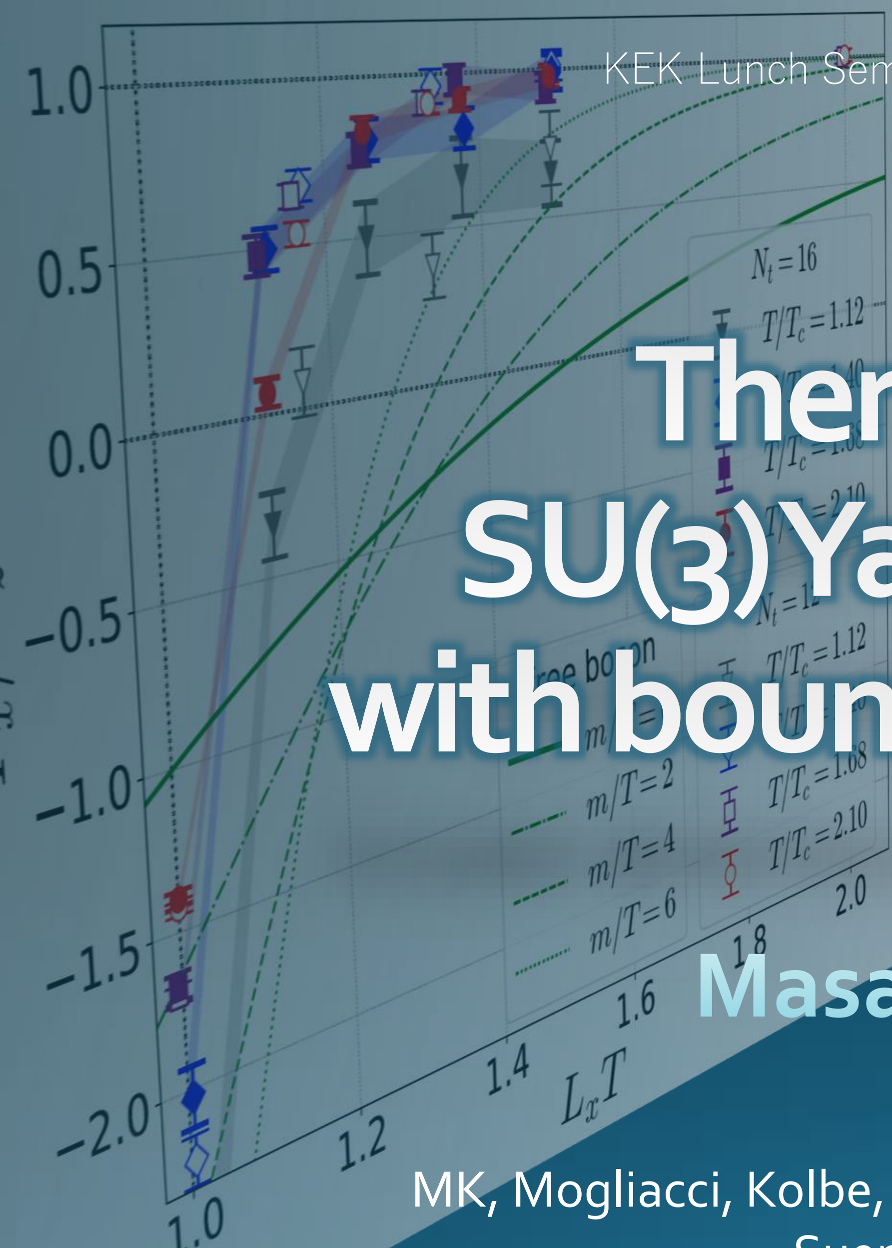
KEK Lunch Seminar, 2023/09/29, KEK



Thermodynamics of SU(3) Yang-Mills theory with boundary conditions

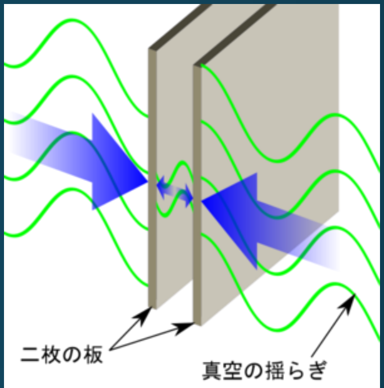
Masakiyo Kitazawa (YITP)

MK, Mogliacci, Kolbe, Horowitz, Phys. Rev. **D99** (2019) 094507
Suenaga, MK, Phys. Rev. **D107** (2023) 074502
D. Fujii, A. Iwanaka, D. Suenaga, MK, in prep.

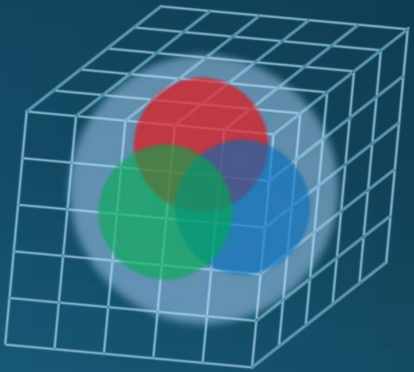
Boundary Conditions in QFT

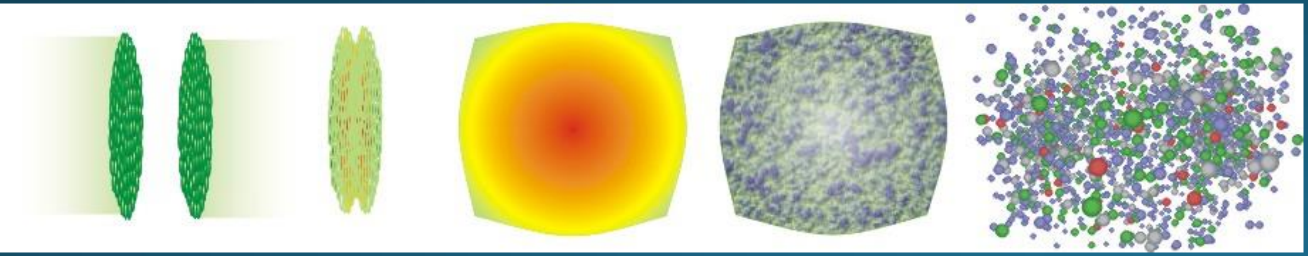
Many motivations

- ☐ Casimir effect
- ☐ Relativistic heavy-ion collisions
- ☐ Numerical simulations (ex. lattice QCD)
- ☐ Matsubara formalism for thermal systems



from wikipedia





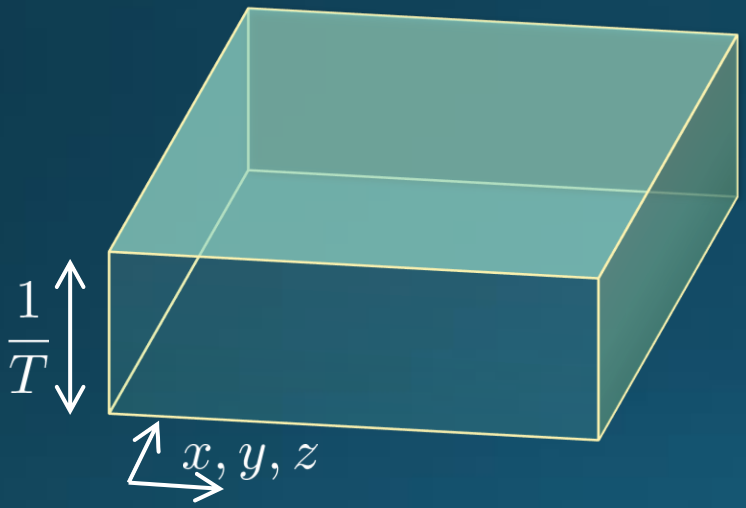
$$L_\tau = \frac{1}{T}$$

Matsubara Formalism

Thermal Field Theory

PBC for imaginary-time

for bosons



$$T^{44} = -\epsilon$$

$$T^{11} = T^{22} = T^{33} = p$$

Matsubara Formalism

Thermal Field Theory

PBC for imaginary-time

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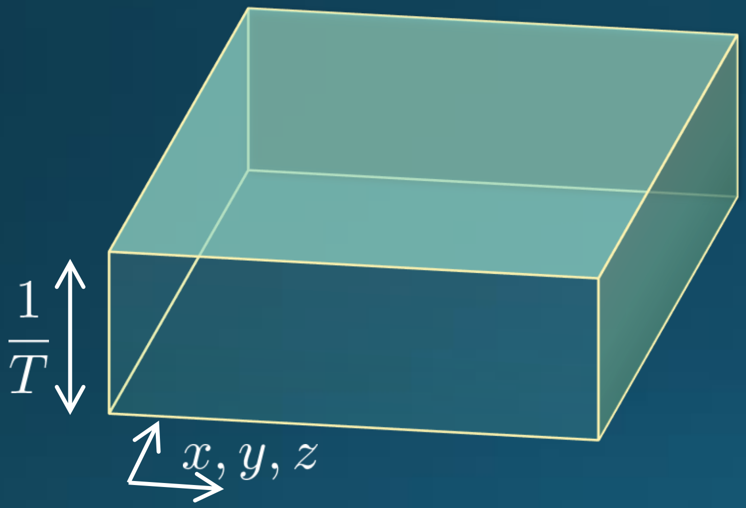
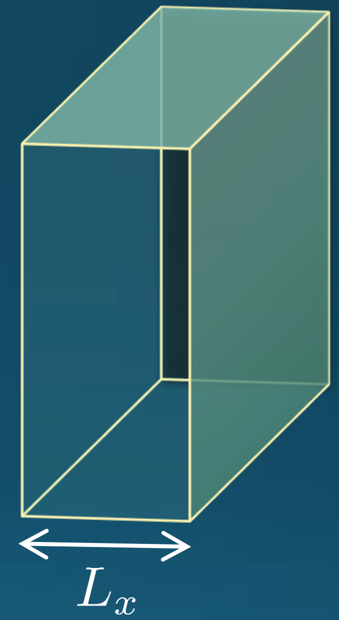


$$T^{44} = -\epsilon$$

$$T^{11} = T^{22} = T^{33} = p$$

PBC along x dir. at $T = 0$



$$\tilde{T}^{11} = T^{44} = -\epsilon$$

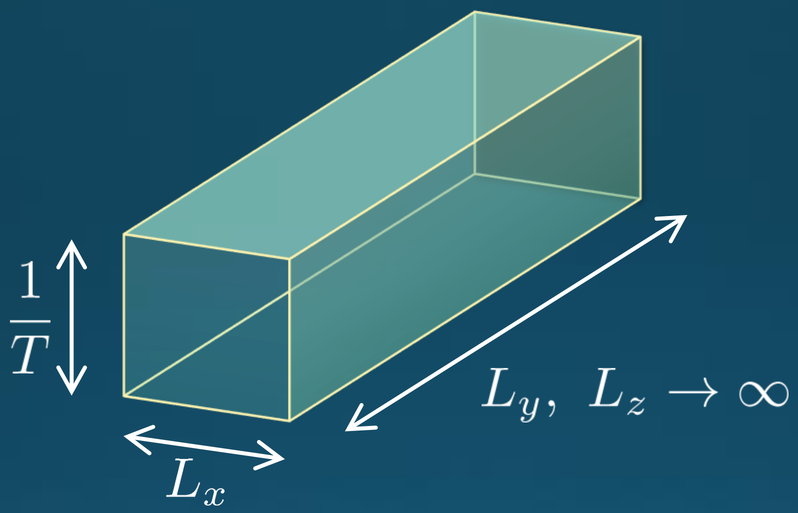
$$\tilde{T}^{44} = \tilde{T}^{22} = \tilde{T}^{33} = p$$

$$\tilde{\epsilon} = -p$$

$$\tilde{p}_x = -\epsilon$$

Purpose

Thermal SU(3) YM with PBC along x direction



QFT on $\boldsymbol{T}^2 \times \boldsymbol{R}^2$

How does thermodynamics behave w.r.t. T and L_x?

- ☐ Thermal Casimir effect in a non-perturbative system
- ☐ QCD phase diagram as a function of L_x
- ☐ 2 Polyakov loops will play important roles

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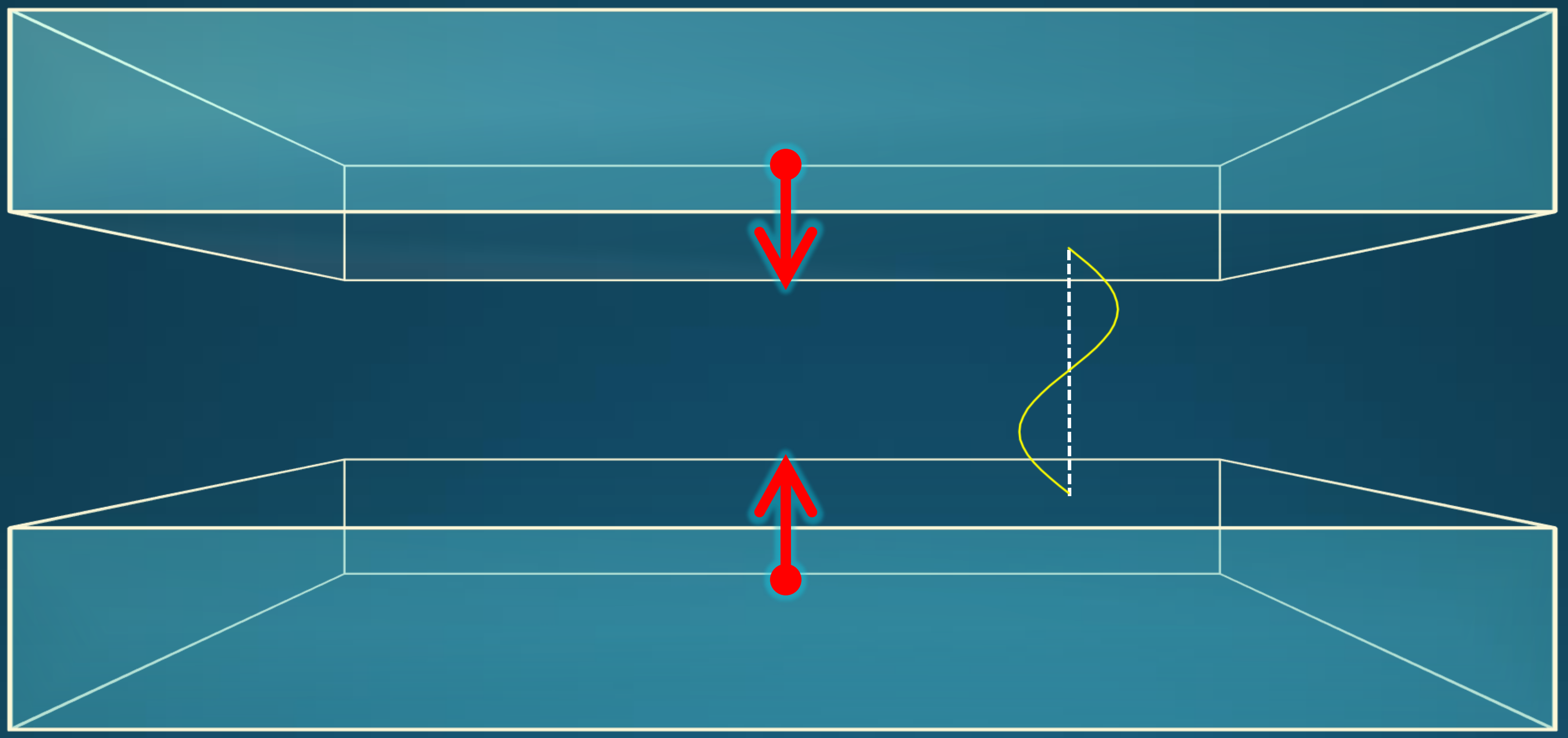
Casimir Effect

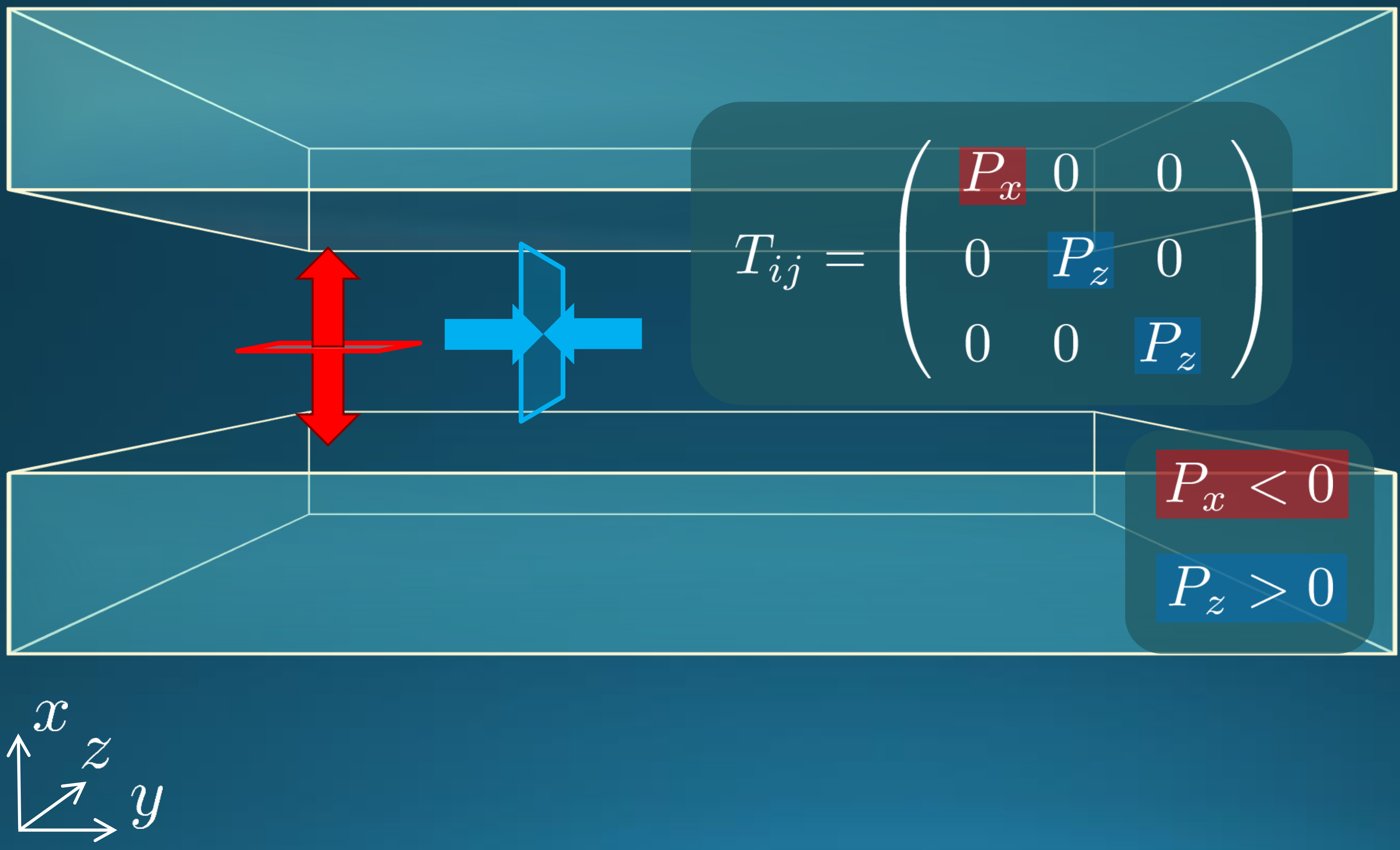
Casimir Effect



attractive force between two conductive plates

Casimir Effect

Brown, Maclay
1969



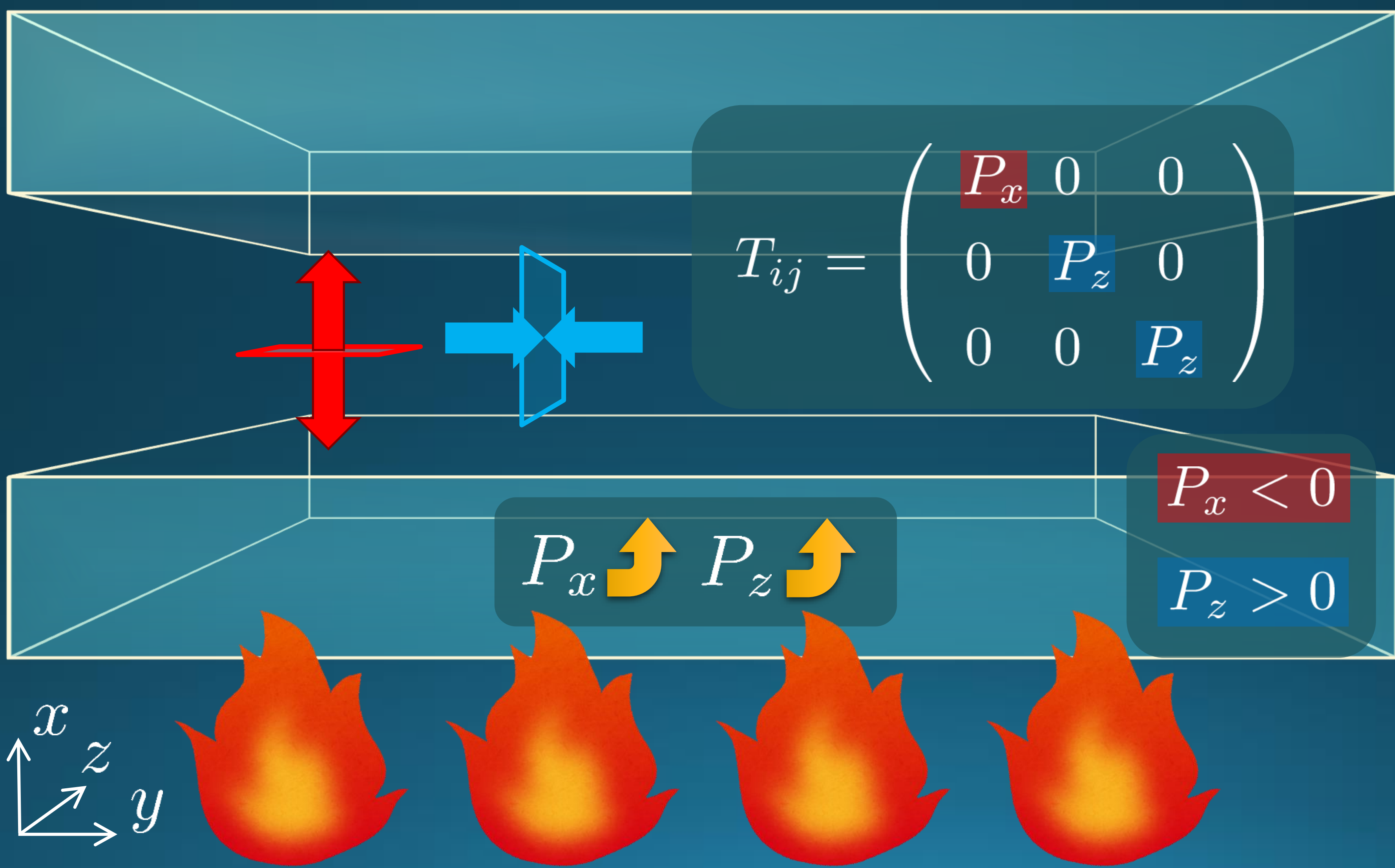
$$T_{ij} = \begin{pmatrix} P_x & 0 & 0 \\ 0 & P_z & 0 \\ 0 & 0 & P_z \end{pmatrix}$$

$$P_x < 0$$

$$P_z > 0$$

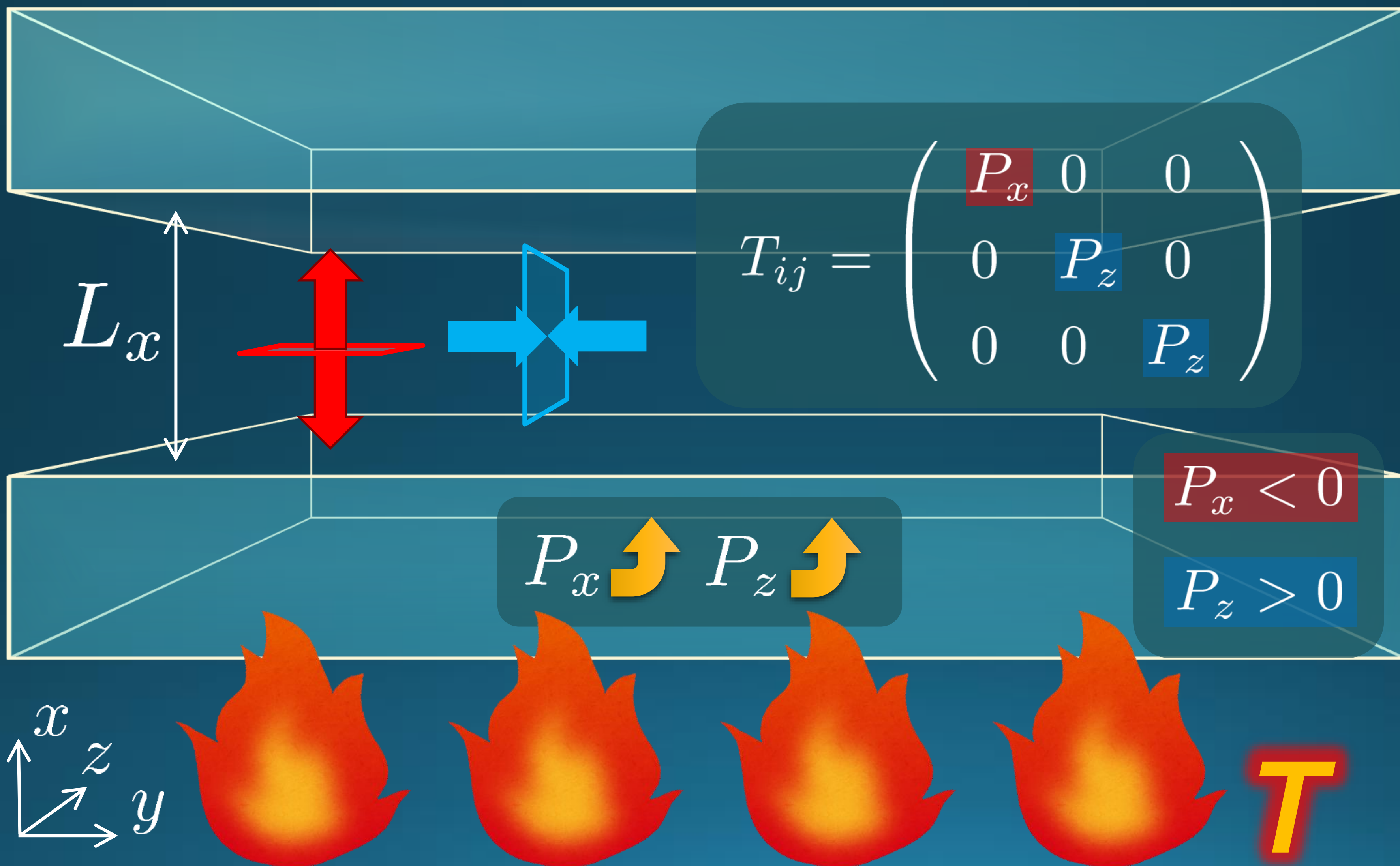
Casimir Effect

Brown, Maclay
1969



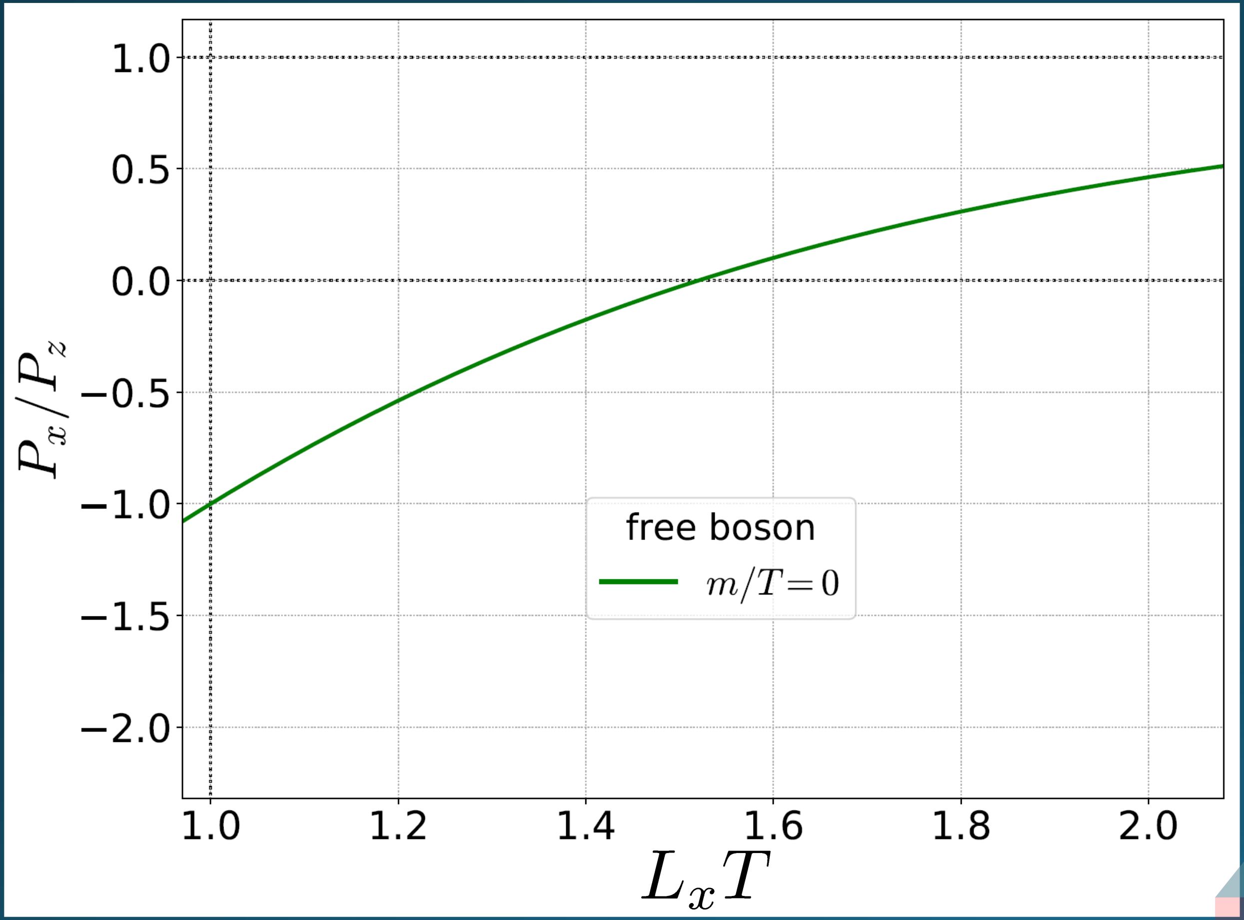
Casimir Effect

Brown, Maclay
1969



Pressure Anisotropy @ $T \neq 0$

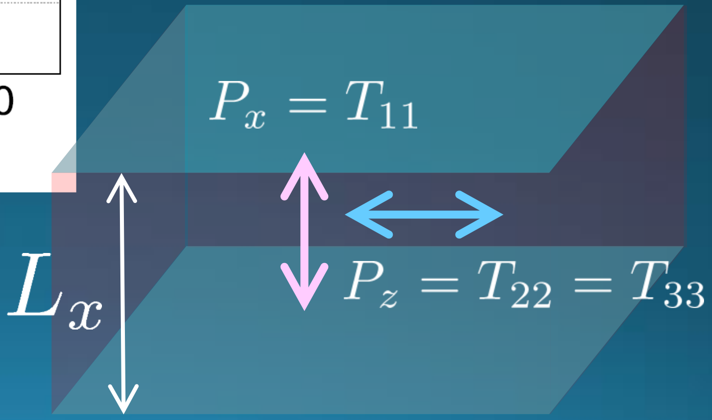
MK, Mogliacci, Kolbe, Horowitz, in prep.



Free scalar field

- $L_2 = L_3 = \infty$
- Periodic BC

Mogliacci+, 1807.07871



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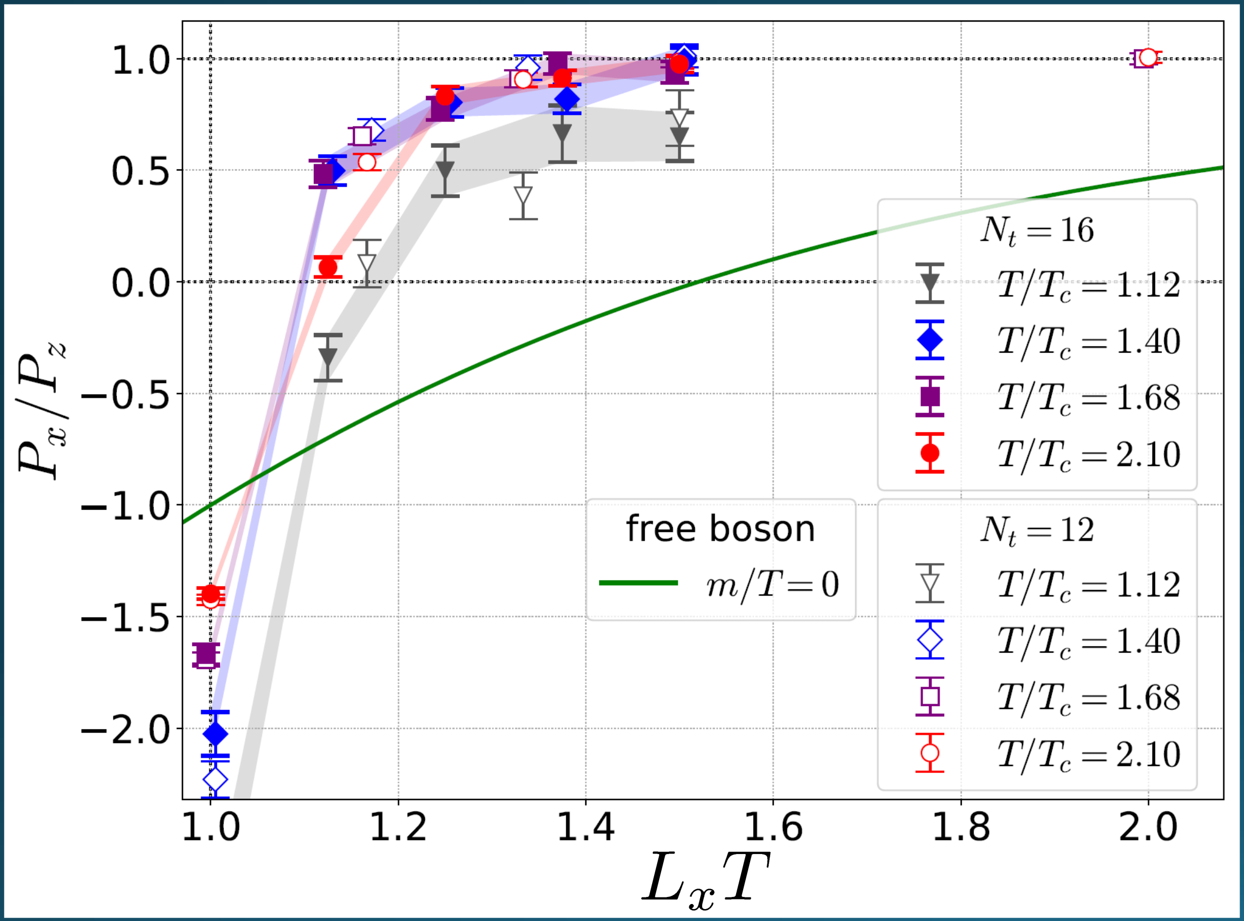


Free scalar field

- $L_2=L_3=\infty$
- Periodic BC

Mogliacci+, 1807.07871

Lattice result

- Periodic BC
- Only t→0 limit
- Error: stat.+sys.

Medium near T_c is remarkably insensitive to finite size!

Thermodynamics on the Lattice

Various Methods

- **Integral**, differential, moving frame, non-equilibrium, ...
- rely on thermodynamic relations valid in V→∞

$$P = \frac{T}{V} \ln Z \qquad sT = \varepsilon + P$$

→ **Not applicable to anisotropic systems**

- We employ **Gradient Flow Method**

$$\varepsilon = \langle T_{00} \rangle \quad P = \langle T_{11} \rangle$$

Components of EMT are directly accessible!

Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

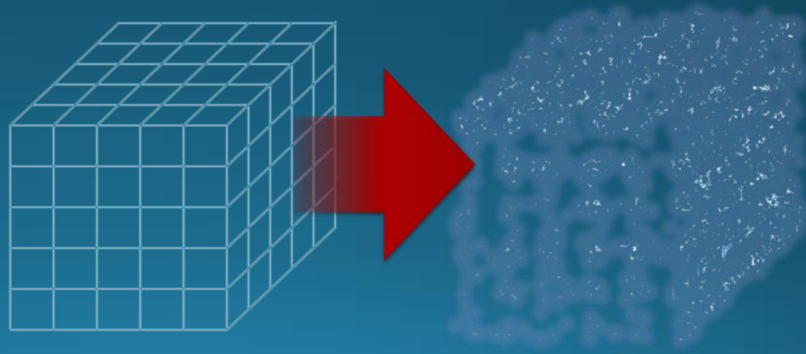
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length²]

leading

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at t>0



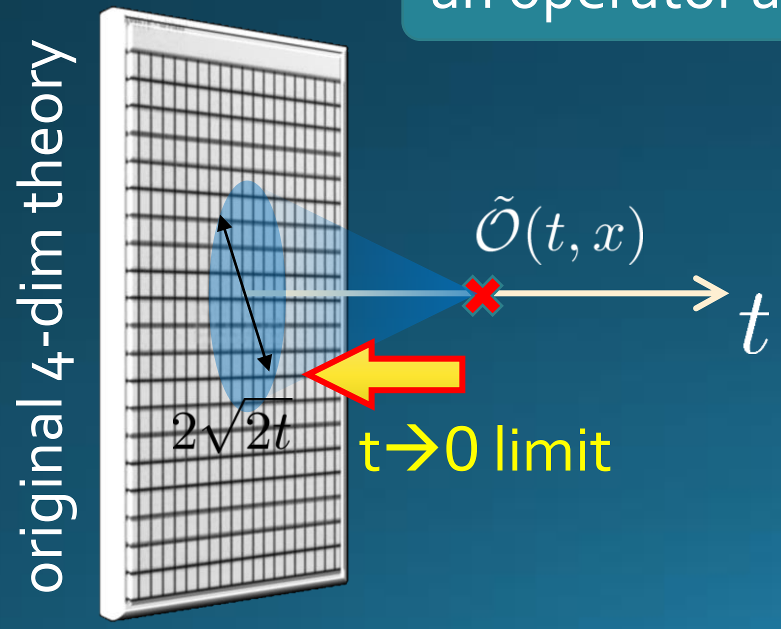
Small Flow-Time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t \to 0]{} \sum_i c_i(t) \mathcal{O}_i^R(x)$$

an operator at t>0

remormalized operators of original theory

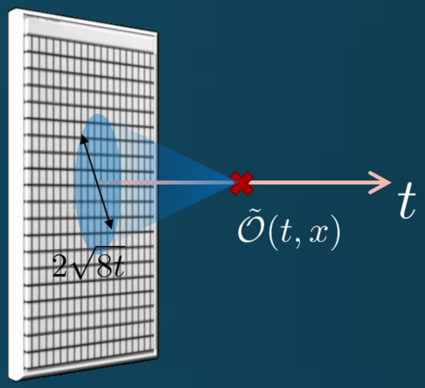


Constructing EMT

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$



vacuum subtr.

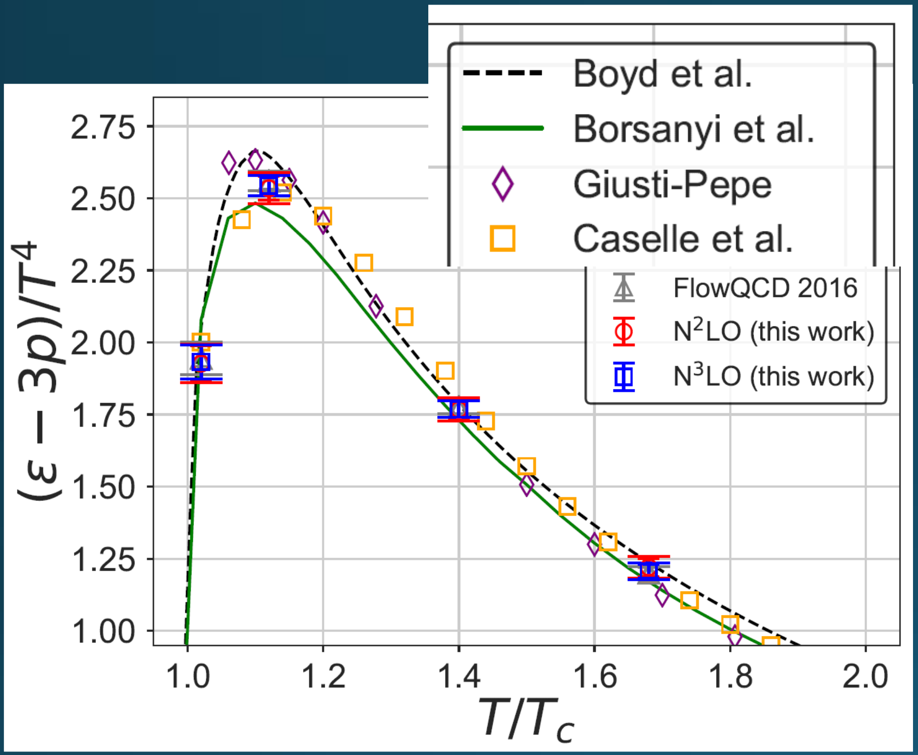
Remormalized EMT

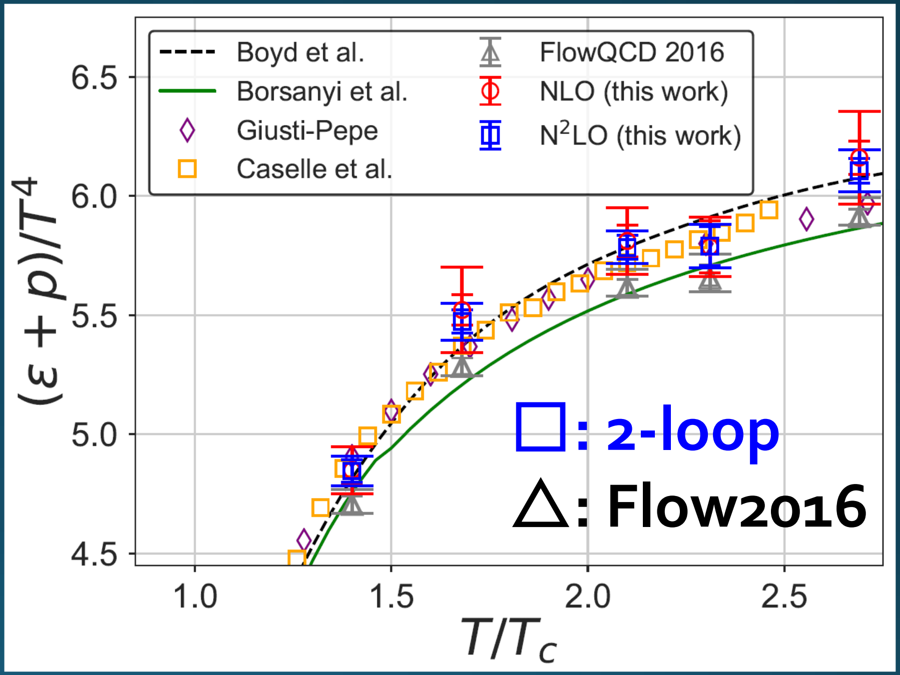
$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\text{subt.}}\right]$$

Perturbative coefficient:
Suzuki (2013); Makino, Suzuki (2014); Harlander+ (2018); Iritani, MK, Suzuki, Takaura (2019)

Thermodynamics

Iritani, MK, Suzuki, Takaura, 2019





Systematic error: μ_0 or μ_d, Λ, t→0 function, fit range

- Good agreement between different methods
- Our method can investigate finite systems with BCs

Numerical Setup

- SU(3) YM theory
- Wilson gauge action

- N_t = 16, 12
- N_z/N_t=6
- 2000~4000 confs.
- Even N_x

- No Continuum extrap.

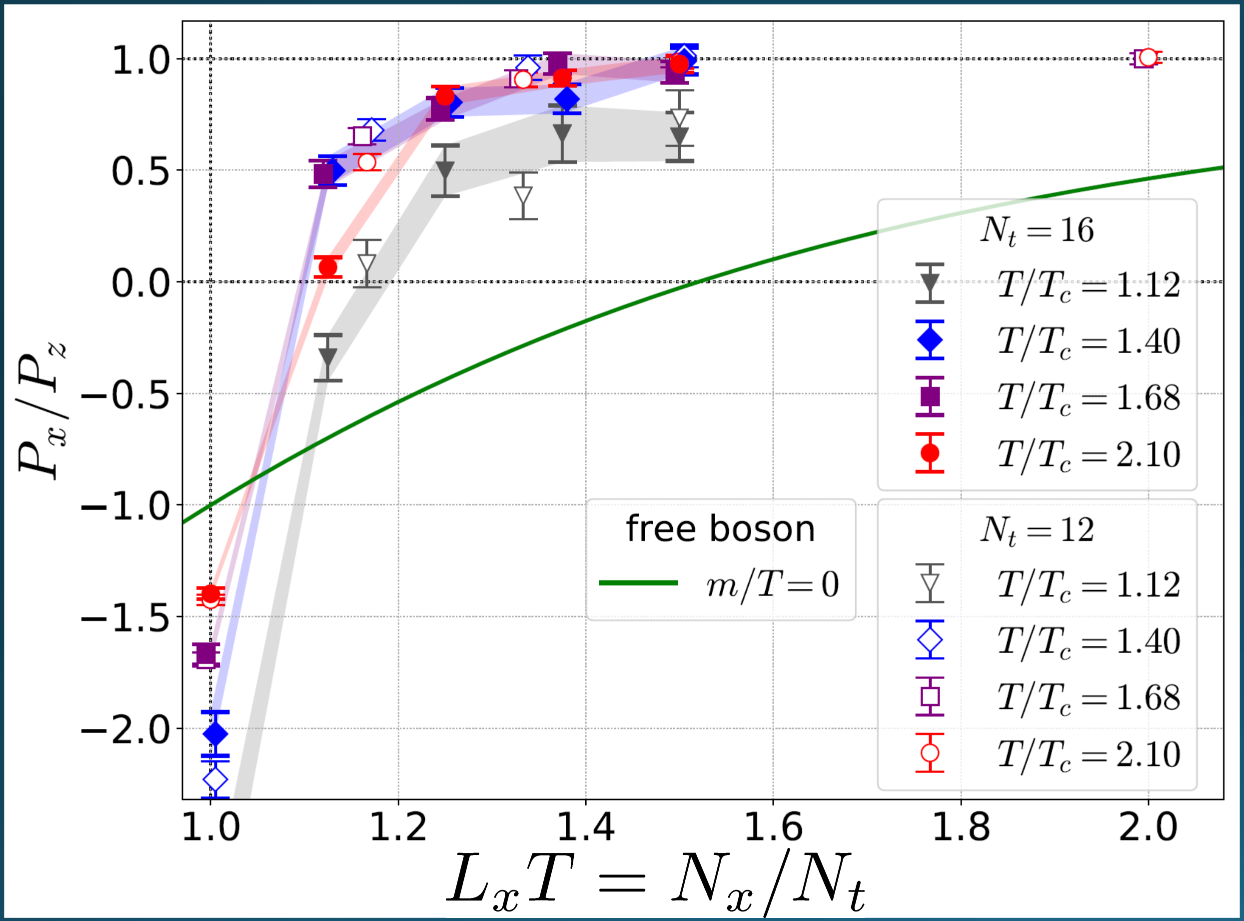
T/T_c	β	N_z	N_τ	N_x	N_{vac}
1.12	6.418	72	12	12, 14, 16, 18	64
	6.631	96	16	16, 18, 20, 22, 24	96
1.40	6.582	72	12	12, 14, 16, 18	64
	6.800	96	16	16, 18, 20, 22, 24	128
1.68	6.719	72	12	12, 14, 16, 18, 24	64
	6.719	96	12	14, 18	64
	6.941	96	16	16, 18, 20, 22, 24	96
2.10	6.891	72	12	12, 14, 16, 18, 24	72
	7.117	96	16	16, 18, 20, 22, 24	128
2.69	7.086	72	12	12, 14, 16, 18	-
$\simeq$ 8.1	8.0	72	12	12, 14, 16, 18	-
$\simeq$ 25	9.0	72	12	12, 14, 16, 18	-

- Same Spatial volume
  - $12 \times 72^2 \times 12 \sim 16 \times 96^2 \times 16$
  - $18 \times 72^2 \times 12 \sim 24 \times 96^2 \times 16$

Simulations on OCTOPUS/Reedbush

Pressure Anisotropy @ $T \neq 0$

MK, Mogliacci, Kolbe, Horowitz, in prep.



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Mogliacci+, 1807.07871

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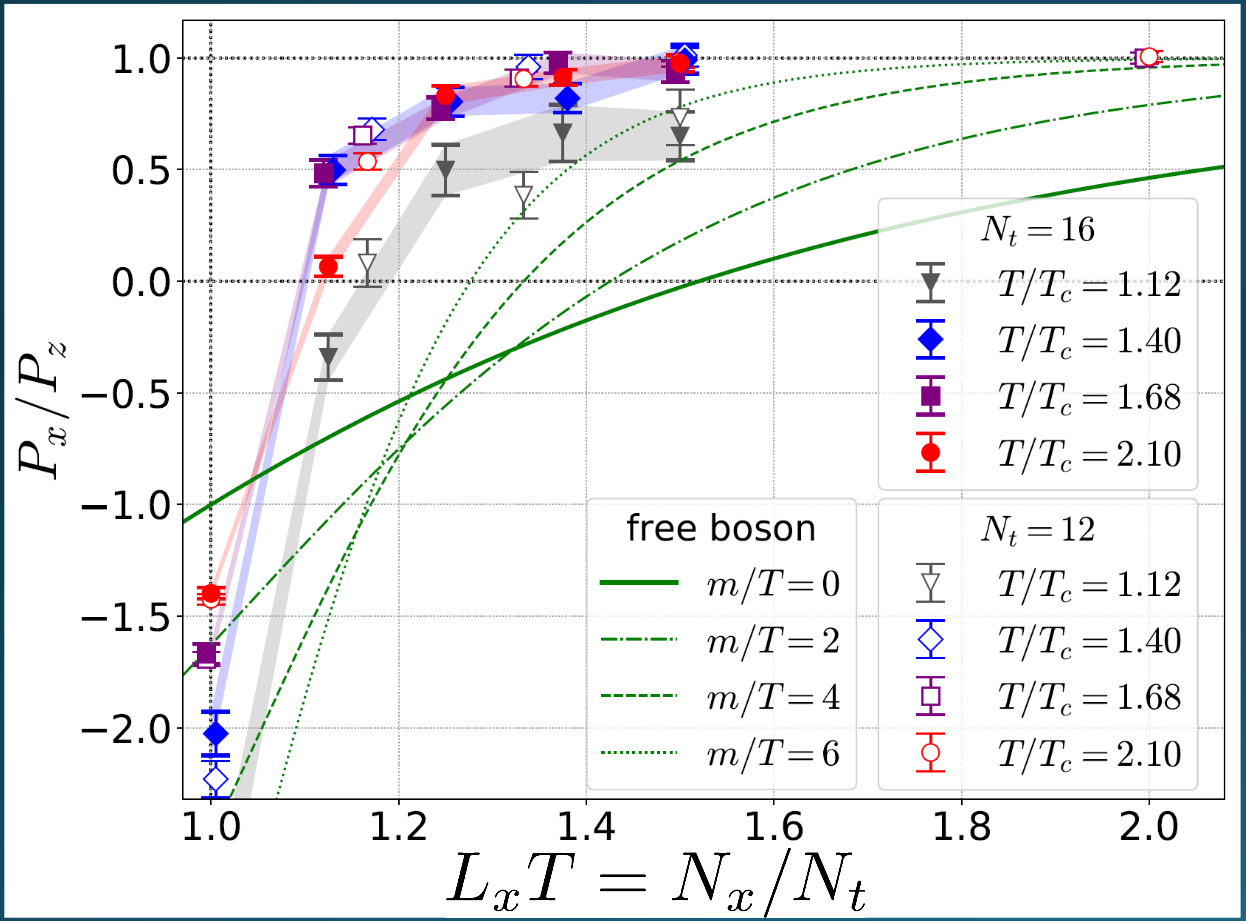

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Higher T

High-T limit: massless free gluons

How does the anisotropy approach this limit?

Difficulties

- Vacuum subtraction requires large-volume simulations.
- Lattice spacing not available → $c_1(t)$, $c_2(t)$ are not determined.

Higher T

High-T limit: massless free gluons
How does the anisotropy approach this limit?

Difficulties

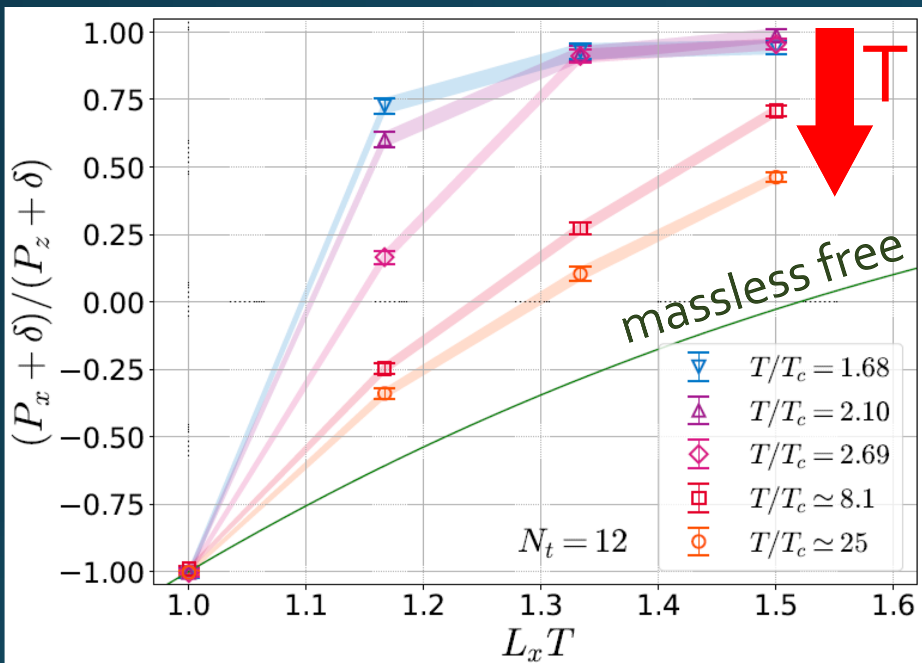
- ☐ Vacuum subtraction requires large-volume simulations.
- ☐ Lattice spacing not available → $c_1(t)$, $c_2(t)$ are not determined.

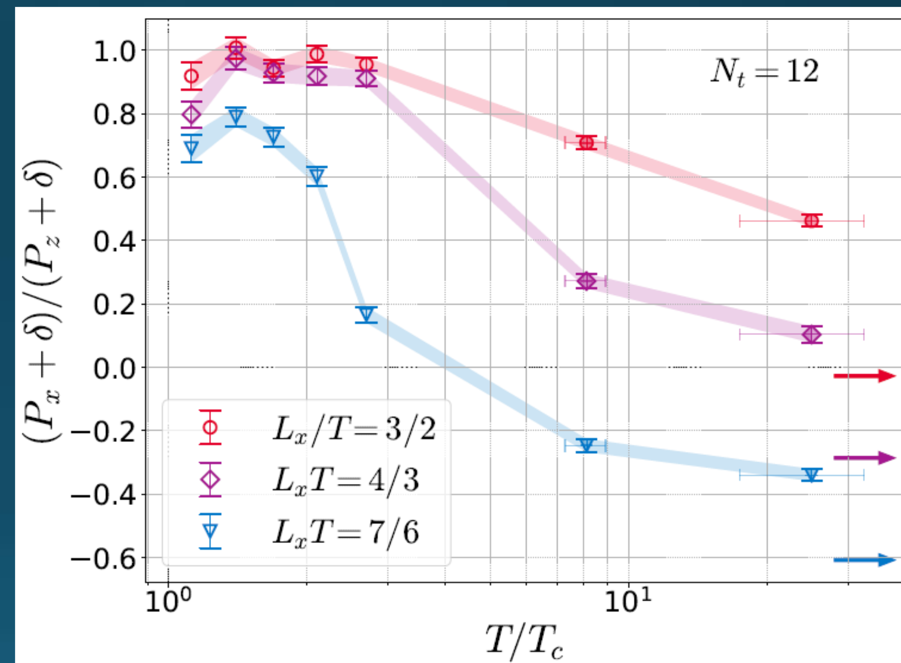
We study

$$\frac{P_x + \delta}{P_z + \delta} \qquad \delta = -\frac{1}{4}\sum_{\mu} T^{\mathrm{E}}_{\mu\mu}$$

No vacuum subtr. nor Suzuki coeffs. necessary!

$$\frac{P_x + \delta}{P_z + \delta}$$





$T/Tc \cong 8.1\ (b = 8.0)\ /\ T/Tc \cong 25\ (b = 9.0)$

- ☐ Ratio approaches the asymptotic value for large T.
- ☐ But, large deviation exists even at $T/Tc \sim 25$.
- ☐ 1st-order phase transition??

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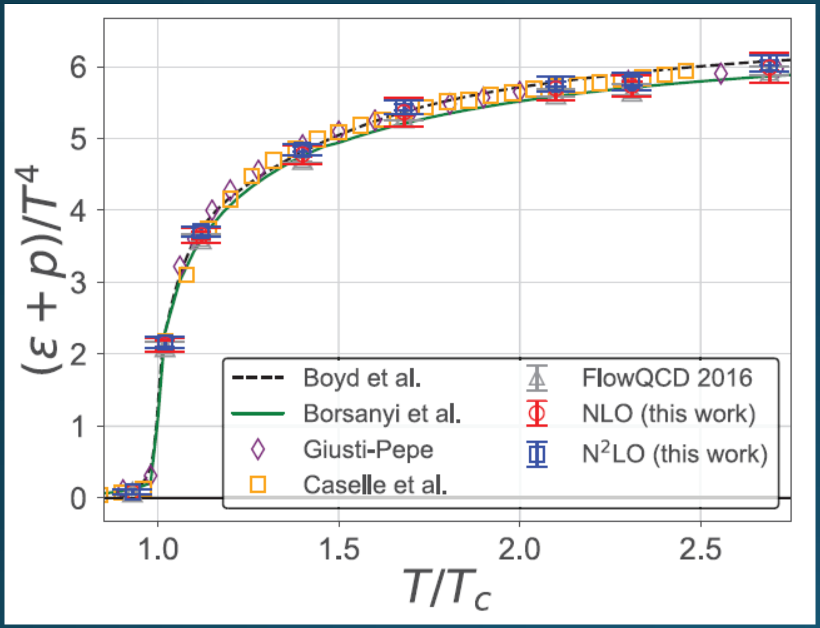
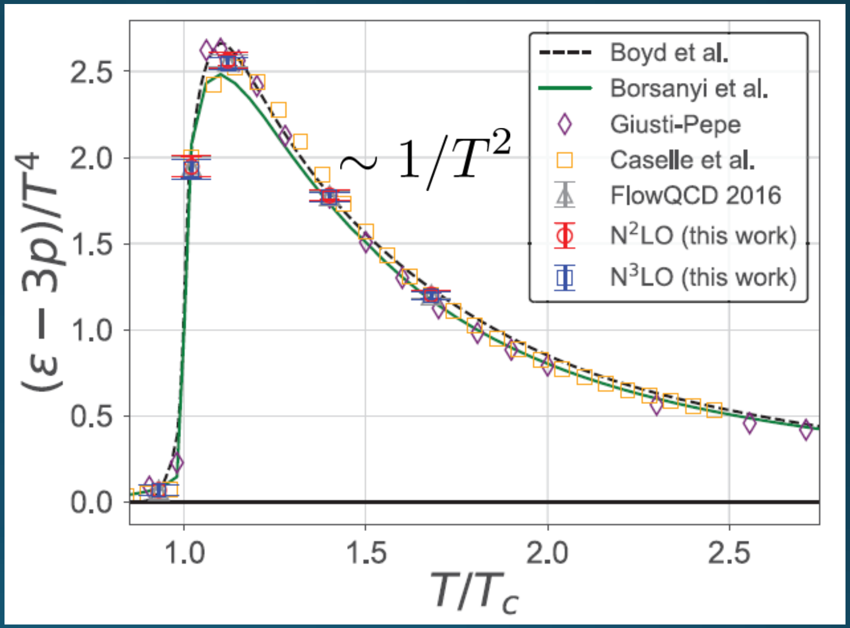
Thermodynamics of SU(3) YM on $S^1 \times R^3$





- ☐ Deconfinement phase transition = 1st-order PT
- ☐ Stefen-Boltzmann limit for $T \to \infty$

➡ How to describe these features quantitatively?

Polyakov-loop Effective Models

Meisinger+, PRD (2003)

General Idea

Introduce Polyakov loop P through a constant background field A_0

$$P = \mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^{L_\tau} A_\tau d\tau\right)\right] \Longleftarrow A_\tau(x) = \begin{pmatrix} \theta_1 & 0 & 0 \\ 0 & \theta_2 & 0 \\ 0 & 0 & \theta_3 \end{pmatrix}$$

Note:

- $P = 0$: confinement / Z(3) symmetric
- $P \neq 0$: deconfinement / Z(3) broken

Polyakov-loop Effective Models

Meisinger+, PRD (2003)

Free Energy

$$F(T;P) = F_{\mathrm{pert.}}(T;P) + F_{\mathrm{pot.}}(T;P)$$

Perturbative term

free energy of massless free gluons with a constant A_τ

$$f_{pert}(\theta) = \sum_{n \in Z} \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}_A \ln\left[\left(\frac{2\pi n}{\beta} - A_0\right)^2 + \vec{k}^2\right]$$

Polyakov-loop potential

Phenomenological free energy that makes P nontrivial

The value of P is determined so as to minimize $F(T;P)$.

$$F_{\mathrm{pot}}(T, P)$$

Meisinger+, PRD (2003)

□Model A

gluon mass term at the leading order

$$F_{\mathrm{gluons}} = F_{\mathrm{pert}} + m_g^2 \tilde{F} + \cdots$$

employ this term as F_{pot}
with a parameter m_g

□Model B

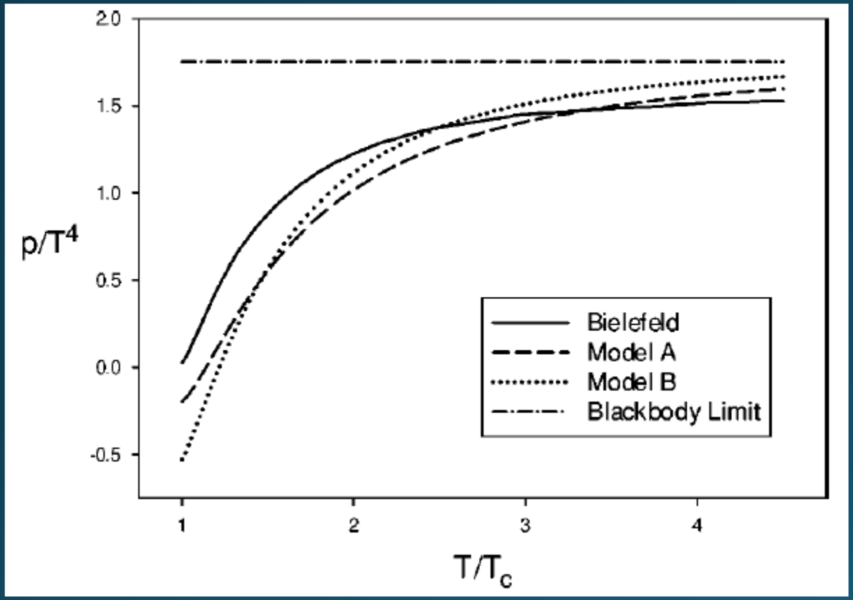
term inspired by the Haar measure

$$F_{\mathrm{pot}} = -\frac{T}{R^3} \ln \prod_{j<k} \sin^2\left(\frac{\theta_j - \theta_k}{2}\right)$$

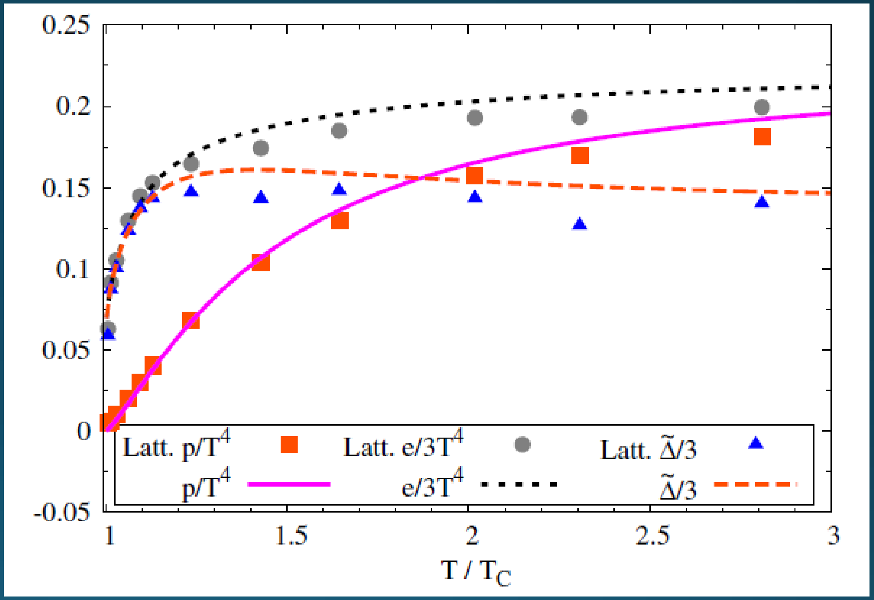
R: phenomenological parameter

Results

Meisinger+, PRD ('03)



Dumitru+, PRD ('12)



Qualitative behavior of lattice thermodynamics near and above T_c is well explained.

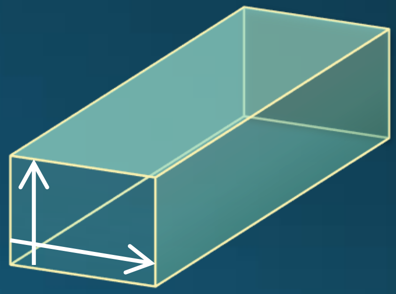
Extension to $\boldsymbol{T}^2 \times \boldsymbol{R}^2$

Suenaga, MK ('23); Fujii+, in prep.

2 Polyakov loops along τ and x directions

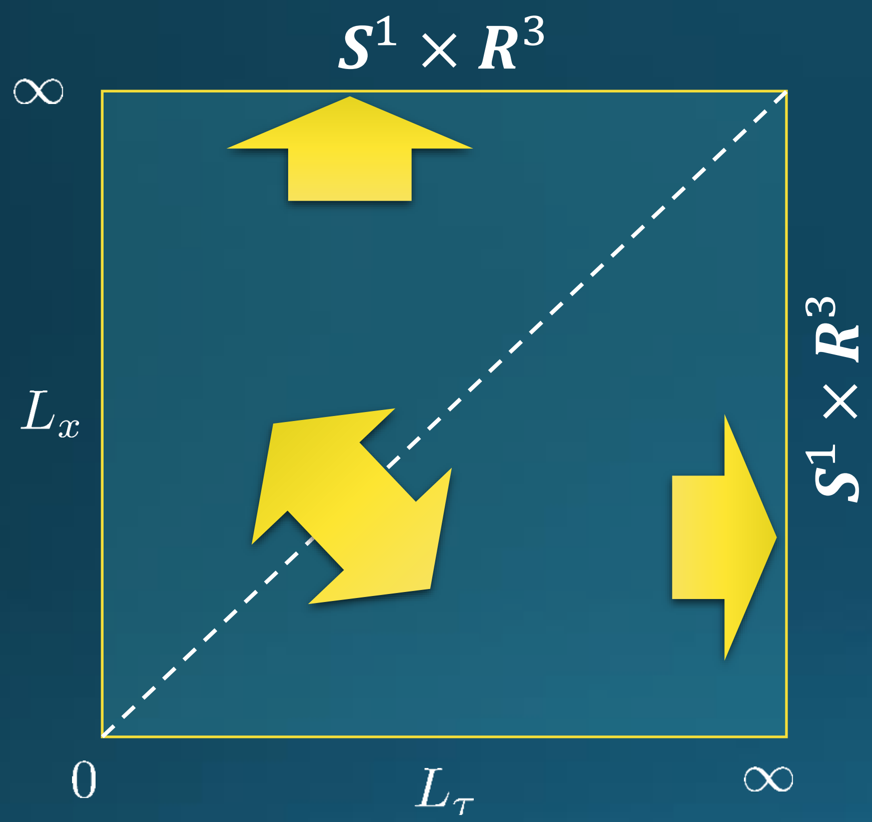
$$P_\tau = \mathrm{Tr}\Big[\mathcal{P}\exp\Big(i\int_0^{L_\tau} A_\tau d\tau\Big)\Big] \qquad P_x = \mathrm{Tr}\Big[\mathcal{P}\exp\Big(i\int_0^{L_\tau} A_x d\tau\Big)\Big]$$

□ Diagonal ansatz: $A_i = \begin{pmatrix} \theta_1 & 0 & 0 \\ 0 & \theta_2 & 0 \\ 0 & 0 & \theta_3 \end{pmatrix}$



$$F_{\mathrm{pert.}} = F_{\mathrm{pert}}(L_\tau, L_x; \vec{\theta}_\tau, \vec{\theta}_x)$$
$$F_{\mathrm{pot.}} = F_{\mathrm{pot}}(L_\tau, L_x; \vec{\theta}_\tau, \vec{\theta}_x)$$

Constraints on F_{pot}



Invariance w.r.t. exchange of τ, x axes

$$F(\vec{\theta}_\tau, \vec{\theta}_x; L_\tau, L_x) = F(\vec{\theta}_x, \vec{\theta}_\tau; L_x, L_\tau)$$

Thermodynamics on $\boldsymbol{S}^1 \times \boldsymbol{R}^3$ must be reproduced at $L_x \to \infty$

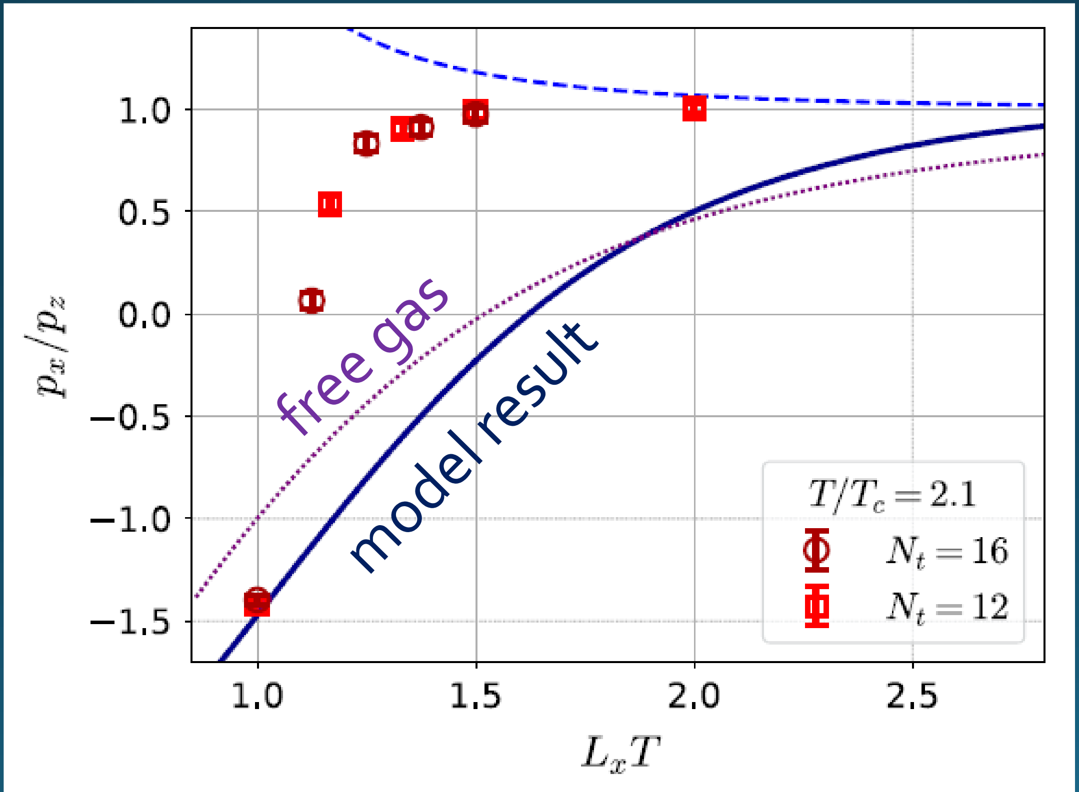
$$F_{\text{pert}}(L_\tau, L_x) \xrightarrow[L_x \to \infty]{} \mathcal{O}(L_x^{-3})$$

$F_{\text{pot}}(L_\tau, L_x)$ must disappear faster

Separable Ansatz

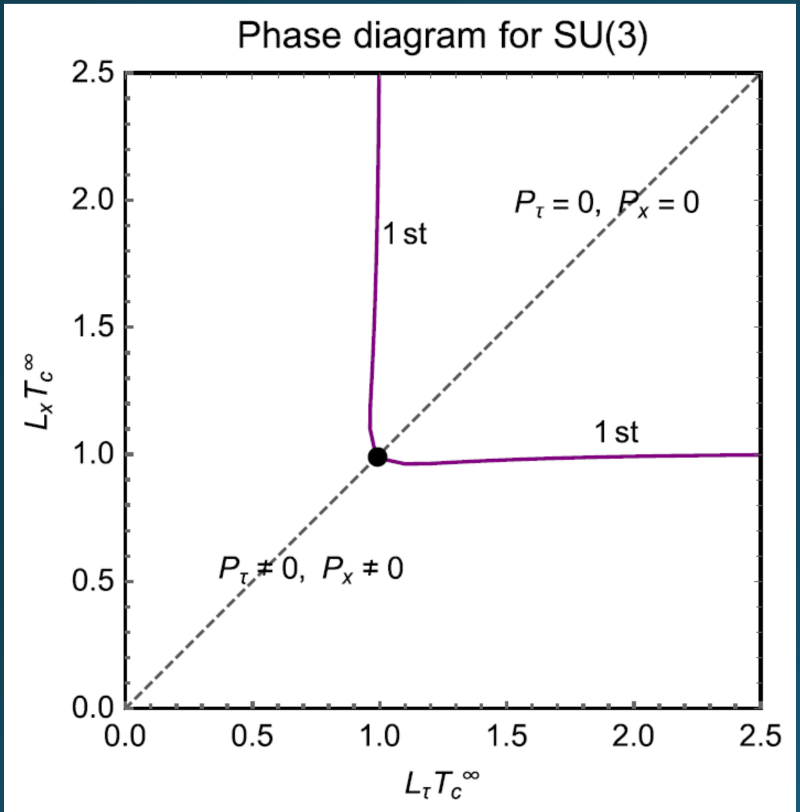
Suenaga, MK ('23)

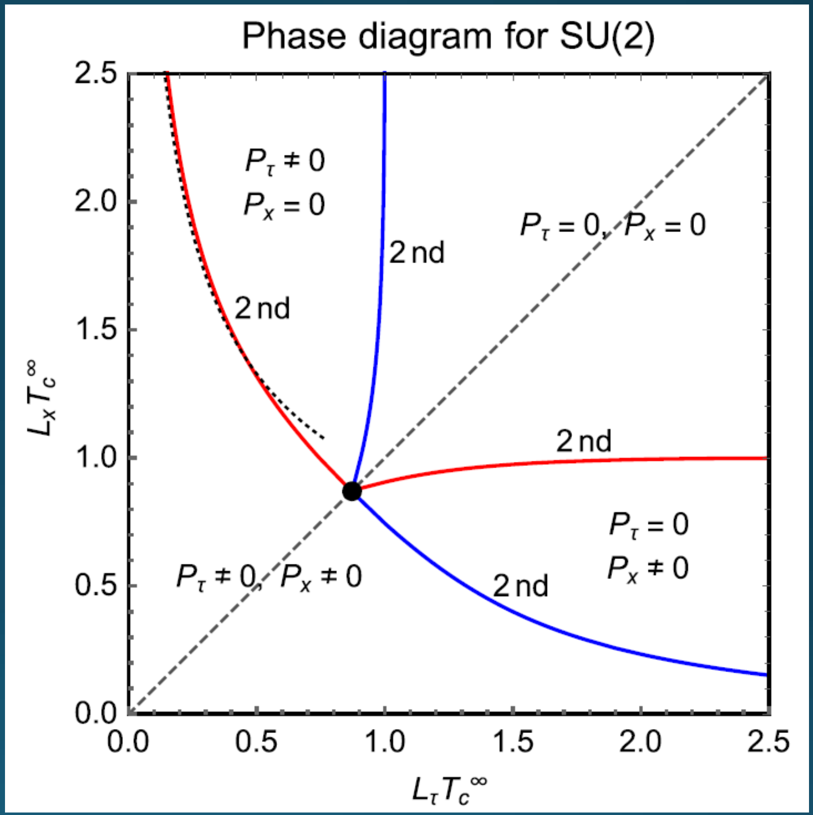
$$F_{\rm pot} = F_{\rm pot}^{S^1\times R^3}(L_\tau;\vec{\theta}_\tau) + F_{\rm pot}^{S^1\times R^3}(L_x;\vec{\theta}_x)$$



Lattice result is not reproduced even qualitatively.

Phase Diagram





Introducing Cross Term

Fujii+, in prep.

Free Energy

$$F = F_{\mathrm{pert.}} + F_{\mathrm{pot}} + F_{\mathrm{cross}}$$

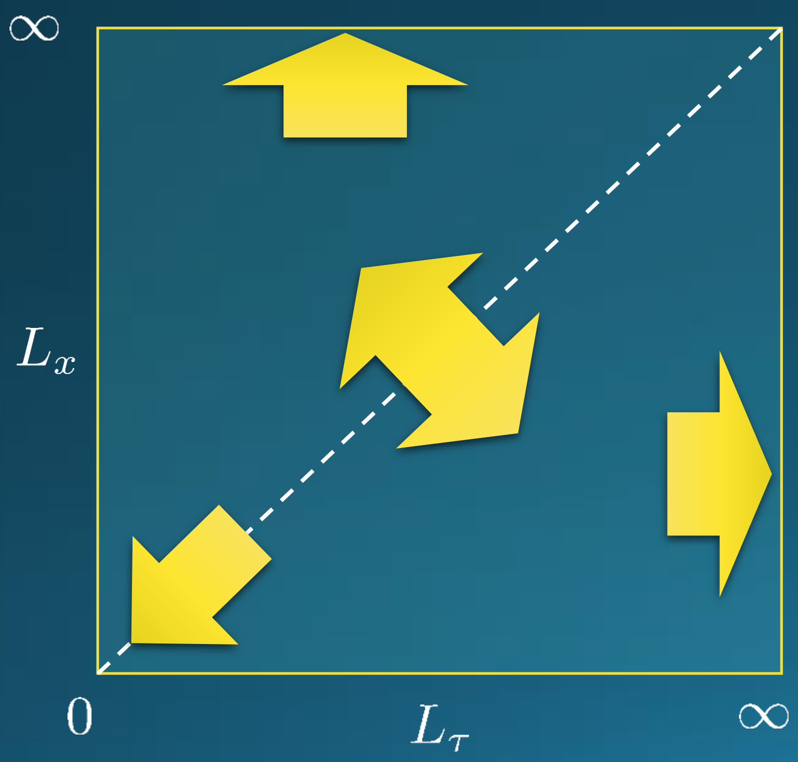
cross term of P_τ and P_x

$$\begin{aligned} F_{\mathrm{cross}} = g(L_\tau, L_x)\Big[& c_4 \mathrm{Tr}[P_\tau^\dagger P_\tau]\mathrm{Tr}[P_x^\dagger P_x] \\ & + c_5 (\mathrm{Tr}[P_\tau^\dagger P_\tau]\mathrm{Tr}[P_x^3] + \mathrm{Tr}[P_\tau^3]\mathrm{Tr}[P_x^\dagger P_x]) \\ & + c_6 \mathrm{Tr}[P_\tau^3]\mathrm{Tr}[P_x^3]\Big] \end{aligned}$$

$c_4,\ c_5,\ c_6$: parameters in the model

Cross Term: coefficient

$$F_{\text{cross}} = g(L_\tau, L_x)\Big[c_4 \mathrm{Tr}[P_\tau^\dagger P_\tau]\mathrm{Tr}[P_x^\dagger P_x] + c_5\left(\mathrm{Tr}[P_\tau^\dagger P_\tau]\mathrm{Tr}[P_x^3] + \mathrm{Tr}[P_\tau^3]\mathrm{Tr}[P_x^\dagger P_x]\right) + c_6 \mathrm{Tr}[P_\tau^3]\mathrm{Tr}[P_x^3]\Big]$$



$$g(L_\tau, L_x) = T_c^4\Big((T_c L_\tau)^2 + (T_c L_x)^2\Big)^{-n}$$

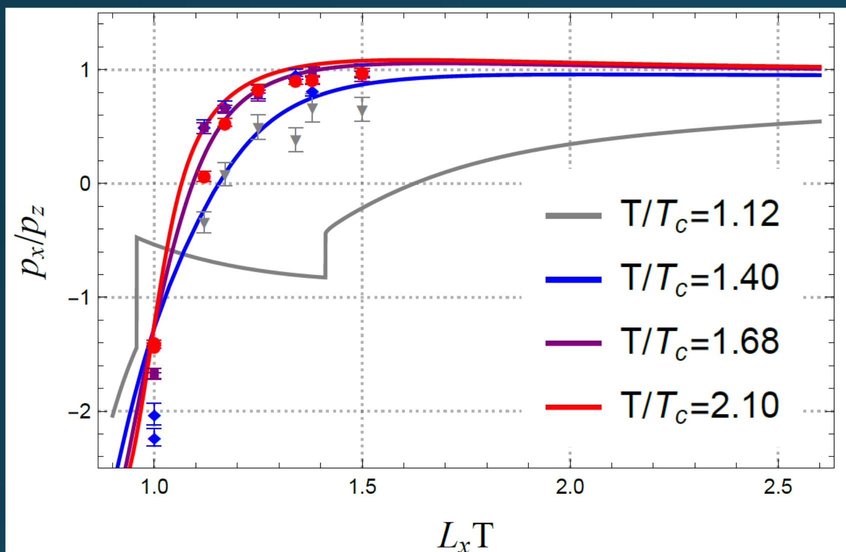
$$1.5 < n < 2.0$$

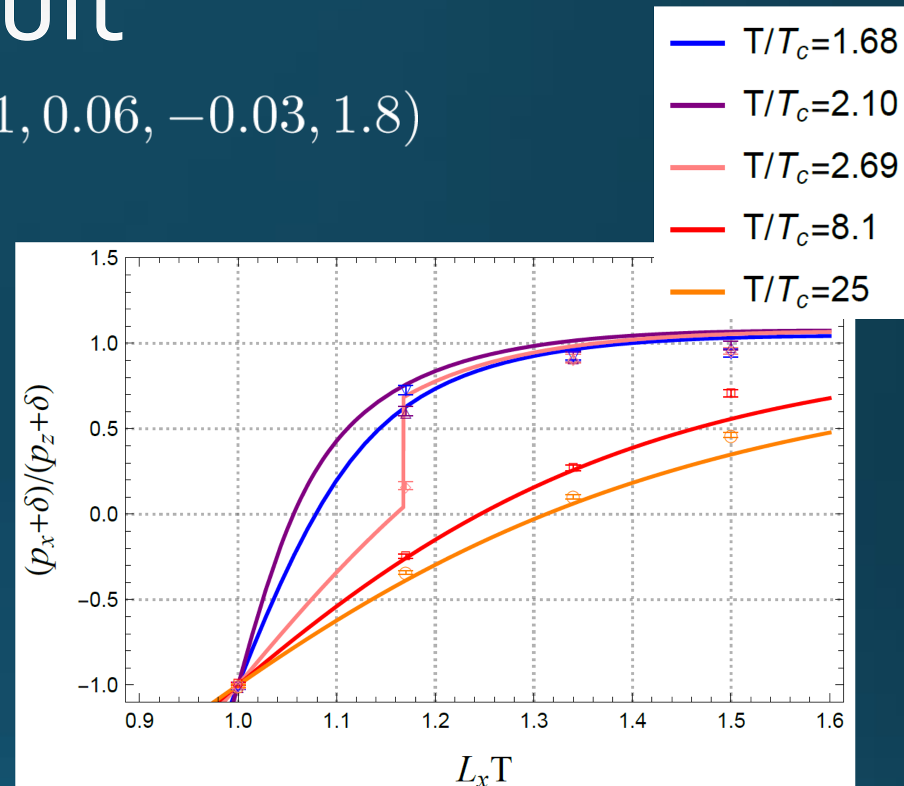
$$F_{\text{pot}}^{S^1\times R^3} \xrightarrow[L_\tau, L_x \to 0]{} \mathcal{O}(L_c^{-2})$$

$$F_{\text{pert}} \xrightarrow[L_c \to \infty]{} \mathcal{O}(L_c^{-3})$$

Result

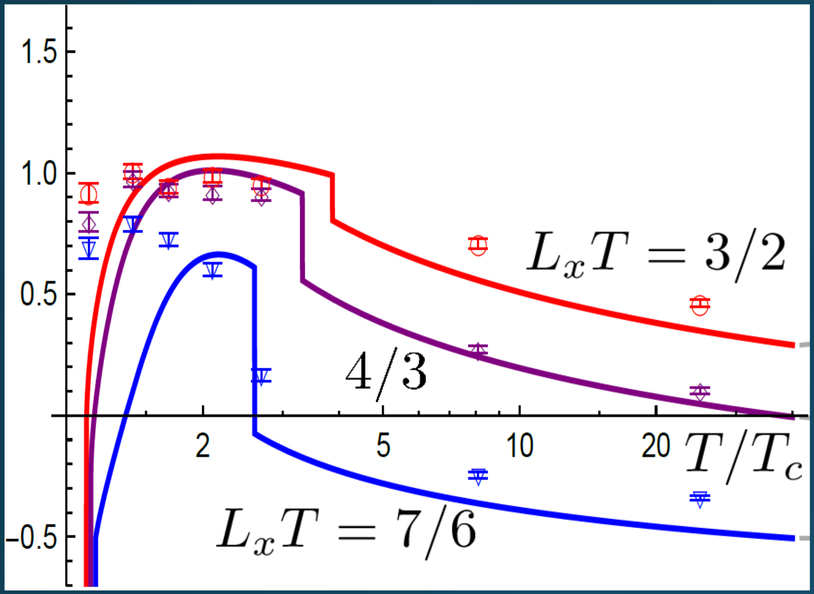
$$(c_4, c_5, c_6, n) = (0.11, 0.06, -0.03, 1.8)$$

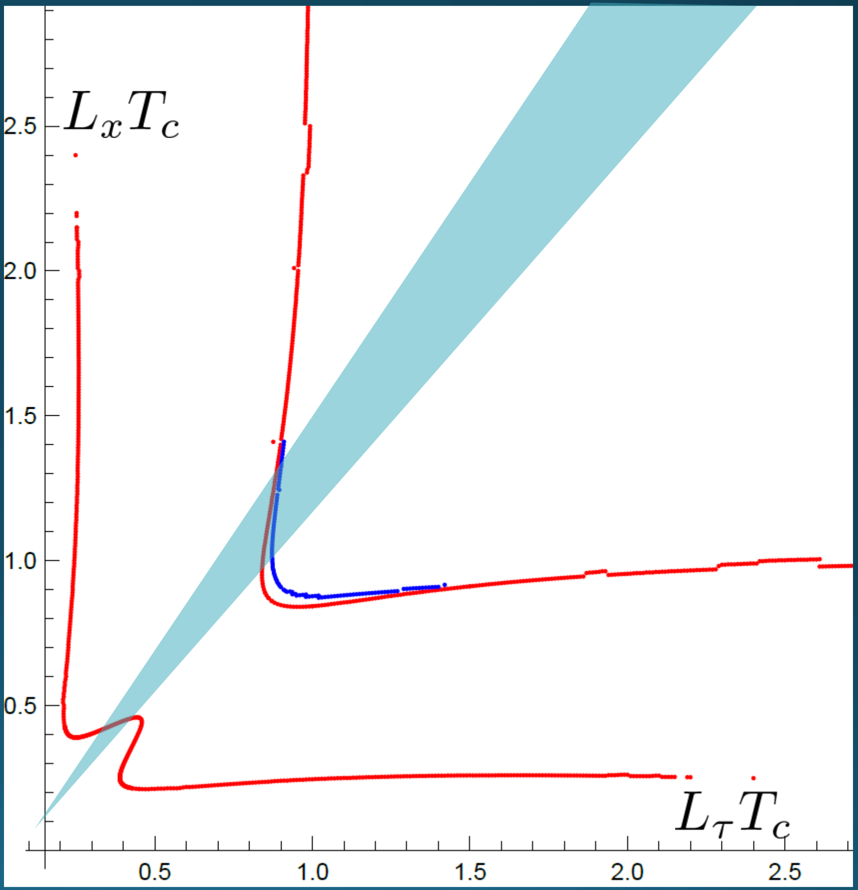




- ☐ Lattice results for $T/T_c > 1.5$ are well reproduced.
- ☐ No parameters to fit the results for $T/T_c = 1.4, 1.12$.
- ☐ Appearance of 1st-order PT?

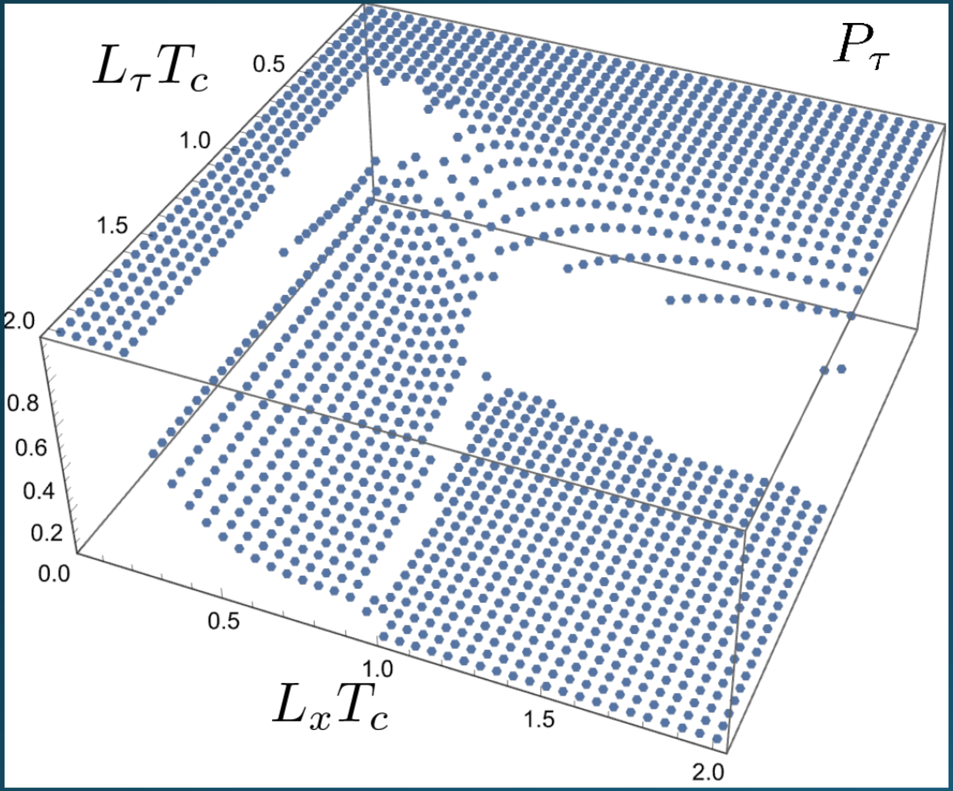
Phase Diagram



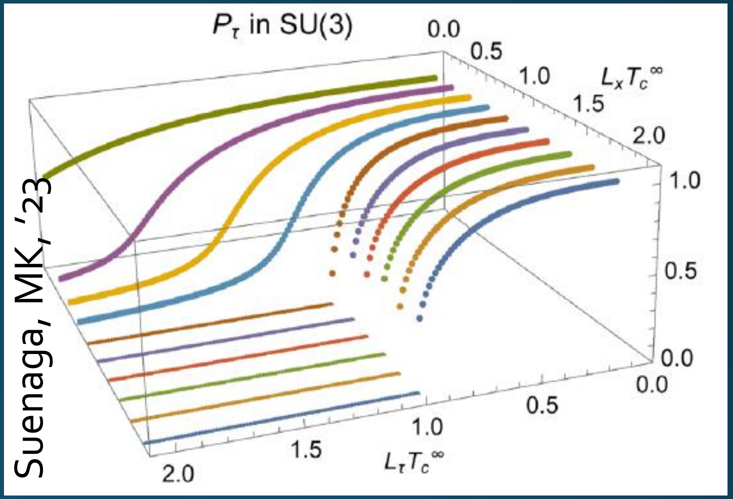


- Appearance of a new 1st PT and critical points in the confined region
- How do it affect lattice simulations?

Polyakov Loops

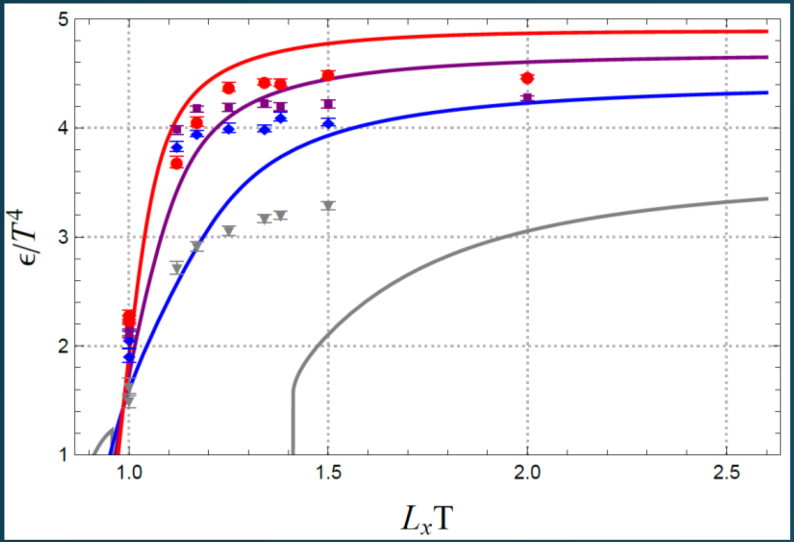


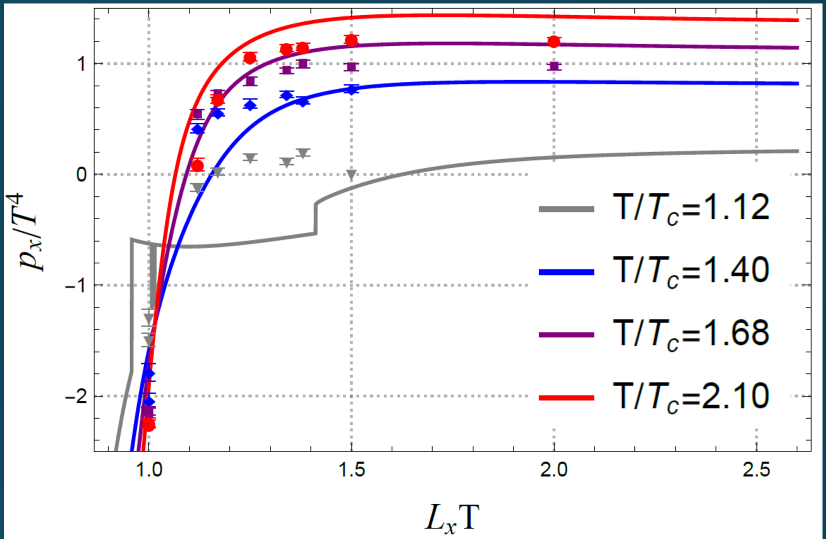
without cross term

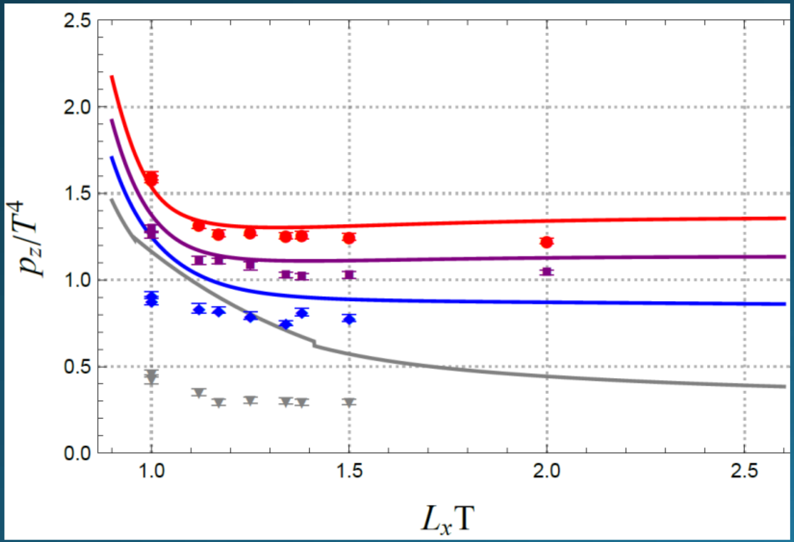


Competition between P_τ and P_x leads to their sudden changes around $L_\tau \simeq L_x$.

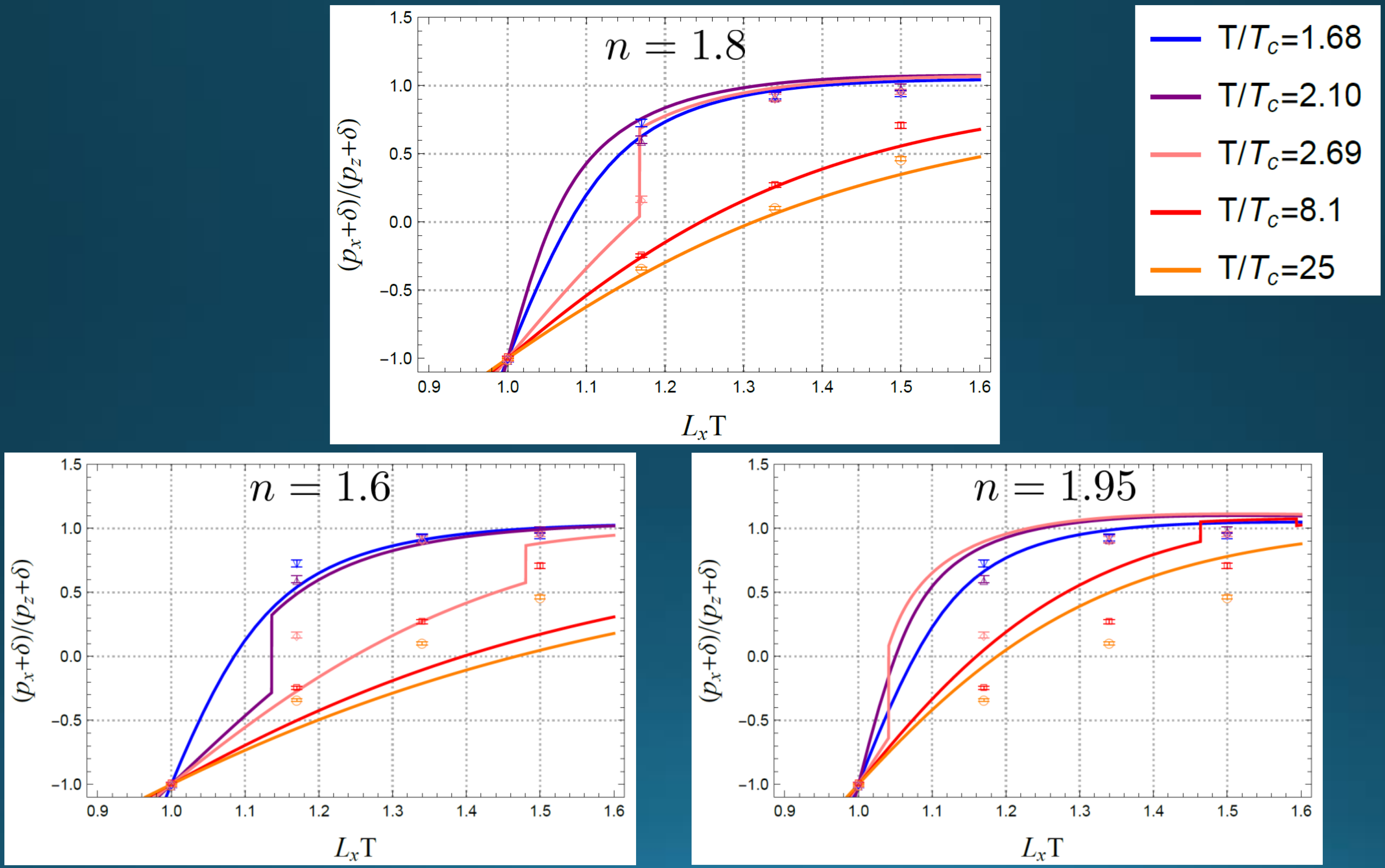
Thermodynamics







n Dependence



The value of n can be constrained by the high-T behavior.

Summary

Lattice thermodynamics in SU(3) YM with periodic BC have peculiar behaviors:

- Medium at $1.4<T/T_c<2.1$ is remarkably insensitive to the boundary.
- Slow approach to the SB limit at small L_τ, L_x.

Our **model** with two Polyakov loops explains the lattice results for $T \geq 1.5T_c$ qualitatively:

- Interplay b/w two Polyakov loops plays a crucial role.
- Appearance of new 1st-PT & CP is predicted.

Future

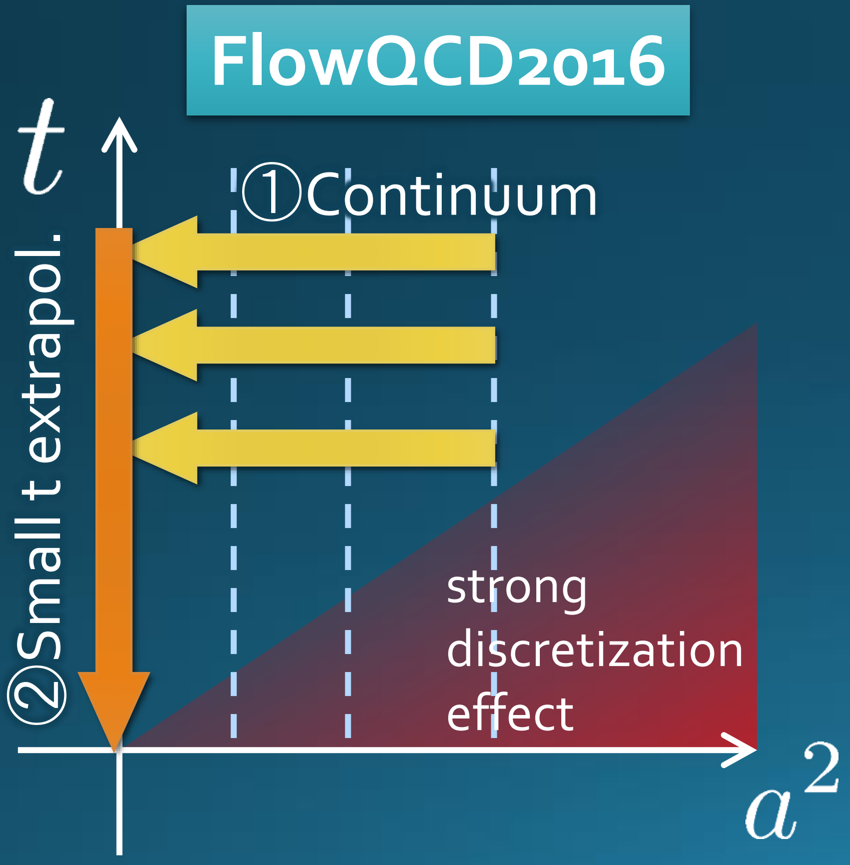
- More lattice results
- Anti-periodic / Dirichlet BCs, BC for two directions, below T_c, …

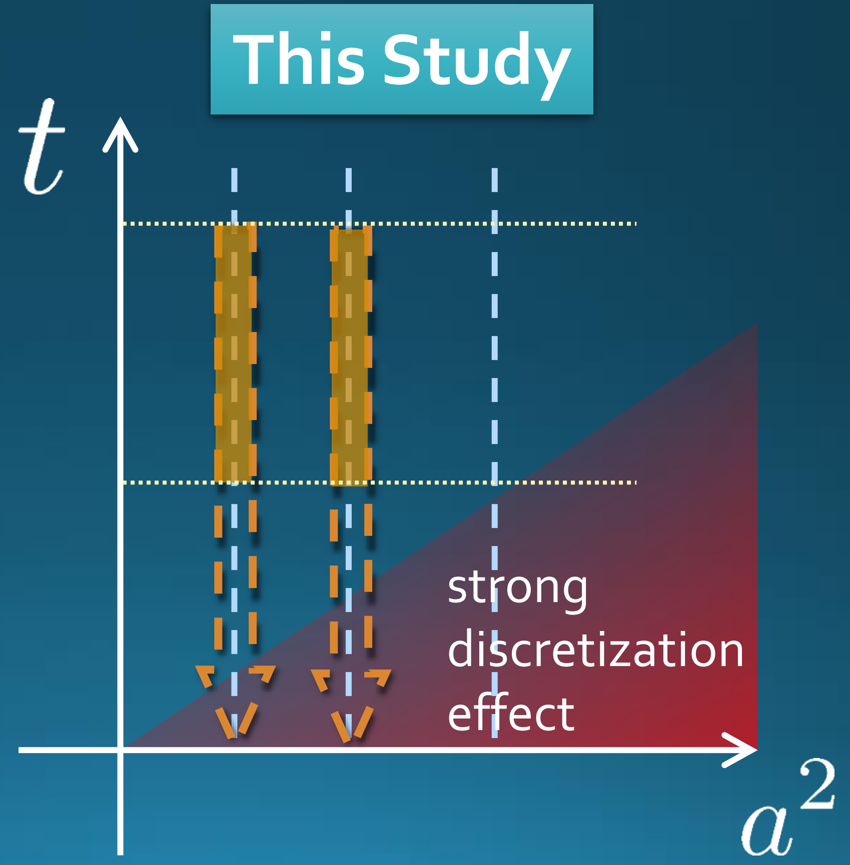
backup

Extrapolations t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

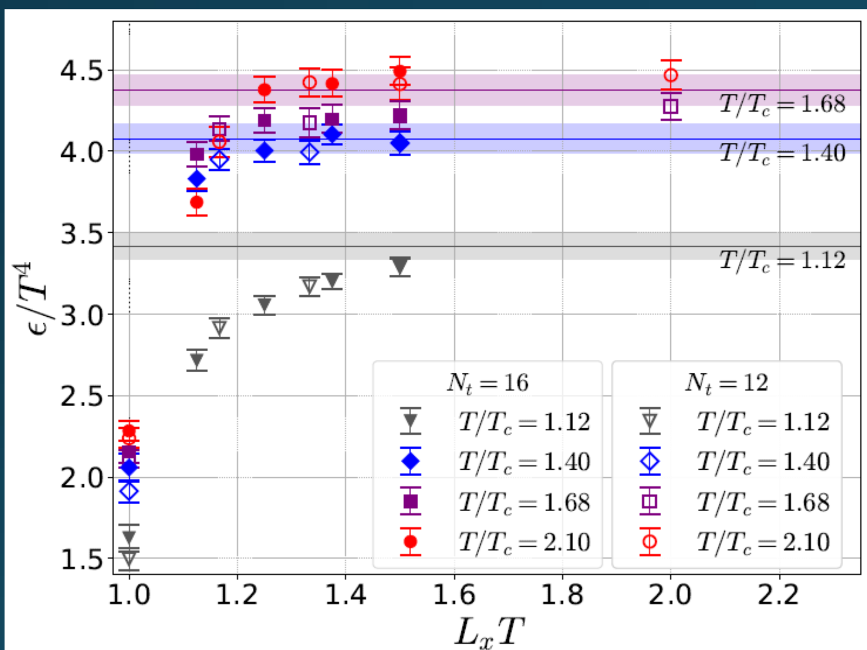
O(t) terms in SFTE lattice discretization



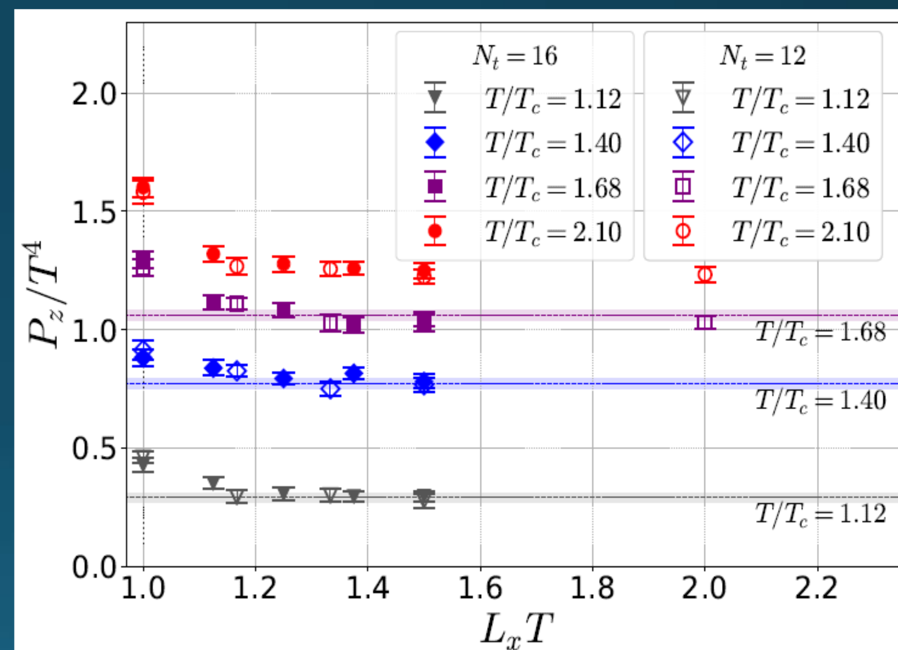


energy densty / transverse P

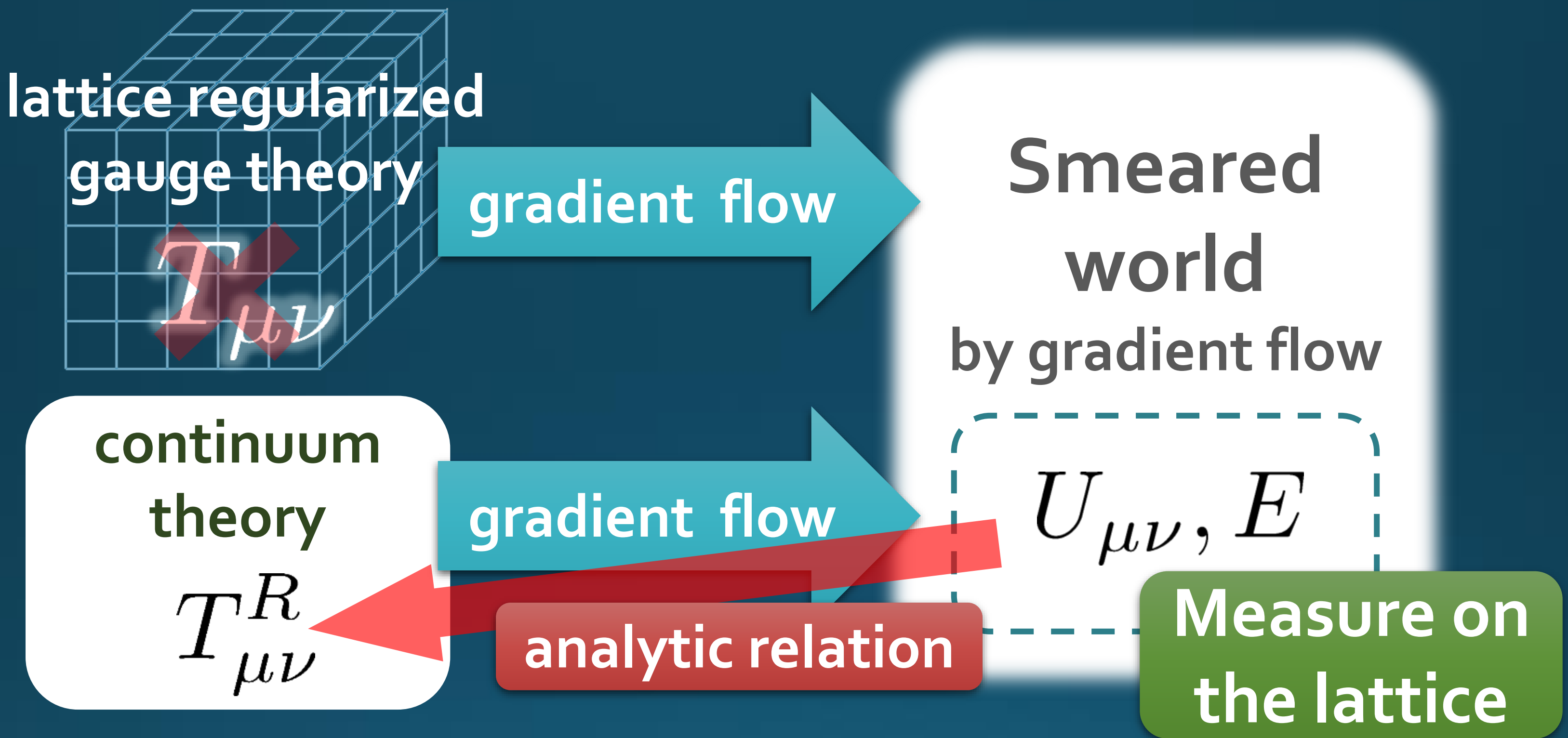
Energy Density



Transverse Pressure P_z



Gradient Flow Method



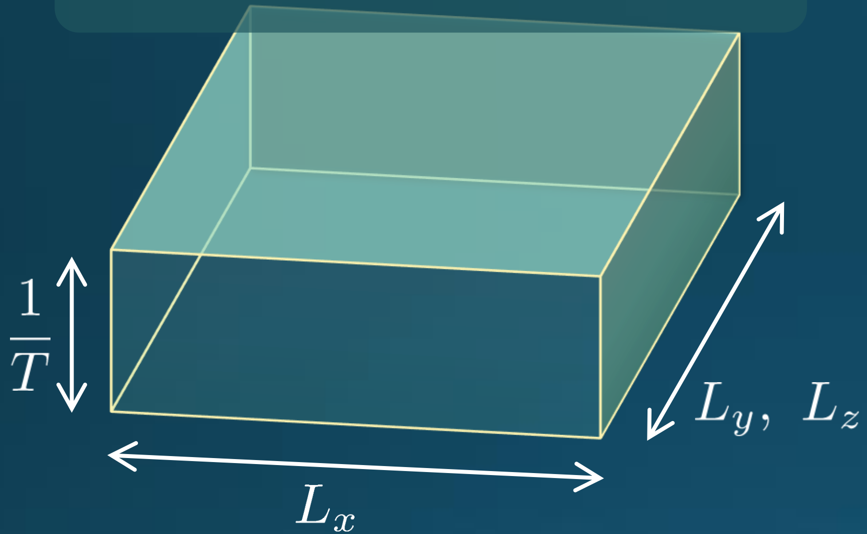
Take Extrapolation (t,a)→(0,0)

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}\frac{a^2}{t} + \cdots$$

O(t) terms in SFTE lattice discretization

Two Special Cases with PBC

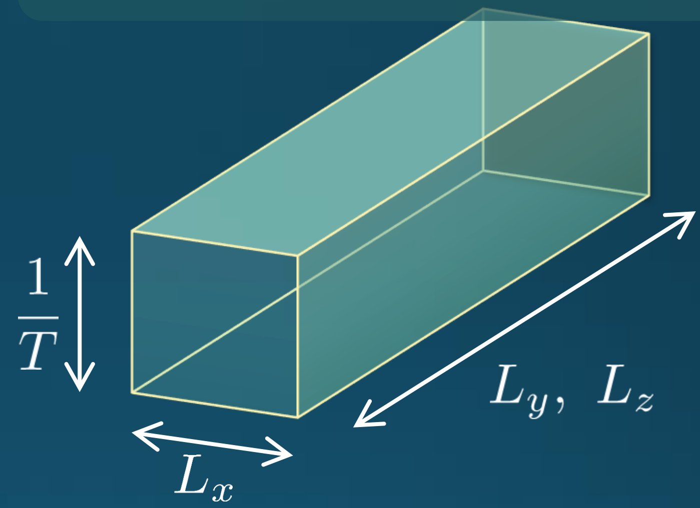
$$1/T \ll L_x = L_y = L_z$$



$$T_{11} = T_{22} = T_{33}$$

$$\frac{p_1}{p_2} = 1$$

$$1/T = L_x, \ L_y = L_z$$



$$T_{44} = T_{11}, \ T_{22} = T_{33}$$

In conformal ($\Sigma_\mu T_{\mu\mu}$=0)

$$\frac{p_1}{p_2} = -1$$

EMT on the Lattice: Conventional

Lattice EMT Operator Caracciolo+, 1990

$$T_{\mu\nu} = Z_6 T^{[6]}_{\mu\nu} + Z_3 T^{[3]}_{\mu\nu} + Z_1 \left(T^{[1]}_{\mu\nu} - \langle T^{[1]}_{\mu\nu} \rangle \right)$$

$$T^{[6]}_{\mu\nu} = (1-\delta_{\mu\nu}) F^a_{\mu\rho} F^a_{\nu\rho}, \;\; T^{[3]}_{\mu\nu} = \delta_{\mu\nu} \left(F^a_{\mu\rho} F^a_{\nu\rho} - \frac{1}{4} F^a_{\rho\sigma} F^a_{\rho\sigma} \right), \;\; T^{[1]}_{\mu\nu} = \delta_{\mu\nu} F^a_{\rho\sigma} F^a_{\rho\sigma}$$

- Fit to thermodynamics: Z_3, Z_1
- Shifted-boundary method: Z_6, Z_3 — Giusti, Meyer, 2011; 2013; Giusti, Pepe, 2014~; Borsanyi+, 2018

Multi-level algorithm

- effective in reducing statistical error of correlator — Meyer, 2007; Borsanyi, 2018; Astrakhantsev+, 2018

Perturbative Coefficients

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

$$T_{\mu\nu}(t) = c_1(t)U_{\mu\nu}(t) + \delta_{\mu\nu}c_2(t)E(t)$$

	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○	○	
$c_2(t)$	× zero	○	○	○

Suzuki (2013) Harlander+(2018)

Iritani, MK, Suzuki, Takaura, 2019

□ Choice of the scale of g²

$$c_1(t) = c_1\left(g^2(\mu(t))\right)$$

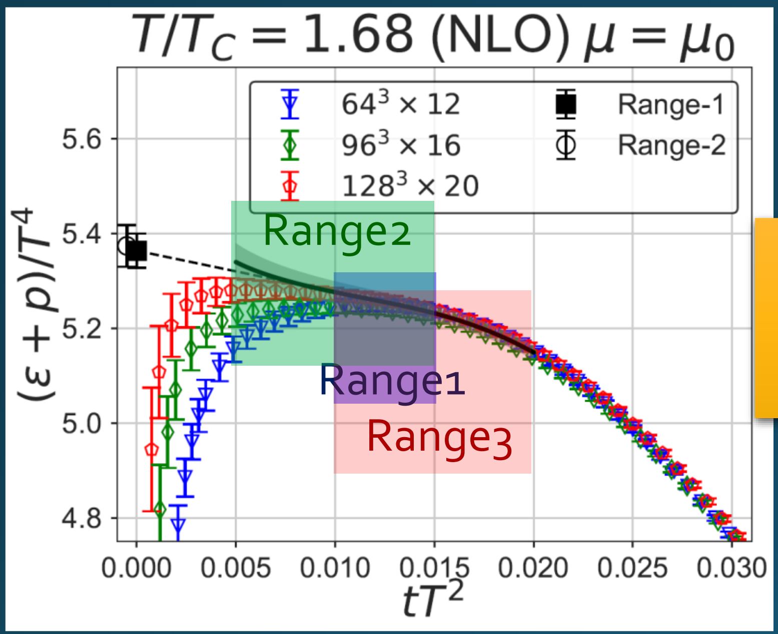
Previous: $\mu_d(t) = 1/\sqrt{8t}$

Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E}t}$

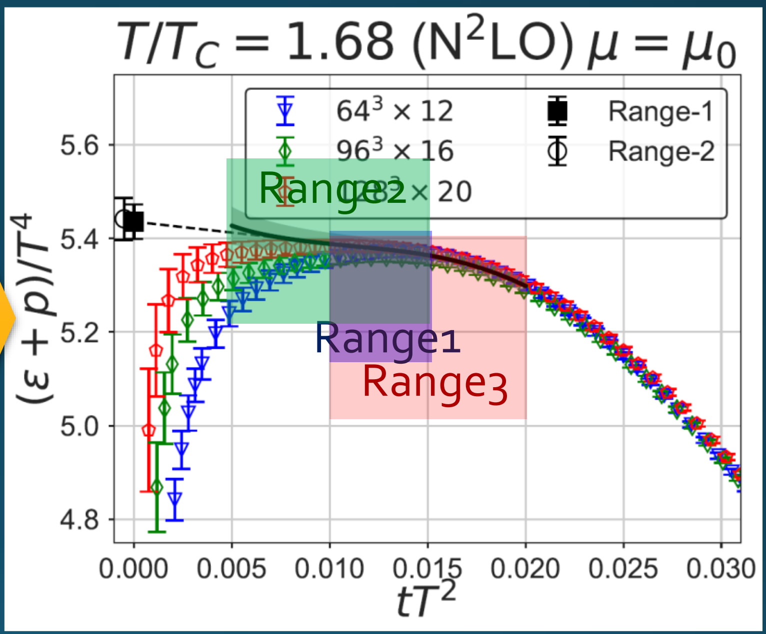
Harlander+ (2018)

Higher Order Coefficient: ε+p

NLO (1-loop)



N²LO (2-loop)



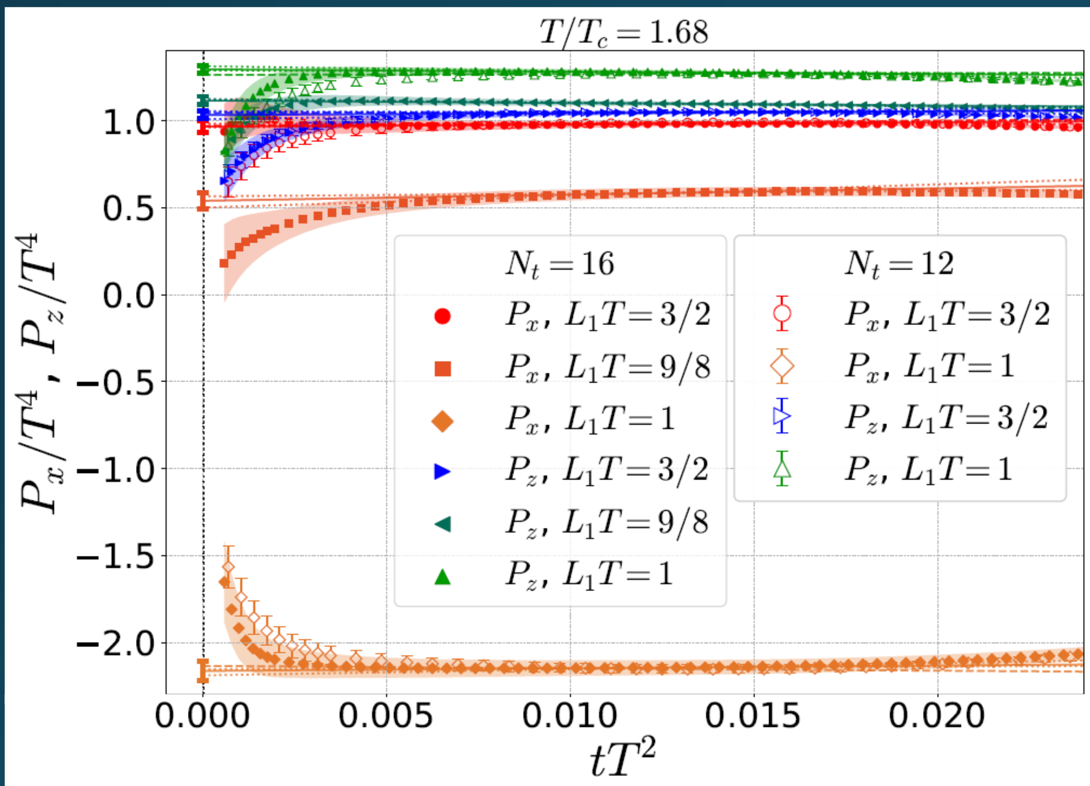
Iritani, MK, Suzuki, Takaura, PTEP 2019

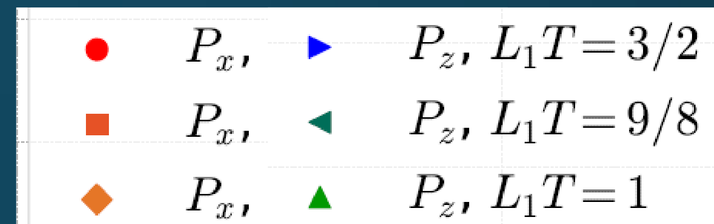
- ☐ t dependence becomes milder with higher order coeff.
- ☐ Better t→0 extrapolation

- ☐ Systematic error: μ_0 or μ_d, uncertaintyof Λ (±3%), fit range
- ☐ Extrapolation func: linear, higher order term in c_1 ($\sim g^6$)

Small-t Extrapolation

$T/T_c = 1.68$





Filled: N_t=16 / Open: N_t=12

Small-t extrapolation

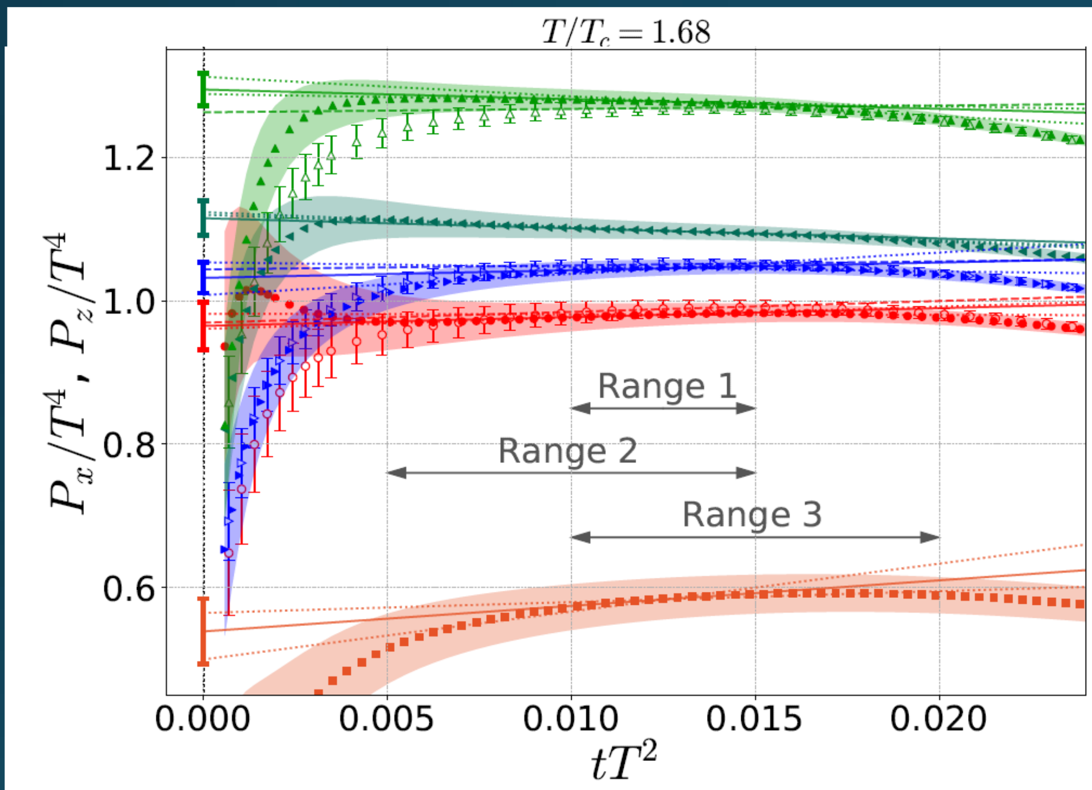
- Solid: N_t=16, Range-1
- Dotted: N_t=16, Range-2,3
- Dashed: N_t=12, Range-1

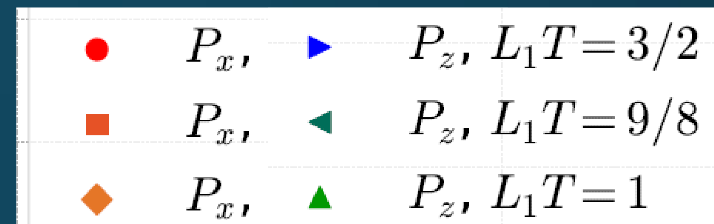
☐ Stable small-t extrapolation

☐ No N_t dependence within statistics for L_xT=1, 1.5

Small-t Extrapolation

$T/T_c = 1.68$





Filled: N_t=16 / Open: N_t=12

Small-t extrapolation

- Solid: N_t=16, Range-1
- Dotted: N_t=16, Range-2,3
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- ☐ Stable small-t extrapolation
- ☐ No N_t dependence within statistics for L_xT=1, 1.5