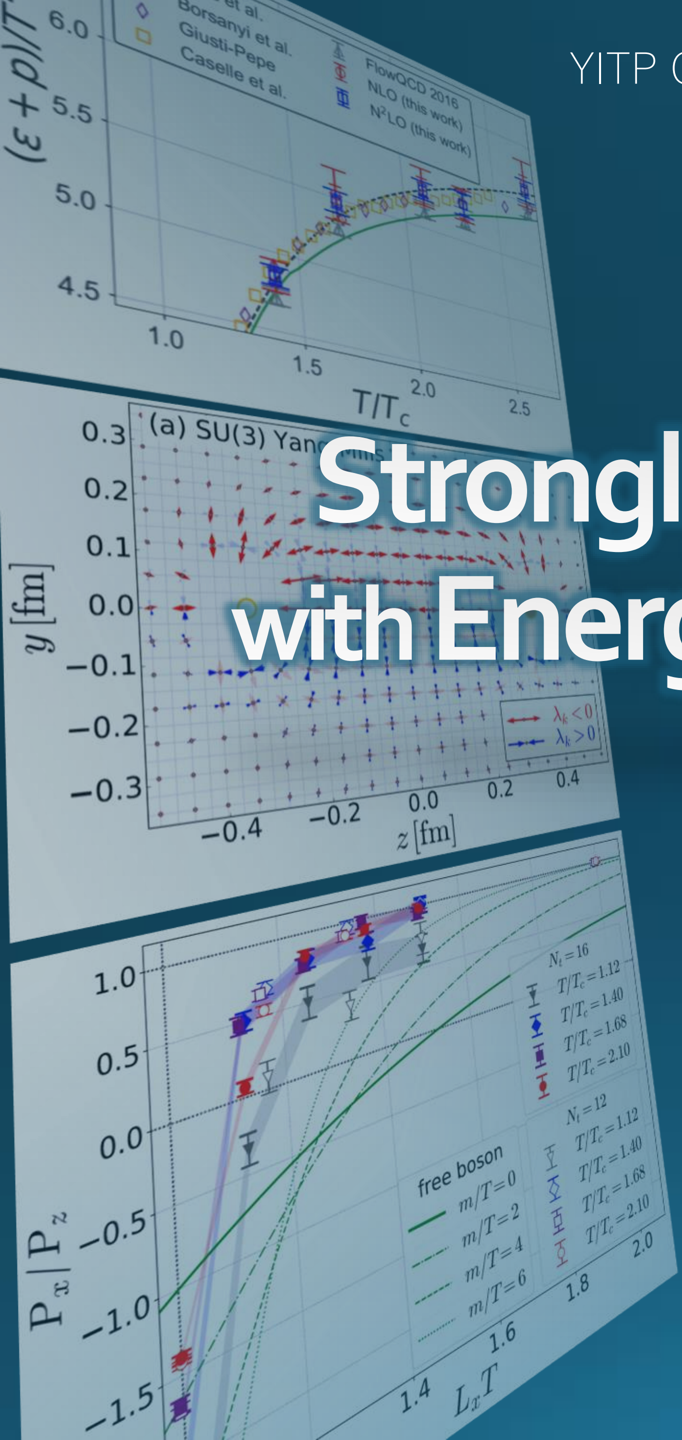
YITP Colloquium, YITP, Kyoto, 2023/01/10



Exploring Strongly-Interacting Systems with Energy-Momentum Tensor

Masakiyo Kitazawa

(YITP)

MK, Mogliacci, Kolbe, Horowitz, PRD**99**, 094507 (2019)
FlowQCD, PLB**789**, 210 (2019)
Yanagihara, MK, PTEP**2019**, 093B02 (2019)
FlowQCD, PRD**102**, 114522 (2020)
Suenaga, MK, 2210.09363
Ito, MK, to appear

Energy-Momentum Tensor

energy momentum

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

pressure

stress

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \qquad \partial_\mu T_{\mu\nu} = 0$$

- The most fundamental quantity in physics.
- All components are important observables.

Gravitational Form Factors

$$\langle \mathrm{hadron}, p' | T_{\mu\nu}(0) | \mathrm{hadron}, p \rangle$$

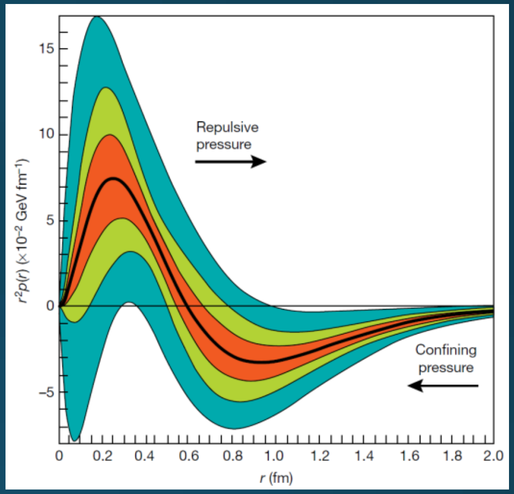
- (partially) accessible with hard exclusive processes

- Mass distribution
- Mechanical structure inside hadrons
- D-term: the last global unknown
- Mass decomposition

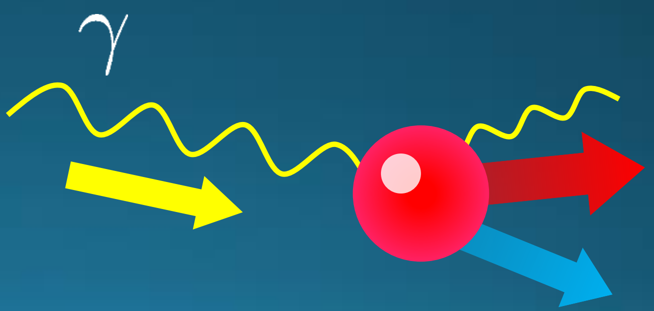
Polyakov(2003); Kumano, Song, Teryaev (2018); Ji (1995); Locre (2018); Hatta, Rajan, Tanaka (2018); …

$$\langle p' | T^a_{\mu\nu}(0) | p \rangle = \bar{u}' \Big[A^a(t) \frac{P_\mu P_\nu}{M} + J^a(t) \frac{i P_{\{\mu} \sigma_{\nu\}\rho} \Delta^\rho}{2M} + D^a(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{4M} + M \bar{c}^a(t) g_{\mu\nu} \Big]$$

Pressure inside proton



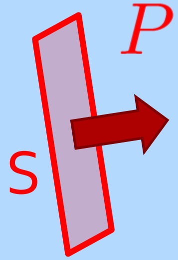
Burkert+, Nature 557, 396 (2018)



Stress

Stress = Force per Unit Area

Pressure

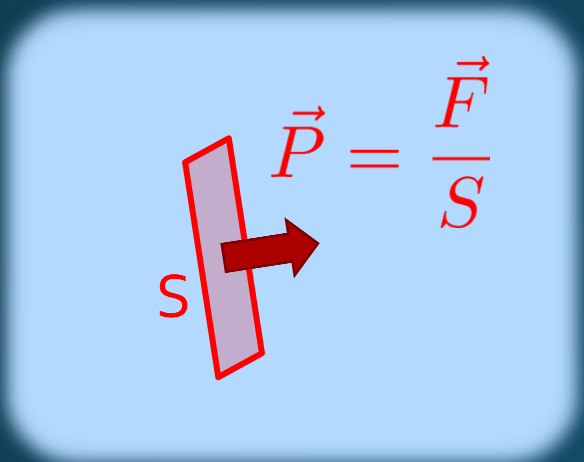


$$\vec{P} = \frac{\vec{F}}{S}$$

$$\vec{P} = P\vec{n}$$

Stress = Force per Unit Area

Pressure

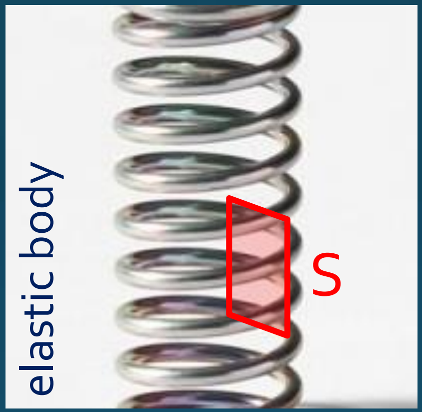


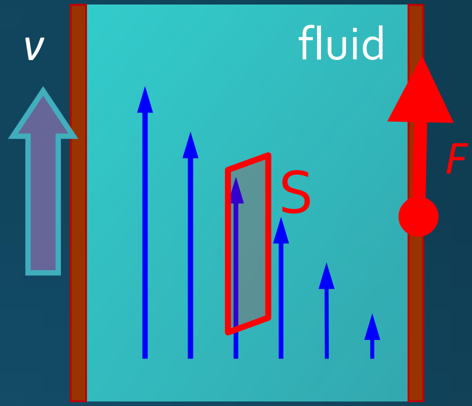
$$\vec{P} = P\vec{n}$$

In thermal medium

$$T_{ij} = P\delta_{ij}$$

Generally, F and n are not parallel





$$\frac{F_i}{S} = \sigma_{ij} n_j$$

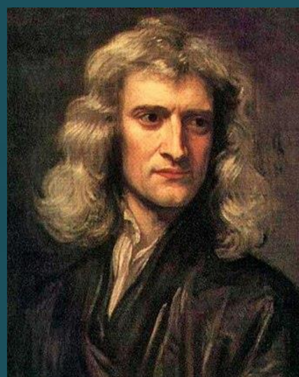
Stress Tensor

$$\sigma_{ij} = -T_{ij}$$

Landau
Lifshitz

Force

Action-at-a-distance



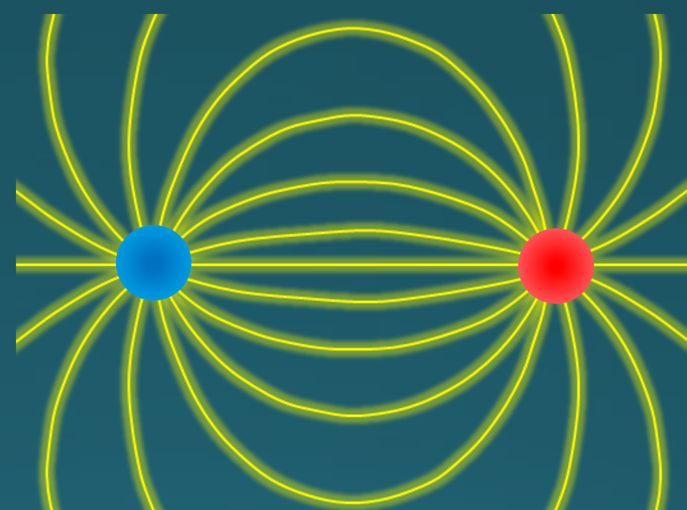
Newton
1687

$m_1,\ q_1$ $m_2,\ q_2$

$$F = -G\frac{m_1 m_2}{r^2} \qquad F = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Local interaction

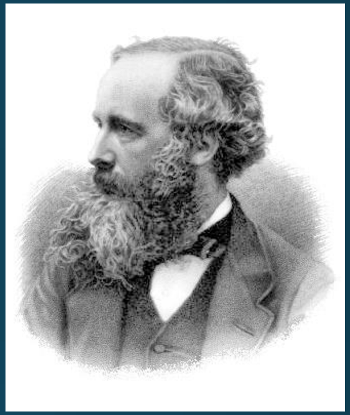
Faraday
1839



Maxwell Stress

(in Maxwell Theory)

$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

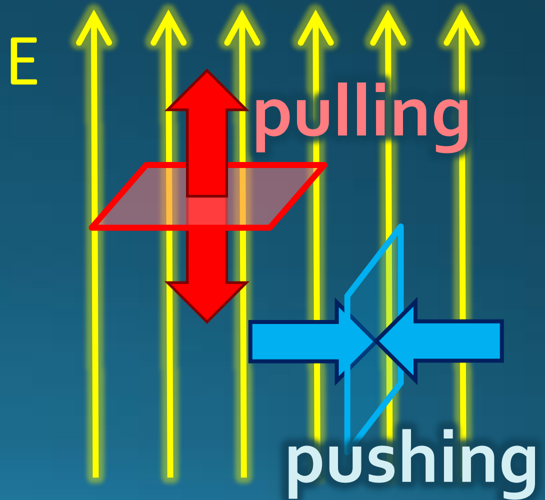


Maxwell

$$\vec{E} = (E, 0, 0)$$

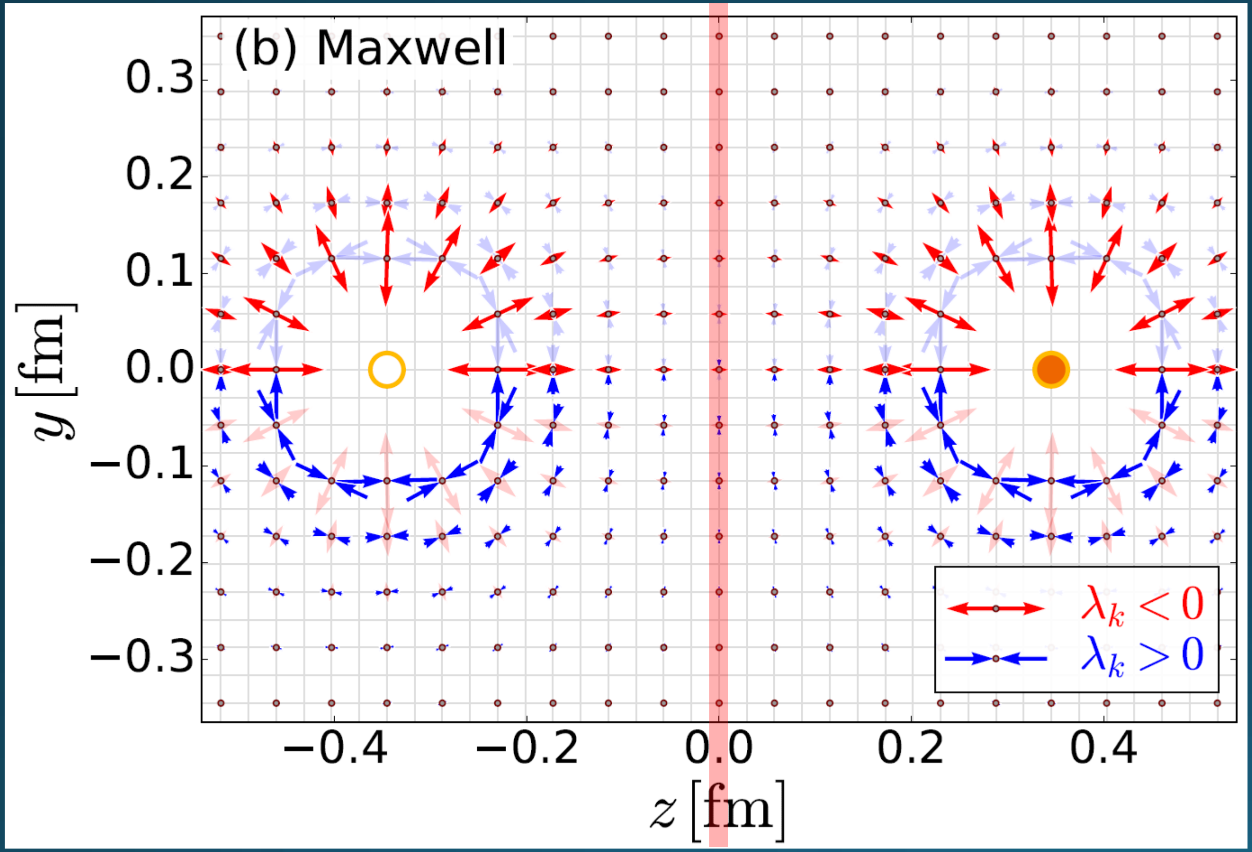
$$T_{ij} = \begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: **Pulling**
- Vertical to field: **Pushing**



Maxwell Stress

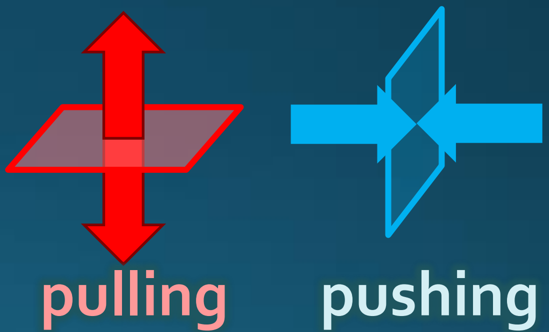
(in Maxwell Theory)



$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$$(k = 1, 2, 3)$$

length: $\sqrt{|\lambda_k|}$

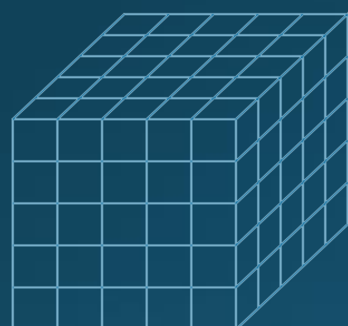


Definite physical meaning

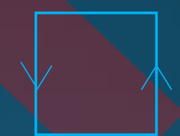
- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction

$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial
because of the explicit breaking of translational invariance



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

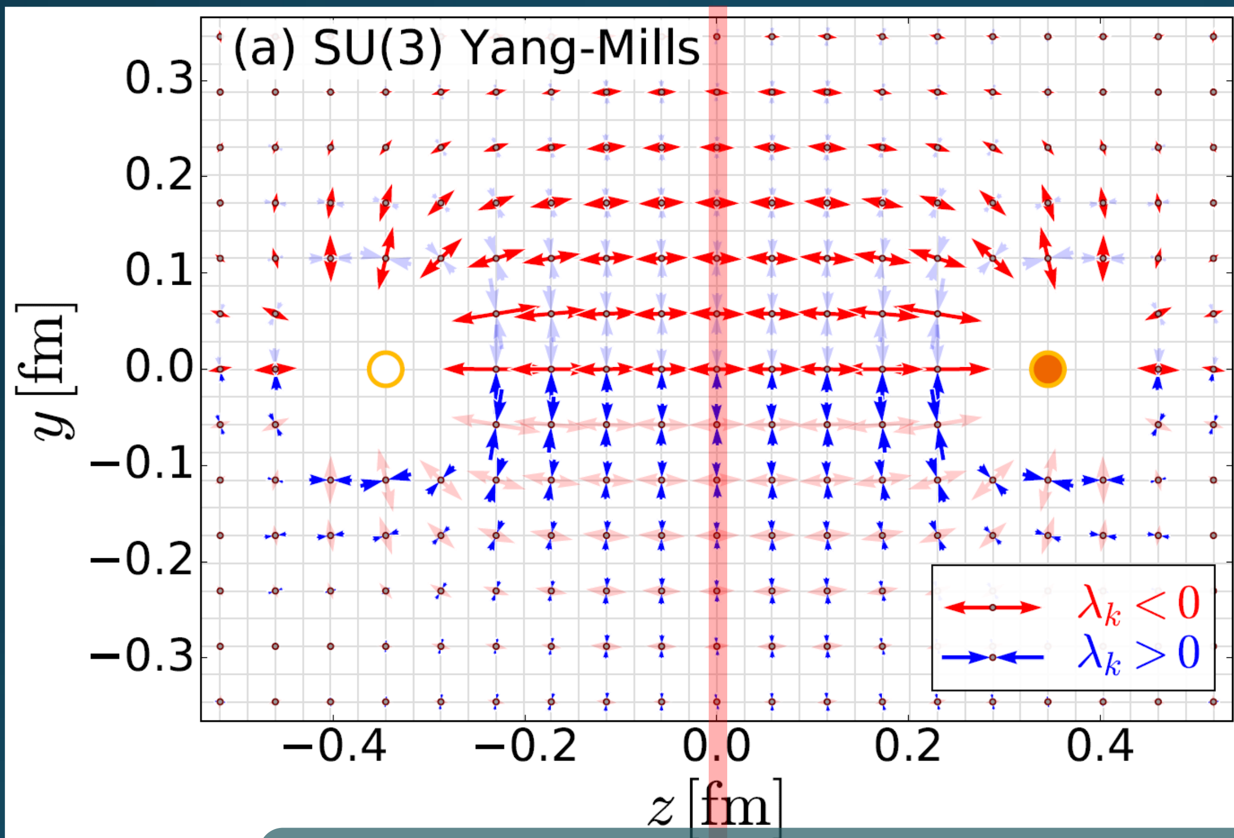
② Measurement is extremely noisy
due to high dimensionality and etc.

→ Solved by the SF*t*X method

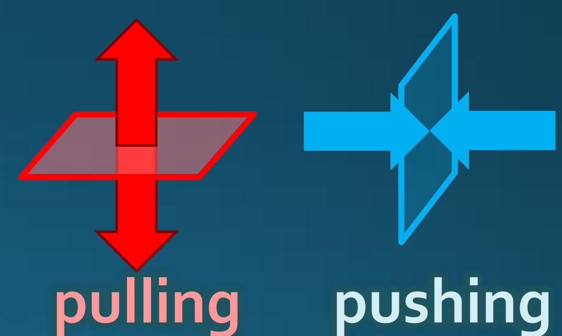
Suzuki ('13); FlowQCD('14); …

Stress Tensor in $Q\bar{Q}$ System

FlowQCD, PLB (2019)



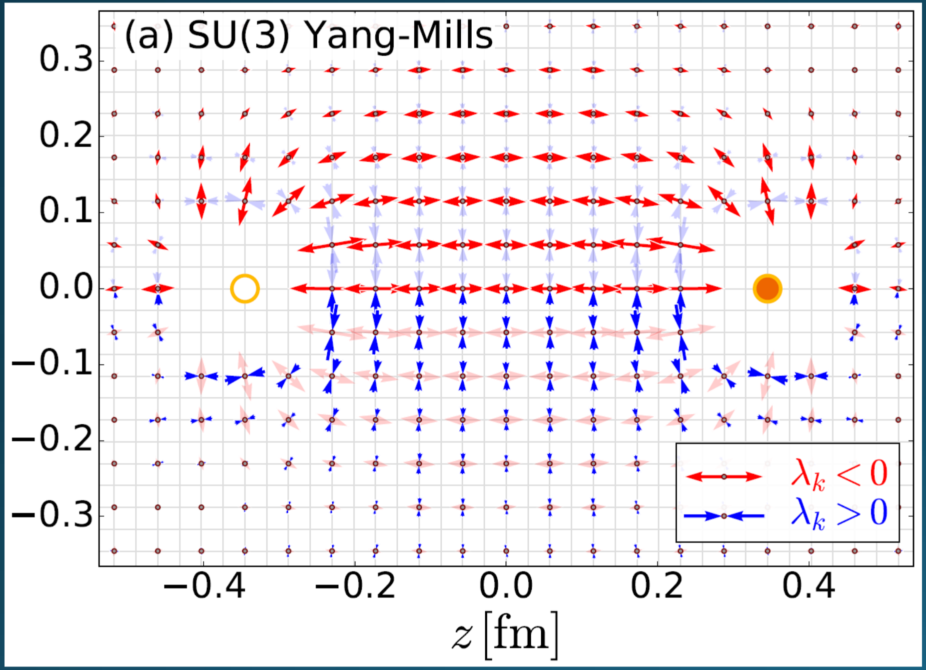
Lattice simulation
SU(3) Yang-Mills
a=0.029 fm
R=0.69 fm
t/a^2=2.0



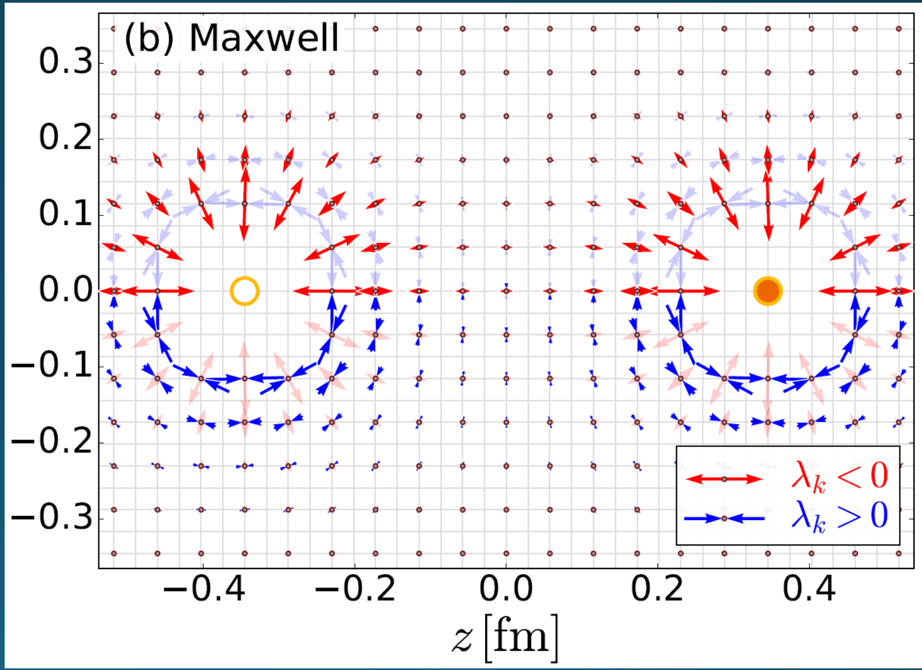
- ☐ **Flux-tube formation**
- ☐ **Definite physical meaning**
  - Distortion of field
  - Propagation of the force as local interaction

SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)



Maxwell
(classical)



Propagation of the force is clearly different in YM and Maxwell theories!

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3. Static-Quark Systems

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4. Model Calculations of EMT

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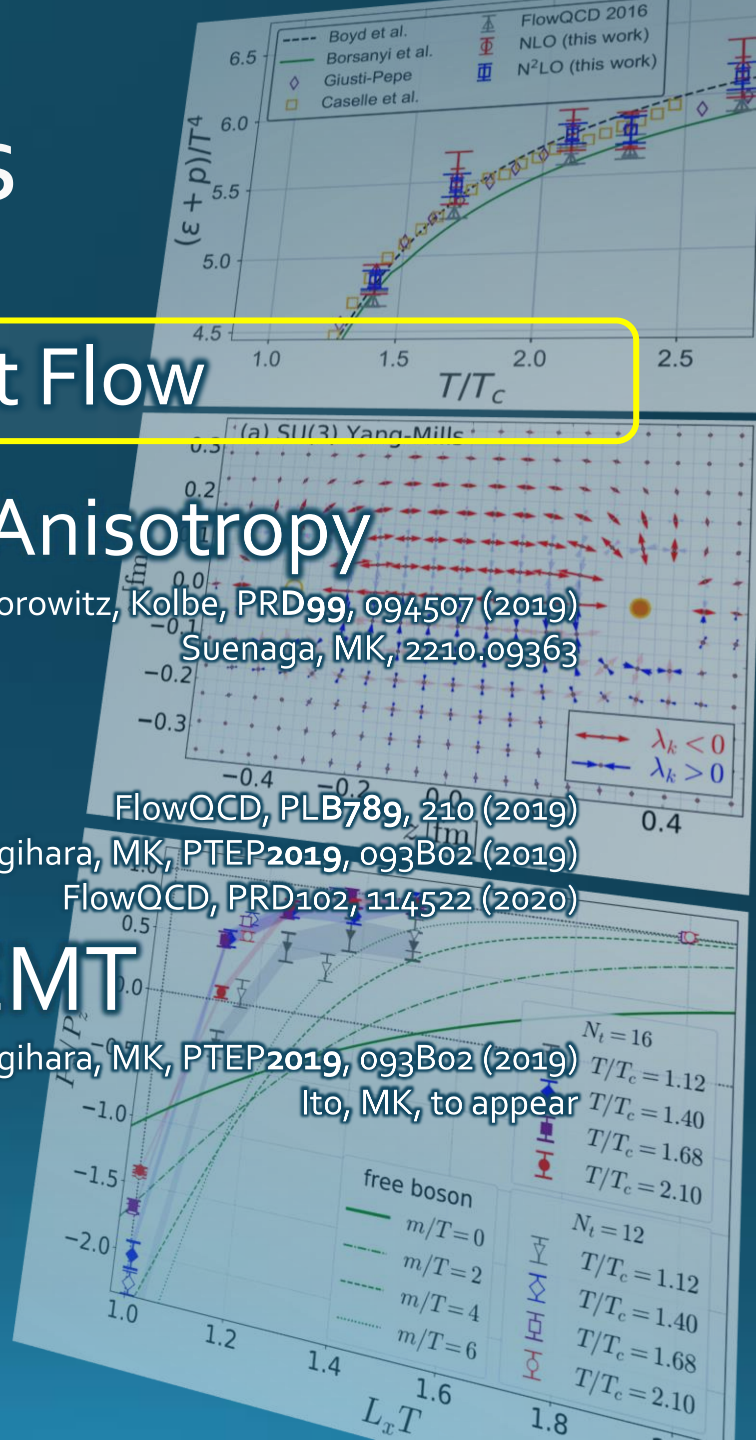


Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

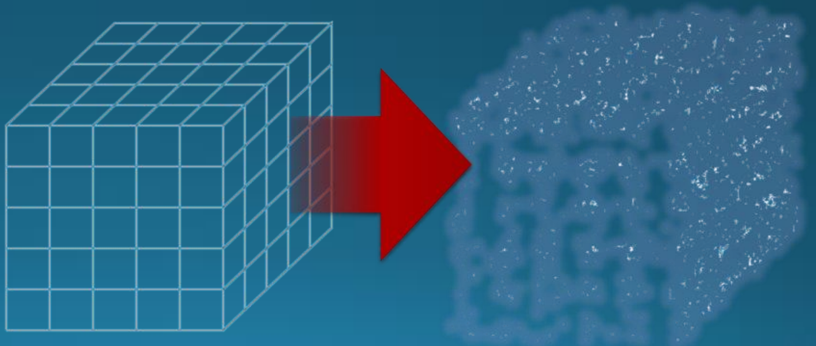
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length²]

leading

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at $t > 0$



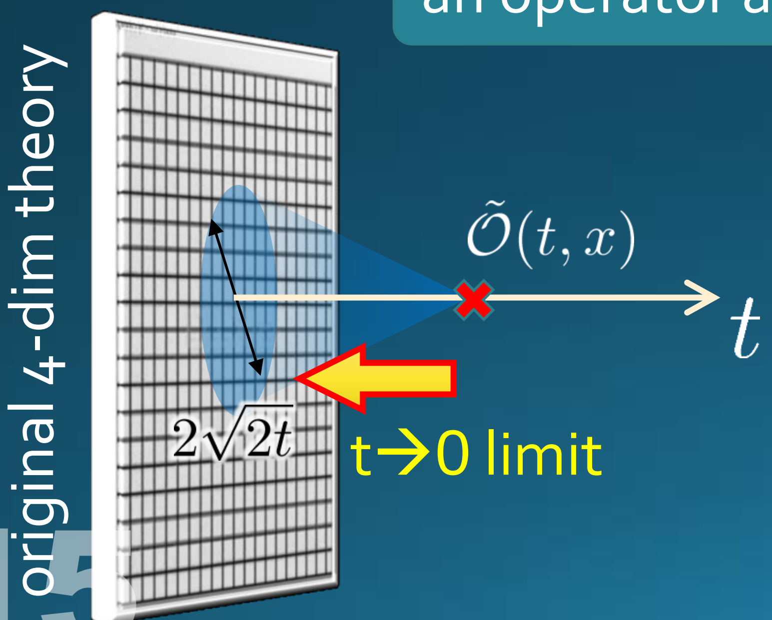
Small Flow-Time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at t>0

remormalized operators of original theory



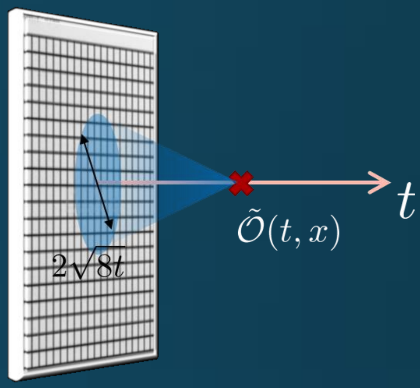
Constructing EMT

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t) T^R_{\rho\rho}(x) + \mathcal{O}(t)$$

vacuum subtr.



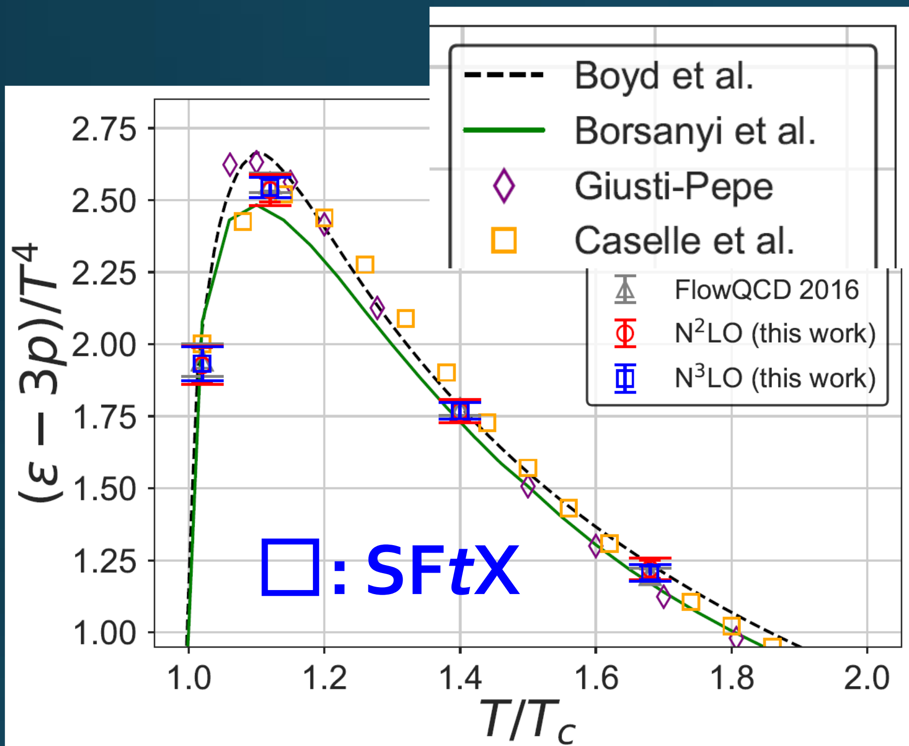
Remormalized EMT

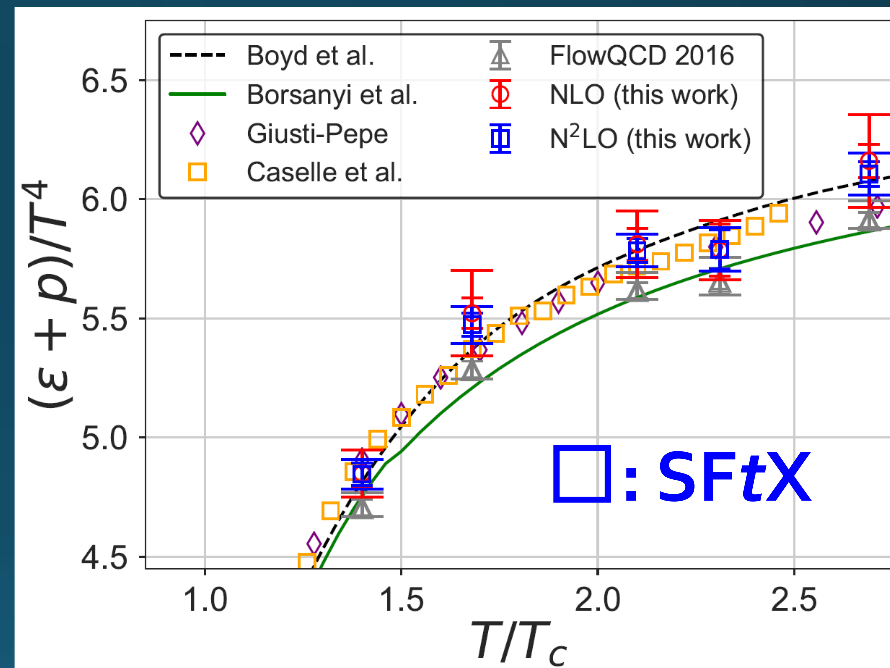
$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\mathrm{subt.}}\right]$$

"SF*t*X method" (Small Flow *t*ime eXpansion)

Thermodynamics: $\varepsilon = \langle T_{00} \rangle,\ p = \langle T_{11} \rangle$

Iritani, MK, Suzuki, Takaura, 2019



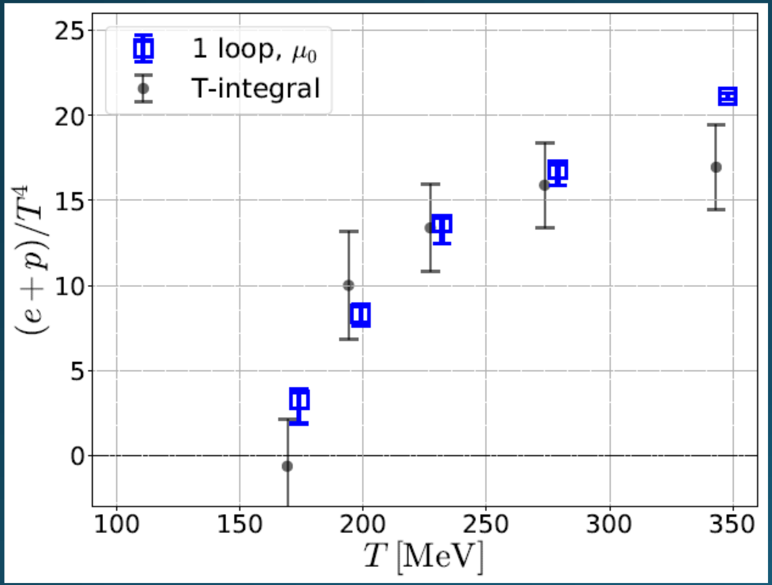


- ☐ Agreement with other methods within 1% level!
- ☐ Smaller statistics thanks to smearing by the flow

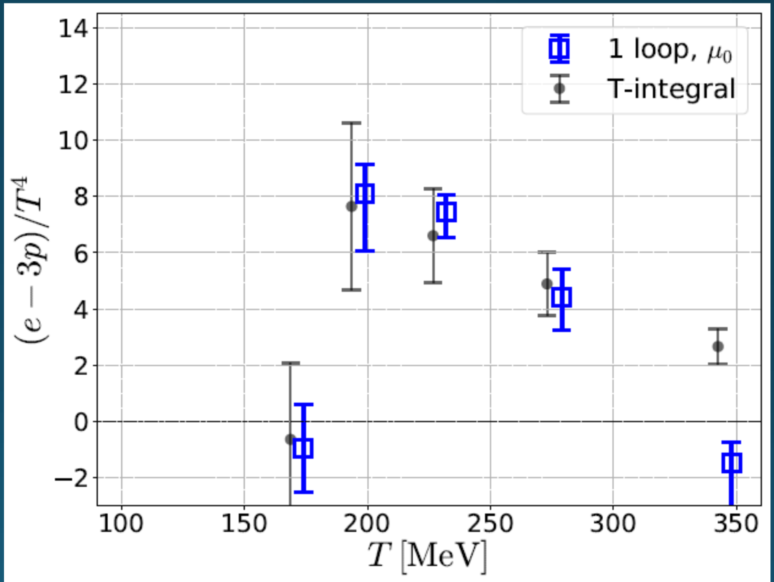
2+1 QCD EoS from Gradient Flow

WHOT-QCD, PR**D96** (2017); PR**D102** (2020)

$$\langle T_{00}\rangle + \langle T_{11}\rangle$$



$$\langle T_{00}\rangle - 3\langle T_{11}\rangle$$



- ☐ Agreement with integral method
- ☐ Substantial suppression of statistical errors

$m_{PS}/m_V \approx 0.63$

Physical mass: Kanaya+ (WHOT-QCD), 1910.13036

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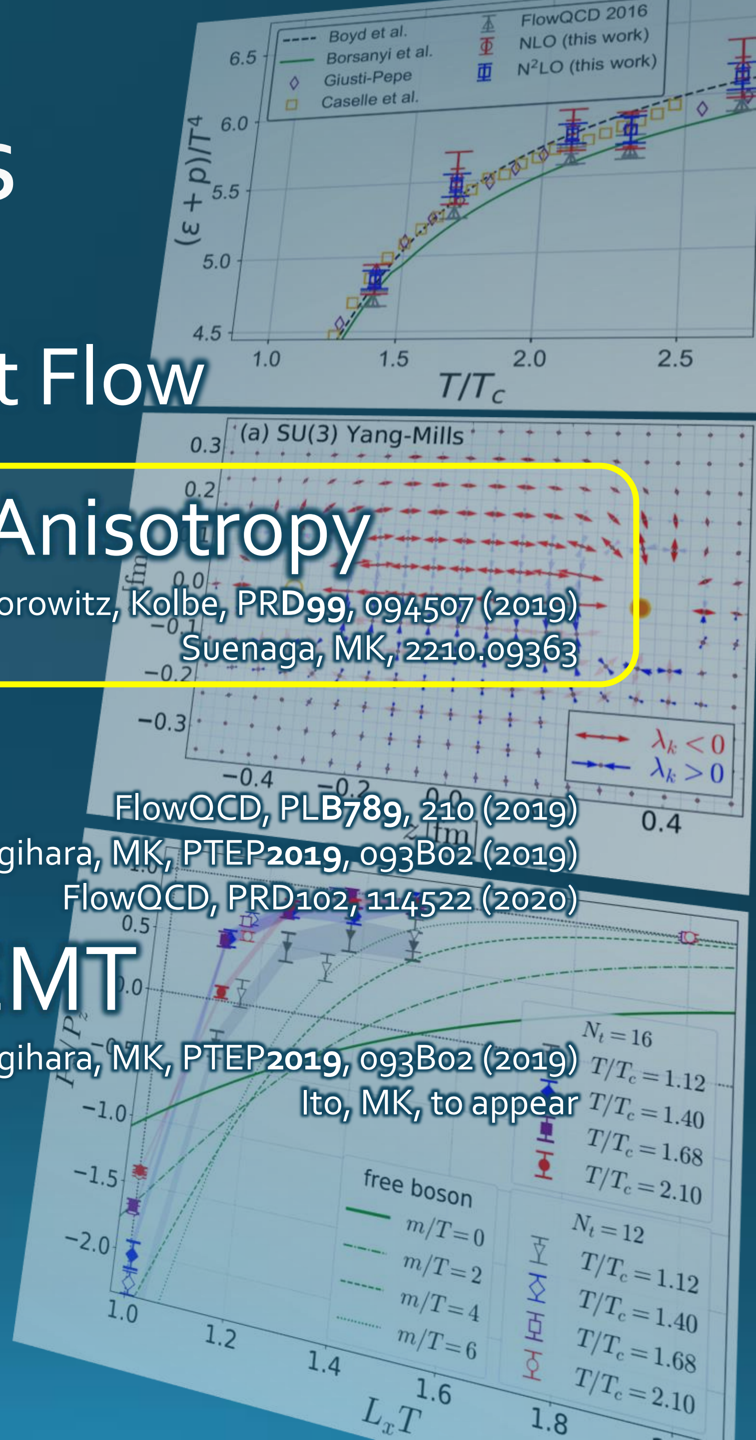
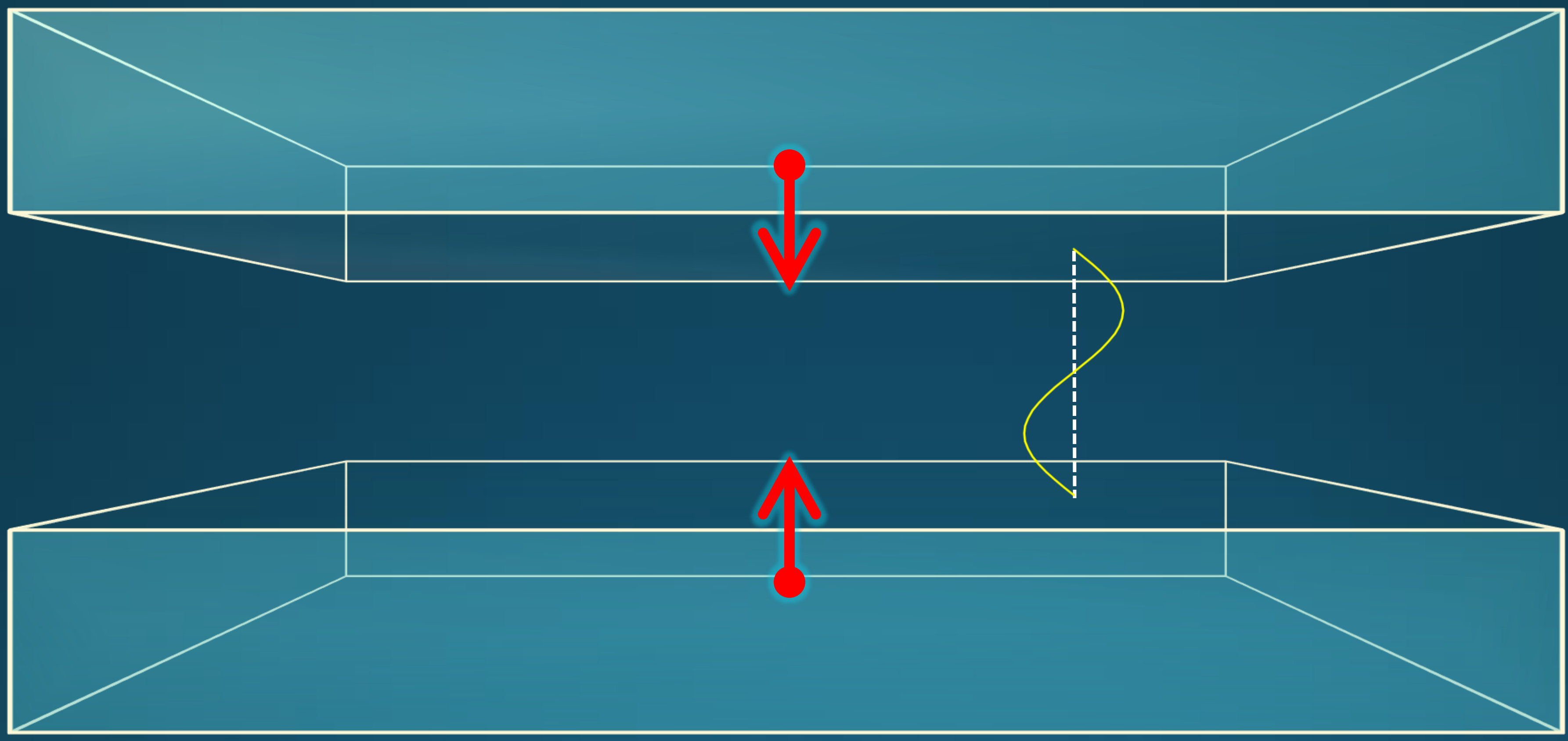


Casimir Effect

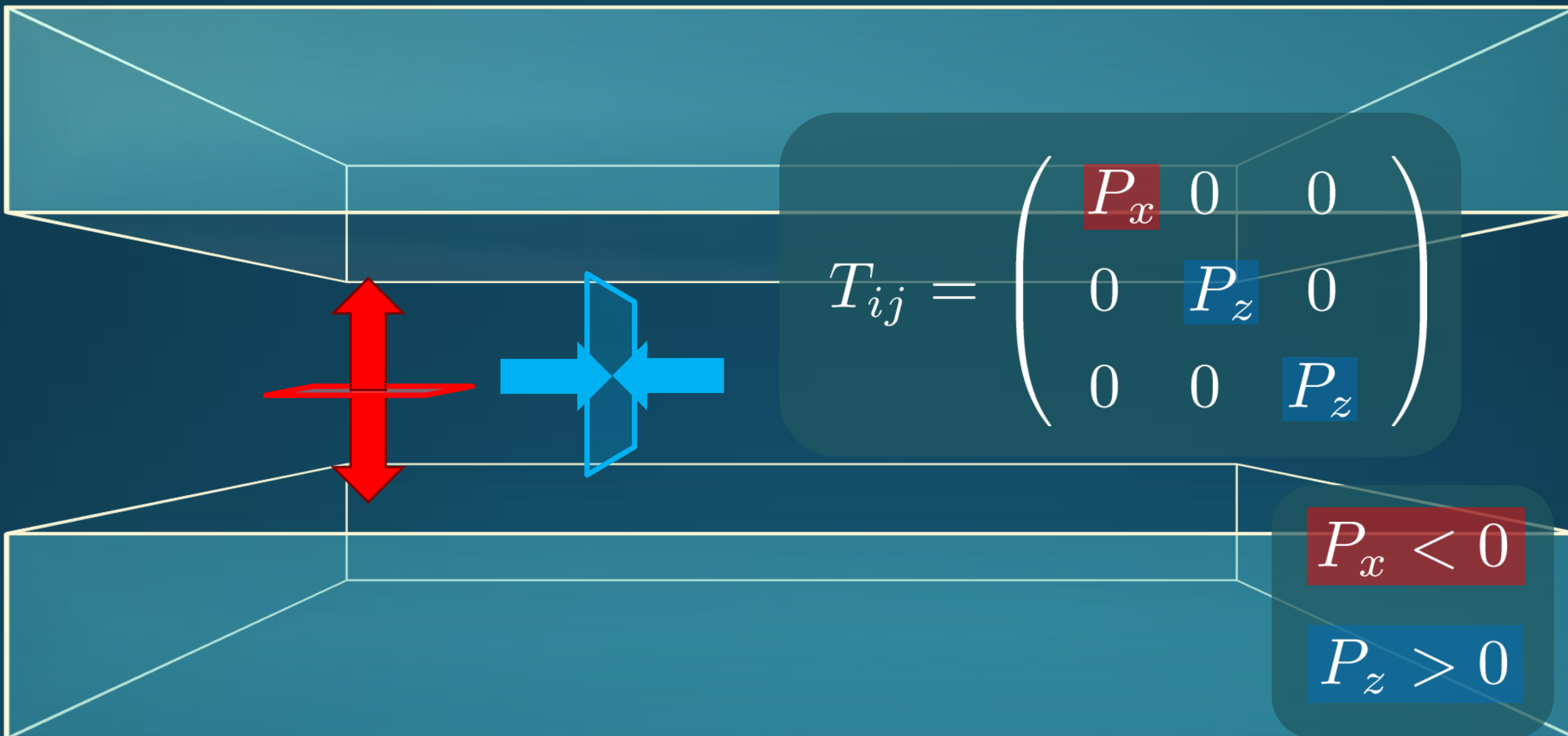
Casimir Effect



attractive force between two conductive plates

Casimir Effect

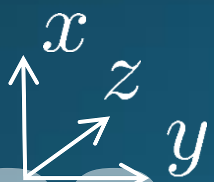
Brown, Maclay
1969



$$T_{ij} = \begin{pmatrix} P_x & 0 & 0 \\ 0 & P_z & 0 \\ 0 & 0 & P_z \end{pmatrix}$$

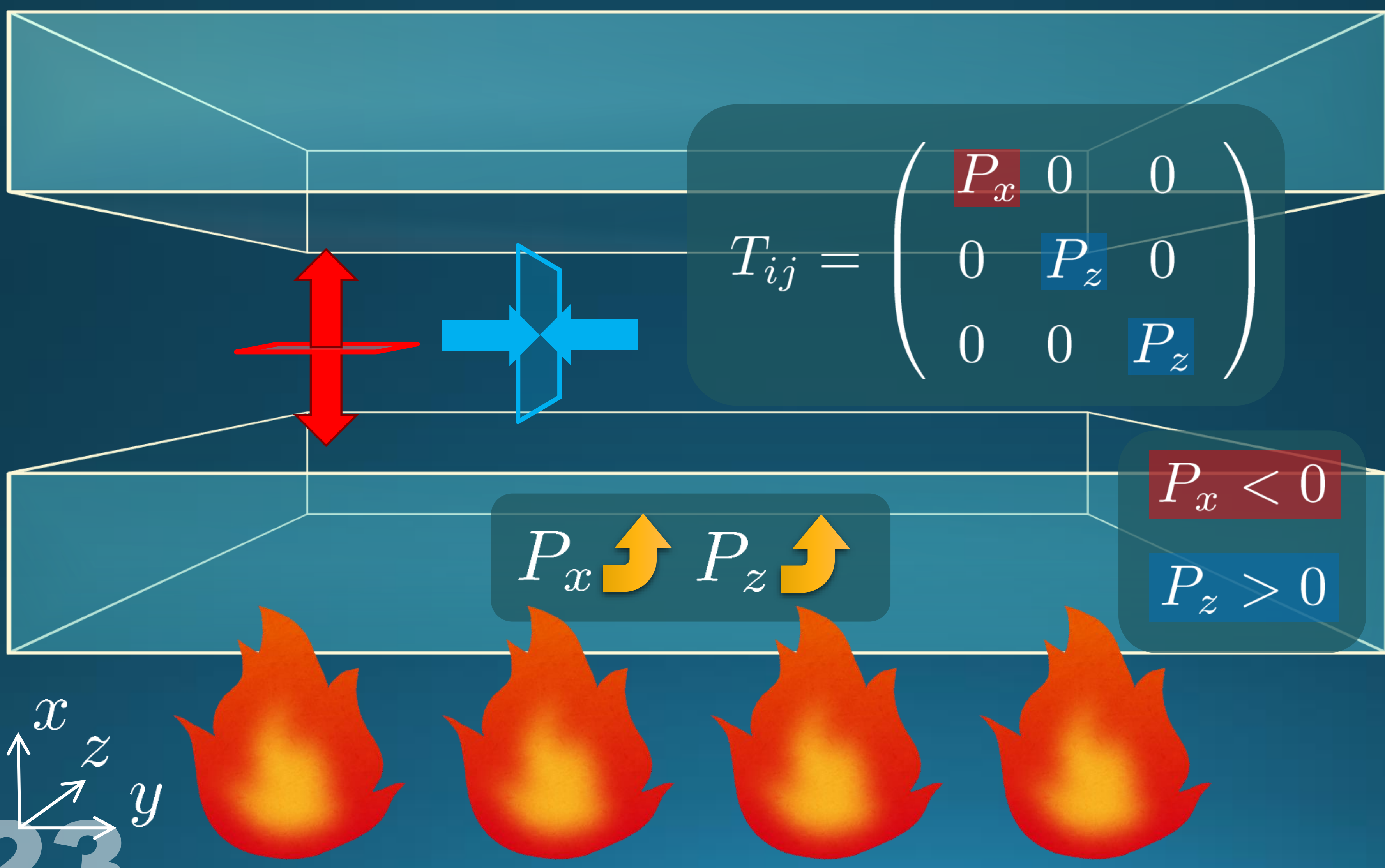
$$P_x < 0$$

$$P_z > 0$$



Casimir Effect

Brown, Maclay
1969



$$T_{ij} = \begin{pmatrix} P_x & 0 & 0 \\ 0 & P_z & 0 \\ 0 & 0 & P_z \end{pmatrix}$$

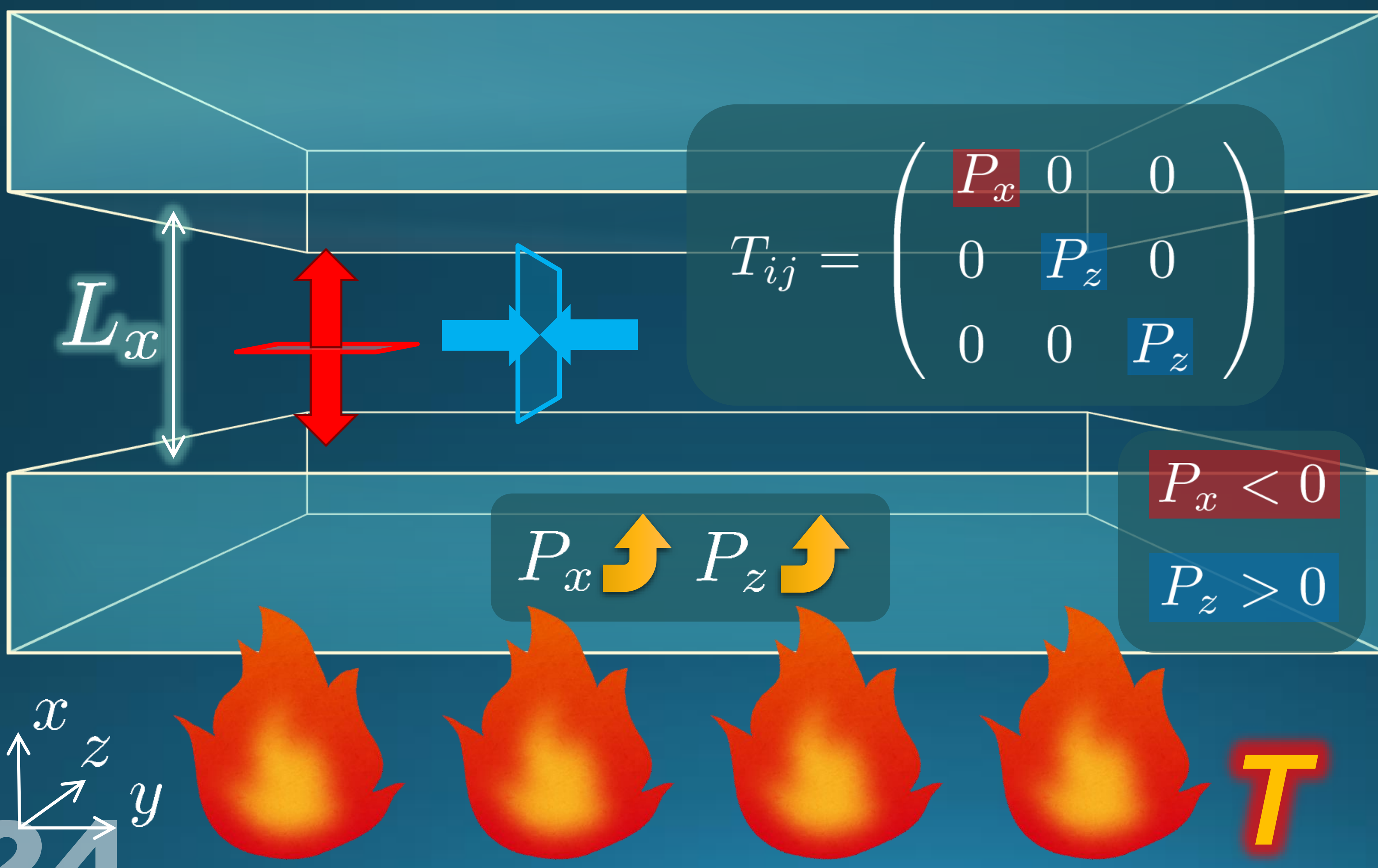
P_x ↑ P_z ↑

$P_x < 0$

$P_z > 0$

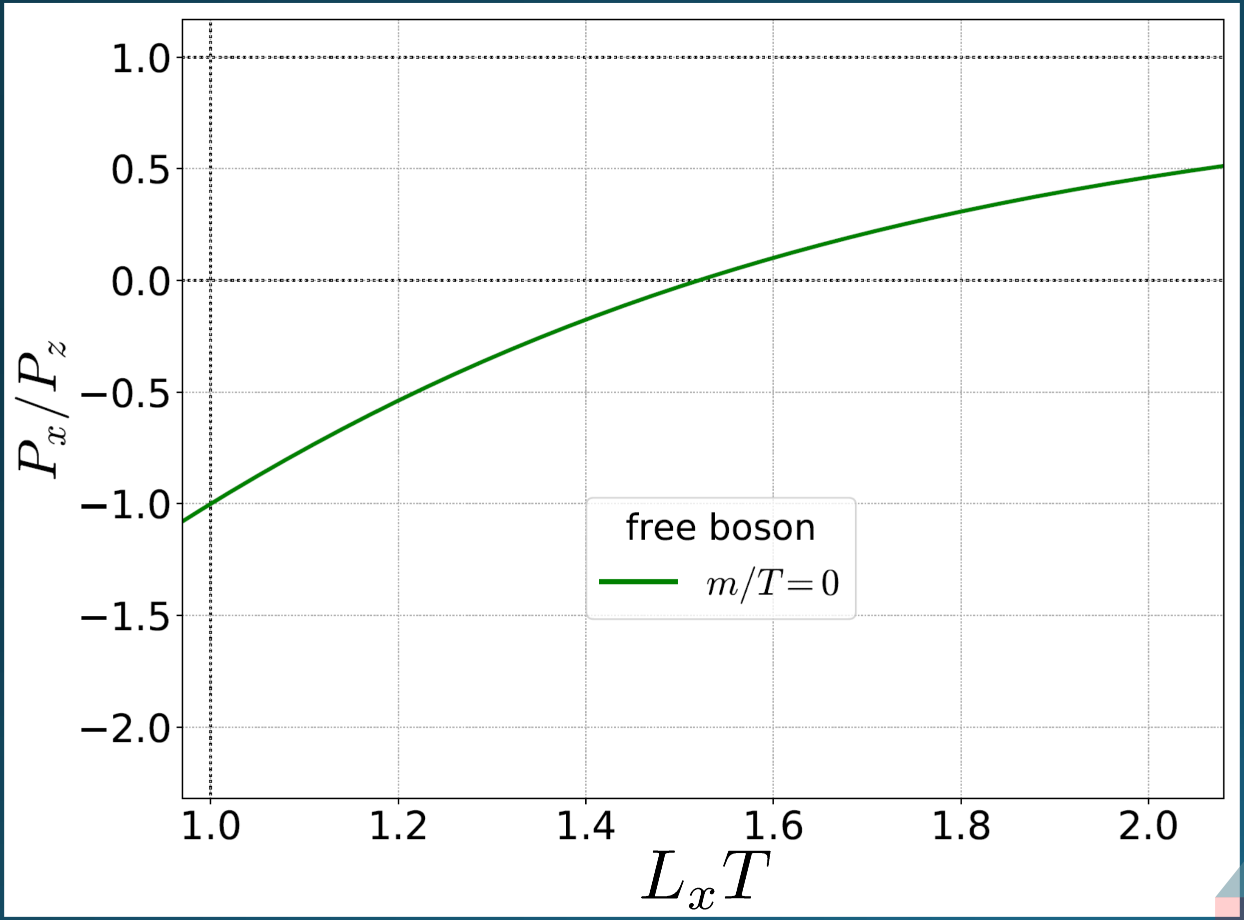
Casimir Effect

Brown, Maclay
1969



Pressure Anisotropy @ T≠0

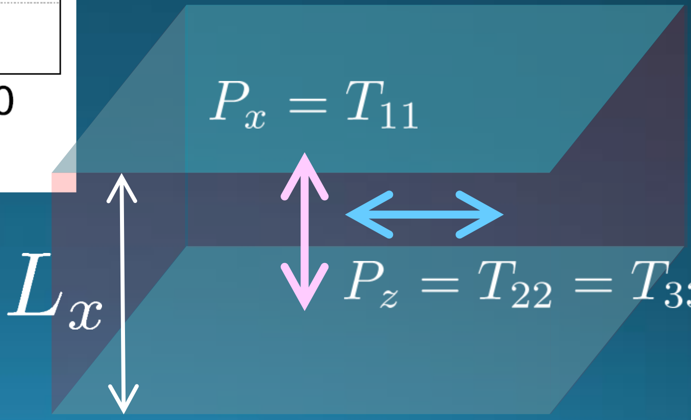
MK, Mogliacci, Kolbe, Horowitz, PRD (2019)



Free scalar field

- $L_2=L_3=\infty$
- Periodic BC

Mogliacci+, 1807.07871



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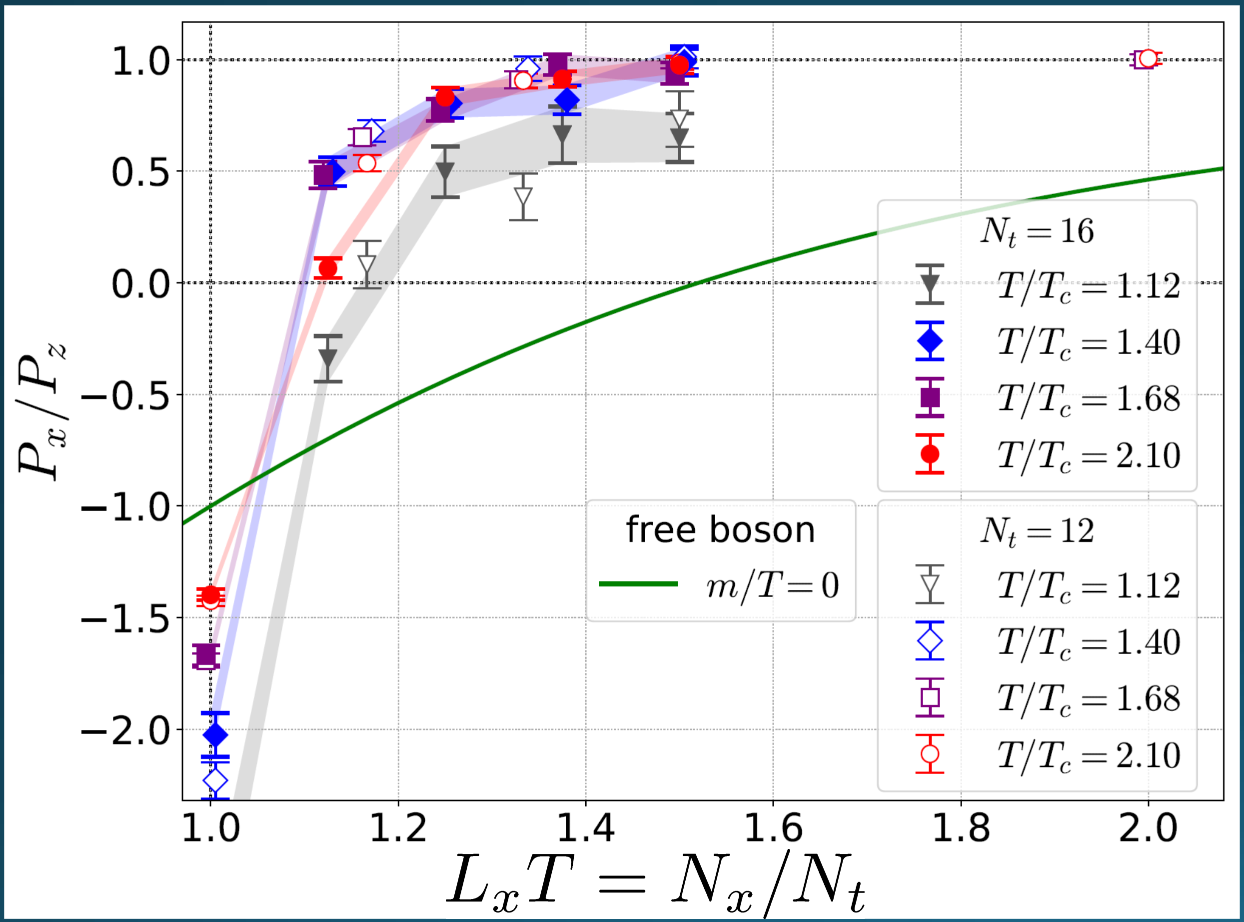
MK, Mogliacci, Kolbe, Horowitz, PRD (2019)



Free scalar field

- $L_2=L_3=\infty$
- Periodic BC

Mogliacci+, 1807.07871

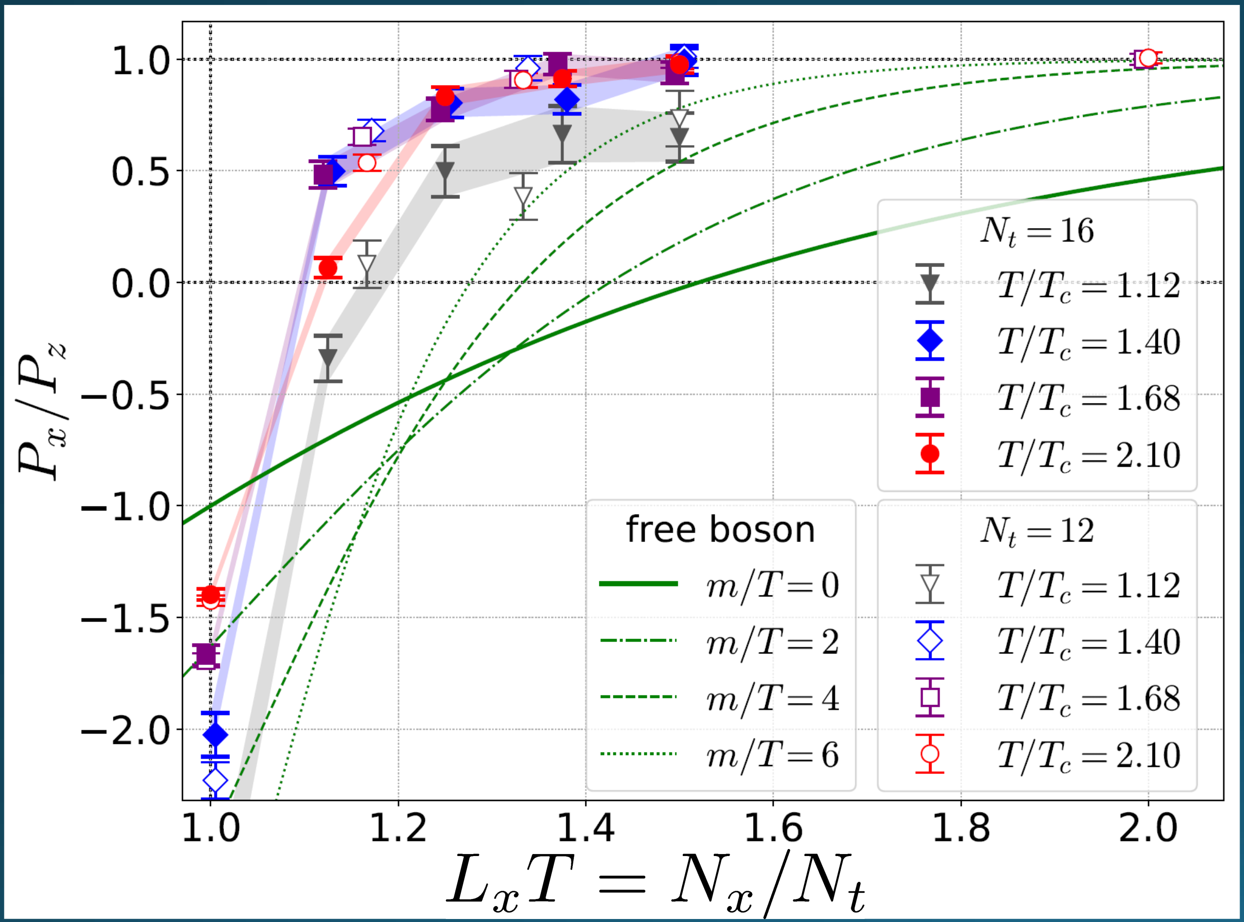
Lattice result

- Periodic BC
- Only t→0 limit
- Error: stat.+sys.

Medium near T_c is remarkably insensitive to finite size!

Pressure Anisotropy @ T≠0

MK, Mogliacci, Kolbe, Horowitz, PRD (2019)



Free scalar field

- $L_2=L_3=\infty$
- Periodic BC

Mogliacci+, 1807.07871

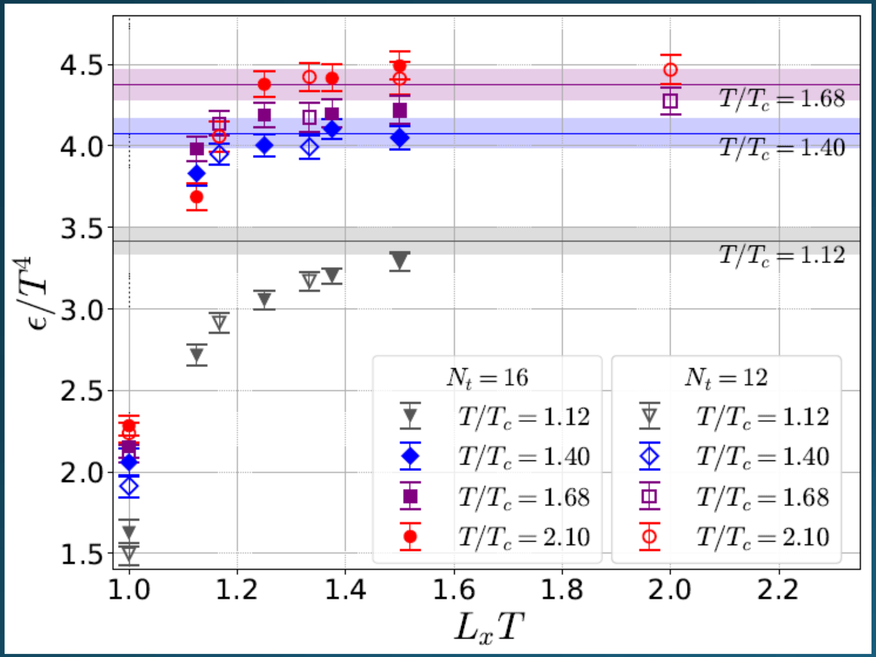
Lattice result

- Periodic BC
- Only t→0 limit
- Error: stat.+sys.

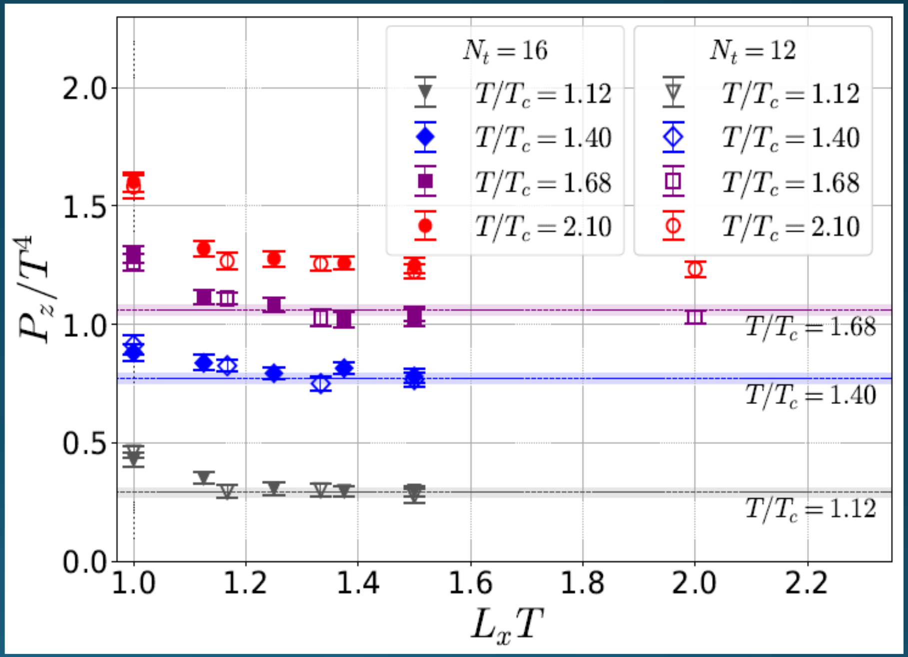
Medium near T_c is remarkably insensitive to finite size!

Energy densty / transverse P

Energy Density



Transverse Pressure P_z



Thermodynamics on the Lattice

Various Methods

- **Integral**, differential, moving frame, non-equilibrium, …
- rely on thermodynamic relations valid in V→∞

$$P = \frac{T}{V} \ln Z \qquad sT = \varepsilon + P$$

→ **Not applicable to anisotropic systems**

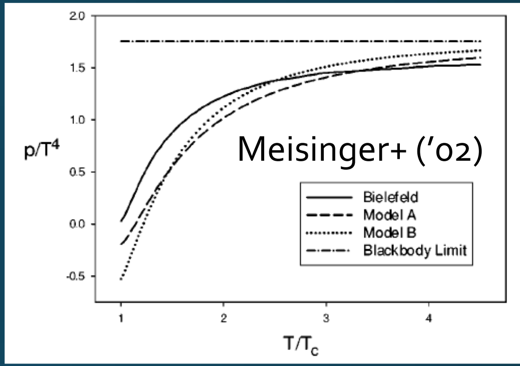
- We employ **SF*t*X Method**

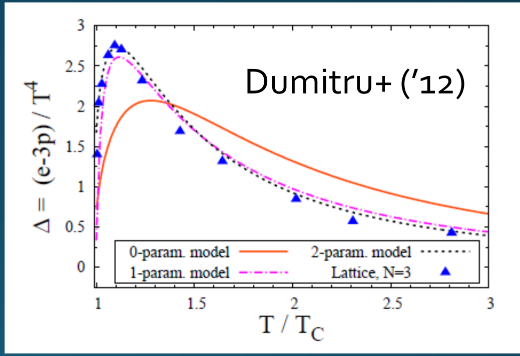
$$\varepsilon = \langle T_{00} \rangle \quad P = \langle T_{11} \rangle$$

Components of EMT are directly accessible!

Interpreting Response against BC

Suenaga, MK ('22)





☐ Remnant of Confinement?

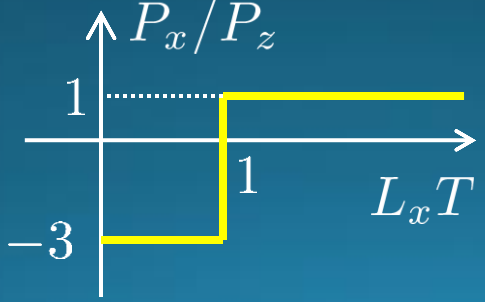
"Intermediate" Polyakov loop P for $T \sim T_c$

- Treat P as a dynamical variable
- Background const gauge field $A^0(x)$
- Potential term to realize $\langle P \rangle = 0$

➡ Thermodynamics near T_c is well described. Meisinger+ ('02); ...

☐ AdS/CFT

- AdS soliton
- N=4 SYM with S^1
- confined phase



Horowitz, Myers ('98)
Myers ('99)
Balasubramanian, Kraus ('99)

Thanks to M. Natsuume

Extension to $\mathbf{T}^2 \times \mathbf{R}^2$

- **Two Polyakov loops**

Suenaga, MK ('22)

$$P_\tau = \frac{1}{N}\mathrm{Tr}[\mathcal{P}e^{\int d\tau A_0}]$$ temporal

$$P_x = \frac{1}{N}\mathrm{Tr}[\mathcal{P}e^{\int dx A_1}]$$ x-direction

treated as dynamical vals.

- **Free Energy**

$$T = \frac{1}{L_\tau}$$

$$f(P_\tau, P_x; L_\tau, L_x) = f_{\mathrm{pert}} + f_{\mathrm{pot}}$$

free gauge field with background field A_τ, A_x

potential term: separable ansatz

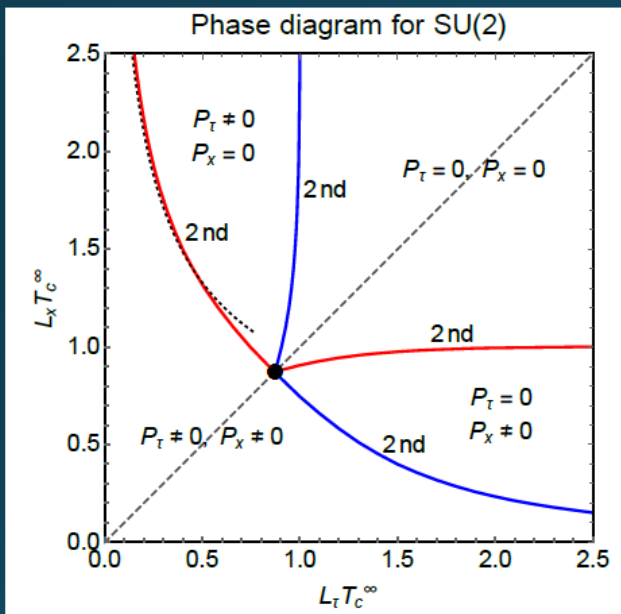
$$f_{\mathrm{pot}}(P_\tau, P_x; L_x, L_\tau)$$
$$= f_{\mathrm{pot}}^{\mathbb{S}^1\times\mathbb{R}^3}(P_\tau; L_\tau) + f_{\mathrm{pot}}^{\mathbb{S}^1\times\mathbb{R}^3}(P_x; L_x)$$

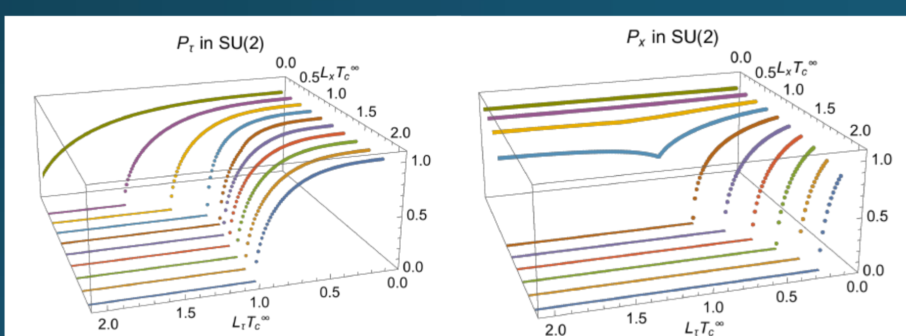
$f_{\mathrm{pot}}^{\mathbb{S}^1\times\mathbb{R}^3}$: from Meisinger+('02)

Phase Diagram on $L_x - L_\tau$ Plane

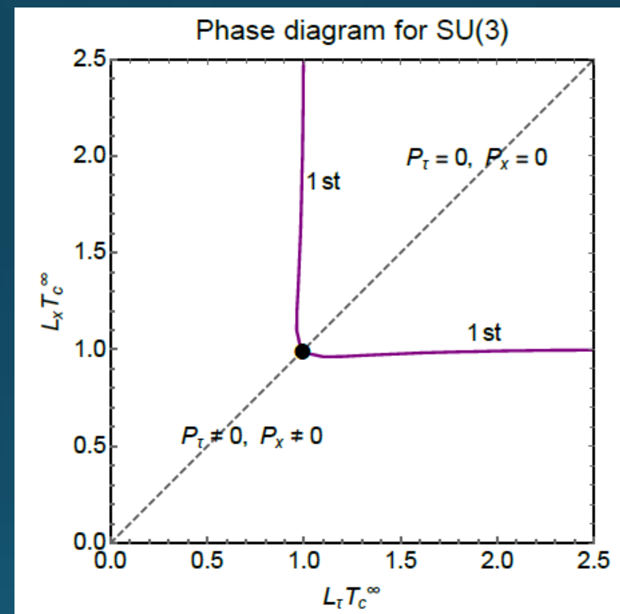
Suenaga, MK ('22)

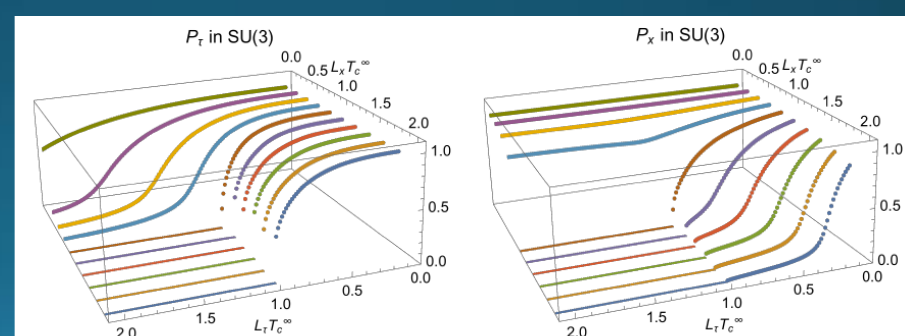
$N = 2$





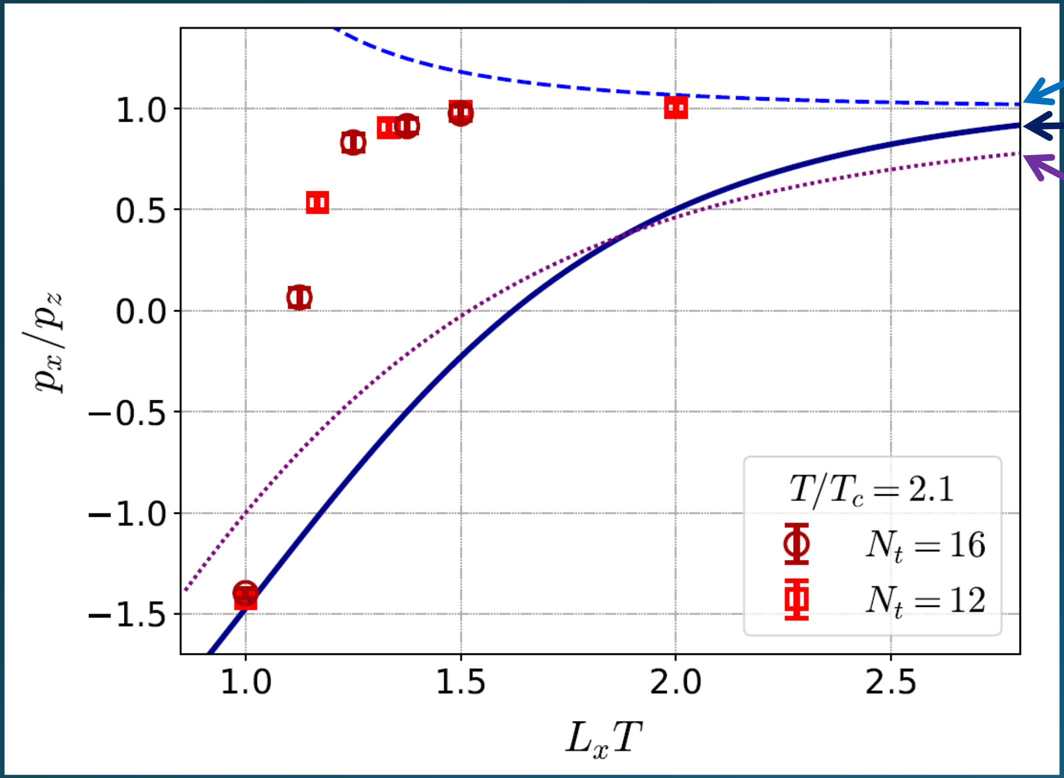
$N = 3$





Thermodynamics

$T/T_c = 2.1$



- fixed A_0, A_1
- model result
- massless free theory

- Anisotropic thermodynamics is strongly affected by A_0, A_1.
- Our simple model is inconsistent with the lattice result.
- Modification of the model will reproduce it.

Contents

1. SFtX: EMT through Gradient Flow

2. Casimir Effect & Pressure Anisotropy

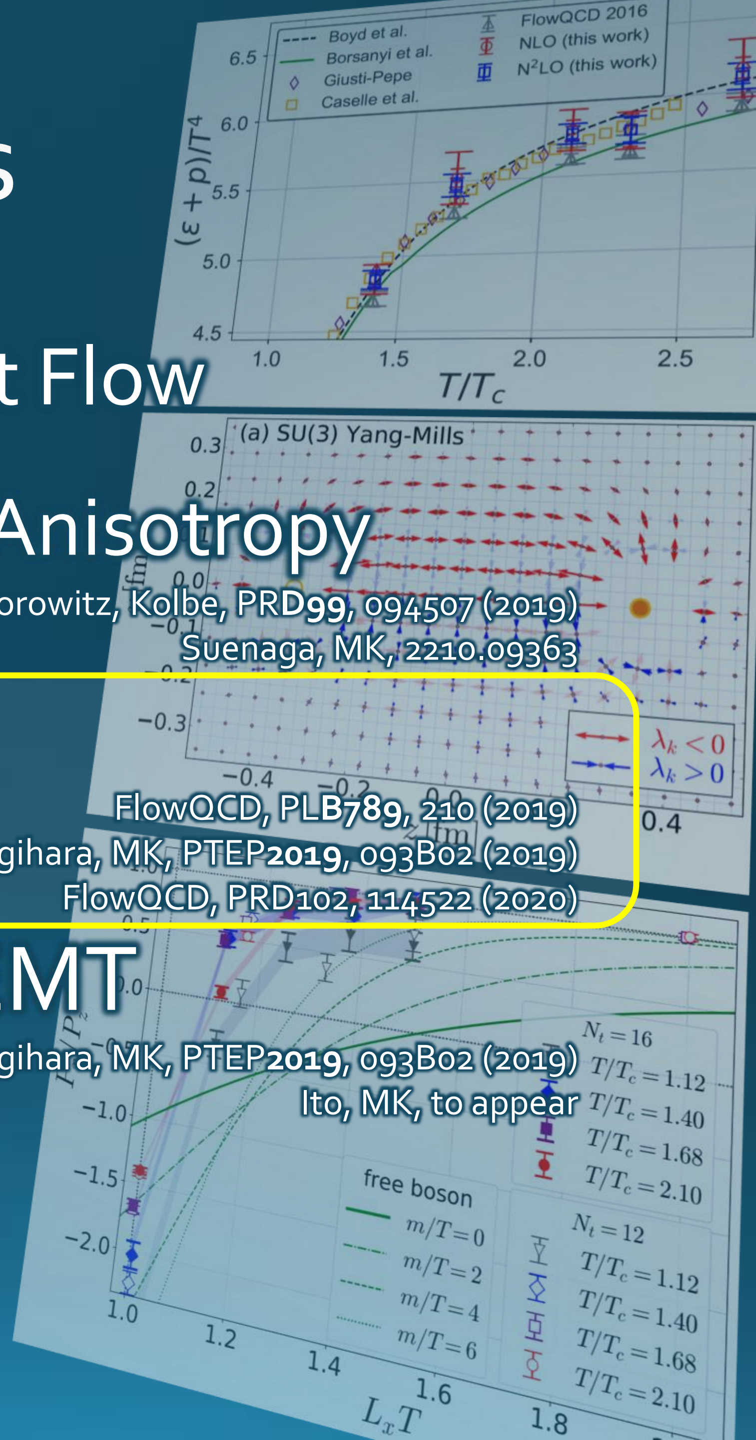
MK, Mogliacci, Horowitz, Kolbe, PRD**99**, 094507 (2019)
Suenaga, MK, 2210.09363

3. Static-Quark Systems

FlowQCD, PLB**789**, 210 (2019)
Yanagihara, MK, PTEP**2019**, 093B02 (2019)
FlowQCD, PRD102, 114522 (2020)

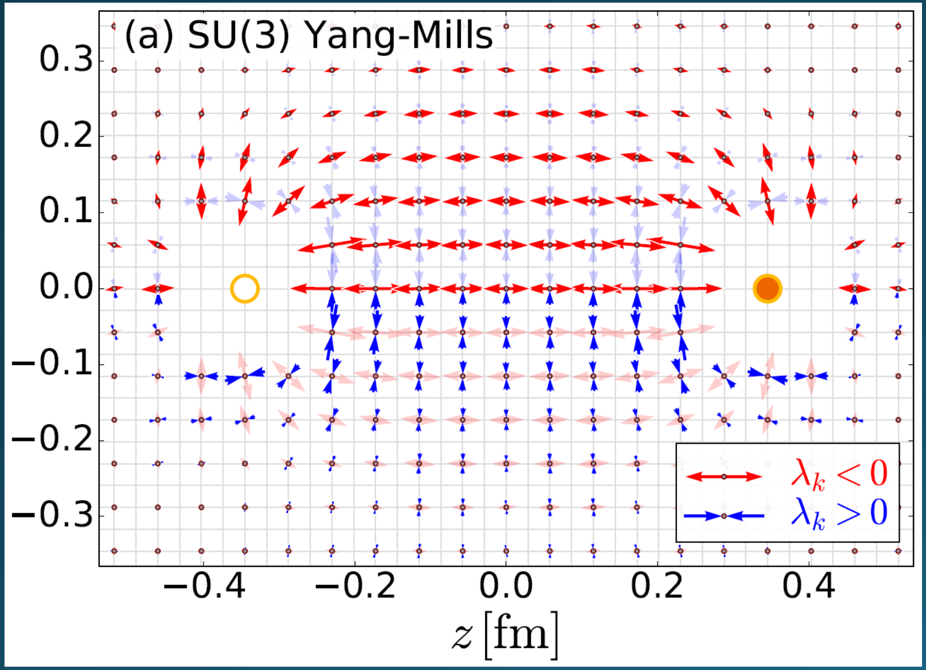
4. Model Calculations of EMT

Yanagihara, MK, PTEP**2019**, 093B02 (2019)
Ito, MK, to appear

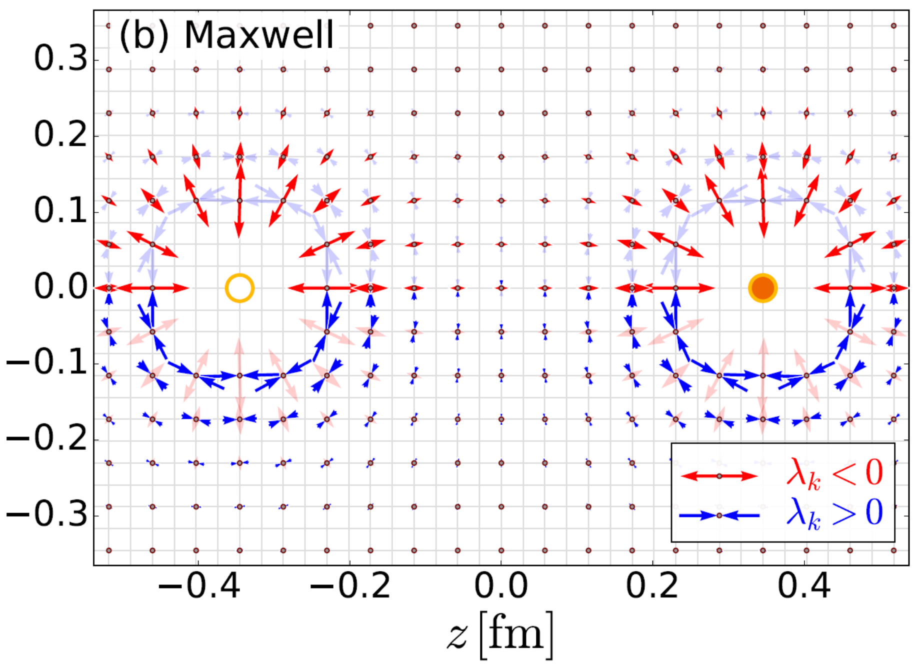


SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)



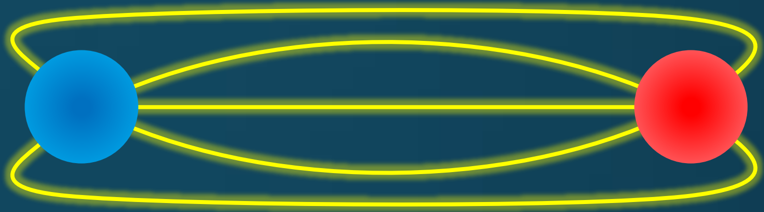
Maxwell
(classical)



Propagation of the force is clearly different in YM and Maxwell theories!

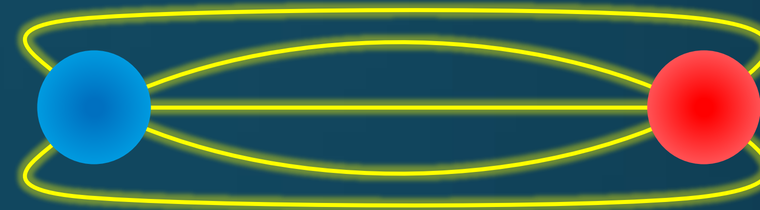
Flux Tube

- ☐ Quark confinement
- ☐ Non-pert. dynamics
- ☐ Linear potential
- ☐ String theory



Flux Tube

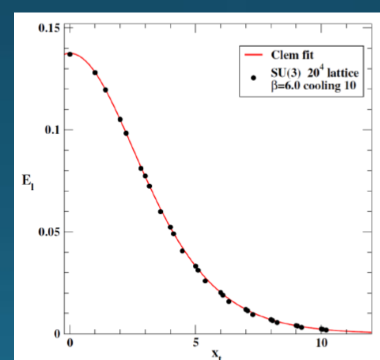
- Quark confinement
- Non-pert. dynamics
- Linear potential
- String theory



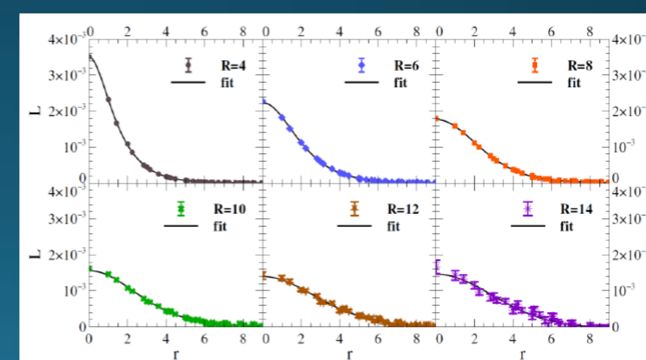
Many Studies on Flux Tube

- Potential
- Color-electric field
- Action density

so many studies…



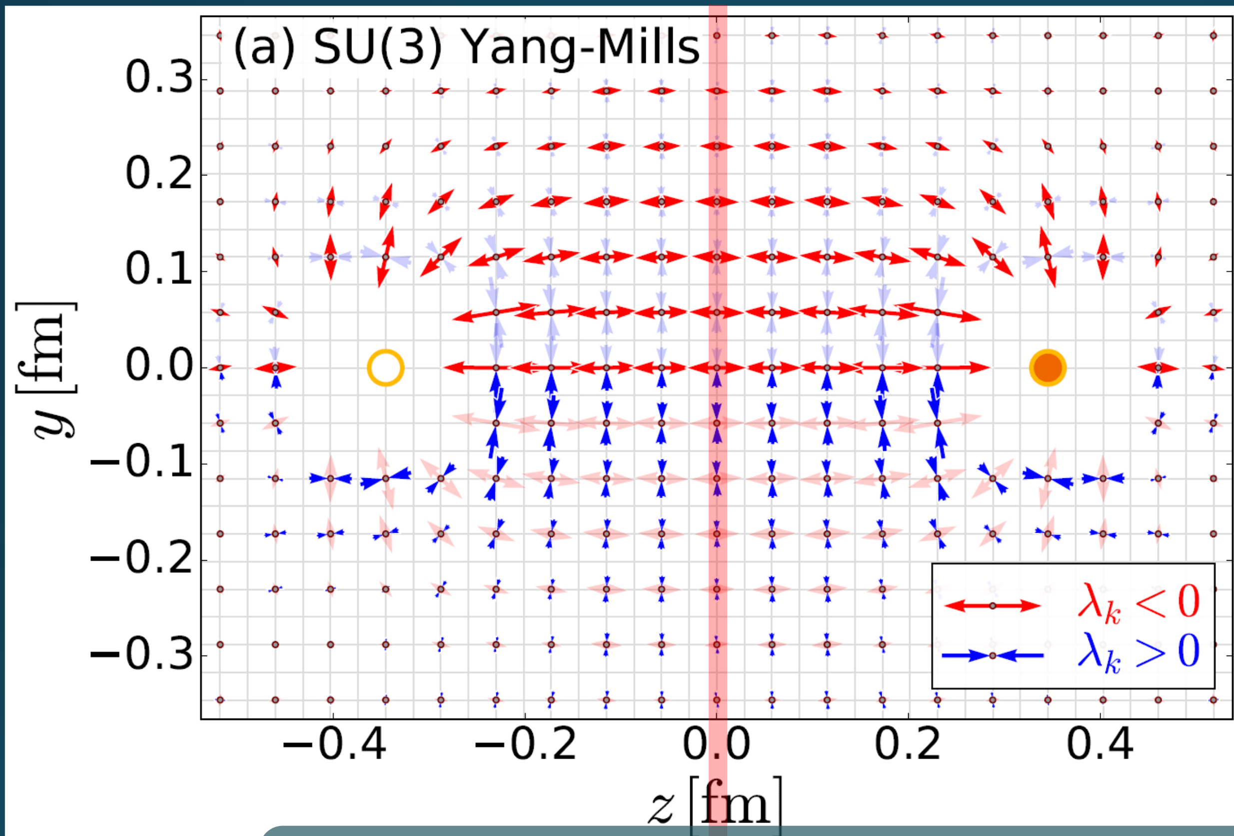
Cea+ (2012)



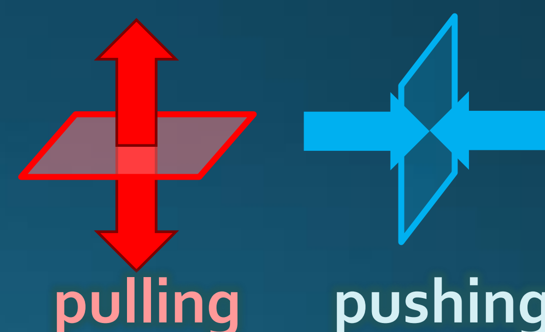
Cardoso+ (2013)

Stress Tensor in $Q\bar{Q}$ System

FlowQCD, PLB (2019)



Lattice simulation
SU(3) Yang-Mills
a=0.029 fm
R=0.69 fm
t/a^2=2.0



- **Flux-tube formation**
- **Definite physical meaning**
  - Distortion of field
  - Propagation of the force as local interaction

Lattice Setup

FlowQCD, PLB (2019)

- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

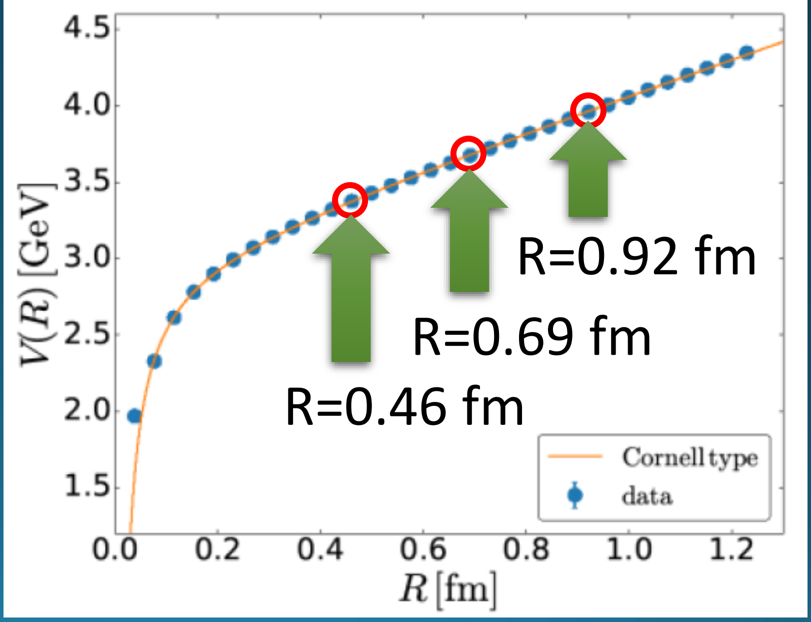
- EMT around Wilson Loop
- APE smearing / multi-hit

- fine lattices (a=0.029-0.06 fm)
- **continuum extrapolation**

- Simulation: bluegene/Q@KEK

$$\langle O(x)\rangle_{Q\bar{Q}} = \lim_{T\to\infty} \frac{\langle \delta O(x)\delta W(R,T)\rangle}{\langle W(R,T)\rangle}$$

β	a [fm]	N_{size}^4	N_{conf}	R/a		
6.304	0.058	48^4	140	8	12	16
6.465	0.046	48^4	440	10	–	20
6.513	0.043	48^4	600	–	16	–
6.600	0.038	48^4	1,500	12	18	24
6.819	0.029	64^4	1,000	16	24	32
			R [fm]	0.46	0.69	0.92



Stress Distribution on Mid-Plane

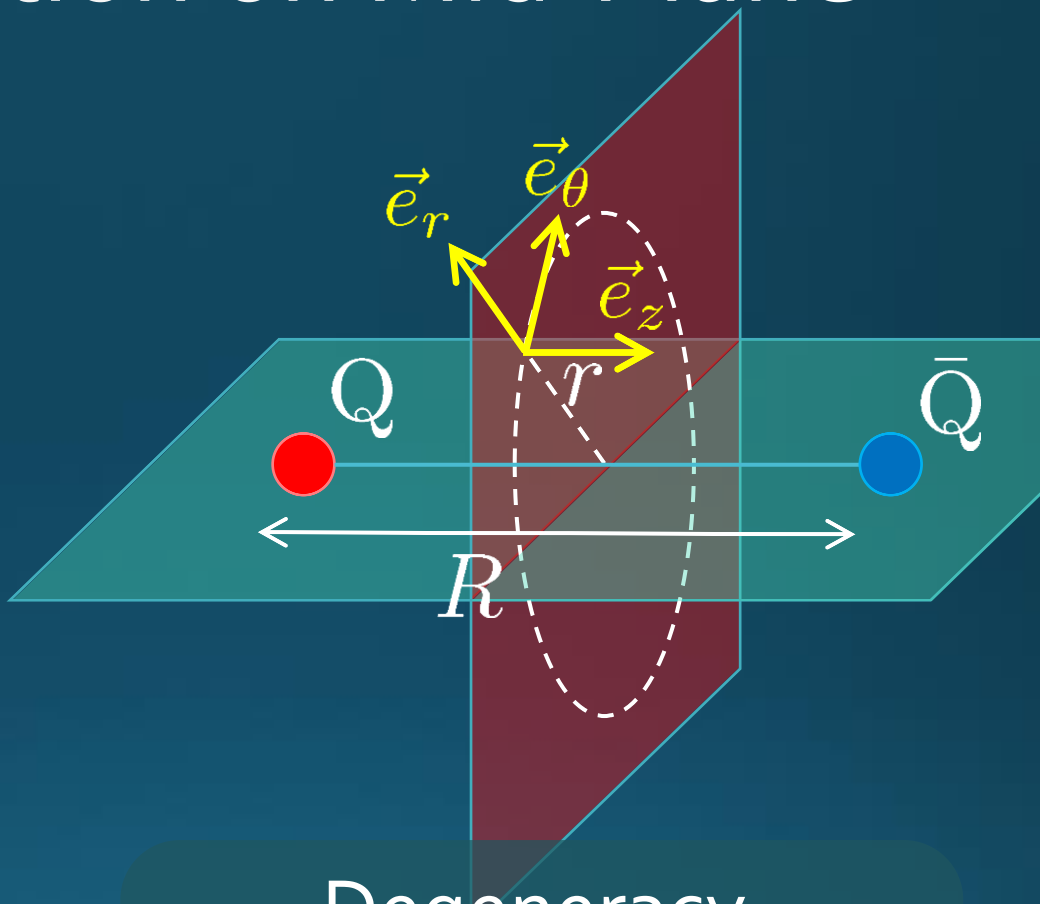
From rotational symm. & parity

EMT is diagonalized
in Cylindrical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{zz} & \\ & & & T_{44} \end{pmatrix}$$

$$T_{rr} = \vec{e}_r^{\,T} T \vec{e}_r$$

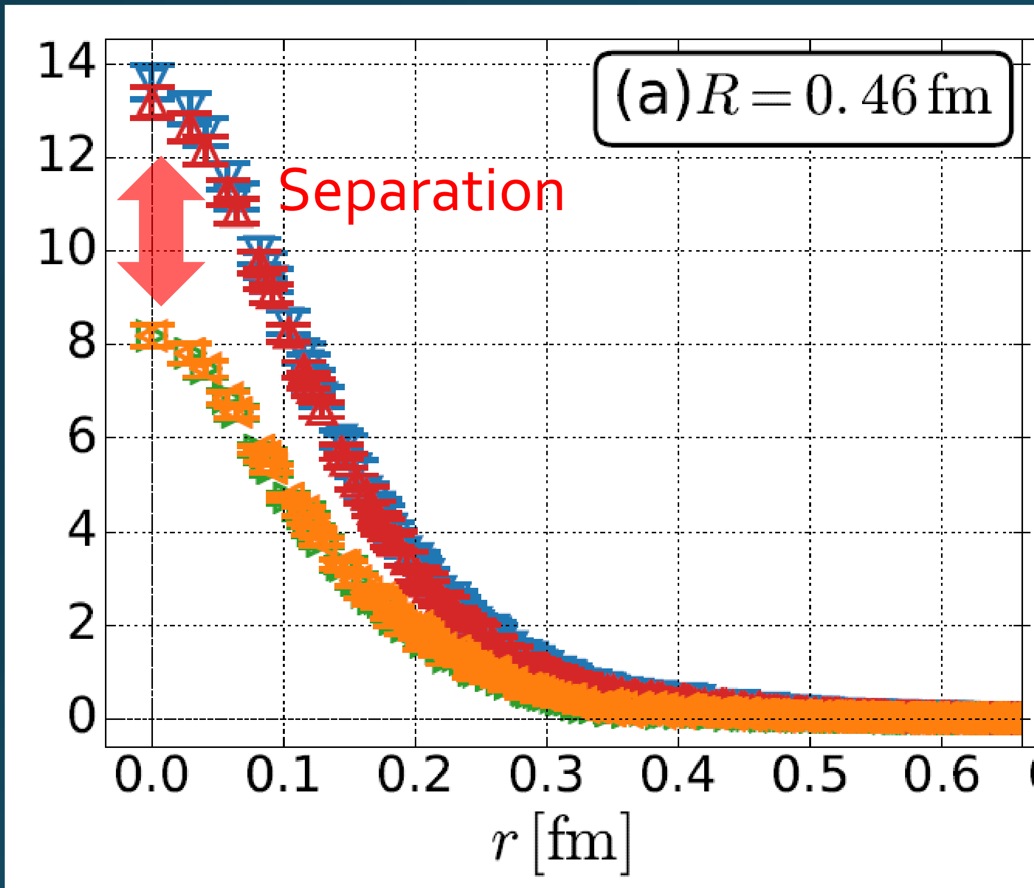
$$T_{\theta\theta} = \vec{e}_\theta^{\,T} T \vec{e}_\theta$$

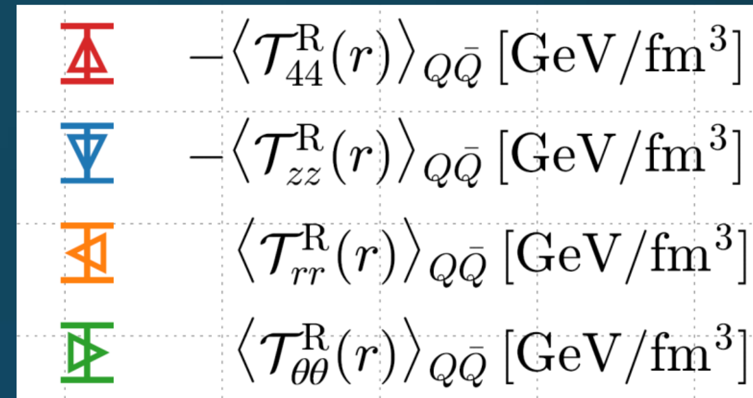


Degeneracy
in Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

Mid-Plane





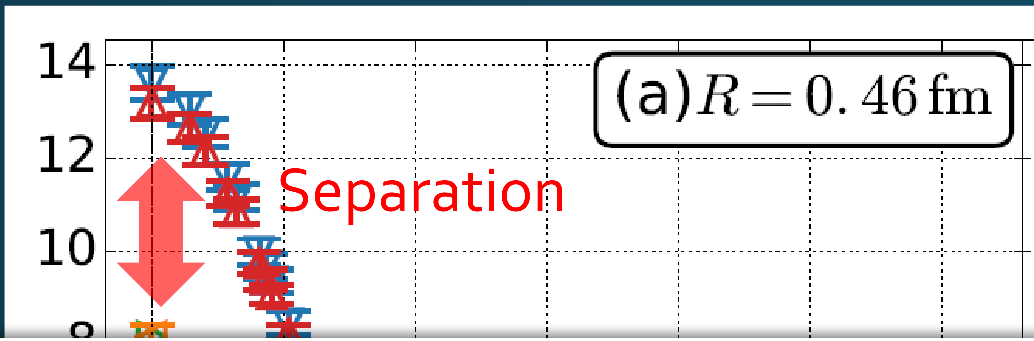
Continuum Extrapolated!

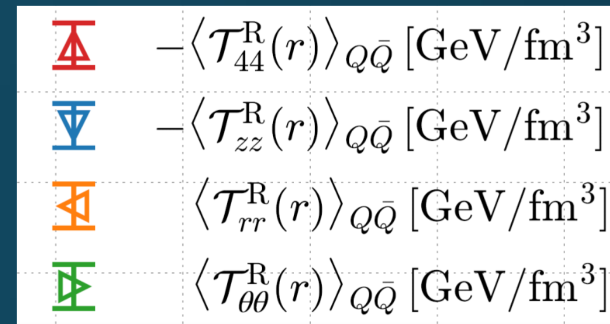
In Maxwell theory

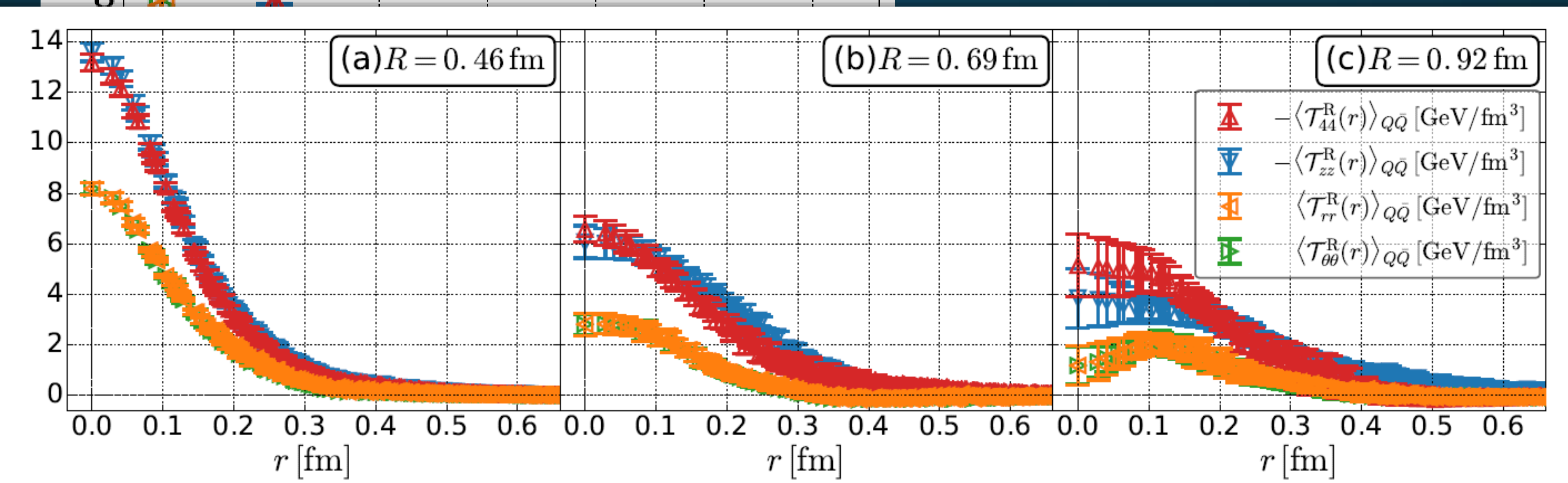
$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

- Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- Separation: $T_{zz} \neq T_{rr}$
- Nonzero trace anomaly $\sum T_{cc} \neq 0$

Mid-Plane

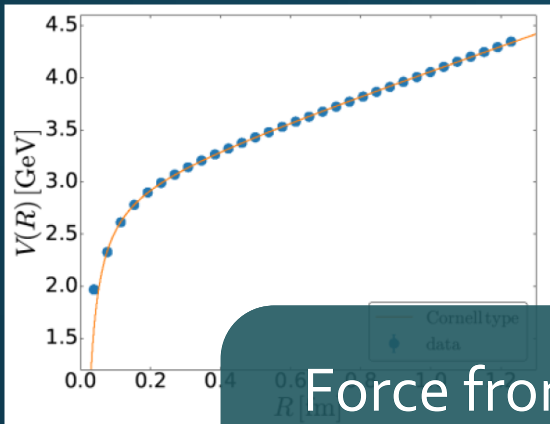


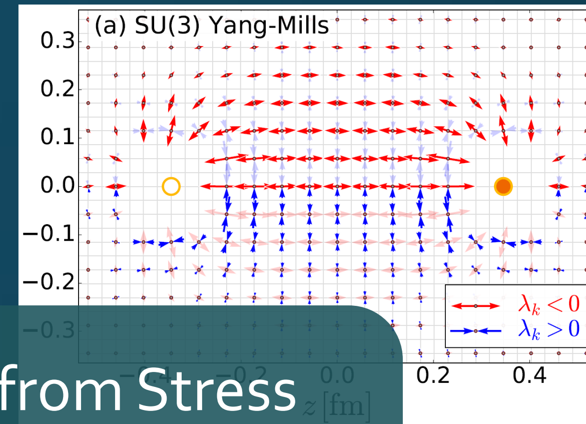




- Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- Separation: $T_{zz} \neq T_{rr}$
- Nonzero trace anomaly $\sum T_{cc} \neq 0$

Force





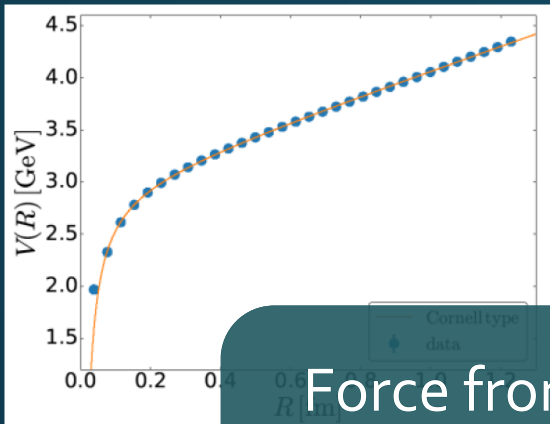
Force from Potential

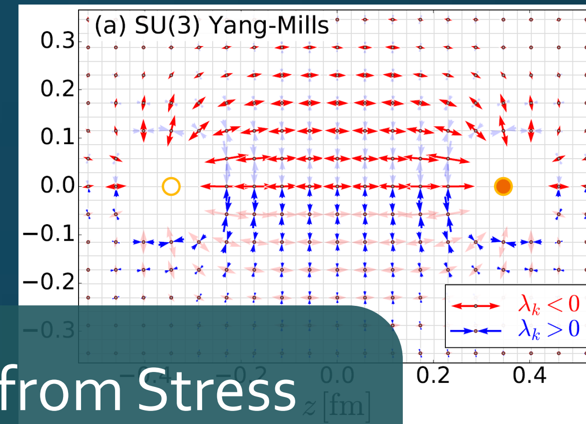
$$F_{\text{pot}} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\text{stress}} = \int_{\text{mid.}} d^2x T_{zz}(x)$$

Force



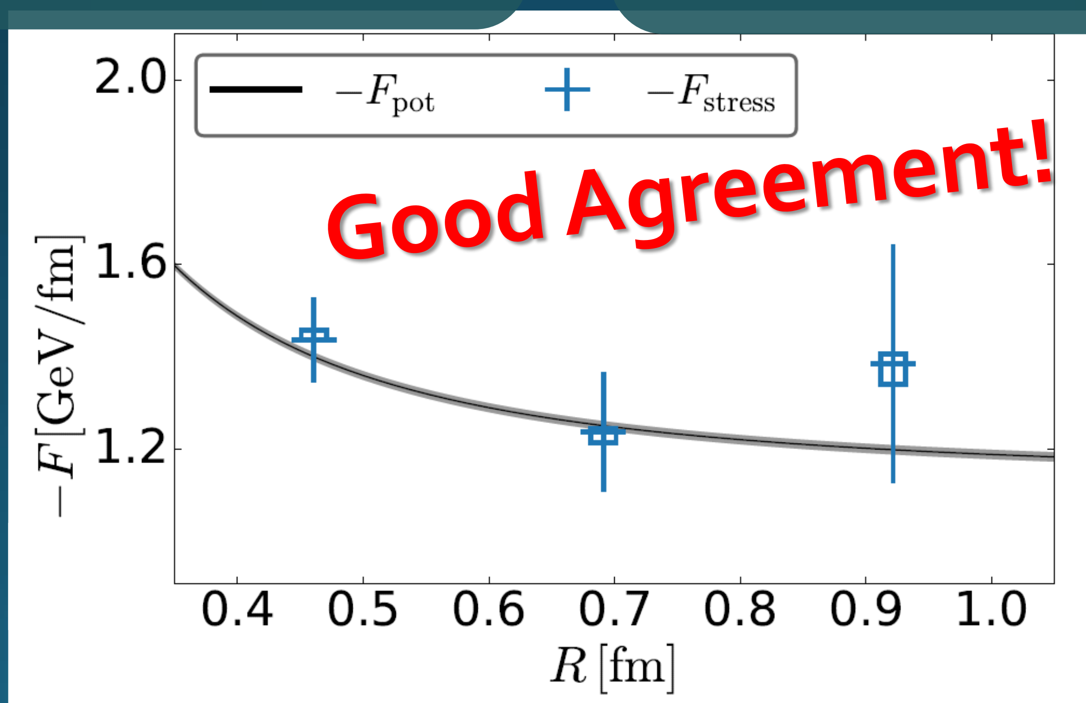


Force from Potential

$$F_{\text{pot}} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\text{stress}} = \int_{\text{mid.}} d^2x T_{zz}(x)$$



Momentum Conservation

Yanagihara, MK, PTEP2019

- **In cylindrical coordinates,**

$$\partial_i T_{ij} = 0 \Rightarrow \partial_r(rT_{rr}) = T_{\theta\theta} - r\partial_z T_{rz}$$

For infinitely-long tube

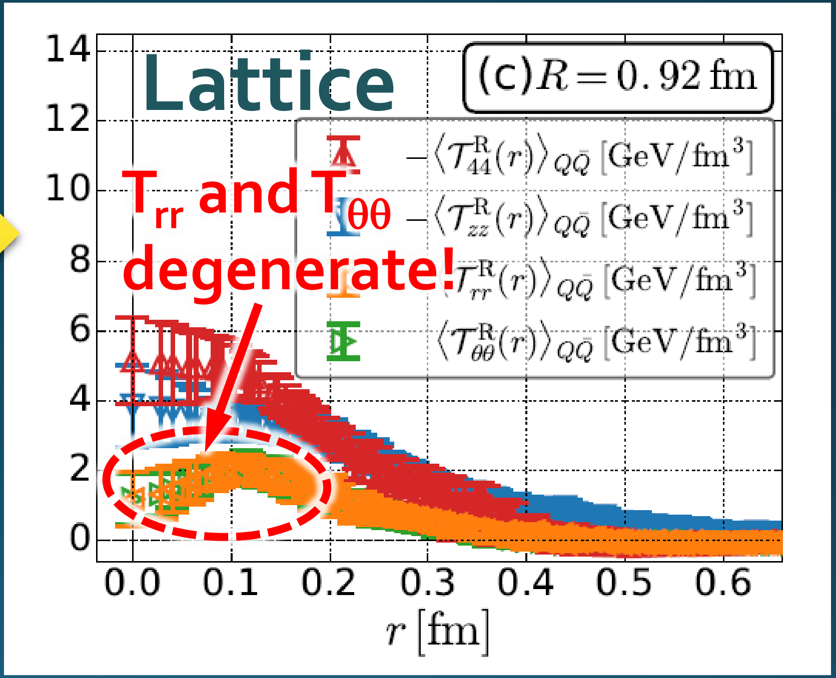
$$\partial_r(rT_{rr}) = T_{\theta\theta} \Rightarrow \int_0^\infty dr T_{\theta\theta}(r) = \Big[rT_{rr}\Big]_0^\infty = 0$$

- T_{rr} and $T_{\theta\theta}$ must separate!
- $T_{\theta\theta}$ must change sign!

Momentum Conservation

Yanagihara, MK, PTEP2019

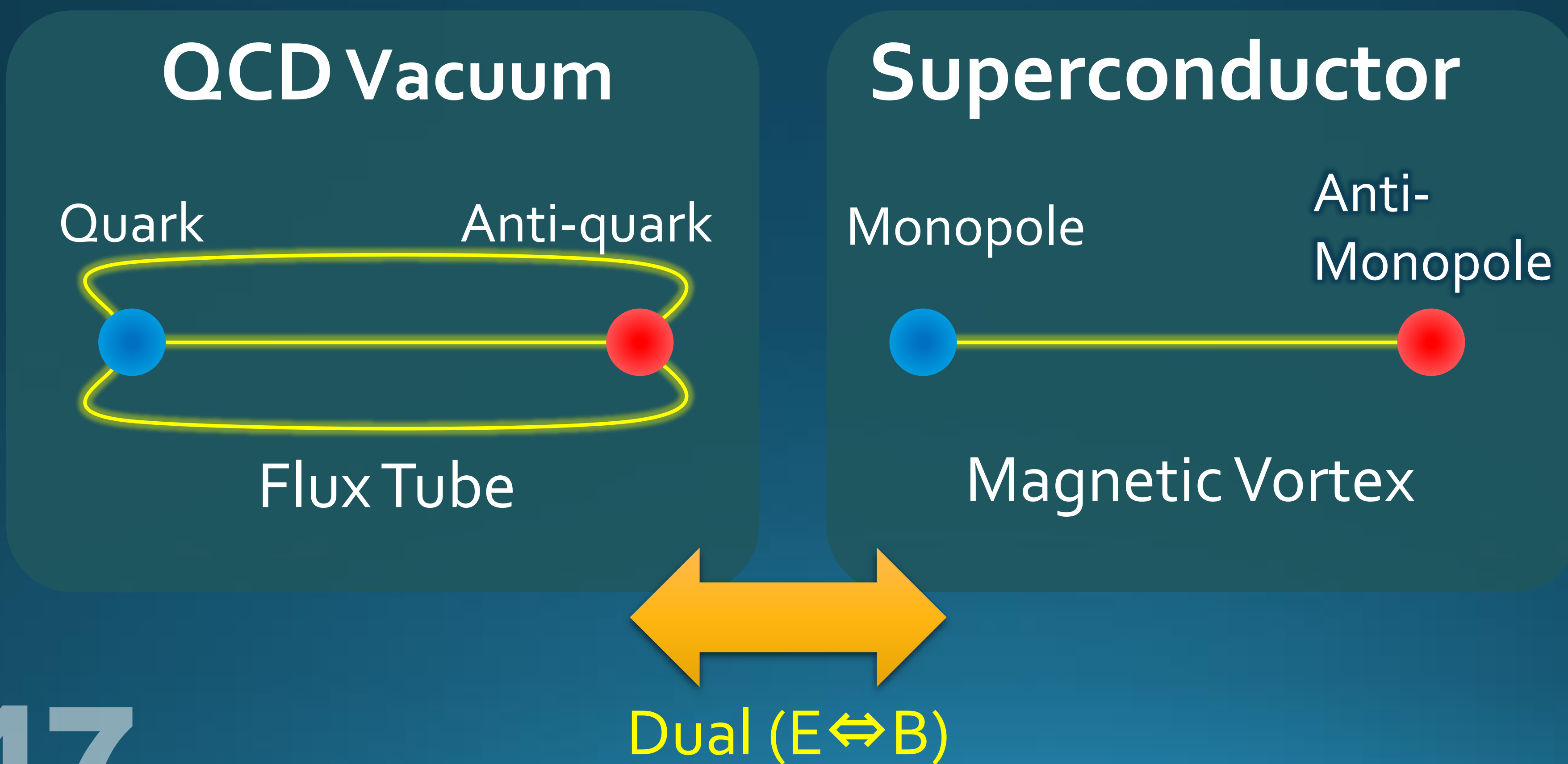
- **Infinitely-long system**
  - T_{rr} and $T_{\theta\theta}$ must separate
  - $T_{\theta\theta}$ must change sign



Effect of boundaries is important for the flux tube at R=0.92fm

Dual Superconductor Picture

Nambu, 1970
Nielsen, Olesen, 1973
t 'Hooft, 1981
...



Abelian-Higgs Model

Yanagihara, MK, 2019

Abelian-Higgs Model

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

GL parameter: $\kappa = \sqrt{\lambda}/g$

- type-I : $\kappa < 1/\sqrt{2}$
- type-II : $\kappa > 1/\sqrt{2}$
- Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

Infinitely long tube

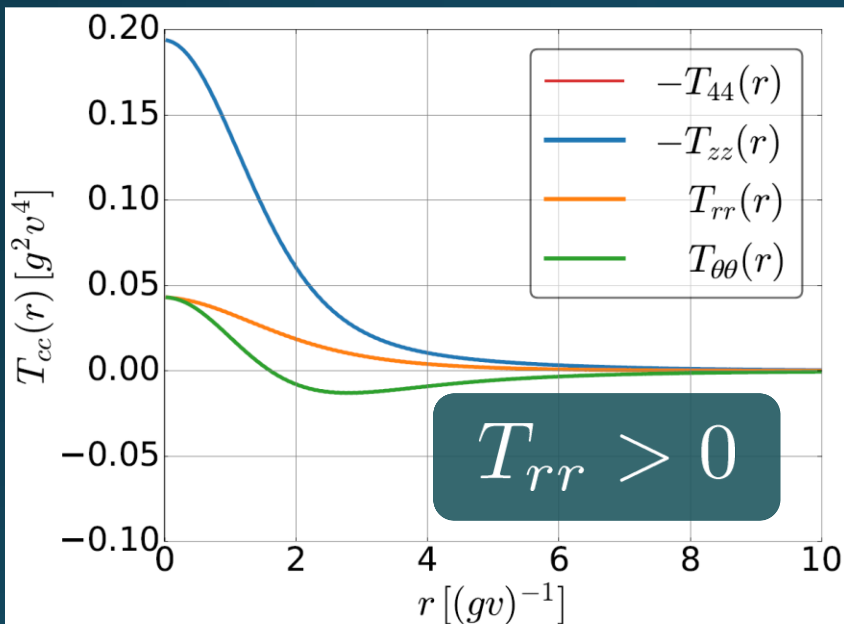
- degeneracy
  $T_{zz}(r) = T_{44}(r)$ Luscher, 1981
- momentum conservation
  $$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

Stress Tensor in AH Model

infinitely-long flux tube

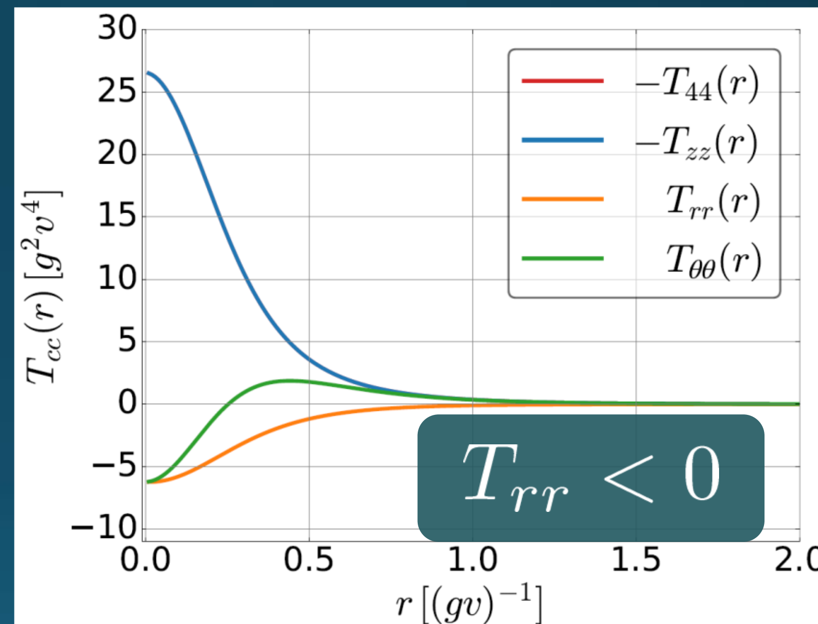
Type-I

$\kappa = 0.1$



$\kappa = 3.0$

Type-II



- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

conservation law

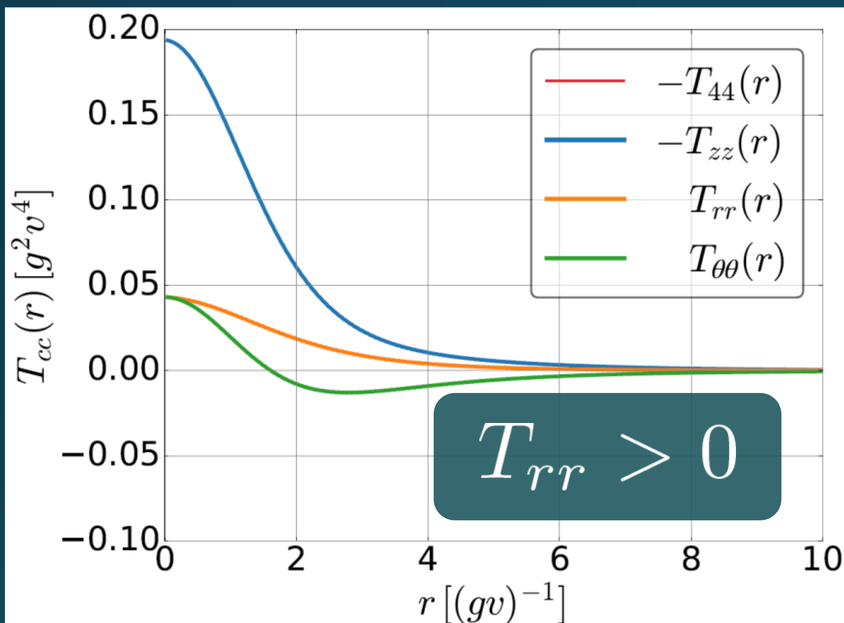
$$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

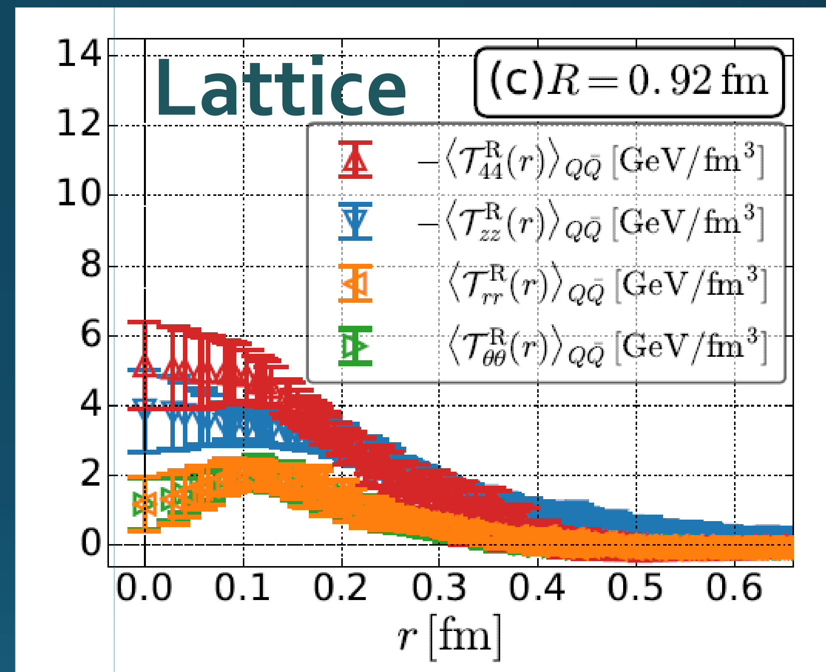
Stress Tensor in AH Model

infinitely-long flux tube

Type-I

$\kappa = 0.1$





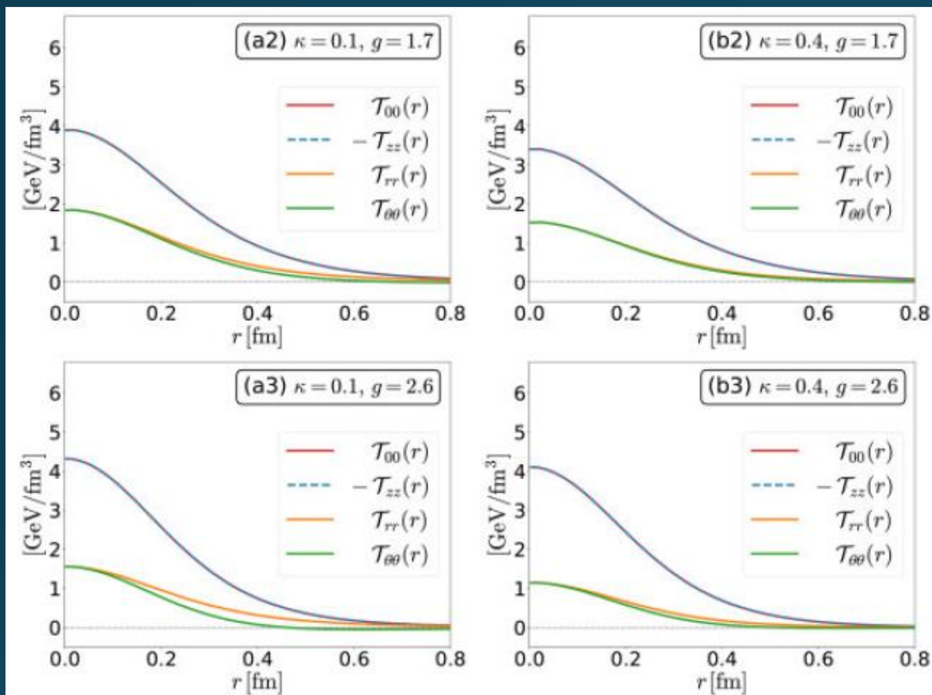
- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

Inconsistent with lattice result

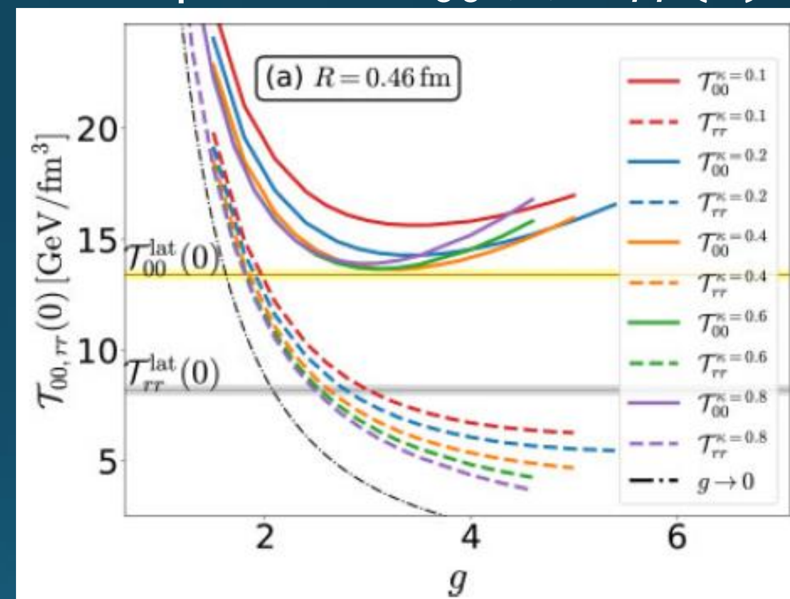
$$T_{rr} \simeq T_{\theta\theta}$$

Flux Tube with Finite Length

Yanagihara, MK (2019)



Comparison: $T_{00}(0), T_{rr}(0)$



- ☐ AH model can reproduce lattice results **qualitatively** by tuning parameters.
- ☐ But, **quantitatively** all parameters are rejected.

Quantum Effects?

- Classical vortex is unstable against quantum fluctuations
- Quantum effects give rise to
  - Luscher term in potential Luscher (1981)
  - Fattening of the tube Luscher, Munster, Weisz (1981)

$$w^2 \sim \log r$$

How do these effects modify EMT distribution?

Single Static-Quark System

□ $T < T_c$: Heavy-light meson

- EMT distribution in the meson

□ $T > T_c$: Single charge

- Screening
- Running coupling

□ $T \approx T_c$

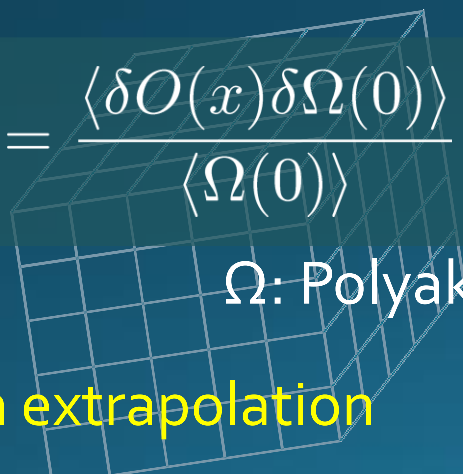
- Confinement transition

This study:
$T > T_c$ in pure YM

Lattice Setup

- ☐ SU(3) Yang-Mills (Quenched)
- ☐ Wilson gauge action
- ☐ Clover operator

- ☐ Analysis above Tc
- ☐ Simulation on a Z_3 minimum
- ☐ EMT around a Polyakov loop



$$\langle O(x)\rangle_{\mathrm{Q}} = \frac{\langle \delta O(x)\delta\Omega(0)\rangle}{\langle \Omega(0)\rangle}$$

Ω: Polyakov loop

- ☐ continuum extrapolation

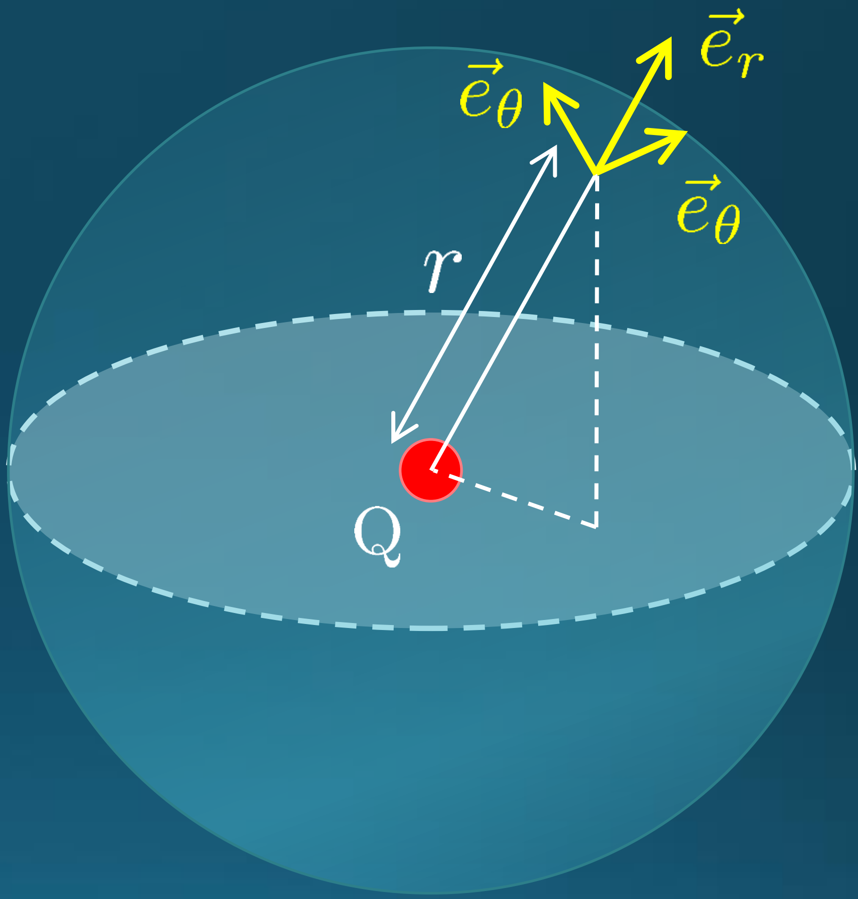
T/T_c	N_s	N_τ	β	a [fm]	N_{conf}
1.20	40	10	6.336	0.0551	500
	48	12	6.467	0.0460	650
	56	14	6.581	0.0394	840
	64	16	6.682	0.0344	1,000
	72	18	6.771	0.0306	1,000
1.44	40	10	6.465	0.0461	500
	48	12	6.600	0.0384	650
	56	14	6.716	0.0329	840
	64	16	6.819	0.0288	1,000
	72	18	6.910	0.0256	1,000
2.00	40	10	6.712	0.0331	500
	48	12	6.853	0.0275	650
	56	14	6.973	0.0236	840
	64	16	7.079	0.0207	1,000
	72	18	7.173	0.0184	1,000
2.60	40	10	6.914	0.0255	500
	48	12	7.058	0.0212	650
	56	14	7.182	0.0182	840
	64	16	7.290	0.0159	1,000
	72	18	7.387	0.0141	1,000

FlowQCD, PRD **102**, 114522 (2020)

Spherical Coordinates

EMT is diagonalized
in Spherical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{\theta\theta} & \\ & & & T_{44} \end{pmatrix}$$

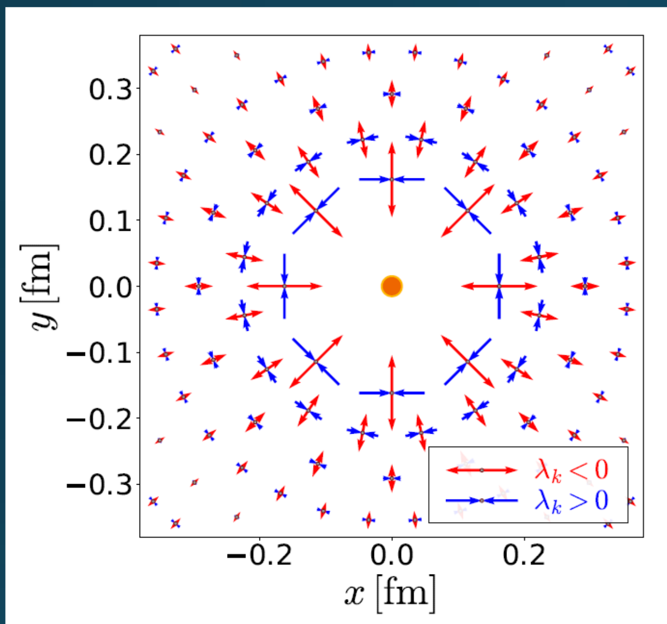


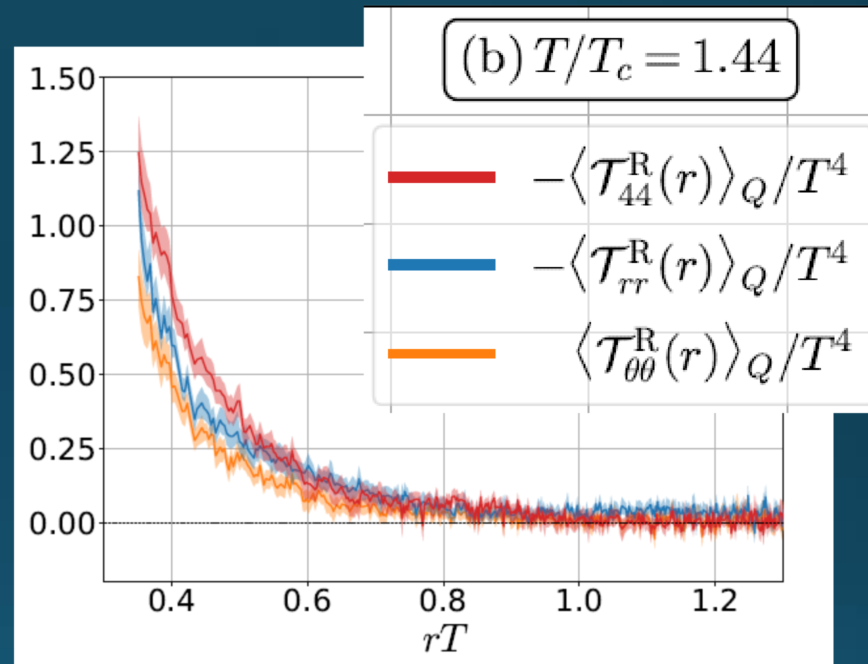
- Maxwell theory

$$T_{44} = T_{rr} = -T_{\theta\theta} = -\frac{|\boldsymbol{E}|^2}{2} = -\frac{\alpha}{8\pi}\frac{1}{r^4}$$

Stress Tensor Around a Quark

$T=1.44T_c$





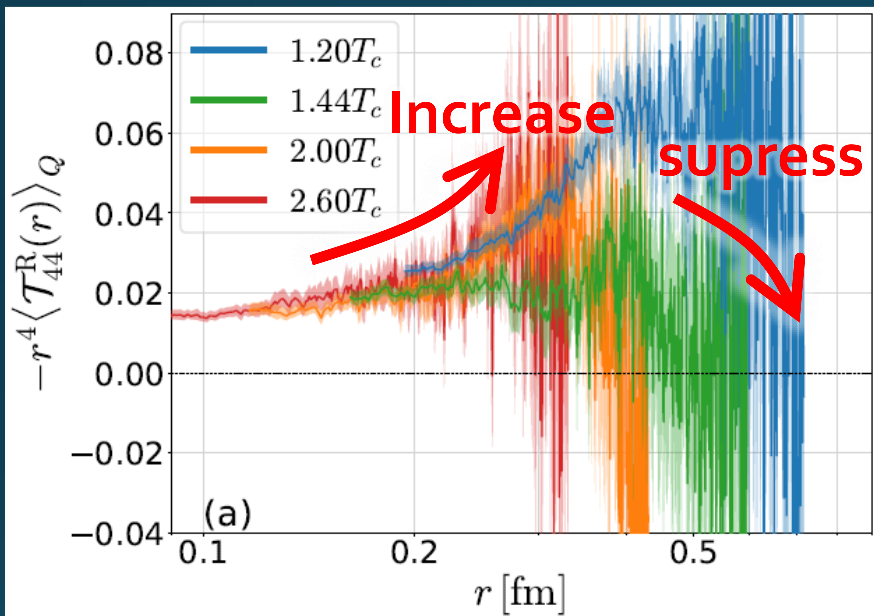
pulling

pushing

- ☐ Suppression at large distance
- ☐ Separation of different channels

r Dependence

$r^4\langle T_{00}(r)\rangle$



Leading order perturbation

$$\langle \mathcal{T}_{44}(r)\rangle = \langle \mathcal{T}_{rr}(r)\rangle = -\langle \mathcal{T}_{\theta\theta}(r)\rangle$$

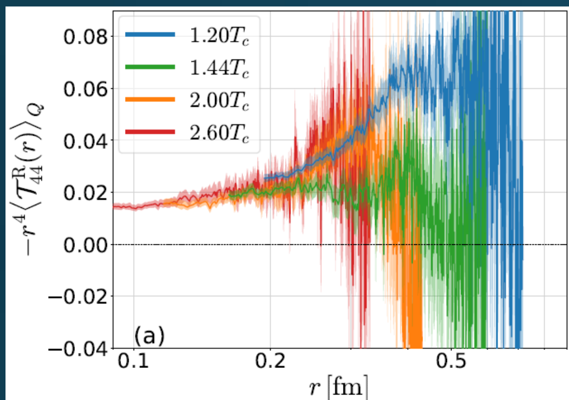
$$= -\frac{C_F}{8\pi}\alpha_s \frac{(m_{\mathrm{D}} r + 1)^2}{r^4} e^{-2m_{\mathrm{D}} r}$$

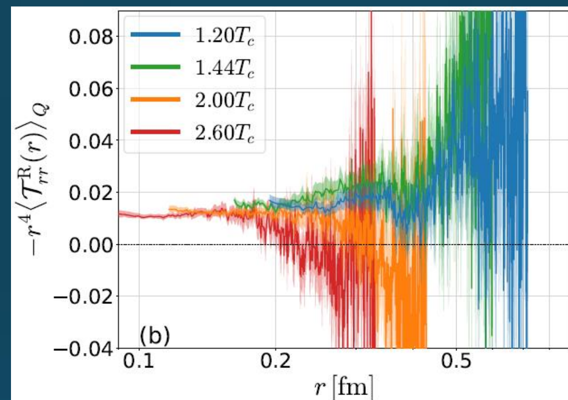
Higher order terms:
M. Berwein, in progress

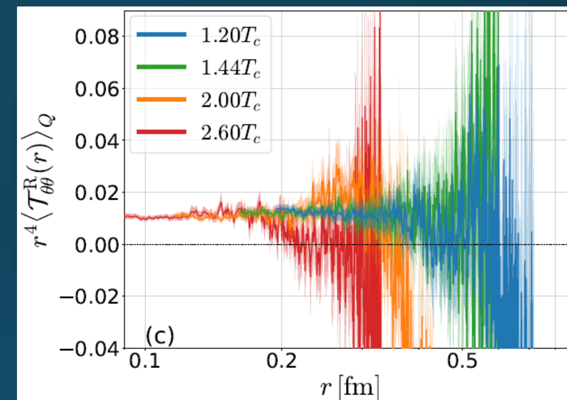
- ☐ Increase at short r / suppression at larger r
- ☐ T dependence is suppressed at $r < 1/T$
- ☐ Too noisy at large r for extracting screening mass m_{D}

Channel Dependence

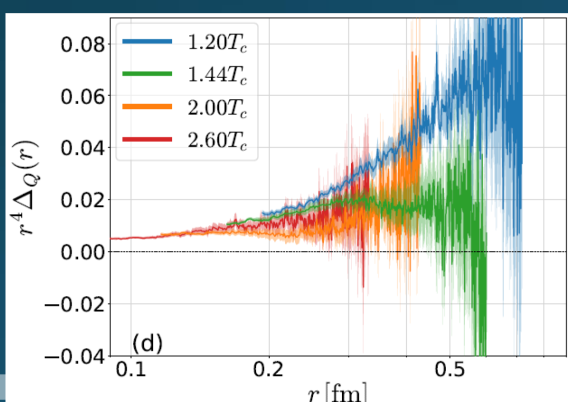
$$r^4\langle T_{00}(r)\rangle \qquad -r^4\langle T_{rr}(r)\rangle \qquad r^4\langle T_{\theta\theta}(r)\rangle$$







$$r^4\Delta(r) = -r^4\langle T_{\mu\mu}\rangle$$



- Separation b/w channels becomes clearer for smaller *T*

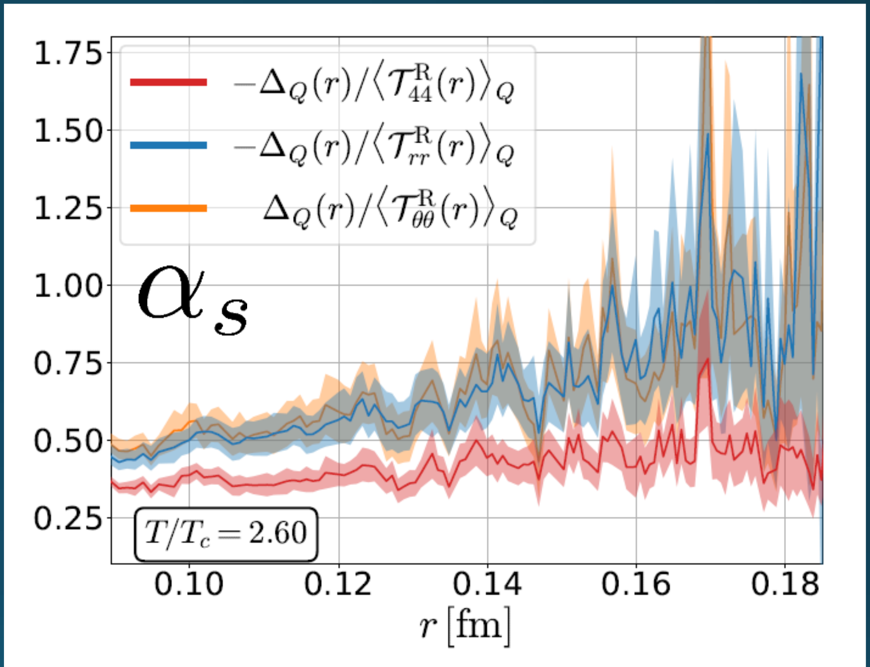
FlowQCD, PRD **102**, 114522 (2020)

Running Coupling

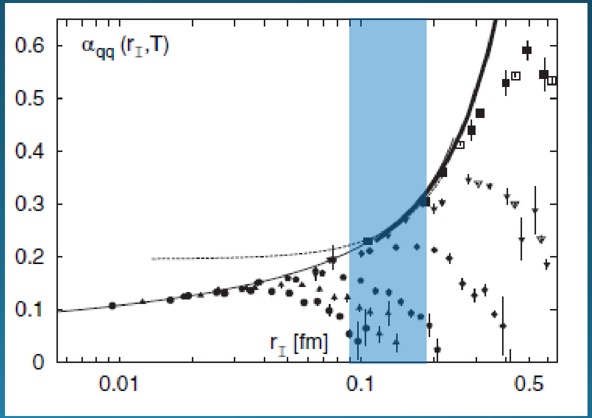
- **Estimate of α_s**

$$\left|\frac{\langle T_{\mu\mu}\rangle}{\langle \mathcal{T}_{44,rr,\theta\theta}(r)\rangle}\right| = \frac{11}{2\pi}\alpha_s + \mathcal{O}(g^3),$$

  - by the formula at the leading-order perturbation theory
  - channel dependent



- Consistent with the estimate from $Q\bar{Q}$ potential



Kaczmarek, Karsch, Zantow, 2004

Contents

1. SFtX: EMT through Gradient Flow

2. Casimir Effect & Pressure Anisotropy

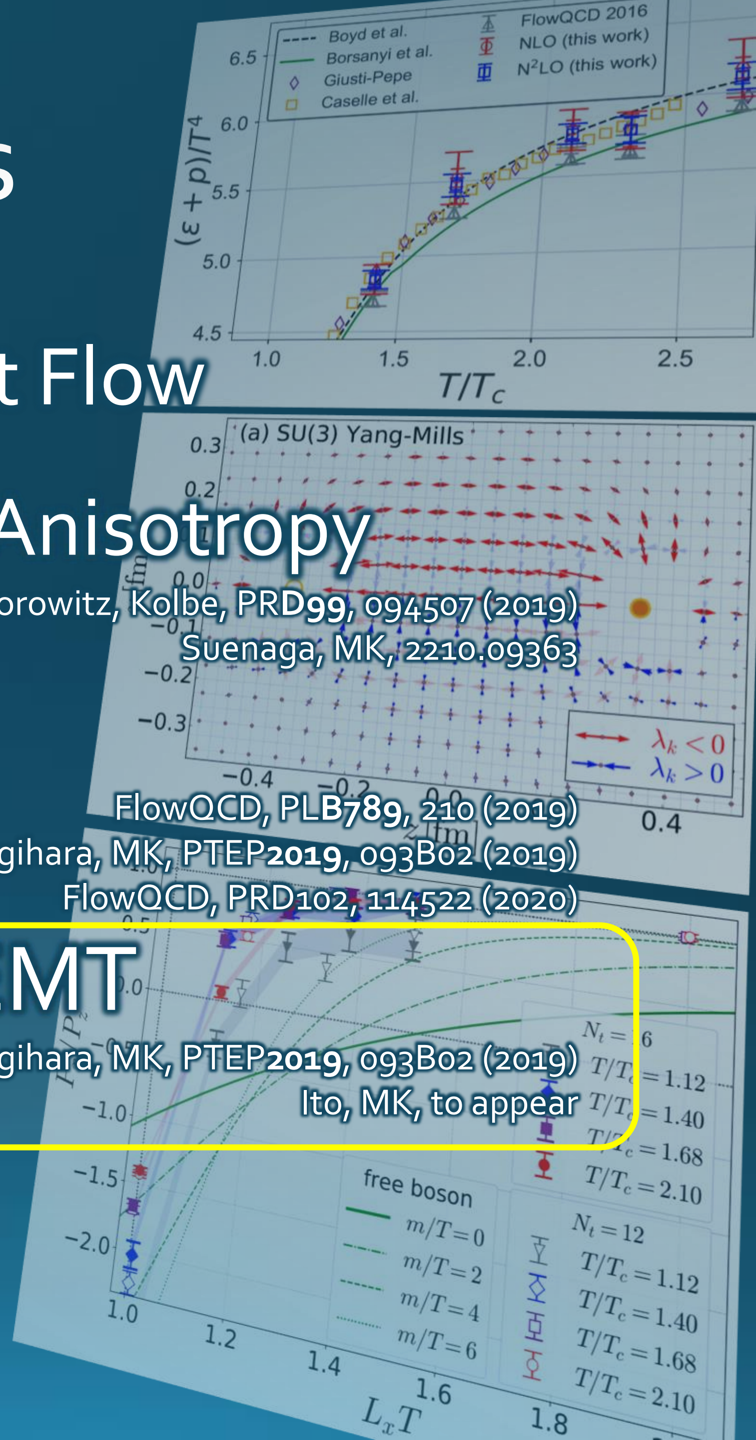
MK, Mogliacci, Horowitz, Kolbe, PRD**99**, 094507 (2019)
Suenaga, MK, 2210.09363

3. Static-Quark Systems

FlowQCD, PLB**789**, 210 (2019)
Yanagihara, MK, PTEP**2019**, 093B02 (2019)
FlowQCD, PRD102, 114522 (2020)

4. Model Calculations of EMT

Yanagihara, MK, PTEP**2019**, 093B02 (2019)
Ito, MK, to appear



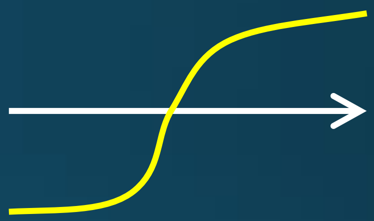
EMT Distr. in Simple Systems

ϕ^4 Theory in 1+1d

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2$$

- **Soliton (kink)**

$$\phi(x) = \frac{m}{\sqrt{\lambda}} \tanh \frac{mx}{\sqrt{2}}$$



- **EMT**

Classical

$$\begin{cases} T_{00} = \dfrac{m^4}{\lambda} \cosh^{-4} \dfrac{m(x-X)}{\sqrt{2}} \\ T_{01} = T_{11} = 0 \end{cases}$$

How do quantum effects modify this result?

Ito, MK, to appear

cf) total energy: Dashen+ ('74)

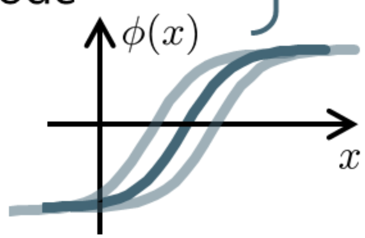
Fluctuation around the Kink

$$\phi(x) = \phi_{\mathrm{kink}}(x) + \eta(x)$$

$$S[\eta] = S_{\mathrm{cl}} + \int d^2x \Big\{ \underbrace{\frac{1}{2}(\partial_0\eta)^2 - \frac{1}{2}\eta\big(-\partial_x^2 - m^2 + 3\lambda\phi_{\mathrm{kink}}^2\big)\eta}_{\text{quadratic} \;\Rightarrow\; \text{diagonalize}} - \lambda\phi_{\mathrm{kink}}\eta^3 - \frac{\lambda}{4}\eta^4 \Big\}$$

$\lambda\phi_{\mathrm{kink}}\eta^3$: $\mathcal{O}(\lambda^{1/2})$

Eigenvals	Eigenfuncs	
$\omega_q^2 = q^2 + 2m^2$	$\psi_q(x) \to \exp\left[i\left(qx \pm \frac{1}{2}\delta\right)\right]$ (δ: phase shift)	scattering states
$\omega_1^2 = \frac{3}{2}m^2$	$\psi_1(x)$: vibrational mode	bound states
$\omega_0^2 = 0$	$\psi_0(x) \sim \partial_x\phi_{\mathrm{kink}}(x)$: translational mode (zero mode)	bound states



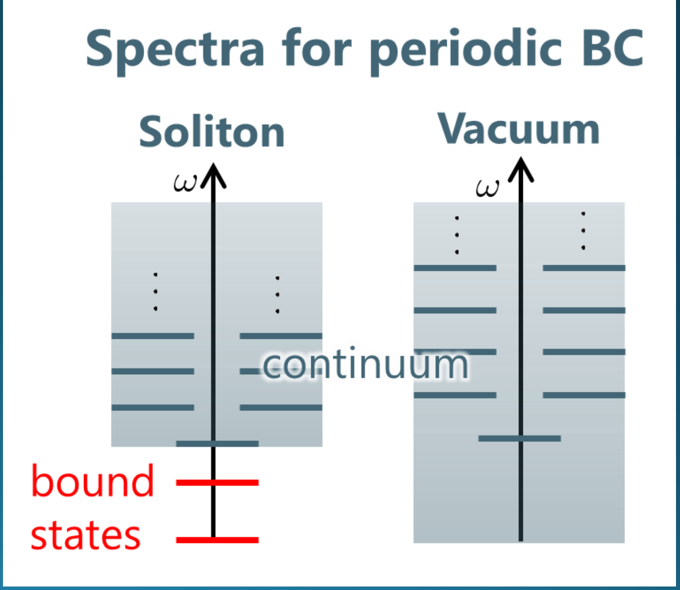
Removal of Divergences

- Zero mode → IR divergence
  - The divergence comes from the translational mode.
  - **Collective coordinate method**: Eliminate it by promoting the position of the kink X as a dynamical variable.

Gervais, Sakita '74; Gervais, Jevicki, Sakita, '75
Tomboulis, '75; Christ, Lee, '75

- UV divergences
  - Mass renormalization
  - Vacuum subtraction by the **mode-number cutoff**

1) Take the length of the system L finite.
2) Subtract with the same mode numbers.
3) Take $L \to \infty$ limit.



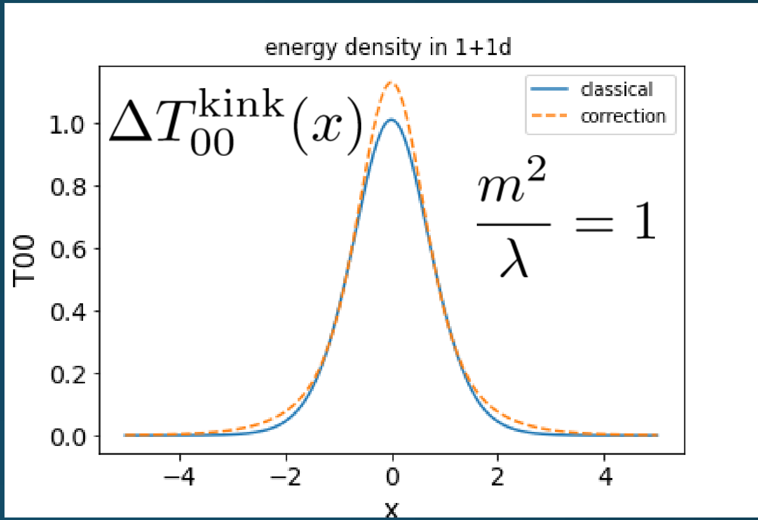
Rebhan and Nieuwenhuizen ('97)
Rajaraman, "solitons & instantons"

Results

$$T_{00}(x) = \Delta T_{00}^{\text{kink}}(x) - \frac{3\sqrt{2}m}{2\pi}\frac{1}{L}$$

$$T_{11}(x) = -\frac{3\sqrt{2}m}{2\pi}\frac{1}{L}$$

L: spatial length



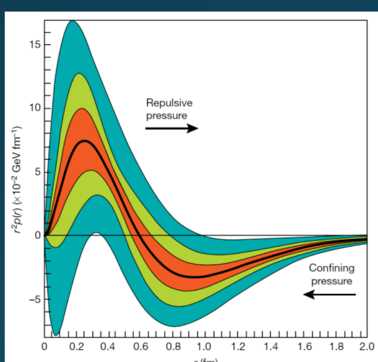
$$\Delta T^{00}_{\text{kink}}(x) = \frac{\sqrt{3}}{6} m^2 \text{sech}^2 \frac{mx}{\sqrt{2}} - \left(\frac{3}{2\pi} + \frac{7\sqrt{3}}{12}\right) m^2 \text{sech}^4 \frac{mx}{\sqrt{2}} + 5\left(\frac{3}{8\pi} + \frac{\sqrt{3}}{12}\right) m^2 \text{sech}^6 \frac{mx}{\sqrt{2}} + \frac{3\sqrt{6}}{8} m^3 x \tanh\frac{mx}{\sqrt{2}} \text{sech}^4 \frac{mx}{\sqrt{2}}$$

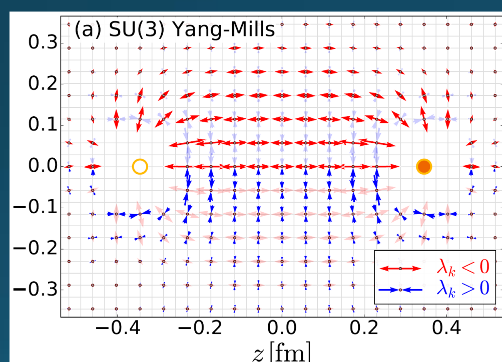
- **Note:**
- Constant term $\sim 1/L$ that vanishes at $L \to \infty$: potential term?
- $E = \int_{-L/2}^{L/2} dx\, T_{00}(x)$ reproduces Dashen+.
- $T_{11}(x)$ is x independent. ➔ consistent with EM conservation

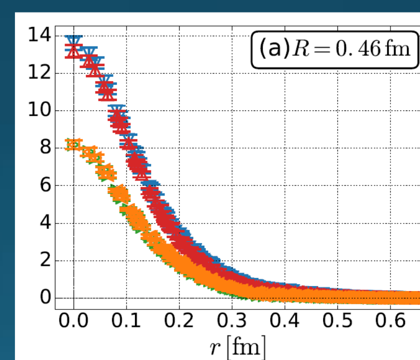
$$\partial_0 T_{01} - \partial_1 T_{11} = 0$$

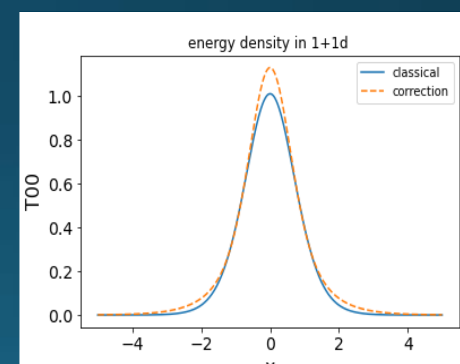
Summary

- Now, EMT distribution in localized systems in QCD is accessible by the experimental and numerical analyses.
- These information will enrich our understanding on the strongly-interacting systems.
- Further lattice and experimental investigations, as well as theoretical studies are called for.









- **Future studies:**

  Hadrons (GFF), single-Q in QCD / QQQ, QQ, etc. / *T* dependence / Solitons / GFF of nuclei / …

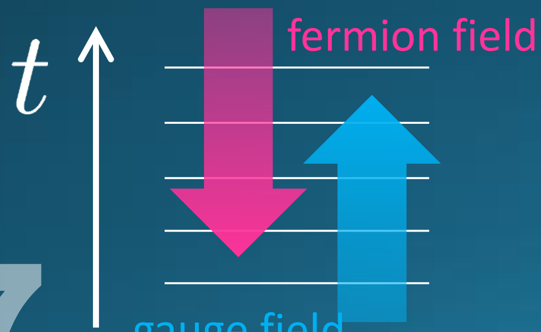
backup

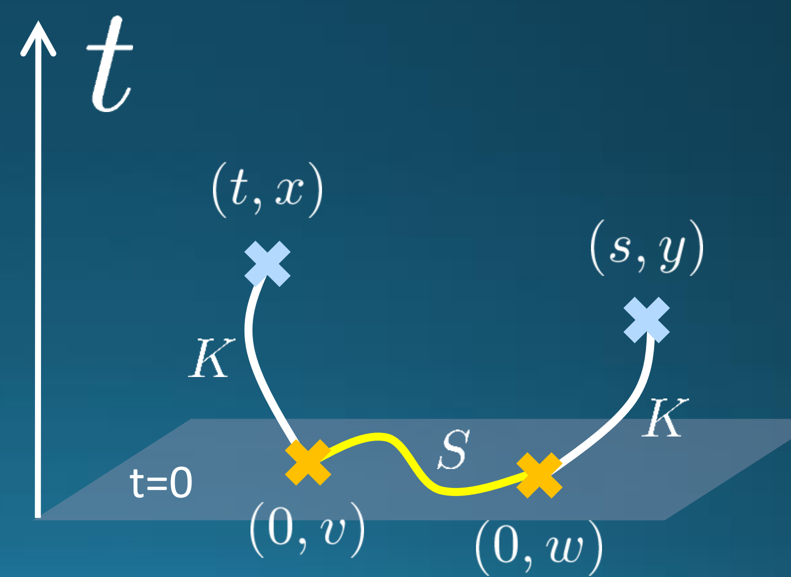
Fermion Propagator

$$S(t,x;s,y) = \langle \chi(t,x)\bar{\chi}(s,y)\rangle$$
$$= \sum_{v,w} K(t,x;0,v)S(v,w)K(s,y;0,w)^{\dagger}$$

$$\Big(\partial_t - D_\mu D_\mu\Big)K(t,x) = 0$$

- propagator of flow equation
- Inverse propagator is needed





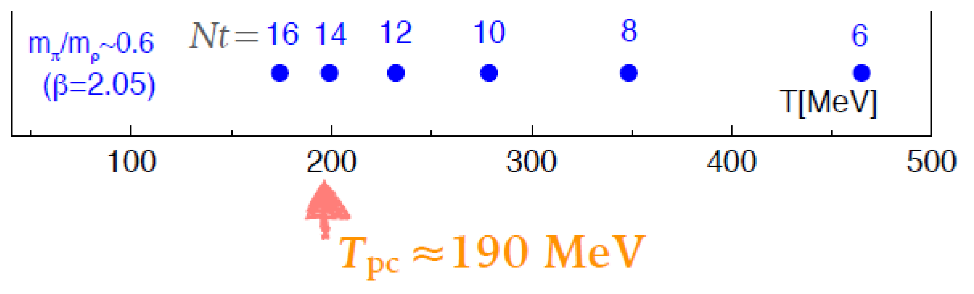
N_f=2+1 QCD Thermodynamics

Taniguchi+ (WHOT-QCD),
PRD**96**, 014509 (2017)

- N_f=2+1 QCD, Iwasaki gauge + NP-clover
- m_{PS}/m_V ≈0.63 / almost physical s quark mass

- T=0: CP-PACS+JLQCD (ß=2.05, 28^3x56, a≈0.07fm)
- T>0: 32^3xN_t, N_t = 4, 6, … , 14, 16):
- T≈174-697MeV

- t→0 extrapolation only (No continuum limit)



Perturbative Coefficients

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

$$T_{\mu\nu}(t) = c_1(t)U_{\mu\nu}(t) + \delta_{\mu\nu}c_2(t)E(t)$$

	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○	○	
$c_2(t)$	× zero	○	○	○

Suzuki (2013) Harlander+(2018)

Iritani, MK, Suzuki, Takaura, 2019

☐ Choice of the scale of g²

$$c_1(t) = c_1\left(g^2(\mu(t))\right)$$

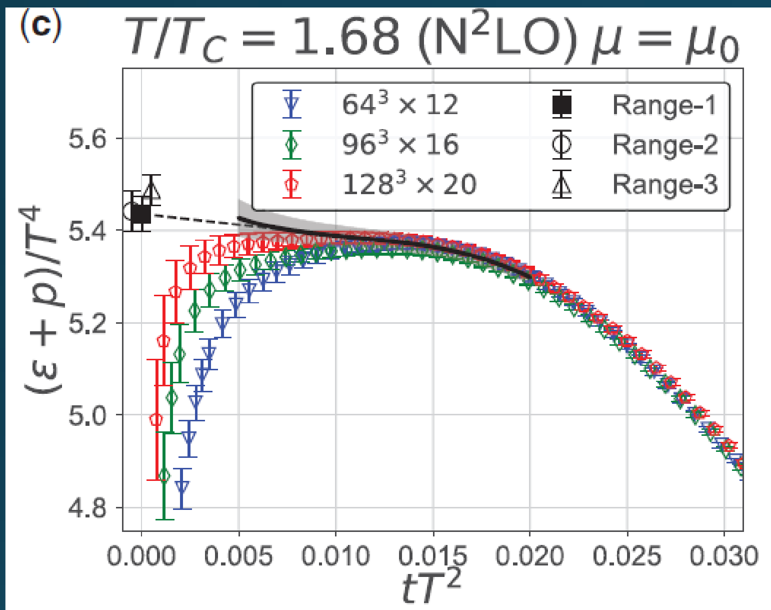
Previous: $\mu_d(t) = 1/\sqrt{8t}$

Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E}t}$

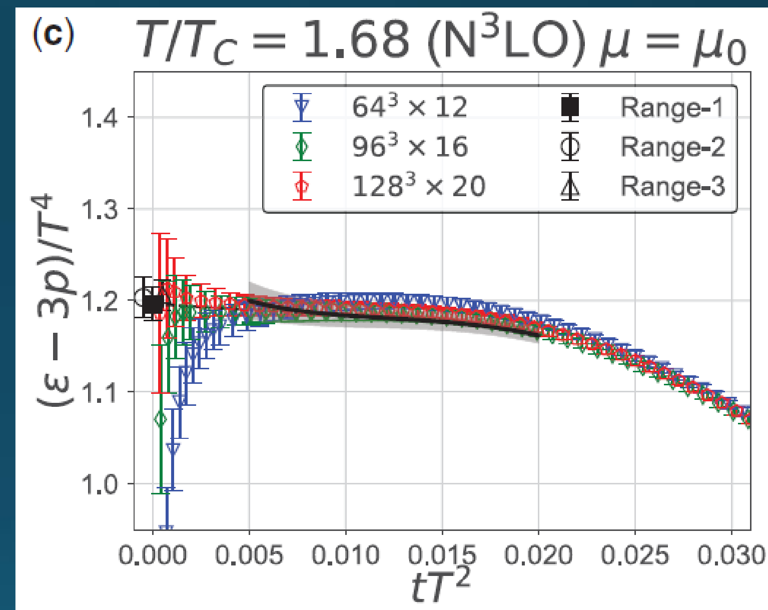
Harlander+ (2018)

t Dependence

$$\langle T_{00} \rangle + \langle T_{11} \rangle$$



$$\langle T_{00} \rangle - 3\langle T_{11} \rangle$$



Iritani, MK, Suzuki, Takaura, PTEP 2019

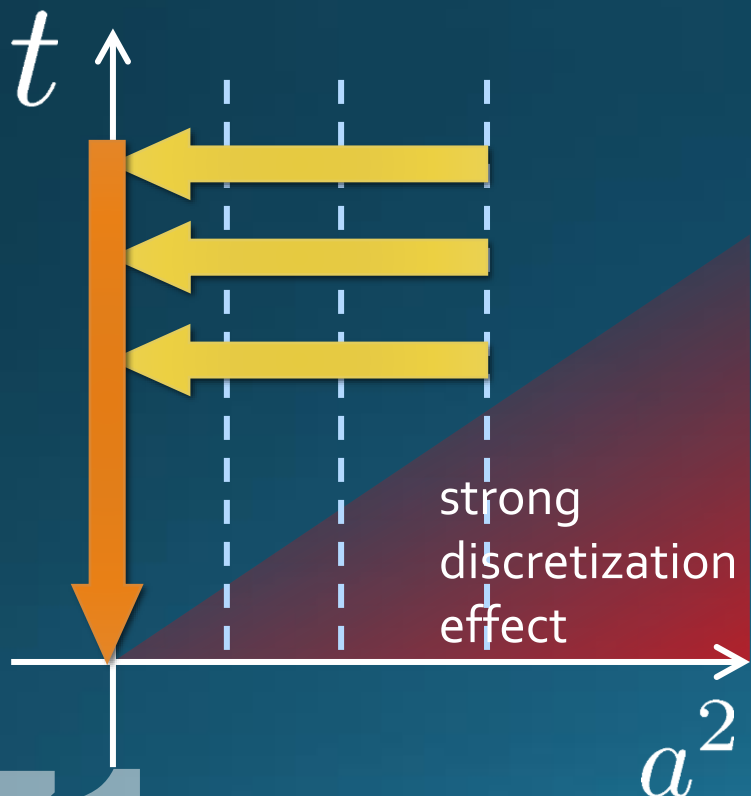
- ☐ Existence of “linear window” at intermediate t

Double Extrapolation

t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu} t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE   lattice discretization



Continuum extrapolation

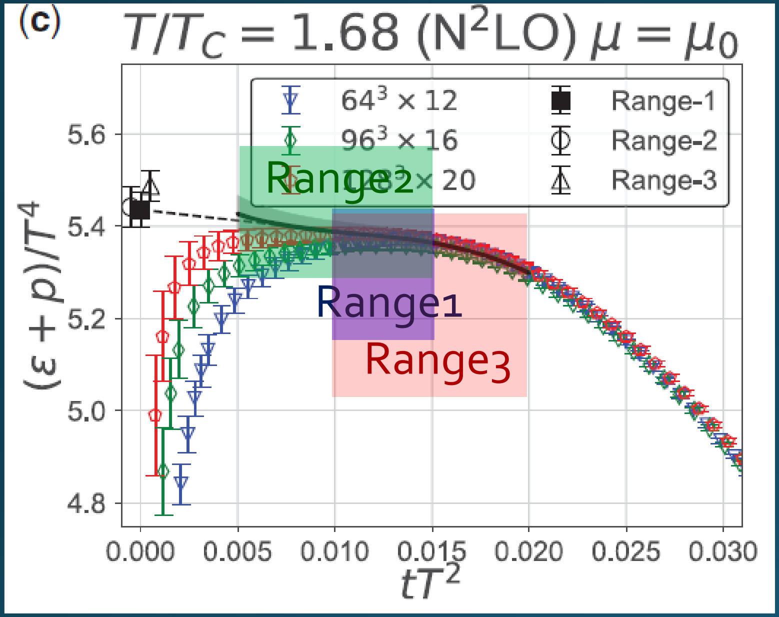
$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

Small t extrapolation

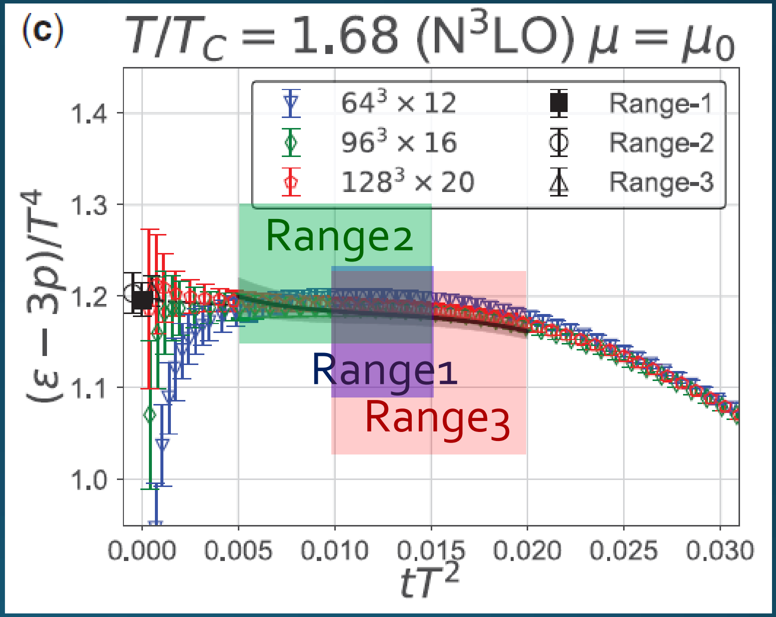
$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$

Thermodynamics: ε+p & ε-3p

$\langle T_{00} \rangle + \langle T_{11} \rangle$



$\langle T_{00} \rangle - 3\langle T_{11} \rangle$



Iritani, MK, Suzuki, Takaura, PTEP 2019

- ☐ Existence of "linear window" at intermediate t
- ☐ Stable t→0 extrapolation
- ☐ Systematic errors: fit range, uncertaintyof Λ (±3%), …

EMT on the Lattice: Conventional

Lattice EMT Operator Caracciolo+, 1990

$$T_{\mu\nu} = Z_6 T^{[6]}_{\mu\nu} + Z_3 T^{[3]}_{\mu\nu} + Z_1 \left(T^{[1]}_{\mu\nu} - \langle T^{[1]}_{\mu\nu} \rangle \right)$$

$$T^{[6]}_{\mu\nu} = (1-\delta_{\mu\nu}) F^a_{\mu\rho} F^a_{\nu\rho}, \quad T^{[3]}_{\mu\nu} = \delta_{\mu\nu} \left(F^a_{\mu\rho} F^a_{\nu\rho} - \frac{1}{4} F^a_{\rho\sigma} F^a_{\rho\sigma} \right), \quad T^{[1]}_{\mu\nu} = \delta_{\mu\nu} F^a_{\rho\sigma} F^a_{\rho\sigma}$$

- Determination of Zs are necessary.
- Non-pert. Determination of Zs
  - Shifted-boundary method
  - Full QCD with fermions

Giusti, Pepe, 2014~; Borsanyi+, 2018
Brida, Giusti, Pepe, 2020

Higher T

High-T limit: massless free gluons

How does the anisotropy approach this limit?

Difficulties

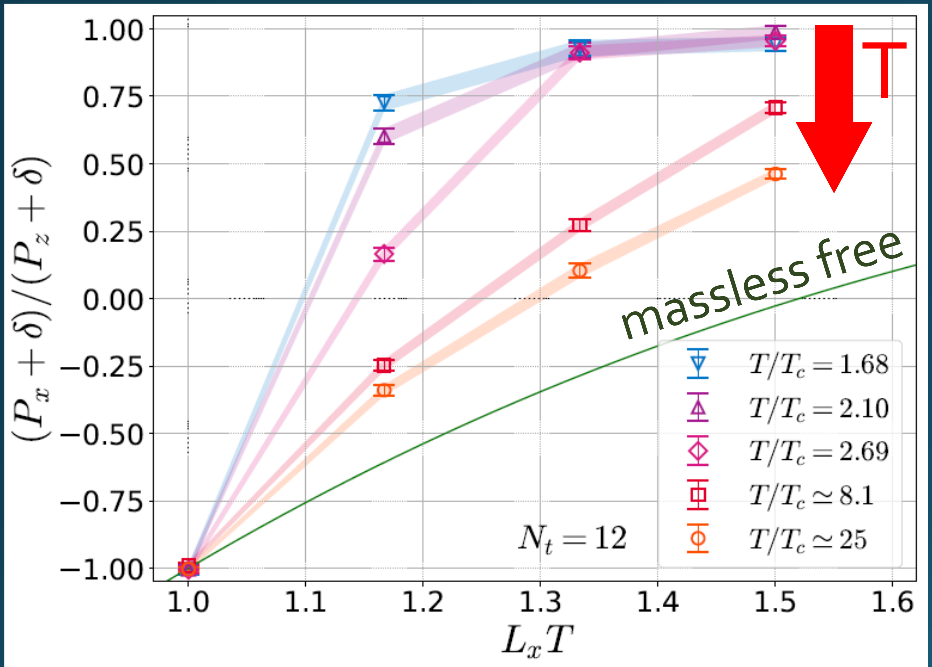
- ☐ Vacuum subtraction requires large-volume simulations.
- ☐ Lattice spacing not available → $c_1(t)$, $c_2(t)$ are not determined.

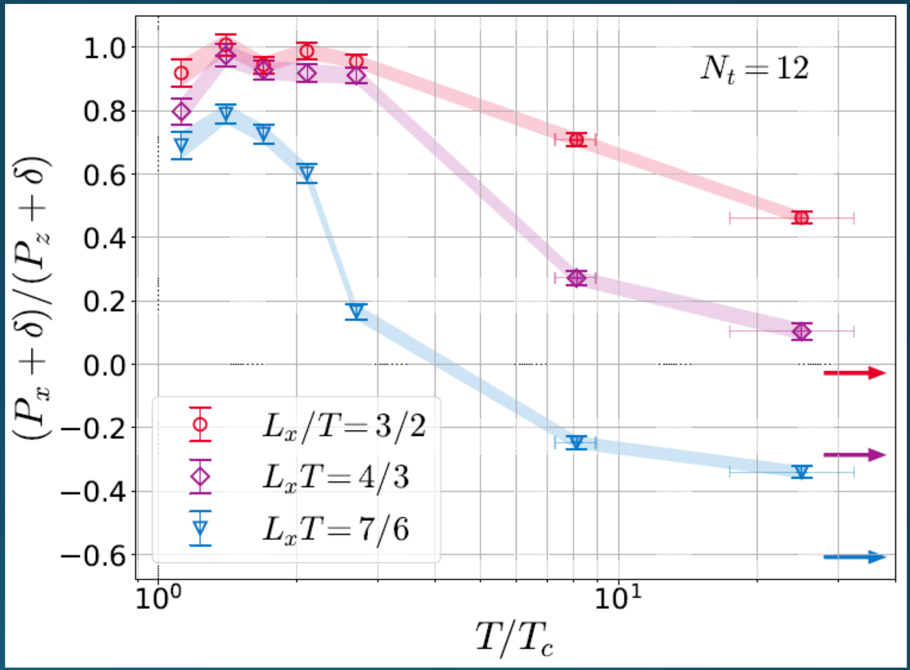
Ratio of Traceless Parts

$$\frac{P_x+\delta}{P_z+\delta} \qquad \delta=-\frac{1}{4}\sum_{\mu}T^{\mathrm{E}}_{\mu\mu}$$

Neither vacuum subtr. nor Suzuki coeffs. necessary!

$$\frac{P_x + \delta}{P_z + \delta}$$





$T/T_c \cong 8.1$ (β=8.0) / $T/T_c \cong 25$ (β=9.0)

- Ratio slowly approaches the asymptotic value.
- But, large deviation still exists even at $T/T_c \sim 25$.