Infinite and Finite Nuclear Matter (INFINUM-2023), 2023/Mar./1, online

Dilepton Production as a Signal to Explore QCD Phase Diagram

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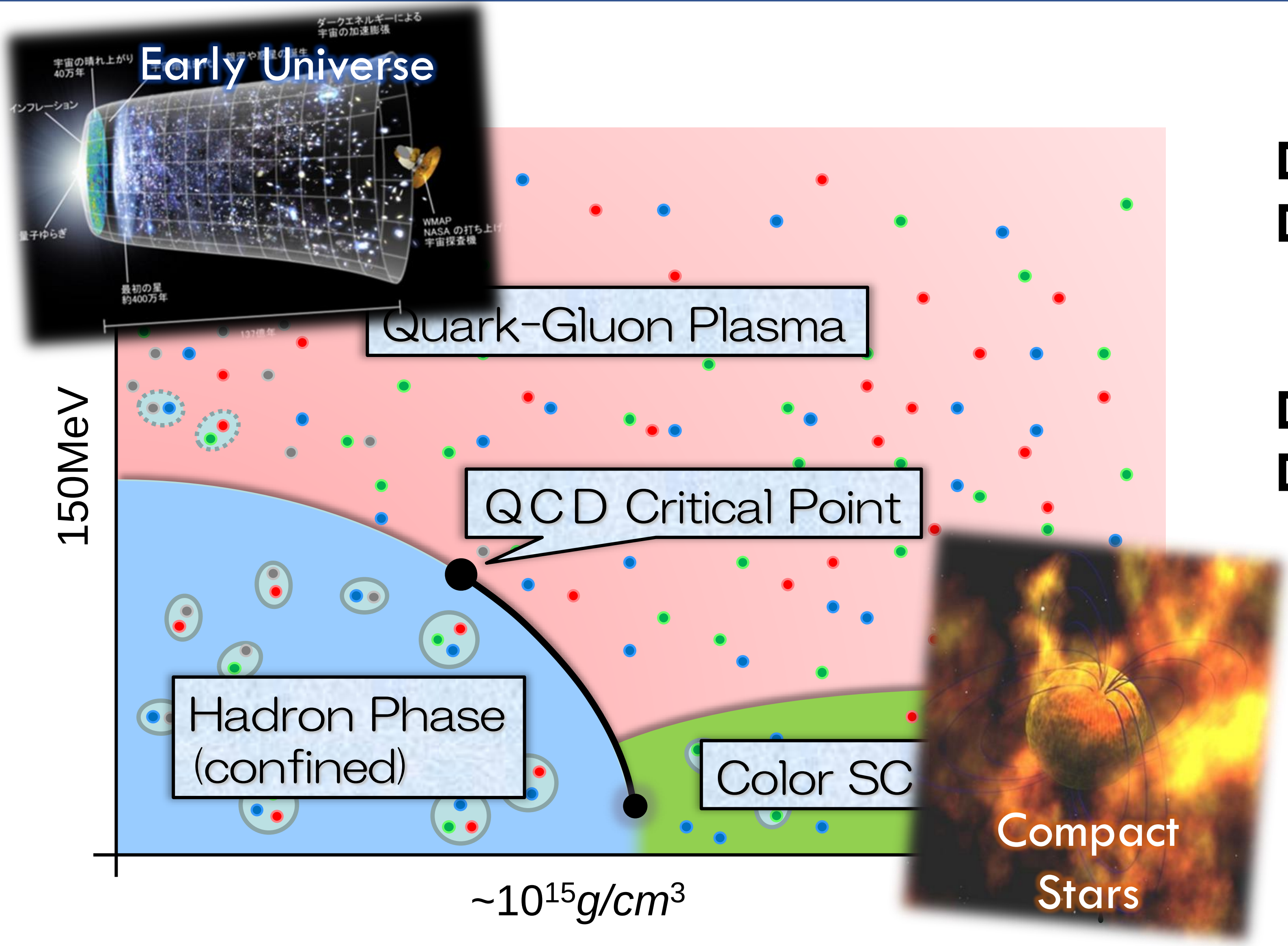
with Toru Nishimura and Teiji Kunihiro

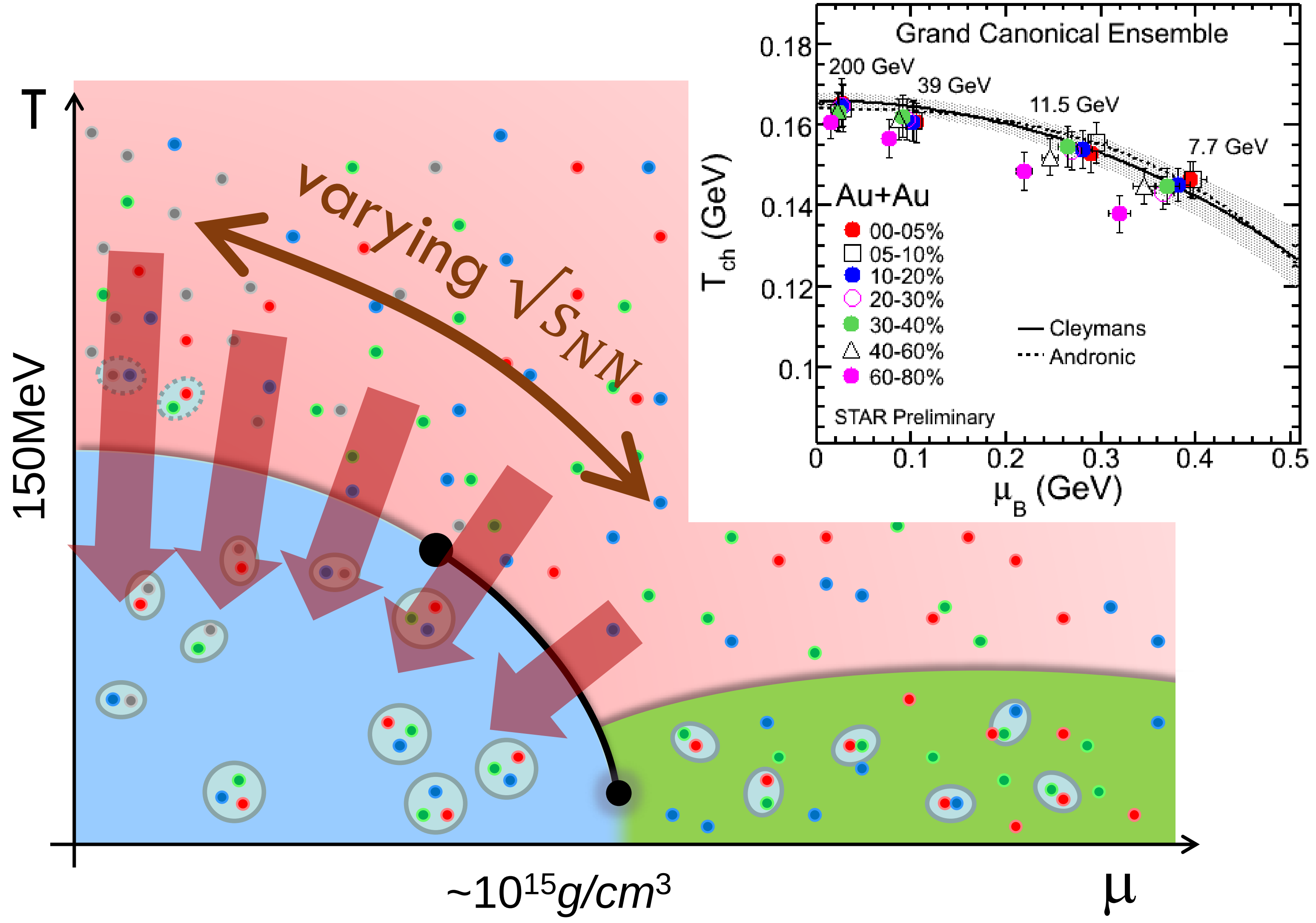
Nishimura, MK, Kunihiro, PTEP2022 (2022) 093D02; ibid., arXiv:2302.03191 [hep-ph]

QCD Phase Diagram



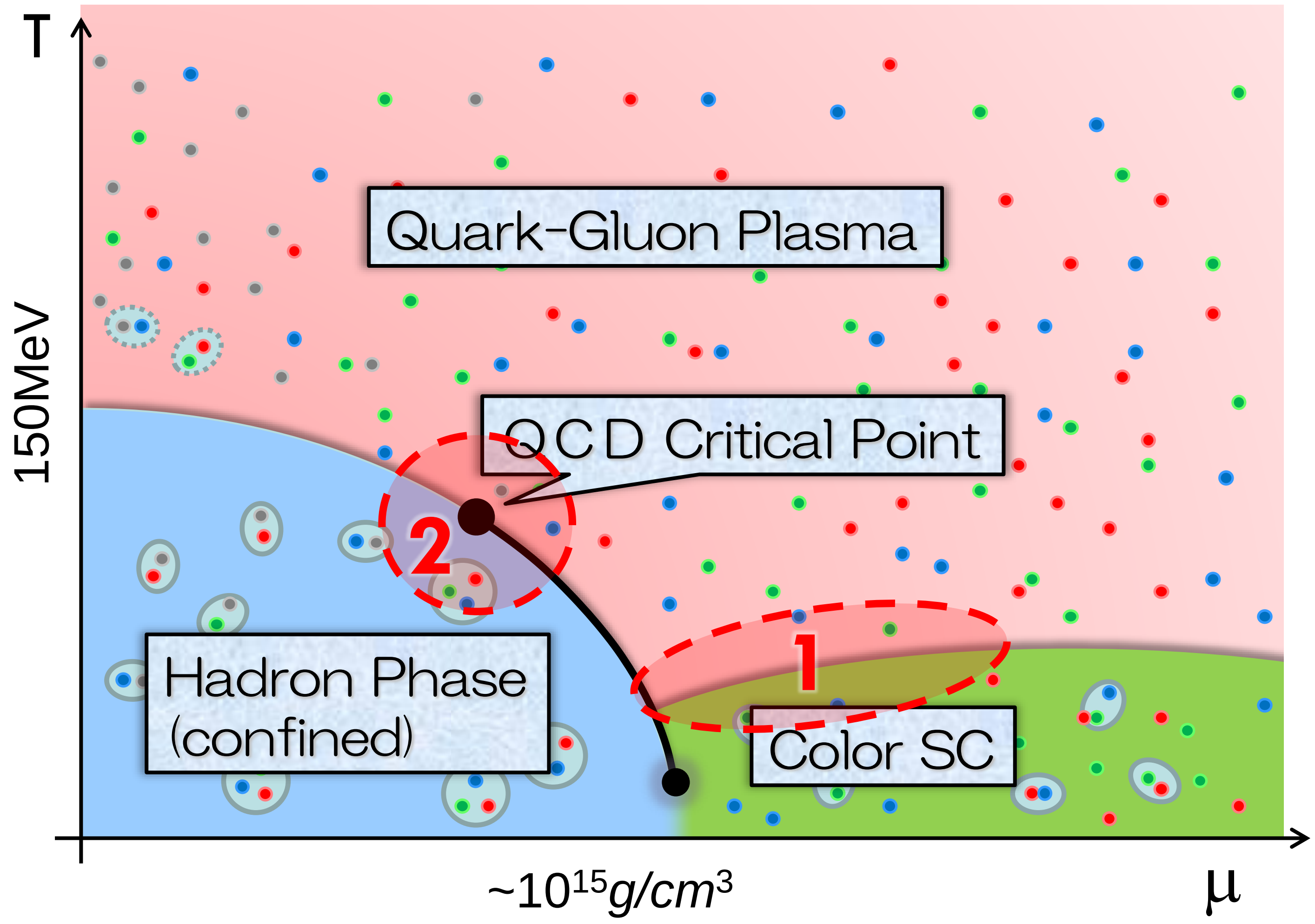
- ☐ Crossover at $\mu = 0$
- ☐ Possible first-order transition and QCD critical point in dense region
- ☐ Multiple QCD-CP? MK+ ('02)
- ☐ Color superconducting phases in dense and cold quark matter

Beam-Energy Scan in Heavy-Ion Collisions



- In HIC, T, μ can be changed by varying the collision energy.
- The "beam-energy scan" program is ongoing all over the world.
  - present: RHIC-BES, GSI-HADES, NA61/SHINE, …
  - future: NICA-MPD, GSI-FAIR, J-PARC-HI

Purpose of This Talk



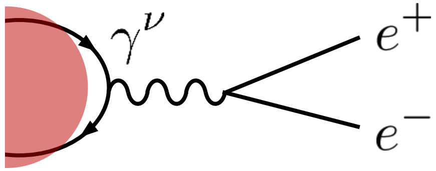
Explore

1. color superconductivity
   Nishimura+, PTEP2022, 093D02 ('22)
2. QCD critical point
   Nishimura+, arXiv:2302.03191

in heavy-ion collisions

using

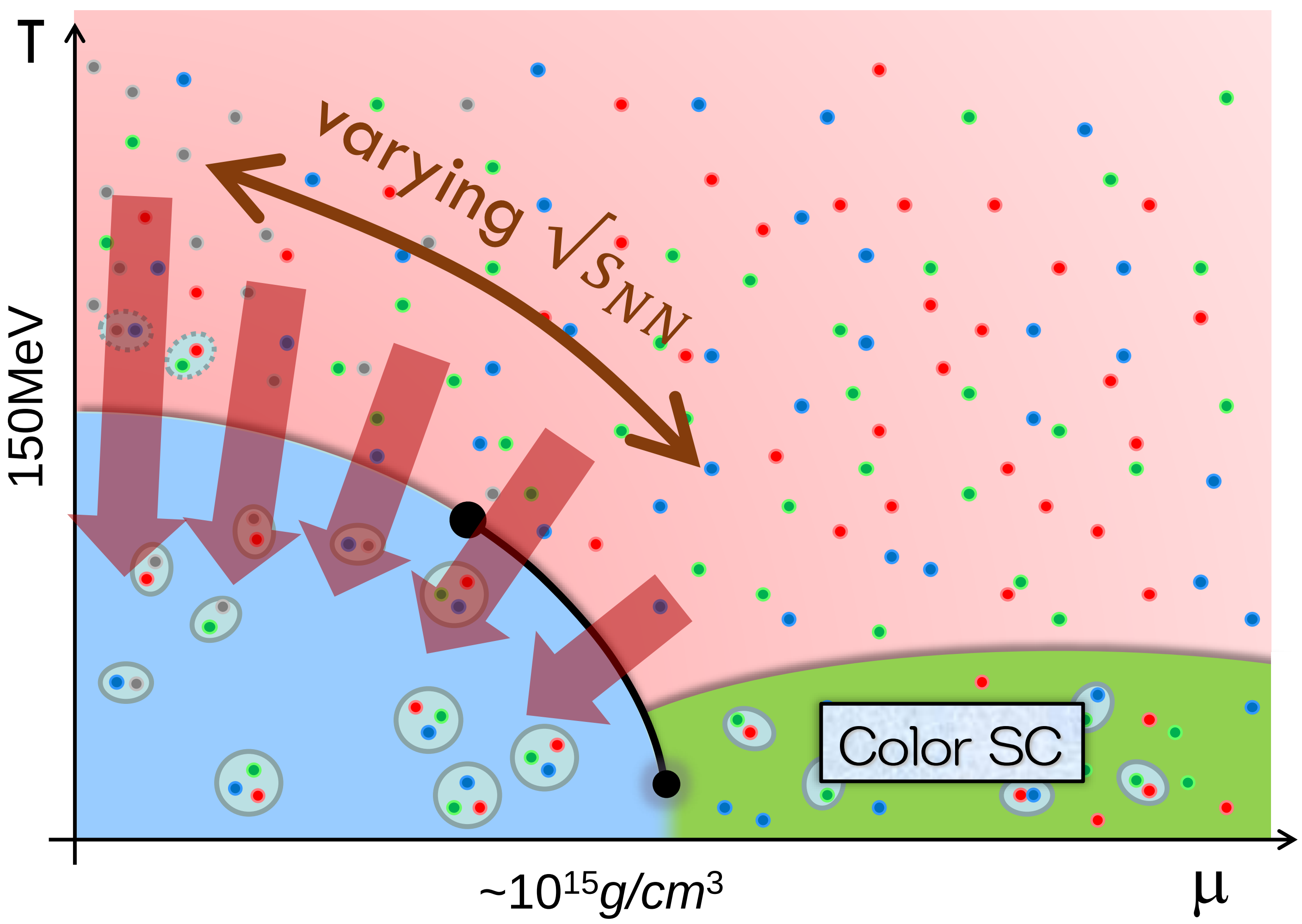
dilepton production rate



Anomalous Dilepton Production as a Precursor of CSC

Nishimura, MK, Kunihiro, PTEP2022, 093D02 ('22)

Color SuperConductiviting Phases (CSC)



- Attractive qq interaction in $\bar{3}$ channel in one-gluon exchange
- Cooper instability at sufficiently low T
- Various phases due to color/flavor d.o.f.
  - CFL, 2SC, …
- SC in a strongly coupled system
  - BCS-BEC crossover

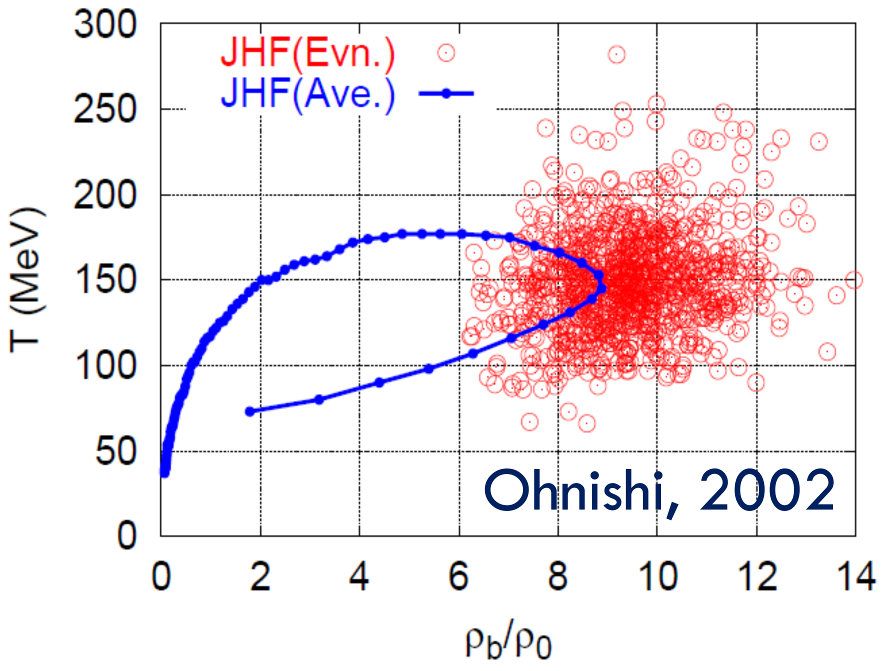
    Abuki, Hatsuda, Itakura ('02)
    MK, Rischeke, Shovkovy ('08)
  - diquark fluctuations

    MK+ ('02)
    Voskresensky ('03)
  - "pseudogap" region

    MK, Koide, Kunihiro, Nemoto ('03)

Observing CSC in HIC

- **Difficulties**
  - CSC would not be created if Tc is not high enough.
  - Even if created, its lifetime would be short.
  - Since CSC is created in the early stage, its signal would be blurred during the evolution in later stage.



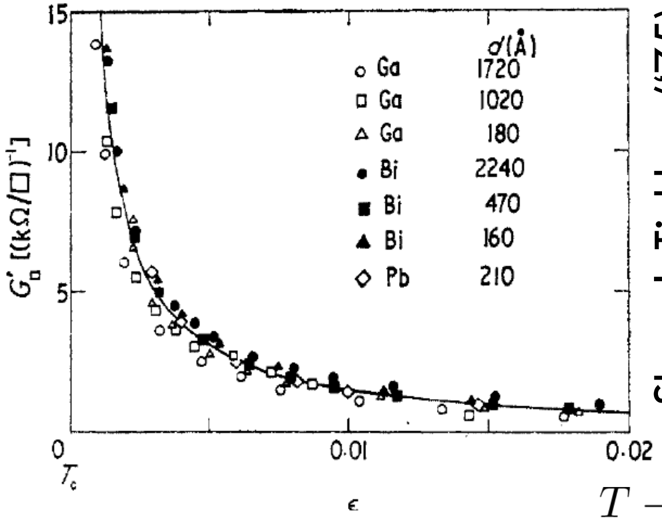
- **Strategy** in the present study:
  - Focus on precursory phenomena of CSC
  - Use dilepton production as an observable

Precursor of CSC

- **Anomalous behavior of observables near but above Tc of SC**
  - electric conductivity
  - magnetic susceptibility
  - pseudogap

- Enhanced pair fluctuations is one of the origins of precursory phenomena.
- More significant phenomena in strongly-coupled systems.

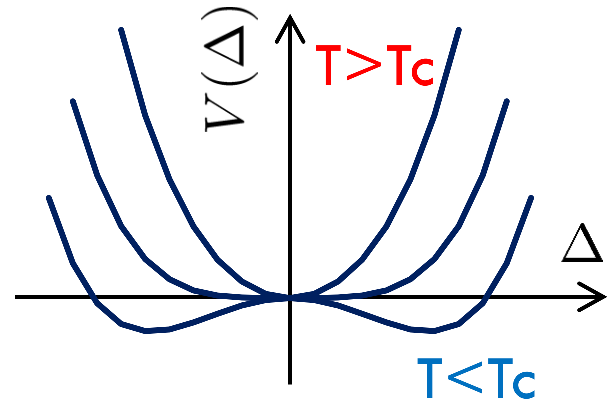
Electric conductivity



Skopal, Tinkham('75)

$$\varepsilon = \frac{T - T_c}{T_c}$$

Landau's free energy



Model

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\mathcal{L}_S = G_S \left((\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \tau \psi)^2 \right)$$

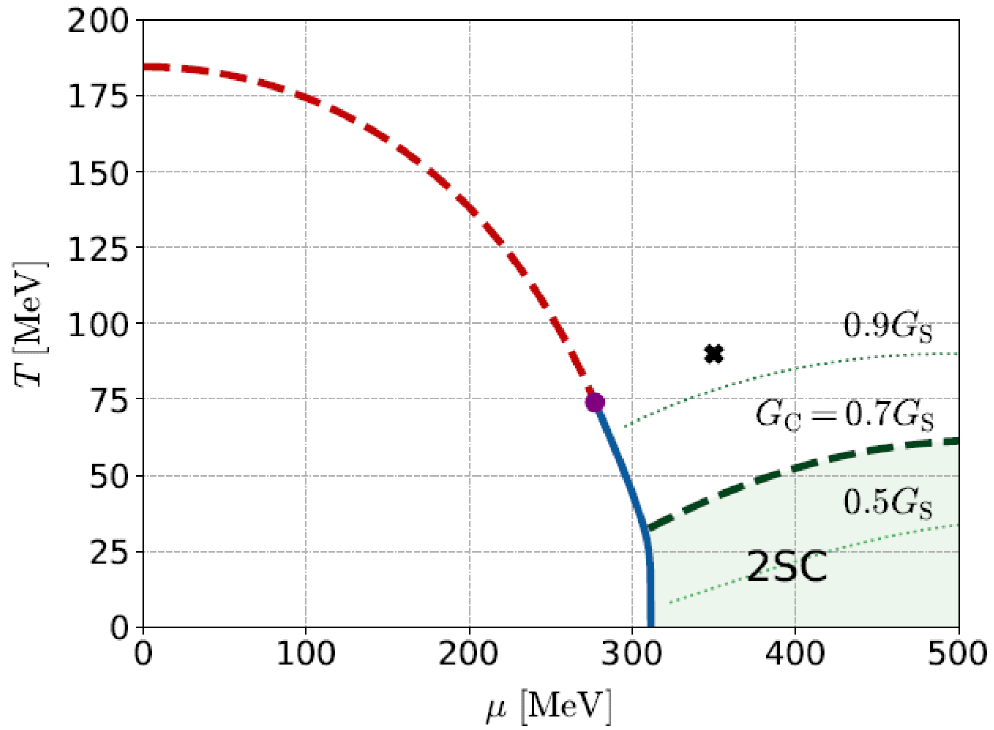
$$\mathcal{L}_C = G_C \left((\bar{\psi} i \gamma_5 \tau_A \lambda_A \psi^C)(\text{h.c.}) \right)$$

diquark interaction

Parameters

$$G_S = 5.01\ \text{GeV}^{-2}, \quad \Lambda = 650\text{MeV}, \quad m_q = 0$$

Phase Diagram in MFA



- Order of phase transition
  - 2nd in the MFA
  - can be 1st due to gauge fluctuation

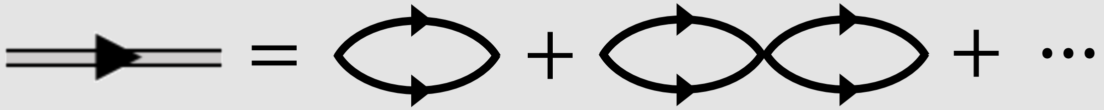
Matsuura+('04), Giannakis+('04)
Noronha+('06), Fejos, Yamamoto('19)

Di-quark Fluctuations

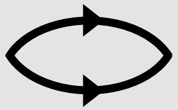
☐ Diquark Propagator

$$D^R(x) = \langle [\Delta^\dagger(x), \Delta(0)] \rangle \theta(t) =$$ 

☐ Random Phase Approximaion



$$= \frac{Q^R(\boldsymbol{k},\omega)}{1 + G_C Q^R(\boldsymbol{k},\omega)}$$

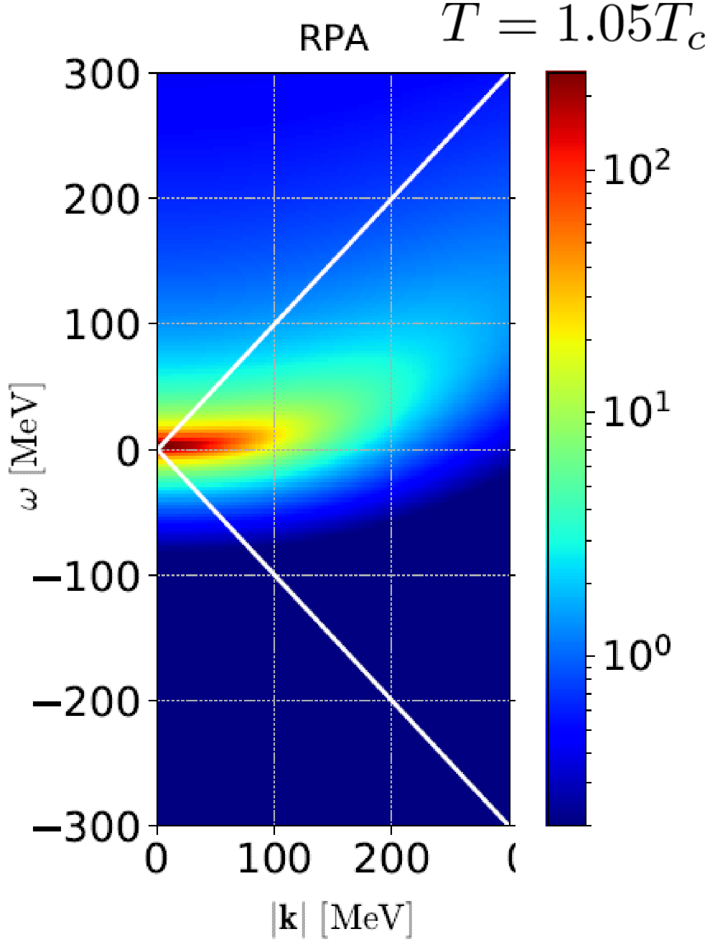
$$Q^R(\boldsymbol{k},\omega) =$$ 

- ☐ Diquark field becomes massless at T=Tc
- ☐ Soft mode of CSC transition
- ☐ Strength in the space-like region

MK, Koide, Kunihiro, Nemoto, '01,'05

Dynamical Structure Factor

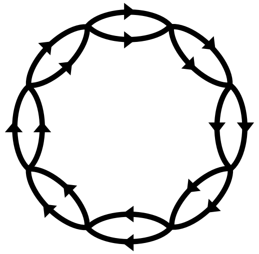
$$S(\boldsymbol{k},\omega) = -\frac{1}{\pi}\frac{1}{1-e^{-\beta\omega}}\mathrm{Im}\mathrm{D}^{\mathrm{R}}(\mathrm{k},\omega)$$



Effects on Observables

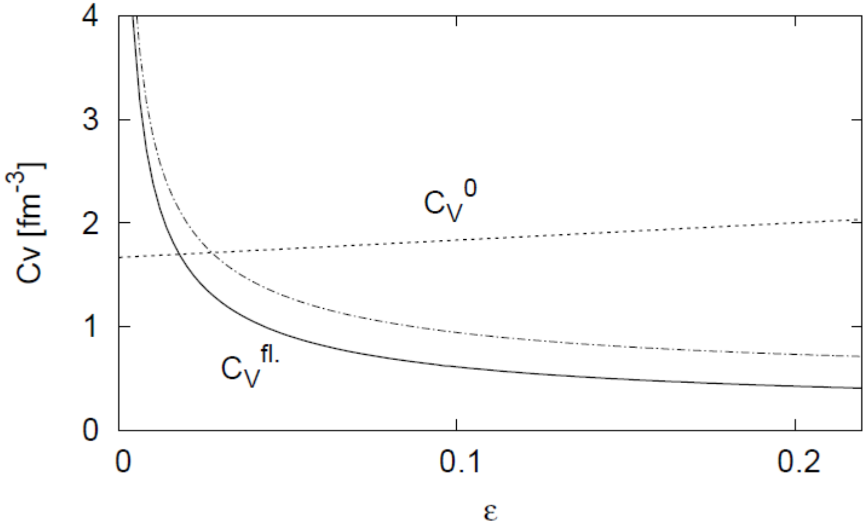
MK, Koide, Kunihiro, Nemoto, '03, '05

□Thermodynamic Potential

$\Omega =$ 

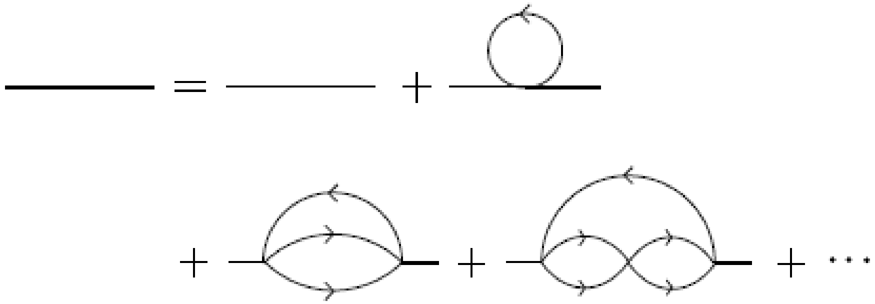
Specific heat

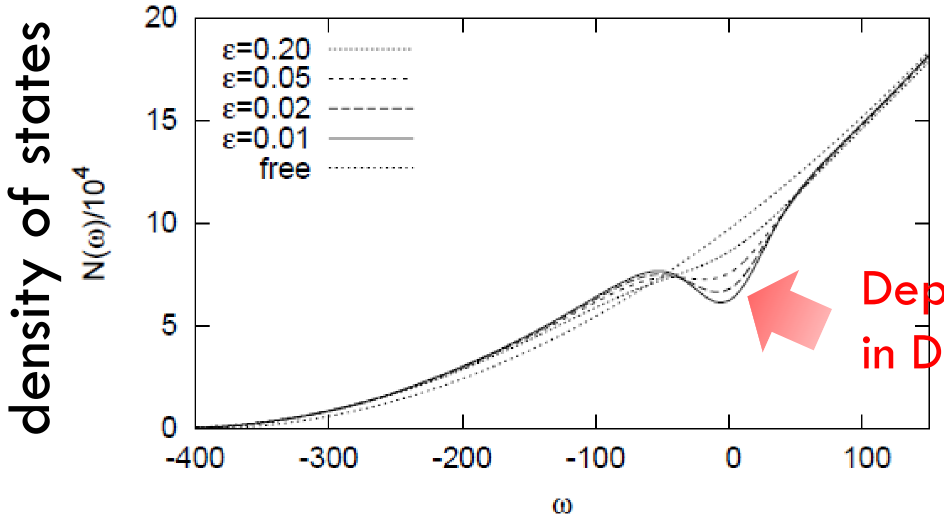
$$c = -T\frac{\partial^2\Omega}{\partial T^2}$$



$$\varepsilon = \frac{T - T_c}{T_c}$$

□Pseudogap



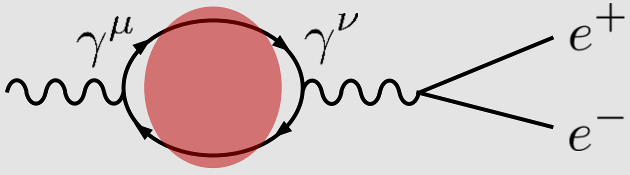


Depression
in DoS above Tc

Photon Self-Energy: Precursor of CSC

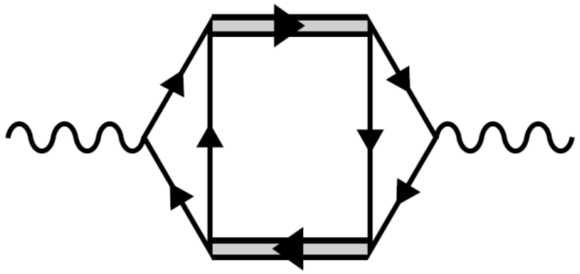
□ Dilepton Production Rate

$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\beta\omega-1}}\mathrm{Im}\Pi^{R\mu}_{\mu}(k)$$

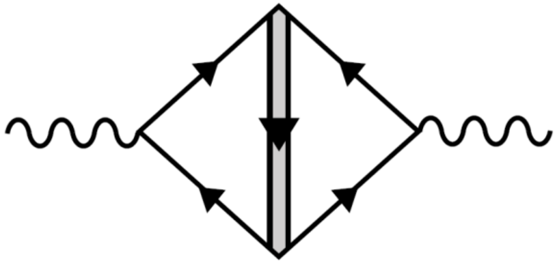


□ Effect of Di-quarks on $\Pi^{\mu\nu}(k)$

Aslamasov-Larkin term



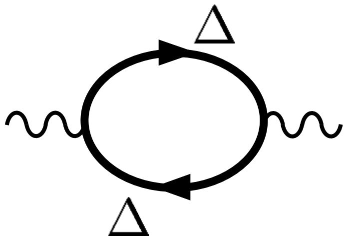
Maki-Thompson term

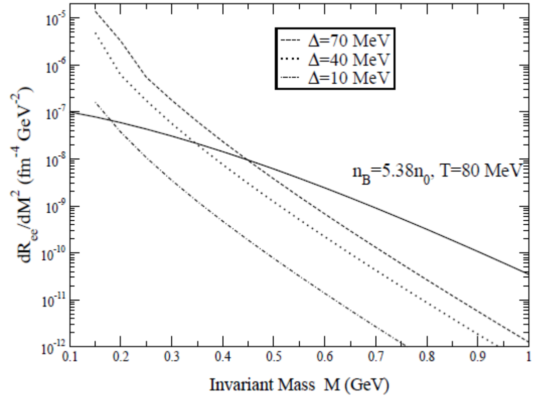


Well-known diagrams in metallic SC
for describing paraconductivity

□ DPR from CFL phase

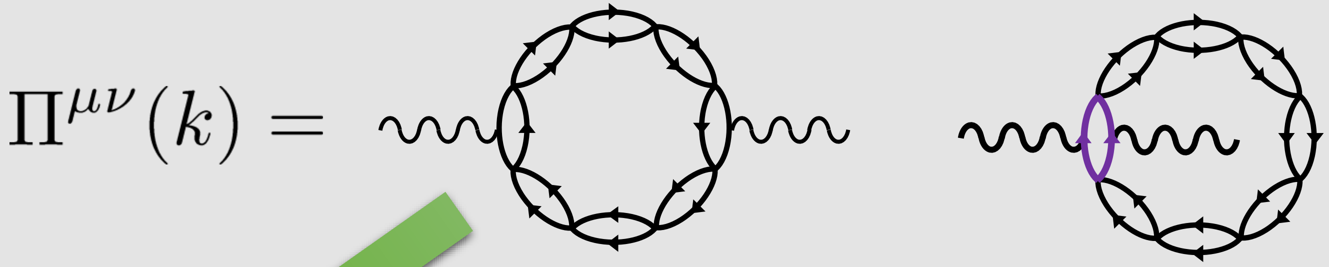
Jaikumar, Rapp, Zahed ('02)

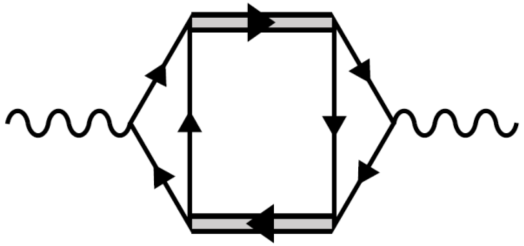


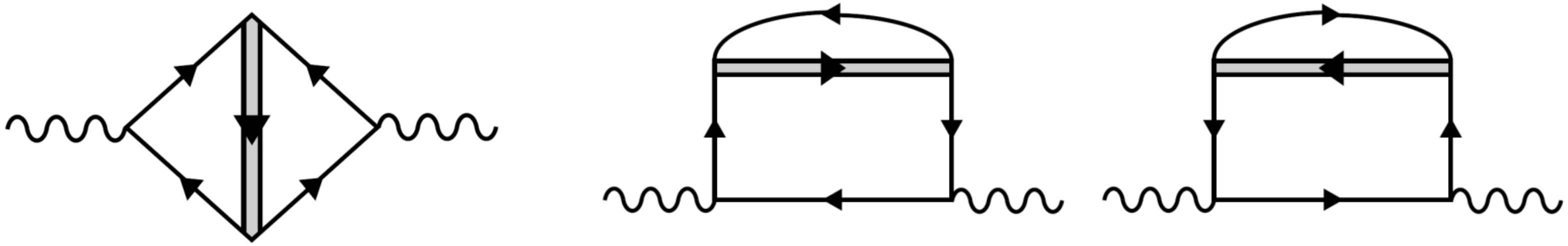


Gauge-Invariant Construction of $\Pi_{\mu\nu}(k)$

Insert two photon vertices in thermodynamic potential







- WT identity $k_\mu \Pi^{\mu\nu}(k) = 0$ is satisfied with AL, MT and DoS terms.

(Modified) Time-Dependent Ginzburg-Landau Approximation

TDGL approximation for T-matrix

$$\Xi^R(\boldsymbol{k},\omega) = \frac{G_C}{1+G_C Q^R(\boldsymbol{k},\omega)} \simeq \frac{1}{c\omega + \Xi^R(\boldsymbol{k},0)^{-1}} \qquad c = \left.\frac{\partial(\Xi^R)^{-1}}{\partial\omega}\right|_{\omega=0}$$

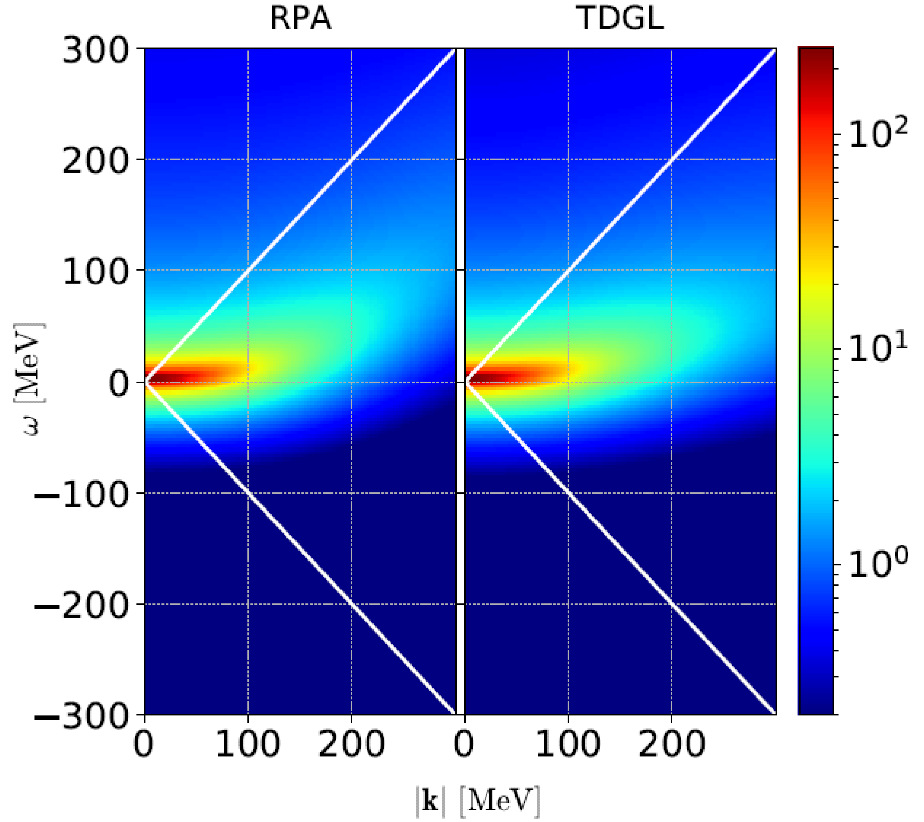
$$\Xi^R(\boldsymbol{k},\omega) = \frac{G_C}{Q^R(\boldsymbol{k},\omega)} D^R(\boldsymbol{k},\omega)$$

Note:

- Valid in low energy region
- $[\Xi^R(0,0)]^{-1} = 0$ at $T = T_c$
- We do not expand w.r.t. k

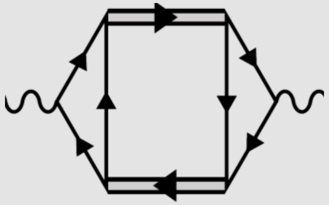
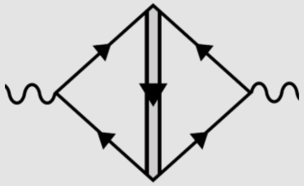
$$\Xi^R(\boldsymbol{k},\omega) \simeq \frac{1}{c\omega + \Xi^R(\boldsymbol{k},0)^{-1}} \simeq \cancel{\frac{1}{c\omega + a + b\boldsymbol{k}^2}}$$

⟺ TDGL equation: $ic\frac{\partial}{\partial t}\Delta + a\Delta - b\nabla^2\Delta = 0$



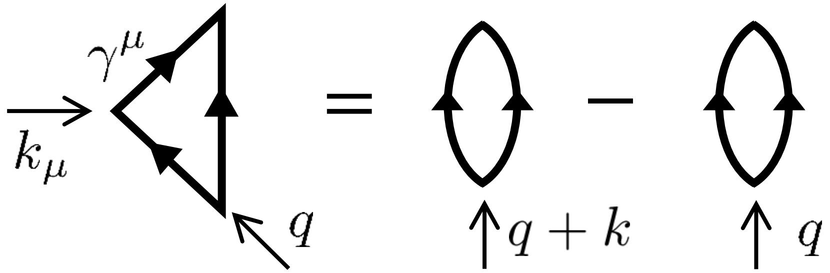
Vertices

Vertices must be determined to be consistent with the TDGL approx.

$\Pi^{\mu\nu}_{\mathrm{AL}}(k) =$  $\Pi^{\mu\nu}_{\mathrm{MT}}(k) =$ 

☐ WT identity for AL vertex

$$k_\mu \Gamma^\mu(q, q+k) = \Xi^{-1}(q+k) - \Xi^{-1}(k)$$

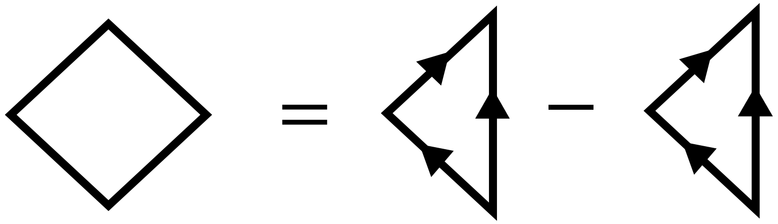


At the lowest order in k

$$\begin{cases} \Gamma^0 = e_\Delta c \\ \Gamma^i = e_\Delta \dfrac{\partial^2 \Xi(q)^{-1}}{\partial q^2}(2q^i + k^i) \end{cases}$$

e_Δ: electric charge of diquarks

☐ MT+DoS



Similar formula for MT+DoS vertex

Photon Self-Energy

Temporal Component

$$\Pi^{00}(k) = \frac{\boldsymbol{k}^2}{k_0^2}\Pi_{\mathrm{L}}(k)$$

➡ $\Pi^{00}(k)$ is obtained from spatial components.

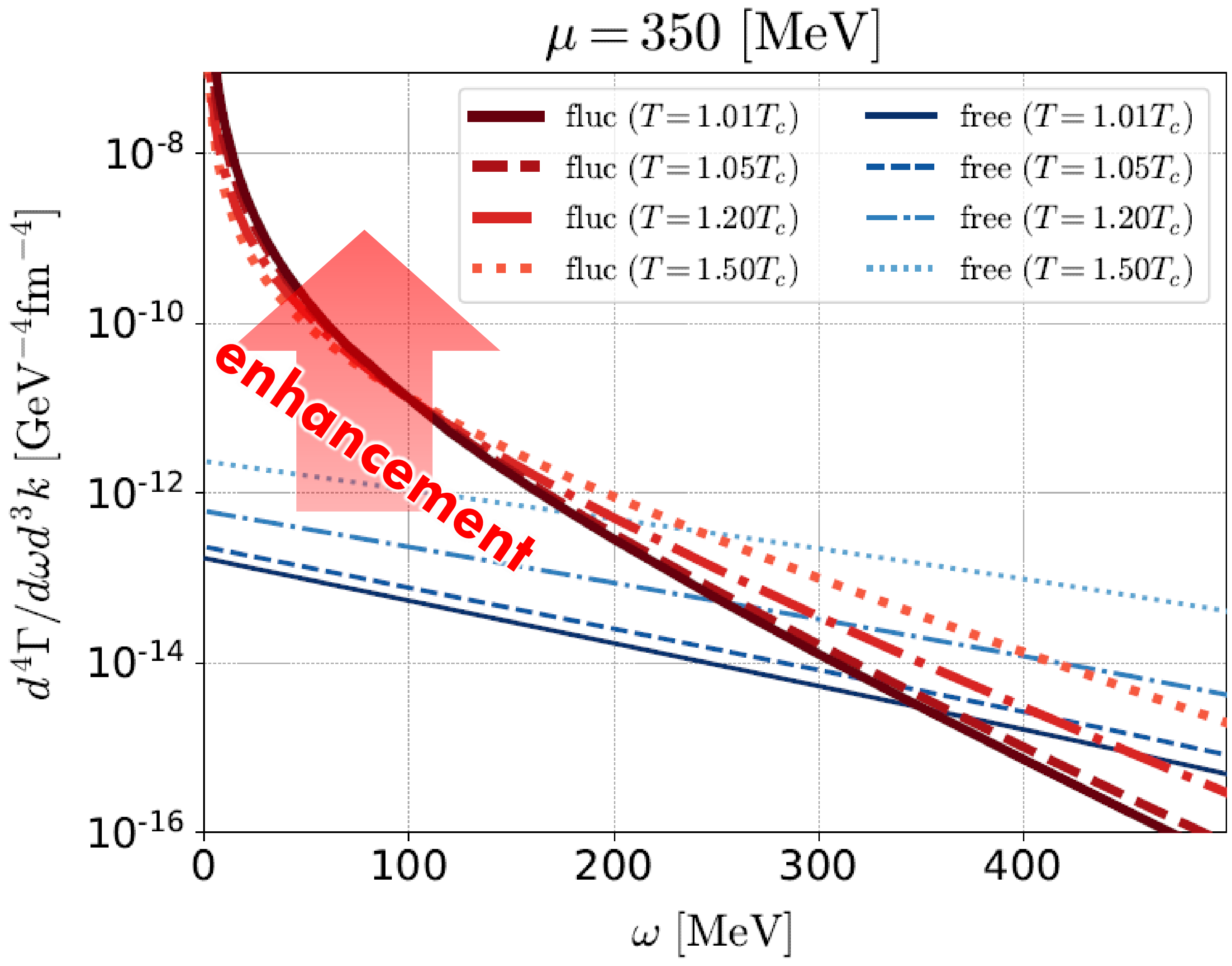
Cancellation of MT+DoS

$$\mathrm{Im}\Pi^{Rij}_{\mathrm{MT+DoS}}(k) = 0$$

➡ Calculation of AL term is sufficient to obtain $\mathrm{Im}\Pi^{\mu\nu}(k)$

$$\mathrm{Im}\Pi^{R\mu}_{\mu}(k) = \frac{\boldsymbol{k}^2 - k_0^2}{k_0^2}\mathrm{Im}\Pi^{R\mu}_{\mathrm{AL,L},\mu} + 2\mathrm{Im}\Pi^{R\mu}_{\mathrm{AL,T},\mu}$$

Production Rate at $\boldsymbol{k} = 0$



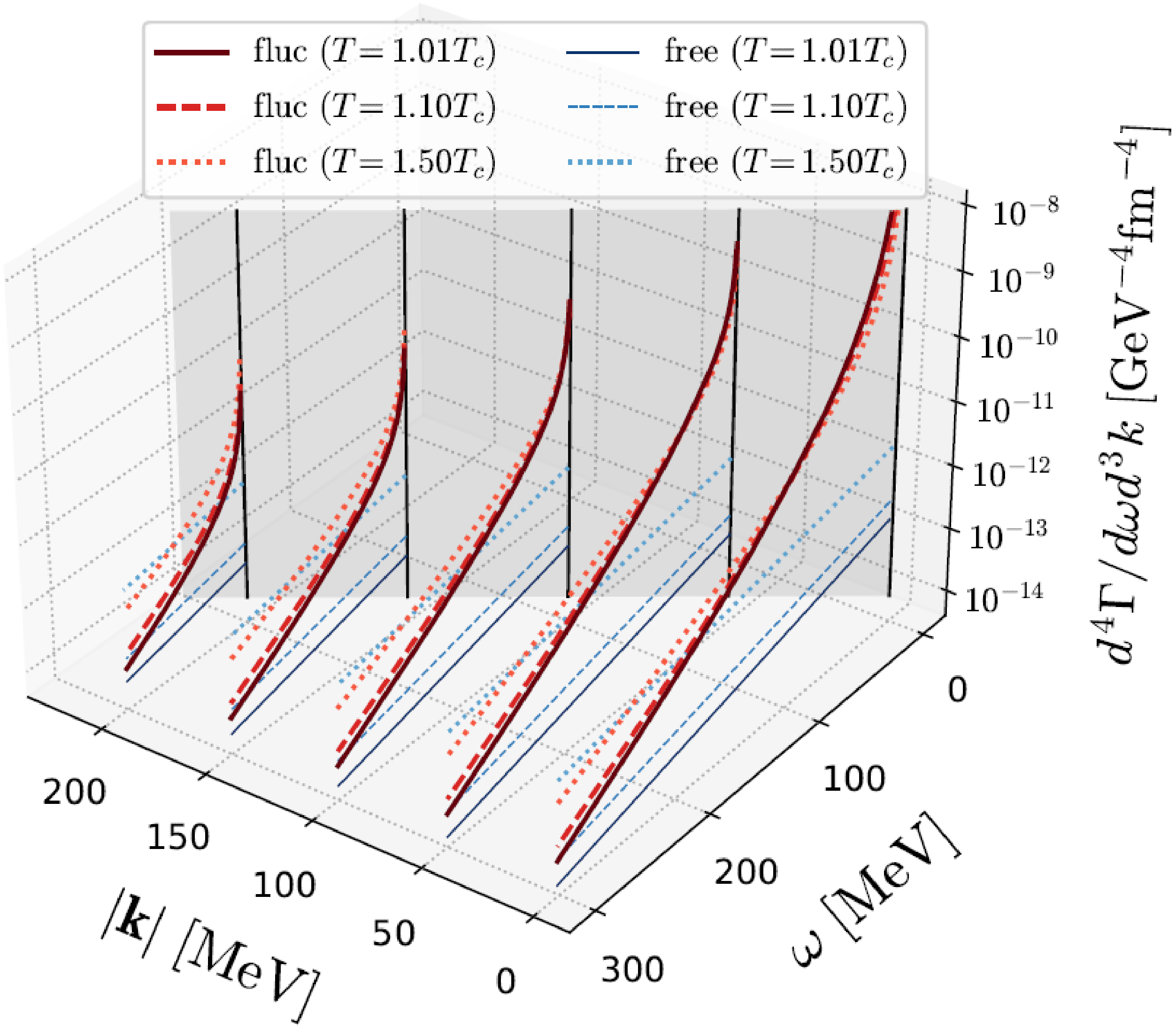
Red: fluctuation contribution

Blue: free quarks

$G_C = 0.7G_S,\ T_c \simeq 45$ MeV

- Di-quark fluctuations give rise to large enhancement in the low energy region $\omega < 200$ MeV and $T < 1.5T_c$.
- Anomalous enhancement is not sensitive to T.

Energy-Momentum Dependence



Red: fluctuation contribution

Blue: free quarks

$G_C = 0.7G_S, T_c \simeq 45$ MeV

- Enhancement due to diquark fluctuations is more suppressed for larger k.

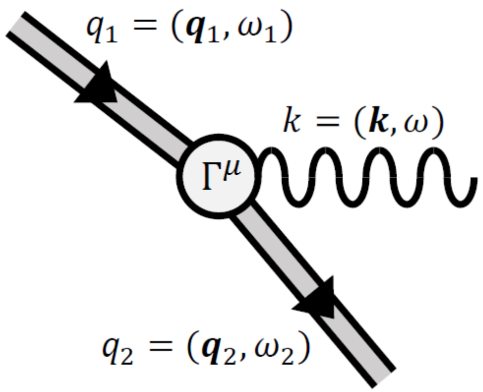
Production Mechanism of Virtual Photons

Production mechanism

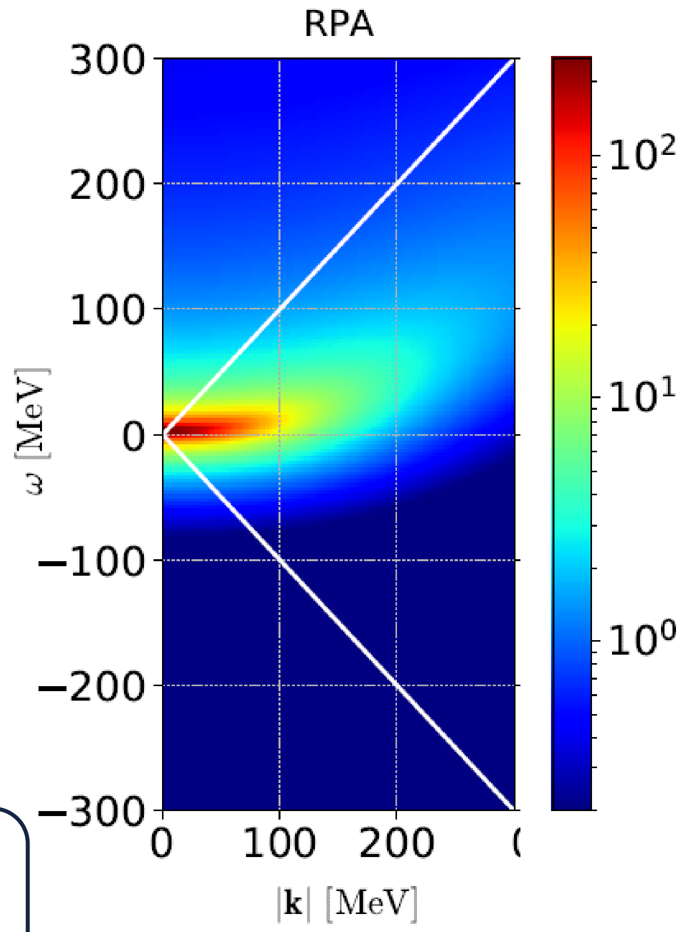
- scattering of diquarks
- diquarks: space-like region

$$\omega = \omega_1 - \omega_2$$

$$\boldsymbol{k} = \boldsymbol{q}_1 - \boldsymbol{q}_2$$

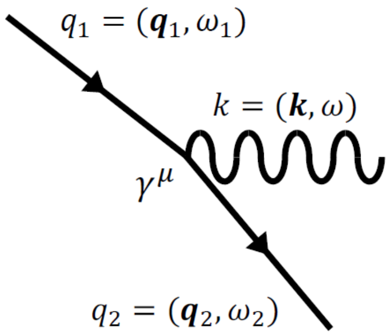


Production in the time-like region is possible.



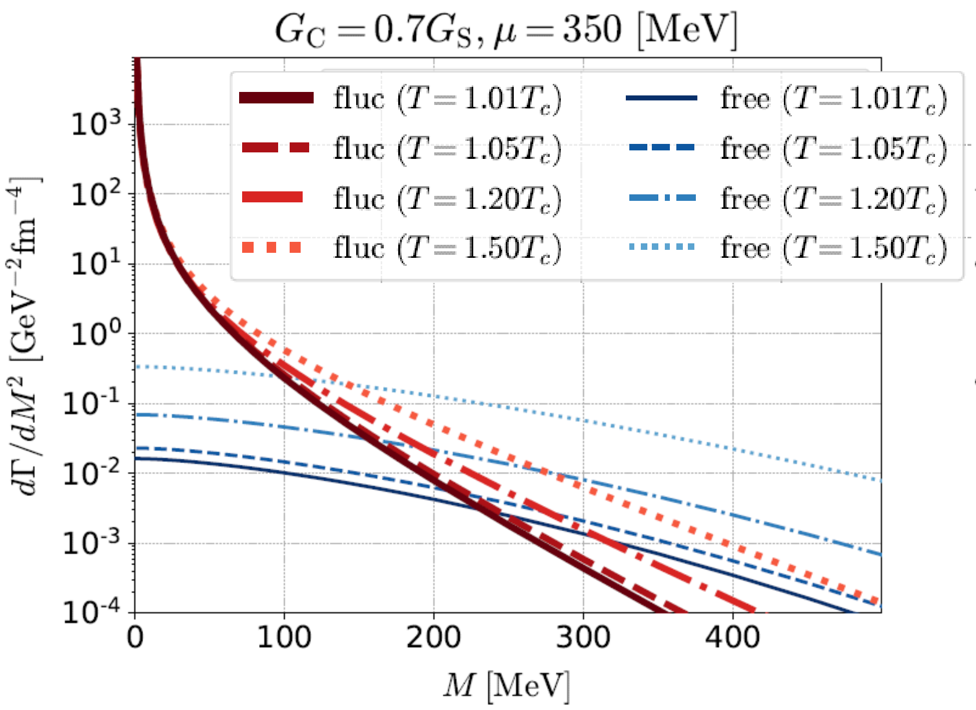
- C.f.) Scattering of free quarks produces virtual photons only in the space-like region.

$$|\boldsymbol{q}_1 - \boldsymbol{q}_2| \geq \omega_1 - \omega_2$$

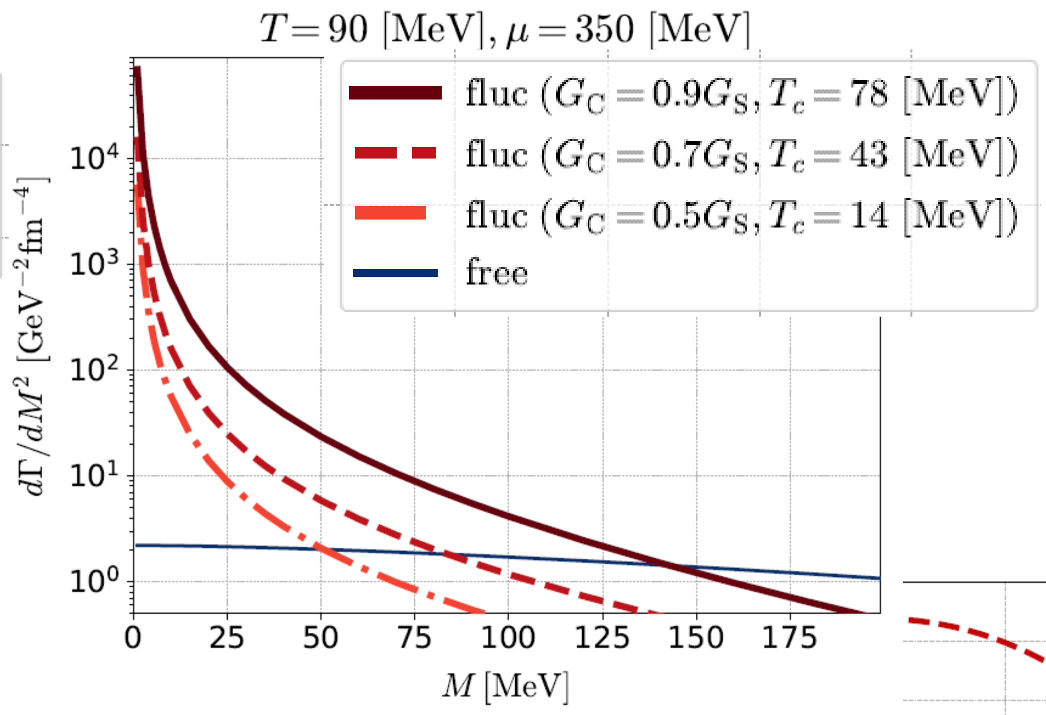


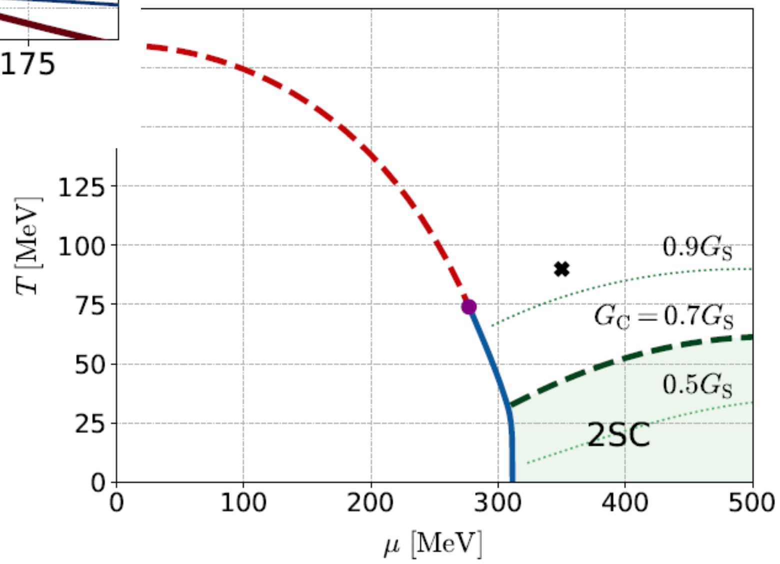
Invariant-Mass Spectrum

Fixed G_C



Fixed Temperature





- ☐ Strong enhancement at low invariant mass, though the range of M is narrower than the previous results.
- ☐ **Observable in the HIC?**

Dilepton Production from QCD Critical Point

Nishimura, MK, Kunihiro, arXiv: 2302.03191

Model

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G_S\big((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\big)$$

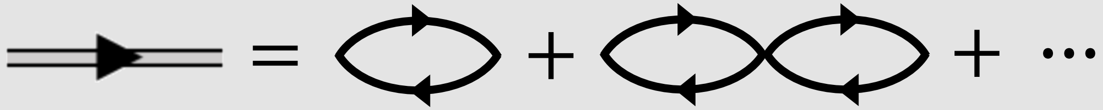
Parameters

$$G_S = 5.5\ \mathrm{GeV}^{-2},\ \Lambda = 631\mathrm{MeV},\ m_q = 5.5\ \mathrm{MeV}$$

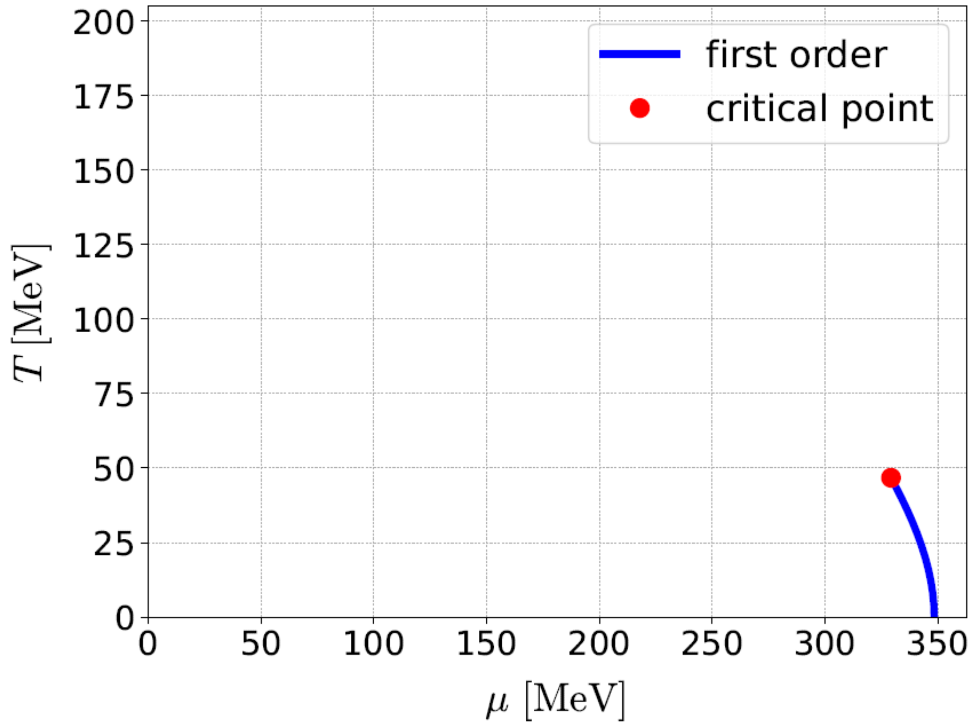
☐ Soft Mode in Sigma ($\overline{q}q$) Channel

$$D^R(x) = \langle[\bar{\psi}\psi(x), \bar{\psi}\psi(0)]\rangle\theta(t) =$$

☐ Random Phase Approximaion



Phase Diagram in MFA

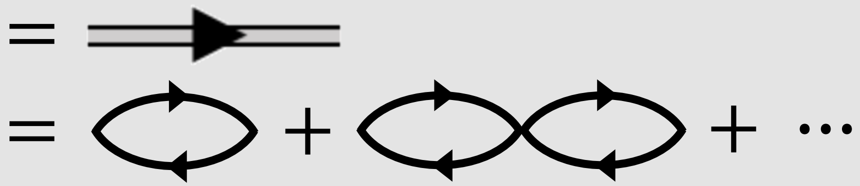


Critical point at:

$$(T, \mu) = (46.757, 329.30)\ \mathrm{MeV}$$

Soft Mode of QCD-CP = Scalar-Density Fluctuations

$$D^R(x) = \langle[\bar{\psi}\psi(x), \bar{\psi}\psi(0)]\rangle\theta(t)$$



- ☐ Soft mode lives in the space-like region.
- ☐ σ mesonic mode does not become massless.
- ☐ D^R has a discontinuity on the light cone.

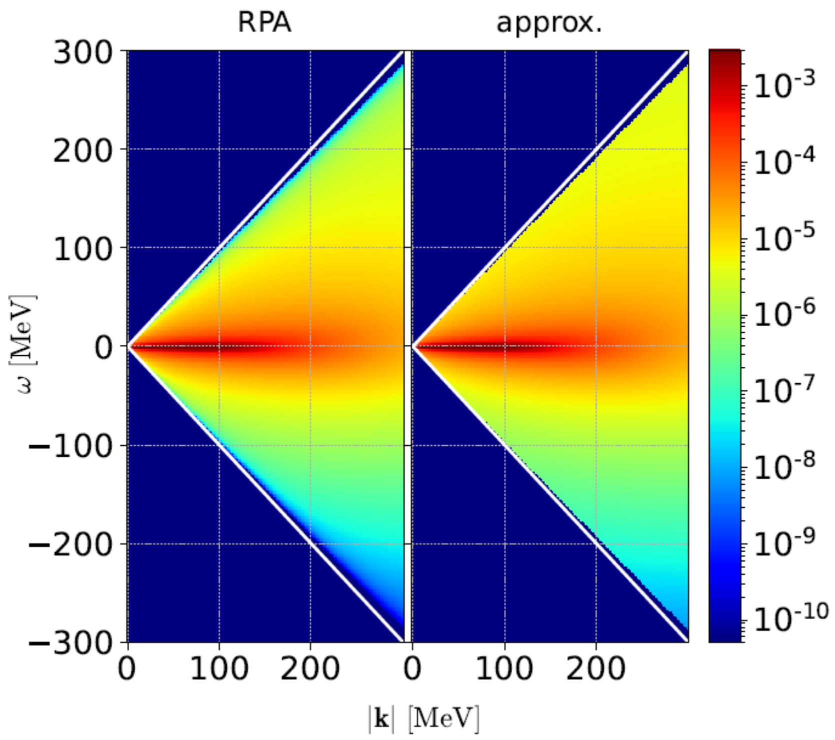
➡ Simple TDGL approx. breaks down. $C(\boldsymbol{k}) \sim 1/k$

$$D^R(\boldsymbol{k},\omega) \simeq \frac{1}{C(\boldsymbol{k})\omega + D^R(\boldsymbol{k},0)^{-1}} \simeq \frac{1}{a + \cancel{c\omega} + b\boldsymbol{k}^2}$$

We use this approximation

Dynamical Structure Factor

$$S(\boldsymbol{k},\omega) = -\frac{1}{\pi}\frac{1}{1-e^{-\beta\omega}}\mathrm{ImD^R(k,\omega)}$$



Formulation

☐ Diquark Fluctuations

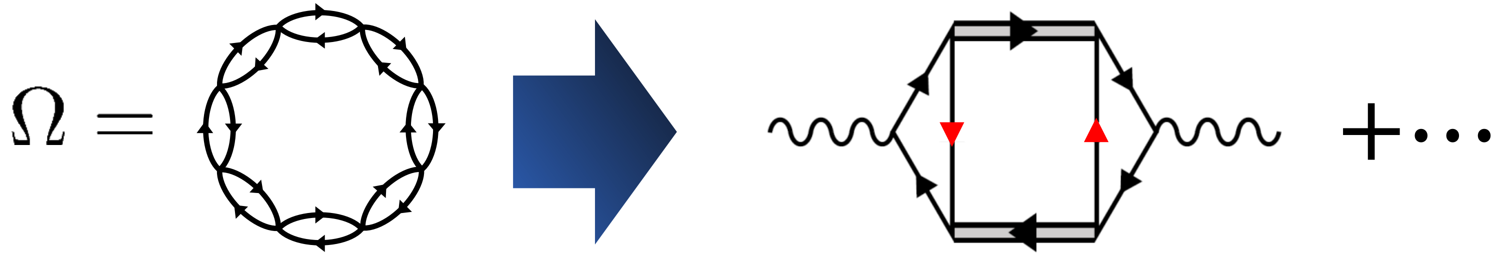
AL term

MT, DoS

Attach 2 photons

☐ Scalar Fluctuations

AL term

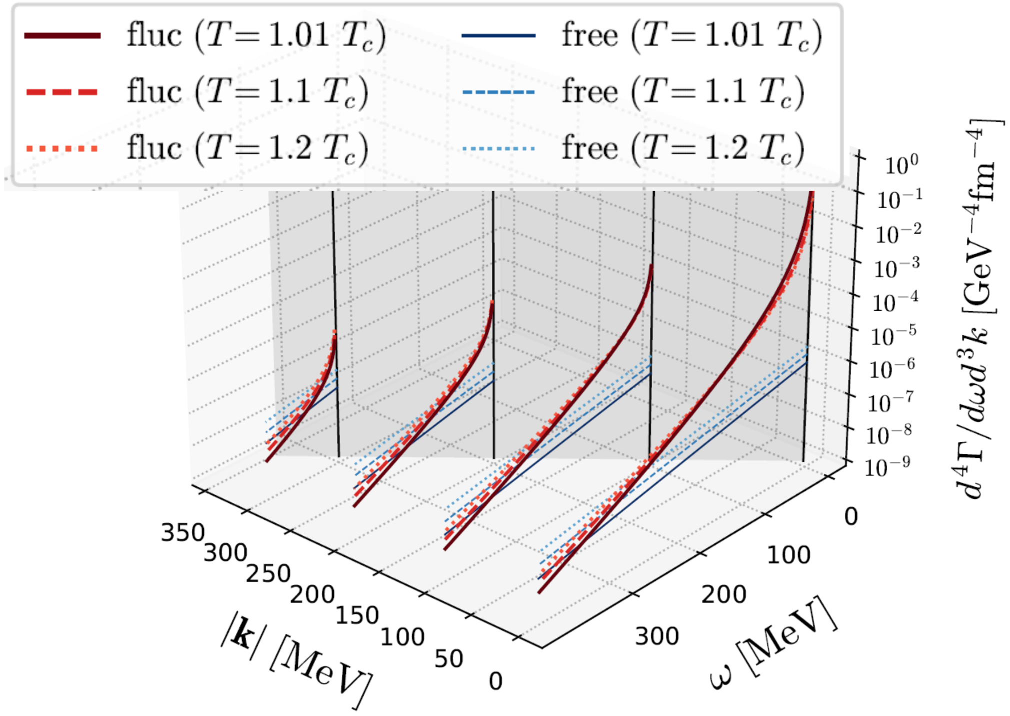


MT, DoS

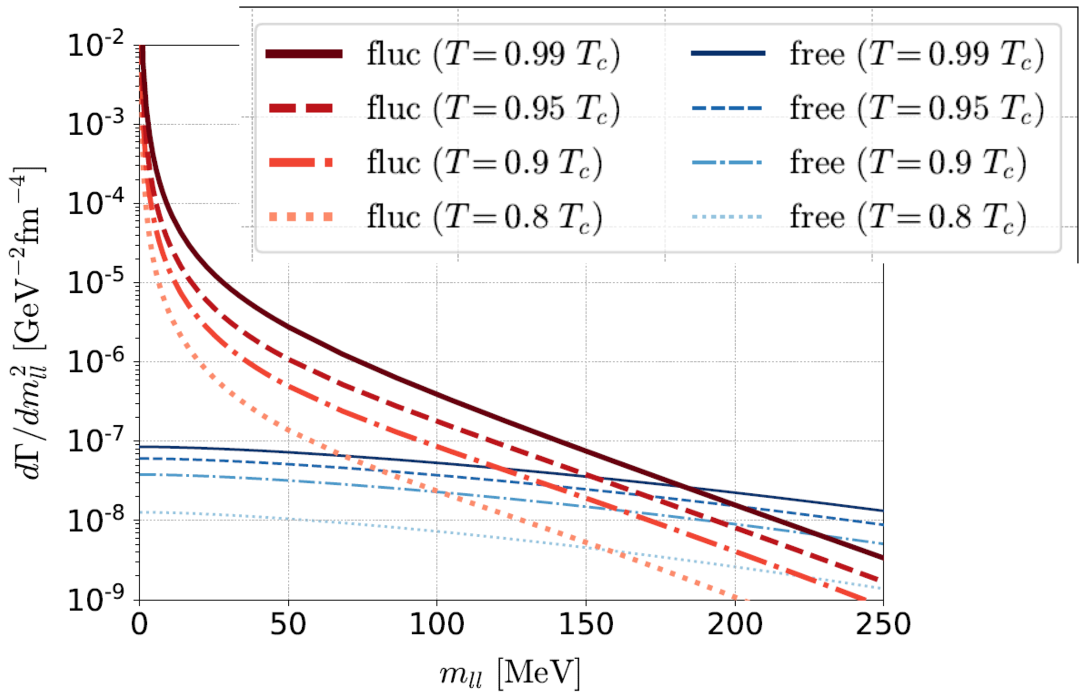
Photon self-energy including the soft mode of QCD-CP can be constructed in a similar manner as before.

Dilepton production rate near QCD-CP

☐ $\omega - k$ plane



☐ Invariant mass spectrum

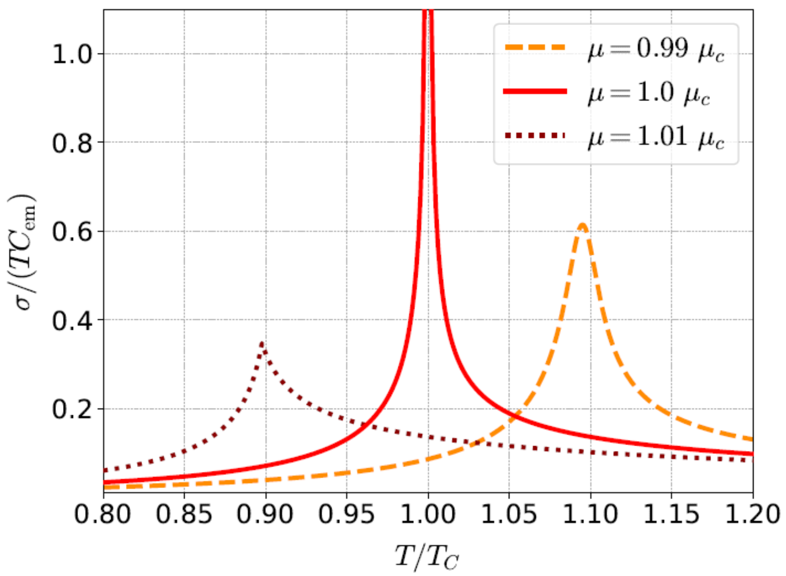


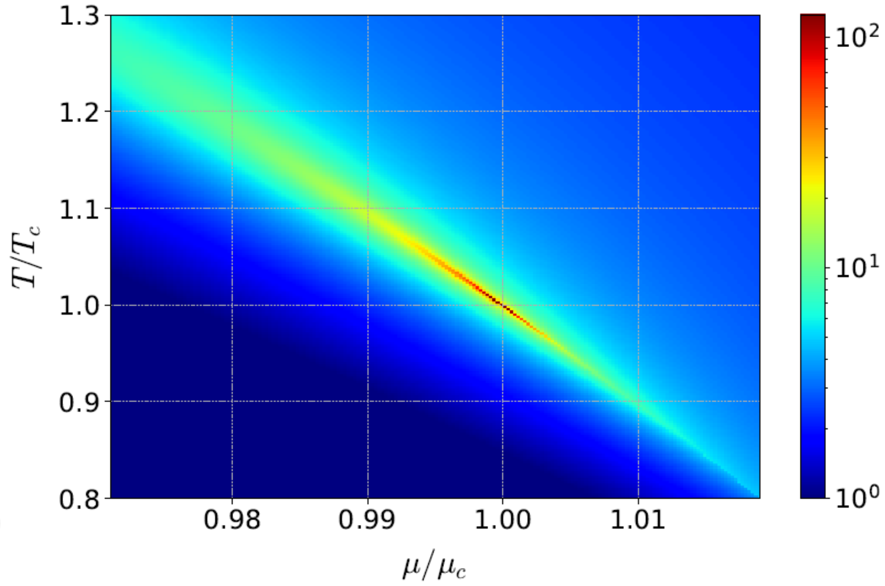
chemical potential: $\mu = \mu_c$

- ☐ Enhancement at low ω, k, m_{ll} regions near QCD-CP
- ☐ Distinguishment from diquark soft mode may be difficult.

Electric Conductivity

- Soft mode leads to enhancement of conductivity σ.





- **Note:**

Both DPR and σ are given from photon self-energy.

$$\frac{d^4\Gamma}{d^4k} = -\frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\omega/T}-1} g_{\mu\nu}\mathrm{Im}\Pi^{R\mu\nu}(\boldsymbol{k},\omega),$$

$$\sigma = \frac{1}{3}\lim_{\omega\to 0}\frac{1}{\omega}\sum_{i=1,2,3}\mathrm{Im}\Pi^{Rii}(\boldsymbol{0},\omega).$$

Critical Exponents

	QCD-CP	CSC
σ	$\lvert T-T_c\rvert^{-2/3}$	$\lvert T-T_c\rvert^{-1/2}$
τ	$\lvert T-T_c\rvert^{-1}$	$\lvert T-T_c\rvert^{-1}$

- Conductivity diverges with different critical exponents in QCD-CP & CSC.
- Can they distinguishable in HIC??

Nishimura, MK, Kunihiro, in prep.

Summary

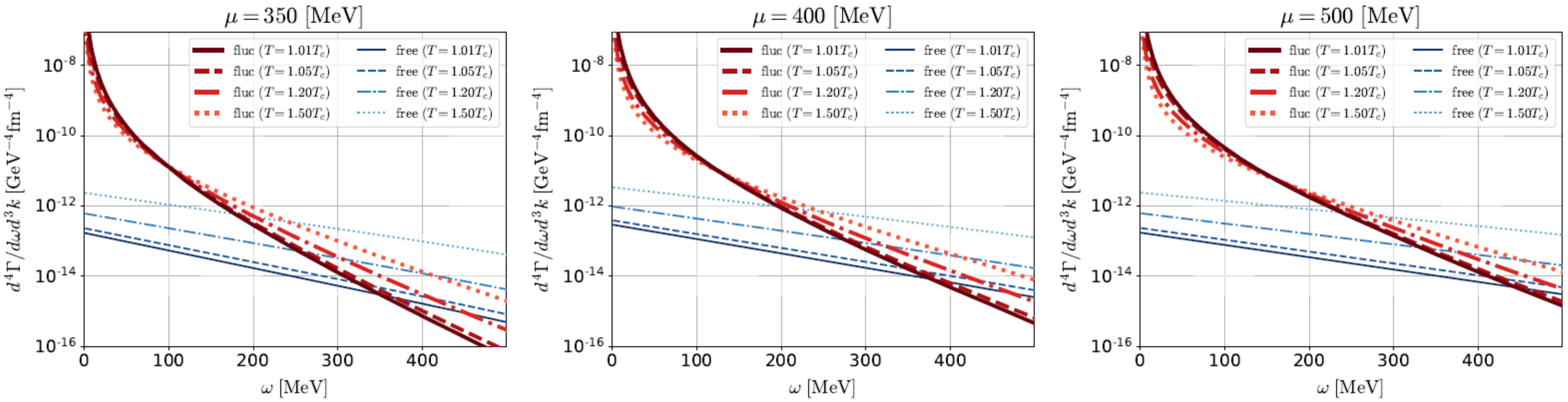
- We calculated dilepton production rates near
  - phase boundary of color superconductivity
  - QCD critical point

  incorporating effects of the soft modes.
- The photon self-energy is constructed from AL, MT and DoS terms in a gauge-invariant manner.

- Dilepton production rate is enhanced significantly near both phase transitions at low invariant-mass region.
- **Signal for existence of QCD-CP and/or CSC phase transition in HIC?**

- **Future**
  - Comparison with bremsstrahlung in QGP, hadronic effects, Dalitz decays, etc.
  - Quantitative estimate on dynamical models

μ Dependence



- The enhancement from di-quark fluctuations is more pronounced at the higher density region.