An efficient numerical solver for relativistic hydrodynamics with an implicit Runge-Kutta method

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Computational relativistic hydrodynamics

Core of dynamical modeling of HIC ▶ **Higher** efficiency/stability/accuracy is **highly** desirable.

Our strategy: Use implicit Runge-Kutta for time integrator

Implicit RK

- stable (A-stability)
- non-linear equations

ex) $\dot{x} = f(x)$

Implicit Euler

$x(t+\Delta t) = x(t) + \Delta t f(x(t+\Delta t))$

Euler (explicit)

$x(t+\Delta t) = x(t) + \Delta t f(x(t))$

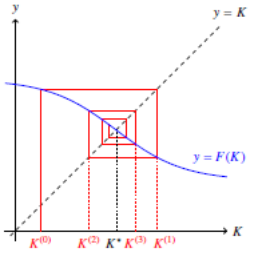
To solve non-linear eqs.

Fixed-point method
— initial guess: previous result

Local optimization
— check convergence locally

RK schemes

Implicit: 1st Gauss-Legendre
Explicit: Heun's method



Space scheme:
KT with MUSCL
Butcher tables:

GL1

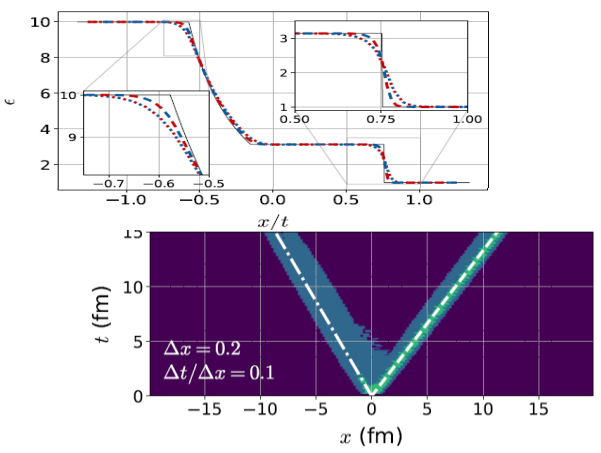
0	0.5
	1

Heun

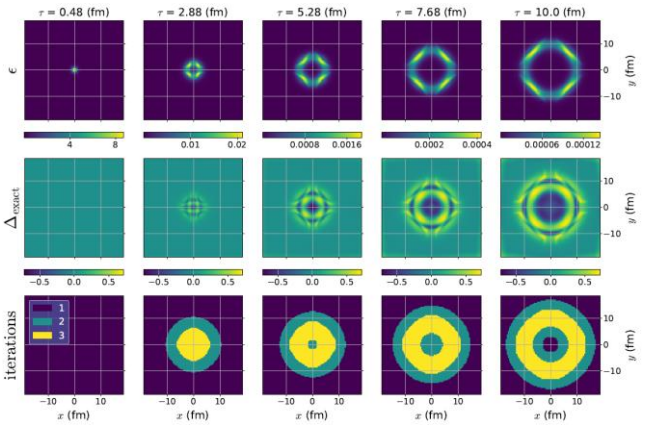
0	0	0
1	1	0
	0.5	0.5

Results

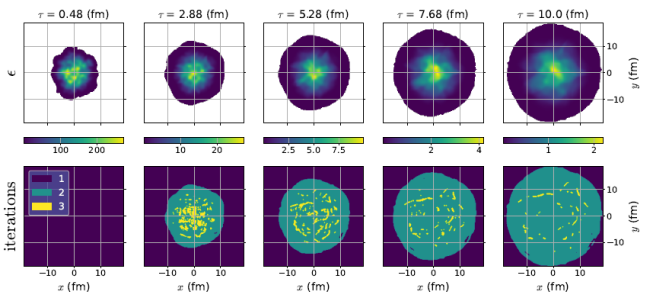
Riemann Problem (1+1d)

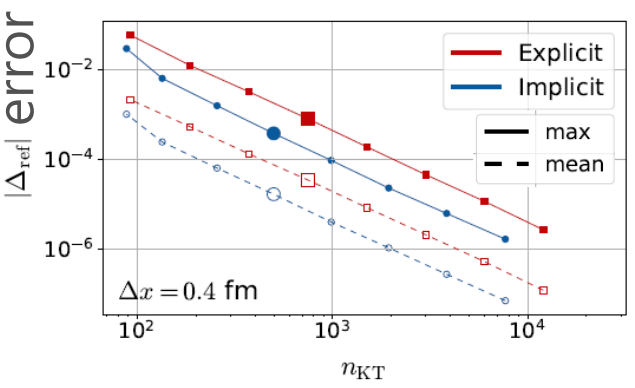


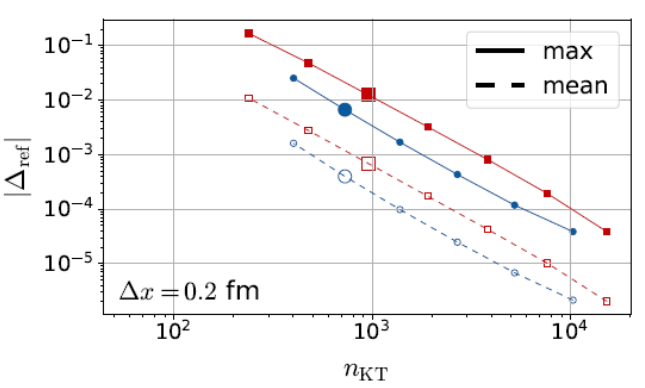
Gubser Flow (2+1d)

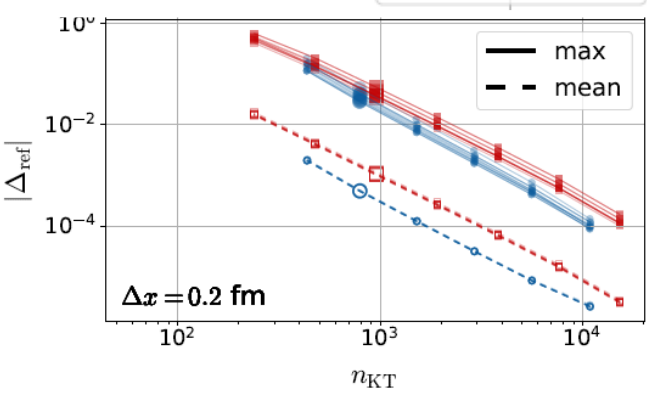


HIC (T_RENTo, 2+1d)









Implicit solver is more efficient than **explicits**!

Conclusion

— New method to solve relativistic hydrodynamics using an implicit RK time integrator.
— Fixed-point method with local optimization to solve non-linear RK equaitons.
— Our solver is more efficient than conventional methods based on explicit RK methods.