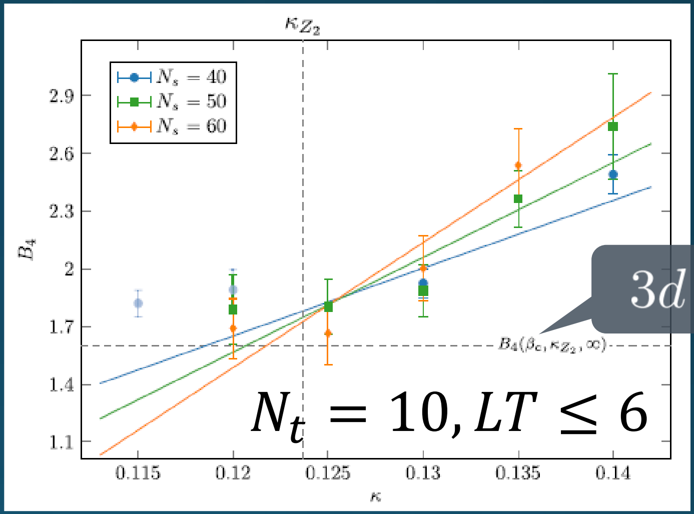
- ☐ **Heavy-quark region**

  Cuteri , Philipsen , Schön, Sciarra, ’21



$3d - Z_2$

$N_t = 12, LT < 2.7$

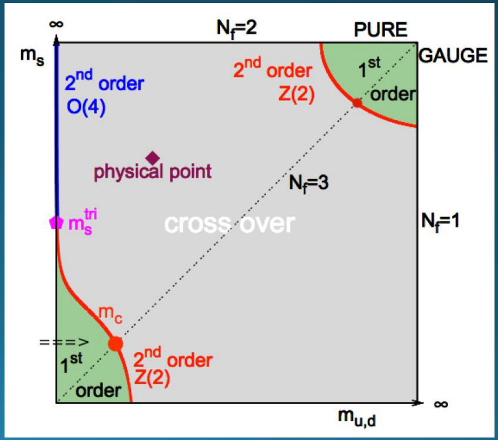


$3d - Z_2$

$N_t = 10, LT \leq 6$

- ☐ Statistically-significant deviation of the crossing point from the 3d-Ising value.

  ➡ Large non-singular contribution?

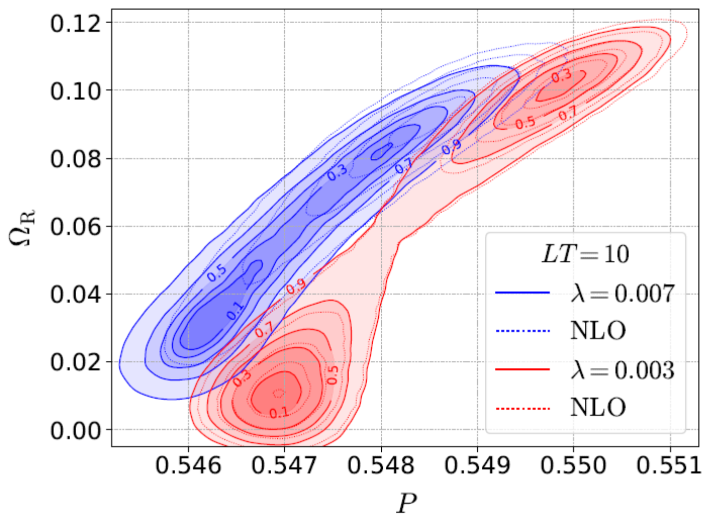


Numerical Simulation

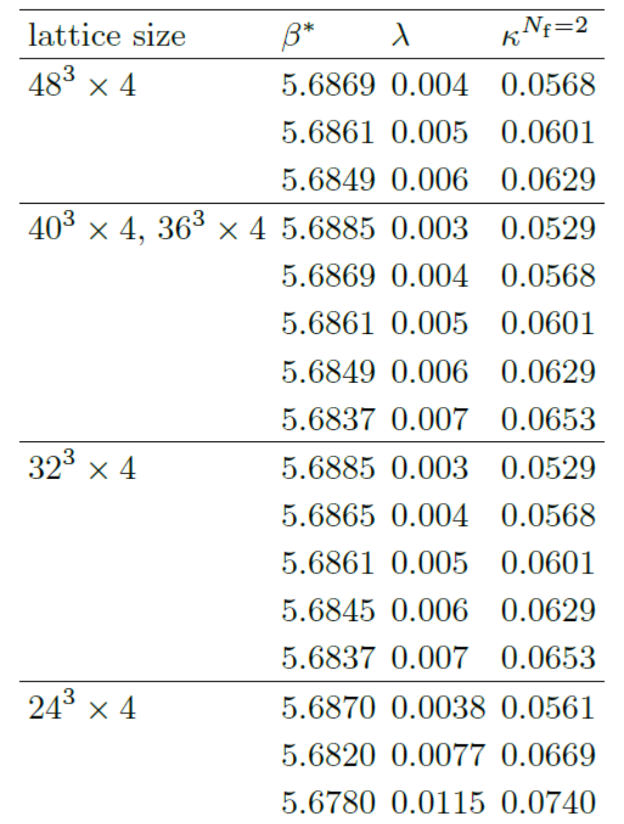
Kiyohara, MK, Ejiri, Kanaya, PRD, 2021

- ☐ **Coarse lattice:** $N_t = 4$
- ☐ But **large spatial volume**:
  $LT = N_s \,/\, N_t \leq 12$

- ☐ Hopping-param. (~$1/m_q$) expansion
- ☐ Monte-Calro with LO action
- ☐ High statistical analysis



Simulation params.

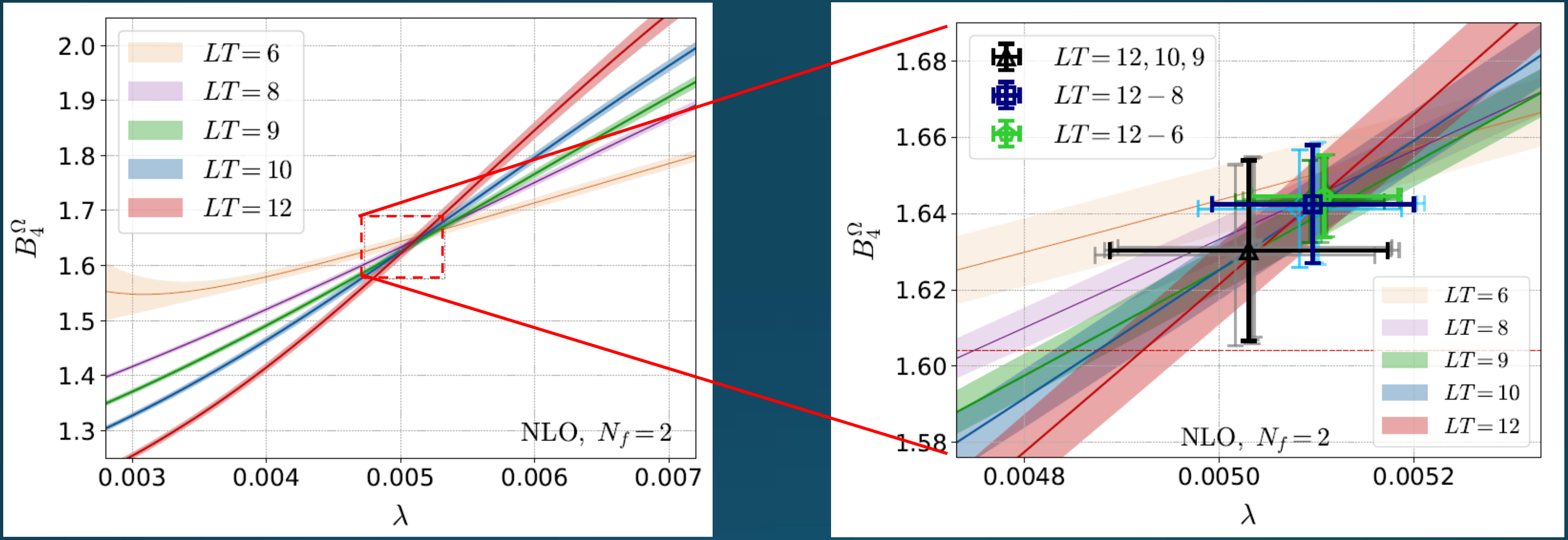


lattice size	β^*	λ	$\kappa^{N_f=2}$
$48^3 \times 4$	5.6869	0.004	0.0568
	5.6861	0.005	0.0601
	5.6849	0.006	0.0629
$40^3 \times 4$, $36^3 \times 4$	5.6885	0.003	0.0529
	5.6869	0.004	0.0568
	5.6861	0.005	0.0601
	5.6849	0.006	0.0629
	5.6837	0.007	0.0653
$32^3 \times 4$	5.6885	0.003	0.0529
	5.6865	0.004	0.0568
	5.6861	0.005	0.0601
	5.6845	0.006	0.0629
	5.6837	0.007	0.0653
$24^3 \times 4$	5.6870	0.0038	0.0561
	5.6820	0.0077	0.0669
	5.6780	0.0115	0.0740

Binder-Cumulan

Seminar in Kyoto-NT group
15:00 Today, Physics bldg. 5F

Kiyohara, MK, Ejiri, Kanaya, PRD, 2021



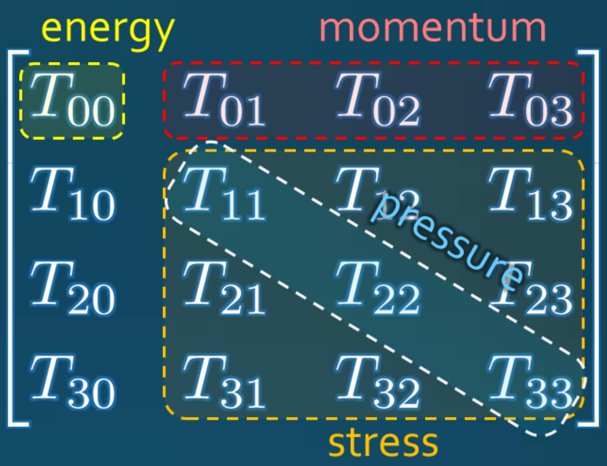
Z2	$B_4 = 1.604$	$\nu = 0.630$
$LT \geq 9$	$B_4 = 1.630(24)(2),$	$\nu = 0.614(48)(3)$
$LT \geq 8$	$B_4 = 1.643(15)(2),$	$\nu = 0.614(29)(3)$

☐ B_4 and ν are consistent with Z_2 universality class only when $LT \geq 9$ data are used for the analysis.

Topic 3

Lattice Simulations with Energy-Momentum Tensor

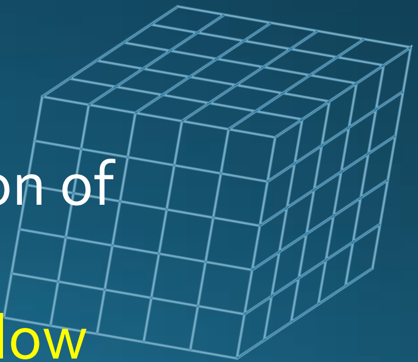
Eneargy-Momentum Tensor



$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

- ☐ **EMT operator on the lattice**
  - ☐ Naïve definition is problematic due to violation of translational symmetry on the lattice.
  - ☐ EMT operator defined through the gradient flow

Suzuki, 2013; FlowQCD, 2014~; WHOT-QCD, 2016~



Thermodynamics: $\varepsilon = \langle T_{00} \rangle,\ p = \langle T_{11} \rangle$

FlowQCD, 2014; 2016;
Iritani, MK, Suzuki, Takaura, 2019

☐ Conventional

from thermodynamic relations

$$p = \frac{T}{V} \ln Z$$

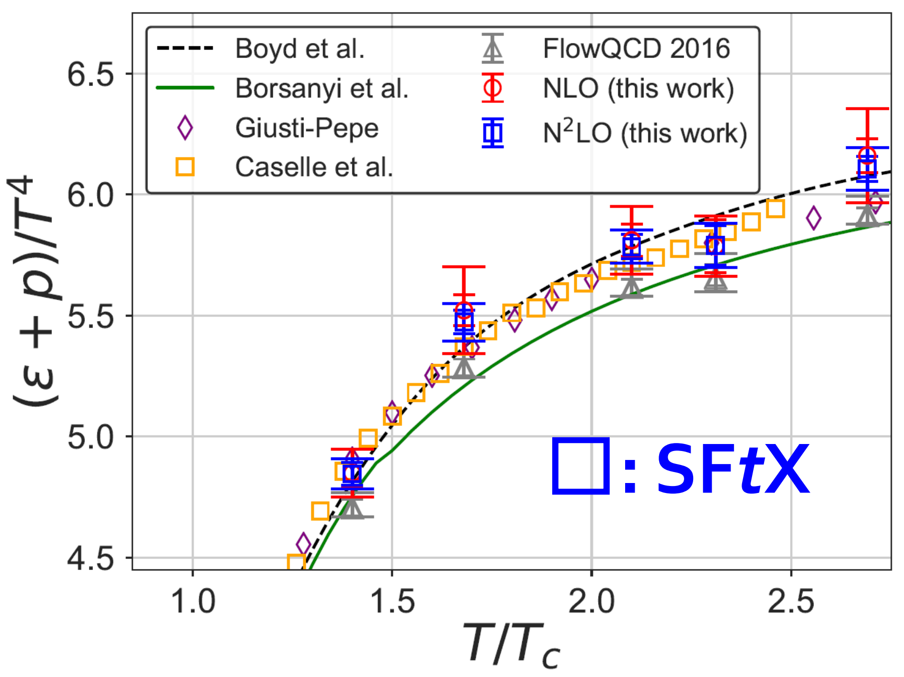
$$T \frac{\partial (p/T^4)}{\partial T} = \frac{\varepsilon - 3p}{T^4}$$

☐ Our method (SFtX)

expectation value of $T_{\mu\nu}$

$$\varepsilon = \langle T_{00} \rangle$$

$$p = \langle T_{11} \rangle$$



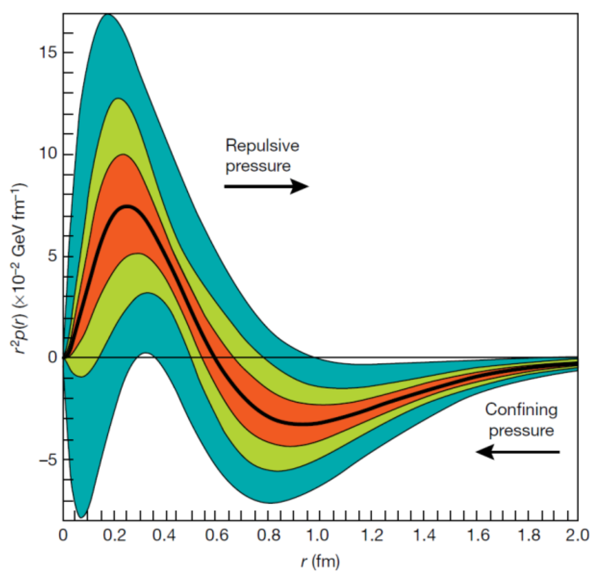
Thermodynamics quantities measured by completely different method agrees within 1% level.

EMT Distribution in Localized Systems

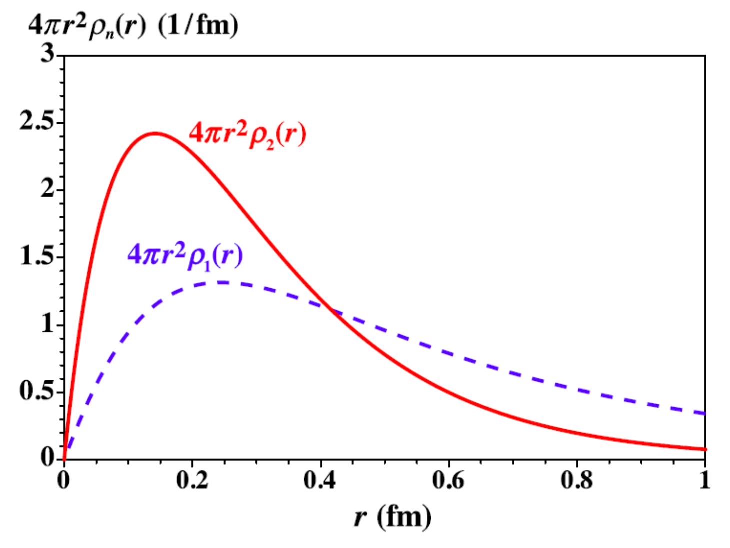
EMT distribution inside hadrons now accessible??

Pressure @ proton

EMT distribution @ pion



Nature, 557, 396 (2018)

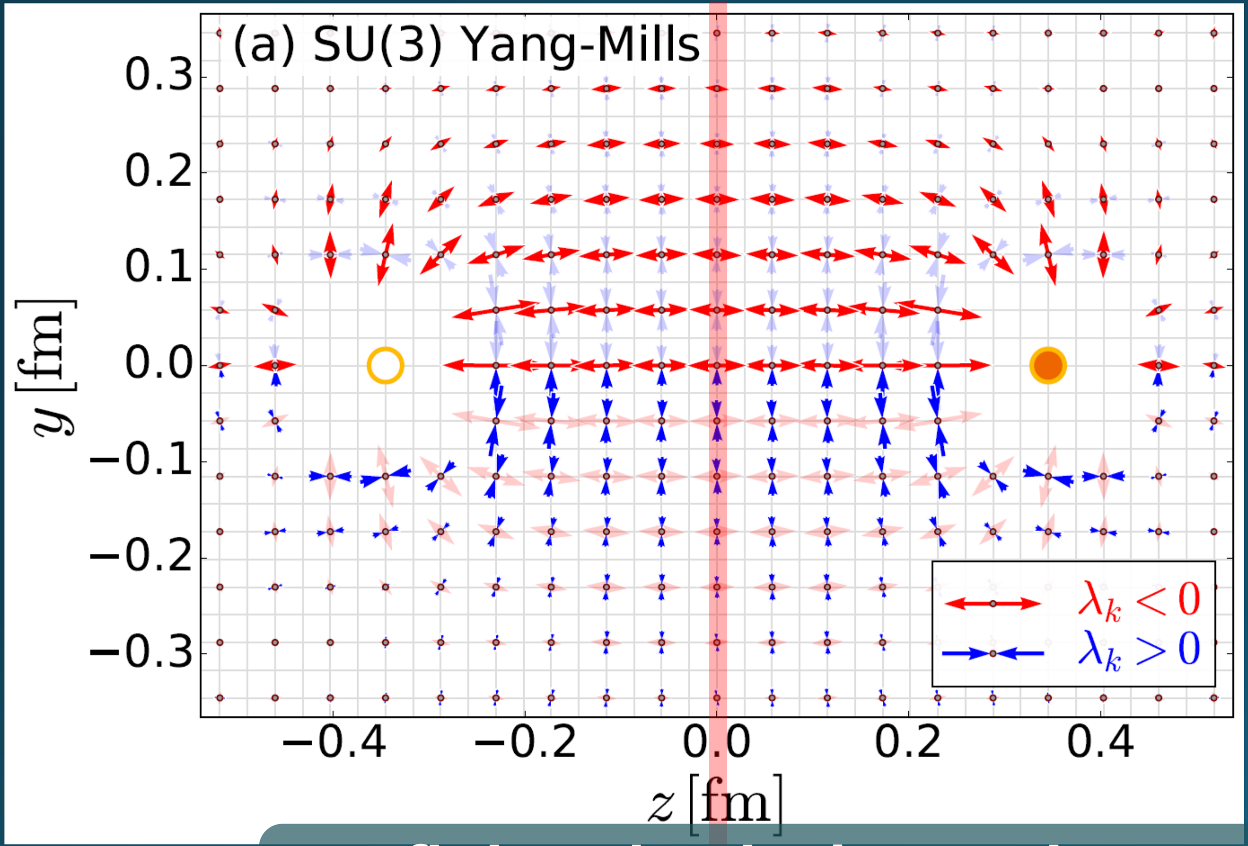


Kumano, Song, Teryaev (2018)

☐ Measurement will be refined at the EIC.

Stress Tensor in $Q\bar{Q}$ System

FlowQCD, PLB (2019)



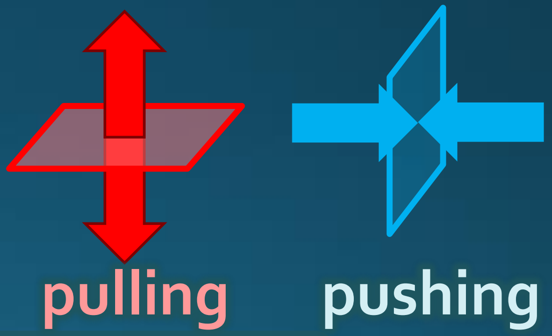
Lattice simulation

SU(3) Yang-Mills

a=0.029 fm

R=0.69 fm

$t/a^2=2.0$



Definite physical meaning

- ☐ Distortion of field, line of field
- ☐ Propagation of the force as local interaction
- ☐ Manifestly gauge invariant

Maxwell Stress

(in Maxwell Theory)

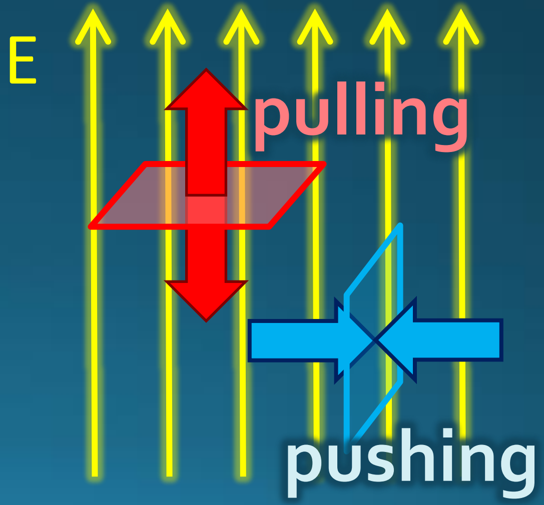
$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

Maxwell

$$\vec{E} = (E, 0, 0)$$

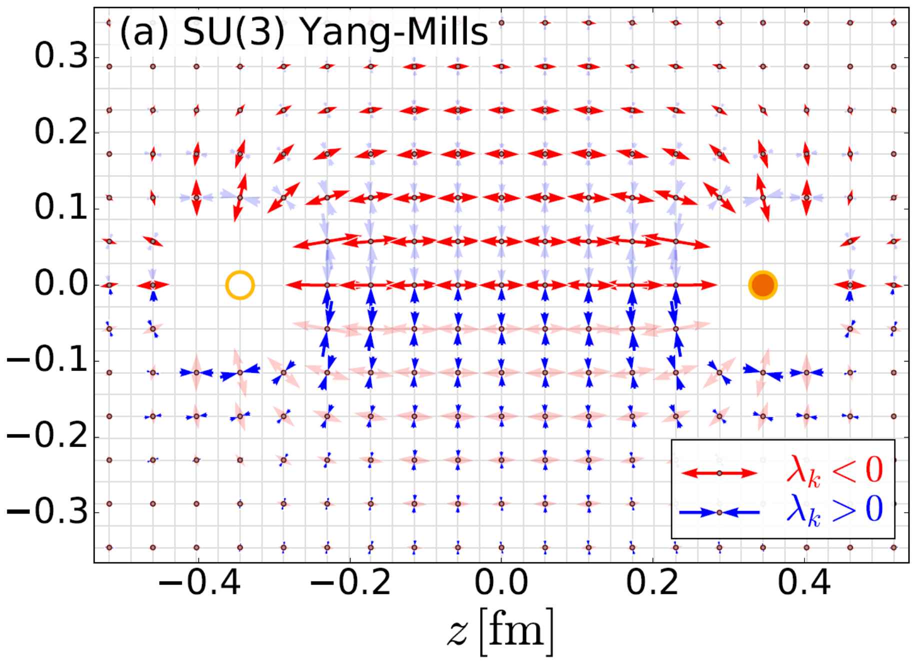
$$T_{ij} = \begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: **Pulling**
- Vertical to field: **Pushing**

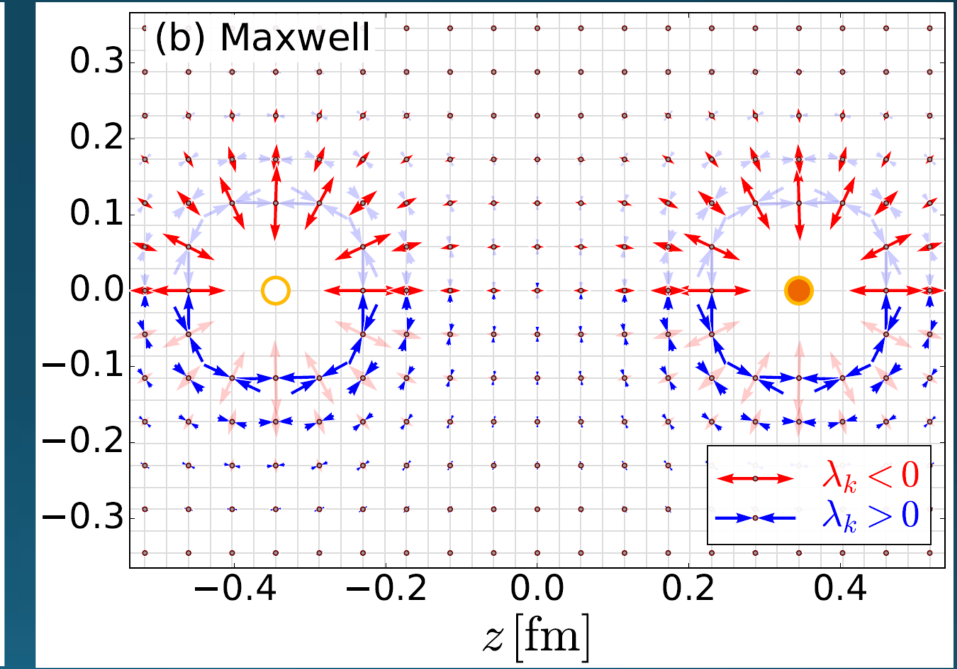


SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)

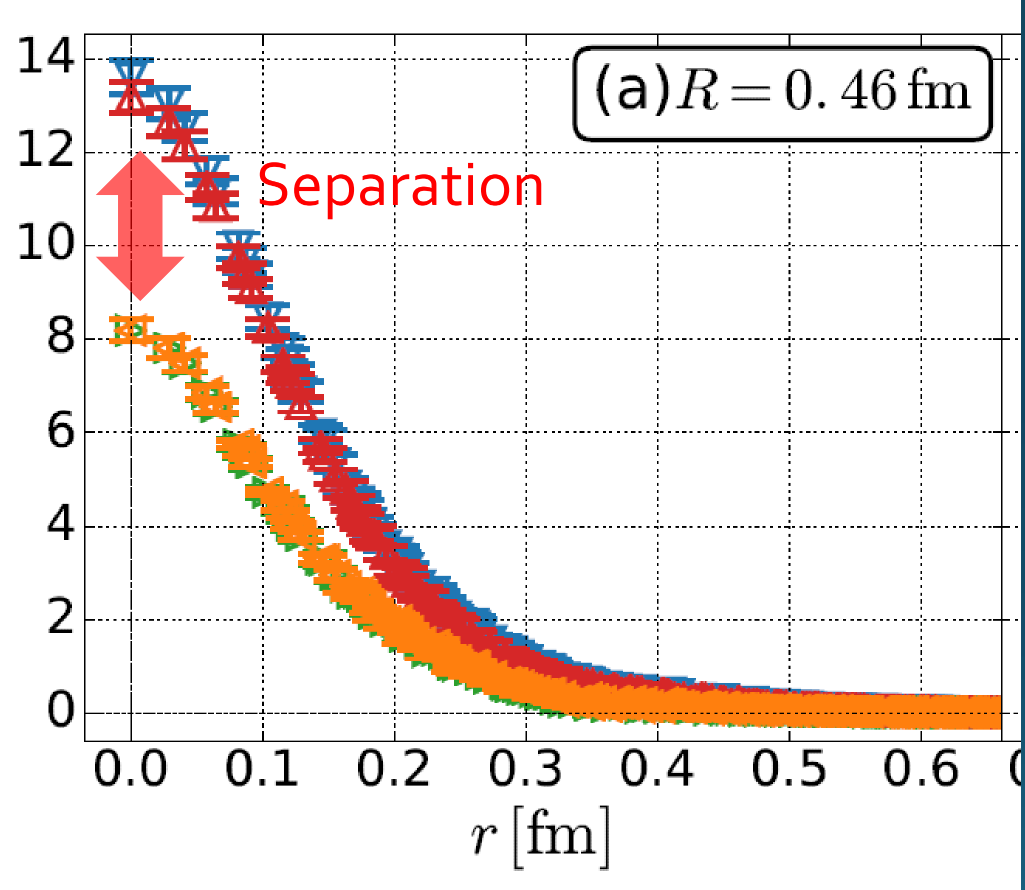


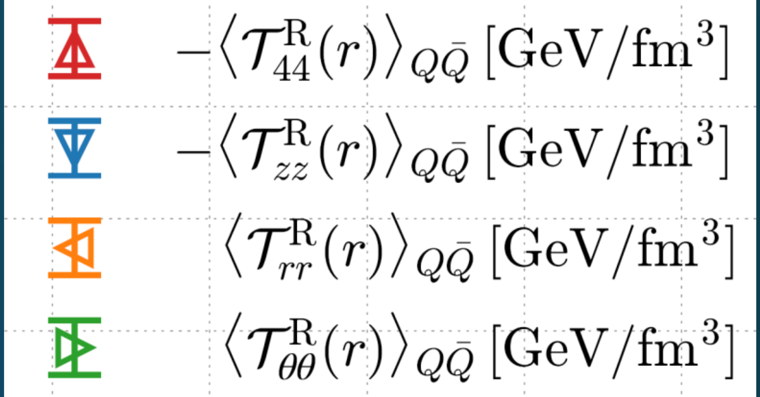
Maxwell
(classical)



Propagation of the force is clearly different
in YM and Maxwell theories!

Mid-Plane





Continuum Extrapolated!

In Maxwell theory

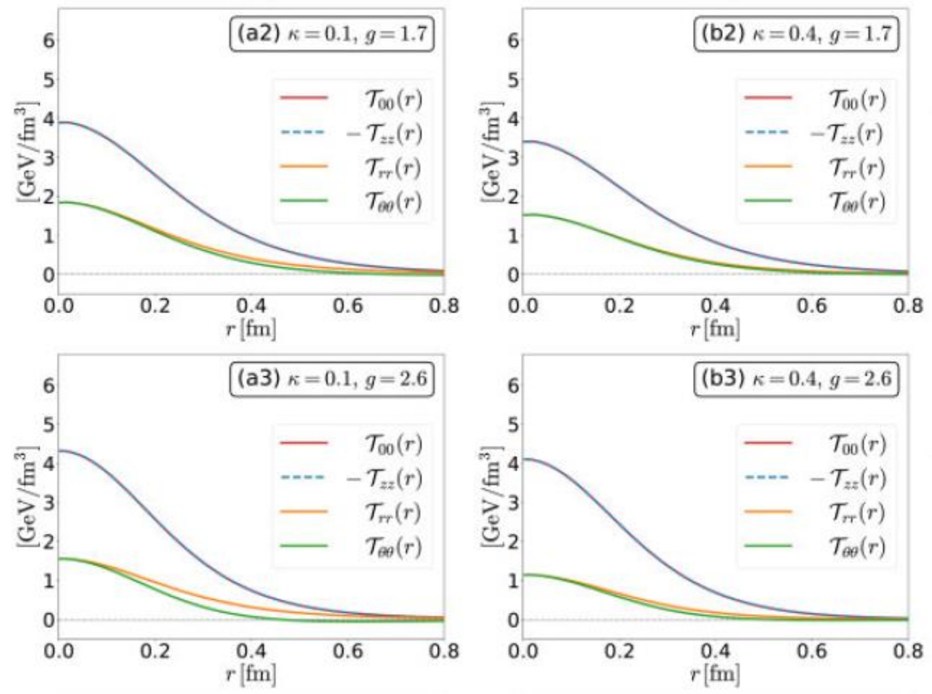
$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

- ☐ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- ☐ Separation: $T_{zz} \neq T_{rr}$
- ☐ Nonzero trace anomaly $\sum T_{cc} \neq 0$

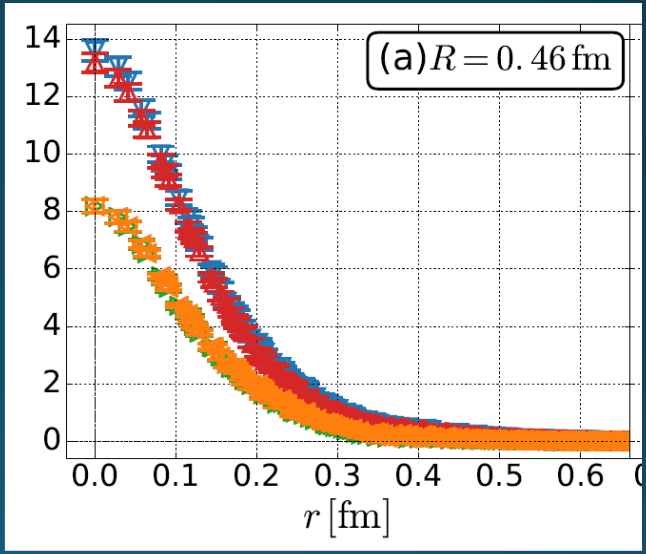
Flux Tube in Dual SC Picture

Yanagihara, MK (2019)

EMT in Abelian-Higgs model



Lattice



- ☐ AH model can reproduce lattice results **qualitatively**.
- ☐ But, all parameters are rejected **quantitatively**.

Quantum Effects?

- ☐ Classical vortex is unstable against quantum fluctuations
- ☐ Quantum effects give rise to
  - ☐ Luscher term in potential Luscher (1981)
  - ☐ Fattening of the tube Luscher, Munster, Weisz (1981)

$w^2 \sim \log r$

How do these effects modify EMT distribution?

EMT Distr. in Simple Systems

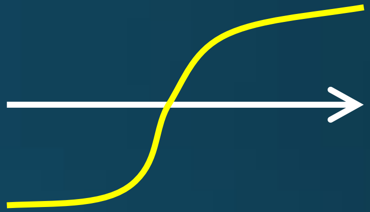
Ito, MK, in prep.

ϕ^4 Theory in 1+1d

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2$$

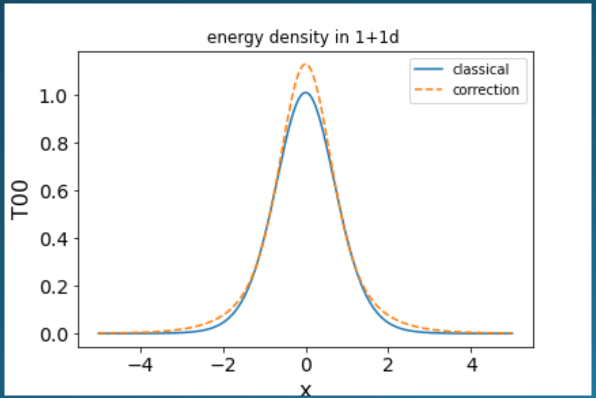
☐ **Soliton (kink)**

$$\phi(x) = \frac{m}{\sqrt{\lambda}} \tanh \frac{mx}{\sqrt{2}}$$



☐ **Quantum effect on EMT at 1-loop order**

$T_{00}(x)$



Confirmation of EMT conservation

$$\partial_x T_{11}(x) = 0$$

Final Comment

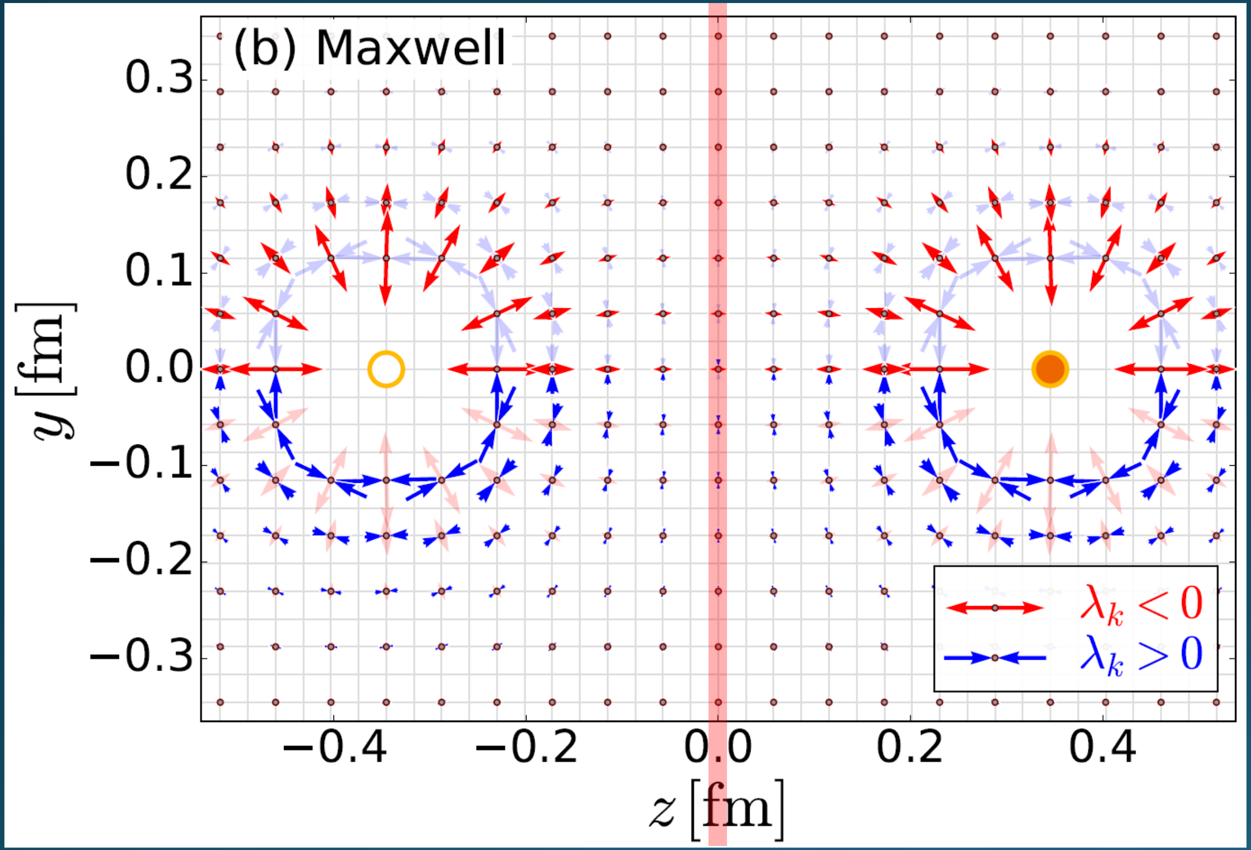
- ☐ Relativistic HIC and lattice simulations are invaluable tools for revealing non-perturbative aspects of QCD. Active researches are ongoing.

- ☐ Looking forward to exchanging research ideas with you!

Backup

Maxwell Stress

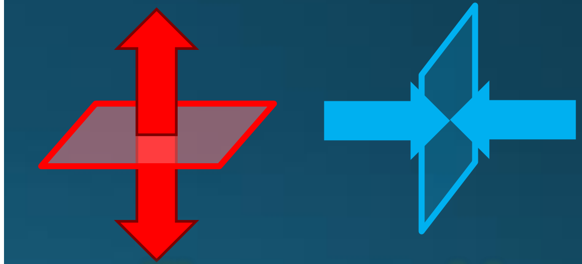
(in Maxwell Theory)



$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$$(k = 1, 2, 3)$$

length: $\sqrt{|\lambda_k|}$



pulling **pushing**

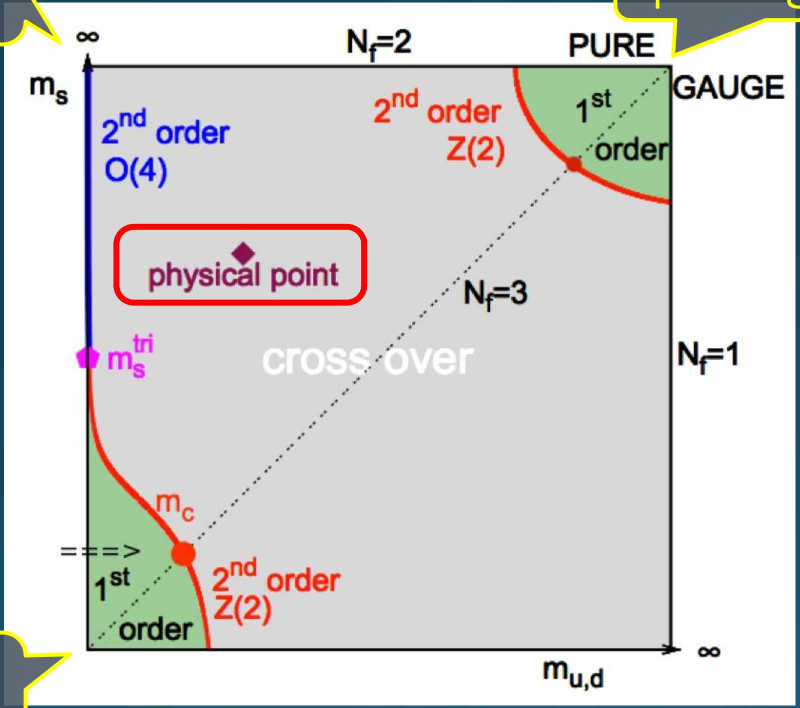
Definite physical meaning

☐ Distortion of field, line of the field

☐ Propagation of the force as local interaction

Columbia Plot

= order of phase tr. at $\mu = 0$



massless
2-flavor

pure gauge

massless
3-flavor

s quark mass

u,d (degenerate) quark mass